Basic Electrical Engineering

THIRD EDITION

D P KOTHARI

I J NAGRATH

The McGraw-Hill Companies

Basic Electrical Engineering
Third Edition

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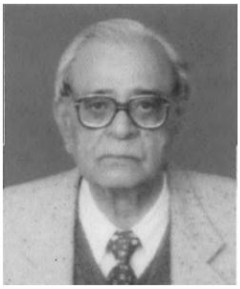

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Third Edition

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Preface to the Third Edition

The excellent response to the First and Second Editions of this book by students and faculty of Indian and foreign universities, and the practising engineers has motivated the authors to venture for the Third Edition. The main aim was to include the missing links, and update and upgrade the material of the Second Edition as per the current needs.

The objective of this edition, like the earlier ones, is to give an exhaustive exposition of the fundamental concepts, techniques and devices in electrical engineering. The basic concepts in Electric Circuit Theory and Electric Machines have remained more or less the same over the years since the First Edition of this text appeared in 1991.

This edition covers the Basic Electrical Engineering Course of almost all the Indian Technical Universities and some foreign universities as well. It is particularly well suited for undergraduate students of all engineering disciplines. Diploma students of EEE and ECE will find this book useful for circuit theory and electric machines courses. The speciality of this edition is that it covers both—the very basic and advanced topics in electric circuits, electric machines and power systems. The book can be used for a single course in electric machines.

The highlights of this edition are large number of solved problems that have been added in all the chapters. The AC Machine chapter is re-written completely and splitted into two chapters, namely, Synchronous Machines and Induction Motor. Two new chapters, namely, Domestic Wiring and Power Systems have been added. The Power Systems chapter gives a brief idea of the entire Power System concepts especially for non-electrical engineering students.

We have deleted two chapters, viz., Laplace Transform and Electric Energy from this edition. If required, these may be referred to from the other books written by the authors.

Almost in all chapters, various sections have been re-written and objectives, summary, review questions and additional solved problems have been added. In addition to this, complex numbers have been added in Appendix C. The book has a rich pool of pedagogical features with 785 solved problems and students' practice problems.

The revised edition, spanning over 16 chapters and 4 appendices, has been structured to provide in-depth information of all the concepts with appropriate pedagogy. **Chapter 1** introduces the Elementary Concepts and Definitions. **Chapters 2 and 3** cover the Fundamentals of Resistive and Reactive Circuits respectively. Steady State Analysis for Sinusoidal Excitation, Frequency Response and Three-Phase Circuits are dealt with in **Chapters 4, 5 and 6** respectively. A detailed exposition of Magnetic Circuits is provided in **Chapter 7** whereas

Chapter 8 focuses on Transformers. EMF and Torque in Electric Machines are explained in **Chapter 9. Chapters 10, 11, 12** and **13** provide detailed information on DC Machines, Synchronous Machines, Induction Motors and Fractional-kW Motors respectively. **Chapter 14** discusses Measurements Techniques and Electric and Electronic Instrumentation. **The last two chapters** provide in-depth understanding of Power Systems and Domestic Wiring. Useful **Appendices** on Graph Theory, Resistance and Complex Numbers have been provided for the easy reference of students.

The book is accompanied by an Online Learning Centre http://www.mhhe.com/kothari/bee3 that offers valuable resources for instructors and students.

For Instructors:

- Solution manual
- Chapter wise powerpoint slides with diagram and notes for effective presentation

For Students:

- A Sample chapter
- Links to reference materiel
- Chapter outline for quick revision during exams

Tata McGraw Hill and the authors would like to thank the following reviewers of this edition.

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While revising the text, we have had the benefit of valuable advice and suggestions from many teachers, students and other readers who used the earlier editions of this book. All these individuals have influenced this edition. We express our thanks and appreciation to them. We hope this support/response would continue in future also.

We are grateful to the authorities of VIT University for providing all the facilities necessary for writing the book.

One of us (D P Kothari) wishes to place on record thanks to his PhD student, Mr K Palanisamy for help in preparing rough drafts of certain portions of the manuscript and in the solution of examples and unsolved problems of certain chapters.

We also thank the TMH team and our families who have supported us during this period and given all possible help so that this book can see the light of the day.

We look forward to receiving suggestions and constructive criticism from teacher and students at the following email-id—tmhcorefeedback@gmail.com (*kindly mention the title and author name in the subject line*).

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Preface to the First Edition

Upon the introduction of four-year courses for the bachelor of engineering degree based on the (10 + 2) year pattern, AICTE (The All India Council of Technical Education) came up with a model curriculum. Following this curriculum two compulsory courses, namely Electrical Science I and II, have been introduced in most engineering institutions and colleges across the country for students of all engineering disciplines. These courses form the foundation for higher level courses in electrical and electronics engineering degrees, and also serve to introduce electrical sciences to students of other disciplines. Electrical Science-I comprises electric circuit theory and electromechanical energy conversion, while Electrical Science-II covers electronics and instrumentation.

There are some foreign books which cover the above four topics of electrical sciences. However, these are largely beyond the reach of Indian students because of their high cost. Further these books are found unsuitable for the syllabus as defined in the AICTE model because of which these have not appeared in cheap editions. So for quite some time a strong need has been felt for textbooks in this area.

This book is designed to fulfil the need of the Electrical Science-I course. It covers all the essential topics in electric circuit theory and electromechanical energy conversion. It follows the pattern suggested by the Teachers' Manual for Electrical Science-I, April, 1984, Indian Society of Technical Education, IIT Campus, New Delhi. It is written in a style which lends itself to easy adaptation to the exact syllabi of various universities, and teaching plans of individual teachers. Certain topics can be altogether left out without loss of continuity and others can be partly covered without going into all the details.

Since Electrical Science-I is the only electrical course offered to non-electrical students, it is desirable that the course encompasses the total span of the book. This would in my view requires 5 lectures/week plus preferably a tutorial hour. Where such time is not warranted by the curriculum design of a discipline, a judicious choice of topics must be made.

Electric circuit theory is covered in Chapters 1 to 7 and it encompasses all the basic techniques of circuit analysis. The first chapter introduces circuit elements including sources; elemental relationships and the two basic laws of circuit theory namely Kirchhoff's Voltage and Current Laws (KVL and KCL). Various types of excitations namely, dc, sinusoidal, step function and other periodic wave shapes are presented in the text as and when the need arises. The techniques of circuit reduction and mesh and nodal analysis are treated through simple resistive circuits (in Chapter 2), and later extended to the more general circuits. A similar approach is used to introduce various network theorems.

In the third chapter, simple techniques of obtaining natural and forced response are illustrated through first and second order circuits. At this stage the forced response is restricted to the case of step functions (dc excitation). The total circuit response is then obtained by combining the two responses with application of initial conditions.

The phasor representation of sinusoidal function is introduced in Chapter 4, before presenting the steady state sinusoidal analysis of circuits. This is followed by a section explaining the transformation from a time domain circuit to a frequency domain circuit. Mesh and nodal analysis and various network theorems are then applied to frequency domain circuits. Concepts of active and reactive power are illustrated here. Response of a circuit to sudden application of sinusoidal excitation is also obtained.

Certain miscellaneous topics—frequency response, Bode diagrams, resonance, Fourier series and analysis, two-port networks—which are a prerequisite for Electrical Science-II are covered in Chapter 5. Chapter 6 discusses the sinusoidal steady state analysis of three-phase balanced circuits and three-phase power. Using these techniques the student can analyse any circuit for determining its transient and/or steady state response.

A full chapter is then devoted to the analysis of circuits using Laplace transformation with various theorems and inversion techniques. The circuit analysis follows the method of transformed networks. The general concept of network function (transfer function) is introduced and the steady state sinusoidal response obtained. This chapter is included for completeness of the text but may be altogether skipped in teaching Electrical Science-I.

Electromechanical energy conversion is covered in Chapters 8 to 13 which treat the basic concepts, transformers and complete range of electric machines including single-phase motors. Steady-state sinusoidal circuit analysis, active and reactive powers and 3-phase circuits discussed in the earlier chapters are used here. The circuit model of each device (static/rotational) is introduced which is then employed in performance calculations.

Beginning with the treatment of magnetic circuits and magnetic materials in Chapter 8, Faraday's laws are discussed before presenting the generalised article on electromechanical energy conversion. Wherever the teacher is pressed for time he may altogether skip this article. It is independent and is not needed at a later stage.

Chapter 9 covers the circuit model of the transformer, determination of model parameters, voltage regulation and efficiency. Articles on autotransformer and three-phase transformers are also included.

Basic groundwork for rotating machines is laid in the chapter on emf and torque. While the derivation of emf equation is sufficiently detailed, the treatment of mmf is brief and comprises a section on the rotating magnetic field. These ideas are then extended to torque development in a rotating machine with illustrations of the synchronous machine, induction machine and dc machine. If the teacher so desires, he may take up this portion along with the relevant machine chapters. Losses, efficiency, rating and cooling are touched upon briefly and from a general viewpoint.

Chapter 11 discusses dc machines with stress on motors. Armature reaction and commutation are discussed and so are the methods of excitation. Characteristics and speed control of shunt and series type dc motors immediately follow and are treated through the assumption of linear magnetization. Starting and efficiency are also touched upon.

The twelfth chapter discusses ac machines—both synchronous and induction machines. For the synchronous machine it is suggested that the teacher need follow only the heuristic treatment for the circuit model in the class. Experimental determination of synchronous reactance and its use in voltage regulation, synchronisation of a machine to mains and operating characteristics of the machine are covered here.

For the induction machine the circuit model is derived through ideas presented earlier and the concepts illustrated in the transformer. To save time, the teacher may skip the derivational steps and directly introduce the circuit model. Stress is laid upon the concept of power across the gap. The complete torque-speed characteristic and its dependence on rotor resistance is explained with the help of several solved examples. No-load and blocked rotor tests for parameter determination are also discussed. Methods of starting and speed control are briefly discussed. A short account is given of the double-cage induction motor. Some of the solved examples on induction motor may be taken up partly or completely ignored.

At the end, a brief descriptive chapter is devoted to single-phase induction and synchronous motors, servomotor, ac series motor and stepper motor.

As an aid to both students and teachers, the book gives as many as 110 typically illustrative solved examples, and 204 unsolved problems with answers.

This book is based upon my experience of teaching similar courses and writing books in allied areas. My efforts have immensely benefitted from the opportunities for classroom experimentation provided by the flexible system of the Birla Institute of Technology & Science (BITS), Pilani.

I J Nagrath

Chapter 1

ELEMENTARY CONCEPTS AND DEFINITIONS

MAIN GOALS AND OBJECTIVES

- *Definition of electrical quantities and their units*
- *Understanding the relationship between charge, voltage, current and power*
- *Acquiring capability to work with sign convention—voltage and current*
- *DC and AC current and voltage; characteristics of sinusoidal waveform universally used for AC currents and voltages*
- *Resistor and Ohm's law, vi relationship of inductor and capacitor*
- *Acquiring ability to employ basic circuit laws—Kirchhoff's Current Law* [KCL] *and Kirchhoff's Voltage Law* [KVL]
- *Ability to deal with independent and dependent source*

1.1 INTRODUCTION

Electric energy is convenient and efficient for production of light, mechanical energy and in information processing. For the first two uses, it can be transported in a clean fashion (as compared to transporting coal, for example) and economically over long distance lines so as to be available at the point of use. Electric energy also can transport information over tremendous distances, with or without wires, equally efficiently and economically. There is almost no competitor to electric energy in these fields.

Electric energy does not occur naturally in usable form and must therefore be centrally generated and instantly transported to myriad points of use spread geographically over vast areas, even beyond state or national boundaries. It cannot be stored in large enough quantities for any major use. Electric energy generation is generally a three-step process—naturally occurring chemically-bonded energy (as in fossil fuels—coal and oil) or nuclear energy is converted to heat form by combustion/nuclear fission, the thermodynamic cycle converts it to mechanical form (rotational) which then is employed to run an electric energy generator. For limited use, electric energy is directly obtained from chemical energy, as in batteries, or solar energy is converted to electric energy as in a solar cell. The trend in electric energy generators is towards *mega* sizes, due to economy in large scales.

Information, usually visual or audio signals or coded messages, have to be processed and/or transported by the intermediate form of electric energy. Speed of processing and economy dictate that electric energy for these purposes must be in minutest possible quantities—in either continuous form or bit form (modern trend). Hence the trend is towards *micro* sizes. Range and variety of such use of electric energy is varied and wide—video and audio systems, control processors, computers, etc.

Fibre optics using light signals is beginning to offer stiff competition to electric energy for purposes of information processing. The end-use energy form of such systems would, for a long time to come, continue to be electric.

This being the first chapter, it begins by introducing the fundamental laws of electricity and conservation of energy. The concepts of electric charge, current, voltage and electric sources and power are clarified alongwith the sign convention. Idealized circuit elements—resistance, capacitance and inductance—are dwelled upon along with basic laws that govern their terminal behaviour. The practical circuit elements—resistor, capacitor and inductor—are introduced.

Interconnection of circuit elements leads to the concept of electric circuit. The two fundamental circuit laws lay the foundation stone of the circuit theory to which the first six chapters of this book are devoted. The chapter ends on the principle of superposition, homogeneity and concept of linearity.

The importance of circuit theory can be judged from the fact that almost all electric and electronic devices, transducers, transmission lines, energy and information processing systems, etc. are modelled in the form of a circuit with sources for the purpose of their analysis and design. Circuit modelling cannot be applied as such to very high frequency devices (microwave equipment etc.), where travelling wave concept is a must for their modelling.

In view of the above account, the electric circuit theory is fundamental to all fields of electrical engineering. Even some mechanical systems can be modelled by an electric circuit on an analogic basis.

1.2 WORK, ENERGY AND POWER

Work Work is done whenever an object moves in a field of force, F (unit of force is newton, N). If the object moves in the direction of force, work is done by the force. However, if the object moves in a direction opposite to the force, work is done by an external agency that moves the object.

Energy It is the capacity for doing work. When a mass is lifted against gravity, work done by an external agency in lifting it gets stored in the mass as potential energy (by virtue of its position in the gravitational force field). If the mass is now allowed to fall, the potential energy will get converted to kinetic energy (associated with velocity). Further, if the mass were to fall on a wedge, it will drive the wedge into a piece of wood (say), thereby doing work. The process of doing work in some sense is a process of energy transfer. In the example cited above, energy is transferred from the external agency to the mass and gets stored as potential energy. When the mass falls, potential energy still remains stored in the mass but gets converted to kinetic energy. Upon hitting the wedge, some of the kinetic energy in the mass gets transformed to heat which is generated while driving the wedge against wood friction.

Energy W = force N × distance moved (m)

Therefore, energy, W is measured in unit of joules, J or newton–metres, N–m.

Principle of Conservation of Energy

In non-relativistic processes, energy (w) never gets destroyed; it gets converted from one form to another as illustrated in the above example.

Power It is the rate of doing work (i.e., rate of transferring energy). Instantaneous power is

$$p = \frac{dw}{dt} \text{ J/s or watts (W)} \tag{1.1}$$

Average power is given by

$$P\text{ (av)} = \frac{W}{T};\ T = \text{time (in seconds) during which energy } W \text{ flows} \tag{1.2}$$

The integral forms of Eq. (1.1) are

$$W = \int_{t_0}^{t_0+T} p\, dt \tag{1.3}$$

$$P\text{ (av)} = \frac{1}{T}\int_{t_0}^{t_0+T} p\, dt \tag{1.4}$$

1.3 BASIC MANIFESTATIONS OF ELECTRICITY

Fundamental electric charge carrying particles are electrons (negative charge) and protons (positive charge). The unit of electric charge is a coulomb, C. In terms of this unit, the electronic charge is 1.602×10^{-19} C (–ve for electron and + ve for proton). Since coulomb is a large unit, it is more practical to use micro-coulomb, μC.

An *electric field* is established in the space surrounding an electric charge and is manifested in the form of force exerted on another charge brought into the field. This force is given by *Coulomb's law* as

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon d^2} \text{ N} \tag{1.5}$$

where Q_1, Q_2 are charges in coulomb, d the distance between them in metres and ε is the *permittivity* of the medium (= $\varepsilon_r\varepsilon_0$; ε_r = *relative permittivity* of medium and ε_0 = *permittivity of free space* = 8.85×10^{-12}).

When we are dealing here with a stationary charge, the field is called *electrostatic field*. The field at any point is measured quantitatively in terms of the force exerted on a unit positive charge and is called *electric field intensity* **E** which is a directed quantity (vector).

Potential Difference It is the work (J) done when a unit positive charge is moved from a point b in the field to another point a. As in the gravitational field, the potential difference or *voltage difference* between two points is a *scalar* quantity independent of the path chosen. The unit of potential difference is *volts* (V), 1V = 1 J/unit positive charge. The symbol of potential difference (or voltage) is as v or V.

If work must be done on the charge (energy input to the charge) as it moves from b to a, the voltage of a is higher than that of b (voltage rises from b to a) and is indicated as v_{ab} (a above b) (Fig. 1.1). In this case, if the charge moves from a to b, energy is output. Obviously

$$v_{ba} = -v_{ab}$$

i.e. the voltage drops in going from a to b.

There are two ways of indicating the voltage difference on a diagram, as shown in Fig. 1.1. It can be indicated by a line with an arrow pointing towards the point whose voltage is higher than that of the other point (no arrow) by the symbol indicated on the arrow as in Fig. 1.1(a), or by arrows at both ends with + and – sign placed at the ends (points) as in Fig. 1.1(b).

Fig. 1.1 Potential difference (voltage difference)

The transfer of electric energy is associated with the motion of charges. In our circuit study, we will consider motion of charges confined to a definite path constituted of materials that are good *conductors of electricity* (aluminium, copper). Poor conductors of electricity are known as insulators and are used to wrap the conductors to prevent the charge from leaking away.

Current Electric current is the rate of flow of charge through a conducting path as shown in Fig. 1.2. The *positive direction* of current is the direction in which *positive charge flows*; this direction is opposite to that in which *electrons* flow. Unit of current is *ampere,* A. One ampere is the charge flow rate of 1 C / sec. The symbol used for current is i or I. The symbol for charge is q.

Average current over a period of time is

$$i\,(avg) = \frac{\Delta q}{\Delta t} \tag{1.6a}$$

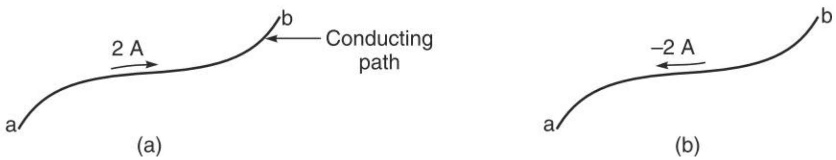


Fig. 1.2 Current

while the instantaneous current is defined as

$$i = \frac{\mathrm{d}q}{\mathrm{d}t} \tag{1.6b}$$

The paths of interest along which charges move (flow) are known as *circuits.*

The charge transferred from time t_0 to t is

$$q = \int_{t_0}^{t} i \, \mathrm{d}t \tag{1.7}$$

As in Fig (1.2)(a), a reference positive direction is chosen for the current. The current in the opposite direction would then be negative as in Fig. 1.2(b). If a current is flowing from a point a to b, it may be indicated by the symbol i_{ab} (a to b). Obviously

$$i_{ba} = -\, i_{ab}$$

We shall generally avoid such double suffix symbolization.

Unidirectional current is known as *direct current* (dc) and, unless otherwise indicated, it is assumed to have constant value with time as shown in the graphical representation of Fig. 1.3.

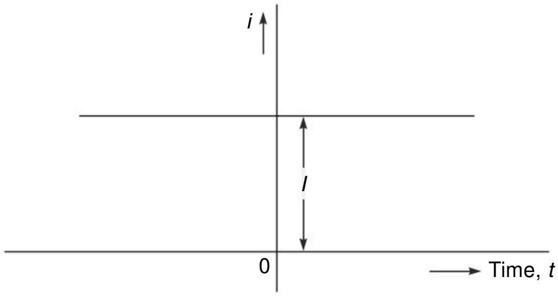


Fig. 1.3 Direct current (dc)

Alternating current or ac, is cyclic in nature, with current flowing in positive direction in half the cycle and in negative direction in the other half, as shown in Fig. 1.4. The current wave shape shown in Fig. 1.4 is sinusoidal which is a very common occurrence in circuits. It can be expressed as

$$i(t) = I_m \cos \frac{2\pi}{T} t \tag{1.8}$$

where T = time period of a one cycle in s

I_m = maximum (peak) current

It easily follows that the frequency

$$f = \frac{1}{T} \text{ Hz (cycles/s); Hz = hertz} \tag{1.9}$$

Equation (1.8) can now be rewritten as

$$i(t) = I_m \cos 2\pi f t \tag{1.10}$$

We can express angular frequency ω as

$$\omega = 2\pi f;\ \text{frequency in rad/s} \tag{1.11}$$

so that

$$i(t) = I_m \cos \omega t \tag{1.12}$$

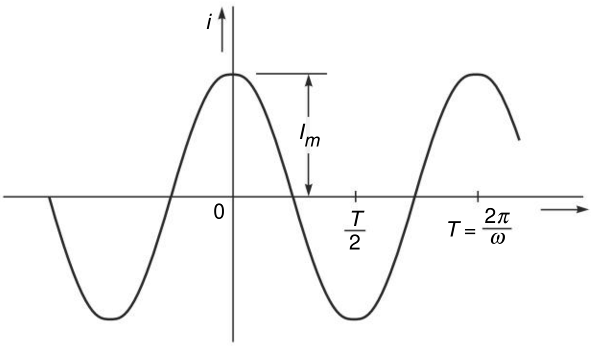


Fig. 1.4 Alternating current (ac)—sinusoidal

It is convenient to plot alternating current in from of Eq. (1.12) with ordinate ωt as in Fig. 1.5.

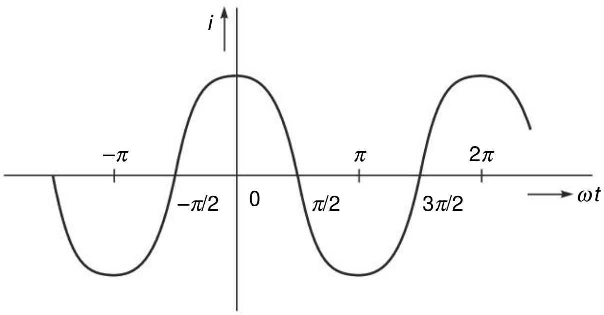


Fig. 1.5 Alternating current—sinusoidal

Mathematically, Eq. (1.12) (as well as Eq. (1.8)) can have any reference point so that in general

$$i(t) = I_m \cos(\omega t + \theta) \tag{1.13}$$

where θ can be either positive or negative as shown in Fig. 1.6(a) and (b). θ is known as the *phase angle* of the current and causes the current wave to shift to right (lag) or left (lead) in time.

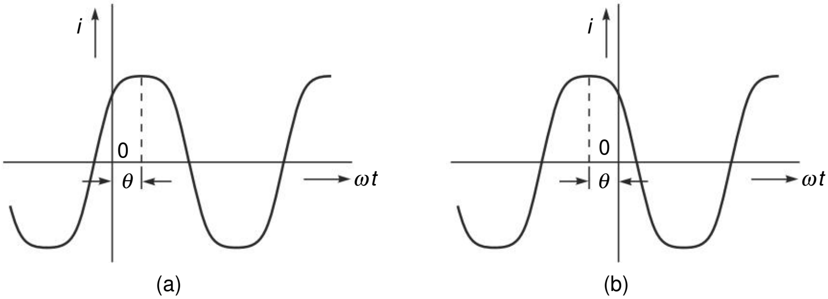


Fig. 1.6 (a) $i(t) = I_m \cos(\omega t - \theta)$; lagging phase angle
(b) $i(t) = I_m \cos(\omega t + \theta)$; leading phase angle

Relative Phase Angle Consider the sinusoidal voltage

$$v_1(t) = V_m \sin \omega t \tag{1.14}$$

and

$$v_2(t) = V_m \sin (\omega t + \theta) \tag{1.15}$$

For phase angle comparison, we choose any corresponding points on the two waves. It is convenient to choose first zero value of $v_1(t)$ which occurs at $\omega t = 0$. The corresponding zero value of $v_2(t)$ occurs at $(\omega t + \theta) = 0$ or at $\omega t = -\theta$. We find that the zero point of $v_2(t)$ occurs earlier in angle by θ rad or in time by θ/ω seconds from the corresponding point of $v_1(t)$. It means that wave $v_2(t)$ *leads* $v_1(t)$ by angle θ rad or time θ/ω seconds. The two waves are sketched in Fig. 1.7 and the angle θ is indicated therein

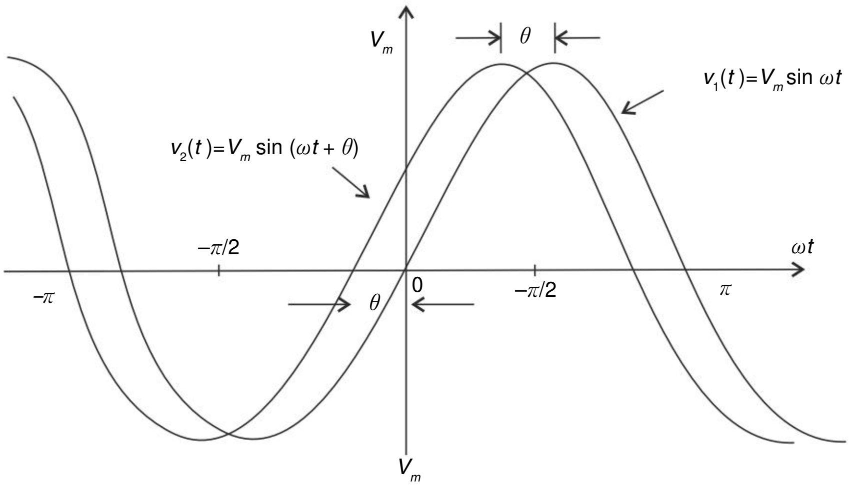


Fig. 1.7 $v_2(t)$ leads $v_1(t)$ by angle θ

It is not necessary to sketch the sine wave to determine the phase angle. Consider

$$v_3(t) = V_m \sin (\omega t - \theta)$$

The first zero occurs at $(\omega t - \theta) = 0$ or $\omega t = \theta$ i.e., later than the *reference sine wave* by angle θ. The reader may sketch v_3 (t) and observe.

For determination of relative phase angle, it is convenient to express both the waves as sines or cosines. For consine waves, convenient corresponding points are where the cosine has unit value.

It is essential that the sinusoidal waves, being phase compared, have the same frequency.

In engineering applications, the phase angle θ is expressed in degrees. If the corresponding time is to determined, it should be converted to rad.

Converting Sines to Cosines and vice versa The sine and cosine are essentially the same function except for 90° phase difference. Thus

$$\cos (\omega t - 90°) = \sin \omega t \tag{i}$$

It means that cos ωt with a lagging angle of 90° is sin ωt. In other words, cos ωt leads sin ωt by 90°. Add 90° on both sides of Eq. (i), we find

$$\sin(\omega t + 90°) = \cos \omega t$$

which means sine with 90° lead is cosine.

Adding or subtracting 360° to the argument of cosine and sine does not cause any change in their value.

Adding or subtracting 180° to the argument of cosine and sine causes a sign reversal. Thus

$$\cos(\omega t - 90° + 180°) = -\cos(\omega t - 90°) = -\sin \omega t$$

Example

$$v_1(t) = V_{m1} \cos(15t + 20°)$$
$$= V_{m1} \sin(15t + 90° + 20°);\ \text{sine with 90° lead is cosine}$$
$$= V_{m1} \sin(15t + 110°)$$

Compare it with

$$v_2(t) = V_{m2} \sin(15t - 30°)$$

We find v_1 leads v_2 by (110° + 30°) = 140°. If we subtract 360° from argument of v_1, we have

$$v_1 = V_{m1} \sin(15t - 250°)$$

which equivalently means v_1 is lagging v_2 by (–250° + 30° = –220°)

It is preferable to express the phase angle between sinusoids as an angle less than 180°.

Example 1.1 Find the angle by which i_1 lags v_1 if $v_1 = 100 \cos(100t - 40°)$V and i_1 equals (a) $4 \cos(100t + 20°)$ A and (b) $1.5 \sin(100t - 60°)$ A.

Solution

(a) Both are cosines. From Fig. 1.8, we can see i_1 lags v_1 by – (20° + 40°) = – 60°

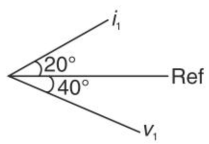


Fig 1.8(a)

(b) For phase comparison we convert i_1 from sine to cosine form

$$1.5 \sin(100t - 60°) = 1.5 \cos(100t - 60° - 90°)$$
$$= 1.5 \cos(100t - 150°)$$

Compare with

$$v_1 = 100 \cos(100t - 40°)$$

i_1 lags v_1 by (150° – 40°) = 110° [Fig. 1.8(b)]

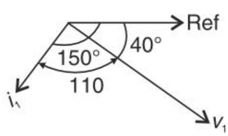


Fig 1.8(b)

Example 1.2 Convert $50 \cos(120t - 45°) - 30 \sin(120t + 160°)$ to the following forms (*a*) $A \cos 120t + B \sin 120t$ (*b*) $C \cos(120t + \phi)$

Solution

(a) $50 \cos(120t - 45°) = 50[\cos 120t \cos 45° + \sin 120t \sin 45°]$

$$= 35.36 \cos 120t + 35.36 \sin 120t \qquad \text{(i)}$$

$$30 \sin(120t + 160°) = 30 [\sin 120t \cos 160° + \cos 120t \sin 160°]$$
$$= -28.19 \sin 120t + 10.26 \cos 120t \quad \text{(ii)}$$

Subtracting Eq. (ii) from eqn. (i), we get the result as

$$25.10 \cos 120t + 63.55 \sin 120t \quad \text{(iii)}$$

(b) We want Eq. (iii) in cosine form. So let

$$\left.\begin{array}{l} 25.10 = C \cos \phi \\ 63.55 = C \sin \phi \end{array}\right\} \Rightarrow C = 68.33,\ \phi = 68.5°$$

Substituting in Eq. (iii)

$$\text{C} \cos \phi \cos 120t + \text{C} \sin \phi \sin 120t = \text{C} \cos(120t - \phi)$$

or $$68.33 \cos(120t - 68.5°) \quad \text{(iv)}$$

Magnetic Effect of Currents

A current-carrying conductor exerts forces on other current carrying-conductors and on magnetic materials in its vicinity which is explained by the presence of a magnetic field. A magnetic field by virtue of this force acts as a medium of energy transfer and is commonly employed for interconversion of electrical and mechanical energy. These ideas will be pursued in Chapter 7.

Electric and magnetic fields both exist simultaneously whenever moving charges are present—the electric field is caused by the presence of the charge, and magnetic field, by virtue of motion of the charge.

1.4 ELECTRIC ENERGY AND POWER

According to the definition of voltage, a charge q Coulombs as it moves through voltage v volt from – sign to + sign (see Fig. 1.9), it acquires energy

$$w = vq \quad J \tag{1.16}$$

As it moves out of + terminal, it carries (or delivers) the energy to the circuit beyond. The electric power is the rate of delivering the energy and is given by the time derivative of Eq. (1.16). Assuming v to be constant, the instantaneous power delivered is

$$p = \frac{dw}{dt} = v\frac{dq}{dt}$$

or $$p = vi \ W \tag{1.17}$$

This is illustrated in Fig. 1.9(a) where the current i is flowing out of the + terminal of voltage v. In other words it means that the circuit is delivering power p.

If the current i flows into the + terminal of v the power p is input to circuit or the circuit is receiving power as illustrated in Fig. 1.9(b).

The average power over a period of time T is given by

$$P = \frac{1}{T}\int_0^T p \, dt = \frac{1}{T}\int_0^T vi \, dt \tag{1.18}$$

when V and I are both constant (dc), the energy transferred (either way) in time T is

$$w = VIT \tag{1.19}$$

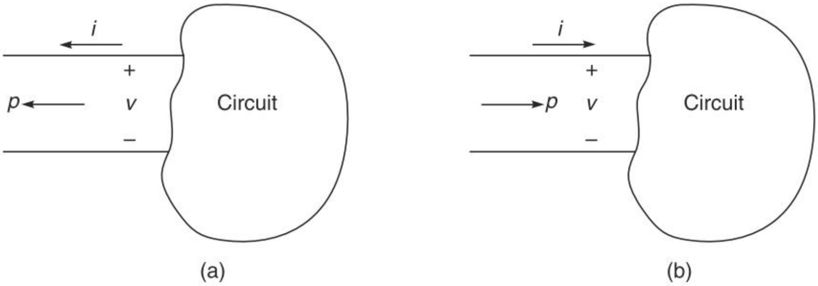


Fig. 1.9 (a) Power output of circuit $p = vi$ (b) Power input to circuit $p = vi$

1.5 SUPERPOSITION AND HOMOGENEITY

Principle of Superposition

An element or circuit obeys the principle of superposition if the net effect of the sum of causes equals the sum of their individual effects.

Mathematically, let cause x and effect y be related as

$$f(x) = y;\ f(\cdot) = \text{function} \tag{1.20}$$

Let the cause be scaled by a sector α. Then the functional relationship obeys **homogeneity**, if

$$f(\alpha x) = \alpha f(x) = \alpha y \tag{1.21}$$

Consider two causes x_1 and x_2, then

$$f(x_1) = y_1$$
$$f(x_2) = y_2$$

Let the combined effect of these two causes be scaled by α_1 and α_2 respectively. The principle of superposition then yields

$$f(\alpha_1 x_1 + \alpha_2 x_2) = f(\alpha x_1) + f(\alpha_2 x_2) \tag{1.22a}$$

If homogeneity is also satisfied, then

$$\begin{aligned} f(\alpha_1 x_1 + \alpha_2 x_2) &= \alpha_1 f(x_1) + \alpha_2 f(x_2) \\ &= \alpha_1 y_1 + \alpha_2 y_2 \end{aligned} \tag{1.22b}$$

A functional relationship is said to be **linear** if it obeys both superposition and homogeneity. Any element (of a circuit or system in general) governed by such a functional relationship is *linear*. A circuit composed of such elements would also be linear.

1.6 IDEAL CIRCUIT ELEMENTS

The general representation of a circuit element is drawn in Fig. 1.10. It has voltage

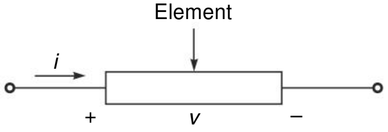


Fig. 1.10 Circuit element

and current associated with it. The voltage is an *across variable* (AV) and the current is a *through variable* (TV). Any one of these variables could be regarded as independent variable, and the other as dependent variable.

A circuit element is *ideal*, when its voltage and current are related by

- constant of proportionality or
- a differential or integral relationship

These relationships can be shown to be linear which means that an ideal circuit element has linear behaviour.

Resistance*

When a voltage is applied across a metallic conductor, (say copper), the electric field created accelerates the *conduction* (*free*) electrons. These electrons collide with metal ions of the crystal lattice (of the metal) and lose part of their energy as heat. Repeated accelerations and collisions cause two components of electron motion, the *drift* (average velocity) and the *random motion*. It is the drift of electrons that constitutes the electric current in the conductor (the conventional current flows in the opposite direction) which is associated with energy loss in the form of heat.

Therefore, the resistance is a *dissipative element*, which converts electric energy into heat, when the current flows through it in any direction. This process of energy conversion is irreversible.

Figure 1.11 shows the schematic representation of resistance. The element has *two terminals* (also called *nodes*). It conducts current from any one node to the other and in the process, voltage drop occurs across the element in the direction of current flow (the terminal at which the current enters acquires positive polarity with respect to the terminal at which the current exits).

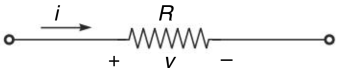


Fig. 1.11 Schematic representation of resistance

Ohm's Law It states that the voltage across the two terminals of a conducting material is proportional to the current flowing through it as

$$v = Ri \tag{1.23}$$

The constant of proportionality R is the *resistance,* represented in Fig. 1.11. The unit of resistance is *ohm* (V/A = 1), abbreviated as capital omega, Ω.

By virtue of the polarities indicated on the resistance of Fig. 1.11, the charge loses energy in passing through the resistance which appears in form of heat. A practical element that possesses the property of resistance is called a *resistor*.

Power dissipated by the resistance (Fig. 1.11) is

$$p = vi = i^2 R = \frac{v^2}{R} \text{ W} \tag{1.24}$$

Equation (1.23) can also be written as

$$i = Gv \tag{1.25}$$

*Refer Appendix B

where $G = 1/R$ = *conductance* in units of mhos (℧) or in units of siemens (S). We shall use the symbol ℧.

Power dissipated can then be expressed in the alternative form

$$p = \frac{i^2}{G} = Gv^2 \tag{1.26}$$

The resistance of a resistor is *temperature dependent* and rises* with temperature. Temperature rise must be limited to a specified value by conducting away the heat generated. Every resistor has therefore a *specified wattage*. Manufacturing limitations impose a tolerance band. A resistor is therefore specified as, for example, 2 W, 20 Ω (nominal) ± 10% at 25°C. It is only within specified limits that a resistor can be regarded as linear, outside which the behaviour becomes nonlinear.

Capacitance

It is a two-terminal element that has the capability of energy storage in electric field. The stored energy can be fully retrieved. Figure 1.12 is the schematic representation of a capacitance. There is a voltage drop in the direction of current with the terminal where the current flows in acquiring positive polarity with respect to the terminal at which the current leaves the element. The law governing the v–i relationship of capacitor is

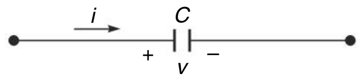


Fig. 1.12 Schematic representation of a capacitance

$$i = C\frac{dv}{dt} \text{ A} \tag{1.27}$$

where C has the units of *farads*, F; the practical unit being a microfarad or μF

Intregrating Eq. (1.27)

$$v = \frac{1}{C}\int_0^t i\,dt + v_C(0) \quad \text{where } v_C(0) = \text{capacitance voltage at } t = 0 \tag{1.28}$$

For an initially uncharged capacitor, $v_C(0) = 0$, so that

$$C = \frac{q}{v}\,;\ \text{as } q = \int_a^t i\ dt \tag{1.29}$$

As per Eq. (1.28) *the voltage* (*or charge*) *of a capacitance cannot change instantly* as it would require infinite current. Linearity of a capacitance element is easily demonstrated.

Let, $\quad v = \alpha_1 v_1 + \alpha_2 v_2$

Substituting in Eq. (1.27)

$$i = C\frac{d}{dt}[\alpha_1 v_1 + \alpha_2 v_2]$$

$$= \alpha_1 C\frac{dv_1}{dt} + \alpha_2 C\frac{dv_2}{dt}$$

*Resistance of carbon resistors decreases with temperature.

$$= \alpha_1 i_1 + \alpha_2 i_2$$

Energy Stored in Capacitance The power fed into capacitance is

$$p = vi = C\, v \frac{\mathrm{d}v}{\mathrm{d}t} \tag{1.30}$$

Integrating Eq. (1.30) and assuming initially uncharged capacitance ($v = 0$ at $t = 0$), the stored energy is found to be

$$w_C = \int_0^t p\, \mathrm{d}t = C \int_0^t v \frac{\mathrm{d}v}{\mathrm{d}t}\, \mathrm{d}t$$

or

$$w_C = C \int_0^v v\, \mathrm{d}v = \frac{1}{2} C\, v^2\ J \tag{1.31}$$

Observe that the energy stored in a capacitance is a function of its (instantaneous) voltage magnitude and is independent of the history of how this voltage is reached.

As the voltage is reduced to zero, all the energy stored in the capacitor is returned to the circuit in which the capacitor is connected.

A practical element possessing the property of capacitance is known as a *capacitor*. It is constructed of two parallel plates (in various forms) with an intervening dielectric.

The capacitance of a parallel plate capacitor is given by

$$C = \varepsilon_0 \varepsilon_r \left(\frac{A}{d}\right), \text{ with units Farads (F)} \tag{1.32}$$

ε_0 = permittivity of free space = 8.85×10^{-12}
ε_r = relative permittivity of medium

where

A = area of each plate (m^2); d = distance between plates (m). To get a capacitance of 1 F, let us calculate the value of (A/d) for $\varepsilon_r = 1$ (in air). From Eq. (1.32)

$$\frac{A}{d} = \frac{1}{8.85 \times 10^{-12}} = 0.113 \times 10^{12}$$

If we assume d = 1 mm, then

$$\begin{aligned} A &= 0.113 \times 10^{12} \times 10^{-3}\ \text{m}^2 \\ &= 113\ \text{km}^2 \end{aligned}$$

It is obvious from this figure that it is not practicable to construct a capacitor of 1F value. It is still an impossibility even if we use $\varepsilon_r = 8$.

In view of the above, the practical value of a capacitor is in units of microfarad (μF) = 10^{-6} F. Capacitors with some μF value are made from two thin aluminium films wrapped round a dielectric material. Larger values are obtained with electrolyte dielectric medium. Cepacitors needed for solid-state microchips are built from silicons or other semiconductors. These may be in ranges, of nanofarad nF = 10^{-9} F or even picofarad pF = 10^{-12} F.

As an example, consider a capacitor of 10 μF to which is applied 50 mA rectangular pulse of duration 0.4 ms as shown in Fig. 1.13(a). The capacitor is given by

$$q = \int_0^t i \; dt = 50 \int_0^0 dt = 50 \; dt \qquad \text{(i)}$$

The charge increase linearly till at $t = 0.4$ ms

$$Q = 50 \text{ mA} \times 0.4 \text{ ms} = 20 \; \mu\text{C}$$

as shown in Fig. 1.13(b)

The capacitor voltage $v = \dfrac{q}{C}$ also rises linearly till at 0.4 ms

$$v = \frac{20 \; \mu\text{C}}{10 \; \mu\text{F}} = 2 \text{ V}$$

as shown in Fig. 1.13(c)

Energy stored in capacitor is

$$w = \frac{1}{2} \; C v^2$$

increase by square law to a value

$$w = \frac{1}{2} \times (10 \; \mu\text{F}) \times (2)^2 = 20 \; \mu\text{J}$$

as shown in Fig. 1.13(d)

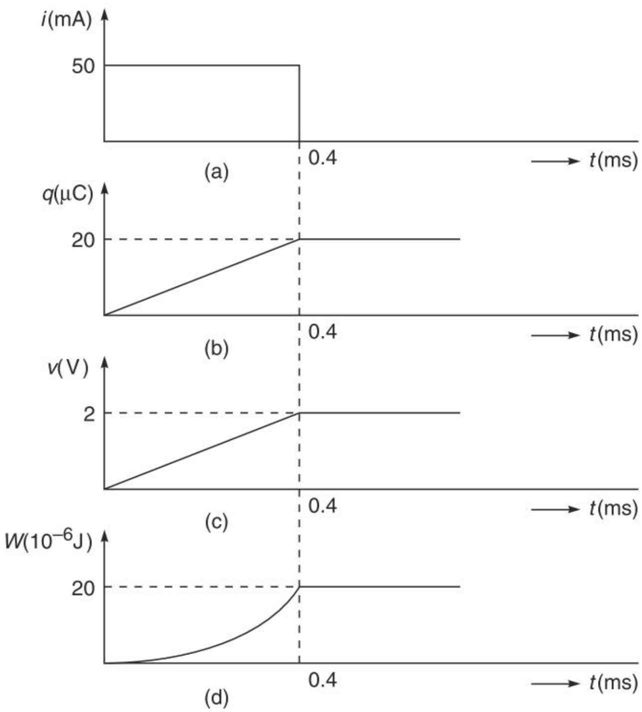


Fig. 1.13 Waveforms of (a) current, (b) charge, (c) voltage and (d) energy in a capacitor

Inductance

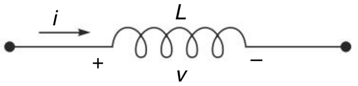


Fig. 1.14 Schematic representation of inductance

It is a two-terminal storage element in which energy is stored in the magnetic field. The schematic representation of the inductance is shown in Fig. 1.14. The changing magnetic field set up by the time varying current through the inductance reacts to induce voltage in it to oppose the change of current (see polarity shown in Fig. 1.14). The voltage as in the case of previous two elements is shown to drop in the direction of current. The v–i relation of an inductance is

$$v = L\frac{di}{dt} \tag{1.33}$$

where L = inductance in henrys, H.

Integrating Eq. (1.33)

$$i = \frac{1}{L}\int_0^t v \; dt \; + \; i(0) \tag{13.4}$$

where $i(0)$ = inductance current at $t = 0$.

According to Eq. (1.34) *current through an inductance cannot change instantly* (compare with capacitance voltage) as it would require infinite voltage.

Let us establish the linearity of an inductance from Eq. (1.34). For convenience, we assume $i(0) = 0$. If

$$v = \alpha_1 v_1 + \alpha_2 v_2$$

Substituting in Eq. (1.34) gives

$$\begin{aligned} i &= \frac{1}{L}\int_0^t [\alpha_1 v_1 + \alpha_2 v_2] \; dt \\ &= \alpha_1 \frac{1}{L}\int_0^t v_1 \, dt + \alpha_2 \frac{1}{L}\int_0^t v_2 \; dt \\ &= \alpha_1 i_1 + \alpha_2 i_2 \end{aligned} \tag{1.35}$$

Energy Stored in Inductance The power fed to inductance is

$$p = vi = Li\frac{di}{dt}$$

Assuming $i\,(t = 0) = 0$, the stored energy is found by integrating power (p) as

$$w_L = \int_0^t p\,dt = \int_0^t L\frac{di}{dt}\,dt$$

or

$$w_L = L\int_0^t i\,di = \frac{1}{2}\;Li^2 \;\; J \tag{1.36}$$

The energy stored in the inductance depends upon the instantaneous current and is independent of the history of the current. As the current reduces to zero, the energy stored in the inductance is returned to the circuit in which it is connected.

An Observation Let us juxtapose the differential-integral equations governing the elemental behaviour of a capacitance and inductance.

$$\text{Capacitance } i = C\frac{dv}{dt};\ v = \frac{1}{C}\int i\ dt$$

$$\text{Inductance } v = L\frac{di}{dt};\ i = \frac{1}{L}\int v\ dt$$

It is seen from these relationships that one set can be obtained from the other by replacing

$$v \leftrightarrow i$$

$$L \leftrightarrow C$$

This interchangeability is known as the concept of *duality*.

Let us now examine a resistance element for which

$$v = Ri$$

$$i = Gv$$

Duality is extended to resistance by interchanging

$$R \leftrightarrow G$$

This concept will by applied to *RLC* circuits in Chapter 3.

A practical inductance is called an *inductor*. It is a coil wound on a magnetic core (may be air core for small values of inductance). A magnetic core inductor has constant inductance only in a limited range of current (at high current the core saturates and inductance reduces, see Chapter 7).

This means that it is linear in a limited range of currents. In electronic circuits the use of inductor is avoided except in high power circuits. In fact, inductance cannot be fabricated as such in semiconductor integrated circuits. Consider that an inductance of 1 H is excited with current waveform sketched in Fig 1.15(a).

- During voltage waveform upto 1s, the $i(t)$ rises from 0 to 4 A at rate 4 A/s. So the voltage $v = L\,\dfrac{di}{dt} = 1.4 = 4$V is constant. From 1 to 3*s*, the rate of change of current is zero and so is the voltage. From 3*s* to 4*s*, the current reduces to zero at rate of 4 A/s. So the voltage is constant –4 V. The voltage waveform is sketched in Fig. 1.15(b).
- Power waveform is the product v_i and is sketched in Fig. 1.15(c). The maximum power is 4 × 4 = 16 W.
- Stored energy waveform is $w_L = \frac{1}{2}Li^2 = \frac{1}{2}i^2$. It rises by square law upto $1/2 \times 4^2 = 8$J at 1s, remains constant from 1*s* to 3*s* as *i* is constant. It then decays to zero from 3*s* to 4*s* (square law) as shown in Fig. 1.15(d).

Independent Source

While the two storage elements (capacitance and inductance) studied earlier can absorb energy and deliver back the same, an independent source can deliver or absorb energy continuously (without any limit). Such elements are called *active* elements.

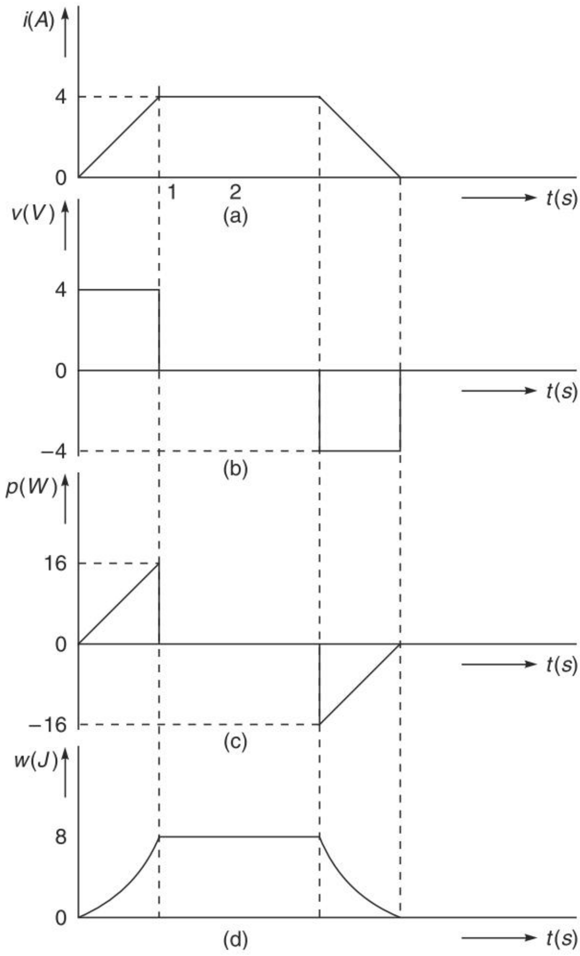


Fig. 1.15 Waveforms of (a) current, (b) voltage, (c) power and (d) energy in an inductance

An independent voltage source is shown in Fig. 1.16(a) and (b) along with its polarity markings. The source voltage is assumed to be completely independent of the current (ideal source). If current flows out of the positive terminal, the source is delivering energy (and power) to the circuit in which it is connected. On the other hand, if current flows into the positive terminal of the source it absorbs power. There is no limit to the power that an ideal source can deliver or absorb.

In a practical voltage source, energy is obtained or absorbed by a conversion process from another energy form. For example, in a battery source, chemical energy is converted to electrical form when the source is delivering, and vice-versa when the source is absorbing. In a practical source, the terminal

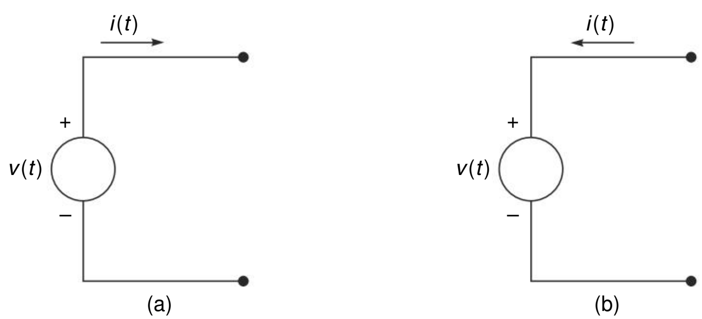


Fig. 1.16 Independent voltage source (two-terminal device) (a) delivering power (b) absorbing power

voltage is current dependent, though in the useful current range this dependence is limited. Further, a practical source can handle only a certain maximum power called *rated power*.

Similarly, an ideal *independent current source* can supply (or receive) a specified current independent of its voltage. Such a source is represented in Fig. 1.16(c). Its terminal voltage is determined by the conditions in the circuit to which it is connected. Such sources occur in electronic circuits. In a practical current source, current is voltage dependent but is practically independent of it in the useful range.

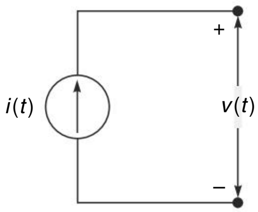
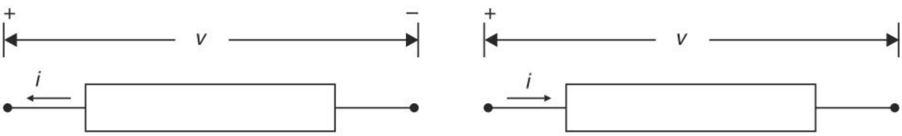


Fig. 1.16(c) Independent current source

Supplying/Absorbing Power It is not only sources that can supply or absorb power, any element can in fact supply or absorb power. In Fig. 1.17(a) below, the element is supplying power and in Fig. 1.17(b), it is absorbing power.

Capacitor and inductor can absorb or supply power, and thereby store or give away stored energy to the circuit to which their terminals are connected.

If the element is a resistor, it absorbs power only for positive value of resistance. However, an active electronic circuit can exhibit negative resistance and so supply power.



(a) Element supplying power

(b) Element absorbing power

Fig. 1.17

Dependent Sources (Ideal)

The voltage/current of a *dependent voltage/current source* is determined by voltage/current at another point in the circuit. The law of dependence is linear, say, a constant of proportionality. Figure 1.18 shows the representation of such dependent source*. Such sources are encountered in modelling of electronic devices.

SUMMARY

Ideal circuit elements we have studied are of two kinds:

Passive Elements (R, L, C) Resistance is the dissipative element. Capacitance and inductance can store and deliver energy without any loss of energy in

*The concept of dependent sources is useful in modelling transistors and other active devices in electronics.

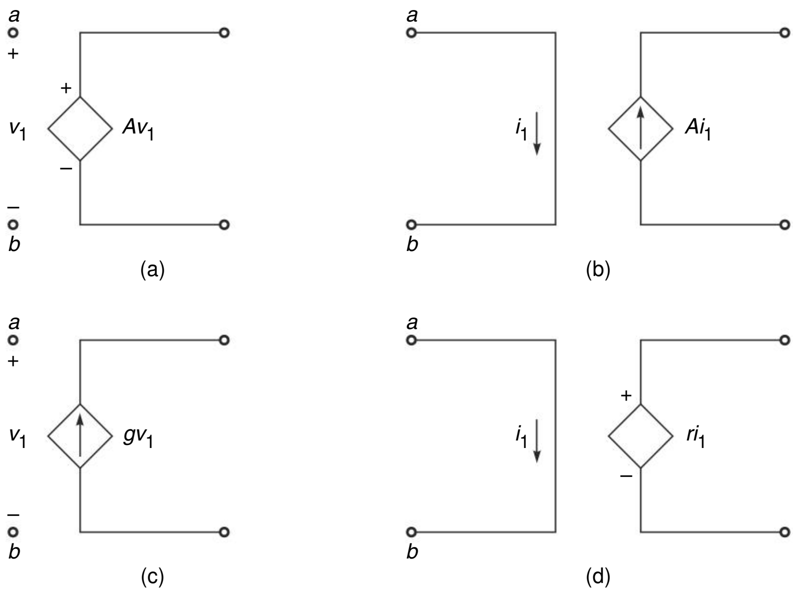


Fig. 1.18 (a) Voltage dependent voltage source
(b) Current dependent current source
(c) Voltage dependent current source
(d) Current dependent voltage source

the process, but their energy handling capacity is limited. Practical passive elements would possess all the three properties (resistance, capacitance and inductance) but depending upon their design one of these properties will predominate. A practical element can be modelled using ideal *R, L* and *C*.

Active Element (Sources) Ideal sources (independent, dependent) can handle infinite power and energy. But practical sources can handle finite (rated) power but infinite energy.

Certain Properties of Ideal Circuit Elements (R, L, C) These elements are:

- Linear (already explained)
- Time invariant
- Bilateral
- Lumped

Bilateral and Unilateral *R, L, C* ideal elements' behaviour is independent either of the terminal (node) at which current is fed in or of the direction of voltage applied at the terminals. If the terminal connections of an element in a circuit are reversed, it would not make any difference to the circuit response. This is the *bilateral* property of *R, L, C* elements.

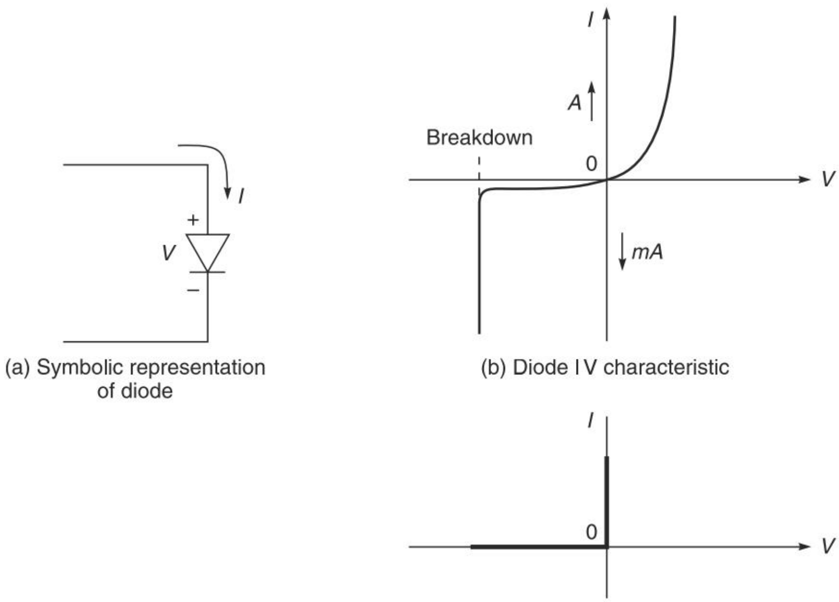
If an element or circuit does not possess the above property, it is said to be *unilateral*. For example, for the dependent voltage source of Fig 1.18(a) the input voltage controls the output voltage. However, a voltage applied at the output terminal would not control the voltage at the input terminals. This is the behaviour of an active device represented by this source which is inherently unilateral.

Nonlinear Linear behaviour of an element in an approximation (idealization) of a general nonlinear behaviour in a limited range of variables (voltage/current). Consider, for example, an electric lamp. As it is switched on, the element heats up and its resistance increases. It stabilizes at a particular temperature at which the element glows and gives off light (and some heat). This kind of behaviour is not only nonlinear but also *time-varying*.

Consider another element, a solid state diode shown schematically in Fig. 1.19(a). The I-V characteristic of an Si-diode in governed by the relation

$$I = I_s\,(e^{\rho v/nV_T} - 1)$$

or $$I = \phi(V)$$



(a) Symbolic representation of diode

(b) Diode I V characteristic

(c) Ideal diode characteristic

Fig. 1.19

It is plotted in Fig 1.19(a) and is highly nonlinear with directional behaviour. When V is positive, only a small value of this voltage can cause the diode to conduct large current, whose value is determined by the circuit in which the diode is connected. For negative values of V, the diode conducts negligible current and the diode breaksdown above a certain (large) negative voltage.

We see from the above account that a diode is a *nonlinear unilateral* element.

Distribution Effects In the ideal model for R, L and C, assume that there are no distribution effects. So in that sense, these are *lumped elements*. A word about distribution: consider a transmission line (say electric). It has resistance, inductance and capacitance all along the line, spread out in length. Except for a short line, we cannot lump these elements. But in the circuit we shall deal with, the element's physical dimensions are small enough compared to voltage/current wave length so that lumping is valid.

Statement The *R, L, C* ideal elements that we shall deal with are *linear, bilateral lumped* and *time-invariant*. This statement will not be repeated in the text but it applies to all the circuits we shall deal with as well as electric machines.

1.7 FUNDAMENTAL LAWS FOR ELECTRIC CIRCUITS

A circuit is in general composed of several ideal elements whose nodes (terminals) are connected in various ways. Several (two or more) elemental nodes may merge to form a single *node*. For convenience of tracking, nodes are labelled (usually by numbers). Shown in Fig. 1.20 is a circuit composed of five resistances and two voltage sources. We shall now define certain terms.

Node is a junction where two or more elemental points meet.

Path is the traversal through elements from one node to another without going through the same node twice.

Branch is a path between two adjoining nodes.

Loop is a closed path where the transversal ends upon the starting node.

Mesh is a loop that does not contain any other loop or within it. Figure 1.20 shows three meshes identified by mesh currents i_1, i_2 and i_3.

Independent Meshes M

$$M = B - N + 1 \tag{1.37}$$

where

B = number of branches
N = number of circuit nodes

In the circuit of Fig. 1.20
$M = 7 - 5 + 1 = 3$

Apart from Ohm's law already stated for a resistive element, we shall now enunciate two fundamental circuit laws (Kirchhoff's laws) which follow rationally from the nature of electrical quantities.

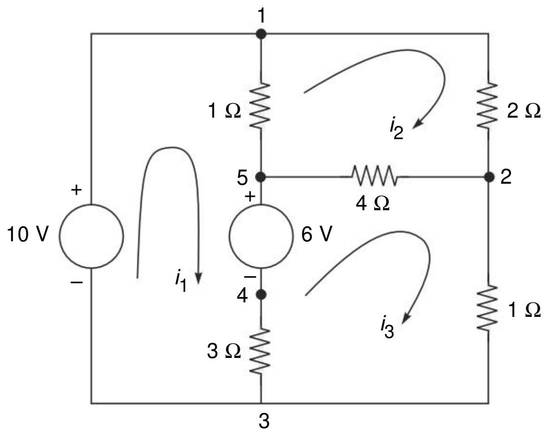


Fig. 1.20 A circuit

Kirchhoff's Current Law (KCL)

Algebraic sum of currents going away from and *coming towards a node is zero.* If the current going away from the node is taken as positive, current coming towards the node is negative or vice versa.

This law is a simple and obvious consequence of the fact that no charge can accumulate at a node. Consider the node in Fig. 1.21. It immediately follows that

$$i_1 - i_2 + i_3 - i_4 + i_5 = 0$$

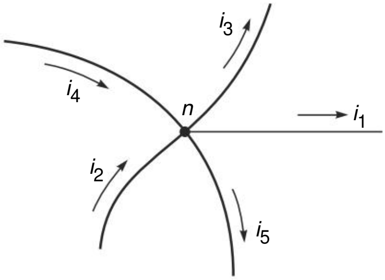


Fig. 1.21

Example 1.3 Consider the 2-node circuit of Fig. 1.22. Note that 0-node is the *reference node*. Write the KCL equations for the two nodes.

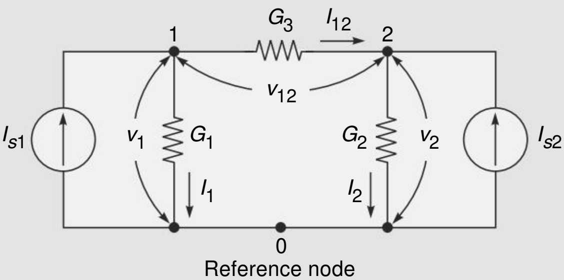


Fig. 1.22

Solution

G_1, G_2, G_3 are the conductances of the three branches.

At node 1

$$I_{s1} - I_1 - I_{12} = 0$$

or $$I_{s1} - G_1 v_1 - G_3 v_{12} = 0$$

or $$G_1 v_1 + G_3 v_{12} = I_{s1} \quad \text{(i)}$$

At node 2

$$I_{s2} - I_2 + I_{12} = 0$$

or $$I_{s2} - G_2 v_2 + G_3 v_{12} = 0$$

or $$G_2 v_2 - G_3 v_{12} = 0 \quad \text{(i)}$$

Going round the inner loop, it is seen that

$$v_{12} = v_1 - v_2 \quad \text{(iii)}$$

Substituting the value of v_{12} in Eqs. (i) and (ii), we get

$$G_1 v_1 + G_3(v_1 - v_2) = I_{s1}$$
$$G_2 v_2 - G_3(v_1 - v_2) = I_{s2}$$

Rearranging, we have

$$(G_1 + G_3)v_1 - G_3\, v_2 = I_{s1} \quad \text{(iv)}$$
$$-\, G_3\, v_1 + (G_2 + G_3)\, v_2 = I_{s2} \quad \text{(v)}$$

These are the two simultaneous algebraic equations to determine v_1 and v_2.

Remark

(i) In writing KCL equation, there is always a reference node w.r.t. which all other node voltages are defined.
(ii) KCL equations number is the same as number of nodes other than the reference node.
(iii) KCL equations solution gives the unknown node voltages.
(iv) Currents in resistance can be obtained from nodal voltage by Ohm's law.
(iv) Reader should observe and discover that KCL equations have a symmetry. This shall be taken up in Chapter 2.

Example 1.4 With reference to Fig. 1.23, we are given

$$i_1 = -\frac{1}{2}e^{-2t}, \quad v_3 = 2e^{-2t}, \quad v_4 = 5e^{-2t}$$

Find v_2.

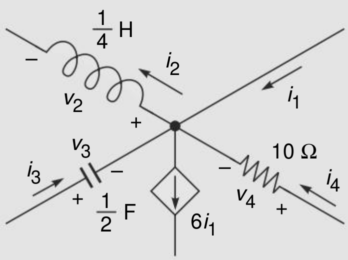


Fig. 1.23

Solution

$$i_3 = \frac{1}{2}\frac{d}{dt}(2e^{-2t}) \qquad \text{from Eq. (1.27)}$$

$$= -2e^{-2t}$$

$$i_4 = \frac{1}{10} \times 5e^{-2t}$$

$$= 0.5e^{-2t}$$

KCL equation at the node is

$$-i_1 + i_2 - i_3 + 6i_1 - i_4 = 0$$

$$5i_1 + i_2 - i_3 - i_4 = 0$$

Substituting values, we get

$$-\frac{5}{2}e^{-2t} + i_2 + 2e^{-2t} - \frac{1}{2}e^{-2t} = 0$$

or $\quad i_2 = +e^{-2t}$

$\therefore \quad v_2 = \frac{1}{4}\frac{d}{dt}(+e^{-2t}) = \frac{1}{2}e^{-2t}$ from Eq. (1.33)

Kirchhoff's Voltage Law (KVL)

The algebraic sum of voltage drops (or rises) round a loop (closed path) or mesh in a specified direction is zero. If voltage drop is taken as positive, voltage rise is negative or vice versa.

This law is a consequence of the fact that in a loop transversal, we return to the starting node.

Example 1.5 Consider a two-mesh network of Fig. 1.24. Express the currents in the three resistances in terms of mesh currents and then write the KVL equation.

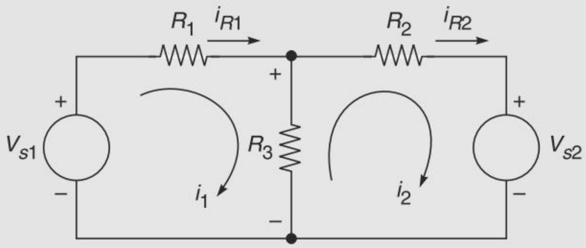


Fig. 1.24

Solution

The two mesh currents are indicated in Fig. 1.24. The currents in the resistances are

$$i_{R1} = i_1, \qquad i_{R2} = i_2, \qquad i_{R3} = i_1 - i_2 \tag{i}$$

The polarity of voltages across the resistances are also shown in Fig. 1.24.

The KVL equations for the two meshes can now be written down (sum of voltage drops)

Mesh 1

$$v_{s1} + R_1 i_{R1} + R_3 i_{R3} = 0 \tag{ii}$$

Mesh 2

$$v_{s2} + R_2 i_{R2} - R_3 i_{R3} = 0 \tag{iii}$$

These equations are expressed in terms of mesh currents (as per Eq. (i)) as

$$v_{s1} + R_1 i_1 + R_3(i_1 - i_2) = 0 \tag{iv}$$

$$v_{s2} + R_2 i_2 - R_3(i_1 - i_2) = 0 \tag{v}$$

These equations are reorganized below

$$v_{s1} = (R_1 + R_3)i_1 - R_3 i_2 \tag{vi}$$

$$-v_{s2} = -R_3 i_1 + (R_2 + R_3)i_2 \tag{vii}$$

These are the requisite mesh equations. Notice that v_{s2} has negative sign as its polarities are in opposition to the mesh current i_2.

Remark

1. The two mesh equations can be solved for mesh currents.
2. Currents in resistances can be obtained from the mesh current.
3. The reader should observe a symmetry in mesh currents, so the mesh equations can be written down by inspection of the circuit. This will be elaborated in Chapter 2.

Example 1.6 Consider the mesh of Fig. 1.25 which forms part of a larger circuit. Given $v_1 = 10 \sin t$, $i_2 = 4 \sin t$, $i_3 = 2 \cos t$

Determine i_4.

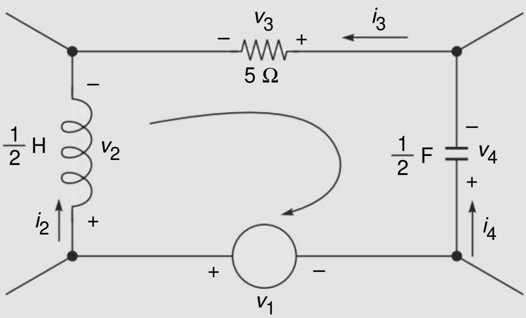


Fig. 1.25

Solution

Going round the mesh in clockwise direction and applying KVL, we get

$$-v_1 + v_2 - v_3 - v_4 = 0 \quad \text{(i)}$$

$$v_2 = \frac{1}{2}\frac{d}{dt}(4 \sin t) \quad \text{using Eq. (1.33)}$$

$$= 2 \cos t \quad \text{(ii)}$$

$$v_3 = 5 \times 2 \cos t = 10 \cos t \quad \text{(iii)}$$

Substituting in Eq. (i)

$$-10 \sin t + 2 \cos t - 10 \cos t - v_4 = 0$$

or

$$v_4 = -8 \cos t - 10 \sin t$$

$$i_4 = \frac{1}{2}\frac{d}{dt}(-8 \cos t - 10 \sin t) \text{ using Eq. (1.27)}$$

$$= -5 \cos t + 4 \sin t$$

Let

$$4 = A \cos \theta$$

$$5 = A \sin \theta$$

Solving

$$A = \sqrt{41},\ \theta = \tan^{-1} 5/4$$

$$i_4 = \sqrt{41} \cos (t + \tan^{-1} 5/4)$$

Example 1.7 In the circuit of Fig. 1.26 find R and V_s. Also find the power output of the source V_s.

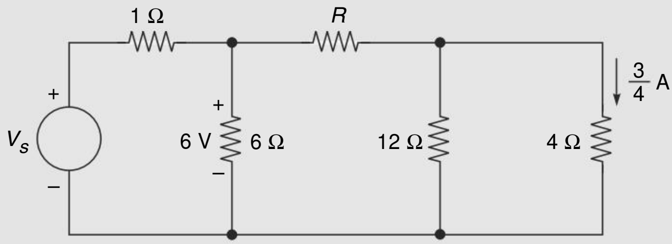


Fig. 1.26

Solution

By application of Ohm's laws, we get voltage across 4 Ω resistance (3 V) which also appears across 12 Ω resistance (these are in *parallel*) and as a consequence current through the 12 Ω resistance is 1/4 A (Fig. 1.27).

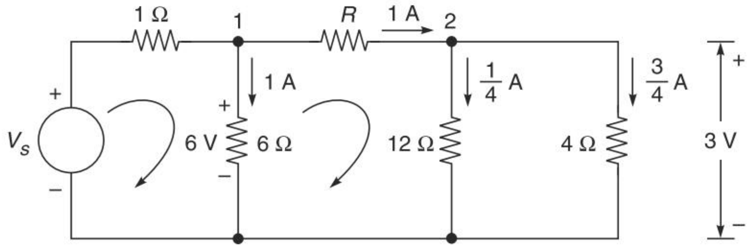


Fig. 1.27

Apply KCL at node 2:

$$\text{Current through } R = \frac{3}{4} + \frac{1}{4} = 1 \text{ A}$$

Applying KVL around the middle mesh;

$$-6 + 1 \times R + \frac{1}{4} \times 12 = 0$$

or $$R = 3\ \Omega$$

Applying KCL at node 1, current through 1 Ω resistance = 1 + 1 = 2 A. Applying KVL round the left mesh,

$$-V_s + 1 \times 2 + 6 = 0$$

or $$V_s = 8 \text{ V}$$

Current out of the positive terminal of V_s (apply KCL at node 1) = 2 A. Power output of V_s source = 8 × 2 = 16 W.

Example 1.8 For the circuit of Fig. 1.28, $i_C = -2e^{-t}$ A, Find V_s.

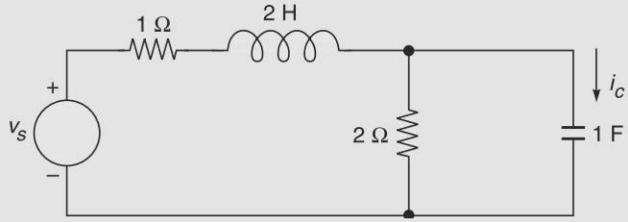


Fig. 1.28

Solution

$$V_C = \frac{1}{C}\int_0^t i_c \; dt = -\int_0^t 2e^{-t} \; dt = 2e^{-t} \text{ V}; v_c(0) = 0$$

This voltage appears also across 2 Ω resistance (*R* and *C* in parallel). Therefore

$$i_{(2\,\Omega)} = \frac{2e^{-t}}{2} = e^{-t} \text{ A}$$

By applying KCL current through *RL* series, we get

$$i = -2e^{-t} + e^{-t} = -e^{-t} \text{ A}$$

$$v_i = L\frac{di}{dt} = 2\frac{d}{dt}(-e^{-t}) = 2e^{-t} \text{ V}$$

$$v_R = Ri = 1 \times (-e^{-t}) = -e^{-t} \text{ V}$$

Applying KVL round the mesh, we have

$$-v_s - e^{-t} + 2e^{-t} + 2e^{-t} = 0$$

or

$$v_s = 3e^{-t} \text{ V}$$

1.8 CONCLUSION

Circuits are composed of six basic types of ideal elements—three passive R, L, C, and three active elements, voltage/current sources and dependent source. A circuit is used to model the behaviour of physical electrical/electronic systems. The laws governing the ideal elemental behaviour (v–i relationships) are proportional, differential and integral relationships or merely constant values. These laws obey the principle of *superposition* and *homogeneity* and are hence *linear*.

Circuits which are composed of elements which obey the principles of superposition and homogeneity are *linear circuits.*

Since currents and voltages are set up in circuit elements because of the presence of sources, the source voltage/current is called *excitation* (something that excites the circuit elements).

Observation It is seen from Example 1.6 that the voltages and currents in all the elements and the source are sinusoidal of the same frequency. This in fact depends upon the excitation, which is voltage v_1 in this example. This is a property of linear circuits.

Also, in Example 1.8 that the voltages and currents in the circuit elements are all exponential, i.e. same as that of the excitation voltage v_s.

Example 1.9 Figure 1.29 shows one node of an electric circuit. Using KCL, find v_2. Given:

$$i_1 = 4 \text{ A}, \qquad v_3 = 3V, \qquad v_4 = 8V$$

Solution

Using Ohm's law, we get

$$i_3 = 3/3 = 1 \text{ A}, \quad i_4 = 8/4 = 2 \text{ A}$$

Applying KCL at the node, we get

$$i_1 - 3i_2 + i_2 - i_3 - i_4 = 0$$

or

$$i_1 - 2i_2 - i_3 - i_4 = 0$$

Substituting different values, we get

$$4 - 2i_2 - 1 - 2 = 0$$

or

$$i_2 = 0.5 \text{ A}$$

$\therefore$

$$v_2 = 2 \times 0.5 = 1 \text{ V}$$

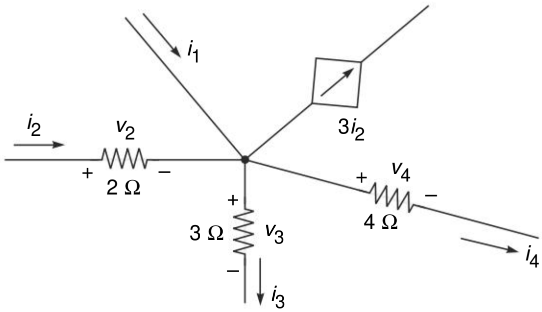


Fig. 1.29

Example 1.10 Consider the loop of Fig. 1.30 which forms part of an electric circuit. Find i. Given

$$v_1 = 6 \text{ V}, \qquad i_2 = 2 \text{ A}, \qquad i_3 = 4 \text{ A}$$

Solution

Using Ohm's law

$$v_2 = 2 \times 2 = 4 \text{ V}, \qquad v_3 = 2 \times 4 = 8 \text{ V}$$

Applying KVL around the loop in the indicated direction, we have

$$v_1 + v_4 - 4i_2 - v_3 + v_2 = 0$$

or $$6 + v_4 - 8 - 8 + 4 = 0 \text{ or } v_4 = 6 \text{ V}$$

∴ $$i_4 = 6/3 = 2 \text{ A}$$

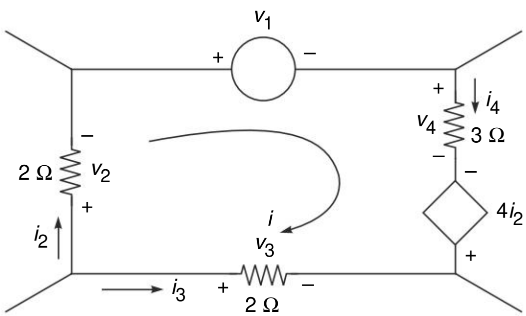


Fig. 1.30

Example 1.11 For the resistive circuit shown in Fig. 1.31, find (a) I_1 (b) V_s.

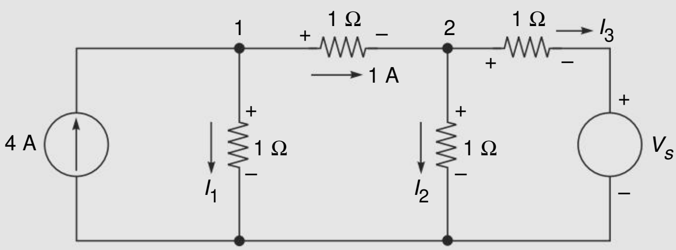


Fig. 1.31

Solution

(a) Applying KCL at node 1, we get

$$I_1 = 4 - 1 = 3 \text{ A}$$

(b) $V_1 = 3 \times 1 = 3$ V, $V_{12} = 1 \times 1 = 1$ V, $V_2 = 3 - 1 = 2$ V, $I_2 = 2/1 = 2$A

KCL at node 2 gives

$$I_3 = 1 - 2 = -1 \text{ A}$$

$$\therefore \quad V_s = V_2 - 1 \times I_3 = 2 - (1 \times -1) = 3 \text{ V}$$

Example 1.12 For the resistive circuit of Fig. 1.32, find i_x and v_x. Also find R_1 and R_2.

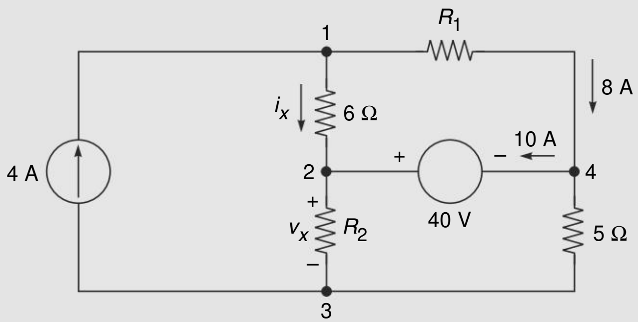


Fig. 1.32

Solution

$i_{34} = 10 - 8 = 2$A $v_{34} = 5 \times 2 = 10$ V

$v_x = v_{23} = 40 - 10 = 30$ V $i_x = 4 - 8 = -4$ A

$i_{23} = i_x + 10 = -4 + 10 = 6$A, $R_2 = v_x / i_{23} = 30/6 = 5\Omega$

$v_{14} = 40 + 6i_x = 40 + 6 \times -4 = 16$ V

$R_1 = v_{14}/8 = 16/8 = 2\Omega$.

Example 1.13 For the circuit of Fig. 1.33 $v_s = 0.01\cos 1000t$ V. Determine v_0.

Solution

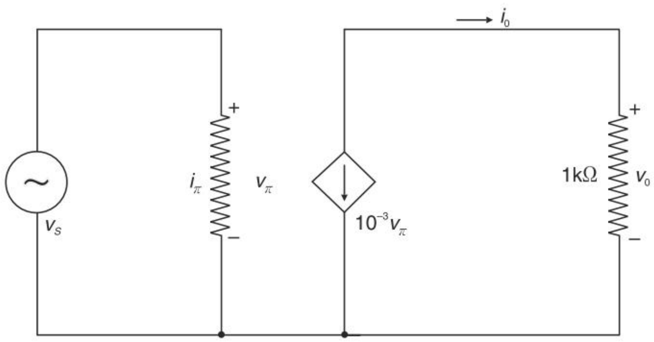


Fig. 1.33

$v_s = 0.01\cos 1000t$ V

$i_0 = -10^{-3}\, v_s = -0.01 \times 10^{-3} \cos 1000t$ V

$v_0 = 1 \times 10^3\, i_0$

$= -0.01 \times 10^3 \times 10^{-3} \cos 1000\, t$

$= -0.01 \cos 1000\, t$ V

Example 1.14 A voltage of $200\sqrt{2}\ \sin(314t)$ V is applied across a 10 μF capacitor shown in the adjoining figure.

(a) Determine the capacitance current i as a function of time

(b) Sketch voltage and current waveforms. What conclusions do you draw?

(c) Sketch instantaneous power as a function of time. What do you observe? Also calculate the value of average power.

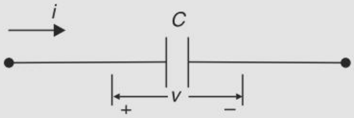


Fig. 1.34

Solution

(a) $$i = C\,\frac{dv}{dt} = 10 \times 10^{-6}\,\frac{d}{dt}[200\sqrt{2}\sin 314t]$$

$= 10 \times 10^{-6} \times 200\sqrt{2} \times 314 \cos 314\, t$

$= 0.888 \cos 314\, t$

(b) Voltage and current waveforms are plotted in figure as below. It is observed that the positive peaks of the current occurs 90° earlier than the positive peaks of the voltage. It means that current *leads* the voltage by 90°.

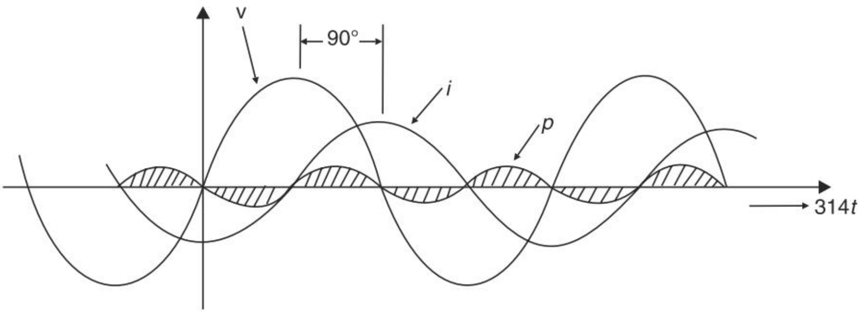


Fig. 1.35

c) Instantaneous power

$p = vi = 200\sqrt{2} \times 0.888 \sin(314\, t) \cos(314t)$

Using trignometric identity, $2 \sin\theta \cos\theta = \sin 2\theta$, we can write

$p = 125.56 \sin(628\text{t})$ W

We find that the instantaneous power oscillate at twice the frequency (i.e. 628 rad/s) of the voltage/current frequency. The average power is zero.

Conclusions

1. The current in a capacitor leads the applied voltage by 90°.
2. Instantaneous power drawn by a capacitor oscillates at twice the frequency of applied voltage.
3. Average power drawn by a capacitor is zero. The energy oscillates back and forth between capacitor and voltage source.

ADDITIONAL SOLVED PROBLEMS

1.1 A resistor R = 800 Ω is shown in the figure. Calculate the power absorbed by R at $t = 0.1s$ when (a) $i = 50\ e^{-10t}$ mA, (b) $v = 50 \cos 25\,t$ V (c) $vi = 10\ t^{2.5}$ VA.

Fig. 1.36

Solution

(a) At $t = 0.1$s

$i = 50\ e^{-1} = 18.4$ mA

$P = i^2 R = [1.84 \times 10^{-3}]^2 \times 800 \times 10^3 = 271$ mW

(b) At $t = 0.1$s

$v = 50 \cos (2.5 \text{ rad}) = 50 \cos 143.2° = -40$ V

$$P = \frac{v^2}{R} = \frac{(-40)^2}{800} = 2W \text{ or } 2{,}000 \text{ mW}$$

(c) $vi = 10 \times (0.1)^{2.5} = 10 \times 3.16 \times 10^{-3}$ W

$= 31.6$ mW

1.2 For the circuit of Fig. 1.37 with voltage dependent current source v_2 = 5V, determine v_s.

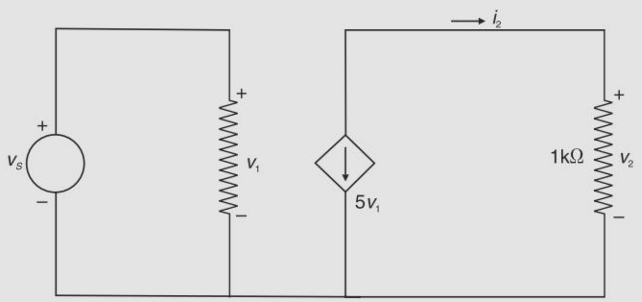


Fig. 1.37

Solution

$$i_2 = \frac{v_2}{1} = 5 \text{ mA} = 5 \times 10^{-3}\ A$$

$5v_1 = -\,i_2 = -5 \times 10^{-3}$

or $v_1 = -10^{-3}\ V = v_s$

or $v_s = -1$ mV

1.3 The inductor of the figure has L = 20 mH.

(a) Find v at t = 10 *ms* if $i = 8e^{-100t}$.

(b) Find i at t = 0.1*s* if $v = 6e^{-12t}$ and i (0) = 8A

(c) If $i = 10\,(1-e^{-50t})$ find the power being delivered to the inductor at t = 40*ms* and the energy stored at t = 50*ms*.

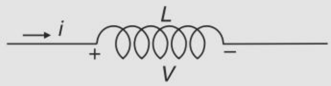


Fig. 1.38

Solution

(a) $$v(t) = L\frac{di}{dt} = 20\times 10^{-3}\frac{d}{dt}(8e^{-100t})$$

$$= 20 \times 10^{-3} \times 8 \times (-100)e^{-100t} = -16e^{-100t}$$

$$v(10 \text{ ms}) = -16e^{-100\times 10\times 10^{-3}} = -16e^{-1}$$

$$= -5.89 \text{ V}$$

(b) $$i(t) = \frac{1}{L}\int v\, dt + K$$

$$i(t) = \frac{10^3}{20}\int 6e^{-12t} dt + K$$

$$= 50 \times \frac{6}{-12}e^{-12t} + K\,. = -25e^{-12t} + K$$

At $t = 0$

$$8 = -25 + K \Rightarrow K = 33$$

Thus

$$i(t) = -25e^{-12t} + 33 \text{ A}$$

$$t = 0.1\text{s}$$

$$i(0.1s) = -25e^{-1.2} + 33 = 25.47 \text{ A}$$

(c) $$i(t) = 10\,(1-e^{-50t})$$

$$v(t) = L\frac{di}{dt} = 20\times 10^{-3} \times 500e^{-50t}$$

$$= 10e^{-50t} \text{ V}$$

At t = 40 ms

$$i\,(40 \text{ ms}) = 10\,(1-e^{-50\times 40\times 10^{-3}}) = 8.65 \text{ A}$$

$$v(40 \text{ ms}) = 10e^{-2} = 1.35 \text{ V}$$

Power being delivered to inductor is given by

$$p\,(40 \text{ ms}) = 1.35 \times 8.65 = 13.03 \text{ W}$$

At t = 50 ms

$$i(50 \text{ ms}) = 10\,(1-e^{-2.5}) = 0.82 \text{ A}$$

Then $$w_L(50 \text{ ms}) = \frac{1}{2}\times\frac{10^3}{20}\times(0.82)^2$$

$$= 6.724 \text{ J}$$

1.4 The current through a capacitor is $i(t) = 9 \sin \pi t$ μA. If the energy stored at t = 200 ms is 3 μJ, what is the value of the capacitor?

Solution

Stored energy $$w_C = \frac{1}{2}Cv^2 \qquad \text{(i)}$$

$$v = \frac{1}{C}\int i\,dt = \frac{1}{C}\int (9\sin \pi t)\times 10^{-6}dt$$

or

$$v = -9.\frac{1}{C}.\frac{1}{\pi}\cos \pi\, t \times 10^{-6}$$

Substituting v in Eq. (i)

$$w_C = \frac{1}{2}\cdot C\cdot \frac{81}{C^2}\cdot\frac{1}{\pi^2}\cos^2 \pi t \times 10^{-12}$$

$\pi t = \pi \times 200 \times 10^{-3} = 0.628$ rad or 36°; cos 36° = 0.809

Substituting values

$$3\times 10^{-6} = \frac{81}{2C}\cdot\left(\frac{0.809}{\pi}\right)^2 \times 10^{-12}$$

or C = 0.895 μF

1.5 The voltage of 25 cos 500t V is applied to 25 mH inductor. The inductor current is zero at t = 0. Find the power being absorbed by the inductor and the energy stored in it at t = 5ms. Find the first time (t > 0) at which the power absorbed is zero and the time when energy stored is zero.

Solution

$$i(t) = \frac{1}{L}\int v(t)dt; i(0) = 0$$

$$i(t) = \frac{1}{L}\int 25\cos 500t\, dt$$

$$= \frac{10^3}{25}\cdot 25\cdot\frac{1}{500}\sin 500t$$

or $i(t) = 2\sin 500t$ A

At t_1 = 5 ms, $500t_1 = 500 \times 5 \times 10^{-3}$ = 2.5 rad or 143.2°

Cos 143.2° = –0.8, sin 143.2 = 0.6

$v(t_1) = 25 \times (-0.8) = -20$ V

$i(t_1) = 2 \times 0.6 = 1.2$ A

Power being absorbed

$p(t_1) = -20 \times 1.2 = -24$ W

1.6 A capacitor is fabricated from two thin aluminium discs with 1 cm diameter separated by 150 μm, with air in between. Calculate its capacitance. What voltage should be applied across this capacitor to store 1 mJ of energy?

What should be the relative dielectric constant of the material to be placed in between the disc so as to store 2 μJ of energy at 100 V?

Solution

The capacitance of the parallel plate capacitor is

$$C = \varepsilon_r\varepsilon_0\left(\frac{A}{d}\right) F \qquad \text{(i)}$$

$\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, ε_r = 1 for air

Substituting values, we get

$$A = \pi \times \left(\frac{1\times 10^{-2}}{2}\right) = \left(\frac{\pi}{4}\right)\times 10^{-4} m^2$$

$d = 150 \times 10^{-6}$ m

$$C = \left[8.85\times 10^{-12}\times \frac{(\pi/4)\times 10^{-4}}{150\times 10^{-6}}\right]$$

$= 4.6$ pF

Energy stored

$$w_C = \frac{1}{2}Cv^2$$

$$1\times 10^{-3} = \frac{1}{2}\times 4.6\times 10^{-12} v^2$$

or $v = 6.59$ kV

To store 2μJ at 100 V, we have to find ε_r

From Eq (i) capacitance would now be

$$C = \varepsilon_r \times 4.6 \text{ pF} \quad \text{(ii)}$$

Substituting values in Eq. (ii)

$$2\times 10^{-6} = \frac{1}{2}\varepsilon_r \times 4.6\times 10^{-12}\times (100)^2$$

or $\varepsilon_r = 87$

1.7

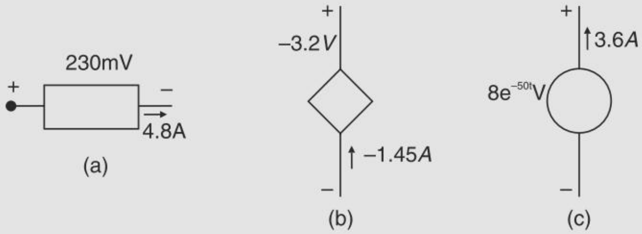


Fig. 1.39

(a) Is the power being absorbed or generated by the circuit element of Fig. 1.39(a) and what is the value of this power?

(b) Is the power being absorbed or generated in the circuit element of Fig. 1.39(b) and what is the value of the power?

(c) Find the power being absorbed or supplied by the circuit element of Fig. 1.39(c) at $t = 10$ ms.

Solution

(a) The current is flowing into positive terminal. So power is being absorbed is

$$p(\text{absorbed}) = 230\times 10^{-3}\times 4.8 = 11.04 \text{ W}$$

(b) The current is flowing out of positive terminal. So power generated is

$$p\text{ (gen)} = -3.2\times(-1.45) = +4.64 \text{ W}$$

(c) The current is flowing out of positive terminal. So

$$p\text{ (supplied)} = 8e^{-50t}\times 3.6 = 28.8\, e^{-50t} \text{ W}$$

At $t = 10$ ms

$$p\text{ (supplied)} = 28.8\, e^{-0.5} = 17.45 \text{ W}$$

1.8 For the circuit of Fig. 1.40, write the expressions for v_R and v_L.

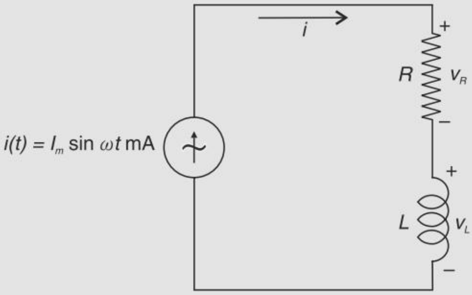


Fig. 1.40

Solution

$$v_R(t) = Ri = I_m R \sin \omega t \qquad \text{(i)}$$

$$v_L(t) = L\frac{di}{dt} = L\frac{d}{dt}[J_m \sin \omega t]$$

$$= I_m (L\omega)\cos\omega t \qquad \text{(ii)}$$

1.9 In example 1.8, if we are given that

I_m = 250 mA, ω = 100 rad/s, R = 4Ω, L = 50 mH

Then at t = 25 ms, find

(a) the power dissipated in resistor
(b) the power absorbed or given out by inductor
(c) The energy stored in inductor
(d) the minimum and maximum energy stored in inductor.

Solution

We will use the expressions derived in Problem 1.8.

At t = 25 ms, $\omega t = 100 \times 25 \times 10^{-3}$ = 2.5 rad or 143.2°

$$i\text{ (20 ms)} = 250\sin 143.2° = 149.6 \text{ mA}$$

$$v_R\text{ (20 ms)} = 149.6\ 10^{-3} \times 4 = 0.6 \text{ V}$$

$$v_L\text{ (200 ms)} = 250 \times 10^{-3} \times 100 \times 50 \times 10^{-3} \cos 143.2°$$

$$= -1 \text{ V}$$

(a) $p_R = i^2 R = (0.1496)^2 \times 4 \times 10^3 = 89.5$ mW

(b) $p_L = -1 \times 149.6 = -149.6$ mW

The negative sign mean the inductor is supplying power.

(c) Energy stored in inductor is

$$w_L = \frac{1}{2}Li^2 = \frac{1}{2} \times 50 \times 10^{-3} \times (0.1496)^2 \times 10^6$$

$$= 559.5 \ \mu\text{J}$$

(d) Minimum stored energy (at i = 0) = 0

Maximum stored energy is at i (max) = I_m = 250 mA

$$w_L\text{ (max)} = \frac{1}{2} \times 50 \times 10^{-3} \times (0.25)^2 \times 10^6 = 1562.5 \ \mu\text{J}$$

1.10 A plot of voltage v_s and time is drawn to scale in Fig. 1.41. This voltage is applied to a capacitor of 2μF capacitance. Plot to scale the capacitor current v_s time. Also plot energy stored in capacitor v_s time.

Solution

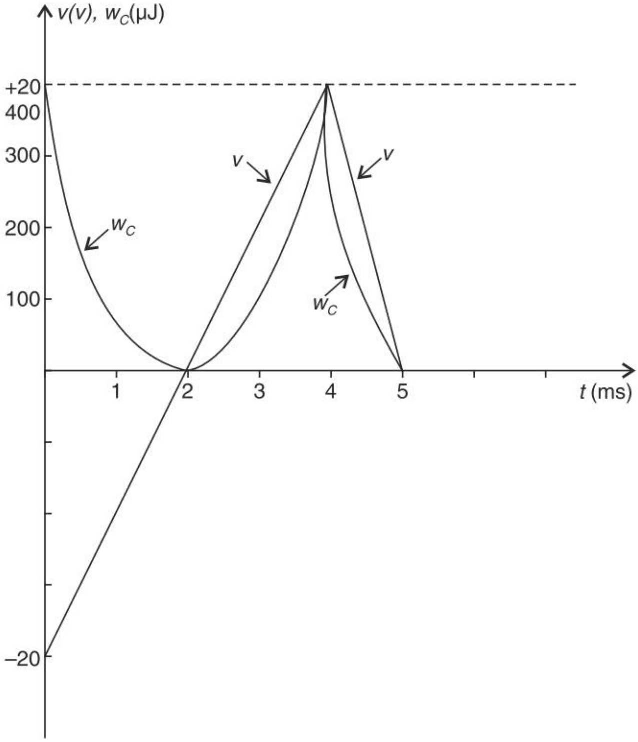


Fig. 1.41

$$i_C = C\frac{dv}{dt}$$

Units

$$i = \frac{F \times 10^{-6} \times V}{T \times 10^{-3}} = \frac{FV}{T} \times 10^{-3} = A \times 10^{-3}\ \text{mA}$$

where voltage V changes with time T linearly

From the figure,

$$i_C(0 \text{ to } 4 \text{ ms}) = 2 \times \frac{40}{4} = 20 \text{ mA}$$

$$i_C(4 \text{ to } 5 \text{ ms}) = 2 \times \frac{-20}{1} = -40 \text{ mA}$$

Stored energy is

$$w_C = \frac{1}{2}\ Cv_2$$

$\underline{0 - 4 \text{ ms}}$

$$v = 10t - 20$$

$$w_C = \frac{1}{2} \times 2 \times (10t - 20)^2\ \mu\text{J}$$

4 – 5 ms

$v = -20t + 100$

$$w_C = \frac{1}{2} \times 2 \times (-20t + 100)\ \mu J$$

w_C is plotted in Fig. 1.41

SI UNITS—INTERNATIONAL STANDARD OF UNITS

Quantity	Symbol	Unit	Symbol
SI Base Units			
Length	-	meter	m
Mass	-	kilogram	kg
Time	-	seconds	s
Temperature	-	Kelvin (°C + 273)	K
Current	i, I	ampere	A
Derived Units			
Charge	q	Coulomb	C
Energy	w	Joule	J
Power	p, P	Watt	W
Voltage	v, V	Volt	V
Circuit elements			
Resistance	R	Ohm	Ω
Conductance	$G = \frac{1}{R}$	Siemen, Mho	S, ℧
Capacitance	C	Farad	F
Inductance	L	Henry	H

The SI system uses decimal system to relate smaller and larger unit values to the basic unit, powers of 10 are prefixed to the basic unit. A list of prefixes, their symbols and names are given in Table 1.1.

Table 1.1 SI Prefixes

Factor	Name	Symbol	Factor	Name	Symbol
10^{-12}	Pico	p		-	- -
10^{-9}	Nano	n	10^{9}	Giga	G
10^{-6}	Micro	μ	10^{6}	Mega	M
10^{-3}	Milli	m	10^{3}	Kilo	k
10^{-2}	Centi	c	10^{2}	Hector	h
10^{-1}	Deci	d	10^{1}	Deka	da

For terms of distance, it is much more common to use 'micron (μm)' than micro; also the unit of an angstrom (Å) is used for 10^{-10} m.

The student is well advised to memorize these prefixes as these are used in this book and all other technical work.

Engineering Units. A quantity is represented by a number 1 to 999 and appropriate metric unit using a power divisible by 3.

Example: $1.2 \times 10^{-5} s$ is expressed as 12μs
13,560,000 Hz as 13.56 MHz
49,000 Ω as 49 kΩ

SUMMARY

- The direction in which positive charges move shows the direction of positive current; alternatively, positive current flow is in the direction opposite to that in which electrons are moving.
- Any element or source is said to supply power if the positive current flows out of positive terminal. Any element or source absorbs (receives) power if the positive current flows into its positive terminal.
- There are six sources: the independent voltage source, independent current source and four possible dependent sources.
- Ohm's law states that the voltage across a resistor is directly proportional to the current flowing through it. i.e., $v = iR$.
- Power dissipated in resistance is given by $p = vi = i^2R = \dfrac{v^2}{R} = Gv^2$ Watts
- Capacitor v - i relationship $i = C\dfrac{dv}{dt}$; integral form can also be used; initial voltage to be accounted for. Stored energy, $w_c = \dfrac{1}{2}Cv^2$ Joules
- Inductance v - i relationship $v = L\dfrac{di}{dl}$; integral form can also be used; initial current to be accounted for stored energy; $w_L = \dfrac{1}{2}Li^2$ Joules
- Kirchhoff's Current Law [KCL] states that algebraic sum of current leaving (or entering) a node are zero.
- Kirchhoff's Voltage Law [KVL] states that algebraic sum of voltage drops (or rises) around any closed path is zero.
- Sinusoidal waveform (voltage or current) $v = V_m \cos \dfrac{2\pi}{T} t$.

  V_m = maximum or peak value
  T = time period (usually in seconds)

  or

  $v = V_m \cos 2\pi f t$

  $f = \dfrac{1}{T}$ frequency in Hertz [Hz] i.e., cycles/s

  or

  $v = V_m \cos \omega t$
  $\omega = 2\pi f$; frequency in rad/s

- For comparing the phases of two sinusoidal waveforms, it is convenient to express both in cosine or both in sine form with positive sign. Of course, both must have same frequency.

REVIEW QUESTIONS

1. Derive the expression for potential difference between two point charges (q Coulomb) of opposite sign displaced by distance d apart.
2. Sketch the current and voltage waveform if
$$i = I \sin(\omega t - 60°)$$
$$v = V \cos(\omega t + 30°)$$
What is the phase difference between the two and which one leads the other?
3. A capacitor of C Farads is charged with q Coulomb. What is the stored energy?
4. State the principles of superposition and homogeneity.
5. Show that an inductor is a linear circuit element.
6. In an inductor L Henry, charge is flowing at the rate of q Coulombs/sec. What is its stored energy?
7. State the conditions for a source or element to supply or absorb power.
8. A capacitor C Farads at an instant has voltage V which is reducing at the rate $\frac{dv}{dt}$. Is it supplying or absorbing power and how much?
9. State Ohm's law.
10. State KCL and underlying logic.
11. State KVL and underlying logic.
12. Define a node and mesh.

PROBLEMS

1.1 Consider a 230 V, 100 W incandescent lamp. Determine:
(a) the lamp resistance,
(b) the lamp current, and
(c) the energy consumed by the lamp in 8h.

1.2 Figure 1.42(a) shows a black box that contains a single ideal circuit element. Three voltage–current relationships for this black box are shown in Fig. 1.42(b), (c) and (d).
(i) Identify the circuit element in each case and its value in appropriate units.
(ii) Find the peak energy storage/power dissipation.
(iii) In resistive case find also the total energy consumed.

1.3 Voltage $200\sqrt{2}$ sin 314 t is applied across a capacitor of 100 μF.
(a) Determine capacitor current as function of time.
(b) Sketch voltage and current waveforms. What conclusion do you draw?
(c) Sketch instantaneous power (vi) as function of time. What do you observe?

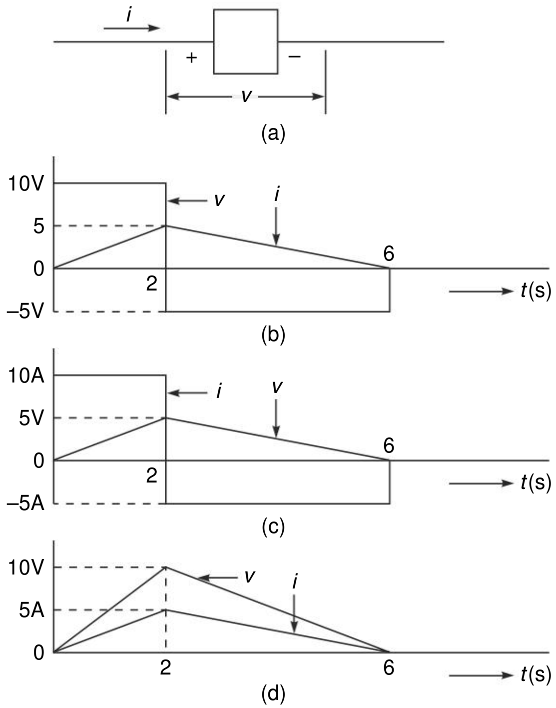


Fig. 1.42

1.4 Repeat Problem 1.3 for an inductance of 500 mH.

1.5 Repeat Problem 1.3 for a resistance of 1 kΩ.

1.6 Current $i = I_0e^{-}$ passes through a resistance R from $t = 0$ onward. What is the total energy dissipated in the resistance up to $t = \infty$?

1.7 Examine Fig. 1.43.

(a) Express the voltage across each resistor in terms of V_s, V_1 and V_2.

(b) Write the necessary KCL equation at the nodes in terms of V_s, V_1 and V_2.

(c) Solve for V_1 and V_2.

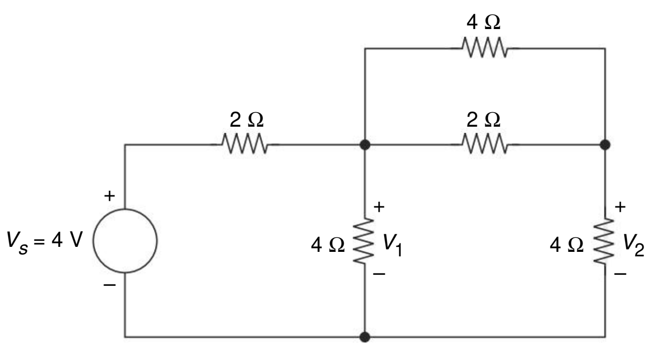


Fig. 1.43

1.8 For the resistive circuit shown in Fig. 1.44, $I_s = 4$ A, $I_2 = 1$ A, $R_1 = R_2 = R_3 = R_4 = 1\Omega$.

(a) Determine the current I_1.
(b) Determine the currents and voltages across R_3 and R_4.
(c) Hence, find V_s.

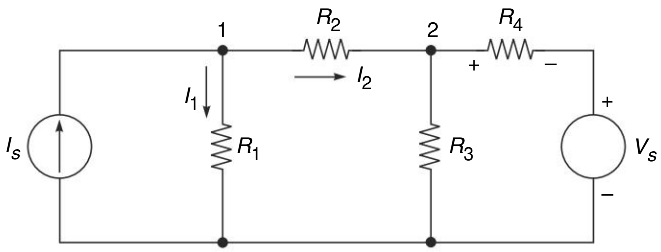


Fig. 1.44

1.9 In the resistive circuit of Fig. 1.45, voltages of the nodes 1, 2 and 3 are V_1, V_2 and V_3 respectively, with respect to node N.
(a) Express the voltages across all resistances in terms of V_1, V_2, V_3 and V_s. Mark the polarities of the voltages so determined.
(b) Write KCL equations at the nodes 1, 2 and 3.

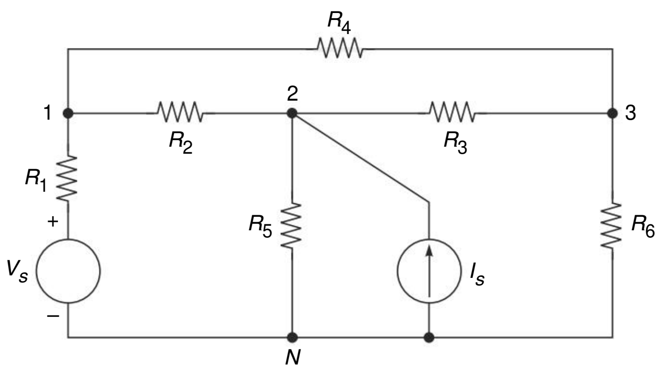


Fig. 1.45

1.10 In the circuit of Fig. 1.46 find i_s. Given $i_L = e^{-2t}$.

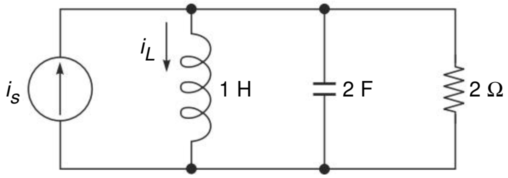


Fig. 1.46

1.11 In the circuit of Fig. 1.47 find i_s. Given $i = \sin 2t$.

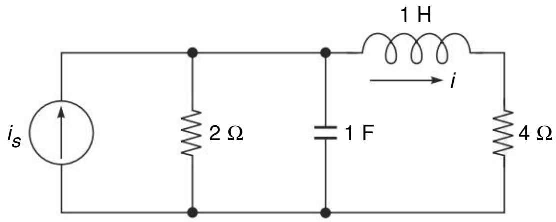


Fig. 1.47

1.12 In the circuit of Fig. 1.48 V_0 = 16 V, find I_s.

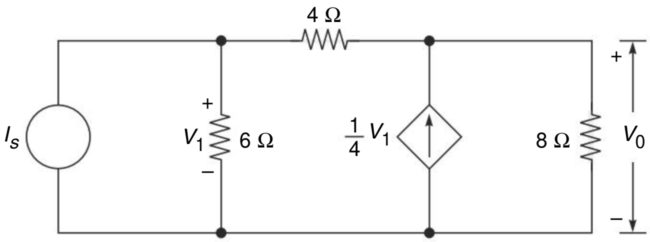


Fig. 1.48

1.13 For the *network* (circuit) shown in Fig. 1.49, two voltage measurements taken are

$$V_{ab} = 12 \text{ V} \quad \text{and} \quad V_{ac} = 20 \text{ V}$$

Find the values of V_1 and V_2.

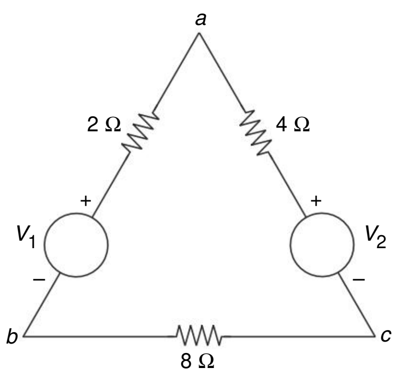


Fig. 1.49

1.14 For Fig. 1.50, find the values of currents I_1 and I_2. What is the power supplied by the two current sources?

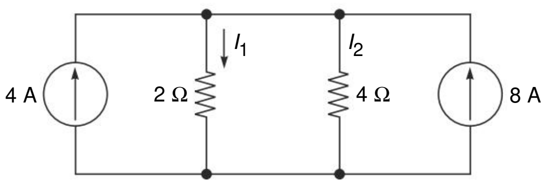


Fig. 1.50

1.15 In Fig. 1.51, $v(t) = 10\sqrt{2}$ sin 314 t. Calculate the value of $i(t)$.

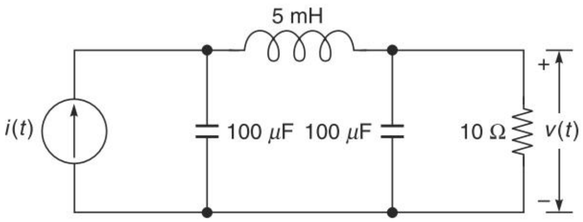


Fig. 1.51

1.16 For the circuit of Fig. 1.52.

(a) Given V_s = 12 V, determine I_s so that I = 0.

(b) Given V_s = 14 V, and I = 1A, find I_s.

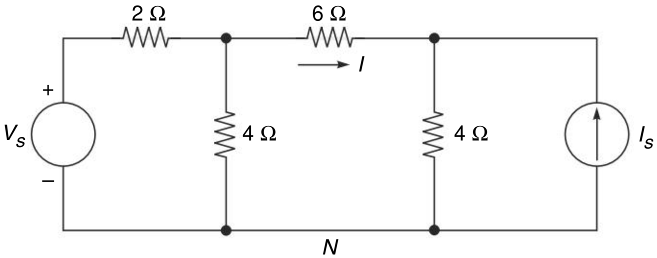


Fig. 1.52

1.17 For the circuit of Fig. 1.53, find the voltage V.

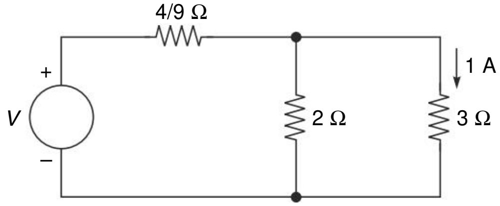


Fig. 1.53

Chapter 2

FUNDAMENTALS OF RESISTIVE CIRCUITS

MAIN GOALS AND OBJECTIVES

- *all the methods of circuit analysis and theorems through resistive network*
- *reduction of passive resistive network by series, parallel combinations and star/delta conversion*
- *practical source conversion—voltage to current source and vice versa*
- *nodal method of circuit analysis*
- *mesh method of circuit analysis*
- *ability to choose between nodal and mesh methods*
- *network theorems, their basis and applications*
- *Thevenin and Norton equivalents*

2.1 INTRODUCTION

Having enunciated the elemental laws and the two fundamental circuit laws, viz. KCL and KVL, governing equations can be written for any circuit. Certain organized techniques that mechanize circuit analysis will be presented here. Also, we shall present some simple techniques of network reduction and certain fundamental theorems that reduce any complex circuit to a simple form from which the circuit behaviour w.r.t. external elements can be visualized. All these techniques and theorems will be presented through resistive circuits which have the simplicity that only algebraic governing equations are involved. These, in later chapters, would be easily extended to circuits containing storage elements and hence integro-differential governing equations.

2.2 SERIES AND PARALLEL COMBINATIONS OF RESISTANCES

Resistance in Series A set of resistances are in series when the same current circulates through them as illustrated in Fig. 2.1(a).

Applying KVL around the loop, we get

$$V = V_1 + V_2 + ... + V_n$$
$$= R_1 I + R_2 I + ... R_n I$$
$$= (R_1 + R_2 + ... + R_n) I = R_{eq} I$$

where $$R_{eq} = R_1 + R_2 + ... + R_n \tag{2.1}$$

The equivalent circuit is shown in Fig. 2.1(b).

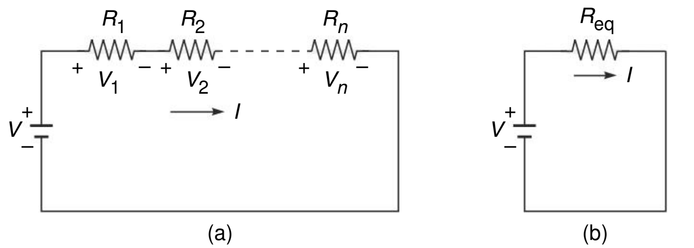


Fig. 2.1 (a) Resistances in series (b) Equivalent resistance

Note: If the loop contains a number of voltage sources in series, the resultant voltage for the equivalent circuit (Fig. 2.1(b)) is their algebraic sum.

Resistances in Parallel

A set of resistances are in parallel when the same source voltage is applied across these as illustrated in Fig. 2.2(a). Applying KCL at the top node, we have

$$I = I_1 + I_2 + ... + I_n$$
$$I = \frac{V}{R_1} + \frac{V}{R_2} + ... + \frac{V}{R_n}$$
$$I = \left(\frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n}\right) \mathrm{V}$$
$$= \left(\frac{1}{R_{eq}}\right) \mathrm{V}$$

where $$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_n} \tag{2.2a}$$

or $$G_{eq} = G_1 + G_2 + ... + G_n \tag{2.2b}$$

The equivalent circuit is shown in Fig. 2.2(b).

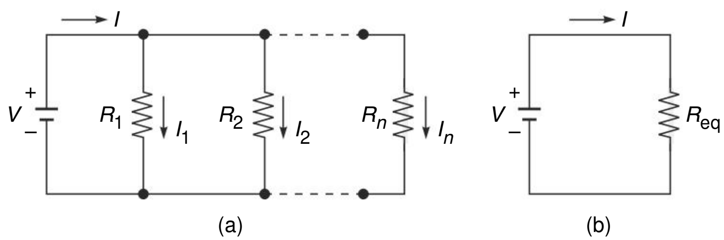


Fig. 2.2 (a) Resistances in parallel (b) Equivalent resistance

For the special case of two resistances in parallel, we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = R_1 \parallel R_2 \tag{2.3}$$

Example 2.1 For the resistive circuit (network) of Fig. 2.3, find the resistance *seen* between nodes *ab* and *bc*.

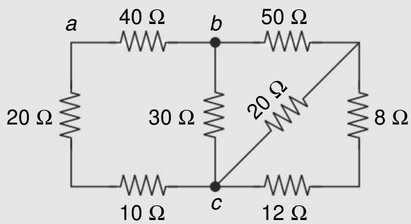


Fig. 2.3

Solution

Proceeding by series–parallel combinations:

R_{ab} (eq) = {[(12 + 8) || 20 + 5) || 30] + 10 + 20} || 40

$$\left(\frac{15 \times 30}{15 + 30} + 30\right) \parallel 40 = 20\ \Omega$$

The reader should calculate R_{bc} (eq).

Example 2.2 In the single loop circuit of Fig. 2.4, find *I*.

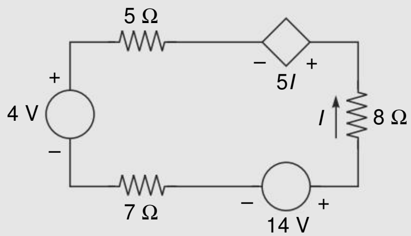


Fig. 2.4

Solution

Writing KVL equation in counter-clockwise direction

$$8I + 5I + 5I + 4 + 7I - 14 = 0$$

$$\therefore \quad I = \frac{10}{25} = \frac{2}{5} \text{ A}$$

2.3 VOLTAGE AND CURRENT DIVISION

Voltage Division

Voltage can be *reduced* by a specified factor by dividing it across two resistors in series as shown in Fig. 2.5. It easily follows that

$$v_2 = R_2 i = R_2 \times \frac{v}{R_1 + R_2}$$

$$= \left(\frac{R_2}{R_1 + R_2}\right)v = Kv \qquad (2.4)$$

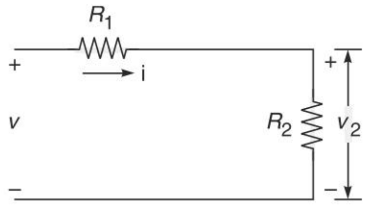


Fig. 2.5

Voltage of any wave shape can be reduced by this method.

Example 2.3 By using the voltage divider circuit of Fig. 2.5, it is desired to obtain 3/4v. Find R_2, given R_1= 100 Ω.

A load resistance R_L is now connected in parallel with R_2. What will be the percentage change in output voltage if (i) R_L = 10 kΩ and (ii) R_L = 1 kΩ.

Solution

$$\frac{R_2}{R_1 + R_2} = \frac{3}{4}$$

$$\frac{R_2}{100 + R_2} = \frac{3}{4}$$

or $$R_2 = 300\ \Omega$$

Circuit with load R_L is drawn in Fig. 2.6.

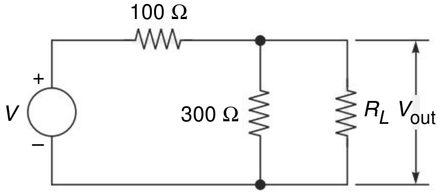


Fig. 2.6

(i) When R_L = 10 kΩ

$$R_2(\text{eq}) = \frac{300 \times 10 \times 10^3}{10300} = 291.26\ \Omega$$

$$K = \frac{291.26}{391.6} = 0.744$$

$$\text{Change in output voltage} = (0.75 - 0.744)\ v = 0.006v$$

$$\%\ \text{change} = \frac{0.006}{0.75} \times 100 = 0.8$$

(ii) For R_L = 1 kΩ

$$R_2(\text{eq}) = \frac{300 \times 1000}{1300} = 230.77\ \Omega$$

$$K = \frac{230.77}{330.77} = 0.698$$

Change in output voltage = (0.75 – 0.698) v = 0.052v

$$\%\text{ change} = \frac{0.052}{0.75} \times 100 = 6.93$$

Observation: As the load resistance is reduced to 1/10, the percentage change in output voltage rises from 0.8 to 6.93.

Current Division

Current i in Fig. 2.7 can be suitably divided into two parts by resistances R_1 and R_2 in parallel.

$$i_1 = \frac{v}{R_1} = \frac{1}{R_1}\left(\frac{R_1R_2}{R_1+R_2}\,i\right)$$

$$= \frac{R_2}{R_1+R_2}\,i \quad (2.5)$$

Similarly,

$$i_2 = \frac{R_1}{R_1+R_2}\,i \quad (2.6)$$

As per Eqs. (2.5) and (2.6), current entering the node of two resistances in parallel divides among them in the inverse ratio of their resistances.

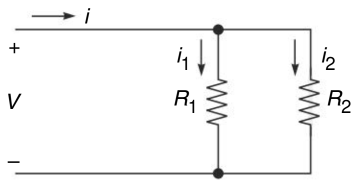


Fig. 2.7

Example 2.4 In the resistance circuit of Fig. 2.8, find v_1 and i_2.

Solution

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{5} + \frac{1}{4} = 0.55$$

$$R_{eq} = 1.82\text{ k}\Omega$$

$$i_1 = \frac{150}{2+0.2+1.82} = 37.3\text{ mA}$$

$$v_1 = 37.3 \times 0.2 = 7.46\text{ V}$$

$$\text{Voltage across } R_{eq} = 1.82 \times 37.3 = 67.886\text{ V}$$

$$i_2 = \frac{67.886}{4} = 16.97\text{ mA}$$

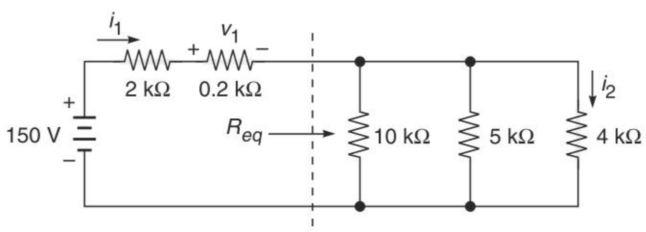


Fig. 2.8

2.4 STAR (*Y*)–DELTA (Δ) CONVERSION

Certain network configurations cannot be resolved by series–parallel combinations alone. Such configurations are handled by *Y–Δ transformations*.

Figure 2.9(a) shows three Δ–connected resistances connected between three nodes *a*, *b* and *c*.

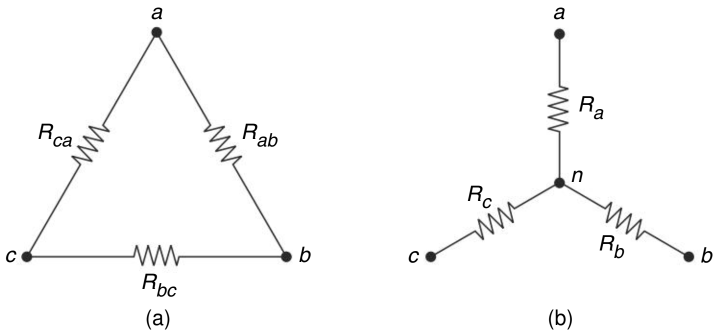


Fig. 2.9 (a) Delta-connected and (b) Star-connected networks

On the other hand in Fig. 2.9(b), there are three *Y*-connected resistances. Notice that *Y*-connection has an extra node *n* that gets eliminated upon converting it to Δ. *Y*–Δ conversion is therefore a *node reduction* technique.

Equating resistance between node pairs:

Node pair *ab*

$$R_a + R_b = R_{ab} \parallel (R_{bc} + R_{ca}) \tag{2.7}$$

Node pair *bc*

$$R_b + R_c = R_{bc} \parallel (R_{ca} + R_{ab}) \tag{2.8}$$

Node pair *ca*

$$R_c + R_a = R_{ca} \parallel (R_{ab} + R_{bc}) \tag{2.9}$$

Solving Eqs. (2.7), (2.8) and (2.9)

Y–Δ Conversion

$$R_{ab} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_c} \tag{2.10a}$$

$$R_{bc} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_a} \tag{2.10b}$$

$$R_{ca} = \frac{R_aR_b + R_bR_c + R_cR_a}{R_b} \tag{2.10c}$$

Δ–Y Conversion

$$R_a = \frac{R_{ab}R_{ac}}{R_{ab} + R_{bc} + R_{ca}} \tag{2.11a}$$

$$R_b = \frac{R_{bc}R_{ba}}{R_{ab} + R_{bc} + R_{ca}} \tag{2.11b}$$

$$R_c = \frac{R_{ca}R_{cb}}{R_{ab} + R_{bc} + R_{ca}} \tag{2.11c}$$

Balanced Y–Δ

A balanced *Y* ($R_a = R_b = R_c = R_Y$) leads to balanced Δ ($R_{ab} = R_{bc} = R_{ca} = R_\Delta$) wherein

$$R_\Delta = 3R_Y \tag{2.12}$$

Example 2.5 Reduce the network of Fig. 2.10 to obtain the equivalent resistance as seen between nodes *ad*.

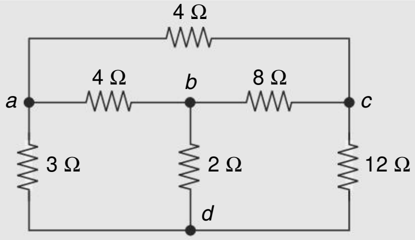


Fig. 2.10

Solution

Converting the Y at node b to Δ,

$$R_x = \frac{4 \times 8 + 8 \times 2 + 2 \times 4}{2} = 28\ \Omega$$

$$R_y = \frac{4 \times 8 + 8 \times 2 + 2 \times 4}{8} = 7\ \Omega$$

$$R_z = \frac{4 \times 8 + 8 \times 2 + 2 \times 4}{4} = 14\ \Omega$$

By series–parallel combination of Fig. 2.11, we get

$$R_{ad}\,(\text{eq}) = \frac{2.1 \times (3.5 + 6.46)}{2.1 + 3.5 + 6.46} = 1.734\ \Omega$$

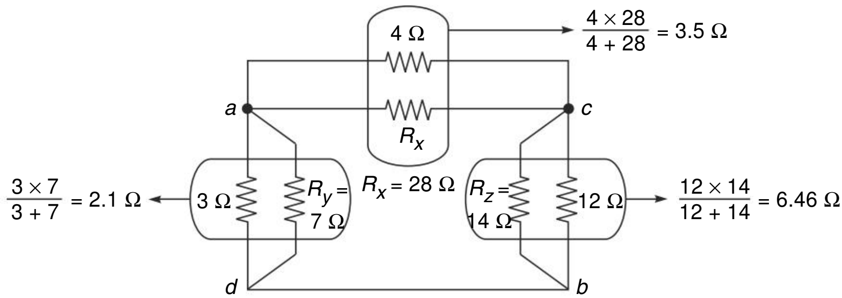


Fig. 2.11

2.5 SOURCE REPRESENTATION AND CONVERSION

In a practical voltage source, the voltage reduces as the load current is increased (by reducing load resistances, as in a battery connected to a resistance load). A practical source can be approximated by an ideal voltage source with a resistance in series as in Fig. 2.12.

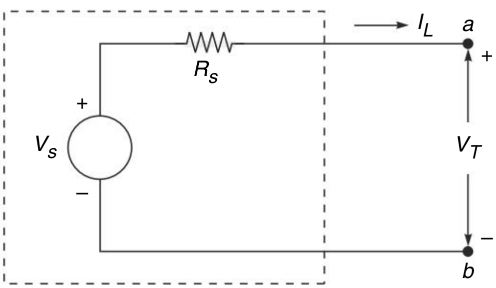


Fig. 2.12 Practical voltage source

The V–I characteristic of the source is represented by the equation

$$-V_S + R_S I_L + V_T = 0 \tag{2.13}$$

or
$$I_L = -\frac{1}{R_S} V_T + \frac{1}{R_S} V_S \tag{2.14}$$

This characteristic ($V_T - I_L$) is represented graphically in Fig. 2.13 wherein

I_{SC} = short-circuit current, i.e. source terminals shorted through zero resistance load ($V_T = 0$)

V_{OC} = open-circuit voltage, i.e. source terminals open ($I_L = 0$).

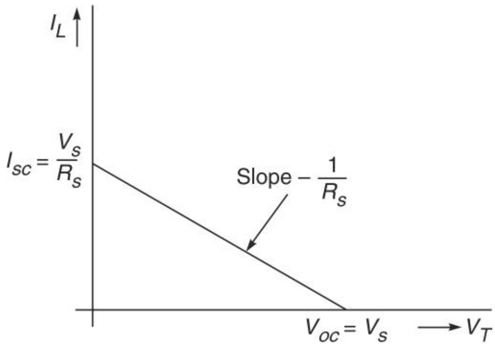


Fig. 2.13 *V–I* characteristic of a practical voltage source

A practical current source feeds a reducing current to a load resistance as its resistance is increased. It can be represented by an ideal current source with a resistance in parallel with it as in Fig. 2.14.

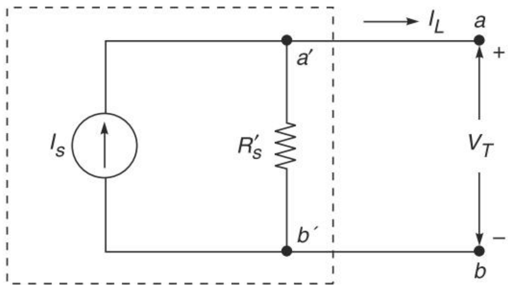


Fig. 2.14 Practical current source

Writing KCL equation at the node a'

$$-I_s + \frac{V_T}{R'_S} + I_L = 0$$

or
$$I_L = -\frac{1}{R'_S} V_T + I_s \tag{2.15}$$

The *V–I* characteristic as per Eq. (2.15) is drawn in Fig. 2.15, wherein

I_{SC} = short-circuit current, i.e. $V_T = 0$

V_{OC} = open-circuit voltage, i.e. $I_L = 0$

For the two practical source representations to be equivalent (comparing Eqs. (2.14) and (2.15))

$$R'_S = R_S$$

$$V_S = R_S I_S = V_{OC} \text{ (open-circuit, i.e. source terminals open)}$$

Also $$I_S = \frac{V_S}{R_S} = I_{SC} \text{ (short-circuit, i.e. current through zero resistance load connecting source terminals)}$$

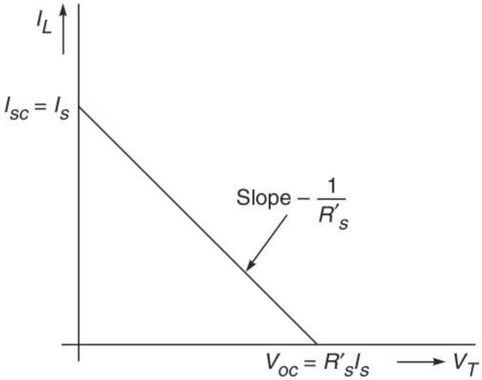


Fig. 2.15 *V-I* characteristic of a ractical current source

Thus, source equivalence is also arrived at by matching open-circuit voltages and short-circuit currents. This result is the consequence of the Thevenin and Norton theorems (Sec. 2.10).

It must be observed here that the current direction of the equivalent current source must be such as to produce the same open-circuit terminal voltage polarity as in the voltage source.

It must be observed here that an ideal voltage source (R_s = 0) cannot be converted to current source (it would mean infinite current with a short-circuit in front). Similarly, an ideal current source cannot be converted to a voltage source.

General Methods of Circuit Analysis In Chapter 2, we presented two basic laws of circuit theory—Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL). Based on these two laws, there are two general methods of circuit analysis; the nodal method based on KCL and the mesh method based on KVL. Before applying these methods, the circuit if possible is reduced in complexity by applying the reduction techniques presented in Sections 2.2 and 2.4

Further in applying nodal and mesh method to analyze a circuit, inter-conversion of voltage and current sources may be needed. This conversion has been presented in Section 2.5.

Certain terms used in circuit analysis defined in Chapter 1 are repeated below:

Node: It is a circuit point where ends (terminals) of two or more circuit elements meet.

Path: It is a traversal through elements from one node to another.

Branch: It is a path between two adjoining nodes.

Loop: It is a closed path starting and ending at the same node without going through the same node more than once.

Mesh: It is a loop that does not contain any other loop within it.

Junction: It is a node where three or more circuit elements (or branches) meet.

Remark: At a node where only two circuit element need not be considered as this node can always be eliminated by combination of elements or if one of these is a voltage source, the voltage at the node is known, it is not an unknown to be determined.

Therefore in circuit analysis we are concerned with junction but we shall loosely use the term 'node' for a junction.

We will now proceed with nodal and mesh circuit analysis which follows by their comparison and choice thereof.

2.6 NODAL ANALYSIS

It shall be assumed here that all sources are current sources and practical voltage sources,* if any, have been converted to equivalent current source form.

Let the circuit have N nodes in all. One of these nodes is chosen as the *reference* (*datum*) node. The voltages of the remaining $(N - 1)$ nodes with respect to the reference node form an independent set of variables that implicitly satisfy KVL equations (we shall observe in what follows that the voltage of any component is the differences of the voltages of the two nodes to which it is connected). $(N - 1)$ KCL equations are written down at the nodes. For a resistive network this step results in $(N - 1)$ simultaneous algebraic equations in $(N - 1)$ nodal voltages. Once the nodal voltages are obtained, any voltage and current in the network can be obtained from these. In writing the nodal equations, it is convenient but not necessary to convert all resistance values to conductances before proceeding with the analysis.

For demonstration, consider the circuit of Fig. 2.16(a). The lower two nodes are identical and can be merged for clarity as in Fig. 2.16(b).

Applying KCL at nodes 1 and 2, respectively,

Node 1: $$-i_1 + G_2 v_1 + G_1(v_1 - v_2) = 0 \tag{2.16}$$

Node 2: $$i_2 + G_3 v_2 + G_1(v_2 - v_1) = 0 \tag{2.17}$$

* The case where an ideal voltage source is connected between two nodes is dealt within Appendix A.

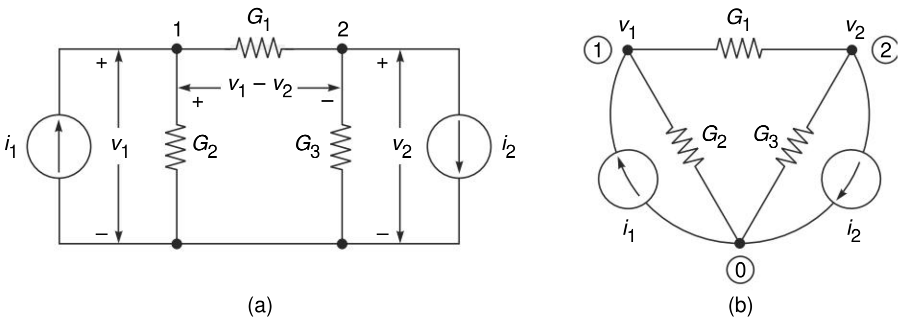


Fig. 2.16

Rearranging,

Node 1: $$(G_1 + G_2)v_1 - G_1v_2 \quad = i_1 \tag{2.18}*$$

Node 2: $$-G_1v_1 + (G_1 + G_3)\, v_2 \quad = -i_2 \tag{2.19}*$$

With given values of i_1 and i_2 (source currents), Eqs (2.18) and (2.19) can be solved for v_1 and v_2. It is possible to generalize these equations for an N-node system which can be written by inspection.

Let $$i_1 = 2\text{ A},\ i_2 = -3\text{ A}$$

$$G_1 = 0.2,\ G_2 = 1,\ G_3 = 0.5$$

Plugging in the values, we have

$$1.2v_1 - 0.2v_2 = 2$$

$$-0.2v_1 + 0.7v_2 = 3$$

Solving, we get

$$v_1 = 5/2\text{ V} \quad v_2 = 5\text{ V}$$

Example 2.6 For the circuit of Fig. 2.17(a), find all the node voltages and the currents in resistances 0.25Ω and 1/3Ω. Use the nodal method.

---

* Equations (2.18) and (2.19) can be written in matrix form as

$$\begin{bmatrix} G_1 + G_2 & -G_1 \\ -G_1 & (G_1 + G_3) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \begin{bmatrix} i_1 \\ -i_2 \end{bmatrix}$$

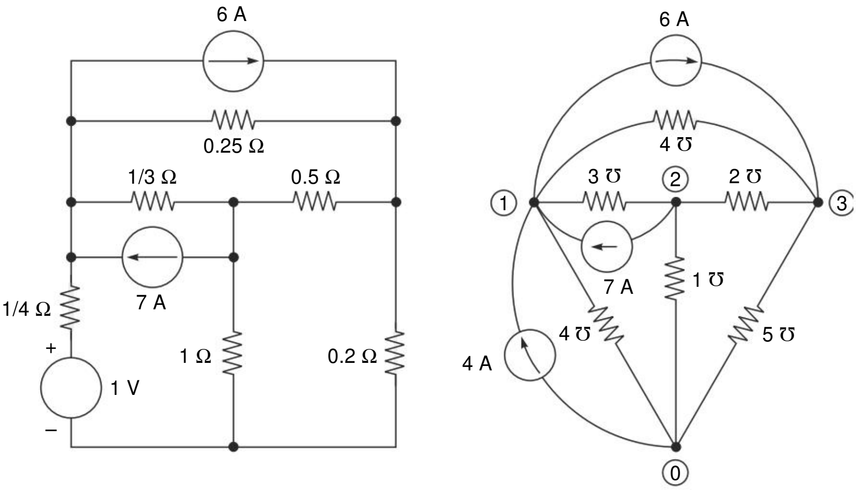
or $$[\boldsymbol{G}][\boldsymbol{v}] = [\boldsymbol{i}]$$

where $[\boldsymbol{G}]$ = node admittance matrix whose

diagonal elements = sum of all admittances connected at the node; off-diagonal elements = minus the sum of all admittances connected between the two nodes (ij); node admittances matrix is a symmetric matrix

[**v**] = vector of node voltages

[**i**] = vector of currents of all current sources entering at each node.



(a) (b)

Fig. 2.17

Solution

Since one of the sources is a voltage source with series resistance, it is first converted to a current source and all resistances are converted to conductances. The circuit is redrawn in Fig. 2.17(b) where 4 nodes are identified and labelled 1, 2, 3 and the datum node 0.

Writing KCL equations at the three nodes:

Node 1: $$(6 - 4 - 7) + 4v_1 + 3(v_1 - v_2) + 4(v_1 - v_3) = 0$$

or $$11v_1 - 3v_2 - 4v_3 = 5 \qquad \text{(i)}$$

Node 2: $$7 + v_2 + 3(v_2 - v_1) + 2(v_2 - v_3) = 0$$

or $$-3v_1 + 6v_2 - 2v_3 = -7 \qquad \text{(ii)}$$

Node 3: $$-6 + 5v_3 + 2(v_3 - v_2) + 4(v_3 - v_1) = 0$$

or $$-4v_1 - 2v_2 + 11v_3 = 6 \qquad \text{(iii)}$$

Using Cramer's rule*

$$v_1 = \frac{\begin{vmatrix} 5 & -3 & -4 \\ -7 & 6 & -2 \\ 6 & -2 & 11 \end{vmatrix}}{\begin{vmatrix} 11 & -3 & -4 \\ -3 & 6 & -2 \\ -4 & -2 & 11 \end{vmatrix}}$$

$$= \frac{5\begin{vmatrix} 6 & -2 \\ -2 & 11 \end{vmatrix} + 7\begin{vmatrix} -3 & -4 \\ -2 & 11 \end{vmatrix} + 6\begin{vmatrix} -3 & -4 \\ 6 & -2 \end{vmatrix}}{11\begin{vmatrix} 6 & -2 \\ -2 & 22 \end{vmatrix} + 3\begin{vmatrix} -6 & -4 \\ -2 & 11 \end{vmatrix} - 4\begin{vmatrix} -3 & -4 \\ 6 & -2 \end{vmatrix}}$$

* An efficient method of solving large number of linear algebraic equations is Gaussian Elimination [V1]. On the other hand Cramer's rule is a very inefficient method as it requires computation of determinants.

$$= \frac{5(62) + 7(-41) + 6(30)}{11(62) + 3(-41) - 4(30)}$$

$$= \frac{203}{439} = 0.462 \text{ V}$$

Similarly,

$$v_2 = \frac{\begin{vmatrix} 11 & 5 & -4 \\ -3 & -7 & -2 \\ -4 & 6 & 11 \end{vmatrix}}{439} = \frac{-326}{439} = -0.743 \text{ V}$$

and

$$v_3 = \frac{\begin{vmatrix} 11 & -3 & 5 \\ -3 & 6 & -7 \\ -4 & -2 & 6 \end{vmatrix}}{439} = \frac{254}{439} = -0.579 \text{ V}$$

$i_{13}(0.25\ \Omega) = 4(0.462 - 0.579) = -0.468$ A

$i_{12}(1/3\ \Omega) = 3(0.462 + 0.743) = 3.615$ A

Example 2.7 For the circuit of Fig. 2.18(a) determine v_x using nodal analysis

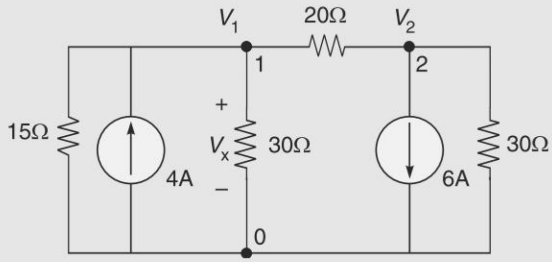


Fig. 2.18 (a)

Solution

The two nodes are identified as 1 and 2 with *datum* node as 0.

Combining resistances 15 Ω and 30 Ω which are in parallel gives the equivalent resistance

$$\frac{15 \times 30}{15 + 30} = 10\Omega$$

The circuit is redrawn in Fig. 2.18(b) The nodal equations are:

Node 1

$$-4 + \frac{v_1}{10} + \frac{v_1 - v_2}{20} = 0$$

$$\left(\frac{1}{10} + \frac{1}{20}\right)v_1 - \frac{1}{20}v_2 = 4$$

$3v_1 - v_2 = 80$ (i)

Node 2

$$6 + \frac{v_2 - v_1}{20} + \frac{v_2}{30} = 0$$

$$-\frac{v_1}{20} + \left(\frac{1}{20} + \frac{1}{30}\right)v_2 = -6$$

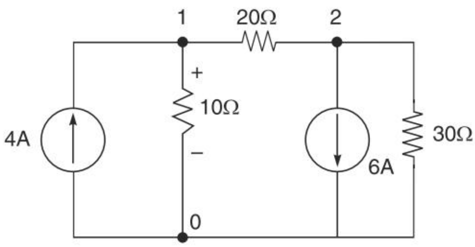


Fig. 2.18(b)

$$-3v_1 + 5v_2 = 360 \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$V_1 = 3.333 \text{ V}, V_2 = -70 \text{ V}$$

$$v_x = v_1 = 3.33 \text{ V}$$

Example 2.8 For the circuit of Fig. 2.19(a) determine the nodal voltages and current through 2Ω resistance.

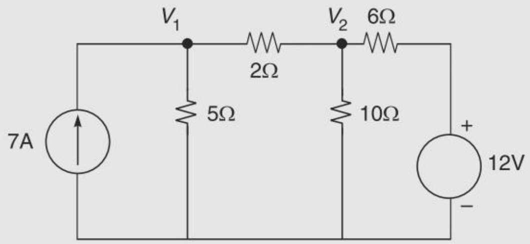


Fig. 2.19 (a)

Solution

Converting voltage source (practical) to current source, the circuit modifies as in Fig. 2.19(b) Nodal equations are

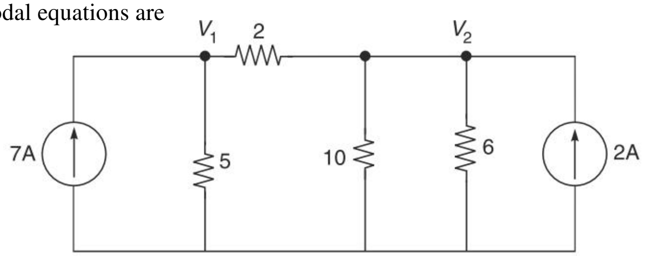


Fig. 2.19(b)

$$\left(\frac{1}{5}+\frac{1}{2}\right)V_1 - \frac{1}{2}V_2 = 7 \qquad \text{(i)}$$

$$-\frac{1}{2}V_1 + \left(\frac{1}{10}+\frac{1}{6}+\frac{1}{2}\right)V_2 = 2 \qquad \text{(ii)}$$

or

$$0.7\ V_1 - 0.5\ V_2 = 7$$

$$-0.5\ V_1 + 0.767\ V_2 = 2$$

Solving we get

$$V_1 = 22.2V, \qquad V_2 = 17.1 \text{ V}$$

$$I_{12} = (22.2 - 17.6)/2 = 2.55 \text{ A}$$

Example 2.9 For the circuit of Fig. 2.20 find the value of V_1 and V_2. Also find the power input/output of the current and voltage sources.

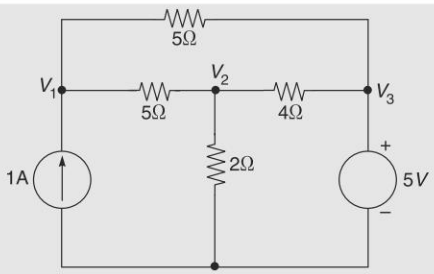


Fig. 2.20

Solution

To apply nodal analysis, 5V source cannot be and need not be converted to current source as voltage V_3 is known to be + 5V

Node V_1

$$-1+\left(\frac{1}{5}+\frac{1}{5}\right)V_1-\frac{1}{5}V_2-\frac{1}{5}\times 5=0$$

Node V_2

$$-\frac{1}{5}V_1+\left(\frac{1}{2}+\frac{1}{5}+\frac{1}{4}\right)V_2-\frac{1}{4}\times 5=0$$

These equations are rewritten as

$$2V_1 - V_2 = 10$$

$$-4V_1 + 19V_2 = 25$$

Solving we get

$V_1 = 6.325\text{V}, \quad V_2 = 2.65$ V

Current source output = 1× 6.325 = 6.325 W

Current output of voltage source

$= (5 - 2.65) / 2 + (5 - 6.325) / 5$

$= 0.91$ A

Output of voltage source = 5 × 0.91 = 4.55 W

Example 2.10 Using nodal analysis determine V_x in the circuit of Fig. 2.21(a)

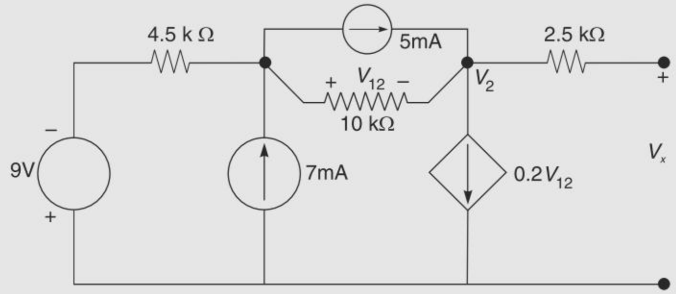


Fig. 2.21(a)

Solution

Convert the voltage source of 9V to current source as drawn below:

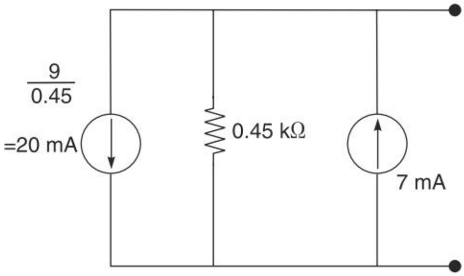


Fig. 2.21(b)

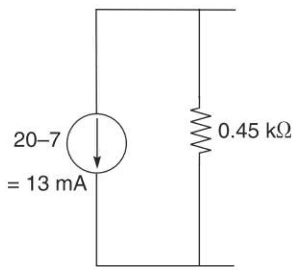


Fig. 2.21(c)

Combining it with current source of 7 mA, the net current source is drawn in Fig. 2.2(c). The complete circuit is drawn in Fig. 2.21(d).

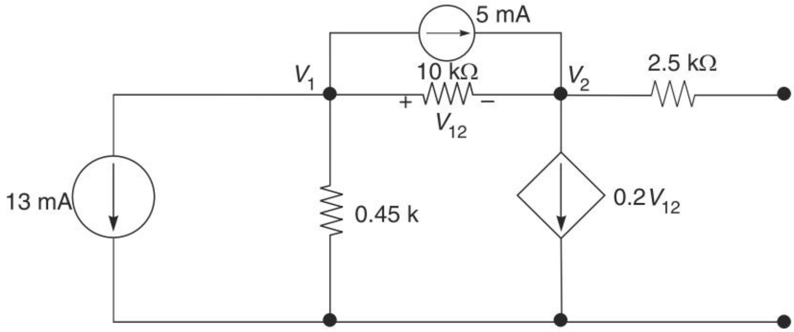


Fig. 2.21(d)

Writing nodal equations

Nodal 1

$$13 + 5 + \frac{V_1}{0.45} + \frac{V_1 - V_2}{10} = 0$$

$$V_{12} = (V_1 - V_2) \qquad \text{(i)}$$

Node 2

$$0.2V_{12} - 5 + \frac{V_2 - V_1}{10} = 0$$

$$0.2\,(V_1 - V_2) - 5 + \frac{V_2 - V_1}{10} = 0 \qquad \text{(ii)}$$

Rearranging Eqs (i) and (ii)

$$\left(\frac{1}{0.45} + \frac{1}{10}\right)V_1 - \left(\frac{1}{10}\right)V_2 = -18 \qquad \text{(iii)}$$

$$\left(0.2 - \frac{1}{10}\right)V_1 + \left(\frac{1}{10} - 0.2\right)V_2 = 5 \qquad \text{(iv)}$$

Solving Eqs (iii) and (iv), we find

$V_1 = -10.35\text{V}$ $\qquad V_2 = -60.35$ V

Now $V_x = V_2 = -60.35\text{V}$ $\qquad$ It is open circuit

Example 2.11 For the circuit of Fig. 2.22(a) use nodal analysis to determine V_x and V_y. What is the power consumed by 6Ω resistance?

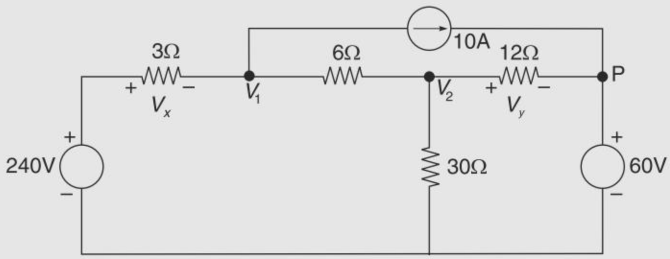


Fig. 2.22 (a)

Solution

At node P the voltage is fixed at +60V, so there are two independent nodes V_1 and V_2. Convert voltage source 240V to current source resulting in circuit presented in Fig. 2.22(b).

At Node V_1

$$-80 + \frac{V_1}{3} + 10 + \frac{V_1 - V_2}{6} = 0 \qquad \text{(i)}$$

At Node V_2

$$\frac{V_2 - V_1}{6} + \frac{V_2 - 60}{12} + \frac{V_2}{30} = 0 \qquad \text{(ii)}$$

Rewriting

$$\left(\frac{1}{3} + \frac{1}{6}\right)V_1 - \left(\frac{1}{6}\right)V_2 = 70 \qquad \text{(iii)}$$

$$-\left(\frac{1}{6}\right)V_1 + \left(\frac{1}{6} + \frac{1}{12} + \frac{1}{30}\right)V_2 = 5 \qquad \text{(iv)}$$

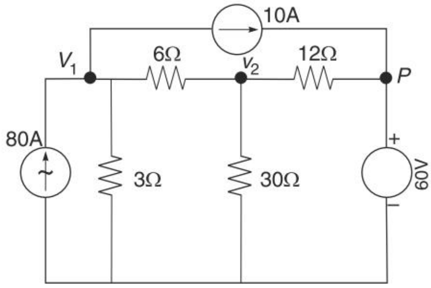


Fig. 2.22 (b)

Solving we get

$V_1 = 181.5$ V, $\quad V_2 = 124.4$ V

$V_x = V_1 - 181.5$ V

$V_y = V_2 - 60 = 64.4$ V

Power consumed in 6Ω resistance

$V(6\Omega) = V_2 - V_1 = 57.1$ V

$$\text{Power } (6\Omega) = \frac{(57.1)^2}{6} = 543.4 \text{ W}$$

Example. 2.12

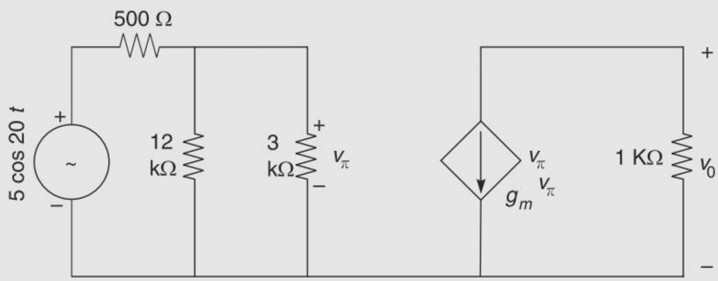


Fig. 2.23

The *ac* circuit model of a bipolar junction transistor amplifier is drawn in Fig. 2.23. If transconductance $g_m = 35$ m ℧, calculate the output $v_0(t)$.

Solution

v_π is obtained from parallel-series voltage divider.

$R_p = 12 \parallel 3 = 2.4$

$$v_\pi = (5 \cos 20t) \times \frac{2.4}{0.5 + 2.4} = 4.14 \cos 20t \text{ V}$$

$g_m v_\pi = 35 \times 4.14 \cos 20\,t = 144.8 \cos 20t$ mA

$v_0 = (g_m\, v_\pi) \times 1 = 144.3 \cos 20t$ V

2.7 MESH ANALYSIS

This is an alternative method of circuit analysis. Mesh analysis algorithm is given below and is explained through the simple circuit of Fig. 2.24.

– Identify independent circuit meshes. There are two such meshes in the circuit of Fig. 2.24.

– Assign a circulating current to each mesh (i_1, i_2 in Fig. 2.24). As each mesh current enters as well as leaves the mesh elements, the mesh currents implicity satisfy KCL. It is preferable to assign the same direction to the mesh currents—usually clockwise.

– Write KVL equations for each mesh (as many as mesh currents). It is observed here that no circuit branch can carry more than two mesh currents.

– It is assumed that all circuit sources are voltage sources. Practical current sources, if any, are first converted to equivalent voltage sources.

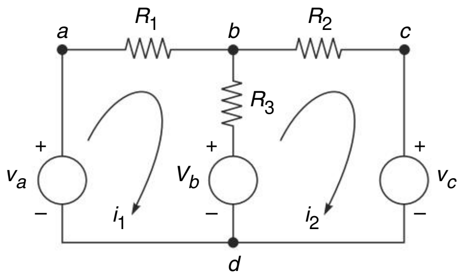


Fig. 2.24

Let us write KVL equations for the two meshes of Fig. 2.24.

Mesh 1: $$R_1 i_1 + R_3(i_1 - i_2) + v_b - v_a = 0 \quad (2.20)$$

Mesh 2: $$R_3(i_2 - i_1) + R_2 i_2 + v_c - v_b = 0 \quad (2.21)$$

These equations can be organized in the form below:

Mesh 1: $$(R_1 + R_3)i_1 - R_3 i_2 = v_a - v_b \quad (2.22)$$

Mesh 2: $$-R_3 i_1 + (R_2 + R_3)i_2 = v_b - v_c \quad (2.23)$$

Equations (2.22) and (2.33)* can be generalized and written down by inspection.

---

* Equations (2.22) and (2.23) can be written in the matrix form

$$\begin{bmatrix} (R_1 + R_3) & -R_3 \\ -R_3 & (R_2 + R_3) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} (v_a - v_b) \\ (v_b - v_c) \end{bmatrix}$$

or $[\mathbf{R}]\,[\mathbf{i}] = [\mathbf{v}]$

where $[\mathbf{R}]$ = mesh resistance (in general impedance) matrix whose

diagonal elements = sum of all resistances round the loop

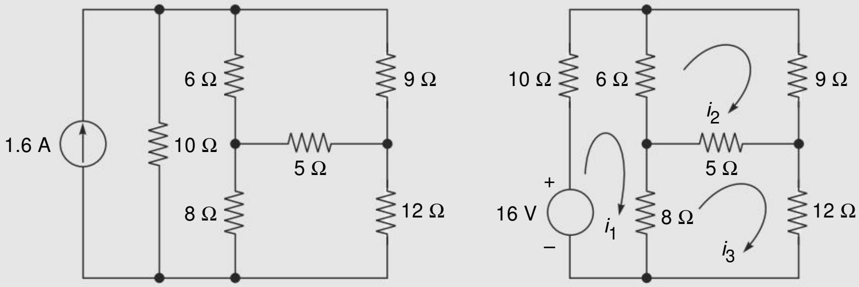
off-diagonal elements = minus the sum of all resistances common to the loops (ij)

mesh resistance matrix is symmetric matrix

$[\mathbf{i}]$ = mesh currents vector

$[\mathbf{v}]$ = vector of algebraic sum of voltages of all voltage sources round the loop.

Example 2.13 Analyse the circuit of Fig. 2.25(a) by the mesh method. From the results, calculate the current in the 5Ω resistance.



(a) (b)

Fig. 2.25

Solution The practical current source of Fig. 2.25(a) is first converted to voltage source as in Fig. 2.25(b). Three meshes are immediately identified with associated currents i_1, i_2 and i_3. KVL equations for the three meshes are written as follows (directly in organized form).

Mesh 1: $$(10 + 6 + 8)i_1 - 6i_2 - 8i_3 = 16$$

or $$24i_1 - 6i_2 - 8i_3 = 16 \qquad \text{(i)}$$

Mesh 2: $$-6i_2 + (6 + 9 + 5)i_2 - 5i_3 = 0$$

or $$-6i_1 + 20i_2 - 5i_3 = 0 \qquad \text{(ii)}$$

Mesh 3: $$-8i_3 - 5i_2 + (8 + 5 + 12)\, i_3 = 0$$

or $$-8i_1 - 5i_2 + 25i_3 = 0 \qquad \text{(iii)}$$

Solving Eqs. (i), (ii) and (iii)

$$i_1 = 0.869 \text{ A},\ i_2 = 0.348 \text{ A and } i_3 = 0.348 \text{ A}$$

$$\text{Current through } 5\Omega \text{ resistance} = i_2 - i_3$$

$$= 0 \text{ A}$$

Resistances 6Ω, 8Ω, 9Ω and 12Ω form a *bridge*. When any resistance is connected across a *balanced bridge*, it will not carry any current. Also observe

$$\frac{6\Omega}{8\Omega} = \frac{2}{3} = \frac{9\Omega}{12\Omega} = \text{(equal bridge arms ratio)}$$

Example 2.14

For the circuit of Fig. 2.26 (a) determine the voltage v across 20Ω resisitance using mesh analysis.

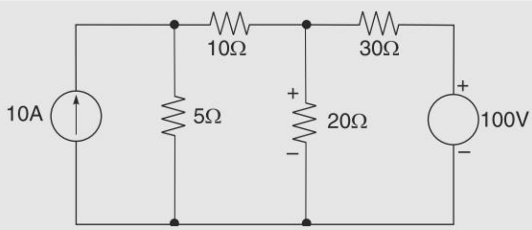


Fig. 2.26(a)

Solution

Converting the practical current to a voltage source, we draw the circuit as in Fig. 2.26(b). Writing the two mesh equations.

$$-50 + 15i_1 + 20\,(i_1 - i_2) = 0 \qquad \text{(i)}$$

Or $$35i_1 - 20i_2 = 50 \qquad \text{(ii)}$$

$$20\,(i_2 - i_1) + 30i_2 + 100 = 0 \qquad \text{(iii)}$$

Or $$-20\,i_1 + 50i_2 = -100 \qquad \text{(iv)}$$

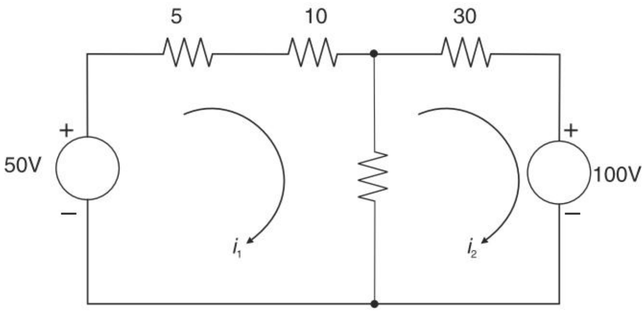


Fig. 2.26(b)

Solving Eqs. (ii) and (iv), we get

$$i_1 = 0.37\,A, \quad i_2 = -1.85 \text{ A}$$

Voltage across 20Ω resistance

$$v = 20\,(i_1 - i_2) = 20 \times 2.22 = 44.4 \text{ V}$$

Example 2.15 For the circuit of Fig 2.27 determine the value of resistance R such that $i_1 = 0.37$ A.

Solution

Mesh equations

$-50 + Ri_1 + 20\,(i_1 - i_2) = 0$

$20\,(i_2 - i_1) + 30i_2 + 100 = 0$

Or

$(R + 20)i_1 - 20i_2 = 50$

$-20i_1 + 50i_2 = -100$

Eliminating i_2,

substituting $i_1 = 0.37$ A, we get

$$R = 15\Omega$$

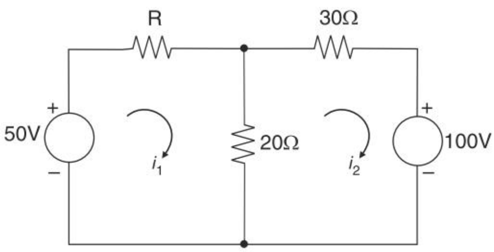


Fig. 2.27

Example 2.16 For the circuit of Fig. 2.28(a) with a dependent source, find the value of v_x using mesh analysis.

Solution

Converting current sources to voltage sources, the circuit modifies to Fig.

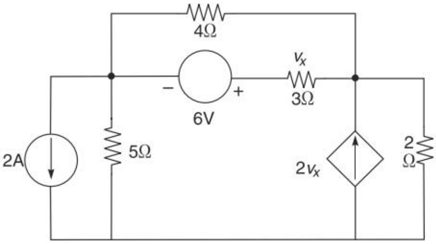


Fig. 2.28 (a)

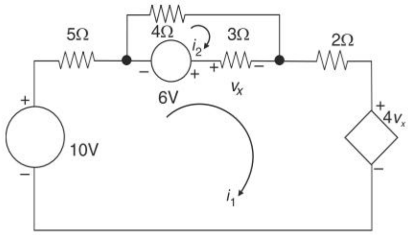


Fig. 2.28(b)

2.28(b). The two mesh equations are:

$$-10 + 5i_1 - 6 + 3\ (i_1 - i_2) + 2i_1 + 4v_x = 0 \qquad \text{(i)}$$

$$6 + 4i_2 + 3\ (i_2 - i_1) = 0 \qquad \text{(ii)}$$

But $$v_x = 3\ (i_1 - i_2) \qquad \text{(iii)}$$

Substituting for v_x in Eq. (i)

$$-10 + 7i_1 + 15(i_1 - i_2) = 0$$

$$6 - 3i_1 + 7i_2 = 0$$

Or

$$22\ i_1 - 15\ i_2 = 16$$

$$-3\ i_1 + 7\ i_2 = -6$$

Solving we get

$$i_1 = 11.\ 6\ A, \quad i_2 = 4.1\ \text{A}$$

Then

$$v_x = 3\ (i_1 - i_2) = 22.5\ \text{V}$$

Example 2.17 Using mesh analysis find currents I_1, I_2 and I_3 in the circuit of Fig. 2.29(a). Find also the power supplied by each ideal current source.

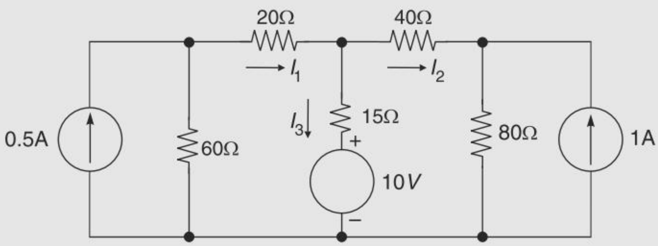


Fig. 2.29(a)

Solution

Converting current sources to voltage source the circuit diagram is drawn in Fig. 2.29(b).

The mess equations are:

$$-30 + 80\ i_1 + 15\ (i_1 - i_2) + 10 = 0$$

$$-10 + 15\ (i_2 - i_1) + 120\ i_2 + 80 = 0$$

or $\quad 95\, i_1 - 15\, i_2 = 20$

$-15\, i_1 + 135\, i_2 = -70$

Solving we get

$i_1 = 0.13095$ A

$i_2 = -0.504$ A

$I_1 = i_1 = 0.13095$ A

$I_2 = i_2 = -0.504$ A

$I_3 = I_1 - I_2 = 0.63492$ A

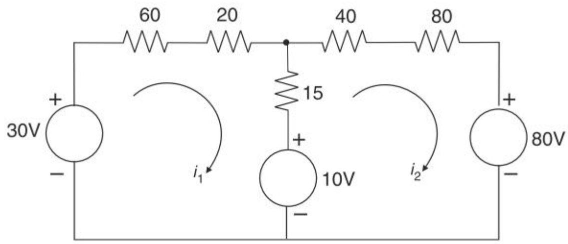


Fig. 2.29(b)

Power supplied by current sources

0.5A source

Figure 2.29(a) current through 60Ω is 0.5 – 0.13095 = 0.36905

Power = (60 × 0.36905) × 0.5 = 11.0715 W

IA source

Figure 2.29(a) current through 80Ω is $(1 + I_2) = 1 - 0.504 = 0.496$ A

Power = (80 × 0.52) × 1 = 39.68 W

2.8 DEPENDENT SOURCES

The techniques of nodal and mesh analysis apply equally for circuits in which dependent sources are present. This will be demonstrated through two examples.

Example 2.18 For the circuit of Fig. 2.30, find the voltage v_{12} by the technique of nodal analysis.

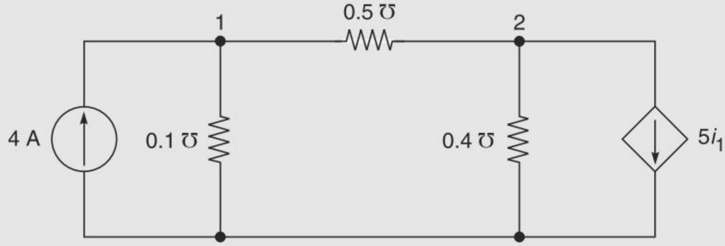


Fig. 2.30

Solution Writing nodal equations for nodes 1 and 2:

Node 1: $\quad -4 + 0.1v_1 + 0.5(v_1 - v_2) = 0$

or $\quad 0.6v_1 - 0.5v_2 = 4 \qquad$ (i)

Node 2: $\quad 5i_1 + 0.4v_2 + 0.5(v_2 - v_1) = 0$

But $\quad i_1 = 0.5\,(v_1 - v_2)$

$$2.5\,(v_1 - v_2) + 0.4v_2 + 0.5\,(v_2 - v_1) = 0$$

or $\quad 2v_1 - 1.6v_2 = 0 \qquad$ (ii)

Solving Eqs. (i) and (ii)

$$v_1 = -160\text{ V} \quad v_2 = -200\text{ V}$$
$$v_{12} = -160 - (-200) = 40\text{ V}$$

Example 2.19 For the circuit of Fig. 2.31 find v_1 using mesh analysis technique.

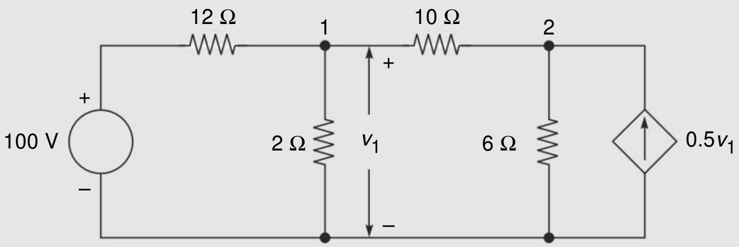


Fig. 2.31

Solution Converting the dependent current source with parallel resistance 6Ω to dependent voltage source, we can redraw the circuit of Fig. 2.31 as in Fig. 2.32.

Writing mesh equations for the circuit of Fig. 2.32,

Mesh 1: $$14i_1 - 2i_2 = 100 \quad \text{(i)}$$

Mesh 2: $$-2i_1 + 18i_2 = -3v_1$$

But $$v_1 = 2(i_1 - i_2)$$

$$\therefore \quad -2i_1 + 18i_2 = -6(i_1 - i_2)$$

or $$4i_1 + 12i_2 = 0 \quad \text{(ii)}$$

Solving Eqs (i) and (ii)

$$i_1 = 6.82\text{ A} \quad i_2 = -2.27\text{ A}$$
$$v_1 = 2(i_1 - i_2)$$
$$= 2(6.82 + 2.27) = 18.2\text{ V}$$

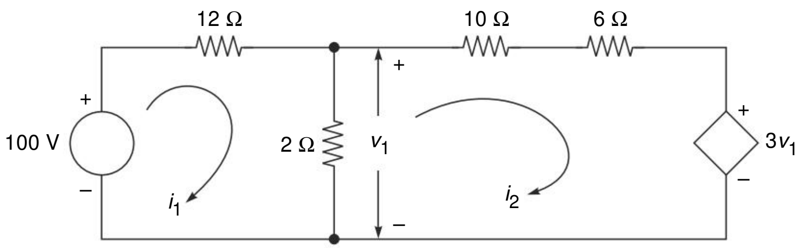


Fig. 2.32

Comparison—Nodal vs Mesh Analysis The choice between nodal or mesh analysis of a circuit depends upon the following factor:

- Number of simultaneous equations to be solved
- If we need in the answer in terms of voltage, the choice would be the nodal analysis. When answer is need in terms of current obvious choice is the mesh

analysis. This not a rigid rule as by use of Ohm's law the answer in voltage can be converted to desired currents and vice versa.

We will illustrate by an example.

Example 2.20 Analyse the circuit of Fig. 2.33 by (a) nodal method and (b) by mesh method. Compare the two.

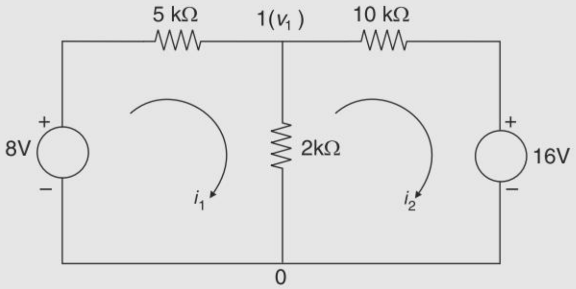


Fig. 2.33

Solution

(a) Nodal Method This is only one independent node (1), the other (0) being the *datum* node. Note that the connections between voltage source and resistance need not be considered as node as voltages at these are known

Nodal equation

$$\frac{(v_1 - 8)}{5} + \frac{v_1}{2} + \frac{v_1 - 16}{10} = 10$$

$$\left(\frac{1}{5} + \frac{1}{2} + \frac{1}{10}\right) v_1 = \frac{8}{5} + \frac{16}{10}$$

Or

$$(2 + 5 + 1)\, v_1 = 16 + 16 = 32$$

$$v_1 = \frac{32}{8} = 4V$$

If we need the current through (say) 5 kΩ resistance, it found by Ohm's law

$$i(5\text{k}\Omega) = \frac{8 - 4}{5} = 0.8\text{mA (source to node 1)}$$

(b) Mesh Method There are two meshes as shown in Fig. 2.33.

Mess equation

$$-8 + 5i_1 + 2\,(i_1 - i_2) = 0$$
$$16 + 2\,(i_2 - i_1) + 10i_2 = 0$$

Or

$$7i_1 - 2i_2 = 8$$
$$-2i_1 + 12i_2 = -16$$

Solving

$$i_1 = 0.8 \text{ mA}, \quad i_2 = -1.2 \text{ mA}$$

Let us say we need the voltage at node 1

$$v_1 = 2 \times (i_1 - i_2) = 2 \times 2 = 4 \text{ V}$$

Based on the number of equations the choice here is in favour of the nodal method.

2.9 NETWORK THEOREMS—SUPERPOSITION THEOREM

Certain network theorems are very helpful in circuit analysis and give a simplified way of visualising the response of a complex network when connected to another network (usually simpler of the two).

Superposition Theorem

The response of a network (voltage across or current through an element) with several independent sources can be obtained as the sum of the responses to sources, taken one at a time as a consequence of circuit linearity. In removing sources other than one, voltage sources are short circuited and current sources are open circuited.

This will be illustrated with the help of an example. It may also be noted that sources are often called *forcing functions* (excitations or generators) and voltage and/or current desired are known as *responses*.

Example 2.21 In the circuit of Fig. 2.34(a) find the node voltages v_1 and v_2 using the *superposition theorem.*

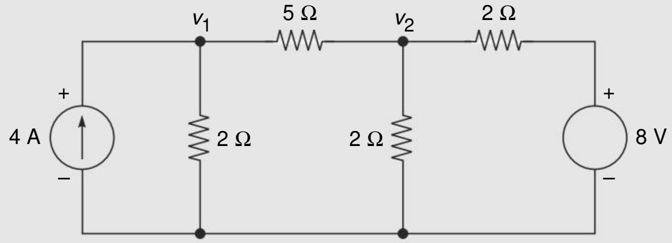


Fig. 2.34(a)

Solution

(i) Only the current source is applied and the voltage source is shorted. The resultant circuit is presented in Fig. 2.34(b). Solving, we get

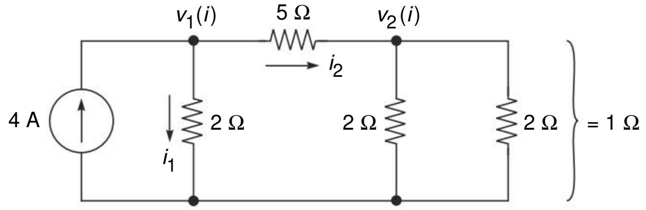


Fig. 2.34(b)

$$i_1 = 4 \times \frac{6}{8} = 3 \text{ A}, \quad v_1\,(\text{i}) = 3 \times 2 = 6\text{V}$$

$$i_2 = 4 \times \frac{2}{8} = 1 \text{ A}, \quad v_2\,(\text{i}) = 1 \times 1 = 1\text{V}$$

(ii) Voltage source only is applied with the current source open-circuited. The resultant circuit is represented in Fig. 2.35. Using series–parallel combination, the equivalent resistance in series with the source is

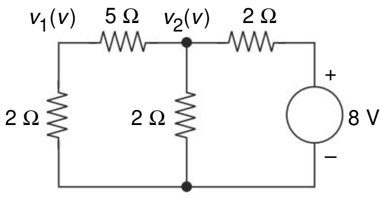


Fig. 2.35

$$\frac{2 \times 7}{2 + 7} = \frac{14}{9}$$

$$v_2\,(\text{ii}) = 8 \times \left(\frac{14/9}{2 + 14/9}\right) = \frac{7}{2} \text{ V}$$

$$v_1\,(\text{ii}) = \frac{7}{2} \times \frac{2}{2 + 5} = 1 \text{ V}$$

Using the superposition theorem

$$v_1 = v_1\,(\text{i}) + v_1\,(\text{ii}) = 6 + 1 = 7 \text{ V}$$

$$v_2 = v_2\,(\text{i}) + v_2\,(\text{ii}) = 1 + \frac{7}{2} = \frac{9}{2} \text{ V}$$

2.10 THEVENIN AND NORTON THEOREMS

These theorems reduce a complex network as seen from two terminals into a simple circuit so that when another network (load) is connected at these terminals, its responses can be easily determined.

This situation is illustrated in Fig. 2.36. *Thevenin* and *Norton* equivalents of N_1 as seen from terminals *ab* are given in Fig. 2.37(a) and (b) respectively. Here

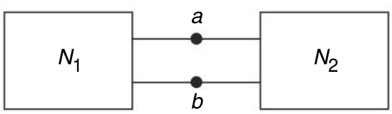


Fig. 2.36

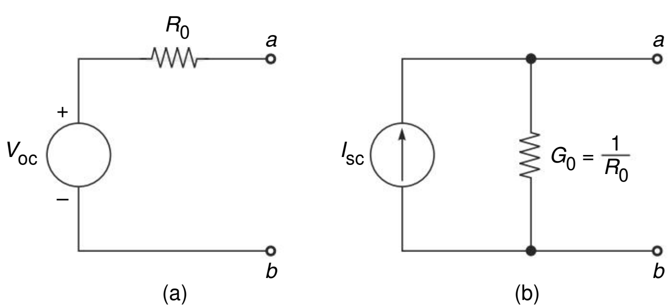


Fig. 2.37 (a) Thevenin equivalent, (b) Norton equivalent

V_{OC} = *open-circuit* voltage at *ab* (when network N_2 is disconnected); voltage of *a* with respect to *b*.

R_0 = equivalent resistance (of N_1) as seen from *ab* with all voltage sources short-circuited and all current sources open-circuited.

I_{SC} = *short-circuit* current which flows from *a* to *b* when terminals *ab* are shorted after disconnecting N_2. Observe that the direction of current in Norton equivalent is such as to produce the same open-circuit polarity as in the Thevenin's equivalent.

It is easily seen that the Norton equivalent follows from the Thevenin equivalent by source conversion and also vice versa. In the Thevenin equivalent we will write

$V_{OC} = V_{TH}$, the *Thevenin Voltage*

$R_0 = R_{TH}$, the Thevenin resistance

In the Norton equivalent we will write

$R_0 = R_N$, the Norton resistance

Obviously

$R_N = R_{TH}$

From the source conversion results of Section 2.5, it immediately follows that

$$I_{SC} = \frac{V_{OC}}{R_0} = \frac{V_{TH}}{R_{TH}} \tag{2.24}$$

When N_2 is connected at terminals *ab* of the Thevenin/Norton equivalent, it will yield an identical response. However, the information on currents and voltages in the elements of N_1 prior to connecting N_2 is lost.

When dependent sources are present, the above equivalents assume that a dependent source and its associated current/voltage are both located in N_1. The Thevenin theorem is particularly useful when the load is to take on a series of values.

When dependent sources are present in N_1, it is convenient to obtain R_{TH} from Eq (2.24) as

$$R_{TH} = \frac{V_{OC}}{I_{SC}} \tag{2.25}$$

Thus in a practical circuit R_{TH} can be obtained from two measurements, open-circuit voltage and short-circuit current. However, care should be taken in short-circuiting the output terminals?

Example 2.22 Find the Thevenin and Norton equivalents of the circuit of Fig. 2.38 as seen at terminal *ab*.

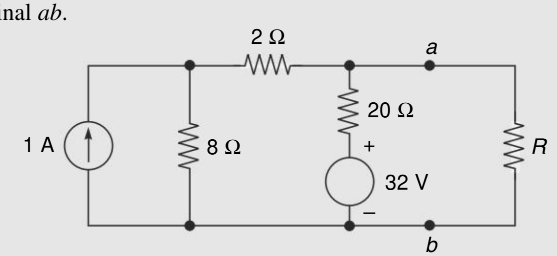


Fig. 2.38

* Thevenin resistance is found experimentally on similar lines. Where the output cannot be short-circuited some external resistance may be used. Thevenin resistance is the output resistance of some circuits like amplifiers.

Solution

Thevenin Equivalent

– Remove load resistance R causing open-circuit at ab.
– Replace 1A source and 8Ω resistance in parallel with it by its equivalent voltage source. The circuit is drawn in Fig. 2.39.

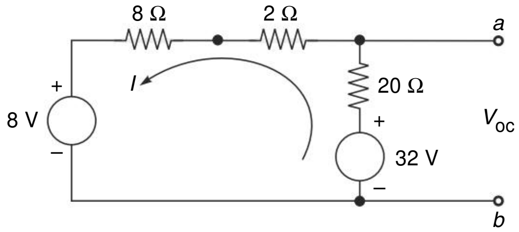


Fig. 2.39

In Fig. 2.39

$$I = \frac{32 - 8}{30} = 0.8 \text{ A}$$

$$V_{oc} = 32 - 20 \times 0.8 = 16 \text{ V}$$

Open-circuit 1A source and short-circuit 32 V source. The resulting circuit is drawn in Fig. 2.40.

$$R_0 = \frac{20 \times 10}{20 + 10} = \frac{20}{3} \ \Omega$$

Thevenin equivalent is drawn in Fig. 2.41.

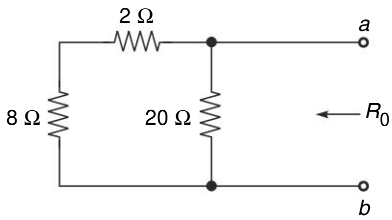


Fig. 2.40

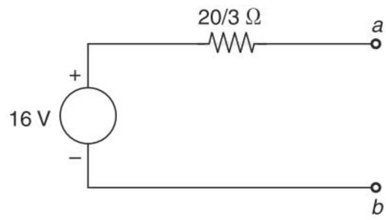


Fig. 2.41 Thevenin equivalent

Norton Equivalent

– Short-circuit at ab as shown in Fig. 2.42.

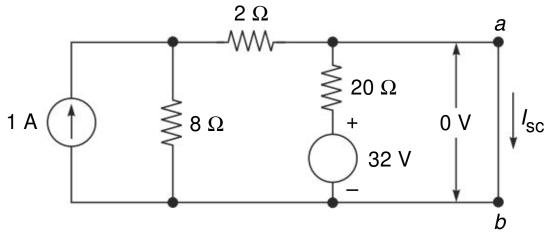


Fig. 2.42

– As before the circuit of Fig. 2.42 modifies to that of Fig. 2.43.

$$I_{SC} = \frac{32}{20} + \frac{8}{10} = \frac{48}{20} = 2.4 \text{ A}$$

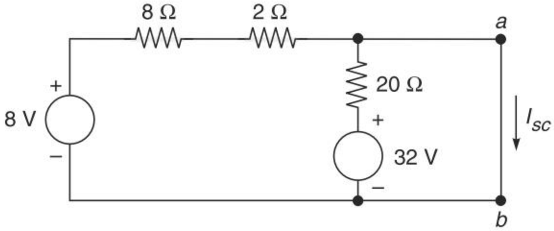


Fig. 2.43

– As calculated in Thevenin equivalent

$$R_0 = \frac{20}{3} \ \Omega$$

– Norton equivalent is drawn in Fig. 2.44.

Check: $V_{OC} = 2.4 \times 20/3 = 16$ V (same as in Thevenin equivalent)

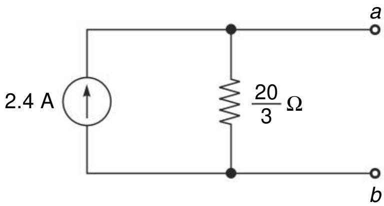


Fig. 2.44

Example 2.23 Find the Thevenin equivalent of the circuit of Fig. 2.45 which includes one dependent current generator.

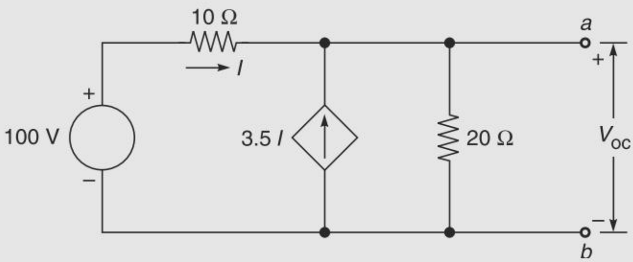


Fig. 2.45

Solution Converting the current source into a voltage source, we get the circuit of Fig. 2.46.

From KVL for the single loop

$$-100 + 10I + 20I + 70I = 0$$

or $\qquad I = 1\text{A}$

$\therefore \qquad V_{OC} = 100 - 10 \times 1$

$\qquad\qquad = 90$ V

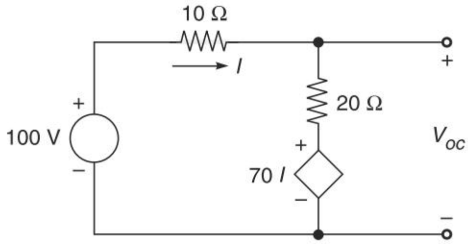


Fig. 2.46

It is obvious that R_0, the Thevenin's resistance, cannot be found by short circuiting the independent source (100 V) as the dependent current source also gets eliminated. So we find short-circuit current in Fig. 2.47.

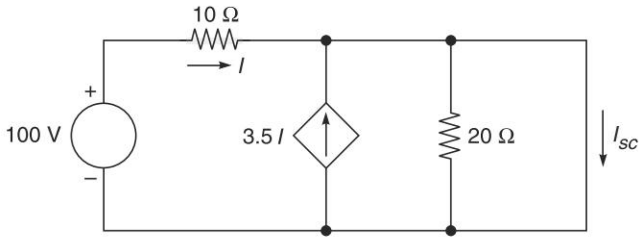


Fig. 2.47

$$I = \frac{100}{10} = 10\text{A}$$

$$\therefore \quad I_{SC} = I + 3.5I = 45 \text{ A}$$

Hence

$$R_0 = \frac{V_{OC}}{I_{SC}} = \frac{90}{45} = 2\ \Omega$$

The Thevenin equivalent is drawn in Fig. 2.48.

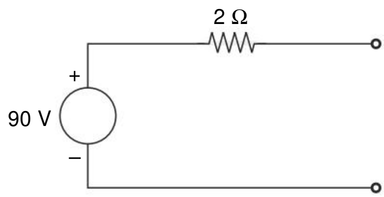


Fig. 2.48

Example 2.24 For the circuit of Fig. 2.38, find the Thevenin voltage and Norton current using the principle of superposition.

Solution Thevenin voltage source

(i) Open-circuit current source. Circuit is now drawn in Fig. 2.49(a). With terminals *a, b* open

$$V_{OC_1} = \frac{32}{20 + 2 + 8} \times (8 + 2) = \frac{32}{3} \text{ V}$$

With terminals *a, b* shorted

$$I_{SC_1} = \frac{32}{20} = 1.6 \text{ A}$$

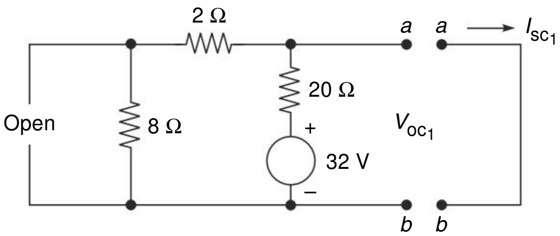


Fig. 2.49(a)

(ii) Short-circuit voltage source. Circuit is now drawn in Fig. 2.49(b).

$$V_{OC_2} = 1 \times \frac{8}{8+2+20} \times 20 = \frac{16}{3}\text{V}$$

$$I_{SC_2} = 1 \times \frac{8}{8+2} = 0.8 \text{ A}$$

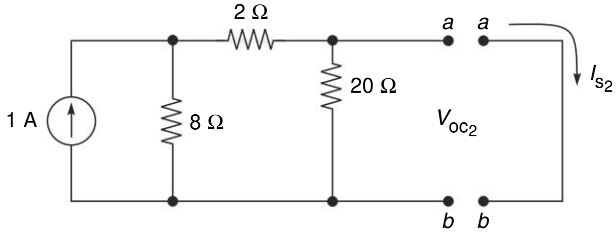


Fig. 2.49(b)

By superposition

$$V_{OC} = V_{OC_1} + V_{OC_2} = \frac{32}{3} + \frac{16}{3} = 16 \text{ V}$$

$$I_{SC} = I_{SC_1} + I_{S_2} = 1.6 + 0.8 = 2.4 \text{ A}$$

2.11 MAXIMUM POWER TRANSFER THEOREM

When a network is loaded by a resistance at two of its terminals, the situation can be represented by the Thevenin equivalent as in Fig. 2.50. We want to investigate the power delivered to load resistance R_L as it is varied.

$$I = \frac{V_{OC}}{R_0 + R_L}$$

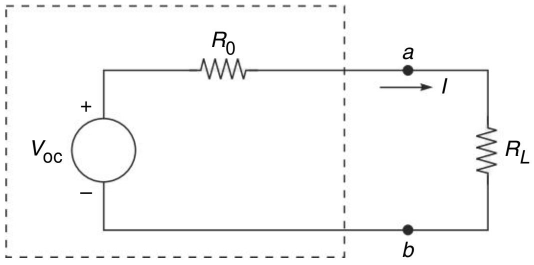


Fig. 2.50

Load power, $$P_L = I^2R_L$$

$$= \left(\frac{V_{OC}}{R_0 + R_L}\right)^2 R_L$$

Maximum power delivered to the load is found from the condition

$$\frac{dP_L}{dR_L} = 0$$

which gives the result

$$R_L = R_0 \tag{2.26}$$

The condition of Eq. (2.25) which states that *power delivered (transferred) to load is maximum when load resistance equals the Thevenin resistance of the source is known as the maximum power transfer theorem.*

With reference to the circuit of Fig. 2.50, under conditions of maximum power transfer.

Power output by source $$P_S = V_{OC} \times \frac{V_{OC}}{2R_L}$$

$$= \frac{V_{OC}}{2R_L}$$

But $$P_L = \left(\frac{V_{OC}}{2R_L}\right)^2 R_L = \frac{V_{OC}^2}{4R_L}$$

Hence, the efficiency of power transfer

$$\eta = \frac{P_L}{P_S} = \frac{1}{2} \quad \text{or } 50\%$$

This is too low an efficiency for energy conversion devices; such devices must have load resistance far larger than that corresponding to the condition of maximum power transfer. However, in electronic devices the objective is to obtain maximum power output irrespective of device efficiency and hence the condition is always used at the power stage of an electronic system. Of course the 50% power lost in the system (devices and components) must be suitably dissipated to limit temperature rise.

ADDITIONAL SOLVED PROBLEMS

Example 2.25 For the network of Fig. 2.51, find the value of the battery current (I) using network reduction techniques.

Solution With reference to Fig. 2.51, convert delta ABD to star, so that the circuit takes the form of Fig. 2.52, where

$$R_{AN} = (8 + 12)/(8 + 12 + 4) = 4\ \Omega$$

$$R_{BN} = (8 + 4)/24 = 1.33\ \Omega$$

$$R_{DN} = (4 \times 12)/24 = 2\Omega$$

Using the series-parallel reduction technique, we get

$$R_{eq} = 10 + 4 + (1.33 + 15) \parallel (2 + 16) = 22.56\ \Omega$$

$$I = 12/22.56 = 0.532\ \text{A}$$

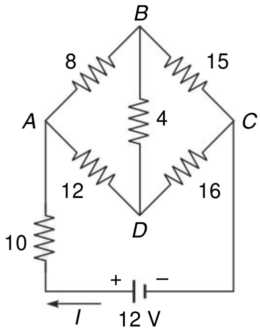


Fig. 2.51

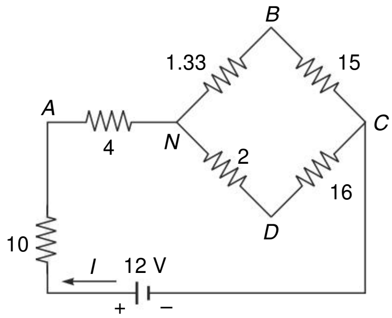


Fig. 2.52

Example 2.26 In the resistive circuit of Fig. 2.53 with a dependent source find the value of v_x.

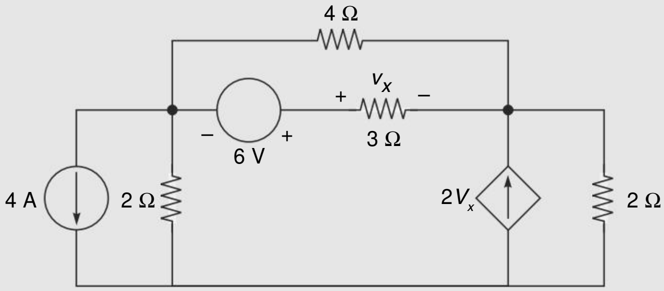


Fig. 2.53

Solution Convert the current sources to voltage sources as shown in Fig. 2.54. Now apply the mesh analysis.

Mesh 1: $\quad -6 + 3(i_1 - i_2) + 4i_1 = 0$

or $\quad 7i_1 - 3i_2 = 6 \qquad$ (i)

Mesh 2: $\quad -4v_x + 2i_2 + 3(i_2 - i_1) + 6 + 2i_2 + 8 = 0$

But $\quad v_x = 3(i_1 - i_2)$

Then,

$$-12(i_1 - i_2) + 2i_2 + 3(i_2 - i_1) + 6 + 2i_2 + 8 = 0$$

$$-15i_1 + 19i_2 = -14 \qquad \text{(ii)}$$

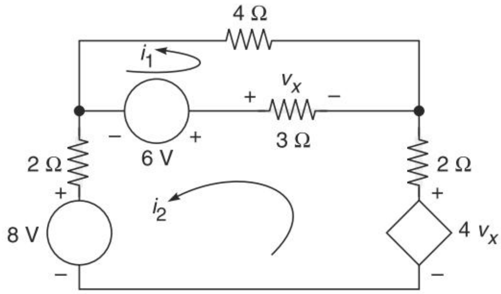


Fig. 2.54

Solving Eqs. (i) and (ii), we get

$$i_1 = 0.818 \text{ A}, i_2 = -0.091 \text{ A}$$

$$\therefore \quad v_x = 3\,(0.818 + 0.091) = 2.727 \text{ V}$$

Example 2.27 Find the Norton equivalent of the circuit of Fig. 2.55 at *ab*.

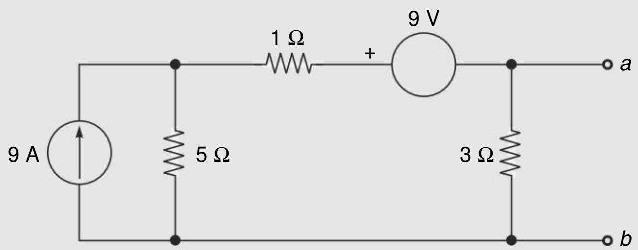


Fig. 2.55

Solution Short-circuiting *ab* and converting the current source to a voltage source as in Fig. 2.56(a), we get

$$I_{SC} = (45 - 9)/6 = 6\text{A}$$

Short-circuiting the voltage source and open-circuiting the current source as in Fig. 2.56(b), we obtain $R_0 = (5 + 1) \parallel 3 = 2\Omega$.

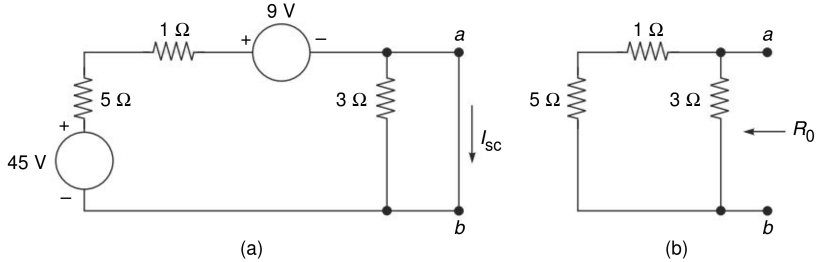


Fig. 2.56

The Norton equivalent is drawn in Fig. 2.57. Notice that the direction of the current in the current source in the equivalent is such as to cause the short-circuit current at *ab* in the same direction as shown in Fig. 2.56(a).

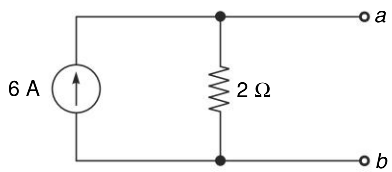


Fig. 2.57

Example 2.28 The circuit shown in Fig. 2.58 is the representation of a transistor amplifier. Using mesh analysis, find

(a) I_0

(b) the power supplied by the 1 mV source.

(c) the power dissipated by the 2.5 kΩ resistance, and

(d) the power absorbed/supplied by the dependent source.

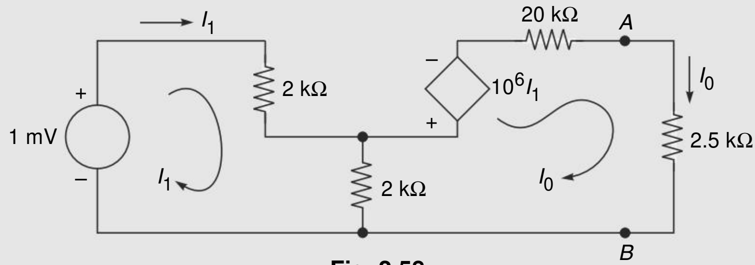


Fig. 2.58

Solution

Mesh 1: $4\,I_1 - 2I_0 = 1 \times 10^{-3}$ V (i)

Mesh 2: $-2I_1 + (2 + 20 + 2.5)\,I_0 = -10^6 \times I_1 \times 10^{-3}$ V

or $-2I_1 + 24.5\,I_0 = -10^3\,I_1$

or $998\,I_1 + 24.5\,I_0 = 0$ (ii)

Note: Currents are in mA. Solving Eqs (i) and (ii), we get

(a) $I_1 = 11.7 \times 10^{-3}$ μA, $I_0 = -447 \times 10^{-3}$ μA

(b) P (2.5 kΩ) = $(0.447 \times 10^{-6})^2 \times 2.5 \times 10^3 = 0.5 \times 10^{-3}$ μW

(c) P (1 mV source) = $10^{-3} \times 0.0117 \times 10^{-6} = 11.7 \times 10^{-6}$ μW

(d) Power supplied by the dependent source

$= 10^6 \times 0.0117 \times 10^{-6} \times 0.447 \times 10^{-6}$

$= 5.23 \times 10^{-3}$ μW

ADDITIONAL SOLVED PROBLEMS

2.29

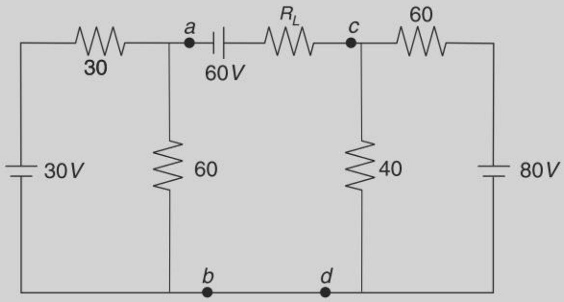


Fig. 2.59(a)

Using the Thevenin theorem reduce the circuit of Fig. 2.59 (a) to a single loop preserving the identity of the load R_L. All resistance value are in ohms.

Solution

As seen from terminals '*ab*'

$$V_{TH\,1} = 30 \times \frac{60}{30 + 60} = 20 \text{ V}$$

$$R_{TH\,1} = 30 \,\|\, 60 = 20$$

As seen from terminals *cd*

$$V_{TH\,2} = 80 \times \frac{40}{40 + 60} = 32 \text{ V}$$

$$R_{TH\,2} = 40 \,\|\, 60 = 24\ \Omega$$

The circuit with Thevenin equivalent is drawn in Fig. 2.59(b). Net voltage round the loop

$$V_{net} = 60 + 20 - 32$$
$$= 48V$$

$$R_{net} = 20 + 24 = 44$$

The reduced circuit is drawn in Fig. 2.59(c).

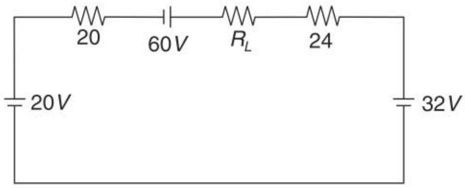


Fig. 2.59(b)

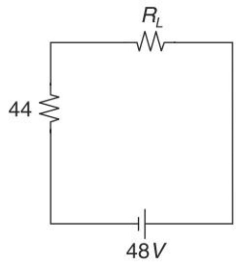


Fig. 2.59(c)

2.30 In the circuit of Fig. 2.60(a) determine the current through 60Ω resistor using mesh analysis. All resistance values are in ohms.

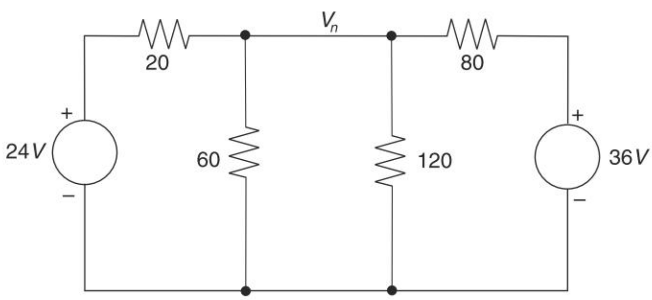


Fig. 2.60(a)

Solution

$60 \,\|\, 120 = 40$

The circuit is now drawn in Fig. 2.60(b) with mesh currents indicated. The two mesh equations are written below.

$$-24 + 20\,I_1 + 40\,(I_1 - I_2) = 0$$
$$+\,40\,(I_2 - I_1) + 80\,I_2 + 36 = 0$$

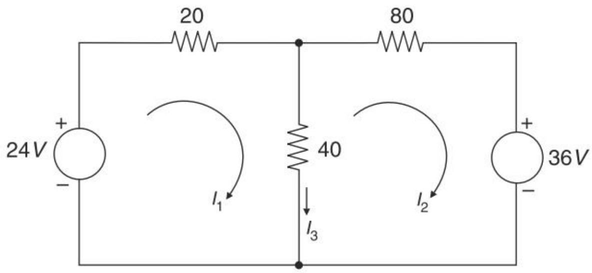


Fig. 2.60 (b)

Rearranging

$$60I_1 - 40I_2 = 24 \qquad \text{(i)}$$

$$-40I_1 + 120I_2 = -36 \qquad \text{(ii)}$$

or

$$6I_1 - 4I_2 = 2.4 \qquad \text{(iii)}$$

$$-4I_1 + 12I_2 = -3.6 \qquad \text{(iv)}$$

From above equations

$$I_1 = 0.257 \text{ A} \quad I_2 = -0.214 \text{ A}$$

Then

$$I_3 = I_1 - I_2 = 0.471 \text{ A}$$

By current division

$$I\,(60\Omega) = 0.471 \times \frac{120}{60 + 120}$$

$$= 0.314\ A \text{ (downwards)}$$

2.31 Solve Problem 2.30 by the nodal method.

Solution

There are two nodes. So we need only one nodal equation. Let the nodal voltage be V_n. The nodal equation is

$$\frac{V_n - 24}{20} + V_n\left(\frac{1}{60} + \frac{1}{120}\right) + \frac{V_n - 36}{80} = 0$$

$$\left(\frac{1}{20} + \frac{1}{60} + \frac{1}{120} + \frac{1}{80}\right)V_n = \frac{24}{20} + \frac{36}{80}$$

$$(6 + 2 + 1 + 1.5)V_n = 144 + 54$$

or

$$V_n = \frac{148}{10.5} = 18.86 \text{ V}$$

Then

$$I\,(60\Omega) = \frac{18.86}{60} = 0.314 \text{ A}$$

Observation: In this problem, the nodal method needs only one equation to be solved, while the mesh method requires two simultaneous to be solved. So obvious choice is the nodal method.

2.32 For the circuit of Fig. 2.61, determine v_x, the voltage across current source.

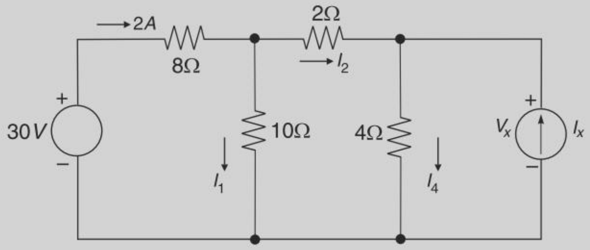


Fig. 2.61

Soultion

Voltage across 8Ω resistance by Ohm's law $V_8 = 2 \times 8 = 16$ V

In the left side mesh two out of three voltages are known. So applying KVL

$$-30 + 16 + 10\,I_1 = 0$$

Or
$$I_1 = \frac{14}{10} = 1.4\text{A}$$

Applying KCL at left node

$$-2 + 1.4 + I_2 = 0$$

Or
$$I_2 = 0.6 \text{ A}$$

Applying KVL around middle mesh (clockwise)

$$-\,10 \times 1.4 + 2 \times 0.6 + 4\,I_4 = 0$$

Or
$$I_4 = 3.2 \text{ A}$$

Now
$$V_x = 4\,I_4 = 4 \times 3.2 = 12.8 \text{ V}$$

From the right node

$$I_x = I_4 - I_2 = 3.2 - 0.6$$
$$= 2.6 \text{ A}$$

2.33 The circuit model of a battery charger and battery is drawn in Fig. 2.62. The variable resistance R controls the charging current.

Determine R for

(a) Charging current of 5A
(b) Power delivered to battery of 12.5 W
(c) Voltage at battery terminals of 13 V

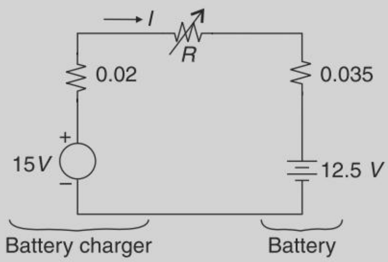


Fig. 2.62

Solution

(a) By KVL

$$-15 + (0.02 + R + 0.035)\, I + 12.5 = 0$$

$$(R + 0.055)\, I = 15 - 12.5 = 2.5$$

$$I = 5\text{A}$$

Then

$$5R + 0.275 = 2.5$$

Or $$R = 0.445\ \Omega$$

(b) Power delivered at battery terminals

$$P = 12.5\, I + 0.035\, I^2 = 25\ \text{W}$$

$$0.035\, I^2 + 12.5\, I - 25 = 0$$

$$I = \frac{-12.5 \pm \sqrt{(12.5)^2 + 4 \times 25 \times 0.035}}{2 \times 0.035} = 1.99\ \text{A}$$

Solving we get

$I = 1.99$ A, negative value is rejected.

(c) Voltage at battery terminals = 13 V

$$I = \frac{13 - 12.5}{0.035} = 14.28\ \text{A}$$

KVL round the loop

$$15 - (0.02 + R) \times 14.28 = 13$$

Solving we get

$$R = 0.12\ \Omega$$

2.34 In the circuit Fig. 2.63 given $i(t) = \sin(t)$, find $i_s\,(t)$.

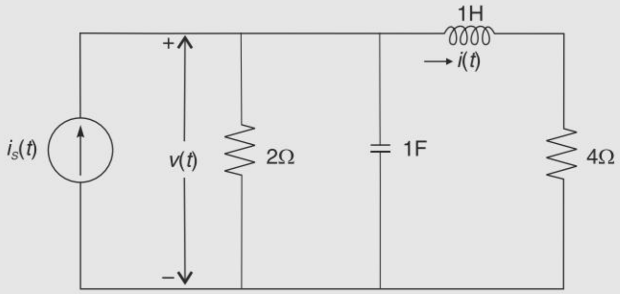


Fig. 2.63

Solution

$$i = \sin 2\,t$$

$$v_L = 1 \times \frac{d}{dt} \sin 2t = 2 \cos 2t$$

$$v_L\,(4\Omega) = 4 \sin 2t$$

Then

$$v = v_L + v\,(4\Omega) = 2 \cos 2t + 4 \sin 2t$$

$$i\,(2\Omega) = v/2 = \cos 2t + 2 \sin 2t$$

If $$1 \times \frac{dv}{dt} = -4 \sin 2t + 8 \cos 2t$$

We then have

$$\begin{aligned} i_s &= i + i\,(2\Omega) + i\,(1F) \\ &= \sin 2t + \cos 2t + 2 \sin 2t - 4 \sin 2t + 8\cos 2t \\ &= \sin 2t + 9 \cos 2t \qquad \text{(i)} \end{aligned}$$

Let $A\cos\theta = 1$, $\quad A \sin\theta = 9$

$$A^2 = 1 + 9^2 = 82, \quad A = \sqrt{82} = 9.06$$

$$\tan\theta = \frac{9}{1}, \quad \theta = 8.36°$$

Substituting in Eq. (i)

$$\begin{aligned} i &= A\cos\theta \sin(2t) + A\sin\theta\cos(2t) \\ &= A\sin(2t + \theta) \\ &= 9.06 \sin(2t + 83.6°) \end{aligned}$$

2.35 For the circuit of Fig. 2.64, determine the relationship between v and i.

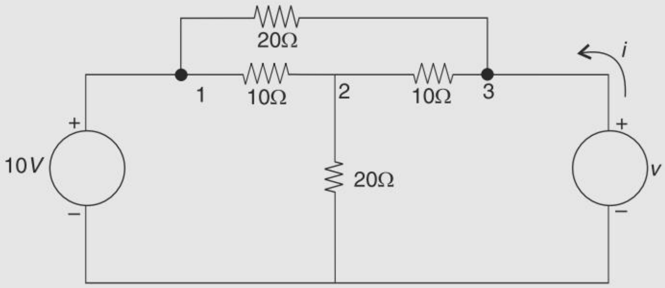


Fig. 2.64

Solution

It is seen that we can find v_2 by applying KCL at node 2

$$\frac{v_2}{20} + \frac{v_2 - v}{10} + \frac{v_2 - 10}{10} = 0 \qquad \text{(i)}$$

$$\left(\frac{1}{20} + \frac{1}{10} + \frac{1}{10}\right)v_2 = \left(\frac{1}{10}\right)v + 1$$

$$0.25\, v_2 = 0.1v + 1$$

$$v_2 = 0.4v + 4 \qquad \text{(ii)}$$

Now applying KCL at node 3

$$v_3 = v$$

$$\frac{v - v_2}{10} + \frac{v - 10}{20} - i = 0$$

$$0.15v - 0.1v_2 - 1 - i = 0$$

Substituting for v_2

$$0.15v - 0.1\,(0.4v + 4) - 1 - i = 0$$

$$0.19v - 0.4 - 1 - i = 0$$

or

$$v = 5.26i + 3.16; \text{ required relationship}$$

2.35 For the circuit of Fig. 2.65(a).

(a) Determine R such that the source delivers a power of 4 mW.

(b) Determine R that results in 15 kΩ resistor to absorb 1.6 mW of power

(c) Replace R by a voltage source that will cause no power to be absorbed by any resistor.

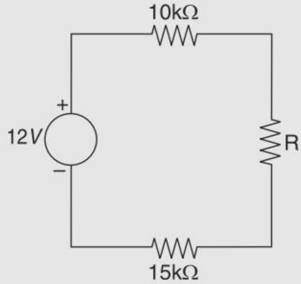


Fig. 2.65(a)

Solution

(a) Total series resistance, R_t = 10+15+R = 25 + R kΩ.

Power delivered by the source

$$P = \frac{V^2}{R_t} = \frac{(12)^2}{25+R} = 4$$

Solving we get R = 11 kΩ

(b) P (10 kΩ) = 10 I^2 = 1.6 mW, I = 0.4 mA

$$\frac{12}{10 + R + 15} = 0.4 \text{ mA}; R = 5\text{k}\Omega$$

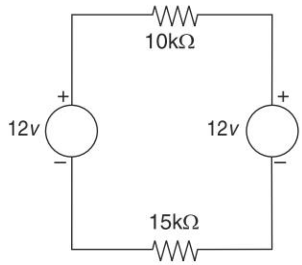


Fig. 2.65(b)

(c) No power absorbed means I = 0. Therefore voltage source is 12 V in opposition as in Fig. 2.65(b).

2.36 Compute the equivalent resistance as indicated in Fig. 2.66 if each resistor is 1 kΩ.

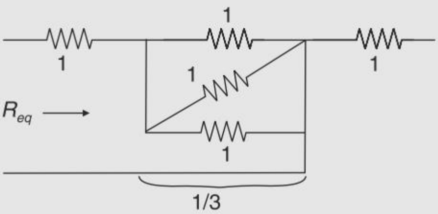


Fig. 2.66

Solution

It is easy to see from the circuit that three 1 kΩ resistances are imparallel. From the figure

$$R_{eq} = 1 + \frac{1}{3} = \frac{4}{3} \text{ k}\Omega$$

The right-most 1 kΩ resistance does not have its one end connected so it has no contribution in R_{eq}.

2.37 Find R_{eq} in the resistance network shown in Fig. 2.67.

Solution

We find the three resistance are in parallel

$$R_p = 40 \parallel (80 + 40) \parallel 60$$

$$= \frac{1}{\frac{1}{40} + \frac{1}{120} + \frac{1}{60}}$$

$$= \frac{120}{3 + 1 + 2} = 20$$

$$30 \parallel (10 + 20) = 15$$

Finally

$$R_{eq} = 20 + 15 = 35 \text{ k}\Omega$$

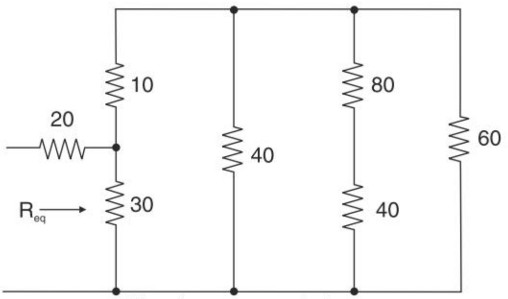


Fig. 2.67

2.38 In the circuit of Fig. 2.68

(a) Determine i_x

(b) Power dissipated in 10 kΩ resistor

All resistances are in kΩ.

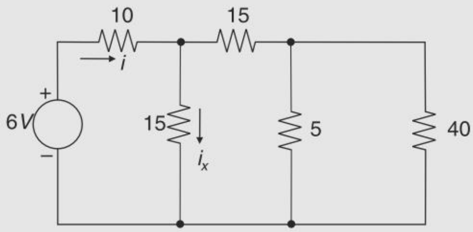


Fig. 2.68

Solution

(a) By series-parallel combinations, we find the equivalent resistance (R_{eq}) across 6V—source

$$R_{eq} = \{[(5 \parallel 40) + 15] \parallel 15\} + 10 \text{ k}\Omega$$

$$\underbrace{19.44 \parallel 15 + 10}$$

$$= 8.47 + 10 = 18.47 \text{ k}\Omega$$

$$i = \frac{6 \times 10^{-3}}{18.47} = 324.8 \ \mu\text{A}$$

By current division

$$i_x = 324.8 \times \frac{19.44}{19.44 + 15} = 183.3$$

(b) $P\,(10 \text{ k}\Omega) = i^2 \times 10 = (183.3)^2 \times 10 \times 10^{+3} = 336 \ \mu\text{W}$

2.39 In the circuit of Fig. 2.69(a) determine the power dissipated in resistor R.

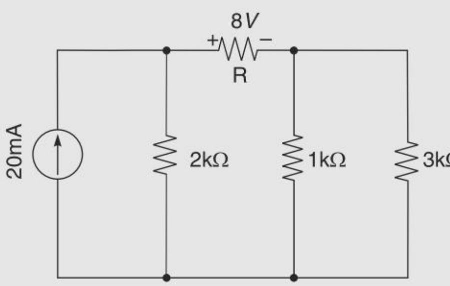


Fig. 2.69(a)

Solution

To begin with we will take two steps

$\Rightarrow 1 \parallel 3 = 0.75\ k\Omega$

$\Rightarrow$ Convert current source to voltage source

$V_s = 20 \times 2 = 40$ V

The circuit is now drawn in Fig. 2.69 (b)

$$I = \frac{40 - 8}{2 + 0.75} = 11.64 \text{ mA}$$

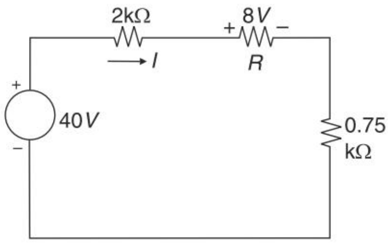


Fig. 2.69(b)

Power dissipated in R

$$P = VI = 8 \times 11.64 = 93.12 \text{ mW}$$

2.40 In the resistance circuit of Fig. 2.70, find

(a) I_x if I_1 = 12mA

(b) I_x if I_2 = 20mA

(c) I_2 if I_x = 6 mA

(d) I_x if I_s = 45 mA

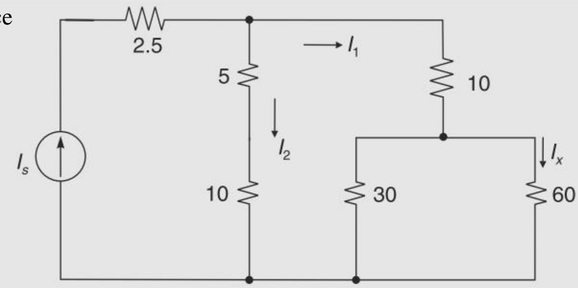


Fig. 2.70

Solution

(a) By current division

$$I_x = 12 \times \frac{30}{30 + 60} = 4 \text{ mA}$$

(b) Voltage across (5 + 10) = 15Ω

$$V_2 = 15\ I_2 = 15 \times 20 = 300 \text{ mV}$$

$$R_1 = (30 \parallel 60) + 10 = 30\ \Omega$$

$$\therefore\ I_1 = \frac{V_2}{R_1} = \frac{300}{30} = 10 \text{ mA}$$

Then $\quad I_x = 10 \times \dfrac{30}{30 + 60} = \dfrac{10}{3} = 10$ mA

(c) $\quad I_x = I_1 \times \dfrac{30}{30 + 60}$

or $\quad I_x = \dfrac{90}{30}\ .\ I_x = 3 \times 6 = 18$ mA

(d) The current I_s flows through 2.5Ω and then divides as I_1 and I_2

$$I_1 = I_s \times \frac{R_2}{R_1 + R_2};\ R_1 = 5 + 10 = 15\Omega,\ R_2 = 30\Omega$$

$$I_1 = 45 \times \frac{30}{15 + 30} = 30 \text{ mA}$$

Again by current division

$$I_x = 30 \times \frac{30}{30 + 60} = 10 \text{ mA}$$

SUMMARY

- The equivalent resistance of N series resistors is $R_{eq} = R_1 + R_2 + ... + R_N$
- The equivalent resistance of N parallel resistors is $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + ... + \frac{1}{R_N}$
- Star/delta conversion of three resistors aids in reduction of passive resistor network.
- Before applying general methods of network analysis, if possible, the passive part of the network should be reduced by above three ways.
- The general methods of circuit analysis are the nodal method and mesh method. These have been well summarized in the text.
- Superposition is often used when the contribution of individual sources to the complete response of the circuit is required.
- Circuit model of a practical voltage source is an ideal voltage source with a series resistance.
- Circuit model of a practical current source is an ideal current source with a resistance in parallel to it.
- A practical voltage source can be converted to practical current source and vice versa. The same is not possible for ideal voltage and current sources.
- Inter-conversion of sources (some time repeatedly) greatly simplifies circuit analysis.
- The Thevenin equivalent of a network at two of its terminals is an independent voltage source with a resistance in series with it.
- The Norton equivalent of a network at two of its terminals is an independent current source with a resistance in parallel with it.
- At two terminals of a network $V_{TH} = V_{OC}$ and R_{TH} is the resistance seen from the terminals with all voltage in the network short-circuited and all current sources open-circuited. The dependent sources are left as they are.
- The Norton equivalent current is I_{SC}, the short-circuit current at the terminals. The Norton resistance $R_N = R_{TH}$. Also $I_{SC} = V_{TH} / R_{TH}$.
- $R_{TH} = \dfrac{V_{OC}}{I_{SC}}$
- Maximum power is transferred to a load at two of a network when $R_L = R_{TH}$.

REVIEW QUESTIONS

1. In the voltage dividing circuits of Fig. 2.4, investigate the effect of load R_L across R_2 on the voltage division ratio K.
2. In the current dividing circuit of Fig. 2.7, investigate the effect of connecting load resistance R_L across the current divider.
3. Define a mesh.

4. A node where only two elements meet does not require a nodal equation. Explain why?
5. Give one important basis for the choice between nodal and mesh analysis of a circuit.
6. Superposition theorem is applicable only for ________ circuits. (Fill in the blank).
7. As seen from two terminals of a circuit, how are V_{OC} and I_{SC} related to the Thevenin resistance R_{TH}?
8. Show that the Thevenin and Norton equivalents of a network have the same value of the resistance.
9. Explain how a practical voltage source is converted to a current source.
10. Explain how a practical current source is converted to a voltage source.
11. Prove the maximum power theorem.

PROBLEMS

2.1 For the resistive circuit of Fig. 2.71, using the method of series–parallel combination, find V_1 and I_2.

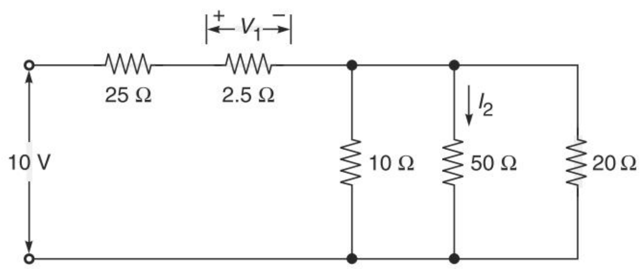


Fig. 2.71

2.2 In the circuit of Fig. 2.72, each resistance is 1Ω. Find the value of V_1.

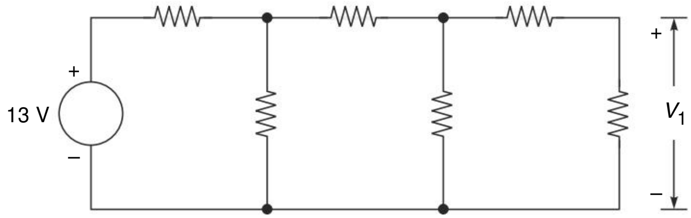


Fig. 2.72

2.3 In Fig. 2.73, find the value of R_{eq}.

2.4 For the circuit shown in Fig. P2.4, find the equivalent resistance between (i) *A* and *B* and (ii) *A* and *N*.

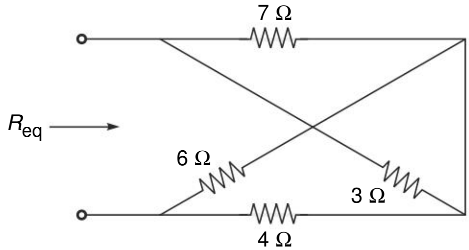


Fig. 2.73

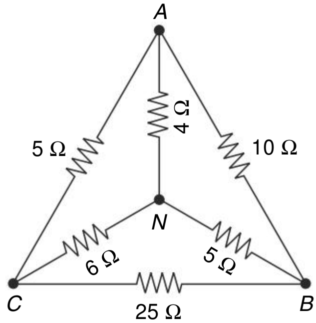


Fig. 2.74

2.5 For the circuit of Fig. 2.75, find the nodal voltages and the current through the 2Ω resistance.

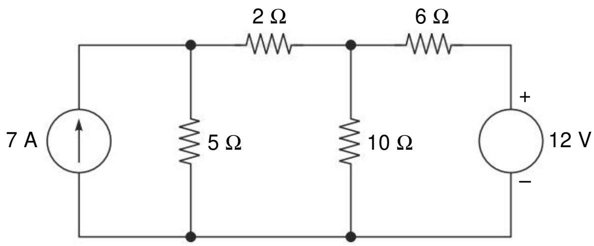


Fig. 2.75

2.6 For the circuit of Fig. 2.76, find the nodal voltages. From the symmetry of nodal equations, attempt to draw generalized conclusions.

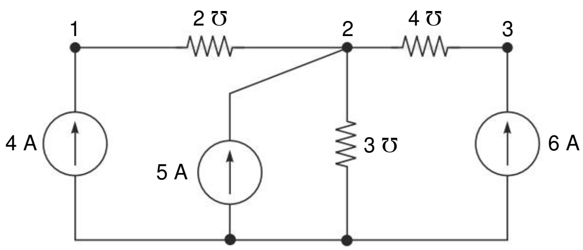


Fig. 2.76

2.7 In the circuit of Fig. 2.77, find the voltage V_1 across the 6Ω resistance using (i) nodal method, and (ii) mesh method of circuit analysis.

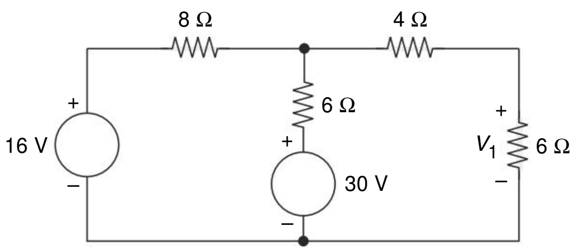


Fig. 2.77

2.8 In the bridge circuit of Fig. 2.78, find the current through 10Ω resistance across *BD*. It is suggested that you use mesh method of analysis.

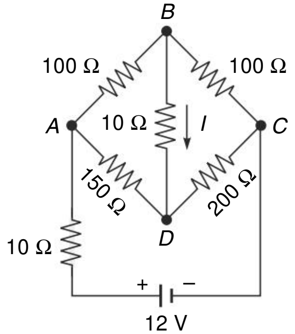


Fig. 2.78

2.9 For the circuit of Fig. 2.79, determine the currents in all the elements.

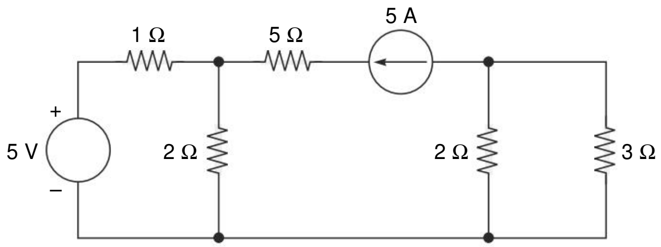


Fig. 2.79

2.10 For the circuit of Fig. 2.80, write the nodal equations in terms of node to datum voltages V_1 and V_2. Solve for V_1 and V_2. Hence, find:
(i) Direction and magnitude of current through 5Ω resistance.
(ii) Power output/input to the current and voltage sources.
Hint: Voltage source being ideal cannot be converted to current source.

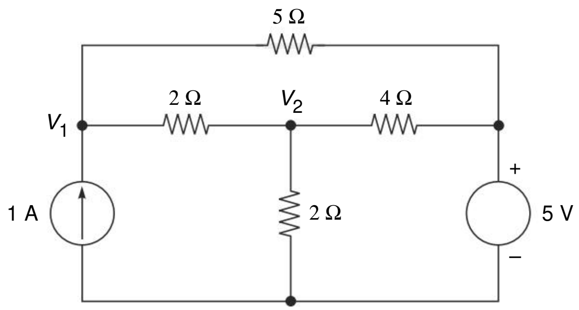


Fig. 2.80

2.11 Using mesh analysis, find currents I_1, I_2 and I_3 in the circuit of Fig. 2.81. Also find the power supplied by the two current sources.

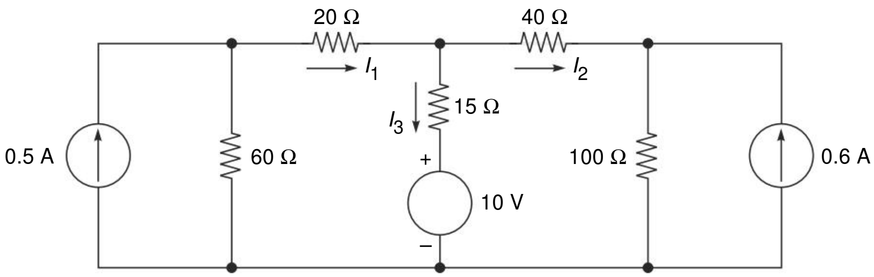


Fig. 2.81

2.12 Find the voltage across 10Ω resistance in Fig. 2.82 using (i) nodal method, and (ii) mesh method.

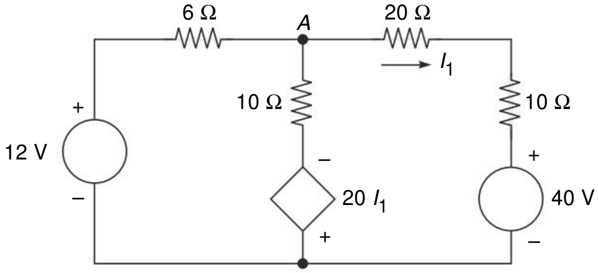


Fig. 2.82

2.13 Using nodal technique, determine I_1 in the circuit of Fig. 2.83.

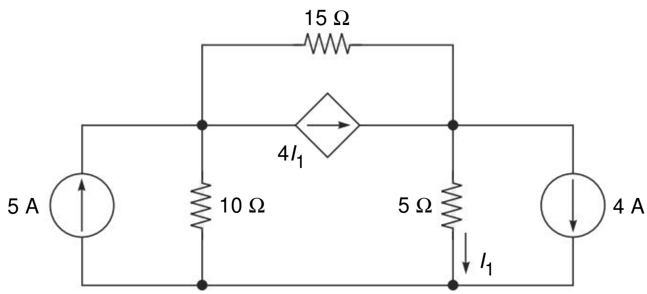


Fig. 2.83

2.14 The circuit of Fig. 2.84 represents a transistor amplifier. Find

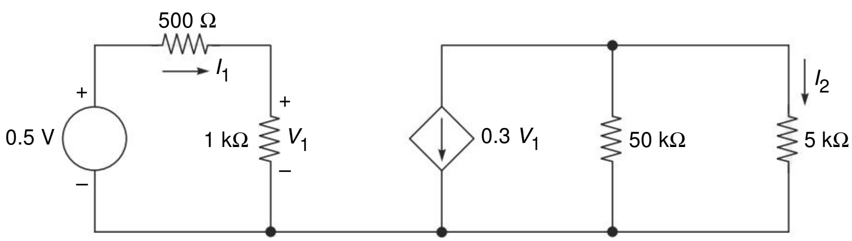


Fig. 2.84

(i) I_2 / I_1

(ii) Ratio of the power consumed by 5 kΩ resistance to the power supplied by 0.5 V source.

2.15 The circuit of Fig. 2.85 represents a transistor amplifier. Find the expression for

(i) current gain $A_I = \dfrac{I_2}{I_1}$

(ii) voltage gain $A_V = \dfrac{V_2}{V_1}$

(ii) input impedance $Z_i = \dfrac{V_1}{I_1}$

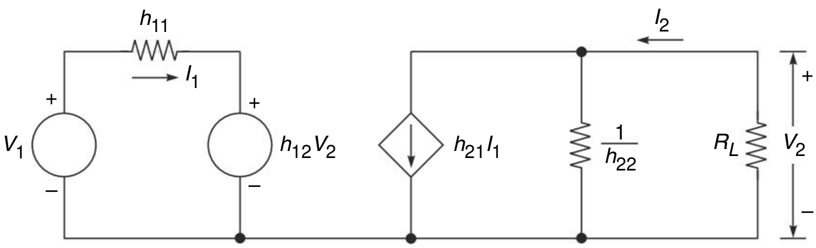


Fig. 2.85

2.16 In the circuit of Fig. 2.86

(i) $V_S = 16$ V, find I_S for $I = 0$.

(ii) $I_S = 16$ A, find V_S for $I = 0$.

Use the principle of superposition.

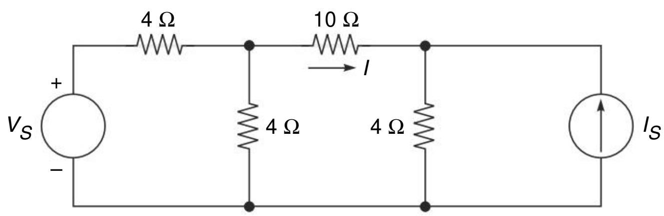


Fig. 2.86

2.17 For the circuit of Fig. 2.87, find V and I by using the principle of superposition.

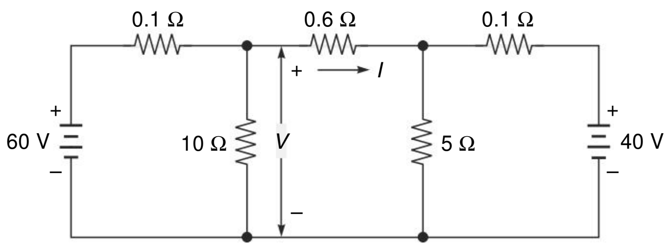


Fig. 2.87

2.18 For the bridge circuit shown in Fig. 2.88, find the galvanometer (G) current using Thevenin equivalent as seen at BD.

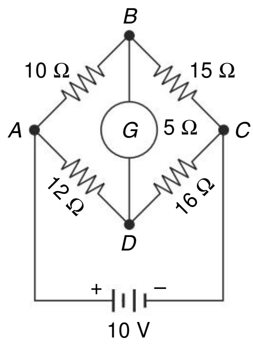


Fig. 2.88

2.19 In the circuit of Fig. 2.89, find the value of I to reduce the node–datum voltage V_1 to zero.

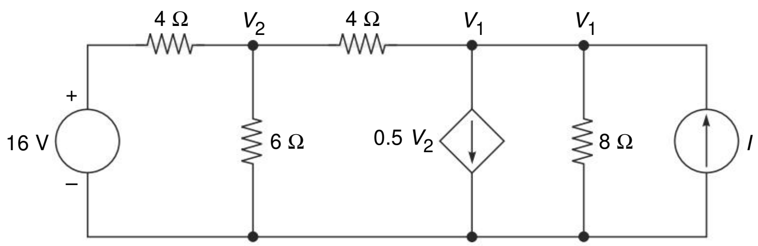


Fig. 2.89

2.20 For the circuit of Fig. 2.90, find the Thevenin equivalent as viewed by the resistance R. Find the value of R for maximum power dissipation in it and the value of this power.

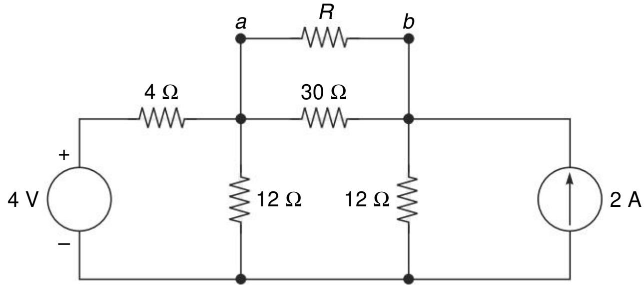


Fig. 2.90

2.21 Find the Thevenin equivalent of the circuit of Fig. 2.91 as seen at terminals XY.

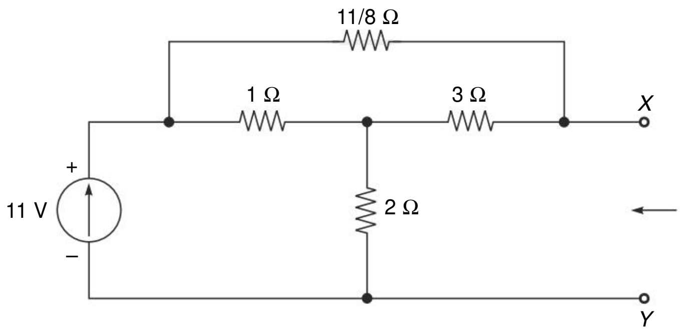


Fig. 2.91

2.22 In the circuit of Fig. 2.92, find the current I_1 using the principle of superposition.

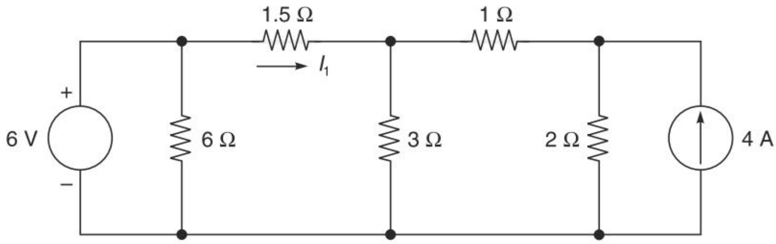


Fig. 2.92

2.23 Find the Thevenin equivalent of the circuit of Fig. 2.93 to the left of *XY*.

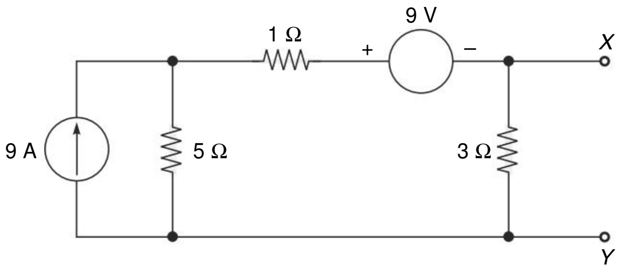


Fig. 2.93

Chapter 3

FUNDAMENTALS OF REACTIVE CIRCUITS

3.1 INTRODUCTION

In Chapter 2, generalized techniques of circuit analysis were discussed in detail but their application so far has been restricted to resistive circuits, because of which, describing equations were algebraic in nature. In this chapter, we shall introduce the analysis of simple circuits with storage elements (reactive circuits). The describing circuits equations as obtained by KCL/KVL now acquire integro–differential form. We will begin with source-free circuits first, with a single storage element (leading to single energy transient). The effect of suddenly applied forcing function (source) will then be considered and the idea of *natural* and *forced response* will be exposed. These techniques will then be extended to circuits with two storage elements (inductance and capacitance) leading to double energy transients. The natural response of *RLC* circuits will be investigated in detail.

Sinusoidal forcing function response will be the subject matter of Chapter 4.

3.2 INDUCTANCE AND CAPACITANCE COMBINATIONS

In Section 2.2, we considered series/parallel combination of resistances. Here we shall see how to combine series/parallel inductances and capacitances.

Series Inductances

Figure 3.1 shows two inductances L_1 and L_2 in series. As per KVL

$$v_s = v_1 + v_2$$

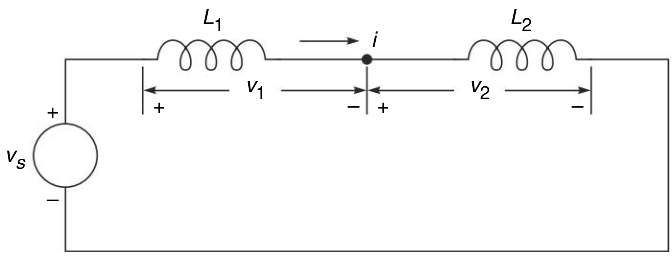


Fig. 3.1 Inductances in series

Using Eq. (1.33)

$$v_s = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

or

$$= (L_1 + L_2) \frac{di}{dt} = L_{eq} \frac{di}{dt}$$

$\therefore$

$$L_{eq} = L_1 + L_2 \tag{3.1}$$

or in general

$$L_{eq} = \sum_i L_i \tag{3.2}$$

Parallel Inductances

Now consider two inductances in parallel as in Fig. 3.2. As per KCL

$$i_s = i_1 + i_2$$

Using Eq. (1.34)

$$i_s = \frac{1}{L_1}\int_{t_0}^{t} v\, dt + i_1(t_0) + \frac{1}{L_2}\int_{t_0}^{t} v_2 dt + i_2(t_0)$$

$$= \left(\frac{1}{L_1} + \frac{1}{L_2}\right)\int_{t_0}^{t} v\ dt + i_1(t_0) + i_2(t_0)$$

$$= \frac{1}{L_{eq}}\int_{t_0}^{t} v\ dt + i_s(t_0)$$

$\therefore$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} \tag{3.3}$$

or

$$L_{eq} = \frac{L_1 L_2}{L_1 + L_2} \tag{3.4}$$

In general

or

$$\frac{1}{L_{eq}} = \sum_i \frac{1}{L_i} \tag{3.5}$$

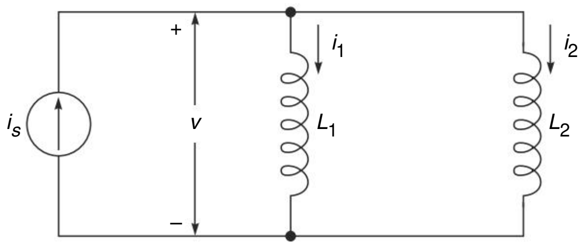


Fig. 3.2 Inductances in parallel

Capacitances in Parallel

Applying KCL to Fig. 3.3

$$i_s = i_1 + i_2$$

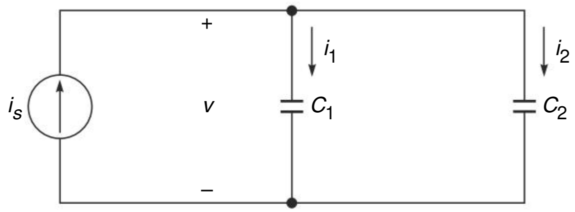


Fig. 3.3 Capacitances in parallel

Using Eq. (1.27)

$$i_s = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$

$$= (C_1 + C_2) \frac{dv}{dt} + C_{eq} \frac{dv}{dt}$$

$\therefore$
$$C_{eq} = C_1 + C_2 \tag{3.6}$$

or in general

$$C_{eq} = \sum_i C_i \tag{3.7}$$

Observe that capacitances in parallel combine in the same way as inductances in series.

Capacitances in Series

Applying KVL to Fig. 3.4

$$v_s = v_1 + v_2$$

As per Eq. (1.28)

$$v_s = \frac{1}{C_1}\int_{t_0}^{t} i\,dt + v_1(t_0) + \frac{1}{C_2}\int_{t_0}^{t} i\,dt + v_2(t_0)$$

$$= \left(\frac{1}{C_1} + \frac{1}{C_2}\right)\int_{t_0}^{t} i\,dt + v_1(t_0) + v_2(t_0)$$

$$= \frac{1}{C_{eq}}\int_{t_0}^{1} i\,dt + v_s(t_0)$$

$\therefore$
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \tag{3.8}$$

or
$$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} \tag{3.9}$$

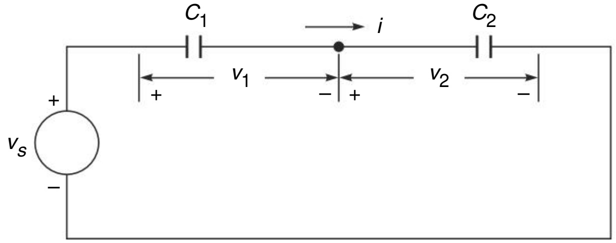


Fig. 3.4 Capacitances in series

In general

$$\frac{1}{C_{eq}} = \sum_i \frac{1}{C_i} \tag{3.10}$$

Observe that capacitances in series combine in the same way as inductances in parallel.

Using the above result, a circuit with series/parallel inductances and capacitances can be easily reduced to a single inductance and a single capacitance.

Example 3.1 For the capacitor circuit of Fig. 3.5 determine C_{eq} if each capacitor is 10 μF.

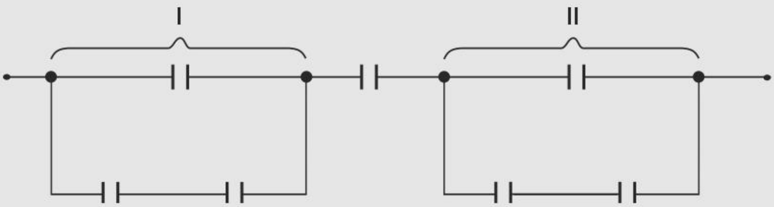


Fig. 3.5

Solution

For Part I of the circuit

$$C_{eq}(\text{I}) = 10 + \left(\frac{10 \times 10}{10 + 10}\right) = 15\ \mu\text{F}$$

Similarly

$$C_{eq}(\text{II}) = 15\ \mu\text{F}$$

The circuit at this stage is drawn in Fig. 3.5(a)

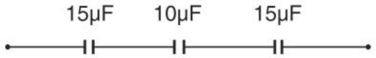


Fig. 3.5(a)

$$\frac{1}{C_{eq}} = \frac{1}{15} + \frac{1}{10} + \frac{1}{15} = \frac{2+3+2}{30}$$

$$C_{eq} = (30/7)\ \mu\text{F}$$

Example 3.2 Find L_{eq} for the inductor circuit of Fig. 3.6.

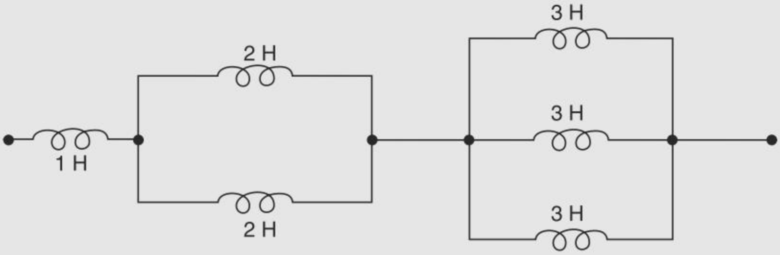


Fig. 3.6

Solution

$$2\text{H} \parallel 2\text{H} = \frac{2 \times 2}{2+2} = 1\text{H}$$

$$3\text{H} \parallel 3\text{H} \parallel 3\text{H} = \frac{1}{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = 1\text{H}$$

Thus

$$L_{eq} = 1 + 1 + 1 = 3\text{ H}$$

Example 3.3 For the capacitor circuit of Fig. 3.7 determine

(a) the charge on each capacitor

(b) the stored energy in each capacitor and total stored energy

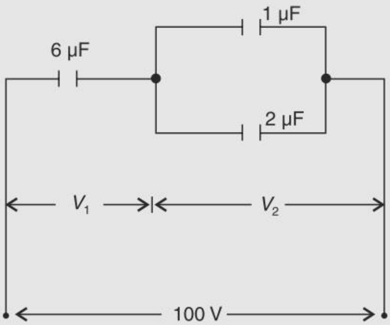


Fig. 3.7

Solution

(a) Combining the two parallel capacitors

$$C_P = 1 + 2 = 3\ \mu\text{F}$$

The circuit is drawn in Fig. 3.7(a)

In series capacitors at any instant the charge is same as

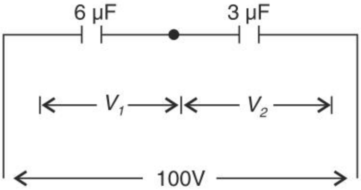


Fig. 3.7(a)

$$q = \int_0^t i\,d(t) \text{ with zero initial charge}$$

$$V_1 = \frac{q}{6 \times 10^{-6}}, \qquad V_2 = \frac{q}{3 \times 10^{-6}}$$

$$V_1 + V_2 = q\left[\frac{1}{6 \times 10^{-6}} + \frac{1}{3 \times 10^{-6}}\right] = 100$$

$$q \times 10^6 \left(\frac{1}{6} + \frac{1}{3}\right) = 100$$

$$\frac{1}{2}\ q \times 10^6 = 100$$

$$q = 200 \times 10^{-6}\text{ C} = 200\ \mu\text{C}$$

Charge on 6 μF, $= q = 200 \times 10^{-6}$ C = 200 μC

$$V_2 = \frac{200 \times 10^{-6}}{3 \times 10^{-6}} = \frac{200}{3}\text{V}, \qquad V_1 = 100 - \frac{200}{3} = \frac{100}{3}\text{V}$$

$$q(1\ \mu\ F) = 1\times10^{-6}\times\frac{200}{3} = \frac{200}{3}\ \mu C \qquad \text{(i)}$$

$$q(2\ \mu\ F) = 2\times10^{-6}\times\frac{200}{3} = \frac{400}{3}\ \mu C \qquad \text{(ii)}$$

$$q(6\ \mu\ F) = 200\ \mu C \qquad \text{(iii)}$$

Stored energy is given by

$$w_C = \frac{1}{2}\ CV^2$$

$$V = \frac{q}{C}$$

$$\therefore \qquad w_C = \frac{1}{2}C\left(\frac{q}{C}\right)^2 = \frac{1}{2}\left(\frac{q^2}{C}\right)$$

1 μ F

$$w_{C1} = \frac{1}{2}\frac{\left(\frac{200}{3}\times10^{-6}\right)^2}{1\times10^{-6}} = 2.22\ \text{mJ}$$

2 μ F

$$w_{C2} = \frac{1}{2}\frac{\left(\frac{400}{3}\times10^{-6}\right)^2}{2\times10^{-6}} = 4.44\ \text{mJ}$$

4 μ F

$$w_{C3} = \frac{1}{2}\frac{\left(\frac{400}{3}\times10^{-6}\right)^2}{2\times10^{-6}} = 4.44\ \text{mJ}$$

Example 3.4 Two inductors when connected in series have an inductance of 4H and 3/4H when connected in parallel. Determine the inductance of each inductor.

Solution

In series

$$L_1 + L_2 = 4\ \text{H} \qquad \text{(i)}$$

In parallel

$$\frac{L_1 L_2}{L_1 + L_2} = \frac{3}{4}\ \text{H} \qquad \text{(ii)}$$

Multiplying Eqs. (i) and (ii)

$$L_1\ L_2 = 3$$

or $\qquad L_2 = \dfrac{3}{L_1}$

Substituting in Eq. (i)

$$L_1 + \frac{3}{L_1} = 4$$

$$L^2_1 - 4L_1 + 3 = 0$$

Solving

$$L_1 = \frac{4 \pm \sqrt{16-12}}{2} = \frac{4 \pm 2}{2} = 3, 1$$

$$L_2 = \frac{3}{L_1} = 1 \text{ or } 3$$

Thus, one inductor is 3 H and other is 1 H.

Example 3.5 Find C_{eq} for the capacitor circuit of Fig. 3.8

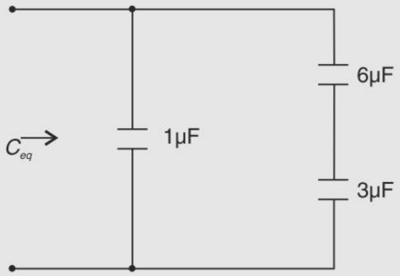


Fig. 3.8

Solution

Capacitance of the two series capacitors

$$\frac{1}{C_{se}} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

or

$$C_{se} = 2\ \mu\text{F}$$

This is in parallel with 1 μF . Thus

$$C_{eq} = 1 + 2 = 3\ \mu\text{F}$$

Example 3.6 There inductors are connected in series-parallel as shown in Fig. 3.9.

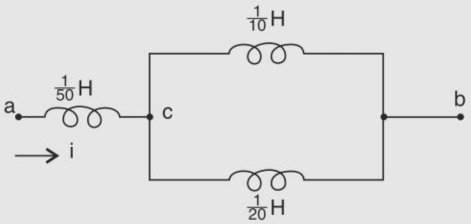


Fig. 3.9

(a) Calculate L_{eq} between terminals '*ab*'

(b) If the current *i* is changing at the rate of 150 A/s, what is the voltage across terminal *bc*.

Solution

(a) Inductance between *b* and *c* (two inductors in parallel)

$$\frac{1}{L_{bc}} = 10 + 20 = 30$$

or

$$L_{bc} = \frac{1}{30}\ \text{H}$$

L_{bc} is in series with L_{ac}. Therefore

$$L_{eq} = \frac{1}{50} + \frac{1}{30} = \frac{3+5}{150} = \frac{8}{150}\text{H}$$

Given (b), $\frac{di}{dt} = 150$ A/s

then

$$V_{bc} = L_{bc}\frac{di}{dt} = \frac{1}{30}\times 150 = 5\text{V}$$

Example 3.7 Three capacitors of capacitance 10, 20 and 40 μF are connected in series across 280 V.

(a) Calculate the equivalent capacitance,

(b) the charge of each capacitor, and

(c) the voltage across each capacitor.

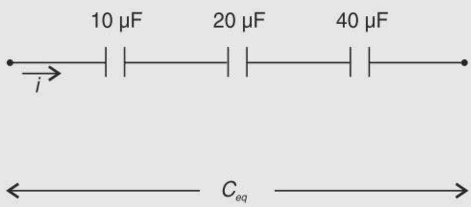


Fig. 3.10

Solution

(a) $$\frac{1}{C_{eq}} = \frac{1}{10} + \frac{1}{20} + \frac{1}{40} = \frac{4+2+1}{40}$$

$C_{eq} = (40 / 7)$; μF

(b) In charging the series capacitors, the charging current is same. Therefore at any instant they have the same charge q on each capacitor.

$$\text{Total charge} = 3q = C_{eq}\,V = \frac{40}{7}\times 10^{-6}\times 280$$

$$q = 1600\times 10^{-6} = 1.6\times 10^{-3}\ \text{C}$$

(c) $$V\,(10\ \mu\text{F}) = \frac{q}{C_1} = \frac{1.6\times 10^{-3}}{10\times 10^{-6}} = 160\ \text{V}$$

$$V\,(20\ \mu\text{F}) = \frac{q}{C_2} = \frac{1.6\times 10^{-3}}{20\times 10^{-6}} = 80\ \text{V}$$

$$V\,(40\ \mu\text{F}) = \frac{q}{C_3} = \frac{1.6\times 10^{-3}}{40\times 10^{-6}} = 40\ \text{V}$$

$$V\,(\text{total}) = 160 + 80 + 40 = 280\ \text{V}$$

Example 3.8 Three capacitors of 10 μF, 20 μF and 40 μF are connected in parallel across 200 V.

(a) Calculate equivalent capacitor.

(b) What is the charge on each capacitor and the total charge?

Solution

(a) For parallel capacitor

$$C_{eq} = 10 + 20 + 40 = 70 \ \mu\text{F}$$

(b) Voltage across each capacitor is 200 V

$$q = \text{C V}$$

$$q_1 = 10 \times 10^{-6} \times 200 = 2 \times 10^{-3} \text{ C}$$

$$q_2 = 20 \times 10^{-6} \times 200 = 4 \times 10^{-3} \text{ C}$$

$$q_3 = 40 \times 10^{-6} \times 200 = 8 \times 10^{-3} \text{ C}$$

Total charge

$$q = q_1 + q_1 + q_3 = (2 + 4 + 8) \times 10^{-3} \text{ C}$$

or

$$q = 14 \times 10^{-3} \text{C}$$

3.3 SOURCE-FREE RL AND RC CIRCUITS

Initial Conditions

Initial (or final) conditions must be known to evaluate the arbitrary constants present in the general solution of differential equations. It also tells us the behaviour of the elements at the instant of switching. Conditions existing, at the instant network equilibrium is changed by switching action, are known as *initial conditions*. This is the *initial state*. Sometimes we may use conditions at $t = \infty$; these are called *final conditions*.

At the reference time, $t = 0$, one or more switches operate. Normally it is assumed that switches act in zero time. To differentiate between the time immediately before and immediately after the operation of a switch, we will use – and + signs. Thus, conditions existing just prior to the operation of a switch will be defined as $i(0^-)$, $v(0^-)$, etc.; and conditions after as $i(0^+)$, $v(0^+)$, etc.

Initial conditions in an electric circuit depend on the state of the network prior to $t = 0^+$, and the network structure at $t = 0^+$, after switching. The evaluation of all voltages and currents and their derivatives at $t = 0^+$ is the evaluation of initial conditions.

Simple *RL* Circuit

Consider the simple *RL* circuit of Fig. 3.11. We shall assume that the value of $i(t)$ at $t = 0$ is prescribed by I_0. This fact is known as an *initial condition* which implies that the inductance has an initial energy storage of (1/2) $L I^2_0$. As per KVL

$$v_R + v_L = R_i + L\frac{di}{dt} = 0$$

or $$\frac{di}{dt} + \frac{R}{L}i = 0 \tag{3.11}$$

which has to be solved to obtain the expression of $i(t)$ such that it satisfies the condition $i(0) = I_0$.

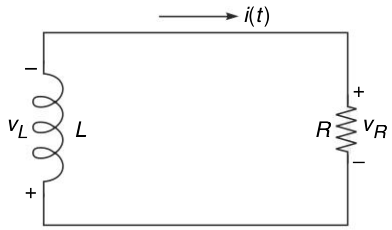


Fig. 3.11 Simple *RL* circuit (source free)

It is immediately recognized that Eq. (3.11) is a homogeneous (no forcing function (source free)) linear differential equation with constant coefficients. It can be solved in various ways. Its solution ($i(t)$) is mathematically known as the *complimentary function*. Observing that upon differentiation and integration the exponential form of $e^{s_1 t}$ is preserved, we assume a solution of the general form

$$i(t) = Ae^{s_1 t} \tag{3.12}$$

where A and s_1 are constants to be determined. Plugging the assumed solution (Eq. (3.12)) into the differential Eq. (3.11), we have

$$A\,s_1\,e^{s_1 t} + \frac{R}{L}Ae^{s_1 t} = 0$$

or $$\left(s_1 + \frac{R}{L}\right)Ae^{s_1 t} = 0 \tag{3.13}$$

Assuming $A = 0$ or $s_1 = -\infty$ leads to a trivial solution (response is zero). A realistic solution is given by

$$s_1 + \frac{R}{L} = 0 \tag{3.14}$$

or $$s_1 = -\frac{R}{L} \tag{3.15}$$

Hence the solution is

$$i(t) = Ae^{-(R/L)t} \tag{3.16}$$

We still need to determine that constant A which can be found by using the initial condition, i.e. at $t = 0$, $i(0) = I_0$. Putting $t = 0$ in Eq. (3.16)

$$A = I_0$$

Hence the final solution is

$$i(t) = I_0 e^{-(R/L)t} \tag{3.17}$$

We shall denote

$$\frac{L}{R} = \tau = \textit{time constant;}\ \text{units } s \tag{3.18}$$

whose significance will soon be clear. Thus, the response (current) can be written as

$$i(t) = I_0 e^{-t/\tau};\ t > 0 \tag{3.19}$$

Properties of Exponential Response

It is seen from Eq. (3.19) that the response $i(t)$ is a decaying exponential as plotted in Fig. 3.12. At $t = \tau$ (one time constant), the value of $i(t)$ reduces to

$$i(\tau) = I_0 e^{-1} = 0.368\, I_0$$

i.e. 36.8% of the initial value. At $t = 5\, t$

$$i(5\tau) = I_0 e^{-5} = 0.0067\, I_0 = 0.67\% \text{ of } I_0$$

i.e. $i(t)$ is practically reduced to zero value (theoretically $i(t)$ vanishes at $t = \infty$).

Consider now the initial rate of decay of $i(t)$. From Eq. (3.19)

$$\frac{di}{dt} = -\frac{I_0}{\tau}\, e^{-t}/\tau$$

or

$$\frac{di}{dt}(0) = -\frac{I_0}{\tau} \tag{3.20}$$

i.e. the initial normalized slope ($di/dt\,(0)/I_0$), is $-1/\tau$. At this initial rate the response would fully decay in time τ (dotted line in Fig. 3.12).

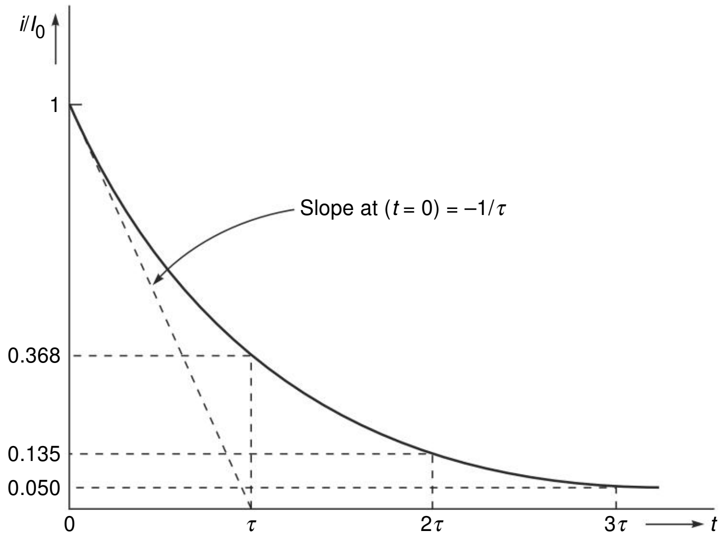


Fig. 3.12 Current response of a simple *RL* circuit excited by initial condition (source free)

We conclude from the above that the rate of decay of current in *RL* circuit, initial condition excited, is wholly governed by τ, the time constant: larger time constant means slower decay and vice versa.

The source free response of a circuit is known as its *natural response*. It gets excited by the initial condition. A forcing function will also excite the natural response of a circuit but, as we shall see later, it also gives rise to another response term known as *forced response*, which must have the same functional form as the forcing function.

Equation (3.14), which is called the *characteristic equation,* has the solution $|s_1| = R/L = 1/\tau$ (dimension s^{-1}), also known as *natural frequency*. Alternatively, it is the natural frequency that determines the circuit's natural response.

The current flowing in the *RL* circuit of Fig. 3.5 causes energy dissipation in the resistance. At $t = \infty$ when $i(t)$ reduces to zero, all the initial inductance energy $(1/2)\ LI_0^2$ would have been dissipated in the resistance (see Problem 1.6).

Simple RC Circuit

Consider now the simple *RC* circuit of Fig. 3.13 wherein the capacitor is charged to a voltage V_0 at $t = 0$. Applying KCL to the circuit node

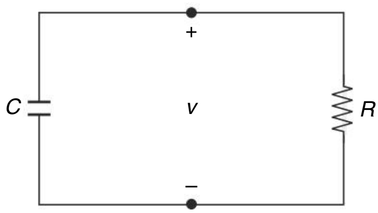


Fig. 3.13 Simple *RC* circuit (source free)

$$C\frac{dv}{dt} + \frac{v}{R} = 0$$

or

$$\frac{dv}{dt} + \frac{v}{RC} = 0 \tag{3.21}$$

Equation (3.21) is identical in form to Eq. (3.11) of the simple *RL* circuit. Hence its solution (natural response) must be given by

$$v(t) = V_0 e^{-t/\tau};\ t > 0 \tag{3.22}$$

where $\tau = RC$ = *time constant;* units, *s*

$|s_1| = 1/RC$ = *natural frequency;* dimension s^{-1}

In this case the capacitor voltage reduces to 36.8% of its initial value in one time constant. All other properties of the time constant also apply. At $t = \infty$ all the initial stored energy of the capacitor $(1/2)\ CV_0^2$ would have been dissipated in the resistance.

Determination of Time Constant of RL/RC Circuits

Source Free Circuits Separately combine all resistances and all inductances (capacitances in *RC* circuits) to yield R_{eq} and L_{eq} (or C_{eq}), thus reducing the circuit to the simple form of Figs. 3.11 (*RL*) and Fig. 3.13 (*RC*). The time constant is then given by

$$\tau = \frac{L_{eq}}{R_{eq}};\ RL \text{ circuit}$$

and

$$\tau = R_{eq} C_{eq};\ RC \text{ circuit}$$

Sources Present Short-circuit all voltage sources and open-circuit all current sources and then proceed as in source free circuit.

The above procedure is now illustrated by examples.

Example 3.9 Find the time constant of the circuits of Figs. 3.14(a) and (b).

Solution Open-circuiting current sources and short-circuiting voltage sources, the circuits become as in Figs. 3.15(a), (b). Combining resistances we get:

(a) $R_{eq} = \dfrac{6 \times 12}{(6+12)} + 4 = 8\ \Omega$ (Thevenin resistance between nodes where inductance is connected)

$$\tau = \frac{L}{R_{eq}} = \frac{2}{8} = \frac{1}{4}\ \text{s}$$

(b) $R_{eq} = 2 + \dfrac{6 \times 12}{(6+12)} + 2 = 8\ \Omega$ (Thevenin resistance between nodes where capacitance is connected).

$$\tau = R_{eq}\,C = 8 \times \frac{1}{2} = 4\ \text{s}$$

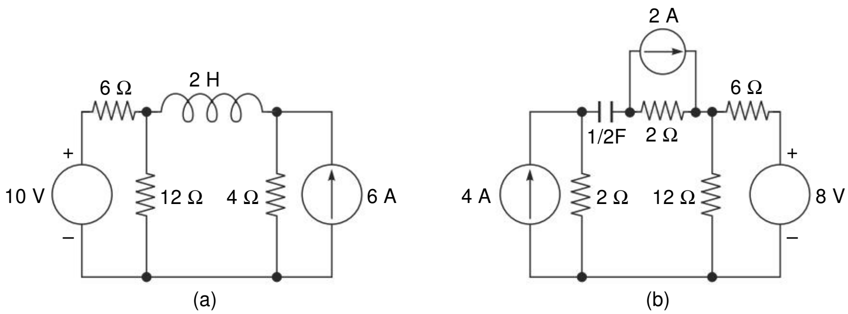


Fig. 3.14

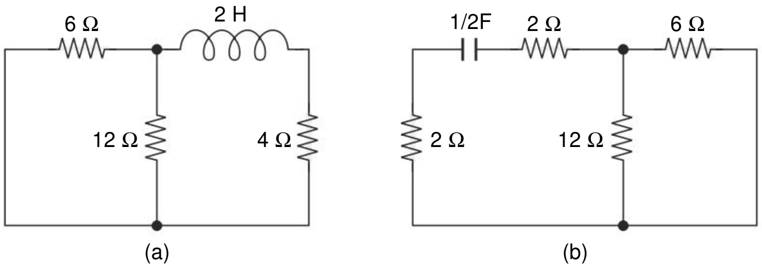


Fig. 3.15

3.4 UNIT STEP FORCING FUNCTION

The unit step is a *singularity function* and represents sudden change or discontinuity (occurring in zero time). It is functionally defined as

$$\begin{aligned} u(t) &= 0 \quad t < 0 \\ &= 1 \quad t > 0 \end{aligned} \tag{3.23}$$

and is graphically represented as in Fig. 3.16. At $t = 0$, $u(t)$ is undefined, and implies

$u(0^-) = 0$ and $u(0^+) = 1$, whereas $t = 0^-$ defines time just before zero and 0^+ as just after zero. A mathematical idealization, $u(t)$ represents closely certain switching operations.

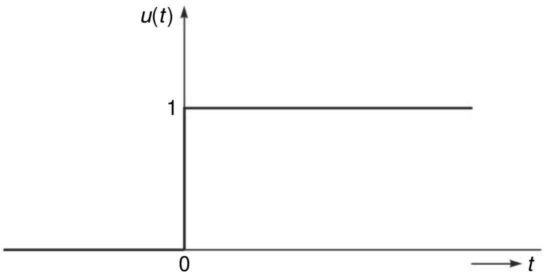


Fig. 3.16 Unit step function

The unit step translated (delayed) in time is

$$u(t - t_0) = 0; \; t < t_0$$
$$\text{and} \quad = 1; \; t > t_0 \tag{3.24}$$

This is graphically shown in Fig. 3.17.

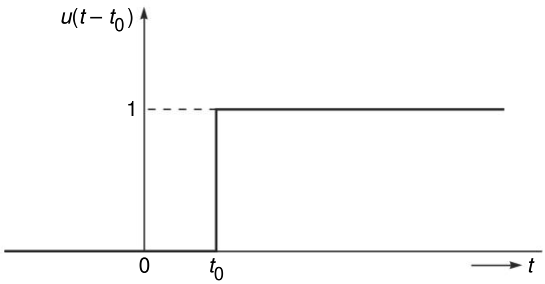


Fig. 3.17 Unit step delayed by time t_0

Step Voltage and Current

$Vu(t - t_0)$ represents a network switching operation as in Fig. 3.18(a). Alternatively, it can be represented as in Fig. 3.18(b) provided initial conditions do not get disturbed. The representation of Fig. 3.18(b) will be commonly used.

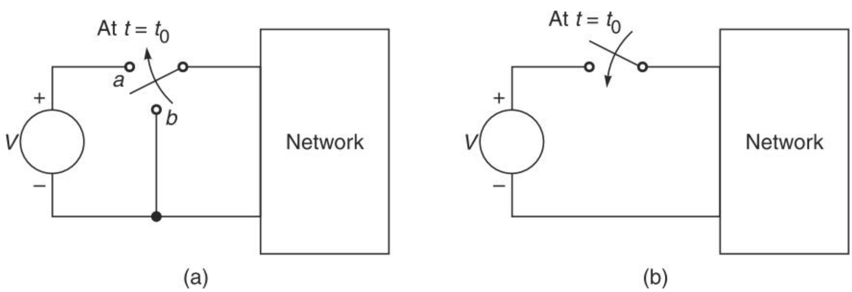


Fig. 3.18 $Vu(t - t_0)$ operation
(a) Exact equivalent, and
(b) Nonexact equivalent

Network representation of a step current $Iu\,(t - t_0)$ is shown in Fig. 3.19.

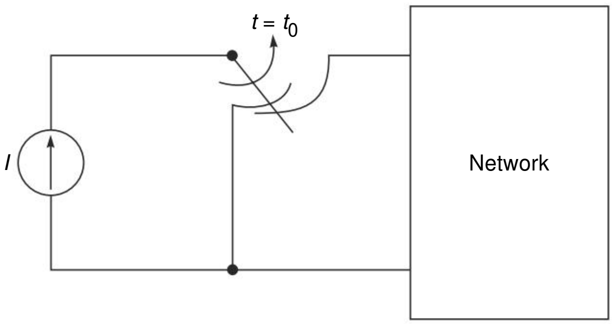


Fig. 3.19 $Iu(t - t_0)$ operation

3.5 STEP RESPONSE OF *RL* / *RC* CIRCUITS

Inductance and Capacitance Behaviour at $t = 0^+$

Inductance

$$i_L = \int_0^t v_L \, dt + i(0)$$

Inductance current, therefore, cannot change suddenly (see Sec. 1.6) because it would require infinite voltage. Thus at $t = 0^+$, inductance acts as an ideal current source of strength $i_L(0^+) = i_L(0^-)$. If $i_L(0^-) = 0$, the inductance acts as an open-circuit. This is illustrated in Fig. 3.20.

Capacitance

$$v_C = \int_0^t i \;\, \mathrm{d}t + v_c(0)$$

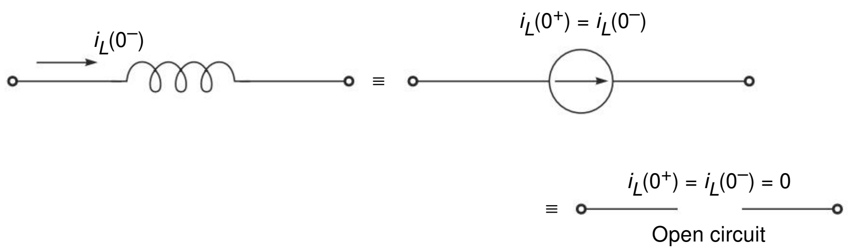


Fig. 3.20 Inductance behaviour at $t = 0^+$

Capacitance voltage, therefore, cannot change suddenly (Sec. 1.6) because of which it acts as an ideal voltage source of strength $v_C(0^+) = v_C(0^-)$. If $v_C(0^-) = 0$ it would act as a short-circuit. This is illustrated in Fig. 3.21.

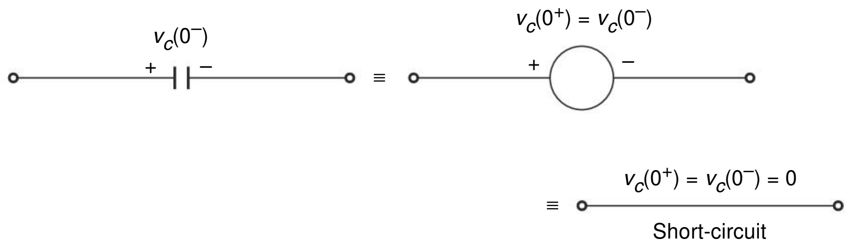


Fig. 3.21

Inductance and Capacitance Behaviour at $t = \infty$ (Steady State) for dc Excitation

Induction Under steady state; the inductance current reaches a constant value I_L. The inductance voltage is then

$$v_L = \frac{d\,I_L}{d\,t} = 0$$

Therefore, the inductance acts as short-circuit.

Capacitance Under steady-state, capacitance voltage reaches a constant value V_C. The capacitance current is then

$$i_C = C\frac{dV_C}{dt} = 0$$

Therefore, the capacitance acts as an open-circuit.

It is immediately concluded that steady state inductance current and capacitance voltage are determined by the resistive circuit after all inductances have been short-circuited and capacitances open-circuited.

Step Voltage Response of *RL* Series Circuit

Consider the *RL* series circuit of Fig. 3.22. Just before the application of voltage (step) the circuit history is represented by the inductor current $i_L(0^-)$ which in this circuit is zero because the circuit is open before the switching-on operation.

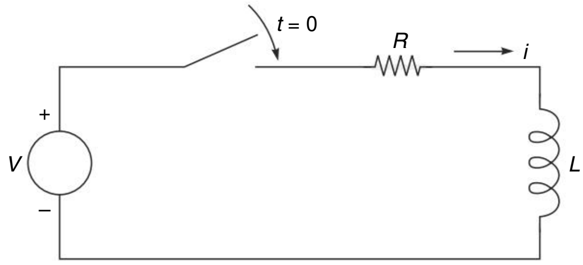


Fig. 3.22

The KVL equation of the circuit is

$$R_i + L\frac{\mathrm{d}i}{\mathrm{d}t} = V; \quad t > 0 \tag{3.25}$$

This is a *nonhomogeneous linear* differential (of first order) with the excitation term appearing on its right hand side. The solution to Eq. (3.25) will have two components, viz. *complimentary function* (natural response, i_n) which should satisfy the equation

$$\frac{\mathrm{d}i}{\mathrm{d}t} + \frac{R}{L}i = 0 \tag{3.26}$$

and the *particular integral* (forced response, i_f) which should satisfy Eq. (3.25). Thus, the complete solution (response) is

$$i = i_n + i_f \tag{3.27}$$

We already know that the natural response is (Eq. (3.16))

$$i_n = Ae^{-(R/L)\,t} = \mathrm{A}e^{-t/\tau} \tag{3.28}$$

Let us now discover the forced response (particular integral of Eq. (3.25)). Since the excitation is constant it is intuitively expected that $i_f = I$ (a constant) would satisfy Eq. (3.25). It leads to

$$RI + L\frac{\mathrm{d}I}{\mathrm{d}t} = \mathrm{V}$$

But

$$L\frac{\mathrm{d}I}{\mathrm{d}t} = 0$$

∴

$$I = \frac{V}{R} = i_f \tag{3.29}$$

The complete response is then

$$i(t) = Ae^{-(R/L)t} + \frac{V}{R}; \quad t > 0 \tag{3.30}$$

in which we must determine the arbitrary constant A such that it satisfies the initial condition on inductance current. As already stated above

$$i(0^+) = i(0^-) = 0$$

Substituting in Eq. (3.30)

$$0 = A + \frac{V}{R}$$

or

$$A = -\frac{V}{R}$$

Hence

$$i(t) = \frac{V}{R}\left(1 - e^{-(R/L)t}\right); \quad t > 0 \tag{3.31a}$$

$$= \frac{V}{R}\left(1 - e^{-t/\tau}\right); \quad t > 0 \tag{3.31b}$$

The response is plotted in Fig. 3.17 where the current rises exponentially from 0 to $I = V/R$ in accordance with the time constant (or natural frequency). It is noticed that the initial (at $t = 0^+$) rate of rise of current is I/τ and the current reaches a value of 63.2% of final value ($I = V/R$) in time of one time constant.

Inductance behaviour at $t = 0^+$ and $t = \infty$ stated earlier is confirmed by the response of Eq. (3.31) from which it follows that

$$i(0^+) = 0$$

i.e. the inductance acts as an open-circuit at $t = 0^+$,

and

$$i(\infty) = \frac{V}{R}$$

i.e. inductance acts as a short-circuit at $t = \infty$ (for dc excitation).

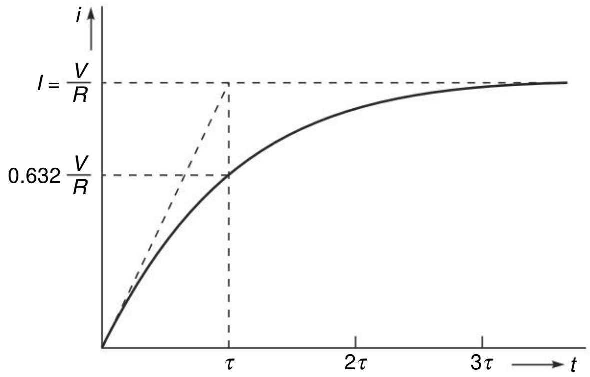


Fig. 3.23

Consider now the general case with **non-zero initial condition**. Then

$$i\,(0^+) = i\,(0^-), \text{ non-zero}$$

From Eq (3.30) at $t = 0^+$, we find

$$i(0^+) = A + \frac{V}{R} \text{ or } A = i(0^+) - \frac{V}{R}$$

Substituting in Eq (3.30), we have the solution

$$i\,(t) = \left[i(0^+) - \frac{V}{R}\right]e^{-t/\tau} + \frac{V}{R};\ t > 0 \tag{3.31c}$$

We have already shown that

$$i\,(\infty) = \frac{V}{R}$$

Then Eq. (3.31c) is written in the **general form**

$$i\,(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau},\ t > 0 \tag{3.31d}$$

In the **general functional form**, we replace $i(t)$ by $f(t)$. The complete response is

$$f\,(t) = f\,(\infty) + [f(0^+) - f\,(\infty)]e^{-t/\tau},\ t > 0 \tag{3.31e}$$

From Eq. (3.31d) at $t = 0^+$

$$i\,(t = 0^+) = i\,(0^+)$$

which mean that inductance acts as a current source

At $t = \infty$

$$i(t = \infty) = \frac{V}{R}$$

which means that inductance acts as short-circuit in steady-state.

Example 3.10 For the circuit shown in Fig. 3.24, find $i(t)$ after the switch is closed at $t = 0$.

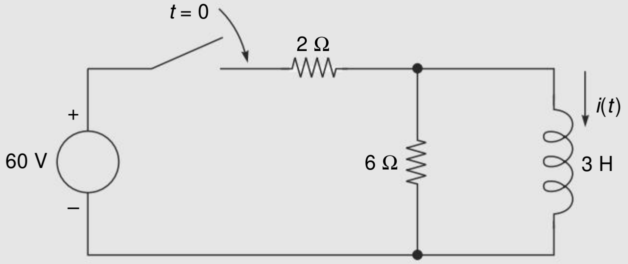


Fig. 3.24

Solution

Natural Response After the switch is closed, short-circuiting the voltage source yields the circuit of Fig. 3.25 from which

$$\tau = L/R_{eq} = 3/1.5 = 2 \text{ s}$$

$$i_n = Ae^{-t/2} \quad \text{(i)}$$

Forced Response With switch closed and $t \to \infty$, the inductance behaves as a short-circuit. The resultant circuit is shown in Fig. 3.26 from which it follows that

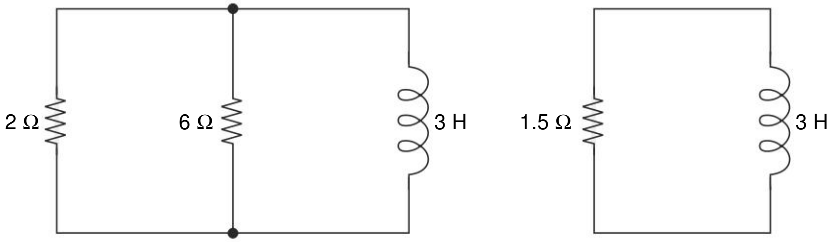


Fig. 3.25

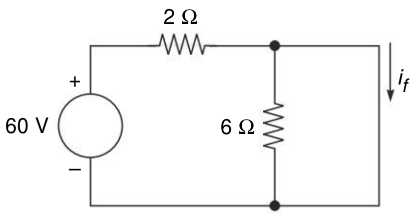


Fig. 3.26

$$i_f = \frac{60}{2} = 30 \text{ A} \quad \text{(ii)}$$

Combining

$$i = i_n + i_f$$

$$i(t) = Ae^{-t/2} + 30; \quad t > 0 \quad \text{(iii)}$$

Initial Condition before closing the switch $i(0^-) = 0$

$\therefore$ $$i(0^+) = i(0^-) = 0$$

Substituting in Eq. (iii)

$$0 = A + 30 \quad \text{or} \quad A = -30$$

Hence

$$i(t) = 30(1 - e^{-t/2}); \quad t > 0 \quad \text{(iv)}$$

$$= 30(1 - e^{-t/2})\, u(t) \quad \text{(v)}$$

Example 3.11 In the circuit of Fig. 3.21, the switch S has been in position '1' for a long time. It is thrown to position '2' at $t = 0$.

(a) Find $i(t)$ for $t > 0$

(b) Find $v_L(0^-)$, $v_L(0^+)$ and $\frac{di}{dt}(0^+)$.

Solution In position '1', steady state has been reached; inductance acts as a short. Then

$$i(0^-) = \frac{24}{24} = 1 \text{ A}$$

$$v_L\,(0^-) = 0 \text{ V}$$

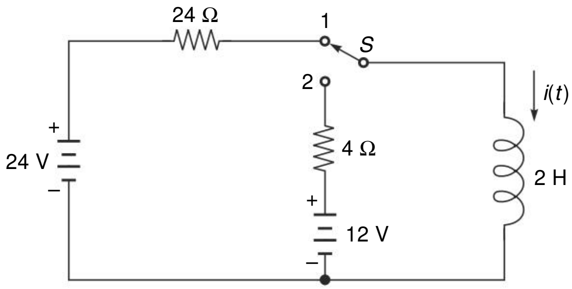


Fig. 3.27

(a) Switch thrown to position '2'

$$\tau = \frac{L}{R} = \frac{2}{4} = 0.5 \text{ s}$$

$\therefore$
$$i_n(t) = Ae^{-2t} \tag{i}$$

$$i_f = \frac{12}{4} = 3 \text{ A}$$

Hence

$$i(t) = Ae^{-2t} + 3 \tag{ii}$$

$$i(0^+) = i(0^-) = 1 \text{ A}$$

Substituting in Eq. (ii)

$$1 = A + 3 \qquad \text{or} \qquad A = -2$$

Hence
$$i(t) = (3 - 2e^{-2t}); \quad t > 0$$
$$= (3 - 2e^{-2t})\,u(t) \tag{iii}$$

which is plotted in Fig. 3.28.

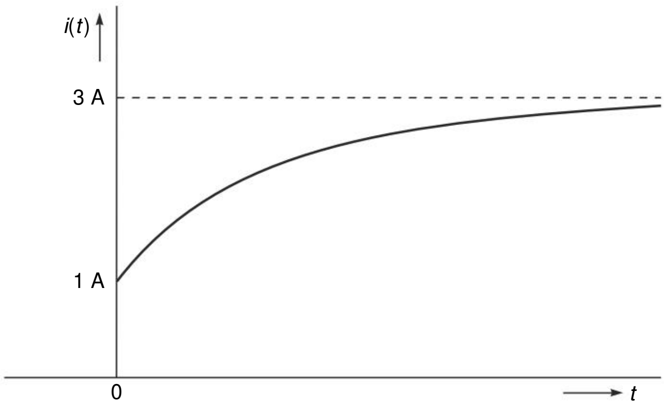


Fig. 3.28

(b) $v_L(0^-) = 0$, under steady-state inductance acts as a short-circuit. Applying KVL at $t = 0^+$

$$v_L(0^+) + 4i(0^+) = 12$$

$$v_L(0^+) + 4 \times 1 = 12$$

or
$$v_L(0^+) = 8 \text{ V}$$

As
$$v_L = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$v_L(0^+) = L\frac{\mathrm{d}i}{\mathrm{d}t}(0^+)$$

$\therefore$
$$\frac{\mathrm{d}i}{\mathrm{d}t}(0^+) = \frac{v_L(0^+)}{L}$$

$$= \frac{8}{2} = 4 \text{ A/s}$$

Step Voltage Response of *RC* Series Circuit

Consider the *RC* series circuit of Fig. 3.29. Just before the switch is closed, the capacitance is charged to a voltage of V_0. After the switch is closed, the differential equation governing the capacitance voltage is

$$Ri + v_C = V; \quad t > 0$$

but
$$i = C\frac{\mathrm{d}v_C}{\mathrm{d}t}$$

$\therefore$
$$RC\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C = V; \quad t > 0 \tag{3.32}$$

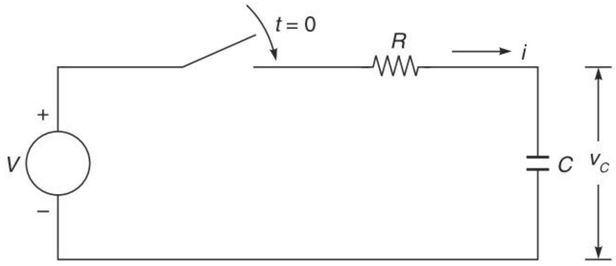


Fig. 3.29

Natural Response It is the solution of

$$RC\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C = 0$$

which gives (compare with Eqs. (3.21) and (3.22))

$$v_{Cn} = Ae^{-t/\tau}; \quad \tau = RC \tag{3.33}$$

Forced Response

$$RC\,\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C \;= V$$

Response will have the same form as excitation. Let it be

$$v_{Cf} = V_{Cf}\ \text{(constant)}$$

Substituting in Eq. (3.32)

$$V_{Cf} = V \tag{3.34}$$

Combining Eqs. (3.33) and (3.34), the complete response is

$$V_C(t) = Ae^{-t/\tau} + V \tag{3.35}$$

Substituting the initial condition $v_C\,(0^+) = V_0$

$$V_0 = A + V \quad \text{or} \quad A = V_0 - V$$

Hence

$$\begin{aligned} v_C\,(t) &= (V_0 - V)\,e^{-t/\tau} + V \\ &= V_0\mathrm{e}^{-t/\tau} + V\,(1 - e^{-t/\tau});\quad t > 0 \end{aligned} \tag{3.36}$$

The plot $v_C\,(t)$ is shown in Fig. 3.30.

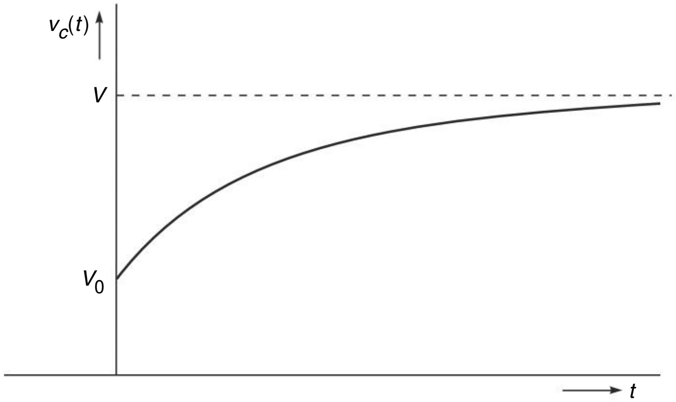


Fig. 3.30

Observe that $v_C(t)$ of Eq. (3.36) has the general form

$$f\,(t) = f\,(\infty) + [f(0^+) - f\,(\infty)]e^{-t/\tau}\,;\ t > 0$$

The expression for current can be obtained from Eq. (3.36) as below.

$$i\,(t) = C\ \frac{dv_C}{dt} = \left(\frac{V - V_0}{R}\right)e^{-t/\tau};\quad t > 0 \tag{3.37}$$

from which if follows that

$$i(0^+) = \frac{V - V_0}{R};\ \text{capacitance acts as a source of voltage} \tag{3.38a}$$

$$i(\infty) = 0;\ \text{capacitance acts as an open-circuit} \tag{3.38b}$$

Example 3.12 In the circuit of Fig. 3.31, the switch has been closed for a long time. Find the expression for v_C as the switch is thrown open. What is the rate of energy consumption in the 400 Ω resistance at $t = 0^+$?

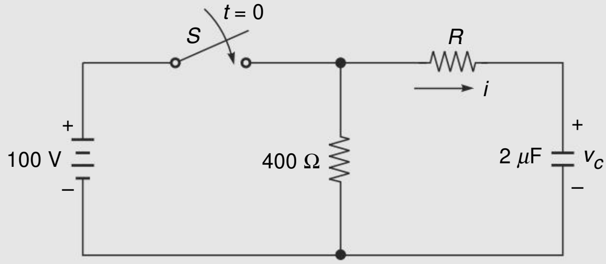


Fig. 3.31

Solution It is obvious that $v_C(0^+) = v_C(0^-) = 100$ V

After the switch is opened

$$\tau = RC = (400 + 100) \times 2 \times 10^{-6} = 10^{-3} \text{ s}$$

$$\therefore \quad v_C(t) = Ae^{-10^3 t}$$

$$v_C(0^+) = A = 100 \text{ V}$$

Hence

$$v_C(t) = 100\, e^{-10^3 t} \text{ V}$$

$$i(0^+) = \frac{100}{500} = 0.2 \text{ A (capacitance acts as a source of 100 V)}$$

$$p(0^+) \text{ in } 400\ \Omega \text{ resistance} = (0.2)^2 \times 400 = 16 \text{ W}$$

Step Current Response of *RL* Parallel Circuit

Consider the circuit of Fig. 3.32.

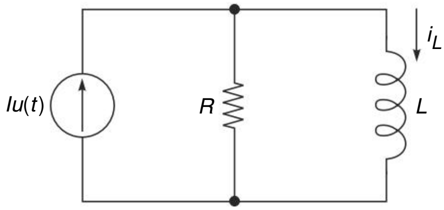


Fig. 3.32

$$\tau \text{ (with current source open-circuited)} = L/R.$$

$$\therefore \quad i_{Ln} = Ae^{-t/\tau}$$

$$i_{Lf} \text{ (inductance acts as short-circuit)} = I$$

Hence

$$i_L = Ae^{-t/\tau} + I; \quad t > 0$$

But

$$i_L(0^+) = 0$$

$$\therefore \quad A = -I$$

Finally

$$i_L(t) = I(1 - e^{-t/\tau}); \quad t > 0 \tag{3.39}$$

Step Current Response of *RC* Parallel Circuit

Consider the circuit of Fig. 3.33.

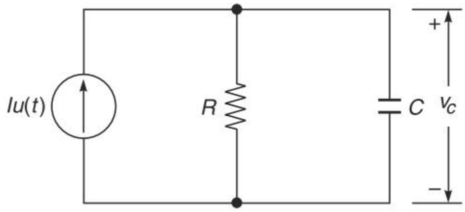


Fig. 3.33

τ (with current source open-circuited) $= RC$.

$\therefore \quad v_{Cn} = Ae^{-t/\tau}$

Under steady state, capacitance acts as an open-circuit so that all the current passes through R. Therefore

$$v_{Cf} = RI$$

Hence

$$v_C(t) = Ae^{-t/\tau} + RI; \quad t > 0$$

But

$$v_C(0^+) = V_0 \text{ (say)}$$

$\therefore \quad V_0 = A + RI \quad$ or $\quad A = V_0 - RI$

Hence

$$v_C(t) = (V_0 - RI)e^{-t/\tau} + RI$$

$$= V_0 e^{-t/\tau} + RI(1 - e^{-t/\tau}); \quad t > 0 \tag{3.40}$$

Example 3.13 In the circuit of Fig. 3.34, the switch S_1 has been closed for a long time. At $t = 0$, the switch S_2 is closed.

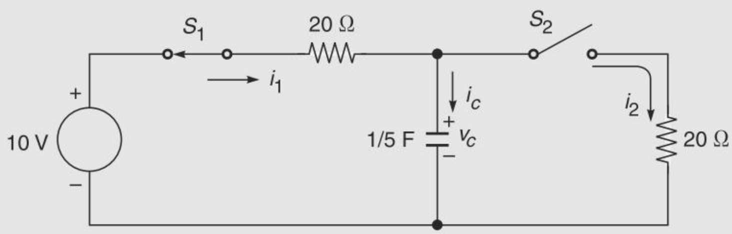


Fig. 3.34

(a) Without solving for $v_C(t)$, find $v_C(0^+)$, $i_C(0^+)$, $v_C(\infty)$ and $i_C(\infty)$.
(b) Derive an expression for $v_C(t)$ and check the results of part (a).

Solution

(a) Before S_2 is closed, the capacitance is fully charged.

$\therefore \quad v_C(0^+) = v_C(0^-) = 10$ V

Applying KVL and KCL at $t = 0^+$,

KVL (left loop): $-10 + 20\, i_1(0^+) + v_C(0^+) = 0$

or $\quad i_1(0^+) = 0$

KVL (right loop): $-v_C(0^+) + 20\, i_2(0^+) = 0$

or $$i_2(0^+) = \frac{10}{20} = 0.5 \text{ A}$$

Using KCL at the node

$\therefore$ $$i_C(0^+) + i_2(0^+) = i_1(0^+)$$

$$i_C(0^+) = 0 - 0.5 = -0.5 \text{ A}$$

At $t = \infty$, capacitance acts as open-circuit, therefore

$$i_C(\infty) = 0$$

$$v_C(\infty) = 10 \times \frac{20}{40} = 5 \text{ V}$$

(b) After closure of S_2

To find τ, short-circuit voltage source. The circuit is shown in Fig. 3.35. Then

$$\tau = RC = 10 \times \frac{1}{5} = 2 \text{ s}$$

$\therefore$ $$v_{Cn} = Ae^{-t/2}$$

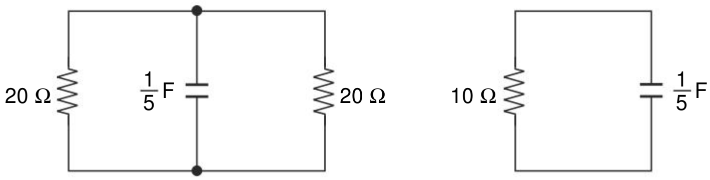


Fig. 3.35

To find forced response, assume capacitance as open-circuit.

$$v_{Cf} = 10 \times \frac{20}{40} = 5 \text{ V}$$

Hence

$$v_C(t) = Ae^{-t/2} + 5; \quad t > 0$$

But $$v_C(0^+) = 10 \text{ V}$$

$$10 = A + 5$$

or $$A = 5$$

$\therefore$ $$v_C(t) = 5\,(1 + e^{-t/2}); \quad t > 0 \qquad \text{(i)}$$

Also $$i_C(\text{t}) = \frac{1}{5} \times 5 \frac{d}{dt}(1 + e^{-t/2}) = -0.5e^{-t/2}; \quad t > 0 \qquad \text{(ii)}$$

All the results of part (a) are borne out by Eqs. (i) and (ii).

SUMMARY

1. Inductors in series combine as

$$L_{eq} = L_1 + L_2 + \ldots + L_N$$

Inductors in parallel combine as

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \dots + \frac{1}{L_N}$$

2. Capacitors in series combine as

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C_N}$$

Capacitors in parallel combine as

$$C_{eq} = C_1 + C_2 + \dots + C_N$$

3. The response of circuits wherein sources are switched in or out always comprises two parts: **natural response** and **forced response.**
4. The natural response is independent of sources and depends only on the value of curcuit components and how they are connected.
5. The forced response always has the same form as the forcing (excitation) function. For *dc* forcing function, it is a constant value.
6. A circuit reduced to a single equivalent inductance L and a single equivalent resistance R will have a natural response of the form $i(t) = I_0 e^{-t/\tau}$ where $\tau = L/R$ is the circuit **time constant**.
7. A circuit reduced to a single equivalent capacitance C and a single equivalent resistance R will have a natural response of the form $v(t) = V_0 e^{-t/\tau}$ where $\tau = RC$ is the circuit **time constant**.
8. Unit step function is the proper way to model switching actions (opening or closing) in a circuit.
9. The **complete response** of an *RL* or *RC* circuit excited by a *dc* source has the form $f(t) = f(\infty) + [f(0^+) - f(\infty)]e^{-t/\tau}$.

3.6 *RLC* CIRCUIT

Source Free Circuit (Natural Response)

We shall begin by considering source free response of *RLC* series and parallel circuits shown respectively in Figs 3.30(a) and (b).

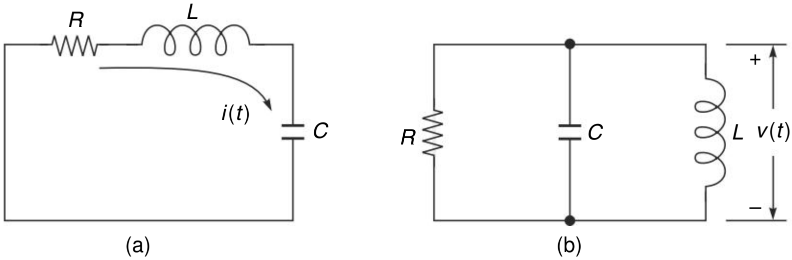


Fig. 3.36 *RLC* circuit (a) Series circuit (b) Parallel circuit

Series Circuit The describing integro-differential equation obtained by applying KVL around the loop is

$$Ri(t) + L\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{C}\int i(t)\,\mathrm{d}t = 0$$

Differentiating once and rearranging gives

$$\frac{\mathrm{d}i^2(t)}{\mathrm{d}t^2} + \frac{R}{L}\frac{\mathrm{d}i(t)}{\mathrm{d}t} + \frac{1}{LC}i(t) = 0 \tag{3.41}$$

Parallel Circuit Apply KCL

$$\frac{1}{R}v(t) + C\frac{\mathrm{d}v(t)}{\mathrm{d}t} + \frac{1}{L}\int v(t)\,\mathrm{d}t = 0$$

Differentiating once and rearranging gives

$$\frac{\mathrm{d}^2v(t)}{\mathrm{d}t^2} + \frac{1}{RC}\frac{\mathrm{d}v(t)}{\mathrm{d}t} + \frac{1}{LC}v(t) = 0 \tag{3.42}$$

It is seen that Eqs (3.41) and (3.42) have identical form. Such circuits are called "dual"* of each other and this concept is known as *duality*. The reader must have already observed duality in Sec. 3.5.

---

* Dual of a network can be easily identified by the procedure explained through the example of the network of Fig. (a) whose dual is drawn in Fig. (b).

– Draw a dotted enclosure to the original network of Fig. (a). This identifies the datum node of the dual.

– Locate a node in each mesh of the original network.

– Draw dotted lines connecting each node with other nodes and cutting across elements of the mesh to which the node belongs. Identify the dual of these elements as per the following rules.

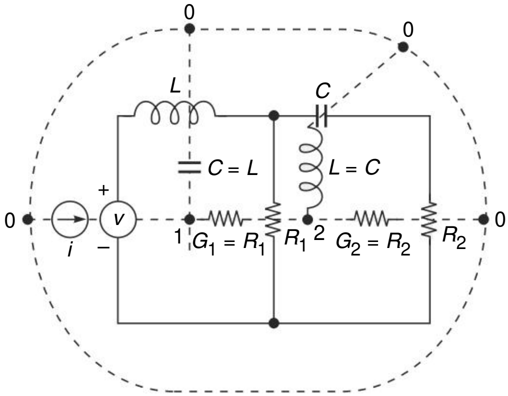
Voltage source ↔ current source

$R \leftrightarrow G$

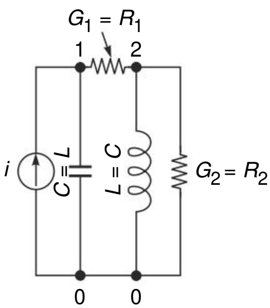
$L \leftrightarrow C$

This correspondence is in terms of the value of elements and not units.

– Draw the dual network (Fig. (b))



(a)



(b)

Circuit Analysis RLC Parallel

We now proceed to solve Eq. (3.42) for the *RCL* parallel circuit. The solution of Eq. (3.41) for the *RLC* series circuit will follow by changing

$$R \text{ to } \frac{1}{R} = G, L \text{ to } C \quad \text{and} \quad C \text{ to } L$$

Let the solution be

$$v(t) = Ae^{st} \tag{3.43}$$

Substituting in Eq. (3.42), we get

$$Ae^{st}\left(s^2 + \frac{1}{RC}s + \frac{1}{LC}\right) = 0 \tag{3.44}$$

Condition for nontrivial response is given by

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \tag{3.45}$$

This equation is known as the *characteristic equation* of the circuit. The solution of the quadratic Eq. (3.45) gives two roots

$$s_1, s_2 = -\frac{1}{2RC} \pm \sqrt{\left(\frac{1}{2RC}\right)^2 - \frac{1}{LC}} \tag{3.46}$$

The natural response is then given by

$$v(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \tag{3.47}$$

Two initial conditions ($v(0^+)$, $dv/dt\ (0^+)$) would now be needed to obtain the two arbitrary constants A_1 and A_2.

Let us define two new terms

$$\alpha = \frac{1}{2RC} \tag{3.48a}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{3.48b}$$

The two roots can now be expressed as

$$s_1, s_2 = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \tag{3.48c}$$

As in the expression v (t) of Eq. (3.47), $s_1 t$ and $s_2 t$ must be dimensionless, s_1, s_2 must have dimension s^{-1} (second^{-1}) or per second. Units of this type are called **frequencies**.

Depending on the values of R, L and C, three types of responses are obtained.

1. $\alpha^2 - \omega_0^2 > 0$; roots are real and unequal; *over damped* response
2. $\alpha^2 - \omega_0^2 = 0$; roots are real and equal; *critically damped* response
3. $\alpha^2 - \omega_0^2 < 0$; roots are complex conjugate; *under damped* response

Case 3– Under-damped The roots can be expressed as

$$s_1, s_2 = -\alpha \pm j\sqrt{\omega_0^2 - \alpha_0^2} \tag{3.49a}$$

We define

$$\omega_0 = \textit{resonance frequency} \text{ (rad/s)}$$

$$\alpha = \xi\omega_0, \quad or \quad \xi = \frac{\alpha}{\omega_0}$$

where ξ = *damping factor*

$$\zeta < 1, \text{ as } \omega_0 > \alpha$$

The roots can be now written as

$$s_1, s_2 = -\zeta\omega_0 \pm j\omega_0\sqrt{1-\xi^2} \tag{3.49b}$$
$$= -\zeta\omega_0 \pm j\omega_d$$

where

$$\omega_d = \omega_0\sqrt{1-\xi^2} = \textit{damped resonant frequency}$$

The complex plane representation of these two roots is shown in Fig. 3.37.

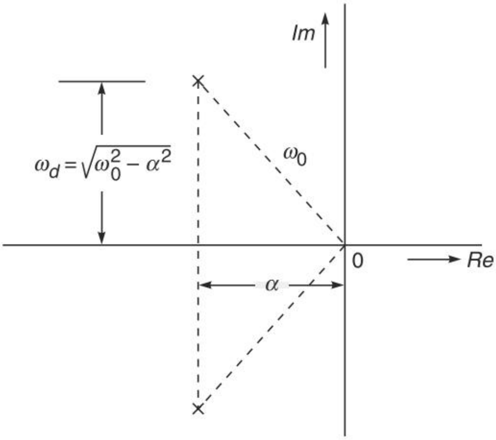


Fig. 3.37 Complex conjugate roots of characteristic equation

Substituting s_1 and s_2 from Eq. (3.49a) in Eq. (3.47)

$$v(t) = e^{-\alpha t}(A_1 e^{j\omega_d t} + A_2 e^{-j\omega_d t})$$

This, in the equivalent form, can be written as

$$v(t) = e^{-\alpha t}[A_1(\cos\omega_d t + j\sin\omega_d t) + A_2(\cos\omega_d t - j\sin\omega_d t)]$$

or
$$v(t) = e^{-\alpha t}[(A_1 + A_2)\cos\omega_d t + j(A_1 - A_2)\sin\omega_d t)]$$
$$= e^{-\alpha t}(B_1\cos\omega_d t + B_2\sin\omega_d t) \tag{3.50}$$

Assume for the parallel circuit an *RLC* combination such that

$$\alpha = \frac{1}{2RC} = 2$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{13}$$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2} = 3 \text{ rad/s}$$

Substituting in Eq. (3.50)

$$v(t) = e^{-2t}(B_1\cos 3t + B_2\sin 3t) \tag{i}$$

Assume initial conditions $v_C(0) = 10$, $i_L(0) = 0$

Substituting in Eq. (i)

$$B_1 = 10$$

$$\therefore \quad v(t) = e^{-2t}\,(10 \cos 3t + B_2 \sin 3t) \tag{ii}$$

Differentiating

$$\frac{dv}{dt} = -\,2e^{-2t}\,(10 \cos 3t + B_2 \sin 3t) + e^{-2t}(-30 \sin 3t + 3B_2 \cos 3t)$$

$$\frac{dv}{dt} = (0^+) = -20 + 3B_2$$

As $i_l(0^+) = 0$, the inductance acts as an open circuit. The capacitance charge $v_C(0^+) = 10V$ causes a current to flow from positive terminal of the capacitance to the resistance R as shown in Fig. 3.37a. As seen from the capacitance this is negative current. Thus

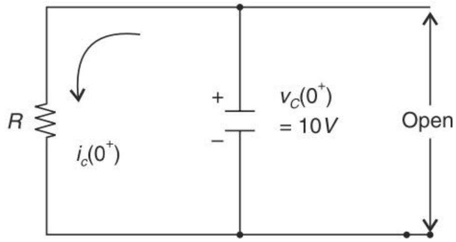


Fig. 3.37a

$$i_C(0^+) = -\frac{10}{R} = C\frac{dv_C}{dt}(0^+)$$

$$\frac{dv_C}{dt}(0^+) = -\frac{10}{RC} = -\,20 + 3B_2; \;\; RC = \frac{1}{4}$$

which yields

$$B_2 = -\frac{20}{3}$$

Hence

$$v(t) = e^{-2t}\,(10 \cos 3t - 20/3 \sin 3t)$$

$$= 12e^{-2t} \cos (3t + 33.7°);\; t > 0 \tag{iii}$$

It is observed here that $v(t)$ as given by Eq. (iii) is an *exponentially decaying sinusoid*.

LC Circuit If R is assumed infinite in the parallel circuit of Fig. 3.30(b) or R is assumed zero in the series circuit of Fig. 3.30(a), the describing equation of the resulting LC circuit (Fig. 3.38) is a second order differential equation in which the first order derivative term is absent (case of zero damping). It immediately follows from Eq. (3.42) (with $R = \infty$) that

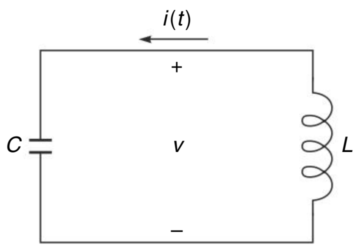


Fig. 3.38

$$\frac{d^2v(t)}{dt^2} + \frac{1}{LC}v(t) = 0 \tag{3.51}$$

Let

$$v(t) = Ae^{st}$$

Substitution in Eq. (3.48) gives

$$Ae^{st}\left(s^2 + \frac{1}{LC}\right) = 0 \tag{3.52}$$

Non-trivial solution is given by

$$s^2 + \frac{1}{LC} = 0 \text{ (compare with Eq. (3.45))} \tag{3.53}$$

From which

$$s = \pm j \frac{1}{\sqrt{LC}} = \pm j\omega_0 \tag{3.54}$$

where

$$\omega_0 = \frac{1}{\sqrt{LC}} = \text{resonant frequency}$$

It is seen that in contrast to Eq. (3.49) the term α which causes exponential decay of the response is absent.

The natural response is now given by

$$\begin{aligned} v(t) &= A_1 e^{j\omega_0 t} + A_2 e^{-j\omega_0 t} \\ &= (B_1 \cos \omega_0 t + B_2 \sin \omega_0 t) \end{aligned} \tag{3.55}$$

Thus $v(t)$ (and so also $i(t)$) is a continuous (non-decaying) sinusoidal oscillation. In practical circuits, R is never zero so that dissipation occurs and the oscillation cannot be sustained but decays slowly. First few cycles would correspond to nearly sinusoidal oscillation.

Assume LC combination such that

$$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{13} \tag{i}$$

Then

$$v(t) = B_1 \cos \sqrt{13}\, t + B_2 \sin \sqrt{13}\, t \tag{ii}$$

Let the capacitor be initially charged so that

$$v_C(0^+) = V_C \tag{iii}$$

Because the inductance acts as an open-circuit

$$i(0^+) = 0 \tag{iv}$$

Substituting Eq. (iii) in Eq. (i)

$$V_C = B_1 \tag{v}$$

From Eq. (ii)

$$i(t) = C\frac{dv(t)}{dt} = C(-\sqrt{13}B_1 \sin\sqrt{13}\, t + \sqrt{13}\; B_2 \cos \sqrt{13}\; t) \tag{vi}$$

By use of Eq. (iv) it follows that

$$B_2 = 0 \tag{vii}$$

Hence

$$v(t) = V_C \cos \sqrt{13}\; t \tag{viii}$$

Then

$$\begin{aligned} i(t) &= C\frac{d}{dt}(V_C \cos \sqrt{13}\; t) \\ &= -\sqrt{13}\; CV_C \sin \sqrt{13}\; t,\ \sqrt{13} = \frac{1}{\sqrt{LC}} \end{aligned}$$

$$= -\frac{V_C}{\sqrt{L/C}} \sin \sqrt{13}\, t \tag{ix}$$

Step Response This will be illustrated by means of examples.

Example 3.14 Solve for $v_C(t)$ in the circuit of Fig. 3.39. The circuit is initially quiescent (zero initial conditions).

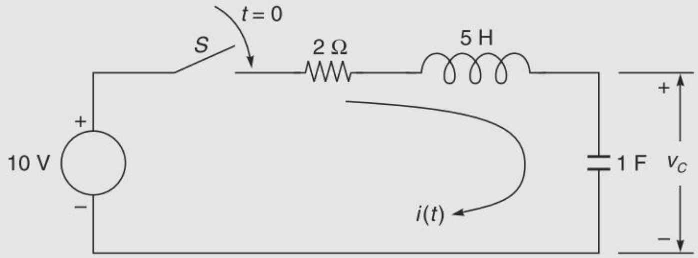


Fig. 3.39

Solution The describing differential equation of the circuit is

$$2i + 5\frac{di}{dt} + v_C = 10;\ t > 0 \tag{i}$$

But

$$i = 1 \times \frac{dv_C}{dt}$$

and

$$\frac{di}{dt} = \frac{d^2v_C}{dt}$$

Substituting in Eq. (i)

$$2\frac{dv_C}{dt} + 5\frac{d^2v_C}{dt^2} + v_C = 10$$

or

$$\frac{d^2v_C}{dt} + \frac{2}{5}\frac{dv_C}{dt} + \frac{1}{5}v_C = 2 \tag{ii}$$

$$\frac{d^2v_C}{dt^2} + \frac{2}{5}\frac{dv_C}{dt} + \frac{1}{5}v_C = 0$$

RLC Series Circuit (Natural Response) Refer. Fig. 3.36(a)

Changing R to $\frac{1}{R}$ changing C to L and L to C in Eqs. (3.48a) and (3.48b)

$$\alpha = \frac{R}{2L};\qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

Natural response, under damped

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$i(t) = e^{-\alpha t}\,[B_1 \cos \omega_d t + B_2 \sin \omega_d t]$$

It has the same form as $v(t)$ for the parallel circuit. In fact, it is to be noted here that the natural response of any variable in under-damped series or parallel RLC circuit has the same form.

In over-damped case, the natural response has the form

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$$

where s_1, s_2 are real unequal roots of the circuit's characteristic equation.

In critically damped case, the natural response is

$$i(t) = (A_1 t + A_2)e^{st}$$

where $s = s_1 = s_2$ are equal roots

Let $$v_{Cn} = Ae^{st}$$

Substituting in Eq. (iii)

$$Ae^{st}\left(s^2 + \frac{2}{5}s + \frac{1}{5}\right) = 0$$

From which

$$s^2 + \frac{2}{5}s + \frac{1}{5} = 0$$

and

$$s = \frac{-\frac{2}{5} \pm \sqrt{\frac{4}{25} - \frac{4}{5}}}{2}$$

$$= -\frac{1}{5} \pm j\frac{2}{5}$$ *underdamped* case (Case 1 of Eq. (3.46)), decaying oscillatory response

Natural response can then be written as

$$v_{cn}(t) = A_1 e^{\left(-\frac{1}{5}+j\frac{2}{5}\right)t} + A_2 e^{\left(-\frac{1}{5}-j\frac{2}{5}\right)t} \tag{iv}$$

Using Euler's theorem

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$v_{cn}(t) = e^{-t/5}\left(B_1 \cos\frac{2}{5}t + B_2 \sin\frac{2}{5}t\right) \tag{v}$$

We can directly write the result from the circuit parameters without the need to write the differential equation describing the circuit.

From Eqs. (3.48a) and (3.48b)

$$\alpha = \frac{R}{2L} = \frac{2}{2 \times 5} = \frac{1}{5}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5}} < \alpha, \text{ under-damped case}$$

$$\omega_0 = \sqrt{\omega_0^2 - \alpha^2} = \frac{2}{5}$$

Hence as per Eq. (3.50), we get the same result as in Eq. (iv).

Forced Response It follows from the differential Eq. (ii) with V_{ef} = constant or directly from the circuit.

At $t = 0$, inductor acts as short circuit and capacitor as open circuit. So

$$V_{cf} = 10 \text{ V}$$

Total Response

$$v_C(t) = e^{-t/5}\left(B_1 \cos\frac{2}{5}t + B_2 \sin\frac{2}{5}t\right) + 10 \quad \text{(v)}$$

Initial Conditions

$$v_C(0^+) = 0,\ i(0^+) = 0$$

$$\frac{dv_C(0^+)}{dt} = \frac{i(0^+)}{C} = 0$$

Substituting initial conditions in Eq. (v)

$$0 = B_1 + 10$$

or $$B_1 = -10$$

$$\frac{dv_C}{dt} = -\frac{1}{5}e^{-t/5}\left(B_1 \cos\frac{2}{5}t + B_2 \sin\frac{2}{5}t\right) + e^{-t/5}\left(-\frac{2}{5}B_1 \sin\frac{2}{5}t + \frac{2}{5}B_2 \cos\frac{2}{5}t\right)$$

or $$0 = -\frac{1}{5}B_1 + \frac{2}{5}B_2$$

or $$B_2 = -5$$

Hence

$$v_C(t) = 10 - e^{-t/5}\left(10 \cos\frac{2}{5}t + 5 \sin\frac{2}{5}t\right)$$

$$= 10 - 11.18\, e^{-t/5} \cos\left(\frac{2}{5}t - 26.6^\circ\right);\ t > 0 \quad \text{(vi)}$$

The response $v_C(t)$ as per Eq. (vi) is damped sinusoidal superimposed on $v_C(\infty) = 10\ V$, the steady-state value. It is plotted in Fig. 3.40(a).

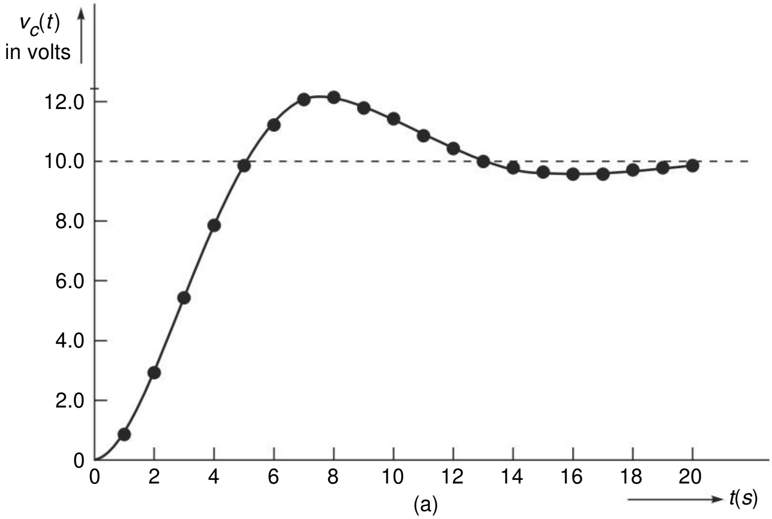


Fig. 3.40 (a) Damped sinusoidal response

For this circuit, as from above calculations

$$\zeta\omega_0 = \frac{1}{5},\ \omega_0\sqrt{1-\zeta^2} = \frac{2}{5}$$

which yields

$$\zeta = 0.447$$

The peak of the oscillatory response of Fig. 3.40(a) is related to ζ the damping factor.

It can be derived that the peak overshoot above the steady state response, which corresponds to the first overshoot, is given by

$$M\text{ (peak)} = M_p = e^{-(\zeta/\sqrt{1-\zeta^2})\pi} \quad \text{(normalized)}$$

For the value of $\zeta = 0.447$ in Fig. 3.40(a)

$$M_p = e^{-(0.447\sqrt{1-0.447^2})\pi} = 0.2078$$

Its value as calculated from the Fig. 3.40(a) is

$$M_p = \frac{2.08}{10} = 0.208$$

The nature of circuit response (in this case $v_C(t)$) to a unit step input for various values of ζ is plotted in Fig. 3.40(b). It is observed from this figure that for

- $\zeta < 1$, the response is damped (decaying) oscillatory settling to the steady value of unity
- $\zeta = 1$, the response is just nonoscillatory (critically damped)

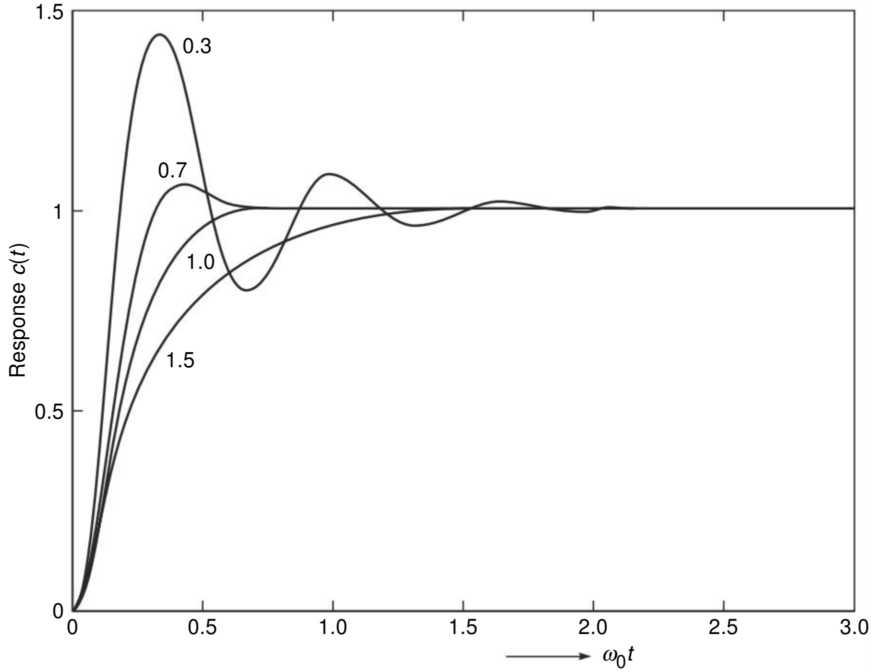


Fig. 3.40 (b) Response vs $\omega_0 t$ (normalized time) of various values of ζ

- $\zeta > 1$, the response is overdamped (and sluggish)
- $\zeta = 0$, the response is non-damped oscillatory (not shown in the figure)

Example 3.15 In the circuit of Fig. 3.41, find $i_L(t)$ after the switch is closed at $t = 0$. The circuit is quiescent before the switch closure (there is no energy storage).

(a) $C = \dfrac{1}{108}$ F

(b) $C = \dfrac{1}{81}$ F

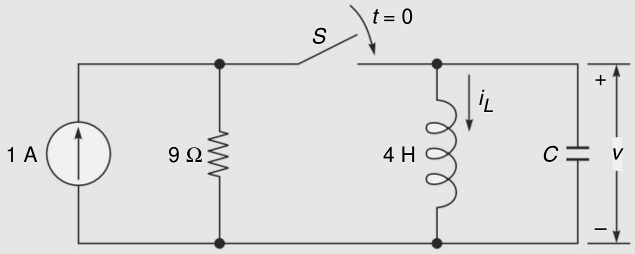


Fig. 3.41

Solution The KCL equation after the switch closure is

$$\frac{v}{9} + \frac{1}{4}\int v \, dt + C\frac{dv}{dt} = 1; \; t > 0 \qquad \text{(i)}$$

Differentiating and rearranging gives

$$C\frac{d^2v}{dt^2} + \frac{1}{9}\frac{dv}{dt} + \frac{1}{4} v = 0$$

or
$$\frac{d^2v}{dt^2} + \frac{1}{9C}\frac{dv}{dt} + \frac{1}{4C} v = 0 \qquad \text{(ii)}$$

(a) $C = \dfrac{1}{108}$ F

$$\frac{d^2v}{dt^2} + 12\frac{dv}{dt} + 27 v = 0 \qquad \text{(iii)}$$

The characteristic equation is

$$s^2 + 12s + 27 = 0 \qquad \text{(iv)}$$

or $s = -9, -3$; *overdamped* case (Case 1 of Eq. (3.46)), non-oscillatory response

Therefore the natural response is

$$v_n(t) = K_1e^{-9t} + K_2e^{-3t}$$

Note from Eq. (ii) that there is no forced response, i.e. $v_f = 0$, as the inductance acts as a short-circuit. Hence

$$v(t) = K_1e^{-9t} + K_2e^{-3t} \qquad \text{(v)}$$

To determine K_1 and K_2:

$$v(0^+) = 0 \qquad \text{(vi)}$$

At $t = 0^+$, the inductance acts as open-circuit and capacitance as short-circuit. Therefore,

$$i_C(0^+) = C\frac{dv}{dt}(0^+) = 1$$

or $$\frac{dv}{dt}(0^+) = \frac{1}{C} = 108 \text{ V/s} \quad \text{(vii)}$$

Substituting in Eq. (v)

$$0 = K_1 + K_2 \quad \text{(viii)}$$

Differentiating Eq. (v)

$$\frac{dv(t)}{dt} = 9K_1e^{-9t} - 3K_2e^{-3t}$$

At $t = 0^+$ $$108 = -9K_1 - 3K_2 \quad \text{(ix)}$$

Solving Eqs (viii) and (ix)

$$K_1 = -18, K_2 = 18$$

Hence $$v(t) = -18e^{-9t} + 18e^{-3t}; \quad t > 0 \quad \text{(x)}$$

(b) $C = \frac{1}{81} F$

$$\frac{d^2v}{dt^2} + 9\frac{dv}{dt} + \frac{81}{4} = 0$$

It gives

$s_1, s_2 = -9/2$; *critically damped* case (Case 2 of Eq. (3.46)), non-oscillatory response.

It is a special case of *equal roots*. Its solution without *proof* is given below

$$v(t) = v_n(t) = K_1te^{-9t/2} + K_2e^{-9t/2} \quad \text{(xi)}$$

Since $$v(0^+) = 0$$

$$0 = K_2$$

$\therefore$ $$v(t) = K_1te^{-9t/2}$$

$$\frac{dv(t)}{dt} = K_1e^{-9t/2} - K_2e^{-9t/2}$$

At $t = 0^+$

$$\frac{dv}{dt}(0^+) = \frac{1}{C} = 81 \text{ V/s}$$

Substituting

$$81 = K_1$$

Hence

$$v = v_n(t) = 81te^{-9t/2}; \quad t > 0 \quad \text{(xii)}$$

SUMMARY

1. Circuits having both types of energy storage elements, which cannot be combined by series/parallel techniques, are governed by second-order differential equation.

2. *RLC* series and parallel circuits are categorized as having three types of responses depending upon the values of *R*, *L* and *C*. These types are:

   Over-damped : $\alpha > \omega_0$; non-oscillatory response
   Critically damped : $\alpha = \omega_0$; just non-oscillatory response
   Under-damped : $\alpha < \omega_0$; decaying oscillatory response

3. *RLC* Parallel circuit: $\alpha = \dfrac{1}{2RC}, \quad \omega_0 = \dfrac{1}{\sqrt{LC}}$

   RLC Series circuit: $\alpha = \dfrac{R}{2L}, \quad \omega_0 = \dfrac{1}{\sqrt{LC}}$

4. Over-damped response is sum of two decaying exponentials; one decaying faster than the other.

   e.g. $A_1 e^{-t} + A_2 e^{-3t}$

   - Critically damped response is of the form $e^{-\alpha t}(A_1 t + A_2)$
   - Under-damped response is exponentially decaying sinusoid of the form $e^{-\alpha t}(B_1 \sin \omega_\alpha t + B_2 \cos \omega_\alpha t)$.

5. During the transient of an *RLC* circuit, the stored energy transfer back and fourth between the two storage elements continuously decaying due to the loss in the resistance element.

3.7 CIRCUIT RESPONSE TO PULSE AND IMPULSE EXCITATIONS

Pulse and Pulse Response

Figure 3.42(a) shows a voltage pulse of strength V_0 and time duration ΔT (a current pulse will be similarly represented). As is seen from Fig. 3.42(b), a pulse can be represented alternatively as the difference of two steps displaced in time by the duration (width) of the pulse. Thus

$$V_0 P(t, \Delta T) = V_0 u(t) - V_0 u(t - \Delta T) \tag{3.56}$$

where $P(t, \Delta T)$ = unit pulse of duration ΔT.

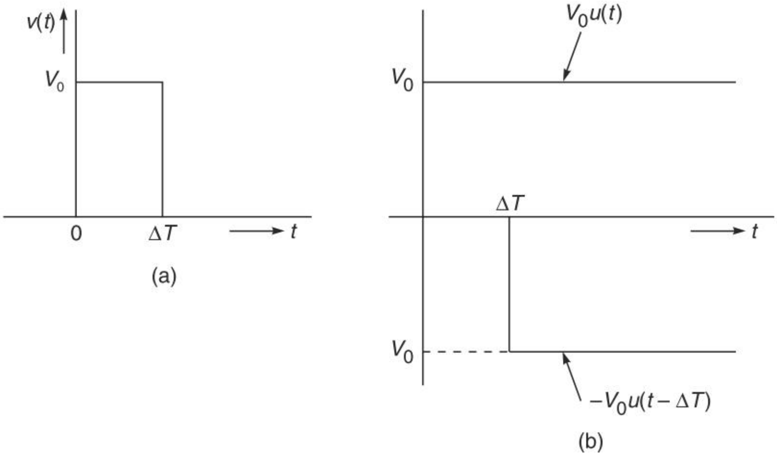


Fig. 3.42 Pulse voltage

A voltage pulse is generated by throwing the switch in Fig. 3.12(a) to position 'a' at $t = 0$ and throwing it back to position 'b' at $t = \Delta T$.

The pulse response can be obtained by superimposition of two step responses. This is illustrated in the following example.

Example 3.16 A series circuit of resistance 250 Ω and inductance 0.25 H is excited from a pulse voltage of strength 10 V and of duration 1 ms. Find the value of the current at 0.5 ms and 2 ms.

Solution

$$\tau = \frac{L}{R} = \frac{0.25}{250} = 10^{-3} \text{ s}$$

Consider step voltage of strength 10 V. Response is given by

$$i(t) = \frac{10}{250}(1 - e^{-1000\,t})\, u(t); \text{ (see Eq. 3.31(b))}$$

$$= 0.04\,(1 - e^{-1000}\ t)\, u(t)$$

$$10\, P\,(t, 10^{-3}) = 10u(t) - 10\, u(t - 10^{-3})$$

Pules response is given by

$$i(t) = 0.04\,(1 - e^{-1000\,t})\, u(t) - 0.04\,(1 - e^{-1000}(t - 10^{-3}))\, u(t - 10^{-3})$$

(i) $t = 0.5 \times 10^{-3}$ s

$$i(0.5 \times 10^{-3}) = 0.04\,(1 - e^{-1000} \times 0.5 \times 10^{-3}) = 0.04(1 - e^{-0.5}) = 0.0157 \text{ A}$$

(ii) $t = 2 \times 10^{-3}$ s

$$i(2 \times 10^{-3}) = 0.04\,(1 - e^{-1000} \times 2 \times 10^{-3}) - 0.04\,(1 - e^{-1000\,(2-1)\,10^{-3}})$$

$$= 0.04\,(1 - e^{-2}) - 0.04\,(1 - e^{-1})$$

$$= 0.0093 \text{ A}$$

Impulse

Consider a pulse of fixed unit area but variable duration and strength (voltage), i.e.

$$V_0 \Delta T = 1$$

As $\Delta T \to 0$, $V_0 \to \infty$. Mathematically, it approaches a pulse of zero duration and infinite strength (voltage) but finite (unit) area. Such an ideal mathematical function (singularity function) is called a *unit impulse* represented as $\delta(t)$ (also called *delta function*). This property of a unit impulse is mathematically expressed as

$$\delta(t) = 0 \qquad t \neq 0$$

$$\int_{-\infty}^{\infty} \delta(t)\, \mathrm{d}t = 1 \tag{3.57}$$

The graphical representation of an impulse is shown in Fig. 3.37. An impulse, of course, could be current or voltage.

It can also be shown that $\delta(t)$ is the derivative of $u(t)$, unit step or

$$\delta(t) = \frac{d}{\mathrm{d}t} u(t) \tag{3.58}$$

This is easily visualized from the fact that the derivative of unit step at $t = 0$ (the point of discontinuity) is infinite but lasts for zero time.

Consider

$$\int_{-\infty}^{\infty} v(t)\,\delta(t)\,dt = \int_{-\infty}^{\infty} v(0)\,\delta(t)\,dt$$

$$= v(0) \tag{3.59}$$

It implies that a unit impulse extracts the value of a function at $t = 0$.

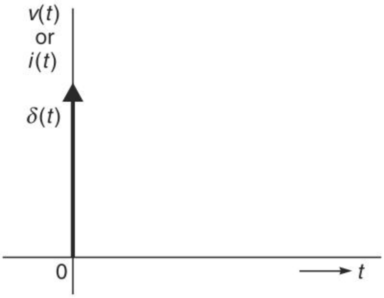


Fig. 3.43 Impulse voltage/current

Also

$$\delta(t - t_0) = \text{unit impulse occurring at } t = t_0 \tag{3.60}$$

An impulse is a good approximation to a short duration pulse and this approximation results in considerable simplification in pulse circuit analysis. The strength of the equivalent impulse equals the pulse area.

Circuit Response to Impulse An impulse current applied to a capacitance transfers charge to it instantly. Thus

$$q = \int_{-\infty}^{\infty} I\,\delta(t)\,dt = I;\; I = \text{impulse strength}$$

Capacitor voltage at $t = 0^+$

$$v(0^+) = \frac{q}{C} = \frac{I}{C} \tag{3.61}$$

Consider now an impulse voltage applied to an inductance

$$i(0^+) = \frac{\int_{-\infty}^{\infty} V\,\delta(t)\,dt}{L}$$

$$= \frac{V}{L};\; V = \text{impulse strength} \tag{3.62}$$

An impulse current source acts as an open-circuit except for the time the impulse appears. Similarly an impulse voltage source acts as a short-circuit except when the impulse appears.

Consider now the impulse voltage excited *RL* series circuit of Fig. 3.44. It is equivalently the case of an *RL* series circuit with an initial inductance current of $i(0^+) = V/L$. Using Eq. (3.19)

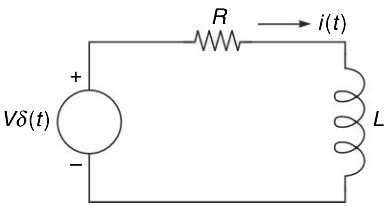


Fig. 3.44

$$i(t) = \frac{V}{L}e^{-t/\tau};\quad t > 0;\quad \tau = L/R \tag{3.63}$$

Figure 3.45 is a current impulse excited *RC* parallel circuit

$$v_C(0^+) = \frac{I}{C}$$

$$v(t) = \frac{I}{C} e^{-t/\tau}; \ t > 0; \ \tau = RC \tag{3.64}$$

RLC series and parallel circuits excited by impulse source could be similarly treated.

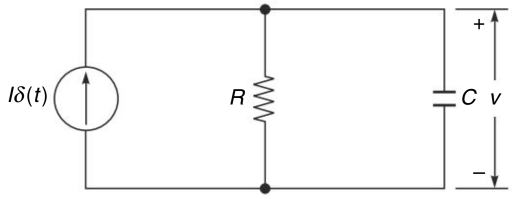


Fig. 3.45

As per Eq. (3.58), impulse is the derivative of a step function. In a linear circuit, impulse response could, therefore, be obtained by taking the derivative of the step response.

The step voltage response of a series *RL* circuit is (Eq. (3.31a)).

$$i(t)\big|_{\text{step}} = \frac{V}{R}(1 - e^{-Rt/L})\, u(t) \tag{3.65}$$

Differentiating Eq. (3.65), we get the impulse voltage response of the circuit as

$$i(t)\big|_{\text{impulse}} = \frac{d}{dt}\, i(t)\big|_{\text{step}}$$

$$= -\frac{V}{R} \times -\frac{R}{L} e^{-Rt/L} u(t) + \frac{V}{R}(1 - e^{-Rt/L})\,\delta(t)$$

$$= \frac{V}{L} e^{-Rt/L} u(t) \tag{3.66}$$

Observe that as per Eq. (3.66), the second term yields zero when evaluated at $t = 0$.

Equation (3.66) checks the result of Eq. (3.63).

It must also be remarked here that the reverse is equally true in the above case, i.e. step response is the integral of the impulse response. The reader should check this by integrating Eq. (3.66).

ADDITIONAL SOLVED PROBLEMS

3.17 For the circuit of Fig. 3.46, find the expression for $v_C(t)$ after the switch is closed. The initial value of $v_C = 4$ V.

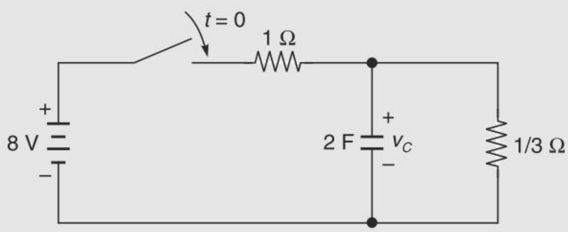


Fig. 3.46

Solution

Time Constant Close the switch and short-circuit the voltage source as in Fig. 3.41 (a). The Thevenin resistance seen by the capacitance is

$$R_{TH} = (1 \times 1/3) / (1 + 1/3) = 1/4\ \Omega$$
$$\tau = R_{TH}\, C = (1/4) \times 2 = 0.5 \text{ s}$$

Natural Response

$$v_{Cn}(t) = Ae^{-2t}$$

Forced Response At $t = \infty$, the capacitor acts as an open-circuit, as in Fig. 3.47(b) from which we have $v_{cf} = 8 \times (1/3) / (1 + 1/3) = 2$ V.

Complete Response

$$\begin{aligned} v_C(t) &= v_{Cn}(t) + v_{cf} \\ &= A\, e^{-2t} + 2 \\ v_C(0^+) &= v_C(0^-) = 4 = A + 2 \Rightarrow A = 2 \\ \therefore \quad v(t) &= 2\,(1 + e^{-2t}) \quad t > 0 \end{aligned}$$

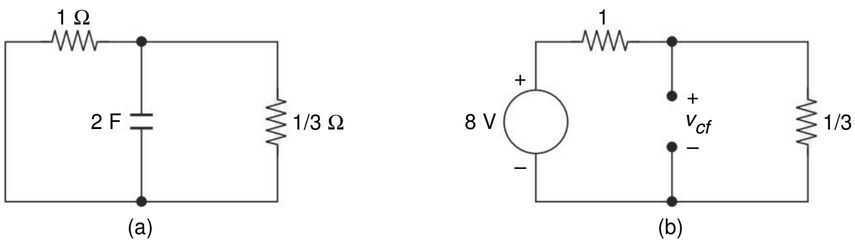


Fig. 3.47

3.18 The *RC* parallel circuit of Fig. 3.48 is initially quiscent and is excited by a current pulse of strength 1 mA and duration 10 ms. Find the capacitor voltage $v_C(t)$ and sketch the same along with the current pulse. Find the value of v_C at t = 10 ms and 20 ms.

Solution

The circuit time constant, $\tau = RC = 5 \times 10^3 \times 1 \times 10^{-6} = \dfrac{1}{200}$ s

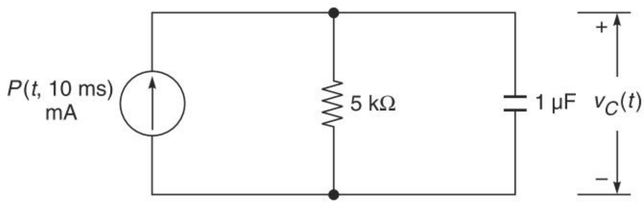


Fig. 3.48

For unit step current, $u(t)$ *mA*

$$v_{Cn}(t) = A\,e^{-200t}$$

$$v_{Cf} = 5 \times 10^3 \times 1 \times 10^{-3} = 5 \text{ V (the capacitor acts as an open-circuit in steady state)}$$

Then $\quad v_C(t) = A\,e^{-200t} + 5$

At $\quad t = 0^+,\ v_C(0^+) = 0;$

$\therefore \quad 0 = A + 5 \quad \text{or} \quad A = -5$

$\therefore \quad v_C(t) = 5\,(1 - e^{-200t})\,u(t)\ V \qquad$ (i)

Now current pulse is expressed as

$$P(t, 10 \text{ ms}) = u(t) - u(t - 0.01) \text{ mA}$$

Then $v_C(t)$ is the superimposition of the two-step responses. Thus

$v_C(t) = 5\,(1 - e^{-200t})\,u(t) - 5\,(1 - e^{-200(t-0.01)})\,u(t - 0.01)$ V $\qquad$ (ii)

The current and voltage responses are plotted in Fig. 3.49.

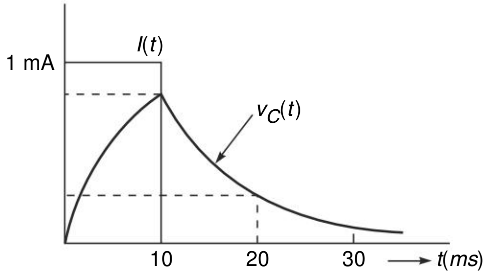


Fig. 3.49

Form Eq. (ii), we observe that

$$v_C(10 \text{ ms}) = 5\,(1 - e^{-200 \times 0.01}) = 5\,(1 - e^{-2}) = 4.32 \text{ V}$$

$$v_C(20 \text{ ms}) = 5\,(1 - e^{-200 \times 0.02}) - 5(1 - e^{-200 \times 0.01})$$

$$= 5\,(e^{-2} - e^{-4}) = 0.585 \text{ V}$$

3.19 For the circuit of Fig. 3.50 (a), find the expression for $i(t)$ at $t > 0$.

Solution

With the switch open [Fig. 3.50 (b)]

$$i_L(0^-) = 6/2 = 3 \text{ A}$$

$$i_L(0^+) = i_L(0^-) = 3 \text{ A}$$

With the switch closed, the initial conditions are shown in Fig. 3.50 (c).

Using the superposition theorem, we get

$$i(0^+) = 6/2 - 3 \times 1/2 = 3/2 \text{ A}$$

Time Constant Short-circuiting voltage source as in Fig. 3.50 (d), we obtain

$$R_{eq} = 1 + 1/2 = 3/2\ \Omega;\ \tau = 1/(3/2) = 2/3 \text{ s}$$

$i_n(t) = A\,e^{-1.5t} \qquad$ (i)

Forced Response The inductance acts as a short-circuit as in Fig. 3.50 (e)

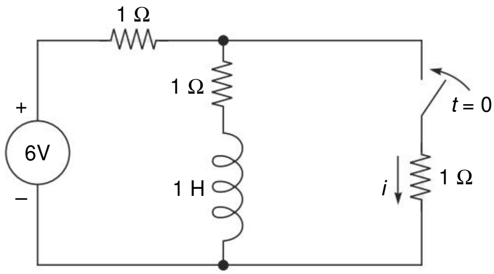
$$i_f = 6/(1 + 1/2) \times 1/2 = 2 \text{ A} \qquad \text{(ii)}$$

Thus

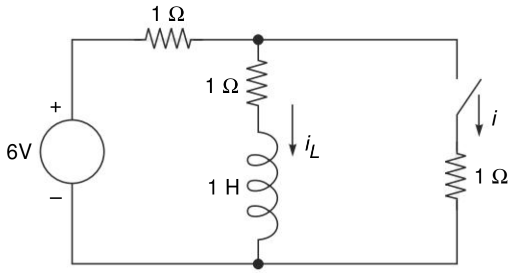
$$i(t) = A\, e^{-3t/2} + 2,\ t > 0 \qquad \text{(iii)}$$

$$i(0^+) = 3/2 = A + 2 \text{ or } A = -1/2$$

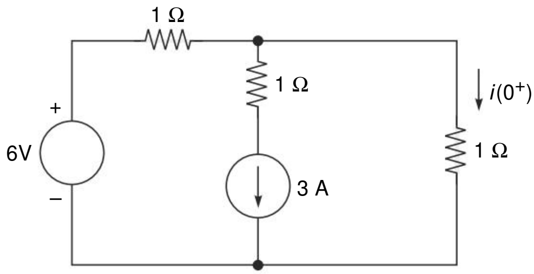
$$\therefore \quad i(t) = -0.5\, e^{-1.5t} + 2,\ t > 0 \qquad \text{(iv)}$$



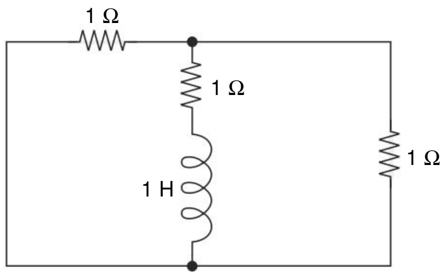
(a)



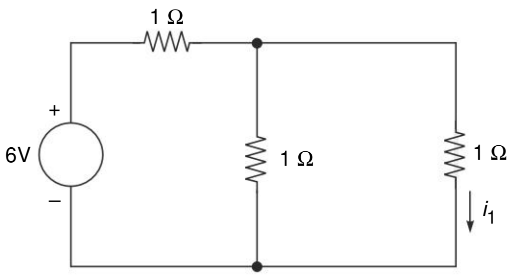
(b)



(c)



(d)



(e)

Fig. 3.50

3.20 For the op-amp circuit of Fig. 3.51(a), find $v_0(t)$ for $t > 0$.

Solution

On an ideal op-amp basis, the circuit is redrawn as in Fig. 3.51 (b). Now

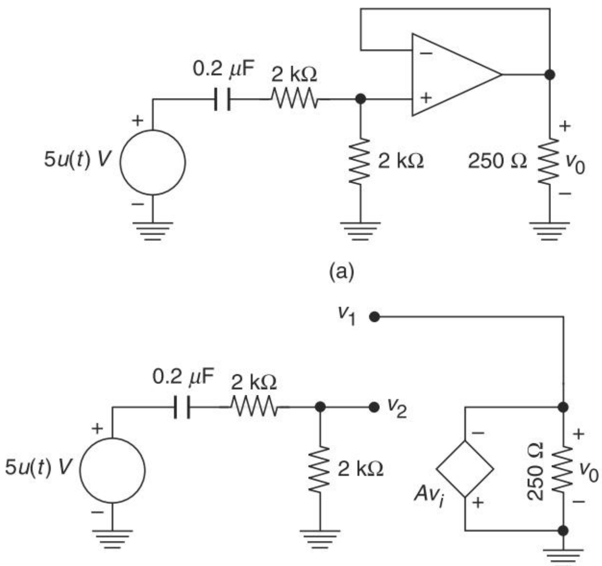
$$v_0 = -Av_i = -A(v_1 - v_2)$$
$$v_1 = v_0$$
$$v_0 = A(v_2 - v_0)$$

or

$$v_0 = [A/(1 + A)]\, v_2 = v_2 \qquad \text{(i)}$$
$$A \to \infty$$

Hence we need to find v_2.

$$\tau = 0.2 \times 10^{-6} \times 4 \times 10^{-3} = 0.8 \times 10^{-3} = 1/1250 \text{ s}$$
$$v_{2n}(t) = Ae^{-1250t} \qquad \text{(i)}$$



(b)

Fig. 3.51

At $t = 0^+$, the capacitance acts as a short-circuit.

$\therefore$ $v_2(0^+) = 5/2 = 2.5$ V

At t = ∞ the capacitance acts as an open-circuit.

$\therefore$ $v_{2f} = 0$. Thus,

$v_2(t) = Ae^{-1250t}$, $t > 0$, $2.5 = A$

$\therefore$ $v_2(t) = 2.5\, e^{-1250t}\, u(t)$.

3.21 For the circuit of Fig 3.52(a) find $v_c(t)$ at t(a) 0^- (b) 0^+ (c) ∞ and (d) 0.05 s.

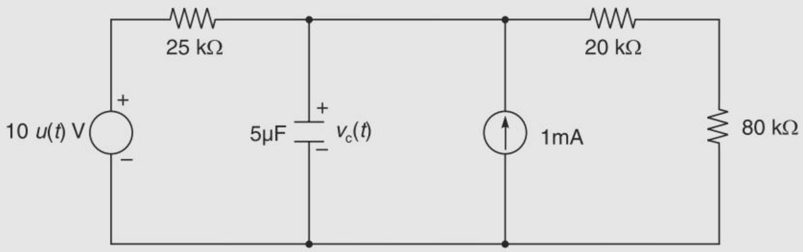


Fig. 3.52a

Solution

We will proceed by replacing the current source by a voltage using Thevenin equivalent. The circuit is redrawn in Fig. 3.52b

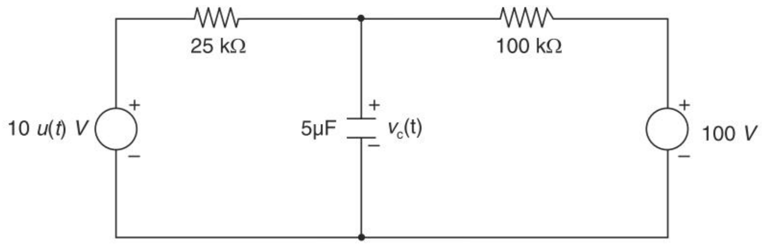


Fig. 3.52b

(a) At $t = 0^-$

$10\,u\,(0^-) = 0$, a short-circuit capacitor acts as open-circuit.

The two resistance act as voltage divider. Therefore

$$v_c\,(0^-) = \frac{25}{100 + 25} \times 100 = 20 \text{ V}$$

(b) At $t = 0^+$, $10u\,(t) = 10$V

The capacitance voltage cannot change instantaneously, therefore

$v_c\,(0^+) = 20V$

(c) At $t = \infty$, the capacitor gets fully charged and it does not draw any current and acts like open-circuit. Therefore

$$v_c(\infty) = (100 - 10) \times \frac{25}{125} = 18\text{V}$$

(d) We need the solution for $v_c(t)$. Short-circuit the two voltage sources, then the circuit becomes as in Fig. 3.46c

$$R_{eq} = \frac{25 \times 100}{125} = 20\ k\Omega$$

The time constant is

$$\tau = R_{eq}C = 20 \times 5 \times 10^{-3} = 0.1s$$

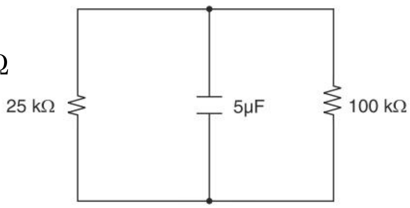


Fig. 3.52c

The solution is then given by

$$\begin{aligned} v_C(t) &= v_c(\infty) + [v_c(0^+) - v_c(\infty)]e^{-t/\tau} \\ &= 18 + (20 - 18)\, e^{-t/0.1} \\ &= 18 + 2e^{-10t} \end{aligned}$$

At $t = 0.05$s

$$v_c(0.05) = 18 + 2e^{-0.5} = 19.2\text{V}$$

3.22 For the circuit of Fig. 3.53 find R_1 and R_2 so that $v_{R2}(0^+) = 10$ V and V_{R1} (1ms) $= 5V$

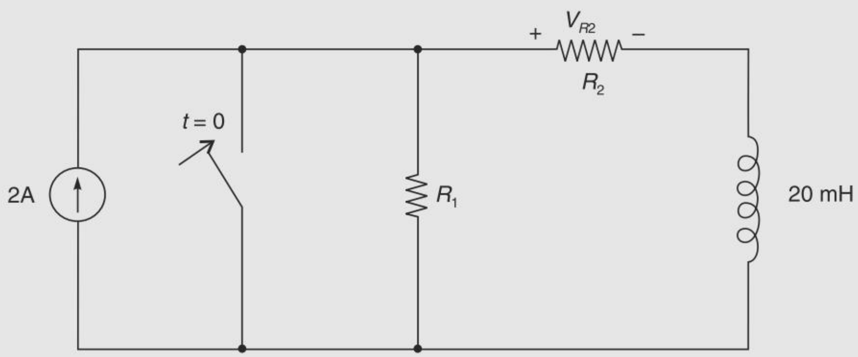


Fig. 3.53

Solution

At $t = 0^-$, inductor is short-circuit. Therefore by current division

$$i_{R2}(0^-) = \left(\frac{R_1}{R_1 + R_2}\right) \times 2 = \text{inductor current at } 0^- = i_L(0^-)$$

Then

$$v_{R2}(0^-) = i_{R2}(0^-)\, R_2 = \frac{2\, R_1 R_2}{R_1 + R_2} \qquad \text{(i)}$$

The switch is closed at $t = 0$

At $t = 0^+$, the inductor current cannot change suddenly.

So

$$v_{R2}(0^+) = \frac{2\, R_1 R_2}{R_1 + R_2} = 10\text{ V} \quad \text{(given)} \qquad \text{(ii)}$$

With switch closed, R_1 is shorted. Therefore

$$\tau = \frac{L}{R_2} = (20 \times 10^{-3})/R_2 \qquad \text{(iii)}$$

At $t = \infty$, inductor current reduces to zero as there is no source in the circuit. Therefore

$$v_{R2}(\infty) = 0$$

We can write

$$v_{R2}(t) = v_{R2}(\infty) + [v_{R2}(0^+) - v_{R2}(\infty)]e^{-t/\tau}$$
$$= 10\, e^{-t/\tau} \qquad \text{(iv)}$$

At $t = 1$ ms

$$v_{R2}(1\text{ ms}) = 10e^{-10^{-3}/\tau} = 5V \text{ (given)}$$

or

$$e^{-10^{-3}/\tau} = 0.5$$
$$-\frac{10^{-3}}{\tau} = \ln 0.5 = -0.693$$
$$\tau = 1.44 \times 10^{-3}$$

From Eq. (iii)

$$\tau = \frac{20 \times 10^{-3}}{R_2} = 1.44 \times 10^{-3}$$

or

$$R_2 = 13.9\Omega$$

From Eq. (i)

$$\frac{2R_1R_2}{R_1 + R_2} = 10 = \frac{2 \times 13.9R_1}{13.9 + R_1}$$

or

$$R_1 = 7.8\Omega$$

3.23 In the circuit of Fig. 3.54(a)

$i_s = 1$ mA, $t < 0$
$= 0$; $t > 0$

Find $v_s(t)$ for (a) $t < 0$ and (b) $t > 0$

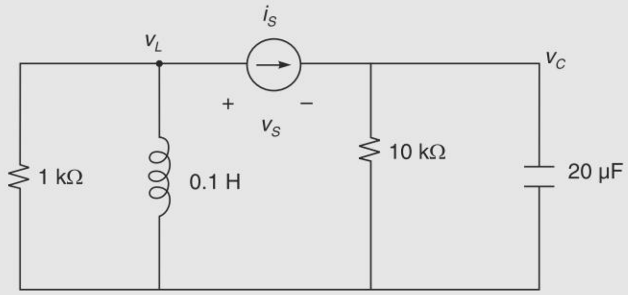


Fig. 3.54 (a)

Solution

(a) $t < 0$

i_S = 1 mA has been flowing for long time, steady-state has been reached. Capacitor acts an open-circuit and inductor as short-circuit. So all the current flows through inductor and none through 2 kΩ resistor. The circuit at $t = 0^-$ is drawn in Fig. 3.48(b).

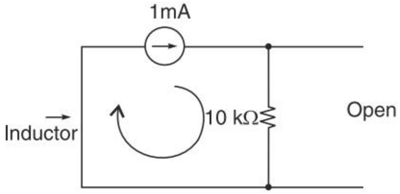


Fig. 3.54(b)

So at $t = 0^-$

$$v_C(0^-) = 1 \times 10 = 10\text{V}$$

$$i_L(0^-) = -1\text{mA (downwards)}$$

$$v_L(0^-) = 0$$

From Fig. 3.48(c)

$$v_S = v_L - v_C = 0 - 10\text{V} = -10\text{V}$$

(b) $t > 0$

Capacitor voltage and inductor current can not change suddenly. So

$$v_C(0^+) = 10V, \quad i_L(0^+) = -1 \text{ mA}, \quad v_L(0^+) = 0$$

As $i_S = 0$, current source acts as open circuit. So *RC* and *RL* operate independently.

RC Circuit

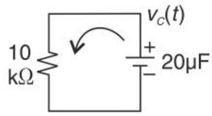


Fig. 3.54(c)

$$\tau = 10 \times 10^{-3} \times 20 \times 10^{-9}$$

$$= 0.2 \times 10^{-3} \text{ s}$$

Capacitor will fully discharge through resistor. So

$$v_C(\infty) = 0$$

Therefore

$$v_C(t) = v_c(0^+)e^{-t/\tau} = 10\, e^{-5 \times 10^3 t}$$

RL Circuit The circuit is drawn in Fig. 3.48(d)

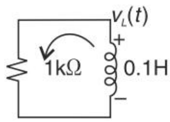


Fig. 3.54 (d)

$$i_L(0^+) = -1\text{mA}$$

$$i_L(\infty) = 0; \text{ current decays to zero}$$

$$\tau = \frac{L}{R} = \frac{0.1}{10^3} = 0.1 \times 10^{-3} \text{ s}$$

Then

$$i_L(t) = -1e^{-t/\tau} = -e^{-t/(0.1\times 10^{-0.3})} = -e^{-10\times 10^3 t}$$

It gives

$$v_L(t) = L\frac{di}{dt} = 0.1\left[\frac{d}{dt}(-e^{-10\times 10^3 t})\right] \text{ mV}$$

$$= 0.1 \times 10 \times 10^3\, e^{-10\times 10^3 t} \text{ mV}$$

$$= e^{-10\times 10^3 t} \text{ V}$$

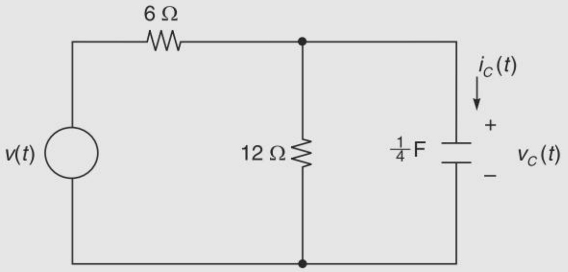
We then have

$$v_S = v_L(t) - v_c(t)$$

or

$$v_s = e^{-10\times10^3 t} - 10e^{-5\times10^3 t}\ \text{V}$$

3.24 In the circuit of Fig. 3.55(a)



3.55(a)

$$v(t) = 12, \quad t < 0$$
$$= 6\cos t\,;\ t > 0$$

Find $v_C(0^+)$, $i_C(0^+)$ and $\dfrac{dv_C}{dt}(0^+)$

Solution

At $t = 0^-$, the capacitor has been fully charged and acts as an open circuit

$v_e(0^-)$ = voltage across 12Ω

$$= \left(\frac{12}{6+12}\right)\times 12 = 8\text{V}$$

$t > 0$

At $v_C(0^+) = 8\text{V}$ acts like a voltage source

$$v(0^+) = 6\cos t\,\big|_{t=0} = 6\text{V}$$

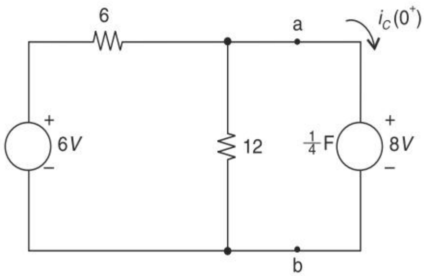


Fig. 3.55 (b)

The equivalent circuit at $t = 0^+$ is drawn in Fig. 3.55(b). We find Thevenin equivalent at *ab*.

$$V_{TH} = \left(\frac{12}{6+12}\right).6 = 4\text{V},\ R_{TH} = \frac{6\times12}{6+12} = 4\ \Omega$$

The circuit is redrawn in Fig. 3.49(c) from which we find

$$i_C(0^+) = \frac{4-8}{4} = -1\text{A}$$

Capacitor current is given by

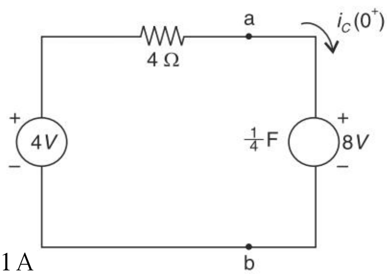


Fig. 3.55 (c)

$$i_C(t) = C\frac{dv_C(t)}{dt} \Rightarrow i_C(0^+) = \frac{1}{4}\frac{dv_C}{dt}(0^+) = -1\ \text{A}$$

$$\therefore \frac{dv_C}{dt}(0^+) = 4\ V/s$$

3.25 In the circuit of Fig. 3.56(a) the switch S is closed at $t = 0$. Find $v_C(t)$, $t > 0$.

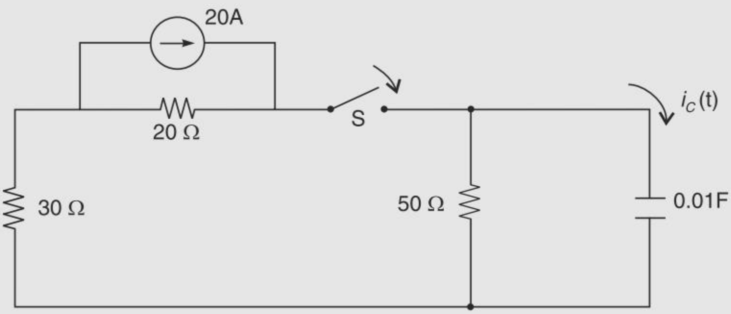


Fig. 3.56 (a)

Solution

Switch Closed $t = 0$

The capacitor acts as short-circuit at $t = 0^+$ The corresponding circuit is shown in Fig. 3.56(b) $i_C\,(0^+)$ is the same as current through 30Ω. We find from the circuit

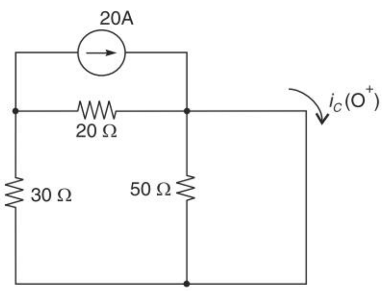


Fig. 3.56 (b)

$$i_C\,(0^+) = 20 \times \frac{20}{20 + 30} = 8\text{A} \qquad \text{(i)}$$

At $t = \infty$, capacitor gets fully charged and so acts as open-circuit. So

$$i_C(\infty) = 0 \qquad \text{(ii)}$$

For the switch-closed circuit with current source open

$$R_{eq} = \frac{50 \times 50}{100} = 25\Omega$$

$$C = 0.01 \text{ F}$$

Then

$$\tau = R_{eq}C = 25 \times 0.01 = 0.25 \text{ s} \qquad \text{(iii)}$$

From results of Eqs. (i), (ii) and (iii), we can write

$$v_C\,(t) = v_C\,(\infty) + [v_C\,(0^+) - v_C\,(\infty)]e^{-t/\tau}$$

or

$$v_C\,(t) = 8e^{-t/\,0.25} = 8e^{-4t} \text{ V}$$

3.26 In the circuit of Fig. 3.57 the switch has been closed for time. It is opened at $t = 0$. Find the values of i_L and v just after the switch close.

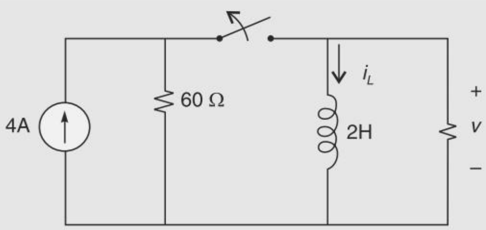


Fig. 3.57

Solution

$t = 0^-$ $\quad i_L(0^+) = 4$ A, $\quad v(0^-) = 0$

$t = 0^+$ $\quad i_L(0^+) = 4$ A, $\quad v(0^+) = -4 \times 20 = -80$ V

3.27 In the circuit of Fig. 3.58, the switch has been closed for a long time. It is opened at $t = 0$. Find i_L and i_x at (a) $t = 0^-$, (b) $t = 0^+$ (c) $i_L(t)$ and $i_x(t)$ and their value at $t = 0.2$ ms.

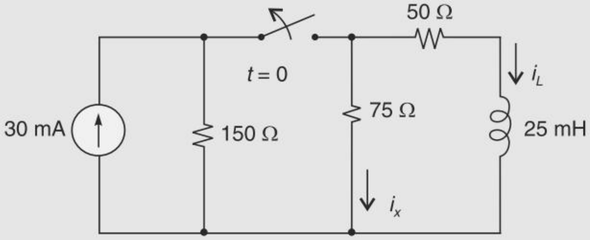


Fig. 3.58

Answer: a) 15 mA, b) 10 mA, c) $i_L = 15e^{-5\times10^3 t}$ mA, $i_x(t) = -i_L(t)$, d) 5.52 mA, –5.52 mA

Solution

$t = 0^-$ $\quad 150 \,||\, 75 = 50\ \Omega$

$i_L(0^-) = 30/2 = 15$ mA

$150 \,||\, 50 = 37.5\ \Omega$

$$i_x(0^-) = \frac{37.5}{37.5 + 75} \times 30 = 10 \text{ mA}$$

$t = 0^+$

$i_L(0^+) = 15$ mA $\qquad i_L(\infty) = 0 \qquad \tau = \dfrac{L}{R} = \dfrac{25 \times 10^{-3}}{75 + 50} = 0.2 \times 10^{-3}$ s

$i_L(t) = 0 + (15 - 0)e^{-t/0.2\times10^{-3}}$

$= 15e^{-5\times10^3 t}$ mA $\qquad i_x(t) = -i_L(t) = -15e^{-5\times10^3 t}$

$t = 0.2$ ms $\quad i_L(0.2) = 5.52$ mA $= -i_x(0.2)$

3.28 In the circuit of Fig. 3.59 the switch has been closed for long time. It is opened at $t = 0$, determine v (t) and the time t_1 at which $v(t_1) = 20$ V.

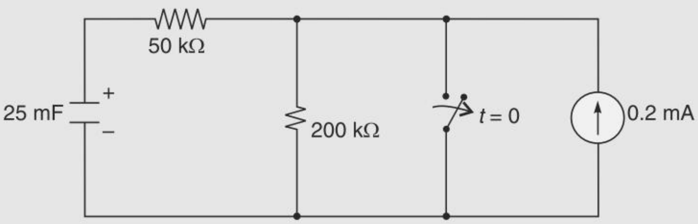


Fig. 3.59

Answer: $40\, e^{-160t}$, $t > 0$, 0.433 ms

Solution

$t = 0^-$ $v(0^-) = 200 \times 0.2 = 40$ V

$t = 0^+$, $v(0^+) = 40$ V

$\tau = RC = 250 \times 25 \times 10^{-6} = 6.25 \times 10^{-3}$ s

$v(t) = 40\ e^{-t/6.25 \times 10^{-3})} = 20$ V

$$\frac{t_1}{0.25 \times 10^{-3}} = -0.693, \quad t_1 = 4.33 \text{ ms}$$

3.29 In the circuit of Fig. 3.60 the switch has been closed for long time. It is opened at $t = 0$. Find $v_C(t)$ and $v_R(t)$, $t > 0$.

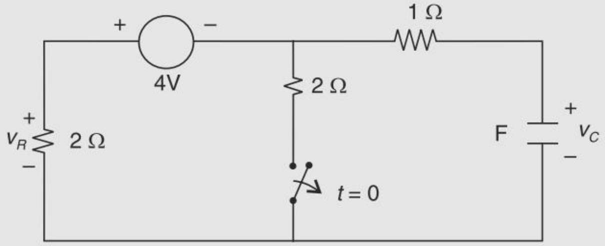


Fig. 3.60

Answer: $v_c\ (t) = 2(2 - e^{-1/15})u(t), \quad v_R(t) = \left(\frac{4}{3}\right)e^{-t/15}u(t)$

3.30 In the circuit of Fig. 3.61 the switch has been closed for a long time. It is opened for a Long time. It is opened at $t = 0$. Determine $v_c\ (t)$.

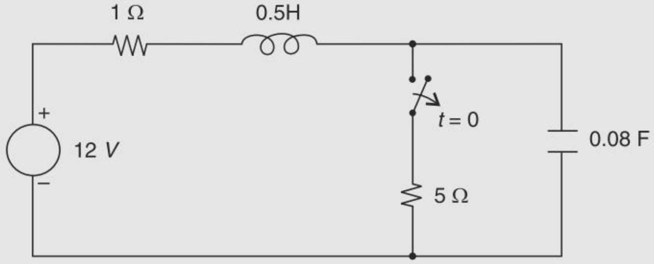


Fig. 3.61

Answer: $12 - e^{-t}\,[2 \cos 4.89t + 0.41 \sin 4.89t]$

Solution

At $t = 0^-$ inductor short and capacitor open

$$i_L(0^-) = \frac{12}{1+5} = 2A \qquad v_c(0^-) = 2 \times 5 = 10 \text{ V}, \quad i_c\ (0^-) = 0$$

At $t = 0^+$, series RLC, $v_c\ (0^+) = 10$ V, $i_c\ (0^+) = i_L\ (0^-) = 2$A, as inductor current cannot change

$$\alpha = \frac{R}{2L} = \frac{1}{2 \times 0.5} = 1 \qquad \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5 \times 0.08}} = 5$$

$$w_d = \sqrt{(5)^2 - 1^2} = 4.89$$
$$v_n(t) = e^{-t}[B_1 \cos 4.89t + B_2 \sin 4.89t]$$
$$v_f = 12V$$
$$v(t) = 12 + e^{-t}[B_1 \cos 4.89t + B_2 \sin 4.89t]$$
$$v(0) = 12 + B_1 = 10, \quad B_1 = -2$$
$$v(t) = 12 + e^{-t}[-2 \cos 4.89t + B_2 \sin 4.89t]$$
$$i_C = -C\frac{dv}{dt}, \quad \frac{dv}{dt}(0^+) = \frac{i_c(0^+)}{C} = \frac{2}{0.08} = 25 \text{ V/s}$$
$$\frac{dv(t)}{dt} = -e^{-t}[-2\cos 4.89t + B_2 \sin 4.89t] + e^{-t}[4.89\times 2\sin 4.89\,t + 4.89B_2 \cos 4.89t]$$
$$25 = 2 + 4.89\,B_2 \qquad B_2 = -4.7$$
$$v(t) = 12 - e^{-t}[2\cos 4.89t - 4.7 \sin 4,89t]$$

3.31 Determine the expression for $v_c(t)$ when the switch is closed at $t = 0$ having been open for long time.

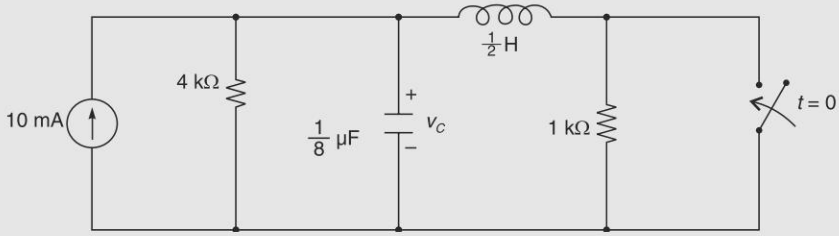


Fig. 3.62

Solution

At $t = 0^-$ $\quad i_L(0^-) = \frac{4}{1+4} \times 10 = 8$ mA

$$v_c(0^-) = (10 - 8) \times 4 = 8 \text{ V}$$

At $t = 0^+$ $\quad i_L(0^+) = 8$ mA $\quad v_R(0^+) = 8$ V

At node left of inductance

$$10 = \frac{8}{4} + i_c(0^+) + 8 \text{ mA} \qquad i_c(0^+) = 0, \frac{dv_c(0^+)}{dt} = 0$$

$$\alpha^2 = \frac{1}{2RC} = \frac{1}{2\times 4\times 10^3 \times \frac{1}{8}\times 10^{-6}} = 10^3 \qquad \alpha = 10\sqrt{10} = 31.62$$

$$\omega_0^2 = \frac{1}{LC} = \frac{1}{\frac{1}{2}\times\frac{1}{8}\times 10^{-6}} = 16\times 10^6 \qquad \omega_0 = 4\times 10^3$$

$$\omega_d = \sqrt{16\times 10^6 - 10^3} \approx 4$$

$$v_n(t) = e^{-31.6\,t}\left[B_1 \cos 4\times 10^3 t\right]$$

$$v_f = 4V \text{ (inductor } sc)$$

$$v_c(t) = e^{-31.6t}[B_1 \cos 4 \times 10^3 t + B_2 \sin 4 \times 10^3 t]$$
$$v_c(0) = 8 = B_1 \rightarrow B_1 = 8$$
$$v_c(t) = e^{-31.6t}[8 \cos 4 \times 10^3 t + B_2 \sin 4 \times 10^3 t]$$
$$\frac{dv_c(t)}{dt} = -31.6\, e^{-31.6}[8 \cos 4\times10^3 t + B_2 \sin 4 \times 10^3 t]$$
$$+ e^{-31.6t}[-8 \times 4 \times 10^3 \sin 4 \times 10^3 t + 4 \times 10^3 B_2 \cos 4 \times 10^3 t]$$
$$0 = -31.6 \times 8 + 4 \times 10^3,\ B_2 \Rightarrow B_2 = 0.0632$$
$$v_c(t) = e^{-31.6t}[8 \sin 4\times10^3 t + 0.0632 \cos 4 \times 10^3 t]$$

3.32 In the circuit of Fig. 3.63 the switch has been closed for long time. It is opened at $t = 0$. Find (a) i_L (10 ms) and (b) t_1 for $i_L(t_1) = 0.5\, i_L(0^+)$

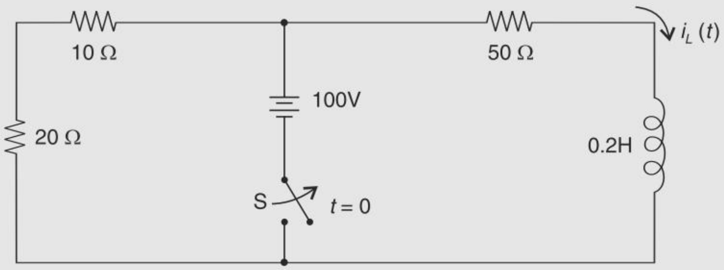


Fig. 3.63

Solution

For $t < 0$, switch has been closed for a long time.

Inductor acts as short circuit.

$$i_L(0^-) = \frac{100}{50} = 2 \text{ A}$$

Switch is opened, inductor current cannot change suddenly.

$$i_L(0^+) = 2 \text{ A}$$

At $t = \infty$, inductor acts as short circuit.

$i_L(\infty) = 0$, inductor current decays to zero.

$$\tau = L/R = \frac{0.2}{50 + 30} = 2.5 \times 10^{-3} \text{ s}$$

Then

$$i_L(t) = 2e^{-t/(2.5\times10^{-3})} = 2e^{-400t}\text{V}$$

(a) i_L (10 ms) $= 2e^{-4} = 0.0366$ A

(b) $i_L(t_1) = 0.5 \times 2 = 1 = 2e^{-400t_1}$

$$e^{-400t_1} = 0.5 \text{ or } -400t_1 = \ln 0.5$$

or

$$t_1 = 1.73 \text{ ms}$$

3.33 In the circuit of Fig. 3.64, find $i_L(t)$ at (a) –0.5*s* (b) 0.5s (c) 2s

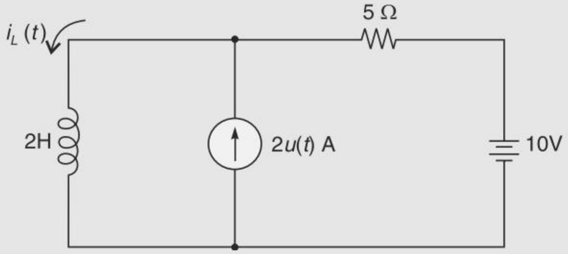


Fig. 3.64

Solution

Current source is switched in only at $t = 0$, For $t < 0$, steady-state exists, inductor acts as short circuit.

$$i_L^-(t) = \frac{10}{5} = 2 \text{ A constant}$$

(a) $i_L(-0.5s) = 2\text{A}$

Switch Closed Opening current source and shorting voltage source

$$R = 5\Omega \qquad L = 2 \text{ H}$$

$$\tau = \frac{L}{R} = \frac{2}{5} = 0.4 \text{ s}$$

$$i_L(0^+) = i_L(0^-) = 2 \text{ A}$$

At $t = \infty$, inductor acts as short circuit, so

$$i_L(\infty) = 2 + \frac{10}{5} = 4 \text{ A}$$

We can directly write

$$i_L(t) = 4 + (2 - 4)\, e^{-t/0.4}$$
$$= 4 - 2e^{-2.5}t \text{ V}$$

(b) $t = 0.5\text{s}$

$$i_L(0.5\text{s}) = 4 - 2e^{-1.25} = 3.71 \text{ V}$$

(c) $t = 1.5\text{s}$

$$i_L(1.5\text{s}) = 4 - 2e^{-3.75} = 3.98 \text{ V}$$

3.34 For the circuit of Fig. 3.65(a), find $i(t)$ after the switch is closed.

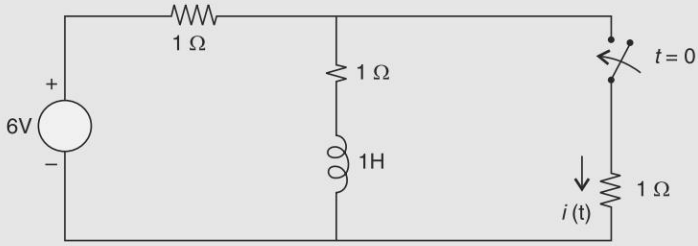


Fig. 3.65(a)

Solution

With switch open; inductor acts a short circuit.

$$i_L(0^-) = \frac{6}{1+1} \quad 3\text{A}$$

With switch closed

$i_L(0^+)$ = 3A, acts as a 3A source

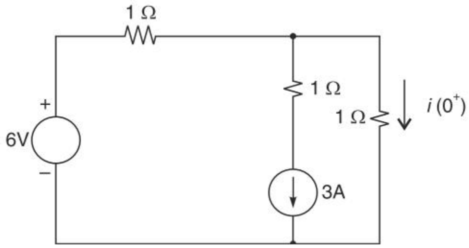


Fig. 3.65(b)

The circuit at t= 0^+ is drawn in Fig. 3.65(b).

By superposition theorem

$$i(0^+) \;=\; \frac{6}{2} - 3 \times \frac{1}{2} = \frac{3}{2}\ \text{A}$$

At $t = \infty$, inductor as short circuit. The circuit is drawn in Fig. 3.65(c).

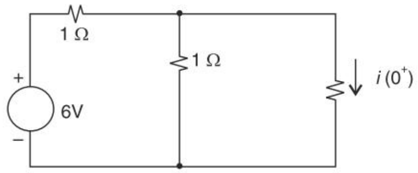


Fig. 3.65(c)

$$i(\infty) = \frac{6}{1+\frac{1}{2}} \times \frac{1}{2} = 2\text{A}$$

Time constant

With switch closed, short circuit the 6V source. The circuit is drawn in Fig. 3.65(d).

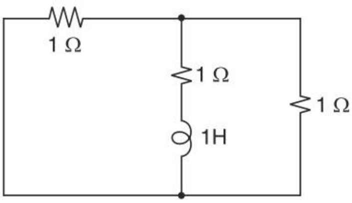


Fig. 3.65(d)

$$R_{eq} = \left(1 + \frac{1}{2}\right) = \frac{3}{2}\Omega$$

$$L = 1\text{H}$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{3/2} = \frac{2}{3}\ \text{s}$$

We can now directly write down

$$i(t) = i(\infty) + [i(0^+) - i(\infty)]e^{-t/\tau}$$

or

$$i(t) = 2 + \left(\frac{3}{2} - 2\right)e^{-3t/2}$$

or

$$i(t) = 2 - \frac{1}{2}\, e^{-3t/2}$$

3.35 For the circuit of Fig. 3.66, find $i_L(t)$.

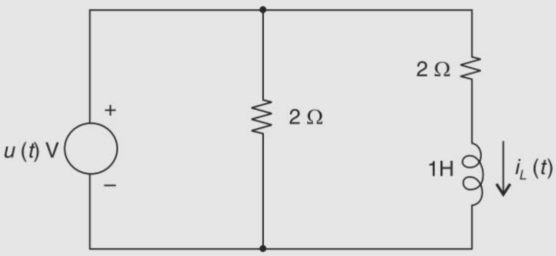


Fig. 3.66

Solution

At $t = 0^-$, the inductor acts as an open circuit

$$i(0^-) = 0$$

$$\therefore \quad i(0^+) = 0$$

At $t = \infty$, the inductor acts as short circuit

Therefore

$$i_L(\infty) = \frac{1}{2} = 0.5\text{A}$$

With voltage source shorted, time constant

$$\tau = \frac{1}{2}\ \text{s}$$

Finally

$$i_L(t) = 0.5 + (0 - 0.5)e^{-2t}$$
$$= 0.5 - 0.5e^{-2t}$$

3.36 In the circuit of Fig. 3.67(a), the switch is closed at $t = 0$, Find (a) $i_L(t)$ and (b) is $\omega(t)$ and their value at $t = 5$ μs.

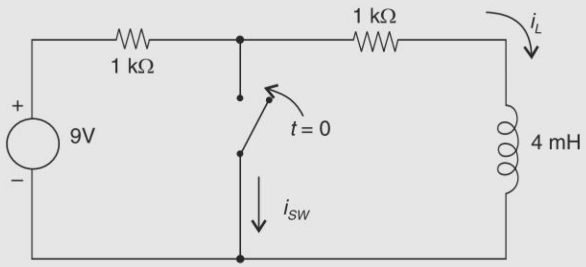


Fig. 3.67(a)

Solution

Switch open—has been in this state for a long time

$$i_L(0^-) = \frac{9}{2} = 4.5\ \text{mA}$$

Switch closed, $t = 0$.

$$i_L(0^+) = 4.5\ \text{mA; inductor current cannot change}$$
$$i_{sw}(0^+) = 0\text{, opens suddenly}$$

At $t = \infty$, inductor act as short circuit. The circuit is drawn in Fig. 3.67(b)

$$i_L(\infty) = 0$$

$$i_{sw}(\infty) = \frac{9}{1} = 9\ \text{mA}$$

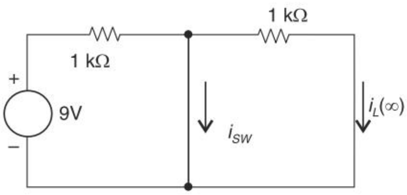


Fig. 3.67(b)

Time constant

Voltage source short circuit drawn in Fig. 3.67(c).

$$\tau = \frac{4\text{ mH}}{1\text{ k}\Omega} = \frac{4\times10^{-3}}{10} = 4\times10^{-6}\text{ s}$$

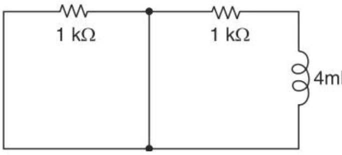


Fig. 3.67(c)

We then have

$$i_L(t) = 4.5e^{-t/(0.4\times10^{-6})}\text{ mA} \quad \text{(i)}$$

$$i_{sw}(t) = 9 + (0\text{–}9)e^{-t/(4\times10^{-6})}$$
$$= 9 - 9e^{-t/(4\times10^{-6})}\text{ mA} \quad \text{(ii)}$$

At $t = 5$ μs

$$i_1(5\mu s) = 4.5e^{-1.25} = 1.29\text{ mA}$$
$$i_{sw}(5\mu s) = 9\text{–}9e^{-1.25} = 6.42\text{ mA}$$

3.37 In the circuit of Fig. 3.68(a), find for the passive elements.

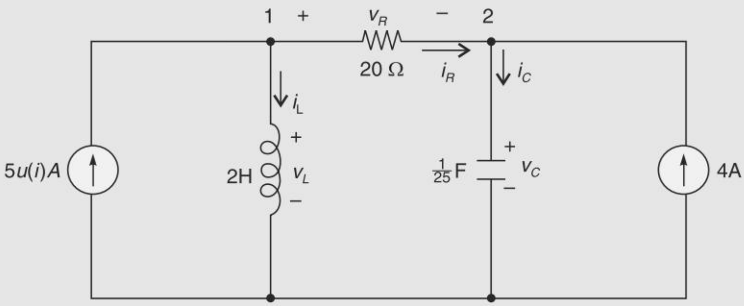


Fig. 3.68(a)

(a) At $t = 0^-$, all voltages and currents

(b) At $t = 0^+$, all voltages and currents

Solution

(a) 5 $u(t)A$ current source is not in action for $t < 0$, so it is open circuit as drawn in Fig. 3.68(b)

At $t = 0^-$

Inductor act as short circuit, capacitor acts as open circuit. So 4A current flows through 20Ω.

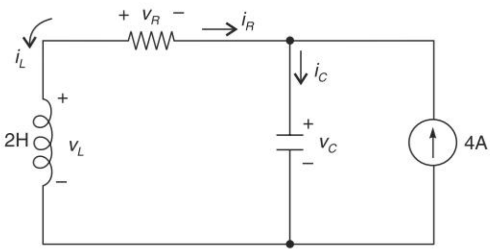


Fig. 3.68(b)

We then have

$$i_R(0^-) = -4\text{ A}, \quad i_L(0^-) = 4\text{ A}$$
$$i_c(0^-) = 0$$
$$v_R(0^-) = -20\times4 = -80\text{ V}$$
$$v_c(0^-) = +80\text{ V}$$
$$v_L(0^-) = 0$$

(b) $$i_L(0^+) = i_L(0^-) = 4\text{ A}$$
$$v_c(0^+) = v_c(0^-) = 80\text{ V}$$

Applying KCL at node 1

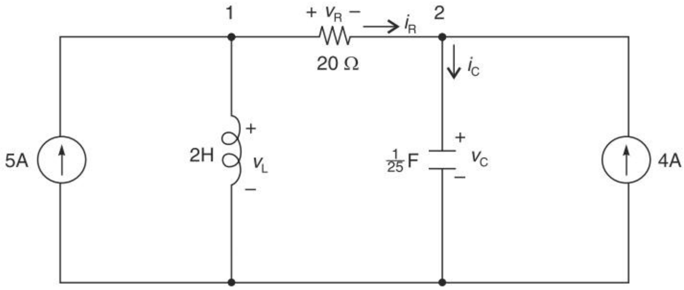


Fig. 3.68(c)

$$-5 + 4 + i_R(0^+) = 0$$

or

$$i_R(0^+) = 1 \text{ A}$$

Then

$$v_R(0^+) = 20\times 1 = 20 \text{ V}$$

Applying KCL at node 2

$$-1 + i_c(0^+) - 4 = 0$$

or $$i_c(0^+) = 5 \text{ A}$$

Applying KVL round the inner mesh

$$-v_c(0^+) - v_R(0^+) + v_L(0^+) = 0$$

$$-80 - 20 + v_L(0^+) = 0$$

or $$v_L(0^+) = 100 \text{ V}$$

Observe that values of inductor and capacitor do not play any role.

3.38 In the circuit of Fig. 3.68a, find the derivatives of the three current and voltage at $t = 0^+$

Solution

$$v_L(0^+) = L\frac{\mathrm{d}\, i_L(0^+)}{\mathrm{d}t}$$

or $$\frac{di_L(0^+)}{dt} = \frac{1}{L}v_L(0^+) = \frac{100}{2} = 50 \text{ A/s}$$

$$v_C(0^+) = C\frac{\mathrm{d}v_C(0^+)}{\mathrm{d}t}$$

or $$\frac{\mathrm{d}\, v_C(0^+)}{\mathrm{d}t} = \frac{1}{C}\, i_C(0^+) = 25\times 5 = 125 \text{ V/s}$$

In Fig. 3.68(a) at node 1

$$-5 + i_L + i_R = 0$$

Taking derivative

$$\left.\frac{\mathrm{d}\, i_R}{\mathrm{d}t}\right|_{t=0} = \left.\frac{di_L}{dt}\right|_{t=0^+} \quad \text{or} \quad \frac{di_R(0^+)}{dt} = -\ 50 \text{ A/s}$$

Similarly at node 2

$$-4 + i_c - i_R = 0$$

Taking derivate and evaluating at $t = 0^+$

$$\frac{d\,i_C(0^+)}{dt} = \frac{d\,i_R(0^+)}{dt} = -50 \text{ A/s}$$

Now $v_R = 2\,i_R$

$$\therefore \quad \frac{d\,v_R(0^+)}{dt} = 20\,\frac{d\,i_R(0^+)}{dt} = 20 \times (-50) = -1000 \text{ V/s}$$

Similarly other derivates at $t = 0^+$ can be found. You may need the KVL equation for the mesh to find $\frac{d\,v_L(0^+)}{dt}$.

3.39 In the circuit of Fig. 3.69, the switch is thrown from '*a*' to '*b*'. Find the expression for $i(t)$.

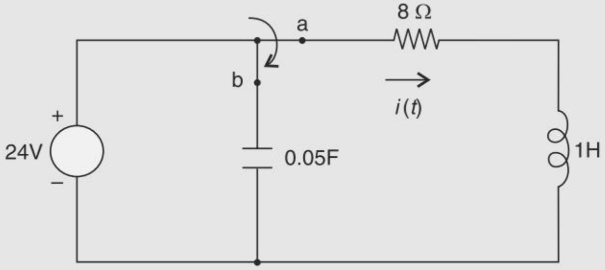


Fig. 3.69

Solution

At $t = 0^-$, switch contact at '*a*'. Inductor acts as short circuit.

$$\therefore \quad i(0^-) = \frac{24}{8} = 3\text{A}$$

As this current flows through inductor,

$$i(0^+) = i(0^-) = 3\text{A}$$

The circuit with switch contact at '*b*' is source-free with initial condition

$$i(0^-) = 3A,\ v_c(0^+) = 0$$

Writing KVL round the mesh (we write $i(t)$ as i)

$$8i + 1\frac{di}{dt} + \frac{1}{0.05}\int i\ dt = 0 \qquad \text{(i)}$$

Taking derivate

$$8\,\frac{di}{dt} + \frac{d^2i}{dt^2} + 20i = 0$$

or

$$\frac{d^2i}{dt^2} + 8\,\frac{di}{dt} + 20i = 0 \qquad \text{(ii)}$$

The characteristic equation is

$$s^2 + 8s + 20 = 0$$

whose roots are

$s = -4 \pm j\,2$; complex conjugate

The solution for the current is then

$$i(t) = e^{-4t}(B_1 \cos 2t + B_2 \sin 2t) \quad \text{(iii)}$$

At $t = 0^+$

$$i(0^+) = 3 = B_1 \text{ or } B_1 = 3$$

Then

$$i(t) = e^{-4t}(3 \cos 2t + B_2 \sin 2t) \quad \text{(iv)}$$

We need another initial condition $\dfrac{\mathrm{d}\,i(0^+)}{\mathrm{d}t}$

Writing the KVL equation around the loop at $t = 0^+$

$$8i(0^+) + 1\cdot\frac{\mathrm{d}\,i(0^+)}{\mathrm{d}t} + v_c(0^+) = 0$$

or

$$\frac{\mathrm{d}i\,(0^+)}{\mathrm{d}t} = -8 \times 3 = -24 \text{ V/s}$$

Taking derivative of Eq. (iv)

$$\frac{\mathrm{d}i}{\mathrm{d}t} = -4e^{-4t}(3\cos 2t + B_2 \sin 2t) + e^{-4t}(-6 \sin 2t + 2B_2 \cos 2t)$$

At $t = 0^+$

$$-24 = -4 \times 3 + 2B_2 \quad \text{or} \quad B_2 = -6$$

Hence

$$i(t) = e^{-4t}(3 \cos 2t - 6 \sin 2t),\ t > 0$$
$$= 6.71 \sin(2t + 63.4°)\,u(t) \quad \text{(v)}$$

Notice that multiplying by $u(t)$ means 'for $t > 0$'.

3.40 In the circuit of Fig. 3.70, the switch has been open for long time. It is closed at $t = 0$. Find the expressions for $v_c(t)$ and $i_{sw}(t)$.

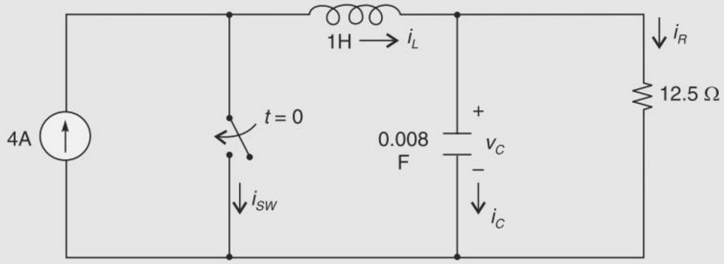


Fig. 3.70

Solution

Switch open; capacitor gets fully charged and acts as open circuit, inductor acts as short circuit. Then

$$v_c(0^-) = 4 \times 12.5 = 50 \text{ V}, \qquad i_L(0^-) = 4 \text{ A}$$
$$i_c(0^-) = 0 \qquad i_R(0^-) = 4 \text{ A}$$

Switch Closed The circuit is now source-free with initial conditions

$$v_c\,(0^+) = 50\text{ V}, \qquad i_L(0^+) = 0$$

It is a parallel *RLC* circuit. In terms of

$$\frac{1}{12.5}v_C + 0.008\frac{d\,v_C}{dt} + \frac{1}{1}\int (v_L = v_C)\,dt = 0 \tag{i}$$

Taking derivative

$$0.008\frac{\mathrm{d}^2 v_C}{\mathrm{d}t^2} + \frac{1}{12.5}\frac{\mathrm{d}v_C}{\mathrm{d}t} + v_C = 0 \tag{ii}$$

or

$$\frac{\mathrm{d}^2 v_C}{\mathrm{d}t^2} + 10\frac{\mathrm{d}v_C}{\mathrm{d}t} + 125v_C = 0 \tag{iii}$$

Its characteristic equation is

$$s^2 + 10s + 125 = 0$$

$$s = -5 \pm j10$$

The solution is then given by

$$v_C(0^+) = e^{-5t}(B_1\cos 10t + B_2\sin 10t) \tag{iv}$$

At $t = 0^+$

$$v_C(0^+) = 50 = B_1$$

Then

$$v_C(t) = e^{-5t}(50\cos 10t + B_2\sin 10t) \tag{v}$$

We need $\dfrac{\mathrm{d}v_C\,(0^+)}{\mathrm{d}t}$. At right side node, we have

$$-i_L\,(0^+) + i_C\,(0^+) + i_R(0^+) = 0$$

$$-4 + i_C(0^+) + 4 = 0$$

$$i_C\,(0^+) = 0 = C\frac{\mathrm{d}v_C\,(0^+)}{\mathrm{d}t}$$

$$\therefore \qquad \frac{\mathrm{d}v_C\,(0^+)}{\mathrm{d}t} = 0$$

Taking derivate of Eq. (v)

$$\frac{\mathrm{d}v_C}{\mathrm{d}t} = -5e^{-5t}(50\cos 10t + B_2\sin 10t) + e^{-5t}(-500\sin 10t + 10B_2\cos 10t)$$

At $t = 0^+$

$$0 = -5 \times 50 + 10\,B_2 \Rightarrow B_2 = 25$$

Hence

$$v_C(t) = e^{-5t}(50\cos 10t + 25\sin 10t)\text{ V}$$
$$= 55.9\,e^{-5t}\sin(10t + 63.4°)\text{ V}$$

To Find $i_{sw}(t)$

$$i_{sw} = -(i_C + i_R)$$

$$i_C = C\frac{dv_C}{dt} = 0.008 \times 55.9 \times (-5)e^{-5t}\sin(10t + 63.4°) + 0.008 \times 55.9\, e^{-5t}[10\cos(10t + 63.4°)]$$

$$i_C = 0.45\, e^{-5t}[10\cos(10t + 3.4°) - 5\sin(10t + 63.4°)]$$

or

$$i_C(t) = 5.03\, e^{-5t}\cos(10t + 26.6°)\ A$$

$$i_R(t) = \frac{v_C}{12.5}\ 4.47\, e^{-5t}\sin(10t + 63.4°)$$

Hence

$$i_{SW}(t) = e^{-5t}[5.03\cos(10t + 26.6°) + 4.47\sin(10t + 26.6°)]$$

The source current of 4A also flows through the switch.

3.41 In the circuit of Fig. 3.71 $V_s = 1V$, Find $v(t)$. The circuit is initially quiescent.

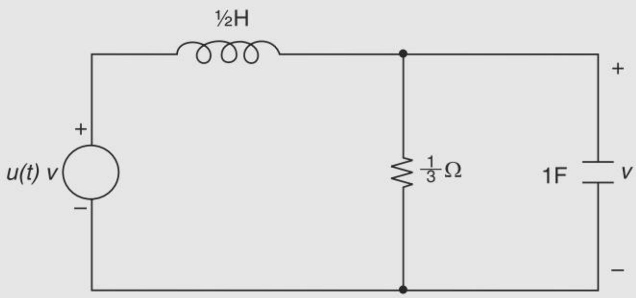


Fig. 3.71

Solution

Natural Response Short circuit voltage source. It is an RLC parallel circuit.

$$\alpha = \frac{1}{2RC} = \frac{3}{2 \times 1} = 1.5$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{2} \times 1}} = \sqrt{2}$$

$\omega_0 < \alpha$, so roots are real

$$\text{Roots } -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} = -1.5 \pm 0.5$$

$$-1, -2$$

Therefore

$$v_n(t) = A_1\, e^{-t} + A_1\, e^{-2t} \qquad \text{(i)}$$

Forced Response $(t = \infty)$

Inductance acts as short circuit, capacitor as open circuit

$$v_f = 1\ \text{V}$$

Total Response $v(t) = v_n(t) + v_f$

$$= 1 + A_1 e^{-t} + A_2\, e^{-2t},\ t > 0 \qquad \text{(ii)}$$

Initial Conditions $(t = 0^+)$

Inductance act as open circuit and capacitor as short circuit. Therefore

$$v(0^+) = 0 \text{ and } \frac{dv\,(0^+)}{dt} = 0$$

Taking derivative of Eq. (ii)

$$\frac{dv(t)}{dt} = -A_1\, e^{-t} - 2\, A_2\, e^{-2t} \qquad \text{(iii)}$$

Letting $t = 0^+$ in Eq. (ii) and (iii), and using initial conditions. we get

$$1 + A_1 + A_2 = 0$$
$$-A_1 - 2A_2 = 0$$

which give

$$A_1 = -2 \qquad A_2 = 1$$

Hence

$$v(t) = (1 - 2e^{-t} + e^{-2t})\, u(t) \qquad \text{(iv)}$$

REVIEW QUESTIONS

1. Explain the meaning and significance of the time constant in single energy storage circuit.
2. Explain the how and why of the initials $(t = 0^+)$ behaviour of an inductance/capacitance in a circuit with *dc* source switched in. Assume that the element is quiescent before switching in.
3. In a circuit with *dc* source, how does an inductance/capacitance behave in steady-state?
4. Explain the meaning of the natural response of a circuit. What determines the natural response—interconnection of passive circuit component only or sources only or both?
5. What is meant by the forced response of a circuit? What determines its nature?
6. Explain what is the characteristic equation of a circuit. How is it determined?
7. What are the types of response of an *RLC* circuit (series/parallel) and what determines each type?
8. What is the meaning and significance of the resonant and damped resonant frequencies of an *RLC* circuit (series/parallel)? Write the expression for these for both the circuits; use standard symbols.
9. For an under-damped *RLC* circuit, write the general expression for the natural response; use standard symbols.
10. For an under-damped *RLC* circuit, what is the nature of decay of sinusoidal oscillation? What determines the time constant of the decay envelope?

PROBLEMS

3.1 In the circuit of Fig. 3.72,

$$v(t) = 12 \text{ V}, \ t < 0$$
$$= 6 \cos t, \ t > 0$$

Determine, $v_C(0^+)$, $i_C(0^+)$ and $\dfrac{dv_C(0^+)}{dt}$

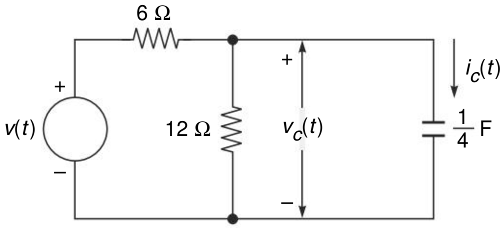


Fig. 3.72

3.2 In the circuit of Fig. 3.73, the switch is closed at $t = 0$. Find $i_C(t)$ for $t > 0$.

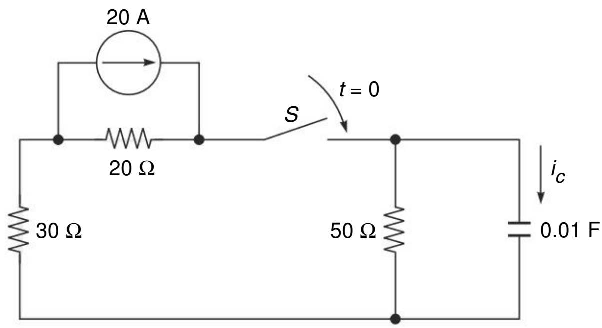


Fig. 3.73

3.3 In the circuit Fig. 3.74, the switch is closed at $t = 0$. Find the expression for $i(t)$ for $t > 0$.

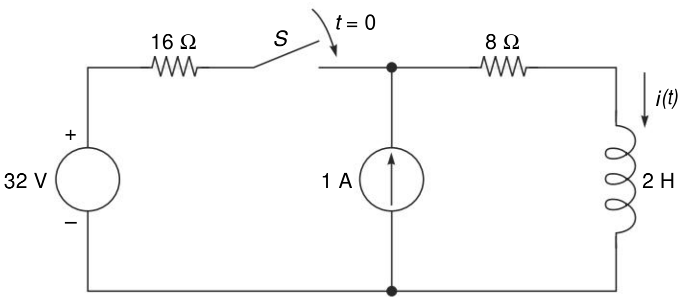


Fig. 3.74

3.4 In the circuit of Fig. 3.75, $v_S = 4u(t)$. Find $v_C(t)$ and $v_R(t)$.

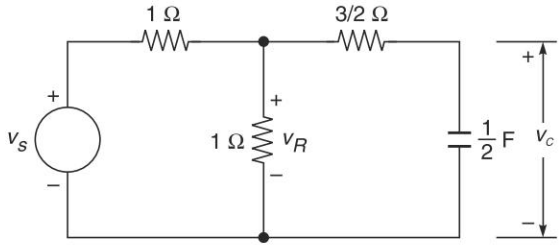


Fig. 3.75

3.5 The circuit of Fig. 3.76 is in steady state when the switch S is closed at $t = 0$. Find $i(t)$, $t > 0$.

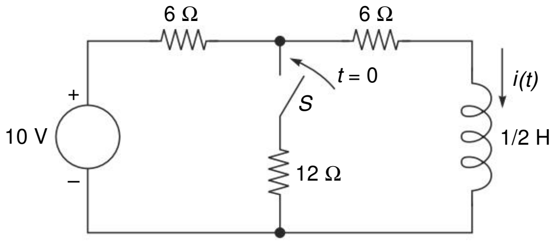


Fig. 3.76

3.6 In the circuit of Fig. 3.77, write the differential equation governing $v(t)$. Find the natural frequencies, resonant frequency and damped resonant frequency.

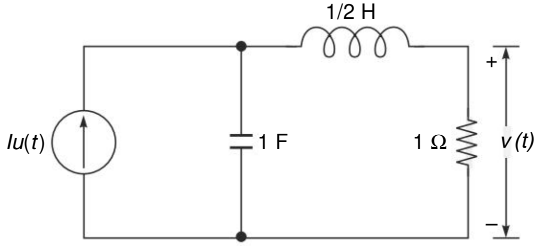


Fig. 3.77

3.7 In the circuit of Fig. 3.78, find the expression for $i(t)$ for $t > 0$.

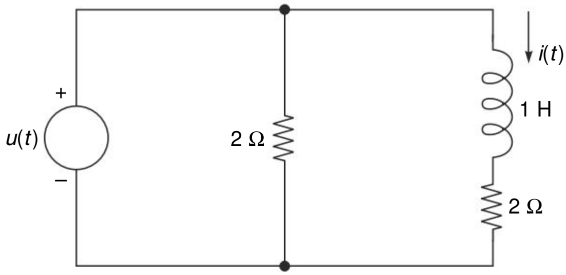


Fig. 3.78

3.8 In the circuit of Fig. 3.79, steady state response has been reached. The switch S is changed to position b at $t = 0$. Determine $v(t)$ and $i_L(t)$.

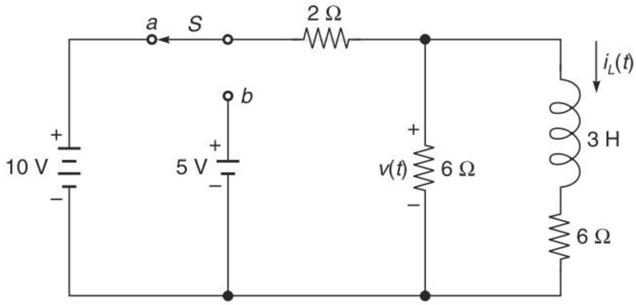


Fig. 3.79

3.9 Steady state is reached in the circuit of Fig. 3.80 with the switch S open. If the switch is closed at $t = 0$, find the current $i(t)$ for $t > 0$.

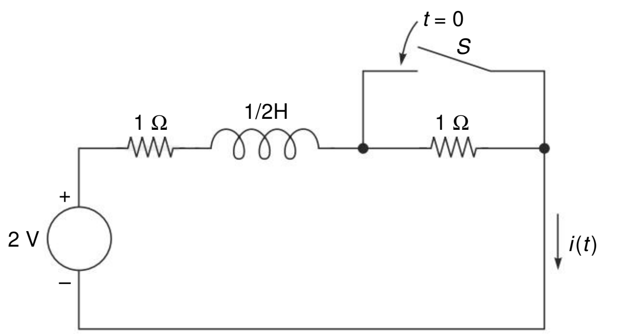


Fig. 3.80

3.10 In the circuit of Fig. 3.81, the switch S has been in position 'a' for the long time. At $t = 0$ the switch is thrown to position 'b'. Determine $i(t)$, $t > 0$ and sketch its waveform.

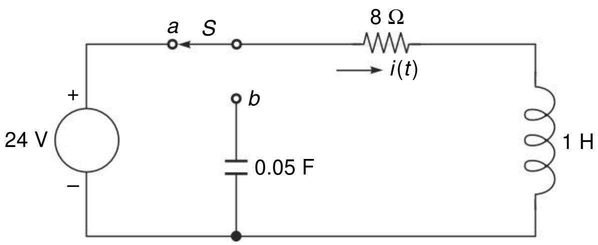


Fig. 3.81

3.11 In the circuit of Fig. 3.82, find the steady state values of i and v after the switch is closed.

3.12 For the circuit of Fig. 3.83, steady state is reached with the switch S open. The switch is closed at $t = 0$.

(a) Express $i(t)$ in terms of $v_C(t)$ and $dv_C(t)/dt$ and hence write the differential equation governing $v_C(t)$.

(b) Find $v_C(0^+)$, $v_C(\infty)$ and $(dv_C/dt)(0^+)$.
(c) Hence determine $v_C(t)$.

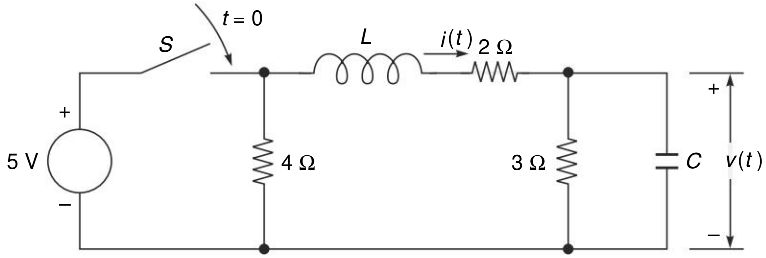


Fig. 3.82

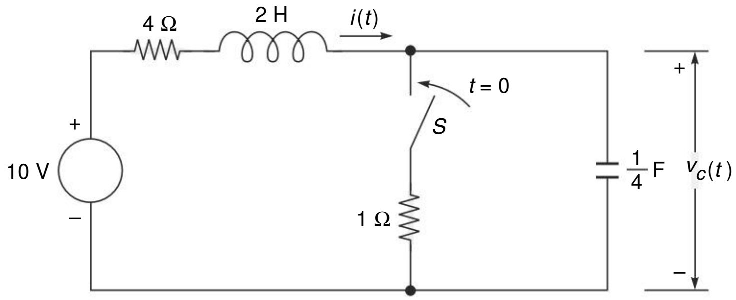


Fig. 3.83

3.13 In the circuit of Fig. 3.84, v_s=0 for $t < 0$ and v_s=1 V for $t > 0$.

(a) Write the differential equation governing $v(t)$ for $t > 0$.
(b) Obtain form part (a) the natural frequencies of the circuit.
(c) Obtain the necessary initial conditions.
(d) Obtain the expression for $v(t)$.

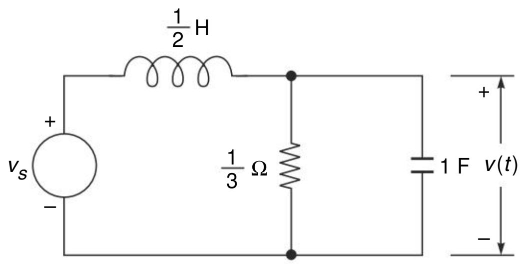


Fig. 3.84

3.14 In the circuit of Fig. 3.85, the switch S is in position 'a' for a long time. At t = 0

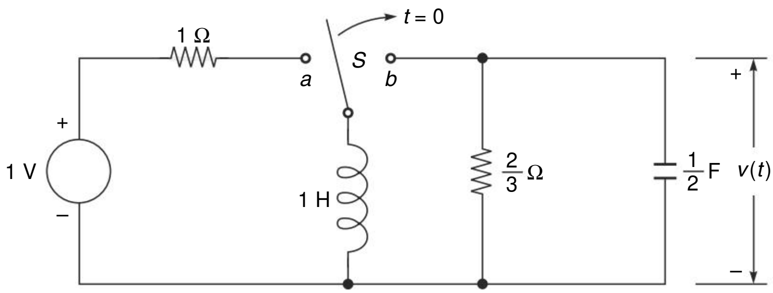


Fig. 3.85

the switch is moved from position '*a*' to position '*b*'. Write the differential equation governing $v(t)$. Determine $v(0^+)$ and $dv/dt(0^+)$. Hence solve for $v(t)$.

3.15 In Fig. 3.87, the initial voltage is $V_0 = 6$ V. Initial current in the inductance is zero. Find i and di/dt at $t = 0^+$.

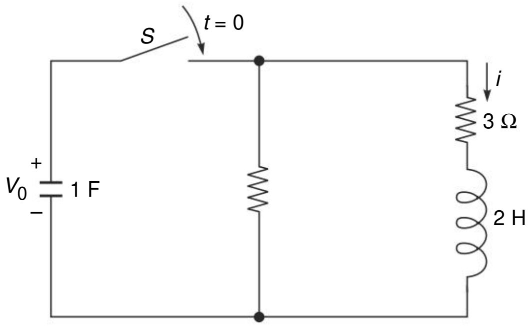


Fig. 3.86

3.16 In the circuit of Fig. 3.88, find the expression for $v(t)$.

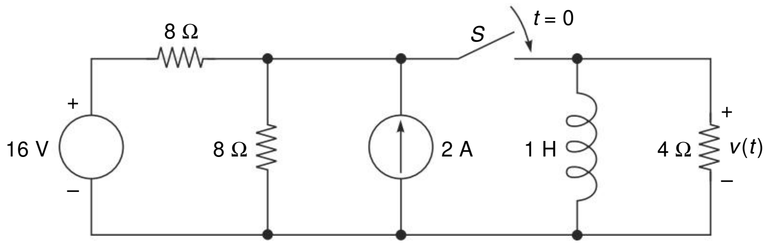


Fig. 3.87

Chapter 4

STEADY STATE ANALYSIS FOR SINUSOIDAL EXCITATION

4.1 INTRODUCTION

So far we have studied the generalized techniques of nodal and mesh analysis of circuits along with certain theorems that help to reduce the work involved in circuit analysis under special situations. The circuits studied were excited from dc (constant) sources. In Chapter 3, we discussed the sudden application of dc sources giving the complete circuit response comprising natural and forced responses upon which initial conditions are superimposed to obtain the response expression. We also discovered that the natural response of dissipative (with a resistive element) circuits vanishes with time wherein the concept of time constant (or natural frequency) was introduced. The techniques of finding initial conditions and steady-state response of circuits were studied by examining the behaviour of reactive elements (inductance, capacitance) at $t = 0^+$ and $t = \infty$. We also studied passive circuit reduction techniques which are helpful in reducing unwanted nodes/meshes.

In this chapter, we shall study the steady state circuit behaviour to sinusoidal excitations. The powerful concept of *phasor* will be introduced and we shall discover that all the techniques of circuit analysis and theorems studied apply to phasor currents and voltages.

4.2 SINUSOIDAL FUNCTION

Waveform of sinusoidal function (voltage/current) was introduced in Chapter 1. Some of its attributes studied so far are summarized below.

- Waveform is periodic, repeats every time period T (second) or after angle 2π. Periodicity is expressed as frequency f in Hz (cycles/second) or in angular frequency $\omega = 2\pi f$ rad/s.
- Sinusoidal waveform has half wave and quarter wave symmetries.
- Sinusoidal waveform has an associated phase which depends upon the reference time selected.

Phase Difference

Consider two sinusoidal waveforms, one voltage and one current.

$$v = v_m \cos(\omega t + \alpha) \tag{4.1}$$

$$i = I_m \cos(\omega t + \beta) \tag{4.2}$$

where V_m and I_m are maximum or *peak* values of respective voltage and current.

These waveforms are sketched in Fig. 4.1 with the assumption of $\beta < \alpha$. From this sketch it is observed that:

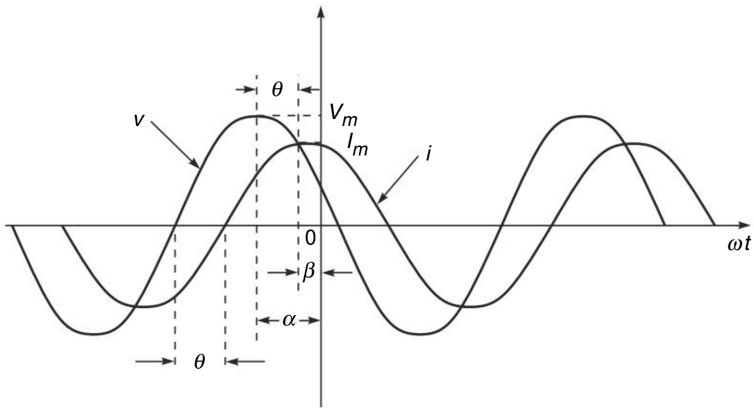


Fig. 4.1 Sinusoidal current and voltage waveforms with a phase difference

Waveform of i is displaced in time (or angle) from that of v, i.e. v and i *differ in phase*. Positive peaks (or other corresponding instantaneous values) of i occur later than those of v by an angle $\theta = \alpha - \beta$. This fact is expressed by stating that i lags v by angle θ or v leads i by angle θ. The situation could also be reversed.

By shifting the reference time (angle), the waveforms of Eqs. (4.1) and (4.2) could be expressed as

$$v = V_m \cos \omega t \tag{4.3}$$

$$i = I_m \cos(\omega t - \theta) \tag{4.4}$$

which also indicate the fact that i lags v by angle ω with the difference that v has a phase angle of zero. Such a waveform is known as a *reference*. It is convenient to use a reference waveform (voltage/current) with respect to which the phases of other voltages/currents in the circuit are expressed.

RMS (Effective) Value

The alternating voltage or current waveshape completes a certain number of cycles in one-second, each cycle comprising an identical positive and negative half-cycle. The value of the quantity varies from instant to instant, peaking at a certain instant only. In specifying a varying voltage or current, its maximum or peak value is normally not used. The *root mean square (rms)* value of the alternating voltage or current is often used in practice to specify the quantity. It is also called the *effective* or *virtual* value of the alternating quantity. During measurements, ammeters and voltmeters register only this quantity for all types of alternating voltage or current waves. The rms value also specifies the 'rating' of an electric motor for a varying duty cycle.

General expression for calculation of rms value of a periodic wave is

$$I_{\text{eff}} = I_{\text{rms}} = \sqrt{\text{average } i^2(t)}$$

$$= \sqrt{\left(\frac{i_1^2 + i_2^2 + \cdots + i_n^2}{n}\right)}$$

= Square root of the mean of the squares of the instantaneous currents over one cycle

General expression for the average value of current wave is

$$I_{\text{av}} = \frac{i_1 + i_2 + \cdots + i_n}{n}, \text{ over positive half cycle}$$

$$i = I_m \cos \omega t = I_m \cos \frac{2\pi}{T} t$$

Instantaneous power dissipation in a resistance is

$$p = i^2 R$$

Average power dissipation over one cycle (time period T) is

$$P = \left(\frac{1}{T}\int_0^T i^2 R \, \mathrm{d}t\right)$$

$$= I^2 R \tag{4.5}$$

where

$$I = I_{\text{(rms)}} = \left[\frac{1}{T}\int_0^T i^2 \, \mathrm{d}t\right]^{1/2} = \text{root mean square (rms) current}$$

Consider an alternating current (ac) of sinusoidal waveform.

$$i = I_m \cos \omega t = I_m \cos \frac{2\pi}{T} t$$

$$I_{\text{(rms)}} = \left[\left(\frac{1}{T}\int_0^T I_m^2 \cos^2 \frac{2\pi}{T} t\right) \mathrm{d}t\right]^{1/2}$$

$$= \frac{I_m}{\sqrt{2}} \tag{4.6a}$$

$I_{\text{av}} = 2\, I_m/\pi$ (average* is taken over positive half-cycle) (4.6b)

The *form factor* $= I\,(\text{rms})/I_{\text{av}}$; indicative of any periodic waveform

$= \pi/2\sqrt{2} = 1.11$ for the sinusoidal current/voltage

The *peak factor* $= I_m/I\,(\text{rms})$; indicative of any periodic waveform

$= \sqrt{2}$ for the sinusoidal current/voltage

This is also called *crest factor* or *amplitude factor.*

In fact as per Eq. (4.5), an alternating current will deliver to a resistance the same power as a direct current of value equal to the rms (or *effective*) value of the alternating current.

The rms value of an alternating voltage is similarly defined and the average power delivered to a resistance would be

* The average of a sinusoidal current over one cycle is zero. The average over half positive cycle is meaningful when we deal with rectification of sinusoidal current.

$$P = \frac{V^2}{R}$$

where $$V = \frac{V_m}{\sqrt{2}};\ v = V_m \cos \omega t \tag{4.7}$$

Because of the average power relationship of Eqs. (4.6) and (4.7), it is customary to express the magnitude of alternating current/voltage in terms of rms value. An ac ammeter or voltmeter would read such a value.

In terms of rms values, the instantaneous current and voltage are expressed as

$$i = \sqrt{2}\, I \cos(\omega t + \alpha) \tag{4.8}$$

$$v = \sqrt{2}\, V \cos(\omega t + \beta) \tag{4.9}$$

Example 4.1 Controlled rectifiers are employed to convert ac to dc. The output of a controlled rectifier has waveform as shown in Fig. 4.2. Find
(a) the average value, and (b) the rms value.

Solution

(a)
$$I_{av} = \frac{1}{2\pi} \int_{\pi/3}^{\pi} 20 \sin \omega t \, d(\omega t)$$
$$= 4.775 \text{ A}$$

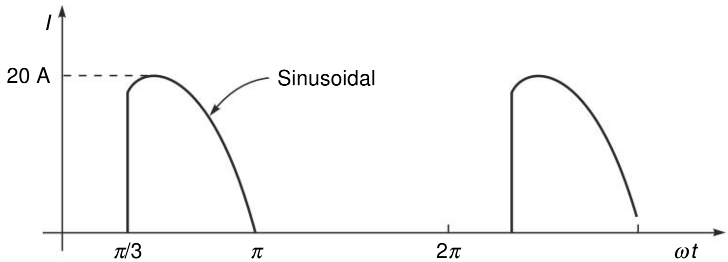


Fig. 4.2

(b)
$$I_{rms} = \left[\frac{1}{2\pi} \int_{\pi/3}^{\pi} 20^2 \sin^2 \omega t \, d(\omega t)\right]^{1/2}$$
$$= 8.97 \text{ A}$$

Example 4.2 Find the average value of the full wave rectified sine wave shown in Fig. 4.3.

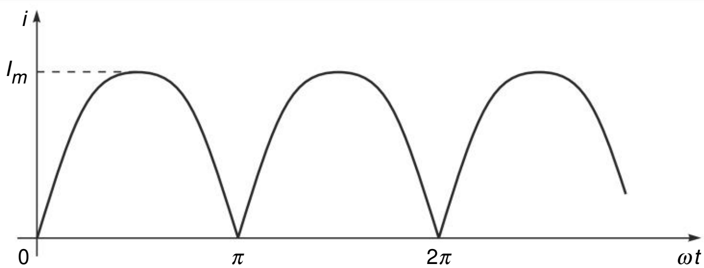


Fig. 4.3

Solution

$$i = I_m \sin \omega t \quad 0 < \omega t < \pi$$
$$= -I_m \sin \omega t \quad \pi < \omega t < 2\pi$$
$$I_{av} = \frac{1}{\pi}\int_0^{\pi} I_m \sin \omega t \, d(\omega t)$$
$$= \frac{I_m}{\pi} \tag{4.10}$$

Phasor Representation

Consider a sinusoidal waveform represented as

$$a = \sqrt{2} A \cos(\omega t + \theta) \tag{4.11}$$

where A = rms amplitude, $\omega = 2\pi f$ rad/s (frequency),
θ = phase angle

Equation (4.11) can be written as*

$a = \text{Re}\,[\sqrt{2}\, A e^{j\theta} e^{j\omega t}]$; Re means take the real part of

or $$a = \text{Re}\,[\sqrt{2}\, \bar{A} e^{j\omega t}] \tag{4.12}$$

where $$\bar{A} = A e^{j\theta} = A \angle \theta \tag{4.13}$$

The representation $\bar{A}$ (as in Eq. (4.13) from which sinusoidal waveform can be reconstructed as per operation of Eq. (4.12) (multiply by $\sqrt{2}\, e^{j\omega t}$ and take real part

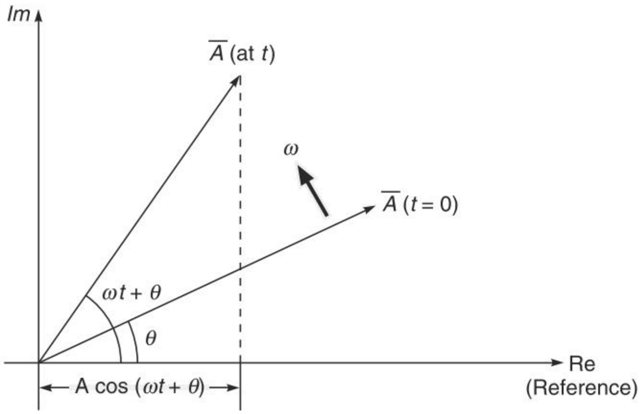


Fig. 4.4 Phasor representation of sine wave

* If sine wave is used as a reference say

$$a = \sqrt{2} A \sin(\omega t + \theta)$$

we can also write it as

$a = Im\,[\sqrt{2} A e^{j\theta} e^{j\omega t}]$; where Im means take the imaginary part of

or $$a = Im\,[\bar{A} e^{j\omega t}]$$

where $$\bar{A} = A e^{j\theta} = A \angle \theta$$

Instantaneous value could now correspond to projection of phasor tip on the imaginary axis. We shall mostly employ cosine as reference.

of) is known as a *phasor*. The expression $e^{j\omega t}$ imparts rotation to the phasor in the complex plane as shown in Fig. 4.4. The projection of the phasor tip on the real axis when multiplied by $\sqrt{2}$ yields the instantaneous value of the original sine wave.

When dealing with addition and subtraction operation of sine waves of the same frequency represented as phasors, it is immediately seen that these would remain fixed relative to each other so that the rotation idea can be kept in the background.

Consider the addition of two waves

$$\sqrt{2}A_1 \cos(\omega t + \theta_1) + \sqrt{2}A_2 \cos(\omega t + \theta_2) = \sqrt{2}\ A \cos(\omega t + \theta) \tag{4.14}$$

In phasor form

$$\bar{A}_1 e^{j\theta_1} + \bar{A}_2 e^{j\theta_2} = \bar{A} e^{j\theta} \tag{4.15}$$

or

$$\bar{A}_1 + \bar{A}_2 = \bar{A} \tag{4.16}$$

This operation is shown in Fig. 4.5(a). Operation $\bar{A}_1 - \bar{A}_2$ is shown in Fig. 4.5(b).

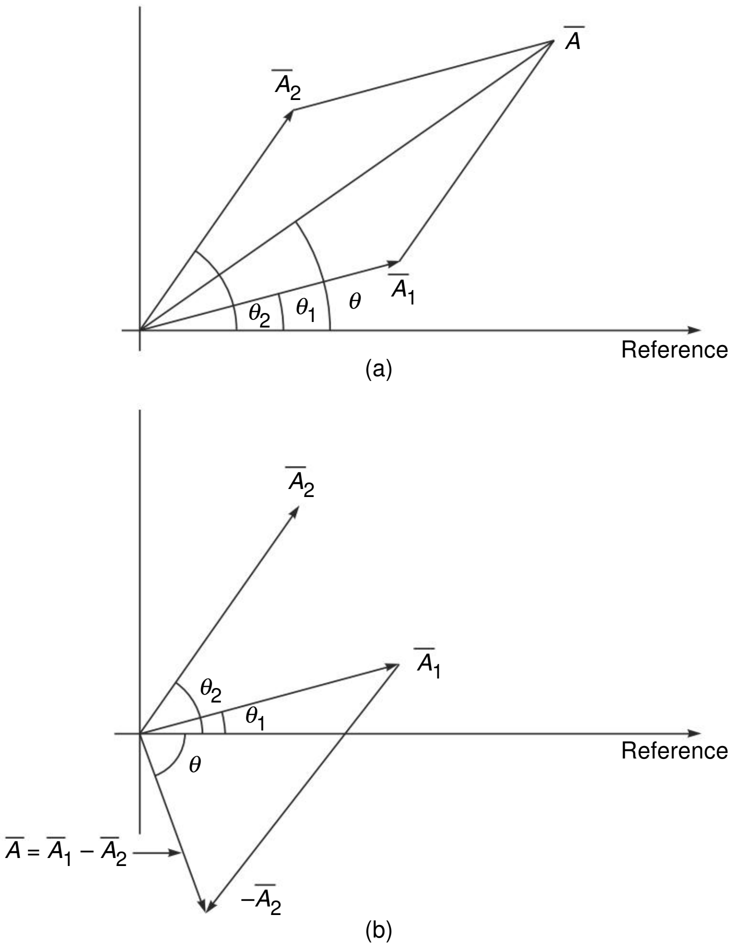


Fig. 4.5 (a) Phasor addition, (b) Phasor subtraction

Example 4.3 Evaluate the expression below using the phasor method

$$e(t) = 100\sqrt{2}\cos(314t - 30°) \pm 200\sqrt{2}\sin(314t - 60°)$$

Solution

$$e(t) = 100\sqrt{2}\cos(314t - 30°) + 200\sqrt{2}\cos(314t - 60°)$$
$$= \sqrt{2}\,E\cos(314t + \theta)$$

We need to find E and θ. Representing the sine waves as phasors

$$\bar{E} = 100\angle -30° + 200\angle -60°$$
$$= (86.6 - j50) + (100 - j173.2)$$
$$= 186.6 - j223.2$$
$$= 290.9\angle -50.1°$$

Hence

$$e(t) = 290.9\sqrt{2}\cos(314t - 50.1°)$$

In the second case

$$\bar{E} = 100\angle -30° - 200\angle -60°$$
$$= (86.6 - j50) - (100 - j173.2)$$
$$= -13.4 + j123.2$$
$$= 123.9\angle 96.2°$$

Hence

$$e(t) = 123.9\sqrt{2}\cos(314t + 96.2°)$$

4.3 SINUSOIDAL STEADY-STATE ANALYSIS

With sinusoidal excitation (forcing function) applied to a circuit, as in the case of dc excitation, the response comprises two parts: *natural response* and *forced response*. The natural or transient response is shortlived which is of no interest to us. We are mainly concerned with the long term steady-state (forced) response. The steady-state response of a circuit (linear) has the same form as the forcing function, that is, it is sinusoidal of the same frequency though the amplitude and phase may change. In other words, the voltage and currents at all points in the circuit are sinusoidal of excitation frequency.

As the derivative and integration of a sinusoid are also sinusoidal, the steady-state solution can be directly obtained from the differential equation of a circuit by substituting in it an assumed sinusoidal solution with unknown amplitude and phase. The amplitude and phase of the solution can then by determined by trigonometric combinations. The process though through straight forward is quite cumbersome and becomes intractable even for any reasonable size network.

We shall study here the phasor method of steady-state sinusoidal circuit analysis, which in effect reduces the circuit differential equation to algebraic form (with complex numbers); which can be easily manipulated to obtain the desired solution.

We shall consider one by one the ideal circuit element R, L, C excited by voltage

$$v(t) = \sqrt{2}\ V \cos(\omega t + \theta) \tag{4.17}$$

We can write

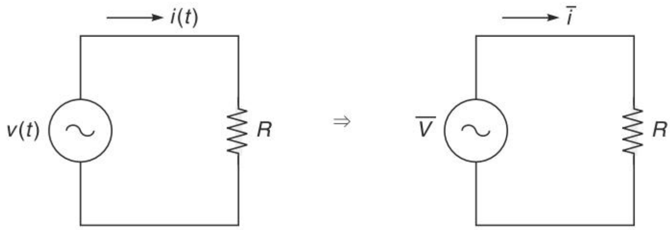
$$v(t) = \text{Re}\,[\sqrt{2}\ V e^{j\theta} e^{j\omega t}],\ \text{Re is real part of} \tag{4.18}$$

Operator Re can be performed on the end result to write the excitation voltage or response current in time form, i.e. in time domain. Thus

$$\underset{\text{Time domain}}{v(t)} \xrightarrow{\text{transforms to}} \underset{\text{Frequency domain}}{\sqrt{2}\ V e^{j\theta} e^{j\omega t} = \overline{V} e^{j\omega t}} \tag{4.19}$$

where $$\overline{V} = \sqrt{2}\, V e^{j\theta};\ \text{a phasor} \tag{4.20}$$

Consider now a resistance, excited by sinusoidal voltage as in Fig. 4.6(a)



(a) In time domain (b) In frequency domain—phasor form

Fig. 4.6

In time domain

$$v(t) = Ri\,(t) \tag{4.21a}$$

In frequency domain

$$\overline{V} e^{j\omega t} = R\overline{I} e^{j\omega t}$$

or in phasor form $$\overline{V} = R\overline{I} \tag{4.21b}$$

or $$\overline{I} = G\overline{V} \tag{4.21c}$$

Its graphical representation called *phasor diagram* is shown in Fig. 4.7 wherein it is obvious that $\overline{V}$ and $\overline{I}$ are in phase.

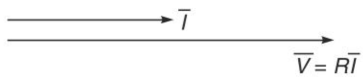
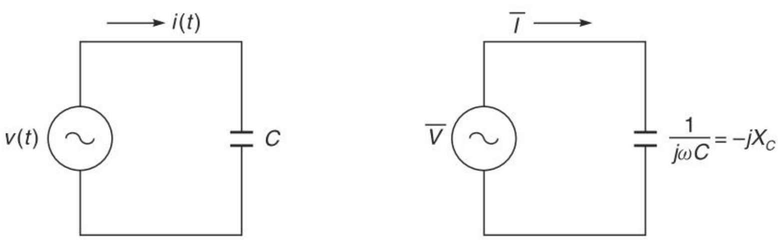


Fig. 4.7 $\overline{V}$–$\overline{I}$ relationship for resistance

Consider now a capacitance *excited* by sinusoidal voltage as in Fig 4.8(a).



(a) In time domain (b) In frequency domain-phasor form

Fig. 4.8

$$v(t) = \frac{1}{C}\int i(t)\,\mathrm{d}t \tag{4.22}$$

In frequency domain

$$\bar{V}e^{j\omega t} = \frac{1}{C}\int \bar{I}\,e^{j\omega t}\ \mathrm{d}t = \frac{1}{j\omega C}\ \bar{I}e^{j\omega t} \tag{4.23}$$

In phasor from

$$\bar{V} = \frac{1}{j\omega C}\bar{I} = -j\frac{1}{\omega C}\ \bar{I} = -jX_C\bar{I} \tag{4.24}$$

where

$$X_c = \frac{1}{\omega C};\ \textit{capacitive reactance}\ (\Omega) \tag{4.25}$$

Equation (4.24) is also expressed as

$$\bar{I} = j\omega C\,\bar{V} = jB_C\,\bar{V}$$

where $\quad B_c = \omega C = \textit{capacitive susceptance}$ (℧) (4.26)

The phasor diagram of Eqs. (4.25) and (4.26) are drawn in Fig. 4.9. Observe that $\bar{I}$ leads $\bar{V}$ by 90°.

Consider now an inductance excited by sinusoidal voltage as in Fig. 4.10(a).

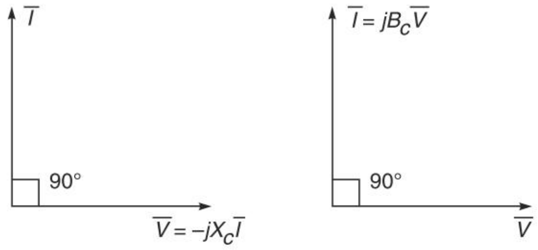
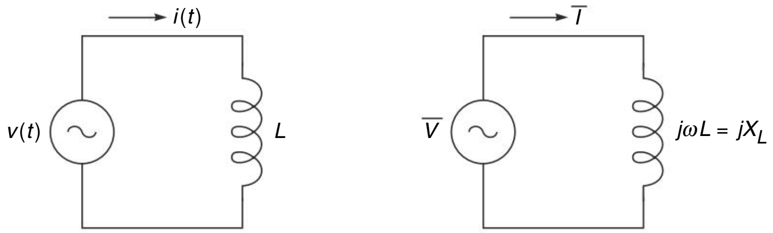


Fig. 4.9 $\bar{V}$–$\bar{I}$ relationship for capacitance



(a) In time domain (b) In frequency domain-phasor form

Fig. 4.10

In time domain

$$v(t) = L\frac{\mathrm{d}i}{\mathrm{d}t}$$

In frequency domain

$$\bar{V}e^{j\omega t} = L\frac{\mathrm{d}}{\mathrm{d}t}[\bar{I}e^{j\omega t}] = j\omega L\bar{I}e^{j\omega t} \tag{4.27}$$

In phasor form

$$\bar{V} = j\omega L\ \bar{I} = jX_L I \tag{4.28}$$

where $\quad X_L = \omega L = 2\pi f L$; inductive reactance Ω (4.29)

In alternative form

$$\bar{I} = \frac{1}{j\omega L}\bar{V} = -j\,\frac{1}{X_L}\,\bar{V} = -\,jB_L\bar{V} \tag{4.30}$$

where $\quad B_L = \dfrac{1}{\omega L} = \dfrac{1}{X_L}$ = *inductive susceptance* (℧) (4.31)

The phasor diagrams for Eqs. (4.28) and (4.30) are drawn in Fig. 4.11.

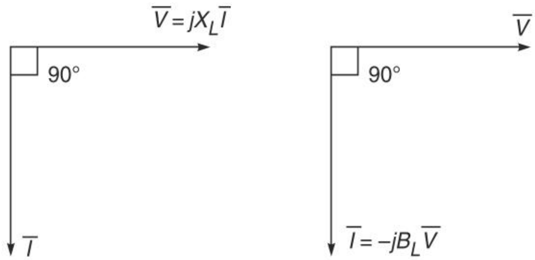


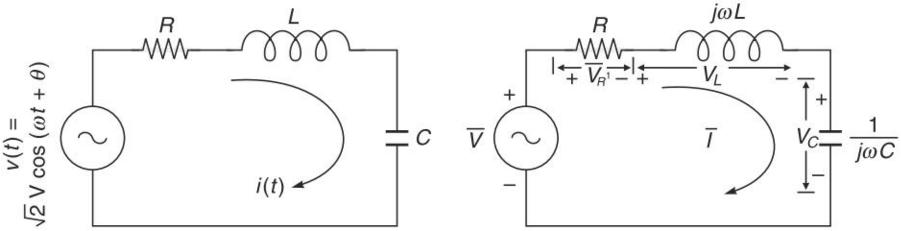
Fig. 4.11 $\bar{V}-\bar{I}$ relationship for inductance

For the three circuit elements the time-domain *v*-*i* relationships and corresponding frequency domain $\bar{V}-\bar{I}$ relationships are summarized in Table 4.1

Table 4.1 Voltage-current relationships in time and frequency domains

Time domain		Frequency domain	
i → R, + v −	$v = Ri$	$\bar{V} = R\bar{I}$	$\bar{I}$ → R, + $\bar{V}$ −
i → L, + v −	$v = L\dfrac{di}{dt}$	$\bar{V} = j\omega L\bar{I}$	$\bar{I}$ → $j\omega L$, + $\bar{V}$ −
i → C	$v = \dfrac{1}{C}\int i\ dt$	$\bar{V} = \dfrac{1}{j\omega C}\,I$	$\bar{I}$ → $1/j\omega C$, + $\bar{V}$ −

Consider now an RLC series circuit excited by sinusoidal voltage as shown in Fig. 4.12(a).



(a) Time domain circuit (b) Frequency domain circuit in phasor form

Fig. 4.12

The Fig. 4.12(a) is redrawn in phasor form (element by element) in Fig. 4.12(b). We can write the phasor equation of the circuit as

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

$$\bar{V} = R\bar{I} + j\omega L\,\bar{I} + \frac{1}{j\omega C}\,\bar{I} \qquad (4.32)$$

or
$$\bar{V} = (R + jX_L - jX_C)\,\bar{I}$$

or
$$\bar{V} = [R + j\,(X_L - X_C)]\,\bar{I} \qquad (4.33)$$

Let
$$X = X_L - X_C$$

Then
$$\bar{V} = (R + jX)\,\bar{I} \qquad (4.34)$$

or
$$\bar{V} = \bar{Z}\,\bar{I} \text{ (\textit{Ohm's law} in phasor form)} \qquad (4.35)$$

Impedance/Admittance

Define

$$\bar{Z} = R + jX = \textit{impedance} \text{ (complex number)} \qquad (4.36a)$$
$$= Z \angle \theta \qquad (4.36b)$$

where

$$Z = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1}\frac{X}{R}$$

The impedance triangle corresponding to Eq. (4.36a) is drawn in Fig. 4.13.

It must be observed here that *impedance is a complex number and not a phasor*.

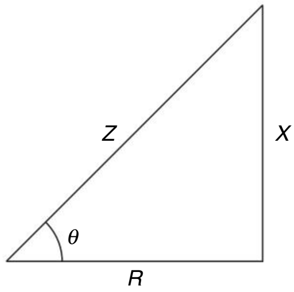


Fig. 4.13

Equation (4.35) can also be written as

$$\bar{I} = \bar{Y}\,\bar{V} \qquad (4.37)$$

where

$$\bar{Y} = \frac{1}{\bar{Z}} = \textit{admittance}\ (℧) \qquad (4.38)$$

$$= \frac{1}{R + jX} = \frac{R}{R^2 + X^2} - j\,\frac{X}{R^2 + X^2}$$

$$= G - jB \qquad (4.39)$$

where G = *conductance* and B = *susceptance*

The phasor diagram of the circuit of Fig. 4.12 is drawn in Fig. 4.14. This corresponds to KVL Eq. (4.33).

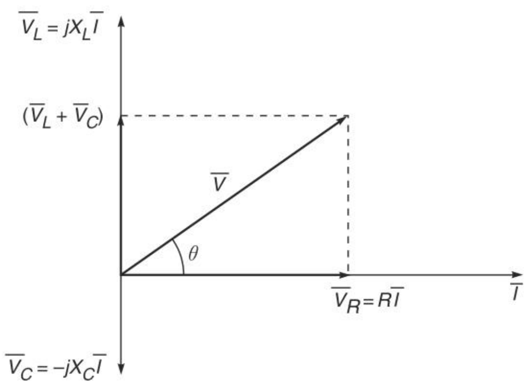


Fig. 4.14 Phasor diagram of RLC series circuit of Fig. 4.12(b)

Series/Parallel Combination of Impedances/Admittances

The same rules apply as for resistances/conductances except that complex number computation is involved.

Impedances in Series

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2 + \ldots \text{ (convenient form)} \tag{4.40}$$

or

$$\frac{1}{\bar{Y}} = \frac{1}{\bar{Y}_1} + \frac{1}{\bar{Y}_2} + \ldots \tag{4.41}$$

Admittances in Parallel

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2 + \ldots \text{ (convenient form)} \tag{4.42}$$

or

$$\frac{1}{\bar{Z}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \ldots \tag{4.43}$$

KVL and KCL Equation (4.32) is the KVL phasor equation for the circuit of Fig. 4.12. Similarly KCL applies in phasor form.

Parallel RLC Circuit Figure 4.15 shows a parallel *RLC* circuit with element values expressed in admittance form. Applying KCL at the single node

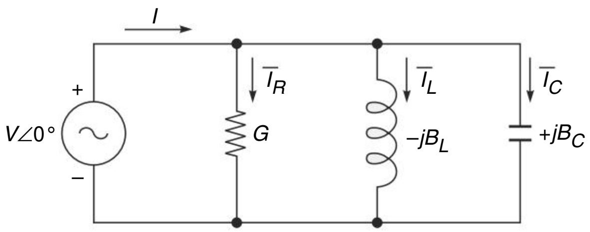


Fig. 4.15 Parallel RLC circuit

$$\begin{aligned}\bar{I} &= \bar{I}_R + \bar{I}_L + \bar{I}_C \\ &= G\bar{V} - jB_L\bar{V} + jB_C\bar{V} \\ &= [G - j(B_L - B_C)]\bar{V}\end{aligned} \tag{4.44}$$

The corresponding phasor diagram is drawn in Fig. 4.16. It follows that

$$\theta = \tan^{-1}\left(\frac{B_L - B_C}{G}\right)$$

The current would lag voltage if $B_L > B_C$ and would lead if $B_C > B_L$, i.e. according to whether inductive susceptance or capacitive susceptance predominates.

Example 4.4 For the circuit of Fig. 4.17, $\bar{I} = 5 \angle 0°$

(a) Find C if $\bar{V} = 100 + j200$ V, $\omega = 1.2$ k rad/s

(b) Find C if $\omega = 200$ rad/s and V = 100 V.

Solution

(a)

$$\omega = 1.2 \text{ k rad/s}$$

$$X_L = 1200 \times 0.05 = 60\ \Omega$$

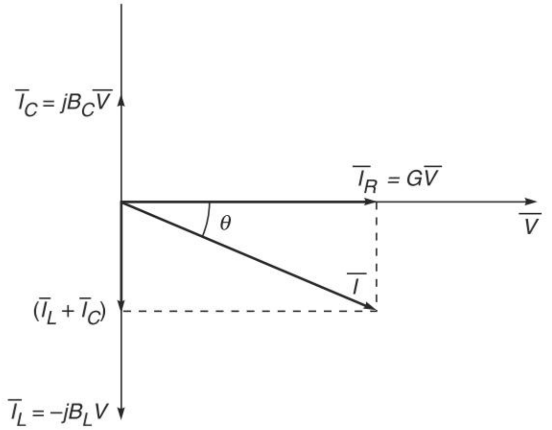


Fig. 4.16 Phasor diagram of *RLC* parallel circuit

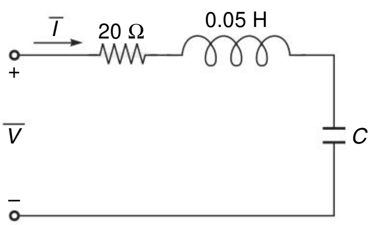


Fig. 4.17

$$X_C = \frac{1}{1200\ C}$$

Now

$$\bar{Z} = \frac{\bar{V}}{\bar{I}}$$

$$20 + j60 - jX_C = \frac{100 + j200}{5\angle 0°}$$

$$20 + j60 - jX_C = 20 + j40$$

or $$X_C = 20 = \frac{1}{1200\ C}$$

or $$C = \frac{1}{24000} = 41.67\ \mu\text{F}$$

(b) $$\omega = 200\ \text{rad/s}$$

$$X_L = 200 \times 0.05 = 10\ \Omega$$

$$V = 5Z\ \text{(Ohm's law in magnitude form)}$$

or $$100 = 5[(20)^2 + (10 - X_C)^2]^{1/2}$$

$$400 = 400 + (10 - X_C)^2$$

$$X_C = 10 = \frac{1}{200\ C}$$

or $$C = 500\ \mu\text{F}$$

Example 4.5 In the circuit of Fig. 4.18, find R and C. Given $V_b = 3V_a$ and V_b and V_a are in quadrature. Find also the phase relation between V, V_b, V_a and I.

Solution

$$X_L = \omega_L = 2\pi fL = 314 \times 0.0255 = 8\ \Omega$$

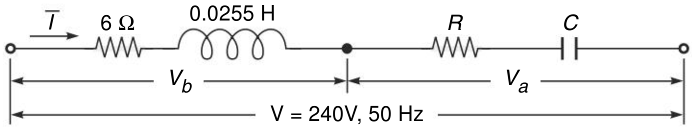


Fig. 4.18

The frequency domain circuit is drawn in Fig. 4.19. Since V_b and V_a are in quadrature

$$V_a^2 + V_b^2 = V^2 = (240)^2$$

or $$V_a^2 + (3V_a)^2 = (240)^2$$

or $$V_a = 75.9 \text{ V}$$

or $$V_b = 227.7 \text{ V}$$

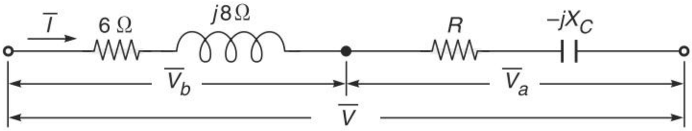


Fig. 4.19

Taking $\bar{I}$ as the reference phasor, the phasor diagram is drawn in Fig. 4.20.

$$\theta_1 = \tan^{-1} 8/6 = 53.1°$$

$$\theta_2 = 90° - 53.1° = 36.9°$$

Also $$(6I)^2 + (8I)^2 = (227.7)^2$$

or $$I = 22.8 \text{ A}$$

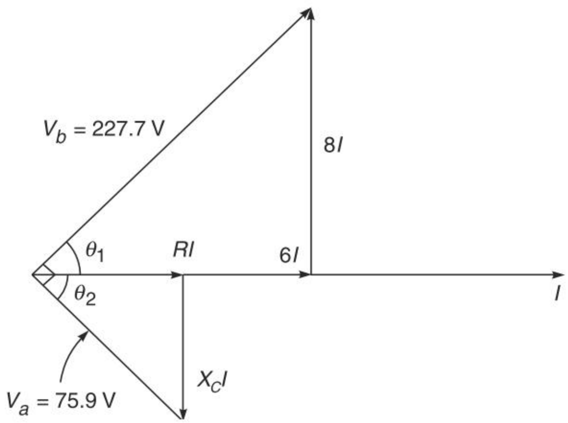


Fig. 4.20

For the *RC* part of the circuit

$$\frac{X_C}{R} = \tan\theta_2 = \tan 36.9° = 0.75 \qquad \text{(i)}$$

Also $\sqrt{[R^2 + X_C^2} \times 22.8 = 75.9$

or $$R^2 + X_C^2 = \left(\frac{75.9}{22.8}\right)^2 = 11.09 \qquad \text{(ii)}$$

Solving Eqs. (i) and (ii)

$$R^2 + (0.75)^2 R^2 = 11.09$$

or $$R = 2.664\ \Omega$$

$$X_C = 2.664 \times 0.75 = \frac{1}{314\ C}$$

or $$C = 1.594\ \text{mF}$$

Example 4.6 Two circuits with impedances of $\bar{Z}_1 = 10 + j15\ \Omega$ and $\bar{Z}_2 = 6 - j8\ \Omega$ respectively are connected in parallel. If the total current supplied is 15 A, find each branch current and their phase angle w.r.t. the total current. What is the voltage across the combination and its phase angle?

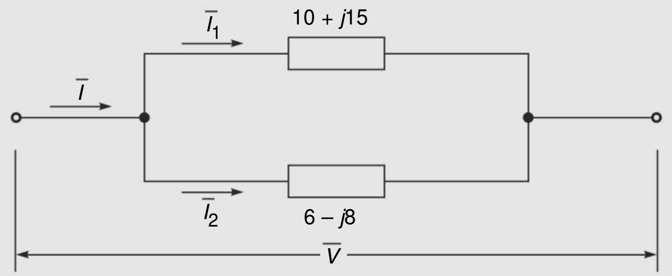


Fig. 4.21

Solution

$$\bar{I} = 15\angle 0°$$

The current will divide in the inverse ratio of the branch impedances.

$$\bar{I}_1 = 15\angle 0° \times \frac{(6 - j8)}{(10 + j15) + (6 - j8)}$$

$$= 15 \times \frac{(6 - j8)}{(16 + j7)}$$

$$= 15 \times \frac{10\ \angle -53.1°}{17.46\ \angle\ 23.6°}$$

$$= 8.59\ \angle -76.7°\ \text{A}$$

$$\bar{I}_2 = 15\ \angle 0° \times \frac{(10 + j15)}{(16 + j7)}$$

$$= 15 \times \frac{18.03\ \angle\ 56.3°}{17.46\ \angle\ 23.6°}$$

$$= 15.49 \angle 32.7° \text{ A}$$

$$\overline{V} = (10 + j15) \times 8.59 \angle -76.7°$$
$$= 18.03 \angle 56.3° \times 8.59 \angle -76.7°$$
$$= 154.9 \angle -20.4 \text{ V}$$

The phasor diagram showing $\overline{I} = \overline{I}_1 + \overline{I}_2$ and $\overline{V}$ is drawn in Fig. 4.22.

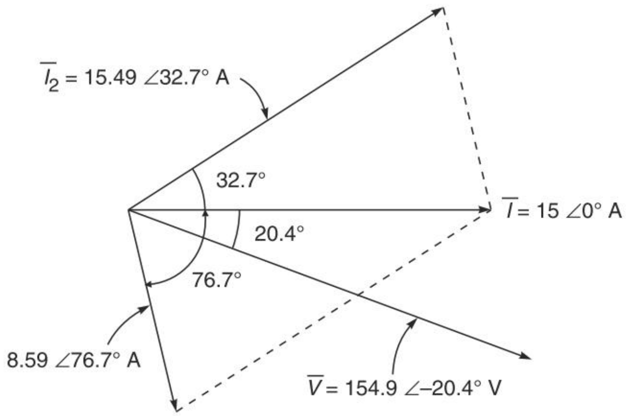


Fig. 4.22

Example 4.7 For the circuit of Fig. 4.23, find $\overline{Z}_{in}$ (input impedance or *driving point* impedance) with

(a) *AB* open-circuited,
(b) *AB* short-circuited, and
(c) *AB* connected through 10 Ω resistance.

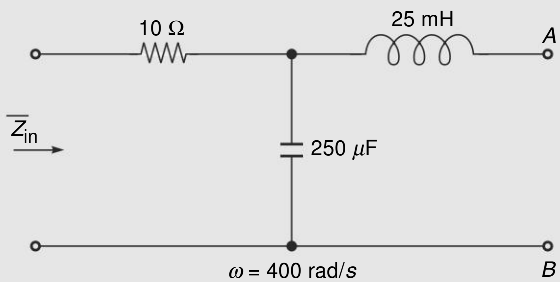


Fig. 4.23

Solution

$$X_L = 400 \times 25 \times 10^{-3} = 10\ \Omega$$

$$X_C = \frac{10^6}{400 \times 250} = 10\ \Omega$$

(a) *AB* open-circuited (Fig. 4.24)

$$\overline{Z}_{in} = (10 - j10) = 14.14 \angle -45°\ \Omega$$

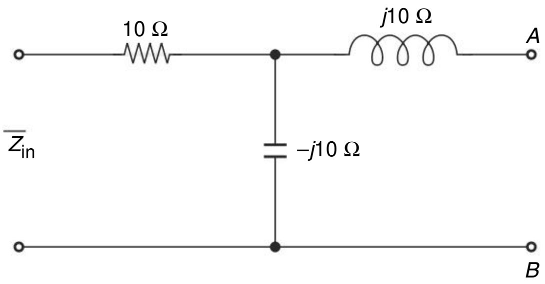


Fig. 4.24

(b) *AB* short-circuited (Fig. 4.25)

$$\overline{Z}_{in} = 10 + \frac{j10 \times -j10}{(j10 - j10)} = 10 + j\,\infty = \infty\ \angle 90°$$

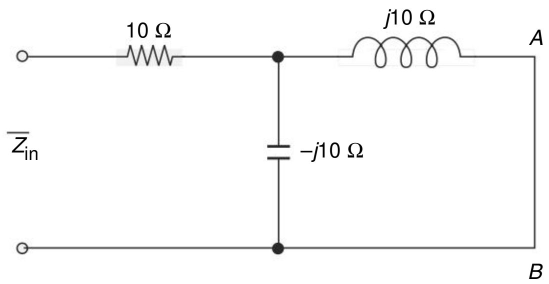


Fig. 4.25

(c) *AB* connected through 10 Ω resistance (Fig. 4.26)

$$\overline{Z}_{in} = 10 + \frac{(10 + j10)\,.\,(-j10)}{10 + j10 - j10}$$

$$= 20 - j10 = 22.36\ \angle -26.6°$$

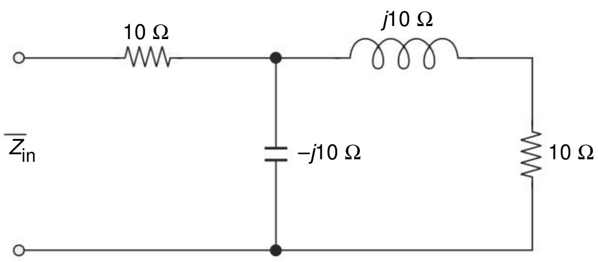


Fig. 4.26

Example 4.8 In the parallel circuit of Fig. 4.27, find

(a) $\overline{I}$ if $\overline{I}_R = 0.02\ \angle 30°$ A, and

(b) $\overline{I}_R$ if $\overline{I} = 2\ \angle -40°$; also find the applied voltage.

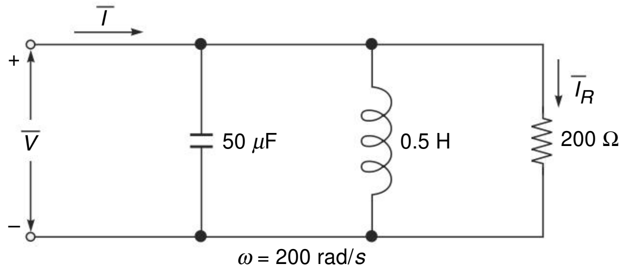


Fig. 4.27

Solution

(a)

$$\overline{I}_R = 0.02 \angle 30° \text{ A}$$

$$\overline{V} \text{ (applied voltage)} = 200 \times 0.02 \angle 30°$$

$$= 4 \angle 30° \text{ V}$$

$$\overline{I}_L = \frac{4 \angle 30° \text{ V}}{j200 \times 0.5} = 0.04 \angle -60° \text{ A}$$

$$\overline{I}_C = \frac{4 \angle 30°}{\dfrac{10^6}{j200 \times 50}} = 0.04 \angle 120° \text{ A}$$

$$\overline{I} = \overline{I}_R + \overline{I}_L + \overline{I}_C$$

$$= 0.02 \angle 30° + 0.04 \angle -60° + 0.04 \angle 120°$$

$$= (0.0173 + j0.01) + (0.02 - j0.0346) + (-0.02 + j0.0346)$$

$$= (0.0173 + j0.01) = 0.02 \angle 30° \text{ A}$$

The phasor diagram showing currents in the three elements, total current and applied voltage is drawn in Fig. 4.28. Observe that $\overline{I}_L$ and $\overline{I}_C$ cancel out so that $\overline{I} = \overline{I}_R$.

(b)

$$\frac{1}{Z_{\text{in}}} = \frac{1}{200} + \frac{1}{j200 \times 0.5} + \frac{1}{\dfrac{10^6}{j200 \times 50}}$$

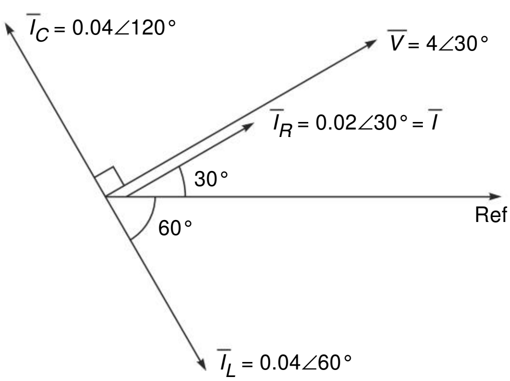


Fig. 4.28

$$= 0.005 - j0.01 + j0.01 = 0.005$$

or $$Z_{in} = 200\ \Omega$$

$$\bar{V} = 200 \times 2\ \angle -40° = 400\ \angle -40°\ \text{A}$$

$$\frac{\bar{I}}{R} = \frac{400\ \angle -40°}{200} = 2\ \angle -40°\ \text{A}$$

4.4 POWER IN SINUSOIDAL STEADY STATE

Figure 4.29 shows an ac source (*single phase*) supplying a load of impedance $Z\angle\theta$. Then in instantaneous form

$$v = \sqrt{2}\,V \sin \omega t$$

$$i = \sqrt{2}\ I \sin (\omega t - \theta)$$

where $I = V/Z$

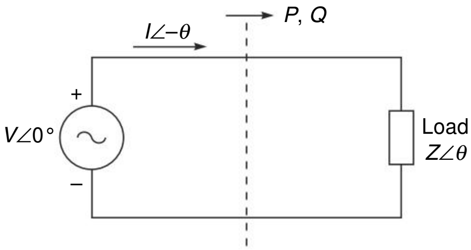


Fig. 4.29

The instantaneous power delivered to the load (or supplied by the source) is given by

$$p = vi = 2V\,I \sin \omega t \sin (\omega t - \theta)$$

$$= VI\,[\cos \theta - \cos (2\omega t - \theta)] \tag{4.45}$$

The above equation can be written in the form

$$p = V\,[I \cos \theta\,(1 - \cos 2\omega t) + I \sin\theta \sin 2\omega t] \tag{4.45a}$$

The above equation identifies two components of current, which are marked on the phasor diagram of Fig. 4.31(a).

$I \cos \theta$; current component *in phase* with voltage

and $I \sin \theta$; current component *in quadrature* (at 90°) to voltage

The in phase component $I \cos \theta$ is feeding *real* (or *active*) average power to load given by

$$P = V\,I \cos \theta\ \text{W} \tag{4.46i}$$

Power Factor $\cos \theta$ is defined as the power factor abbreviated as *pf*. Real power can then be expressed as

$$P = V\,I \times pf \tag{4.46ii}$$

$S = VI$, the volt-amperes fed to the load is called *apparent power*. We can then write

$$pf = \frac{P}{V\,I} = \frac{real\ power}{apparent\ power} \tag{4.46iii}$$

pf is *lagging* when $\bar{I}$ lags $\bar{V}$ and *leading* when $\bar{I}$ leads $\bar{V}$.

It is seen from Eq. 4.45(a) that $I \cos \theta$ has associated with it any oscillating component of power of frequency 2ω (twice the excitation frequency) with zero average value.

The quadrature component of current $I \sin \theta$ as per Eq. 4.45(a) feeds only oscillating power (frequency 2ω) to the load with zero average value. It is the *reactive power.*

$$Q = VI \sin \theta \quad \text{VAR} \tag{4.47}$$

the units being VAR, volt-ampere reactive.

The waveforms of $v(t)$, $i(t)$ and instantaneous power $p = vi$ are sketched in Fig. 4.30. The average real power is indicated by constant value P. It is easy to observe the total oscillating component of power having frequency 2ω.

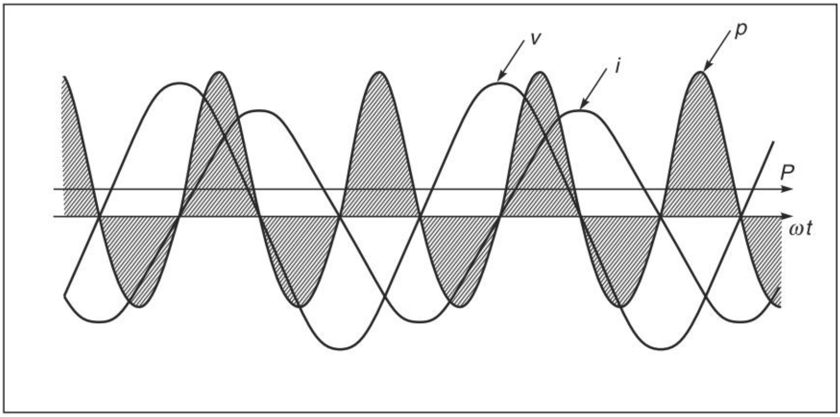


Fig. 4.30

Reactive power will be taken as *positive* for *lagging* and *negative* for *leading* *pf* (power factor) load. The complex power phasor diagram is drawn in Fig. 4.31(b) wherein

$$\bar{S} = P + jQ$$

Units for S are VA. Compare the complex power phasor diagram of Fig. 4.31(b) with the voltage and current phasor diagram of Fig. 4.31(a).

For large powers, unit for active power are kW/MW and those for reactive power are kVAR/MVAR and the complex power has the units of kVA/MVA.

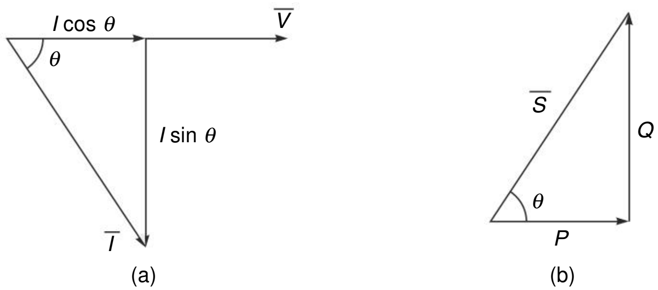


Fig. 4.31 (a) In phase and quadrature current components (b) complex power

Complex Power

It is convenient to express power in complex form:

$$\bar{S} = \bar{V}\,\bar{I}^* \tag{4.48}$$

Let $\quad \bar{V} = Ve^{j0},\ \bar{I} = Ie^{j(0-\theta)};\ \bar{I}$ lags $\bar{V}$ by θ

Then

$$\bar{S} = Ve^{j0}\, Ie^{-j(0-\theta)}$$

$$= V\,I\,e^{j\theta} = VI\cos\theta + j\,VI\sin\theta \tag{4.49}$$

$$= P + jQ \tag{4.50}$$

It may be seen that Q is positive for lagging *pf*.

The phasor diagram of complex power corresponding to Eq. (4.50) is drawn in Fig. 4.31(b) wherein θ is reversed w.r.t. that of Fig. 4.31(a) because of conjugating of current in Eq. (4.48). It follows from the phasor diagram (or Eq. (4.50)) that

$$S = \sqrt{P^2 + Q^2}\ \text{VA} \tag{4.51}$$

and $\quad \text{pf} = \cos\theta = \cos\tan^{-1} Q/P$; lagging for positive Q, leading for negative Q (4.52)

For a series *RL* circuit (Fig. 4.32)

$$\bar{S} = \bar{V}\,\bar{I}^*$$

$$= \bar{Z}\,\bar{I}\,\bar{I}^* = \bar{Z}\,\bar{I}^2$$

$$= (R + jX_L)\,I^2 = RI^2 + jX_L I^2 \tag{4.53}$$

Hence

$P = RI^2$ = real power consumed in resistive element

$Q = X_L I^2$ = reactive power consumed in reactive (inductive) element

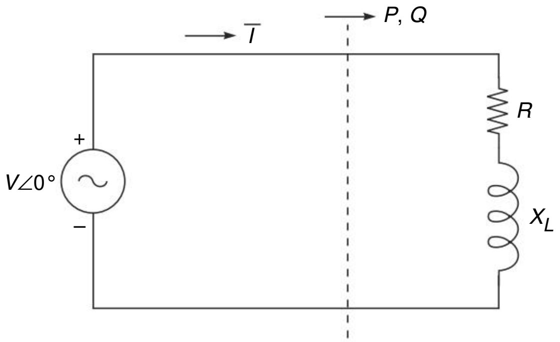


Fig. 4.32

Analyzing circuit in terms of complex powers is known as the *volt-ampere method* (Example 4.11).

Example 4.9 For the circuit of Fig. 4.33.

$$v_S(t) = \cos\omega t$$

Find

(a) the *driving point admittance* $\bar{Y}(j\omega)$, and

(b) the value of the frequency at which the pf of the circuit would be unity.

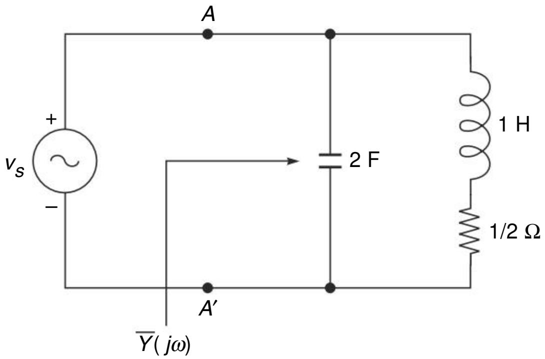


Fig. 4.33

Solution

(a) $$\overline{Y}(j\omega) = \frac{1}{\frac{1}{2} + j\omega} + 2\omega$$

(b) $$\overline{I} = \overline{Y}(j\omega)\, V_s \angle 0°$$

For unity power factor, the imaginary part of $\overline{Y}$ $(j\omega)$ should be zero.

$$\overline{Y}(j\omega) = \frac{\frac{1}{2} - j\omega}{\frac{1}{4} + \omega^2} + j2\omega$$

$$= \frac{2}{1 + 4\omega^2} - j\left[\frac{4\omega}{1 + 4\omega^2} - 2\omega\right]$$

$\therefore$ $$\frac{4\omega}{1 + 4\omega^2} = 2\omega$$

$$2 = 1 + 4\omega^2$$

$$\omega = \frac{1}{2} \text{ rad/s}$$

Example 4.10 Find the average power fed to the circuit of Fig. 4.34 by the current source and the power factor.

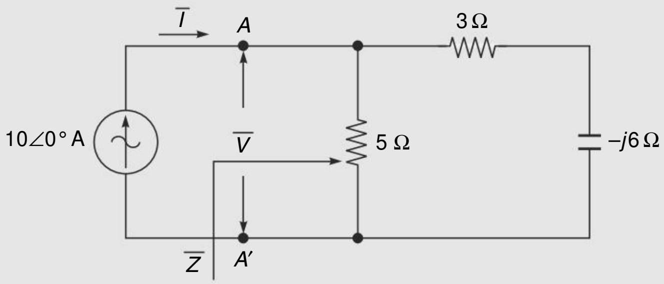


Fig. 4.34

Solution

Driving point impedance

$$\bar{Z} = \frac{5 \times (3 - j6)}{5 + 3 - j6}$$

$$= \frac{5 \times (3 - j6)}{8 - j6} = 5\,\frac{(3 - j6)\,(8 + j6)}{100}$$

$$= \frac{60 - j30}{20} = 3 - j1.5$$

$$\bar{V} = (3 - j1.5) \times 10 \angle 0°$$

$$= 10(3 - j1.5) = 33.54 \angle -26.6° \text{ V}$$

$\bar{I}$ leads $\bar{V}$ by 26.6°. Therefore

$$\text{pf} = \cos 26.6° = 0.894 \text{ leading}$$

$$P = V\,I \cos\theta$$

$$= 33.54 \times 10 \times 0.894 = 300 \text{ W}$$

Example 4.11 Two generators (single phase ac) feed a load of 15 kW, 0.8 pf lagging through two transmission lines. G_1 feeds 8 kW at 0.8 pf lagging into its line at 430 V. The connection and line impedances are shown in Fig. 4.35. Using the volt-ampere method, find

(a) load voltage,

(b) Q_1 supplied by G_1, and

(c) P_2, Q_2 supplied by G_2, pf and voltage V_2.

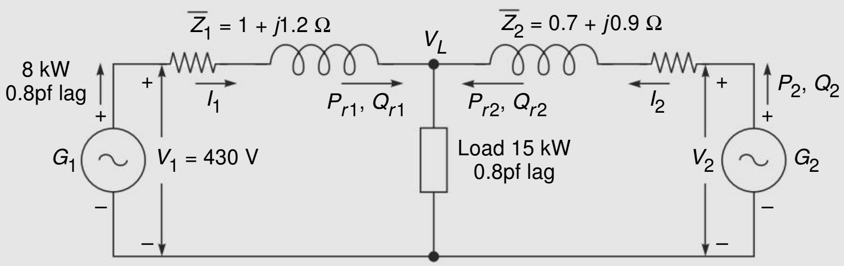


Fig. 4.35

Solution

(a) With reference to Fig. 4.35

$$I_1 = \frac{8 \times 1000}{430 \times 0.8} = 23.26 \text{ A}$$

$$P_{r1} = P_1 = I^2{}_1 R_1 = 8000 - (23.26)^2 \times 1$$

$$= 7459 \text{ W}$$

$$pf = \cos \tan^{-1} \frac{Q_1}{P_1} = 0.8$$

or $$Q_1 = P_1 \tan \cos^{-1} 0.8$$

$$= 8000 \times 0.75 = 6000 \text{ VAR}$$

$$Q_{r1} = Q_1 - I_1^2 \times 1.2 = 6000 - (23.26)^2 \times 1.2$$

$$= 5351 \text{ VAR}$$

$$S_{r1} = [(7459)^2 + (5351)^2]^{1/2} = 9180 \text{ VA}$$

$$V_L = \frac{Sr_1}{I_1} = \frac{9180}{23.26} = 395 \text{ V}$$

$$P_L = 15000 \text{ W}$$

$$Q_L = 15000 \tan \cos^{-1} 0.8 = 11250 \text{ VAR}$$

$$P_{r2} = P_L - P_{r1} = 15000 - 7459 = 7541 \text{ W}$$

$$Q_{r2} = Q_L - Q_{r1} = 11250 - 5351 = 5899 \text{ VAR}$$

$$Q_{r2} = [(7541)^2 + (5899)^2]^{1/2} = 9574 \text{ VA}$$

$$I_2 = \frac{9574}{395} = 24.24 \text{ A}$$

$$P_2 = P_{r2} + I_2^2 R_2 = 7541 + (24.24)^2 \times 0.7$$

$$= 7952 \text{ W}$$

$$Q_2 = Q_{r2} + I_2^2 X_2 = 5899 + (24.24)^2 \times 0.9$$

$$= 6428 \text{ VAR}$$

$$\text{pf} = \cos \tan^{-1} \frac{6428}{7952} = 0.778 \text{ lagging}$$

$$S_2 = \sqrt{(7952^2 + (6428)^2} = 10.225 \text{ kVA}$$

$$V_2 = \frac{10225}{24.24} = 422 \text{ V}$$

Power Factor Improvement

Most electric loads are reactive in nature and have lagging power factor less than unity. Particularly the industrial loads (with induction motor drives) have low pf which may even be less than 0.8 if motors are not fully loaded. Transmission lines, transformers and generators of the electric power utility have to carry the lagging reactive power of the load so that their full real power capability is not exploited and further reactive current causes additional ohmic losses and large voltage drops. These factors cause operational financial loss to the utility. The utility therefore induces its industrial consumers to improve their power factor by imposing penalty through tariff for the reactive component of the consumer's load.

Industrial consumers thus find it economical to improve the pf of their individual motors and/or the total installation by installing shunt capacitors (static) which draw compensating leading current. The limit to which the pf must be improved is dictated by the balance of the yearly tariff saving against the yearly interest and depreciation cost of installing the capacitors. While these details are beyond the scope of this text, the effectiveness of shunt capacitors in pf improvement is illustrated in the Example 4.12.

Example 4.12 A 10 kVA load at 0.8 pf lagging is fed from 231 V, 50 Hz supply. Calculate the kVA capacity and the capacitance value of the shunt capacitor required to improve the overall pf (load + shunt capacitor) to 0.95 lagging. Compare the current drawn from the supply before and after installing the capacitors.

Solution

Figure 4.36 shows the capacitor in parallel (shunt) to the load.

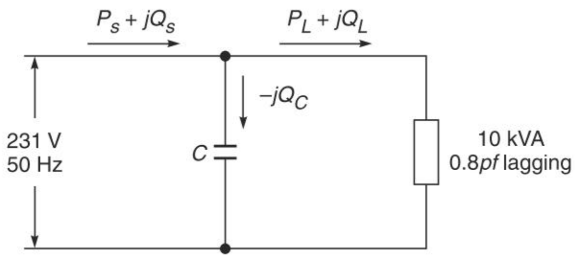


Fig. 4.36 PF improvement by shunt capacitor

Load: $P_L + jQ_L = 10 \times 0.8 + j10 \times \sin\cos^{-1} 0.8$

$= 8 + j6$

Capacitor draws $= -jQ_C$ kVAR

Supply: $P_S + jQ_S = (8 + j6) - jQ_C$

$= 8 + j(6 - Q_C)$

New pf $= \cos\tan^{-1} \dfrac{6 - Q_C}{8} = 0.95$ lagging

$\therefore \quad Q_C = 3.37$ kVAR

$$3.37 = \frac{(231)^2 \times 314 \times C}{1000}$$

or $\quad C = 201\ \mu\text{F}$

Supply current before pf improvement

$$10 = \frac{231 \times I_S}{1000}$$

or $\quad I_S = I_L = 43.29$ A

Supply current after pf improvement

$$P_S + jQ_S = 8 + j(6 - 3.37) = 8 + j2.63 = 8.42\ \angle 18.2°$$

$$I_S = \frac{8.42 \times 1000}{231} = 36.45 \text{ A}$$

Alternatively:

$$\bar{I}_L = 43.29 \times 0.8 - j43.29 \times 0.6$$

$$= 34.63 - j25.97 \text{ A}$$

$$\bar{I}_C = j\frac{3.37 \times 1000}{231} = j14.59 \text{ A}$$

$$\bar{I}_S = \bar{I}_L + \bar{I}_C = 34.63 - j(25.97 - 14.59)$$
$$= 34.63 - j11.38$$

or $$I_S = 36.45 \text{ A}$$

It is seen that because of power factor improvement, the current drawn from the supply reduces from 43.29 A to 36.45. This benefits the supplier as line real and reactive power loss reduces as real power is $I_L^2 R_L$ and reactive loss is $I_L^2 X_L$.

4.5 NODAL AND MESH METHODS OF ANALYSIS

Nodal and mesh methods of analysis equally apply to sinusoidal steady state analysis by converting the circuit into its frequency domain equivalent—elements as impedances (admittances) in complex number form and source voltages/currents as phasors. These methods are best illustrated by examples.

Example 4.13 Figure 4.37 is the frequency domain representation of a circuit. Determine the values (magnitude and angle) of the phasor voltages $\bar{V}_1$ and $\bar{V}_2$.

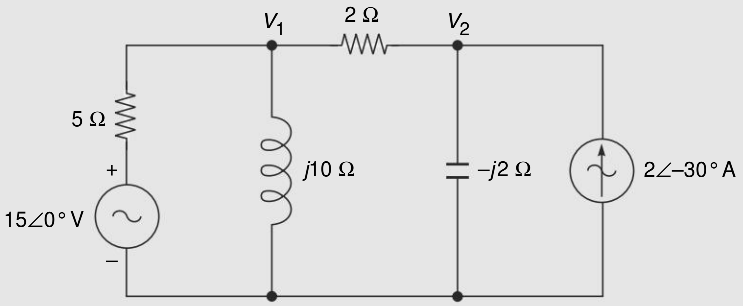


Fig. 4.37

Solution Converting the voltage source (15 ∠0°) to current source form, the circuit from changes to that in Fig. 4.38. Observe that all elemental values are in impedance form. Applying KCL at the two nodes:

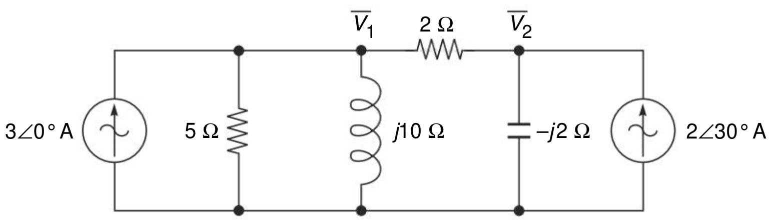


Fig. 4.38

Node 1: $$\frac{\bar{V}_1}{5} + \frac{\bar{V}_1}{j10} + \frac{\bar{V}_1 - \bar{V}_2}{2} = 3$$

or $$(0.7 - j0.1)\,\bar{V}_1 - 0.5\,\bar{V}_2 = 3 \qquad \text{(i)}$$

Node 2: $$\frac{\bar{V}_2}{-j^2} + \frac{\bar{V}_2 - \bar{V}_1}{2} = 2\ \angle -30°$$

or $\qquad -0.5\bar{V}_1 + (0.5 + j0.5)\,\bar{V}_2 = 2\,\angle -30° = (1.732 - j1)$ (ii)

Let us now solve simultaneously Eqs. (i) and (ii)

$$\Delta = \begin{vmatrix} (0.7 - j0.1) & -0.5 \\ -0.5 & (0.5 + j0.5) \end{vmatrix} = (0.15 + j0.3) = 0.335\,\angle 63.4°$$

$$\Delta_1 = \begin{vmatrix} 3 & -0.5 \\ (1.732 - j1) & (0.5 + j0.5) \end{vmatrix} = (2.366 + j1) = 2.569\,\angle 22.9°$$

$$\Delta_2 = \begin{vmatrix} (0.7 - j0.01) & 3 \\ -0.5 & (1.732 - j1) \end{vmatrix} 2.613 - j0.873 = 2.754\,\angle -18.5°$$

$$\bar{V}_1 = \frac{\Delta_1}{\Delta} = \frac{2.569\angle 22.9°}{0.335\angle 63.4°} = 7.669\,\angle -40.5°\text{ V}$$

$$\bar{V}_2 = \frac{\Delta_2}{\Delta} = \frac{2.754\angle -18.5°}{0.335\angle 63.4°} = 8.221\,\angle -81.9°\text{ V}$$

Currents in all the components can now be calculated. It is assumed above that both sources have the same frequency. It is only then that these can be represented as phasors with fixed phase difference (15 $\angle 0°$ V and 2 $\angle -30°$ A).

Example 4.14 Draw the frequency domain equivalent of the circuit of Fig. 4.39 at $\omega = 600$ rad/s. Using the mesh method of analysis, find $\bar{V}_R$ and v_R, i.e. phasor voltage across resistance and its time domain expression.

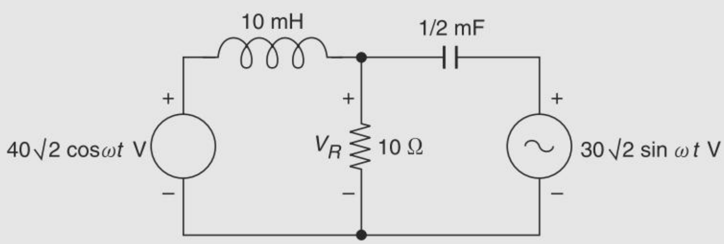


Fig. 4.39

Solution

$$X_L = \omega L = 600 \times 10 \times 10^{-3} = 6\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{12 \times 10^3}{600} = 20\ \Omega$$

$$40\sqrt{2}\cos\omega t = \text{Re}\,[\sqrt{2} \times 40\, e^{j0}\, e^{j\omega t}]$$

or $\qquad \bar{V}_1 = 40\,\angle 0°$ V

$$30\sqrt{2}\,\sin\omega t = 30\sqrt{2}\cos(\omega t - 90°) = \text{Re}\,[\sqrt{2} \times 30\, e^{-j90°}\, e^{j\omega t}]$$

or $\qquad \bar{V}_2 = 30\,\angle -90°$

The frequency domain equivalent of the circuit is drawn in Fig. 4.40. Writing down mesh equations:

$$j6\bar{I}_1 + 10(\bar{I}_1 - \bar{I}_2) = 40 \qquad \text{(i)}$$

$$-j20\,\bar{I}_2 + 10(\bar{I}_2 - \bar{I}_1) = -j30 \qquad \text{(ii)}$$

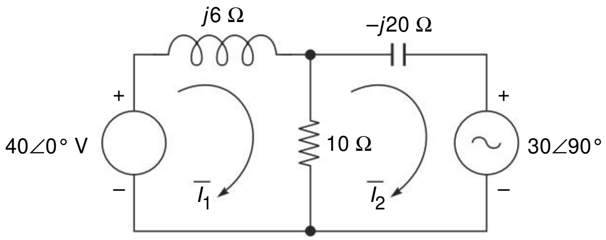


Fig. 4.40

Rearranging

$$(10 + j6)\ \bar{I}_1 - 10\bar{I}_2 \quad = 40 \qquad \text{(iii)}$$

$$-10\bar{I}_1 + (10 - j20)\ I_2 \quad = -j30 \qquad \text{(iv)}$$

Solving Eqs (iii) and (iv) simultaneously

$$\Delta = \begin{vmatrix} 10 + j6 & -10 \\ -10 & (10 - j20) \end{vmatrix} = 120 - j140 = 184.4\ \angle -49.4°$$

$$\Delta_1 = \begin{vmatrix} 40 & -10 \\ -j30 & (10 - j20) \end{vmatrix} = 400 - j1100 = 1170.5\ \angle -70°$$

$$\Delta_2 = \begin{vmatrix} (10 + j6) & 40 \\ -10 & -j30 \end{vmatrix} = 580 - j300 = 653\ \angle -27.3°$$

Now

$$\bar{I}_1 = \frac{\Delta_1}{\Delta} = \frac{1170.5\ \angle -70°}{184.4\ \angle -49.4°} = 6.35\ \angle -20.6° = 5.94\ \angle -j2.23 \text{ A}$$

$$\bar{I}_2 = \frac{\Delta_2}{\Delta} = \frac{653\ \angle -27.3°}{184.4\ \angle -49.4°} = 3.54\ \angle 22.1° = 3.28 + j1.33 \text{ A}$$

Current through resistance $= \bar{I}_1 - \bar{I}_2$

$$= (5.94 - j2.23) - (3.28 + j1.33)$$

$$= 9.22 - j3.56 = 9.88\ \angle -21.1° \text{ A}$$

$$\bar{V}_R = 10 \times 9.88\ \angle -22.1° = 91.54 - j3.72$$

$$v_R = 98.8\sqrt{2}\cos(600\,t - 22.1°) \text{ V}$$

4.6 NETWORK THEOREMS

All the network theorems advanced in Section 2.9 apply to frequency domain circuits with phasor currents/voltages and complex number impedances/admittances. These will be illustrated through examples.

Superposition Theorem

Example 4.15 For the circuit of Fig. 4.41 with impedance values shown, find the

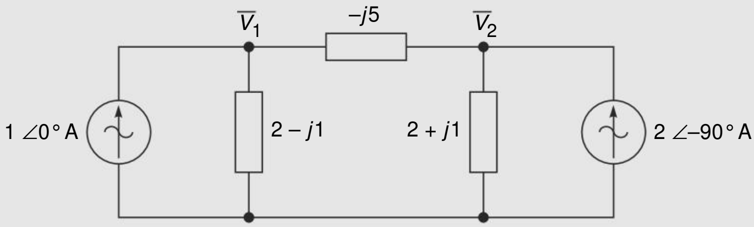


Fig. 4.41

phasor voltages $\overline{V}_1$ and $\overline{V}_2$ by the application of superposition theorem.

Solution

(i) Open-circuit the source 2 ∠– 90° A as in Fig. 4.42. Using the current division technique

$$\overline{V}_{11} = 1\ \angle 0^\circ \times \left(\frac{-j5 + 2 + j1}{2 - j1 - j5 + 2 + j1}\right) \times (2 - j1)$$

$$= \frac{2 - j4}{4 - j5} \times (2 - j1) = \frac{-j10}{(4 - j5)}$$

$$= \frac{-j10(4 + j5)}{41} = \frac{50 - j40}{41}$$

$$\overline{V}_{21} = 1\angle 0^\circ \times \left(\frac{2 - j1}{4 - j5}\right) \times (2 + j1)$$

$$= \frac{5(4 + j5)}{41} = \frac{20 + j25}{41}$$

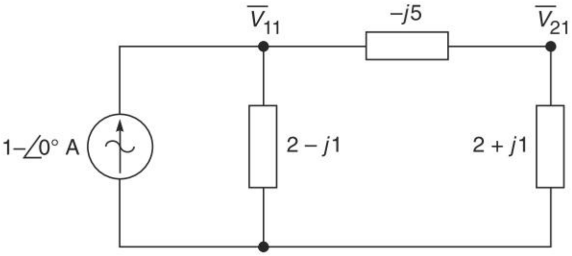


Fig. 4.42

(ii) Open-circuit the source 1 ∠0° A as in Fig. 4.43. Using the current division technique

$$\overline{V}_{22} = -\ j2 \times \left(\frac{-j5 + 2 - j1}{4 - j5}\right) \times (2 + j1)$$

$$= -j2 \times \frac{(2 - j6)\ (2 + j1)\ (4 + j5)}{41} = \frac{20 - j180}{41}$$

$$\overline{V}_{12} = -j2\ \times\ \left(\frac{2 + j1}{4 - j5}\right)\ \times\ (2 - j1)$$

$$= -j2 \times \frac{(2j + j1)(2 - j1)(4 + j5)}{41} = \frac{50 - j40}{41}$$

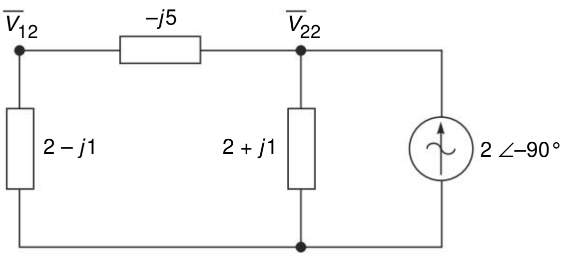


Fig. 4.43

Applying superposition theorem

$$\bar{V}_1 = \bar{V}_{11} + \bar{V}_{12}$$

$$= \frac{50 - j40}{41} + \frac{50 - j40}{41} = \left(\frac{100 - j80}{41}\right)$$

$$\bar{V}_2 = \bar{V}_{21} + \bar{V}_{22}$$

$$= \frac{20 + j25}{41} + \frac{20 - j180}{41} = \frac{40 - j155}{41}$$

Thevenin and Norton Theorems

Thevenin Theorem As in the case of resistive circuits, Thevenin equivalent at two circuit terminals comprises Thevenin Voltage $\bar{V}_{TH} = \bar{V}_0$ open-circuit voltage with Thevenin impedance $\bar{Z}_{TH}$ in series; $\bar{Z}_{TH}$ open-circuit impedance seen from the teminals with source removed (voltage sources short circuited and current sources open circuited). Of course, we are dealing with frequency domain circuit; phasor voltages, phasor currents and impedances/ admittances. We shall illustrate by an example.

Example 4.16 For the circuit of Fig. 4.44, find the Thevenin equivalent as seen across terminals AB.

Solution Frequency domain representation of the circuit of Fig. 4.44 is given in Fig. 4.45.

One way to proceed is to convert the delta comprising 2 Ω, 2 Ω and $-j1$ Ω into star. This is the procedure we shall adopt. The Δ–Y conversion formula of Eq. (2.11) is generalized into complex number form as:

$$\bar{Z}_a = \frac{\bar{Z}_{ab}\ \bar{Z}_{ac}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{4.54a}$$

$$\bar{Z}_b = \frac{\bar{Z}_{ba}\ \bar{Z}_{bc}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{4.54b}$$

$$\bar{Z}_c = \frac{\bar{Z}_{ca}\ \bar{Z}_{cb}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{4.54c}$$

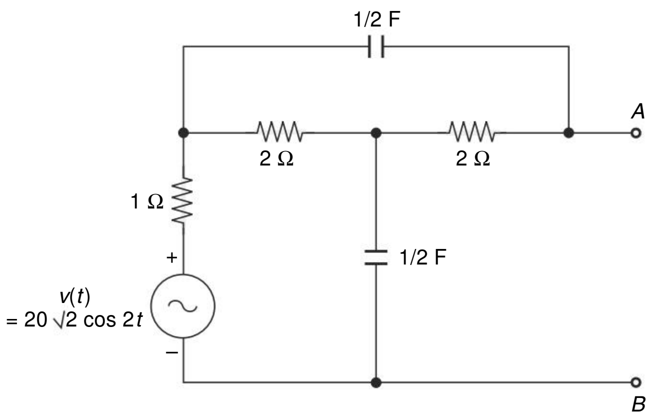


Fig. 4.44

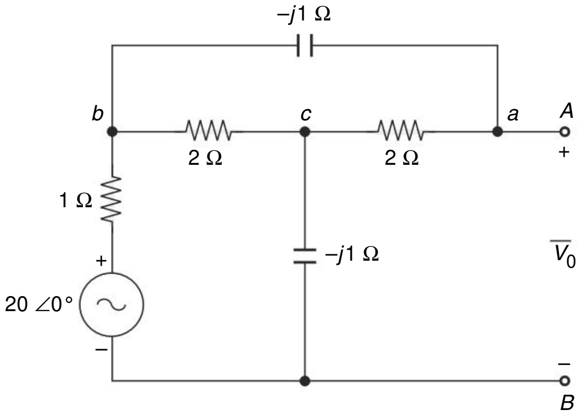


Fig. 4.45

From Fig. 4.43

$$\overline{Z}_{ab} = -j1,\ \overline{Z}_{bc} = 2,\ \overline{Z}_{ca} = 2$$

$$\overline{Z}_{ab} + \overline{Z}_{bc} + \overline{Z}_{ca} = -j1 + 2 + 2 = 4 - j1$$

Substituting in relationships of Eq. (4.54)

$$\overline{Z}_a = \frac{-j1 \times 2}{4 - j1} = \frac{-j2}{4 - j1} = \frac{2 - j8}{17} = 0.118 - j0.471$$

$$\overline{Z}_b = \frac{2 \times -j1}{4 - j1} = \frac{j2}{4 - j1} = \frac{2 - j8}{17} = 0.118 - j0.471$$

$$\overline{Z}_c = \frac{2 \times 2}{4 - j1} = \frac{4}{4 - j1} = \frac{16 - j4}{17} = 0.941 + j0.235$$

The transformed circuit is now drawn in Fig. 4.46.

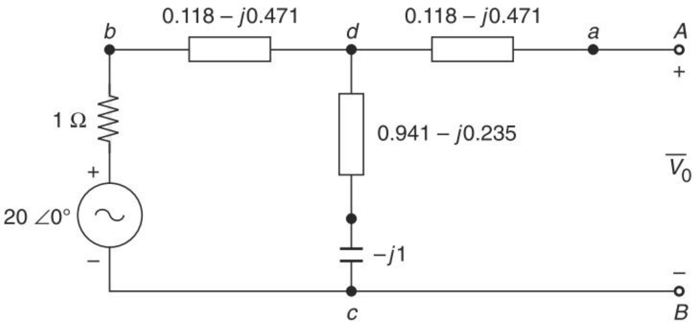


Fig. 4.46

Observe that $\overline{V}_{dc} = \overline{V}_0$ as $\overline{I}_{dA} = 0$. Using voltage division technique (Eq. (2.4)) in complex number form is

$$\overline{V}_0 = \overline{V}_{dc} = 20\ \angle 0° \times \frac{0.941 + j0.235 - j1}{1 + 0.118 - j0.471 + 0.941 + j0.235 - j1}$$

$$= 20\ \angle 0° \times \left(\frac{0.941 - j0.765}{2.059 - j1.236}\right)$$

$$= 20 \times \frac{1.213 \angle -39.1^\circ}{2.402 \angle -31^\circ} = 10.1 \angle -8.1^\circ \text{ V}$$

Short circuiting the source, the Thevenin equivalent impedance is

$$\bar{Z}_{TH} = \frac{(1.118 - j0.471)(0.941 - j0.765)}{2.059 - j1.236} + 0.118 - j\,0.471$$

$$= 0.613 \angle -30.9^\circ + 0.118 - j\,0.471$$

$$= 0.644 - j\,0.786\ \Omega$$

The Thevenin equivalent is drawn in Fig. 4.47 (a).

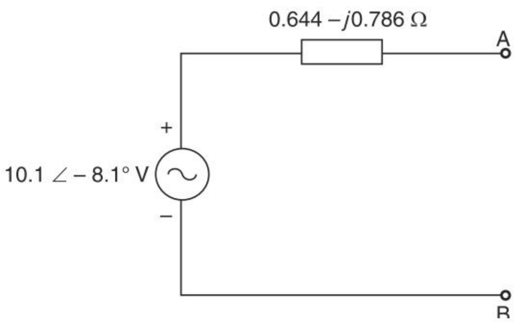


Fig. 4.47(a) Thevenin equivalent circuit

Norton Equivalent All we need is to find is $\bar{I}_{SC}$, the short-circuit current. From Fig. 4.47 (b).

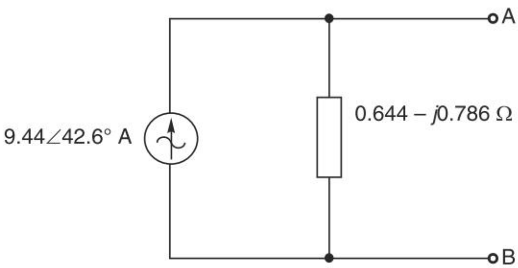


Fig. 4.47(b) Norton equivalent circuit

$$\bar{I}_{SC} = \frac{10.1\angle -8.1^\circ}{0.644 - j0.786} = \frac{10.1\angle -8.1^\circ}{1.016\angle -50.7^\circ}$$

$$= 9.94 \angle 42.6^\circ$$

The Norton equivalent is $\bar{I}_{SC}$ with $\bar{Z}_{TH}$ in parallel with it as drawn in Fig. 4.47(b).

Maximum Power Transfer Theorem

The theorem is presented without proof (proof is not hard to establish). Consider a network represented by its Thevenin equivalent loaded at its terminals *AB* with load impedance $\bar{Z}_L = R_L + jX_L$ as shown in Fig. 4.48. Maximum power is transferred (absorbed) by the load if

$$X_L = -X_{TH}$$

$$R_L = R_{TH}$$

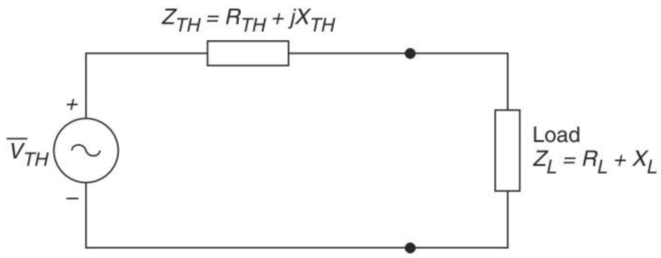


Fig. 4.48 Maximum power transfer

$$\text{or} \qquad \overline{Z}_L = \overline{Z}^*_{TH} \quad \text{*means complex conjugate of} \tag{4.55}$$

This technique of transferring maximum power to load is known as *impedance matching* and is of considerable importance in electronic circuits where output power is the chief concern though the power transfer efficiency is reduced to 50%. On the other hand, efficiency is of paramount importance in power circuits which operate far from the condition of impedance matching.

Special Cases

(i) If only R_L can be varied, the condition of maximum power transfer is

$$R_L = [R^2_{TH} + (X_{TH} + X_L)^2]^{1/2} \tag{4.56}$$

(ii) If only X_L is variable, the condition of maximum power transfer is

$$X_L = -X_{TH} \tag{4.57}$$

(iii) If $\theta_L = \tan^{-1} X_L/R_L$ remains fixed but $|Z_L|$ is varied, the condition of maximum power transfer is

$$|\overline{Z}_L| = |\overline{Z}_{TH}| \tag{4.58}$$

Example 4.17 For the circuit of Fig. 4.49, find the values of R_L and X_L for maximum power absorption and the value of the maximum power.

If X_L is fixed at 4 Ω, find R_L for maximum power absorption and its value.

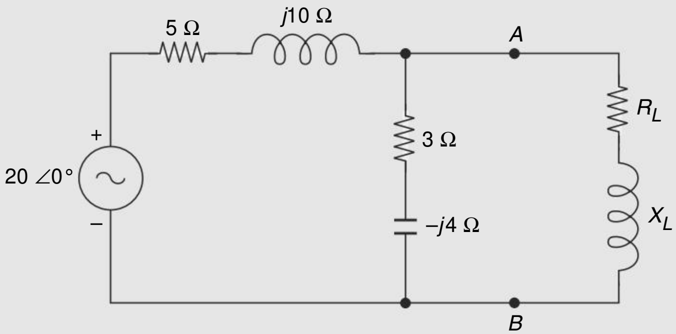


Fig. 4.49

Solution

To begin with, we find the Thevenin equivalent of the circuit to the left of *AB* which is redrawn in Fig. 4.50.

From Fig. 4.50

$$\overline{V}_{TH} = 20\ \angle 0° \times \frac{3 - j4}{(5 + j10) + (3 - j4)}$$

$$= 10\ \angle -90°\ \text{V}$$

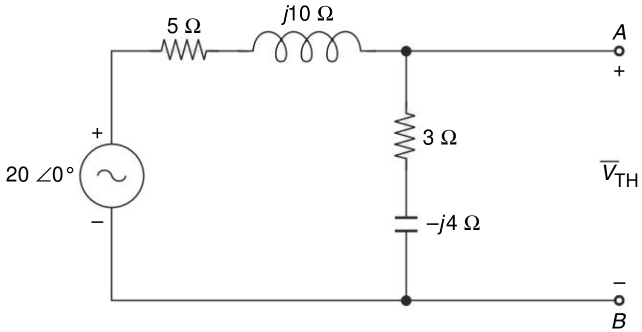


Fig. 4.50

Short-circuiting the voltage source as in Fig. 4.51

$$\overline{Z}_{TH} = \frac{(5 + j10)(3 - j4)}{(8 + j6)} = \left(\frac{55 + j10}{8 + j6}\right)$$

$$= 5 - j2.5\ \Omega$$

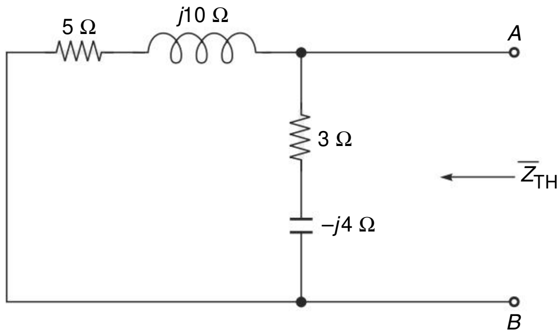


Fig. 4.51

By the maximum power transfer theorem

$$\overline{Z}_L = 5 + j2.5\ \Omega$$

Value of maximum power transferred

$$= \left(\frac{10}{5 + 5}\right)^2 5 = 5\ \text{W}$$

In the second case, $X_L = 4\ \Omega$ (fixed). For maximum power transfer, as per Eq. (4.56).

$$R_L = [R_{TH}^2 + X_{TH} + X_L)^2]^{1/2}$$

$$= [5^2 + (-2.5 + 4)^2)^{1/2}\text{; note the negative sign}$$

$$= 5.22\Omega$$

Maximum power absorbed

$$= \left[\frac{10 \angle -90^\circ}{(5 - j2.5) + (5.22 + j4)}\right]^2 \times 5.22 = 4.89 \text{ W}$$

4.7 SUPERPOSITION OF AVERAGE POWER IN AC CIRCUITS

Consider the circuit of Fig. 4.52 with two voltage sources of different frequencies in series. The instantaneous voltages of the sources are expressed as

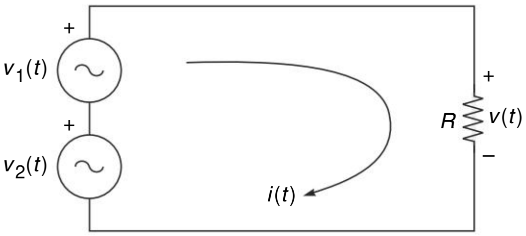


Fig. 4.52

$$v_1(t) = \sqrt{2}\, V_1 \sin(\omega_1 t + \phi_1)$$
$$v_2(t) = \sqrt{2}\, V_2 \sin(\omega_2 t + \phi_2)$$

Then $\qquad v(t) = v_1(t) + v_2(t)$

The instantaneous power fed into the resistance R is

$$p(t) = \frac{v^2(t)}{R} = \frac{v_1^2(t) + 2v_1(t)v_2(t) + v_2^2(t)}{R}$$

Substituting values of $v_1(t)$ and $v_2(t)$, we get

$$p(t) = \frac{2}{R}[V_1^2 \sin^2(\omega t + \phi_1] + V_1 V_2 \sin(\omega_1 t + \phi_1)\sin(\omega_2 + \phi_2)$$
$$+ V_2^2 \sin^2(\omega_2 t + \phi_2)]$$
$$= \frac{2}{R}\left\{\frac{1}{2}V_1^2[1 - \cos(2\omega_1 t + 2\phi_1) + \frac{1}{2}V_1 V_2[\cos(\overline{\omega_1 - \omega_2}t + \overline{\phi_1 - \phi_2})]\right.$$
$$+ \frac{1}{2}V_2^2[1 - \cos(2\omega_2 + 2\phi_2)]$$

Average power is given by

$$P = av\, p(t) = \frac{V_1^2}{R} + \frac{V_2}{R_2}$$
$$= P_{av1} + P_{av2}$$

The result means that *average powers supplied by different frequency sources in a circuit can be superimposed.*

However, if

$$\omega_1 = \omega_2 = \omega$$

Then $\qquad P_{av} \neq P_{av1} + P_{av2}$

The reader can easily establish this result.

Example 4.18 In the circuit of Fig. 4.53

$$v(t) = 4\sqrt{2}\sin(3t + 30°)\ \text{V}$$
$$i(t) = 0.8\cos(5t + 10°)\ \text{A}$$

Compute the average power consumed by the load connected at the terminals '*ab*'.

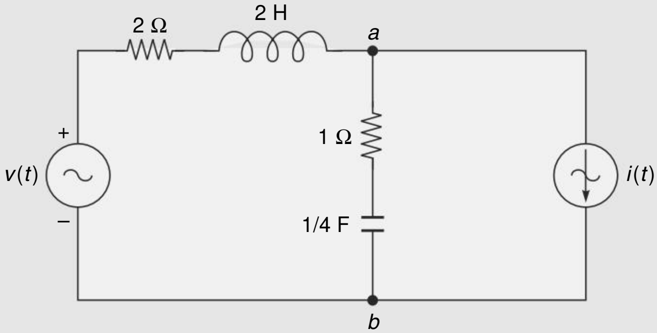


Fig. 4.53

Solution

Let us carry out the superposition of average powers.

(i) Open circuit the current source.

The frequency domain circuit at 3 rad/s is drawn in Fig. 4.54(a)

$$\bar{I}_{L1} = \frac{4\angle 30°}{6 + j6 + 1 - j1} = 0.465 \angle -5.5° \text{A}$$

$$P_{av1} = (0.465)^2 \times 1 = 0.216 \text{ W}$$

(ii) Short-circuit voltage source

The frequency domain circuit at 5 rad/s is drawn in Fig. 4.54(b)

$$I_{L2} = 0.8 \angle -10° \times \frac{6 + j10}{(6 + j10 + 1 - j0.6)}$$
$$= 0.796 \angle -4.8° \text{A}$$
$$P_{av2} = (0.797)^2 \times 1 = 0.634 \text{ W}$$

Hence

$$P_{av} = P_{av1} + P_{av2}$$
$$= 0.216 + 0.634 = 0.85 \text{ W}$$

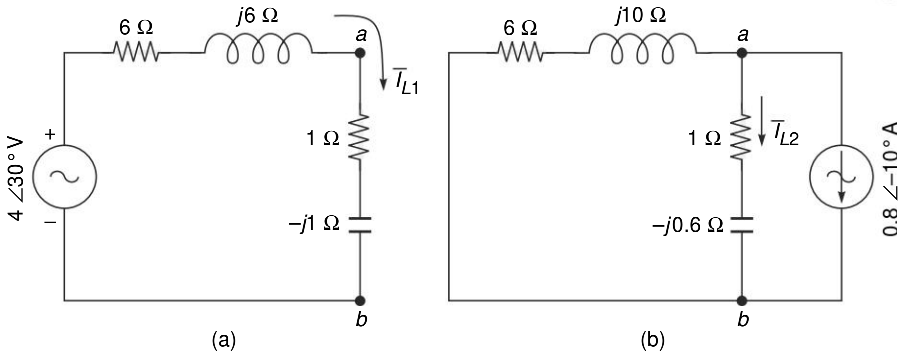


Fig. 4.54

4.8 SUDDEN APPLICATION OF SINUSOIDAL EXCITATION

Having learnt sinusoidal steady state analysis, the stage is set for treatment of sudden application of sinusoidal excitation. Both natural and forced responses would be present, with the forced response being the steady state response of the circuit to sinusoidal excitation. The arbitrary constant(s) would then be determined from the initial conditions. This is best illustrated by an example.

Example 4.19 For the circuit shown in Fig. 4.55, find $i(t)$ after the switch is closed at $t = 0$. Assume zero initial condition.

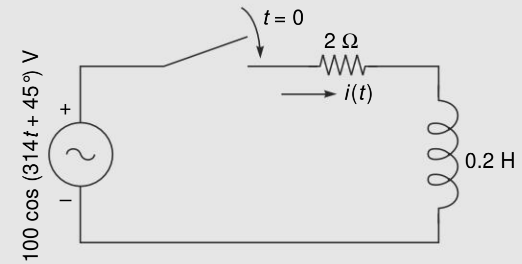


Fig. 4.55

Solution

Natural Response Short-circuit the voltage source

$$\tau = L/R = 0.1 \text{ s}$$

$$\therefore \quad i_n(t) = Ae^{-10t}$$

Forced Response It is the steady-state sinusoidal response. Convert the circuit to its frequency domain form as in Fig. 4.56.

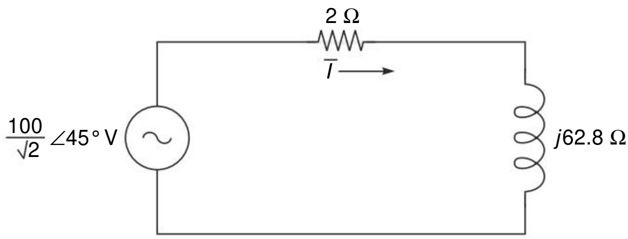


Fig. 4.56

$$\bar{I} = \frac{\dfrac{100}{\sqrt{2}}\angle 45^\circ}{2 + j62.8} = \frac{159}{\sqrt{2}}\angle -43.2^\circ \qquad \text{(ii)}$$

In time domain form

$$i_f = \sqrt{2} \times \frac{1.59}{\sqrt{2}} \cos(314t - 43.2^\circ)$$

$$= 1.59 \cos(314t - 43.2^\circ) \qquad \text{(iii)}$$

Total Response Add the two responses

$$i(t) = Ae^{-10t} + 1.59 \cos(314t - 43.2°) \quad \text{(iv)}$$

At $t = 0^+$, $i(t) = 0$, then

$$0 = A + 1.59 \cos(-43.2°)$$

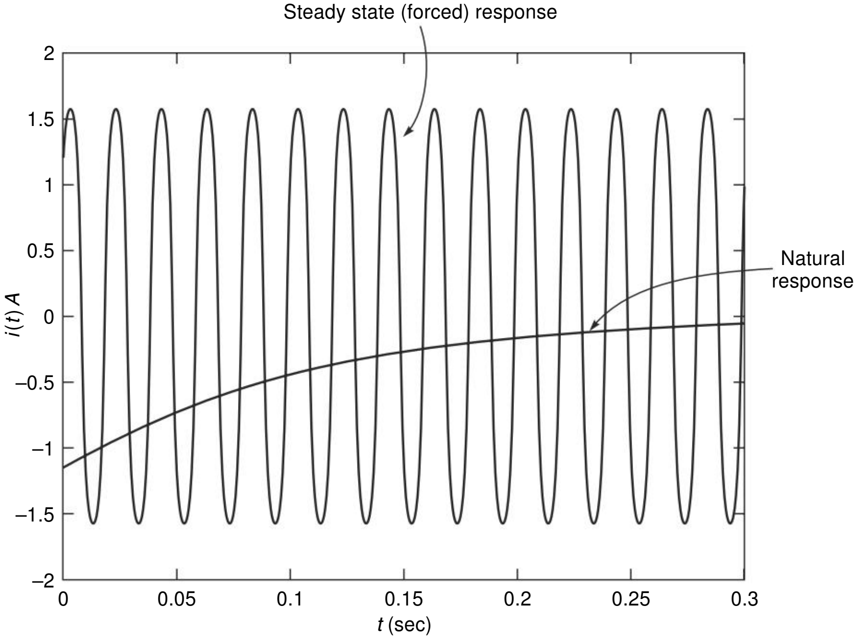


Fig. 4.57(a) Natural and steady-state response

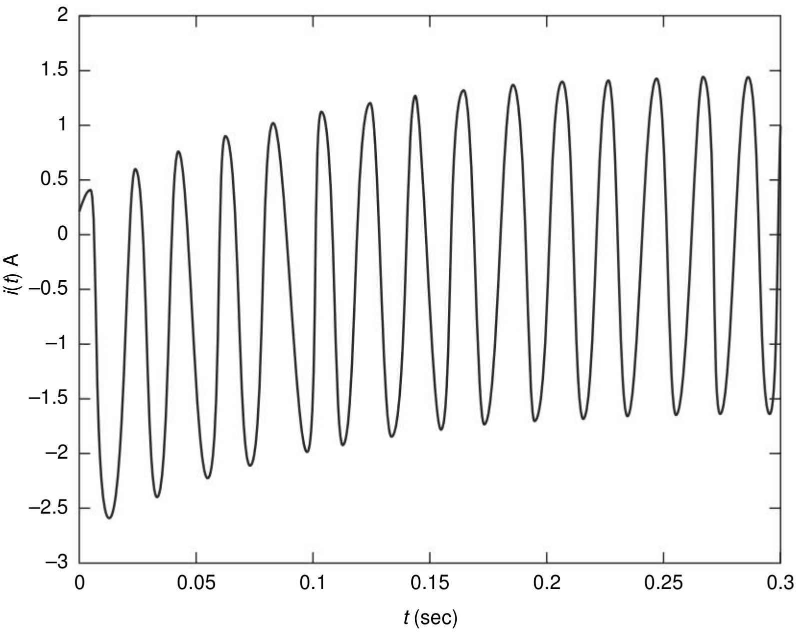


Fig. 4.57(b) Total response

or $\quad A = -1.16$

Hence

$$i(t) = \underbrace{-1.16e^{-10t}}_{\text{Natural response}} + \underbrace{1.59\cos(314t - 43.2^\circ)}_{\text{Steady state (forced response)}} \tag{v}$$

Natural and steady-state responses are respectively plotted in Fig. 4.57(a) and their sum $i(t)$ is plotted in Fig. 4.57(b). It is seen that $i(t)$ is shifted down by the decaying natural response. After the natural response has died out, the steady-state response is an symmetrical sinusoid.

Example 4.20 For the circuit of Fig. 4.58, the switch is thrown open at $t = 0$. Find $v(t)$ given: $v(0) = 20$ V.

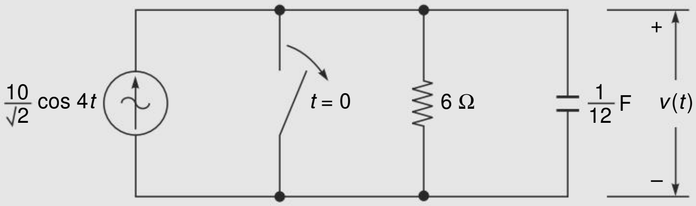


Fig. 4.58

Solution

Natural Response Open circuiting current source

$$\tau = RC = 6 \times \frac{1}{12} = 1/2 \text{ s}$$

$$v_n(t) = Ae^{-t/2} \tag{i}$$

Forced (Steady State) Response Frequency domain circuit is drawn in Fig. 4.59.

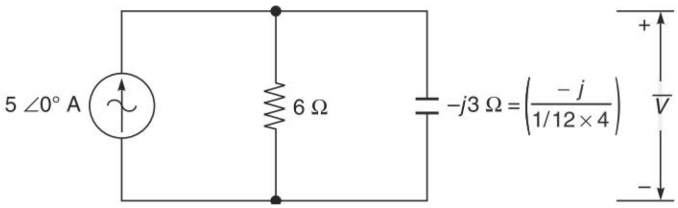


Fig. 4.59

$$\bar{Z} = \frac{6 \times -j3}{6 - j3} = 2.68 \angle -63.4^\circ \ \Omega$$

$$\bar{V} = 5 \angle 0^\circ \times 2.68 \angle -63.4^\circ = 13.4 \angle -63.4^\circ \text{ V}$$

or $\quad v_f(t) = 13.4\sqrt{2}\cos(4t - 63.4^\circ)$

$$= 18.95 \cos(4t - 63.4^\circ) \text{ V} \tag{ii}$$

Total Response

$$v(t) = Ae^{-t/2} + 18.95\cos(4t - 63.4^\circ);\ t > 0 \tag{iii}$$

Substituting initial conditions

$$20 = A + 18.95 \cos(-63.4°)$$

or $$A = 11.5$$

$\therefore$ $$v(t) = 11.5e^{-t/2} + 18.95 \cos(4t - 63.4°);\ t > 0 \quad \text{(iv)}$$

Example 4.21 For the series circuit of Fig. 4.60, with the current and voltage indicated, find the values of *R, r, L* and the frequency of the applied voltage and its magnitude.

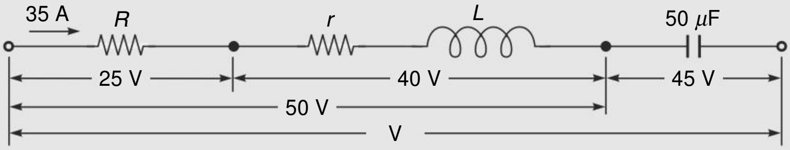


Fig. 4.60

Solution

$$45/X_C = 35 \text{ A or } X_C = 1.286 = 10^6/(\omega \times 50)$$

$\therefore$ $$f = 10^6/(1.286 \times 2\pi \times 50) = 2.475 \text{ kHz}$$

The voltage phasor triangle is drawn in Fig. 4.61(a).

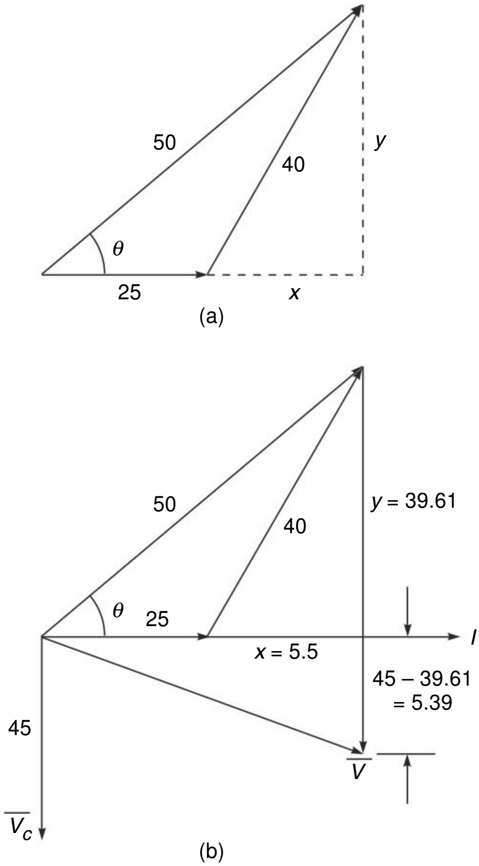


Fig. 4.61

$$\cos\theta = [(25)^2 + (50)^2 - (40)^2]/(2 \times 25 \times 50) = 0.61$$

or $$\theta = 52.4°$$

$$x = 50 \cos 52.4° - 25 = 5.5 \text{ V}$$

$$y = 50 \sin 52.4° = 39.61 \text{ V}$$

From the circuit diagram of Fig. 4.60, we have

$$35 \times r = 5.5$$

or $$r = 0.157\ \Omega$$

and $$35 \times (2\pi \times 2475\, L) = 39.61$$

or $$L = 0.073 \text{ mH}$$

The complete phasor diagram is drawn in Fig. 4.61(b), form which

$$V\text{(applied)} = [(25 + 5.5)^2 + (5.39)^2]^{0.5} = 31 \text{ V}$$

$$R = 25/35 = 0.714\ \Omega$$

Example 4.22 Convert the time domain circuit of Fig. 4.62 to the frequency domain and solve for i_1 using mesh analysis

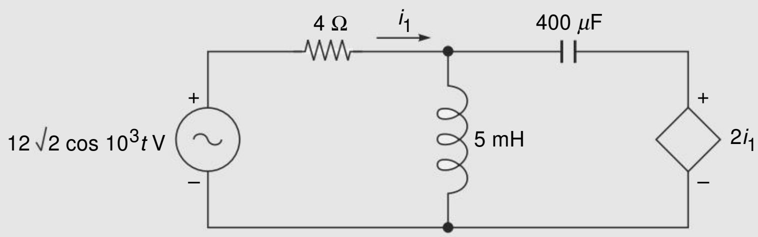


Fig. 4.62

Solution

The frequency domain circuit is drawn in Fig. 4.63 for $\omega = 10^3$ rad/s. The mesh equations are as follows:

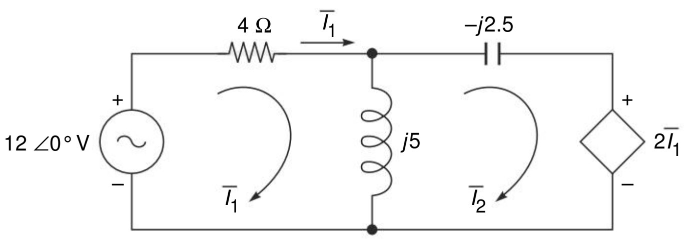


Fig. 4.63

Mesh 1: $$(4 + j5)\,\bar{I}_1 - j5\,\bar{I}_2 = 12 \quad \text{(i)}$$

Mesh 2: $$-j5\,\bar{I}_1 + j2.5\,\bar{I}_2 = -2\bar{I}_1 \quad \text{(ii)}$$

or $$(2 - j5)\,\bar{I}_1 + j2.5\,\bar{I}_2 = 0 \quad \text{(iii)}$$

Solving Eqs. (i) and (ii), we get

$$\Delta = \begin{vmatrix} (4+j5) & -j5 \\ (2-j5) & j2.5 \end{vmatrix} = 12.5 + j20 = 23.58 \angle 58°$$

$$\Delta_1 = \begin{vmatrix} 12 & -j5 \\ 0 & j2.5 \end{vmatrix} = j30 = 30 \angle 90°$$

$$\Delta_2 = \begin{vmatrix} (4+j5) & 12 \\ (2-j5) & 0 \end{vmatrix} = -24 + j60 = 64.42 \angle 111.8°$$

$$\bar{I}_1 = \Delta_1 | \Delta_2 = 30 \angle 90° | 64.42 \angle 111.8° = 0.46 \angle -21.8° \text{ A}$$

$$i_1 = 0.46\sqrt{2} \cos(10^3 t - 21.8°) \text{ A}$$

Example 4.23 For the circuit of Fig. 4.64, find the current $i(t)$ using the superposition theorem.

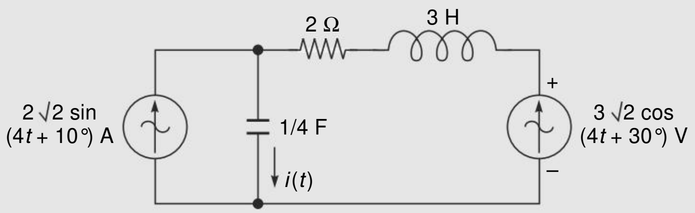


Fig. 4.64

Solution

$$\omega = 4 \text{ rad/s}$$

$$3H \to j4 \times 3 = j12\ \Omega$$

$$1/4\ F \to -j(1/(1/4) \times 4) = -j1\ \Omega$$

$$2\sqrt{2} \sin(4t + 10°) \to 2 \angle 10° \text{ A}$$

$$3\sqrt{2} \cos(4t + 30°) \to 3 \angle(90° + 30°) = 3 \angle 120° \text{ V}$$

The frequency domain circuit is drawn in Fig. 4.65 (sine is taken as reference).

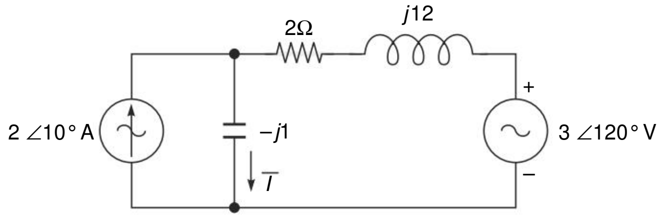


Fig. 4.65

Employing superposition, the circuit as an excited current/voltage source is redrawn in Fig. 4.66(a) and 4.66(b). Calculations of $\bar{I}_1$ and $\bar{I}_2$ are carried out as follows:

$$\bar{I}_1 = \left(\frac{2 + j2}{2 + j2 - j1}\right) \times 2 \angle 10° = 2.828 \angle 45° \times 2 \angle 10°/2.236 \angle 26.6°$$

$$= 2.53 \angle 28.4° = 2.23 + j1.20 \text{ A}$$

$$\bar{I}_2 = 3\ \angle 120°/(2 + j2 - j1) = 3\ \angle 120°/2.236°\ \angle 26.6°$$
$$= 1.342\ \angle 93.4° = -0.08 + j1.34 \text{ A}$$
$$\therefore \quad \bar{I} = \bar{I}_1 + \bar{I}_2 = 2.14 + j2.55 = 3.33\ \angle 50° \text{ A}$$
$$i(t) = 3.3\sqrt{2}\ \sin(4t + 50°) \text{ A}$$

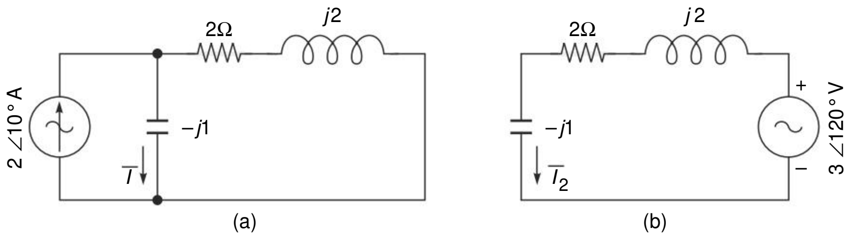


Fig. 4.66

Example 4.24 A coil of resistance 8Ω and inductance 0.1H are connected in series with a condenser of 160 μF capacitance across a 230 V, 50 Hz supply (Fig. 4.67). Calculate (a) the inductive reactance (b) the capacitive reactance (c) the circuit impedance, current and the pf, and (d) the coil and condenser voltages respectively.

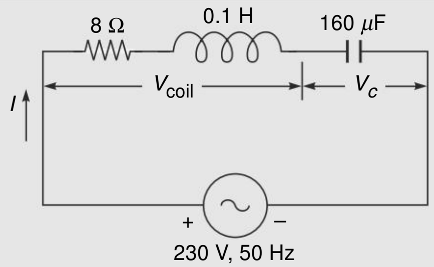


Fig. 4.67

Solution

(a) $X_L = 314 \times 0.1 = 31.4\ \Omega$

(b) $X_C = 10^6/314 \times 160 = 19.9\ \Omega$

(c) $Z = \sqrt{[8^2 + (31.4 - 19.9)^2]} = 14\ \Omega$

$I = V/Z = 230\ /14 = 16.43$ A

$\text{pf} = \cos \tan^{-1} (31.4 - 19.9)/8 = 0.572$ lagging $(X_L > X_C)$

(d) $V_{\text{coil}} = 16.43\ \sqrt{8^2 + 31.4^2} = 532.4$ V

$V_C = 16.43 \times 19.9 = 327$ V

Remark Observe that coil voltage and condenser voltages are more than the applied voltage. This phenomenon will be studied is detail in Chapter 5.

ADDITIONAL SOLVED PROBLEMS

4.25 In the circuit of Fig. 4.68 find the values of I_1, I_2, and I_3

Solution Impedance seen by the source

$$\bar{Z}_1 = -j5 + 5 \parallel j\,5$$

$$5 \parallel j5 = \frac{5 \times j5}{5 + j5} = \frac{j25}{5\sqrt{2}\ \angle 45°} = \frac{5}{\sqrt{2}}\ \angle 45°$$

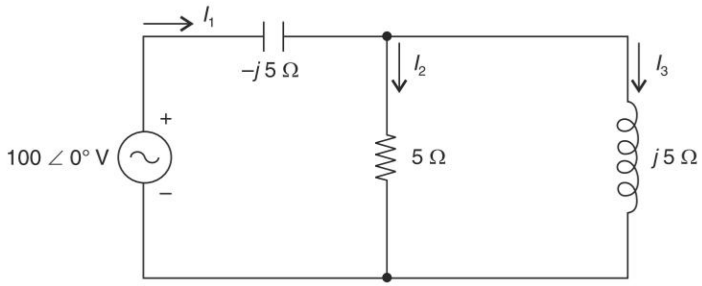


Fig. 4.68

$$= \frac{5}{\sqrt{2}}(\cos 45° + j\sin 45°) = 2.5 + j2.5\ \Omega$$

$$\bar{Z}_1 = -j5 + (2.5 + j\,2.5) = 2.5 - j\,2.5$$

$$= 3.536\ \angle -45°\ \Omega$$

$$\therefore \qquad \bar{I}_1 = \frac{100\ \angle 0°}{3.536\ \angle -45°} = 28.28\ \angle\ 45°$$

We now use the current division (between 5 Ω and j 5 Ω)

$$\bar{I}_2 = \frac{j5}{5+j5} \times 28.28\ \angle\ 45° \qquad \bar{I}_3 = \frac{j5}{5+j5} \times 28.28\ \angle 45°$$

$$= \frac{j1}{\sqrt{2}\ \angle 45°} \times 28.28\ \angle\ 45° \qquad = \frac{1}{\sqrt{2}\ \angle 45°} \times 28.28\ \angle 45°$$

$$= j\,20 = 20\angle 90°\ A \qquad = 20\ A$$

4.26 In the circuit of Fig. 4.69 is $i_s(t) = 0.5\cos 400\,t\ V$. Determine (a) $i_L(t)$ and $i_x(t)$.

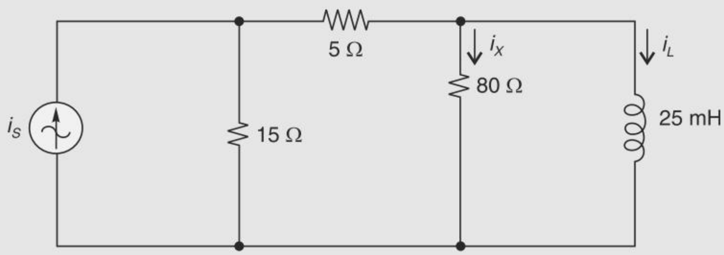


Fig. 4.69

Solution

Replace the practical current source to voltage source. The circuit is drawn in Fig. 4.69 (a).

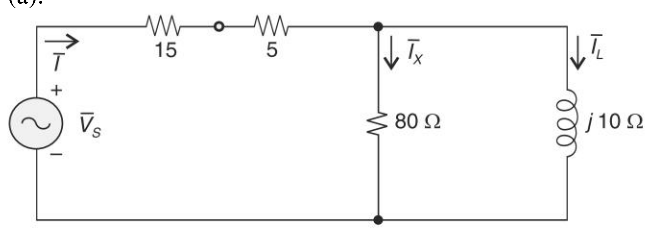


Fig. 4.69a

$$v_s(t) = 15 \times 0.4 \cos 400t$$
$$= 6 \cos 400t$$

Converting to phasor form

$$\bar{V}_S = 6 \angle 0° \; V$$
$$X_L = 400 \times 25 \times 10^{-3} = 10 \; \Omega$$

Let us calculate the impedance as seen by the source.

$$\bar{Z} = 20 + 80 \parallel j\,10$$
$$= 20 + \frac{j800}{80 + j10} = \frac{1600 + j200 + j800}{80 + j10}$$
$$= \frac{1600 + j1000}{80 + j10} = \frac{160 + j100}{8 + j1} = \frac{188.6\angle 32°}{8.06\angle 7.1°}$$
$$= 23.4 \angle 24.9° \; \Omega$$

Then

$$\bar{I} = \frac{\bar{V}_S}{\bar{Z}} = \frac{6}{23.4 \angle 24.9} = 0.256 \angle -24.9°$$

By current division in a parallel circuit

$$\bar{I}_L = 0.256\angle -24.6° \times \frac{80}{80+j10} = 0.256 \angle -24.6° \times \frac{80}{80.6\angle 7.1°} = 0.25 \angle -31.7° \text{A}$$

$$\bar{I}_L = 0.256 \angle -24.6° \times \frac{10 \angle 90°}{80.6 \angle 7.1°} = 0.032 \angle 58.3° \text{ A}$$

In time domain form

$$i_L(t) = 0.25 \cos(400t - 31.7°)$$
$$i_x(t) = 0.032 \cos(400t - 58.3°)$$

4.27 The time domain circuit of Fig. 4.70 is excited with a voltage source $v_s(t) = 4\sqrt{2} \cos 2t$.

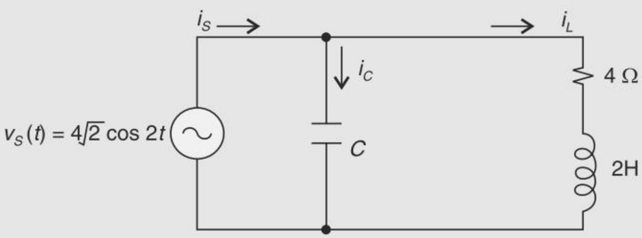


Fig. 4.70 Time domain circuit

(a) Draw the frequency-domain circuit labeling current and voltages.
(b) Determine the value of the capacitor C such that $\bar{I}_s$ is in phase with $\bar{V}_s$
(c) Draw the phasor diagram showing voltage and currents.

Solution

(a) The frequency-domain circuit is drawn in Fig. 4.70 (a) wherein

$$\bar{V}_s = 4 \angle 0°, X_L = \omega L = 2 \times 2 = 4 \; \Omega \qquad X_C = \frac{1}{\omega C} \text{ to be determined}$$

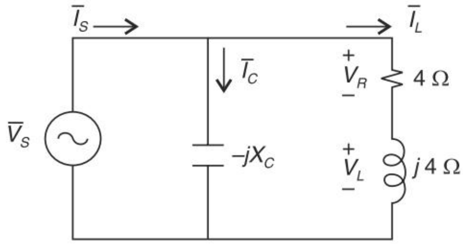


Fig. 4.70 (a)

(b) $\bar{I}_s = \bar{I}_L + \bar{I}_C$

For $\bar{I}_s$ to be in phase with $\bar{V}_s$

$\bar{I}_s = \bar{I}_s \angle 0°$

which means that its imaginary party is zero.

For this the imaginary parts of $\bar{I}_L$ and $\bar{I}_C$ should be cancel out

$$I_L = \frac{4}{4+j4} = \frac{1}{1+j1} = \left(\frac{1}{2} - j\frac{1}{2}\right) = \frac{1}{\sqrt{2}}\angle -45°\text{A}$$

Its imaginary part is $\left(-j\frac{1}{2}\right)$ A

$$\bar{I}_C = \frac{4}{jX_C} = j\frac{4}{X_C} \text{ it imaginary only}$$

Then

$$j\frac{4}{X_C} - j\frac{1}{2} = 0 \quad or \quad X_C = 8\Omega, \qquad X_C = \frac{1}{\omega C} = \frac{1}{2C} = 8 \Rightarrow C = \frac{1}{16} = 0.0625\text{ F}$$

$\bar{I}_C = j\dfrac{1}{2} = j\,05$

(c) Phase diagram is drawn in Fig. 4.71 (b)

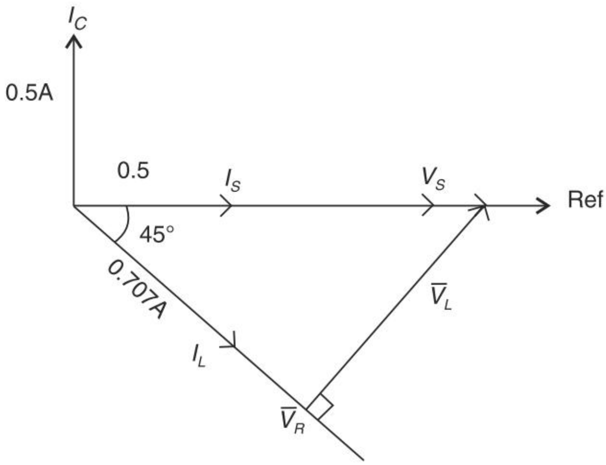


Fig. 4.71 (b)

$$\overline{V}_R = \frac{1}{\sqrt{2}} \times 4 = 2\sqrt{2}\ V$$

$$\overline{V}_L = \frac{1}{2} \times 4 = 2\sqrt{2}\ V$$

$\overline{V}_L$ leads $\overline{V}_R$ by 90°

4.28 In the circuit of Fig. 4.72, find $v_x(t)$.

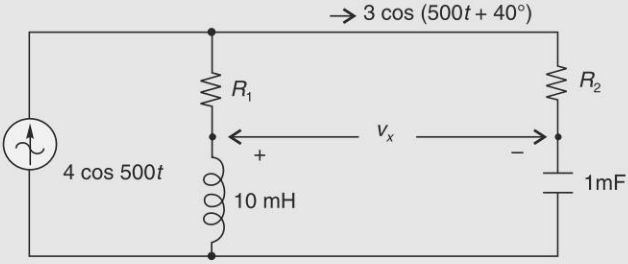


Fig. 4.72

Solution

$$\omega = 500 \text{ rad/s}$$

Converting currents to phasor form; we will use peak amplitude

$$4 \cos 500t \;\rightarrow\; 4 \angle 0° \text{ A}$$

$$3 \cos (500t + 40°) \;\rightarrow\; 3 \angle 40° \text{ A}$$

The frequency domain circuit is drawn in Fig. 4.72 (a)

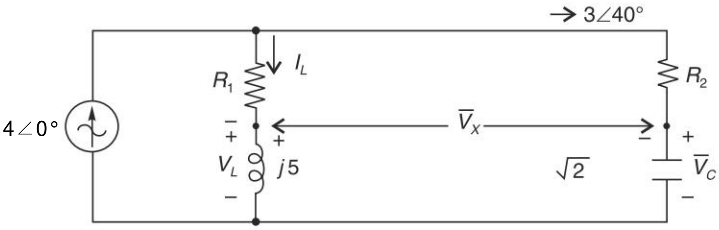


Fig. 4.72(a)

$$X_L = 500 \times 10 \times 10^{-3} = 5\ \Omega$$

$$X_C = \frac{1}{500 \times 1 \times 10^{-3}} = 2\ \Omega$$

At node

$$\overline{I}_L = 4 \angle 0° - 3 \angle 40° = 4 - (2.3 + j1.93)$$

$$= 1.7 - j1.93 \text{ A}$$

$$\overline{I}_C = 3 \angle 40° = 2.3 + j1.93$$

$$\overline{V}_C = -j2\ \overline{I}_C = -j2\ (2.3 + j\ 1.93) = 3.86 - j\ 4.6 \text{ V}$$

$$\overline{V}_L = j5\ \overline{I}_L = j\ 5\ (2.3 + j\ 1.93) = -9.65 + j\ 11.5$$

From Fig. 4.72 (a)

$$\overline{V}_X = \overline{V}_L - \overline{V}_C$$

Substituting values, we find

$$\overline{V}_X = -(13.51 - j\,16.1) = -\,21.0\ \angle\ -50°\ \text{A}$$

4.29 In the circuit of Fig. 4.73. R = 2Ω, L = 0.3 H and $i_R = 10\sqrt{2}\ \cos\,(10t + 45°)$ A with v as reference. Draw the phasor diagram showing $\overline{V}_C$, $\overline{V}$ and $\overline{I}_L$. Determine there from

(a) value of capacitor

(b) $\overline{I}_L$ and $i_L(t)$

(c) $\overline{V}$ and $v\,(t)$

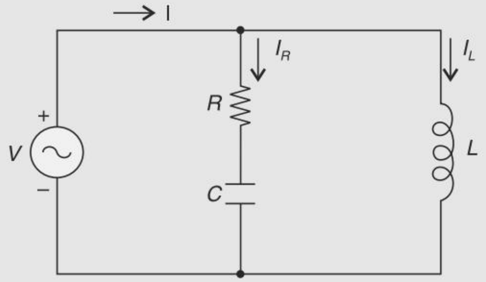


Fig. 4.73

Solution

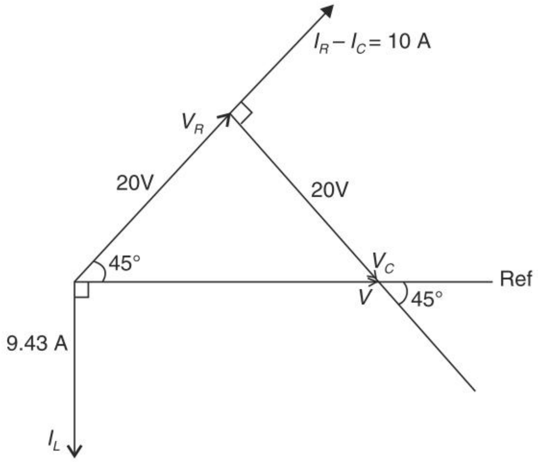


Fig. 4.73(a)

The phasor diagram is drawn in Fig. 4.73a

$$\overline{I}_R = 10\ \angle\ 45°$$

$$\overline{V} = V\ \angle\ 0°$$

As $\qquad R = 2\ \Omega$

$$R\overline{I}_R = 2 \times 10\ \angle 45° = 20\ \angle 45°\text{V}$$

$\overline{V}_C$ being capacitor voltage lags $\overline{I}_R = \overline{I}_C$ by 90°

From the voltage triangle, we find

$$\overline{V}_C = 20\ \angle\ 45° - 90° = 20\ \angle\ -45°\ \text{V}$$

Also $\qquad \overline{V} = 20\sqrt{2}\ \ \angle 0°\ \ V = 28.28$

Now $\qquad V_C = \omega\, C I_C\ ;\ \omega = 10$ rad/s

$$C = \frac{20}{10 \times 10} = 0.2\ \text{F}$$

$$X_L = \omega\, L = 10 \times 0.3 = 3\ \Omega$$

$$\bar{I}_L = \frac{\bar{V}}{X_L} \angle -90^\circ = \frac{20\sqrt{2}}{3} \angle -90^\circ = 9.43 \angle -90^\circ \text{ V}$$

From $\bar{V}_C$

$$v_c = 28.28 \cos (10\,t - 45^\circ)$$

From $\bar{I}_L$

$$i_L(t) = 9.43 \cos (10t - 90^\circ) = 9.43 \sin 10t \text{ A}$$

4.30 In the circuit of Fig. 4.74, $R = 2$ kΩ, $C = 5$ μF. Determine its series circuit equivalent having the same terminal (driving point) admittance at $\omega = 1000$ rad/s.

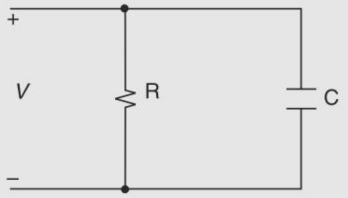


Fig. 4.74

Solution

$$X_C = \frac{1}{\omega C} = \frac{1}{1000 \times 5 \times 10^{-6}}$$

$$= 0.2 \times 10^3 \ \Omega$$

Terminal admittance

$$\bar{Y} = \frac{1}{R} + \frac{1}{-jX_C},$$

$$= (0.5 + j5) \times 10^{-3} \ \mho \qquad \text{(i)}$$

The series equivalent is drawn in Fig. 4.74 (a). For the series circuit to have same admittance, it is convenient to match the impedances.

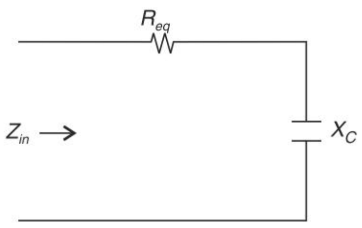


Fig. 4.74a

From Eq. (i)

$$\bar{Z}_i = \frac{1}{\bar{Y}_i} = \frac{10^3}{0.5 + j5} = \frac{0.5 - j5}{0.25 + 25} \times 10^3$$

$$= 19.8 - j198 \ \ \Omega$$

Therefore

$$R_{eq} = 19.8 \ \Omega \qquad X_{Ceq} = 198 \ \Omega = \frac{1}{1000 \times C}$$

$$\text{or} \quad C = \frac{10^6}{1000 \times 198} = 50.5 \ \mu\text{F}$$

4.31 In the circuit of Fig. 4.75, $C = 250$ μ F and $i_c(t) = 6\sqrt{2}\cos 1000t$.

(a) Determine $\bar{I}_c$

(b) If the current input is $\bar{I} = 10 \angle -37^\circ$. Determine R and X.

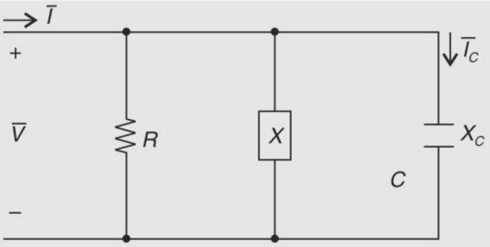


Fig. 4.75

Solution

(a) In phasor form, $\overline{V}_C = 6 \angle 0°$

(b) $$X_C = \frac{1}{\omega C} = \frac{10^6}{1000 \times 250} = 4\Omega$$

$$\overline{I}_C = \frac{\overline{V}_C}{-j\,X_C} = \frac{6}{-14} = 1.5 \angle 90° \text{ A}$$

At the node

$$\overline{I} = \overline{I}_R + \overline{I}_X + \overline{I}_C$$

or $$\overline{I}_R + \overline{I}_X = 10 \angle -37° - 1.5 \angle 90°$$

$$= (8 - j\,6) - j1.5 = 8 - j\,7.5$$

Therefore

$$\overline{I}_X = -j\,7.5$$

As the current $\overline{I}_X$ lags voltage $\overline{V}$, X is inductive.

$$X = \frac{6}{7.5}\,\Omega$$

$$R = \frac{6}{8} = \frac{3}{4}\,\Omega$$

4.32 In the circuit of Fig. 4.76 the following data is given.

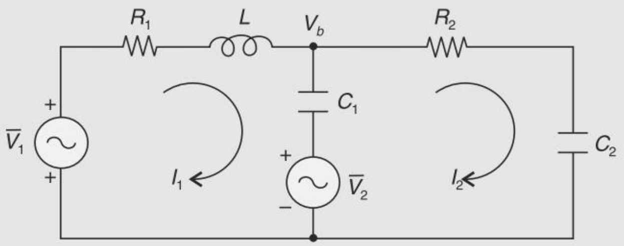


Fig. 4.76

$\overline{V}_1 = 30 + j10$ V

$\overline{V}_2 = 30 + j0$ V

$\omega = 1000$ rad/s

$L = 1$ H, $C_1 = C_2 = 1\ \mu F$, $R_1 = R_2 = 1\text{k}\ \Omega$

(a) Determine mesh currents $\overline{I}_1$ and $\overline{I}_2$. Therefore find the current through C_1.

(b) Use nodal analysis to determine $\overline{V}_b$.

Solution

The circuit is frequency-domain and is drawn in Fig. 4.76 (a) wherein

$$X_L = \omega L = 1000 \times 1 = 1\text{ k}\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{10^6}{1000 \times 1} = 1\text{ k}\Omega$$

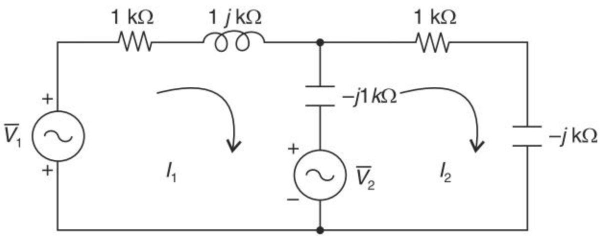


Fig. 4.76(a)

Same values hold for both capacitors.

Current $\bar{I}_1$ and $\bar{I}_2$ are in mA.

Mesh 1

$$-\bar{V}_1 + 1.\bar{I}_1 + j1.\bar{I}_1 - j1.\bar{I}_1 + j\bar{I}_2.1 + \bar{V}_2 = 0$$

$$\bar{I}_1 + j\,\bar{I}_2 = \bar{V}_1 - \bar{V}_2 = 30 + j\,10 - 30 = j10$$

Mesh 2

$$-\bar{V}_2 - 2j\,\bar{I}_2 + 1.I_2 + j\,I_1 = 0$$

$$j\bar{I}_1 + (1 - 2j)\,\bar{I}_2 = \bar{V}_2 = 30 \qquad \text{(ii)}$$

Simultaneous equations to be solved are

$$\bar{I}_1 + j\,\bar{I}_2 = j\,10 \qquad \text{(iii)}$$

$$j\,\bar{I}_1 + (1 - j2)\,\bar{I}_2 = 30 \qquad \text{(iv)}$$

Multiplying Eq. (iii) by j

$$j\,\bar{I}_1 - \bar{I}_2 = -10 \qquad \text{(v)}$$

Eliminating $\bar{I}_1$ from Eqs. (iv) and (v)

$$2\,(1 - j)\,\bar{I}_2 = 40$$

or $$\bar{I}_2 = \frac{20}{1-j} = \frac{20(1+j)}{2} = 10(1+j)$$

$$= 10\sqrt{2}\ \angle\ 45°\text{A}$$

From Eq. (v)

$$jI_1 - 10\,(1 + j) = -10$$

$$j\,I_1 - 10 - 10j = -10$$

$$jI_1 = 10\,j \Rightarrow I_1 = 10$$

Current through C_1

$$\bar{I}_{C1} = \bar{I}_1 - \bar{I}_2 = 10 - 10\,(1 + j) = j10 = 10\ \angle 90°$$

4.33 Find the Thevenin and Norton equivalent as seen from the terminals '*a*' and '*b*' of the circuit of Fig. 4.77. Draw the equivalent circuits.

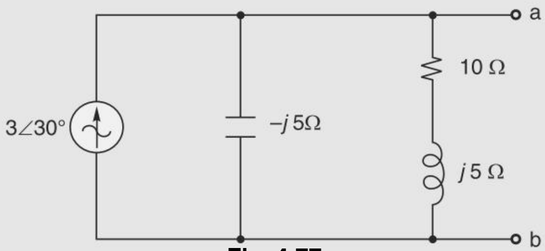


Fig. 4.77

Solution

Impedance of each parallel branch

$$\bar{Z}_C = -j5$$

$$\bar{Z}_{RL} = (10 + j\,5)$$

By current division

$$\bar{I}_C = \frac{10 + j5}{10 + j5 - j5} \times 3 \angle 30° = (1 + j\,0.5) \times 3 \angle 30°$$

$$= 1.12 \angle 26.6° \times 3 \angle 30°$$

$$= 3.36 \angle 56.6°\ V$$

Open-circuit voltage

$$\bar{V}_{OC} = \bar{V}_{TH} = -j\,5 \times \bar{I}_C = -j5 \times 3.36 \angle 56.6°$$

$$= 16.8 \angle -33.4\ V$$

Open current source impedance seen from '*a*', '*b*'

$$\bar{Z}_{TH} = \frac{-j\,5 \times (10 + j5)}{10 + j\,5 - j\,5} = -j5\,(1 + j0.5)$$

$$= 2.5 - j5\ \Omega$$

or $5.59 \angle -63.4°\ \Omega$

The Thevenin equivalent is drawn in Fig. 4.77 (a).

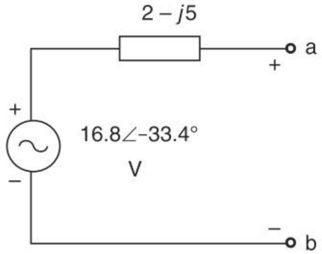


Fig. 4.77(a)

Norton's Equivalent We find short-circuit current to be

$$\bar{I}_{SC} = \frac{V_{TH}}{Z_{TH}} = \frac{16.8\angle -33.4°}{5.59\angle -63.4} = 3\angle 30°\ A$$

The Norton's equivalent is drawn in Fig. 4.77 (b). Observe the direction of I_{SC}. It is such as to produce the same polarity at '*ab*' as $\bar{V}_{OC}$. It is also observed that if '*ab*' are shorted in Norton, I_{SC} would flow in the same direction as in Thevenin on shorting 'ab'.

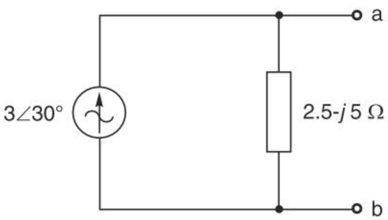


Fig. 4.77(b)

4.34 In the circuit of Fig. 4.78. determine the current $i(t)$ using Superposition Theorem.

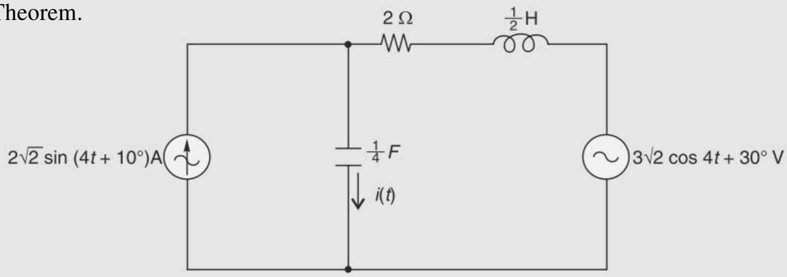


Fig. 4.78

Solution

We convert the circuit to frequency-domain form.

$\omega = 4$ rad / s

$$\frac{1}{3}H \rightarrow j4 \times \frac{1}{2} = j2$$

$$\frac{1}{4}F \rightarrow -j\frac{1}{4 \times \frac{1}{4}} = -j1\Omega$$

$2\sqrt{2}$ sin $(4t + 10°)$ A → $2 \angle 10°$ A; sine reference

$3\sqrt{2}$ cos $(4t + 30°)$ A → $3 \angle 90° + 30° = 3 \angle 120°$ V

The circuit is drawn in Fig. 4.78 (a)

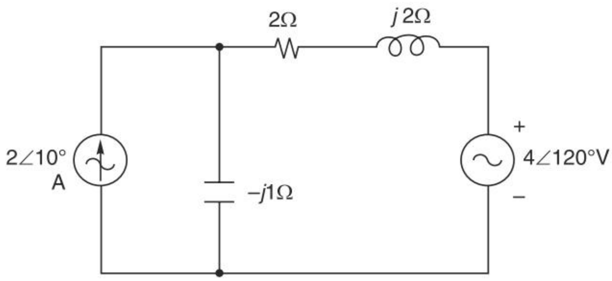


Fig. 4.78(a)

1. Shorting voltage source

By current division, we get

$$\bar{I}_1 = 2\angle 10° \times \frac{2 + j2}{2 + j2 - j1}$$

$$= 2\angle 10° \frac{2(1 + j1)}{2 + j1} = \frac{2\angle 10° \times 2\sqrt{2}\angle 45°}{2.24 \angle 26.6°}$$

$= 2.525 \angle 28.4°$ $A = 2.23 + j\ 1.20\ A$

2. Open circuiting current source

$$\bar{I}_2 = \frac{3\angle 120°}{2 + j2 - j1} = \frac{3\angle 120°}{2 + j1}$$

$= 1.342 \angle 93.4° = -0.08 + j\ 1.34\ A$

Now

$$\bar{I} = \bar{I}_1 + \bar{I}_2 = 2.15 + j\ 2.54 = 3.33 \angle 49.8°$$

$i(t) = 3.33\sqrt{2}$ sin $(4t + 49.8°)$ A

4.35 In the circuit of Fig. 4.79, the steady-state has been reached. Find the power absorbed at t = 1ms by (a) resistor (b) inductor and (c) current source.

Solution

We first convert the circuit to frequency-domain.

$$10\sqrt{2}\cos 2000t, A \xrightarrow{\text{phasor}} 10 \angle 0°\ A, \qquad \omega = 2000 \text{ rad / s}$$

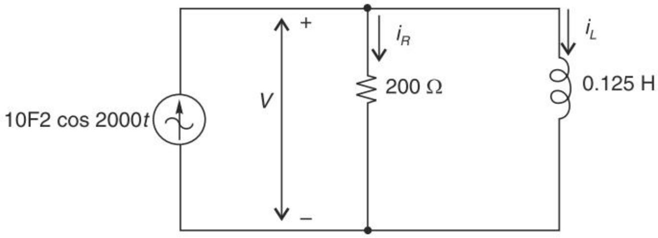


Fig. 4.79

$$200\Omega \rightarrow 200\Omega$$
$$0.125\text{H} \rightarrow j\,2000 \times 0.125 = j\,250$$

The circuit is drawn in Fig. 4.79 (a).

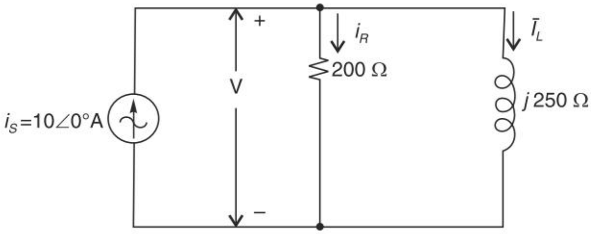


Fig. 4.79(a)

Impedance $\bar{Z} = 200 \parallel j250$

$$= \frac{200 \times j250}{200 + j250}$$

$$= 156.3 \angle 38.6°\Omega$$

$$\bar{V} = \bar{I}_s\,\bar{Z} = 10 \angle 0° \times 156.3 \angle 38.6° \text{ V}$$

$$= 1.563 \angle 38.6° \text{ V} \qquad \text{(i)}$$

$$\bar{I}_L = \frac{\bar{V}}{j250} = \frac{1563 \angle 38.6°}{j250}$$

$$= 6.25 \angle -51.4° \text{ A} \qquad \text{(ii)}$$

$$\bar{I}_R = \frac{\bar{V}}{200} = \frac{1563 \angle 38.6°}{200} = 7.82 \angle 38.6° \text{ A} \qquad \text{(iii)}$$

Converting to time-domain form

$$i_s(t) = 10\sqrt{2} \cos 2000t \text{ A}$$

$$i_L(t) = 6.25\sqrt{2} \cos (2000t - 51.4°) \text{ A}$$

$$i_R(t) = 7.82\sqrt{2} \cos (2000t + 38.6°) \text{ A}$$

$$v(t) = v_s(t) = v_L(t) = 1563\sqrt{2} \cos (2000t + 38.6°) \text{ A}$$

At t = 1ms, $2000 \times 1 \times 10^{-3} = 2$ rad = 114.6°

The values of currents and voltages are obtained below.

$$i_s(t) = 10\sqrt{2} \cos 114.6° \text{ A}$$

$$i_L(t) = 6.25\sqrt{2}\cos(114.6° - 51.4°) \text{ A}$$
$$i_R(t) = 7.82\sqrt{2}\cos(114.6° + 38.6°) \text{ A}$$
$$v = 1563\sqrt{2}\cos(114.6° + 38.6°)$$

or

$$i_s = 5.89 \text{ A},\ i_L = 4.07 \text{ A},\ i_R = -9.87 \text{ A}$$
$$v = v_s = v_L = -1973 \text{ V}$$

Instantaneous power absorbed

$$p\text{ (source)} = -v_s i_s = -[-1973 \times 5.89] / 1000 = -11.62 \text{ kW}$$
$$p(L) = v_L i_L = -1973 \times 407/1000 = -8.03 \text{ kW}$$
$$p(R) = i_R^2 . R = (-9.87)^2 \times 200/1000 = 19.48 \text{ kW}$$

Check

$-(-11.62 - 8.03) = 19.65\ A$ in place of 19.48 kW

The difference is due to computational round-offs.

4.36 A phasor voltage $\overline{V} = 120 \angle 45°\ V$ is applied across an impedance $\overline{Z} = 16.3 \angle 24.5°\ \Omega$. Obtain an expression for instantaneous power and then find the value of average Power fed to the impedance. Given $\omega = 50$ rad/s.

Solution

Current drawn by impedance

$$\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{120 \angle 45°}{16.3 \angle 24.5°} = 7.26 \angle 20.5° \text{ A}$$

Converting to time-domain

$$v(t) = 120\sqrt{2}\sin(50t + 45°) \text{ V} \qquad \text{(i)}$$
$$i(t) = 7.36\sqrt{2}\sin(50t + 20.5°) \text{ A} \qquad \text{(ii)}$$

Instantaneous power

$$p = vi = 120 \times 7.36\,[2\sin(50t + 45°)\sin(50t + 20.5°)] \text{ W}$$

or

$$p = 883\,[\cos(45° - 20.5°) - \cos(100t + 45° + 20.5°)] \text{ W}$$
$$= 883\cos 245° - 883\cos(100t + 66°)$$

Average power $P = 883\cos(24.5°) = 803.5$ W

$\overline{I}$ lead $\overline{V}$ by 24.5°, pf = cos 24.5 = 0.909

4.37 In the circuit of Fig. 4.80. $R = 3$ kΩ, $L = 1$ H, $C = 0.25\ \mu$F. (a) Determine the impedance presented to current $i(t) = 20\sqrt{2}\cos(5000t + 60°)$ A in steady-state. Calculate $v(t)$.

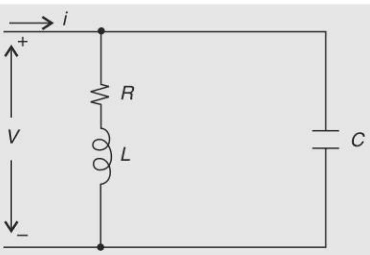


Fig. 4.80

Solution

Converting the circuit and given current to frequency domain form, $\omega = 5000$ rad/s

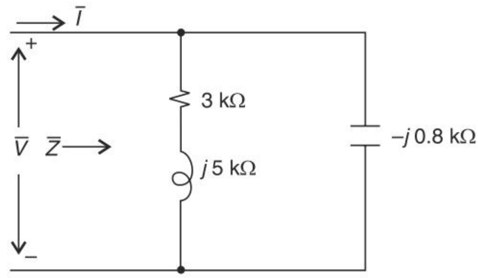


Fig. 4.80a

$$R = 3\text{ k}\Omega \rightarrow 3\text{ k}\Omega$$

$$L = 1\text{H} \rightarrow j\,5000 \times 1$$

$$= j\,5\text{ k}\Omega$$

$$C = 0.25\mu F \longrightarrow -j\frac{10^6}{5000 \times 0.25} = -j\,0.8\text{ k}\Omega$$

(a) Impedance

$$\bar{Z} = (3 + j\,5)\,(-j\,0.8)$$

$$= \frac{-j0.8\,(3 + j5)}{3 + j5 - j0.8}$$

$$= \frac{4 - j2.4}{3 + j4.2} = \frac{4.66\ \angle -31^\circ}{5.16\ \angle 54.5^\circ}$$

$$= 0.903\ \angle -85.5^\circ\ \Omega$$

$$i(t) \xrightarrow{Phasor} 20\ \angle 60^\circ\ A$$

Voltage $\bar{V} = \bar{I}\,\bar{Z} = 20\ \angle 60^\circ \times 0.903\ \angle -85.5 = 18.06\ \angle\ 25.5^\circ$ V

Then

(b) $v(t) = 18.06\sqrt{2}\cos(5000t - 25.5^\circ)$

4.38 For the circuit of Fig. 4.81. Find the Thevenin equivalent to the left of terminals '*ab*'. Find *v* (*t*) and also find the complex power consumed by 0.8 H inductor.

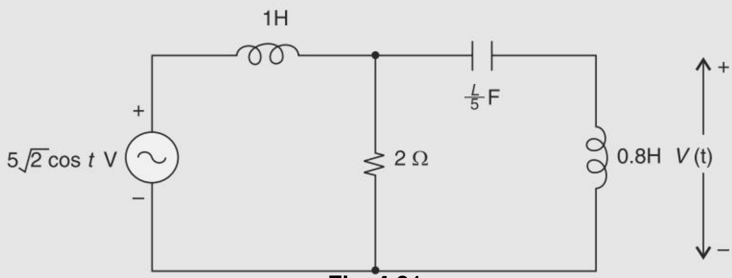


Fig. 4.81

Solution

Transforming to frequency domain

$$5\sqrt{2}\cos 5t \xrightarrow{\text{Phasor}} 5\ \angle 0^\circ\ V,\ \omega = 5\text{ rad / s}$$

$$2\Omega \rightarrow 2\Omega$$

$$\frac{1}{5}\text{F} \rightarrow -j\frac{1}{5 \times \frac{1}{5}} - j1\Omega$$

$0.8 \text{ H} \rightarrow j5 \times 0.8 = +j\,4\Omega$

The corresponding circuit is drawn in Fig. 4.81 (a).

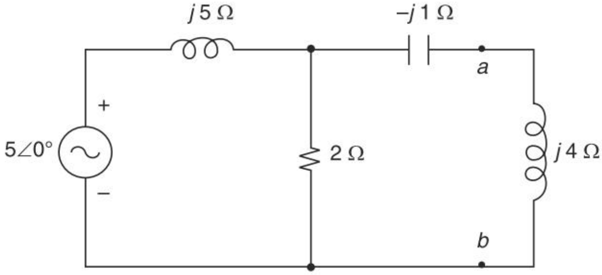


Fig. 4.81(a)

Open circuit voltage, disconnect inductor $j\,4\ \Omega$ as in Fig. 4.81 (b).

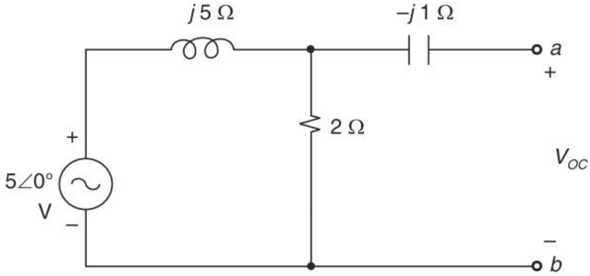


Fig. 4.81(b)

V_{OC} is same as voltage across 2Ω as $-j\ 1\Omega$ does not carry any current. Therefore

$$V_{OC} = V_{TH} = \left(\frac{2}{2+j5}\right)5\angle 0°$$

$$= 1.857 \angle -68.2° \text{ V}$$

Thevenin impedance

Short circuit $5\ \angle 0° V$ as in Fig. 4.81 (c)

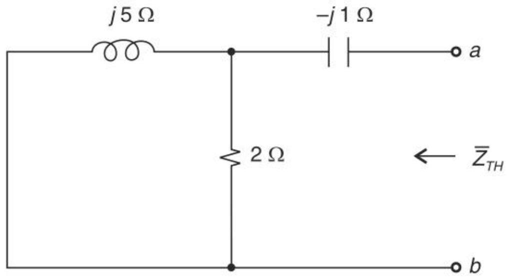


Fig. 4.81(c)

Impedance seen from '*ab*'

$$\overline{Z}_{TH} = -j1 + \frac{j5 \times 2}{2+j5} = -j1 + \frac{j5 \times 2}{2+j5}$$

$$= -j1 + 1.857 \angle 21.8°$$

$$= 1.724 - j\,0.310 = 1.752 \angle -10.2°\ \Omega$$

Complex power drawn by 0.8 H.

The Thevenin equivalent with load 0.8 is drawn in Fig. 4.81 (d). From the figure

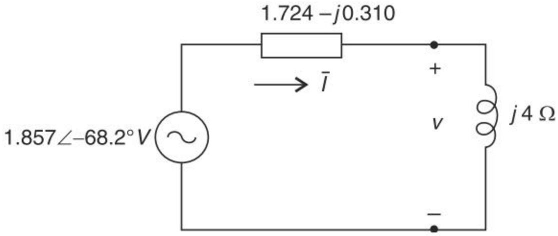


Fig. 4.81(d)

Current, $\bar{I} = \dfrac{1.875 \angle -68.2^\circ}{1.724 - j\,0.310 + j\,4}$

$$\bar{I} = \frac{1.875 \angle -68.2^\circ}{5.737 \angle 3.1^\circ} = 0.327 \angle -71.3^\circ \text{ A}$$

$$\bar{V} = \bar{I}.\,j\,0.8 = 0.327 \angle -71.3^\circ \times j4$$
$$= 1.308 \angle 18.7^\circ \text{ V}$$

Then

$$v(t) = 1.308\sqrt{2}\cos(5 + 18.7^\circ) \text{ V}$$

Complex power consumed by 0.8 H

$$\bar{S} = j\,(0.327)^2 \times 4 = 0 + j\,0.428 \text{ VA}$$

or

$$S = 0.428 \text{ VARs (inductive) Note + sign}$$

Also by relationship

$$\bar{S} = \bar{V}\,\bar{I}^* = 1.308 \angle 18.7^\circ \times 0.327 \angle 71.3^\circ$$
$$= 0.482 \angle 90^\circ = 0 + j\,0.482 \text{ VA}$$

4.39 A power supply feeds an ohmic load. The following information is known about supply voltage

- voltage wave $v(t) = V_m \cos(\omega t + \theta)$
- frequency of supply 15.36 MHz
- maximum power is supplied 600 W to 3 Ω load
- minimum value of voltage occur at 20.3 ms

Find the values of V_m, ω and θ.

Solution

Maximum power, $p_m = \dfrac{V_m^2}{3} = 600$ W

$$V_m = \sqrt{1800} = 42.43V$$

$$\omega = 2\pi f = 2\pi \times 15.36 \times 10^6 = 96.51 \times 10^6 \text{ rad/s}$$

In a cosine wave, minimum voltage recurs at $t = 20.3$ ms. Minimum value of cosine is zero. At minimum voltage.

$$\therefore \quad \cos(\omega t + \theta) = 0$$

$$\text{or} \quad \omega t + \theta = 90°$$

$$\text{At} \quad t = 20.3 \text{ ms}$$

$$\omega t = 96.51 \times 20.3 \times 10^3 \text{ rad / s}$$

$$= 96.51 \times 20.3 \times 10^3 \times 180/\pi \text{ degrees}$$

$$= 112, 251, 198.3°$$

Remove multiples of 360°

112, 251, 198.3° – 360° $x = \omega t = 318.3°$

$\omega t + \theta = 90°$

$$\theta = 90 - 318.3 = -228.3° + 360°$$

$$= 131.7°$$

4.40 A resistance of 6Ω is connected in series with a choke having resistance r and inductance L connected in series across a voltage source of voltage 240 V, 50 Hz. The voltage drop across resistance is 60V and across choke coil is 205 V. Calculate (a) resistance, inductance and impedance of the choke coil (b) its power loss and power factor.

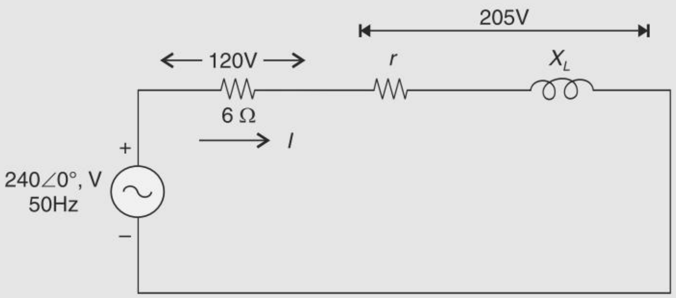


Fig. 4.82

Solution

(a) By Ohm's law

$$I = \frac{120}{6} = 20 \text{ A}$$

$$20\sqrt{(6 + r)^2 + X_L^2} = 240 \text{ V} \tag{i}$$

$$20\sqrt{r^2 + X_L^2} = 205 \text{ V} \tag{ii}$$

Equations (i) and (ii) yield

$$r = 1.5\Omega \qquad X_L = 10.14\Omega \qquad L = \frac{10.14}{2\pi \times 50} = 32.28 \text{ mH}$$

$$Z = \sqrt{(1.5)^2 + (10.14)^2} = 10.25\Omega$$

(b) Power loss in choke = $(20)^2 \times 1.5 = 600$ W

$$\text{Power factor of choke} = \frac{600}{205 \times 20} = 0.146 \text{ lagging}$$

4.41 Find the admittance Y_{ab} if $\omega = 2$ rad / s. Also find its equivalent as R in parallel with L or C.

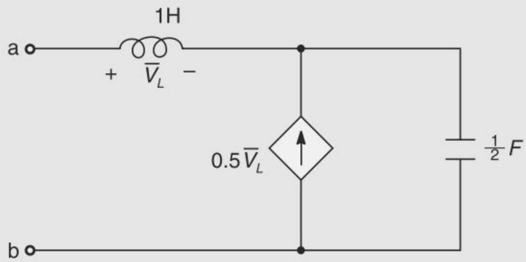


Fig. 4.83

Solution

$\omega = 2$ rad / s

1 H $\rightarrow j\,2\Omega$

$$\frac{1}{2}\text{F} \rightarrow \frac{1}{j2 \times \frac{1}{2}} = -\,j1\,\Omega$$

The frequency domain circuit is drawn in Fig. 4.83(a). Let a voltage $1 \angle 0°\ V$ be applied at terminal '*ab*' and the current shown by the circuit be $\overline{I}$.

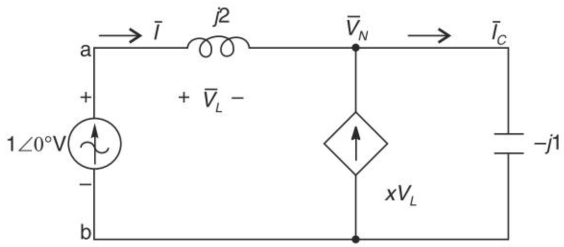


Fig. 4.83(a)

Then

$$\overline{V}_{ab} = \frac{\overline{I}}{1 \angle 0°} = \overline{I}$$

$$\overline{V}_L = j\,2\,\overline{I}$$

To find $\overline{I}_C$ we need to know $\overline{V}_n$.

$$(1-\overline{V}_n) = \overline{V}_L = j\,2\overline{I}$$

or $\overline{V}_n = 1 - j\,2\overline{I}$

Then

$$\overline{I}_C = j\,1 \times \overline{V}_n = j\,(1 - j\,2\overline{I}) = 2\overline{I} + j$$

At node n

$$\overline{I} + x\,\overline{V}_L - \overline{I}_C = 0$$

$$\overline{I} + j\,2x\,\overline{I} - 2\overline{I} - j = 0$$

$$(j\,2x - 1)\overline{I} = j$$

$$\bar{I} = \frac{j}{j\,2x - 1} = \frac{1}{2x + j}$$

We choose $x = \frac{1}{2}$, then

$$\bar{Y}_{ab} = \frac{1}{1 + j}\ \Omega$$

We can write

$$\bar{Y}_{ab} = \frac{1 - j}{1 + 1} = \frac{1}{2} - j\frac{1}{2} = \frac{1}{R} + \frac{1}{j\omega L};\quad \omega = 1\ \text{rad/s}$$

Then $R = 2\Omega$, $L = 2$H in parallel will be have same admittance.

4.42 In the *RLC* series circuit of Fig. 4.84, find the value of capacitor which if connected in shunt across '*ab*' renders $\bar{Z}_{in} = R_{in} + j\,0$.

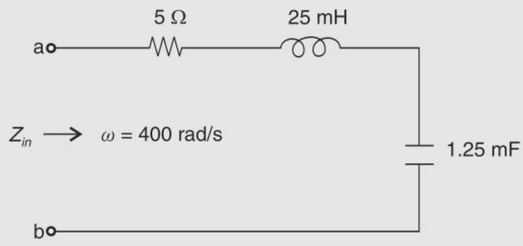


Fig. 4.84

Solution

Converting the circuit element to frequency domain

$$5\Omega \to 5\Omega$$

$$1\ \text{mH} \to j\,400 \times 25 \times 10^{-3} = j10$$

$$1\ \text{mF} \to \frac{1}{j\,400 \times 1.25 \times 10^{-3}} = -j2$$

$$\bar{Z}_{in} = 5 + j\,10 - j2 = 5 + j8\Omega$$

Convert to admittance

$$\bar{Y}_{in} = \frac{1}{5 + j8} = \frac{5 - j8}{25 + 64} = 0.0562 - j0.089$$

Let the shunt capacitane be C mF

Capacitive $= 400C \times 10^{-3}$

$= 0.4\ C$

The circuit in admittance form having shunt capacitance in drawn in Fig. 4.84 (a)

$$\bar{Y}_{in} = 0.0562 - j\,0.0899 + j\,0.4C.$$

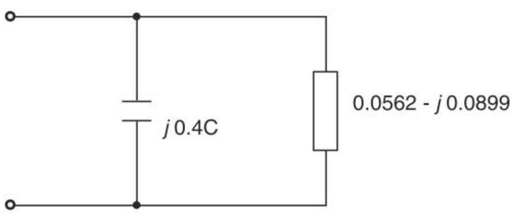


Fig. 4.84(a)

For $\overline{Y}_{in}$ to be resistive only, imaginary part should be zero.

$$j\,0.4\,C = j\,0.0899$$
$$C = 0.225 \text{ mF}$$

Then

$$\overline{Y}_{in} = 0.0562$$
$$R_{in} = \frac{1}{0.0562} = 17.8\Omega$$

4.43 For the circuit of Fig. 4.85, calculate input impedance $\overline{Z}_{in}$ as seen from terminal '*ab*' Given $\omega = 800$ rad/s. If *x* and *y* are shorted what would $\overline{Z}_{in}$?

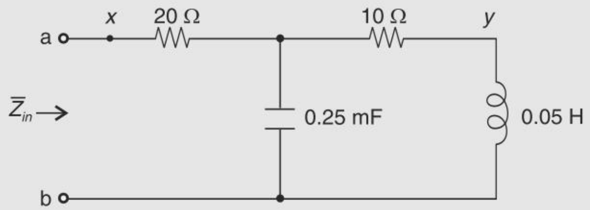


Fig. 4.85

Solution

Converting to frequency domain

$$25 \text{ mF} \rightarrow \frac{1}{j\,800 \times 0.25 \times 10^{-3}} = -j5$$
$$0.05 \text{ H} \rightarrow j\,800 \times 0.05 = j\,40\ \Omega$$

Impedance of parallel circuit

$$(10 + j40) \| (-j5) = \frac{-j5(10 + j40)}{10 + j40 - j5}$$
$$= \frac{200 - j5}{10 + j35} = \frac{200.06\ \angle -1.4^\circ}{36.4\ \angle 74^\circ}$$
$$= 5.5\ \angle -75.4^\circ = 1.386 - j\,5.322\Omega$$

$$\overline{Z}_i = 20 + 1.386 - j\,5.322$$
$$= 21.39 - j\,5.32\Omega$$

x and *y* shorted

The two resistance come in parallel. The circuit is redrawn in Fig. 4.85a. (10 || 20) are in series with $-j5$ and the combination is in parallel with $j\,40$.

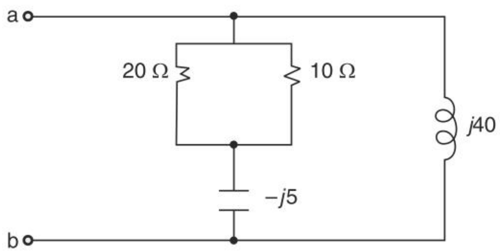


Fig. 4.85a

$$10 \| 20 = \frac{10 \times 20}{10 + 20} = 6.67\Omega$$
$$6.67 - j\,5 = 8.33\ \angle -36.7^\circ$$
$$8.33\ \angle -36.70^\circ \| j40$$

$$\bar{Z}_i = \frac{8.33 \angle -36.7° \times 40 \angle 90°}{6.67 - j\,5 + j\,40 = 35.63\ 79.2} = 9.35 \angle -25.9°\ \Omega$$

4.44 Average power drawn by the load in Fig. 4.86 is 200 kW, $pf = 0.707$ lagging. The source voltage is $v(t) = 2000\sqrt{2}\cos 314t$. Find the value of the shunt capacitor C to as to raise the combined pf to 0.85 lagging.

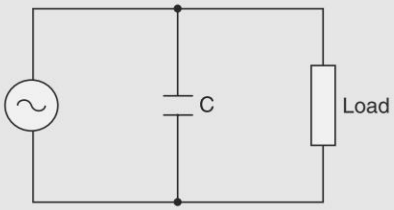


Fig. 4.86

Solution

$$pf = \cos\theta = 0.707 \Rightarrow \theta = 45° \text{ lag}$$

From P, Q, S phasor diagram

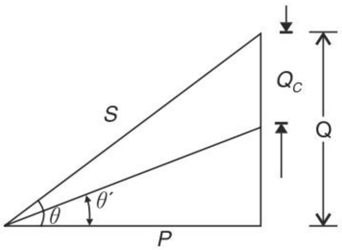


Fig. 4.86a

$$\frac{Q}{P} = \tan\theta = \tan 45° = 1$$

$$Q = P = 200 \text{ KVA (positive)}$$

For pf = 0.85, $\theta' = 32°$, $\tan\theta' = 0.62$

$$\tan\theta' = \frac{Q'(load + cap)}{P} = 0.62$$

$$Q' = 200 \times 0.62 = 124 \text{ KVAR}$$

$$Q' = Q - Q_C$$

$$124 = 200 - Q_C \quad \text{or} \quad Q_C = 76 \text{ KVAR (negative)}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{314C}$$

$$I_C = \frac{2000}{X_C} = 2000 \times 314\ C$$

$$V = 2000\ V$$

$$Q_C = V\,I_C = (2000)^2 \times 314C = 76 \times 10^3$$

$$\text{or}\quad C = \frac{76 \times 10^3}{(2000)^2 \times 314} = \frac{76 \times 10^{-6}}{4 \times 0.314} = 60\ \mu\text{F}$$

4.45 A current source $20\sqrt{2}\cos 2000\,t\ A$, a resistance 250Ω and 0.25 H inductor are connected in parallel and have reached steady-state at $t_s = 1\ ms$. Find the power being absorbed by resistor, inductor and source.

Solution

We will first convert the circuit to frequency domain.

$$10\sqrt{2}\cos 2000\,t \xrightarrow{\text{Phasor}} 20 \angle 0°\ A, \qquad \omega = 2000 \text{ rad/s}$$

$$\text{Resistance } 250\ \Omega \rightarrow 250\ \Omega$$

$$\text{Inductance } 0.25\text{ H} \rightarrow 2000 \times 0.25 = 500\ \Omega$$

The circuit is drawn in Fig. 4.87(a).

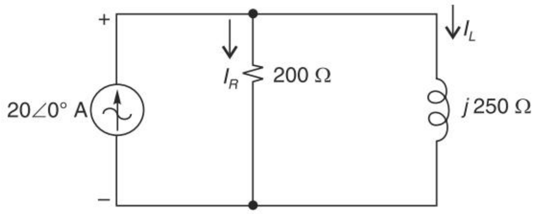


Fig. 4.87(a)

By the method of current division

$$\bar{I}_R = 20 \angle 0° \times \frac{j\,250}{200 + j\,250}$$

$$= \frac{20 \times 250 \angle 90°}{320 \angle 51.3°} = 15.625 \angle 38.7° \text{ A}$$

$$\bar{I}_L = 20 \angle 0° - 15.625 \angle 38.7°$$

$$= 20 - 12.19 - j\,9.77 = 7.81 - j\,9.77 \text{ A}$$

$$= 12.51 \angle -51.3° \text{ A}$$

$$\bar{V}_L = j\,250\,\bar{I}_L$$

$$= j\,250 \times 12.5 \angle 38.7°$$

$$= 31.28 \angle 38.7°$$

Converting to time domain

$$i_R(t) = 15.625\sqrt{2}\cos(2000t + 38.7°)$$

$$i_L(t) = 12.51\sqrt{2}\cos(2000t + 51.3°)$$

$$v_L(t) = 3128\sqrt{2}\cos(2000t + 38.7°)$$

At $t = 1ms$; $2000\,t = 2 \text{ rad} = 114.6°$

$$i_R = 15.625\sqrt{2}\cos(114.6° + 38.7°)$$

$$= -19.23 \text{ A}$$

$$i_L = 12.51\sqrt{2}\cos(114.6° - 51.3°)$$

$$= 7.96 \text{ A}$$

$$v_L = 3128\sqrt{2}\cos(114.6° + 38.7°)$$

$$= -3951\ V = v_s \text{(source voltage)}$$

$$i_s = 20\sqrt{2}\cos(114.6°)$$

$$= -11.77 \text{ A}$$

Instantaneous powers absorbed

$$p(R) = i^2_R\,R = (-19.23)^2 \times 200/1000 = 73.96 \text{ kW}$$

$$p(L) = v_L\,i_L = -3951 \times 7.96/1000 = -31.45 \text{ kW}$$

$$p(S) = v_s i_s = -[-3951 \times -11.77] = -46.50 \text{ kW}$$

4.46 In the circuit of Fig. 4.88 $v_L(t) = \sqrt{2}\cos 2t$

(a) Obtain the current and voltage phasors in all the element and source current.

(b) Find the power absorbed by all the elements and show that their sum equals the power supplied by the current source.

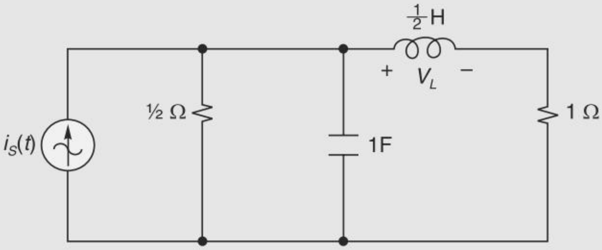


Fig. 4.88

Solution

The frequency domain circuit is drawn in Fig. 4.88 (a).

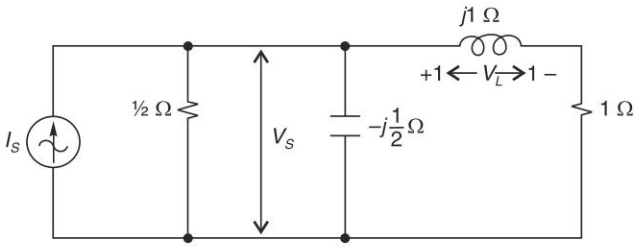


Fig. 4.88a

$$v_L(t)\cos 2t \xrightarrow{Phasor} \overline{V}_L = 1\angle 0°\ V, \qquad \omega = 2 \text{ rad / s}$$

$$1F \rightarrow \frac{1}{j2\times 1} = -j\frac{1}{2}\Omega$$

$$\frac{1}{2}\text{H} \rightarrow j2\times\frac{1}{2} = j1\Omega$$

Computation

(a) $\overline{I}_L = \dfrac{\overline{V}_L}{j1} = \dfrac{1\angle 0°}{j1} = -j1A = \overline{I}(1\Omega)$

Voltage drop in 1Ω resistor $\overline{V}(1\Omega) = \overline{I}_L \times 1 = -j\,1V$

Voltage drop across inductor and resistor in series

$$\overline{V}_S = \overline{V}_L + \overline{V}(1\Omega) = 1\angle 0 - j\,1 = \sqrt{2}\angle -45°\ V$$

$$v_S(t) = 2\cos(2t - 45°)\ V$$

$$\overline{V}\left(\frac{1}{2}\Omega\right) = \overline{V}\left(\frac{1}{2}\text{F}\right) = \overline{V}_S = \sqrt{2}\angle -45° V$$

$$I\left(\frac{1}{2}\Omega\right) = \frac{\sqrt{2}\angle -45°}{1} = 2\sqrt{2}\angle -45°$$

$$\overline{I}\text{ (IF)} = \frac{\sqrt{2}\angle -45°}{-j\frac{1}{2}}$$

$$= 2\sqrt{2}\ \angle\ 45°$$

$$\bar{I}_S^* = \bar{I}\left(\tfrac{1}{2}\Omega\right) + \bar{I}(1\text{F}) + \bar{I}_L$$

$$= 2\sqrt{2}\ \angle -45° + 2\sqrt{2}\ \angle 45° - j\,1$$

$$= 4 - j\,1$$

$$\bar{I}_s^* = 4 + j\,1$$

(b) Power loss

$$P(1\Omega) = -j\,1 \times j1 = 1\text{ W}$$

$$P\left(\tfrac{1}{2}\Omega\right) = \sqrt{2}\angle -45° \times \sqrt{2}\angle 45° = 2\text{W}$$

Power supplied by current source

$$\bar{S}_s = \bar{V}_s\,\bar{I}_s^* = \sqrt{2}\ (-45° \times (4 + j\,1))$$

$$= (1 - j\,1)\,(4 + j\,1) = 5 - j\,3\text{ VA}$$

$$\text{P} = 5\text{ W}$$

4.47 In the circuit of Fig. 4.89. $L_1 = L_2 = 2$ mH, $C = 500$ μF and $R = 2\ \Omega$.

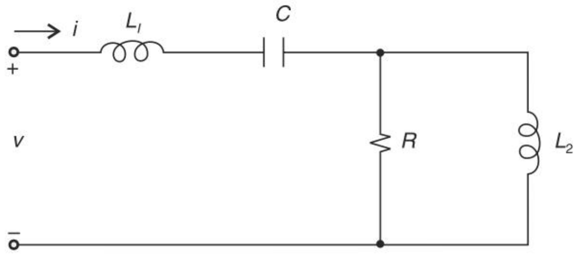


Fig. 4.89

(a) For $v = 10\sqrt{2}\cos(1000t + 90°)V$, determine the input impedance.

(b) Calculate $\bar{I}$, $\bar{I}_R$ and $\bar{I}_{L2}$ show them on a phasor diagram along with $\bar{V}$, $\bar{V}_L$ and $\bar{V}_C$.

Solution

Transforming the circuit to frequency domain

$$\bar{V} = 10\ \angle\ 90°\ V, \qquad \omega = 1000\text{ rad/s}$$

$$C \to \frac{1}{j\omega C} = -j\frac{10^6}{1000 \times 500}$$

$$= -j\,2\ \Omega$$

$$L_1 = L \to j\omega L = j \times 1000 \times 2 \times 10^{-3}$$

$$= j\,2\ \Omega$$

The circuit is redrawn in Fig. 4.89 (a)

(a) Input impedance

R, *L* in parallel

$$\bar{Z}_2 = 2 \parallel (j2) = \frac{j4}{2 + j2} = \frac{j2}{1 + j1} = \frac{j2}{\sqrt{2}\angle 45°} = \sqrt{2}\angle 45°\Omega$$

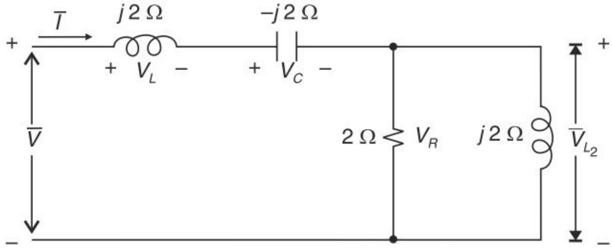


Fig. 4.89(a)

L_1, C in series

$$\bar{Z}_1 = j\,2 - j2 = 0\ \Omega$$

Therefore

$$\bar{Z} = \bar{Z}_2 + \bar{Z}_1 = \sqrt{2}\ \angle\ 45°\ \Omega$$

(b) $$\bar{I} = \frac{\bar{V}}{\sqrt{2}\angle 45°} = \frac{10\angle 90°}{\sqrt{2}\angle 45°} = 5\sqrt{2}\angle 45°$$

$\bar{V}_{L2} = \bar{V}_R = \bar{V} = 10\angle 90°\ V$; $\bar{V}_C = -j2\bar{I} = -j\ 2{\times}5\ \sqrt{2}\ \angle 45° = 10\ \sqrt{2}\ \angle -45° A$

$$\bar{I}_R = \frac{10\ \angle\ 90°}{2} = 5\ \angle\ 90°$$

$$\bar{I}_{L2} = \frac{10\ \angle\ 90°}{2} = 5\ \angle\ 0°$$

Phasor diagram (Fig. 4.83(b))

Reference phasor $\bar{I}_L = 5\ \angle\ 0°$

$$\bar{V}_L = 10\ \angle\ 90°$$

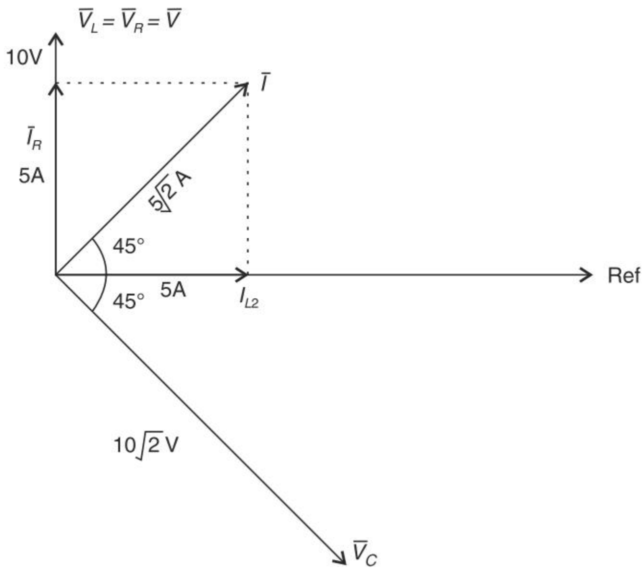


Fig. 4.89(b)

4.48 In the circuit of Fig. 4.90, determine $v_1(t)$, $v_2(t)$ and $v_3(t)$.

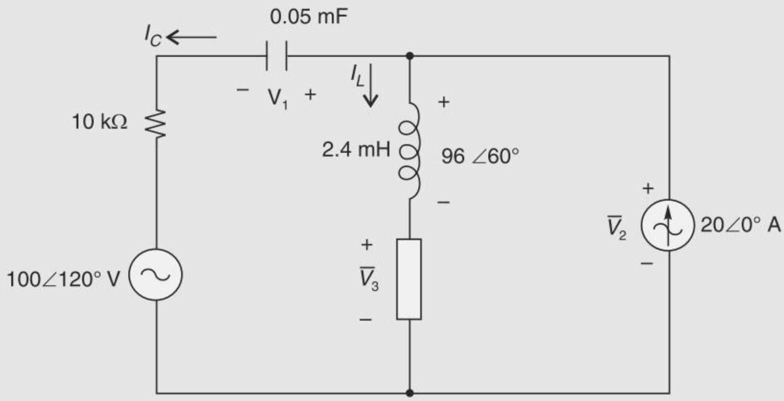


Fig. 4.90

Given: $\omega = 5\ k$ rad/s

Solution

Converting L and C to frequency domain

$$0.05 \text{ mF} \rightarrow -j\frac{10^3}{5 \times 10^3 \times 0.05}$$

$$= -j\,4\Omega$$

$$2.4 \text{ mH} \rightarrow j\,5 \times 10^3 \times 2.4 \times 10^{-3}$$

$$= j\,12\ \Omega$$

Inductor current $\overline{I}_L = \dfrac{96\angle 60^\circ}{j12} = 8\angle -30^\circ$ A

Capacitor current $\overline{I}_C = 20 \angle 0^\circ - 8 \angle -30^\circ$ A

$$= 13.07 - j\,4 = 14.27 \angle -16.2^\circ \text{ A}$$

In order to determine $\overline{V}_3$, it is necessary to find nodal voltage which equals $\overline{V}_2$.

Applying KVL round the outer loop

$$\overline{V}_2 = -j\,4\,\overline{I}_C + 10\,\overline{I}_C + 100 \angle 120^\circ$$

$$= (10 - j\,4)\,\overline{I}_C + 100 \angle 120^\circ$$

$$= 10.77 \angle -21.8^\circ \times 14.27 \angle -16.2^\circ + 100 \angle 120^\circ$$

$$= 153.7 \angle -38^\circ + 100 \angle 120^\circ$$

$$= 71.1 - j\,8.08 = 71.55 \angle -6.4^\circ \text{ V}$$

From the inductive branch

$$\overline{V}_3 = \overline{V}_2 - 96 \angle 60^\circ$$

$$= (71.1 - j\,8.08) - (48 + j\,83.3)$$

$$= 23.1 - j\,91.33 = 94.2 \angle -75.6^\circ$$

Capacitor voltage

$$\bar{V}_1 = -j\,4 \times 14.27 \angle -16.2°$$
$$= 57.1 \angle -106.2° \text{ V}$$

Voltages in time-domain form

$$v_1(t) = 57.1\sqrt{2}\cos(5000t - 106.2°) \text{ V}$$
$$v_2(t) = 71.1\sqrt{2}\cos(5000t - 6.4°) \text{ V}$$
$$v_3(t) = 94.2\sqrt{2}\cos(5000t - 75.6°) \text{ V}$$

4.49 For the circuit of Fig. 4.91 determine the Norton equivalent.

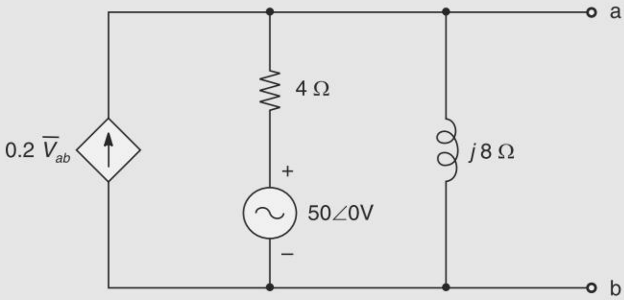


Fig. 4.91

Solution

Terminals '*ab*' are shorted so $\bar{V}_{ab} = 0$, dependent current source opens. When a short current is fed only by 50 ∠0° *V* source, then

$$I_{SC}\,(a \text{ to } b) = \frac{50\angle 0°}{4} = 12.5 \angle 0° \text{ A}$$

Open-circuit impedance

Independent source is shorted. As $\bar{V}_{ab} = 0$, dependent current source opens.

$$\bar{Z}_N = \bar{Z}_{TH} = (4 \,\|\, j\,8) = \frac{4 \times j8}{4 + j8} = \frac{32j}{4 + j8} = \frac{32j\,(4 - j8)}{16 + 64}$$
$$= \frac{256 + j\,128}{80}$$
$$\bar{Z}_N = 3.2 + j\,1.6$$

The Norton equivalent is drawn in Fig. 4.91 (a).

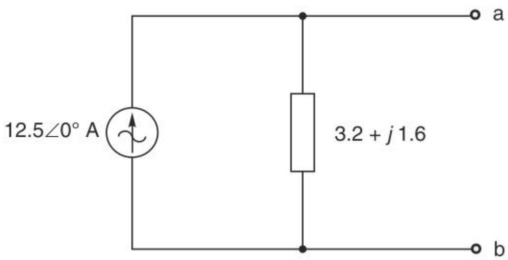


Fig. 4.91(a)

4.50 In the circuit of Fig. 4.92, find the Thevenin equivalent at 'AB' and their from find the complex power fed to the load. What should be the load impedance so that it receives maximum power?

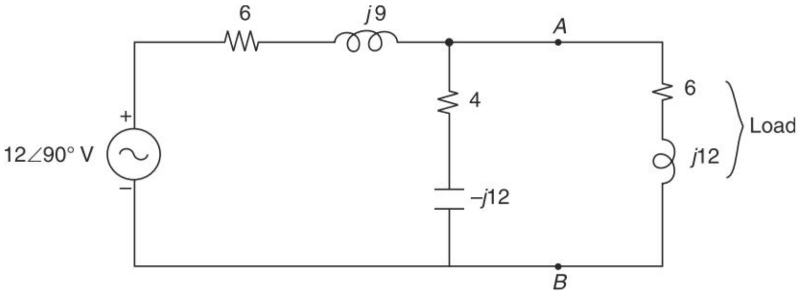


Fig. 4.92

Solution

Thevenin equivalent voltage

$$V_{TH} = V_{OC} = \left[\frac{4 - j12}{(6+4) - j(12-9)}\right] \times 12 \angle 90^\circ$$

$$= \frac{4 - j12 = 12.65\angle -71.6^\circ}{10 - j3 = 10.44\angle -16.7^\circ} \times 12\angle 90^\circ$$

$$= 14.54 \angle 35.1^\circ\ V$$

Short circuit the voltage source and

$$Z_{TH} = (6 + j\,9) \parallel (4 - j\,12)$$

$$= \frac{10.82 \angle 56.3^\circ \times 12.65 \angle -71.6^\circ}{10 - j3 = 10.44 \angle -16.7^\circ}$$

$$= 13.1 \angle -54.9^\circ \Omega = 7.48 - j\,10.64\Omega$$

The circuit in form of Thevenin equivalent is drawn in Fig. 4.92 (a).

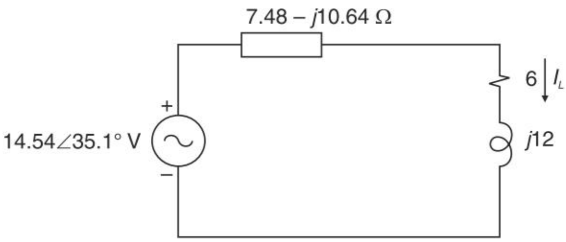


Fig. 4.92(a)

$$\bar{I}_L = \frac{14.54 \angle 35.1^\circ}{(7.48 + 6) + j(12 - 10.64)}$$

$$= \frac{14.54 \angle 35.1}{13.48 + j1.36} = \frac{14.54 \angle 35.1^\circ}{13.54 \angle 5.7^\circ}$$

$$= 1.07 \angle 29.4^\circ\ A$$

Complex power fed to load

$$\bar{S} = \bar{V}\ \bar{I}^* = \bar{Z}\cdot\bar{I}\cdot\bar{I}^* = I^2\ \bar{Z}$$

$$= (1.07)^2 \times (6 + j\,12)$$
$$= 6.86\text{ W} + j\,13.7\text{ VAR (inductive)}$$

Load from maximum power output

$$\bar{Z}_L = \bar{Z}^*_{TH} = 7.48 + j\,10.64\Omega$$

$$\bar{I}_L = \frac{14.54\ \angle\ 35.1^\circ}{\bar{Z}^*_{TH} + \bar{Z}^*_{TH}} = \frac{14.54\ \angle\ 35.1^\circ}{2 \times 7.48} = 0.972\ \angle\ 35.1^\circ\text{ A}$$

Output

$$\bar{S} = (0.972)^2 \times 13.1\ \angle\ 54.9^\circ$$
$$= 12.37\ \angle\ 54.9^\circ\text{ VA}$$
$$= 7.11\text{W} + j\,10.12\text{ VAR (inductive)}$$

Max. real power output = 7.11 W (more than 6.86 W)

4.51 For the network of Fig. 4.93, what load impedance across terminal '*ab*' would absorb maximum average real power? What is the value this real power?

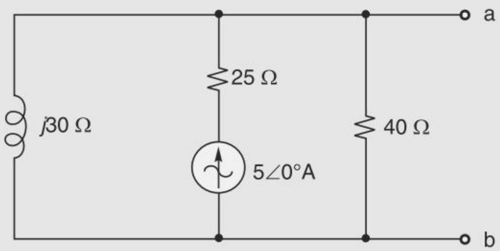


Fig. 4.93

Solution

Thevenin impedance (current source open-circuited) as seen from terminals 'ab'

$$\bar{Z}_{TH} = 40 \parallel (j30) = \frac{j40 \times 30}{40 + j30} = \frac{j120}{4 + j3}$$

$$= \frac{j120\ (4 - j3)}{25} = \frac{360 + j480}{25}$$

$$= \frac{600\ \angle\ 53.1^\circ}{25} = 24\ \angle\ 53.1^\circ = 14.4 + j19.2\ \Omega$$

For maximum power absorption

$$\bar{Z}_L = \bar{Z}^*_{TH} = 24\ \angle\ -5.31^\circ$$

Thevenin voltage

From Fig. 4.51 (a) (i)

$$\bar{V}_{\text{TH}} = 24\ \angle\ 53.1^\circ \times 5 \angle\ 0^\circ$$
$$= 120\ \angle\ 53.1^\circ\ V$$

$$\bar{Z}_{TH} + \bar{Z}^*_{TH} = 2\text{ Re }(\bar{Z}_{TH})$$
$$= 2 \times 14.4 = 28.8\ \Omega$$

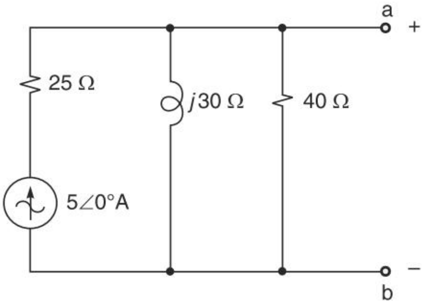


Fig. 4.93a

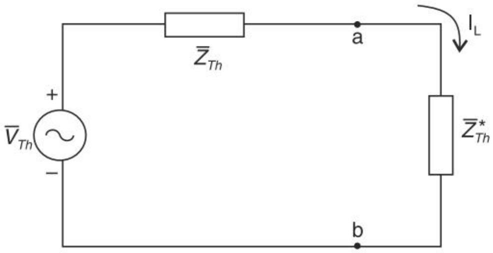


Fig. 4.93b

Therefore

$$\overline{I}_L = \frac{\overline{V}_{TH}}{28.8} = \frac{120 \angle 53.1°}{28.8}$$

$$= 4.17 \angle 53.1° \, A$$

$$I_L = 4.17 \, A$$

Maximum real power absorbed

$$P\,(\max) = I_L^2 \times 14.4; \text{ where Re } (\overline{Z}^*_{TH}) = 14.4 \, \Omega$$

$$= (4.17)^2 \times 14.4 = 250 \text{ W}$$

4.52 In the circuit of Fig. 4.94 $v_s(t) = 4\sqrt{2}\cos 2t \; V$, (a) determine $\overline{I}_L$ (b) find the value of C such that $\overline{I}_s$ is in phase with $\overline{V}_s$ (c) draw the complete phasor diagram.

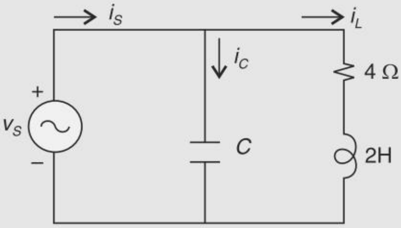


Fig. 4.94

Solution

Transforming the circuit to frequency domain

$$v_s(t) \xrightarrow{\text{Phasor}} \overline{V}_s = 4\sqrt{2} \angle 0° \; V, \; \omega = 2 \text{ rad/s}$$

$$2 \text{ H} \rightarrow j\,2 \times 2 = j\,4\Omega$$

$$C \rightarrow \frac{1}{j\,2\,C}$$

The frequency domain circuit is drawn in Fig. 4.94 (a).

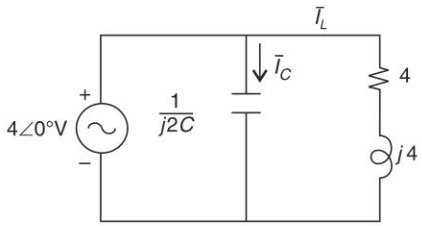


Fig. 4.94(a)

(a) $$\bar{I}_L = \frac{4\sqrt{2}\ \angle\ 0°}{4 + j\,4}$$

$$= \frac{\sqrt{2}\ \angle\ 0°}{1 + j\,1}$$

$$= 1\ \angle -45°\ A$$

$$= \left(\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right) A$$

(b) For $\bar{I}_s$ to be in phase with $\bar{V}_s$, $\bar{I}_C$ should cancel the imaginary part of $\bar{I}_2$. Therefore

$$I_C = j\frac{1}{\sqrt{2}} = \frac{4\sqrt{2}\ \angle 0°}{\dfrac{1}{j2C}} = j\,8\sqrt{2}\,C$$

or

$$C = \frac{1}{16} = 0.0625 \text{ F}$$

(c) The phasor

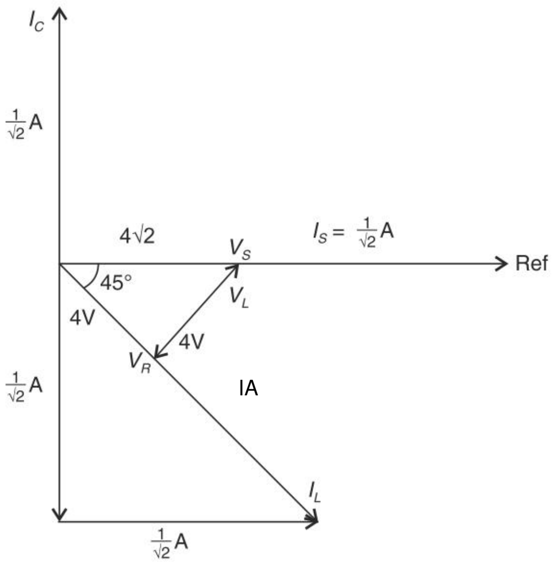


Fig. 4.94(b)

SUMMARY

- Between two sinusoidal waveforms of the same frequency, it is possible to determine the phase angle difference and which one is leading or lagging. All we need to do is to find the angle between consecutive corresponding point say positive peaks.
- The forced response of a circuit to sinusoidal voltage or current is a single sinusoid of the same frequency as the forcing function.
- A sinusoidal function can be transformed to a phasor.
- A phasor can only have magnitude and angle, the frequency is to be recorded separately.
- Time-domain description of a circuit can be transformed to frequency-domain form wherein voltage/current source become phasors, capacitors and inductors take the form of impedance (or admittance), which are complex numbers.
- Resistance transforms to resistance.

  Capacitance transforms to capacitive reactance $\dfrac{1}{j\omega C}$.

  Inductance transforms to capacitive reactance $j\omega L$.

  Combination of resistances and reactance contribute impedance (or its inverse admittance).
- All the analysis techniques and theorems of resistive circuits apply to frequency-domain circuits.
- Phasor analysis can be used only for circuit wherein all sources have the same frequencies.
- Condition for maximum power transfer is $\bar{Z}_L = \bar{Z}^*_{TH}$
- Complex power is $\bar{S} = P + j\,Q$; Q is positive for lagging *pf* and negative for leading *pf*. The unit of P is Watt (W), Q is volt-ampere reactive (VAR) and S is volt-ampere (VA).
- $\textit{Power Factor} = \dfrac{\textit{real power}}{\textit{apparent power}}$ of a load lagging/leading

  $= \cos\theta$, θ = angle between voltage across load and current drawn by load
- Capacitors in shunt across load are employed to improve the combined *pf* thereby reducing reactive power demand power supply.

REVIEW QUESTIONS

1. Explain the meaning of steady-state sinusoidal response of a circuit.
2. Distinguish between time and frequency domain relationship. Take the example of an RL series circuit excited by sinusoidal current.
3. What is a phasor? How does the phasor concept help in addition and

subtraction of sinusoidal quantities? Use a two-term expression as an example.

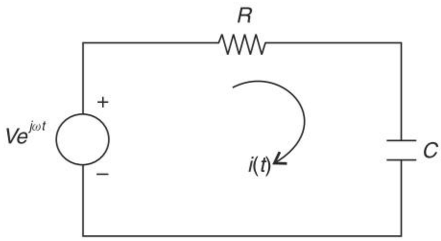
4. How does the sinusoidal response of a circuit differ from the excitation (voltage or current)?
5. Explain what is impedance? What role does it play in phasor diagrams?
6. Distinguish between a phasor and a complex quantity.
7. Can you carry out the following addition using the phasor method?

$$20 \cos (200t + 30°) + 10 \sin (100t - 60°)$$

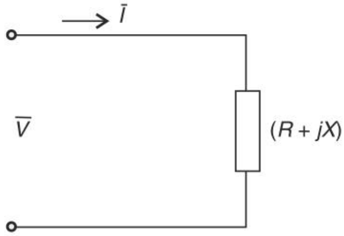
8. Draw the phasor diagram to find

$$\overline{V} = 10 \angle 0° + 10 \angle 60° - 10\sqrt{3} \angle 30°$$

9. From the given circuit, determine the expression for forced response.



10. Convert $\overline{I} = 20 \angle 60°$, $f = 400$ Hz to time domain form.
11. Voltage $\overline{V} = 100 \angle 30° V$ is applied across two terminals of a circuit which draws current $\overline{I} = 10 \angle -30° A$. Is the impedance seen from the terminals inductive or capacitive?
12. Explain the meaning and significance of the power factor of a circuit.
13. A sinusoidally excited circuit applied voltage is V(rms). The circuit draws current I leading voltage by angle θ. What are the reactive volt-ampere (VAR) drawn by the circuit and its sign?
14. A capacitor C is connected across R in series with inductor L. The admittance of this parallel circuit is $\overline{Y} = G + j$ B. Write the expression for G.
15. For the adjoining circuit, write the expression for complex power in terms of current $\overline{I}$.



16. What is the sign convention for reactive power?
17. How does a capacitor in shunt across a lagging pf load improve it power

factor. Can the combined pf be made unity?

18. Write the expression for power factor for complex power $\bar{S} = P + jQ$. When it will be lagging / leading?

19. A load draws real power P at pf $= \cos\theta$. Write the expression for reactive power Q of the load.

PROBLEMS

4.1 An iron-cored choking coil has the circuit equivalent of a series resistance of 5Ω (which represents the iron loss of the coil) and an unknown inductance L. It draws a current of 10 A on applied voltage of 240 V, 50 Hz. Find (a) coil inductance, (b) iron loss and (c) pf of the coil.

4.2 A coil of resistance 8 Ω and inductance 0.1 H is connected in series with a condenser of capacitance 160 μF across 230 V, 50 Hz supply. Calculate (a) the inductive reactance, (b) the capacitive reactance, (c) the circuit impedance, current and *pf*, and (d) the coil and condenser voltages respectively.

4.3 A resistance of 6 Ω is connected in series with an iron-cored choke coil (r in series with L). The circuit draws a current of 5 A at 240 V, 50 Hz. The voltage across resistance is 120 V and across the coil is 200 V. Calculate (a) resistance, reactance and impedance of the coil, (b) the power absorbed by the coil and (c) the overall *pf*.

4.4. The series circuit of Fig. 4.95 carries a current of 35 A. Find the values of R, r and L and the frequency of the applied voltage and its magnitude.

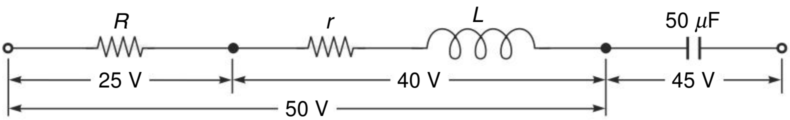


Fig. 4.95

4.5 The average power drawn by the load in Fig. 4.96 is 250 kW at 0.707 lagging pf. The generator voltage is $v(t) = 2300\sqrt{2}\sin 314t$. Find the value of C such that the resultant power factor (load and cap) is 0.866 lagging.

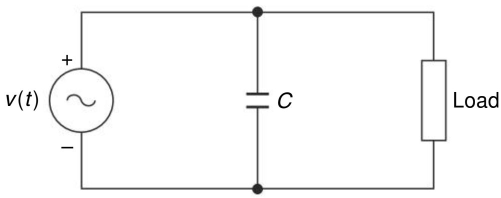


Fig. 4.96

4.6 For the circuit of Fig. 4.97, find the value of R and the *pf* of the circuit.

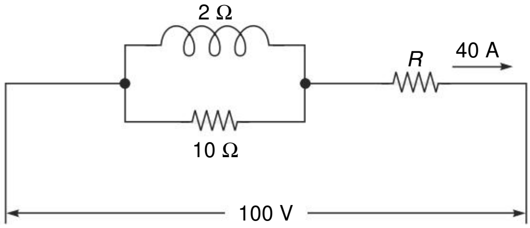


Fig. 4.97

4.7 For the circuit of Fig. 4.98, find the value of V and the circuit pf.

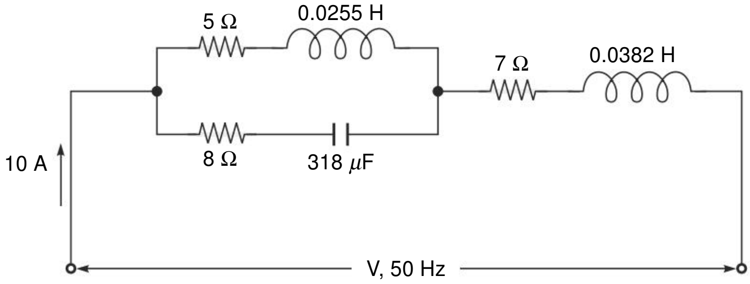


Fig. 4.98

4.8 For the circuit of Fig. 4.99

$$v_s = 4 \cos 2t$$

(a) Determine $\overline{I}_L$.

(b) Determine the value of C such that $\overline{I}_s$ and $\overline{V}_s$ are in phase.

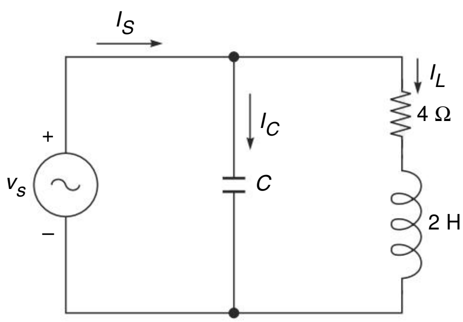


Fig. 4.99

4.9 For the circuit of Fig. 4.100

$$v_1(t) = \sqrt{2} \sin \omega t$$

(a) Determine $\overline{V}_1 / \overline{V}_2(j\omega)$.

(b) Given $\omega = 2$, determine $v_2(t)$.

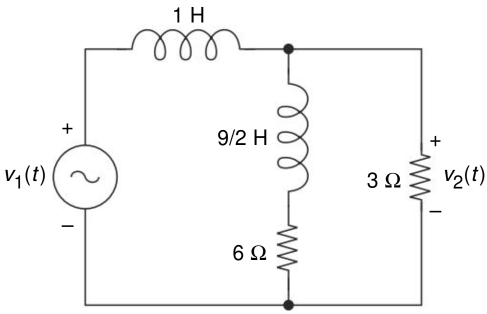


Fig. 4.100

4.10 For the circuit of Fig. 4.101

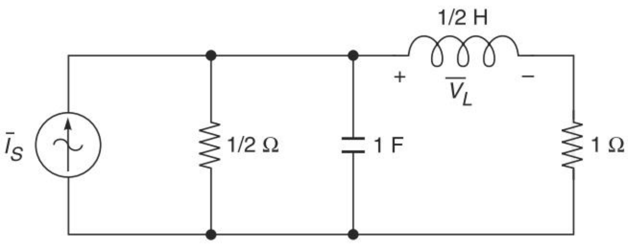


Fig. 4.101

$v_L = \sqrt{2} \cos 2t$ or $\bar{V}_L = 1 \angle 0°$

(a) Obtain the current and voltage phasors in all the elements and $\bar{I}_S$.

(b) Write an expression for $i_S(t)$.

(c) Obtain the complex power of the source $\bar{I}_S$.

(d) Obtain the power absorbed by the resistors.

4.11 For the circuit of Fig. 4.102.

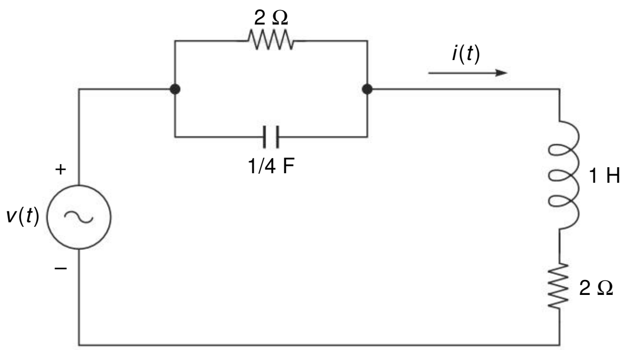


Fig. 4.102

$i(t) = \sqrt{2} \cos 2t$

Find $v(t)$ using phasor diagram.

4.12 For the circuit of Fig. 4.103, find $v(t)$. Hence determine the average power drawn from the source.

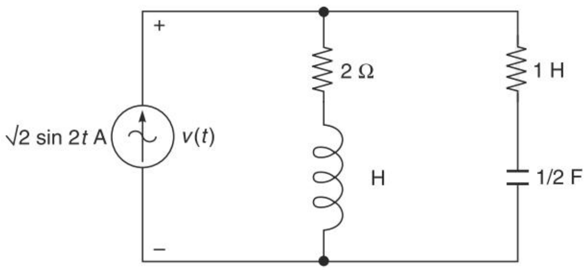


Fig. 4.103

4.13 For the circuit of Fig. 4.104, find voltages $\bar{V}_1$ and $\bar{V}_2$.

4.14 For the circuit shown in Fig. 4.105

$i_L(t) = \sqrt{2} \cos 2t$

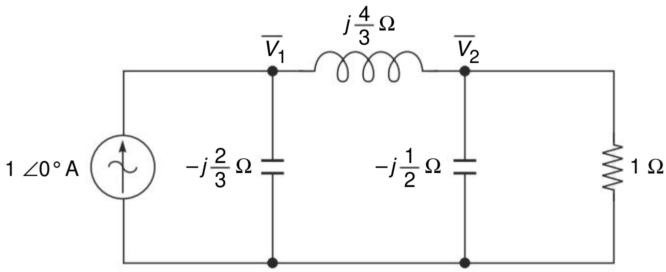


Fig. 4.104

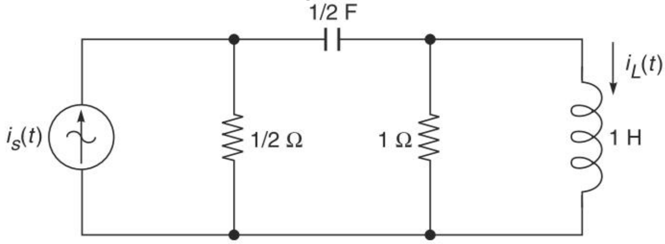


Fig. 4.105

(a) Construct a phasor diagram showing currents and voltages in all the elements and $\overline{I}_s$.
(b) Determine the complex power supplied by the source.
(c) Determine the complex power for each passive element. Also, show that the average real power drawn from the source equals the sum of the average power absorbed by the resistive elements.

4.15 A generator feeds two loads, *A* and *B*, over lines of resistances and reactances as shown in Fig. 4.106. Given: Load *A*, 10 kW at 0.8 *pf* leading; load *B*, 12 kW at 0.707 *pf* lagging at 220 V. Determine:

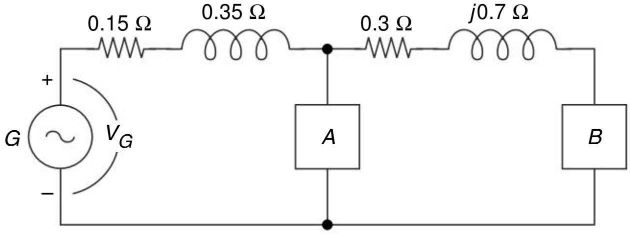


Fig. 4.106

(a) The terminal voltage at load A.
(b) The terminal voltage, *pf* and complex power supplied by the generator.

4.16 For the power distribution system of Fig. 4.107, find

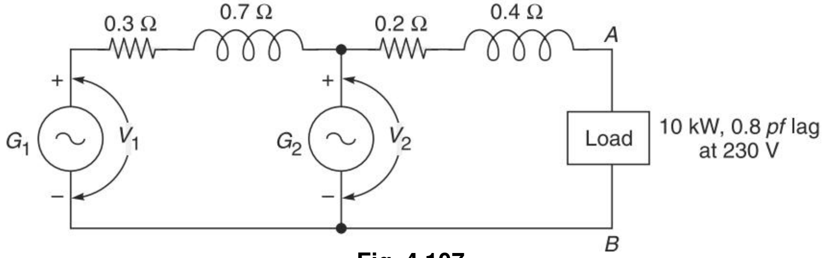


Fig. 4.107

(a) The generator terminal voltages

(b) If G_2 supplies 5 kW at 0.707 *pf* lagging, find the *pf* and complex power supplied by G_1.

4.17 For the circuit of Fig. 4.108 find voltage V_{AB} using the mesh method of analysis.

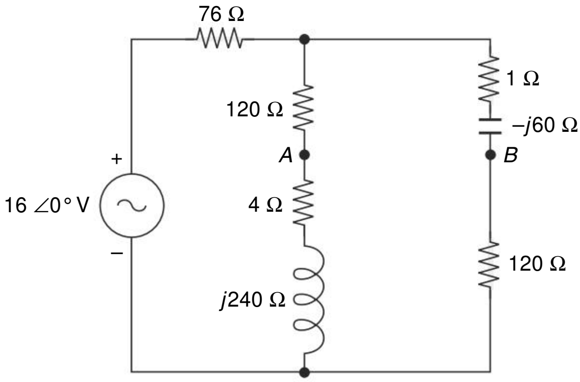


Fig. 4.108

4.18 For the circuit of Fig. 4.109, find $v_2(t)$ by the nodal method.

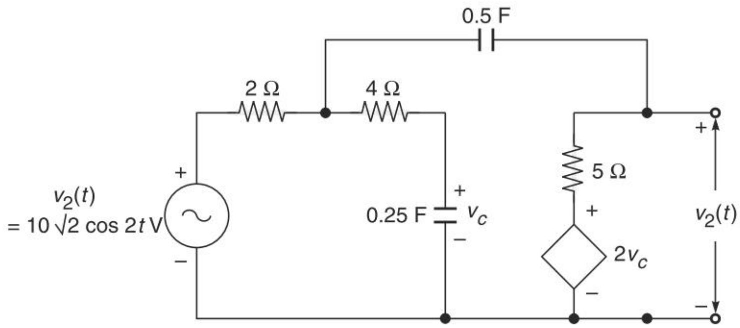


Fig. 4.109

4.19 Solve Example 4.13 using the mesh method of analysis.

4.20 For the circuit shown in Fig. 4.110, find

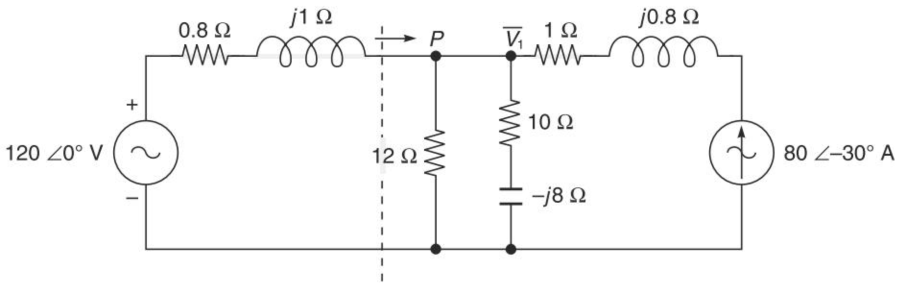


Fig. 4.110

(a) the voltage $\overline{V}_1$

(b) the power P

(c) the power supplied by each source.

4.21 For the circuit of Example 4.16, find R_L and X_L of the load to be connected at AB for maximum power transfer.
If R_L is fixed at 0.8 Ω, find X_L for maximum power transfer.
Find the value of maximum power in the above two cases.

4.22 For the circuit of Fig. 4.111, calculate the complex power absorbed by the load $(4 + j6)$ 0Ω by finding first the Thevenin equivalent of the circuit to the left of AB.

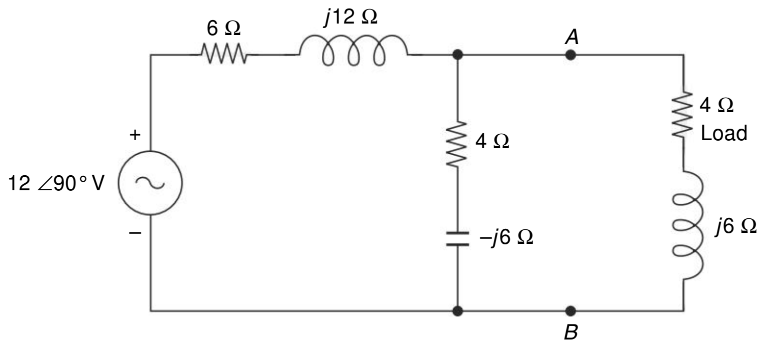


Fig. 4.111

Chapter 5

FREQUENCY RESPONSE

MAIN GOALS AND OBJECTIVES

- *Introducing concepts of frequency response—frequency bands.*
- *RLC resonant circuits, determining resonant frequency, quality factor and bandwidth.*

5.1 INTRODUCTION

Having considered the sinusoidal steady state response of circuits, we shall now generalize by focusing the response at all frequencies (0 to ∞), known as frequency response of circuits. This frequency domain behaviour shall exhibit a direct relationship to the natural frequencies of the circuit, a concept we have so far used in time domain. It can be shown that there is a formal relationship between time domain response and frequency domain response. In fact, an aperiodic signal can be shown to be a collection of all possible frequencies from 0 to ∞ wherein the frequency varies continuously (this will not be proved here). On the other hand, a periodic signal can be shown to comprise discrete frequencies ($0 - \infty$). This we shall elaborate.

It is now obvious that there are two ways of dealing with network response (or system response in general)—time domain and frequency domain. Frequency domain response is a powerful method of treating audio and video signals. In control systems, we will additionally require the time domain response—response to sudden changes like step input.

5.2 FREQUENCY RESPONSE OF SIMPLE CIRCUITS

The frequency response will comprise real and imaginary parts or can be measured as magnitude and phase angle, both, being functions of the frequency parameter. Magnitude and phase angle are the commonly used response terms. The frequency response is often expressed as the ratio of input/output which is known as network function $\overline{H}(j\omega)$.

RC Parallel Network

For the RC parallel network of Fig. 5.1.

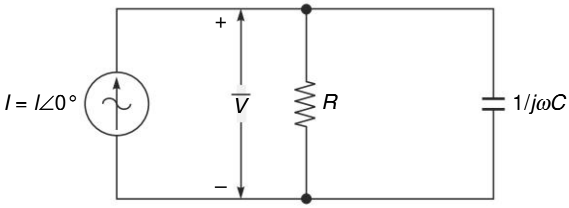


Fig. 5.1

$$\bar{V} = \frac{\bar{I}}{\bar{Y}} = \frac{I \angle 0^\circ}{\left(\frac{1}{R} + j\omega C\right)}$$

$$= \frac{IR}{1 + j\omega RC} \qquad (5.1)$$

In polar form

$$\bar{V} = \frac{IR}{[1 + (\omega RC)^2]^{1/2}} \angle - \tan^{-1}\omega RC$$

$$= V(\omega) \angle -\alpha(\omega) \qquad (5.2)$$

where

$$V(\omega) = \frac{IR}{[1 + (\omega RC)^2]^{1/2}} \qquad (5.3a)$$

$$\alpha(\omega) = \tan^{-1} \omega RC \qquad (5.3b)$$

$V(\omega)$ and $\alpha(\omega)$ are plotted in Fig. 5.2 as functions of frequency. It is seen from these plots that at frequency

$$\omega_1 = \frac{1}{RC}$$

$$V = \frac{IR}{\sqrt{2}}$$

$$= 0.707\ IR;\ \alpha = -45^\circ \qquad (5.4)$$

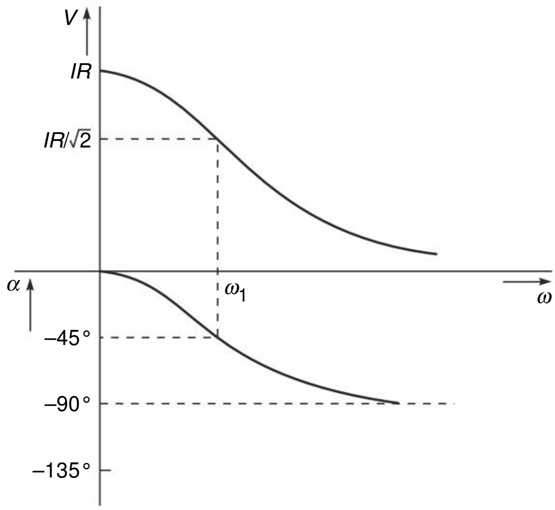


Fig. 5.2 Frequency response of RC parallel network

According to Eq. (5.4), the voltage (response) at ω_1 is reduced to 0.707 ($=1/\sqrt{2}$) of its value at $\omega = 0$, or the power dissipated in the network (V^2/R) is reduced to 1/2 at this frequency, the corresponding phase angle being – 45°. It is also observed that this frequency is the same as the natural frequency (1/time constant) of the network. This is indicative of the link between frequency domain and time domain. This frequency is also known as the *half-power angular frequency*. At frequencies higher than ω_1, the network attenuates the voltage sharply.

The region of frequencies up to ω_1 is known as the *pass band* and beyond is called *stop band* (region of high attenuation). The range of frequencies ($0 - \omega_1$) is called the *bandwidth* of the network.

RL Series Network

Consider the *RL* series network of Fig. 5.3. The network acts as a voltage divider. Thus

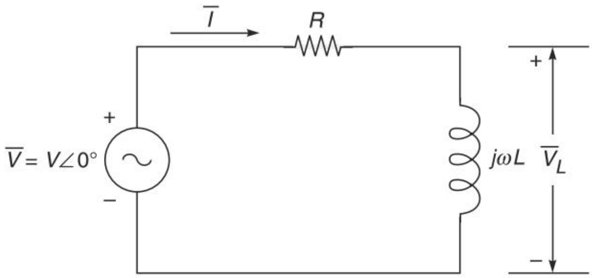


Fig. 5.3

$$\frac{\bar{V}_L}{\bar{V}} = \bar{H}(j\omega) = \frac{j\omega L}{R + j\omega L} \tag{5.5}$$

where $$H(\omega) = \frac{(\omega L/R)}{[1 + (\omega L/R)^2)]^{1/2}} \tag{5.6a}$$

$$\alpha(\omega) = 90° - \tan^{-1}\frac{\omega L}{R} \tag{5.6b}$$

$H(\omega)$ and $\alpha(\omega)$ are plotted in Fig. 5.4. It is easily seen that

$$\omega_1 = \frac{R}{L} = \frac{1}{\tau} \quad (\textit{natural frequency})$$

$H(\omega_1) = 1/\sqrt{2}$ and $\alpha(\omega_1) = 45°$. Here $0 - \omega_1$ is the stop band and above ω_1 is the pass band; just the reverse of the *RC* parallel network.

5.3 RESONANCE

In Section 5.2, the frequency response of *RL* and *RC* circuits was considered and it was found that their frequency response could be divided into two regions, viz. pass band and stop band. Here we shall consider circuits that pass a narrowband of frequencies and reject others. Such circuits are known as resonant circuits and

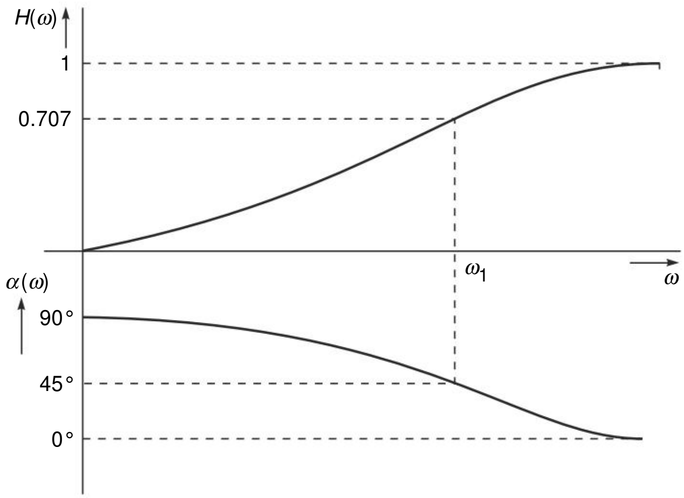


Fig. 5.4 Frequency response of *RL* series network

phenomenon of resonance occurs in all types of physical systems, e.g. in a stringed instrument.

Consider the RLC series and parallel networks of Fig. 5.5(a) and (b).

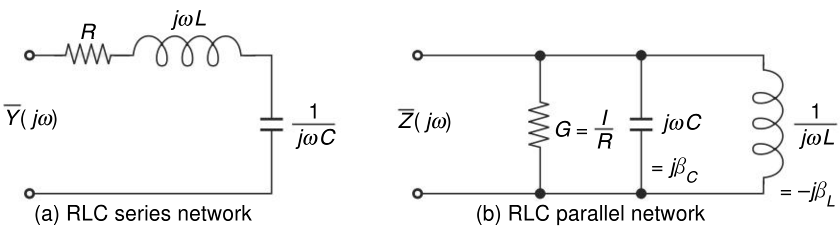


Fig. 5.5

Series Circuit

$$\bar{Y}(j\omega) = \frac{1}{\bar{Z}(j\omega)} = \frac{1}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \tag{5.7}$$

Parallel Circuit

$$\bar{Z}(j\omega) = \frac{1}{\bar{Y}(j\omega)} = \frac{1}{G + j\left(\omega C - \frac{1}{\omega L}\right)} \tag{5.8}$$

Observe that the admittance of the series circuit has the same form as the impedance of the parallel circuit (replace *R* by *G* and *L* by *C*). This is because the two circuits are *dual* of each other. Both circuits can therefore be treated together.

From Eqs (5.7) and (5.8), it immediately follows that admittance of the series circuit and the impedance of the parallel circuits are maximum when the following condition is satisfied

$$\omega L = \frac{1}{\omega C}$$

If frequency is variable (with fixed L and C), this condition (called condition of resonance) is satisfied at a frequency of

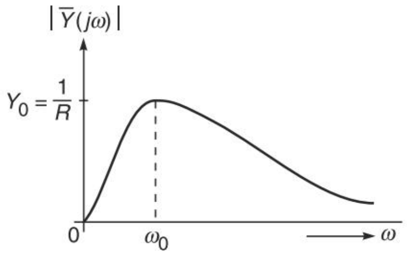
$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ ; } \textit{resonant frequency} \tag{5.9}$$

At the resonant frequency

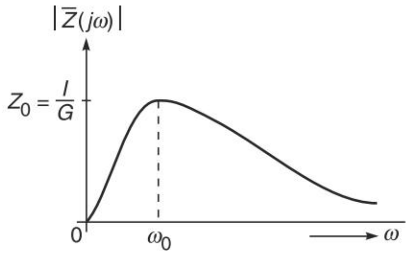
$$\text{Series circuit: } \bar{Y}(j\omega_0) = \frac{1}{R} = \bar{Y}_0 \text{ (maximum admittance)} \tag{5.10}$$

$$\text{Parallel circuit: } \bar{Z}(j\omega_0) = \frac{1}{G} = \bar{Z}_0 \text{ (maximum impedance)} \tag{5.11}$$

The plots of $|\bar{Y}(j\omega)|$ for the series circuit and $|\bar{Z}(j\omega)|$ for the parallel circuit drawn in Figs. 5.6(a) and (b) as ω is varied from 0 to ∞. These plots, are similar as Eqs. (5.7) and (5.8) have the same form. The nature of the plot is induced by the reasoning below.



(c) Series circuit



(d) Parallel circuit

Fig. 5.6

At $\quad \omega = 0$

$$\frac{1}{\omega C} = \infty \Rightarrow |\bar{Y}(j\omega)| = 0; \quad \text{Eq. (5.7)}$$

$$\frac{1}{\omega L} = \infty \Rightarrow |\bar{Z}(j\omega)| = 0; \quad \text{Eq. (5.8)}$$

At $\quad \omega \to \infty$

$$\omega L = \infty \Rightarrow |\bar{Y}(j\omega)| = 0; \quad \text{Eq. (5.7)}$$

$$\omega C = \infty \Rightarrow |\bar{Z}(j\omega)| = 0; \quad \text{Eq. (5.8)}$$

These plots, therefore, start at zero at $\omega = 0$, pass through a maxima and tend to zero asymptotically as $\omega \to \infty$. The magnitudes of the maximum admittance and maximum impedance and the frequency at which these occur are found below.

Consider a fixed amplitude variable frequency voltage excited series circuit and current excited parallel circuit as in Fig. 5.7(a) and (b). It then follows from Eqs. (5.7) and (5.8) that

$$\bar{I} = \frac{V\angle 0^\circ}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \text{ (series circuit)} \tag{5.12}$$

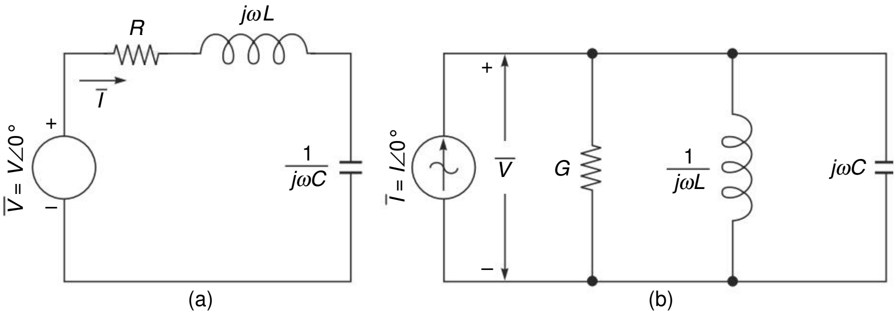


Fig. 5.7 (a) Voltage excited RLC series circuit and (b) Current excited RLC parallel network

$$\bar{V} = \frac{I\angle 0°}{G + j\left(\omega C - \frac{1}{\omega L}\right)} \text{ (parallel circuit)} \tag{5.13}$$

As per Eqs. (5.10) and (5.11)

$$\bar{I}(\max) = \frac{V}{R} \quad \text{(series circuit)} \tag{5.13a}$$

$$\bar{V}(\max) = \frac{1}{G} \quad \text{(parallel circuit)} \tag{5.13b}$$

It follows from Eqs. (5.13a) and (5.13b) that at *resonance both circuits have unity power factor*.

Under resonance condition, the circuit behaviour is described below:

1. Series circuit–unity power factor, admittance (pure conductance) is maximum or impedance (pure resistance) is minimum.
2. Parallel circuit–unity power factor, impedance (pure resistance) is maximum or admittance (pure conductance) is minimum. We can therefore, make the general statement that "at resonance the terminal voltage and input current to the circuit are in phase, i.e. *pf* is unity."

The phasor diagrams at resonance for the series and parallel circuits are drawn in Figs. 5.8(a) and (b) respectively.

It is seen above from Eqs. (5.7) and (5.8) and also Eqs. (5.12) and (5.13) that the equations for admittance of series and impedance of parallel circuit have the same form. So from now onwards we will deal with parallel resonant circuit only as this has more practical applications.

From the phasor diagram of Fig. 5.8(b), it is seen that the inductance and capacitance currents cancel out and the current input equals the conductance current. We can write the expressions for inductance and capacitance currents at resonance in terms of the resonance frequency.

Capacitance current

$$\bar{I}_{C0} = j\omega_0 C\bar{V}_0$$

But $\quad \bar{V}_0 = \bar{I}/G = R\bar{I}; \quad \bar{I} = \bar{I}_{G0}$

Then
$$\bar{I}_{C0} = j\omega_0 CR\bar{I} \tag{5.14}$$

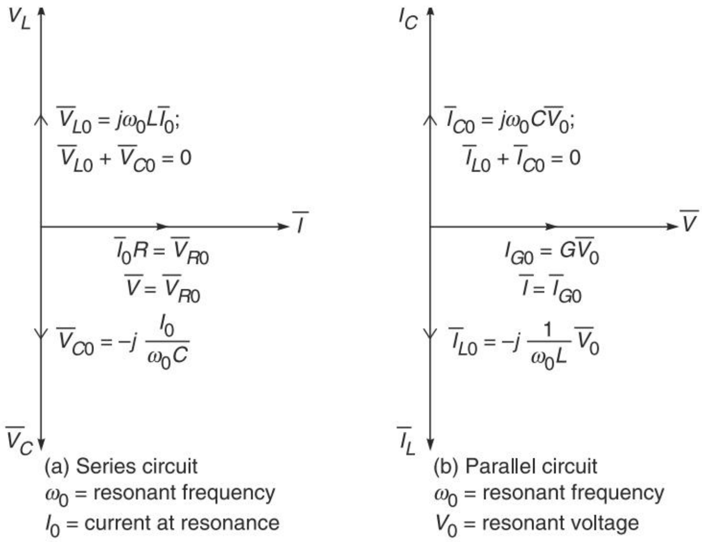


Fig. 5.8 Phasor diagrams at resonance

Inductance current

$$\bar{I}_{L0} = -j\frac{1}{\omega_0 L}\bar{V}_0$$

$$= -j\frac{1}{\omega_0 L}R\bar{I}$$

$$= -j\omega_0 CR\bar{I} \tag{5.15}$$

It then follows that

$$\bar{I}_{C0} + \bar{I}_{L0} = 0 \tag{5.16a}$$

and also

$$|\bar{I}_{C0}| = |\bar{I}_{L0}| = \omega_0\, CR\ \ I\ ;\ \ I\ = \text{input current} \tag{5.16b}$$

It is seen that from the above equations and the phasor diagram that the net current drawn by the LC combination is zero, while all the input current flows through the conductance (G). The voltage across the circuit is then $\bar{V}_0 = \bar{I}/G = \bar{I}R;\ \bar{I} = \bar{I}_{G0}$, which indeed implies unity *pf* for the complete circuit.

The current that circulates through L and C has a magnitude $(\omega_0 CR)I$. In a resonant circuit by design $\omega_0 CR >> 1$ (it is the circuit's *quality factor* as we shall soon see)

$$I_{CO} = I_{LO} >> I \text{ (input current)}$$

It means that resonant LC circulating current is far larger than input current, a sort of current amplifying effect.

Similarly, for the series resonant circuit, we find from the phasor diagram of Fig. 5.8 (a)

$$|\bar{V}_{LO}| = |\bar{V}_{CO}| = \omega_0\, L\, I_0 \tag{i}$$

and

$$\bar{V}_{LO} + \bar{V}_{CO} = 0 \tag{ii}$$

The applied voltage therefore drops across resistance only that is

$$V = R\,I_o \text{ or } I_o = \frac{V}{R} \tag{iii}$$

Equation (i) is then written as

$$V_{LO} = V_{CO} = \left(\omega_0 \frac{L}{R}\right) V \tag{iv}$$

In series resonance, $\left(\omega_0 \frac{L}{R}\right) >> 1$ as it is the *quality factor*.

Thus

$$V_{LO} = V_{CO} >> V \text{ (input voltage)} \tag{v}$$

In other words, the voltage drops across L and C are far larger than the applied voltage.

Frequency Response Consider again the parallel circuit excited by fixed magnitude of input current, but variable frequency, and examine the variation of the magnitude of the circuit terminal voltage (it will of course have the same frequency as the input current). The frequency response of the circuit is sketched in Fig. 5.9.

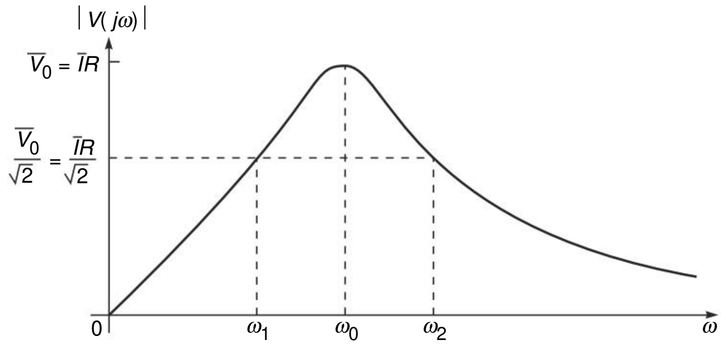


Fig. 5.9 Frequency response of parallel circuit; ω_0 = resonant frequency, $R = 1/G$

The voltage magnitude peaks at ω_0 which corresponds to maximum impedance (resistive) as per Eq. (5.13b).

At frequencies ω_1, ω_2 the voltage magnitude reduces to

$$\frac{V_0}{\sqrt{2}} = \frac{IR}{\sqrt{2}} \text{ : magnitude of circuit impedance reduces to } R/\sqrt{2} \tag{5.17}$$

Power dissipated in the circuit at these frequencies is

$$\frac{(V_0/\sqrt{2})^2}{R} = \frac{1}{2}\left(\frac{V_0^2}{R}\right) = \text{half of power dissipated at resonance} \tag{5.18}$$

Therefore, these frequencies are known as *half-power frequencies*. This concept has already been introduced in Section 5.2 for *RC* and *RL* circuits, but these circuits have a single half-power frequency compared to two in a parallel (or series) *RLC* circuit.

It is observed from Fig. 5.9 that for $\omega < \omega_1$ and $\omega > \omega_2$, the circuit voltage drops sharply. Also at $\omega = 0$, $V = 0$ as inductance acts as a short-circuit.

The range of frequencies between ω_1 to ω_2 is the *pass-band* with *bandwidth*,

$$\omega_b = \omega_1 - \omega_2 \tag{5.19}$$

Before deriving the expression for the circuit bandwidth, it is necessary to define the quality factor which has a direct bearing on the bandwidth.

For the parallel *RLC* circuit of Fig. 5.7(b), how the individual susceptances B_C and B_L, admittance Y and impedance Z vary with frequency are shown in Fig. 5.10(a). The Fig. 5.10(b) is the plot of Z vs ω indicating $Z_{max} = 1/G$, half power frequencies (at $Z = 1/(\sqrt{2}\,G) = 0.707/G$) and the pass-band (bandwidth)

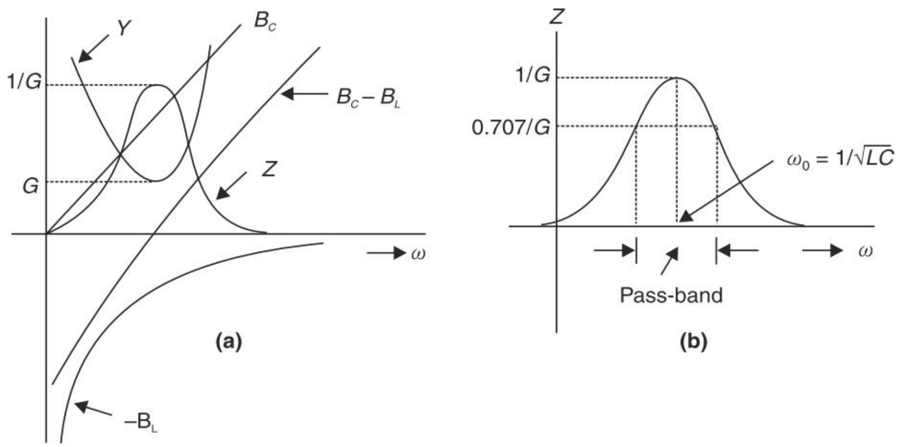


Fig. 5.10 Frequency response in parallel *RLC* circuit

Quality Factor

It is convenient to work in terms of admittance of the parallel circuit (at resonance admittance is minimum). From Eq. (5.8).

$$\bar{Y}(j\omega) = G + j\left(\omega C - \frac{1}{\omega L}\right);\ R = 1/G \tag{5.20}$$

In terms of resonance frequency ω_0, Eq. (5.20) can be written as

$$\bar{Y}(j\omega) = G\left[1 + j\left(\frac{\omega_0 C}{G}\right)\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$$

We define

$$Q_0 = \frac{\omega_0 C}{G} = \omega_0 RC = 2\pi f RC = \text{Quality factor at resonance} \tag{5.21}$$

Then

$$\bar{Y}(j\omega) = \frac{1}{R}\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] \tag{5.22}$$

The quality factor is a dimensionless quantity and it is a determining factor in bandwidth of the circuit as we shall soon see.

It can be proved that

$$Q_0 = 2\pi\left(\frac{\text{maximum energy stored per period}}{\text{total energy lost per period}}\right) \tag{5.23}$$

This is proved later.

For the series *RLC* circuit replacing *C* by *L* and *G* by *R* in Eq. (5.21), the quality factor is given by

$$Q_0 = \omega_0 \frac{L}{R} \tag{5.24}$$

Let us now derive the expression of bandwidth and see how is it affected by the *Q* factor. Reconsider the expression for admittance as in Eq. (5.22), reproduced below

$$\bar{Y}(j\omega) = G\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right] \tag{5.25a}$$

At half-power points, the admittance increases to $\sqrt{2}\,G$ (or impedance reduces to $\frac{R}{\sqrt{2}}$). It is seen from Eq. (5.22) that this can happen only when the imaginary part is ± 1. This occurs at two frequencies given by

$$Q_0\left(\frac{\omega_2}{\omega_0} - \frac{\omega_0}{\omega_2}\right) = -1 \quad \text{and} \quad Q_0\left(\frac{\omega_0}{\omega_0} - \frac{\omega_0}{\omega_1}\right) = 1$$

from which we get the expressions for the two half-power frequencies as

$$\omega_1 = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} - \frac{1}{2Q_0}\right] \tag{5.25b}$$

$$\omega_2 = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} + \frac{1}{2Q_0}\right] \tag{5.26}$$

The bandwidth is then found as

$$\omega_b = \omega_2 - \omega_1 = \frac{\omega_0}{Q_0} \tag{5.27}$$

Multiplying Eq. (5.25a) and (5.25b) it can be shown that

$$\omega_0 = \sqrt{\omega_1 \omega_2} \tag{5.28}$$

which means that the resonance frequency is the geometric mean of the two half-power frequencies.

It is easily observed from the expression for bandwidth (Eq. (5.27)) that higher quality factor means lower bandwidth or greater *frequency selectivity* of the circuit.

For high-*Q* circuit Eqs. (5.25) and (5.26) get simplified to

$$\omega_1, \omega_2 = \omega_0\left(1 \mp \frac{1}{2Q_0}\right); \; 1 + \left(\frac{1}{2Q_0}\right)^2 = 1$$

$$= \omega_0 \mp \frac{\omega_0}{2Q_0} = \omega_0 \mp \frac{1}{2}\omega_0 \tag{5.29}$$

It means that in a high-*Q* circuit bandwidth is symmetrical about the resonant frequency. In other words, half-power points are located half bandwidth away from ω_0 on either side.

Link with Time Domain

The characteristic equation of *RLC* parallel circuit given in Eq. (3.45) is reproduced below.

$$s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0$$

$$s_1, s_2 = -\alpha \pm j\sqrt{\omega_0^2 - \alpha^2}$$

where

$$\alpha = \frac{1}{2RC} = \zeta\omega_0$$

$$\omega_0 = \frac{1}{\sqrt{LC}} = \text{resonant frequency} \tag{5.30}$$

It is these roots which determine the transient response of the circuit, which as shown in Section 3.6 is a damped sinusoid for $\alpha < \omega_0$.

As shown earlier in this section.

$$Q_0 = \omega_0 RC \quad \text{or} \quad RC = \frac{Q_0}{\omega_0}$$

Therefore,

$$\alpha = \frac{\omega_0}{2Q_0} \tag{5.31}$$

The rate of decay of the transient response is determined by the exponential factor $e^{-\alpha t}$. Resonant frequency is of course unique and a common factor in time and frequency domain responses.

Proof of Eq. (5.23) for Quality Factor

With reference to Fig. 5.7(b) under resonant condition, the currents in inductance and capacitance cancel themselves out and the circuit current $\bar{I}$ flows in the conductance. Let

$$i(t) = I_m \cos \omega_0 t$$

The corresponding voltage response (at resonance) is

$$v(t) = \frac{i(t)}{G} = \frac{I_m}{G}\cos \omega_0 t$$

The instantaneous energy stored in the capacitance is

$$w_C(t) = \frac{1}{2}Cv^2 = \frac{I_m^2 C}{2G^2}\cos^2 \omega_0 t$$

The instantaneous energy stored in the inductor is

$$w_L(t) = \frac{2}{2}Li_L{}^2 = \frac{1}{2}L\left(\frac{1}{L}\int_0^t v dt\right)^2$$

$$= \frac{I_m^2 C}{2G^2}\sin^2 \omega_0 t$$

The total instantaneous energy stored (in capacitance and in inductance) is

$$w(t) = \omega_L(t) + \omega_C(t) = \frac{I_m^2 C}{2G^2}$$

which being constant is also the maximum energy stored.

Average power dissipated by the conductance

$$P_G = \frac{I_m^2}{2G}$$

Energy dissipated in one cycle

$$P_G T = \frac{1}{f_0}\frac{I_m^2}{2G}$$

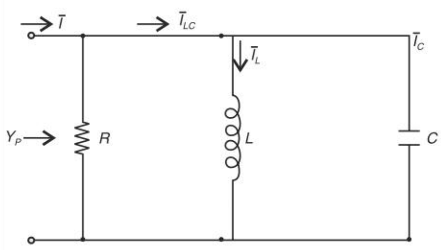
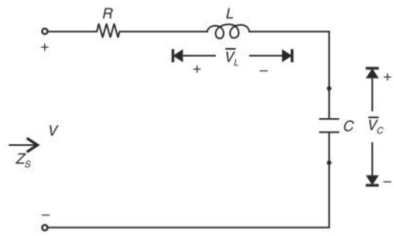
$$= \frac{2\pi}{\omega_0}\frac{I_m^2}{2G}$$

Substituting in Eq. (5.23)

$$Q = 2\pi\left(\frac{I_m^2 C}{2G^2}\right)\Bigg/\left(\frac{2\pi}{\omega_0}\frac{I_m^2}{2G}\right)$$

$$= \frac{\omega_0 C}{G} = \omega_0 RC$$

Table 5.1 Summary of Resonance : $\omega_0 = \frac{1}{\sqrt{LC}}$

	
$Q_0 = \omega_0 RC,\ \alpha = \frac{1}{2RC}$	$Q_0 = \omega_0\frac{L}{R},\quad \alpha = \frac{R}{2L}$
At ω_0, magnitude-wise	At ω_0, magnitude-wise
$I_{LO} = I_{CO} = Q_O I$	$V_{LO} = V_{CO} = Q_O V$
At any frequency	At any frequency
$\overline{Y}_p(j\omega) = \frac{1}{R}\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$	$\overline{Z}_s(j\omega) = R\left[1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]$

Half power frequencies

$$\omega_1, \omega_2 = \omega_0\left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \mp \frac{1}{2Q_0}\right]$$

$$\omega_0 = \sqrt{\omega_1\omega_2}$$

Bandwidth $\omega_b = \omega_2 - \omega_1 \frac{\omega_0}{Q_0} = 2\alpha$

Approximation valid for high Q_0

$$\omega_1, \omega_2 = \omega_0 \mp \frac{1}{2}\omega_b$$

Example 5.1 A parallel *RLC* circuit has Q_0 = 200. Two component values are given. Find the third component value for the following combinations.

(a) $R = 1\Omega$, $C = 2\ \mu F$

(b) $L = 2 \times 10^{-15}$ H, C = 1.2 nF

(c) $R = 118.5$ kΩ, $L = 120$ pF

Solution

(a) $Q_0 = \omega_0 RC; \quad \omega_0 = \dfrac{1}{\sqrt{LC}}$

$$Q_0 = \frac{1}{\sqrt{LC}} \cdot RC = R\sqrt{\frac{C}{L}}$$

$$200 = 1 \times \sqrt{\frac{2\times 10^{-6}}{L}} \Rightarrow L = \frac{2\times 10^{-6}}{(200)^2} = 50 \times 10^{-12} = 50 \text{ pH}$$

(b) $$200 = R\sqrt{\frac{1.2\times 10^{-9}}{2\times 10^{-15}}} \Rightarrow R = 200 \times \sqrt{0.6\times 10^{6}} = 155 \text{ k}\Omega$$

(c) $$200 = 118.5 \times 10^3 \sqrt{\frac{C}{120\times 10^{-12}}}$$

$$\sqrt{C} = \frac{200\sqrt{120\times 10^{-12}}}{118.5\times 10^{3}} = 18.49 \times 10^{-9}$$

$$C = 341.9 \times 10^{-18} = 0.342 \times 10^{-15}$$

$$= 342 \text{ pF}.$$

Example 5.2 A parallel *RLC* circuit has R = 1 kΩ, C = 40 μF and L = 13 mH. Determine ω_0 and Q_0.

Solution

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{13\times 10^{-3}\times 49\times 10^{-6}}}$$

or

$$\omega_0 = 1.253 \times 10^{-3} = 1.253 \text{ kHz}$$

$$Q_0 = \omega_0 RC = 1.253 \times 10^3 \times 1 \times 10^3 \times 49 \times 10^{-6}$$

$$= 61.4$$

Example 5.3 An *RLC* series circuit has R = 5 Ω, L = 5 mH and C = 0.08 μF. Calculate (a) ω_0, f_0, (b) ω_b; ω_1, ω_2 and (c) $\overline{Z}_{in}$ at resonant frequency.

Solution

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{5\times 10^{-3}\times 0.08\times 10^{-6}}} = 50 \text{ k rad/s}$$

$$f_0 = \frac{\omega_0}{2\pi} = \frac{50}{2\pi} = 7.96 \text{ kHz}$$

(b) $$Q_0 = \omega_0 \frac{L}{R} = 50\times 10^3 \times \frac{5\times 10^{-3}}{5} = 50$$

Bandwidth, $= \dfrac{\omega_0}{Q_0} = \dfrac{50 \times 1000}{50}$ =1000 rad/s

It is easily seen that we can express

$$\omega_b = \frac{\omega_0}{Q_0} = \frac{\omega_0 R}{\omega_0 L} = \frac{R}{L} = \frac{5}{5 \times 10^{-3}} = 1000 \text{ rad/s}$$

Half power frequencies

$$\omega_1, \omega_2 = \omega_0 \left[\sqrt{1 + \left(\frac{1}{2Q_0}\right)^2} \pm \frac{1}{2Q_0} \right]$$

$$\frac{1}{2Q_0} = 0.01 . \left(\frac{1}{2Q_0}\right)^2 = 0.1 \times 10^{-3} << 1$$

So, we can use the approximate result

$$\omega_1, \omega_2 = \omega_0 \mp \frac{1}{2}\omega_b = 50{,}000 \mp 500$$

$$= 49{,}500,\ 50{,}500 \text{ rad/s}$$

Example 5.4 A series circuit has an inductor of $L = 40$ μH and $R = 4.02\Omega$. What should be that value C of the capacitor for the circuit to be resonant at 800 kHz. What is the bandwidth of the circuit and $\overline{Z}_m$?

Solution

$$\omega_0 = 2\pi \times 800 \times 10^3 \text{ rad/s}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L}$$

Substituting values

$$C = \frac{1}{(2\pi \times 800 \times 10^3)^2 \times 40 \times 10^{-6}}$$

$$= \frac{10^{-5}}{(2\pi \times 8)^2 \times 4} = 0.99 \text{ nF}$$

Quality factor $Q_0 = \omega_0 \dfrac{L}{R}$

$$Q_0 = (2\pi \times 800 \times 10^3) \times \frac{40 \times 10^{-6}}{4.02} = 50$$

Bandwidth, $f_b = \dfrac{f_0}{Q_0} = \dfrac{800 \times 10^3}{50} = 16$ kHz

Example 5.5 In the circuit of Fig. 5.11, $G = 5$ μ℧, $L = 2$ mH. It draws a minimum current 2 mA at a frequency of 5 k rad/s.

(a) At what value of C the voltage V is maximum and what is its value?

(b) With the value of C as found in part (a) what should be the value of I for the inductor current to be 1 A?

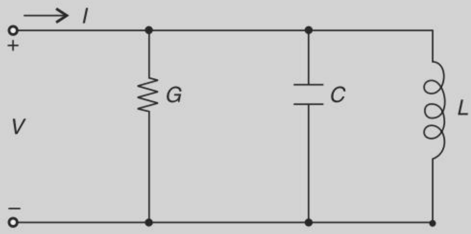


Fig. 5.11

Solution

(a) For V to be maximum the circuit should resonant for which

$$\omega_0^2 = \frac{1}{LC} \quad \text{or} \quad C = \frac{1}{\omega_0^2 L}$$

$$C = \frac{1}{(5\times10^3)^2 \times 2 \times 10^{-3}} = 20\ \mu F$$

Under resonance I flows through G only. So

$$V = \frac{I}{G} = \frac{I}{5\times10^{-6}} = 0.2 \times 10^6\, I\ \text{V}$$

$$V(\max) = \frac{I}{G} = \frac{2\times10^{-3}}{5\times10^{-6}} = 400\ \text{V}$$

$$\omega L = 5 \times 10^3 \times 2 \times 10^{-3} = 10\Omega$$

b) $$I_L = \frac{V}{\omega L} = \frac{0.2 \times 10^6\, I}{10} = 1000\ \text{mA}; \ (1\ \text{A})$$

$$\therefore\ I = \frac{10^4}{0.2 \times 10^6} = 0.05\ \text{mA}$$

Example 5.6 An RLC series circuit has resonance frequency of 1000 Hz. Its power factor reduces to 0.707 at a frequency of 1050 Hz. Calculate its quality factor.

Solution

At half-power frequency, the real and imaginary part of $\overline{Z}_{in}$ of the series circuit are equal. So its power factor is

$$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$$

Thus

$$\omega_2 = 1050\ \text{Hz, higher half power frequency.}$$

Then bandwidth

$$\omega_b = \frac{\omega_0}{Q_0} \quad \text{or} \quad Q_0 = \frac{\omega_0}{\omega_b} = \frac{1000}{100} = 10$$

As $\left(\frac{1}{2Q_0}\right)^2 = \left(\frac{1}{20}\right)^2 = 2.5 \times 10^{-3} <<1$, so it is high-$Q$ circuit and so our assumption is valid.

Example 5.7 An RLC series circuit of Fig. 5.12 has a resonant frequency of 200 k rad/s and a bandwith of 5 k rad/s. The inductor coil has $L = 2.5$ m H and a Q of 65. Calculate the value of coil resistance r.

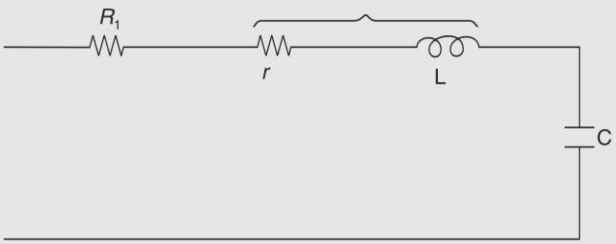


Fig. 5.12

Solution

$$\omega_b = \frac{\omega_0}{Q_0}$$

$$Q_0 = \frac{\omega_0}{\omega_b} = \frac{200}{5} = 40$$

$$Q_0 = \omega_0 \frac{L}{r}$$

$$65 = 200 \times \frac{2.5 \times 10^{-3}}{r}$$

$$r = 7.69 \times 10^{-3}\,\Omega$$

Example 5.8 A capacitor of 12 nF is connected in series with an inductor of 4 mH and 5 Ω resistance as shown in Fig. 5.18(a).

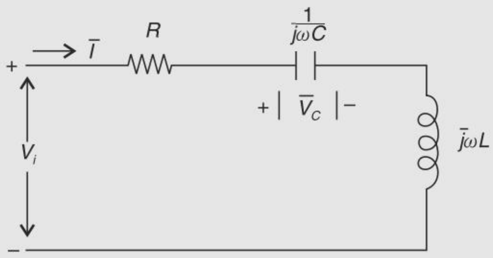


Fig. 5.13(a)

(a) Calculate the resonant frequency, ω_0.

(b) At ω_0 the voltage across the capacitor is required to be 1.5V. What voltage should be applied across the circuit input?

(c) Draw a phasor diagram. Show as to how the capacitor voltage can be larger than the input voltage.

Solution

(a) Resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{4 \times 10^{-3} \times 12 \times 10^{-9}}}$$

or

$$\omega_0 = 0.144 \times 10^6 = 144 \text{ k rad/s}$$

(b) At ω_0, $V_C = 1.5\ V$

$$I = \omega_0 C V_C = 144 \times 10^3 \times 12 \times 10^{-9} \times 1.5$$
$$= 2.592 \text{ mA}$$

At the resonance circuit impedance $Z_i = R = 5\Omega$, (resistive)

$$\therefore\ V_i = RI = 5 \times 2.592 = 12.96 \text{ mV}$$

(c) At ω_0

$\bar{I}$ is in phase with $\bar{V}_i$

$\bar{V}_C$ lags $\bar{I}$ by 90°

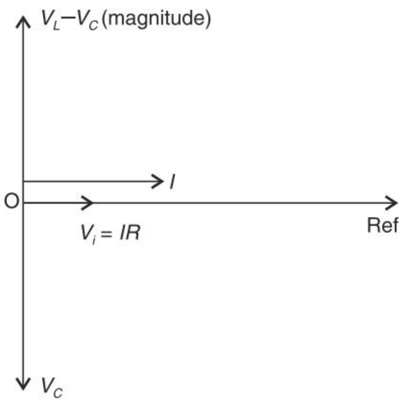


Fig. 5.13(b)

$\overline{V}_L = -\overline{V}_C$ so lead $\overline{I}$ by 90°

Capacitive reactance

$$B_C = \frac{1}{\omega_0 C} = \frac{1}{144 \times 10^3 \times 12 \times 10^{-9}}$$

or $B_C = 579\ \Omega >> 5\Omega$

so $V_C >> V_i$

Example 5.9 A parallel resonant circuit is required to have $\omega_0 = 2.5$ MHz, Z_{in} (at ω_0) = 60 kΩ, $Q_0 = 80$. Determine the value of R, L and C.

Solution

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2.5 \text{ MHz} \quad \text{or} \quad 1.57 \text{ M rad/s} \qquad \text{(i)}$$

In a parallel RLC circuit $Z_I(\omega_0) = R$

$$\therefore \quad R = 60 \text{ k}\Omega \qquad \text{(ii)}$$

$$Q_0 = \omega_0 RC \qquad \text{(iii)}$$

$$80 = 15.7 \times 10^6 \times 60 \times 10^3\ C \qquad \text{(iv)}$$

$$\text{or} \quad C = 84.9\ \mu\text{F} \qquad \text{(v)}$$

From Eq. (i)

$$\frac{1}{LC} = (15.7 \times 10^6)^2$$

$$L = \frac{1}{(15.7 \times 106)^2 \times 84.9 \times 10^{-6}} = 478 \text{ nH}$$

$$R = 60 \text{ k}\Omega, \qquad C = 84.9\ \mu\text{F}, \qquad L = 0.478\ \mu\text{H}$$

Example 5.10 A series resonant circuit has a capacitor of 2.5 μF and a resistor of 8Ω. Its bandwidth is 400 rad/s. Determine (a) L (b) ω_0 (c) Q_0 and (d) ω_1, ω_2.

Solution

(a) Bandwidth, $\omega_b = \dfrac{\omega_0}{Q_0}$, $Q_0 = \omega_0 \dfrac{L}{R}$

Then

$$\omega_b = \frac{R}{L} = 400$$

Or

$$L = \frac{8}{400} \times 10^3 = 20\,\text{mH}$$

(b) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 10^{-3} \times 2.5 \times 10^{-6}}}$$

Or $\omega_0 = 4.47$ k rad/s

(c) $$Q_0 = \omega_0 \frac{L}{R} = 4.47 \times 10^3 \times \frac{20 \times 10^{-3}}{8}$$

or $Q_0 = 11.2$

(d) It is high-Q circuit. So $\omega_1, \omega_2 = \omega_0 \mp \frac{1}{2}\,\omega_b = 4.270, 4.670\ k$ rad/s

Example 5.11 An *RLC* seies circuit has a resonant frequency of 10^6 rad/s and a bandwidth of 1 k rad/s. At resonance an applied voltage of 0.05 V causes a current of 5 mA to flow. Find

(a) the values of R, L and C

(b) the voltage across L and across C. Also find the net voltage across L and C.

(c) frequencies at which current will reduce by a factor of $\frac{1}{\sqrt{2}}$.

Solution

(a) At resonant frequency, the circuit presents only the resistance R as

$$\left(j\omega_0 L - j\frac{1}{\omega_0 C}\right) = 0$$

Thus

$$R = \frac{0.05}{5 \times 10^{-3}} = 10\,\Omega$$

Bandwidth $\omega_b = \dfrac{\omega_0}{Q_0}$ or $Q_0 = \dfrac{\omega_0}{\omega_b} = \dfrac{10^6}{10^3} = 1000$

Also

$$Q_0 = \omega_0\left(\frac{L}{R}\right) \quad \text{or} \quad L = R\left(\frac{Q_0}{\omega_0}\right) = 10 \times \frac{1000}{10^6} \times 10^3$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \quad \text{or} \quad C = \frac{1}{\omega_0^2 L} = \frac{10^{12}}{10^{12} \times 10 \times 10^{-3}}$$

(b) At ω_0, $V_L = V_C = \omega_0 LI = 10^6 \times 10 \times 10^{-3} \times 5 \times 10^{-3} = 50\ V$

(c) Current reduction by $\frac{1}{\sqrt{2}}$ occurs at half-power frequencies

$$\omega_1, \omega_2 = 10^6 \mp \frac{1}{2} \times 10^3 = 500 \times 10^3,\ 1500 \times 10^3 \text{ rad / s}$$

Example 5.12 An *RLC* parallel circuit has $R = 1$ MΩ, $L = 1$H $C = 1$ μF. It is excited by a current $\bar{I} = 10 \angle 0°$ μA. Find the ω_0 and Q_0. If the frequency of current $\bar{I}$ is ω_0, find the voltage across the circuit. At what frequencies would the voltage reduces by a factor of $\frac{1}{\sqrt{2}}$.

Solution

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1 \times 10^{-6}}} = 1000 \text{ rad/s}$$

At resonance, current $\bar{I}$ flows only through R. Therefore circuit voltage is

$$\bar{V} = 1 \times 10^6 \times 10 \times 10^{-6} \angle 0° = 10 \angle 0\ V$$

The voltage will reduce by a factor of $\frac{1}{\sqrt{2}}$ at half-power frequencies, so we must find bandwidth.

$$Q_0 = \omega_0 RC$$

$$\omega_b = \frac{\omega_0}{Q_0} = \frac{1}{RC} = 1 \text{ rad/s}$$

Then

$$\omega_1, \omega_2 = \omega_0 \mp \frac{1}{2}\omega_b$$

$$= 1000 \mp 0.5 = 999.5,\ 1000.5 \text{ rad/s}$$

Example 5.13 A constant voltage of frequency 1 MHz with connected across an inductor (r in series with L) in series with a variable capacitor. The circuit draws maximum current when the capacitance is 500 pF. The current reduces by a factor $\frac{1}{\sqrt{2}}$ when the capacitor is adjusted to 450 *pF*. Determine r, L and Q_0. Given L=10 mH and C=1n F.

Solution

The circuit is drawn in Fig. 5.14

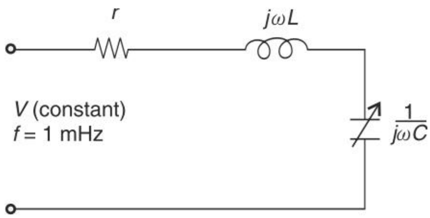


Fig. 5.14

Case 1

$$C = 500 \text{ p F}$$

The current drawn is maximum at 1 MHz. Therefore, resonant frequency is 1 MHz.

$$\omega_0^2 = \frac{1}{LC}$$

$$L = \frac{1}{\omega_0^2 C} = \frac{10^3}{(2\pi \times 10^6)^2 \times 500 \times 10^{-12}} = 0.0633 \text{ mH}$$

Case 2

$$C = 450 \text{ pF}$$

Resonant frequency of the circuit is now

$$(2\pi f_0')^2 = \frac{1}{LC} = \frac{1}{0.0633 \times 10^{-3} \times 450 \times 10^{-12}}$$

$$\omega_0' = 2\pi f_0' = 5.925 \times 10^6 \text{ rad/s}$$

$$f_0' = 0.943 \text{ MHz}$$

As with applied frequency of f_0 = 1 MHz, the current reduces by a factor of $\frac{1}{\sqrt{2}}$. Therefore f_0' is the half-power frequency. It gives us the bandwidth

$$\omega_b' = 2\,(2\pi \times (1 - 0.943) \times 10^6 = 0.717 \times 10^6 \text{rad/s}$$

$$\omega_b = \frac{\omega_0'}{Q_0'}, \quad Q_0' = \omega_0' \frac{L}{r}$$

or $$\omega_b' = \frac{r}{L}$$

$$0.717 \times 10^6 = \frac{r}{0.0633 \times 10^{-3}}$$

$$r = 45\Omega$$

Then

$$Q_0 \text{ (with 400 pF)} = (2\pi \times 10^6) \times \frac{0.0633 \times 10^{-3}}{45} = 8.84\ (>5)$$

Example 5.14 A parallel *RLC* resonant has $R = 10$ kΩ, $L = 50$ mH and $C = 100$ μF. Find (a) ω_0 (b) Q_0 (c) α (d) ξ and (e) ω_d

Solution

(a) $$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{50 \times 10^{-3} \times 100 \times 10^{-9}}} = 14.14 \times 10^3 \text{ rad/s}$$

(b) $$Q_0 = \omega_0 RC = 14.14 \times 10^3 \times 10 \times 10^3 \times 100 \times 10^{-9} = 14.14$$

(c) $$\alpha = \frac{\omega_0}{2Q_0} = \frac{14.14 \times 10^3}{2 \times 14.14} = 500$$

(d) $$\alpha = \xi\omega_0 \ \xi = \frac{\alpha}{\omega_0} = \frac{500}{14.14 \times 10^3} = 0.0354$$

(e) $$\omega_d = \omega_0\sqrt{1-\xi^2} = 14.14\sqrt{1-(0.0354)^2} \times 10^3$$

or $$\omega_d = 14.13 \times 10^3 \text{ rad/s}$$

Example 5.15 In a parallel *RLC* resonant circuit has $\omega_0 = 1000$ rad/s, $\omega_d = 997$ rad/s and $Y_{in} = 1.2$m℧ at resonance. Find the value of *R*, *L* and *C*.

Solution

$$\omega_0 = 1000 = \frac{1}{LC} \rightarrow LC = 10^{-6} \qquad \text{(i)}$$

$$\omega_d = \omega_0\sqrt{1-\zeta^2}$$

$$997 = 1000\sqrt{1-\zeta^2} \rightarrow \zeta = 0.0774$$

$$\alpha = \zeta\,\omega_0 = 0.0774 \times 1000 = 77.4$$

$$\alpha = \frac{1}{2RC} = 77.4 \rightarrow RC = 6.46 \times 10^3 \qquad \text{(ii)}$$

$$\bar{Y}_{in} \text{ (at resonance)} = \frac{1}{R} = 1.2\text{ m℧} \rightarrow R = 833.3\ \Omega$$

From Eq (ii)

$$C = \frac{6.46 \times 10^{-3}}{8333.3} = 7.75 \times 10^{-6} = 7.75\ \mu\text{F}$$

From Eq (i)

$$L = \frac{10^{-6}}{7.75 \times 10^{-6}} = 0.129\ H = 129 \text{ mH}$$

Example 5.16 Consider the parallel circuit of Fig. 5.15 wherein the inductor is represented as a resistance (accounting for inductor iron loss and resistance of the conductor with which it is wound) in series with inductance. Find the condition of resonance and the expression for the quality factor.

Solution

$$\bar{Z}(j\omega) = \frac{1}{j\omega C + 1/(R + j\omega L)}$$

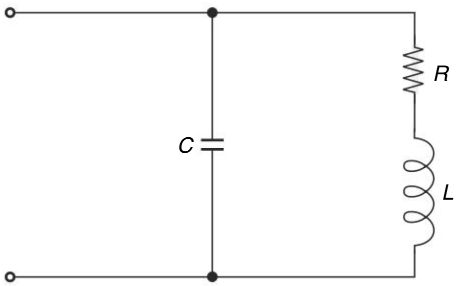


Fig. 5.15

$$= \frac{R + j\omega L}{1 + j\omega RC - \omega^2 LC} \qquad \text{(i)}$$

If $\qquad \omega L >> R \left(\text{or } \frac{\omega L}{R} >> 1\right)$

$$\bar{Z}\,(j\omega) = \frac{j\omega L}{1 + j\omega RC - \omega^2 LC}$$

$$= \frac{1}{(RC/L + j\,(\omega C - 1/\omega L)} \qquad \text{(ii)}$$

It immediately follows that the resonant angular frequency is

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad \text{(iii)}$$

$\bar{Z}(j\omega)$ of Eq. (ii) can then be written as

$$\bar{Z}(\,j\omega) = \frac{1}{(RC/L)\left[1 + j\frac{\omega_0 C}{RC/L}\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right]} \qquad \text{(iv)}$$

The quality factor then is

$$Q = \frac{\omega_0 C}{RC/L} = \frac{\omega_0 L}{R} \qquad \text{(v)}$$

Also

$$Z_{\max} = \frac{L}{RC} = RQ^2 \qquad \text{(v)}$$

Example 5.17 In the parallel *RLC* circuit of Fig. 5.16, determine $i_R\,(t)$, $i_L\,(t)$, $i_C\,(t)$ and $i_{CL}\,(t)$. Determine the phasor diagram showing all currents and voltage.

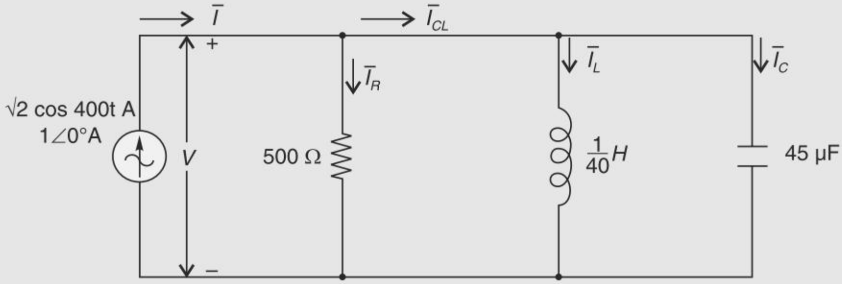


Fig. 5.16

Solution

Frequency of applied current, $\omega_0 = 400$ rad / s.

$$\omega L = 400 \times \frac{1}{40} = 10, \quad \frac{1}{\omega C} = \frac{10^6}{400 \times 250} = 10\Omega$$

As $\omega L = \dfrac{1}{\omega C}$, ω is the resonant frequency $\omega_0 = 400$ rad/s

Therefore

$$\bar{I}_{CL} = 0 \text{ as } \bar{I}_L + \bar{I}_C = 0 \text{ at resonance}$$

Thus

$$\bar{I}_R = \bar{I} = 1\angle 0°A, \quad i_R(t) = \sqrt{2}\cos 400t \text{ A}$$

$$\bar{V} = 500\, I_R = 500\angle 0° V, \quad v(t) = 500\sqrt{2}\cos 400\,t \text{ V}$$

Then

$$\bar{I}_L = \frac{\bar{V}}{j\omega L} = -j\frac{500}{10} = -j\,50 = 50\angle -90° \text{ A}$$

$$i_L(t) = 50\sqrt{2}\cos(400t - 90°)$$

$$\bar{I}_C = j\omega C\bar{V} = j0.1 \times 500 = j50 = 50\angle 90° \text{ A}$$

$$i_C(t) = 50\sqrt{2}\cos(400t + 90°)$$

$$\bar{I}_{CL} = \bar{I}_L + \bar{I}_C = -j\,50 + j\,50 = 0 \text{ A}$$

Therefore circulating current = 50 A

The phasor diagram is drawn in Fig. 5.16(a)

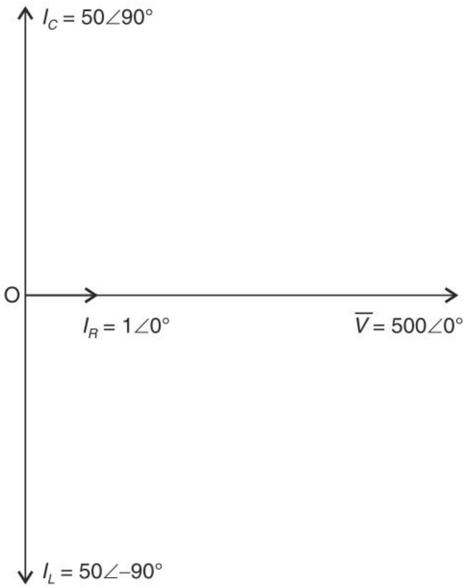


Fig. 5.16(a)

Observation: It is seen that the circulating current is 50 times the source current.

Example 5.18 A voltage of $v(t) = 100\sqrt{2}\cos\omega t$ is applied to a series *RLC* circuit having $R = 10\Omega$, $L = 2$ mH and $C = 200$ μF. Calculate the resonance frequency and the current drawn by the circuit at this frequency. What are the corresponding value of voltages across *L*, *C* and *L–C* series? Draw the phasor diagram showing the current and all the voltages.

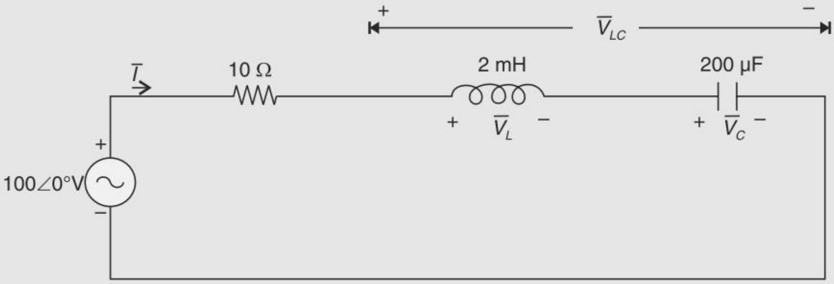


Fig. 5.17

Solution

The circuit is drawn in Fig. 5.17.

Resonance frequency

$$\omega_0 = \frac{1}{\sqrt{2\times 10^{-3}\times 200\times 10^{-9}}}$$

$$= \frac{10^6}{\sqrt{400}} = 50 \text{ kHz}$$

At ω_0, circuit impedance = $10 + j\,0\ \Omega$

$$\therefore\ \bar{I} = \frac{100}{10} = 10\ \angle 0°\ A$$

$$\omega_0 L = 50 \times 10^3 \times 2 \times 10^{-3} = 100\ \Omega$$

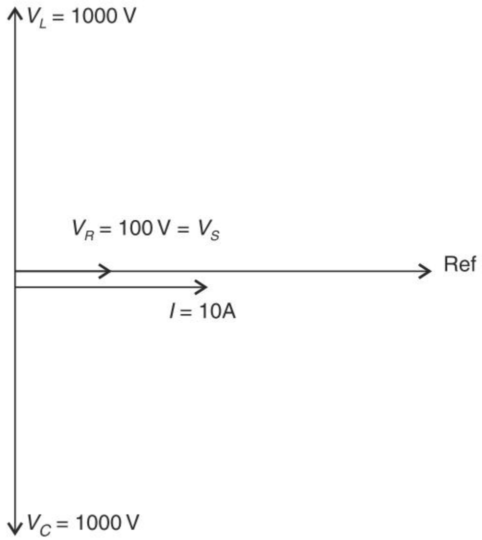


Fig. 5.17(a)

$$\frac{1}{\omega C} = \frac{1}{50\times 10^{3}\times 200\times 10^{-9}} = 100\Omega$$

$\overline{V}_L = j\,100 \times 10\angle 0° = j1000\ V\,;\ V_L = 1000V$

$\overline{V}_C = j\,100 \times 10\angle 0° = -j1000\ V\,;\ V_C = 1000V$

$\overline{V}_{LC} = \overline{V}_L + \overline{V}_C = j\,1000 - j\,1000 = 0\ V$

The phasor diagram is drawn in Fig. 5.17 (b).

Observation: $V_L = V_C = 1000$ V

Example 5.19 In the parallel circuit of Fig. 5.18. $R = 1\Omega$, $L = 10$ μH and $C = 10$ pF, applied voltage $V = 10$. Results of example of Example 5.3, calculate the resonant frequency. Find the following at this frequency (a) Q_0 (b) input current and (c) circulating current.

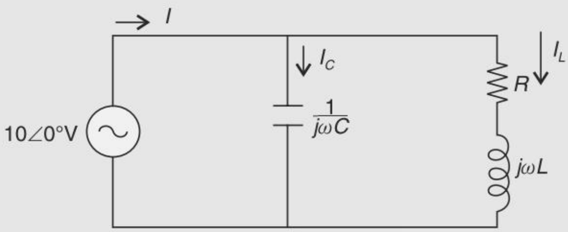


Fig. 5.18

Solution

Resonance frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10\times 10^{-6}\times 10\times 10^{-12}}}$$

or $\quad \omega_0 = 0.1 \times 10^9 = 100 \times 10^6 = 100$ MHz

(a) $\quad Q_0 = \omega_0 \dfrac{L}{R} = 100 \times 10^6 \times \dfrac{10 \times 10^{-6}}{1}$

or $\quad Q_0 = 1000$

(b) $\quad \overline{Z}\,(j\omega_0) = RQ^2 = 1 \times (1000)^2 = 10^6\,\Omega$ (resistance)

$$I_{in}\,(\omega_0) = \frac{10}{10^6} = 10^{-5}\text{ A}$$

(c) Circulating current

$\overline{I}_C = j\,V\,\omega_0\,\text{C} = j10 \times 100 \times 10^6 \times 10 \times 10^{-12}$

$\qquad = j\,0.1$ A

$\overline{I}_C = 0.1\,A >> I_{in} = 10^{-5}$ A

Check

$$\overline{I}_L = \frac{\overline{V}}{R + j\omega_0 L} \approx \frac{100}{j100\times 10^{6}\times 10\times 10^{-6}} = -\,j\,0.1\,A$$

$\overline{I}_L + \overline{I}_C \approx 0$

Example 5.20 A coil of resistance 2Ω and inductance 0.02 H is connected in series with a condenser across 200 V mains. What capacitance must the condenser have in order that maximum current may occur at (a) 25, (b) 50, (c) 100 Hz? Find also the current and the voltage across the condenser in each case.

Solution

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

or $$C = \frac{1}{\omega_0^2 L}$$

(a) $$\omega_0 = 2\pi \times 25 = 50\pi \text{ rad/s}$$

$$C = \frac{10^6}{(50\pi)^2 \times 0.02} = 2026 \text{ μF}$$

(b) $$\omega_0 = 2\pi \times 50 = 100\pi \text{ rad/s}$$

$$C = \frac{10^6}{(100\pi)^2 \times 0.02} = 507 \text{ μF}$$

(c) $$\omega_0 = 2\pi \times 100 = 200\pi$$

$$C = \frac{10^6}{(200\pi)^2 \times 0.02} = 127 \text{ μF}$$

(d) $$f = 25 \text{ Hz}$$

$$I = \frac{200}{2} = 100 \text{ A}$$

$$V_C = \frac{1}{C\omega} = \frac{100}{2026 \times 10^{-6} \times 50\pi}$$

$$= 314.2 \text{ V}$$

$$f = 50 \text{ Hz}$$

$$I = \frac{200}{2} = 100 \text{ A}$$

$$V_C = \frac{1}{C\omega} = \frac{100}{507 \times 100\pi}$$

$$= 627.3 \text{ V}$$

$$f = 100 \text{ Hz}$$

$$I = \frac{200}{2} = 100 \text{ A}$$

$$V_C = \frac{1}{C\omega} = \frac{200}{127 \times 10^{-6} \times 200\pi}$$

$$= 1253.2 \text{ V}$$

Example 5.21 A coil of 10 Ω resistance has an inductance of 0.1 H and is connected in parallel with a condenser of 150 μF capacitance. Calculate the frequency at which the circuit will act as non-inductive resistance *R*. Find also the value of *R*.

Solution

Referring to Fig. 5.15

$$\overline{Z}(j\omega) = \frac{1}{j\omega C + 1/(r + j\omega L)}$$

$$= \frac{r + j\omega L}{(1-\omega^2 LC) + j\omega rC}$$

$$= \frac{(r + j\omega L)\,[(1 - \omega^2 LC) - j\omega rC)]}{(1 - \omega^2 LC)^2 + \omega^2 r^2 C^2} \qquad \text{(i)}$$

For the circuit to act as non–inductive resistance, the imaginary part of the numerator of $\overline{Z}(j\omega)$ should be zero or

$$\omega L(1 - \omega^2 LC) - \omega r^2 C = 0$$

or

$$LC\omega^2 + \frac{r^2 C}{L} - 1 = 0 \qquad \text{(ii)}$$

Substituting the values

$$150 \times 10^{-6}\,\omega^2 + \frac{100 \times 150 \times 10^{-6}}{0.1} - 1 = 0$$

$$150 \times 10^{-6}\,\omega^2 = 1 - 0.15 = 0.85$$

$$\omega = \left(\frac{0.85 \times 10^6}{150}\right)^{1/2} = 75.3 \text{ rad/s}$$

or

$$f = 12 \text{ Hz}$$

At this frequency

$$R = \frac{r(1 - \omega^2 LC) + \omega^2 rLC}{(1 - \omega^2 LC) + \omega^2 r^2 C^2} \qquad \text{(iii)}$$

$$= \frac{10 \times 0.915 + (75.3)^2 \times 10 \times 0.1 \times 150 \times 10^{-6}}{0.915 + (73.3)^2 \times 100 \times (150)^2 \times 10^{-12}}$$

$$= 10.78\ \Omega$$

Example 5.22 A parallel *RLC* circuit has an adjustable capacitor for changing the resonant frequency from 540 to 1610 kHz. The maximum value of Q_0 is to be 50. If $R = 35\Omega$ specify L and C_{max} and C_{min}.

Solution From Eqs. (5.9) and (5.21)

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = \omega_0 RC = R\sqrt{\frac{C}{L}}$$

R is fixed. For a given *Q*, *C* would be maximum at the lower resonant frequency (540 kHz).

$$2\pi \times 540 \times 10^3 = \frac{1}{\sqrt{LC}} \qquad \text{(i)}$$

$$50 = 35\sqrt{\frac{C}{L}} \quad \text{or} \quad C = 2.041\, L \tag{ii}$$

Substituting Eq. (ii) in Eq. (i)

$$2.041\, L^2 = \frac{1}{(2\pi \times 540 \times 10^3)^2}$$

or

$$L = 0.206\ \mu\text{H}$$

$$C_{max} = 2.041 \times 0.206 = 0.42\ \mu\text{F}$$

For C_{min}

$$2\pi \times 1610 \times 10^3 = \frac{1}{\sqrt{LC}}$$

or

$$C_{min} = \frac{1}{0.206 \times 10^{-6} \times (2\pi \times 1610 \times 10^3)^2}$$

$$= 0.0474\ \mu\text{F}$$

5.4 FOURIER SERIES AND FOURIER ANALYSIS

Fourier Series

Any periodic function (sinusoidal or otherwise) satisfies the condition

$$f(t) = f(t + T) \tag{5.32}$$

where T is designated the *time period* and

$$f = \frac{1}{T}\ \text{Hz} = \text{fundamental frequency} \tag{5.33a}$$

or

$$\omega = \frac{2\pi}{T} = 2\pi f \tag{5.33b}$$

A non-sinusoidal periodic function can be split up into a Fourier series—sum of sinusoidal functions whose frequencies are integral multiples of the fundamental frequency (ω). Thus

$$f(t) = a_0 + a_1 \cos \omega t + b_1 \sin \omega t + a_2 \cos 2\omega t + b_2 \sin 2\,\omega t + \ldots \tag{5.34a}$$

or

$$f(t) = \sum_{n=0}^{\infty} a_n \cos n\omega t + \sum_{n=0}^{\infty} b_n \sin n\omega t \tag{5.34b}$$

In the above expansion, a_0 is the average value of $f(t)$ over a time-period given, i.e.

$$a_0 = \frac{1}{T}\int_0^T f(t)\ dt;\ dc \text{ component} \tag{5.35}$$

The *fundamental frequency component* is

$$a_1 \cos \omega t + b_1 \sin \omega t$$

while

$$a_n \cos n\omega t + b_n \sin n\omega t; \quad n > 1$$

are known as the *harmonic* components.

To find a_m, multiply $f(t)$ by cos $m\omega t$ and integrate over 0 to T. Thus

$$\int_0^T f(t)\cos m\omega t\, dt = \sum_{\substack{n=0\\ n\neq m}}^{\infty}\int_0^T a_n \cos n\omega t \cos m\omega t\, dt$$

$$+ \int_0^T a_m \cos^2 m\omega t\, dt$$

$$+ \sum_{n=0}^{\infty}\int_0^T b_n \sin n\omega t \cos m\omega t\, dt$$

$$= \int_0^T a_m \cos^2 m\omega t\, dt;\ \text{all other integrals being zero}$$

$$= \frac{a_m T}{2}$$

or $$a_m = \frac{2}{T}\int_0^T f(t)\cos m\omega t\, dt;\ m \neq 0 \tag{5.36}$$

Similarly

$$b_m = \frac{2}{T}\int_0^T f(t)\sin m\omega t\, dt;\ m \neq 0 \tag{5.37}$$

Using Eqs. (5.35), (5.36) and (5.37), coefficients of the Fourier series terms of a periodic function can be found.

Certain Observations

1. If $f(t)$ has even symmetry, sine terms would be absent.
2. If $f(t)$ has odd symmetry, cosine terms would be absent.

The above results Eqs. (5.35), (5.36) and (5.37) can be expressed in angular form as below:

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(t)\, d(\omega t) \tag{5.38}$$

$$a_m = \frac{1}{\pi}\int_0^{2\pi} f(t)\cos m\omega t\, d(\omega t) \tag{5.39}$$

$$b_m = \frac{1}{\pi}\int_0^{2\pi} f(t)\sin m\omega t\, d(\omega t) \tag{5.40}$$

Example 5.23 Consider full-wave rectified sine wave shown in Fig. 5.19. Find the Fourier series.

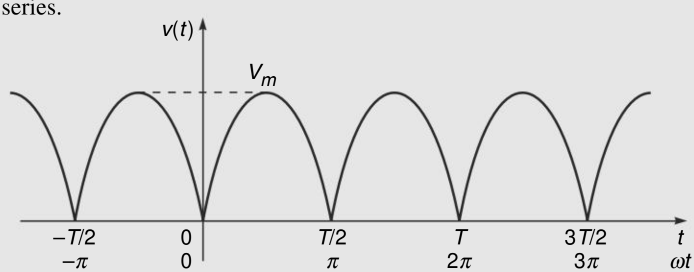


Fig. 5.19 Full-wave rectified sine wave

Solution Rectified full-wave can be expressed as (over one period)

$$v(t) = V_m \sin \omega t;\ 0 < \omega t < \pi;\ \omega = \frac{2\pi}{T}$$

$$= -V_m \sin \omega t;\ \pi < \omega t < 2\pi$$

As per Eq. (5.38)

$$a_0 = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin \omega t\, d(\omega t) - \int_{\pi}^{2\pi} V_m \sin \omega t\, d(\omega t)\right]$$

$$= \frac{2V_m}{\pi}$$

Because of even symmetry, sine terms would be absent. As per Eq. (5.39)

$$a_m = \frac{1}{\pi}\left[\int_0^{\pi} V_m \sin \omega t \cos m\,\omega t\, d(\omega t)\right.$$

$$\left. - \int_{\pi}^{2\pi} V_m \sin \omega t \cos m\,\omega t\, d(\omega t)\right];\ m \neq 0$$

Integrating, we get

$$a_m = 0,\ m = \text{odd}$$

$$= \frac{4m}{(1-m^2)\pi};\ m = \text{even}$$

Hence only even harmonics are present.

Thus according to Fourier series

$$v = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi} \cos 2\omega t - \frac{4V_m}{15\pi} \cos 4\omega t - \frac{4V_m}{35\pi} \cos 6\,\omega t - \ldots$$

Observe that the harmonic amplitudes reduce progressively.

Example 5.24 Find the Fourier Series of the periodic function shown in Fig. 5.20:

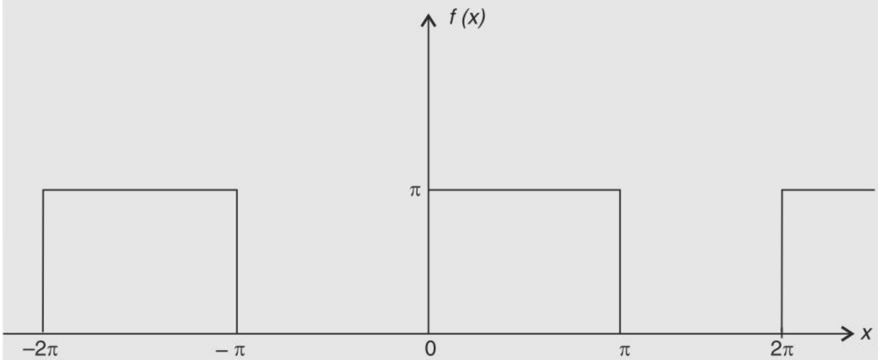


Fig. 5.20

Solution

From the graph

$$f(x) = \begin{cases} 0, & -\pi \le x < 0 \\ \pi, & 0 \le x \le \pi \end{cases}$$

Fourier series for the $f(x)$ is the form of

$$F_n(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos(nx) + b_n \sin(nx))$$

Here the $f(x)$ is periodic function which is integrable on $[-\pi, \pi]$, so

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0} 0\,dx + \int_{0}^{\pi} \pi\,dx\right] = \frac{1}{2\pi}\left[\pi.\int_0^{\pi} dx\right]$$

$$= \frac{\pi\cdot\pi}{2\pi} = \frac{\pi}{2}$$

$$a_n = \frac{1}{\pi}\int_0^{\pi} \pi\cdot\cos(nx)\,dx = 0, n \geq 1$$

and

$$b_n = \frac{1}{\pi}\int_0^{\pi} \pi\cdot\sin(nx)\,dx = \frac{\pi}{\pi\cdot n}[1-\cos n\pi]$$

$$= \frac{1}{n}(1-(-1)^n)$$

For even 'n' $b_n = 0$

$$\therefore\; b_{2n+1} = \frac{2}{2n+1}; n - 0, 1, 2...$$

Therefore, the Fouries series of $f(x)$ is

$$f(x) = \frac{\pi}{2} + 2\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + ...\right)$$

Example 5.25 Find the Fourier series of the function shown in Fig. 5.21.

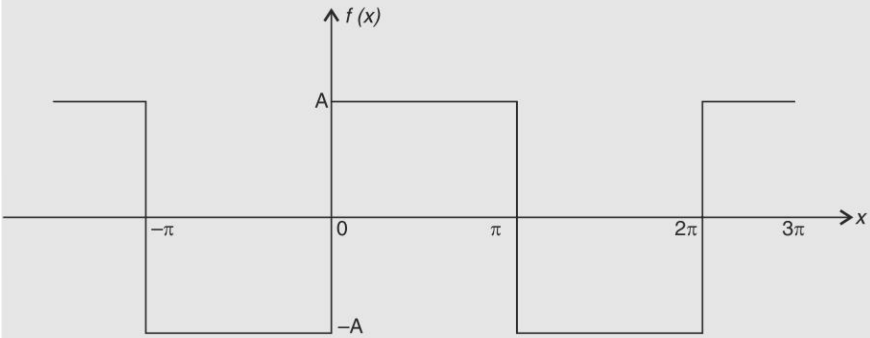


Fig. 5.21

Solution

From the above graph

$$f(x) = \begin{cases} -A, & -\pi \leq x < 0 \\ A, & 0 \leq x \leq \pi \end{cases}$$

$$\therefore \quad a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0}(-A)dx + \int_{0}^{\pi} A dx\right]$$

$$= \frac{1}{2\pi}[-A\cdot\pi + A\pi] = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{0} -A\cos(nx)dx + \frac{1}{\pi}\int_{0}^{\pi} A\cos(nx)dx = 0$$

$$b_n = \frac{1}{\pi}\left[\int_{-\pi}^{0} -A \sin nx\, dx + \int_{0}^{\pi} A \sin nx\, dx\right]$$

$$= \frac{2A}{\pi}\int_{0}^{\pi}\sin nx\, dx = \frac{2A}{\pi n} - \cos nx\Big|_{0}^{\pi}$$

$$= \frac{2A}{\pi n}[1-\cos n\pi] = \frac{2A}{\pi n}[1-(-1)^n]$$

$$= \frac{4A}{\pi(2n+1)}; n = 0, 1, 2$$

$$= \begin{cases} 0 & \\ \dfrac{2A}{\pi(2n+1)} & \text{for values of } n \quad 2n+1 \end{cases}$$

$$\therefore \quad F_n(x) = \frac{4A}{\pi}\left(\sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots\right)$$

Example 5.26 Find the Fourier series of the function shown below:

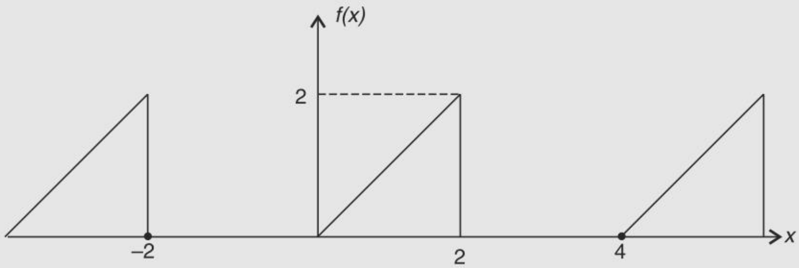


Fig. 5.22

Solution

From the graph

$$f(x) = \begin{cases} 0, & -2 \le x < 0 \\ x, & 0 \le x \le 2 \end{cases}$$

$$a_0 = \frac{1}{2l}\int_0^l f(x)\,dx$$

Since $l = 2$,

$$a_0 = \frac{1}{2.2}\int_0^2 x\,dx = \left[\frac{x^2}{2.2.2}\right]_0^2 = \frac{1}{2}$$

$$a_n = \frac{1}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

Sub $l = 2$, here

$$a_n = \frac{1}{2}\int_0^2 x \cos\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{1}{2}\left(\frac{2}{n\pi}\right)^2 \cdot (\cos(n\pi) - 1)$$

$$= \frac{1}{2}\left(\frac{2}{n\pi}\right)^2 \cdot ((-1)^n - 1)$$

$$b_n = \frac{1}{2}\int_0^2 x \sin\left(\frac{n\pi x}{2}\right)dx$$

$$= \frac{-2\cos n\pi}{n\pi} = \frac{2}{n\pi}(-1)^{n+1}$$

For $n \geq 1$

$$F_n(x) = \frac{1}{2} + \sum_{n=1}^{\infty}\left[\frac{2}{n^2\pi^2}((-1)^n - 1)\cos\left(\frac{n\pi x}{2}\right) + \frac{2}{n\pi}(-1)^{n+1}.\sin\left(\frac{n\pi x}{2}\right)\right]$$

Network Response to Periodic Functions—Fourier Analysis

Steady-state response of a linear network to a periodic function can be found by superposition of the steady-state responses to its Fourier series which must be suitably truncated depending upon the desired accuracy. This is best illustrated by an example.

Example 5.27 Full-wave rectified sinusoidal voltage with $V_m = 200\sqrt{2}$ V and angular frequency 314 rad/s is applied to the filtering circuit of Fig. 5.23. Find the expression for steady-state output voltage $v_0(t)$. Truncate the Fourier series up to 4th harmonic.

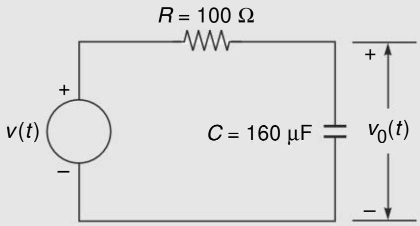


Fig. 5.23

Solution

$$v(t) = \frac{2V_m}{\pi} - \frac{4V_m}{3\pi}\cos 2\omega t - \frac{4V_m}{15\pi}\cos 4\omega t$$

Given

$$V_m = 200\sqrt{2},\ \omega = 314 \text{ rad/s}$$

$$v(t) = 180 - 120\cos 628\,t - 24\cos 1256\,t \qquad \text{(i)}$$

For input sinusoidal phasor $\bar{V}$, the output phasor at frequency ω is

$$\bar{V}_0 = \left(\frac{\frac{1}{jC\omega}}{R + \frac{1}{jC\omega}}\right)\bar{V}$$

$$= \frac{V}{1 + jRC\omega} \tag{ii}$$

$$RC = 100 \times 160 \times 10^{-6} = 16 \times 10^{-3}$$

$$V_0 = \frac{\overline{V}}{1 + j16 \times 10^{-3}} \tag{iii}$$

(i) The dc component is 180 V. For dc case, $\omega = 0$ in Eq. (ii) from which we get

$$V_0\,(\text{dc}) = 180\ \text{V} \tag{iv}$$

(ii) The second harmonic ($\omega = 628$) component is $v(2) = 120 \cos 628\, t$.

$$\overline{V}_0(2) = \frac{120}{\sqrt{2}} \angle\, 0° \tag{v}$$

The corresponding output voltage is

$$\overline{V}_0(2) = \frac{(120/\sqrt{2})\angle 0^0}{1 + j16 + 10^{-3} \times 628}$$

$$= \frac{120/\sqrt{2}}{1 + j10} = (11.94/\sqrt{2}) \angle -84.3° \tag{vi}$$

$$v_0(2) = 11.94 \cos (628t - 84.3°)$$

Observe that the strength of the second harmonic in the output reduces to 11.94 V while the input harmonic strength is 120 V.

(iii) The fourth harmonic ($\omega = 1256$) component is 24 cos 1256 t.

$$\overline{V}_0(4) = \frac{24}{\sqrt{2}} \angle\, 0°$$

The corresponding output voltage is

$$\overline{V}_0(4) = \frac{(24/\sqrt{2})\angle 0°}{1 + j16 \times 10^{-3} \times 1256}$$

$$= \frac{24/\sqrt{2}}{1 + j20} = 1.2\sqrt{2} \angle -37.1° \tag{vii}$$

The output voltage is given by

$$V_0(t) \approx 180 - 11.94 \cos (628t - 84.3°) - 1.2 \cos (1256t - 87.1°) \tag{viii}$$

Comparison of expressions (i) and (viii) reveals that while the dc strength is preserved in the output, the harmonic content is considerably reduced (2nd harmonic 11.94/120 = 0.1 and 4th harmonic 1.2/24 = 0.05). This is the *filtering effect* of the *RC* circuit.

Example 5.28 In the circuit of Fig. 5.24

$$v(t) = 4\sqrt{2} \sin (3t + 30°)\ \text{V}$$
$$i(t) = 0.8\sqrt{2} \cos (5t - 10°)\ \text{A}$$

Compute the average power consumed by the load connected at terminals 'ab'.

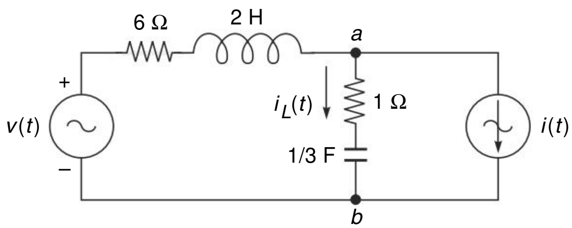


Fig. 5.24

Solution

Open-circuiting the current source, the frequency domain circuit (3 rad/s) is drawn in Fig. 5.25.

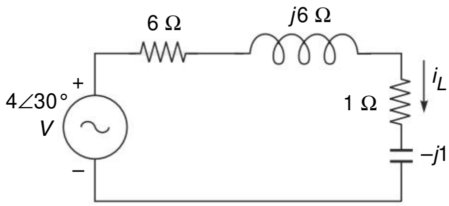


Fig. 5.25

$$\bar{I}'_L = (4 \angle 30°)/(6 + j6 + 1 - j1) = 0.465 \angle -5.5° \text{ A}$$
$$P_{av}' = (0.465)^2 \times 1 = 0.216 \text{ W}$$

Short-circuiting the voltage source, the frequency domain circuit (5 rad/sec) is drawn in Fig. 5.26.

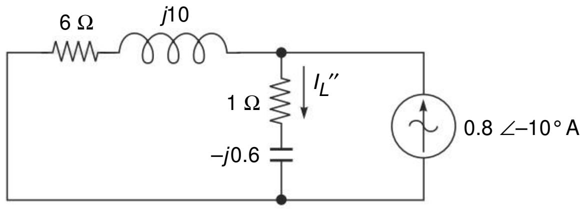


Fig. 5.26

$$I_L'' = 0.8 \angle -10° \times \frac{6 + j10}{(6+j10 + 1-j.06)} = 0.796 \angle -4.29°$$
$$P_{av}'' = (0.796)^2 \times 1 = 0.634 \text{ W}$$
$$P_{av} = P_{av}' + P_{av}'' = 0.216 + 0.634 = 0.85 \text{ W}$$

Example 5.29 For the parallel circuit of Fig. 5.27, show that when voltage V is applied to the circuit, the circulating current is $V\sqrt{C/L}$. Given, $L = 10$ μH, $R = 1\Omega$, $C =$ 10 pF and V = 10 V. Find the current input to the circuit at (a) resonant frequency, and (b) at 90% of the resonant frequency.

Solution

With reference to Fig. 5.27

Circulating current $= V\omega_0 C = VC/\sqrt{LC}$

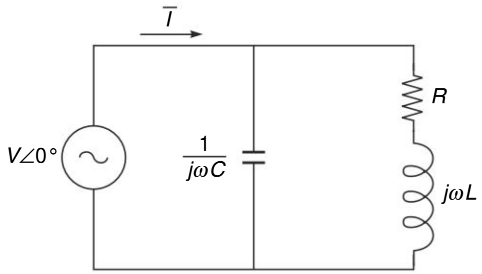


Fig. 5.27

$$= V\sqrt{C/L}$$

At resonant frequency (from Eq. (v) of Example 5.31) $Z\,(\max) = L/RC = 10 \times 10^{-6}/(1 \times 10^4 \times 10^{-12}) = 10^3\ \Omega$

$$I = 10/1000 = 0.01 \text{ A}$$

$$\omega_0 = 1/\sqrt{LC} = 1/\sqrt{10 \times 10^{-6} \times 10^4 \times 10^{-12}} = 3.162 \times 10^6 \text{ rad/s}$$

$$\omega = 0.9\,\omega_0 = 0.9 \times 3.162 \times 10^6 \text{ rad/s}$$

From Eq. (ii) of Example 5.3

$$\bar{Y}\,(j\omega) = L/RC + j\,(\omega C - 1/\omega L)$$

$$= 10^{-3} + j\,[0.9 \times 3.162 \times 10^6 \times 10^4 \times 10^{-12}$$

$$- 1/(0.9 \times 3.162 \times 10 \times 10^6 \times 10^{-6})]$$

$$= (1 - j6.682) \times 10^{-3}$$

or $\quad Y = 6.76 \times 10^{-3}$

$\therefore \quad I = 0.00676 \times 10 = 0.0676$ A

SUMMARY

- Resonance is a condition in which a circuit, when excited by fixed amplitude sinusoidal voltage/ current, produces maximum amplitude current/voltage response.
- An electrical network (circuit) is in resonance when voltage and current at its input are in phase, that is, the power factor is unity.
- The quality factor (Q) of a network is proportional to the maximum energy stored divided by the energy lost per cycle.
- At half-power frequency, the circuit response amplitude reduces to $\frac{1}{\sqrt{2}}$ of its maximum value.
- The bandwidth of a resonant circuit is the difference between upper and lower half-power frequencies.
- In a high Q resonant circuit, the resonant frequency is mid-frequency of the bandwidth.
- At resonant frequency, the admittance of a series circuit is maximum and the impedance of a parallel circuit is maximum.

REVIEW QUESTIONS

1. What are the two kinds of simple resonant circuits?
2. Sketch the variation of impedance/admittance with frequency in *RLC* series and parallel resonant circuits.
3. Explain what is meant by resonance in electrical circuits (simple circuits).
4. What is the power factor of an *RLC* parallel circuit at resonant frequency? Explain the reason.
5. Repeat question 4 for *RLC* series circuit.
6. In an *RLC* parallel circuit, the input current is $i(t) = I_m \cos \omega_0 t$; ω_0 = resonant frequency. Write the expression for the circuit voltage.
7. In an *RLC* series circuit, the applied voltage is $v(t) = V_m \cos \omega_0 t$; ω_0 = resonant frequency. Write the expression for the circuit current.
8. Explain what is quality factor and what is its significance. How does it affect the circuit bandwidth?
9. Under what condition is the resonant frequency mid-between the bandwidth?
10. What is the circulating current in an *RLC* parallel resonant circuit?
11. In an *RLC* series circuit, the net voltage across *L* and *C* at resonant frequency is zero. Explain.
12. Write the expression for quality factor Q_0 at resonant frequency for series and parallel resonant circuits.
13. What is meant by half-power frequency of a resonant circuit? Is there one ot two such frequencies?
14. Cite a practical application of a parallel resonant circuit.

PROBLEMS

5.1 For the lag and lead networks shown in Fig. 5.28(a) and (b) respectively, find

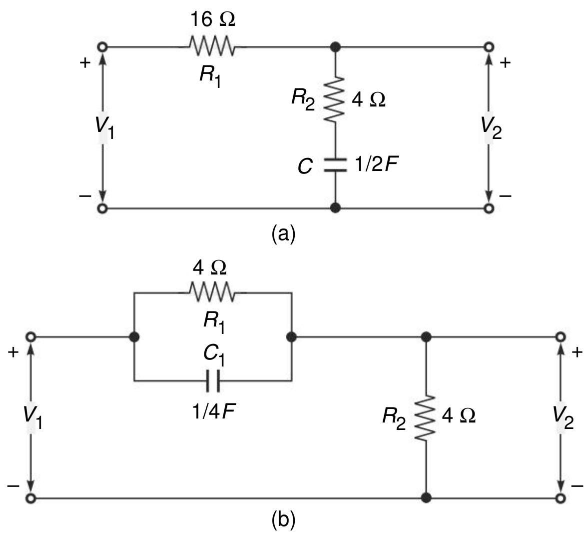


Fig. 5.28 (a) Lag network and (b) lead network

$$\overline{H}(j\omega) = \frac{\overline{V}_2(j\omega)}{\overline{V}_1(j\omega)}$$

This is called the *transfer function*.

5.2 For the network of Fig. 5.29 find $\overline{H}(j\omega) = \overline{V}_2(j\omega/\overline{V}_1(j\omega)$.

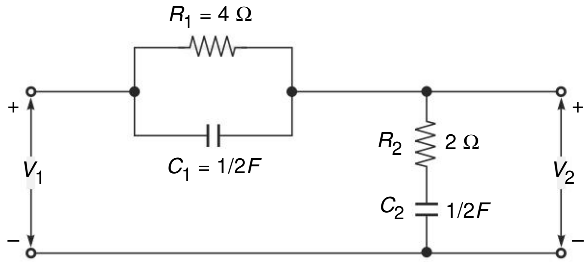


Fig. 5.29

5.3 A constant voltage of frequency 1 MHz is applied to an inductor (r in series with L) in series with a variable capacitor. The current drawn is maximum when the capacitor is set to 400 pF; while the current is reduced to $1/\sqrt{2}$ of that when the capacitance is 450 pF. Find the values of r, L and Q.

5.4 A resistor and a capacitor are in series with a variable inductor. When the circuit is connected to 230 V, 50 Hz mains, the maximum current obtained by varying the inductor is 0.35 A, the voltage across capacitor being 300 V. Calculate the circuit constants.

5.5 A coil of inductance 7.5 H and resistance 40 Ω in series with a condenser is fed from a constant voltage variable frequency source. The maximum current is 1.2 A at 100 Hz. Find the frequency when the current is 0.848 A.

5.6 For the parallel circuit shown in Fig. 5.30, determine i_R, i_L and i_C. Draw the phasor diagram indicating all currents and voltages.

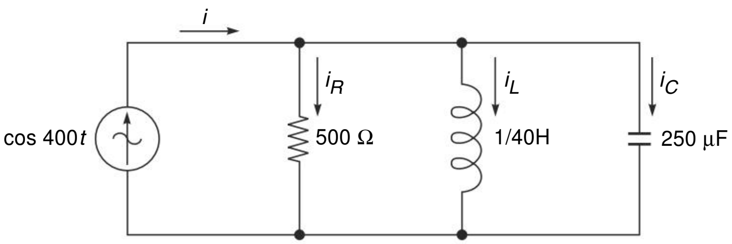


Fig. 5.30

5.7 For the circuit of Fig. 5.31, show that the circulating current is $V\sqrt{C/L}$ provided R is small where V is the applied voltage. Given $L = 10$ mH, $R = 1\ \Omega$ and $C = 104$ pF, find the current input to the parallel circuit at (a) resonant frequency and (b) at 90% of resonant frequency.

5.8 For the circuit shown in Fig. 5.31, find ω_0 and Q_0 at ω_0.

5.9 A parallel resonant circuit has $R = 60$ kΩ, $L = 5$ mH and $C = 50$ pF. Determine f_0, Q_0 and the bandwidth in Hz.

5.10 A coil of 15 Ω resistance and inductance 0.75 H is connected in series with a condenser.

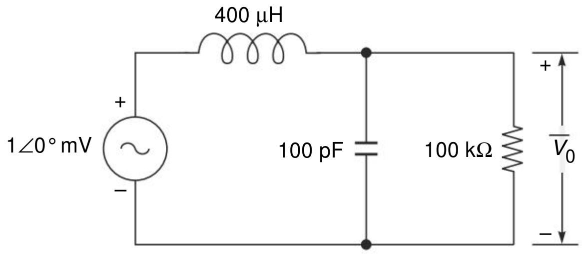


Fig. 5.31

The combination draws maximum current when a fixed amplitude sinusoidal voltage of 50 Hz is applied. A second condenser is now connected in parallel with this circuit. What should be its capacitance for the combined circuit to act as a non-inductive resistance at 100 Hz? Calculate the current drawn by the combined circuit if the applied voltage is 200 V.

5.11 Figure 5.32 shows a half-wave rectified waveform. Find the Fourier series.

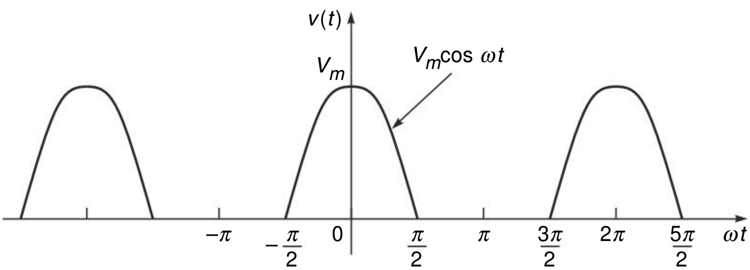


Fig. 5.32

5.12 For the square wave voltage shown in Fig. 5.33, find the Fourier series. Fundamental frequency = ω rad/s.

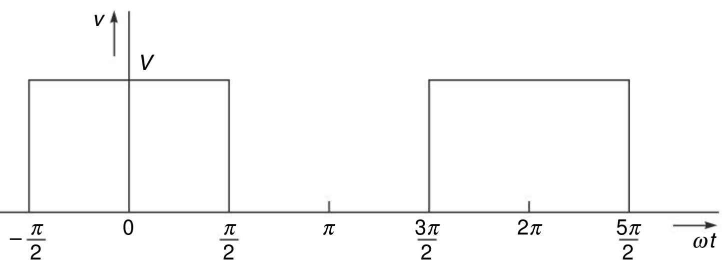


Fig. 5.33

5.13 The voltage waveform of Fig. 5.33 is applied to an *L*-section filter of Fig. 5.34 Find $v_0(t)$. Given $\omega = 314$ rad/s, $L = 1/2$ H, $C = 2$ μF, $R = 1/2$ kΩ.

5.14 For the triangular wave form of Fig. 5.35, find the Fourier series.

5.15 The circuit of Fig. 5.36 $i(t)$ has triangular wave form as shown in Fig. 5.35 with I_m = 10 mA and a period of 1 ms. Find the steady value of $v_0(t)$ using Fourier analysis. Truncate the Fourier series after fourth harmonic.

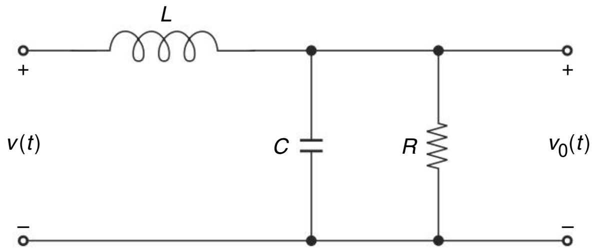


Fig. 5.34

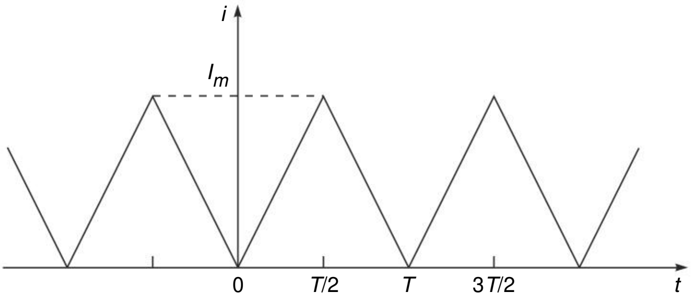


Fig. 5.35

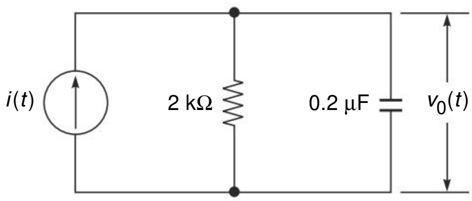


Fig. 5.36

5.16 Half-wave rectified sine wave with V_m = 200 V and angular frequency 314 rad/s is applied to the filtering circuit of Fig. 5.37. Find the expression for steady state output voltage $v_0(t)$. Truncate the Fourier series up to 4th harmonic.

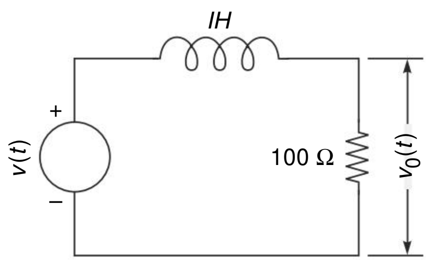


Fig. 5.37

5.17 For the circuit of Fig. 5.38

(a) find the transfer function $\overline{V}_0 / \overline{V}_s$ and

(b) sketch the frequency response of the transfer function of part (a).

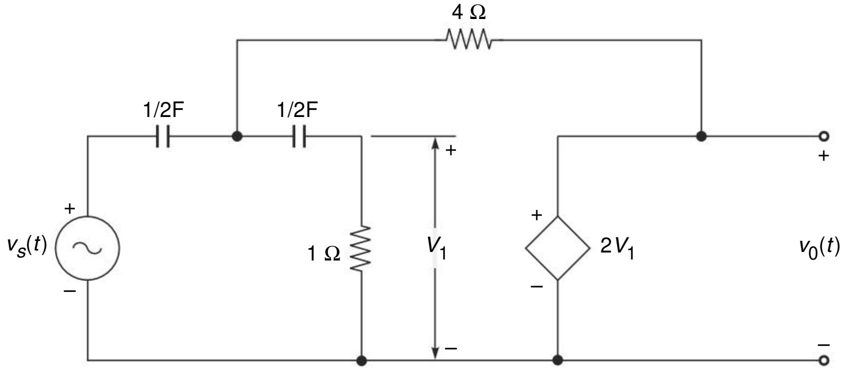


Fig. 5.38

Chapter 6

THREE-PHASE CIRCUITS

6.1 INTRODUCTION

Most generation, transmission and large power utilization is accomplished by means of polyphase systems which have several sources equal in magnitude but with a progressive difference of $2\pi/q$, where q is the number of phases (for $q = 2$, phase difference is $\pi/2$). Such systems have distinct economic and operational advantages over a single-phase system. To avoid undue complexity, three-phase system is almost universally adopted.

6.2 THREE-PHASE VOLTAGES AND CURRENTS

A set of three sinusoidal voltages (or currents) that are equal in magnitude but have a progressive phase difference of $2\pi/3$ (120°) constitute a balanced three-phase voltage (or current) system. The three-phase quantities (voltages/currents) are otherwise said to be unbalanced. Three voltage sources forming a *balanced three-phase system* are shown in Fig. 6.1(a). In instantaneous form these voltages (known as phase voltages) are expressed as

$$v_{aa'} = \sqrt{2}\,V_p \sin \omega t \tag{6.1a}$$

$$v_{bb'} = \sqrt{2}\,V_p \sin (\omega t - 120°) \tag{6.1b}$$

$$v_{cc'} = \sqrt{2}\,V_p \sin (\omega t - 240°) \tag{6.1c}$$

where V_p = rms amplitudes of phase voltage

The wave form of the three-phase voltages is drawn in Fig. 6.1(b).

In phasor form, Eq. (6.1) can be written as

$$\overline{V}_{aa'} = V_p \angle 0° \tag{6.2a}$$

$$\overline{V}_{bb'} = V_p \angle -120° \tag{6.2b}$$

$$\overline{V}_{cc'} = V_p \angle -240° \tag{6.2c}$$

The phasor diagram of the three voltages is drawn in Fig. 6.2(a) where for

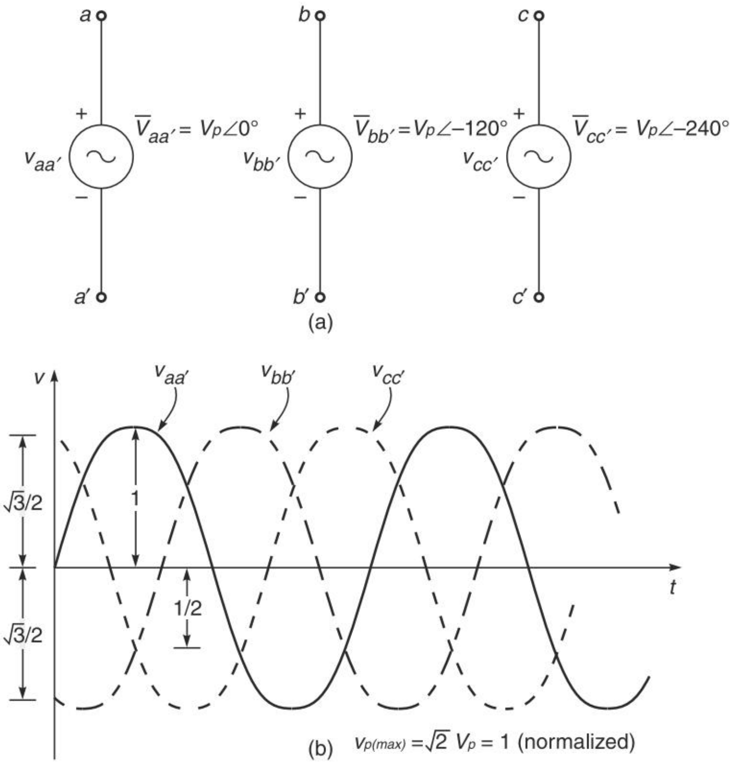


Fig. 6.1 Balanced three-phase system: (a) three-phase voltage sources and (b) voltage waveform of three-phase sources

conveniences we may write $\overline{V}_{aa'}$ as $\overline{V}_a$-phase a, $\overline{V}_{bb'}$ as $\overline{V}_b$-phase b and $\overline{V}_{cc'}$ as $\overline{V}_c$-phase c

Consider now the instantaneous voltage sum

$$\begin{aligned} v_{aa'} + v_{bb'} + v_{cc'} &= \sqrt{2}\, V\,[\sin \omega t + \sin(\omega t - 120°) \\ &\quad + \sin(\omega t - 240°)] = 0 \end{aligned} \tag{6.3}$$

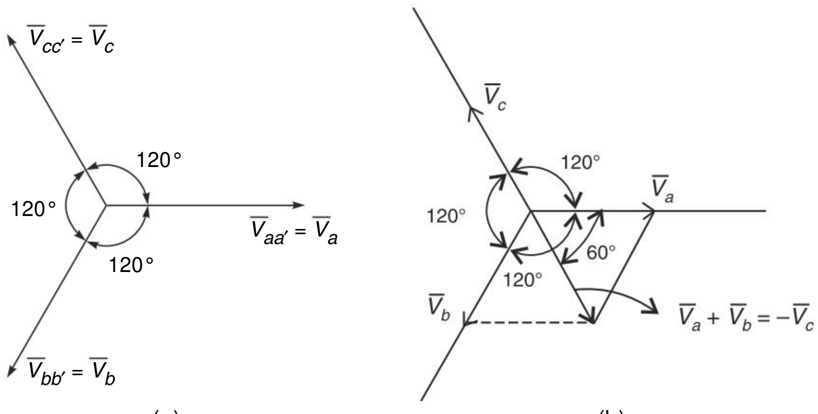


Fig. 6.2 Phasor diagram of balanced three-phase voltages

It is immediately seen that the sum of balanced three-phase voltages (or currents) is zero at all times. In phasor form

$$\bar{V}_{aa'} + \bar{V}_{bb'} + \bar{V}_{cc'} = 0 \tag{6.4}$$

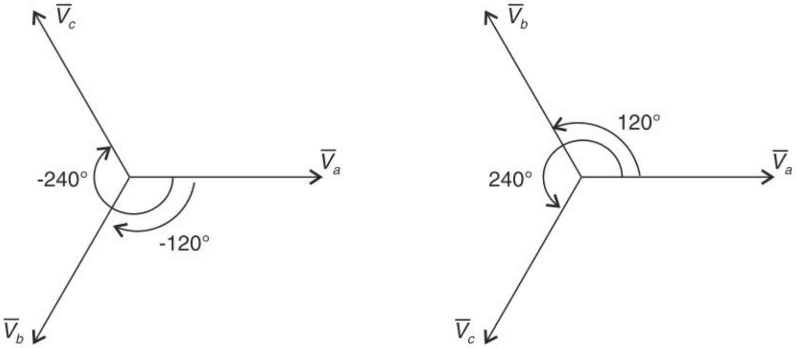
This is verified from the phasor diagram of Fig. 6.2(b).

Phase Sequence

In the balanced 3-phase voltages presented above in phasor form [Eqs. (6.2)] and in phasor diagram of Fig. 6.2, phase ‘a’ leads phase ‘b’ leads phase ‘c’ by 120° This known as positive phase sequence *abc*.The negative phase sequence is *cba* wherein

$$\begin{aligned} \bar{V}_a &= V_p \angle 0° \\ \bar{V}_b &= V_p \angle 120° \\ \bar{V}_c &= V_p \angle 240° \end{aligned} \tag{6.5}$$

The phasor diagrams for both the phase sequences are drawn in Fig. 6.3 (a) and (b) for comparison.



Positive phase sequence *abc* Negative phase sequence *cba*

Fig. 6.3

In actual physical system, the phase sequence depends on arbitrary labelling of the terminals as *a, b* and *c*. We shall assume in our study positive phase sequence.

6.3 STAR (Y) CONNECTION

Both three-phase source and load can be star connected. This source-load connection is labeled as Y-Y connection.

Star (Y) Connected Source

In this connection, the similar polarity ends (say a', b', c') of the three phase voltage sources of Fig. 6.1(a) are joined together to form the *neutral point* and three leads are taken out from the other ends and one lead from the neutral as shown in Fig. 6.4.

Line-to-Line Voltages

These are commonly referred to as Line Voltages. The phasor diagram showing phase and line voltages is drawn in Fig. 6.5

From the Star Connection of Fig. 6.4

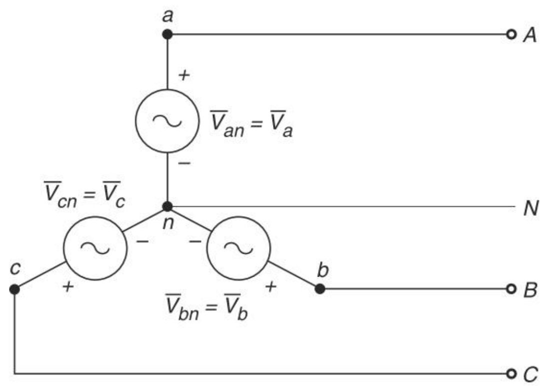


Fig. 6.4 Star connection

The voltage between lines a and b (or A and B) is

$$\overline{V}_{ab} = \overline{V}_{an} - \overline{V}_{bn} \tag{6.6}$$

It immediately follows from the phasor diagram of Fig. 6.5 that

$$\overline{V}_{ab} = \sqrt{3}\ \overline{V}_{an} \angle 30°\text{, reference phase 'a'}$$
$$= \sqrt{3}\ V_p \angle 30°; \qquad 6.6\text{(a)}$$

Similarly

$$\overline{V}_{bc} = \overline{V}_{bn} - \overline{V}_{cn}$$
$$= \sqrt{3}\ \overline{V}_{bn} \angle 30° = \sqrt{3}\ V_p \angle -90°\text{; reference phase 'a'} \tag{6.7}$$

and

$$\overline{V}_{ca} = \overline{V}_{cn} - \overline{V}_{an}$$
$$= \sqrt{3}\ \overline{V}_{cn} \angle 30°; = \sqrt{3}\ V_p \angle -210°\text{; reference phase 'a'} \tag{6.8}$$

It is seen from the phasor diagram of Fig. 6.5 that for balanced phase voltages,

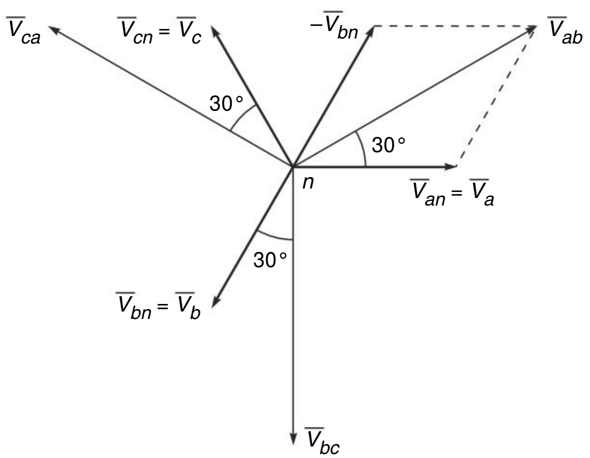


Fig. 6.5 Phase and line voltages-star connection

line voltages also form a balanced set advanced in phase by 30° from the phase voltages. In terms of magnitude

$$V\text{ (line-to-line)} = \sqrt{3}\, V\text{ (phase)}$$

or
$$V_L = \sqrt{3}\, V_P \qquad (6.9)$$

Some of the voltage phasors

$$\overline{V}_{ab} + \overline{V}_{bc} + \overline{V}_{ca} = 0$$

Star connected Source Feeding Star connected Load

Consider now a balanced 3-phase load connected in star fed from a balanced 3-phase star connected source through 3 lines and a neutral connection as shown in Fig. 6.6 It is to be noted that a balanced load means that each of the three *Phase impedances* ($\overline{Z}_p$) are equal.

It is to be recorded here

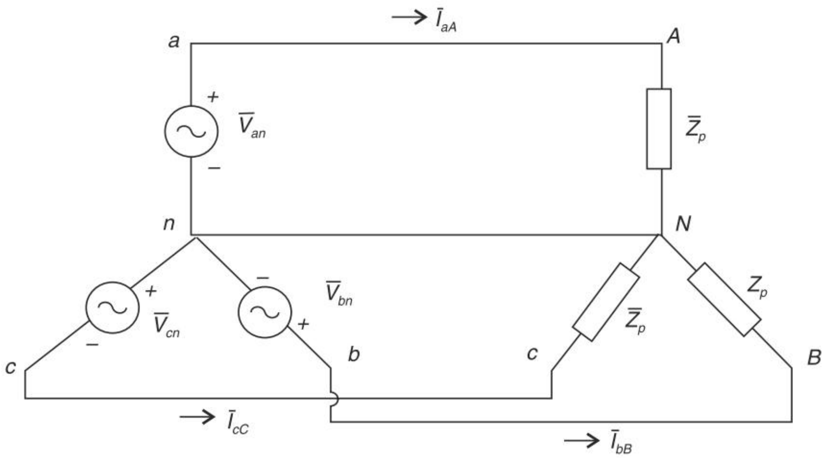


Fig. 6.6 Balanced star connected source feeding balanced star connected load

that the connecting lines have zero impedance

From the loop *na AN*, the line current

$$\bar{I}_{aA} = \frac{\overline{V}_{an}}{\overline{Z}_p} = \frac{V_{an}}{Z_p} \angle 0° \qquad (6.10)$$

Similarly

$$\bar{I}_{bB} = \frac{\overline{V}_{bn}}{\overline{Z}_p} = \left(\frac{V_{an}}{\overline{Z}_p}\right) \angle -120°$$

and
$$\bar{I}_{cC} = \frac{\overline{V}_{cn}}{\overline{Z}_p} = \left(\frac{V_{an}}{\overline{Z}_p}\right) \angle -240°$$

We thus find that the line currents form a balanced set. Therefore

$$\bar{I}_{aA} + \bar{I}_{bB} + \bar{I}_{cC} = 0 \qquad (6.11)$$

As the sum of the three line currents is zero, no current flows in the neutral connection. The source and load neutrals need not be connected; neutral nodes *n* and N have the same potential.

It is seen from the Star-Star connection that the phase and line currents are identical i.e.,

$$I(line) = I(phase)$$

or
$$I_L = I_P \qquad (6.12)$$

The phasor diagram showing phase voltages and line current is drawn in Fig. 6.7. Each line current differs in phase from corresponding phase voltage of the impedance angle $\theta = \angle \bar{Z}_p$.

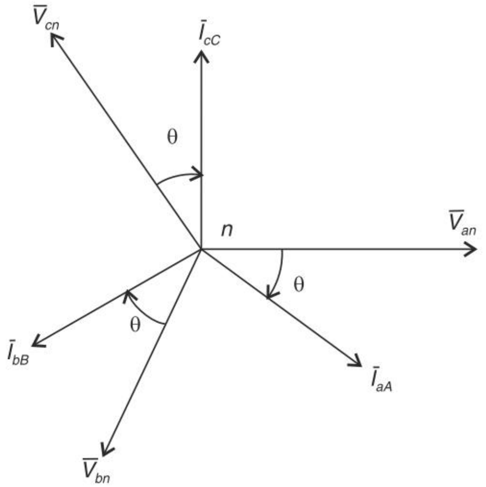


Fig. 6.7 Phasor diagram of balanced Star-Star connection

Power Factor In a balanced source-load system, the power factor cos θ is the same for each phase. It is commonly referred to as the system power factor.

Example 6.1 For the balanced Star-Star connection of Fig. 6.6, the source phase voltage is 200 V and load impedance is 100 ∠ 60° Ω. Calculate all the phase voltage, line voltages and time currents in phasor form.

Solution

Phase voltages

$$\bar{V}_{an} = 200 \angle 0° V, \quad \bar{V}_{bn} = 200 \angle{-120°}, \bar{V}_{cn} = 200 \angle{-240°}$$

Line currents

$$I_L = I_P = \frac{200}{100} = 2\text{A}$$

Phase angle = $\theta = -60°$

$$\bar{I}_{aA} = 2 \angle -60° A, \bar{I}_{bB} = 2 \angle{-120°} - 60° = 2 \angle{-180°} \text{ A}$$

$$\bar{I}_{cC} = 2 \angle{-240°} -60° = 2 \angle{-300°} = 2 \angle 60° \text{ A}$$

The reader is advised to draw a complete phasor diagram.

6.4 DELTA (Δ) CONNECTION

A balanced Star-connected source feeding a balanced delta-connected load is sketched in Fig. 6.8.

It is immediately observed that no neutral connection is possible.

Voltage applied across each delta phase is the line voltage i.e.

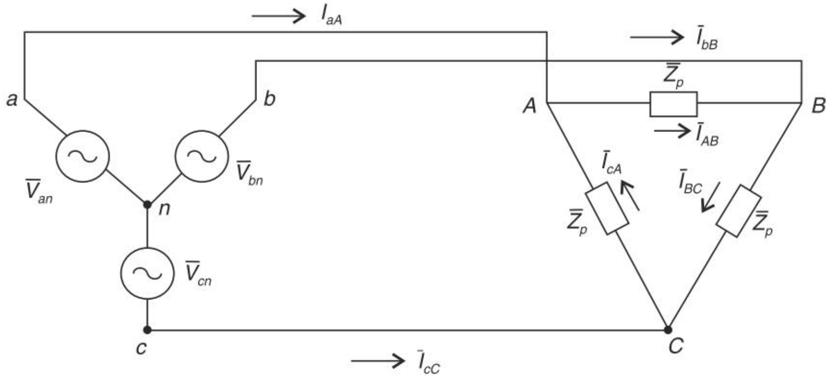


Fig. 6.8 Star-connected source feeding Delta- connected load

$$\text{V(line)} = \text{V(phase } \Delta)$$

or $$V_L = V_P \tag{6.13}$$

The delta phase currents are expressed as

$$I_{AB} = \frac{\bar{V}_{ab}}{\bar{Z}_p},\ \bar{I}_{BC} = \frac{\bar{V}_{bc}}{\bar{Z}_p},\ \bar{I}_{CA} = \frac{\bar{V}_{ca}}{\bar{Z}_p} \tag{6.14}$$

As the delta voltages are a balanced 3-phase set, the delta phase currents also form a balanced set except that these differ by the phase angle $\theta = \angle \bar{Z}_p$

The line currents are flowing to delta terminals (A,B,C) and are found by applying KCL at each node. Thus

$$\begin{aligned} \bar{I}_{aA} &= \bar{I}_{AB} - \bar{I}_{CA} \\ \bar{I}_{bB} &= \bar{I}_{BC} - \bar{I}_{AB} \\ \bar{I}_{cC} &= \bar{I}_{CA} - \bar{I}_{BC} \end{aligned} \tag{6.15}$$

As we know from the phase and line relationship of balanced voltages, the phase and line delta currents are related as

$$I_L = \sqrt{3}\, I_P \tag{6.16}$$

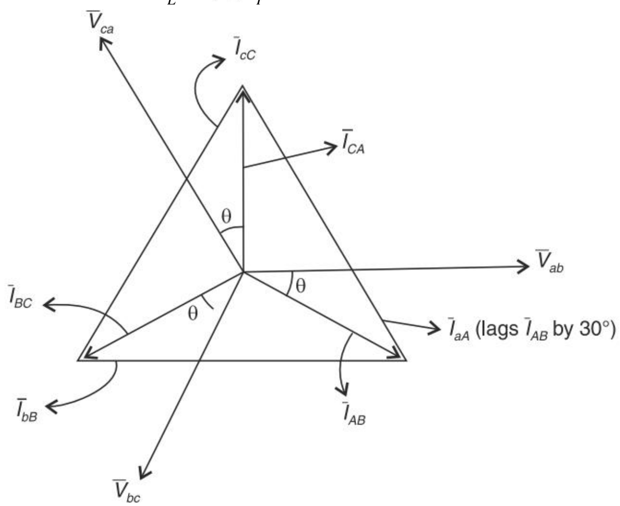


Fig. 6.9 Phasor diagram delta load voltages and currents

The phase relationship is that the line currents lag in phase by 30° from the phase currents.

The above delta relationships are clarified by the phasor diagram of Fig. 6.9.

Power Factor It is cos θ, where θ is the angle between line voltage and corresponding phase current. Also as said already $\theta = \angle \bar{Z}_p$.

Example 6.2 A three-phase power system with a line voltage of 400 V is supplying a delta-connected load of 1500W at 0.8 *pf* lagging. Determine the phase and line currents and also the phase impedance.

Solution

The circuit diagram of the power system is drawn in Fig. 6.10.

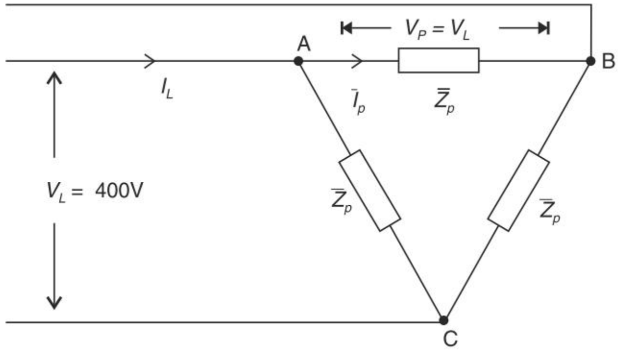


Fig. 6.10

It is a balance 3-phase system. So each load phase draws one-third power. Thus

$$P\ (phase) = \frac{1500}{3} = 500\text{ W}$$

But $\qquad P = V_p I_p \cos\theta$

For Delta-connection

$$V_P = V_L = 400\text{ V}$$

or $\qquad pf, \cos\theta = 0.8$ lag

Substituting values

$$500 = 400\, I_P \times 0.8$$

or $\qquad I_P = 1.56\ A$

In Delta-connection

$$I_L = \sqrt{3}\, I_P = 1.56\sqrt{3}$$
$$= 2.70\ A$$

Now

$$pf \text{ angle } \theta = \cos^{-1} 0.8 = 36.9° \text{ lag}$$

Therefore

$$\bar{I}_p = 2.70 \angle -36.9°$$

Then

$$\overline{Z}_P = \frac{\overline{V}_P}{\overline{I}_P} = \frac{400\angle 0°}{2.70\ \angle -36.9°}$$

or $\overline{Z}_p = 256\ \angle 36.9°\Omega$

The circuit diagram is drawn here for tutorial purpose, otherwise it is not necessary.

Example 6.3 A 3-phase sysetm supplies 1200 W to a Star-commected load at 0.8 *pf* lagging. Determine the amplitude of line and phase current and $\overline{Z}_p$, the phase impedance.

Solution

The circuit diagram is drawn in Fig. 6.11.

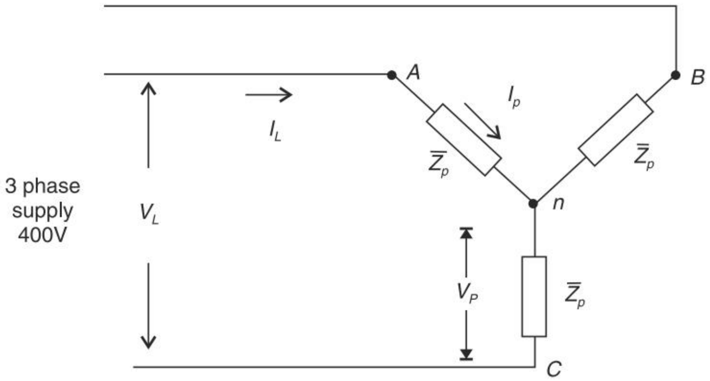


Fig. 6.11

Power drawn by each phase

$$P\ (phase) = \frac{1200}{3}\ 400\ \text{W} \tag{i}$$

In Star-connection

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 231\ \text{V}$$

$$I_L = I_p$$

$$P\ (phase) = V_p I_p \cos\theta$$

$$400 = 231\ I_p \times 0.8$$

or $I_p = 2.16$ A, $\theta = 36.9°$ lag

so $I_L = 2.16$ A

Phasor impedance

$$\overline{Z}_P = \frac{\overline{V}_P}{\overline{I}_P} = \frac{231}{2.16\ \angle -36.9°}$$

or $\overline{Z}_P = 106.9\ \ \angle 36.9°\ \Omega$

It is not necessary to draw the circuit diagram. It is drawn here for tutorial purpose.

Δ–connected Source A 3-phase Delta-connected balanced source feeding a 3-phase balanced load sketched in Fig. 6.12.

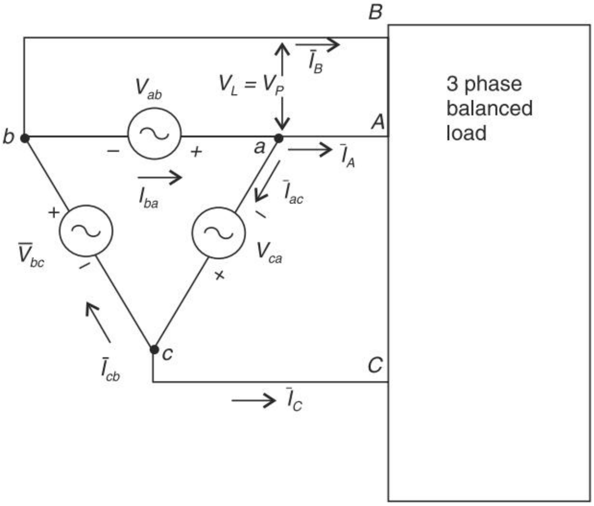


Fig. 6.12 Delta connected source feeding 3 phase load

As the three sources are a balanced set of voltages $\overline{V}_{ab} + \overline{V}_{bc} + \overline{V}_{ca} = 0$

Further, as line and phase voltages are identical, their amplitudes are equal i.e..

$$V_L = V_p \tag{6.17}$$

The three balanced current can be expressed as

$$\begin{aligned}\bar{I}_{ba} &= I_p \angle 0° \\ \bar{I}_{cb} &= I_p \angle -120° \\ \bar{I}_{ac} &= I_p \angle -240°\end{aligned} \quad 6.18)$$

From KCL at the delta nodes, line currents are

$$\begin{aligned}\bar{I}_A &= \bar{I}_{ba} - \bar{I}_{ac} \\ \bar{I}_B &= \bar{I}_{cb} - \bar{I}_{ba} \\ \bar{I}_C &= \bar{I}_{ac} - \bar{I}_{cb}\end{aligned} \quad (6.19)$$

The current phasor diagram is drawn in Fig. 6.13 from which it follows that amplitude of line current = $\sqrt{3}$ times of phase current

$$I_L = \sqrt{3}\, I_p \tag{6.20}$$

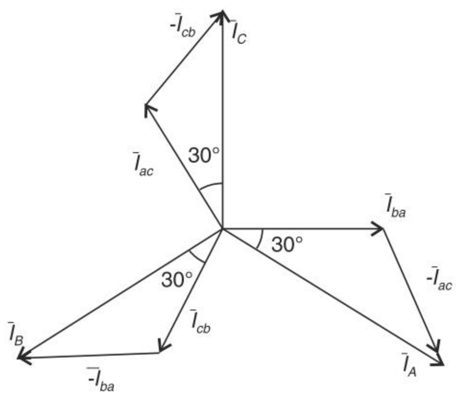


Fig. 6.13

In practice, source is *not delta-connected* as even a slight unbalance of source voltages, causes of the sum of delta voltages to be non-zero. Even a small delta loop voltage causes a large circulating current as the impedances of the three phases is very small. This causes undesirable power loss in the loop and reduces the capacity of delta source to supply load current.

6.5 THREE-PHASE POWER

Phase voltages and currents in a balanced three-phase circuit (star or delta) can be written in the instantaneous form as

$$v_a = \sqrt{2}\, V_P \sin \omega t \tag{6.21a}$$

$$v_b = \sqrt{2}\, V_P \sin (\omega t - 120°) \tag{6.21b}$$

$$v_c = \sqrt{2}\, V_P \sin (\omega t - 240°) \tag{6.21c}$$

and

$$i_a = \sqrt{2}\, I_P \sin (\omega t - \theta) \tag{6.22a}$$

$$i_b = \sqrt{2}\, I_P \sin (\omega t - \theta - 120°) \tag{6.22b}$$

$$i_c = \sqrt{2}\, I_P \sin (\omega t - \theta - 240°) \tag{6.22c}$$

where θ = phase angle between phase voltage and current pair.

The instantaneous power in each phase is

$$p_a = v_a i_a = V_P I_P [\cos \theta - \cos (2\omega t - \theta)] \tag{6.23a}$$

$$p_b = v_b i_b = V_p I_p [\cos \theta - \cos (2\omega t - \theta - 240°)] \tag{6.23b}$$

$$p_c = v_c i_c = V_P I_P [\cos \theta - \cos (2\omega t - \theta - 480°)] \tag{6.23c}$$

The total instantaneous three-phase power is

$$p = p_a + p_b + p_c = 3 V_P I_P \cos \theta \tag{6.24}$$

Notice that the sum of the three second harmonic oscillating terms which have a progressive phase difference of 120° is zero. As a result the instantaneous three-phase power in a balanced system is constant and equal to three times the average power per phase. This is in contrast to power in a single-phase system (Eq. (4.45)) which has a second harmonic oscillating component. Constancy of power in a balanced three-phase system affords the important advantage of uniform torque in three-phase electric machines. *Inter alia* this is an important reason in universal adoption of three-phase system except in low power applications.

Thus in a balanced three-phase system

$$P = 3\, V_P I_P \cos \theta; \quad \cos \theta = pf \tag{6.25}$$

Converting phase values to line values

$$P = \sqrt{3}\, V_L I_L \cos \theta = 3 I_P^2 R_P; \;\; \cos \theta = pf \tag{6.26}$$

where R_P = equivalent per phase series resistance of load. Similarly

$$Q = \sqrt{3}\, V_L I_L \sin \theta = 3 I_P^2 X_p \tag{6.27}$$

where X_P = equivalent per phase series reactance of load. Of course, in a balanced three-phase system each phase has the same power factor.

6.6 THREE-PHASE CIRCUIT ANALYSIS

Figure 6.14 shows a typical three-phase, star-connected voltage source, transmission line and star-connected load. Both, source and load are balanced three-phases. The two neutrals are isolated but were these connected as shown by the dotted line, no current would flow in the neutral connection as $\bar{I}_a + \bar{I}_b + \bar{I}_c = 0$; the currents would form a balanced set in a balanced system. Thus no voltage drop would occur in the neutral connection. For the reference phase a (loop shown).

$$\bar{V}_a = (\bar{Z}_{TL} + \bar{Z}_L)\,\bar{I}_a \tag{6.28}$$

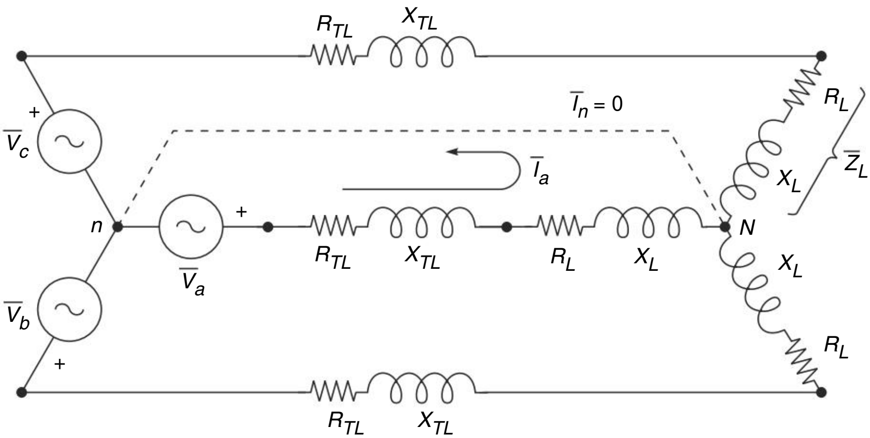


Fig. 6.14 Balanced three-phase system

We need only solve the loop Eq. (6.28) corresponding to phase a. The currents and voltages in other two loops would have the same magnitude but a progressive phase difference of 120°. Equation (6.28) corresponds to the single-phase equivalent circuit of Fig. 6.15 whose solution completely determines the solution of the three-phase circuit of Fig. 6.15.

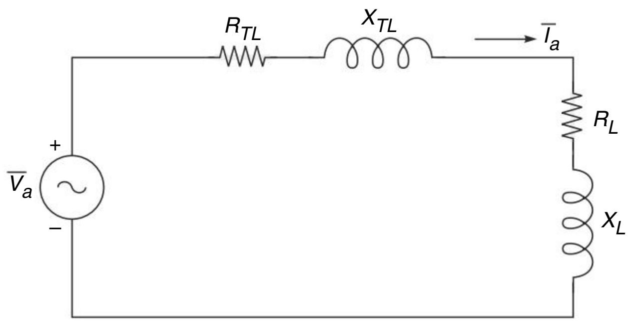


Fig. 6.15 Per phase equivalent of the three-phase circuit of Fig. 6.14

6.7 STAR-DELTA CONVERSION

Figure 6.16 shows three delta connected impedances whose equivalent star (shown dotted) is to be found. Generalization of Eq. (2.11), in complex impedance form, yields

$$\bar{Z}_a = \frac{\bar{Z}_{ab}\,\bar{Z}_{ac}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{6.29a}$$

$$\bar{Z}_b = \frac{\bar{Z}_{bc}\,\bar{Z}_{ba}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{6.29b}$$

$$\bar{Z}_c = \frac{\bar{Z}_{ca}\,\bar{Z}_{cb}}{\bar{Z}_{ab} + \bar{Z}_{bc} + \bar{Z}_{ca}} \tag{6.29c}$$

Similarly in converting star to delta it follows from Eq. (2.10)

$$\bar{Z}_{ab} = \frac{\bar{Z}_a\,\bar{Z}_b + \bar{Z}_b\,\bar{Z}_c + \bar{Z}_c\,\bar{Z}_a}{\bar{Z}_c} \tag{6.30a}$$

$$\bar{Z}_{bc} = \frac{\bar{Z}_a\,\bar{Z}_b + \bar{Z}_b\,\bar{Z}_c + \bar{Z}_c\,\bar{Z}_a}{\bar{Z}_a} \tag{6.30b}$$

$$\bar{Z}_{ca} = \frac{\bar{Z}_a\,\bar{Z}_b + \bar{Z}_b\,\bar{Z}_c + \bar{Z}_c\,\bar{Z}_a}{\bar{Z}_b} \tag{6.30c}$$

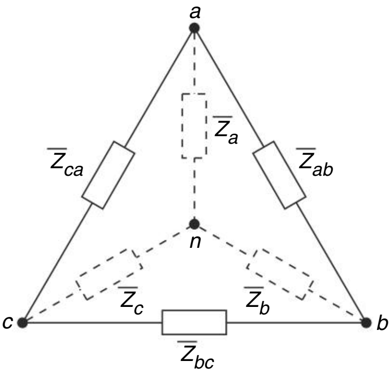


Fig. 6.16 Star-delta conversion

If the impedances are balanced, it follows from the above results that

$$\bar{Z}_\Delta = 3\bar{Z}_Y \tag{6.31}$$

Example 6.4 A balanced star-connected load is supplied from a symmetrical three-phase, 400 V (line-to-line) supply. The current in each phase is 50 A and lags 30° behind the phase voltage. Find (a) phase voltage, (b) phase impedance, and (c) active and reactive power drawn by the load. Also draw the phasor diagram showing phase and line voltages and line currents.

Solution The system circuit diagram is drawn in Fig. 6.17.

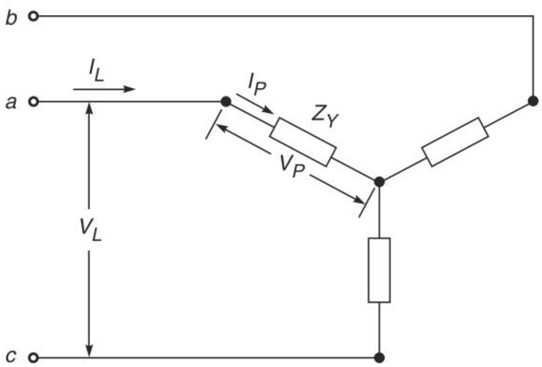


Fig. 6.17

(a) $V_L = 400$ V

$$V_P = \frac{400}{\sqrt{3}} = 231\ V;\ \text{reference phasor}$$

(b) $\bar{I}_L = \bar{I}_P = 50 \angle -30°$

$$\bar{Z}_Y = \frac{231 \angle 0°}{50 \angle -30°} = 4.62 \angle +30°$$

$$= 4 + j2.31$$

(c) $P = \sqrt{3}\, V_L I_L \cos\theta$

$$= \sqrt{3} \times 400 \times 50 \cos 30° = 30 \text{ kW}$$

$$Q = \sqrt{3}\, V_L I_L \sin\theta$$

$$= 3 \times 400 \times 50 \times \sin 30° = 17.32 \text{ kVAR}$$

The phasor diagram is drawn in Fig. 6.18.

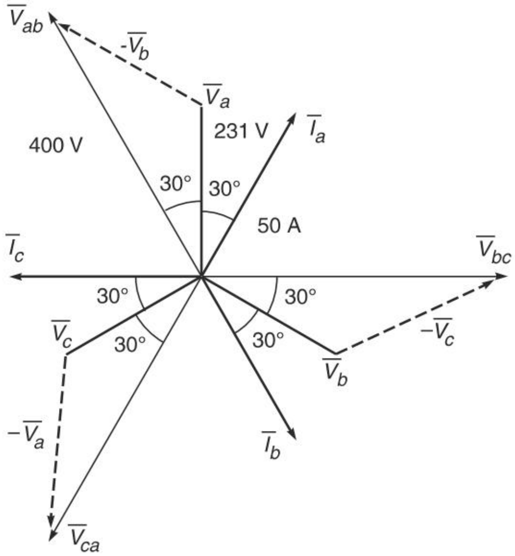


Fig. 6.18

Example 6.5 A balanced delta-connected load of impedance 16 + j 12 Ω/phase is connected to a three-phase 400 V supply. Find the phase current, line current, power factor, power, reactive VA and total VA. Also draw a phasor diagram.

Solution Refer to Fig. 6.19.

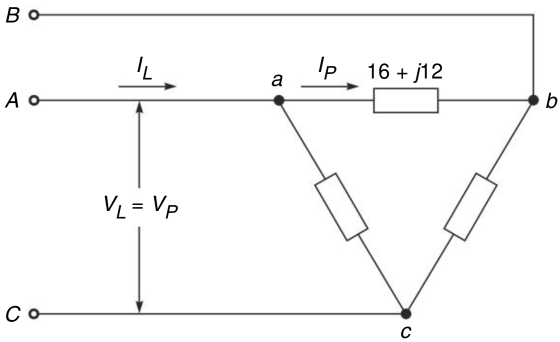


Fig. 6.19

$$V_P = V_L = 400 \text{ V}$$

$$I_P = \frac{400 \angle 0°}{16 + j12} = 20 \angle -36.9° \text{A}$$

$$pf = \cos 36.9° = 0.8 \text{ lagging}$$

$$I_L = \sqrt{3} \times 20 = 34.64 \text{ A}$$

$$P = \sqrt{3} \times 400 \times 34.64 \times 0.8 = 19.2 \text{ kW}$$

$$Q = \sqrt{3} \times 400 \times 34.64 \times \sin 36.9° = 14.4 \text{ kVAR}$$

$$S = [P^2 + Q^2]^{1/2} = 24 \text{ kVA}$$

The phasor diagram is drawn in Fig. 6.20 where 36.9° $\approx$ 37°

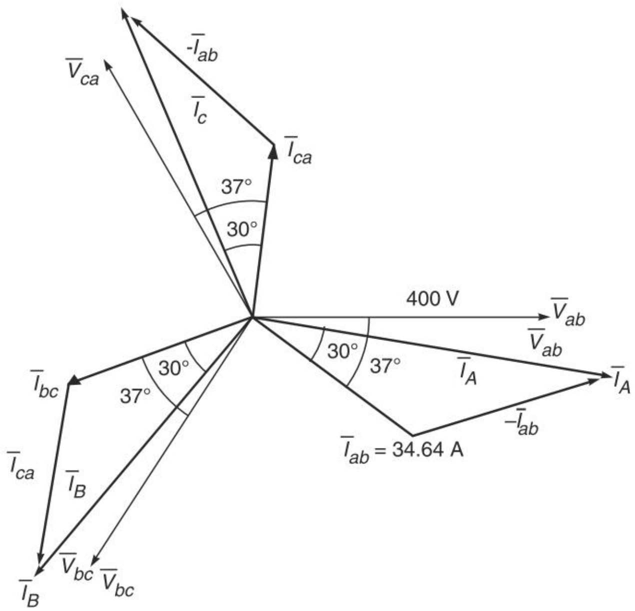


Fig. 6.20

Example 6.6 A three-phase, 400 V source (terminal voltages assumed constant independent of load) supplies a load with an equivalent star impedance of $(60 + j15)$ Ω/phase through a transmission line of impedance $(0.3 + j1.0)$Ω/line (phase). Compute (a) the line current, (b) the load voltage, (c) the power, reactive power and VA consumed by the load, (d) the power and reactive power loss in the line and (e) the power, reactive power and VA supplied by the source.

Solution Figure 6.21 is the per phase circuit of the system (equivalent star basis).

$$V_{\text{source}} = 400/\sqrt{3} = 231 \text{ V (reference phasor)}$$

$$\overline{Z}(\text{total}) = (0.3 + j1.0) + (60 + j15)$$

$$= 60.3 + j16 = 62.4 \angle 14.9°$$

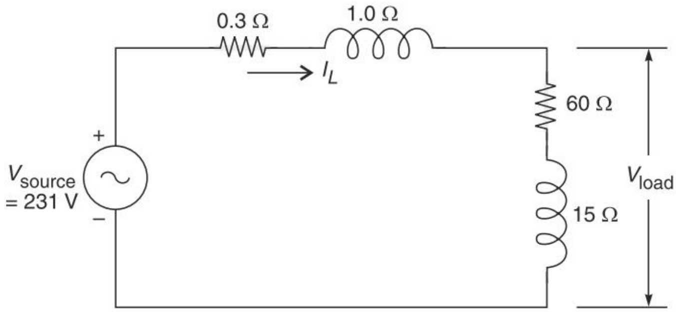


Fig. 6.21

$$I_L = \frac{231}{62.4} = 3.7 \text{ A}$$

$$\bar{Z}_{load} = 60 + j15 = 61.8 \angle 14°$$

$$V_{load} = 3.7 \times 61.8 = 228.7 \text{ V (phase-to-neutral)}$$

$$= 228.7\sqrt{3} = 396 \text{ V (line-to-line)}$$

$$P_{load} = 3 \times (3.7)^2 \times 60 = 2.46 \text{ kW}$$

$$Q_{load} = 3 \times (3.7)^2 \times 15 = 0.616 \text{ kVAR}$$

$$S_{load} = [(2.46)^2 + (0.616)^2]^{1/2} = 2.54 \text{ kVA}$$

$$P_{lineloss} = 3 \times (3.7)^2 \times 0.3 = 12.3 \text{ W}$$

$$Q_{lineloss} = 3 \times (3.7)^2 \times 1 = 41 \text{ VA}$$

$$P_{source} = 2.46 + 0.0123 = 2.47 \text{ kW}$$

$$Q_{source} = 0.016 + 0.041 = 0.057 \text{ kVAR}$$

$$S_{source} = [(2.47)^2 + (0.057)^2]^{1/2} = 2.47 \text{ kVA}$$

Example 6.7 Figure 6.22 shows what is known as one-line diagram of a power system. The impedances indicated on the lines are per phase values. The load requires 400 kW at 0.8 lagging pf. Generator G_1 has a terminal voltage of 11 kV and supplies 200 kW at 0.75 pf lagging. Find the load voltage and terminal voltage and reactive VA supplied by G_2.

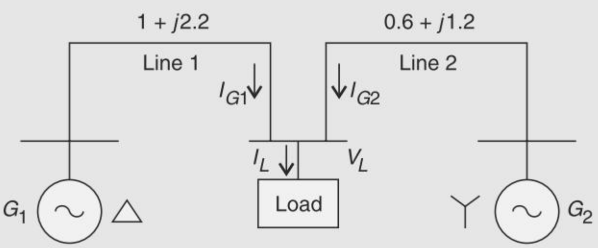


Fig. 6.22

Solution

$$I_{G1} = \frac{200}{\sqrt{3} \times 11 \times 0.75} = 14 \text{ A}$$

$$P_{G1} = 200 \text{ kW}$$
$$Q_{G1} = 200 \tan \cos^{-1} 0.75 = 176.4 \text{ kVAR}$$
$$P_{\text{line loss}} = 3 \times (14)^2 \times 1 = 0.588 \text{ kW}$$
$$Q_{\text{line loss}} = 3 \times (14)^2 \times 2.2 = 1.294 \text{ kVAR}$$
$$P_L(G_1) = 200 - 0.588 = 199.4 \text{ kW}$$
$$Q_L(G_1) = 176.4 - 1.294 = 175.1 \text{ kVAR}$$
$$S_L(G_1) = [(199.4)^2 + (175.1)^2]^{1/2} = 265.4 \text{ kVA}$$
$$V_L = \frac{265.4}{\sqrt{3} \times 14} = 10.94 \text{ kV (line-to-line)}$$
$$P_L = 400 \text{ kW}$$
$$Q_L = 400 \tan \cos^{-1} 0.8 = 300 \text{ kVAR}$$
$$P_L(G_2) = 400 - 199.4 = 200.6 \text{ kW}$$
$$Q_L(G_2) = 300 - 175.1 = 124.9 \text{ kVAR}$$
$$S_L(G_2) = [(200.6)^2 + (124.9)^2]^{1/2} = 236.3 \text{ kVA}$$
$$I_{G2} = \frac{236.3}{\sqrt{3} \times 10.94} = 12.5 \text{ A}$$
$$P_{\text{line loss2}} = 3 \times (12.5)^2 \times 0.6 = 0.281 \text{ kW}$$
$$Q_{\text{line loss2}} = 3 \times (12.5)^2 \times 1.2 = 0.562 \text{ kVAR}$$
$$P_{G2} = 200.6 + 0.281 = 200.9 \text{ kW}$$
$$Q_{G2} = 124.9 + 0.562 = 125.5 \text{ kVAR}$$
$$S_{G2} = [(200.9)^2 + (125.5)^2]^{1/2} = 236.3 \text{ kVA}$$
$$V_{G2} = \frac{236.9}{\sqrt{3} \times 12.5} = 10.92 \text{ kV (line-to-line)}$$

6.8 THREE PHASE POWER MEASUREMENT

Two-Wattmeter Method

A wattmeter has two coils—a current coil and a voltage coil. A single phase wattmeter is connected in the circuit as in Fig. 6.23 to measure the power $P = V I \cos \phi$ wherein the load current passes through the current coil and the voltage is applied across the voltage coil.

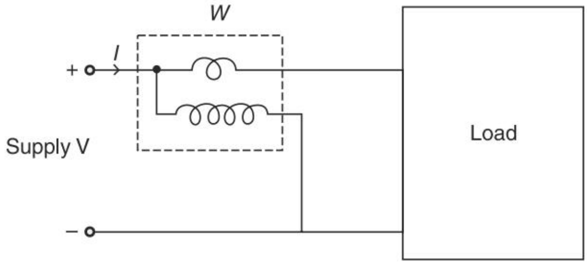


Fig. 6.23

Instead of using three wattmeters to measure 3-phase power, it is possible and economical to use two wattmeters to measure 3-phase power. The connection diagram to measure power to a 3-phase balanced load is drawn in Fig. 6.24. The current coils of the two wattmeters are connected in series with any two lines and the negative ends of the two voltage coils are connected to the third line.

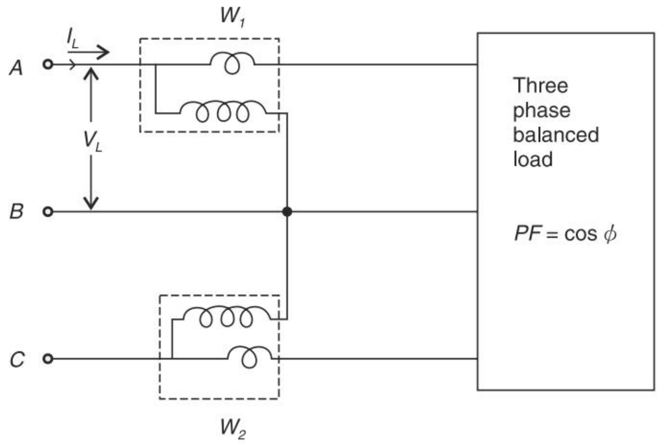


Fig. 6.24

The load may be star or delta connected. We shall assume star but delta load can be converted to equivalent star. The phasor diagram showing relevant voltages and currents is drawn in Fig. 6.25 with lagging phase angle ϕ.

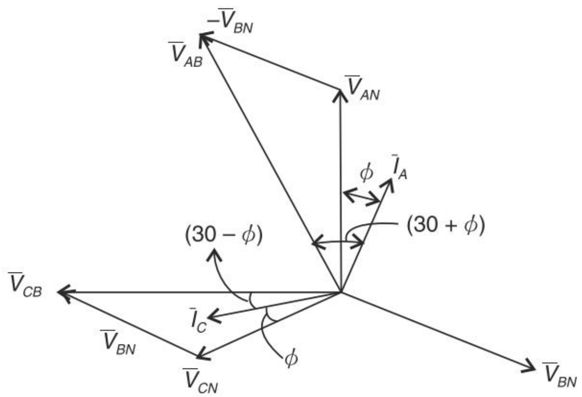


Fig. 6.25

From the phasor diagram power reading of the wattmeter W_1 is

$$W_1 = V_L I_L \cos \angle \text{ between } \overline{V}_{AB} \text{ and } \overline{I}_A$$

$$= V_L I_L \cos (30^\circ + \phi) \quad (6.31)$$

The reading of wattmeter W_2 is

$$W_2 = V_L I_L \cos \angle \text{between } \overline{V}_{CB} \text{ and } \overline{I}_C$$

$$= V_L I_L \cos (30^\circ - \phi) \quad (6.32)$$

The sum of the wattmeter readings

$$W_1 + W_2 = V_L I_L [\cos(30°+\phi) + \cos(30°-\phi)]$$
$$= 2V_L I_L \cos 30° \cos\phi$$
$$= \sqrt{3}\, V_L I_L \cos\phi = P \text{ (three-phase power)} \tag{6.33}$$

Also

$$W_1 - W_2 = V_L I_L \sin\phi \tag{6.34}$$

The ratio

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{1}{\sqrt{3}} \tan\phi$$

The power factor is then given by

$$pf = \cos\phi = \cos\tan^{-1}\left(\frac{W_1 - W_2}{W_1 + W_2}\right) \tag{6.35}$$

Conclusion The three phase power is the algebraic sum of the readings of the two wattmeter. If ($W_1 - W_2$) is positive, the phase angle and so the *pf* are lagging; follows from Eq. (6.35). On the other hand the phase angle and so the *pf* is leading if ($W_1 - W_2$) is negative.

Without proof it can be stated that the two-wattmeter method reads the total power ($W_1 + W_2$) even if the load were unbalanced.

Example 6.8 In Fig. 6.25, find the phase angle between $\bar{V}_{AC}$ and $\bar{I}_A$ if the load *pf* is $\cos\phi$ lagging.

With $\bar{V}_{AN}$ as reference

$$\angle \bar{I}_A = -\phi$$
$$\angle \bar{V}_{AC} = -30°$$
$$\angle \text{ between } \bar{V}_{AC} \text{ and } \bar{I}_A = -30° - (-\phi) = -30° + \phi$$

Example 6.9 A balanced 3-phase 400V supply is connected to balanced 3-phase delta-connected load as shown in Fig. 6.26. It is found by measurement that $\bar{I}_{ab}$ = $20\angle -30°$ A.

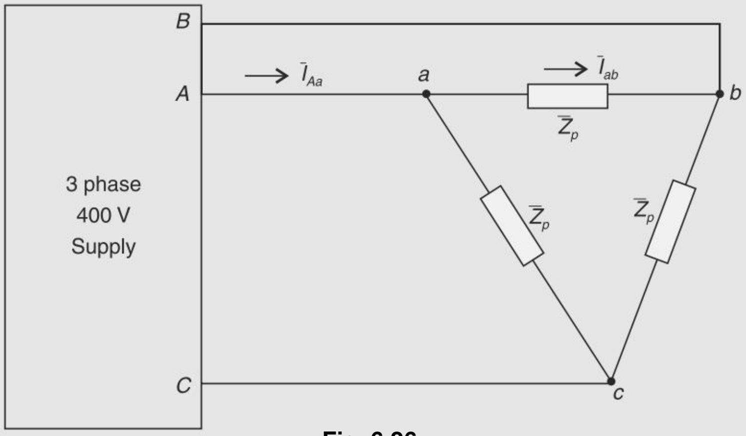


Fig. 6.26

(a) Determine line current $\bar{I}_{Aa}$.
(b) Compute the total value of power received by the load.
(c) Calculate the resistive component of load impedance.
(d) Draw the phasor diagram showing line voltages, phase currents and line currents.

Solution

(a) $\bar{I}_{ab} = 20 \angle -30°$ A Reference $V_{ab} \angle 0°$

For balanced load

$$\bar{I}_{bc} = 20 \angle -30° - 120°$$
$$= 20 \angle -150° \text{ A}$$
$$\bar{I}_{ca} = 20 \angle -30° - 240°$$
$$= 20 \angle -270°$$
$$= 20 \angle -270° + 360°$$
$$= 20 \angle 90° \text{ A}$$

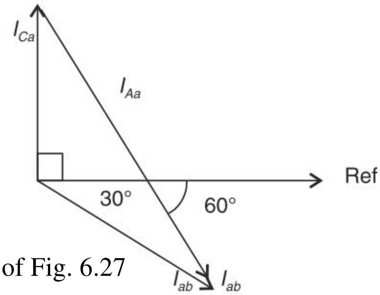


Fig. 6.27

The line current from phasor diagram of Fig. 6.27

$$\bar{I}_{Aa} = \bar{I}_{ab} - \bar{I}_{ca}$$
$$= 20\sqrt{3} \angle -60° \text{ A}$$

(b) Power drawn by phase *ab* of load

$$P_{ab} = 400 \times 20 \cos 30° = 6.928 \text{ kW}$$
$$P\ (total) = 3 \times 6.928 = 20.78 \text{ kW}$$

(c) $\bar{Z}_p = \dfrac{400 \angle 0°}{20 \angle -30°} = 10 \angle 30° \ \Omega$

$= 8.66 + j5$

Then

$R_p = 8.66\ \Omega$

(d)

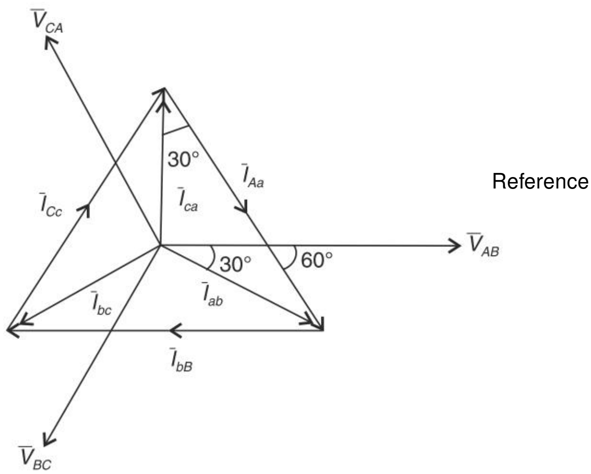


Fig. 6.28 Phasor diagram

Example 6.10 Reconsider the delta connected load of Fig. 6.26 wherein $\overline{Z}_p = 17.32 + j10\,\Omega$. Determine
(a) the phasor current in each line
(b) total power consumed by the load
(c) phasor sum of the three line currents. Why does it have to be this value?

Solution

(a) $\overline{V}_{AB}$ reference, $\overline{Z}_p = 17.32 + j10 = 20 \angle 30°$

$$\overline{I}_{ab} = \frac{\overline{V}_{AB}}{\overline{Z}_p} = \frac{400 \angle 0°}{20 \angle 30°}$$

$$= 20 \angle -30° \text{ A}$$

For balanced load

$$\overline{I}_{ca} = 20 \angle -30° - 120° = 20 \angle -150° \text{ A}$$

The line current

$$\overline{I}_{Aa} = \overline{I}_{ab} - \overline{I}_{ca} = 20\sqrt{3} \angle 0°$$

See the phasor diagram of Fig. 6.29

If then follows that

$$\overline{I}_{Bb} = 20\sqrt{3} \angle -120°$$

and $$\overline{I}_{Cc} = 20\sqrt{3} \angle -240°$$

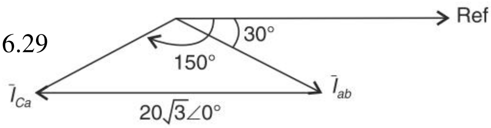


Fig. 6.29

(b) $P_{ab} = R_p \times 17.32 = (20)^2 \times 17.32$

$$= 6.928 \text{ kW}$$

$$P(total) = 3 \times 6.928 = 20.78 \text{ kW}$$

(c) $\overline{I}_{Aa} + \overline{I}_{Bb} + \overline{I}_{Cc} = 20\sqrt{3}\,(1\angle 0 + 1 \angle -120° + 1 \angle -240°)$

$= 0$; sum of balanced 3-phased current

Remark Even if the system in not balanced, sum of three- line line current flow to detla-connected load will be zero as in delta-connection there is no return path.

Example 6.11 A balanced Star-connected load has an impedance of $5 \angle 60°\,\Omega$. The voltage of 'a' to neutral is $\overline{V}_{an} = 25 \angle 30°\ V$. Calculate the phasor currents in phases 'b' and 'c' Write the expression for the phasor voltage 'a' to 'c' i.e., $\overline{V}_{ac}$.

Solution

$$\overline{I}_{am} = \frac{\overline{V}_{an}}{\overline{Z}_P} = \frac{25 \angle 30°}{5 \angle 60°} = 5 \angle -30° \text{ A}$$

Being balanced

$$\overline{I}_{bn} = 5 \angle -30° - 120° = 5 \angle -150° \text{A}$$

and $$\overline{I}_{cn} = 5 \angle -30° + 120° = 5 \angle 90° \text{ A}$$

Given $$\overline{V}_{an} = 25 \angle 30°\ V$$

Then

$$\overline{V}_{cn} = 25 \angle 30° + 120° = 25 \angle 150° \text{ V}$$

$$\overline{V}_{ac} = \overline{V}_{an} - \overline{V}_{cn} = 25\sqrt{3} \angle 0° \text{ V}$$

or $$\overline{V}_{ac} = 43.3\ 0° \text{ V}$$

The phasor diagram is drawn in Fig. 6.30

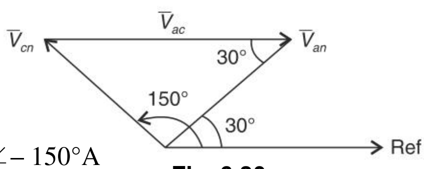


Fig. 6.30

Example 6.12 A 3-phase, 50 Hz, 400 V system feeds a load of 25kW at *pf* = 0.7 lagging as shown in Fig. 6.31 Three capacitors are connected between lines across the load to improve the *pf* to 0.85. Determine the resultant current drawn from the supply and value of the capacitors.

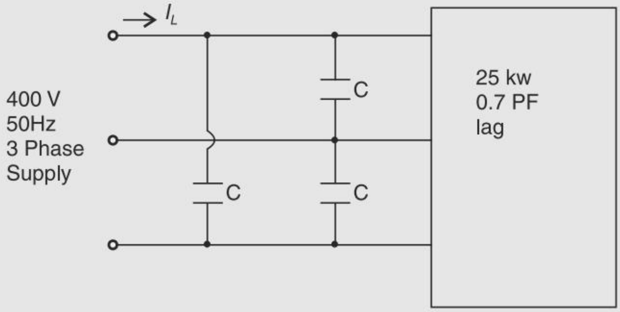


Fig. 6.31

Solution

As the capacitors are delta-connected, we shall take load also as delta-connected. The line current fed to load is

$$25 \times 10^3 = \sqrt{3} \times 400 \times I_L \times 0.7$$

or $\quad I_L = 51.55 \text{ A}, \quad \phi = \cos^{-1} 0.7 = 45.6°$

The phase current is

$$\bar{I}_p = \frac{51.55}{\sqrt{3}} \angle -45.6° = 29.76 \angle -45.6° \text{ A}$$

The system's per phase diagram is drawn in Fig. 6.32. As the capacitor does not draw any real power, the resultant phase current is given by

$$I_p = \left(\frac{25000}{3}\right) \times \frac{1}{400} \times \frac{1}{0.85} = 24.5 \text{ A}, \quad \phi' = \cos^{-1} 0.85 = 31.8° \text{ lag}$$

Resultant line current

$$I_L' = I_p' = 24.5\sqrt{3} = 42.4 \text{ A}$$

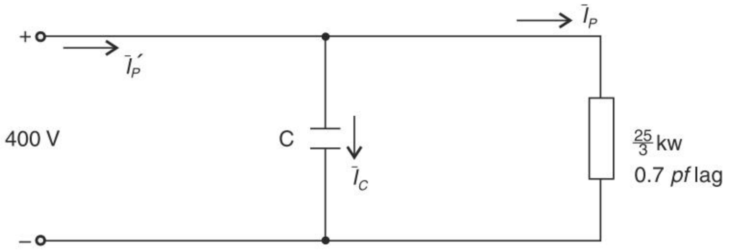


Fig. 6.32

As per KCL at the node in Fig. 6.32

$$\bar{I}_p' = \bar{I}_p + \bar{I}_C$$

$$24.5 \angle -31.8° = 29.76 \angle -45.6° + \bar{I}_C$$

Converting to rectangular form

$$20.8 - j\,12.9 = 20.8 - j\,21.26 + j\,I_C; \quad I_C \text{ leads by } 90°$$

Or $I_C = 10.74$ A

But $I_C = V\omega C$

So we get

$$C = \frac{10.74}{400 \times 2r \times 50} = 85.5\ \mu F$$

Example 6.13 A 400 V, 3-phase, 4 wire system supplies resistive loads between each of the three lines and neutral. Calculate the lines and neutral currents, phase sequence is RYB.

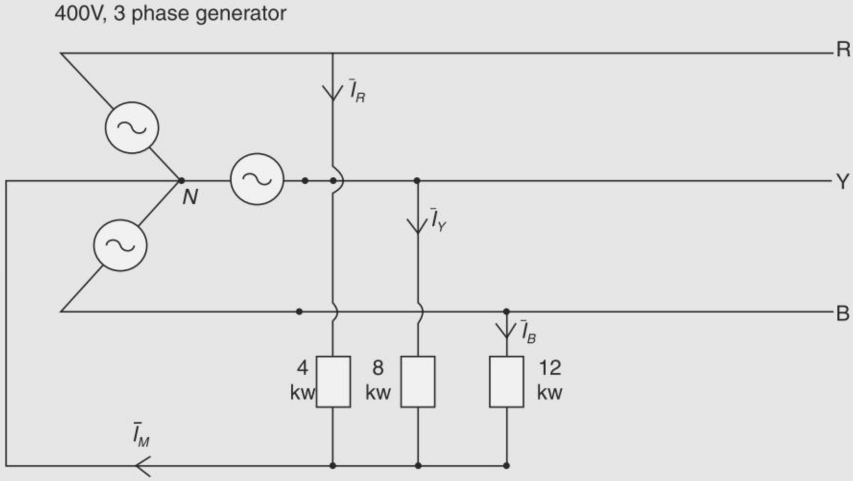


Fig. 6.33

Solution

Phase to neutral voltages

$$\overline{V}_{RN} = \frac{400}{\sqrt{3}} \angle\ 0° = 231\ \angle\ 0° V$$

$$\overline{V}_{YN} = 231 \angle -120° \text{V}$$

$$\overline{V}_{BN} = 231\ \angle -240° \text{V}$$

Line currents

$$\overline{I}_R = \frac{4 \times 10^3}{231} \angle 0° = 17.3\ \angle\ 0°\ \text{A} = 17.3 + j\,0\ \text{A}$$

$$\overline{I}_Y = \frac{8 \times 10^3}{231} \angle -120° = 34.6\ \angle -120°\ \text{A} = -17.3 - j30\ \text{A}$$

$$\overline{I}_B = \frac{12 \times 10^3}{231} \angle -240° = 51.9\ \angle -240°\ \text{A} = -26 + j45\ \text{A}$$

$$\overline{-26 + j\,15\ \text{A}}$$

$$\overline{I}_N = \overline{I}_R + \overline{I}_Y + \overline{I}_B = -26 + j15 = 30\ \angle 150°\ \text{A}$$

Thus

$$I_R = 17.3\ \text{A},\ I_Y = 34.6\ \text{A},\ \overline{I}_B = 51.9\ \text{A},\ \overline{I}_N = 30\ \text{A}$$

Example 6.14 A 3-wire, 3-phase balanced supply sysetm of Fig. 6.34 has phase sequence *abc* and $\overline{V}_{bc} = 400 \angle 60°$ V, R_l = line resistance = 1Ω. It supplies a load of 10 KVA at 0.8 *pf* lagging Calculate (a) total power lost in line resistance (b) $\overline{V}_{bc}$ and $\overline{I}_b$.

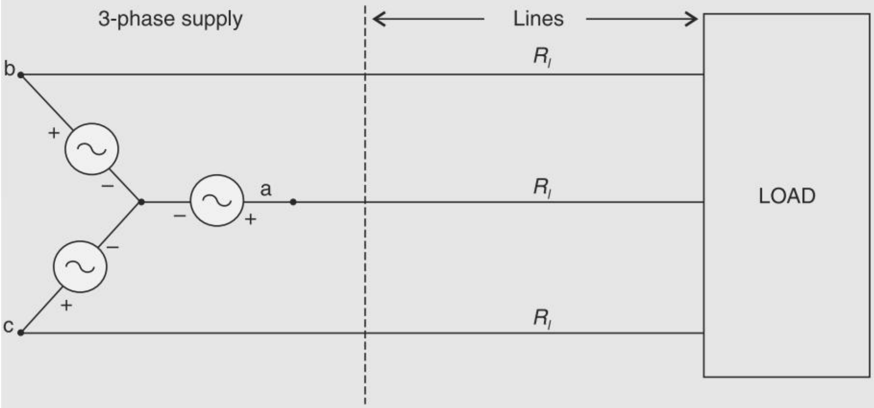


Fig. 6.34

Solution

(a) $V_L = V_{bc} = 400$ V

$$KVA = \sqrt{3}\, V_L I_L \times 10^{-3} = 10$$

$$I_L = \frac{10 \times 10^3}{\sqrt{3} \times 400} = 1.44 \text{ A}$$

Power loss in line resistance = $3 \times (1.44)^2 \times 1$
= 6.22 W

(b) $V_{an} = \dfrac{V_{bc}}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 231$ V

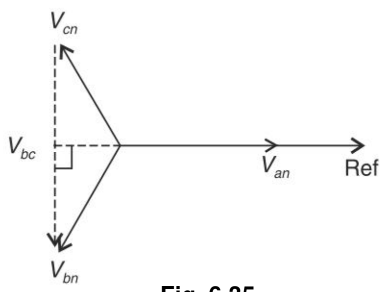


Fig. 6.35

As per phasor diagram (Fig. 6.35)

$$\overline{V}_{bc} = 400 \angle -90° \text{ V}$$

$$pf = \cos\theta = 0.8 \text{ lag}$$

$$\theta = 36.9° \text{ lag}$$

For star connection

$$I_a = I_L = 1.44 \text{ A}$$

$$\overline{V}_{an} = 231 \angle 0°, \quad \overline{I}_a = 1.44 \angle -36.9°$$

$$\overline{I}_b = 1.44 \angle -36.9° - 120° = 1.44 \angle -156.9°$$

Example 6.15 A 3-phase, 3-wire balanced system with Delta-connected load is sketched in Fig. 6.36. Given $\overline{V}_{an} = 231 \angle 60°$, complex power absorbed by each phase of load = 2.5– *j*1.2 KVA. Phase sequence *abc*. Determine $\overline{V}_{bc}$, $\overline{Z}_p$, $\overline{I}_{an}$.

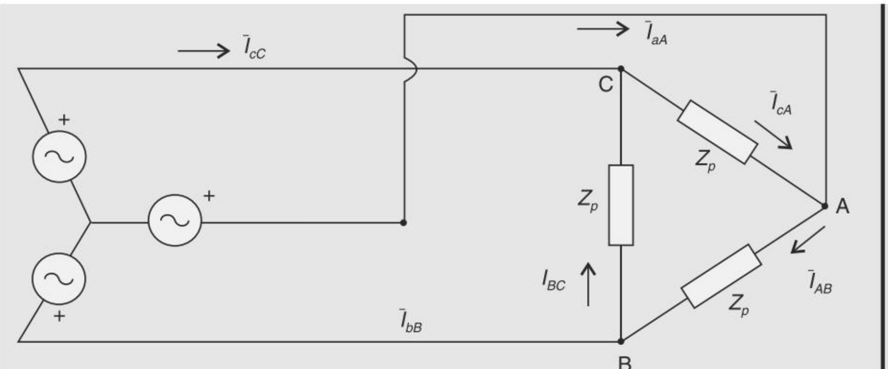


Fig. 6.36

Solution

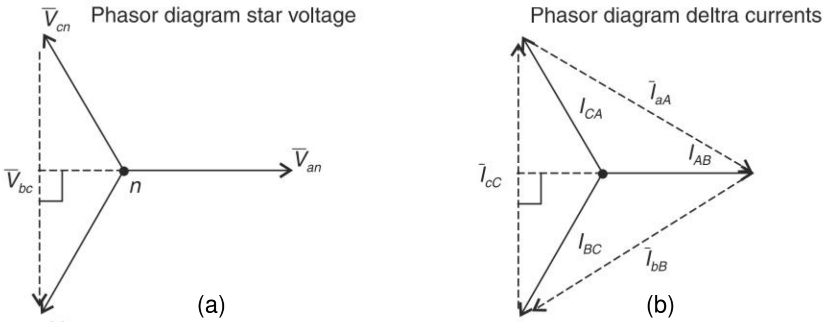


Fig. 6.37

$$\bar{V}_{bc} = \sqrt{3}\,\bar{V}_{an} \angle 60°–90°; \text{ from phasor diagram of Fig. 6.37(a)}$$

$$= \sqrt{3} \times 231 \angle –30°$$

$$= 400 \angle –30° \text{ V} = 0.4 \angle –30° \text{ kV}$$

$$\bar{S}_P = \bar{V}_{BC}\, \bar{I}^*_{BC}$$

$$(2.5 – j1.2) = 0.4 \angle – 30°,\ I^*_{BC} = 2.77 – 25.6°$$

$$\bar{I}^*_{BC} = \frac{2.5 – j\,1.2}{0.4 \angle –30°} = \frac{2.77 \angle –25.6°}{0.4 \angle –30°} = 6.925 \angle 4.4° \text{ A}$$

$$\bar{I}_{BC} = 6.925 \angle – 4.4° \text{ A}$$

From the delta phasor diagram (Fig. 6.37(b))

$$\bar{I}_a = \bar{I}_{aA} = \sqrt{3}\ \bar{I}_{BC} \angle 90° = 12 \angle 85.6° \text{ A}$$

Example 6.16 The unbalanced star-connected load of Fig. 6.38 is fed from a 400 V, three-phase supply. Calculate the line currents and phase-to-neutral load voltages.

Solution

$$\bar{V}_{ab} = 400 \angle 0° \text{ (Reference phasor)},\ \bar{V}_{bc} = 400 \angle –120°,$$

$$\bar{V}_{ca} = 400 \angle – 240° \text{ V}$$

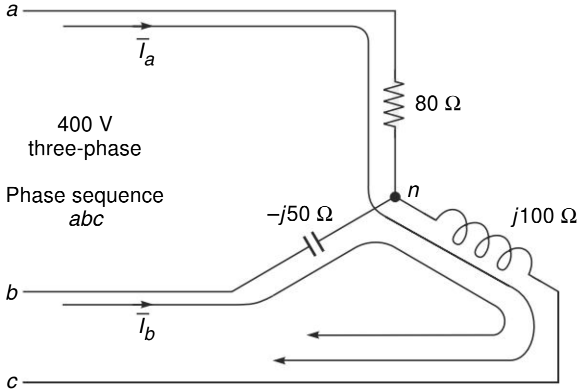


Fig. 6.38

The two mesh currents $\overline{I}_a$ and $\overline{I}_b$ are shown in Fig. 6.38. The corresponding mesh equations are:

$$(80 + j100)\,\overline{I}_a + j100\,\overline{I}_b = \overline{V}_{ac}$$

$$-j50\,\overline{I}_b + j100\,(\overline{I}_a + \overline{I}_b) = \overline{V}_{bc}$$

or
$$(80 + j100)\,\overline{I}_a + j100\,\overline{I}_b = -400\,\angle -240°$$

$$j100\,\overline{I}_a + j50\,\overline{I}_b = 400\,\angle -120°$$

Solving, we get

$$\overline{I}_a = 5.413\,\angle 81.3°\text{A}$$

$$\overline{I}_b = 10.877\,\angle -142°\text{A}$$

$$\overline{I}_c = -(\overline{I}_a + \overline{I}_b) = -(5.413\,\angle 81.3° + 10.877\,\angle -142°)$$

$$= 7.868\,\angle 9.0°$$

$$\overline{V}_{an} = 80 \times \overline{I}_a = 80 \times 5.413\,\angle 81.3°$$

$$= 433.04\,\angle 81.3°$$

$$\overline{V}_{bn} = -j50 \times \overline{I}_b = -j50 \times 10.877\,\angle -142°$$

$$= 543.85\,\angle 128°$$

$$\overline{V}_{cn} = -j100 \times \overline{I}_c = j100 \times 786.8\,\angle 9.90$$

$$= 786.8\,\angle 99.9°$$

Example 6.17 Consider the unbalanced delta-connected load of Fig. 6.39. Find the line currents.

Solution

$$\overline{V}_{ab} = 400\,\angle\,0°,\ \overline{V}_{bc} = 400\,\angle -120°,\ \overline{V}_{ca} = 400\,\angle -240°\text{ V}$$

$$\overline{I}_{ab} = \frac{400\,\angle\,0°}{j100} = -j4\text{ A}$$

$$\overline{I}_{bc} = \frac{400\,\angle -120°}{-j50} = -8\,\angle -30°\text{ A}$$

$$= 6.928 - j4 \text{ A}$$

$$\bar{I}_{ca} = \frac{400 \angle -240°}{80} = 5 \angle -240° \text{ A}$$

$$= -2.5 + j4.33$$

$$\bar{I}_a = \bar{I}_{ab} - \bar{I}_{ca} = (-j4) - (-2.5 + j4.33)$$

$$= 2.5 + j0.33 = 2.522 \angle 7.5° \text{ A}$$

$$\bar{I}_b = \bar{I}_{bc} - \bar{I}_{ab} = (6.928 - j4) - (-j4)$$

$$= 6.928 \angle 0° \text{ A}$$

$$\bar{I}_c - \bar{I}_{ca} - \bar{I}_{bc} = (-2.5 + j4.33) - (6.928 - j4)$$

$$= -9.428 + j8.33 = 12.581 \angle 138.5° \text{A}$$

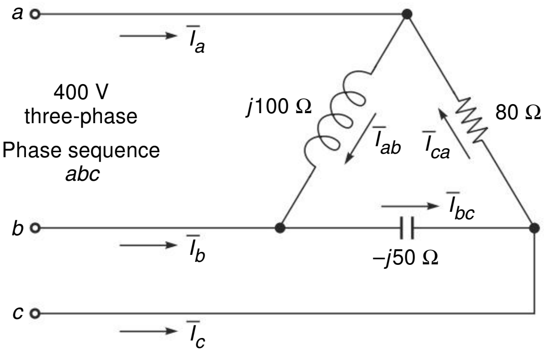


Fig. 6.39

ADDITIONAL SOLVED PROBLEMS

6.18 A 400 V, balanced three-phase supply is connected to a *Y*-connected load of 900 W at a power factor of 0.8 leading. Calculate the line current and the per phase load impedance.

Solution

The per phase power $= 900/3 = 200$ W; $V_{ph} = 400/\sqrt{3}\ V = 231$ V

$$300 = 231 \times I_{ph} \times 0.8$$

$$I_{ph} = I_{\text{line}} = 300/(231 \times 0.8) = 1.623 \text{ A}$$

$$Z_{ph} = V_{ph} / I_{ph} = \frac{231}{1} = 142.3\ \Omega$$

Angle of impedance $= \cos^{-1} 0.8 = -36.9°$

$$\therefore \quad Z_{ph} = 142.3 \angle -36.9°\ \Omega$$

6.19 A 400 V, three-phase source supplies three equal impedance of $(40 + j30)\ \Omega$ in Δ formation. Calculate the phase current, line current and the total power drawn from the source.

Solution

Since the line voltage and phase voltage are equal for a Δ-connected load, the phase current is given by

$$I_p = \frac{400 \angle 0°}{40 + j30} = 8 \angle -36.9°$$

$$I_L = \sqrt{3}\, I_p = \sqrt{3} \times 8 = 13.86 \text{ A}$$

$$P_{\text{total}} = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3} \times 400 \times 13.86 \times 0.8$$

$$= 7680 \text{ W}$$

6.20 Figure 6.40 represents a one-line diagram of a power system (each line representing the three phases). The load requires 30 kW at 0.8 lagging pf. Generator G_1 operates at 800 V (line-to-line) and supplies 15 kW at 0.8 lagging *pf*. Find the load and terminal voltage and active and reactive powers supplied by G_2.

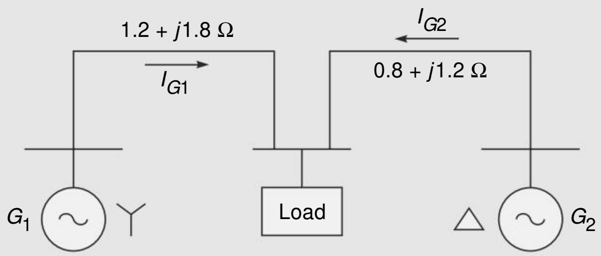


Fig. 6.40

Solution

$$I_{G1} = (15 \times 1000) / (\sqrt{3} \times 800 \times 0.8) = 13.53 \text{ A}$$

$$P_{G1} = 15 \text{ kW},\ Q_{G1} = 15 \tan\cos^{-1} 0.8 = 11.25 \text{ kVAR}$$

$$P_{\text{line1}} = 3 \times 13.53^2 \times 1.2 = 0.695 \text{ kW}$$

$$Q_{\text{line1}} = 3 \times 13.53^2 \times 1.8 = 0.989 \text{ kVAR}$$

$$P_L(G_1) = 15 - 0.695 = 14.34 \text{ kW},\ Q_L(G_1) = 11.25 - 0.989$$

$$S_L(G_1) = \sqrt{\ }\,[(14.34)^2 + (10.26)^2] = 17.63 \text{ kVA}$$

$$V_L = 17.630/(\sqrt{3} \times 13.53) = 752 \text{ V}$$

$$P_L = 30 \text{ kW},\ Q_L = 30 \tan\cos^{-1} 0.8 = 22.5 \text{ kVAR}$$

$$P_L(G_2) = 30 - 14.34 = 15.66 \text{ kW},\ Q_L(G_2) = 22.5 - 10.26$$

$$= 12.24 \text{ kVAR}$$

$$S_L(G_2) = \sqrt{\ }\,[(15.66)^2 + (12.24)^2] = 19.876 \text{ kVA}$$

$$I_{G2} = 19.876/(\sqrt{3} \times 752) = 15.26 \text{ A}$$

$$P_{\text{line2}} = 3 \times (15.26)^2 \times 0.8 = 0.559 \text{ kW}$$

$$Q_{\text{line2}} = 3 \times (15.26)^2 \times 1.2 = 0.838 \text{ kVAR}$$

$$P = 15.66 + 0.559 = 16.22 \text{ kW}$$

$$Q = 12.24 + 0.838 = 13.08 \text{ kVAR}$$

$$S = [(16.22)^2 + (13.08)^2]^{0.5} = 20.84 \text{ kVA}$$

$$V = 20840/(3 \times 15.26) = 788 \text{ V}$$

6.21 This problem will illustrate the three-phase unbalanced circuit analysis.

A delta connected unbalanced load is arranged as shown in Fig. 6.41 and is connected to a balanced supply voltage of 415 V. The phase sequence is RYB. Calculate the phase and line currents.

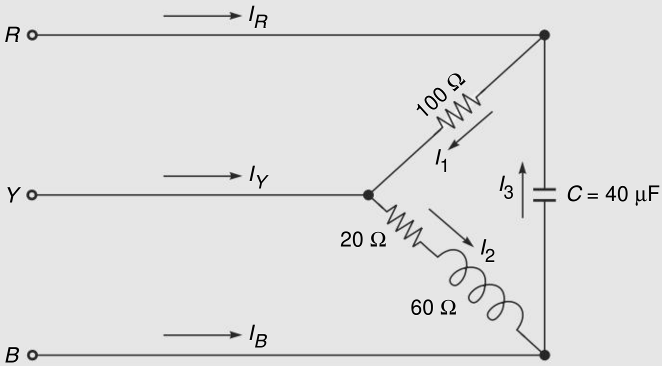


Fig. 6.41

Solution

The three line voltages are:

$$\overline{V}_{RY} = 415 \angle 0°, \; \overline{V}_{YB} = 415 \angle -120° \; \overline{V}_{BR} = 415 \angle -240°$$

$$\overline{Z}_{RY} = 100\ \Omega = 100 \angle 0°\ \Omega \;\; \overline{Z}_{YB} = 20 + j60 = 63.25 \angle 71.6°$$

$$\overline{Z}_{BR} = 10^6/(j314 \times 40) = 1/(j12.56 \times 10^{-3})\ \Omega$$

The phase currents are computed as follows:

$$\overline{I}_1 = 415 \angle 0°/100 = 4.15 \angle 0°\text{ A} = 4.15 + j0$$

$$\overline{I}_2 = 415 \angle -120°/63.25 \angle 71.6°$$

$$= 6.56 \angle -191.6°\text{A} = -6.42 + j1.32$$

$$\overline{I}_3 = 415 \angle -240° \times j12.56 \times 10^{-3}$$

$$= 5.21 \angle -150°\text{ A} = -4.51 - j2.61$$

The line currents are computed as

$$\overline{I}_R = \overline{I}_1 - \overline{I}_3 = (4.15 + j0) - (-4.51 - j2.61)$$

$$= 8.66 + j\,2.61 = 9.04 \angle 16.8°\text{ A}$$

$$\overline{I}_Y = \overline{I}_2 - \overline{I}_1 = (-6.42 + j1.32) - (4.15 + j0)$$

$$= -10.57 + j1.32 = 10.65 \angle 172.9°\text{ A}$$

$$\overline{I}_B = \overline{I}_3 - \overline{I}_2 = (-4.51 - j2.61) - (6.42 + j1.32)$$

$$= 1.91 - j3.93 = 4.37 \angle -64.1°\text{ A}$$

SUMMARY

- All practical power systems are three-phase; generation, transmissions and distribution except domestic wiring which is single-phase inside the building.
- Three-phase connections are star and delta.

- Three-phase operation is arranged to be nearly balanced.
- Balanced three-phase voltages (and currents) have a progressive phase difference of 120°.
- Phase sequence; positive *abc*, phase 'a' leads 'b' by 120° and 'b' leads 'c' by 120°, negative sequence *cba*, phase 'c' leads 'b' by 120° and 'b' leads 'a' by 120°.
- V_L = line to line voltage (or line voltage), V_P = phase voltage, I_L = line current, I_P = phase current
- Star connection $V_L = \sqrt{3}\, V_P$, $I_L = I_P$

  Delta connection $V_L = V_P$; $I_L = \sqrt{3}\, I_P$

1. Three phase power (active)

   $P = \sqrt{3}\, V_L I_L \cos\theta$, W θ = phase angle between phase voltage and phase current, it is same for each phase

   Power factor = $\cos\theta$ lagging or leading; same for each phase. Commonly called 3-phase (balanced) power factor.

   Three-phase reactive power

   $Q = \sqrt{3}\, V_L I_L \sin\theta$, VAR; positive for lagging *pf* and negative for leading *pf*

   Three-phase volt-amperes (VA)

   $$S = \sqrt{P^2 + Q^2}$$

2. Balanced 3-phase Star/Delta load impedance conversion

   $$Z_Y = \frac{1}{3} Z_\Delta \text{ per phase}$$

3. In a balanced 3-phase system, calculations are carried out on per *phase basis*. The system is represented by *one-line diagram*.

REVIEW QUESTIONS

1. A balanced 3-phase star-connected voltage source supplies power to a balanced 3-phase star-connected load. Show by a phasor diagram that if the load neutral is connected to the source neutral, no current will flow in the connecting wire.
2. Write the mathematical expressions for three balanced voltage sources. When one of the voltages has the peak value at an instant, what are the corresponding values of the other two voltages?
3. Show by a phasor diagram that when three-phase balanced voltage sources are connected in delta formation, no current will flow round the loop so formed. Explain why delta-connected source is not used in practice.
4. Draw the phasor diagram of a star connected balanced voltage source, if the relative phase angle between $\bar{V}_{BC}$, and $\bar{V}_{An}$ is 90°.
5. Solve question 4 with the relative phase angle between $\bar{V}_{BC}$ and $\bar{V}_{Bn}$ as 30°.
6. In a 3-phase balanced delta-connected load supplied from a balanced 3-phase voltage source, what is the angle between line and phase currents as two three-phase sets?

7. A three-phase load has a power factor of say 0.8 lagging. What is the meaning of this?
   Is it same for all the three phases or their average value?
8. What is the meaning of 'phase sequence' in a 3-phase voltage source?
9. A balanced 3-phase delta- connected load has phase impedance of $(R + jX)$ and draws line current I. Write the expression for 3-phase active and reactive powers.

PROBLEMS

6.1 A balanced star load of $8 + j6\ \Omega$ per phase is connected to three-phase, 230 V supply. Find the line current, power factor, power, reactive VA and total VA.

6.2 A balanced three-phase, delta-connected load of 160 kW takes a leading current of 100 A with a line voltage of 1100 V, 50 Hz. Find the circuit constants of the load per phase.

6.3 A balanced delta-connected load of $16 + j12\ \Omega$/phase is connected to a three-phase, 230 V supply. Find the line current, power factor, power, reactive VA and total VA.

6.4 A symmetrical three-phase, 400 V system supplies a balanced delta-connected load. The current in each delta branch is 30 A and its phase angle is 37° lagging (w.r.t. voltage across each branch). Find the line current and total power. Draw the phasor diagram showing all currents and voltages.

6.5 Three star-connected impedances $\bar{Z}_1 = 16 + j20\ \Omega$ per phase are in parallel with three delta-connected impedances of $\bar{Z}_2 = 27 + j18\ \Omega$ per phase across a three-phase, 400 V supply. Find the line current, power factor and reactive VA of the combination.

6.6 A 220 V, three-phase voltage is applied to a balanced delta-connected load.

$$\bar{I}_{ab} = 10\ \angle -37°\ \text{A w.r.t}\ \bar{V}_{ab}$$

(a) Find the line current and its phase angle wrt line to neutral voltage.
(b) Compute total power received by the load.
(c) Find the value of the resistive part of the phase impedance.

6.7 Input power to a three-phase motor is measured by the two-wattmeter method. The two wattmeter readings are 4.8 kW and –1.6 kW with a line voltage of 400V. Calculate (a) the total power (active) (b) the power factor and (c) the line current.

Ans: (a) 2.4 kW (b) 0.21 lag (c) 16.5 A

6.8 A star-connected source feeds a delta-connected load through lines of resistance 5Ω each. The sending end complex power is $3.45 + j1.8$ KVA and receiving end complex power is $3 + j1.8$ KVA. Find (a) line current (b) load phase current (c) source phase voltage.

Ans: (a) 5.447A (b) 3.162 A (c) 236.8V

6.9 For the 3-phase system of Fig. 6.42, determine the three line currents $\bar{I}_{aA}$, $\bar{I}_{bB}$ and $\bar{I}_{cC}$ in phasor form. What is the total complex power supplied by the source?

Hint: Complex power $= \dfrac{V_L^2}{R} + j\dfrac{V_L^2}{X_L} - j\dfrac{V_L^2}{X_C}$

Ans: (a) 80.30 ∠ 45° A , 120.9 ∠ – 170.1° A, 72 ∠ – 30°A

6.10 Across a 400V, 3-phase supply, sequence RYB, the following loads are connected in delta

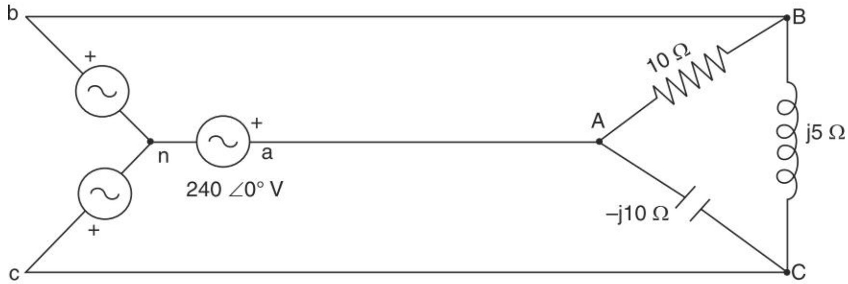


Fig. 6.42

RY.......... 100 Ω resistance
YB.......... 60 Ω inductive reactance
BR..........120 Ω capacitive reactance

Calculate phase and line currents, their magnitude and phase angle. Take reference as $V_{RY} \angle 0°$. Also calculate the total active and reactive power drawn by the delta load.

Ans: $\bar{I}_{RY} = 4\angle 0°$ A, $\bar{I}_{YB} = 5\angle 210°$ A, $\bar{I}_{BR} = 3.2\ \angle -150°$ A, $\bar{I}_{R} = 6.96\ \angle 13.3°$ A, $\bar{I}_{Y} = 8.7\angle 163.3°$ A, $\bar{I}_{B} = 4.38\ \angle -69.3°$A, 1.6kW, + 0.27 KVAR

Chapter 7

MAGNETIC CIRCUITS

MAIN GOALS AND OBJECTIVES

- *Fundamentals laws of electromagnetism and induction also certain rules*
- *Definitions of magnetic quantities*
- *Magnetic circuits, dc analog*
- *Energy stored in magnetic field*
- *Inductance—self and mutual*
- *AC operation of magnetic circuit*
- *Electromechanical energy conversion (EMEC)*

7.1 INTRODUCTION

The electromagnetic system is an essential element of all rotating electric machinery and electromechanical devices as well as static devices like the transformer. The role of the electromagnetic system is to establish and control electromagnetic fields for carrying out conversion of energy, its processing and transfer. Practically all electric motors and generators, ranging in size from fractional-kW units found in domestic appliances to the gigantic several thousand motors employed in heavy industry and several hundred megawatt generators installed in modern generating stations, depend upon the magnetic field as the coupling medium allowing interchange of energy in either direction between electrical and mechanical systems. It is seen that all electric machines including transformers use the medium of magnetic field for energy conversion and transfer.* The study of theses devises essentially involves electric and magnetic circuit analysis and their interaction. Also, several other essential devices like relays, circuit breakers, etc. need the presence of a confined magnetic field for their operation.

The purpose of this chapter is to review the physical laws governing magnetic fields, induction of emf and production of mechanical force, and to develop methods

* Transformers are used in such varied applications as radio and television receivers and electrical power transmission and distribution circuits.

of magnetic circuit analysis.* Simple magnetic circuits and magnetic materials will be briefly discussed. In the chapters to follow, how the concepts of this chapter are applied in the analysis of transformers and machines will be dealt with.

7.2 AMPERES LAW—MAGNETIC QUANTITIES

Consider a long straight conductor carrying current i_1 as shown in Fig. 7.1. As discovered by Ampere, the current creates a field of force in the surrounding medium which is known as the magnetic field. To investigate the field, we place an elemental conductor of length l parallel to the long conductor carrying current i_2 in opposite direction. Of course, the elemental conductor $(i_2\, l)$ is part of a circuit (not shown in figure) which causes the current i_2 to flow. A force of repulsion acts on the elemental conductor at right angle to the long conductor and the element as shown in Fig. 7.1. The magnitude of this force (of magnetic origin) in MKS units is given by

$$F = \left(\frac{\mu i_1}{2\pi r}\right)(i_2 l)\ \text{N} \tag{7.1}$$

where μ is a constant of the medium called *permeability.*

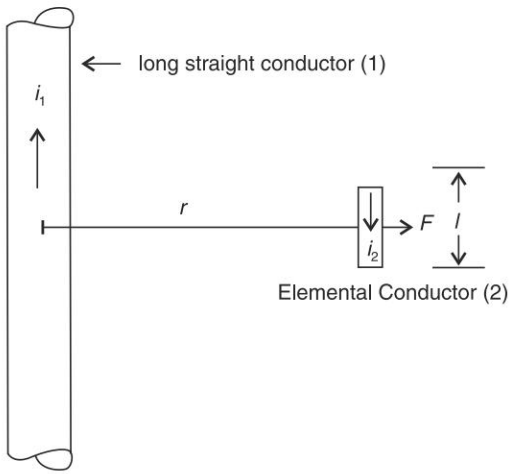


Fig. 7.1 Illustrative of Ampere's Law

As we rotate the elemental conductor (2) about the long conductor (1) in a plane circular path of radius r, the force F on conductor (2) remains constant and outward oriented. This *closed* circuit path is a line of magnetic flux; just called a *flux line.* In the region surrounding the conductors, there are flux lines all along the conductor extending from the conductor surface outwards. The symbol for flux is ϕ and its units are Weber's (Wb).

* The concept of quasi-static magnetic field and the underlying assumptions and their validity, which make the magnetic circuit analysis a simpler exercise and do not require involved magnetic field analysis, will be brought out clearly though briefly.

Flux Density (B)

In any elemental area across flux, it is convenient to work in terms of flux density B as Wb/m^2.

We can write Eq. (7.1) as

$$F = B \cdot (i_2\, l) \tag{7.2 a}$$

where the flux, density B is defined as

$$B = \mu\left(\frac{i_1}{2\pi r}\right), \quad \frac{\text{N}}{\text{Am}}$$

In terms of flux, units of B are Wb/m^2, generally called *tesla* (T).

Magnetic Field Intensity (H) or Magnetizing Force

It is convenient to work in terms of a quantity that is independent of the medium. We define from Eq. (7.2) the magnetic field intensity as

$$H = \frac{i_1}{2\pi r} = \frac{B}{\mu} \text{ A/m} \tag{7.3}$$

It is indeed the consitive force i_1 spread over the length of the flux path

The current i_1 in one conductor many comprise of

$$i_1 = Ni \tag{7.4}$$

where there are N conductors each carrying current i.

As current always flows in a closed path, N conductors indeed are N turns. So we write

$$N\, i = \textit{ampere-turns} \text{ (AT)} \tag{7.5}$$

The ampere-turns in magnetic circuits are referred to as *magnetomotive force.*

$$\mathcal{F} = mmf = Ni \quad \text{AT} \tag{7.6}$$

Direction of Magnetic Flux, B and H

At this stage, we need to examine the direction of the magnetic quantities. The cross-sectional view of a long conductor carrying current is drawn in Fig. 7.2.

The symbol ⊕ on the conductor indicates the direction of current is into the plane of the paper. By virtue of symmetry, the *flux lines* are circular *closed paths.* The magnetic field intensity H is tangential to the flux line at every point and flux density B has the same direction as H. Thus, both H and B are *vectors* and have the same direction.

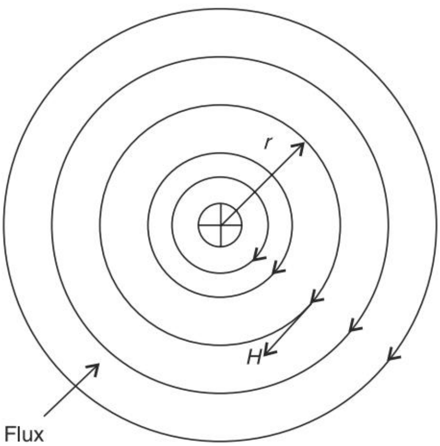


Fig. 7.2 Magnetic field intensity and flux

Right Hand Rule If you grasp the conductor by the right hand such that the thumb points in the direction of current, the flux is established in the direction in which the fingers curl.

It is observed from Fig. 7.2 that the flux lines are more dense near the conductor than far away, which means that the flux density decreases outwards. This results from the fact that

$$H = \frac{i}{2\pi r} \text{ and } B = \mu H$$

Permeability It is the property of the medium which determines the flux density for a given magnetizing force; it is indeed a constant of proportionality. Thus

$$B = \mu H \; ; \; \mu = \text{permeability} \tag{7.7}$$

In *free space*

$$B = \mu_0 H$$

where $\mu_0 = 4\pi \times 10^{-7}$ Wb/Am, permeability of free space (7.8)

We can express

$$\mu = \mu_0 \mu_r$$

where

$$\mu_r = \frac{\mu}{\mu_0}, \text{ the } \textit{relative permeability} \text{ of the medium} \tag{7.9}$$

Magnetic materials, iron, steel and certain alloys, by virtue of their inherent property induce much larger flux density. These magnetic materials have

$$\mu_r = 4000 - 10000$$

Of course, non-magnetic materials have

$$\mu_r = 1$$

Magnetic Flux For uniform flux density, normal to area A of Fig. 7.3 (a), the flux passing through the area is

$$\phi = B A \tag{7.10}$$

If the flux makes an angle θ with respect to the surface normal as in Fig. 7.3 (b), then

$$\phi = B A \cos \theta \tag{7.11}$$

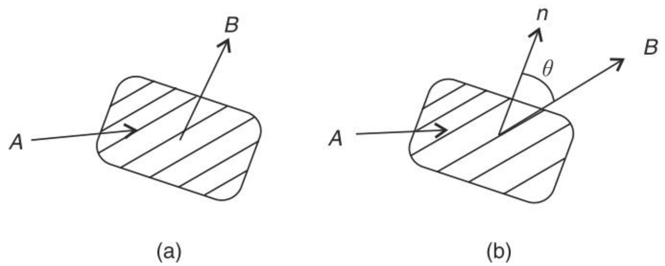


Fig. 7.3

In general, the total flux piercing a surface must be obtained by surface integral of the dot product.

Both H and B are *vectors* (these are directed quantities) and are related by the constant (μ, μ_0), while ϕ is a *scalar* quantity.

As we shall be dealing with magnetic fields of simple geometry, all we need is the magnitudes of H and B.

Ampere's Circuital Law For the current carrying long conductor (1) of Fig. 7.1. both the H and B vectors have circular paths as shown in Fig. 7.2. Let us integrate the H vector along any closed path.

$$\oint H \cdot \mathrm{d}l = \int_0^{2\pi r} \frac{i_1}{2\pi r}\,\mathrm{d}l = i_1 \text{ A} \tag{7.12}$$

In general, i_1 is $F = N\,i$ ampere-turns. Thus

$$\oint H\,\mathrm{d}l = F\ (mmf) \tag{7.13}$$

This is the *Ampere's circuital law*. As H is tangential to a flux line, Eq. (7.13) simplifies as

$$F = H\,l,\ l = \text{length of flux line}$$

This law would be employed in analysis and design of electromagnetic circuits and devices.

Example 7.1 A long straight wire carrying a current of 5 A is placed in air. Assume the relative permeability of air to be unity.

(a) Find the magnetic field intensity at a distance of 0.4 m from the centre of the first wire.

(b) Another long straight wire carrying a current of 2.5 A is placed at a distance of 0.4 m from the first wire. Find the direction and magnitude of the force per meter existing between the two wires.

(c) Repeat parts (a) and (b) if the wires are placed in iron with relative permeability of 8000 and the wire spacing of 0.04 m.

Solution

(a) $H = \dfrac{i}{2\,\pi\,r}$

Substituting values

$$H = \frac{5}{2\pi \times 0.4} = 6.25/\pi \text{ A/m}$$

(b) As per Ampere's law

$$F = \left(\frac{\mu_0 i_1}{2\pi r}\right)(l i_2) \text{ N}$$

For per meter, we take $l = 1$ m

$$F = \left(\frac{4\pi \times 10^{-7} \times 5}{2\pi \times 0.4}\right) \times (1 \times 2.5)$$

or $F = 62.5 \times 10^{-7}$ N

As the current in both the conductors is in the same direction, the force is that of attraction i.e., force on the second wire is directed towards the first wire.

(c) Part (a)

H is unaffected by the medium

$\therefore \quad H \ (6.25/\pi)$ A/m

Part (b)

Medium is iron with $\mu_r = 8000$

Therefore

$$F \text{ (iron)} = \mu_r F \text{ (air)}$$
$$= 8000 \times 62.5 \times 10^{-7} = 0.05 \text{ N}$$

Example 7.2 Two long parallel wires are at a distance of 15 cm as shown in Fig. 7.4. If $I_1 = 50$ A, what should be the value and direction of the current I_2 such that the magnetic field intensity at point P is zero?

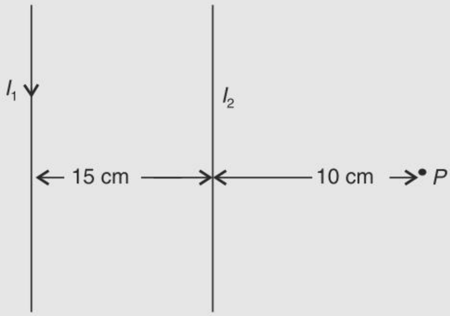


Fig. 7.4

Solution

At point P

$$H_1(I_1) = \frac{50}{2\pi\,(15+10) \times 10^{-2}} \text{ A/m}$$

$$H_2(I_2) = \frac{I_2}{2\pi \times 10 \times 10^{-2}} \text{ A/m}$$

For net H at P to be zero

$$H_1 + H_2 = 0 \quad \text{or} \quad H_2 = -H_1$$

or
$$\frac{I_2}{2\pi \times 10 \times 10^{-2}} = \frac{50}{2\pi \times 10 \times 10^{-2}}$$

or
$$I_2 = -50 \times \frac{10}{25} = -20\,A \text{ (in opposite direction to } I_1)$$

Example 7.3 Two long wires are located in the x-y plane oriented in the z-direction as shown in Fig. 7.5. Calculate the magnitude and direction of magnetic field intensity at point P.

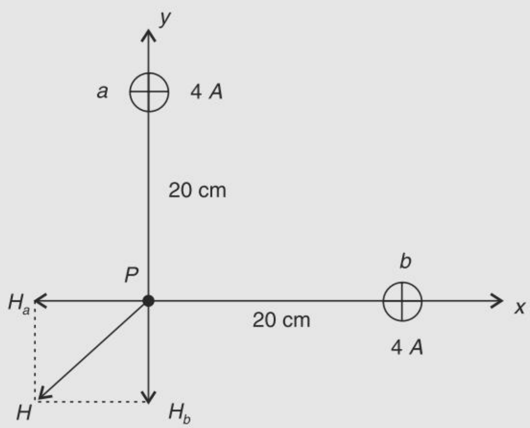


Fig. 7.5

Solution

Due to current in conductor *a*; direction is shown in Fig. 7.4 by the right-hand rule.

$$H_a = \frac{4}{2\pi \times 20 \times 10^{-2}} = \frac{10}{\pi} \text{ A/m}$$

Similarly

$$H_b = \frac{10}{\pi} \text{ A/m}$$

Adding the two vectors

$$\overline{H} = \sqrt{2}\,H_a \angle 90° + 45°$$

$$\overline{H} = \frac{10\sqrt{2}}{\pi} \angle 135° \text{ A/m}$$

7.3 MAGNETIC CIRCUITS

Consider a toroidal ring of *ferromagnetic* material of mean radius *R* and circular cross-section of diameter *d* as shown in Fig. 7.6. The ring termed as *core* is excited by a coil wound round it with *N* turns carrying a current *i*. By virtue of symmetry, flux established in the magnetic core is circular in shape.

Leakage Flux

The flux established along paths that lie mostly in air is very small compared to the core flux as core has a permeability μ_r times that of air. This flux is called *leakage flux*, i.e. it leaks through the core. There are no magnetic insulators to prevent such leakage. Being small in magnitude leakage flux will be neglected here*.

All the flux lines in the core enclose a current of

$$\mathcal{F} = Ni \text{ A} \tag{7.14}$$

* Effect of leakage flux in transformers and electric machines cannot be ignored. It must therefore be computed in machine design or in a built-up machine its effect must be determined experimentally.

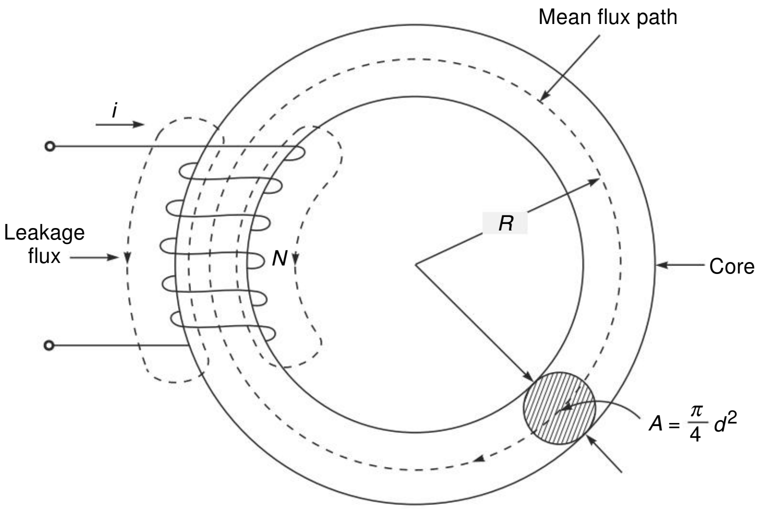


Fig. 7.6 Toroidal ring of ferromagnetic material with exciting coil

which is known as *magnetomotive force* (mmf) and in engineering practice has the units of ampere-turns (AT). It is a kind of magnetic potential difference. Thus

$$\mathcal{F} = Ni \text{ AT} \tag{7.15}$$

The mmf is expended in establishing flux round the core.

By symmetry, H in the core is constant along each flux line and for the mean flux line of radius R shown dotted in Fig. 7.7, the magnetizing force is

$$H = \frac{\mathcal{F}}{2\pi R} = \frac{\mathcal{F}}{l} \text{ AT/m} \tag{7.16}$$

where l is the length of the mean flux path. The length of the flux lines in a toroidal core increases and H reduces as we proceed outwards. For slender core ($d << R$), it is sufficiently accurate to neglect this variation and base calculations on the *mean flux path* and regard H as constant across the core cross-section.

The mean flux density

$$B = \mu H = \frac{\mu \mathcal{F}}{l} \text{ T} \tag{7.17}$$

As the flux in this core geometry is normal to the cross-sectional area, the total flux established round the core is given by

$$\phi = AB = \frac{\mathcal{F}}{l/\mu A} = \frac{\mathcal{F}}{\mathcal{R}} = \mathcal{P}\mathcal{F} \tag{7.18}$$

where $$\mathcal{R} = \frac{l}{\mu A} \text{ AT/Wb} \tag{7.19}$$

= *reluctance* of the magnetic circuit

and $\mathcal{P} = 1/\mathcal{R}$ = *permeance* of the magnetic circuit (7.20)

While units of reluctance (also permeance) have been defined above, these need not be specified every time.

The concept of reluctance lumps the magnetic system into a circuit analogically expressed as a dc electric circuit as shown in Fig. 7.7. In fact Eq. (7.18) is no different from the circuital Ohm's law. In this analog

$\mathcal{F}$ ~ dc voltage (potential)

$\mathcal{R}$ ~ resistance

ϕ ~ current

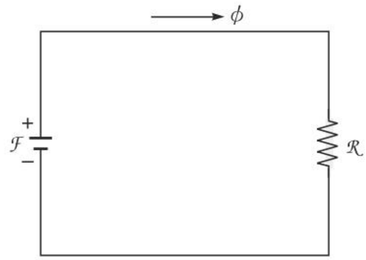


Fig. 7.7 DC circuit analog of magnetic system of Fig. 7.6

The resistance of an electric wire is given by

$$R = \frac{l}{\rho A}$$

an equation analogous to Eq. (7.19) for magnetic reluctance.

The Kirchhoff's two laws of the electric circuit equally apply to the magnetic circuits. It means that the mmf of loop equals the mmf expended in various parts of the loop (KVL) and the incoming and outgoing fluxes are equal at a junction of magnetic elements (KCL). In applying these laws, the circuit is divided into a number of elements so chosen that the flux density is approximately the same all over the element. The length of each element corresponds to its mean path.

The *dc* analog of a magnetic circuit applies only for computation of $\mathcal{F}$, ϕ and $\mathcal{R}$. There is no power loss in $\mathcal{R}$ but power loss $I^2 R$ occurs in *dc* circuit resistance.

Example 7.4 Figure 7.8(a) shows a rectangular magnetic core with an air-gap. Find the exciting current needed to establish a flux density of B_g = 1.2 T in the air-gap. Given N = 400 turns and μ_r(iron) = 4000.

Solution

It is a simple series magnetic circuit with its analog shown in Fig. 7.8(b).

Core length $l_c = 2\,[(20 - 4) + (16 - 4)] - 0.2 = 55.8$ cm

Cross-sectional area of core $A_c = 16$ cm^2

$$\text{Core reluctance} \quad \mathcal{R}_c = \frac{55.8 \times 10^{-2}}{4000 \times 4\pi \times 10^{-7} \times 16 \times 10^{-4}} = 0.694\times 10^5 \text{ AT/Wb}$$

$$= 0.694 \times 10^5 \text{ AT/Wb}$$

Air-gap length $l_g = 0.2$ cm

Area of air-gap $A_g = 16$ cm^2

$$\text{Air gap reluctance } \mathcal{R}_g = \frac{0.2 \times 10^{-2}}{4\pi \times 10^{-7} \times 16 \times 10^{-4}}$$

$$= 9.95 \times 10^5 \text{ AT/Wb}$$

$$\mathcal{R}\text{(total)} = \mathcal{R}_c + \mathcal{R}_g$$

$$= 0.694 \times 10^5 + 9.95 \times 10^5 = 10.64 \times 10^5 \text{ AT/Wb}$$

Flux in the magnetic circuit, $\phi = B\,A = 1.2 \times 16 \times 10^{-4} = 1.92$ mWb

Now $\quad Ni = \mathcal{F} = \phi\mathcal{R}$

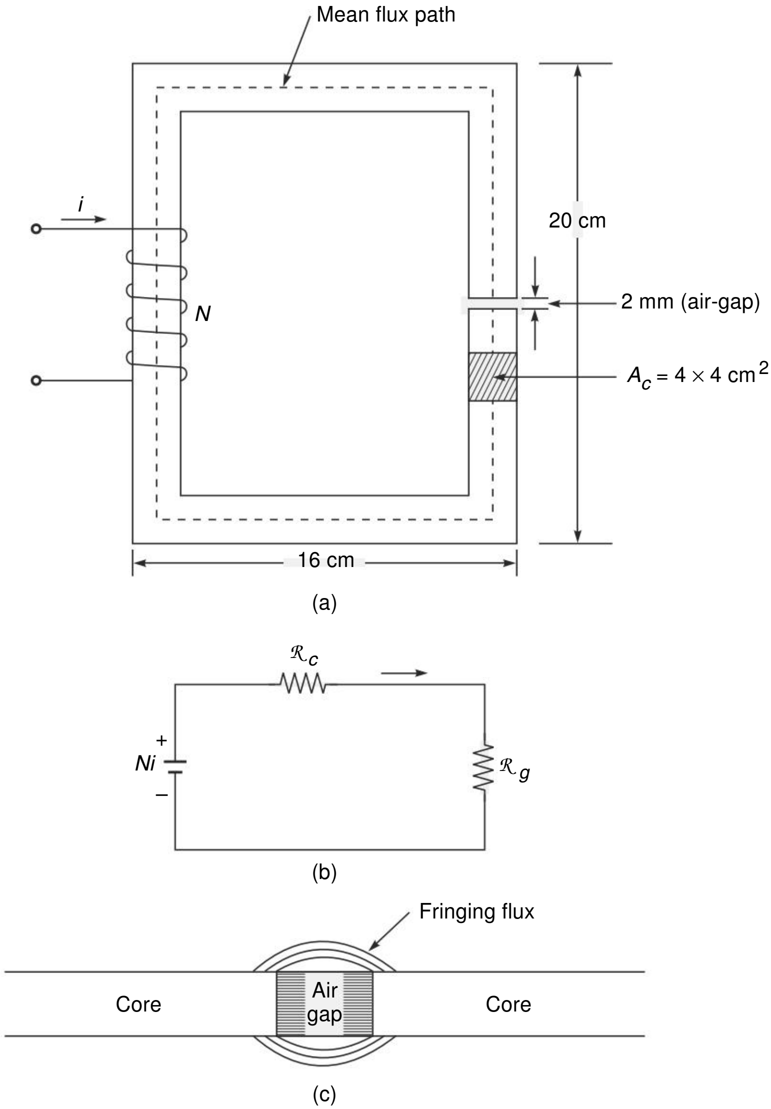


Fig. 7.8 (a) Magnetic core with air-gap, (b) electric circuit analog of the case of Fig. 8.5 (a) and (c) flux fringing

$$= 1.92 \times 10^{-3} \times 10.64 \times 10^{5}$$

$$= 2043 \text{ AT}$$

$\therefore$ Exciting current $\quad i = \dfrac{2043}{400} = 5.11$ A

It is seen above that

$$\mathcal{R}_g / \mathcal{R}_c = 14.34$$

Therefore for simplicity of computation, $\mathcal{R}_c$ (magnetic core reluctance) may be altogether neglected. Then

$$i = 1.2 \times 16 \times 10^{-4} \times 9.95 \times 10^5/400$$
$$= 4.77 \text{ A}$$

This simplification has caused an error of 6.6% which can be easily tolerated in magnetic circuit calculations.

Fringing The flux passing from the core to the air-gap cannot remain confined to the air-gap but would somewhat spread out, an effect called *fringing* as illustrated in Fig. 7.8 (c). This is because the flux paths near the air-gap have length comparable to the proper air-gap length. As a result, the average flux density in the air-gap is slightly less than the flux density in the core i.e., (B_g) $(av) < B_c$. Fringing can be accounted for *emprically* by increasing the linear dimensions of the gap by one gap length.

Let us consider the effect of fringing on the result (exciting current) for the example given below

Air-gap area $A_g = 4 \times 4$ cm^2

Gap length = 0.2 cm

Air-gap area modified to account for fringing

$$A_g \text{ (modified)} = (4 + 0.2) \times (4 + 0.2)$$
$$= 17.64 \text{ cm}^2$$

Now $$\mathcal{R}_g = \frac{0.2 \times 10^{-2}}{4r \times 10^{-7} \times 17.64 \times 10^{-4}} = 9.002 \times 10^5 \text{ AT/Wb}$$

Then

$$\mathcal{R}\text{(total)} = 0.694 \times 10^5 + 9.022 \times 10^5 = 9.72 \times 10^5 \text{ AT/Wb}$$

For the same core flux (1.92 mWb)

$$B_g\text{(av)} = \frac{1.92 \times 10^{-3}}{17.64 \times 10^{-4}} = 1.09 \text{ T } (< B_c = 1.2 \text{ T})$$

Now

$$i = \frac{\phi\mathcal{R}}{N} = \frac{1.92 \times 10^{-3} \times 9.72 \times 10^5}{400}$$
$$= 4.66 \text{ A}$$

It is observed that for the same core flux, the exciting current needed is slightly less than that calculated earlier (5.11 A) as fringing somewhat reduces air-gap reluctance (9.022×10^5 in place of 9.95×10^5).

Example 7.5 The magnetic circuit of Fig. 7.9 has a cast steel core with dimensions as shown. It is required to establish a flux of 0.8 m Wb in the air-gap of the central limb. Determine the mmf of the exciting coil, if for the core material $\mu_r = \infty$. Neglect fringing.

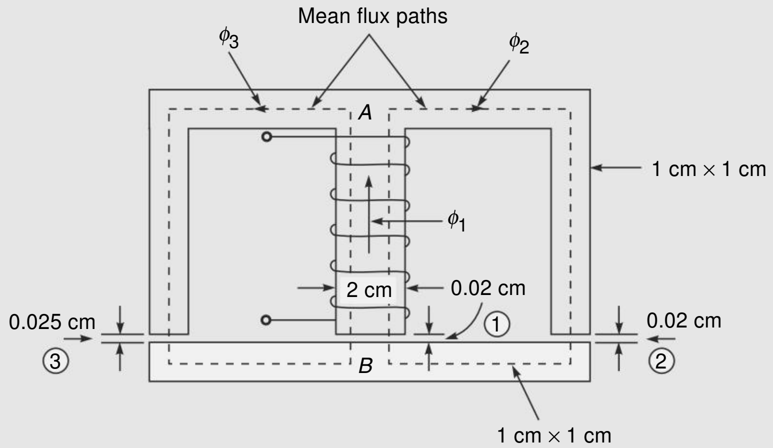


Fig. 7.9

Solution

Since $\mu_r = \infty$. No mmf drops in any member of the core. The analogous electrical circuit is drawn in Fig. 7.10(a). It can be reduced to the circuit of Fig. 7.10(b) by parallel combination for $\mathcal{R}_{g2}$ and $\mathcal{R}_{g3}$. Various gap reluctances are

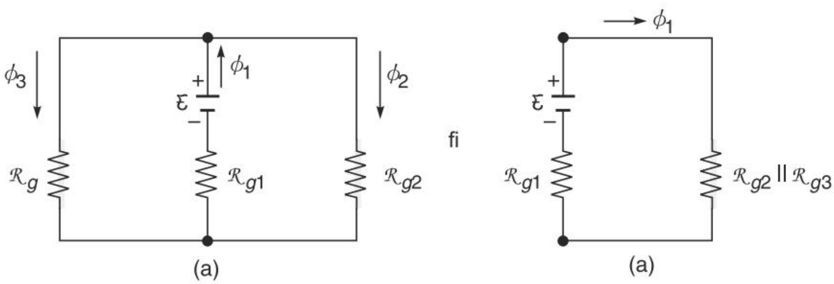


Fig. 7.10

$$\mathcal{R}_{g1} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 2 \times 1 \times 10^{-4}} = 0.796 \times 10^6 \text{ AT/Wb}$$

$$\mathcal{R}_{g2} = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 1 \times 10^{-4}} = 1.592 \times 10^6 \text{ AT/Wb}$$

$$\mathcal{R}_{g3} = \frac{0.025 \times 10^{-2}}{4\pi \times 10^{-7} \times 1 \times 1 \times 10^{-4}} = 1.989 \times 10^6 \text{ AT/Wb}$$

$$\mathcal{R}_{g2} \parallel \mathcal{R}_{g3} = \frac{1.592 \times 1.989}{1.592 + 1.989} \times 10^6 = 0.884 \times 10^6 \text{ AT/Wb}$$

Exciting coil mmf required $\mathcal{F} = \phi_1(\mathcal{R}_{g1} + \mathcal{R}_{g2} \parallel \mathcal{R}_{g3})$

$= 0.8 \times 10^{-3} (0.796 + 0.884) \times 10^6$

$= 1344$ AT

Example 7.6 The core of Fig. 7.11 has a relative permeability of 4000. The central limb is required to carry a flux of 0.01 Wb. Find the current needed for the exciting coil.

Solution

The analogous electrical circuit of Fig. 7.11 is drawn in Fig. 7.12, wherein

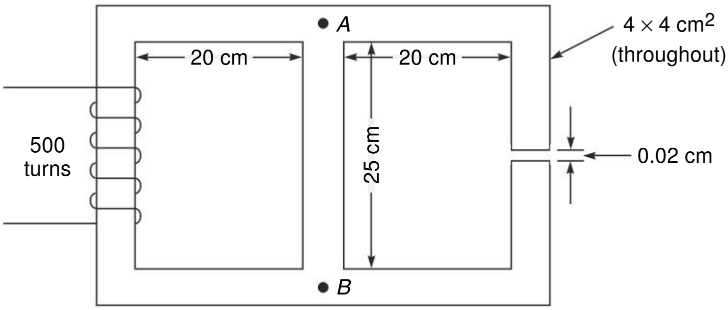


Fig. 7.11

$\mathcal{R}_L$ = reluctance of the left limb (A to B)

$\mathcal{R}_C$ = reluctance of the central limb (A to B)

$\mathcal{R}_R$ = reluctance of the right limb (A to B minus air-gap)

$\mathcal{R}_G$ = reluctance of the air-gap

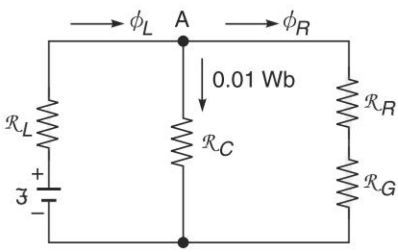


Fig. 7.12

Various reluctances are calculated below:

$l_L = 2 \times (20 + 4) + (25 + 4) = 77$ cm

$l_C = 25 + 4 = 29$ cm

$l_R = 2 \times (20 + 4) + (25 + 4) - 0.02 = 76.98$ cm

$l_G = 0.02$ cm

$$\mathcal{R}_L = \frac{77 \times 10^{-2}}{4\pi \times 10^{-7} \times 4000 \times 16 \times 10^{-4}} = 0.0957 \times 10^6$$

$$\mathcal{R}_C = \frac{29 \times 10^{-2}}{4\pi \times 10^{-7} \times 4000 \times 16 \times 10^{-4}} = 0.0361 \times 10^6$$

$$\mathcal{R}_R = \frac{76.98 \times 10^{-2}}{4\pi \times 10^{-7} \times 4000 \times 16 \times 10^{-4}} = 0.0957 \times 10^6$$

$$\mathcal{R}_G = \frac{0.02 \times 10^{-2}}{4\pi \times 10^{-7} \times 16 \times 10^{-4}} = 0.0995 \times 10^6$$

$$\mathcal{F}_{AB} = 0.01 \times \mathcal{R}_C = 0.01 \times 0.0361 \times 10^6 = 361 \text{ AT}$$

$$\phi_R = \frac{\mathcal{F}_{AB}}{\mathcal{R}_R + \mathcal{R}_G} = \frac{361}{(0.0957 + 0.0997) \times 10^6}$$

$$= 1.8861 \times 10^{-3} \text{ Wb}$$

$$\phi_L = \phi_C + \phi_R = (10 + 1.8861) \times 10^{-3} = 11.8861 \times 10^{-3}$$

Going round the left loop

$$\mathcal{F} = \phi_L \mathcal{R}_L + \mathcal{F}_{AB}$$

$$= 11.8861 \times 10^{-3} \times 0.0957 \times 10^6 + 361$$

$$= 1137 + 361 = 1498 \text{ AT}$$

∴ Exciting coil current $I = \dfrac{1498}{501} = 3$ A

7.4 MAGNETIC MATERIALS AND B–H RELATIONSHIP (MAGNETIZATION CHARACTERISTIC)

Magnetic materials are characterized by high permeability and nonlinear *B–H* relationship (magnetization characteristic) which exhibits *saturation* and *hysteresis.* This type of behaviour is explained by the domain theory of magnetization for which a suitable book on material science may be consulted.

Magnetic materials are classified as *ferromagnetic* and *ferrimagnetic*. Iron and its various alloys are ferromagnetic. Hard ferromagnetic materials include permanent magnetic materials such as alnicos, chrome steels, certain copper–nickel alloys and several other alloys. Ferrimagnetic materials consist of mixed oxides of iron and other metals. The oxide mixture is 'sintered', i.e. heated to a steady temperature of 1300°C which is maintained for several hours. The resulting material known as *ferrite* is chemically homogeneous and extremely hard. It has typically maximum flux density of 0.3–0.5 T, as compared to 2.18 T for pure iron.

Magnetization Characteristic

The *B–H* relationship for cyclic *H* is the *hysteresis loop* exhibited in Fig. 7.13 where the tip of the loop corresponds to the maximum *H* of the cyclic variation. Three hysteresis loops are indicated in this figure. The portions of the loops for decreasing *H* lie above the portions for increasing *H*, which is the hysteresis lag typical of ferro- and ferrimagnetic materials. The dotted curve passing through tips of the hysteresis loops is the *normal magnetization curve* or *B–H curve* of the material. A typical magnetization curve is provided in Fig. 7.14. It is initially nonlinear with a nearly linear portion in the middle and exhibits saturation for high values of *H*. For extremely high values of *H* it possesses a slope corresponding to that of free space ($\mu_r = 1$). It is this *B–H* curve which is used in magnetic circuit calculations and hysteresis effects, where necessary, are accounted for empirically. In fact the *B–H* curve is appreciably affected by heat treatment and mechanical handling. High degree of precision need therefore not be attempted in these calculations.

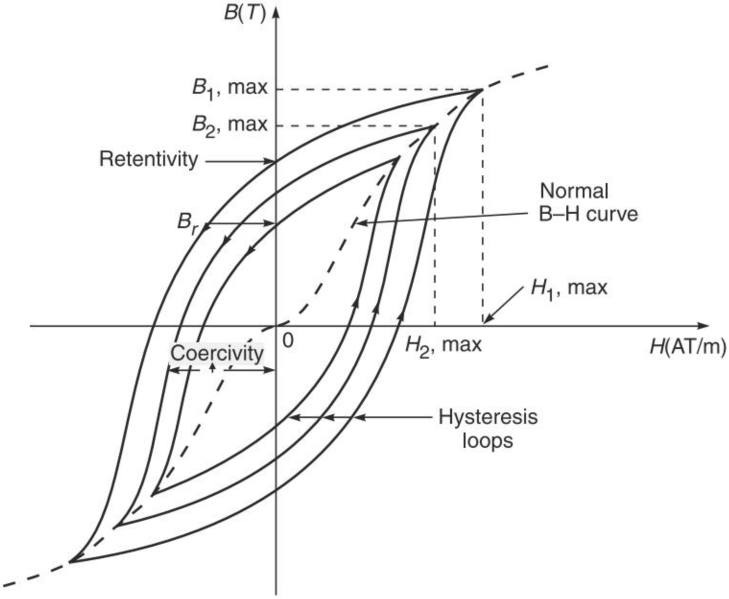


Fig. 7.13 Hysteresis loop and magnetization (B–H) Curve

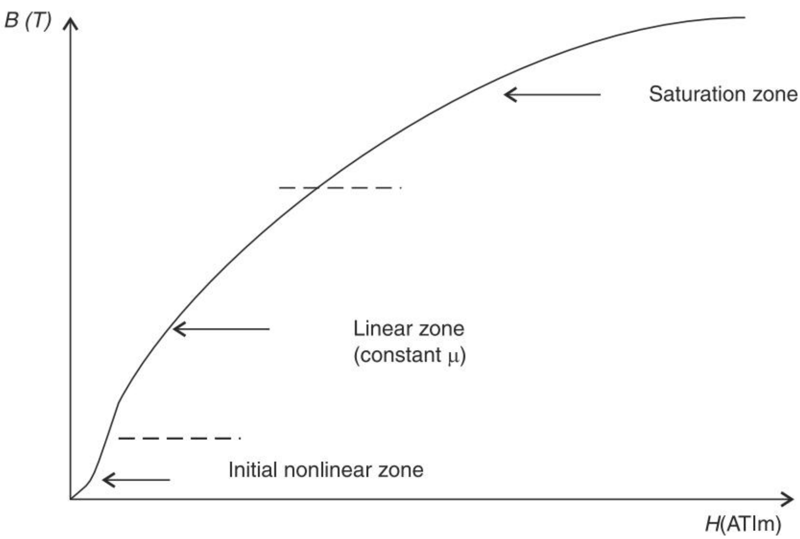


Fig. 7.14 Typical normal magnetization curve of ferromagnetic material

In a B-H curve the value of the flux density at H=0 is known as the *residual flux density* B_r. The value of H to reduce B_r to zero is called the *coercive force* H_c. The maximum possible value of B_r corresponding to deep saturation is know is as *retentivity* and the maximum value of H_0 is the *coercivity.* All these value are indicated in Fig. 7.13.

A ferrite material known as magnetic ceramic has a square hysteresis loop which is substantially magnetically bistable as shown in Fig. 7.15. Square-loop materials are used in switching circuits, as storage elements in computers, and in special type of transformers in electronic circuits.

A small change magnetic force H caused full reversal of the flux density from $+B_r$ to $-B_r$ and vice versa. This can be recognized as I/O switching in computing circuit

Typical magnetization curves for important ferromagnetic materials are shown in Fig. 7.16. For economic reasons, magnetic circuits are designed with magnetic material in a slightly saturated state.

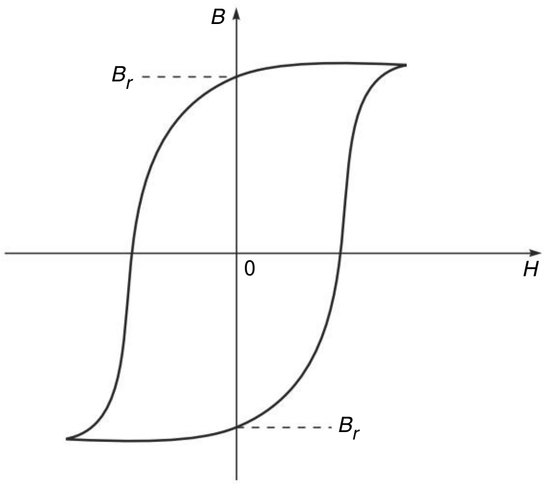


Fig. 7.15 Square-loop magnetic material

Sheet Steels

Transformers and such elements of electric machines that carry alternating flux are constructed from *silicon steel* in the form of thin sheets. Cores constructed from these sheets are called *laminated* cores with laminations of silicon steel. Addition of silicon in steel increases its electric resistivity thereby reducing eddy-current losses (Sec. 7.8). Laminating further greatly reduces eddy-current losses.

Steel has higher permeability in the direction of the edge of the cubic crystal. Steel sheets are therefore cold rolled so that all crystal edges align along the direction of rolling. These sheets are termed as *cold-rolled grain oriented* (CRGO) steel. Laminations must be cut from sheet steel such that the direction of flux these are to carry is oriented along the sheet length.

Example 7.7 The magnetic circuit of Fig. 7.17 has a cast steel core whose dimensions are given below:

Length	$(ab + cd)$ = 50 cm	Cross-sectional area	= 25 cm^2
Length	ad = 20 cm	Cross-sectional area	= 12.5 cm^2
Length	dea = 50 cm	Cross-sectional area	= 25 cm^2

Determine the exciting coil mmf required to establish an air-gap flux of 0.75 mWb. Use the *B–H* curve of Fig. 7.16.

Solution

Assuming no fringing the flux density in the path *abcd* will be same, i.e.

$$B = \frac{0.75 \times 10^{-3}}{25 \times 10^{-4}} = 0.3 \text{ T}$$

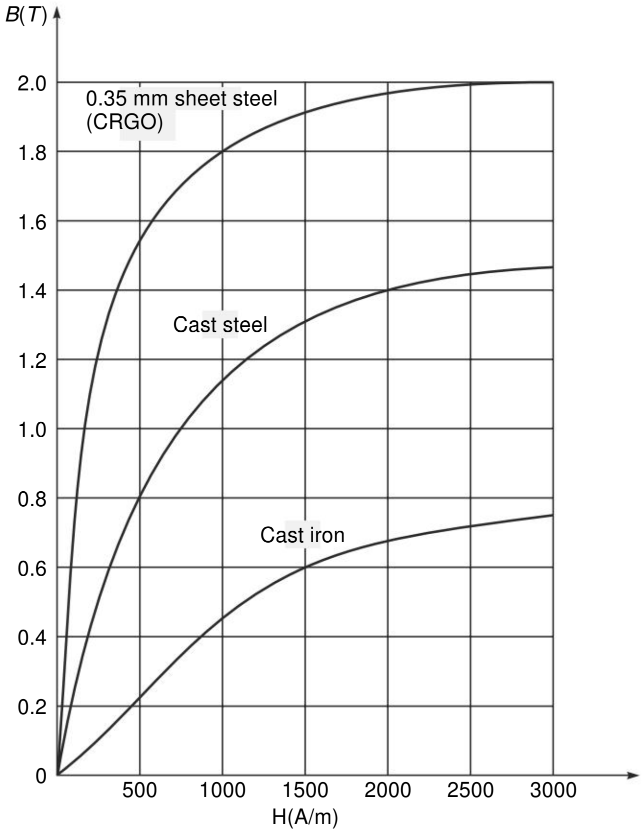


Fig. 7.16 Magnetization curves

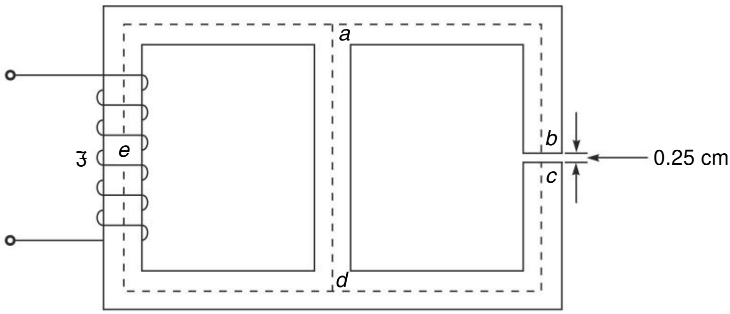


Fig. 7.17

$$\mathcal{F}_{bc} = \frac{B}{\mu_0} l_{bc} = \frac{0.3 \times 0.25 \times 10^{-3}}{4\pi \times 10^{-7}} = 60 \text{ AT}$$

$$H_{ab} = H_{cd} = \text{(form Fig. 7.12 for cast steel for } B = 0.3 \text{ T)}$$

$$= 200 \text{ AT/m}$$

$$\mathcal{F}_{ab+cd} = 200 \times 50 \times 10^{-2} = 100 \text{ AT}$$

$$\therefore \quad \mathcal{F}_{ad} = 60 + 100 = 160 \text{ AT}$$

$$H_{ad} = \frac{160}{20 \times 10^{-2}} = 800 \text{ AT/m}$$

$$B_{ad} \text{ (form Fig. 8.12)} = 1.04 \text{ T}$$

$$\phi_{ad} = 1.04 \times 12.5 \times 10^{-4} = 1.3 \text{ mWb}$$

Then

$$\phi_{dea} = 0.75 + 1.3 = 2.05 \text{ mWb}$$

$$B_{dea} = \frac{2.05 \times 10^{-3}}{25 \times 10^{-4}} = 0.82 \text{ T}$$

$$H_{dea} \text{ (from Fig. 7.16)} = 500 \text{ AT/m}$$

$$\mathcal{F}_{dea} = 500 \times 50 \times 10^{-2} = 250 \text{ AT}$$

$$\mathcal{F} = \mathcal{F}_{dea} + \mathcal{F}_{ad} = 250 + 160 = 410 \text{ AT}$$

7.5 ELECTROMAGNETIC INDUCTION AND FORCE

In this section, certain fundamental laws of electromagnetism will be enunciated and also certain rules will be put forth which are helpful in application of the laws.

Faraday's Law

Flux Linkages If flux ϕ passes through all the N turns of a coil as shows in Fig. 7.18 the flux is said to link the coil.

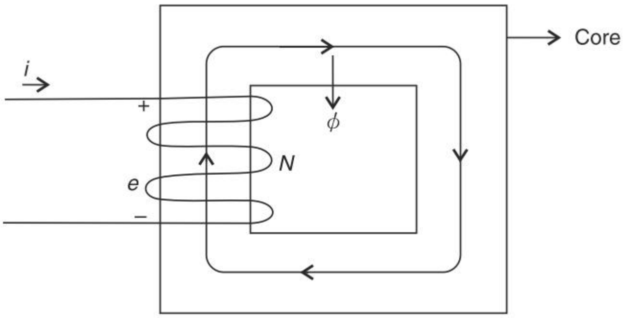


Fig. 7.18

The flux linkage of the coil are

$$\lambda = N\phi \text{ Weber-turns (Wb-T)} \qquad (7.21)$$

The Faraday's law states that if the magnitude of the flux through the coil changes with time, an *emf* is induced in the coil which is given by

$$e = -\frac{d\lambda}{dt} = -N\frac{d\phi}{dt} \; V \qquad (7.22)$$

Lenz's Law

The negative sign in Eq. (7.22) means that the induced *emf* would tend to cause a current flow in the coil which would oppose the change in flux (the original cause of *emf* induction). This statement is known as Lenz's law.

If the opposing polarity of *emf* is indicated on the coil terminals as in Fig. 7.19, then the Faraday's law need to give only the *emf* magnitude as

$$e = \frac{d\lambda}{\mathrm{d}t} = N\frac{d\phi}{\mathrm{d}t}\ V \tag{7.23}$$

Change in flux linkages of a coil may occur in two ways:

- The coil remains stationary and the flux through it changes with time. The *emf* so inducted is known as *statically induced emf (transformer emf).*
- Flux density distribution remains constant and stationary in space but the coil moves relative to it so as to change the flux linkages of the coil. The *emf* so inducted is known as *dynamically inducted emf* (*motional emf*).

Both the above processes of induction may occur simultaneously in a coil.

The dynamically induced *emf* in a conductor of length l(m) placed at angle θ to a stationary magnetic field of flux density B(T) cutting across it at speed v(*m/s*) is given by

$$\begin{aligned} e &= |\boldsymbol{v} \times \boldsymbol{B}|\, l \quad V \\ &= Blv \sin\theta\ V \end{aligned} \tag{7.24}$$

where θ is the angle between the direction of flux density and conductor velocity. In electric machines $\theta = 90°$, so that

$$e = Blv\ V \tag{7.25}$$

This is known as the *flux-cutting rule* with the direction of *emf* given by $\boldsymbol{v}\times\boldsymbol{B}$ or by the well-known *Fleming's right-hand rule.*

Extend the thumb, first and second fingers of the right hand mutually at right angles to each other. If the thumb represents the direction of v (motion of conductor with respect to B), first finger the direction of B, then the second finger gives the direction of *emf* along l (the conductor).

Lorentz Force Equation

Force of electromagnetic origin is given by

$$\boldsymbol{F} = l\,\boldsymbol{i} \times \boldsymbol{B}\ \mathrm{N} \tag{7.26}$$

where $\boldsymbol{F}$ is the force acting on a straight conductor of length l (m) carrying current i (A) placed in a uniform field of flux density B(T). The magnitude of force is given by

$$F = Bil \sin\theta\ \mathrm{N} \tag{7.27}$$

where direction is along $\boldsymbol{i} \times \boldsymbol{B}$ and θ is the angle between current direction and flux density. If $\theta = 90°$ as in electric machines

$$F = Bil\ \mathrm{N} \tag{7.28}$$

which is the well-known *Bil rule* or *Biot-Savart Law*

The direction of force can also be found by the *Fleming's left-hand rule:*

Extend the thumb, first and second fingers of the left hand mutually at right angles to each other. If the thumb represents the direction of *B,* the second finger the direction of *I,* then the thumb points in the direction of force on the conductor.

From Eq. (7.28), B can be imagined to have units of N/Am.

7.6 INDUCTANCE: SELF AND MUTUAL

Self Inductance

Consider a coil of N turns wound on an iron core and carrying current I as shown in Fig. 7.19

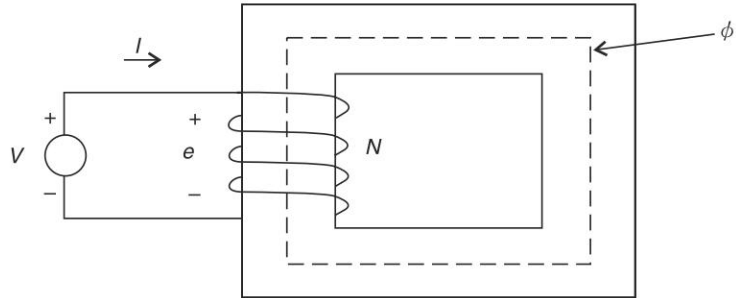


Fig. 7.19 Self inductance

The coil creates flux ϕ in the core called *self-flux,* which is assumed to link all the N turns (no leakage). As the flux varies with time (caused by current varying with time) the *emf* induced in the coil called the *counter emf* is given by

$$e = N\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}\lambda}{\mathrm{d}t} \text{ (opposing the current as per Lenz's law)}$$

$$= N\frac{\mathrm{d}\phi}{\mathrm{d}i}\cdot\frac{\mathrm{d}i}{\mathrm{d}t} \quad V$$

or $$e = L\frac{\mathrm{d}i}{\mathrm{d}t} V$$

where

$$L = N\frac{\mathrm{d}\phi}{\mathrm{d}i} = \frac{\mathrm{d}\lambda}{\mathrm{d}i} \text{ H (henry)} \tag{7.29}$$

is called the *self-inductance* of the coil. The unit of inductance is henry (H) = WbT/A

Equation (7.29) is the general expression for the self-inductance of a coil at any point on the B-H curve of its core material. It indeed is the incremental inductance which corresponds to incremental change around and operating point on the curve.

For a *linear* B-H curve (material operated in the region of constant permeability or when the magnetic circuit has a domanant air-gap), L is constant which can then be expressed as

$$L = \frac{N\phi}{i} = \frac{\lambda}{i} \text{ H} \tag{7.30}$$

The coil flux linkage are then

$$\lambda = Li \tag{7.31}$$

From Eq. (7.30), we can derive alternative expressions for self-inductance.

$$L = \frac{N\phi}{i} = \frac{N^2\phi}{Ni}$$

But $Ni = Hl$ (Ampere's circuital law)

$$\therefore \qquad L = \frac{N^2 BA}{Hl} = N^2 \mu \frac{A}{l}$$

or
$$L = \frac{N^2}{\mathcal{R}} = \mathcal{P} N^2 \ \text{H} \tag{7.32}$$

It is found from Eq. (7.32) that the coil self-inductance is independent of excitation current and depends upon the core geometry, permeability of the core's magnetic material and number of coil turns. In fact, inductance is proportional to the square of the number of turns of the coil.

In the general case when both the configuration and current in an inductive coil are considered then Eq. (7.31) of flux linkages modify to

$$\lambda = L(x)\, i \tag{7.33}$$

where x is a length / angle parameter.

The counter *emf* is then expressed as

$$e = \frac{d\lambda}{dt} = \frac{d}{dt}[L(x)\, i]$$

Or
$$e = L\frac{\partial i}{dt} + i\frac{\partial L}{dt} \tag{7.34}$$

Inductance Core For high inductance value, iron core with high permeability is used; see Eq. (7.32). But the B-H curve non-linearity causes the inductance value to vary for change in the operating point or for larger magnitude change in current value. Therefore, iron-cored inductance operation must be restricted to small current variations about the operating point. It is used in low frequency power circuits.

For low value linear inductance, air core is used for high frequency electronic circuits.

Mutual Inductance

When two coils are wound on a common core or placed close to each other, a part of the flux produced by one coil also links the other coil as shown in Fig. 7.20. This leads to the concept of *mutual inductance* defined as

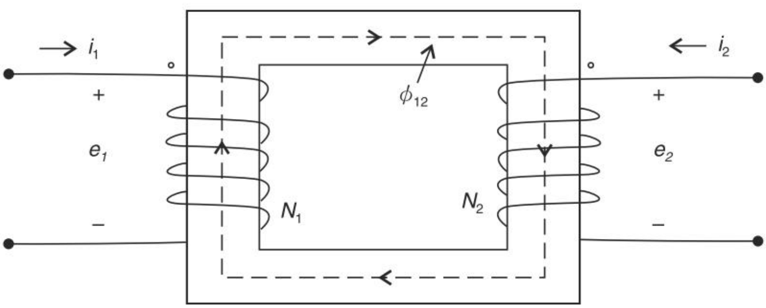


Fig. 7.20 Mutual inductance

$$L_{12}\ (\text{or } M_{12}) = \frac{\lambda_{12}}{i_2}\ H$$

$$L_{21}\ (\text{or } M_{21}) = \frac{\lambda_{21}}{i_1}\ H \tag{7.35}$$

where

λ_{12} = flux linkages of coil 1 due to current in coil 2

λ_{21} = flux linkages of coil 2 due to current in coil 1

For a bilateral magnetic circuit

$$M_{12} = M_{21} = M \tag{7.36}$$

In general in a linear magnetic circuit

$$M = k\sqrt{L_1 L_2} \tag{7.37}$$

where k is the coefficient of coupling (which can at most be unity). For a light coupling,* i.e., all the flux linking both coils (no leakage)

$$M = \sqrt{L_1 L_2} \tag{7.38}$$

Dot Convention Dots are used on the two coupled coils for similar terminals as shown in Fig. 7.21. Current flowing into dotted terminal of each coil produces core flux in the same direction. If the flux varies with time, the dotted terminals have the same polarity of *emf induced*. The reader may check by assuming the flux to be increasing and applying Lenz's law. When both coils are carrying current, the total flux linkages are given by

$$\lambda_1 = L_{11}\, i_1 + L_{12}\, i_2$$

$$\lambda_2 = L_{21}\, i_1 + L_{22}\, i_2 \tag{7.39}$$

where L_{11}, L_{22} are self-inductance of the coils and L_{12}, L_{21} are mutual inductance of the coils (equal in a bilateral circuit).

The induced *emf* in each coil is given by**

---

* For tight coupling there is no leakage

$$\phi_{21} = \frac{N_1 i_1}{\mathcal{R}},\ \mathcal{R} = \text{reluctance of the magnetic circuit}$$

$$M_{21} = \frac{\phi_{21} N_2}{i_1} = \frac{N_1 N_2}{\mathcal{R}} = M_{12} = M$$

and

$$L_1 = \frac{\phi_{12} N_1}{i_1} = \frac{N_1^2}{\mathcal{R}}$$

$$L_2 = \frac{N_2^2}{\mathcal{R}}$$

Hence $\quad M = \sqrt{L_1 L_2}$

**These equations can be written in single suffix form as

$$e_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$e_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

$$e_1 = L_{11}\frac{di_1}{dt} + L_{12}\frac{di_2}{dt} \tag{7.40a}$$

$$e_2 = L_{21}\frac{di_1}{dt} + L_{22}\frac{di_2}{dt} \tag{7.40b}$$

From examination of the form of these equations, it easily follows that a mutually coupled coil can be modeled in the form of the circuit of figure given below with self-inductances only. The reader can verify this by writing the two mesh equations.

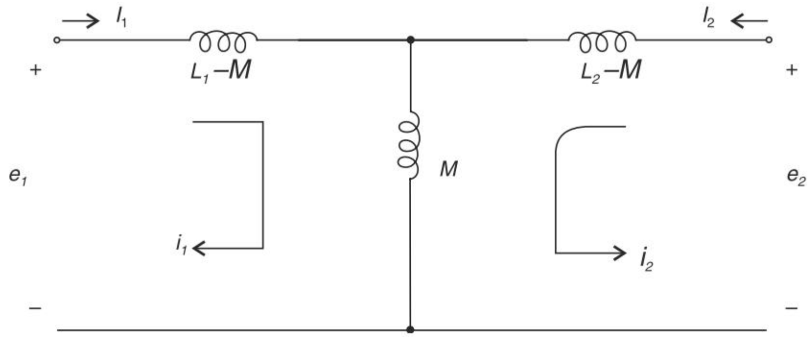


Fig. 7.21

7.7. ENERGY STORED IN MAGNETIC SYSTEMS (LINEAR)

Figure 7.22 shows a self-inductance (iron-cored) when the resistance r of the coil is lumped outside so that L is devoid of any resistance (pure, lossless). The electrical energy drawn from the source gets stored in the magnetic system. In time dt

$$dW_e = ei\,d\,t = dW_f \tag{7.41}$$

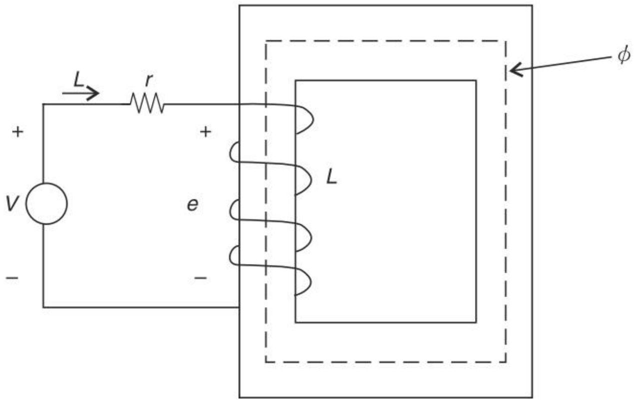


Fig. 7.22 Self inductance

where dW_f is the change in the field energy. As

$$e = \frac{d\lambda}{dt}$$

$$\therefore \qquad dW_f = dW_e = i d\lambda$$

The energy absorbed by the magnetic system in establishing flux linkages λ from zero state is

$$W_1 = \int_0^{\lambda} i \, d\lambda$$

As $\qquad \lambda = Li$

$$\therefore \qquad W_f = L\int_0^{a} i \, dt$$

$$\text{or} \qquad W_f = \frac{1}{2} Li^2 = \frac{1}{2} i\lambda \tag{7.42}$$

This is the energy stored in the magnetic field.

The field energy expression of Eq. (7.42) can be written in term of magnetic system quantities using Eq. (7.32). Thus

$$W_f = \frac{1}{2}\frac{N^2 i^2}{R} = \frac{1}{2}\mathcal{R}\phi^2 \tag{7.43}$$

where $\mathcal{R}$ is the reluctance and ϕ is the flux established in the magnetic circuit.

In case of an air-gap in the core, air-gap reluctance being far larger than that of the core, major portion of the field energy would reside in the air-gap.

Energy Stored in Mutual Inductance

For two coupled coils (both excited), Eq. (7.43) generalizes to

$$W_f = \int_0^{\lambda 1} i_1 \, d\lambda_1 + \int_0^{\lambda 2} i_2 \, d\lambda_2 \tag{7.44}$$

From Eq. (7.39)

$$d\lambda_1 = L_{11} + di_1 + L_{12}\, di_2$$

$$d\lambda_2 = L_{21} + di_1 + L_{22}\, di_2$$

Substituting in Eq. (7.44) and taking $L_{21} = L_{12}$, we get

$$W_f = L_{11}\int i_1 \, di_1 + L_{22}\int i_2 \, di_2 + L_{12}\int (i_1 \, di_2 + i_2 \, di_1)$$

$$= \frac{1}{2} L_{11}\, i_1^2 + \frac{1}{2} L_{22}\, i_2^2 + L_{12}\, i_1\, i_2 \tag{7.45}$$

Example 7.8 A cast steel core shown in Fig. 7.23 has uniform cross-section. It is wound with two coils. The coil 2 carries a current of 2 A.

(a) What should the current of coil 1 and its direction for a core flux density 1.4 T in the direction indicated on the figure. For cast steel, $\mu_r = 3000$.

(b) With both coils carrying currents as found in part (a), find the energy stored in the core.

(c) Find the inductance L_1, L_2 and the mutual inductance between coils. There is no leakage of flux.

Solution

(a) Core cross-sectional area, $A_C = (10 \times 10^{-2})\,(10 \times 10^{-3}) = 10^{-3}\,\text{m}^2$

$B_C = 1.4$ T required

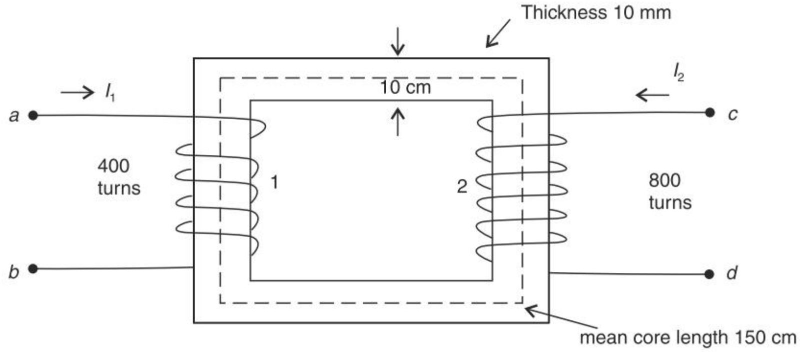


Fig. 7.23

Field intensity, $H_C = \dfrac{B_C}{\mu_0 \mu_r} = \dfrac{1.4}{4\pi \times 10^{-7} \times 3000}$ AT/m

Total *mmf* needed, F (total) $= H_C l_C$

$$= \frac{1.4 \times 10^{-2}}{4\pi \times 10^{-7} \times 3000} = 5.57 \text{ AT}$$

Coil 2 produces $F_2 = 2 \times 800 = 1600$ AT

$$F \text{ (total)} = F_1 + F_2$$

$$557 = F_1 + 1600$$

or $F_1 = -1043$ AT

It is found from the right rule that F_2 produces flux in the desired direction. So F_1 must oppose the flux.

Required $I_1 = \dfrac{1043}{400} = 2.61\ A$

The current direction is out of the terminal '*a*'.

(b) With coil currents as found in part (a)

$$B_c = 1.4 \text{ T}$$

$$\phi_C = B_C A_C = 1.4 \times 10^{-3} \text{ Wb}$$

Core reluctance $R_C = \dfrac{l_C}{\mu A_C} = \dfrac{150 \times 10^{-2}}{4\pi \times 10^{-7} \times 3000 \times 10^{-3}}$

$$= 398 \times 10^3$$

Energy stored in core

$$W_f = \frac{1}{2} R_C \phi_C^2$$

$$= \frac{1}{2} \times 398 \times 10^3 \times (1.4 \times 10^{-3})^2$$

or $W_f = 390$ mJ

(c) Self-inductance coil 1

$$L_1 = \frac{N_1^2}{R_C} = \frac{(400)^2}{398 \times 10^3} = 0.4 \text{ H}$$

Self-inductance coil 2

$$L_2 = \frac{N_2^2}{R_C} = \frac{(1600)^2}{398 \times 10^3} = 1.6 \text{ H}$$

As there is no leakage, mutual inductance is

$$M = \sqrt{L_1 L_2} = \sqrt{0.4 \times 1.6} = 0.8\text{H}$$

7.8 AC OPERATION OF MAGNETIC CIRCUITS

Consider an N-turn iron-core coil of Fig. 7.24 with ac excitation. The coil is assumed to be ideal with zero resistance. The induced emf in the coil must be sinusoidal for it to balance the ac applied voltage (KVL). This constrains the flux in the core to be sinusoidal. Let

$$\phi = \phi_{\max} \sin \omega t \tag{7.46}$$

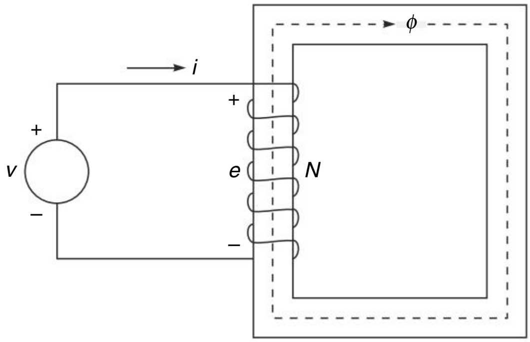


Fig. 7.24 Magnetic circuit with ac excitation

where

$\phi_{\max}$ = maximum core flux

ω = $2\pi f$ rad/s

f = frequency (of excitation) in Hz

The coil induced emf as per Faraday's law is

$$v = e = N \frac{d\phi}{dt} = \omega N \phi_{\max} \cos \omega t \tag{7.47}$$

and its rms value is

$$V = E = \frac{2\pi}{\sqrt{2}} f N \phi_{\max} = 4.44 f N \phi_{\max} \tag{7.48}$$

or $$V = E = 4.44 f N A_c B_{\max} \tag{7.49}$$

where A_c is the core area of cross-section.

It is seen from Eqs. (7.46) and (7.47) that the flux phasor lags the induced emf phasor by 90° as illustrated in phasor diagram of Fig. 7.25. This is because cos ωt leads sin ωt by 90°.

It is easily observed from Eqs. (7.47)/(7.48) that the maximum value of the core flux (sinusoidal) or flux density is dictated by the voltage applied to the coil (ideal). The excitation current* can then be found from the B–H curve of the core material. On the other hand, in dc excitation, the coil resistance determines the excitation current, which in turn decides the core flux density as per the B–H curve.

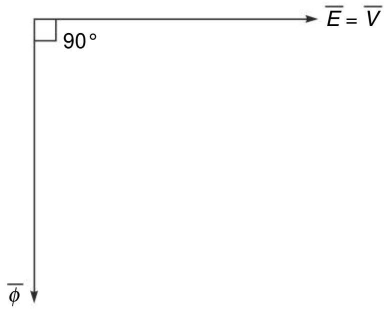


Fig. 7.25 Phasor relationship of $\bar{E}$ (= $\bar{V}$) and $\bar{\phi}$

Example 7.9 A ring of magnetic material has rectangular cross-section. The inner diameter of the ring is 20 cm and the outer diameter 25 cm, its thickness being 2 cm. An air-gap of 1 mm length is cut across the ring. The ring is wound with 500 turns carrying a current of 2 A. The permeability of the magnetic material is 6000. Find the following:

(a) flux density in the air-gap,

(b) inductance of the coil,

(c) energy stored in the magnetic material and in the air-gap,

(d) rms emf induced in the coil when carrying alternating current of 2 sin 314 t, and

(e) the excitation current when the coil is excited with a voltage of 100 sin 314 t V.

Neglect fringing and leakage.

Solution

Area of cross-section of ring $= \dfrac{5}{2} \times 2 = 5 \text{ cm}^2$

Mean length of ring $= \pi \times \left(\dfrac{20+25}{2}\right) - 0.1 = 70.6$ cm

Length of air-gap $= 0.1$ cm

Reluctance of ring $\mathcal{R}_r = \dfrac{70.6 \times 10^{-2}}{4\pi \times 10^{-7} \times 6000 \times 5 \times 10^{-4}}$

$= 0.187 \times 10^6$

Reluctance of air-gap $\mathcal{R}_g = \dfrac{0.1 \times 10^{-2}}{4\pi \times 10^{-7} \times 5 \times 10^{-4}}$

$= 1.59 \times 10^6$

$\mathcal{R}(\text{total}) = (0.187 + 1.59)\, 10^6 = 1.78 \times 10^6$

Coil mmf $\mathcal{F} = 2 \times 500 = 1000$ AT

$\phi = \mathcal{F}/\mathcal{R}(\text{total}) = \dfrac{1000 \times 10^3}{1.78 \times 10^6}$

$= 0.562$ mWb

* Excitation current would be sinusoidal for linear core (unsaturated) otherwise it would contain strong third harmonic (40%).

(a) $$B\ (\text{air-gap}) = \frac{0.562 \times 10^{-3}}{5 \times 10^{-4}} = 1.124\ \text{T}$$

(b) $$L = \frac{\lambda}{i} = \frac{0.562 \times 10^{-3} \times 500}{2} = 140.5\ \text{mH}$$

(c) From Eq. (7.43), $W_f\,(\text{magnetic material}) = \frac{1}{2}\,\mathcal{R}_r \phi^2$

$$= \frac{1}{2} \times 0.187 \times 10^6 \times (0.562)^2 \times 10^{-6}$$

$$= 0.03\ \text{J}$$

$$W_f\,(\text{air-gap}) = \frac{1}{2}\,\mathcal{R}_g \phi^2$$

$$= \frac{1}{2} \times 1.59 \times 10^6 \times (0.562)^2 \times 10^{-6}$$

$$= 0.25\ \text{J}$$

Observe that energy stored in the magnetic ring is only 11% of the total energy stored in the magnetic circuit (ring + air-gap).

(d) Since $i_{\max} = 2$ A, as calculated in part (a),

$$\phi_{\max} = 0.562\ \text{mWb}$$

From Eq. (7.48)

$$E = 4.44\, f N \phi_{\max}$$

$$= 4.44 \times \frac{314}{2\pi}\, 500 \times 0.562 \times 10^{-3}$$

$$= 62.35\ \text{V}$$

(e) From Eq. (7.48)

$$\frac{100}{\sqrt{2}} = 4.44 \times \left(\frac{314}{2\pi}\right) \times 500 \times \phi_{\max} \times 10^{-3}$$

or $$\phi_{\max} = 0.637\ \text{mWb}$$

Now $$\mathcal{F}_{\max} = 0.637 \times 10^{-3} \times 1.78 \times 10^6$$

$$= 1134\ \text{AT}$$

$$i_{\max} = \frac{1134}{500} = 2.27\ \text{A}$$

$$i\ (\text{excit}) = 2.27 \sin (314\, t - 90°)$$

The excitation current tags applied voltage by 90°.

7.9 HYSTERESIS AND EDDY-CURRENT LOSSES

When magnetic materials undergo cyclic variations of flux density, hysteresis and eddy-current power losses occur in them, which are together known as *core loss* and appear in the form of heat. The core loss is important in determining temperature rise, rating and efficiency of transformers, machines and other ac-operated electromagnetic devices.

Hysteresis Loss

With reference to the hysteresis loop of Fig. 7.26, the energy absorbed by a ferromagnetic material per unit volume* as H is raised from zero to H_{max} is

$$\int_{-B_r}^{B_{max}} H \, dB$$
$$= \text{area of } bgo$$

As H is reduced to zero, energy from the magnetic field is returned to the source of excitation as dB is now negative. Per unit value of energy returned is

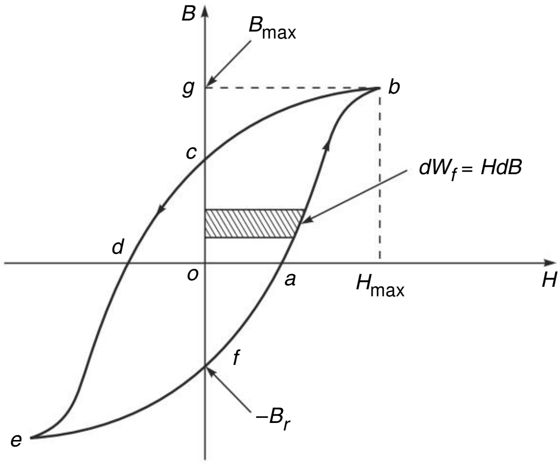


Fig. 7.26 Hysteresis loss

$$\int_{B_{max}}^{0} H \, dB = \text{area } cbgc$$

In half cycle of H variation energy not recovered from the material is the area *ofbco*. In one complete cycle of variation, energy lost per unit volume is the area of the hysteresis loop.

It is established empirically that for a given volume of material, power loss on account of hysteresis is

$$P_h = k_h f B^n_{max} \text{ V W} \tag{7.50}$$

where

k_h = characteristic constant of core material

n = *Steinmetz exponent*; range 1.5–2.0; typical value 1.6

V = volume of the material (m^3)

Eddy-Current Loss

Since core is made of conducting material, voltages induced in it by alternating flux produce circulating currents in iron. These are called eddy-currents and are accompanied by i^2r loss which is called eddy-current loss. As induced voltages and currents are proportional to frequency and flux density (Eq. 7.49), it is reasonable to expect that the power loss will vary as

$$P_e = k_e f^2 B^2_{max} \text{ V W} \tag{7.51}$$

---

* $$W_f = \int_0^{\lambda} i \, d\lambda$$
$$= \int_0^{B} \left(\frac{Hl}{N}\right)(AN)\,dB = Al \int_0^{B} H \, dB$$
$$W_t \text{ (energy/unit volume)} = W_f/Al = \int_0^{B} H \, dB \text{ J/m}^3$$

when flux varies sinusoidally; k_e, being the characteristic constant of the core, depends upon effective resistance and length of eddy-current paths.

Thus there are two methods used in practice to reduce the eddy current loss considerably.

1. Silicon Steel Silicon Steel produced by adding 4% silicon to iron has much higher resistivity. The increased resistance of eddy current paths reduces the resistive loss which is $i^2r = v^2/r$ for given induced voltage.

2. Laminating Steel To increase the path lengths of eddy currents, the steel is cut into thin *laminations* (0.35 mm) along the flux paths. The laminations are lightly insulated from each other by varnish. This restricts the eddy currents to individual lamination resulting in very much elongated eddy paths and consequent reduction in eddy current loss.

Screening Effect The eddy currents produce their own flux in the core which by Lenz's law pushes the main flux away from the core centre making the flux density at the centre lower than that near the core surface. This screening effect of eddy currents is negligible at lower frequencies but may be of great importance at high frequencies as it effectively reduces the core cross-section.

7.10. ELECTROMAGNETIC ENERGY CONVERSION (EMEC)

How the stored magnetic field energy gets converted to mechanical work is the subject matter of this section. Mechanical movement in an electromagnetic system requires the presence of air-gap. Therefore, it is sufficiently accurate to assume the magnetization curve to be linear. In place of *B-H* coordinates, it is convenient to use λ (flux linkages) and i (excitation current) as coordinates as sketched in Fig. 7.27.

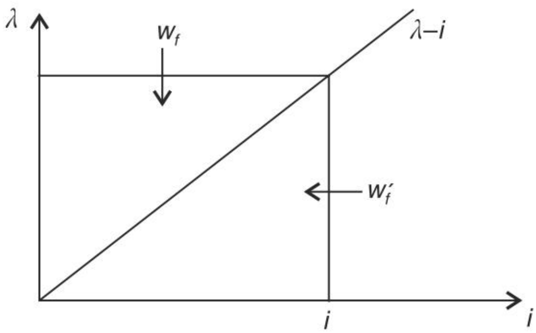


Fig. 7.27

The energy stored in the magnetic field is given by

$$W_f = \int_0^{\lambda} i\, d\lambda$$

As i is a function of λ, we write it as

$$W_f = \int_0^{\lambda} i(\lambda)\, d\lambda \tag{7.52a}$$

It is the area above λ–i curve as indicated in Fig. 7.27.

We define *co-enrgy* as

$$W_f' = \int_0^{\lambda} \lambda(i)\, di \tag{7.52b}$$

It is the area below $i - \lambda$ curve.

It is easily seen from Fig. 7.27 that

$$W_f + W_f' = i\lambda \tag{7.53}$$

From linear $\lambda - i$ curve, it follows that

$$W_f' = W_f = \frac{1}{2}\, i\,\lambda \tag{7.54}$$

Also from Eq. (7.53)

$$W_f' = i\lambda - W_f \tag{7.55}$$

Therefore

$$dW_f' = i\, d\lambda - dW_f \tag{7.56}$$

To illustrate the conversion of field energy to mechanical energy, consider the attracted armature relay of Fig. 7.28.

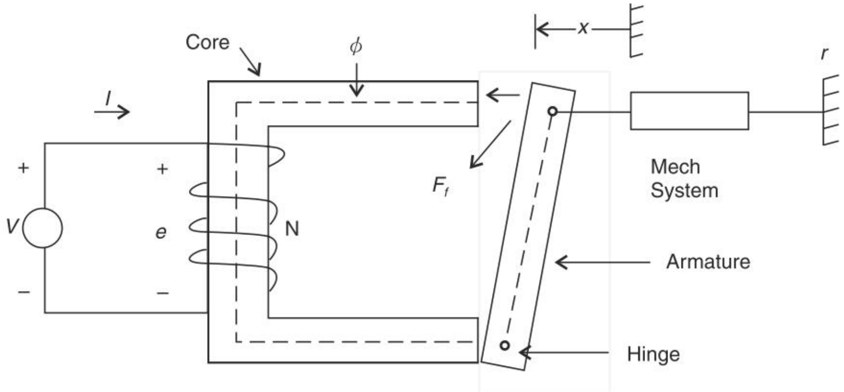


Fig. 7.28 Attracted armature relay

The relay armature move by distance dx, as per the law of conservation of energy, Mechanical energy output = electrical energy input–increase in field energy

or

$$F_f\, dx = i\, d\lambda - dW_f;\ F_f = \text{mechanical force} \tag{7.57}$$

Using Eq. (7.56), we get

$$F_f\, dx = dW_f' = dW_f\,(i,x), \text{ as per Eq. (7.54)}$$

Observe that for the relay of Fig. 7.27, $W_f\,(i, x)$ is a function of i and the air-gap determine by distance parameter x. It then follows that

$$F_f = \frac{\partial W_f\,(i, x)}{\partial x} \tag{7.58}$$

But $W_f(i, x) = \dfrac{1}{2}\, i^2\, L(x) = \dfrac{1}{2}\, \phi^2\, \mathcal{R}(x)$

Therefore the expression from the force becomes

$$F_f = \frac{1}{2}\ \ i^2 \frac{\partial L(x)}{\partial x} \tag{7.59}$$

$$= -\frac{1}{2}\phi^2\frac{\partial \mathcal{R}}{\partial x} \tag{7.60}$$

We find that the *force acts in the direction to increase inductance or to reduce reluctance.*

Example 7.10 The energy stored per unit volume of magnetic field in air is $B^2/2\mu_0$. Derive the expression for the force of attraction between two magnetized surface with air in between.

A U-shaped lifting magnet made of cast steel is wound with an exciting coil of 1000 turns. It is required to lift a steel mass of 160 kg at an air distance of 0.1 mm. The mean length of the magnetic path is 7.5 cm and its cross-sectional area is 24 cm^2. Neglecting reluctance of the mass to be lifted and fringing. Calculate the minimum exciting current needed.

The B-H curve data for cast steel are

B (T)	1.81	1.82	1.83
H (AT/m)	2808	3000	3500

Solution

$$W_f(air) = \frac{B^2}{2\mu_0} \quad \text{J/m}^3$$

For cross-section 1 m^2 and length l of air distance

$$W_f(B, l) = \left(\frac{B^2}{2\mu_0}\right) l,$$

Force of attraction

$$F = \frac{\partial W_f}{\partial l} = \frac{B^2}{2\mu_0} \quad \text{N/m}^2$$

U-shaped magnet plus mass to be lifted

$$A_C = 12 \text{ cm}^2, \text{ air-gap length} = 0.1 \text{ mm}$$

Minimum force needd for lifting

$$= 160 \times g \text{ N}$$

$$\left(\frac{B^2}{2\mu_0}\right) A_C = 160 \text{ g}$$

$$B^2 = \frac{320 \times 9.81 \times 2 \times 4\pi \times 10}{24 \times 10^{-4}} \quad \text{or} \quad B = 1.81 \text{ T}$$

For $B = 1.81$ T, $\quad H = 2800$ AT/m

Mean length of magnetic path = 75 cm

$$F \text{ (lifting magnet)} = 2800 \times 0.75 = 2100 \text{ AT}$$

Air-gap length = 0.1 mm

$$A_C = 24 \text{ cm}^2$$

$$\mathcal{R}_g = \frac{l}{\mu_0 A} = \frac{0.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 24 \times 10^{-4}}$$

$$= 33.2 \times 10^3$$

Flux

$$\phi_g = BA_C = 1.81 \times 24 \times 10^{-4} = 4.344 \times 10^{-3} \text{ Wb}$$

$$\text{AT}_g = \phi_g \mathcal{R}_g = 33.2 \times 4.344$$
$$= 144$$

Total mmf needed

$$F = 2100 + 144 = 2244 \text{ AT}$$

Minimum exciting current

$$i\,(\text{min}) = \frac{2244}{1000} = 2.244 \text{ A}$$

Example 7.11 An attracted armature relay shown in Fig. 7.29 (a) has an air-gap of $x = 0.05$ mm. It magnetization curve in open and closed position is given in Fig. 7.29 (b). Observe that it is linear.

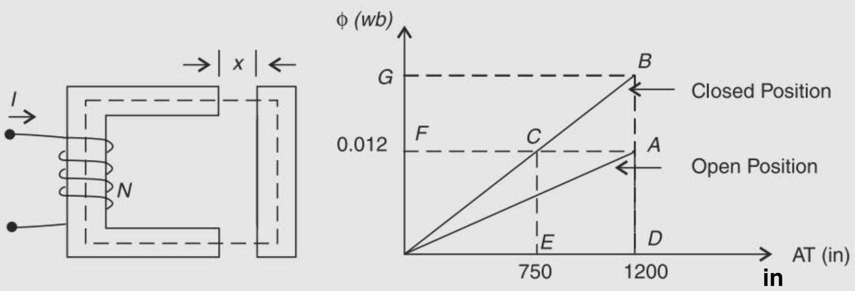


Fig. 7.29

(a) If the relay armature move very fast from open to closed position, find the mechanical work done. Where does this energy come from?

(b) If in part (a) the relay armature moves very slow, find the work done. Where does this energy come from?

(c) Calculate the force on the armature in open and closed positions.

Solution

Note: AT vs ϕ plot will be the same result as i vs λ plot as iN vs λ/N is AT vs ϕ

(a) As the armature moves very fast, flux has no time to change. It remain 0.012 Wb. There is no induced emf in the excitation coil, so current changes instantaneously and AT reduces from 1200 to 750 corresponding to the magnetization curve of closed position.

Thus

$$\Delta W_e = 0$$

$$\Delta W_f = \text{area } OAF - \text{area } OCF$$
$$= -\text{ area } OAC\text{, a reduction}$$

By energy balance

$$\Delta W_e = \Delta W_f + \Delta W_m$$

$$0 = -\text{ area } OAC + \Delta W_m$$

or $$\Delta W_m = \text{area } OAC \frac{1}{2} = \text{area } EDAC$$

$$= \frac{1}{2}(1200 - 750) \times 0.012 = 2.7 \text{ J}$$

The mechanical energy output is provided by conversion of the field energy.

(b) As the armature move very slowly, the flux increase while excitation current remains constant. Therefore the magnetic circuit state corresponds to point B.

$$\Delta W_c = \Delta\phi \cdot F_0 = \text{area } FABG$$

$$\Delta W_f = \text{area } OBG - \text{area OAF } \frac{1}{2} = \text{area } FABG$$

$$= \frac{1}{2}\Delta W_e$$

So $$\Delta W_m = \Delta W_e - \Delta W_f = \frac{1}{2}\Delta W_e$$

$$= \frac{1}{2} \text{ area } FABG$$

$$\phi_G = \frac{0.012}{750} \times 1200 = 0.0192 \text{ Wb}$$

Flux corresponding to point G is $\phi_G = \dfrac{0.012}{750} \times 1200 = 0.0192$ wb

Hence $$\Delta W_m = \frac{1}{2}(0.0192 - 0.012) \times 1200$$

$$= 4.32 \text{ J}$$

Half the electrical energy input is converted into mechanical energy and half is stored in the field.

(c) We know that

$$F_f = -\frac{1}{2}\phi^2 \ \partial R / \partial x$$

We need to find $R(x)$. From Fig. 7.29

$$R_A \text{ (relay open)} = \frac{F}{\phi} = \frac{1200}{0.012} = 100000$$

$$R_B \text{ (relay open)} = \frac{1200}{0.0192} = 62500$$

Thus

$$R(\text{total}) = R(\text{core}) + R(\text{air-gap})$$

$$= R_C + \frac{(10000 - 62500)}{2 \times 0.05} \cdot 2x$$

$$= R_C + 0.75 \times 10^6 x$$

Relay open

$$F_0 = -\frac{1}{2} \times (0.012)^2 \times \partial R/\partial x$$

$$= \frac{1}{2} \times (0.012)^2 \times 0.75 \times 106 = -54 \text{ N}$$

The minus sign means that the force tends to reduce the air-gap.

Relay closed

$$F_C = -\frac{1}{2} (0.0192)^2 \times 0.75 \times 10^6 = -138.24 \text{ N}$$

Example 7.12 For the magnetic circuit of Fig. 7.30, find $\bar{I}_2$, $\bar{V}_2$ and $\bar{V}_2/\bar{V}_1$.

Solution

Writing the mesh equation for the two meshes, we have

$$(1 + j10 \times 1)\ \bar{I}_1 - j10 \times 8\ \bar{I}_2 = 10\ \angle 0° \text{ (Mesh 1)}$$

$$(1 + j10)\ \bar{I}_1 - j80\ \bar{I}_2 = 10\ \angle 0° \qquad \text{(i)}$$

$$-j10 \times 8\ \bar{I}_1 + (500 + j10 \times 80)\ \bar{I}_2 = 0 \quad \text{(Mesh 2)}$$

or

$$j80\ \bar{I}_1 - (500 + j800)\ \bar{I}_2 = 0 \qquad \text{(ii)}$$

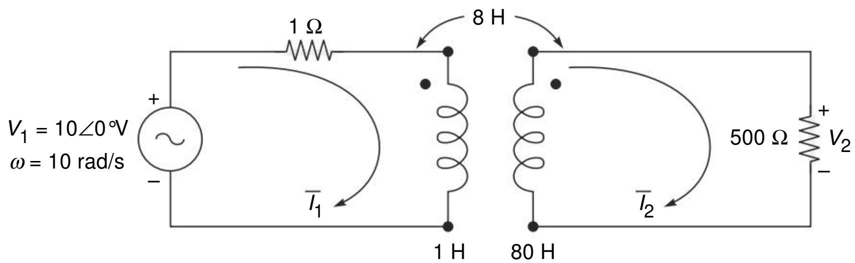


Fig. 7.30

From Eq. (ii), we get

$$\bar{I}_1 = [(500 + j800)/(j80)]\ \bar{I}_2 = (10 - j6.25)\ \bar{I}_2 \qquad \text{(iii)}$$

Substituting Eq. (ii) in Eq (i), we get

$$[(1 + j10)\ (10 - j\ 6.25) - j80]\ \bar{I}_2 = 10\ \angle 0°$$

or

$$\bar{I}_2 = 0.135\ \angle -10.7° \text{ A} \qquad \text{(iv)}$$

$$\therefore \quad \bar{V}_2 = 500\ \bar{I}_2 = 68\ \angle -10.7° \qquad \text{(v)}$$

$$\therefore \quad \bar{V}_2/\bar{V}_1 = 6.8\ \angle -10.7°$$

Example 7.13 A square loop of sides $2d$ is placed with its sides parallel to an infinitely long conductor carrying current I. The centre line of the square is at a distance b form the conductor. Determine the expression for the total flux passing through the loop. What would be the loop flux if the loop is placed such that the conductor is normal to the plane of the loop? Does the loop flux in this case depend upon its relative location wrt the conductor?

Solution

With reference to Fig. 7.31, at a distance r from the conductor.

$$H = I/(2\pi\, r) \text{ A/m} \qquad \text{(Ampere's law)}$$

$$B = \mu_0 H = \mu_0\ I/(2\pi r) \text{ T}$$

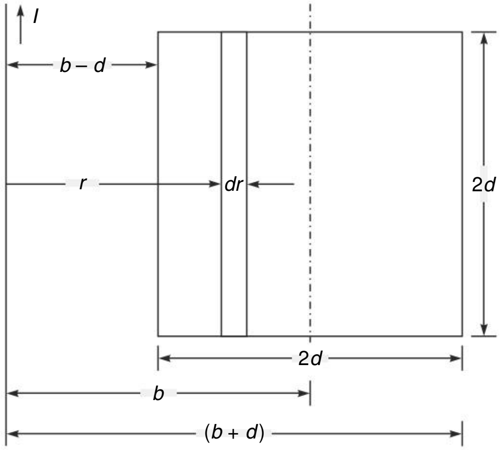


Fig. 7.31

The flux passing through elemental strip (width dr)

$$d\phi = B\, dA = (\mu_0 I/2\pi r) \times (2d)\, dr$$

Integrating, we get

$$\phi = (\mu_0 Id/\pi) \int_{b-d}^{b+d} \frac{dr}{r} = \frac{\mu_0 Id}{\pi} \ln\left\{\frac{b+d}{b-d}\right\}$$

If the conductor is normal to the plane of the loop, (regardless of its relative location) the flux through the loop is zero.

Example 7.14 A magnetic circuit has a mean core length of 160 cm and uniform cross-section of 5 cm^2. It has an air-gap of 0.8 mm and is wound with a coil of 1200 turns. Determine the self-inductance of the coil if the core material has a relative permeability of 1600.

Solution

The core reluctance is

$$\mathcal{R}_c = l_c/\mu_0\mu_r A = (160 \times 10^{-2})/(4\pi \times 10^{-7} \times 1600 \times 5 \times 10^{-4})$$
$$= 1.59 \times 10^6$$

The air-gap reluctance is

$$\mathcal{R}_g = l_g/\mu_0 A = (0.8 \times 10^{-3})/(4\pi \times 10^{-7} \times 5 \times 10^{-4})$$
$$= 1.27 \times 10^6$$

$$\mathcal{R}(\text{total}) = (1.59 + 1.27)\, 10^6 = 2.86 \times 10^6$$

Coil inductance $L = N^2/\mathcal{R} = (1200)^2/(2.86 \times 10^6) = 0.5$ H

Example 7.15 A toroid is composed of three parts of different materials but of uniform cross-sectional area. Their mean length and relative permeability are:

$l_1 = 0.15$ m, $\mu_{r_1} = 1447$ (cast steel)

$l_2 = 0.30$ m, $\mu_{r_2} = 5969$ (mild silicon steel)

$l_3 = 0.45$ m, $\mu_{r_3} = 47750$ (nickel iron)

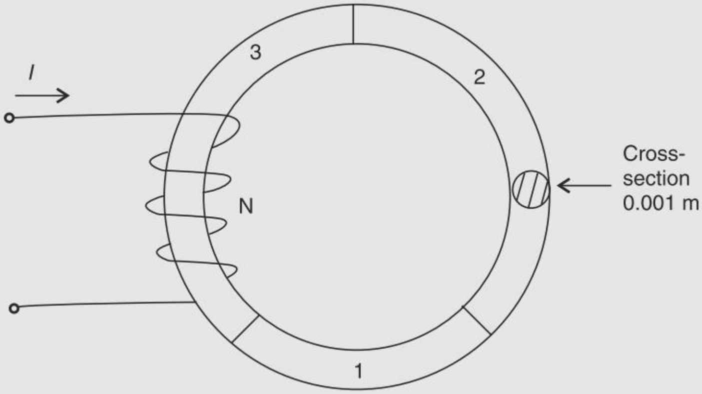


Fig. 7.32

It is required to establish a flux of 0.6 m Wb in the toroid. Calculate.

(a) the magnetic field intensity in each part

(b) the mmf required

(c) the excitation current of the coil

Solution

(a) Flux density $B = \dfrac{\phi}{A} = \dfrac{0.6 \times 10^{-3}}{0.001}$ = 0.6T uniform in all the parts

Magnetic field intensities

$$H_1 = \frac{B}{\mu_0 \mu_{r1}} = \frac{0.6}{4\pi \times 10^{-7} \times 1447} = 330 \text{ AT/m}$$

$$H_2 = \frac{B}{\mu_0 \mu_{r2}} = \frac{0.6}{4\pi \times 10^{-7} \times 5969} = 80 \text{ AT/m}$$

$$H_3 = \frac{B}{\mu_0 \mu_{r3}} = \frac{0.6}{4\pi \times 10^{-7} \times 47750} = 10 \text{ AT/m}$$

(b) $F = H_1 l_1 + H_2 l_2 + H_3 l_3$

$= 330 \times 0.15 + 80 \times 0.30 + 10 \times 0.45 = 78$ AT

(c) Excitation current

$$I = \frac{F}{N} = \frac{78}{100} = 0.78 \text{ A}$$

Example 7.16 The total core loss of a specimen of silicon steel is found to be 1500 W at 50 Hz. Keeping the flux density constant, the loss becomes 3000 W when the frequency is raised to 75 Hz. Calculate separately the hysteresis and eddy-current loss at each of these frequencies.

Solution

$$P_L = Af + Bf^2 = P_h + P_e \text{ ; } B_{max} \text{ constant}$$

$$P_L / f = A + Bf$$

$$1500/50 = A + 50\,B \quad \text{(i)}$$

$$3000/75 = A + 75\,B \quad \text{(ii)}$$

Solving Eqs. (i) and (ii), we get

$$A = 10, B = 2/5$$

At 50 Hz,

$$P_h = 10 \times 50 = 500 \text{ W}, P_e = (2/5) \times (50)^2 = 1000 \text{ W}$$

At 75 Hz

$$P_h = 10 \times 75 = 750 \text{ W}, P_e = (2/5)\,(75)^2 = 2250 \text{ W}$$

Example 7.17 A rectangular ring ferromagnetic material has a 16-cm inner diameter a 20-cm outer diameter and 2.5-cm thickness. A coil of 750 turns is wound on the ring. It is found that to produce a flux of 1.25 m Wb, the coil must carry 1 A. Find the following quantities:

(a) mmf (b) magnetic field intensity (c) flux density (d) reluctance (e) permeability (f) relative permeability

Solution

(a) *mmf*, $F = 750 \times 1 = 750$ AT

(b) Mean dia $= \dfrac{16 + 20}{2}$ 18 cm

Mean ring length, $l_C = 18\pi = 56.55$ cm

Magnetic field intensity $H_C = \dfrac{750}{56.55 \times 10^{-2}} = 1326$ AT/m

(c) Core cross-sectional area

$A_C = (20 - 16) \times 2.5 = 10 \text{ cm}^2$

Flux density $B_C = \dfrac{1.25 \times 10^{-3}}{10 \times 10^{-4}} = 1.25$ T

(d) Reluctance, $R = \dfrac{F}{\phi} = \dfrac{750}{1.25 \times 10^{-3}} = 600 \times 10^3$

(e) Permeability, $\mu = \dfrac{B_C}{H_C} = \dfrac{1.25}{1.326}\ 0.943 \times 10^{-3}$

(f) Relative permeability, $\mu_r = \dfrac{\mu}{\mu_0} = \dfrac{0.943 \times 10^{-3}}{4\pi \times 10^{-7}} = 75.04$

Example 7.18 Three coils are wound on a cast steel core as shown in Fig. 7.33. The current directions in coils are indicated in the figure core data.

Cross-sectional area, $A_c = 12 \text{ cm}^2$

Mean path length, $I_c = 30$ cm

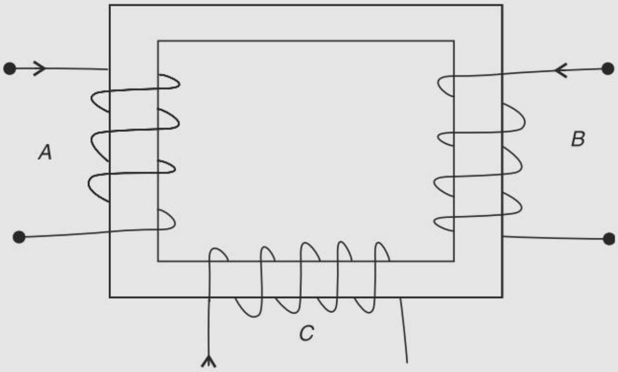


Fig. 7.33

Coils data

A	200 turns,	1A
B	600 turns,	0.75 A
C	?	0.5 A

How many turns should coil C have create core flux of 1.5 mWb.

Solution

Core flux = 1.5 mWb

Core cross-section = 12 cm^2

Core flux density, $B_C = \dfrac{1.5 \times 10^{-3}}{12 \times 10^{-4}} = 1.25$ T

From the magnetization curve of cast steel of Fig. 7.33 corresponding to $B_c = 1.25$ T

$$H_C = 1250 \text{ AT/m}$$

Total *mmf* needed $F = H_C\, l_C = 1250 \times 0.30 = 375$ AT

$$AT_a = 200 \times 1 = 200$$

$$AT_b = 600 \times 0.75 = 450$$

As per right hand rule AT_a and AT_b oppose each other. So

$$AT_b - AT_a = 450 - 200 = 250 \text{ (downwards)}$$

AT_c aids AT_b, so

$$F = AT_b - AT_a + AT_c = 375$$

or $\quad AT_c = 375 - 250 = 125$

Number of turns of coil C

$$N_C = \frac{125}{0.5} = 250$$

Direction of flux is clockwise.

ADDITIONAL SOLVED PROBLEMS

7.19 In the magnetic circuit of Fig. 7.34, the relative permeability of the core steel is 2000.

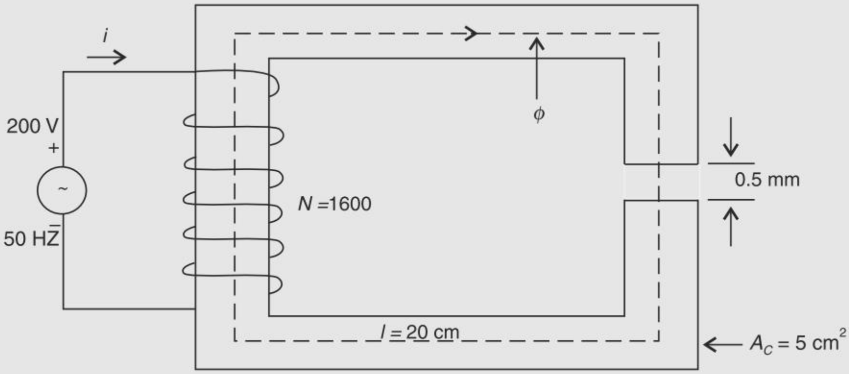


Fig. 7.34

Calculate

(a) the maximum flux density

(b) the maximum value of exciting current

(c) the maximum value of the energy stored in the magnetic circuit. What percentage of it resides in the air-gap?

Assume zero coil resistance and no flux leakage. Neglect fringing.

Solution

(a) The coil induced *emf* equals applied voltage

$$V = E = 4.44\, f\, \phi_{max}\, N \text{ rms}$$

$$200 = 4.44 \times 50\phi_{max} \times 1600$$

or $$\phi_{max} = 0.563 \text{ mWb}$$

$$B_{max} = \frac{\phi_{max}}{A_C} = \frac{0.563 \times 10^{-3}}{5 \times 10^{-4}}\ 1.126 \text{ T}$$

(b) Core reluctance

$$\mathcal{R}_C = \frac{20 \times 10^{-2}}{4\pi \times 10^{-7} \times 2000 \times 5 \times 10^{-4}} = 159 \times 10^3$$

Air-gap reluctance

$$\mathcal{R}_g = \frac{0.5 \times 10^{-3}}{4\pi \times 10^{-7} \times 5 \times 10^{-4}} = 159 \times 10^3$$

$$\mathcal{R}\text{(total)} = (159 + 796) \times 10^3 = 955 \times 10^3$$

Now $$F_{max} = N\, i_{max}\,;\ \phi_{max} = \frac{N\, i_{max}}{R\text{(total)}}$$

Thus

$$i_{max} = \frac{\phi_{max} R\text{(total)}}{N} = \frac{0.563 \times 10^{-3} \times 955 \times 10^3}{1600}$$

$$= 0.336 \text{ A}$$

(c) $W_f\,(\max) = \dfrac{1}{2}\,R\,(\text{total}) \times \phi^2_{\max} = \dfrac{1}{2} \times 955 \times 10^3 \times (0.563 \times 10^{-3})^2$

$= 151$ mJ

Percent energy in air-gap $= \dfrac{\mathcal{R}_g}{R(\text{total})} \times 100 = \dfrac{796 \times 10^3}{955 \times 10^3} \times 100 = 83.3$

SUMMARY

- Ampere's law states that

  $F = \left(\dfrac{\mu i_1}{2\pi r}\right) i_2 l$ N; MKS units

  i_1 = current in very long conductor

  $i_2 l$ = elemental current

  μ = permeability of medium

  r = distance of $i_2 l$ from the long conductor

  F acts redically outwards

- Flux Density (B) is given by

  $B = \mu \left(\dfrac{i_1}{2\pi r}\right)$ N/Am

  Then $F = B\, i_2 l$ N

- Magnetic Filed Intensity (H) is given by

  $H = \left(\dfrac{i_1}{2\pi r}\right)$ A/m

  $i_1 = N\, i$, (AT) N = number of turn

  In engineering units

  $H = \dfrac{Ni}{l}$ AT/m

  where l = length of closed flux path.

- Flux is a closed magnetic line around a current such that H is tangential to it at every point.
- Direction of flux is determine by right hand rule.
- Flux Density (B) is given by

  $B = \mu H$ Wb/m^2, tesla (T)

  where permeability of medium $\mu = \mu_0\, \mu_r$

  μ_0 = permeability of free space

  $= 4\pi \times 10^{-7}$ Wb/Am

  μ_r = relative permeability of medium

- Both B and H are vectors.

  In magnetic fields of simple geometry, we deal only with their magnitudes.

- Magnetic Flux piercing a surface is given by $\phi = BA$; B normal to area A

  In general $\phi = BA \cos\theta$; B makes an angle θ with normal to A

- Magnetomotive force (mmf) –Ampere-turns (AT) is also called magnetic potential; Symbol $\mathcal{F}$

  $\mathcal{F} = Hl$ AT

  l = length of flux line

- Flux around a core is given by $\phi = \dfrac{\mathcal{F}}{\mathcal{R}}$ Wb

  where

  Reluctance, $\mathcal{R} = \dfrac{l}{\mu A}$

  Permeance, $p = \dfrac{1}{\mathcal{R}} = \dfrac{\mu A}{l}$

  Its *dc* analog is

  $\mathcal{F}$ ~ *dc* voltage (potential)

  ϕ ~ current

  $\mathcal{R}$ ~ resistance

- *B*-*H* characteristic of magnetic material; exhibits saturation and hysteresis. Normal magnetization curve—used in magnetic circuit calculations
- Terms of importance in *B*-*H* characteristic

  residual flux, coercive force

  retentivity ; coercivity

- Faraday's Law

  Flux linkages $\lambda = \phi N$ Wb – T

  $$e = -\frac{d\lambda}{dt} = -N\frac{d\phi}{dt} \quad V$$

- Lenz's Law

  emf in Faraday's law opposes the change in flux ϕ

- Flux cutting rule

  $e = Blv \quad V$

  Apply Fleming right hand rule for direction of *emf*

- Lorentz Force Equation

  $F = B\,il$ N

  Bil rule for apply Fleming left hand rule for direction of force

- Self inductance

  $$L = N\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{\mathrm{d}\lambda}{\mathrm{d}t} \quad \mathrm{H}$$

  $$e = N\frac{\mathrm{d}i}{\mathrm{d}t} \quad V$$

  For linear case

  $$L = \frac{N\phi}{i} = \frac{\lambda}{i} \quad \mathrm{H}$$

  Also $L = \dfrac{N^2}{\mathcal{R}}$ H

- Mutual Inductance

$$M_{12} = \frac{\lambda_{12}}{i_2}, \quad M_{21} = \frac{\lambda_{21}}{i_1} \ H$$

For bilateral magnetic circuit

$$M = M_{12} - M_{21} = k\sqrt{L_1 L_2}, \ k = \text{coupling coefficient}$$

For tight coupling

$$M = \sqrt{L_1 L_2}$$

- Energy stored in magnetic field

$$W_f = \frac{1}{2} i\lambda = \frac{1}{2} Li^2 \ \text{J}$$

or $$W_f = \frac{1}{2}\frac{N^2 i^2}{\mathcal{R}} = \frac{1}{2}\phi^2 \mathcal{R} \ \text{J}$$

In a magnetic circuit with air-gap, major portion of the field energy resisdes in the air-gap.

- AC operation of magnetic circuits

Induced emf in excitation coil is $E = V = 4.44 f N \phi_{max}$ V.

REVIEW QUESTIONS

1. State Ampere's law. How do you use the law to define a flux line and flux density?
2. Define the main magnetic quantities needed to deal with magnetic circuits. How are these inter-related?
3. What is the magnetic force which creates magnetic flux density? What name is used for it?
4. Write the expression for B at distance r from a long conductor carrying current i. What is the path along which B is constant and its direction?
5. In question 4 a conductor of length l carries current i_2 in the same direction as the long conductor and is parallel to it. What is the force on it and its direction? Use the left hand rule.
6. What is permeability and relative permeability? What are the magnetic quantities it relates?
7. State Ampere's circuit law. How is it used in magnetic circuit analysis?
8. State the Ohm's law of magnetic circuit.
9. Explain what is magnetomotive force and compare it with electromotive force.
10. Define magnetic circuit reluctance and how is it analogous to electric circuit resistance? Can we use series / parallel combinations of reluctance?
11. In a magnetic circuit, the core has an air-gap. Why is the reluctance of the air-gap much higher than the rest of the circuit?
12. For a magnetic core with air-gap, linear analysis yields acceptable result. Justify.

13. Write the expression for energy stored magnetic field. Convert the expression to energy density form.
14. In a magnetic core with air-gap carrying flux ϕ, why does major portion of the stored energy reside in the air-gap.
15. State the dot convention for mutually coupled coil in terms of the flux direction and also in terms of emf induced if the flux is varying.
16. For the couple circuit of the figure below place the dot on each coil.

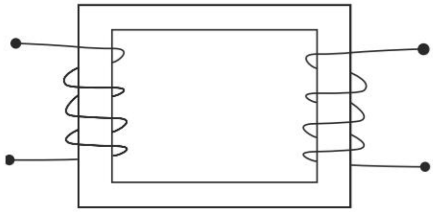


Fig. 7.35

17. Distinguish between leakage flux and fringing flux. How are these accounted for empirically?
18. A coil wound on an iron core is excited from an ac source at voltage V_{rms}. Write the expression for ϕ_{max} in the core. Why is it independent of the core reluctance?

PROBLEMS

7.1 A coil of 1000 turns is wound on a laminated core of sheet steel having a cross-section of 25 cm^2 and a mean length of 50 cm. The stacking factor is 0.90. What is the current required to produce a core flux of 3 mWb. Use the magnetization curve of Fig. 8.10.
Hint: Stacking factor = net area of cross-section of steel/gross area of core cross-section.

7.2 An air-gap of 2 mm length is cut at right angle to the core cross-section in the case of Prob. 7.1. What should be the value of coil current to maintain the same flux density? Neglect fringing.

7.3 The coil current in Prob. 7.2 is adjusted to 1.5 A. Find the total core flux.

Hint: $AT = \dfrac{B_c}{\mu_0} l_g + H_c l_c$; $(B_g = B_c)$; $AT = 1000 \times 1.5 = 1500$

$B_c = f(H_c)$, the magnetization curve

Solve these two equations numerically or graphically.

7.4 Solve Prob. 7.2 if the air-gap of 2 mm length is cut at an angle of 45°.

7.5 In the magnetic circuit of Fig. 7.36 the coil F_2 is supplying 500 AT in the direction indicated. Find the AT (in magnitude and direction) that the coil F_1 must provide to produce a flux of 4 mWb in the air-gap in the central limb from A to B. The relative permeability of the core is 4500.

7.6 For the magnetic circuit shown in Fig. 7.37, the magnetization curve of the core is as follows:

H (AT/m)	200	400	500	600	800	1000	1400
B (T)	0.46	0.87	0.98	1.08	1.23	1.33	1.48

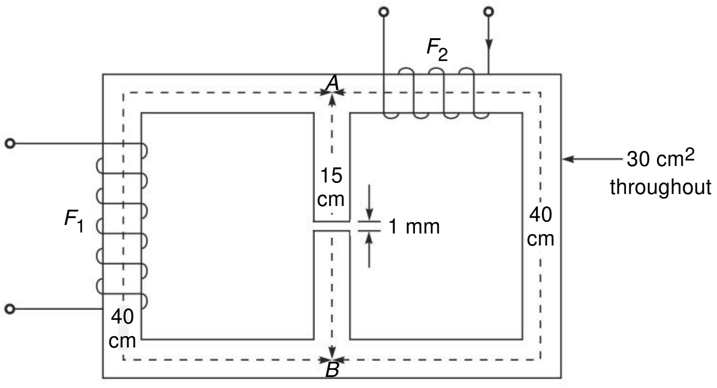


Fig. 7.36

Calculate the exciting current required to create a flux of 0.25 mWb in the air-gap. What is the flux in the central limb?

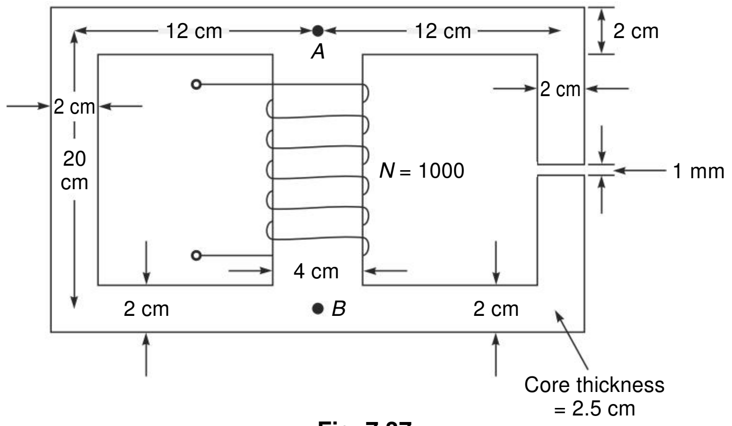


Fig. 7.37

7.7 In the magnetic circuit shown in Fig. 7.38 the area of cross-section of the central limb is 12 cm^2 and that of each outer limb (*A* to *B*) is 6 cm^2. A coil current 0.5 A produces 0.5 mWb in the air-gap. Find the relative permeability of the core material.

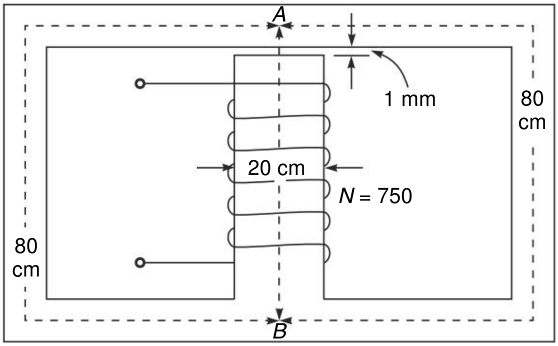


Fig. 7.38

7.8 For the magnetic circuit of Fig. 7.39.

(a) Calculate the energy stored in the core and in the air-gap for a coil current of 4 A. What will these values be if $\mu_r = \infty$?

(b) Calculate the excitation current and induced emf in the coil to produce a flux of 0.4 sin 314 t mWb in the air-gap.

(c) Calculate the inductance of the coil. What will be its value for $\mu_r = \infty$?

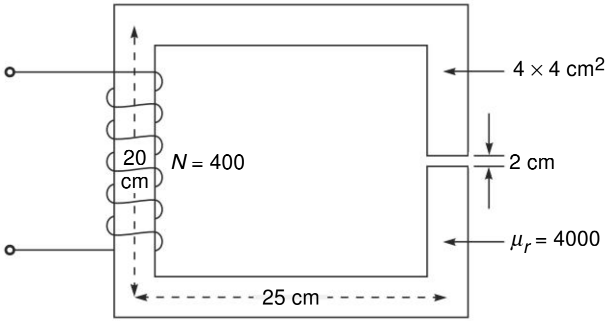


Fig. 7.39

7.9 For the electromagnetic relay of Fig. 7.40 calculate the maximum force on the armature if the saturation flux density in the iron part is 1.8 T.
Given: Cross-sectional area of the core = 5 cm × 5 cm; coil turns 10000
Ans : 6.45×10^3 N, attraction.

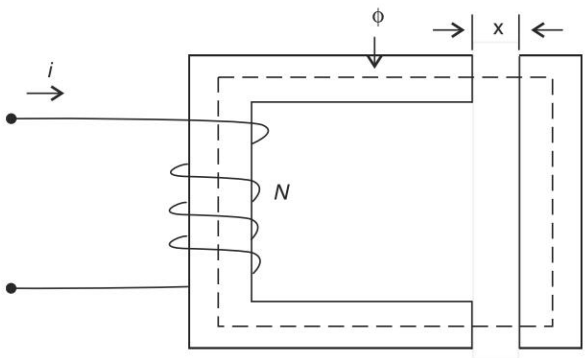


Fig. 7.40

Chapter 8

TRANSFORMERS

MAIN GOALS AND OBJECTIVES

- *Transformer, static device, transformation of voltage / current levels and need thereof.*
- *Constructional features, materials*
- *Ideal transformer, relationships, impedance transformation*
- *Real transformer, development of circuit model (equivalent circuit), referring circuit parameter from one to the other side*
- *Determination of circuit model parameters*
- *PU system—choice of bases*
- *The performance indices—voltage regulation, efficiency*
- *Autotransformer connection, application, advantages*
- *Three phase transformer connection*

8.1 INTRODUCTION

Economical and technologically feasible voltage levels at which large chunks of electric power can be generated are typically 11-37 kV, while the most convenient utilization voltages are 230/400 V for industrial, commercial and domestic purposes. Large industrial motors may be run at 3.3, 6.6 or 11 kV. It is impossible to transmit directly, even over modest distances, the electric power as it is generated (11-37 kV). Unacceptably large power losses and voltage drops would result. As a rule of thumb, economical transmission voltage is 0.625 kV/km line-to-line, e.g. 400 kV for a line of about 640 km. It is therefore essential to *step-up voltages* at the sending (generating) end and to *step-down* at the receiving end. Usually more than one step of step-down may be necessary. Step-up and step-down of voltage levels is accomplished by means of static electromagnetic devices called *transformers*.

It was seen in Sec. 7.8 that alternating flux is set up in a core by a coil excited with ac voltage (Fig. 7.24), which in turn induces coil emf* of excitation frequency

* The magnitude of flux is determined by the fact that the coil emf must equal the excitation voltage (KVL).

proportional to the number of coil turns (Eq. (7.48)). If another coil is wound on the same core, the *mutual flux* (alternating) would induce emf in it also of the same frequency and of magnitude proportional to its coil turns. The ratio of the voltage of the two coils can be easily adjusted by means of their *turn-ratio*. Such a device, which indeed is a mutually coupled circuit, is called a transformer and is exhibited digrammatically in Fig. 8.1. The coil excited from the ac source is called the *primary* and receives electric power from the source. The other coil is called the *secondary* and the voltage induced in it could be used to feed a load. The subscript '1' will be associated with the primary and '2' with the secondary. Primary and secondary roles in a transformer are easily reversed by the prevailing electrical conditions at the two ports. To avoid confusion in practice the two transformer coils are known as *HV* (*high-voltage*) and *LV* (*low-voltage*) windings.

Also shown in Fig. 8.1 are the mutual and leakage flux paths. Since a significant part of the leakage flux paths is through air, leakage fluxes ϕ_{l1} and ϕ_{l2} are much less than the mutual flux ϕ.

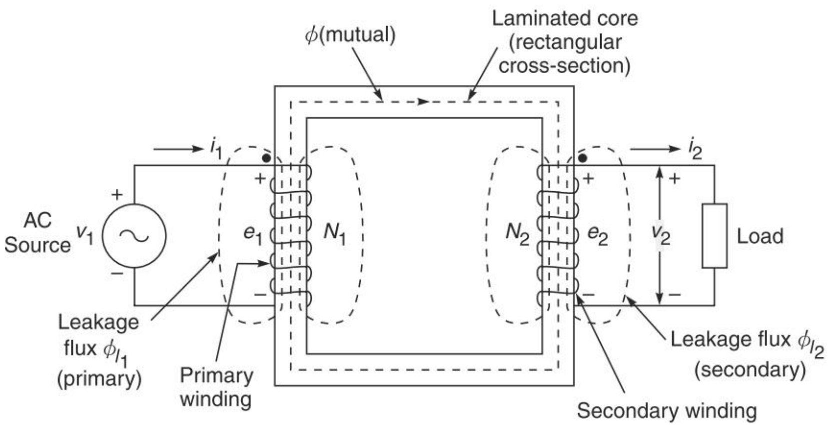


Fig. 8.1 A two winding transformer

The *dots* indicated on the two coils (windings) are the polarity marks. As the mutual flux alternates, these coil ends simultaneously acquire the same polarity. Also current into the dot in one coil and out of the dot in the other coil would tend to produce core flux in the opposite directions.

The primary and secondary windings are made of *copper/aluminum* conductors.

Transformer Core

The transformer core is made of highly permeable iron. It is essential so that excitation current required to establish core flux ϕ is a small percentage (2 - 4) of the primary current i_1, the rest being the useful component which corresponds to the load current. Further high permeable core provides light coupling between the two windings, i.e., leakage flux is kept very low.

As the core carries alternating flux to keep low the eddy current loss, the core is constructed with silicon steel laminations lightly insulated in form of rectangular strips (0.35 mm thickness for 50 Hz). This type of construction is used for power transformers operated at 25-400 Hz. Core type transformer construction is shown in Fig. 8.2(a). With some refinements, this core construction is used for audio transformers, (20 to 2000 Hz.) For transformers in electronic circuit operating at hundreds of kHz, powdered iron 'slug' is used as core with loose coupling as shown in Fig. 8.2(b). *Air-cored* transformers are used for radio devices and certain type of measuring and testing instruments.

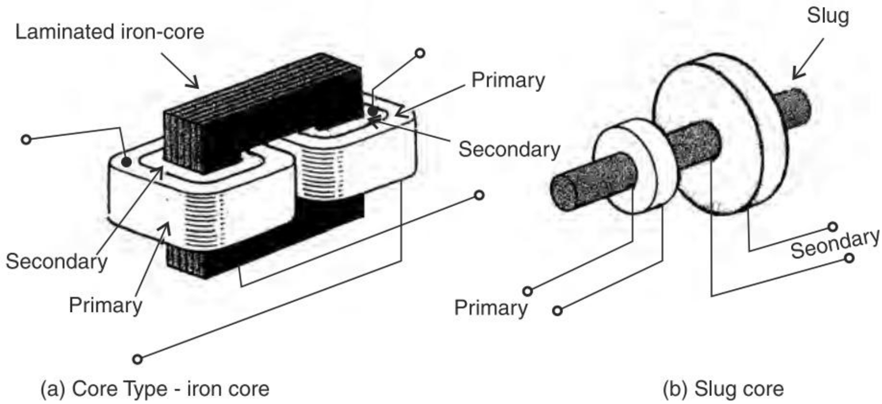


Fig. 8.2 Transformer core types—single phase

Core Types

Two types of geometrical core shapes and winding arrangements are used practically —*core type* and *shell type* as shown in cross-sectional views of Fig. 8.3 (a) and (b) of a single phase transformer. It is easily seen that core type has a longer mean flux path but a shorter mean length of coil turn.

The winding arrangement must be such as to reduce the leakage flux, which we will see later causes reactive voltage drops in both primary and secondary. In a core type transformer to reduce the leakage flux, half-LV and half-HV are wound on each limb as shown in Fig. 8.3 (a). For economical insulation, the LV coils are placed inside (adjoining the core) and HV coils are placed outside.

In a shell type transformer, there are three limbs with both windings placed on the central limb as shown in Fig. 8.3 (b). Half of the flux of the central limb is returned though each outer limb. To reduce leakage, LV and HV coil packets are sandwiched.

As per the statement in first para above, the core type construction requires more iron but less copper. So construction type is preferred, while shell is used for special requirements (explanation beyond the scope of this book).

Housing and Cooling To prevent ingress of moisture and deterioration of winding insulation, the built-in core and windings are placed in a steel tank filled with *transformer oil* as shown in Fig. 8.4. Oval or circular tubes are provided on the outside surfaces of the transformer tank, aiding in natural circulations of oil, which removes the heat of core and winding (I^2R) losses and transports it to the tank surfaces for cooling purpose. Oil circulation removes the heat generated by iron losses in the

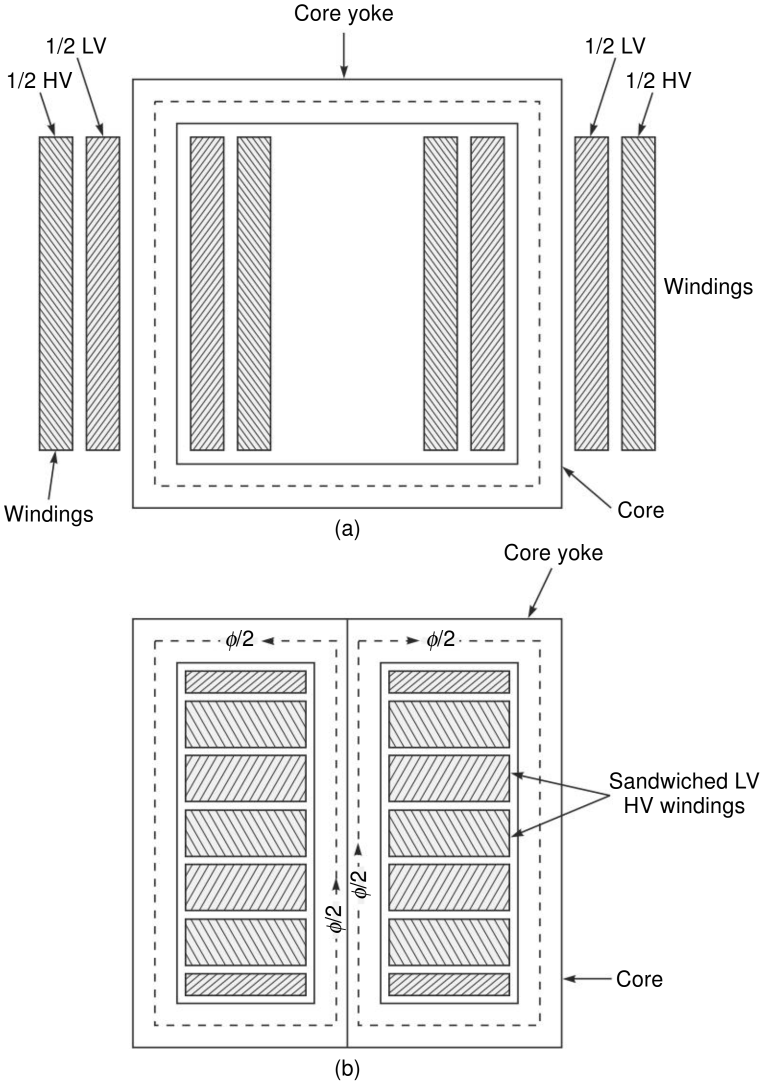


Fig. 8.3 (a) Core type transformer, (b) Shell type transformer

core. To prevent the coil from absorbing moisture from air and from being oxidized, the tank must be sealed and connected to the atmosphere through a narrow passage for breathing purposes. Inside this passage is placed silica gel for drying the air that the transformer breathes in.

Insulation

Windings made of copper/aluminium conductor (round/strip) are insulated by braided cotton, cotton tape, empire tape etc. and then impregnated with varnish under vacuum to dispace air packets inside the insulation. Windings are insulated from the

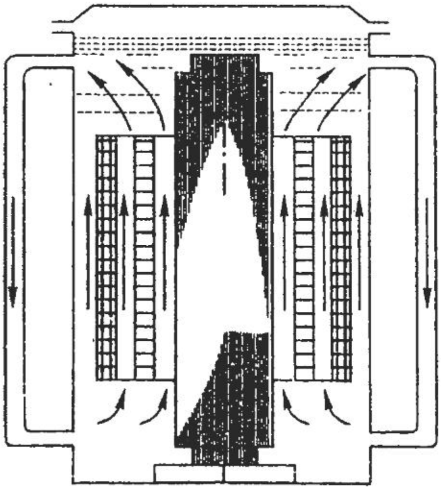


Fig. 8.4 Natural cooling in transformers

core by means of bakelite cylinders, and plastic spacers are also insulated from each other. Such insulation must be spaced to allow free space for circulation of coil.

8.2 IDEAL TRANSFORMER (IT)

In order to develop the mathematical model of a transformer, it is convenient to visualize a circuit element termed the "ideal" transformer by making certain assumptions in the realistic transformer. These assumptions only introduce insignificant model errors and are as follows:

- The transformer windings are resistanceless. This in effect means that ohmic power losses and resistance voltage drops in the actual transformer are neglected.
- The transformer core material has infinite permeability so that it requires zero mmf to create flux in the core.
- The leakage flux is negligible, i.e. no reactive voltage drops in windings.
- The transformer core losses are negligible.

Figure 8.5 is a diagrammatic representation of an ideal transformer.

The diagrammatic representation of an ideal transformer is provided in Fig. 8.5 showing the core to carry mutual flux, primary winding connected to sinusoidal voltage and secondary winding connected to a load. As the source voltage applied to the primary is sinusoidal, all voltage and current in this electromagnetic device are sinusoidal whose instantaneous values are indicated on the diagram. Using standard symbols, we can write these as *rms* value or as phasor.

Voltages and *emfs*

As the primary and secondary circuits are linked through the mutual flux, we begin with flux which is expressed as

$$\phi = \phi_{\max} \sin \omega t; \;\; \omega = 2\pi f \text{ rad/s} \tag{8.1}$$

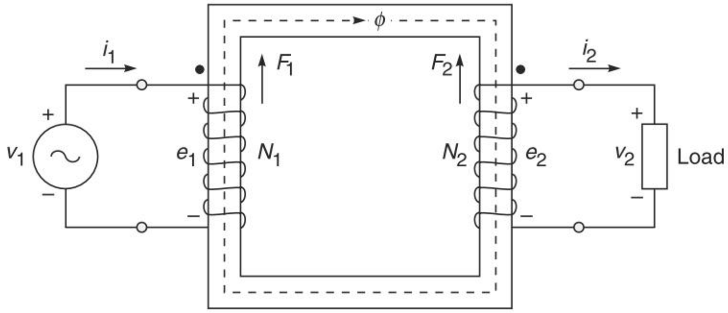


Fig. 8.5 Ideal transformer (*IT*)

The *emf* induced in primary winding balances the applied voltage as per KVL. Thus

$$v_1 = e_1 = N_1 \frac{d\phi}{dt} = \omega N_1 \phi_{max} \cos \omega t \tag{8.2}$$

The secondary induces *emf* which equals the load voltage and is similarly given by

$$v_2 = e_2 = \omega N_2 \phi_{max} \cos \omega t \tag{8.3}$$

In terms of *rms* values

$$V_1 = E_1 = \sqrt{2}\,\pi f N_1 \phi_{max} \tag{8.4}$$

and $$V_1 = E_2 = \sqrt{2}\,\pi f N_2 \phi_{max} \tag{8.5}$$

We find that the voltage and *emf*s are in phase and the flux lags by 90° (sin ωt lags cos ωt by 90°)

Transformation Ratio

It is found from the above equations that the voltage transformation ratio of *rms* values is

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = a \text{ (turn ratio)} \tag{8.6}$$

Also, in terms of phasors we have

$$\frac{\bar{V}_1}{\bar{V}_2} = \frac{\bar{E}_1}{\bar{E}_2} = a \text{ (turn ratio)} \tag{8.7}$$

as the voltages are in phase.

It is seen from these equations that in an ideal transformer, the voltages are in direct ratio of turns with no change in phase angle.

The maximum value of the flux is found from Eqs (8.4) and (8.5) to be

$$\phi_{max} = \frac{E_1}{\sqrt{2}\pi f N_1} = \frac{E_2}{\sqrt{2}\pi f N_2}\,; \sqrt{2}\pi = 4.44 \tag{8.8}$$

It is seen that ϕ_{max} is determined by the applied voltage and its frequency and is independent of current. This is a general result and, as we shall see in later chapter, applies also to *ac* machines.

The phasor diagram showing the primary and secondary voltage including *emf*s and flux is drawn in Fig. 8.6 wherein, as said already, the flux phasor $\varPhi$ lags $\bar{E}_1$ and $\bar{E}_2$ by 90°.

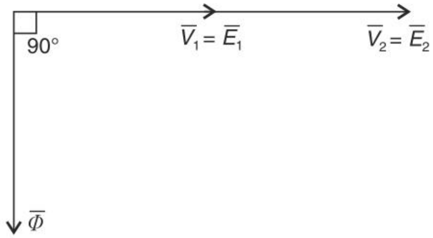


Fig. 8.6 Phasor diagrams of ideal transformer, currents not shown

Currents

The current drawn by the secondary load is

$$i_2 = \sqrt{2}\, I_2 \cos(\omega t - \theta);\ \theta = \text{phase angle assumed lagging} \tag{8.9}$$

By Lenz's law, this current causes *mmf* to oppose the core flux ϕ. In phasor terms, secondary *mmf* is

$$F_2 = \bar{I}_2 N_2,\quad \bar{I}_2 = I_2 \angle -\theta \tag{8.10}$$

As the core flux cannot change being governed by primary applied voltage and frequency, a circuit is drawn from the source to cause *mmf* $\bar{F}_1$ equal and opposite to $\bar{F}_2$. Thus

$$\bar{F}_1 = \bar{F}_2$$

or

$$\bar{I}_1 N_1 = \bar{I}_2 N_2 \tag{8.11}$$

which yield the *current transformation ratio*

$$\frac{\bar{I}_1}{\bar{I}_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{8.12}$$

We find that an ideal transformer *transforms the current in the inverse ratio of turns* and the phase is preserved. In terms of *rms* values

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{8.13}$$

The complete phasor diagram of the ideal transformer is drawn in Fig. 8.7 showing currents as well.

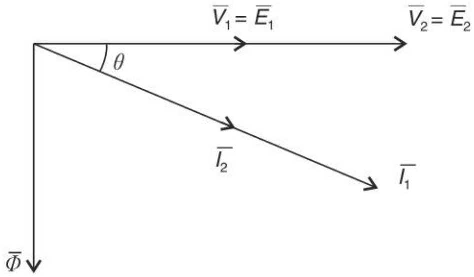


Fig. 8.7 Phasor diagram of ideal transformer

Important Note The transformation ratio holds for *emfs* induced by the mutual flux. This equals the voltage ratio for the ideal transformer because there is no voltage drop. The current transformation applies to the secondary current and its primary current equivalent. These concept will get clarified in Sections 8.3 and 8.4.

Volt-Amperes

Multiplying Eq. (8.7) with Eq. (8.13) yields

$$\frac{\bar{V}_1 \bar{I}_1}{\bar{V}_2 \bar{I}_2} = 1 \quad \text{or} \quad \bar{V}_1 \bar{I}_1 = \bar{V}_2 \bar{I}_2 \tag{8.14}$$

It means that VA in equals VA out, that is, there is no loss in the ideal transformer. Now take complex conjugate of Eq. (8.13) and multiply with Eq. (8.7). We get

$$\frac{\bar{V}_1 \bar{I}_1^*}{\bar{V}_2 \bar{I}_2^*} = 1 \text{ ; as 'a' is a real constant} \tag{8.15a}$$

Therefore

$$\bar{V}_1 \bar{I}_1^* = \bar{V}_2 \bar{I}_2^* = \bar{S} = P + jQ \tag{8.15b}$$

It indicates the balance of real and reactive power, a statement of no loss in an ideal transformer.

It is to be noted here that equality of phasor VA of Eq. (8.14) means equality of instantaneous power

$$p = v_1 i_1 = v_2 v_2$$

That is, there is no loss of instantaneous power.

Equivalent Circuit The equivalent circuit of an ideal transformer is drawn in Fig. 8.8.

Here $$\bar{V}_2' = a \bar{V}_2 \tag{8.16}$$

and $$\bar{I}_2' = \frac{1}{a} \bar{I}_2 \tag{8.17}$$

The three vertical lines are indicative of the core. It is not necessary to draw these in every diagram.

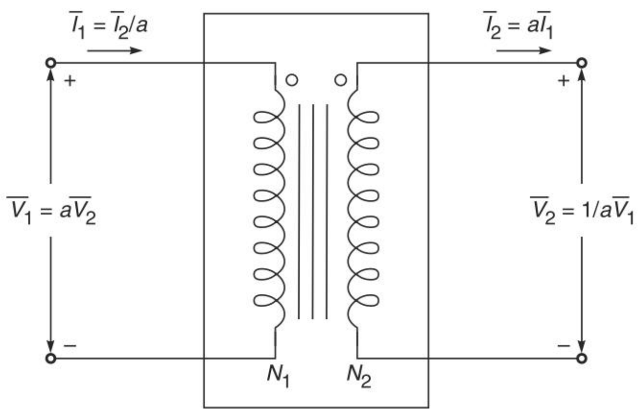


Fig. 8.8 Equivalent circuit of IT ($a = N_1/N_2$)

are called 'the secondary voltage and current referred to the primary'. Similarly we define

$$\bar{V}_1' = \frac{1}{a}\,\bar{V}_2 \tag{8.18}$$

$$\bar{I}_1' = a\,\bar{I}_1 \tag{8.19}$$

as 'the primary voltage and current referred to the secondary'.

It may be remarked here that V_2 applied to the secondary winding of an ideal transformer produces the same maximum core flux as V_1 applied to the primary winding (Eq. (8.8).

Impedance Transformation

In Fig. 8.9; an impedance $\bar{Z}_2$ is connected on the secondary side of the ideal transformer. The impedance as seen in the primary side is found as

$$\frac{\bar{V}_2}{\bar{I}_2} = \bar{Z}_2 \tag{8.20a}$$

or

$$\frac{(N_2/N_1)\bar{V}_1}{(N_1/N_2)\bar{I}_1} = \bar{Z}_2$$

$$\frac{\bar{V}_1}{\bar{I}_1} = \left(\frac{N_1}{N_2}\right)^2 \bar{Z}_2 \tag{8.20b}$$

$$= a^2\bar{Z}_2 = \bar{Z}_2' \tag{8.21}$$

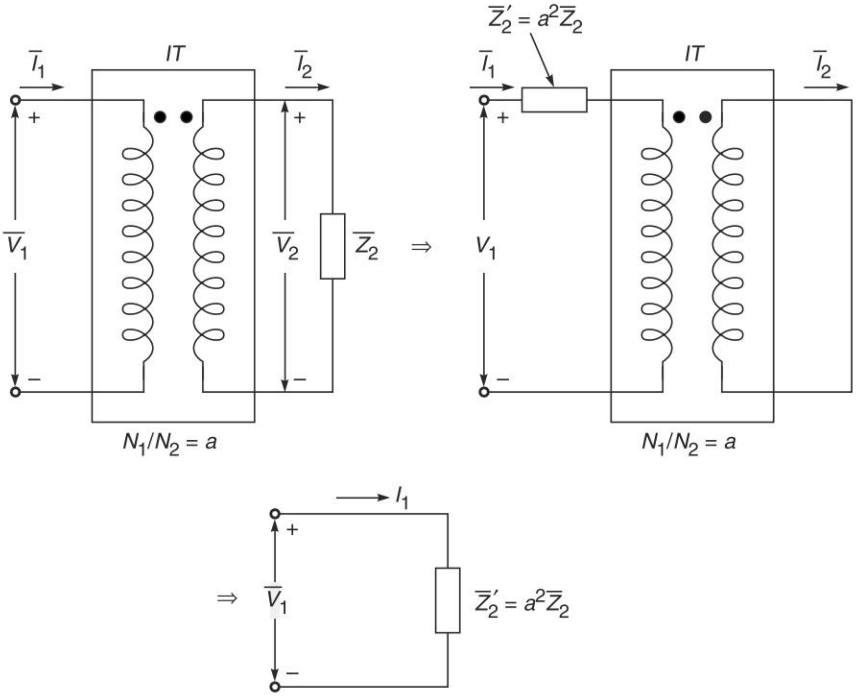


Fig. 8.9 Impedance transforming property of *IT*

The impedance transformation property (Eq. (8.21)) is illustrated diagrammatically in Fig. 8.9. $\bar{Z}'_2$ is called 'the secondary impedance referred to the primary', vice versa applies equally, i.e. $\bar{Z}'_1 = (1/a^2)\ \bar{Z}'_1$. The *impedance transforms from one side of the ideal transformer to the other in the direct square ratio of turns.*

Equation (8.21) can be put in the admittance form as

$$\bar{Y}'_2 = \frac{1}{a^2}\ \bar{Y}_2 \tag{8.22}$$

i.e., the admittance transforms from one side of the ideal transformer to the other in the inverse square ratio of turns.

The impedance transforming property of the transformer is employed in impedance matching in electric circuits (Eq. (4.48)).

Example 8.1 An ideal transformer has a turn-ratio of 100/300. The LV winding is connected to a source of 3.3 kV, 50 Hz. An impedance of $(100 + j\,35)\ \Omega$ is connected across the secondary terminals. Calculate (a) the value of maximum core flux, (b) the primary and secondary currents, (c) the real and reactive powers supplied by the source to the transformer primary, and (d) the value of impedance which connected directly across the source would draw the same real and reactive power as in (c).

Solution

(a) From Eq. (8.8)

$$\phi_{max} = \frac{V_1}{\sqrt{2}\,\pi f N_1} = \frac{3.3 \times 1000}{\sqrt{2}\,\pi \times 50 \times 100}$$
$$= 0.149 \text{ Wb}$$

(b)
$$V_2 = 3.3 \times (300/100) = 9.9 \text{ kV}$$
$$\bar{I}_2 = \frac{9.9 \times 1000}{(100 + j35)} = 93.44\ \angle -19.3° \text{ A}$$
$$\bar{I}_1 = (300/100) \times 93.44\ \angle -19.3°$$
$$= 280.3\ \angle -19.3° \text{ A}$$

(c)
$$\bar{S} = \bar{V}_1 \bar{I}_1^* = \bar{V}_2 \bar{I}_2^*$$
$$= 9.9 \times 93.44\ \angle 19.3° \text{ kVA}$$
$$= (873.1 + j\,305.7)$$

Hence $P_1 = 873.1$ kW, $Q_1 = 305.7$ kVAR

(d)
$$\bar{Z}_1 = \bar{Z}'_2 = a^2 \bar{Z}_2$$
$$= (100/300)^2\,(100 + j\,35) = 11.11 + j\,3.89\ \Omega$$

8.3 ACCOUNTING FOR FINITE PERMEABILITY AND CORE LOSS

In a real transformer, the core has finite permeability and to establish flux in the core the primary winding would draw a current component called *magnetizing current* from the source over and above the load current. Assuming the core to be linear, let the core reluctance be $\mathcal{R}$. The magnetizing current is then given by

$$i_m = \frac{\mathcal{R}\phi}{N_1}\text{ A} \; ; \; \phi = \phi_{\max} \sin \omega t \tag{8.23}$$

Substituting for ϕ from Eq. (8.5)

$$i_m = \sqrt{2}\left(\frac{\mathcal{R}V_1}{\omega N_1}\right) \sin \omega t \tag{8.24}$$

It is seen from Eqs (8.24) and (8.1) that the magnetizing current is in phase with the core flux and lags the induced emf by 90° as drawn in the phasor diagram of Fig. 8.10.

In phasor form Eq. (8.24) is written as

$$\bar{I}_m = \frac{\bar{V}_1}{jX_m} = -jB_m\bar{V}_1; \; B_m = \frac{1}{X_m} = \frac{\mathcal{R}}{\omega N_1} \tag{8.25}$$

In circuit model of the magnetizing current, it is the current drawn by a magnetizing reactance X_m (or magnetizing susceptance B_m) from the primary voltage source.

A real core will also have power loss (core loss) because it carries alternating flux. It can be modelled as a resistance R_i (or conductance G_i) across the primary voltage source. This is a sufficiently accurate representation for constant frequency operation. This power loss is $G_i\ V_1^2$ while the actual core loss has two components, viz. the eddy-current loss proportional to V_1^2 ($\phi_{\max} \propto V_1$) and hysteresis loss proportional to $V_1^{1.6}$; but square law assumption does not cause any significant error.

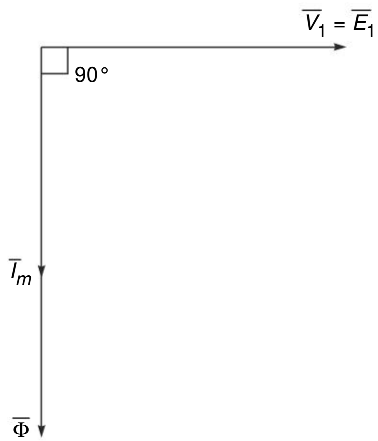


Fig. 8.10 Magnetizing current phasor diagram

The net exciting current* drawn by the primary to create core flux is then

$$\bar{I}_0 = \bar{I}_m + \bar{I}_i \tag{8.26}$$

where $\bar{I}_m = -jB_m\bar{V}_1$ = magnetizing current

$\bar{I}_i = G_i\bar{V}_1$ = core (iron) loss current

The phasor diagram of the exciting current is drawn in Fig. 8.11. While the magnetizing the current I_m lags the voltage V_1 (primary applied voltage), the core loss current $\bar{I}_1$ is in phase with it. Further, the core loss is quite small in transformers as the core is constructed with laminated silicon steel. So magnitude-wise $I_i << I_m$. Therefore, the angle θ_0 in the phasor diagram is close to 90°. It also means that the *exciting current I_0 has a low lagging power factor.*

The circuit model of the transformer at this stage of development is drawn in Fig. 8.12. The only assumption that is still made is that the windings are resistanceless and their leakage flux is negligible.

* Loosely the term magnetizing current will be used to mean exciting current.

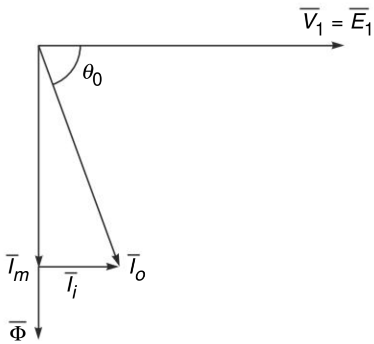


Fig. 8.11 Exciting current phasor diagram

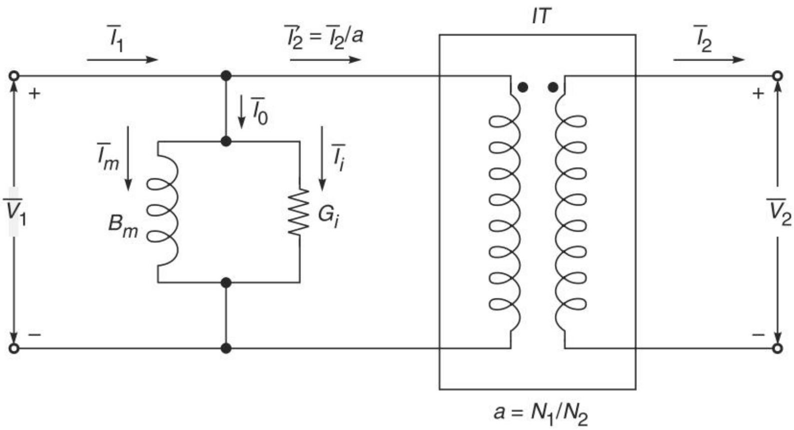


Fig. 8.12 Circuit model of transformer (resistance and leakage neglected)

In the above development the core has been assumed to be linear. In the real core with hysteresis, to produce sinusoidal core flux, the exciting current i_0 will be periodic but nonsinusoidal with a predominant third harmonic.* Except for special effects, this fact is usually ignored and i_0 is taken as the equivalent sinusoidal current with the same rms value.

It is seen from Fig. 8.12 that the resultant current (under load) drawn from the primary is

$$\bar{I}_1 = \bar{I}_0 + \bar{I}'_2 \tag{8.27}$$

The exciting current (also called magnetizing current) is predominantly reactive and is essential but not a load-delivering component of the primary current. It must therefore be kept as low as possible. This is why transformer cores are constructed of high-permeability sheet steel. The magnetizing current in a transformer is in the range 2–5% of the rated current. Further, it being mainly reactive (θ_0 close to 90°), rms magnitudewise

$$I_1 \approx I'_2 = I_2/a \tag{8.28}$$

* Taking the hysteresis curve of Fig. 7.13 and assuming B to be sinusoidal, the reader may find out by point-by-point method the waveshape of the exciting current.

It is also clear from Fig. 8.12 that on no-load ($I'_2 = I_2/a = 0$) the transformer primary would draw only the exciting current from the source, which therefore is synonymous with the term *no-load current* (hence the symbol I_0).

8.4 CIRCUIT MODEL OF TRANSFORMER

Both primary and secondary of a transformer have winding resistances. Apart from this the two windings have leakage flux; ϕ_{l1} linking only the primary and ϕ_{l2} linking only the secondary (see Fig. 8.1). These leakage fluxes do not contribute in the process of energy transfer, which takes place via the mutual flux ϕ_m, but these cause the primary and secondary windings to possess leakage inductances and, therefore, leakage reactances at steady sinusoidal operation. The winding resistances and leakage reactances can be lumped in series with the ideal windings (resistance and leakage-less) in a circuit model. The ideal primary and secondary windings along with the core (which now carries only the mutual flux ϕ_m) indeed constitute the ideal transformer. Let windings resistances be r_1, r_2 and winding reactances (inductive) be x_1 and x_2.

The complete circuit model (commonly called *equivalent circuit*) is drawn in Fig. 8.13. It comprises the following circuit elements.

1. Magnetizing shunt branch—B_m and G_i in parallel
2. Primary resistance r_1 and leakage reactance x_1 in series
3. Ideal transformer (turn ratio $N_1/N_2 = a$)
4. Secondary resistance r_2 and leakage reackage x_2 in series

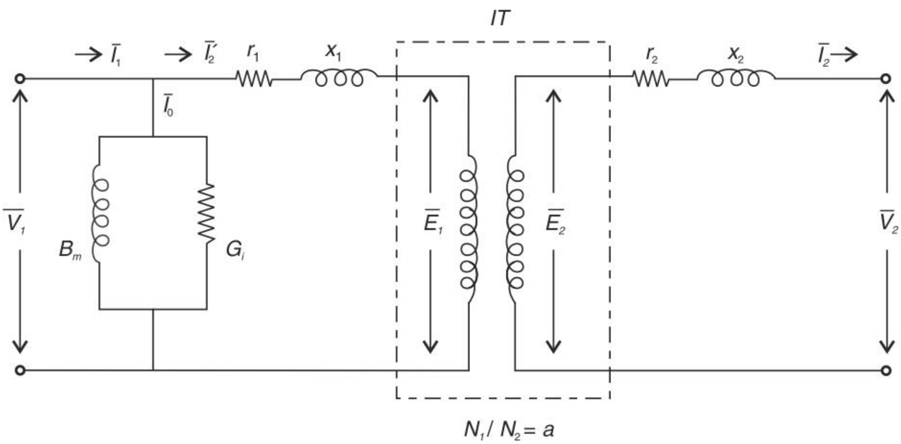


Fig. 8.13

By the technique of impedance transformation, these can be transferred to one side of the transformer say the primary. Then equivalent series resistance and reactance of the transformer referred to the primary side are

$$\text{Equivalent resistance } R = r_1 + r'_2 = r_1 + a^2 r_2 \tag{8.29}$$

Equivalent resistance $X = x_1 + x'_2 = x_1 + a^2 x_2$ (8.30)

The transformer circuit model (equivalent circuit) of Fig. 8.13 with secondary resistance and reactance, referred to the primary side, gets modified to the form shown in Fig. 8.14 where

$$\bar{I}'_2 = \bar{I}_2/a \tag{8.31}$$

$$\bar{V}'_2 = a\bar{V}_2 \tag{8.32}$$

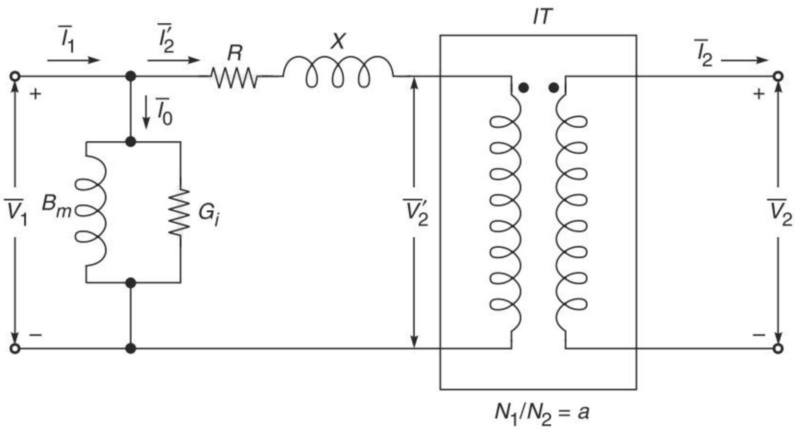


Fig. 8.14 Circuit model of transformer referred to the primary side

In the circuit model of a transformer, it is not necessary to carry the ideal transformer as these voltage and current conversions (Eqs (8.31) and (8.32) can always be carried out computationally. The transformer circuit model with ideal transformer left out is drawn in Fig. 8.15.

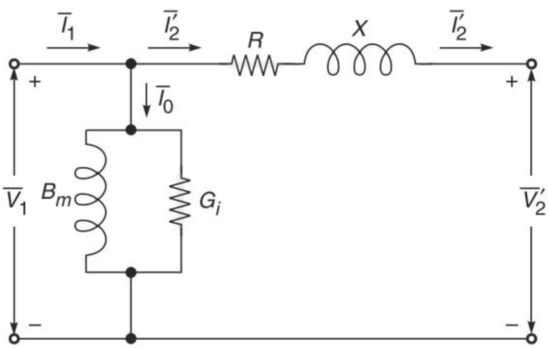


Fig. 8.15 Circuit model of transformer (*IT* left out)

The magnetizing shunt branches in the circuit model of Fig. 8.15 do not affect voltage computation and may therefore be ignored. Further, since R is much smaller in a transformer than X, R may also be ignored. These two steps lead to the simplified circuit models of Fig. 8.16. It is also unnecessary to carry the superscript 'dash' on current and voltage.

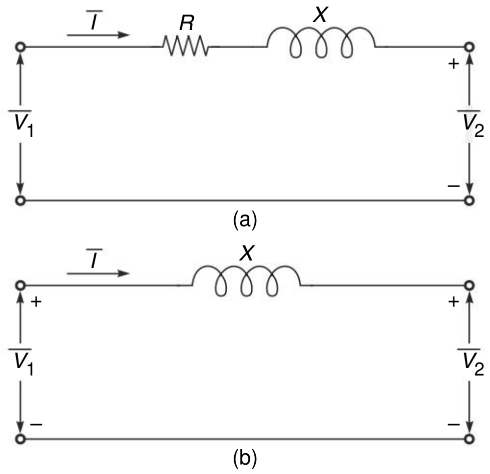


Fig. 8.16 Simplified circuit model of transformer

SUMMARY

1. It is seen from the complete circuit model of a transformer, as in Fig. 8.13 that because of voltage drops in the primary and secondary resistances and leakage reactances, we have

$$\frac{\bar{V}_1}{\bar{V}_2} \approx \frac{\bar{E}_1}{\bar{E}_2} = \frac{N_1}{N_2} = a$$

as the turn ratio is the ratio of induced *emfs*. However, as the series voltage drops are very small, it is sufficiently accurate to assume that

$$\frac{\bar{V}_1}{\bar{V}_2} \approx \frac{N_1}{N_2} = a$$

like in an ideal transformer.

2. We observe from the circuit model of Fig. 8.14 that it has a shunt branch (B_m and G_i) across the source voltage $\bar{V}_1$ and a series branch (R and X) between $\bar{V}_1$ and $\bar{V}_2$. Therefore we can treat these independently—shunt branch for finding exciting current $\bar{I}_0$ and the series branch for calculating voltage drop $(\bar{V}_1 - \bar{V}_2)$. The input current can then be found as the sum of the two currents - exciting current plus load current i.e. $\bar{I}_1 = \bar{I}_0 + \bar{I}_2$.

Example 8.2 A 150 kVA, 2400/240/240 V single-phase transformer has the following parameters of the equivalent circuit (circuit model as seen of 2400 V side.)

$$r_1 = 0.2\ \Omega \qquad r_2 = 2 \times 10^{-3}\ \Omega$$
$$x_1 = 0.6\ \Omega \qquad x_2 = 6 \times 10^{-3}$$

and

$$R_1 = 10\ \text{k}\Omega \qquad X_m = 1.6\ \text{k}\Omega$$

(a) Calculate the equivalent resistance and leakage reactance as seen of the HV side.

(b) Convert the resistance and reactance values referred to LV side.

(c) At rated current, calculate the impedance voltage drop on the HV side. Also calculate the voltage drop as percentage of the rated voltage.

(d) With the secondary open (no load), what current will be drawn from HV side (2400 V source). What is its pf?

Solution

We will take the turn ratio same as the ratio of rated voltage. Thus

$$\text{Turn ratio, } a = \frac{2400}{240} = 10$$

(a) As seen on HV side

$$R = 0.2 + (10)^2 \times 2 \times 10^{-3}$$
$$= 0.2 + 0.2 = 0.4\ \Omega$$
$$X = 0.6 + (10)^2 \times 6 \times 10^{-3} = 1.2\ \Omega$$

(b) As referred to the LV side

$$R(LV) = \frac{1}{(10)^2} \times 0.4 = 4 \times 10^{-3}\ \Omega$$

$$X(LV) = \frac{1}{(10)^2} \times 1.2 = 12 \times 10^{-3}\ \Omega$$

(c) With reference to Fig. 8.8

$$I_2' = \frac{150 \times 1000}{2400} = 62.5 \text{ A}$$

Series impedance

$$\bar{Z} = R + jX = 0.4 + j\,1.2$$

or $\quad Z = 1.265\ \Omega$

Voltage drop = $I_2' Z = 62.5 \times 1.265 = 79$ V

$$\text{Percentage voltage drop} = \frac{79}{2400} \times 100 = 3.29\%$$

(d) With secondary open (no load), only the shunt magnetizing branch will draw current from 2400 V source.

$$\therefore \quad I_m = \frac{2400}{X_m} = \frac{2400}{1.6 \times 10^3}$$
$$= 1.5 \text{ A}$$

$$\therefore \quad I_i = \frac{2400}{R_i} = \frac{2400}{10 \times 10^3}$$
$$= 0.24 \text{ A}$$

$$\bar{I}_0 = 0.24 - j\,1.5 \text{ A} = 1.52 \angle -81° \text{ A}$$

$$\text{pf} = \cos 81° = 0.156 \text{ lagging}$$

Example 8.3 A 200 kVA, 1100/415 V, 50 Hz single-phase transformer has 80 turns of secondary. Calculate

(a) approximate value of primary turns
(b) the full-load secondary current and approximate value of primary current,
(c) maximum core flux
(d) the secondary impedance which would fully load the transformer and its value as seen on the primary side.

Solution

Assumption: We take the transformer to be ideal and turn ratio to be the same as the voltage ratio.

(a) $\dfrac{N_1}{N_2} = \dfrac{11000}{415}$, $\qquad N_1 = \dfrac{11000}{415} \times 80 = 2120.48$

As turns must be integral, we take $N_1 = 2120$

$$a = \frac{N_1}{N_2} = \frac{2120}{80}$$

(b) Full load = 200 kVA

Voltage applied to primary $V_1 = 11000$ V

Secondary voltage, $V_2 = 11000 \times \dfrac{80}{2120} = 415.9 \approx 415\text{V}$

$$I_2 = \frac{200 \times 10^3}{415} = 481.9 \text{ A}$$

$$I_2 = 481.9 \times \frac{80}{2120} = 18.19 \text{ A}$$

The primary current value is approximate because we have taken the transformer to be ideal which means that the exciting current drawn from the source has been neglected. As the exciting current is very small and has a phase angle close to 90°, this is a fair approximation.

(c) Full load secondary kVA = 200; series impedance of transformer ignored

$$\frac{(415)^2}{Z_2} \times 10^{-3} = 200$$

Z_2 (for full loading) = 0.861Ω

As seen on primary

$$Z_2' = a^2 Z_2 = \left(\frac{2120}{80}\right)^2 \times 0.861 = 6046\ \Omega$$

Example 8.4 The core of a transformaer, having dimension as in Fig. 8.17, is made of silicon steel (B-H curve of Fig. 8.17). Calculate the no-load current when the primary is excited at 200 V, 50 Hz. Assume the iron loss to be 23 kW/m^3 of core volume. What is the *pf* of the no-load current and the value of no-load power drawn from the source.

Solution

To find magnetizing current:

$$\phi_{max} = \frac{V_1}{\sqrt{2}\ \pi f N_1} = \frac{200}{\sqrt{2}\ \pi \times 500 \times 150}$$

$$= 6 \text{ mWb}$$

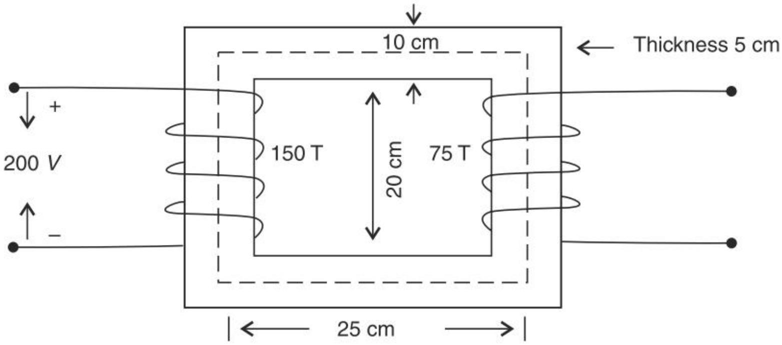


Fig. 8.17

Cross-sectional area of core, $A_c = 10 \times 5 = 50\ \text{cm}^2$

So $$B_{\max} = \frac{\phi_{\max}}{A_c} = \frac{6 \times 10^{-3}}{50 \times 10^{-4}} = 1.2\ \text{T}$$

From the *B-H* curve of Fig. 8.17, we find

$H\ (\text{max}) = 250\ AT/\text{m}$

Mean flux path length, $l = 2 \times (25 + 10) + 2\ (20 + 10)$

$= 130\ \text{cm}$

$$AT\ (\text{max}) = 250 \times 130 \times 10^{-2} = 325$$

$$i_m\ (\text{max}) = \frac{325}{150} = 2.17\ \text{A}$$

$$I_m\ (\text{rms}) = \frac{2.17}{\sqrt{2}} = 1.535$$

To find core loss current:

Core volume $= 2\ (25 + 2 \times 10) \times 10 \times 5 + 2 \times 20 \times 10 \times 5$

$= 6500\ \text{cm}^3$

Core loss, $P_n = 23 \times 6500 \times 10^{-6} = 0.15\ \text{kW} = 150\ \text{W}$

$P_i = V_1 I_1$

Core loss current, $I_i = \dfrac{150}{200} = 0.75\ \text{A}$

Example 8.5 A transformer has 150 primary turns and 75 secondary turns. Its primary is excited at 200 V and the secondary is loaded with an impedance of 5<30° Ω. Calculate the primary current and its pf. State assumptions made.

Solution

Assumption: Transformer is ideal

Given $\bar{Z}_2 = 5\ \angle 30°\ \Omega$

$$\bar{I}_2 = \frac{200\ \angle 0°}{5\ \angle 30°} = 40\ \angle -30°\ \text{A}$$

Primary current, $I_1 = \left(\frac{N_2}{N_1}\right) I_2$

or $$I_1 = \frac{75}{100} \times 40 \angle -30° = 20 \angle -30° \ \Omega$$

Power factor = cos 30° = 0.860 lagging

Example 8.6 An audio-frequency transformer is employed to couple a 60Ω resistive load to a source of 6 V in series with a resistance of 2400 Ω.

(a) Determine the transformer turn-ratio to ensure that maximum power is transferred to the load.

(b) Calculate the value of the maximum power and corresponding load current and voltage.

Solution

As per maximum power theorem, the load resistance reflected to the source side must equal source resistance. Thus

(a) $$\left(\frac{N_1}{N_2}\right)^2 \times 60 = 2400$$

Or $$\frac{N_1}{N_2} = \sqrt{40} = 6.325$$

(b) Maximum load power, P_L (max) $= \frac{1}{2}\left[\frac{(60)^2}{2400 + 2400}\right] = 0.375$ W

Current drawn from source, $I_1 = \frac{6}{2400 + 2400} = 1.25 \times 10^{-3}$ A

Load current, $I_L = I_2 = \left(\frac{N_1}{N_2}\right) I_1 = 1.25 \times 10^{-3} \sqrt{40} = 0.79$ mA

Load voltage $V_L = V_2 = \left(\frac{N_2}{N_1}\right) \times \frac{1}{2} \times 6 = \frac{3}{6.325} = 0.474$ V

Since there is no-load, the current drawn from the mains is

$$\bar{I}_0 = \bar{I}_i - j\, I_m = 0.75 - j\, 1.535$$
$$= 1.71 \angle -64°$$

No- load pf = cos 64° = 0.44 lagging

Power drawn from mains = core loss

= 150 W

Check: $P_0 = V_1 I_0 \cos \theta_0 = 200 \times 1.71 \times 0.44 = 150$ W

8.5 PER UNIT SYSTEM

It is often convenient to scale electrical quantities in per unit of the base or reference values of these quantities. The basic per unit (pu) scaling equation is

$$\text{Per unit value} = \frac{\text{Actual value}}{\text{Base value}} \tag{8.33}$$

The pu system offers the advantage that the device parameters tend to fall in a relatively narrow range, making the erroneous values conspicuous. Also in computations, one does not have to deal with very small and very large numbers. In a power system (with many transformers of different voltage ratio), ideal transformers are no longer necessary in the model.

Base values are related to each other by the usual electrical laws. For a single-phase system,

$$P_B, Q_B, (\text{VA})_B = V_B I_B \tag{8.34a}$$

$$R_B, X_B, Z_B = \frac{V_B}{I_B} \tag{8.34b}$$

$$G_B, B_B, Y_B = \frac{I_B}{V_B} \tag{8.34c}$$

$(VA)_B$ and V_B are first to be selected, then it follows from Eq. (8.34) that

$$Z_B = \frac{V_B^2}{(\text{VA})_B} = \frac{1000(\text{kV})_B^2}{(\text{kVA})_B} \tag{8.35}$$

Then $$Z\,(pu) = \frac{Z(\Omega)\times(\text{VA})_B}{V_B^2} \tag{8.36}$$

In large devices and systems, it is more practical to use base values in kVA/MVA and kV. Equation (8.36) can then be written as

$$Z(\text{pu}) = \frac{Z(\Omega)\times(\text{kVA})_B}{1000(\text{kV})_B^2} \tag{8.37a}$$

or $$Z(\text{pu}) = \frac{Z(\Omega)\times(\text{MVA})_B}{(\text{kV})_B^2} \tag{8.37b}$$

In changing Z(pu) from one set of base values to another,

$$Z(\text{pu})_{\text{new}} = Z(\text{pu})_{\text{old}} \times \frac{(\text{MVA})_{B,\,\text{new}}}{(\text{MVA})_{B,\,\text{old}}} \times \frac{(\text{kV})_{B,\,\text{old}}^2}{(\text{kV})_{B,\,\text{new}}^2} \tag{8.38}$$

In a 3-phase star-connected system (equivalent star can always be found)

$(\text{MVA})_{P,\,B}$ = MVA base per phase

$(\text{MVA})_{3P,\,B}$ = MVA base 3-phase

$(\text{kV})_{P,\,B}$ = kV base line-to-neutral

$(\text{kV})_{L,\,B}$ = kV base line-to-line

Then

$$(\text{kV})_{L,\,B} = \sqrt{3}\,(\text{kV})_{P,\,B} \tag{8.39}$$

$$(\text{MVA})_{3P,\,B} = 3\,(\text{MVA})_{P,\,B} \tag{8.40}$$

It can be easily shown that

$$V_P\,(\text{pu}) = V_L\,(\text{pu});\ \text{no factor of } \sqrt{3} \tag{8.41}$$

$$(\text{MVA})_P\,(\text{pu}) = (\text{MVA})_{3P}(\text{pu});\ \text{no factor of } 3 \tag{8.42}$$

$$I_{P,\,B} = I_{L,\,B} = \frac{(\text{MVA})_{3P,\,B}}{\sqrt{3}\,(\text{kV})_{L,B}} \tag{8.43}$$

Now

$$Z_B = \frac{((\mathrm{kV})_{L,B}/\sqrt{3})^2}{(1/3)(\mathrm{MVA})_{3P,B}} = \frac{(\mathrm{kV})^2_{L,B}}{(\mathrm{MVA})_{3P,B}} \tag{8.44}$$

$$Z(\mathrm{pu}) = \frac{Z(\Omega)\times(\mathrm{MVA})_{3P,B}}{(\mathrm{kV})^2_{L,B}} \tag{8.45}$$

By definition

$$Z_{\Delta,B} = 3\,Z_{Y,B} \tag{8.46}$$

It then follows that

$$Z_Y(\mathrm{pu}) = Z_\Delta(\mathrm{pu}) \tag{8.47}$$

Since it is a common practice to use 3-phase MVA, and line-to-line kV bases, suffixing can be simplified as

$$\begin{aligned}(\mathrm{MVA})_{3P,\,B} &\to (\mathrm{MVA})_B\\ (\mathrm{kV})_{L,\,B} &\to (\mathrm{kV})_B\end{aligned} \tag{8.48}$$

Advantages of PU System

1. Choice of Bases

Voltage bases on the two sides of a transformer should be in direct ratio of transformation (and current bases in the inverse ratio). Its advantage is that the *pu* values of transformer parameters on either side are the same. The reason is not hard to find as shown below.

$$\begin{aligned}Z(pu) &= \frac{I(HV)Z(HV)}{V(HV)}\\ &= \frac{I(LV)}{V(LV)}\times\left[\frac{N(LV)}{N(HV)}\right]^2\times(HV)\\ &= \frac{I(LV)\,Z(LV)}{V(LV)}\end{aligned} \tag{8.49}$$

This eliminates the need of ideal transformers for the circuit models of transformers with different transformation rations in a power system.

It is convenient to choose the primary voltage as base on the primary side and the secondary voltage as the base on secondary side. The rating of the transformer is taken as its VA/kVA/MVA base.

2. The transformer or, in general, the device parameters in pu lie in a narrow range; in some characteristic of the device. There is no need to deal with very small and very large numbers.

Example 8.7 A 50 kVA, 1100/220 V has primary and secondary resistance and leakage reactance as below:

	Resistance	Leakage reactance
Primary (HV)	0.125 Ω	0.625 Ω
Secondary (LV)	0.005 Ω	0.025 Ω

(a) Calculate the impedance of the transformer referred to *HV* and *LV*.

(b) Find the pu impedance of the transformer both from *HV* side and *LV*. Are both these values equal?

Solution

(a) Referred to HV side:

$$\bar{Z}_1 = (0.125 + j\,0.625) + \left(\frac{1100}{220}\right)^2 (0.005 + 0.025)$$

$$= 0.25 + j1.25\ \Omega$$

(b) Referred to *LV* side:

$$\bar{Z}_2 = (0.005 + j\,0.025) + \left(\frac{220}{1100}\right)^2 (0.125 + j0.625)$$

$$= (0.01 + j0.05)\ \Omega$$

(c) $(\text{kVA})_B = 50$ or $(MVA)_B = 0.05$

HV side $(\text{kV})_B = 1.1$

LV side $(\text{kV})_B = 0.22$

As found from HV side:

$$\bar{Z}\ (\text{pu}) = (0.25 + j\,1.25) \times \frac{0.05}{(1.1)^2}$$

$$= 0.01 + j\,0.052$$

As found from LV side:

$$\bar{Z}\ (\text{pu}) = (0.01 + j\,0.05) \times \frac{0.05}{(0.22)^2}$$

$$= 0.01 + j\,0.052$$

Example 8.8 A single- phase, 600 kVA, 2400/600 V transformer has the following circuit model parameters:

$r_1 = 0.05\ \Omega$ $r_2 = 0.004\ \Omega$

$x_1 = 0.025\ \Omega$ $x_2 = 0.016\ \Omega$

$R_1 = 1667\ \Omega$ $X_m = 417\ \Omega$ (as seen on HV side)

(a) Draw its equivalent circuit as seen from LV side.

(b) Convert the circuit of part (a) to pu form.

Solution

As seen on LV side:

$$R + j\,X = (0.004 + j\,0.016) + \left(\frac{600}{2400}\right)^2 (0.05 + j\,0.025)$$

$$= 0.007125 + j\,0.01756\ \Omega$$

Shunt branch as seen on LV side:

$$R_i\,(\text{LV}) = 1667 \times \left(\frac{600}{2400}\right)^2 = 104.2\ \Omega$$

$$X_m\,(\text{LV}) = 417 \times \left(\frac{600}{2400}\right)^2 = 26.1\ \Omega$$

The circuit is drawn in Fig. 8.18 (a)

It may be noted that because of voltage drops in $(R + j\,X)$

$$V_1'\left(\frac{2400}{600}\right) \neq V_1 = 2400$$

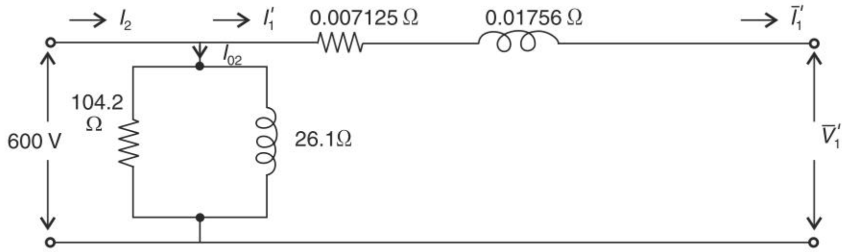


Fig. 8.18

Further turn ratio, $\frac{N_1}{N_2} \approx \frac{2400}{600}$

PU parameter values:

$$(MVA)_B = 0.6, \; (kV)_B = 0.6$$

$$(R + j\,X) = (0.007125 + j0.01756) \times \frac{0.6}{(0.6)2}$$

$$= 0.012 + j\,0.0293 \text{ pu}$$

$$R_i = 104.2 \times \frac{0.6}{(0.6)^2} = 173.7 \text{ pu} \qquad X_m = 26.1 \times \frac{0.6}{(0.6)^2} = 43.5 \text{ pu}$$

(b) Circuit in pu is drawn in Fig. 8.19.

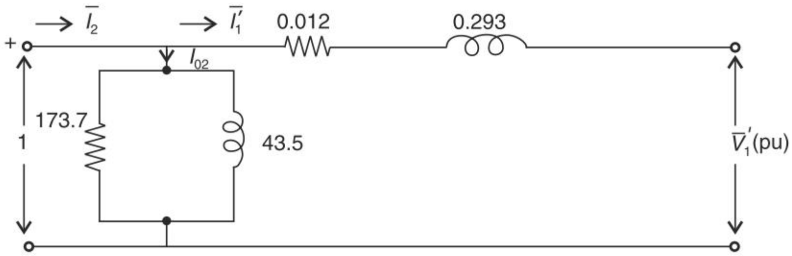


Fig. 8.19

Example 8.9 A 50 kVA, 2200/220 V transformer when tested gave the following results

OC test, measurements on LV side: 405 W, 5 A, 220 V

SC test, measurements on HV side: 805 W, 20.2 A, 95 V

(a) Draw the circuit model of the transformer referred to the HV and LV sides. Label all the parameters.

(b) Calculate the pu parameters of the transformer in LV sides.

Solution

OC test (LV side):

$$Y_0 = \frac{5}{220} = 0.0227 \text{ ℧}$$

$$G_i = \frac{405}{(220)^2} = 0.0084 \text{ ℧}$$

$$B_m = [(0.0227)^2 - (0.0084)^2]^{1/2} = 0.021 \text{ ℧}$$

SC test (HV side):

$$Z = \frac{95}{20.2} = 4.7\ \Omega$$

$$R = \frac{805}{(20.2)^2} = 1.97\ \Omega$$

$$X = [(4.7)^2 - (1.97)^2]^{1/2} = 4.27\ \Omega$$

(a) Circuit model referred to HV side:

$$a = \frac{2200}{220} = 10$$

$$G_i = 0.0084 \times \frac{1}{(10)^2} = 0.084 \times 10^{-3}\ \mho$$

$$B_m = 0.021 \times \frac{1}{(10)^2} = 0.21 \times 10^{-3}\ \mho$$

$$R = 1.97\ \Omega$$

$$X = 4.27\ \Omega$$

The circuit model is drawn in Fig. 8.20.

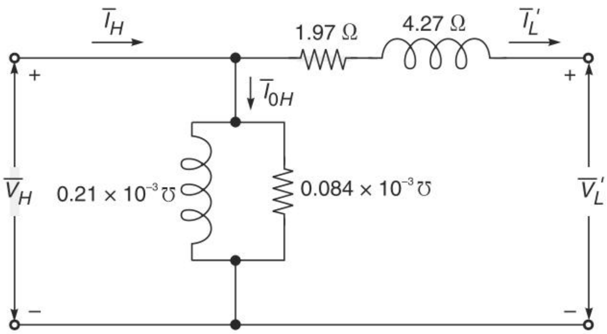


Fig. 8.20

Circuit model referred to LV side:

$$G_i = 8.4 \times 10^{-3}$$

$$B_m = 21 \times 10^{-3}$$

$$R = 1.97 \times \frac{1}{(10)^2} = 0.02\ \Omega$$

$$X = 4.27 \times \frac{1}{(10)^2} = 0.043\ \Omega$$

The circuit model is drawn in Fig. 8.20.

We then have pu values, so

$$G_i = \frac{0.084 \times 10^{-3}}{0.0103} = 8.2 \times 10^{-3}\ \mho\ \text{(pu)}$$

$$B_m = \frac{0.21 \times 10^{-3}}{0.0103} = 20.4 \times 10^{-3}\ \mho\ \text{(pu)}$$

$$R = \frac{1.97}{96.8} = 0.02\ \Omega\ \text{(pu)}$$

$$X = \frac{4.27}{96.8} = 0.044\ \Omega\ \text{(pu)}$$

(b) The pu parameter can be found from any side of transformer (as these have the same value). Let us calculate from the HV side.

$$(\text{kVA})_B = 50$$

$$(\text{kV})_B = 2.2\ \text{kV (HV)}$$

From Eq. (8.35),

$$Z_B = \frac{1000 \times (\text{kV})_B^2}{(\text{kVA})_B} = \frac{1000 \times (2.2)^2}{50}$$

$$= 96.8\ \Omega$$

$$Y_B = \frac{1}{Z_B} = \frac{1}{96.8} = 0.0103\ ℧$$

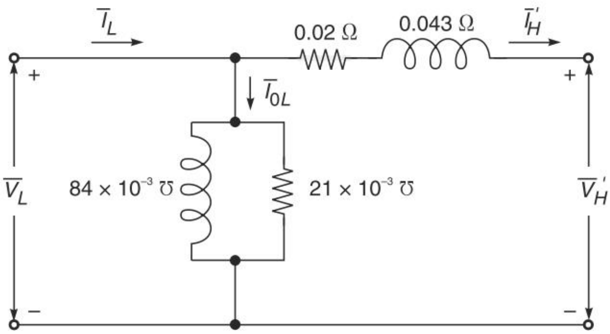


Fig. 8.21

8.6 DETERMINATION OF PARAMETERS OF CIRCUIT MODEL OF TRANSFORMER

It is not practical to test a transformer for its voltage drop characteristic and its efficiency by a direct loading test. Such a test would suffer from three disadvantages, viz.

(i) loss of energy during testing,

(ii) it may not be practical to arrange for load except for small size transformers

(iii) losses as administered by direct loading would be in serious error as these are found by the difference of the input and output power readings which are close to each other, the losses being very small. It is well known that errors in meter readings add up and become much larger per cent of the difference. It is therefore standard practice in transformer testing to determine the transformer losses and the parameters of the circuit model by means of *nonloading* tests. The transformer performance is then computed from the circuit model.

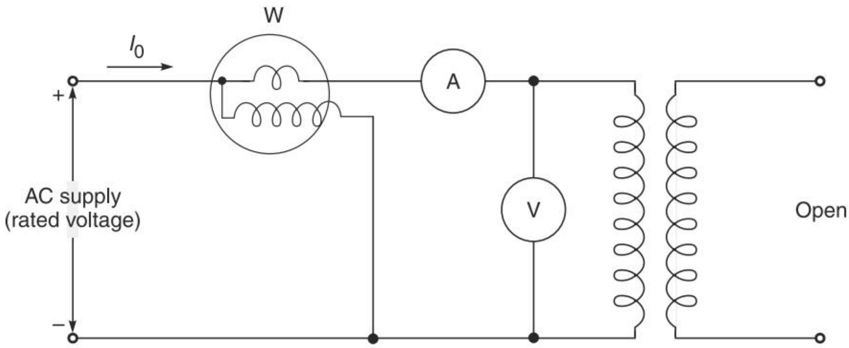
Transformer parameter determination necessitates two tests, viz. open-circuit test and short-circuit test.

Open-Circuit (OC) or No-Load Test

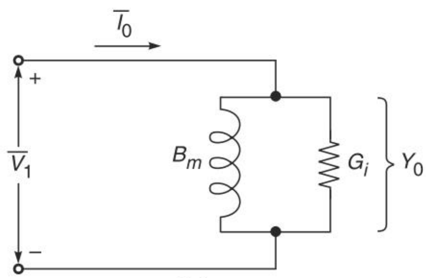
The transformer is excited at rated voltage (and frequency) from one side while the other side is kept open-circuited as shown in Fig. 8.22(a). It is usually convenient to conduct such a test from the LV side. The circuit model under open-circuit is drawn in Fig. 8.22(b); it follows from Fig. 8.15 by setting $I_2' = 0$.

Let the meter readings be

$$\begin{aligned} \text{voltage (V)} &= V_1 \\ \text{current (A)} &= I_0 \\ \text{and} \qquad \text{power (W)} &= P_0 = \text{core loss } (P_i) \end{aligned} \tag{8.50}$$



(a)



(b)

Fig. 8.22 (a) Circuit diagram for *OC* test
(b) Circuit model as seen on open-circuit

It then follows that

$$Y_0 = \frac{I_0}{V_1} \tag{8.51}$$

$$G_i = \frac{P_0}{V_1^2} \tag{8.52}$$

and

$$B_m = \sqrt{Y_0^2 - G_i^2} \tag{8.53}$$

By connecting a voltmeter on the secondary side, the OC test also yields the voltage ratio of the transformer, which is practically its turn ratio a.

The values of G_i and B_m as computed can be transferred to the other side of the transformer, if so desired.

It is seen that the OC test yields (i) core loss and (ii) parameters of the shunt branch of the transformer model.

The OC test is usually conducted from the LV side as low voltage small current supply is needed for the test.

Example 8.10 A 50 kVA, 2200/110 V transformer is connected to 110 V supply with metering. With 2200V side open circuited, the motor readings are 110V, 10 A, 400 W. Compute the parameters of the shunt branch of the equivalent circuit as seen from the LV and HV sides. Also compute the pu values.

Solution

OC test on LV side—shunt branch praameters:

$$Y_0 = \frac{10}{110} = 0.091\ \mho$$

$$G_i = \frac{400}{(110)^2} = 0.033\ \mho$$

$$B_m = (Y_0^2 - G_i^2)^{2/2} = 0.085\ \mho$$

As seen on HV side:

$$G_i\,(HV) = 0.033 \times \left(\frac{110}{2200}\right)^2 = 8.25 \times 10^{-3}\ \mho$$

$$B_m\,(HV) = 0.085 \times \left(\frac{110}{2200}\right) = 21.25 \times 10^{-3}\ \mho$$

PU value :

$$(VA)_B = 50 \times 10^3$$

$$V_B\,(LV \text{ side}) = 110$$

Then

$$G_i\,(\text{pu}) = 0.091 \times \frac{(VA)_B}{V_B^2} = 0.033 \times \frac{50 \times 10^3}{(110)^2}$$

$$= 0.136\ \mho\ (\text{pu})$$

$$B_m\,(\text{pu}) = 0.085 \times \frac{50 \times 10^3}{(110)^2}$$

$$= 0.351\ \mho\ (\text{pu})$$

Short-Circuit (SC) Test

This test determines the series parameters of the transformer circuit model. The transformer is shorted on one side and is excited from a reduced voltage (rated frequency) source from the other side as shown in Fig. 8.22. The transformer circuit model under short-circuit conditions is drawn in Fig. 8.24(a). As the primary current is limited only by the resistance and leakage reactance of the transformer, V_{SC} needed to circulate full-load current is only of the order of 5–8% of the rated voltage. At this reduced voltage, the exciting I_0 which is 2 to 5% of the rated current gets reduced to 5% of 2% which is 0.1% to 8% of 5% = 0.4% of the rated current. The magnetizing

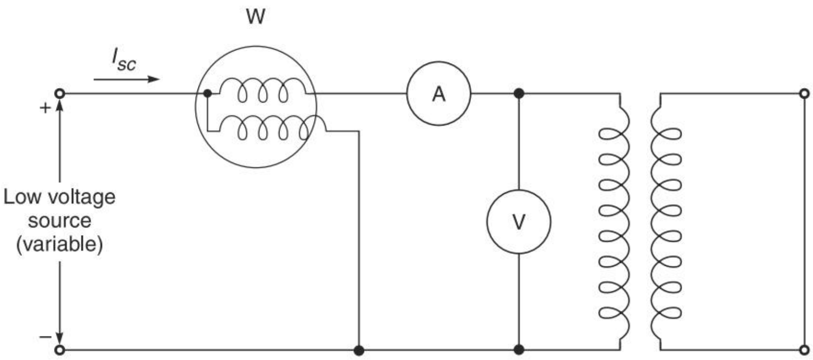


Fig. 8.23 Short-circuit (SC) test on transformer

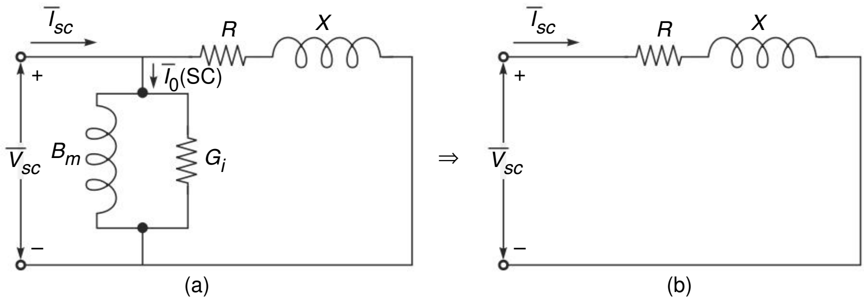


Fig. 8.24 Circuit model under SC conditions

shunt branch of the circuit model can therefore be conveniently dropped resulting in the circuit of Fig. 8.24(b).

For convenience of the supply voltage and current needed, the SC test is usually conducted from the HV side and the LV side is short circuited.

In conducting the SC test, as in Fig. 8.23, the source voltage is gradually raised till the transformer draws *full-load* current. The meter readings under these conditions are

$$\begin{aligned} \text{voltage (V)} &= V_{SC} \\ \text{current (A)} &= I_{SC} \qquad (8.54a) \\ \text{power input (W)} &= P_{SC} = I^2R \text{ loss* or copper loss } P_c \end{aligned}$$

(total in the two windings)

From the circuit model of Fig. 8.24(b)

$$Z = \frac{V_{SC}}{I_{SC}} = \sqrt{R^2 + X^2} \qquad (8.54b)$$

* As the transformer is excited at 5–8% of rated voltage, the core flux gets reduced by the same percentage and the core losses being proportional to square of core flux are reduced to 0.25–0.64% of that at rated voltage and are hence negligible. The power drawn by the transformer under SC condition is therefore wholly I^2R loss for all practical purposes.

Equivalent resistance $R = \frac{P_{SC}}{(I_{SC})^2}$ (8.55)

Equivalent reactance $X = \sqrt{Z^2 - R^2}$ (8.56)

It is thus seen that the SC test yields information about (i) full-load copper loss and (ii) equivalent resistance and reactance of the transformer.

Together OC and SC tests determine all the four parameters of the transformer circuit model of Fig. 8.15—two shunt parameters (G_i, B_m) and two series parameters (R, X).

Example 8.11 A 25 kVA, 2200/220 V, 50 Hz single-phase transformer is found to have the following paramaters:

$r_1 = 2\Omega$ $\quad r_2 = 0.025\Omega$

$x_1 = 7\Omega$ $\quad x_2 = 0.07\Omega$

$X_m = 16000\ \Omega$

Find the following:

(a) No-load current, its *pf* and power when excited from the LV side.
(b) With the LV side shorted, the HV side voltage needed to circulate full-load current. What is exciting current compared to full-load current?
(c) What is the power factor under part (b)?
(d) Convert the given parameter values of the transformer to *pu*.

Solution

(a) No-load, LV excited at 220 V:

Only magnetizing current will be drawn. The core loss current is zero (negligible).

$$I_m = \frac{220}{16000} \times 10^3 = 13.75 \text{ mA, } 90° \text{ lagging}$$

$$pf = 0, \ P_0 = 0$$

Note: There be a small amount of core loss power draw which is being ignored here.

(b) Turn ratio, $a = \frac{2200}{220} = 10$

Referred to HV side:

$$R = 2 + (10)^2 + 0.025 = 4.5\ \Omega$$

$$X = 7 + (10)^2 + 0.07 = 14\ \Omega$$

$$Z = \sqrt{R^2 + X^2} = \sqrt{(4.5)^2 + (14)^2} = 14.7\ \Omega$$

Full-load current, $I\,(f\,l) = \frac{25 \times 10^3}{2200} = 11.36$ A

$$V_{SC} = Z\,I_{SC} = 14.7 \times 11.36 = 167 \text{ V}$$

It is $\frac{167}{2200} \times 100 = 7.59\%$ of rated voltage.

At this voltage, magnetizing current $= \frac{167}{16000} = 10.44$ mA

It is $\dfrac{10.44 \times 10^{-3}}{11.36} \times 100 = 0.09\%$. So magnetizing current can be neglected in the SC test.

(c) SC $pf = \cos\left(\tan^{-1}\left(\frac{X}{R}\right)\right) = \cos\left(\tan^{-1}\left(\frac{14}{4.5}\right)\right)$

$= 0.31$

(d) Parameters in pu

$$(\text{kVA})_B = 25 \ (\text{kV})_B \ (\text{HV}) = 2.2$$

$$Z_B = \frac{1000 \times (2.2)^2}{25} = 193.6$$

$$r_1 = \frac{2}{193.6} = 0.0103, \qquad r_2 = \frac{(10)^2 \times 0.025}{193.6} = 0.0129$$

$$x_1 = \frac{7}{193.6} = 0.0362, \qquad x_2 = \frac{(10)^2 \times 0.025}{193.6} = 0.0362$$

$$R = r_1 + r_2 = 0.0103 + 0.0129 = 0.0232$$

$$X = x_1 + x_2 = 0.0362 + 0.0362 = 0.0724$$

$$X_m = \frac{16000}{193.6} = 82.64\ \Omega$$

8.7 VOLTAGE REGULATION

Domestic, commercial and industrial loads demand a nearly constant voltage supply. It is, therefore, essential that the output voltage of a transformer stays within narrow limits as load and its power factor vary. The leakage reactance is the chief cause of voltage drop in a transformer and must be kept as low as possible by design and manufacturing techniques.

The voltage regulation of a transformer at a *given power factor* is defined as

$$\%\ \text{Voltage regulation} = \frac{V_{2,0} - V_{2,\text{fl}}}{V_{2,\text{fl}}} \times 100 \tag{8.57}$$

where $V_{2,\text{fl}}$ is the full-load secondary voltage (it is assumed to be adjusted to the rated secondary voltage) and $V_{2\text{o}}$ is the secondary voltage when the load is thrown off.

Examination of the circuit model of the transformer reveals that for voltage computation, the shunt branch can be left out, resulting in simple series circuit drawn in Fig. 8.25 (a). The phasor diagram showing circuit current and voltage is drawn in Fig. 8.25 (b) for *lagging power factor.* Some explanation is in order:

- Resistive voltage drop $\bar{I}R$ is parallel to load current $\bar{I}$
- Reactive voltage drop $j\bar{I}X$ lead $\bar{I}$ by 90°
- Input voltage $\bar{V}_1$ leads load voltage $\bar{V}_2$ by angle δ, which if necessary can be determined from the phasor diagram
- Additional geometrical construction is shown dotted in the figure

The corresponding phasor equation of the circuit is $\bar{V}_1 = \bar{V}_2 + \bar{I}R + j\bar{I}X$

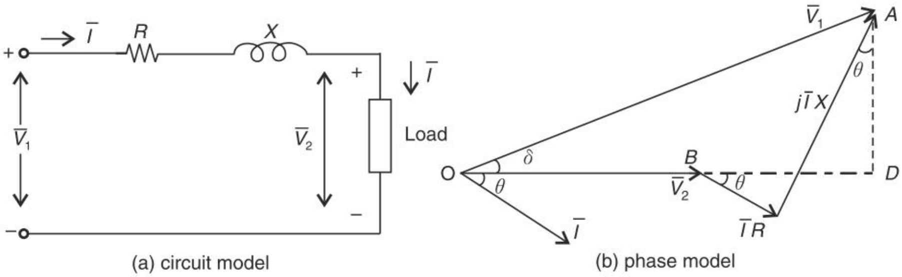


Fig. 8.25 (a) Circuit model, (b) Phasor diagram (not proportional)

As in a transformer, IR and IX voltage drops are much smaller in magnitude compared to V_1 and V_2, the angle between $\bar{V}_1$ and $\bar{V}_2$ in Fig. 8.25(b) is only a few degrees such that

$$V_1 \approx OD$$

$$V_1 - V_2 \simeq BD$$

$$= I(R \cos \theta + X \sin \theta);\ \theta \text{ lagging} \tag{8.58a}$$

$$= I(R \cos \theta - X \sin \theta);\ \theta \text{ leading} \tag{8.58b}$$

The reader is advised to draw the phasor diagram for leading pf.

When the load is thrown off

$$V_{20} = V_1$$

$$\therefore \quad V_{20} - V_2 = V_1 - V_2$$

Then

$$\%\ \text{Voltage regulation, Reg} = \frac{V_{20} - V_2}{V_2} \times 100$$

$$= \frac{I(R \cos \theta + X \sin \theta)}{V_2} \times 100 \tag{8.59}$$

Now $$\frac{IR}{V_2} = R\ (\text{pu}), \frac{IX}{V_2} = X\ (\text{pu})$$

assuming full-load (rated) current I and full-load (rated) voltage V_2 to be the base values. Therefore

$$\text{Reg (pu)} = R\ (\text{pu}) \cos \theta \pm X\ (\text{pu}) \sin \theta \tag{8.60}$$

For maximum voltage regulation (from Eq. (8.59))

$$\frac{d\,(\text{Reg})}{d\theta} = 0 = -R \sin \theta + X \cos \theta$$

or $$\tan \theta = \frac{X}{R}$$

or $$\text{pf} = \cos \theta = \frac{R}{(R^2 + X^2)^{1/2}};\ \text{lagging} \tag{8.61}$$

From Eq. (8.58b) voltage regulation is zero when

$$R \cos \theta - X \sin \theta = 0; \quad \theta \text{ leading}$$

or $$\tan\theta = \frac{R}{X}$$

or $$\text{pf} = \cos\theta = \frac{X}{(R^2 + X^2)^{1/2}}; \text{ leading} \tag{8.62}$$

Leading θ larger than that given in Eq. (8.62) would result in negative voltage regulation, i.e. secondary voltage on full load is higher than the no-load voltage.

8.8 NAME PLATE RATING

The voltage ratio of a transformer is specified as V_1 (rated)/V_2 (rated), where V_1 (rated) and V_2 (rated) are the primary and secondary voltage at full load and specified pf. Since the voltage drop in a transformer is only a few per cent, this ratio is also taken as the turns-ratio N_1/N_2 for all practical purposes, i.e.

$$\frac{V_1 \text{ (rated)}}{V_2 \text{ (rated)}} \approx \frac{N_1}{N_2}$$

A transformer depending upon its size can carry only a certain current, called full-load current, without overheating. The transformer rating is then

$$\text{kVA (rating)} = \frac{V \text{ (rated)} \times I \text{ (full-load)}}{1000}$$

It could also be expressed as VA (rated) for small transformers and in MVA (rated) for very large size transformers.

The pu impedance of a transformer on its rated voltage and kVA bases is given by

$$Z\,(\text{pu}) = \frac{I \text{ (full-load) } Z\,(\Omega)}{\text{V (rated)}}$$

$$= \frac{\text{kVA (rated) } Z\,(\Omega)}{1000\,(\text{kV (rated)})^2}$$

where I (full load), V (rated) and Z (Ω) pertain to any side of the transformer.

The percentage impedance is defined as

$$\%\,Z = Z\,(\text{pu}) \times 100$$

$$= \frac{I \text{ (full-load) } Z\,(\Omega)}{V \text{ (rated)}} \times 100$$

Obviously it has also the meaning of per cent voltage drop under full-load.

Example 8.12 The resistances and leakage reactances of a 10 kVA, 50 Hz, 2300/230 V distribution transformer are: $r_1 = 3.96\ \Omega$ and $r_2 = 0.0396\ \Omega$, $x_1 = 15.8\ \Omega$ and $x_2 = 0.158\ \Omega$; subscript 1 refers to HV and 2 to LV winding.

(a) The transformer delivers rated kVA at 0.8 pf lagging to a load on the LV side. Find the HV-side voltage necessary to maintain 230 V across load terminals. Also find the percentage voltage regulation.

(b) If a capacitor bank is connected across the load, what should be the kVA capacity of the bank to reduce the voltage regulation to zero? What should be the HV-side voltage under these circumstances?

Solution

(a) Referred to HV side:

equivalent resistance $R = 3.96 + 0.0396 \times (10)^2 = 7.92\ \Omega$

equivalent reactance $X = 15.8 + 0.158 \times (10)^2 = 31.6\ \Omega$

With reference to Fig. 8.26(a)

$$V_2 = 230 \times 10 = 2300 \text{ V (referred to HV)}$$

At rated kVA load

$$I = \frac{10 \times 1000}{2300} = 4.35 \text{ A}, \cos\theta = 0.8 \text{ lagging}$$

$$V_1 - V_2 = I(R\cos\theta + X\sin\theta)$$

$$= 4.35\,(7.92 \times 0.8 + 31.6 \times 0.6) = 110 \text{ V}$$

$$V_1 = 2300 + 110 = 2410 \text{ V}$$

$$V_{20} = V_1 = 2410 \text{ V}$$

$$\text{Voltage regulation} = \frac{2410 - 2300}{2300} \times 100\%$$

$$= 4.78\%$$

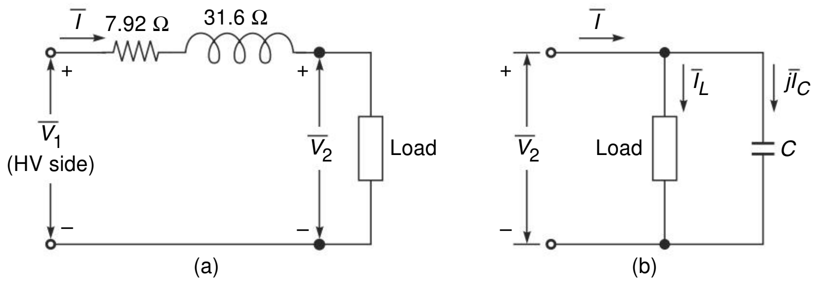


Fig. 8.26

(b) For zero voltage regulation:

$$\text{pf} = \cos\theta = \frac{X}{(R^2+X^2)^{1/2}}$$

$$= \frac{31.6}{[(7.92)^2 + (31.6)6]^{1/2}} = 0.97 \text{ leading}$$

or $\theta = 14.1°$ leading

A capacitor C is placed in parallel with the load to improve the power factor to 0.97 leading (Fig. 8.26(b))

$$\bar{V}_2 = 2300 \angle 0° \text{ V}$$

Load current $\bar{I}_L = 4.35\,(0.8 - j\,0.6) = 3.48 - j\,2.61$

$$\bar{I} = \bar{I}_L + j\,I_C = 3.48 - j\,2.61 + j\,I_C$$

For $\theta = 14.10°$

$$\tan 14.1° = \frac{I_C - 2.61}{3.48}$$

or $I_C = 3.48$ A

$$\text{Rating of capacitor bank} = \frac{2300 \times 3.48}{1000} = 8 \text{ kVA}$$

Since voltage regulation is zero

$$V_1 = 2300 \text{ V}$$

Example 8.13 A 100 kVA transformer has primary and secondary turns 400 and 100 respectively. Its primary and secondary resistance and reactances are:

$$r_1 = 0.3\ \Omega \qquad r_2 = 0.015\ \Omega$$
$$x_1 = 1.1\ \Omega \qquad x_2 = 0.055\ \Omega$$

The supply voltage is 2400 V.

Calcutate:

(a) Equivalent resistance and reactance on the primary side

(b) Voltage regulation and secondary voltage at a power factor of (i) 0.8 lagging and (ii) 0.8 leading

(c) The power factor for zero voltage regulation.

Solution

$$\text{Turn ratio,} \qquad a = \frac{400}{100} = 4$$

(a) Referred to primary :

$$R = 0.3 + (4)^2 \times 0.015 = 0.54\ \Omega$$
$$X = 1.1 + (4)^2 \times 0.055 = 1.98\ \Omega$$

(b) For calculating full-load current, we do not know the secondary voltage. We approximate it as the turn-ratio equivalent of the primary voltage; a departure from the strict definition of voltage regulation. So on primary side

$$I_1 = \frac{100 \times 1000}{2400} = 41.67 \text{ A}$$

(i) 0.8 lagging pf :

$$\text{Voltage drop} = 41.67\,(0.54 \times 0.8 + 1.98 \times 0.6)$$
$$= +\,67.5 \text{ V}$$

$$\text{Voltage regulation} = \frac{+67.5}{2400} \times 100 = +\,2.81\%$$

$$\text{Secondary voltage} = \frac{2400 - 67.5}{4} = 583 \text{ V}$$

(ii) 0.8 leading pf *:*

$$\text{Voltage drop} = 41.67\,(0.54 \times 0.8 - 1.98 \times 0.6)$$
$$= -31.5 \text{ V voltage rise}$$

$$\text{Voltage regulation} = \frac{-31.5}{2400} \times 100 = -1.31\%$$

$$\text{Secondary voltage} = \frac{2400 - 31.5}{4} = 607.9 \text{ V}$$

(c) For zero voltage regulation :

$0.54 \cos \theta - 1.98 \sin \theta = 0$

$$\text{or } \tan \theta = \frac{1.98}{0.54} = 3.67$$

$$pf = \cos \theta = 0.263 \text{ leading}$$

8.9 EFFICIENCY

The efficiency of transformer (or in fact any other device) is

$$\eta = \frac{\text{output power}}{\text{input power}} \tag{8.63}$$

or

$$\eta = \frac{\text{output}}{\text{output + losses}} = 1 - \frac{\text{losses}}{\text{output + losses}} \tag{8.64}$$

The transformer has two losses:

– Core (iron) loss P_i which is a constant loss*

– Copper (I^2R) loss, P_c (both windings), a variable loss.

Other transformer losses, which are insignificant for efficiency computation, are

– Load (stray) loss which results from leakage fields inducing eddy currents in tank walls and conductors

– Dielectric loss caused by leakage current in the insulating materials

With reference to Fig. 8.27 of a transformer on load,

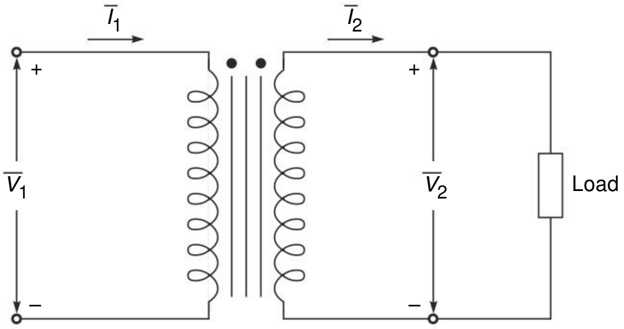


Fig. 8.27 Transformer on load

$$\text{output} = V_2 I_2 \cos \theta_2$$

where $\cos \theta_2$ = load pf

From Eq. (8.63)

$$\eta = \frac{V_2 I_2 \cos \theta_2}{V_2 I_2 \cos \theta_2 + P_i + I_2^2 R_2} \tag{8.65}$$

where R_2 is the equivalent transformer resistance referred to secondary.

* Operation at constant voltage and frequency.

Reorganizing Eq. (8.65)

$$\eta = \frac{V_2 \cos \theta_2}{V_2 \cos \theta_2 + (P_i / I_2 + I_2 R_2)} \tag{8.66}$$

For a given pf the efficiency varies with the load current, maximum efficiency is achieved when Eq. (8.66) has minimum denominator, i.e.

$$I_2^2 R_2 = P_i \tag{8.67}$$

or copper loss = iron loss

or variable loss = constant loss

From Eq. (8.67) the efficiency is maximum a load current of

$$I_2 = \sqrt{\frac{P_i}{R_2}}$$

and at a load of $V_2 I_2 \cos \theta$

1. *Power transformers:* These are the transformers employed at transmission level where the load throughout the day is nearly constant. These are designed to have η_{max} at full load.
2. *Distribution transformers:* The load at distribution level has considerable variations during the day. So these transformers are designed to have η_{max} at about 3/4th full-load.

Example 8.14 Reconsider the transformer of Example 8.13 rated 10 kVA, 50 Hz. 2300/230 V. Its equivalent resistance and reactance on the HV sides are 7.92 Ω and 31.6 Ω respectively. It has a core loss of 75W at rated voltage of 2300V.

(a) It is supplying a load of 10 kVA at 0.8 pf lagging at rated voltage 230 V. For this, the supply voltage on the HV side is 2410 V.

Compute its efficiency of operation. Assume the core loss to be proportional to the square of the applied voltage.

(b) By shunt capacitor, the power factor of the load in part (a) is informed to 0.97 lagging. As a result, the supply voltage on the HV side needed is 2300 V, which means zero voltage regulating. Compute the transformer efficiency under the operating condition.

(c) Find the maximum efficiency of the transformer for a load of 0.8 pf.

Solution

(a) HV side voltage, $V_1 = 2410$ V

Core loss at this voltage, $P_i = 75 \times \left(\frac{2410}{2300}\right)^2 = 82.2$ W

Load current, $I = \frac{10 \times 10^{-3}}{2300} = 4.35$ A, HV side

Copper loss, $P_C = (4.35)^2 \times 7.92 = 150$ W

Total loss, $P_L = 82.3 + 150 = 232.3$ W

Power output, $P_0 = 10 \times 0.8 = 8$ kW

The efficiency is then found to be

$$\eta = \frac{8}{8 + 0.232} \times 100 = 97.2\%$$

(b) HV voltage, $V_1 = 2300$ V

$P_i = 75$ W

Power output, $P_o = 8$ kW at 0.97 lagging

$$I = \frac{8 \times 10^3}{2300 \times 0.97} = 3.59 \text{ A}$$

$$P_c = (3.59)^2 \times 7.92 = 102 \text{ W}$$

$$P_L = 75 + 102 = 177 \text{ W}$$

$$P_0 = 8 \text{ kW}$$

$$= \frac{8}{8.177} \times 100 = 97.8\%$$

Remarks Notice that because of pf improvement (0.8 lag to 0.97 lead), the load current has reduced in magnitude (4.35 to 3.59 A); therefore reducing the copper loss and raising the efficiency from 97.2% to 97.8%. Even this small efficiency improvement would result in significant saving in energy over a period of time, say one year. The expenditure incurred in the capacitor bank may be worthwhile.

(c) For maximum efficiency

$$I^2R = P_i$$

or $$I^2 \times 7.92 = 75$$

It is assumed above that $P_i = 75$ W remains constant as the voltage drop in the transformer is small and V_1 and V_2 are both close to the rated values.

Now

$$I = \left(\frac{75}{7.92}\right)^{1/2} = 3.08 \text{ A}$$

$$\text{Load} = 2300 \times 3.08 = 7.08 \text{ kVA at 0.8 pf}$$

$$= 88.5\% \text{ of full load}$$

$$P_0 = 3.08 \times 2300 \times 0.8 = 5.667 \text{ kW}$$

$$P_L = 2 \times 75 = 150 \text{ W}$$

$$\eta_{max} = \frac{5.667}{5.817} = 97.4\%$$

Example 8.15 (Comprehensive Example): The test data obtained on a 15 kVA, 3000/250V, 50 Hz distribution transformer are:

OC test (LV side)	250 V	0.62 A	105 W
SC test (HV side)	157 V	5.2 A	360 W

(a) Determine the equivalent circuit parameters referred to the HV side.
(b) Convert the circuit parameters determined in parta) into pu values.
(c) The transformer is carrying full-load at 250V, 0.8 pf leading. Calculate the

transformer (i) voltage regulation, and (ii) efficiency.

(d) Find the pf of full-load for zero voltage regulation.

(e) Find the pu load at 0.8 pf for maximum transformer efficiency and its value.

Solution

(a) OC test (LV side) :

$$Y_0 = \frac{I_0}{V_1} = \frac{0.62}{250} = 2.48 \times 10^{-3} \ ℧$$

$$G_i = \frac{P_0}{V_1^2} = \frac{105}{(250)^2} = 1.68 \times 10^{-3} \ ℧$$

$$B_m = \sqrt{Y_0^2 - G_i^2} = 10^{-3}\sqrt{(2.48)^2 - (1.68)^2}$$

$$= 1.82 \times 10^{-3}$$

Converting to HV side:

Turn ratio $a = \dfrac{3000}{250} = 12$

SC test (HV side):

$$G_1 = \frac{1}{(12)^2} \times 1.68 \times 10^{-3} = 11.67 \times 10^{-6}$$

$$B_m = \frac{1}{(12)^2} \times 1.82 \times 10^{-3} = 12.64 \times 10^{-6}$$

SC test (HV side):

$$Z = \frac{157}{5.2} = 30.2 \ \Omega$$

$$R = \frac{360}{(5.2)^2} = 13.3 \ \Omega$$

$$X = [(30.2)^2 - (13.3)^2]^{1/2} = 27.1\Omega$$

(b) pu values :

$$(\text{kVA})_B = 15, (\text{kV})_B = 3 \ kV \text{ (HV side)}$$

$$Z_B = \frac{1000 \times (3)^2}{5.2} = 600 \ \Omega, \ y_B = \frac{1}{600} \ ℧$$

$$G_1 = 1.68 \times 10^{-6} \times 600 = 1 \times 10^{-3} \text{ pu}$$

$$B_m = 2.48 \times 10^{-6} \times 600 = 1.49 \times 10^{-3} \text{ pu}$$

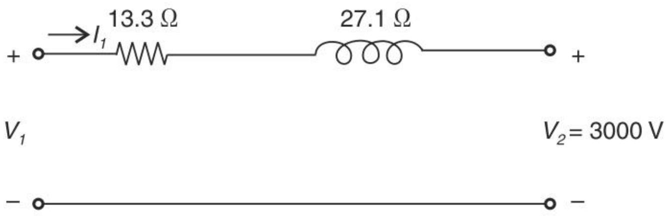
$$R = \frac{13.3}{600} = 0.022 \text{ pu}, \qquad X = \frac{27.1}{600} = 0.045 \text{ pu}$$

(c) Full-load = 15 kVA, 0.8 pf leading :

(i) Computation on HV side (see figure)

$$V_2 = 250 \times 12 = 3000 \text{ V}$$

Load current, $I = \dfrac{15 \times 1000}{3000} = 5 \text{ A}$



$$\text{Voltage drop } = I\,(R\cos\theta - j\sin\theta)$$

$$\text{V (drop) } = 5\,(13.3 \times 0.8 - 27.1 \times 0.6)$$

$$= 28.1 \text{ V}$$

$$\text{Voltage regulation} = \frac{-28.1}{3000} \times 1000 = -\,0.937\%$$

(ii) Efficiency:

Core loss, $P_1 = 105$ V

Full-load Copper loss, $P_C = (5)^2 \times 13.3 = 332.5$

Total loss, $P_L = 105 + 332.5 = 437.5$ W

Power output, $P_0 = 15 \times 0.8 = 12$ kW

(d) For zero voltage regulation:

$R\cos\theta - X\sin\theta = 0$

$$\tan\theta = \frac{R}{X} = \frac{13.3}{27.1} = 0.49$$

$$\text{pf } = \cos\,(\tan^{-1} 0.49) = 0.9 \text{ leading}$$

Observe that condition of zero voltage regulation is independent of load.

(e) Load in pu for maximum efficiency (0.8 pf) :

Condition $P_i = P_c$

$$(\text{kVA})_B = 15,\ (\text{kV})_B = 0.3$$

$$P_i = \frac{205}{15 \times 10^3} = 7 \times 10^{-3} \text{ pu}$$

$$P_C = I^2 \times R\,(\text{pu}) = I^2 \times 0.022$$

$$0.022\,I^2 = 7 \times 10^{-3}$$

or $I = 0.564$ pu, pu load $= 0.564$

$$\eta_{max} = 0.564$$

8.10 AUTOTRANSFORMER

A two-winding transformer when electrically connected as shown in Fig. 8.28 is known as an autotransformer. Unlike a two-winding transformer, the two windings of an autotransformer are not electrically isolated.

Let the two-winding transformer connected as an autotransformer be regarded as ideal. With this assumption, in Fig. 8.28 all voltages will be in phase and so will be

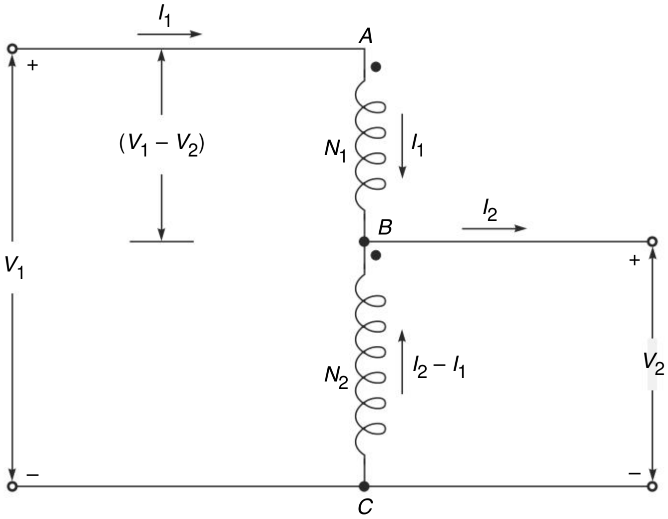


Fig. 8.28 Autotransformer

all currents. The two-winding voltage ratio is

$$a = \frac{V_1 - V_2}{V_2} = \frac{N_1}{N_2} \tag{8.68}$$

The autotransformer voltage ratio is

$$a' = \frac{V_1}{V_2} = \frac{(V_1 - V_2) + V_2}{V_2}$$

or
$$a' = 1 + a \tag{8.69}$$

Now
$$(\text{VA})_{\text{TW}} = (V_1 - V_2)I_1 = (I_2 - I_1)V_2 \tag{8.70}$$

$$(\text{VA})_{\text{Auto}} = V_1 I_1 = V_2 I_2$$

But
$$\frac{I_2 - I_1}{I_1} = \frac{N_1}{N_2} = a$$

or
$$\frac{I_1}{I_1} = \frac{1}{1 + a} \tag{8.71}$$

Substituting Eq. (8.71) in Eq. (8.70),

$$(\text{VA})_{\text{TW}} = \left(1 - \frac{1}{1 + a}\right) V_2 I_2$$

$$= \left(1 - \frac{1}{a'}\right)(\text{VA})_{\text{Auto}}$$

or
$$(\text{VA})_{\text{Auto}} = \left(\frac{1}{1 - 1/a'}\right)(\text{VA})_{\text{TW}} \tag{8.72}$$

or
$$(\text{VA})_{\text{Auto}} > (\text{VA})_{\text{TW}} \tag{8.73}$$

It is easily seen from Eq. (8.72) that the nearer a' is to unity, the larger is $(\text{VA})_{\text{Auto}}$ compared to $(\text{VA})_{\text{TW}}$. *An autotransformer is therefore applied for voltage ratios close to unity.*

The explanation of Eq. (8.73), lies in the fact that in an autotransformer, part of VA is conducted electrically, whereas in a two-winding transformer, all VA is transferred magnetically.

Example 8.16 A 2500/250 V, 25 kVA transformer has a core loss of 130 W and full-load copper loss of 320 W. Calculate its efficiency at full load, 0.8 pf.

The transformer is now connected as an autotransformer to give 2500/2750 V. Calculate its kVA rating and efficiency at full load, 0.8 pf. Compare with the two-winding kVA rating and efficiency.

Solution

(i) Two-winding Transformer:

$$\text{Power output} = 25 \times 0.8 = 20 \text{ kW}$$

$$\text{Losses} = 130 + 320 = 450 \text{ W}$$

$$\eta = \frac{20}{20.45} \times 100 = 97.8\%$$

(ii) Autotransformer: With reference to Fig. 8.29.

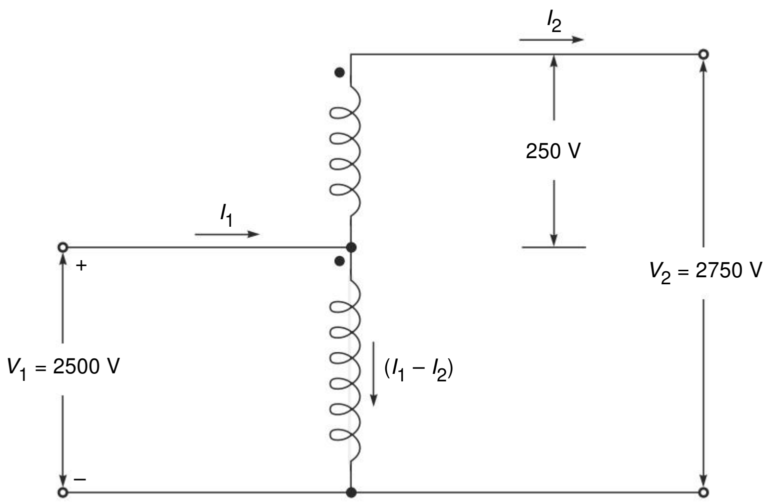


Fig. 8.29

$$I_2 = \frac{25 \times 1000}{250} = 100 \text{ A}$$

$$I_1 - I_2 = \frac{25 \times 1000}{2500} = 10 \text{ A}$$

$$\therefore \quad I_1 = 110 \text{ A}$$

$$\text{kVA rating} = \frac{2500 \times 110}{1000} = \frac{2750 \times 100}{1000} = 275$$

$$\text{Power output} = 2.75 \times 0.8 = 220 \text{ kW}$$

$$\eta = \frac{220}{220 + 0.45} = 99.8\%$$

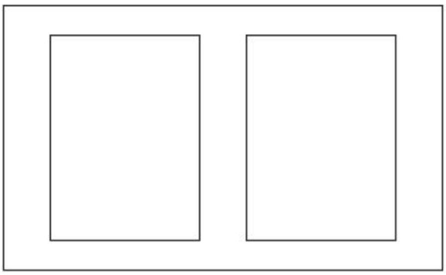
It is seen that when a two-winding transformer is connected as an autotransformer, its rating goes up from 25 kVA to 275 kVA and its efficiency from 97.8% to 99.8%. This is possible because a large part of its kVA is transported conductively.

8.11 THREE-PHASE TRANSFORMERS

Three identical single-phase transformers can be connected to form a 3-phase bank. Primary and secondary sides of the bank can be connected in star/delta with various possible arrangements as

- star/star
- delta/delta
- star/delta or delta/star

Instead of three single-phase transformers, it costs about 15% less to have a single 3-limb core as shown in Fig. 8.30 with primary and secondary of a phase wound on each limb. Like the sum of the currents in 3-phase in zero, the sum of the fluxes in the three limbs at any instant is zero providing for csontinuous flux paths. For reasons of economy, this arrangement (3-limb core) is popularly used. Of course, if one phase is out, the complete transformer must be replaced.



Core type (commonly used)

Fig. 8.30 Three-phase transformer core

In finding voltages and currents in a 3-phase transformer along with the ratio of transformation between the coupled windings, one must employ the line and phase relationship of star/delta connections (Secs. 6.3 and 6.4) with the assumption that the transformer is feeding a balanced load. Figure 8.31 shows a 3-phase transformer connected in delta on the primary side and star on the secondary side. In this figure, the coupled windings are drawn parallel to each other for ease of identification. Various line and phase voltages and currents are indicated on the figure (these follow easily). For a phase-to-phase transformation ratio of a : 1 (delta/star)

$$\frac{V_{\text{line}}(\text{star})}{V_{\text{line}}(\text{delta})} = \frac{\sqrt{3}\,V/a}{V} = \frac{\sqrt{3}}{a}$$

and

$$\frac{I_{\text{line}}(\text{star})}{I_{\text{line}}(\text{delta})} = \frac{aI/\sqrt{3}}{I} = \frac{a}{\sqrt{3}}$$

Phase Shift

In star/star and delta/delta connection, the line voltages and currents are in phase on the primary and secondary sides. However, in a delta/star connection the line

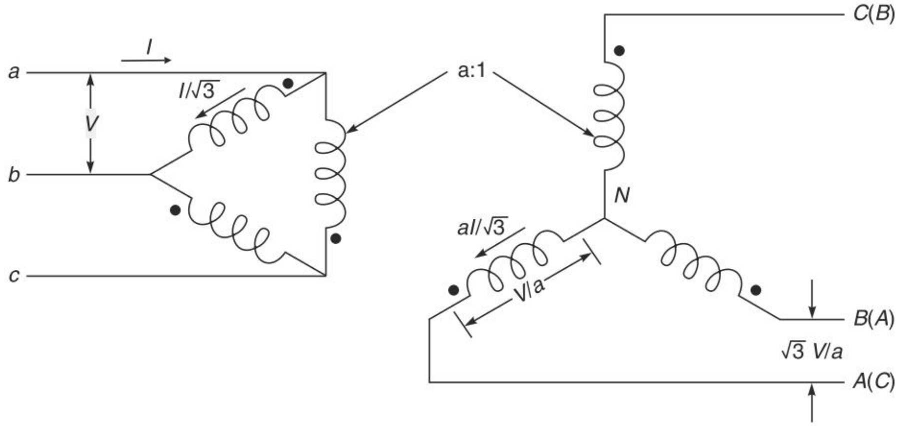


Fig. 8.31 Delta/star transformer connection (phase shift + 30°)

voltages and currents undergo a shift in phase which can be ±30° or ± 90° depending upon the connections.

The delta/star connection of Fig. 8.31 with polarity marks indicated is a commonly used connection. The phasor diagram for voltages is shown in Fig. 8.32. The phase sequence is assumed to be *abc*/*ABC*.

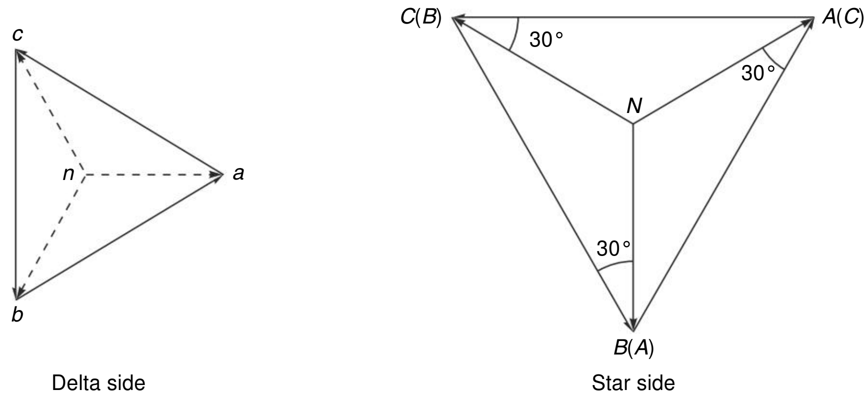


Fig. 8.32 Voltage phasor diagram of delta/star connection of Fig. 8.32

It is observed from above that the line voltages on star side lead the line voltages on delta side by 30°, viz. V_{ab} by 30°. The phase shift would become –30° by changing the phase sequence to *acb*/ACB, viz. V_{AC} by 30°. Relabeling the terminals on the star side, as shown within brackets, would make the phase shift –90° viz. V_{AB} lags V_{ab} by 90°. The reader may relabel to make the phase shift +90°. The line currents would undergo the same phase shift as line voltages in balanced 3-phase loading.

In power system applications of transformers, it is standard practice to connect the transformers (Δ/*Y*) such that the phase shifts by + 30° in going from LV side to HV side.

Star/Delta Connection It is the most commonly used connection as the delta side provides a low impedance path for third harmonic current to flow, thereby reducing third harmonic voltage on the lines. At transmission level, the low voltage side is connected delta and the high voltage side is connected. This provides for neutral grounding connection for high voltage transmission. However, at distribution level, the delta-star with star connection on low voltage side is employed to provide a neutral wire for feeding 3-phase and single-phase loads.

Example 8.17 A 3-phase transformer consisting of three 1-phase transformers with turn ratio of 10 : 1 (primary : secondary) is used to supply a 3-phase load of 120 kVA at 400 V on the secondary side. Calculate the primary line current and voltage if the transformer is connected (a) Δ/Y (b) Y/Δ. What is the line-to-line transformation ratio in each case?

Solution

(a) Δ/Y-connection (Fig. 8.33(a)):

$$I = \frac{120 \times 1000}{\sqrt{3} \times 400} = 173.2 \text{ A}$$

$$\text{Primary line-to-line voltage} = \frac{aV}{\sqrt{3}} = 10 \times \frac{400}{\sqrt{3}} = 2309 \text{ V}$$

$$\text{Primary line current} = \frac{\sqrt{3}I}{\text{a}} = 1.732 \times 173.2 \times \frac{1}{10} = 30 \text{ A}$$

Line-to-line transformation ratio (primary/secondary):

$$= \frac{aV/\sqrt{3}}{V} = \frac{a}{\sqrt{3}} = \frac{10}{\sqrt{3}}$$

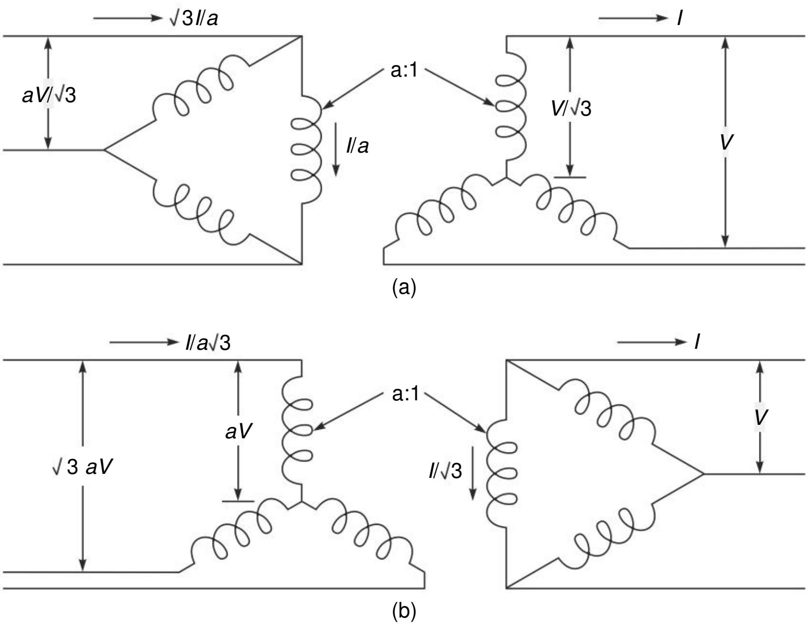


Fig. 8.33

(b) Y/Δ-connection (Fig. 8.31(b))

$$I = 173.2 \text{ A}$$

$$\text{Primary line-to-line voltage} = \sqrt{3}\,aV = \sqrt{3} \times 10 \times 400 = 6928 \text{ V}$$

$$\text{Primary line current} = \frac{1}{a\sqrt{3}} = \frac{173.2}{10 \times 1.732} = 10 \text{ A}$$

$$\text{Line-to-line transformation ratio} = \frac{\sqrt{3}\,aV}{V} = \sqrt{3}\,a = 10\sqrt{3}$$

8.12 SPECIAL TRANSFORMERS

Audio-Frequency Transformer

It is used at the output stage of audio frequency electronic amplifier for matching the load to the output impedance of the power amplifier stage. Here the load is fixed but the frequency is variable over a band (audio, 20 Hz to 20 kHz), the response being the ratio V_2/V_1 (Sec. 5.2). A flat frequency response over the frequency band of interest is most desirable. The corresponding phase angle (angle of V_2 wrt V_1) is called phase response. A small angle is acceptable.

Figure 8.34 shows the more exact circuit model of a transformer with frequency variable over a wide range. Here the magnetizing shunt branch is drawn between primary and secondary impedances (resistance and leakage reactance). Also represented is the shunting effect of transformer windings' stray capacitance C_s. In the intermediate frequency (IF) range, the shunt branch acts like an open circuit and series impedance drop is also negligibly small such that V_2/V_1 remains fixed (flat response) as in Fig. 8.35.

In the LF (low frequency) region, the magnetizing susceptance is low and draws a large current with a consequent large voltage drop in $(r_1 + j\omega L_1)$. As a result, V_2/V_1 drops sharply to zero (at dc $B_m = 0$) (Fig. 8.35). In the HF (high frequency) region, $B_s = 1/\omega C_s$ (stray capacitance susceptance) has a strong shunting effect and V_2/V_1 drops off as in Fig. 8.35, which shows the complete frequency response of a transformer on logarithmic frequency scale.

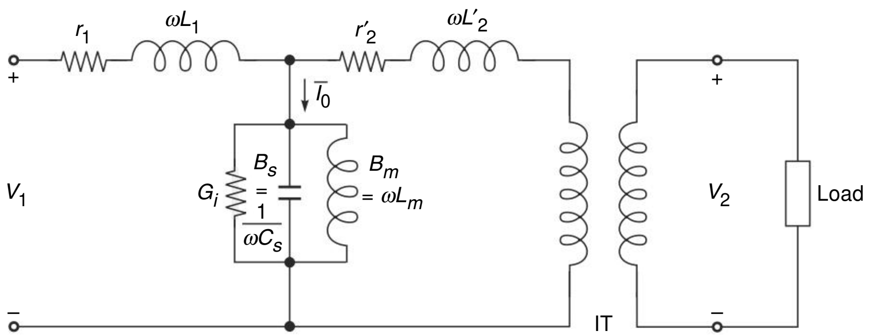


Fig. 8.34

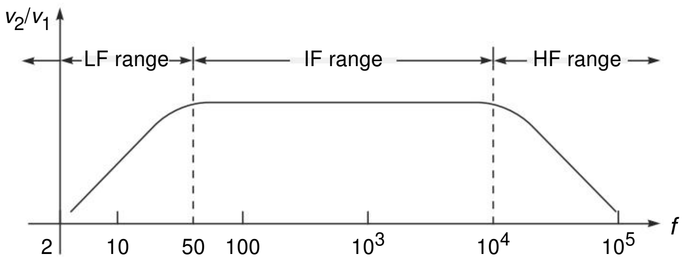


Fig. 8.35 Frequency response (V_2/ V_1 vs log (f)) of a transformer

Current Transformer (CT)

It is a two-winding transformer whose primary is current excited and secondary is shorted (through an ammeter or current coil of a relay) to produce current proportional to the primary current (in the inverse ratio of turns). In power system use, primary may be a single turn, i.e. the line itself. The secondary is usually rated for 1–5 A and certain VA; the VA of the load (ammeter) is known as the *burden*. The current transformer is used to step-down large currents for measurement and relaying purposes.

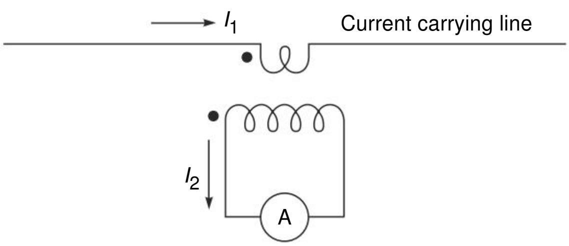


Fig. 8.36 Current transformer

Errors in the current measuring ratio of the CT are caused by (i) magnetizing current and (ii) voltage drops in resistances and leakage reactances. Burden should be such that the core does not get saturated otherwise it will draw an abnormally large magnetizing current introducing an intolerably large error. It may be mentioned here that in relay applications a CT is called upon to measure the short circuit current of the power system.

The secondary of a CT should not be allowed to become open circuited (even inadvertently), otherwise the whole of the primary current acts as a magnetizing current causing extreme permanent magnetization of the core, rendering the CT useless as a current ratio transducer.

ADDITIONAL SOLVED PROBLEMS

8.18 A 50 Hz transformer has 500 turn primary which on no-load takes 60 W power at a current 0.4 A at an input voltage of 220 V. The resistance of the winding is 0.8 Ω. Calculate: (a) the core loss, (b) the magnetizing reactance X_m and (c) the core loss resistance R_i. Neglect leakage reactance.

Solution

$$P\,(\text{in}) = 60\text{ W}, I_0 = 0.4\text{ A}$$

Power loss in winding resistance, $P_{ci} = (0.4)^2 \times 0.8 = 0.128$ W

(a) Core loss $P_i = 60 - 0.128 = 59.88\text{ W} \approx 60$ W

Observe that the winding resistance loss at no-load can be easily ignored.

(b) $\cos\theta_0 = 60/(220 \times 0.4) = 0.682$ lagging. $\theta_0 = 47°$

$$I_m = 0.4\sin\theta_0 = 0.293\text{ A}$$

$$220/X_m = I_m = 0.293\text{ A}$$

or $X_m = 751\ \Omega$

(c) $I_{i0} = 0.4\cos\theta = 0.273$ A

$$220/R_i = 0.273$$

or $R_i = 806\ \Omega$

8.19 A 15 kVA, 2200/220 V, 50 Hz transformer gave the following test results:

OC (LV side): 220 V, 2.72 A, 185 W

SC (HV side): 112 V, 6.3 A, 197 W

Compute the following:

(a) core loss, (b) full-load copper loss, (c) efficiency at full load, 0.85 lagging pf and (d) voltage regulation at full-load, 0.8 lagging/leading pf.

Solution

$$\text{Turn ratio} = 2200/220 = 10$$

(a) Core loss $P_i = 185$ W

(b) $I_{HV}\,(\text{fl}) = 15{,}000/2200 = 6.82$ A

Full-load copper loss

$$P_C\,(\text{fl}) = (6.82/6.3)^2 \times 197 = 231\text{ W}$$

(c) $P\,(\text{out}) = 15 \times 0.85 = 12.75$ kW

$$P_L = P_i + P_c\,(\text{fl}) = 185 + 231 = 416\text{ W}$$

$$\eta = 12.75/(12.75 + 0.416) = 96.8\%$$

(d) $Z(\text{HV}) = 112/6.3 = 17.78\ \Omega$

$$R(\text{HV}) = 197/(6.3)^2 = 4.96\ \Omega$$

$$X(\text{HV}) = 17.07\ \Omega$$

$$\text{Voltage drop} = 6.82\,(4.96 \times 0.8 \pm 17.07 \times 0.6)$$

$$= 96.92\text{V}, -42.76\text{ V}$$

$$\text{Voltage regulation} = +96.92/2200 = +4.41\%\ (0.8\text{ lagging pf})$$

$$= -42.76/2200 = -1.94\%\ (0.8\text{ leading pf})$$

8.20 The maximum efficiency of a 50 kVA transformer is 97.4% and occurs at 90% of the full load at unity pf. Calculate the efficiency of the transformer at (a) full-load, 0.8 pf, and (b) (1/2) full load at 0.9 pf.

Solution

The copper loss (I^2R) is proportional to square of load (load current). At 90% full-load (50 kVA) copper loss to

$$P_C = (0.9)^2\, P_C\,(\text{fl}) = 0.81\, P_C\,(\text{fl})$$

At maximum efficiency

$$P_i = 0.81\, P_C\,(\text{fl})$$

$$\text{Total loss, } P_L = 2\, P_i$$

$$\text{Power output, } P_0 = 50 \times 1 \times 0.9 = 45 \text{ kW}$$

$$\eta = \frac{P_0}{P_0 + P_L} = \frac{1}{1 + \frac{P_0}{P_L}} = 0.974$$

or

$$P_1 = \left(\frac{1 - 0.974}{0.974}\right) \times 45 = 1.33 \text{ kW}$$

$\therefore$

$$P_1 = \frac{1}{2}\, P_1 = 0.665 \text{ kW}$$

and

$$P_c(\text{fl}) = \frac{0.665}{0.81} = 0.82 \text{ kW}$$

(a) Full-load 0.8 pf:

$$P_0 = 50 \times 0.8 = 40 \text{ kW}$$

Then

$$P_L = 0.665 + 0.82 = 1.485 \text{ kW}$$

$$\eta = \frac{40}{41.485} \times 100 = 96.4\%$$

(b) Half full-load, 0.9 pf:

$$P_0 = 25 \times 0.9 = 22.5 \text{ kW}$$

$$P_L = 0.665 + \left(\frac{1}{2}\right)^2 \times 0.82 = 0.87 \text{ kW}$$

$$\eta = \frac{22.5}{23.31} = 96.3\%$$

8.21 A 500 kVA transformer has 95% efficiency at full load and also at 60% of full load both at upf.

(a) Separate out the transformer losses.

(b) Determine the transformer efficiency at 75% full load, upf.

Solution

(a) $$\frac{500}{500 + P_i + P_c} = 0.95 = \frac{300}{300 + P_i + 0.36P_c}$$

which gives the following two equations

$$P_i + P_c = 26.32$$

$$P_i + 0.36\, P_c = 15.79$$

Solving, we get

$$P_c = 16.45 \text{ kW}, \quad P_i = 9.87 \text{ kW}$$

(b)

$$P_L = 9.87 + (0.75)^2 \times 16.45 = 19.12 \text{ kW}$$

$$\eta = (500 \times 0.75)/(500 \times 0.75 + 19.12) = 95.15\%$$

8.22 A 2200/220 V, 50 Hz single-phase transformer has *emf* per turn of approximately 12 V. Calculate (a) the number of primary and secondary turns (b) the cross-sectional area of the core if the maximum flux density is limited to 1.5 T.

Solution

(a) *emf/turn* = 12

$$\text{Primary turns, } N_1 = \frac{2200}{12} = 183.3 = 183 \text{ (integral)}$$

$$\text{Secondary turns, } N_2 = \frac{200}{12} = 183 \ = 18 \text{ (integral)}$$

(b)

$$\phi_{\max 1} = \frac{V(\text{turn})}{4.44\, f \times 1} = \frac{12}{4.44 \times 50}$$

$$= 0.054\ W_b$$

$$B_{\max 1} = \frac{\phi_{\max}}{A_c}$$

$$1.5 = \frac{0.054}{A_c} \text{ or } A_C = 0.036 \text{ m}^2$$

8.23 A 330/600V, 50Hz transformer has silicon laminated steel (B-H value of Fig. 7.16) and a maximum flux density of 1.27. The core dimensions are cross- sectional area 24 cm^2 and mean length 120 cm. Calculate the primary and secondary turns.

The transformer secondary feeds a load of 20A at 0.8 pf lagging. Calculate the primary current assuming the transformer to be ideal.

Now calculate the magnetizing current assuming resistances and leakage reactance to be negligible and ignore core loss. Calculate the net current drawn from the supply. Comment on the results.

Solution

(a) Maximum flux density, $B_{\max} = 1.2T$

$$\phi_{\max 1} = 1.2 = A_C = 1.2 \times 25 \times 10^{-4} = 3 \text{ mWb}$$

$$\phi_{\max 1} = \frac{v_1}{4.444\ fN_1} = \frac{v_1}{4.444\ fN_2} = 3 \times 10^{-3}$$

$$\frac{3300}{4.44\ 50\ N_1} = \frac{600}{4.44\ fN_2} = 3 \times 10^{-3}$$

We get

$$N_1 = 4955 \qquad N_2 = 901$$

Secondary load, $I_2 = 20$ A, 0.8 lagging :

Assuming IT (Ideal transformer)

$$I_1 = I_2' = \left(\frac{901}{4955}\right) \times 20 = 3.637A, 0.8 \text{ lagging}$$

$$\bar{I}_2 = 2.91 - j2.19 \text{ A}$$

To find magnetizing current (core loss negligible) :

For $B_{\max} = 1.2$, it is found from the *B-H* curve of Fig. 7.16.

$$H_{max} = 250 \text{ AT/m}$$

Mean core length, $l_c = 120$ cm

$$AT_{max} = 250 \times 120 \times 10^{-2} = 300$$

Then

$$i_m(\max) = \frac{300}{4955} = 0.061 \text{ A}$$

$$i_m(\max) = \frac{0.061}{\sqrt{2}} = 0.043 \text{ A}$$

It is 90° lagging, $I_i = 0$

From the circuit model of Fig. 8.8 :

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2' = -j\,0.043 + 2.91 - j\,2.18$$
$$= 2.91 - j\,2.22 = 3.66\ \angle -37.3°$$

or $I_1 = 3.66$ A, pf = cos 37.3° = 0.795 lagging

We find that consideration of magnetizing current has made very little difference

$$\frac{3.66 - 3.637}{3.637} \times 100 = 0.63\% \text{ increase}$$

Note: In Power System Analysis magnetizing current is normally ignored.

8.24 A 50 kVA, 2400/240V, 50 Hz transformer draws 375 W at 0.4 power factor when 2400V is applied to the HV winding and the LV winding is open-circuited. If 240V is applied to the LV winding with HV open-circuited, what current will flow into the LV winding and at what pf ?

Solution

Turn ratio, $a \approx \dfrac{2400}{240} = 10$

HV side:

$$VI_0 \cos\phi_0 = 375,\ I_0 = \frac{375}{2400 \times 0.4} = 0.391 \text{ A}$$

LV side:

$$I_0' = 0.391 \times 10 = 3.91\text{A}$$

Power factor is the same = 0.4 lagging

8.25 A 220/110 V transformer has an impedance of 0.32 + j0.85 Ω in the 220 V winding and an impedance of 0.11 + j0.27 Ω in the 110 winding. With 220 V applied to the HV side, a short circuit occurs on the 110 V. What currents will flow in the two windings?

Solution

Turn ratio, $a \approx \dfrac{220}{110} = 2$

Total impedance as seen on HV side

$$= (0.32 + j\,0.85) + 4\,(0.11 + j\,0.27)$$

$$= 0.76 + j\,1.93 = 2.07 < 68.5° \text{ W}$$

Before short circuit, LV voltage = 110 V and HV voltage = 220 V.

Short circuit current:

$$I\,(HV) = \frac{220}{2.07} = 106.3 \text{ A}$$

$$I\,(LV) = 106.3 \times 2 = 212.6 \text{ A}$$

8.26 A 100 kVA transformer is rated 11 kV/230V, 50 Hz. It requires 310V to be applied to the primary to circulate full-load current with short on the secondary side absorbing 5.21 kW. Determine its per cent voltage regulation and the primary voltage for power factors of (i) unity (ii) 0.8 lagging and (iii) 0.8 leading.

Solution

Secondary shorted:

Primary side data

$$V_{SC} = 320\text{V},\ P_{SC} = 5.21 \text{ kW}$$

$$I_{SC} = I\,(\text{fl}) = \frac{100}{11} = 90.9 \text{ A}$$

Equivalent impedance

$$Z = \frac{310}{90.9} = 3.41\ \Omega$$

$$R = \frac{5210}{(90.9)^2} = 0.63\ \Omega$$

$$x = [(3.41)^2 - (0.63)^2]^{1/2} = 3.35\ \Omega$$

Voltage drop, $V\,(\text{drop}) = I\,(R\cos\theta + X\sin\theta)$

(i) *Upf,* $\theta = 0$

$$V\,(\text{drop}) = 90.9 \times 0.63 = 57.3 \text{ V}$$

$$\text{Voltage regulation} = \frac{57.3}{11\times10^3} \times 100 = 0.52\%$$

Primary voltage, $V_1 = 11000 + 57 = 11.057$ kV

(ii) pf 0.8 lagging

$$V\,(\text{drop}) = 90.9\,(0.63 \times 0.8 + 3.35 \times 0.6) = 228 \text{ V}$$

$$\text{Voltage regulation} = \frac{228}{11\times10^3} \times 100 = 2.07\%$$

Primary voltage, $V_1 = 11000 + 228 \approx 11.23$ kV

(iii) *pf* 0.8 lagging

$$V\,(\text{drop}) = 90.9\,(0.63 \times 0.8 - 3.35 \times 0.6) = -183\text{V}$$

$$\text{Voltage regulation} = \frac{-183}{11\times10^3} \times 100 = -1.66\%$$

Primary voltage, $V_1 = 11000 - 228 \approx 10.82$ kV

8.27 The resistances and leakage reactance of a 10 kVA, 2200/220 V, distribution transformer are as follows:

HV	$r_1 = 4\ \Omega$	$x_1 = 5\ \Omega$
LV	$r_2 = 0.04\ \Omega$	$x_2 = 0.05\ \Omega$

The transformer is supplying rated kVA at 0.8 lagging power factor to a load of rated voltage. Find the voltage required on the HV side and the voltage regulation.

At what power factor would the voltage regulation be zero? What are the real and reactive component of the load?

Solution

(a) Transfer impedance as seen on HV side :

$$\text{Turn ratio, } a \approx \frac{2200}{220} = 10$$

$$R_1 = 4 + (10)^2 \times 0.04 = 8\ \Omega$$

$$X_1 = 5 + (10)^2 \times 0.05 = 10\ \Omega$$

$$\text{Load current, } I_1 = \frac{10 \times 1000}{2200} = 4.545\ \text{A}$$

$$\text{pf} = 0.8 \text{ lagging}$$

Load voltage (referred to HV), $V_2 = 220 \times 10 = 2200$

$$\begin{aligned}\text{Voltage drop } &= I_1(R_1 \cos\theta + X_1 \sin\theta)\\ &= 4.545\,(8 \times 0.8 + 10 \times 0.6)\\ &= +\,56.36\text{V}\end{aligned}$$

$$\text{Voltage regulation } = \frac{+56.36}{2200} \times 100 = +\,2.56\%$$

Voltage to be applied to HV side

$$V_1 = 2200 + 56.36 = 2256\ \text{V}$$

For zero voltage regulation

$$8\cos\theta - 10\sin\theta = 0,\ \theta \text{ leading}$$

$$\tan\theta = \frac{10}{8},\ \theta = 51.3°$$

$$\text{pf} = \cos 51.3° = 0.62 \text{ leading}$$

Load:

Real power, $P = 10 \times 0.62 = 6.2$ kW

Reactive power, $Q = -10 \sin 51.3° = -7.8$ kVAR (leading)

8.28. A 200/400V, 20 kVA, 50 Hz transformer is connected as an autotransformer to transformer from 600 V to 200V.

(a) Determine the autotransformer ratio 'a'.

(b) Determine the kVA rating of the autotransformer.

(c) With a load of kVA as found in part (a) connected to 200 V terminals, determine the currents in the load and the two transformer windings.

(d) In part (a) find the kVA transformer and kVA conducted.

Solution

The autotransformer connections are drawn in

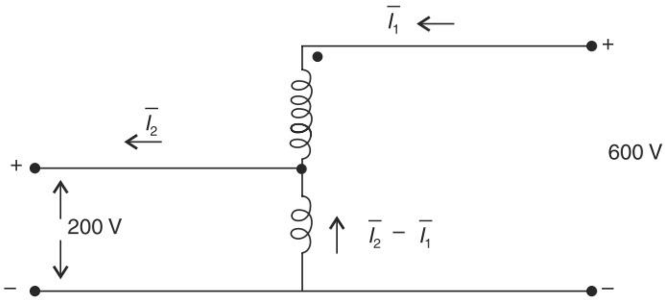


Fig. 8.37

(a) Autotransformer ratio

$$a = \frac{600}{200} = 3$$

(b) Current handling capacity of 400 V winding

$$I_1 = \frac{20 \times 10^3}{400} = 50 \text{ A}$$

As an autotransformer rated current on 600 V side, $I_1 = 50$ A

Auto kVA rating = $600 \times 50 \times 10^{-3} = 30$

(c) With 30 kVA load :

$$I_1 = 50 \text{ A}, \qquad I_2 = \frac{30 \times 10^3}{200} = 150 \text{ A}$$

Or directly $I_2 = a\, I_1 = 3 \times 50 = 150$ A

Current in transformer secondary

$$I_2 - I_1 = 150 - 50 = 100 \text{ A}$$

(d) kVA transformed = $400 \times 50 \times 10^{-3} = 20$

kVA conducted = $30 - 20 = 10$

8.29 A 100 kVA, 11 kV/400 V distribution is connected delta-star. It is fully loaded. Assume the three transformer to be ideal.

Calculate

(a) phase-to-phase transformation ratio, and

(b) line and phase currents on both primary and secondary sides.

Solution

Secondary star connected:

$$V_L = 400 \text{ V}, \qquad V_P = \frac{400}{\sqrt{3}} = 231 \text{ V}$$

$$I_L = \frac{100 \times 10^3}{\sqrt{3} \times 400} = 144.3 \text{ A} = I_P$$

Primary delta-connected

$$V_L = V_P = 11 \text{ kV}$$

(a) phase transformation ratio :

$$\frac{HV}{LV}\text{(Phase)} = a = \frac{11000}{231} = 47.6, \quad \text{so} \quad a = 48 \text{ integral}$$

(b) Star side :

$$I_L = \frac{100 \times 10^3}{\sqrt{3} \times 400} = 144.3\text{A} = I_P$$

Delta side :

$$I_L = \frac{100}{\sqrt{3} \times 11} = 5.25 \text{ A}$$

$$I_P = \frac{5.25}{\sqrt{3}} = 3.03\text{A}$$

SUMMARY

- A transformer comprises of two mutually coupled coils wound on a magnetic core to carry the mutual flux, thereby making the coupling very light.
- A transformer which can raise the voltage level of the source and correspondingly bring down the current level is called a step-up transformer OR if it can bring down the voltage level and raise the current level then it is called a *step-down* transformer.
- The windings are made of conducting wires, copper or aluminium and the core is made of lightly insulated *laminated* (thin sheet joined together) *silicon steel*.
- The winding connected to the source is called the *primary* and the second winding that feeds the load is called the *secondary*.
- To avoid confusion, the windings are referred to as HV and LV windings.
- The transformers have two types of cores—core type and shell type; the core type is most common in use.
- Core type: core is rectangular with half LV and half HV on opposite limbs with LV placed inside next to core. This split winding arrangement reduce leakage flux and is economical insulation-wise.
- Ideal Transformer

  It transformer voltage in direct ratio of primary and secondary turns and transforms currents in the inverse ratio of turns. Symbolically

  $$\frac{V_1}{V_1} = \frac{N_1}{N_1} = \frac{I_2}{I_2} = a \text{ (turn ratio)}$$

  It is transforms impedance from one side to other in direct square ratio of turns and admittance in inverse square ratio of turns. Thus

$$Z_1 = \left(\frac{N_1}{N_2}\right)^2 Z_2, \qquad Y_1 = \left(\frac{N_2}{N_1}\right)^2 Y_2$$

Volt-amperes (VA) are preserved i.e.,

$$\bar{V}_1 \bar{V}_1 = \bar{V}_2 \bar{V}_2 \text{ and } S = \bar{V}_1 \bar{V}_1^* = \bar{V}_2 \bar{V}_2^*$$

- The transformer core flux is

$$\phi_{\max} = \frac{V_1}{\sqrt{2}\,\pi f N_1^2} = \frac{V_2}{\sqrt{2}\,\pi f N_2^2}; \quad \sqrt{2}\,\pi = 4.44$$

  In a real transformer, the relationship holds for induced *emfs* but is approximately acceptable for voltages. Observe that the relationship is independent of primary and secondary currents.
- Core flux lags induced emfs by 90°.
- In a real transformer, *magnetizing current* (I_m) is drawn by the primary (which is connected to the source). It lags the applied voltage by 90°.
- Magnetizing current is accompanied by the core loss current (I_i) in phase with the applied voltage.
- Exciting current of a transformer is

  $\bar{I}_0 = \bar{I}_m + \bar{I}_i$; lags applied voltage by slightly less than 90° as $I_i << I_m$
- Exciting current of a transformer is far less than its rated load current.
- Power (active and reactive) is transferred from the primary to secondary side by the load current and not by the exciting current. In fact, the exciting current produces flux in the core so that the transformer can carry load current.
- Secondary AT = N_2I_2 are equal and opposite the load component of primary AT = N_1I_1 such that core flux does not change.
- Each transformer winding has resistance and leakage reactance (caused by leakage flux).
- Transformer resistance and reactance can be referred to any one side as

$$R_1 = r_1 + a^2 r_2, \qquad X_1 = x_1 + a^2 x_2; \quad a = \text{turn ratio} = \frac{N_1}{N_2}$$

$$R_2 = r_2 + \frac{1}{a^2} r_1, \qquad X_2 = x_2 + \frac{1}{a^2} x_1$$

- Transformer circuit model (also called equivalent circuit) comprises
  1. magnetizing branch ($G_i \parallel B_m$) in shunt across the applied voltage;
  2. series branch ($R + jX$) connecting the source to the ideal transformer;
  3. ideal transformer; it may not be shown in the circuit diagram; usually omitted.
- Transformer losses:
  1. Core loss (P_i); constant for constant applied voltage
  2. Copper loss (P_C); variable proportional the square of the load current
- Determination of circuit model (equivalent circuit) parameters by two test OC and SC
- OC (Open Circuit) Test: One side excited at rated voltage, the other side left open.

Three meter readings taken

V_1, I_0, $P_0 = P_i$ (core loss)

$$Y_0 = \frac{V_1}{I_0}, \quad G_i = \frac{P_0}{I_0^2}, \quad B_m = \sqrt{Y_0^2 - G_i^2}$$

- SC (Short Circuit) Test: One side shorted, low voltage applied on the other side. Three meter readings taken. V_{SC}, I_{SC}, P_{SC} (copper loss)

$$Z = \frac{V_{SC}}{I_{SC}}, \quad R = \frac{P_{SC}}{I_{SC}^2}, \quad X = \sqrt{Z^2 - R^2}$$

There are equivalent values on the test side.

- Per Unit System

$(KVA)_B$ or $(MVA)_B$ taken as transformer rating (convenient) voltage bases must be in ratio of transformation. It is convenient to use rated voltage on each side.

$(KV)_B$ (HV) and $(LV)_B$ (LV)

$$Z_B = \frac{1000(KV)_B^2}{(KVA)_B} = \frac{(KV)_B^2}{(MVA)_B}$$

$$Y_B = \frac{1}{Z_B}$$

Then

$$Z(\text{pu}) = \frac{Z(\Omega)}{Z_B}, \quad Y(\text{pu}) = \frac{Y(\Omega)}{Y_B}$$

For three phase system, use three phase (KVA) or (MVA) as base and line-to-line kV as base. The above relations apply. Z (pu) and Y (pu) are per phase values on equivalent star has

- $$\% \text{ Voltage regulation} = \frac{V_{20} - V_{21}\text{fl}}{V_{20}\text{fl}} \times 100$$

V_{20} = secondary voltage when full-load is thrown off with circuit represented on any side

$$\text{Voltage regulation} = \frac{V_2 - V_1}{V_2} \times 100$$

$V_2 - V_1$ = voltage drop $\approx I(R\cos\theta \pm X\sin\theta)$; + for lagging, – for leading

- Efficiency (η)

$$\eta = \frac{P_o(V_2 I_2 \cos\theta)}{P_o + P_i + P_c}$$

For η (max)

$$P_C = I_2^2 R_2 = P_i \text{ or } I_2 = \sqrt{\frac{P_i}{R_2}}$$

- Autotransformer—it is a single winding transformers with windings connected electrically. It is used for transformer ratio close to unity. Its kVA rating and efficiency is greatly enhanced compared to the two-winding transformer.
- Three-phase transformer—three identical single-phase transformer connected in star / delta, the most common configuration at transmission level LV side, is delta connected and HV side star connected; neutrals on HV side are grounded.

At distribution level, HV side is delta connected and LV side is star connected; neutral of star is used to supply single-phase load.

- Depending upon line labeling and phase sequence, the line voltage on the two sides of a star/delta transformer got shifted by ± 30° or ± 90°. The labeling used in practice is 30°.

REVIEW QUESTIONS

1. What is a transformer? Explain the function it fulfils as an element of a power system.
2. Explain the constructional differences between core and shell type transformers.
3. Explain briefly the ideal transformer as a circuit element. Can voltage and current rations be adjusted independently?
4. Explain the operation and application of the impedance transforming property of an ideal transformer.
5. State how the LV and HV windings are arranged in a core-type transformer. Explain the reason.
6. What is the phase relationship between the core flux, the magnetizing current and the induced *emfs* in the primary and secondary winding of a transformer? Draw the phasor diagram.
7. What is the transformation ratio of a transformer? Why is it not identical to voltage ratio of a transformer?
8. What determines the maximum value of flux in a transformer core when excited from the primary side? Does the value of flux change substantially when the secondary is loaded? Explain the reason.
9. Explain how the OC / SC tests separate out the core loss and copper loss.
10. A transformer is excited from the primary side at rated voltage but with secondary open. Would it draw any current? If so, what is action of this current and its components?
11. Why cannot the SC test separate out the primary and secondary resistances and leakage inductances?
12. Justify that under SC test the core loss is negligible.
13. Prove that in the system, if the voltage bases are selected in the ratio of transformation, the *pu* impedance of the transformer is same as either side.
14. State and prove the condition from maximum efficiency of a transformer.
15. Draw the phasor diagram of a transformer as seen from any one side for zero voltage regulation.
16. From the phasor diagram of question 15, derive the approximate condition for zero voltage regulation.
17. Justify the statement that in the circuit model of a transformer in a power system, the magnetizing branch can be ignored.

18. Explain the meaning of all the items in the name plate of a transformer.
19. How can we refer the transformer winding resistance and leakage reactance from one side to the other?
20. From the percentage impedance given as the name plate, how can you find the voltage to be applied for full load current to flow in SC test?
21. Draw the equivalent circuit of a transformer. Identify the test by which the value of each circuit element can be found.

PROBLEMS

8.1 A single-phase transformer is rated 600/200 V, 25 kVA, 50 Hz.

(a) Calculate the magnitude of primary and secondary currents when the transformer is fully loaded (use IT model).

(b) What should be the impedance of the load in ohms to fully load the transformer when connected on (i) 600 V side and (ii) 200 V side (use IT model)?

(c) What would be the value of the maximum core flux when the transformer is excited at rated voltage on either side, given $N_1 = 60$ turns?

(d) If the transformer is operated from a 60 Hz, source, what should be its voltage rating for the maximum core flux to stay at the same value as in part (c)?

(e) If the 600 V side is excited at 600 V, 40 Hz, what would be the core flux and the secondary voltage? What effect do you expect to observe in the core under these conditions?

8.2 A 25 kVA, 600/200 V transformer is subjected to an SC test. The voltage applied on one side, with the other shorted, is 5.2% of the rated voltage. The transformer draws rated current and a power of 242 W during the test.

(a) Compute equivalent resistance and leakage reactance of the transformer in ohms on either side and in pu.

(b) Compute the core flux as a percentage of core flux at rated voltage.

(c) From part (b) justify that all the 242 W constitute ohmic losses.

8.3 A 25 kVA, 600/200 V transformer is found by the SC test from the 600 V side to have equivalent resistance 0.139 Ω and equivalent reactance 0.735 Ω as seen from the HV side. A load impedance of $\bar{Z}_L = 1.48 + j\,1.04\ \Omega$ is connected across the secondary.

(a) Find currents in both windings assuming transformer to be ideal.

(b) Solve part (a) again by taking the transformer impedance into account.

(c) Calculate voltage regulation of the transformer.

8.4 The 25k VA, 600/200 V transformer when subjected to SC test from the 600 V side is found to have equivalent resistance of 0.139 Ω and equivalent reactance of 0.735Ω. The transformer is now OC tested from a 600 V source with the secondary open. The transformer draws a power of 195 W.

Compute the efficiency of the transformer when loaded with with $\bar{Z}_L = 1.48 + j\,1.04\ \Omega$ on the secondary side.

8.5 A 50 kVA, 1100/220 V, 50 Hz transformer has an HV winding resistance of 0.125 Ω and a leakage reactance of 0.625 Ω. The LV winding has corresponding values of 0.005 Ω and 0.025 Ω respectively. Find the equivalent impedance of the transformer referred to HV and LV sides. Find the pu impedance of the transformer.

8.6 Consider the transformer of Prob. 8.5 to give its rated output at (a) 0.8 lagging pf and (b) 0.8 leading pf on the LV side. Find the HV terminal voltage and % regulation. Use pu system.

8.7 The transformer of Prob. 8.5 has a core loss of 580 W. Find its efficiency at 3/4th full load, 0.8 lagging pf.

8.8 A 50 kVA, 1100/220V, 50Hz transformer has an HV winding resistance of 0.125Ω and a leakage reactance of 0.625Ω. The LV winding has comes ponding values of 0.005W and 0.025W regpectively. Wehn excuited from 1100V source gets shorted at LV terminals. Find the steady-state current which would be drawn by the HV if the source voltage is assumed to remain constant.

8.9 The transformer of Prob. 8.5 is fully loaded on the secondary side at (a) 0.8 lagging, (b) 0.8 leading pf while it is fed on the primary side at 1100 V. Calculate the voltage at the secondary terminals.

8.10 The circuit model of a 5 kVA, 200/400 V, single-phase transformer, referred to the LV side, is shown in Fig. 8.38.

(a) An OC test is conducted from the HV side at 400 V. Calculate the power input, power factor and current (magnetizing) drawn by the transformer.

(b) An SC test is conducted from the LV side by allowing full-load current to flow. Calculate the voltage required to be applied, the power input and power factor.

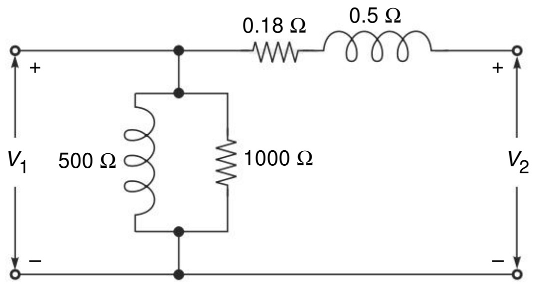


Fig. 8.38

8.11 The following test results were obtained on a 20 kVA, 2200/220 V transformer:

OC test (LV): 220 V, 1.1 A, 125 W

SC test (HV): 52.7 V, 8.4 A, 287 W

(a) The transformer is loaded at unity pf on secondary side with a voltage of 220 V. Determine the maximum efficiency and the load at which it occurs.

(b) The transformer is fully loaded. Determine the load pf for zero voltage regulation.

8.12 A 1000/200 V, 25 kVA transformer is connected as an autotransformer to yield a transformation ratio of 1000/1200 V. Calculate its kVA rating. Calculate also the currents in the two windings when the autotransformer is fully loaded.

8.13 A variable-voltage laboratory transformer is shown in Fig. 8.39. With the transformer fully loaded, compute the ratio of I_1/I_2 when

(a) the sliding contact is adjusted to 50% of input voltage,

(b) when it is placed at 10% voltage.

(c) Is it permissible to place the sliding contact at the extreme positions?

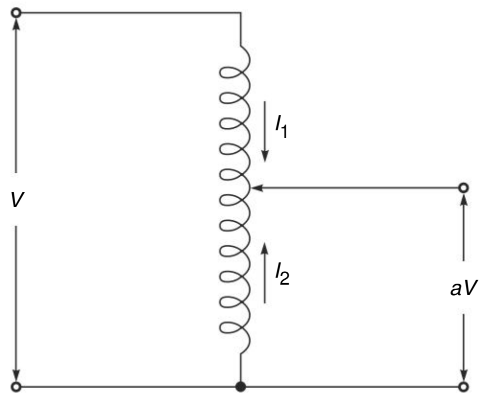


Fig. 8.39

8.14 A 20 kVA, 2400/240 V, two-winding transformer has an efficiency of 97.5% at full load, 0.8 pf. It is connected as a 2400/2640 V autotransformer. At full load calculate the kVA output, kVA transformed and kVA conducted. Find also the efficiency at full load, unity power factor.

8.15 A Δ/Y connected 3-phase transformer as shown in Fig. 8.39 has a voltage ratio of 22 kV (Δ)/345 kV (*Y*) (line-to-line). The transformer is feeding 500 MW and 100 MVAR to the grid (345 kV). Determine the kVA and voltage rating of each unit and compute all currents and voltages in both magnitude and phase in lines and all the windings (3 primaries and 3 secondaries). Assume the transformer units to be ideal.

8.16 Three identical transformers each rated 6.6/22 kV, 3 MVA, are connected in Y/Y. The transformer bank is fed from a source of line voltage $6.6\sqrt{3}$ kV. The secondary side feeds a delta-connected load composed of three equal impedances. Assuming the individual transformers to be ideal find

(a) the value of *Z* in ohms to fully load the bank (i.e. 9 MVA),

(b) the current in each leg of the load (Δ connected) and

(c) the current in each transformer primary and secondary.

8.17 The three transformers of Prob. 8.16 are connected in Δ/*Y* and are fed from 6.6 kV (line-to-line) source on Δ side. The load comprises three Δ-connected impedances. Assuming all the three transformers to be ideal solve for all parts of Prob. 8.16. Also find the primary and secondary side line currents.

Chapter 9

EMF AND TORQUE IN ELECTRIC MACHINES

MAIN GOALS AND OBJECTIVES

- *To familiarize with the general constructional features of electric machines*
- *To know stator, rotor, field poles, armature windings, concept of electrical angle and mechanical angle*
- *What is synchronous speed, its determination from the frequency and number of machine poles*
- *To learn about ac armature windings, distributed full-pitched, short-pitched, the general emf formula*
- *Armature coil mmf fundamental, standing pulsating field*
- *To understand the rotating magnetic field; conditions, speed and direction*
- *Interaction torque of rotating magnetic field, conditions for steady torque*
- *Learning about electric machine type and their operation*
  - synchronous machine, induction machine: their distinguishing features
- *The dc machine, elementary treatment, commutator action*
- *General approach to machine losses and efficiency, cooling*

9.1 INTRODUCTION

The underlying concepts of electromechanical machines and devices are common. This chapter is therefore written to explain these concepts and their imaginative application to a variety of machines, mainly dc machines and two types of ac machines—synchronous and induction machine.

Electromechanical energy conversion takes place whenever a change in flux linkages is associated with mechanical motion. Speed voltage is generated in a coil when there is relative movement between the coil and magnetic field. Alternating emf is generated if the change in flux linkage of the coil is cyclic. The *field windings*, which are the primary source of flux in a machine, are, therefore, arranged to produce cyclic north–south space distribution of poles. A cylindrical structure is a natural choice for such a machine. Coils, which are seats of induced emf's, are several in number in practical machines and are suitably distributed and connected in series/parallel and in star/delta 3-phase connection to give the desired voltage and to supply the rated current: this arrangement is called the *armature winding*. When the armature coils carry currents, they produce their own magnetic field, which interacting with the magnetic field of the field winding produces electromagnetic torque tending to align the two magnetic fields.

The field winding and armature winding are appropriately positioned on a common magnetic circuit composed of two parts—the *stator* (stationary member) and the *rotor* (rotating member). The stator is the annular portion of a cylinder in which a cylindrical rotor rotates, there being an appropriate clearance (*air-gap*) between the two. The rotor axle is carried on two bearings which are housed in two end covers bolted on the two sides of the stator as shown in Fig. 9.4. The stator and rotor are made of high permeability magnetic material, e.g. silicon steel. Further, the member in which the flux rotates is built up of thin insulated laminations to reduce eddy-current loss.

This introductory chapter deals with the basis of operation of the common electromechanical conversion devices. It also deals with energy/power loss in electric and magnetic materials used in machines. Origin of these losses is common in all machine types. Losses are also clarified as to their nature—constant, variable.

The concepts discussed in this chapter from a general viewpoint will be applied to the analysis and performance of electric machines in the three chapters that follow.

9.2 ROTATING MACHINES

An electromechanical energy conversion device is a link between electrical and mechanical systems. Conversion of energy either way takes place whenever change of flux linkages is associated with mechanical motion. While certain transducers may have translatory motion, rotary motion is invariably employed for continuous

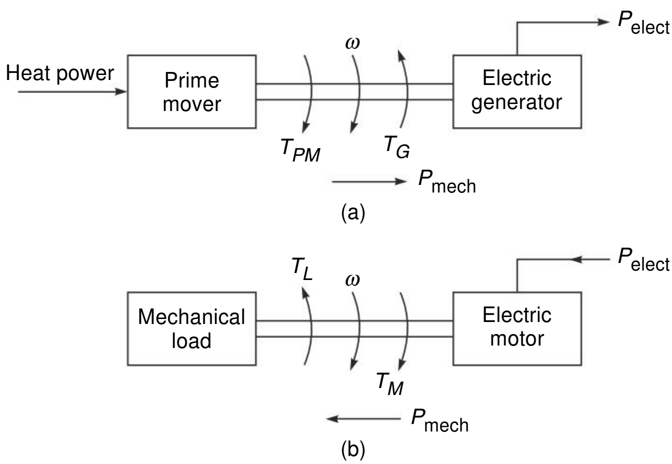


Fig. 9.1 (a) Electric generator ($T_{PM} = T_G$) where T_{PM} = Prime move torque, T_G = Generator torque
(b) Electric motor ($T_M = T_L$) where T_M = motor torque, T_L = load torque

conversion of energy (power conversion). When power is converted from mechanical to electrical form, the device is called a *generator* as shown in Fig. 9.1(a) and when the conversion is from electrical to mechanical form the device is called a *motor* as shown in Fig. 9.1(b). As the electromechanical energy conversion process is a *reversible* one, the same device could act as a generator or motor depending upon the electrical and mechanical conditions at its two ends (ports). In general, such a rotary device is called an *electric machine*.

Constructional Features

An electric machine has a cylindrical structure with an annular stationary member called the *stator* and a cylindrical rotating member called the *rotor*. The stator and rotor with an air-gap in between form the field structure of the machine and are made of high permeability steel; the member carrying alternating or rotating flux being in laminated form. The *field windings* placed on one of these members are the primary source of flux in the machine. These are dc excited and are arranged to produce alternate north–south poles in pairs (*even number of poles*). The other member houses a set of coils in slots uniformly cut around the periphery and are connected to form a winding called *armature winding*. In a 3-phase machine, the armature windings are connected in star or delta. The relative motion between the two members causes emf to be induced in the armature winding, which exchanges current and power with the external electrical system. The interaction between the primary field and current-carrying armature conductors causes production of torque of electromagnetic origin resulting in conversion of energy. From a field viewpoint, the current-carrying armature winding produces its own field, which interacts with the primary field to produce torque.

A *synchronous machine* is an machine in which the field windings are placed on the rotor and the stator carries ac armature windings as shown in Fig. 9.2. While the reverse structure is possible (field on stator and armature winding on rotor), the arrangement of Fig. 9.2 is almost universally adopted in a synchronous machine because of ease of mechanical construction and of insulating a high-voltage stationary winding. The field poles are made projecting or *salient type* (Fig. 9.2). For high-speed machines *nonsalient* or *cylindrical* poles are preferred (Fig. 9.6). The field windings are excited through *slip rings* form a dc source as shown in Fig. 9.4. Generally a small dc generator called the *excitor* is coupled to the shaft of the synchronous machine for this purpose. In case the armature windings are placed on the rotor, load current at rated voltage has to be conducted through three (3-phase case) slip rings. This is another important reason for preferring the arrangement shown in Fig. 9.2. This type of machine must run at a definite fixed speed (called *synchronous* speed) corresponding to the frequency and number of poles.

In a dc *machine,* the field windings are on the stator and armature winding on the rotor as shown in Fig. 9.3. This arrangement is mandatory as alternating voltages and currents induced in the armature winding have to be converted to dc form via a rotating mechanical converter called the *commutator* (not shown in this figure).

In an *induction machine,* the stator carries a 3-phase winding, which when excited form a 3-phase ac source draws a current component that sets up a rotating

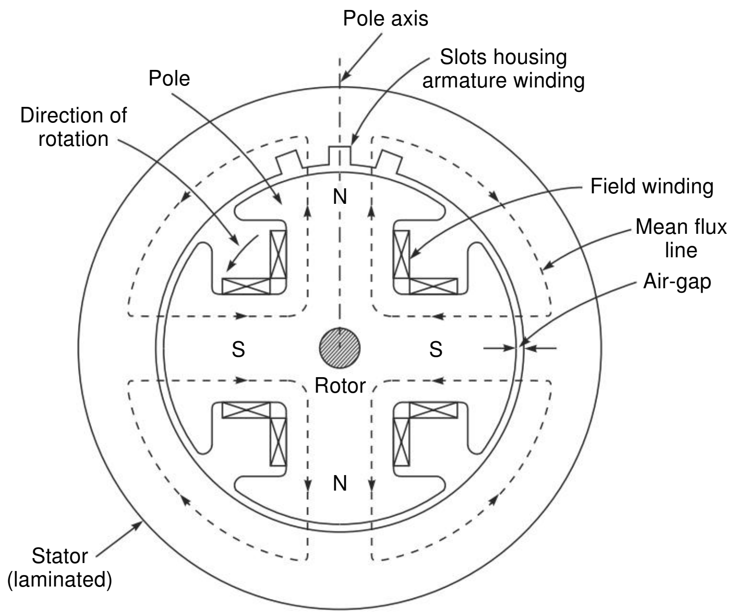


Fig. 9.2 Salient-pole synchronous machine (cross-sectional view); 4 poles

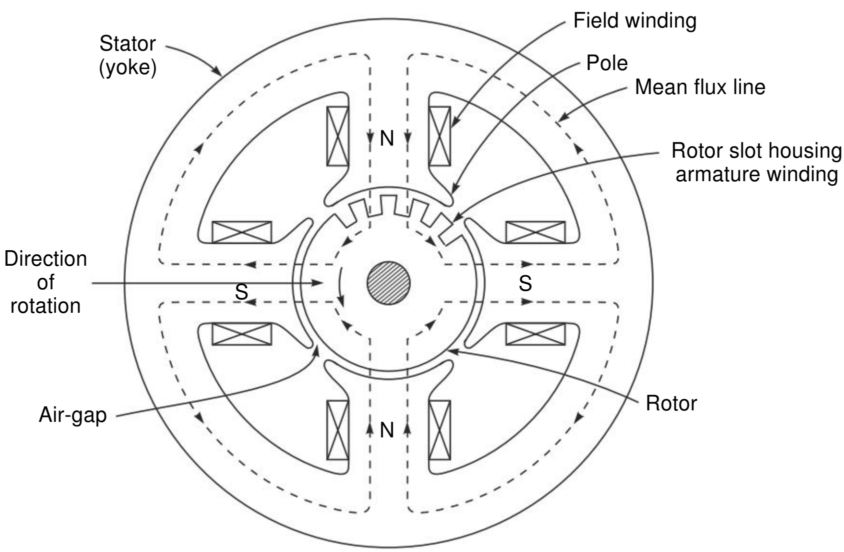


Fig. 9.3 DC machine (cross-sectional view); 4 poles

flux pattern with alternate north–south in the air-gap rotating at synchronous speed corresponding to the supply frequency and number of poles of the machine. The rotating field causes induced currents in the short-circuited rotor windings (or short-circuited conducting bars). The stator- and rotor-caused fields interact to produce machine torque. The machine (motor) runs at speeds somewhat below synchronous speed.

The rotor in an electric machine is carried on a shaft, which is supported in a bearing housed in two *end covers* bolted on to the stator. One of the shaft ends extends out of the bearings to which the mechanical system (prime mover in case of generator and load in case of motor) is coupled. The general arrangement is illustrated in the half cross-sectional view for a synchronous machine as shown in Fig. 9.4.

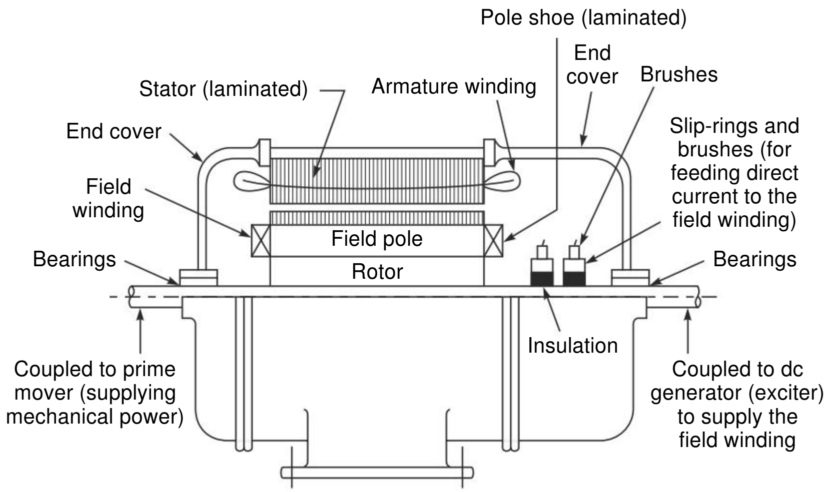


Fig. 9.4 AC machine–synchronous type

In this chapter, induced emf and torque in an electric machine will be treated form a general point of view and certain mathematical relations established.

9.3 ELEMENTARY SYNCHRONOUS MACHINE

In the 4-pole structure exhibited in Fig. 9.2, the field poles have odd symmetry, viz. alternate north–south. The field coils are of *concentrated* kind. If the effect of armature slotting (a secondary affect) is ignored and the air-gap is assumed uniform over the pole faces, it is obvious that the air-gap flux density will be constant over a major part of the pole face, gradually reducing because of fringing at the pole tips and finally becoming zero in the interpolar region. The air-gap flux density wave (around the stator's inner periphery) is therefore a *flat-topped wave* as shown in Fig. 9.5 (a).

This is not the right kind of flux density distribution as it is desirable to generate sinusoidal emf. The pole faces in such a machine structure (*salient-pole* kind) are therefore *chamfered* at the edges so that the air-gap increases towards pole edges causing the flux density wave to become nearly sinusoidal. It shall be assumed* form now on that the air-gap flux density is sinusoidal as shown in Fig. 9.5(b).

The mechanical angle θ_m round the machine periphery is always 2π but it is more convenient to designate the complete angle of one cycle of the

* The harmonic emfs generated by the slightly nonsinusoidal B-wave are filtered out by the machine's leakage inductances and the inductive transmission lines.

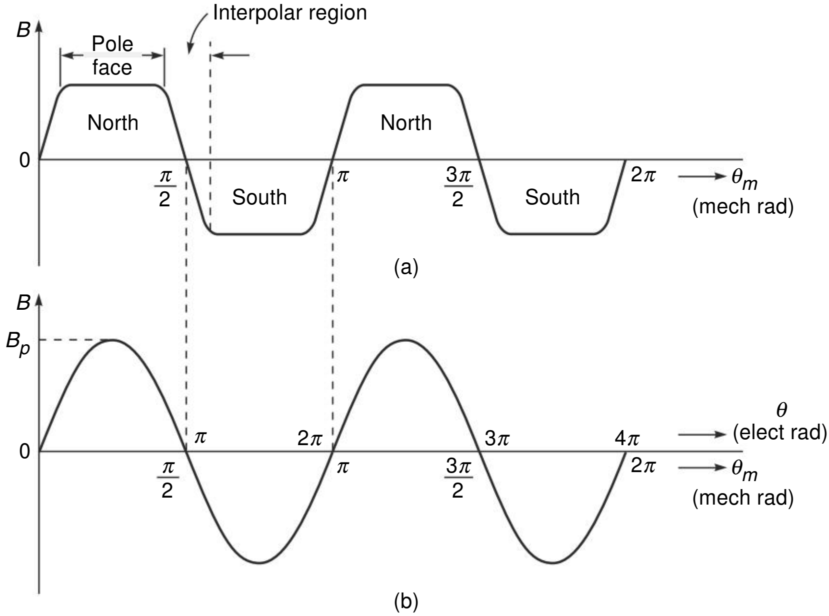


Fig. 9.5 Flux density wave in electric machines (4-pole structure)

B-wave (north–south) as 2π, which to distinguish it is called the *electrical angle* θ. For a P-pole machine, the relationship between these two angles is expressed as

$$\frac{\theta}{\theta_m} = \frac{2\pi \times (P/2)}{2\pi} = \frac{P}{2} \tag{9.1}$$

Another method of achieving a nearly sinusoidal B-wave is to use a cylindrical (nonsalient) rotor structure with a uniform air-gap but distributed field windings as depicted in Fig. 9.6. In this arrangement, as one moves away from the pole-axis, the flux paths link progressively smaller number of field ampere-turns and hence produces the nearly sinusoidal B-wave. The actual B-wave will have a stepped shape corresponding to the rotor slots. The high frequency harmonics pertaining to these steps will be ignored in machine analysis here.

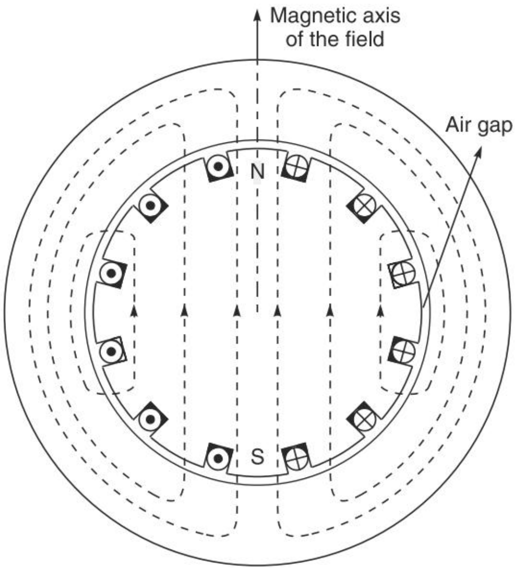


Fig. 9.6 Nonsalient pole (cylindrical) rotor

It is easily observed from Figs. 9.2 and 9.3 that the B-wave repeats itself every one pole

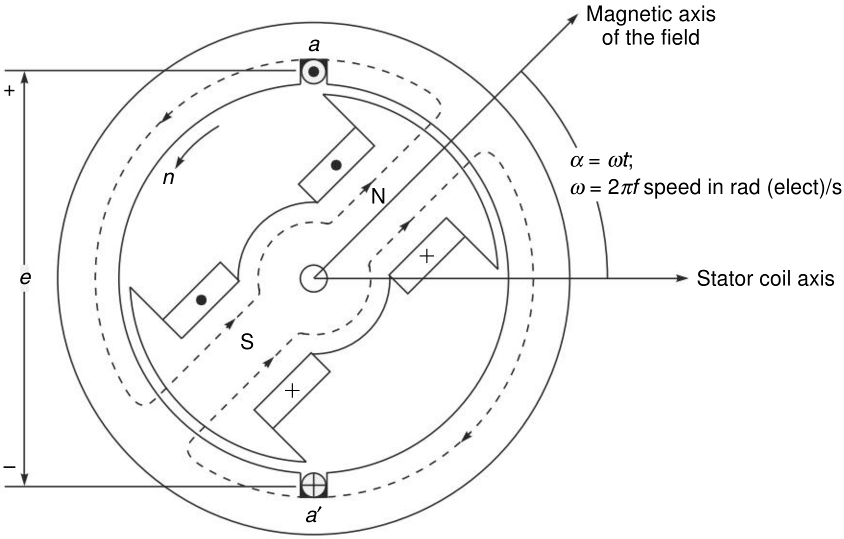


Fig. 9.7 Elementary generator (synchronous)—salient-pole, 2-pole rotor

pair; so does the electrical conditions on the stator (this follows as a consequence). We can therefore picturise and model the machine on a one pole-pair basis in the form of the elementary machine shown in Fig. 9.7.

Figure 9.7 shows an elementary 2-pole generator with a single armature coil. The coil may be single or multiturn and has generally a *diamond shape* as shown in Fig. 9.8. The two *coil-sides* (a, a') are located in stator slots spaced 1/2 B-wavelength or π (180^0) electrical apart. Such a coil is known as *full-pitch* coil. Let the rotor rotate at a constant speed of n rpm or revolution per minute

$$\omega_m = \frac{2\pi n}{60} \text{ rad (mech)/s}$$

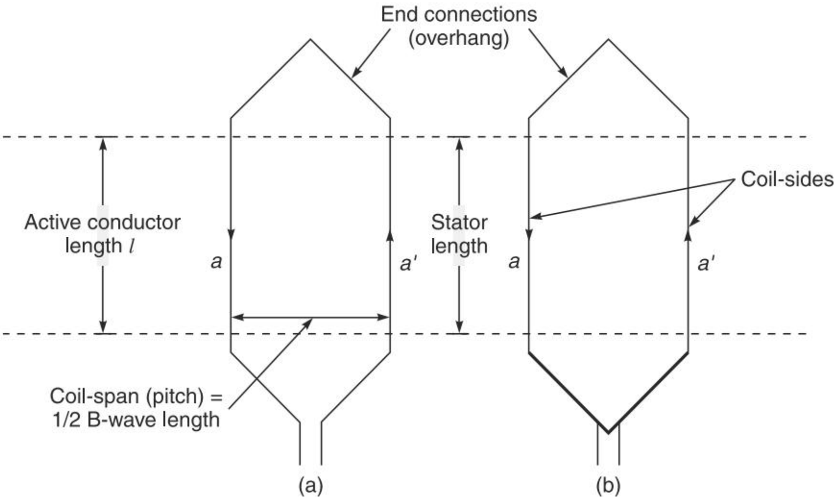


Fig. 9.8 (a) Single-turn coil, (b) Multiturn coil

or $$\omega = \left(\frac{P}{2}\right)\omega_m = \frac{\pi n P}{60} \text{ rad(elect)/s} \quad (9.2)$$

Figure 9.9(a) shows the cross-sectional developed view of the stator along with sinusoidal *B*-distribution (*B*-wave in space). This *B*-wave is present all along the axial length of the machine and the end effects are ignored. As the rotor rotates, the *B*-wave moves with it gliding past the coil-sides. As a result, emf of the same wave shape as the *B*-wave, i.e. sinusoidal, is generated in each coil-side (*Blv* rule). At any time, the emfs induced in the coil-side a, a' are of the same magnitude but of opposite signs; therefore, around the coil these emfs add up so that the coil emf is twice the coil-side emf. This is in consequence of the fact that the two coil-sides are π rad (elect) apart which means that if one coil side is under north pole the other is under south pole. The sinusoidal wave of the coil emf generated as a result of the rotor motion is depicted in Fig. 9.9(b).

One cycle of emf is generated when the rotor moves through the angle corresponding to one pole-pair (2π rad (elect)). Thus the frequency of emf generated is

$$f = \frac{\omega}{2\pi} = \frac{nP}{120} Hz\,;\ \omega = \frac{\pi n P}{60} \quad (9.3)$$

It is seen from Eq. (9.3) that for a given number of poles, a certain frequency (power frequency is 50 Hz) is obtained at a definite speed called the *synchronous speed* and hence the name *synchronous generator*. For high speed steam turbines, usually 2 poles at 3000 rpm are employed. High centrifugal forces at such high speeds demand cylindrical pole construction. For low speed hydroturbines, say 16 poles, 375 rpm, salient pole is ideal construction.

Three-Phase Generator (Alternator)

Practical synchronous generators are always of the 3-phase kind in which three coils are placed in slots around the stator with a relative electrical spacing of 120° ($2\pi/3$) between the coil axes as illustrated in Fig. 9.10(a). As a consequence the emfs of the three coils (phases) differ relatively in time phase by 120°. The three coils are usually connected in star in a synchronous generator. It is observed from Fig. 9.10 that each phase occupies 60° elect/pole on the stator one after another with coil-side sequence $ac'b$.

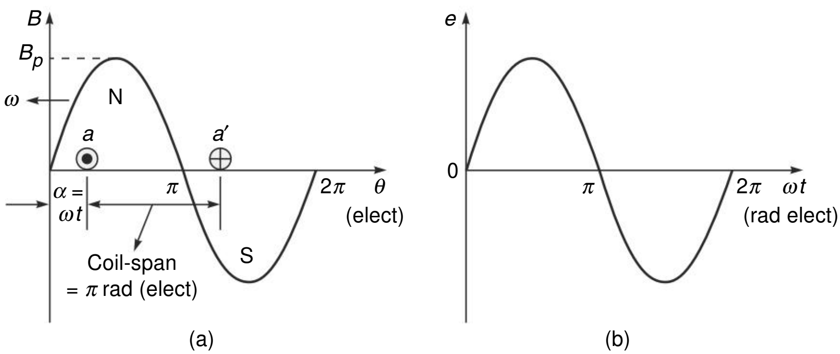


Fig. 9.9 (a) B-wave and coil sides at time t, (b) Coil emf

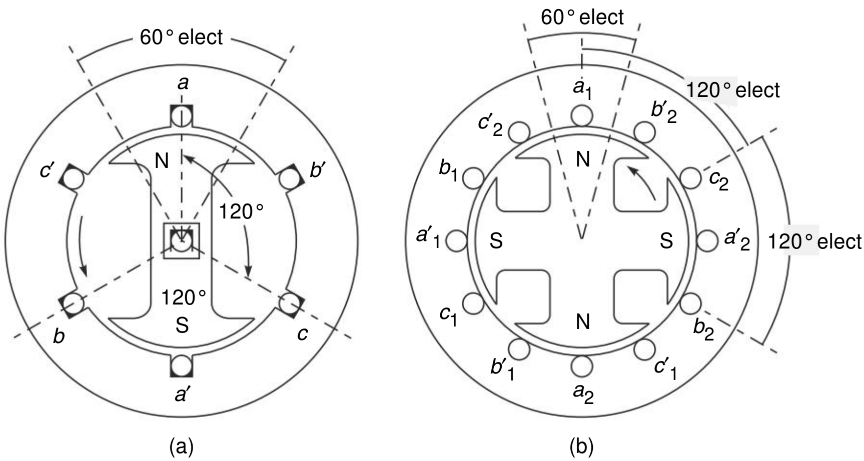


Fig. 9.10 (a) Three-phase, 2-pole synchronous generator
(b) Three-phase, 4-pole synchronous generator

Figure 9.10(b) shows the corresponding arrangement for a 4-pole generator. Since the total electrical angle of the machine is 4π rad, each phase now has two coils with coil axis spacing of 2π. The two coils of each phase can be connected in series or parallel, as shown in Fig. 9.11(a) and (b), depending upon voltage and current requirement of the machine. Series connection means current corresponding to coil current (limited from heating point of view) and voltage twice that of coil voltage. In parallel connection, voltage is same as coil voltage while current is twice that of coil current. The three phases so connected are connected in star.

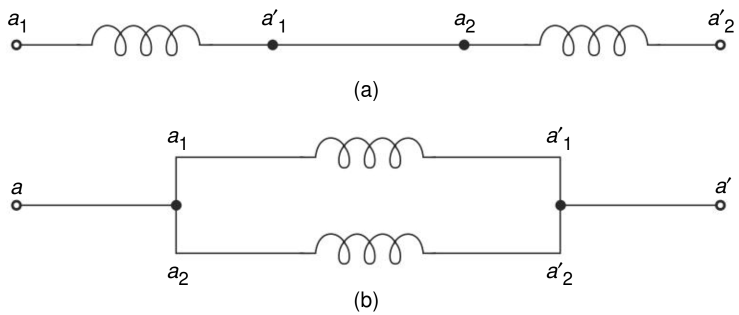


Fig. 9.11

9.4 GENERATED EMF OF AC WINDINGS

The B-wave (assumed sinusoidal) of the elementary machine depicted in Fig. 9.7 is drawn in Fig. 9.9(a). It can be expressed as

$$B = B_p \sin \theta \tag{9.4}$$

where B_p is the peak flux density and θ is the electrical angle. The flux contained in an angle $d\theta$ is

$$d\phi = Blr\left(\frac{2}{P}\right)d\theta\,;\ d\theta_m = \left(\frac{2}{P}\right)d\theta$$

where l is the stator length (axial) or active conductor length and r is the mean air-gap radius of machine.

The flux linking the coil (single turn) is given by

$$d\phi = \int_{\alpha}^{\alpha+\pi} lr\left(\frac{2}{P}\right) B_P \sin\theta\, d\theta$$

$$= \left(\frac{4}{P}\right) B_P\, lr \cos\alpha$$

But

$$\alpha = \omega t$$

$\therefore$

$$\phi = \left(\frac{4}{P}\right) B_P\, lr \cos\omega t \qquad (9.5)$$

The flux/pole is then given by

$$\Phi = B_{av} \times 2\pi\frac{lr}{p}$$

For siusoidal flux density distribution

$$B_{av} = \left(\frac{2}{\pi}\right) B_P \qquad (9.6)$$

Therefore

$$\Phi = \left(\frac{2}{\pi}B_P\right) \times 2\pi\frac{lr}{p}$$

$$= \left(\frac{4}{P}\right) B_P lr$$

Therefore

$$\phi = \Phi\cos\omega t \qquad (9.7)$$

Flux linking an N-turn coil is given by

$$\lambda = N\Phi = N\Phi\cos\omega t \qquad (9.8)$$

Hence the coil emf is given by

$$e = -\frac{d\lambda}{dt} = \omega N\Phi\sin\omega t \qquad (9.9)$$

wherein the assumed positive direction of emf is such that if current were allowed to flow in that direction, it would cause flux to be produced in positive direction along the coil axis. Observe that for convenience we used $e = +\,(d\lambda/dt)$ in the transformer and negative sign in emf was incorporated in the circuit diagram.

The rms value of the coil (phase) emf is

$$E = \sqrt{2}\,\pi f N\Phi = 4.44\, fN\Phi \qquad (9.10)$$

which is the same result as in transformer except that Φ here means flux/pole.

It follows from Eqs. (9.8) and (9.9) that the flux linkage phasor (loosely called flux phasor) leads the phase emf phasor by 90° as depicted in Fig. 9.12. Compare it with the transformer case.

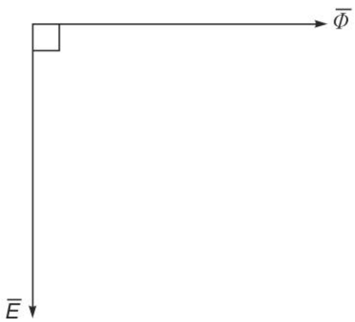


Fig. 9.12

The difference in the two is explained by the negative sign in $e = - d\lambda/dt$.

Distributed Winding

In order to fully utilize the armature peri-phery and further to build higher voltages (it may be as high as $11/\sqrt{3}$ kV or even $37/\sqrt{3}$ kV per phase), more than one coil/pole-pair/phase, i.e. more than one slot/pole/ phase (SPP) are employed in practice. Figure 9.13 shows a generator (2-pole) with SPP = 3. Such an arrangement for armature winding is called distributed winding. The total number of slots *S* are uniformly distributed around the armature. Then

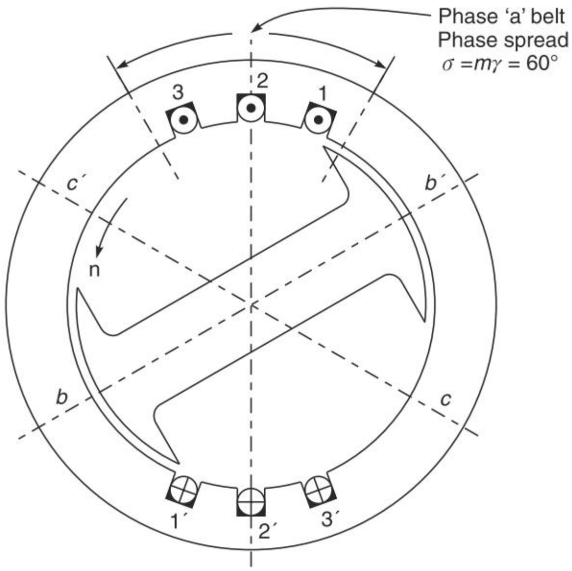


Fig. 9.13 Synchronous generator with distributed winding

$$\text{SPP} = m = \frac{S}{qP};\ q = \text{number of phases (generally } q = 3) \qquad (9.11)$$

The angle between adjacent slots is

$$\gamma = \frac{\pi P}{S} \text{ rad (elect)} \qquad (9.12)$$

The emf induced in the phase coils therefore differs progressively by angle γ (elect) though the amplitude of emf is the same in each coil.

Let all the phase coils be connected in series. The phase *emf* is the addition of the coil. *Emf* phasors as shown in Fig. 914. Wherein the coil emfs phasors form the sides of a regular polygon. From the geometry of the figure

Coil emf $E_C = AB = 2\ OA \sin \gamma/2$ (9.13)

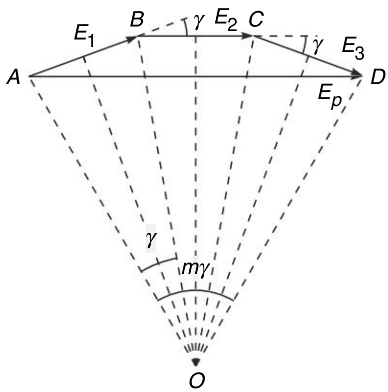


Fig. 9.14 Phasor diagram of coil emfs (m = 3)

$$\text{Phase emf } E_p = AD = 2\, OA \sin m\,\gamma/2 \tag{9.14}$$

Since $E_p < mE_c$, we define

Breadth factor (or *distribution factor*), $K_b = \dfrac{E_p}{mE_c}$

or

$$K_b = \frac{\sin m\gamma/2}{m \sin \gamma/2} < 1 \tag{9.15}$$

The induced emf formula of Eq. (9.10) now gets modified on account of distributed winding. We have

$$E_p = 4.44\, K_b f N_{\text{ph}}\, \Phi \tag{9.16}$$

where N_{ph} is the total number of (series connected) turns/phase.

Because of nonsinusoidal distribution of air-gap flux density, the B-wave contains space harmonics (poles that are odd multiples of fundamental number of poles). This contributes to induction of harmonic voltages in phase emf. For nth harmonic

$$K_b\ (n\text{th harmonic}) = \frac{\sin mn\,\gamma/2}{m \sin n\,\gamma/2} \tag{9.17}$$

Distributed windings are a necessity for accommodating the desired number of turns per phase, but its incidental advantage is that the harmonic content of the phase *emf* gets reduced to much less than the harmonic of the induced *emf*s in the coil as $K_b\ (n) < K_b$ (fundamental). Inparticular, K_b (n) has a low value for the troublesome harmonics like 3rd, 5th, thirteenth etc. This fact will be illustrated in Example 9.2.

Two-Layer Winding

It was so far assumed that only one coil-side is accommodated in each slot (single-layer winding). In modern practice, however, two coil-sides/slot are placed with one side of each coil being in top layer and the other side in the bottom layer of a slot π rad (elect) away for full-pitch coils. The shape of such a coil is shown in Fig. 9.15(a). The winding layout in *developed form* is shown in Fig. 9.15(b) for 4 poles and SPP = 1. The top layer coil-sides are shown in solid lines and the bottom layer coil-sides are in dotted lines. The direction of the induced *emf* of the coil-sides in each slot are indicated in Fig. 9.15(b). We find from the connections indicated that coils 1 and 3 are in series and coils 2 and 4 are in series and these two sets are connected in parallel. Therefore

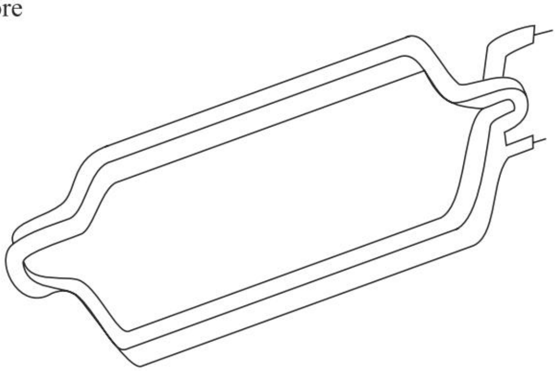


Fig. 9.15 (a) Coil of a double-layer winding

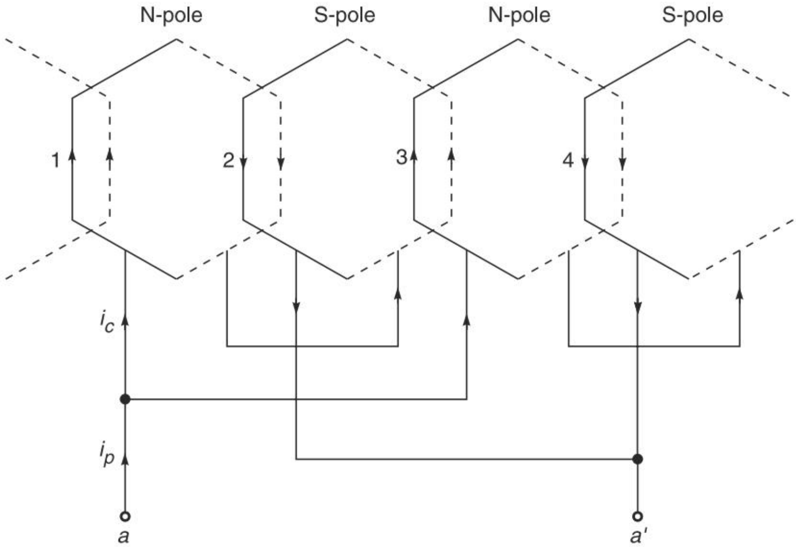


Fig. 9.15 (b) One phase of a double-layer winding (4 poles, SPP = 1)

$$i \text{ (phase)} = 2i_c \text{ (coil)}$$

and $$emf \text{ (phase)} = emf \text{ of two distribited coils in series}$$

It is possible to connect all the four coils in series, which will yield $i_p = i_c$ and double the phase *emf*.

This type of winding offers the advantages that all coils are identical in shape (convenience in the manufacturing process) and that the end portions of the coils (overhang) can be neatly arranged mechanically.

Example 9.1 A 50 Hz, 6-pole synchronous generator has 36 slots. It has a two layer winding with full-pitch coils of 10 turns each. The flux/pole is 0.016 Wb (sinusoidally distributed). Determine the induced emf (line-to-line) if the coils are connected to form (a) 1-phase winding, (b) 2-phase winding and (c) 3-phase star-connected winding. If the current-carrying capacity of conductor composing coils is 10 A, what is the generator kVA capacity in each case?

Solution :

(a) 1-phase winding :

$$m = \frac{S}{P} = \frac{36}{6} = 6$$

$$\gamma = \frac{180 \times 6}{36} = 30^\circ$$

$$K_b = \frac{\sin m\gamma/2}{m \sin \gamma/2} = \frac{\sin 6 \times 30^\circ/2}{6 \sin 30^\circ/2}$$

$$= 0.644$$

$$N_{ph} = \frac{36 \times 0 \times 2}{2} = 360$$

$$E_p = 4.44\, K_b\, f N_{ph} \phi$$

$$= 4.44 \times 0.644 \times 50 \times 360 \times 0.016$$
$$= 823.5 \text{ V}$$

$$\text{kVA capacity} = \frac{823.5 \times 10}{1000} = 8.235$$

(b) 2-phase winding :

$$m = \frac{S}{2P} = \frac{36}{2 \times 6} = 3$$
$$\gamma = 30°$$
$$K_b = \frac{\sin 3 \times 30°/2}{3 \sin 30°/2}$$
$$= 0.91$$
$$N_{ph} = \frac{36 \times 10 \times 2}{2 \times 2\,(\text{phases})} = 180$$

It is assumed that the turns/phase are series connected.

$$E_p = 4.44 \times 0.91 \times 50 \times 180 \times 0.016 = 582 \text{ V}$$
$$E^* \text{ (line)} = 582 \sqrt{2} = 822.7 \text{ V}$$
$$\text{kVA capacity} = \frac{2 \times 10 \times 582}{1000} = 11.64$$

(c) 3-phase winding (star connected):

$$m = \frac{S}{3P} = \frac{36}{3 \times 6} = 2$$
$$\gamma = 30°$$
$$K_b = \frac{\sin 2 \times 30°/2}{2 \sin 30°/2} = 0.966$$
$$N_{ph} = \frac{36 \times (2 \times 10)}{2 \times 3 \text{ (phases)}} = 120$$
$$E_p = 4.44 \times 0.966 \times 50 \times 120 \times 0.016 = 412 \text{ V}$$
$$E \text{ (line)} = 412\sqrt{3} = 713 \text{ V (star connected)}$$
$$\text{kVA capacity} = \frac{3 \times 10 \times 412}{1000} = 12.36$$

It is observed that for the same machine (same amount of iron and copper), the kVA capacity progressively increases as the number of phases is increased from one to two to three. It can be shown that increase beyond three phases would not be significant (but there is a considerable increase in system complexity). Three phase synchronous generator and correspondingly 3-phase power system is universally adopted.

Example 9.2 Calculate the fundamental, third and fifth harmonic breadth factors for a stator wound with 54 slots for 3-phase and 6 poles.

Solution

$$m = \frac{54}{3 \times 6} = 3$$

* Angle between phase voltage for the 2-phase case is 90°. There fore E (Line) = $\sqrt{2}\ E_p$.

$$= \frac{180° \times 6}{54} = 20°$$

(i) K_b (fundamental) $= \dfrac{\sin 3 \times 20°/2}{3 \sin 20°/2} = 0.96$

(ii) K_b (third harmonic) $= \dfrac{\sin 3 \times 3 \times 20°/2}{3 \sin \times 3 \times 20°/2} = 0.667$

(ii) K_b (fifth harmonic) $= \dfrac{\sin 3 \times 5 \times 20°/2}{3 \sin \times 5 \times 20°/2} = 0.218$

These values of K_b indicate the *harmonic reduction property of a distributed winding.*

Short-Pitched (Chorded) Coils

Short-pitched coils with span less than π rad are employed for reduction in harmonic content of the voltage wave and to save in overhang copper. Figure 9.16 shows such a coil.

Assuming the positive direction of emf in each coil side as shown in Fig. 9.16

$$\bar{E}_a = E_{cs}$$

$$\bar{E}_{a'} = \bar{E}_{cs}\, e^{-j(\pi-\beta)} = E_{cs} \angle -\pi + \beta \quad (9.18)$$

where E_{cs} is the coil-side emf and β is the angle by which the coil span is shorter than p (full-pitch).

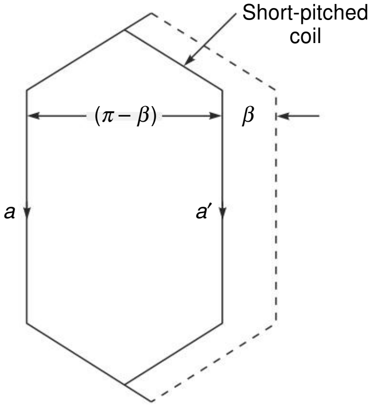


Fig. 9.16 Short-pitched coil

The coil emf is

$$\bar{E}_c = \bar{E}_a - \bar{E}_{a'}$$

$$= E_{cs} \angle 0° - E_{cs} \angle -\pi + \beta \quad (9.18)$$

The phasor diagram of Eq. (9.18) is drawn in Fig. 9.17. It is easily found from the phasor diagram that

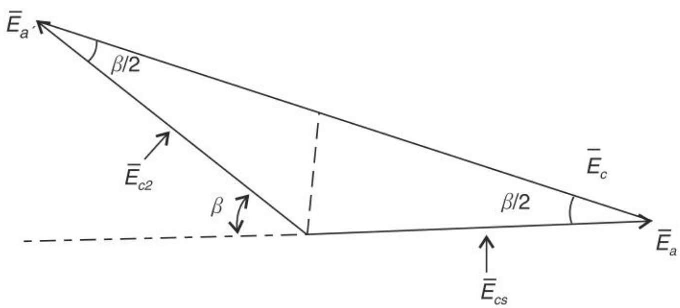


Fig. 9.17 Phasor diagram of short-pitched coil

$$E_c \text{ (short–pitched)} = 2\, E_{cs} \cos \frac{\beta}{2} \quad (9.19a)$$

$$E_c \text{ (full–Pitched) } = 2\, E_{cs} \tag{9.19b}$$

In ratio form

$$\frac{E_c(\text{short–pitched})}{E_c(\text{full–pitched})} = \cos\frac{\beta}{2}$$

This *emf* reduction factor called is *pitch factor*.

$$K_p = \cos \beta/2 \tag{9.20}$$

The induced emf formula now stands modified as

$$E_p \text{ (phase) } = 4.44\, K_w f N_{\text{ph}} \text{ (series) } \phi \tag{9.21}$$

where $K_w = K_b\, K_p = \textit{winding factor}$

For nth harmonic

$$K_p(n) = \cos n\,\beta/2 \tag{9.22}$$

By suitable choice of β, the designer can eliminate some particular harmonic. For example, for elimination of 13th harmonic

$$13\, \beta/2 = 90°$$

or $\beta = 14°$

This value of β will not only eliminate the 13th harmonic but will also reduce the magnitude of other harmonics. For example, for 7th harmonic

$$K_p(7) = \cos\frac{7\times 14°}{2} = 0.656$$

Example 9.3 A 3-phase, 50 Hz, star-connected synchronous generator with double-layer winding runs at 500 rpm. It has 12 turns/coil and 5 slots/pole/phase and coil pitch of 13 slots. If the flux/pole is 0.025 Wb sinusoidally distributed, find the phase and line emfs induced. Assume that the total turns/ phase are series connected.

Solution

$$P = \frac{120f}{n} = \frac{120\times 50}{500} = 12$$

$$S = 5 \times 3 \times 12 = 180$$

Each slot has 2 coil sides and each coil side has 12 conductors.

Total number of conductors = 180 × 2 × 12 turns/phase

$$N_{\text{ph}} = \frac{180\times 2\times 12}{2\times 3} = 720$$

$$m = 5$$

$$\gamma = \frac{180\times 12}{180} = 12°$$

Now $K_b = \dfrac{\sin 5\times 12°/2}{5\sin 12°/2} = 0.957$

$$\text{Pole pitch} = \frac{180}{2} = 15$$

$$\text{Coil pitch} = 13$$

$$\text{Short-pitching angle} = (15 - 13) \times 12° = 24°$$

$$
\begin{aligned}
K_p &= \cos 24°/2 = 0.978 \\
E_p &= 4.44\, K_b K_p f N_{ph} \text{ (series) } \Phi \\
&= 4.44 \times 0.957 \times 0.978 \times 50 \times 720 \times 0.025 \\
&= 3740 \text{ V} \\
E \text{ (line)} &= \sqrt{3} \times 3740 = 6478 \text{ V}
\end{aligned}
$$

9.5 MMF OF AC WINDING

Like in our treatment of induced emf, we shall first consider the mmf distribution caused by a single full-pitched stator coil (a, a') as shown in Fig. 9.18(a). It is assumed that the reluctance of the iron path is negligible so that the coil AT (Ni) is expended equally in the two air-gaps that the flux has to cross. Thus $Ni/2$ ampere-turns are expended to establish flux from rotor through air-gap to stator (north pole on rotor and south on stator) regarded as positive and is constant over the complete pitch (π rad). On the other hand $Ni/2$ ampere-turns establish flux from stator to rotor through the air-gap (south pole on rotor and north on stator) and are therefore considered to have negative sign.

The *mmf* versus angle θ in form of developed diagram is drawn in Fig. 9.18 (c). The *mmf* is constant ($Ni/2$) from conductor a' to a (anti-clockwise in the cross-sectional view of Fig. 9.18a). This *mmf* considered positive established the air-gap flux from rotor to stator (verify by the right hand rule). Correspondingly, the *mmf* ($Ni/2$) which is constant from conductor a to a' established the air-gap flux from stator and is considered negative.

The *mmf* magnitude versus space angle θ is plotted in Fig. 9.18 (b). It is a *periodic rectangular space wave* and would produce a corresponding B-wave in the air-gap. As per the Fourier series, the fundamental of the *mmf* wave is

$$F_{a1}\text{(fundamental)} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta \tag{9.23}$$

shown in dotted line in Fig. 9.18 (b).

We shall proceed on the basis of the fundamental only as justified below.

1. In the practical case of distributed winding, the *mmf* harmonics would be considerably attenuated as the harmonic distribution factor K_b(n) is much less than the fundamental distribution factor K_b. This is similar to the *emf* harmonics as was shown in Section 9.4.
2. The machine leakage reactance reduce the harmonic content of the induced *emf* by their filtering action.

We can write Eq. (9.23) as

$$F_{a1} = Ki \cos\theta$$

or
$$\mathcal{F}_{a1}\text{(fundamental)} = \frac{4}{\pi}\left(\frac{Ni}{2}\right)\cos\theta \tag{9.23}$$

where for a distributed winding K will suitably modify because of appearance of breadth and pitch factors (see following section), so that for one phase

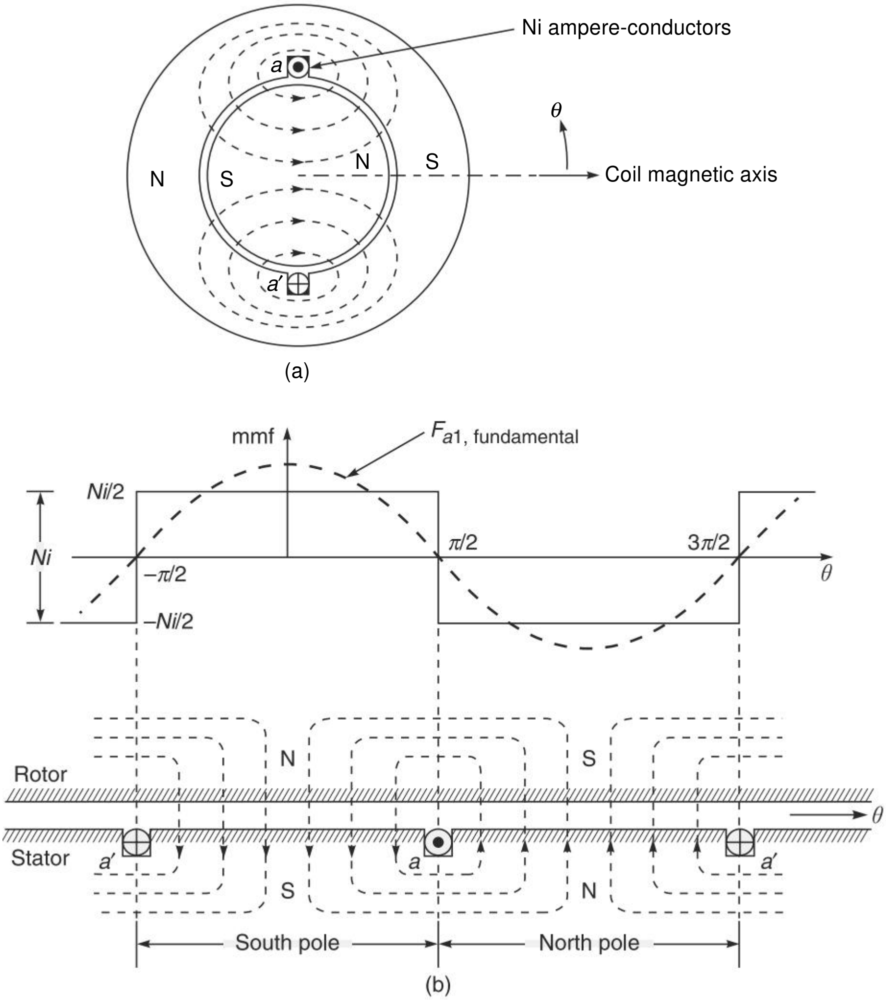


Fig. 9.18 MMF space wave of a single coil

$$\mathcal{F}_{a1}(\text{fundamental}) = K'i_a \cos\theta \tag{9.24}$$

But $\qquad i_a = \sqrt{2}\, I \cos\omega t$

where $\qquad I$ = rms phase current

$$\therefore \quad \mathcal{F}_{a1} = \sqrt{2}\, K'I \cos\omega t \cos\theta$$

$$= F_m \cos\omega t \cos\theta \tag{9.25}$$

The mmf distribution of Eq. (9.25) is a *standing* (*pulsating*) wave which has sinusoidal space distribution and whose amplitude varies sinusoidally with time at fundamental frequency. The pulsating wave at various time instants is illustrated in Fig. 9.19.

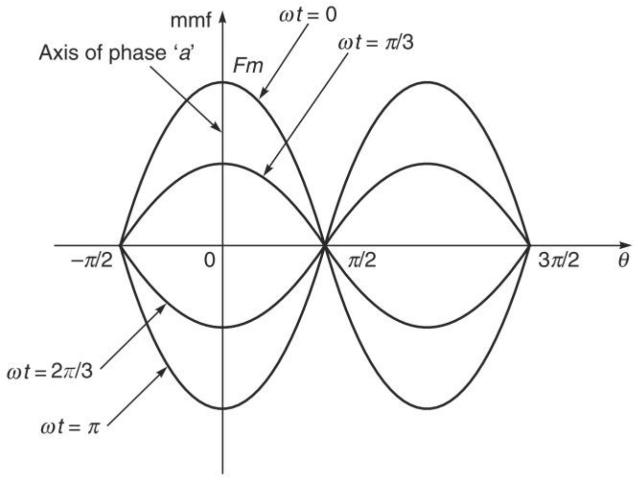


Fig. 9.19 Pulsating mmf of one phase of the stator

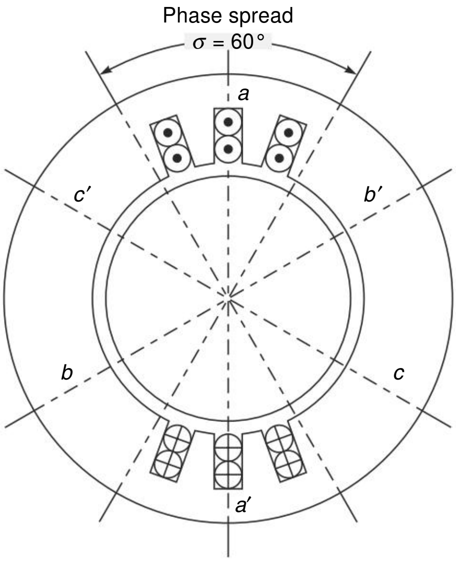


Fig. 9.20 A 3-phase, 2-pole structure with double-layer winding *(m = 3)*

*Mmf Space Wave of One Phase (Distributed Winding)

Figure 9.19 shows the distributed winding of phase *a* of a synchronous generator. Each coil now creates its own rectangular mmf wave with a progressive phase difference of γ. The resultant wave is a stepped and flat-topped wave which is closer to a sinusoid than that of Fig. 9.18(b) (which means reduction in harmonic content). To obtain the fundamental of the stepped wave, we can proceed by adding the fundamentals of the individual rectangular waves with phase difference γ (as in the emf case). Thus

$$\mathcal{F}_{a1} = \frac{4}{\pi} K_w \left(\frac{N_{ph}(series)}{P} \right) i_a \cos\theta \tag{9.27a}$$

* Optional.

where

$$K_w = K_b K_p = \text{winding factor (same as in emf case)}$$
$$i_a = \text{phase current.}$$
$$= \sqrt{2}\, I \cos \omega t \quad (I = \text{rms phase current})$$

It then follows that

$$\mathcal{F}_{a1} = \frac{4\sqrt{2}}{\pi} K_w \left(\frac{N_{\text{ph}}(\text{series})}{P} \right) I \cos \omega t \cos \theta$$

$$= F_m \cos \omega t \cos \theta \tag{9.27}$$

where $$F_m = \frac{4\sqrt{2}}{\pi} K_w \left(\frac{N_{\text{ph}}(\text{series})}{P} \right) I \tag{9.28}$$

9.6 Rotating Magnetic Field

Consider now the three phases of an ac winding carrying balanced alternating currents (phase sequence *abc*). Then

$$\left.\begin{aligned} i_a &= I_m \cos \omega t \\ i_b &= I_m \cos (\omega t - 120°) \\ i_c &= I_m \cos (\omega t - 240°) \end{aligned}\right\} \tag{9.29}$$

Figure 9.21 shows the three phase coils (concentrated) along with their magnetic axes, which are located 120° (elect) apart in space. The three mmfs can be expressed as

$$\left.\begin{aligned} \mathcal{F}_a &= F_m \cos \omega t \cos \theta \\ \mathcal{F}_b &= F_m \cos (\omega t - 120°) \cos (\theta - 120°) \\ \mathcal{F}_c &= F_m \cos (\omega t - 240°) \cos (\theta - 240°) \end{aligned}\right\} \tag{9.30}$$

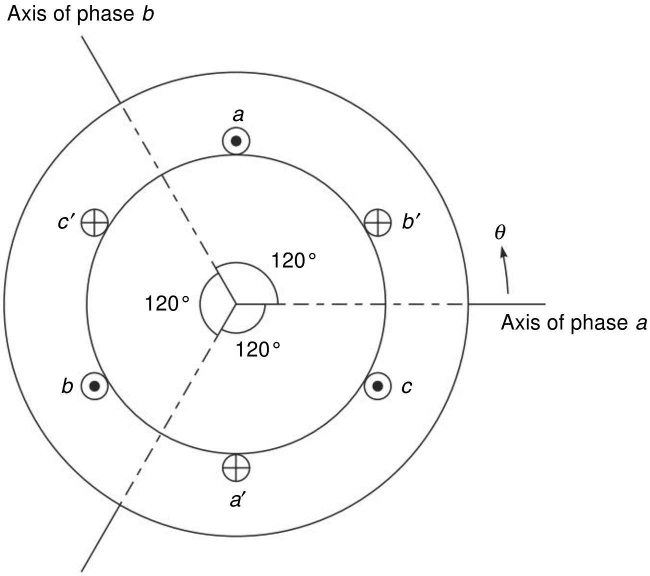


Fig. 9.21 Relative location of the magnetic axes of the three phases

The resulting mmf wave is then given by

$$\mathcal{F} = \mathcal{F}_a + \mathcal{F}_b + \mathcal{F}_c$$
$$= F_m\,[\cos \omega t \cos \theta + \cos (\omega t - 120°) \cos (\theta - 120°)$$
$$+ \cos (\omega t - 240°) \cos (\theta - 240°)]$$

Simplifying trigonometrically

$$\mathcal{F}(\theta, t) = \frac{3}{2} F_m \cos (\omega t - \theta) + \frac{1}{2} F_m\, [\cos (\omega t + \theta)$$
$$+ \cos (\omega t + \theta - 240°) + \cos (\omega t + \theta - 480°]$$
$$= \frac{3}{2} F_m \cos (\omega t - \theta) \tag{9.31}$$

As per Eq. (9.31) the resultant field rotates at speed $\omega = 2\pi f$ rad (elect)/s or $n = 120f/P$ rpm, *synchronous speed*, in the direction form the *leading to the lagging* phase axis; a-axis to be axis in Fig. 9.21. The direction of rotation can be easily reversed by changing the phase sequence of the currents (interchange two stator leads as connected to the source (3-phase)). The peak amplitude of the resultant field is

$$F_{\text{peak}} = \frac{3}{2} F_m \tag{9.32}$$

Conclusion Whenever a 3-phase winding, with 120°(elect) *spatial phase difference* between the axes of the three phases is fed with balanced 3-phase currents with a *time phase difference* of 120°(elect), the resultant mmf (and its associated B-wave) rotates at synchronous speed $\omega_S = 2\pi f$ rad (elect)/s (or $n_s = 120\, f/P$ rpm). The direction of rotation of the mmf wave is from the leading to the lagging phase and can be reversed by changing the phase sequence of currents.

Physical Picture

A sinusoidally distributed (in space) field can be represented by a vector oriented along its peak value, i.e. along the coil axis. The vector oscillates if the coil carries alternating current.

At $\omega t = 0$ (from Eq. (9.29))

$$i_a = I_m,\; i_b = -\frac{1}{2} I_m,\; i_c = -\frac{1}{2} I_m$$

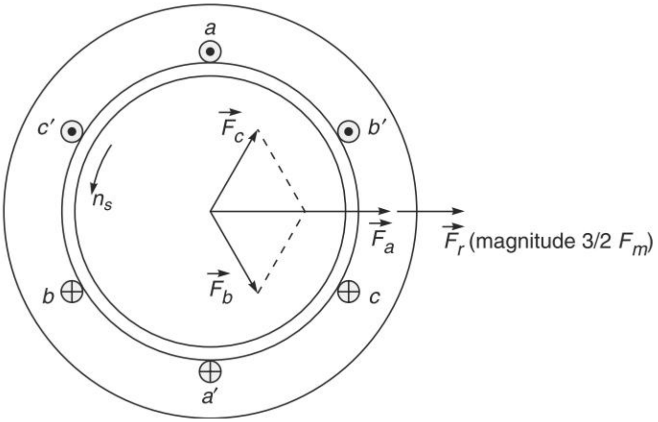


Fig. 9.22 Resultant field at $\omega t = 0$; peak value (3/2) F_m along the axis of phase a

The corresponding three field vectors are drawn in Fig. 9.21 ($F_a = F_m$, $F_b = -1/2\ F_m$ and $F_c = -1/2\ F_m$) wherein it is shown that the resultant field has a peak value of (3/2) F_m and is directed along the axis of phase a.

$$\text{At } \omega t = 120°$$
$$i_a = -\frac{1}{2} I_m,\ i_b = I_m,\ i_c = -\frac{1}{2} I_m$$

It immediately follows that the resultant field would now lie along the axis of phase b (counterclockwise rotation) and will have the same peak value ((3/2) F_m).

Similarly at $\omega t = 240°$, the resultant field will lie along the axis of phase c and so on.

The above illustrates physically the concept of a rotating magnetic field whose speed is $\omega_s = 2\pi f$ rad (elect)/s.

Example 9.4. A 3-phase, 6-pole, 50 Hz synchronous machine is carrying balanced currents. Its peak fundamental field has a value of 450 AT.
(a) Write the expression for F_{a1} in terms of electrical as well as mechanical space angles.
(b) Write the expression for the rotating stator field.
(c) What is its speed in rpm and mechanical rad/s?

Solution

(a) $F_{a1} = 450 \cos \omega t \cos \theta_e$ AT, $\omega = 2\pi f = 314$ rad/sec
$= 450\ \omega t \cos 3\theta_m$

(b) Rotating field
$$F = \frac{3}{2} \times 450 \cos(\omega t - \theta_e) \text{ AT}$$

(c) $$n_s = \frac{120 \times 50}{6} = 1000 \text{ rpm},$$
$$\omega_m = \frac{2\pi}{60} \times 1000 = 104.7 \text{ mech rad/s}$$

9.7 TORQUE IN ROUND ROTOR MACHINE

From a field viewpoint, electromagnetic torque is the result of interaction of two magnetic fields in the air-gap, one $\vec{F}_1$ created by the stator currents and the other $\vec{F}_2$ by the rotor currents. For creation of steady torque, the following two conditions must be met:

– The two fields must be stationary relative to each other
– The two fields must have the same number of poles

The reason for these will become obvious from the discussion that follows.

Figure 9.23(a) shows the cross-sectional view of a 2-pole round rotor machine. The axes of the stator and rotor fields are indicated therein and these have an angle λ between them. It is immediately obvious that the attractive forces between the rotor north and stator south and also that between the rotor south and stator north cause a torque to act on the rotor in the direction indicated tending to reduce λ, i.e. it tends to align the two fields. This torque will be balanced by the prime mover/load torque at a steady value of λ. There will of course be a reaction torque on the stator in the opposite direction which will be balanced by the bolting down of the stator to the foundations. It is assumed that the two fields are stationary relative to each other

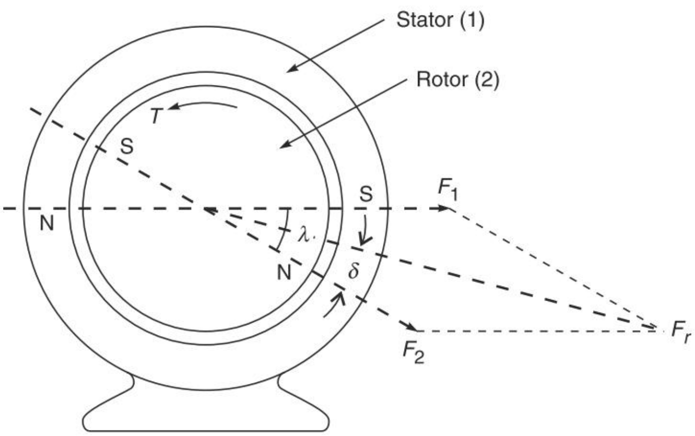


Fig. 9.23(a) Torque in round rotor machine

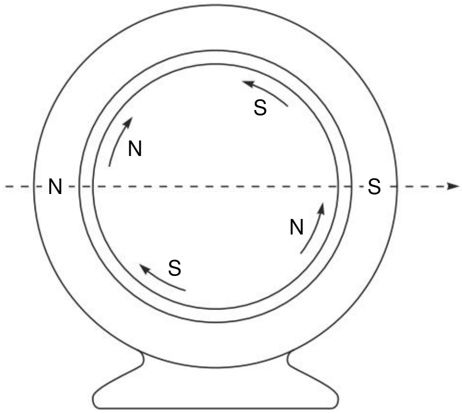


Fig. 9.23(b) Case of 2-pole stator and 4-pole rotor

as only then is a steady torque possible. If the fields were to rotate relative to each other the attractive forces between them will alternate by repelling forces resulting in average zero torque.

Figure 9.23 (b) shows a 2-pole stator and a 4-pole rotor. It is easily seen from this figure that the forces on the rotor surface alternate resulting in net zero torque and hence the condition that both the fields (stator and rotor) must have the same number of poles.

Figure 9.23 (a) shows the vector representation of two sinusoidally distributed fields of peak values F_1 and F_2; F_r is the *resultant field*. Because of sinusoidal flux density distribution caused by these fields, it is intuitive to expect that the interaction torque would be proportional to the sine of λ, the angle of separation of their axes. This torque can be expressed as

$$T = K'F_1 F_2 \sin \lambda$$

or
$$T = K' F_r F_2 \sin \delta \tag{9.33}$$

where δ is the angle between the axes of $\mathbf{F}_r$ and $\mathbf{F}_2$.

The resultant field establishes an *air-gap flux* of

$$\Phi_r = \mathcal{P} F_r/\text{pole} \tag{9.34}$$

where

$$\mathcal{P} = \text{per pole permeance (Wb/AT)}$$

and

$$F_2 \propto I_2 \text{ (rotor current)}$$

Hence the torque expression can be written in the more useful form

$$T = K\Phi_r I_2 \sin \delta \tag{9.35}$$

Example 9.5 In a given electric machine, F_2 = 850 AT and F_1 = 400 AT, λ = 123.6°. If permeance/pole is 1.408×10^{-3} Wb/AT, calculate the resultant air-gap flux/pole. Also calculate the value of angle δ.

Solution

The *mmf* vector diagram to drawn in adjoining figure, from which we find

$$\theta = 180° - 123.6 = 56.4°$$

Then

$$F_r = [(400)^2 + (850)^2 - 2 \times 400 \times 850 \cos 56.4°]^{1/2}$$
$$= 711.5 \text{ AT}$$

Air-gap flux/pole

$$\Phi_r = \frac{2}{\pi} \mathcal{P}F_r,\ F_r \text{ (average)} = \frac{2}{\pi} F_r$$
$$= \frac{2}{\pi} \times 1.408 \times 10^{-3} \times 711.5 = 0.638 \text{ Wb}$$

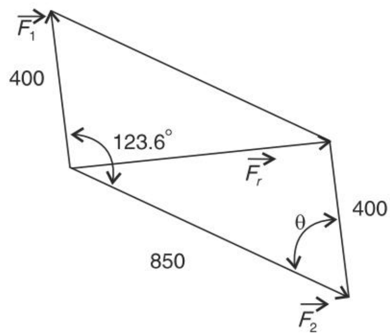


Fig. 9.23(c)

9.8 BASIC MACHINE TYPES

Synchronous Machine

In this machine, $\mathbf{F}_2$ is created by the field current and rotates at synchronous speed n_s while the 3-phase stator currents of frequency $f = n_s P/120$ flowing in the 3-phase stator winding produce synchronously rotating filed $\mathbf{F}_1$ in the same direction as $\mathbf{F}_2$, i.e. $\mathbf{F}_1$ and $\mathbf{F}_2$ are relatively stationary. The two fields are thus *locked* into each other or are *synchronized*. This is the synchronous action of the machine wherein the torque is produced only at synchronous speed $n_s = 120f/P$.

Figure 9.24(a) and (b) show respectively the generating and motoring operation of the synchronous machine. The following observations are made from Fig. 9.24.

Generating Operation [Fig. 9.25 (a)] $\overline{\text{F}}_2$ leads $\overline{\text{F}}_r$ angle δ in the direction of rotation. The prime mover torque T_{PM} causes $\overline{\text{F}}_2$ to drag $\mathbf{F}_1$ along with it. The interaction of fields produces the electromagnetic torque which opposes and balances the prime mover torque ($T = T_{PM}$) resulting in conversion of mechanical power (input) to electrical power (output). Compare with Fig. 9.1 (a).

Motoring Operation [Fig. 9.25 (b)] $\overline{F}_2$ lags $\overline{F}_r$ by angle δ in opposite direction of rotation. $\overline{F}_1$ now pulls $\overline{F}_2$ (the rotor) along with it at synchronous speed n_s. The interaction of the fields produces the electromagnetic torque $T = T_L$ (mechanical load torque) resulting in conversion of electrical power (input) to mechanical power (output).

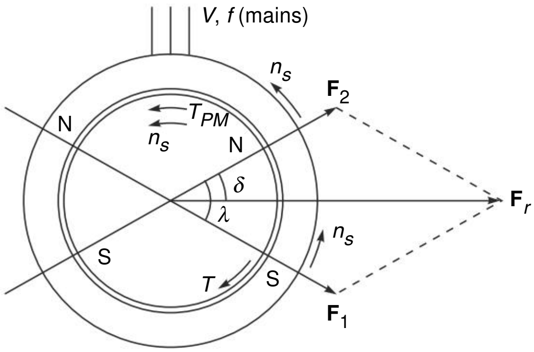
Torque-angle (T-δ) characteristics For a certain stator, terminal voltage V

$$V\text{ (line)} = \sqrt{3} \times 4.44\, K_w f\, \Phi_r N_{ph}\text{ (series)} \tag{9.36}$$

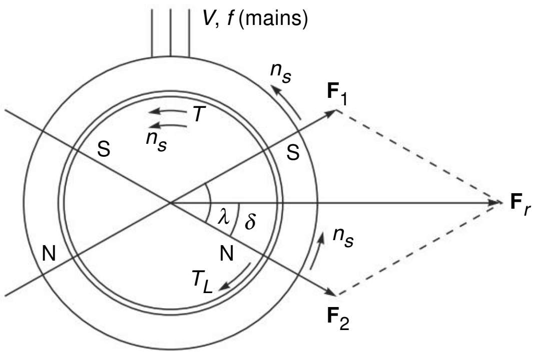
if the stator resistance and leakage reactance (per phase) are ignored. Thus for a given terminal voltage, Φ_r remains constant (as in a transformer). For a given field current ($I_f = I_2$), Eq. (9.33) becomes

$$T = K_T \sin \delta \tag{9.37}$$

The sinusoidal T–δ relationship of Eq. (9.37) is called *torque–angle* (or *power angle* ($P = T\omega_s$)) characteristic diagram and is drawn in Fig. 9.25. Maximum electromagnetic torque called *pull-out* torque, is produced at $\delta = 90°$. At T_{PM} or T_L more than $T_{\text{Pull-out}}$ the angle δ increases without bound and the synchronous link between



(a)



(b)

Fig. 9.24 Torque production in synchronous machine (a) generating operation, (b) motoring operation

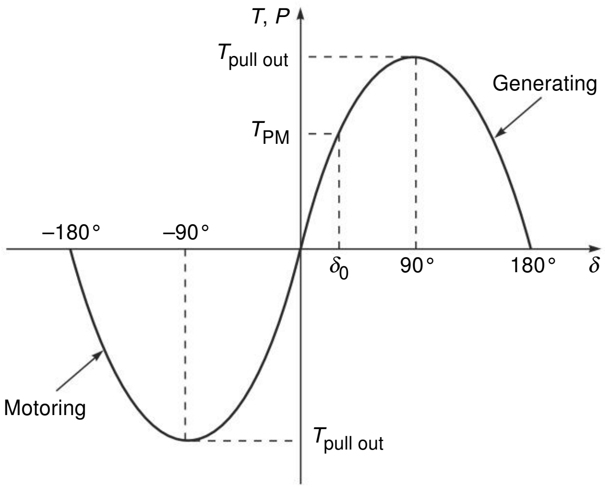


Fig. 9.25 Torque (power)–angle characteristic of synchronous machine

the stator and rotor fields is snapped. The machine is then said to *pull-out* or *lose synchronism*.

Motor Starting A synchronous machine cannot self start as a motor. As the stator is switched on to a 3-phase supply, a synchronously rotating field is produced but the rotor filed being stationary, the condition of net torque production is not met (zero average torque) and therefore the machine cannot start as a motor. The machine rotor with field winding excited must be brought close to synchronous speed by a small auxiliary motor and then the stator should be switched on at the appropriate

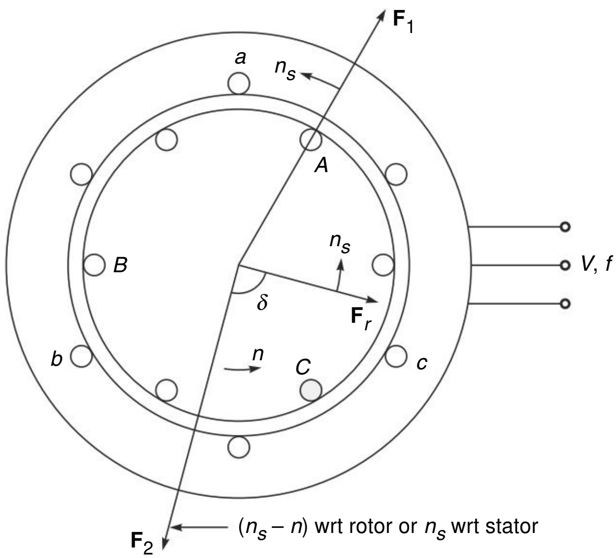


Fig. 9.26 Principle of induction motor

instant synchronized to the mains before the machine can be loaded as a motor. The synchronous motor is usually started by the induction method (induction principle is explained below).

Induction Machine

This machine has a 3-phase stator similar to that of a synchronous machine. It has a rotor which is either wound for 3-phase and short circuited or is a uniformly spaced set of conducting bars placed in rotor slots and shorted at each end by conducting ring. The machine of the former kind of rotor is called a *wound-rotor* machine while that of the latter kind of rotor is called a *squirrel-cage* machine. We shall describe here mainly the motoring operation of the machine.

Figure 9.26 shows the cross-sectional view of a wound-rotor induction motor. When the stator is excited from 3-phase mains (V, f), it draws 3-phase currents setting up a synchronously (n_s) rotating field $\mathbf{F}_1$ which induces 3-phase currents of frequency f (same as the stator frequency) in the stationary short-circuited rotor. The rotor's rotating field $\mathbf{F}_2$ also starts rotating at speed n_s in the same direction producing a starting torque. The motor is therefore *self-starting* unlike the synchronous motor. The rotor picks up speed and reaches a steady speed $n < n_s$ till the electromagnetic torque T balances the load torque T_L. It easily follows that the rotor cannot reach speed n_s as at this speed, the relative speed between stator filed and rotor winding (or conductors) would be zero and no rotor currents would be induced ($\mathbf{F}_2 = 0$) and hence there would be no motoring torque.

Figure 9.26 shows $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_r$ for rotor speed $n < n_s$. It will now be shown that the rotor field $\mathbf{F}_2$ always runs at speed n_s w.r.t. the stator irrespective of the rotor speed and hence the interacting fields are always stationary relative to each other, the necessary condition for electromagnetic torque production as stated earlier.

Speed of stator field w.r.t. rotor conductors = $(n_s - n)$ in the forward direction (that of n_s)

$$\text{Frequency of rotor currents, } f_2 = \frac{(n_s - n)P}{120} = \left(\frac{(n_s - n)}{n_s}\right)\left(\frac{n_s P}{120}\right)$$

$$= sf \text{ (also called } \textit{slip frequency}\text{)} \tag{9.38}$$

where

$$s = \frac{n_s - n}{n_s} = \left\{1 - \frac{n}{n_s}\right\} = \textit{slip} \text{ of rotor (per unit speed at which the rotor falls behind the stator field)} \tag{9.39}$$

Obviously $s = 1$ for $n = 0$, i.e. for the stationary rotor and $s = 0$ for $n = n_s$, i.e. for the motor running at the synchronous speed.

The rotor currents at frequency f_2 cause a rotating field $\mathbf{F}_2$ that runs w.r.t. the rotor surface at speed $(n_s - n)$ in the forward direction, while the rotor itself runs at speed n. Hence the speed of $\mathbf{F}_2$ wrt the stator is always

$$(n_s - n) + n = n_s$$

and it therefore produces a torque by interaction with $\mathbf{F}_1$. Observe that the resultant $\mathbf{F}_r$ leads $\mathbf{F}_2$ by angle δ as it would be in a motoring operation; this angle is close to 90°

Resultant field $\mathbf{F}_r$ and air-gap flux Φ_r per pole must be nearly constant so that stator induced emf balances the applied voltage as stator resistance and leakage reactance are very small. Hence the torque developed is proportional to rotor current (Eq. (9.35)). Also with constant Φ_r the rotor induced emf is proportional to slip.

Induction Motor Connection Diagram While detailed operation of an induction motor is the subject matter of chapter 11, to give a physical feel at this stage the stator, rotor and the connection diagram of a wound rotor induction motor are drawn in Fig. 9.27 (a) and (b). The stator is delta connected and wound rotor star connected and terminals brought out through slip rings and shorted externally. This type of motor is also known as *slip-ring* induction motor.

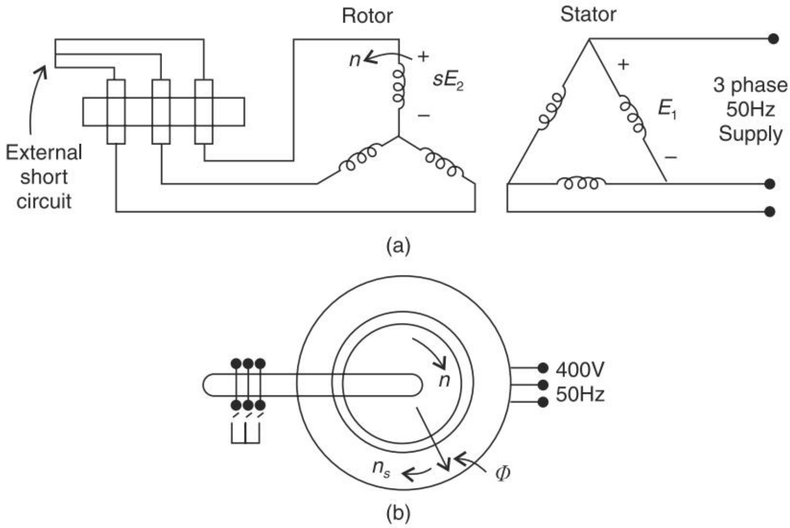


Fig. 9.27 Stator, rotor and connection diagram of induction motor-wound rotor type

In the connection diagram of Fig. 9.27 (a), E_1 and E_2 are the stator and rotor induced *emf*s as per Eq. (9.21). The effective turn ratio is

$$\frac{E_1}{E_2} = \frac{K_{\omega 1}\, f_1 N_1}{K_{\omega 2}\, f_2 N_1} \tag{9.40}$$

Torque-Slip Characteristic Since the machine produces torque at any rotor speed, it is known as an *asynchronous machine*. An important performance measure in a motor is the variation of its speed as the shaft torque (load torque) is increased. The torque–slip (speed) characteristic of induction motor is sketched in Fig. 9.28 form which the following observations are made.

– At low slip*

$$T \propto s \text{ (linear)}$$

* Rotor frequency $f_2 = sf$ is very small at low slip ($s = 0.04$, $f_2 = 2$ Hz). The rotor circuit reactance can therefore be neglected so that rotor circuit is basically resistive. Rotor current is therefore proportional to rotor induced emf, which itself is proportional to slip. Hence $T \propto s$ (Eq. (9.35), $\delta \approx 90°$).

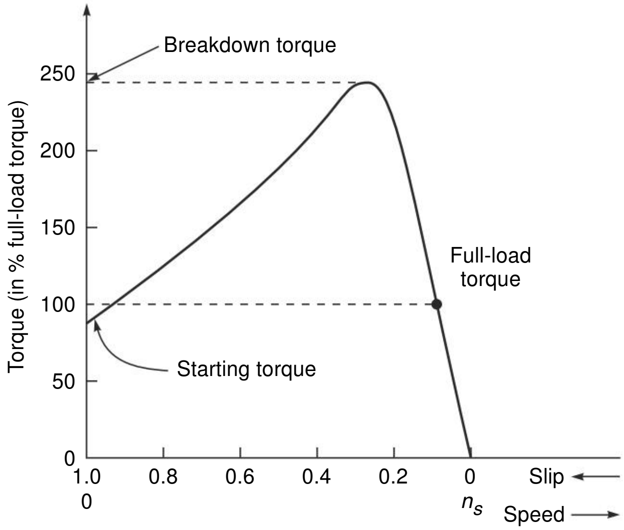


Fig. 9.28 Torque–slip characteristic of induction motor

– At high slip

$$T \propto 1/s \text{ (rectangular hyperbola)}$$

– The motor has a maximum torque called *breakdown* torque (T_{BD}) and cannot run at a load torque more than T_{BD}.

– The motor has a definite starting torque (corresponding to $s = 1$) which is much less than T_{BD}.

– It can be shown that if the stator voltage is changed, the torque would vary directly as square of voltage.

Example 9.6 A 6-pole synchronous generator driven at 1000 rpm feeds a 4-pole induction motor which is loaded to run at a slip of 4%. What is the motor speed?

Solution

Frequency of the synchronous generator,

$$f = \frac{6 \times 1000}{120} = 50 \text{ Hz}$$

Synchronous speed of the induction motor,

$$n_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Motor slip $\quad s = 0.04$

Motor speed $\quad n = (1 - s)\, n_s$ Eq. (9.38)

$$= 0.96 \times 1500 = 1440 \text{ rpm}$$

Example 9.7 A 4-pole, 50 Hz wound-rotor motor when supplied at rated voltage and frequency with slip-rings* open-circuited develops a voltage of 80 V between any two rings. With the same stator excitation, the rotor is now driven by external means at (a) 1500 rpm in opposite direction to the direction of rotation of the stator caused field and (b) 1000 rpm in the same direction. Find the slip-ring voltage and its frequency in each of these two cases.

Solution

$$n_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

With slip rings open, the rotor remains stationary as it does not carry any currents and no torque is developed (s = 1). The corresponding voltage (standstill) is V_2 = 80 V and the frequency is f = 50 Hz.

(a)
$$n_1 = -\ 1500 \text{ rpm}$$
$$s_1 = \frac{1500 - (-\ 1500)}{1500} = 2$$
$$f_2 = s_1 f = 100 \text{ Hz}$$

With given stator excitation, Φ_r in air-gap is fixed. The rotor voltage is proportional to the rotor frequency or the slip. Thus slip-ring voltage is 2 × 80 = 160 V.

(b)
$$n_2 = 1000 \text{ rpm}$$
$$s_2 = \frac{1500 - 1000}{1500} = \frac{1}{3}$$
$$f_2 = s_2 f = \frac{50}{3} = 16\frac{2}{3} \text{ Hz}$$

Slip-ring voltage $= s_2\, V_2 = \frac{1}{3} \times 80 = 26\,\frac{2}{3}$ V

DC Machine

In a dc machine, the field poles are on the stator (and are dc excited) and the rotor constitutes the armature. Figure 9.30 shows the cross-sectional view of a 2-pole elementary dc machine with a single-coil armature. As the armature rotates, an alternating emf is induced in the coil and under load it would carry alternating current. In order to obtain dc voltage and current at the armature terminals, the coil current must be rectified. This is accomplished by means of a *commutator*, which in this elementary case comprises a split copper ring (with mica insulation between two segments) and two carbon brushes to conduct the current from these segments. The coil ends are connected to each segment. It is easily seen that though the coil current and voltage are alternating, the reversal of commutator segments connected to the brushes at appropriate time causes the brush voltage and current to be unidirectional.

The dc brush voltage is thus the average value of the rectified coil voltage. For obtaining a higher dc voltage for a given peak flux density, it is desirable to use a

* In a wound-rotor induction motor three connections from the winding are brought out through slip-rings. Under running condition the slip rings are shorted. At the time of starting the motor external resistances are included in the rotor circuit through the slip-rings. A wound-rotor motor is therefore also known as a *slip-ring* induction motor.

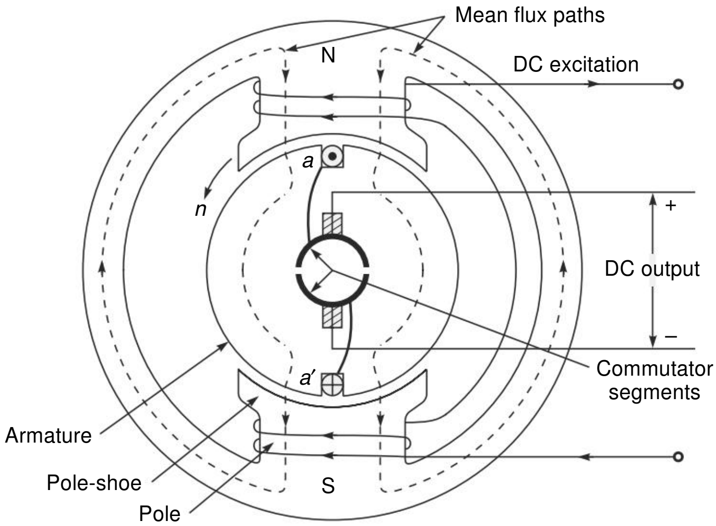


Fig. 9.29 A 2-pole elementary dc machine

flat-topped *B*-wave instead of the nearly sinusoidal wave as in ac machines. For this reason, dc poles are always of the salient-pole kind with each pole face covering about 70% of the pole-pitch (one pole or π rad (elect)) as illustrated in Fig. 9.30. Figure 9.30 shows the typical dc machine *B*-wave and the rectified coil emf (brush voltage). Smooth dc voltage would be obtained when the armature is wound with several coils placed in slots round the armature and as many commutator segments to each of which two coil-ends are connected. It is equivalent to adding several rectified waves of shape as in Fig. 9.31 (b) with a progressive phase difference.

It will be seen in Ch. 10 that when the armature carries current, it would cause a triangularly distributed mmf $\mathbf{F}_2$ with axes at 90° (fixed) to the axis of the main field pole—optimum condition for torque production.

9.9 LOSSES AND EFFICIENCY

Various losses in an electric machine are enumerated and elaborated below.

Constant Losses

These losses remain constant for a machine operated at constant mains voltage and run at substantially constant speed. These losses can be subdivided into two components.

No Load Core (Iron) Loss These losses have their origin in hysteresis and eddy-current phenomena (Sec. 8.8). In a transformer, these arise due to fixed direction alternating flux. In machines, these losses are caused by fixed flux distribution (north–south) in which the rotor moves as in the armature of a dc machine or a rotating flux distribution that sweeps through the stator as in the armature (stator) of a synchronous machine, or the stator of an induction machine. The flux density in such

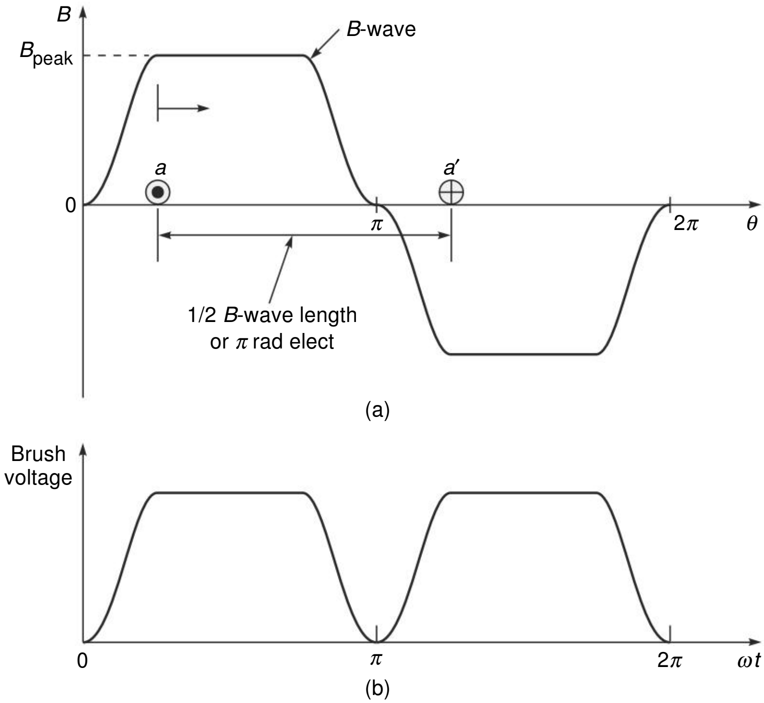


Fig. 9.30 B-wave and brush voltage in a dc machine

a member oscillates in both magnitude and direction resulting in much higher loss density than in a transformer. It is to be noted here that the frequency of flux variation and axis rotation is very low (sf = 1–4 Hz) in the rotor of an induction machine and therefore these kind of losses in that member are small enough to be ignored.

Because of slotting, high frequency flux density oscillations take place in the slotted member and even in the main poles of a dc machine due to the effect of armature slotting ($f_{slot} = (2S/P)f$ (fundamental)). The associated loss is known as *pulsation loss*.

Machine members that are the seat of iron loss are made of silicon steel laminated parallel to the flux path.

Mechanical Losses These comprise brush friction, bearing friction and windage and ventilation system losses and are substantially constant for small variation in speed.

Variable Losses

Copper (I^2R) Loss These are field and armature winding ohmic losses and are computed with dc resistance of winding at 75°C. Field copper loss for dc and synchronous machines is constant for given excitation and can therefore be lumped with constant losses.

The voltage drop at dc machine brushes is fixed (of the order of 1–2 V) as the conduction process is mainly by short ionized gaps rather than by physical contact. Therefore, strictly speaking, brush contact loss is directly proportional to the armature current.

Stray-load loss Under conditions of load, the flux density wave undergoes distortion. This leads to load-dependent losses in armature teeth.

Also when the armature conductors carry load current, this is not uniformly distributed over the conductor cross-section being an alternating current, thereby increasing the effective conductor resistance.

These two loss components together are known as stray-load loss. This loss is difficult to compute and is usually estimated to be about 1% of the machine output.

Machine Efficiency

The machine efficiency is defined as

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}} \text{ (convenient for generators)} \tag{9.41a}$$

or

$$\eta = \frac{\text{Input} - \text{losses}}{\text{Input}} \text{ (convenient of motors)} \tag{9.41b}$$

Because of constant and variable (proportional to square of load current) loss components, the device (machine/transformer) efficiency increases from zero at no load (zero output) to a maximum value occurring at the load where

$$\text{Constant loss} = \text{variable loss}$$

and then drops off. A typical efficiency—load curve is drawn in Fig. 9.31. The load at which maximum efficiency occurs is decided by the daily loading pattern of the device and is adjusted by the designer by relative proportioning of iron (associated with constant loss) and copper (associated with variable loss) employed in the device.

9.10 RATING AND COOLING

Electric machines are rated in terms of operating voltage, frequency, speed and output. For synchronous generators, output is specified as kVA/MVA at a specified power factor (this fixes the size of the turbine to which it will be coupled). In case

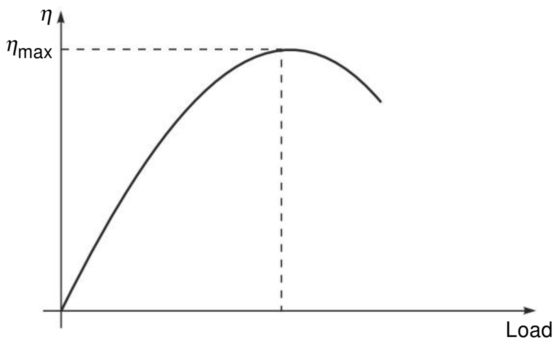


Fig. 9.31 Efficiency-load curve

of motors, output is specified as kW (or hp where hp = 746 W). Frequency is not a pertinent rating in dc machines while for ac machines it is the standard supply frequency of 50 Hz. The rated voltages are the standard values for ac machines, viz. 230 V/400 V/3.3 kV/6.6 kV/11 kV. Different standard values are employed in dc machines in higher voltage ratings. Most manufacturers build small and medium sized motors in standard kW sizes (consult ISI: 325–1970).

Insulation is the most vulnerable component in electric machines. Depending upon the class of insulation, it cannot be stressed to temperatures beyond 130–150°C. The maximum output that a machine can supply without exceeding a specified temperature rise above the ambient (40°C as per the ISI) is known as the *continuous rating* of the machine. The continuous rating for a given frame size is determined by the losses and by the nature of the cooling provided to dissipate the heat of losses so as to limit the temperature rise within the specified value. Also for a given rating, the losses must be kept within limits so as to get acceptable machine efficiency.

Because of the thermal capacity and the overall thermal resistance of its cooling system, which act like an *RC*-circuit, a machine with given continuous rating can withstand much larger loads for a short period of time. The *short-time rating* of a machine is generally specified for a standard period of 5, 15, 20, 30 or 60 minutes.

The continuous rating of a motor for a particular duty cycle (involving on-load and off-load periods) corresponds to the root means square (rms) kW load, which is based upon the assumption that the total motor losses are proportional to the square of its loading. The formula for rms kW is as under:

$$(\text{kW})_{\text{rms}} = \left[\frac{(k\text{W})^2 \ \text{time}}{\text{running time} + (\text{standstill time}/k)}\right]^{1/2} \tag{9.42}$$

where the constant $k(>1)$ accounts for poor ventilation (cooling) during the standstill period(s) where there is no forced cooling. For open-type machines $k = 4$.

Cooling

To prolong the insulation life of an electric machine to an acceptable value of 10–30 years, the heat of losses must be dissipated fast enough so that the temperature of the machine does not exceed the specified value.

In electric machines, the conduction cooling must be strengthened by forced convective cooling for effective loss dissipation. Various methods of forced convective cooling are enumerated below. The reader is advised to study the construction of electric machines in a laboratory to understand the various mechanical means that are employed for this purpose.

Radial Ventilation A rotor fan is used and the airflow paths are radial. It is used for machines up to 20 kW.

Axial Ventilation Here, the centrifugal fan is so located as to cause an axial flow of air currents through the machine.

Combined Radial and Axial Ventilation Because of the combined method, this is more effective and is useful for larger rating particularly in induction motors.

Hydrogen cooling For large turbogenerators, hydrogen in a closed circuit provides an effective method of cooling. Hydrogen is preferred over air cooling because (i) on unit volume basis, it has the same heat storage capacity as air (ii) its heat transfer capability by forced convection is 1.5 times that of air (iii) its thermal conductivity is 7 times that of air and (iv) absence of oxygen in the cooling circuit improves the life of insulation.

Hydrogen circuit should be at a pressure sufficiently higher than one atmosphere so that non-ambient air can leak into it and the explosive hydrogen–oxygen mixture cannot build up. Rotors are usually cooled by circulating hydrogen through hollow conductors. Nowadays direct water cooling of rotor conductors is being employed for very high rating machines.

9.11 MATCHING CHARACTERISTICS OF ELECTRIC MACHINE AND LOAD

Figure 9.32 shows the speed–torque characteristic of an induction motor and a fan-type load. These characteristics intersect at a point P which would correspond to the steady state operation of the motor–load combination. The fact that P is a stable operating point can be established by arguing that on being given a small perturbation, the system returns to the point P.

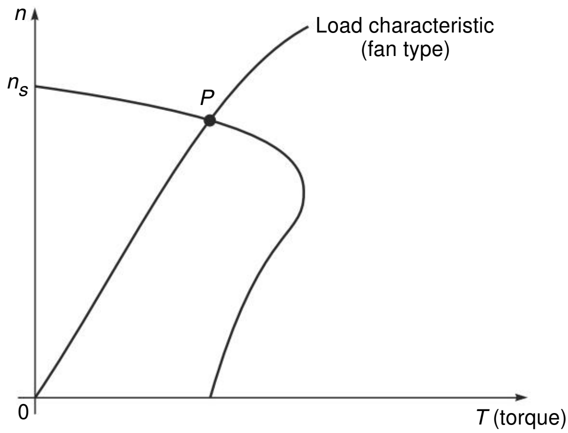


Fig. 9.32 Matching of motor-load torque characteristics

Various types of mechanical loads are enumerated below.

Constant-Speed Load Machine tools, hydraulic pumps, etc. require substantially constant speed. Paper mill drives need dead constant speed.

Variable-speed (constant kW) Load These are traction-type loads requiring high torque at low speeds and low torque at high speeds.

Adjustable-Speed Load like certain type of machine tools, rolling mill drives, etc. The range of speed adjustment can be quite demanding in either direction.

The motor speed–torque characteristics can be classified as

Synchronous type Speed remains constant independent of the load torque (Fig. 9.33(a)).

Shunt type Speed drops by a few points per cent as the load torque increases (Fig. 9.33(b)).

Series type Speed is inversely proportional to the load torque (Fig. 9.33(c))

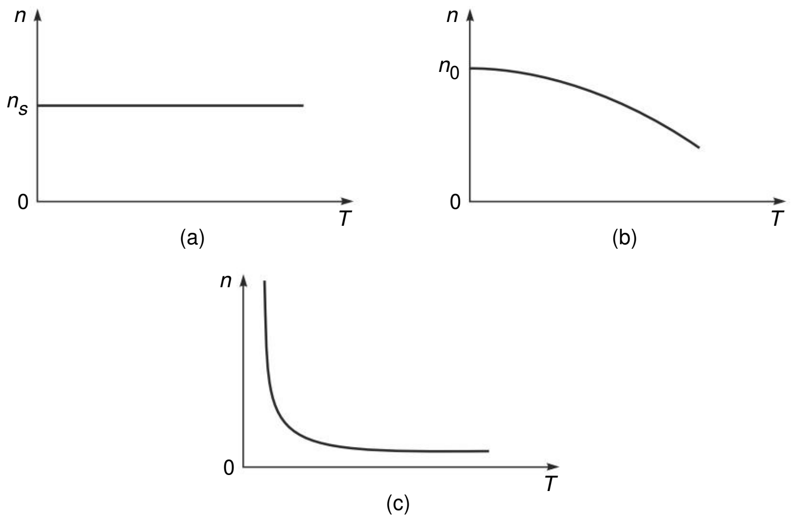


Fig. 9.33 Types of motor characteristics
(a) Synchronous-type characteristic,
(b) Shunt characteristic ($n_0 = n_s$ for induction motor)
(c) Series characteristic

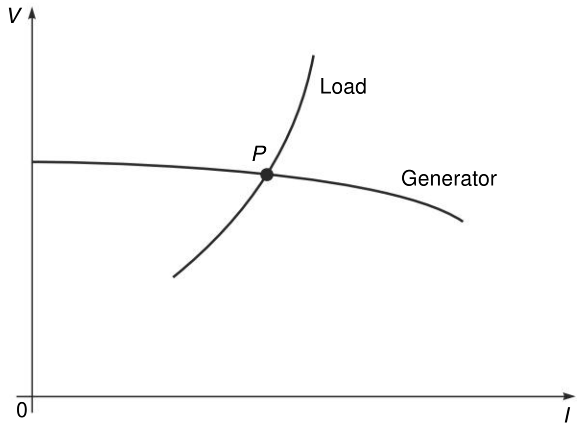


Fig. 9.34 Steady operating of the generator–load system

All these characteristics can be translated up or down by manipulation of certain controlling parameters of electric motors allowing a range of speed adjustment.

Figure 9.34 is illustrative of the steady operating point of a generator-load (electric) combination as the intersection point P of the volt-ampere characteristics of the generator (drooping) and load (rising). The steady load voltage can be controlled by translating up or down the generator V–I characteristic by adjustment of its field excitation.

ADDITIONAL SOLVED PROBLEMS

9.8 Calculate the coil pitch to eliminate the 5th harmonic in the induced emf of a synchronous generator.

For a 6-pole synchronous generator having 72 slots, calculate the nearest coil pitch as above. Calculate the value of k(1), k(5) and k(13) for this coil pitch.

Solution

$$\cos(5\beta/2) = 0 \text{ or } 5\beta/2 = 90° \text{ or } \beta = 36°$$

$$\text{coil pitch} = 180° - 36° = 144°$$

6-pole, 72-slot stator:

$$\text{slot angle } \gamma = (6 \times 180°)/72 = 15°$$

$$\text{coil pitch in slots} = 144/15 = 10 \text{ or } 150°$$

$$\beta = 180° - 15° \times 10 = 30°$$

$$K_p(1) = \cos(30°/2) = 0.966$$

$$K_p(5) = \cos(5 \times 30°/2) = 0.259$$

$$K_p(13) = \cos(13 \times 30°/2) = -0.966$$

Remark This choice of coil pitch does not lead to much reduction in the 13th harmonic though the 5th harmonic is reduced to 25.9% of its value with full pitch coils. Short pitching also reduces coil overhang copper.

9.9 A 2-pole, 3-phase, 50 Hz star-connected synchronous machine has 42 stator slots. Each slot has two conductors in a double-layer winding. The coil pitch is of 17 slots each phase has two parallel paths. Calculate the flux/pole needed to generate a line voltage of 2300 V.

Solution

$$m = 42/(3 \times 2) = 7$$

$$\gamma = (2 \times 180°)/42 = 8.57°$$

$$K_b = \frac{\sin(7 \times 8.57°/2)}{7\sin(8.57°/2)} = 0.952$$

$$\text{Coil pitch} = 17 \text{ slots, pole pitch} = 42/2 = 21 \text{ slots}$$

Short pitching angle,

$$\beta = (21 - 17) \times 8.57° = 34.3°$$

$$K_p = \cos(34.3°/2) = 0.956$$

$$\text{Total conductors} = 42 \times 2 = 84$$
$$\text{Number of coils} = 84/2 = 42$$
$$\text{Coils / phase} = 42/3 = 14$$

The coils are connected two parallel paths. Therefore

$$N_{ph}\text{ (series)} = 14/2 = 7;\text{ each parallel path}$$

Emf is determined by the series coils in a parallel path. Thus

$$E_p = 4.44\, K_b\, K_p\, f\, \Phi\ \ N_{ph}\text{ (series)}$$
$$2300/\sqrt{3} = 4.44 \times 0.952 \times 0.956 \times 50 \times 7 \times \Phi$$

or $$\Phi = 0.94\text{ Wb}$$

9.10 A 3-phase induction motor runs at a speed of 940 rpm at full load when supplied form a 50 Hz, 3-phase mains.
(a) How many poles does the motor have?
(b) What is its per cent slip at full load?
(c) What is the corresponding speed of
(i) the rotor field w.r.t. the rotor surface?
(ii) the rotor field w.r.t. the stator?
(d) What would be the rotor speed at twice full-load slip?

Solution

(a) $n_s = 1000\text{ rpm} \quad P = (120 \times 50)/1000 = 6$

(b) $s = (1000 - 940)/1000 = 6\%$

(c) (i) $1000 - 940 = 60\text{ rpm}$

(ii) $960 + 40 = 1000\text{ rpm}$

(d) $s = 2 \times 6 = 12\%$

$n = 1000 - (12 \times 1000)/100 = 880\text{ rpm}$

9.11 In Problem 9.10, what would be the slip and speed at full-load torque if the total resistance of the rotor is doubled (by addition of external resistance through slip rings)?

Solution

Figure 9.35 shows the rotor circuit with added external resistance. For the same torque, the rotor current must remain the same. ($T = K_T I_2$, for fixed stator voltage). Thus,

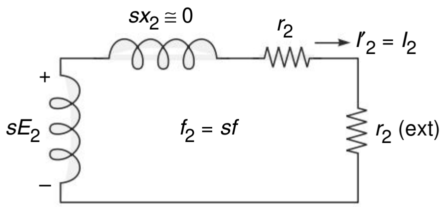


Fig. 9.35 Rotor circuit with load and external rotor resistance

$$\frac{sE_2}{r_2} = \frac{s'E_2}{2r_2}$$

or $$s' = 2s = 12\%$$

$$n = 1000 - (12 \times 1000)/100 = 880 \text{ rpm}$$

Remark This indeed is a method of controlling the speed of a wound rotor induction motor.

9.12 A star connected, 3-phase, 8-pole, 60 H synchronous generator develops a voltage of 13 kV line-to-line for a field current of 5A. The generator is required to generate the same line voltage at 50 Hz. Find the field current required and the synchronous speed.

Solution

E_p (phase) = $4.44\, K_\omega f N_{ph}\, \Phi$

$\Phi \,\alpha\, I_f$ field current

No changes in induced *emf*. Therefore

$$4.44\, K_\omega F_1 N_{ph} F_1 \Phi_1 = 4.44\, K_\omega f_2 N_{ph} \Phi_2$$

which gives

$$\Phi_2 = \left(\frac{f_1}{f_2}\right) \Phi_1 = \left(\frac{60}{50}\right) \Phi_1$$

Or $$I_{f2} = \left(\frac{6}{5}\right) I_{f1} = \frac{6}{5} \times 5 = 6\text{A}$$

The excitation needs to be increase to 6 A.

Synchronous speed $n_A(2) = \dfrac{120 \times 50}{8} = 750$ rpm

9.13 A synchronous motor-generator set links a 3-phase, 60 Hz system to a 3-phase, 50 Hz system as shown in the schematic diagram of Fig. 9.36. Determine

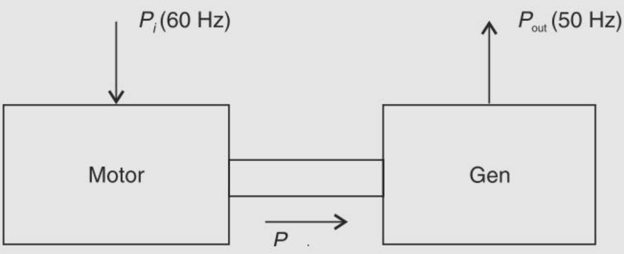


Fig. 9.36

(a) the minimum number of motor poles and corresponding number of generator pole, and

(b) the set speed.

Solution

The set speed must be synchronous speed of both motor and generator.

$$n\text{ (set)} = \frac{120f_1}{P_1} = \frac{120f_2}{P_2}$$

or $$\frac{P_2}{P_1} = \frac{f_2}{f_1} = \frac{50}{60} = \frac{10}{12}$$

$$P_1 = 12,\ P_2 = 10,\ n\text{ (set)} = \frac{120 \times 60}{12} = 600\text{ rpm}$$

9.14 Calculate the distribution factor of a single-phase motor having 6 slot /pole (a) when all the slots are wound and (b) when only 4 adjacent slots are would. Comment on the effective use of copper in the two case.

Solution

$$\text{Slot angle } \gamma = \frac{180^0}{6} = 30°,\ \text{SSP},\ m = 6$$

All 6 slots wound

$$K_b(1) = \frac{\sin 6 \times 30°/2}{6 \sin 30°/2} = 0.644$$

4 slots wound, $m = 4$

$$K_b(1) = \frac{\sin 4 \times 30°/2}{4 \sin 30°/2} = 0.8365$$

Copper used is proportional to slots used. In winding 6 slots rather than 4 slots, 50% more copper is used but the distribution factor reduces from 0.8365 to 0.644. Therefore, it is better to wind less slots in single phase and make up for *emf* by increasing turns / coil.

9.15 A 3-phase, 50 Hz motor runs at 965 *rpm*.

(a) Calculate the number of motor poles.

(b) What is the slip and frequency of the rotor currents?

(c) What is the speed of the stator field with respect to rotor and with respect to rotor field?

Solution

(a) $$\frac{120f}{P} = \frac{120 \times 50}{P} \approx 965$$

or $$P \approx 6.12 \therefore P = 6,\ n_s = 1000\text{ rpm}$$

(b) slip, $$s = \frac{1000 - 965}{1000} = 0.035$$

Rotary Frequency, $f_2 = sf_1 = 0.035 \times 50 = 1.75$ Hz

(c) Speed of stator field relative to rotor surface = 1000–965

= 35 rpm (forward)

Speed of rotor field with respect to rotor surface

$$= \frac{120 \times 1.75}{6} = 35\text{ rpm (forward)}$$

Speed of stator field with respect to rotor field

$$= 1000 - (965 + 35) = 0, \text{ stationary}$$

9.16 A 3-phase, 50 Hz, 4-pole, 400 V wound rotor induction motor has delta-connected stator and star-connected rotor winding. Assume that effective stator to rotor turn ratio of 2:1 For a rotor, speed of 1440 rpm, calculate

(a) the slip,

(b) the rotor frequency and

(c) the rotor induced *emf* line-to-line and sketch the rotor circuit

Solution

(a)
$$n_s = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$n = 1440 \text{ rpm}$$

$$\text{slip} = \frac{1500 - 1440}{1500} = 0.04$$

(b) Rotor frequency, $f_2 = sf = 0.04 \times 50 = 2$ Hz

(c) Stator induced *emf* = applied voltage; E_1 (phase) = 400 V (delta)

Rotor incuded emf (phase) at 50 Hz. As per turn ratio

$$E_2 \text{ (50 Hz)} = \frac{400}{2} = 200 \text{ V (star Phase)}$$

At slip $s = 0.04$ $f_2 = 2$ Hz

$$E_2 \text{ (2 Hz)} = 200 \times \frac{2}{50} = 8 \text{ V}$$

$$E_2 \text{ (line)} = \sqrt{3} \times 8 = 13.856$$

Rotor circuit (phase)

The circuit has rotor resistance in series with reactance sx_2, where x_2 in the rotor reactance at $s = 1$ (50 Hz). Rotor induced *emf* = $s\,E_2$. The circuit is drawn in Fig. 9.37.

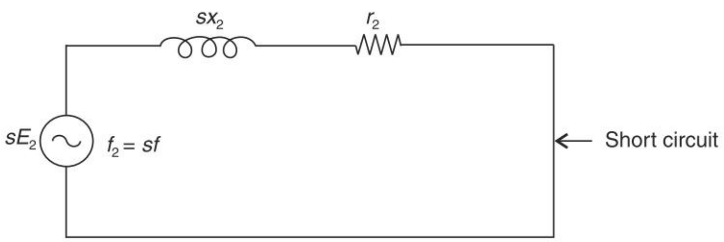


Fig. 9.37

SUMMARY

- Constructional Feature-Electric Machines

  Rotating electric machines have two flux carrying parts which are made of laminated silicon steel. These two are

  Stator : It is a stationary annular cylinder.

Rotor: It rotates within the stator supported by a shaft, ball bearings and *end rings* bolted to the stator. There is a narrow *air-gap* between the stator and rotor.

Windings: There are two windings made of copper. These are placed in stator and rotor *slots* or in one of these wound on projecting poles. In *synchronous* and *dc machines,* the main filed is created by the field poles (even in number) and *dc* excited. The other winding which interchanges electric power with the external circuit and so carries the *load current* is called the *armature winding* and in the *seat of induced emf.*

In a synchronous machine, where the field poles are on the rotor and armature winding on the stator, is the preferred construction universally adopted excitation current is provided to the field poles through *slip ring brush* arrangement. In a *dc* machine, it is a must that the field poles are on the stator and the armature on the rotor. The rotor also carries a *commutator* whose segments are suitably connected to the armature windings and act to convert the alternating armature current to *dc* for the external connection.

In an *induction machine,* both stator and rotor are slotted and carry armature windings; rotor may carry just slot conductors shorted by end rings.

Thus the electric machines are of two types

- AC Machines: Synchronous and induction

- DC Machines

- Field pole types

  Salient or projecting poles, non-salient or cylindrical poles.

  Synchronous machine can have both types, *dc* machine has only salient poles

- Induced AC *emf* in rotating machines. It is the speed *emf.* The relative motion between *B*-wave and coils, which causes change in flux linkage and *emf induction.*

- Mechanical and electrical angles

$$\frac{\theta_e}{\theta_m} = \frac{2}{P} \text{ ; } P = \text{number of pole}$$

- Speed-Poles-Frequency

$$n = \frac{120f}{p} \text{ synchronous speed in } rpm$$

or

$$f = \frac{nP}{120} \text{Hz}$$

- **Armature Coils**

  Could be single-turn or multi-turn with two end connection.

  Coil-side - each active side of a coil

- **Coil span (pitch)** - full-pitched, angle between coil sides is π rad or 180° electrical

  - short-pitched; angle between coil sides is less than π in terms of number of slots

- Two-layer windings—two coil sides per slot

- Induced *emf* of a single N turn full-pitch coil

$$E\ (rms) = \sqrt{2}\,\pi f N \Phi = 4.44 f N \Phi$$

$$\Phi = flux\ /\ pole$$

Induced *emf* phasor lags the flux phasor by 90°

- Distributed winding

More than one coil / phase

Slots / pole / phase, $SPP = m = \dfrac{S}{qp}$; $S = slots$, q = number of phases, generally three

$$\text{Slot angle, } \gamma = \frac{\pi P}{S} \text{ rad (elec.)}$$

$$\text{Phase spread, } \sigma = m\gamma$$

- Breadth factor, K_b

Because of distributed winding the phase *emf* is less than the algebraic sum of series turns/phase by the breadth factor

$$K_b = \frac{\sin m\gamma/2}{m \sin \gamma/2} < 1$$

$K_b\ (harmonics) < K_b\ (fundamental)$

Therefore distributed winding incidentally reduce the harmonic content of *emf* induced.

- Short-pitched (corded) coils

The *emf* of a short-pitched coils is less than that for a full-pitched coil by the Pitch Factor

$$K_p = \cos\frac{\beta}{2} < 1\ ;\quad \beta = \text{short-pitching angle in rad elect.}$$

- Chording of coil saves in overhang copper by proper choice of β any particular harmonic can be eliminated
- Winding Factor

$$K_\omega = K_b K_p < 1$$

- General formula for induced *emf* / phase

$$E_p = \sqrt{2}\ \pi K_\omega f N_{\text{ph}}\ (series)\ \Phi\ V$$

It is applicable for synchronous machine (stator) and induction machine stator and rotor. For 3-phase synchronous and induction machine, the windings of the three phase are laid 120° elect. apart from each other.

- MMF of AC Winding

A single coil produces rectangular *mmf* wave (with north and south poles, strength ($Ni/2$)

Fundamental *mmf* wave

$$F_{a1} = \frac{4}{\pi}(Ni/2)\cos\theta, \qquad \theta = \text{space angle elect.}$$

For sinusoidal current ($i_a = \sqrt{2}\ I \cos \omega t$)

$$F_{a1} = \sqrt{2}K' I \cos\omega t \cos\theta$$

$$= F_m \cos\omega t \cos\theta$$

It is a standing pulsating wave.

For a distributed winding

$$f_{a1} = \frac{4\sqrt{2}}{\pi} K_\omega \left(\frac{N_{ph}(series)}{P}\right) I \cos \omega t \cos\theta$$

$$= F_m \cos \omega t \cos \theta$$

- Rotating Magnetic Field

  A 3-phase winding with their axis located at 120° *elect. phase difference* from each other are fed with 3-phase balanced currents with a *time difference of 120° elect.* from each other, the resultant *mmf* wave and its associated *B*-wave rotates at synchronous speed $\omega_s = 2\pi f$ rad (elect)/s (or $n_s = 120$ *rpm*). The direction of rotation of *mmf* wave from the leading phase current to the lagging phase current. The number of poles of the *mmf* wave is same as that for which the winding is wound.

- Torque in Round Rotor Machine

  Necessary conditions for production of steady torque by two interacting magnetic fields:

  1. The two fields must be relatively stationary
  2. The two field must have the same number of poles

  Torque expression:

  F_1, F_2 are peak values of sinusoidally distributed fields rotating at synchronous speed and F_r the resultant field.

  $$\text{Torque, } T = K F_1 F_2 \sin \lambda, \quad \lambda = \text{angle between } F_1 \text{ and } F_2$$

  $$= K F_r F_2 \sin \delta; \quad \delta = \text{angle between } F_r \text{ and } F_2$$

  It is F_r that produces the *air-gap flux,* Φ_r / pole

- Synchronous Machine

  Generating

  F_2 leads F_r by angle δ. Electromagnetic torque $T = T_{PM}$ in same direction speed synchronous

  Motoring

  F_2 lags F_r by angle δ. Electromagnetic torque $T = T_L$ (load torque) in same direction speed synchronous. Non-self starting

  Terminal voltage (equal to induced *emf*)

  $$V\text{(line)} = \sqrt{3} \times 4.44\, K_w f N_{ph} \text{ (series)}$$

  For fixed terminal voltage, Φ_r is constant; same as in a transformer. Therefore

  $$T = K_T \sin \delta$$

  Pullout torque, $\quad T_{max} = K_T,\ \delta = 90°$

  Higher torque load causes *loss of synchronism* or the machine *pullout*

- Induction Machine

  Two types : (1) *wound rotor,* Terminals shorted externally (2) *squirrel cage* rotor, copper or aluminium bars in slots shorted by conducting end rings.

  Operation

  When the stator is excited from 3-phase mains (V, f), the rotating field induces currents in shorted windings and their interaction produces torque. The machine is therefore *self-starting*. The steady rotor speed n must be less n_s, the synchronous speed for induction currents in rotor and torque production. The induction motor is therefore *asynchronous motor.*

$$\text{Slip, } s = \frac{n_s - n}{n_s}$$

Rotor frequency, $f_2 = sf$

The air-gap flux Φ_r is determined by the applied voltage. The torque-slip (T– s) characteristic is non-linear and slip at full-load is 2-5%. So speed less than synchronous is nearly constant (shunt characteristic).

Maximum torque is the *breakdown* above which the motor *stalls.*

REVIEW QUESTIONS

1. What measures are adopted to make the B-wave in a synchronous machine nearly sinusoidal? Why should the B-wave be sinusoidal?
2. A synchronous machine has P poles and generates voltage of frequency f Hz. Write the expressions for its speed in rad (elect.) /s, rad (mech) / s and *rpm.*
3. Explain the terms coil span, coil pitch, short-pitching and cording of coils.
4. What is SPP? Write its expressions for a stator having S slots and P poles.
5. In a distributed winding, why is the phase *emf* less than the algebraic sum of phase coils in series?
6. Derive the expression for the breadth factor by means of a phasor diagram.
7. Repeat question 6 for the pitch factor.
8. What is the purpose of using short-pitched coils in *ac* windings?
9. Write the expression for phasor *emf* in a synchronous machine. Use standard symbol and explain what each symbol stands for.
10. Taking the B-wave to be sinusoidal, derive the expression for flux/pole
11. Write the expression for flux linlages of an N-turn coil if the B-wave is sinusoidal and rotating at synchronous speed ω rad/s.
12. Draw the phasor diagram relating *emf* phasor to flux phasor.
13. Sketch the *mmf* wave of an N-turn coil carrying current i. Write the expression for its fundamental if i is sinusoidal. What kind of wave is this?
14. Write the expression for standing pulsating space wave and sketch it at $\omega t = 0, \frac{\pi}{3}, \frac{2\pi}{3}$ and π rad.
15. Relate the peak value of a rotating magnetic field and the maximum value of the fundamental of the *mmf* space wave of one phase.
16. State the conditions for a 3-phase winding of a stator to create a rotating magnetic field and its speed and direction of rotation.

17. State the conditions for two interacting rotating fields to create steady torque.
18. Two interacting fields $\vec{F}_1$ and $\vec{F}_2$ have a resultant field $\vec{F}_r$. Write the expression for torque developed in terms of $\vec{F}_2$ and $\vec{F}_r$. Explain the significance of the angle between them.
19. Write the expression for stator line voltage of a synchronous machine and show that it determines the air-gap flux / pole of the machine.
20. What is the pull-out torque of a synchronous machine and the meaning of loss of synchronism?
21. Explain the reason why a synchronous motor is not self-starting.
22. Explain the process of how an induction motor develops torque when *ac* supply is connected to its stator. Why it cannot develop torque at synchronous speed?
23. Define slip of an induction motor. At full-load, what is the range of the value of slip.
24. What is the frequency of the rotor currents of an induction motor?
25. Why is an induction motor as called asynchronous motor?
26. List the type of losses in as electric machine. What is the nature of each loss?
27. What is the relative speed between stator and rotor rotating fields in an induction motor?
28. Sketch the torque-slip characteristic of an induction motor. Explain the nature of the low slip part of the characteristic. Locate on the characteristic the full-load torque operating point.
29. Explain how an induction motor can self-start but cannot run at synchronous speed.
30. Explain why rotor induced *emf* is proportional to slip.
31. Distinguish between time phase difference and space phase difference.
32. State the condition of maximum efficiency of an electric machine. Compare it with that of a transformer.
33. When a conducting coil is moving past a sinusoidal *B*-wave, what is the relative position of the coil axis when the induced *emf* in it is (i) maximum and (ii) zero?

PROBLEMS

9.1 Determine breadth and pitch factor for 3-phase winding with 3 slots/pole/phase. The coil span is 8 slot pitches.

9.2 A 3-phase, 20-pole synchronous generator has a star-connected winding with 180 slots and 8 conductors/slot. The flux/pole is 0.05 Wb (sinusoidally distributed) and the speed is 300 rpm. Find the frequency and the line-to-line induced emf. The total turns/phase may be assumed to be series connected. Also assume the stator winding to be full-pitched.

9.3 A 3-phase, 50 Hz, star-connected synchronous generator with double-layer winding runs at 750 rpm. It has 8 turns/coil and 4 slots/pole/phase with a coil pitch of 11 slots. If the flux/pole is 0.05 Wb, find the line emf induced. Assume that the total turns/phase are connected in two series circuits connected in parallel. If each turn can carry 10 A, what is the permissible phase current and the machine kVA?

9.4 A 50 Hz salient-pole synchronous generator has 288 stator slots with 8 conductor/slot and the rotor is driven at 250 rpm. Full-pitch coils are used. The mean air-gap diameter is 3.2 m and the stator length is 0.8 m. The peak value of sinusoidally distributed flux density wave is 1.2 T. Permissible conductor current is 10 A.

(a) Calculate the rms voltage that can be induced by connecting all the turns in series (single phase). Also find the machine kVA.

(b) Find the per phase rms voltage and machine kVA when all the turns are series connected in 3-phase.

Comment upon the results of parts (a) and (b).

Hint: $$B_{av} = \frac{2}{\pi} B_{peak}$$

9.5 A 2-pole synchronous generator has a frequency of 50 Hz. The stator has 24 slots with 2 conductors/slot and a permissible conductor current of 8 A. Armature winding has full-pitch coils. The flux/pole is 2.2 Wb. Compute the rms value of the generator emf (line) and machine kVA capacity if the stator winding is connected as (a) a single-phase winding with all coils in series, (b) a 2-phase winding and (c) a 3-phase winding.

9.6 Sinusoidally distributed mmf along the air-gap of a round-rotor machine has peak value of F_r. The machine dimensions are

$$P = \text{poles, } D = \text{mean air-gap diameter}$$

$$l = \text{axial length of stator, } g = \text{air-gap length}$$

The flux/pole in the air-gap set up by this mmf can be expressed as

$$\Phi_r = \mathcal{P}\Phi_r$$

where $\mathcal{P}$ is the effective *permeance/pole*.

Derive the expression for $\mathcal{P}$.

9.7 A 2-pole synchronous motor fed form 50 Hz mains is coupled to a synchronous generator. What should be the number of poles of the generator for it to generate a voltage of 400 Hz?

9.8 An induction motor runs at no-load and full-load speeds respectively of 990 and 950 rpm when fed from a 50 Hz, 3-phase mains.

(a) What is the number of poles of the motor?

(b) Calculate motor slip at no load and full load. Also calculate the frequency of rotor currents under these two conditions.

(c) Calculate at both no load and full load the speed of the rotor field w.r.t. the rotor surface, wrt the stator field and that wrt to the stator.

9.9 A slip-ring induction motor runs at 960 rpm on full load when connected to 50 Hz main. Calculate

(a) slip

(b) number of poles and

(c) slip and speed at full-load with the rotor resistance doubled. Assume the rotor leakage reactance to be negligible at slip frequency.

Chapter 10

DC MACHINES

MAIN GOALS AND OBJECTIVES

- *To familiarize with constructional details, the commutation*
- *Winding types, lap windings connections to commutation and brushes*
- *Derivation and clarity of relationships for EMF and Torque, power conversion*
- *To distinguish between generating and motoring machine*
- *To picturize armature reaction in space relationship to the main field, effects and remedy*
- *To acquire clear understanding of commutation, effect of reactance emf and remedy, the interpoles*
- *The field excitation and excitation types, machine types*
- *Open circuit characteristic of a dc machine*
- *Versatility of dc machine—speed and torque can be controlled by the field (stator) and the armature (rotor)*
- *Speed-torque characteristics of the three motor types and their method of speed control*
- *The starting of dc motors*

10.1 INTRODUCTION

A dc machine is constructed in many forms and for a variety of purposes, from the 3 mm stepper drawing a few μA, 1.5 V in a quartz crystal watch to the giant 100 MW or more rolling mill motor. It is a highly versatile and flexible machine. It can satisfy the demands of load requiring high starting, accelerating and retarding torques. A dc machine is also easily adaptable for drives with a wide range of speed control and fast reversals.

DC motors are used in rolling mills, in traction and in overhead cranes. They are also employed in many control applications as actuators and as speed or position sensors. With ac being universally adopted for generation, transmission and distribution, there are almost no practical uses now of the dc machine as a power generator. Its use as a motor–generator (ac motor–dc generator) for feeding dc drives has also been replaced in modern practice by rectifier units. In certain applications dc motors act as generators for brief time periods in the 'regenerative' or 'dynamic braking' mode, especially in electric traction systems.

With high power, high voltage SCR (silicon controlled rectifier) being now available at low cost, frequency control SCR circuits are used to feed ac induction motors, such that the overall system has the features of a dc motor listed above. It is a strong competitor of dc motor in traction, particularly because an induction motor is much cheaper than a dc motor. It competes well with dc motor even with the added cost of SCR circuitry.

The basic principles underlying the operation and constructional features of a dc machine were discussed in Sec. 9.8 (refer to Fig. 9.28). The use of an electric field winding, which supplies electric energy to establish a magnetic field in the magnetic circuit, results in great diversity and a variety of performance characteristics. The armature winding is connected to the external power source through a *commutator–brush* system (see Fig. 10.5(a) item 6), which is a mechanical rectifying (switching) device.

The design of electrical machines has become a very interesting and challenging topic and is continuously changing with new improved magnetic, electrical and insulating materials; the use of improved heat-transfer techniques; development of new manufacturing processes; and the use of computers. The objective of this chapter is to analyse the behaviour of the dc machine and present the physical concepts regarding its steady state performance.

10.2 CONSTRUCTIONAL AND OPERATIONAL FEATURES

For proper connectivity and clarity of understanding, the brief account of the dc machine presented in Section 9.8 will be repeated with added details and explanation.

A *dc* machine has field poles on the rotor which are *dc* excited. The rotor constitutes the armature which carries the armature windings as shown in the cross-sectional view of an elementary 2-pole *dc* machine with a single armature coil in Fig. 10.1 As the rotor rotates, alternating *emf* is induced in the coil which under load would carry alternating current. In order to have *dc* voltage at the terminals and to feed *dc* load current, the alternating armature current has to be *rectified*.

Rectification is accomplished by a mechanical rectifier called the *commutator*, which is carried on the rotor shaft and so rotates along with armature. In the elementary machine, the commutator comprises two *copper segments* insulated from each other by thin mica insulation and are also insulated from the shaft as shown in Fig. 10.1. The coil ends are connected to the segment, and two *carbon brushes* in sliding contact with the segments conduct current to external circuit. It is easily seen that as the armature current and *emf* reverses, the brush contacts reverse simultaneously such that brush voltage and current are unidirectional.

In *ac* machines, design actions are taken to make air-gap *B*-wave nearly sinusoidal so as to induce sine wave *emf*. On the other hand, as constant unidirectional brush voltage is the aim, the *B*-wave is made flat topped by (i) uniform air-gap under the

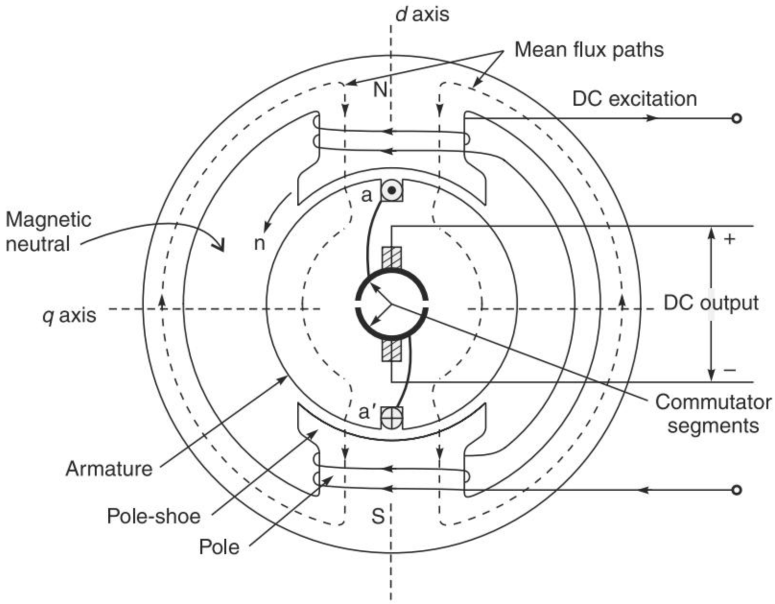


Fig. 10.1 A 2-pole elementary dc machine

poles and (ii) making the pole shoes wider. For this reason, the *dc* poles are always salient kind with each pole shoe covering about 70% of the pole pitch (π rad) as shown in Fig. 10.1. Typical B-wave of a *dc* machine and the rectified coil *emf* wave (brush voltage) are sketched in Fig. 10.2 (a) and (b) respectively.

Smooth *dc* voltage would be obtained when the armature is wound with several coils placed in slots round the armature and as many commutator segments to each of which two coil-ends are connected. It is equivalent to adding several rectified waves of shape as in Fig. 10.2 (b) with a progressive phase difference. The reader is advised to superimpose one more rectified *emf* wave with + 90° phase shift on Fig. 10.2 (b), add the two waves and see the increase in voltage and smoothing effect.

Axes of dc machine It is convenient to identify two axes of a *dc* machine (applies to all electric machine).

The axis of the magnetic pole (middle of the pole) is called the *d*-axis and the axis mid between poles (inter-polar region) is called the *q*-axis which is at 90° elect degrees to the *d*-axis.

The two axes are identified on the 2-pole *dc* machine of Fig. 10.1.

Longitudinal and cross-sectional views of a dc machine are drawn in Fig. 10.5 (a) and (b) respectively. All the important machine parts, their location and general shape are clearly brought out by these figures which would be referred to repeatedly.

10.3 ARMATURE WINDINGS AND COMMUTATION

In a practical dc machine, there may be more than two poles and there are a number

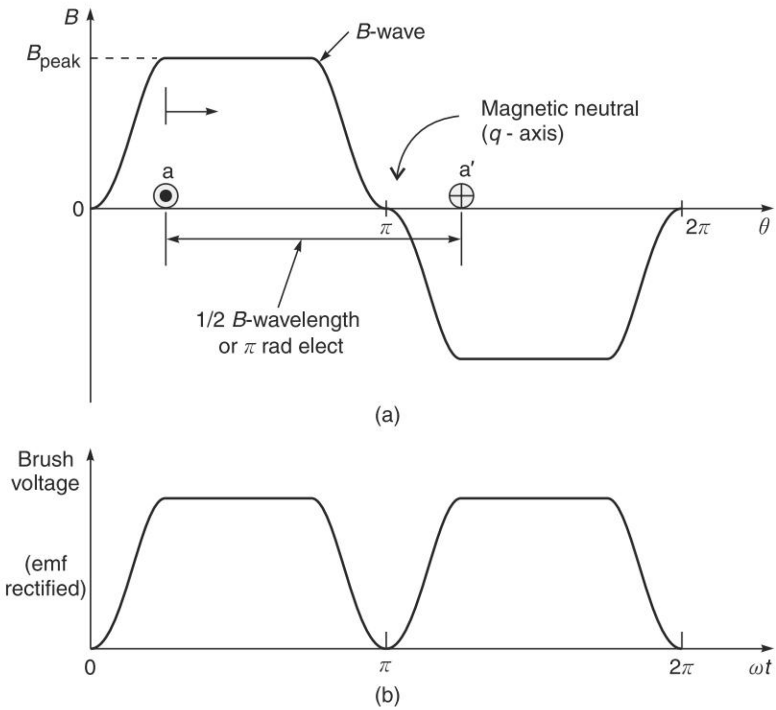


Fig. 10.2 B-wave and brush voltage in a dc machine

of armature coils (single or multi-turn) accommodated in slots in two layers. The coils are full-pitched and one side of a coil is in top in one slot and its second side is in bottom of a slot one pole pitch (π rad) away. To each segment are connected the coil ends of two different coils starting end of one coil and finishing end of the other. Therefore, *there are as many commutator segments as the number of armature coils. The armature coils are connected in two possible ways—lap winding and wave winding*. We shall consider lap winding only.

Lap winding

In a book of first level, the purpose is to explain as to how the coils are wound and how are they connected to the commutator segments.

We shall take the example of *4-pole, 12 armature slots, single-turn coils and 2 coil-sides per slot*. The best way to draw a winding diagram is in developed form explained below.

Developed Diagram Imagine the armature surface to be cut parallel to the axis of rotation and laid out on a plane with poles underneath and slots on top with top layer coil-sides shown by solid lines and bottom layer coil-sides shown by dotted lines, as in Fig. 10.3. The commutator segments appear on the lower side of the diagram numbered consecutively. The coil-sides are numbered top, bottom consecutively. As the last coils on each side cannot be shown connected, the connectivity is indicated by coil-side numbering scheme. We find

Number of coils = number of slots = 12; two coil-sides/slot

Number of commutator segment = 12

Number of slots under each pole = 12/4 = 3

Coil span (full pitch) = 12/4 = 3 slots

The current direction of coil-sides (2 × 3 = 6) over the, poles alternates is as shown in the diagram. In the coil-side numbering scheme adopted, odd-numbered coil-sides are in top layer and even numbered coil-sides are in the bottom layer. In all, the coil-side numbers for the example in hand are 1 to 24.

Coil span in terms of coil-sides = 2 × 3 + 1 = 7

Thus coil-side 1 and 8 form one coil, 3 and 10 form the next coil, and so on. Further, the coils are diamond shaped.

The winding proceeds with the starting end of one coil connected to a commutator segment and the finishing end to the next consecutive commutator segment to which is also connected the starting end of the next consecutive coil. The winding thus progresses with partially overlapping coils till the winding closes onto itself. It is also observed that the winding progresses by one commutator segment for each coil, which means that

commutator pitch = 1 segment

In the winding diagram of Fig. 10.3, coil-side 1 is connected to segment 1 and coil-side 8 (of the same coil) to segment 2 to which is connected coil-side 3 of the next coil and so forth.

Brushes and Parallel Paths The brushes are placed at the commutator segments where the current in both coil ends connected to that segment either comes out or goes in. It is seen from the diagram that this happens at commutator segment numbers 1, 4, 7 and 10. Thus, there are four brushes equal to number of poles and the adjoining brushes are 12/4 = 3 segments apart. Further, the alternate brushes are positive current coming out and negative (current going in). Similar polarity brushes are connected in parallel as shown in the diagram.

Further, the winding can be divided into four parallel paths (equal to number of poles) each parallel path having 12/4 = 3 coils in series spread over two adjoining poles. Also, each parallel path coils are getting connected to opposite polarity brushes. As the armature rotates, one coil goes out of a parallel path and simultaneously another comes in. Thus, at any instant, the same number of series-connected coils are similarly disposed under a given pole pair and hence the brush voltage is d.c. (constant).

One more observation is made from the winding diagram of Fig. 10.3 that because of diamond shaped coils, the brushes are physically located opposite the middle of the poles (the d-axis) but electrically these are connected to the coils in the inter-polar region (the q-axis).

Ring Diagram The ring diagram of the lap winding of Fig. 10.3 is drawn in Fig. 10.4 with commutator in actual circular form while the coils connected between adjacent segement are shown schematically. The location of brushes, their external

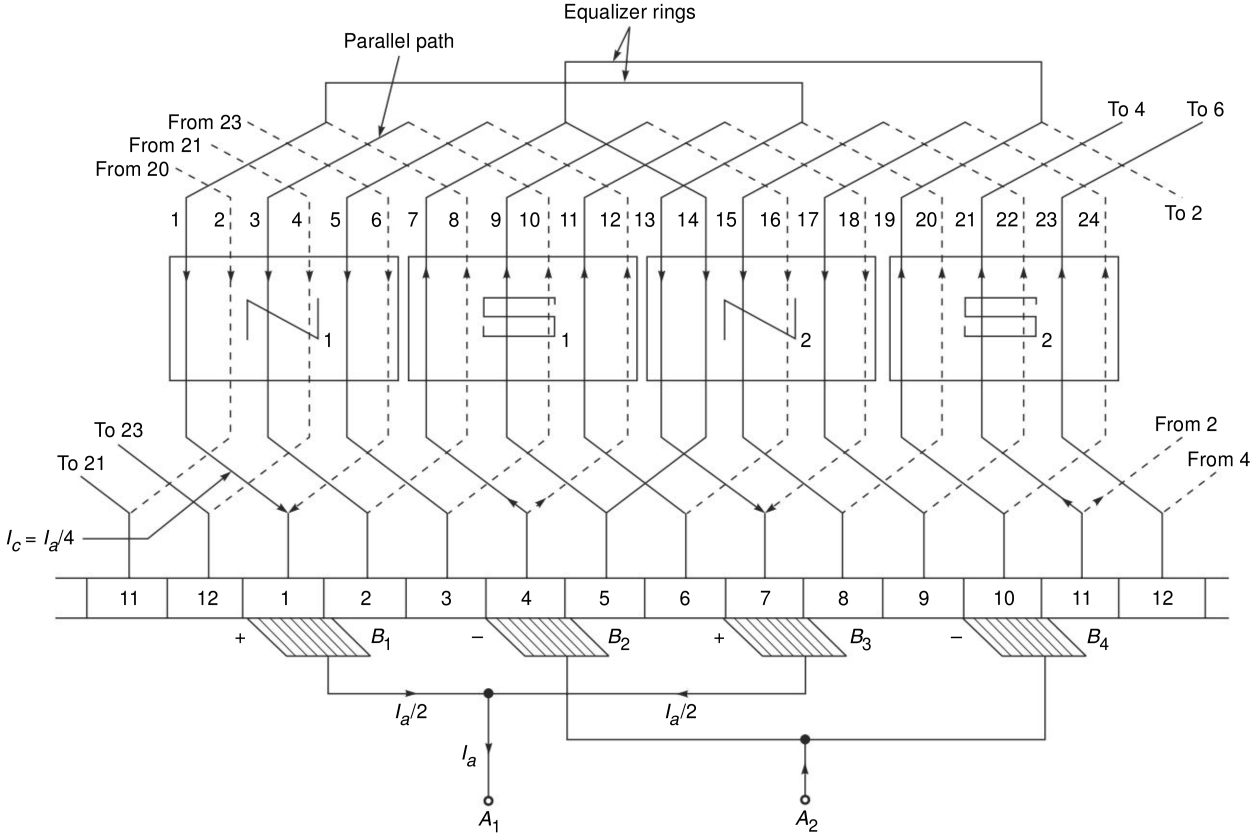


Fig. 10.3 4-pole lap-winding, 12 armature slots; single-turn coils and 2 coil-sides/slot

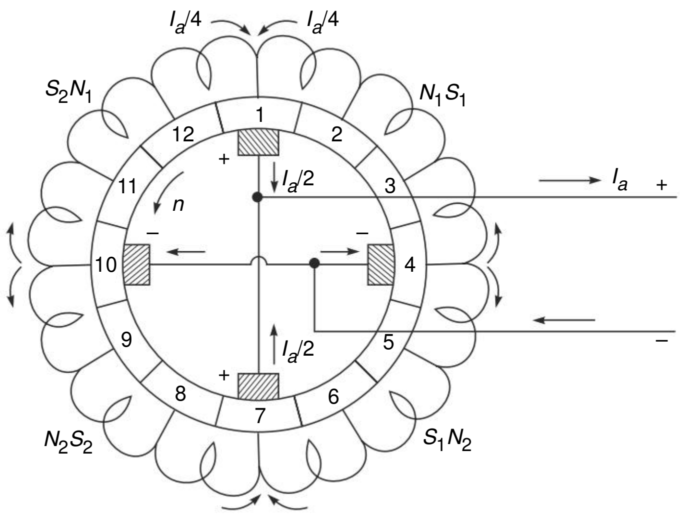


Fig. 10.4 Ring diagram of lap winding of Fig. 10.1

connections and the pole pairs under which the various coil sets (three coils each) lie are also indicated therein. It is seen that the winding is a closed one. As the parallel path emfs are equal and the way they are connected, the sum of emfs is zero and so current would flow in the absence of the load current.

Equalizer rings Irrespective of the statement made above, the sum of emfs may not be exactly zero because of the slight differences in flux/pole of all poles. As the closed path resistance is very small, a large current might flow in the windings. This is prevented by providing equaliser rings shown in diagram of Fig. 10.3. These rings are placed on the other side of the armature connecting the points of equipontential. Any derivation of the potential difference is eliminated as the current flows in the rings of negligible resistance but no current flows in closed armature windings.

To summarize for *lap winding*:

$$\text{No. of parallel paths, } A = P \tag{10.1a}$$

$$\text{No. of brushes } A = P \tag{10.1b}$$

$$\text{Conductor current } I_c = I_a/A \tag{10.1c}$$

where I_a is the armature current (See Figs. 10.3 and 10.4).

In a *wave winding,* the coil connections instead of lapping backward move forward from a coil under one pole pair to a coil under the next pole pair and so forth. The wave winding diagram is not under the scope of this but the conclusions are as under.

$$\text{No. of parallel paths } A = 2 \text{ (independent of number of poles)} \tag{10.2a}$$

$$\text{No. of brushes } = 2 \text{ (minimum needed but } P \text{ brushes are used in practice)} \tag{10.2b}$$

$$\text{Conductor current } I_c = I_a/A = I_a/2 \tag{10.2c}$$

The interested reader may consult reference.

Some further observations can be made from Fig. 10.3. The brushes are located mechanically opposite the main pole centres, i.e. in the magnetic axes of the poles, but because of the diamond shape of armature coils, these get connected to coil sides lying in or close to the interpolar region (magnetic neutral axis). Further, as a coil moves out of the influence of a given pole pair, the current in this coil must reverse. This process of current reversal, which is essential to a dc machine operation is known as *commutation*.

The reversal of current is opposed by the static coil emf ($L\,di/dt$) and therefore must be aided in some fashion for smooth current reversal, which otherwise would result in sparking at the brushes. The aiding emf is dynamically induced into the coils undergoing commutation by means of *compoles* or *interpoles* which are series excited by the armature current. These are located in the interpolar region of the main poles and therefore influence the armature coils only when these undergo commutation will be dealt at length in section 10.6.

Figures 10.5 (a) and (b) give the longitudinal and cross-sectional views of a dc machine, wherein the interpoles are clearly indicated as distinct from the main poles. Also observe in Fig. 10.5(b) the construction of the commutator segments and how these are connected to the armature coils and the brushes.

10.4 EMF AND TORQUE

EMF Equation

Full-pitch armature coils are assumed. Let

$$\Phi = \text{flux/pole}$$

Imagine the coil in Fig. 10.2(a) to lie in the interpolar region linking all the flux of one pole. Thus its flux linkages are

$$\lambda_1 = N_c\Phi$$

where N_c is the number of coil turns. As the coil moves through one pole pitch, its flux linkages change to

$$\lambda_2 = -N_c\,\Phi \text{ (it now links the flux of opposite polarity)}$$

During this movement, change of flux linkages of the coil is

$$\Delta\lambda = -2N_c\Phi \tag{10.3}$$

For a P-pole machine, the time of travel through one pole pitch is

$$\Delta t = \frac{2\pi}{P\omega_m}\ s \tag{10.4}$$

where ω_m is the armature speed in mechanical rad/s. Hence the average coil emf induced is

$$E_c = -\frac{\Delta\lambda}{\Delta t} = \Phi\omega_m N_c P \tag{10.5}$$

Let $\quad C_p$ = coils/parallel path

Then the armature emf is

$$E_a = \frac{\Phi\omega_m(C_pN_c)P}{\pi}$$

But $$C_p N_c = \frac{Z}{2A} = \text{turns/parallel path}$$

where Z is the total number of armature conductors. Hence

$$E_a = \frac{\Phi \omega_m Z}{2\pi}\left(\frac{P}{A}\right) = K_a\, \Phi\, \omega_m \tag{10.6}$$

where $$K_a = \frac{ZP}{2\pi A} = \text{machine constant}$$

Equation (10.6) can also be written as

$$E_a = \frac{\Phi n Z}{60}\left(\frac{P}{A}\right) = \frac{2\pi}{60} K_a \Phi_n \tag{10.7}$$

where n is the armature speed in rpm.

Torque Equation

Average force on an armature conductor is

$$f_{\text{av}} = B_{\text{av}}\, l\, I_c\, r \tag{10.8}$$

where l is the active conductor length, and I_c is the conductor current. Average torque on armature caused by one conductor is

$$t_{\text{av}} = B_{\text{av}}\, l\, I_c\, r \tag{10.9}$$

where r is the mean air-gap radius. Total torque developed by armature conductors is

$$T = B_{\text{av}}\, I_c\, l\, Z\, r \tag{10.10}$$

Now

$$B_{\text{av}} = \frac{\Phi P}{2\pi r l} \tag{10.11}$$

Therefore

$$T = \frac{1}{2\pi}\, \Phi I_c Z P$$

$$= \frac{1}{2\pi}\, \Phi I_a Z\left(\frac{P}{A}\right) \text{ Nm} \tag{10.12}$$

$$= K_a\, \Phi I_a \text{ Nm} \tag{10.13}$$

Notice that K_a is the same constant in the emf Eq. (10.6) and torque Eq. (10.13).

Power Balance

Mechanical power

$$T\omega_m = K_a\, \Phi\, \omega_m I_a$$

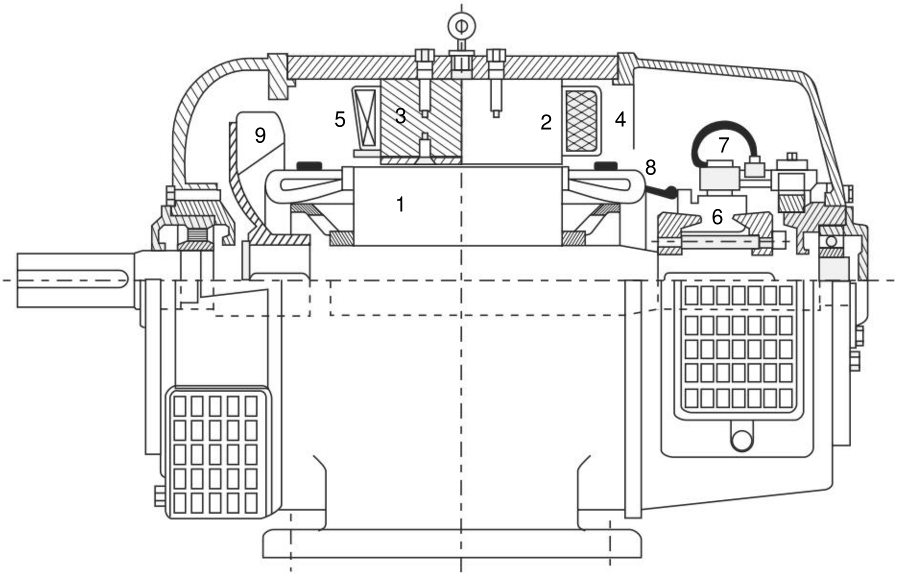
$$= E_a I_a \text{ W} \tag{10.14}$$

Observe that $E_a I_a$ is the electrical power converted to mechanical form and vice versa. From Eq. (10.14)

$$T = \frac{E_a I_a}{\omega_m} \text{ Nm} \tag{10.15}$$

The magnetic circuit of the machine is assumed linear because of the presence of the air-gap whose reluctance is more that the reluctance of the iron path (see Example 8.1) under this assumption

$$\Phi = K_f I_f \tag{10.16}$$



1. Armature core
2. Main field pole
3. Interpole
4. Main pole winding
5. Interpole winding
6. Commutator
7. Brush and brush holder
8. Armature winding overhang
9. Fan

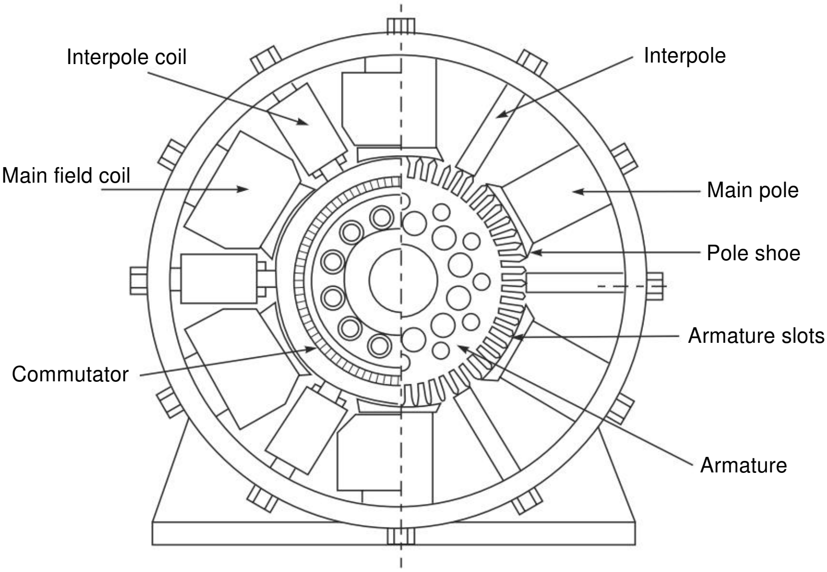


Fig. 10.5 (a) Longitudinal sectional view of a dc machine
(b) Cross-sectional view of a dc machine

where I_f is the field current. Substituting in Eqs. (11.6) and (11.13)

$$E_a = K_a K_f I_f \omega_m = K'_a K_f \omega_m \qquad (10.17)$$

and

$$T = K_a K_f I_f I_a = K'_a I_f I_a \qquad (10.18)$$

where

$$K'_a = K_a K_f$$

Example 10.1 A 6-pole dc machine armature has 36 slots 2 coil-sides/slot, 8 turns/coil and is wave wound. The pole shoe is 18 cm long and the mean air-gap diameter is 25 cm. The average flux density over one pole pitch is 0.8 T. Find the gross torque and mechanical power output when the machine is operating as a motor at 1200 rpm with an armature input current of 10 A.

Solution

$$Z = 36 \times 2 \times 8 = 576$$

$A = 2$ wave winding

$$\Phi = \frac{\pi \times 0.25}{6} \times 0.18 \times 0.8 = 0.0188 \text{ Wb}$$

$$\text{Induced emf } E_a = \frac{\Phi n Z}{60}\left(\frac{P}{A}\right)$$

$$= \frac{0.0188 \quad 1200 \quad 576}{60} \times \left(\frac{6}{2}\right)$$

$$= 650 \text{ V}$$

$$\text{Gross mechanical power developed} = E_a I_a$$

$$= \frac{650 \times 10}{1000} = 6.5 \text{ kW}$$

$$\text{Torque developed} = \frac{6.5 \times 1000}{\left(\frac{2\pi \times 1200}{60}\right)} = 51.72 \text{ Nm}$$

Example 10.2 A 4-pole dc machine has an armature radius of 14.5 cm and active length 21 cm. The pole are is 70% of pole pitch. Average flux density under poles 0.8 T. The armature has 33 slots, 33 coils with a turns / coil. Determine

(a) the armature constant K_a,

(b) the armature induced emf E_a,

(c) the conductor current when the armature carries a current of 240 A, and

(d) the torque and mechanical power developed at the armature current as in part (c).

Solution

One pole of the machine is sketched in the figure.

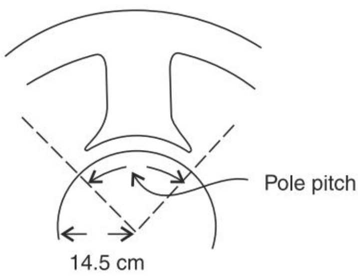


Fig. 10.6

(a) Machine constant

$$K_a = \frac{ZP}{2\pi \times A}$$

$$P = 4,\ A = 4,\ Z = 2 \times 33 \times 11 = 726$$

$$K = \frac{726 \times 4}{2\pi \times 4} = 115.55$$

(b) A_p (pole area) = $\left(\frac{2\pi \times 0.145}{4}\right) \times 0.7 \times 0.21 = 33.5 \times 10^{-2}\ \text{m}^2$

B_{arc} (under pole, assumed zero in inter-polar region) = 0.8 T

Φ (flux/pole) = $33.5 \times 10^{-3} \times 0.8 = 26.8$ mWb

$$E_a = K_z\, \Phi\, \omega_m \quad ; \quad n = 1200 \text{ rpm}$$

$$= 115.55 \times 26.8 \times 10^{-3} \times \left(\frac{2\pi \times 1200}{60}\right) = 389 \text{ V}$$

(c) I_c (conductor current)= $\dfrac{I_a}{A} = \dfrac{240}{4} = 60$ A

(d) T (developed) = $K_a\, \Phi\, I_a = 11.55 \times 26.8 \times 10^{-3} \times 240 = 743$ Nm

P_G (developed) = $E_a\, I_a = 389 \times 240 \times 10^{-3} = 93.86$ kW

10.5 CIRCUIT MODEL

The circuit model of a dc machine is given in Fig. 10.6. The armature circuit has induced emf E_a and a series resistance of R_a (= R_p/A, where R_p is the resistance of a parallel path). The brush voltage drop is constant of the order of 2 V, which is either ignored or its effect included in R_a by linearizing it. The field has a resistance R_f fed with field current I_f (= V_f/R_f) and is shown at 90° to the brush axis (this angle is 90° electrical in an actual machine; Fig. 10.1). The field provides the per pole flux which induces the emf E_a in the armature when it runs at speed n. The power converted (mechanical to electrical or vice versa) is E_aI_a.

Generating Mode

I_a (armature current) flows in the direction of E_a as shown in Fig. 10.7. For the armature circuit

V (terminal voltage) $= E_a - I_aR_a;\ E_a > V$ (10.19)

P_{mech} (in net) $= E_aI_a$ (mechanical power converted to electrical form) (10.20a)

P_{mech} (in gross) $= E_aI_a$ + rotational losses (10.20b)

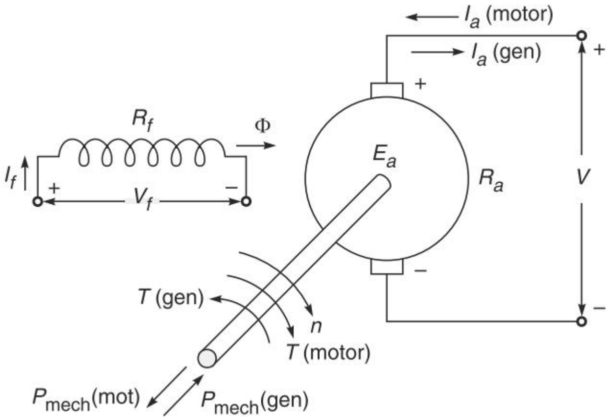


Fig. 10.7 Circuit model of dc machine

where rotational losses = mechanical loss + core loss

$$P_0 \text{ (electrical output)} = E_a I_a - I_a^2 R_a \text{ (armature copper loss)} \quad (10.20c)$$

Motoring Mode

I_a flows in the opposite direction to E_a as shown in Fig. 10.7. E_a in a motor is therefore known as *back emf*. For the armature circuit

$$V \text{ (terminal voltage)} = E_a + I_a R_a; \quad V > E_a \quad (10.21)$$

$$P_i \text{ (electrical input)} = VI_a \quad (10.22a)$$

$$E_a I_a \text{ (electrical power converted to mechanical form)}$$

$$= VI_a - I_a^2 R_a \text{ (armature copper loss)} \quad (10.22b)$$

$$P_{\text{mech}} \text{ (out gross)} = E_a I_a \quad (10.23a)$$

or $$P_{\text{mech}} \text{ (out net)} = E_a I_a - \text{rotational losses} \quad (10.23b)$$

DC Machine Ratings

Generator Output in kW, terminal voltage and prime mover speed.

Motor Output in kW, terminal voltage and speed at full-load.

Example 10.3 A 215 V dc machine has an armature resistance of 0.4 Ω. It is supplying 5 kW as a generator when run at 1000 rpm and is excited to give a terminal voltage of 215 V. At what speed would it run as a motor if it is fed at the same terminal voltage, draws the same armature current but the flux/pole is increased by 10%?

Solution As generator

$$I_a = \frac{5 \times 1000}{215} = 23.26 \text{ A}$$

$$E_{ag} = 215 + 0.4 \times 23.26 = 224.3 \text{ V}$$

or $$224.3 = K_a \Phi_g \times \left(\frac{2\pi \times 1000}{60}\right) \quad \text{(i)}$$

As motor $$E_{am} = 215 - 0.4 \times 23.26 = 205.7 \text{ V}$$

or $$205.7 = K_a \, \Phi_m \times \left(\frac{2\pi n}{60}\right) \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i)

$$\left(\frac{\Phi_m}{\Phi_g}\right) \left(\frac{n}{1000}\right) = \frac{205.7}{224.3}$$

$$n \text{ (motor)} = \frac{205.7}{224.3} \times \frac{1}{1.1} \times 1000$$

$$= 834 \text{ rpm}$$

Example 10.4 A 200 kW, 400 V, separately excited dc motor runs at 600 rpm. It has 864 lap-connected conductors. The full load armature copper loss is 8 kW. Calculate the useful flux/pole.

Solution Figure 10.8 shows the connection diagram of the motor. It is assumed that 200 kW is the armature input at full load.

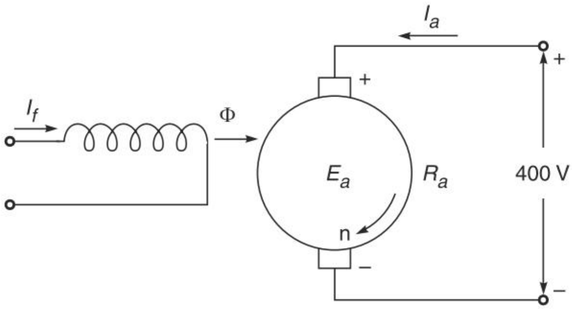


Fig. 10.8 Circuit model of dc machine

$$I_a = \frac{200\times1000}{400} = 500 \text{ A}$$

Full load copper loss $I_a^2 R_a = 8$ kW

$$R_a = \frac{8\times1000}{(500)^2} = 0.032\ \Omega$$

$$E_a = V - I_a R_a$$

$$= 400 - 500 \times 0.032 = 384 \text{ V}$$

But $E_a = ;\ \dfrac{\Phi n Z}{60}\left(\dfrac{P}{A}\right);\ \dfrac{P}{A} = 1$ for lap-connected armature

or $$384 = \frac{\Phi\times600\times864}{60}$$

or $$\Phi = 0.044 \text{ Wb}$$

10.6 ARMATURE REACTION

When the dc machine armature carries current, it causes its own mmf distribution known as armature reaction. Figure 10.9 shows the cross-sectional view of a 2-pole machine. All the conductors under the north pole carry current in one direction and those under the south pole in the opposite direction. As the armature rotates, this pattern of current distribution remains fixed in space (i.e. stationary wrt the main poles as is necessary for torque production (Sec. 9.7)). In Fig. 10.9 the conductors 1, 1′, 2, 2′, etc. form a coil with peak ampere-turns AT_a, whose axis is along the brush axis (q-axis) or at 90° elect to the main pole axis (d-axis) independent of the armature rotation. As many conductors move out of the influence of one pole and the same number moves in. Therefore the conductor current pattern is fixed in space and so the AT_a axis is stationary along the q-axis and the direction of AT_a is also fixed. Such an armature reaction is known as *cross-magnetizing*. AT_a at 90° to AT_f corresponds to λ = 90° in Eq. (9.33), which is the best condition for torque production.

It is seen from Fig. 10.9 that armature reaction AT opposes the main pole AT at one pole tip and strengthens it at the other pole tip (this is the cross-magnetizing effect). The flux density wave in the air-gap therefore gets distorted from the trapezoidal shape at no-load, $I_a = 0$ (Fig. 10.2), such that the flux density increases in one half of the main poles and decreases in the other half as shown in Fig. 10.10.

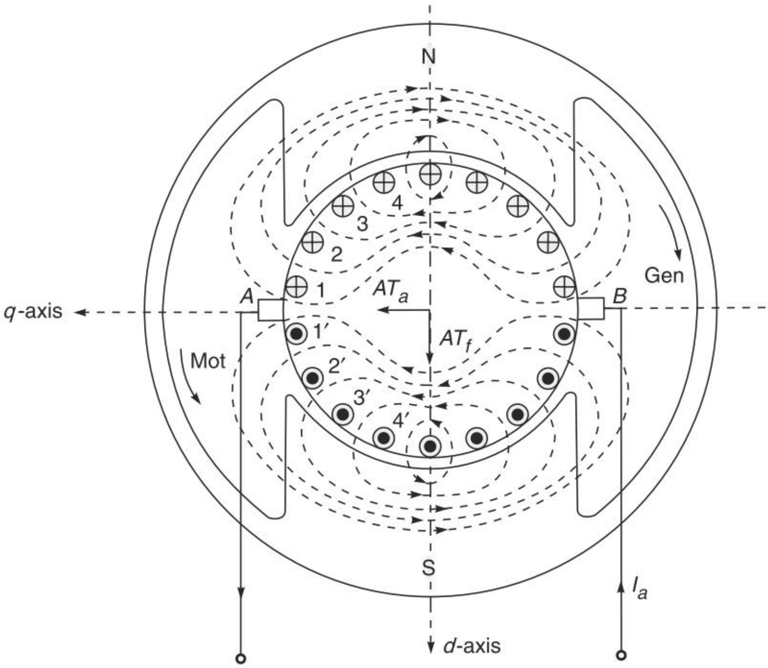


Fig. 10.9 Armature reaction in a dc machine

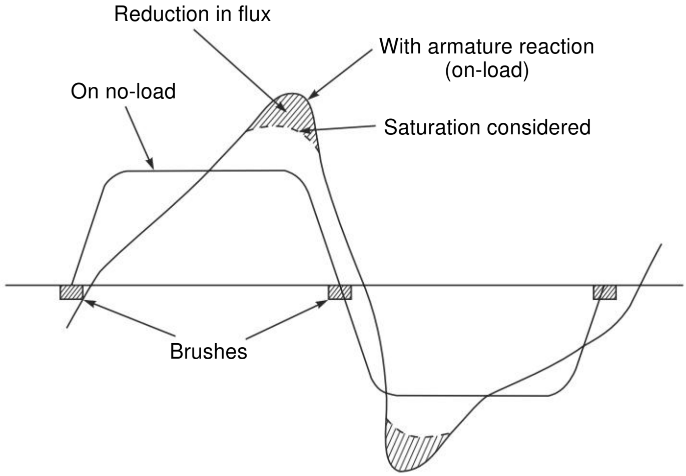


Fig. 10.10 Distortion of *B*-wave in dc machine air-gap

The decrease in flux in one half of the pole is balanced by an equal increase in the other half so long as the magnetic material of the poles is in an unsaturated state. Thus in the linear region of magnetization, the flux/pole remains unaffected by armature reaction even though the *B*-wave gets distorted. In the saturation region of magnetization the increase in flux in one half of the pole is less than the decrease in the other resulting in net reduction in flux/pole.

Compensating Winding

Armature reaction varies with the armature current. In case of a sudden change in motor load, the armature reaction flux Φ_a changes at a sharp rate inducing large statically induced emfs in armature coils which appear across commutator segments causing these to flash over resulting in complete short circuit of the machine commutator. Hence AT_a must be compensated by compensating winding placed in slots cut out in main pole shoes with windings axis along the q-axis. These windings have a few turns connected in series with armature such that these are excited by I_a. This is illustrated in Fig. 10.11.

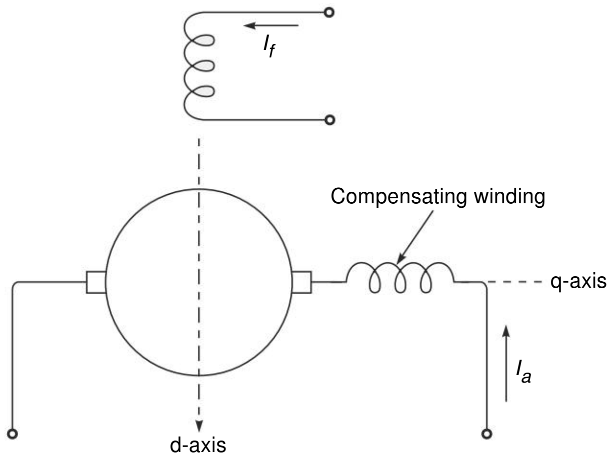


Fig. 10.11 Connections of compensating winding

10.7 COMMUTATION

When an armature coil moves under the influence of one pole-pair, it carries constant current in one direction. As the coil moves into the influence of the next pole-pair, the current in it must reverse (see Figs. 10.4 and 10.5). This reversal of current in a coil is called *commutation*. Several coils (equal to number of poles in a lap winding) undergo commutation simultaneously. During the process of commutation, the coil-sides of the coil lie in the interpolar region and the coil is shorted by the brush (this can be easily observed from Fig. 10.4). The change in current is opposed by the *reactance* emf induced in the coil because of its leakage inductance (Lenz's law). As a result, the coil current does not reverse fully at the end of the commutation period when the coil has moved into the influence of the next pole-pair. The balance current then sparks across the brush. Continuous heavy sparking at the brushes damages the commutator severely reducing, its life span, apart from causing intense radio disturbance in the neighbouring region.

To aid the process of commutation, speed emf (dynamically induced) is injected in the commutating coil to oppose the reactance emf. This speed emf must obviously be in the direction in which the current will flow in the coil after commutation. The speed emf is obtained by placing narrow *series-excited* poles in the interpolar region

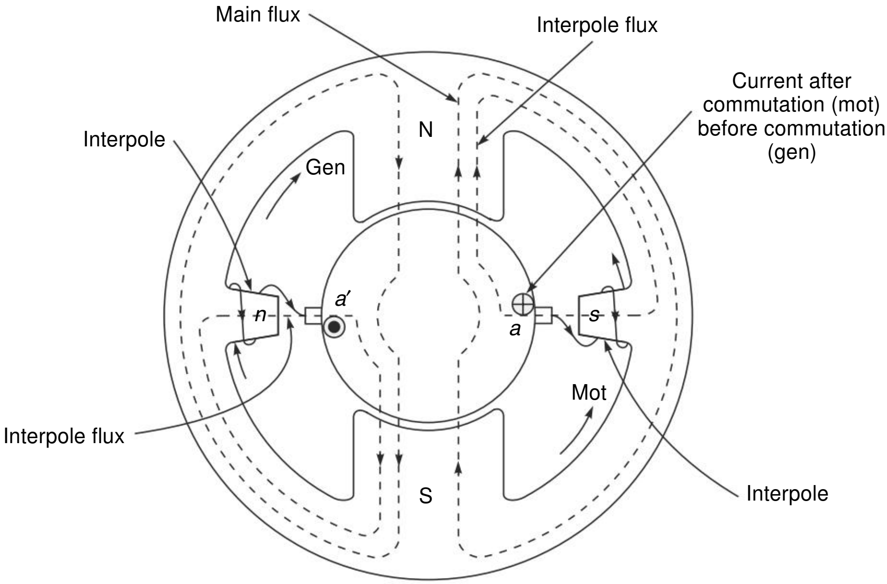


Fig. 10.12 (a) Interpoles in a dc machine

as shown in Fig. 10.12 (a) so as to influence only the commutating coils. These poles are known as *interpoles or compoles*. In Fig. 10.12 (a) the reader should verify (by right-hand rule) that the direction of speed emf induced in the coil-sides of the commutating coils is in the appropriate direction with the polarity of interpoles indicated in the figure.

10.8 METHODS OF EXCITATION AND MAGNETIZATION CHARACTERISTICS

The field of a dc machine is excited by either of the following two methods or of a combination of these.

Voltage Excitation The field winding has a large number of turns of thin wire and is excited from a voltage source (self or separate). The field resistance is therefore high and the field carries a small current. Such a field winding is known as *shunt field winding* illustrated in Fig. 10.12(b).

Current Excitation The field is excited here by a few turns of thick wire (low resistance) connected in series with the armature. This is the *series field wind-*

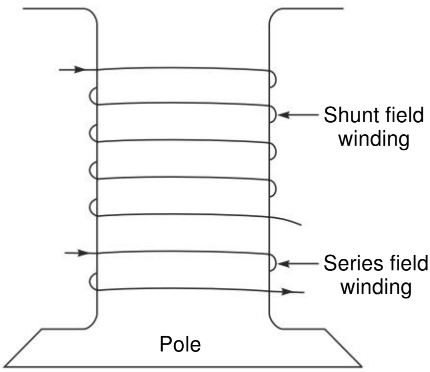


Fig. 10. 12 (b) Shunt and series field windings

ing also shown in Fig. 10.12 (b) As it carries the armature current, the excitation varies with load.

A dc generator can be excited in the following two modes:

– Separate excitation (requiring a separate dc voltage source), and

– Self-excitation (excited from its own voltage/current).

Excitation Methods

DC motors field windings can be excited in several ways

- ***Separately Excited*** The shunt field winding is excited from an independent voltage souce as in Fig. 10.13(a).

- ***Shunt Excitation*** The shunt field winding is connected across the armature as in Fig. 10.13(b).

- ***Series Excitation*** The series field winding is connected in series with the armature as in Fig. 10.13(c).

- ***Compound Excitation*** Both shunt and series field windings are connected is shunt across the armature and in series with the armature respectively as in Fig. 10.13(d).

The compund excitation could be *cummulative* with series field aiding the shunt field or *differential* with series field opposing the shunt field. The differential compound excitation is rarely used.

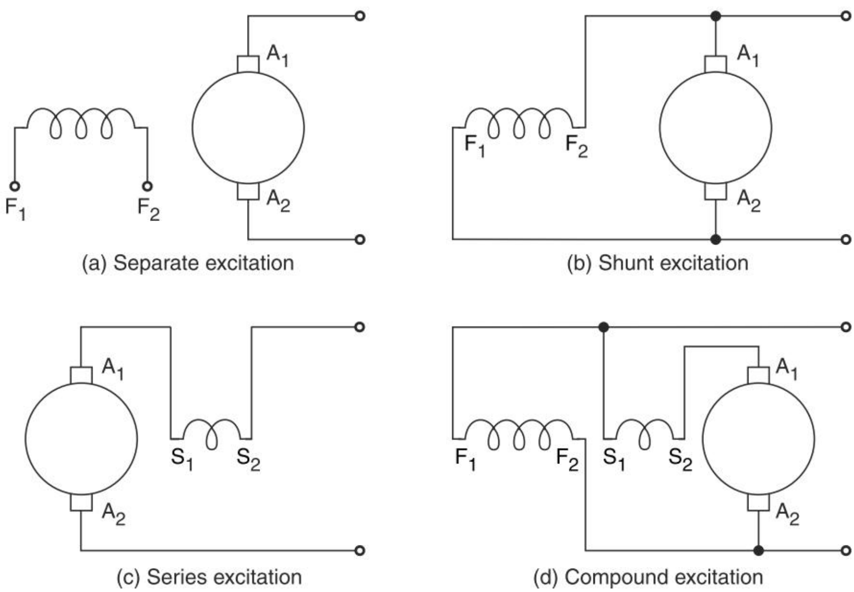


Fig. 10.13 Excitation of dc machine

Magnetization Characteristic

Φ– I_f or E_a–I_f (since $E_a \propto \Phi$) for a given machine speed is the magnetization characteristic of the machine. It can be obtained by separately exciting the machine, as in Fig. 10.14(a), which machine run as a generator by a prime mover at a constant speed (rated speed) with open-circuited armature. Under this condition of operation, $E_a = V_{OC}$. That is why this is also called open-circuited characteristic (OCC). The field current I_f is varied by means of a regulating resistance in series with the field. The readings of V_{oc} and I_f recorded for plotting OCC. Note that R_f is the total resistance in the field circuit. A typical characteristic is exhibited in Fig. 10.14(b). In the initial linear part of the characteristic, the air-gap effect predominates and the dotted tangential line is called the *air-gap line*. At $I_f = 0$ the residual flux in the magnetic circuit causes a small induced voltage, viz. the *residual voltage*. At higher values of I_f, saturation effect in iron of the machine begins to show up. For economic reasons, the machine is normally operated in a slightly saturated state. As the machine speed is changed, every point on the magnetization curve translates proportionally up or down as illustrated.

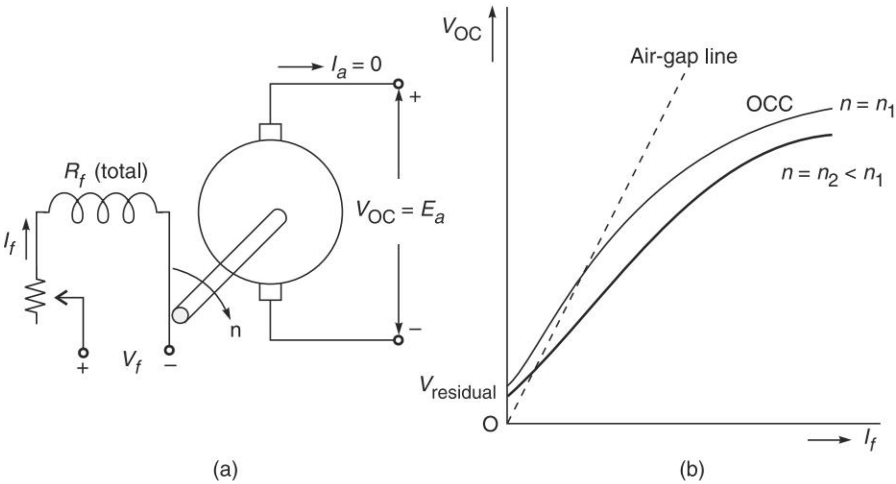


Fig. 10.14 Open-circuit characteristic (OCC) of dc machine

Self-Excited Generator (Shunt Generator)

In a shunt generator, the field is connected across the generator terminals with the generator running at fixed speed (rated speed) as shown in Fig. 10.15(a). At no-load ($I_L = 0$)

$$I_a = I_f \text{ (very small)} \tag{10.24}$$

$$\therefore V \text{ (terminal voltage)} = E_a - I_f R_a \approx E_a \tag{10.25}$$

Thus V–I_f is the magnetization curve. For the field circuit

$$V = I_f R_f \;(R_f\text{- line}) \tag{10.26}$$

The no-load voltage (V_0) is thus given by the intersection of the magnetization characteristic and the R_f-line as shown in Fig. 10.15(b).

The value of R_f when the R_f-line is tangential to the magnetization characteristic (same as the air-gap line) is called the *critical resistance*, R_{fc}. For field resistance more than this value, the no-load voltage of the shunt generator would be very small (close to residual value), which means that the generator fails to excite. For a given R_f, if the machine speed is reduced, the critical operation again visits. Such a speed (for a given R_f) is called the *critical speed*. It also easily follows that for a given speed, the no-load voltage (V_0) can be adjusted by variation of field resistance R_f by means of an external series resistance called the *regulating resistance*.

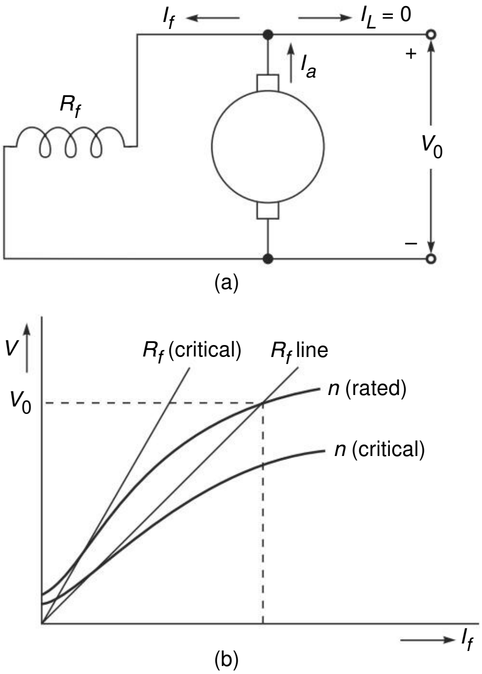


Fig. 10.15 (a) Shunt generator on no-load (b) No-load voltage of shunt generator

Example 10.5 The OCC data of a dc generator at 1800 rpm is as below:

V_{OC}(V)	8	40	74	113	152	213	234	248	266	278
I_f (A)	0	0.5	1.0	1.5	2.0	3.0	3.5	4.0	5.0	6.0

The field of the generator is shunt connected.

Find

(a) the field resistance and the field current for a no-load voltage of 250 V,

(b) the value of the critical field resistance, and

(c) the value of the critical speed.

(d) what external resistance must be added in the field to reduce the terminal voltage to 220 V?

Solution The OCC is drawn in Fig. 10.16.

(a) Draw a line from the 250 V point on the OCC to origin. Reading from the figure,

$$I_f = 4.1 \text{ A}, \ R_f = \frac{250}{4.1} = 61 \ \Omega$$

(b) R_f (critical) is found by drawing a line from the origin such that is parallel to the nearly straight line part of OCC. R_f(critical) is the slope of this line which can be found by reading the co-ordinates of any point on the line. We choose the point P (150 V, 2 A).Therefore

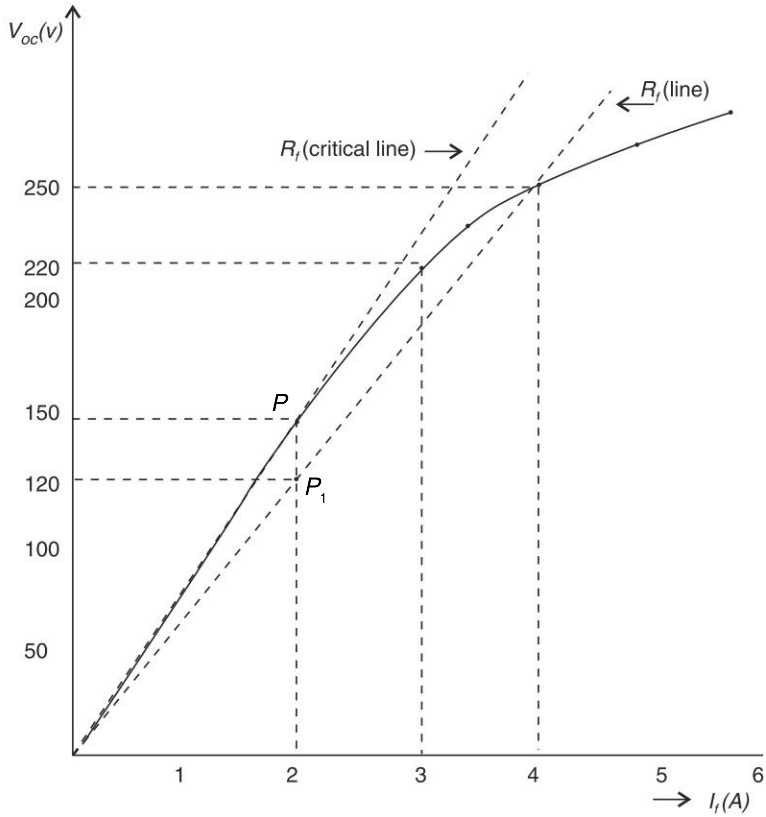


Fig. 10.16

$$R_f(\text{critical}) = \frac{150}{2} = 75\ \Omega$$

(c) To find the critical speed for $R_f = 61\,\Omega$, we need to shift the OCC downwards till $R_f(61\ \Omega)$ line is tangential to it. There is no need to draw the full characteristic. We draw a vertical line from P to locate P_1 on the $R_f\,(61\ \Omega)$ line. This is tangent point of downward shifted OCC. The speed change corresponds to voltage values of P and P_1 .

The voltage corresponding to P_1 is 120 V_3

Therefore,

$$N_{\text{critical}} = 1800 \times \frac{120}{150} = 1440 \text{ rpm}$$

(d) From the figure,

$$R_f\,(V_0 = 220) = \frac{220}{3.2} = 68.8\ \Omega$$

External resistance to be added = 68.8 – 61 = 7.8 Ω

The dc generator will not be discussed any further as these have been superseded by controlled rectifiers. However, the interested reader study solved problems 10.24 and 10.25.

10.9 CHARACTERISTICS OF DC MOTORS AND SPEED CONTROL

The emf and torque equations of a dc machine are reproduced below:

$$E_a = = \frac{\Phi n Z}{60}\left(\frac{P}{A}\right) = K_E\,\Phi n \tag{10.27}$$

$$T = \frac{1}{2\pi}\Phi I_a Z\left(\frac{P}{A}\right) = K_T \Phi I_a \tag{10.28}$$

Equation (10.27) can be written in the form

$$n = K_N\,\frac{E_a}{\Phi};\quad K_N = \frac{1}{K_E} \tag{10.29}$$

Of course K_N and K_T are related ($K_N K_T = 60/2\pi$).

Shunt Motor

Figure (10.17) shows the connections of a dc shunt motor run from a source of voltage V. The per pole flux Φ is governed by the field current I_f by means of the regulating resistance R_r. For the armature circuit

$$E_a = V - I_a R_a \tag{10.30}$$

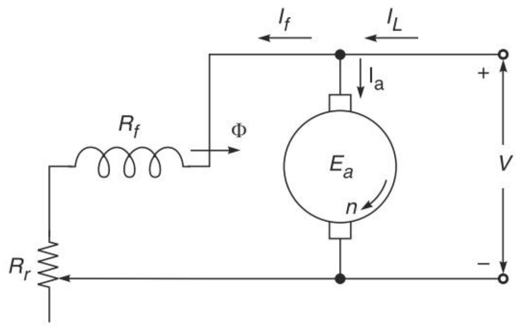


Fig. 10.17 DC shunt motor

Substituting in Eq. (10.29)

$$n = K_N\left(\frac{V - I_a R_a}{\Phi}\right) \tag{10.31}$$

Substituting I_a from the torque Eq. (10.28) in Eq. (11.31), the motor speed can be expressed as

$$n = \frac{K_N V}{\Phi} - \left(\frac{K_N R_a}{K_T \Phi^2}\right)T \tag{10.32}$$

At no load* ($T = 0$)

$$n = n_0 = \frac{K_N V}{\Phi} \tag{10.33}$$

Hence

$$n = n_0 - \left(\frac{K_N R_a}{K_T \Phi^2}\right)T \tag{10.34}$$

For a given field current (Φ fixed), as per Eq. (10.34). the speed–torque characteristic of a shunt motor is linear and the speed drops from the no-load value as the load torque is increased. The speed drop is small (only a few per cent) as shown in Fig. 10.18 because R_a is small (0.01 pu or less); this is the *shunt characteristic*. The torque cannot be allowed to exceed a certain limit (full load or rated value) laid down by the permissible armature current (Eq. (10.28)). The actual motor speed–torque characteristic will exhibit a slight nonlinearity (bends upwards) because of reduction

* Under practical no-load condition the motor has to develop a small torque T_0 to overcome its own windage and friction torque.

in flux caused by armature reaction in presence of saturation (ignored in the above analysis).

Speed Control

As per Eq. (10.32), the speed of a shunt motor for a given torque load can be controlled by

- control of Φ by adjusting I_f, V constant. This is called *field control*;
- control of V, the armature voltage, I_f constant. This is called *armature control*; and
- control of R_a by an adjustable resistance included in the armature circuit. This is used only for small motors as it reduces the motor efficiency because of loss in the resistance added.

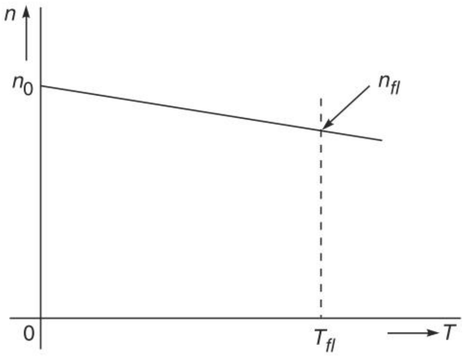


Fig. 10.18 Speed–torque characteristic of shunt motor (for a fixed current)

Field Control

The armature voltage is held constant while the field current and therefore the flux/pole is controlled by the regulating resistance R_r in the field circuit as shown in Fig. 10.17. The no-load speed increases inversely with Φ (or I_f) as per Eq. (10.33). The speed–torque characteristic at each no-load speed (given field current) is linear but its slope increases as Φ is reduced (Eq. (10.32)). Three characteristics at different no-load speeds are drawn in Fig. 10.19. As R_f is fully cut out, no-load speed lower than this value is not feasible.

As the field is weakened, the maximum torque to which the motor can be loaded for rated armature current reduces proportionally (Eq. (10.28)). This torque limit is indicated in dotted line in Fig. 10.19. This is *constant-kW* (*or hp*) *drive* wherein torque load must reduce as speed is increased.

Armature Control The field current I_f is held constant at maximum value (V/R_f).

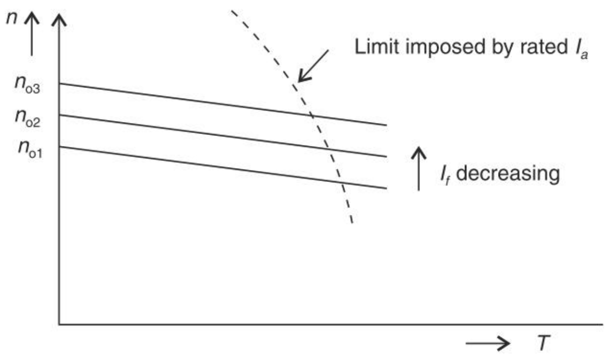


Fig. 10.19 Field control of shunt motor speed (armature voltage constant)

The no-load speed varies directly as the armature voltage as per Eq. (10.33). At a given armature voltage, the speed drops slightly as the load torque is increased. The speed-torque characteristics moves up or down parallel to itself as per Eqs. (10.33), and (10.34) as the armature voltage is varied, shown in Fig. 10.20. At rated current, the torque capability is T_{fl} at any armature voltage. Thus armature control is *constant-torque drive*.

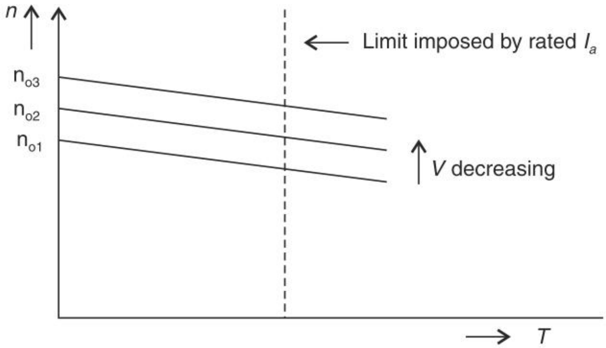


Fig. 10.20 Armature control of shunt motor speed (field current constant)

Aramature and Field Control By varying armature voltage with maximum field current, the motor is increased with constant-torque capability. Having reached the rated voltage, the speed can increase further by reducing the field current but with decreasing torque capability i.e. constant kW drive. In fact, with this arrangement the range of speed control can be made as wide as 1:6.

The armature voltage control requires a variable source with full rated current capability; high power source to match the motor rating. The field control requires an independent voltage source but with small current capability, low power source.

Ward-Leonard speed control This is the scheme which provides the arrangement of armature and field control. An earlier used scheme is presented in Fig. 10.21. It employs an ac motor—dc motor generator set matching the dc motor rating for

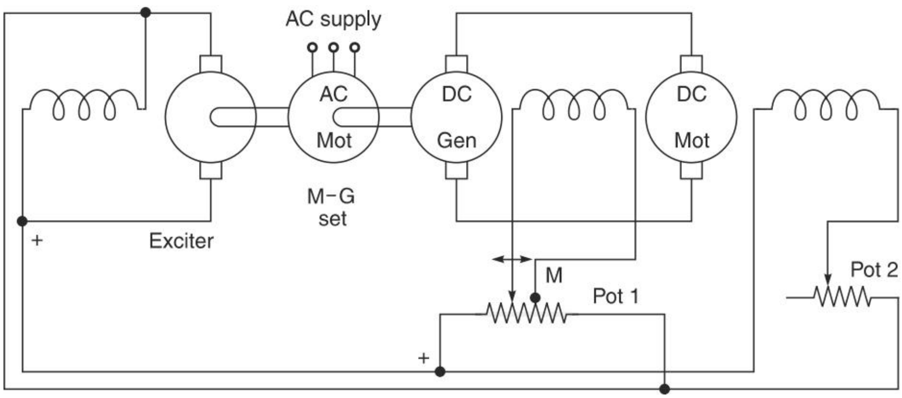


Fig. 10.21 Ward–Leonard speed control system

feeding variable voltage current to the motor. A small dc generator called the exciter is coupled to the motor-generator feeds field current to the dc generator field and dc motor field. As the field requires low power, the two field currents are controlled by potentioneters. The dc generator field current provides control over armature voltage.

With the availability of electronic power devices the M-G set has been superseded by high power Silicon Controlled Rectifier (SCR) and the exciter by a fixed voltage rectifier. The modern scheme is drawn in Fig. 10.22 (a). Its constant-torque and constant-kW regions are identified in Fig. 10.20 (b).

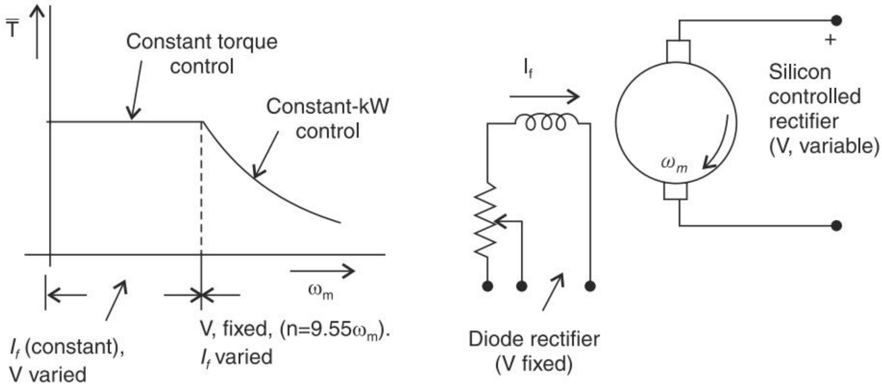


Fig. 10.22 Ward Leonard control of a dc shunt motor

Example 10.6 A 4-pole, 230 V dc shunt motor has 888 wave-connected conductors. It draws a field current of 0.6 A to give a no-load flux of 5.4 mWb. The armature resistance is 0.8 Ω. Calculate the motor speed at a no-load current of 2 A. What would be the motor current (line) and speed when it develops a torque of 29.6 Nm? What is the speed regulation from no load to this torque?

Solution Refer Fig. 10.17, At no load

$$I_L = 2 \text{ A}$$

$$I_f = 0.6 \text{ A}$$

$$\therefore \quad I_a = 2 - 0.6 = 1.4 \text{ A}$$

$$E_a = 230 - 1.4 \times 0.8 = 229 \text{ V}$$

But
$$E_a = \frac{\Phi n Z}{60}\left(\frac{P}{A}\right) \tag{i}$$

or
$$229 = \frac{5.4 \times 10^{-3} \times n \times 888}{60} \times \left(\frac{4}{2}\right)$$

or
$$n = n_0 = 1430 \text{ rpm}$$

At load

Torque developed $T = 29.6$ Nm

$$T = \frac{1}{2\pi} \Phi I_a Z\left(\frac{P}{A}\right) \tag{ii}$$

Substituting values

or
$$29.6 = \frac{1}{2\pi} \times 5.4 \times 10^{-3} \times I_a \times 888 \times \left(\frac{4}{2}\right)$$

or $$I_a = 19.4 \text{ A}$$

$$I_L = 19.4 + 0.6 = 20 \text{ A}$$

$$E_a = 230 - 0.8 \times 19.4 = 214.5 \text{ V}$$

Substituting values

$$214.5 = \frac{5.4 \times 10^{-3} \times n \times 888}{60} \times \left(\frac{4}{2}\right)$$

or $$n = 1340 \text{ rpm}$$

$$\text{Speed regulation} = \frac{1430 - 1340}{1430} \times 100 = 6.3\%$$

Example 10.7 A 230 V dc shunt motor has an armature resistance of 0.1 Ω and a shunt field resistance of 275 Ω. It runs at speed of 1000 rpm when drawing an armature current of 75 A. Calculate the additional resistance to be inserted in the field circuit to raise the motor speed to 1200 rpm at an armature current of 125 A. Assume linear magnetization characteristic.

Solution At

$$n_1 = 1000 \text{ rpm}$$

$$E_{a1} = 230 - 75 \times 0.1 = 222.5 \text{ V}$$

$$I_f = \frac{230}{275} = 0.837 \text{ A}$$

Since the magnetization characteristic is linear, Eq. (11.27) can be written as

$$E_a = K'_E I_f n \qquad \text{(i)}$$

Substituting values

$$222.5 = K'_E \times 0.837 \times 1000 \qquad \text{(ii)}$$

At $$n_2 = 1200 \text{ rpm}$$

$$E_{a2} = 230 - 125 \times 0.1 = 217.5 \text{ V}$$

Substituting in Eq. (i)

$$217.5 = K'_E \times I_{f2} \times 1200 \qquad \text{(iii)}$$

Dividing Eq. (iii) by Eq. (ii)

$$\frac{1200 I_{f2}}{0.837 \times 1000} = \frac{271.5}{222.5}$$

or $$I_{f2} = 0.681 \text{A}$$

$$R_{f2} = \frac{230}{0.681} = 338 \ \Omega$$

$$R_{f,\text{ ext}} = 338 - 275 = 63 \ \Omega$$

Example 10.8 A 115 V dc shunt motor draws an armature current of 25 A when running at 1450 rpm at full load torque. Motor armature circuit resistance is 0.3 Ω. Calculate the resistance to be added in series with motor armature to reduce the speed to 1200 rpm at 3/4$^{\text{th}}$ full load torque. Calculate also the armature circuit efficiency. What would be the motor speed as half full load torque when the resistance as calculated above is included in the armature circuit?

Solution At full load torque, speed $n_1 = 1450$ rpm

$$I_{a1} = 25 \text{ A}$$

$$E_{a1} = 115 - 25 \times 0.3 = 107.5 \text{ V}$$

$$E_a = K_E \Phi n = K'_E n;\ \Phi = \text{constant, field is not changed} \quad \text{(i)}$$

$$T = K_T \Phi I = K'_T I_a \quad \text{(ii)}$$

or $$107.5 = K'_E \times 1450 \quad \text{(iii)}$$

$$T = K'_T \times 25 \quad \text{(iv)}$$

At 3/4th full load, $n_2 = 1200$ rpm (external resistance added in armature circuit (Fig. 10.23)

$$E_{a2} = K'_E \times 1200 \quad \text{(v)}$$

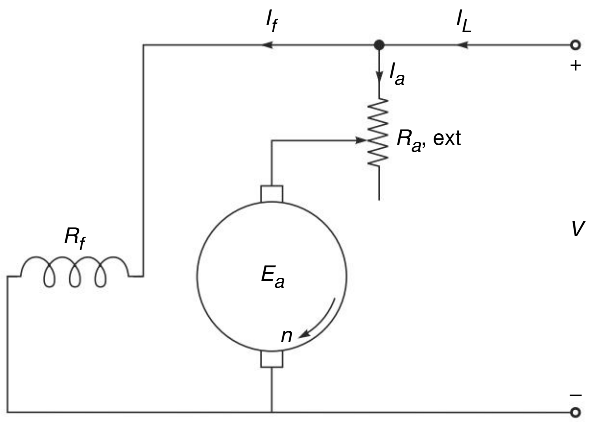


Fig. 10.23

From Eqs. (iii) and (v)

$$E_{a2} = 107.5 \times \frac{1200}{1450} = 89 \text{ V}$$

Also $$\frac{3}{4} T = K'_T I_{a2} \quad \text{(vi)}$$

From Eqs. (iv) and (vi)

$$I_{a2} = \frac{3}{4} \times 25 = 18.75 \text{ A}$$

From KVL equation of the armature circuit

$$89 = 115 - 18.75\,(0.3 + R_{a,\text{ ext}})$$

or $$R_{a,\text{ ext}} = 1.09\ \Omega$$

Armature circuit efficiency, $\eta_a = \dfrac{E_a I_a}{V I_a}$

$$= \frac{89}{115} \times 100 = 82.5\%$$

Thus 17.5% of the power fed to the armature circuit is dissipated in the external resistance (1.09 Ω) and the armature resistance (0.3 Ω).

If instead, armature circuit is fed from a rectifier with voltage

$$V = 89 + 18.75 \times 0.3 = 94.63 \text{ V}$$

Then armature efficiency would be

$$\eta_a = \frac{89}{94.63} \times 100 = 94\%$$

The method of adding series resistance is only employed for very small (fractional-kW) motors. It can be used for larger motors to reduce speed for short time periods only.

Example 10.9 An 8 kW, 230 V dc shunt motor has an armature resistance of 0.5 Ω. It runs at a speed of 1200 rpm on no load. At what speed would the motor run when delivering a gross mechanical power of 8 kW (i.e. inclusive of rotational loss)? Also calculate the armature current and the torque developed. Assume that the field current is maintained constant.

At what speed would the motor run when coupled to a centrifugal pump which requires a torque of

$$T_p = 0.6 \times 10^{-4}\, n^2 \text{ Nm } (n = \text{speed in rpm})$$

inclusive of torque of rotational losses of the motor?

Note: At no load, the armature current would be so small that the armature voltage drop can be ignored.

Solution *At no load*

$$E_a \approx V = 230 \text{ V}$$

Since field current is to be maintained constant

$$230 = K\Phi n = K'_E \times 1200$$

or $$K'_E = 0.1917 \quad \text{(i)}$$

At load

$$E_a I_a = 8000 \text{ W} \quad \text{(ii)}$$

Also $$I_a = \left(\frac{230 - E_a}{0.5}\right) \quad \text{(iii)}$$

Substituting Eq. (iii) in Eq. (ii)

$$E_a (230 - E_a) = 4000$$

or $$E_a^2 - 230\, E_a + 4000 = 0 \quad \text{(iv)}$$

Solving

$$E_a = 211 \text{ V} \quad \text{or} \quad 19 \text{ V}$$

We reject $E_a = 19$ V as it would mean too large I_a (230 – 19)/0.5 = 422 A) well beyond the rated current of the motor (= 8000/230 = 34.8 A).

Now $$211 = K'_E \times n$$

or $$n = \frac{211}{0.1917} = 1100 \text{ rpm}$$

Also $$T = \frac{8000}{\left(\frac{2\pi \times 1100}{60}\right)} = 69.4 \text{ Nm}$$

$$I_a = \frac{230 - 211}{0.5} = 38 \text{ A}$$

But

$$T = K'_T I_a$$

or $$69.4 = K'_T \times 38$$

or $$K'_T = \frac{69.4}{38} = 1.826 \tag{v}$$

Pump Load

$$T = 1.826 \times \left(\frac{230 - 0.1917n}{0.5}\right)$$

or $$T = 840 - 0.7\,n \tag{vi}$$

Pump load torque

$$T_p = 0.6 \times 10^{-4}\, n^2 \tag{vii}$$

At steady speed

$$T = T_p$$

or $$840 - 0.7n = 0.6 \times 10^{-4}\, n^2$$

$$0.6 \times 10^{-4}\, n^2 + 0.7\,n - 840 = 0 \tag{viii}$$

Solving

$$n = 1097 \text{ rpm}$$

Example 10.10 A separately excited dc motor is operating at an armature voltage of 300 V. Its no-load speed is 1200 rpm. When fully loaded, it delivers a motor torque of 350 Nm and its speed drops to 1100 rpm. What is the full load current and power? What is the armature resistance of the motor?

The motor is now fed with armature voltage of 600 V, while its excitation is held fixed as before. It is once again fully loaded. Find the motor torque, power and speed.

Solution *At no load*

At no load, the armature draws a very small current to develop mechanical to supply only its rotational loss. Therefore, armature voltage drop $I_{ao} R_a$ is of negligible order. Thus

$$E_a \approx V = 300 \text{ V}$$

and $$300 = (K_a \Phi)\ \frac{2\pi\ \ 1200}{60}; \qquad \text{Eq. (10.6)}$$

or $K_a \Phi = 2.39$ (remains constant as excitation does not change)

Motor loaded to 350 Nm

$$350 = (K_a\ \Phi) I_a \text{ ; Eq (10.13)}$$

or $$I_a = \frac{350}{2.39} = 146.4 \text{ A (full load)}$$

$$E_a = 300 \times \frac{1100}{1200} = 275 \text{ V}$$

Mechanical power developed

$$= E_a I_a = 275 \times 146.7$$

$$= 40.3 \text{ kW}$$

$$R_a = \frac{300 - 275}{146.4} = 0.171\ \Omega$$

Armature voltage of 600 V

At full load* $\quad I_a = 146.4$ A

$$T = 350 \text{ Nm (field excitation has not changed)}$$

$$E_a = 600 - 146.4 \times 0.171 = 575 \text{ V}$$

$$575 = 2.39 \times \left(\frac{2\pi \times n}{60}\right)$$

or $\quad n = 2297$ rpm

$$\text{Power} = E_a I_a = 575 \times 146.4$$

$$= 84.2 \text{ kW}$$

Observe that while motor speed has more than doubled, torque has remained the same and power has more than doubled.

Example 10.11 For the motor of Ex. 10.10 the armature voltage is held fixed at 300 V while its current is reduced to one half. Calculate the full load torque, power and speed.

Solution As the field excitation is reduced to one half,

$$K_a \Phi = \frac{2.39}{2} = 1.195$$

But $\quad I_a = 146.4$ A (full load)

$$E_a = 300 - 146.4 \times 0.171 = 275 \text{ V (as before)}$$

$$275 = 1.195 \times \frac{2\pi \times n}{60}$$

or $\quad n = 2200$ rpm

$$\text{Power developed} = E_a I_a = 275 \times 146.7$$

$$= 40.3 \text{ kW}$$

$$\text{Torque } T = K_a \Phi I_a$$

$$= 1.195 \times 146.4 = 175 \text{ Nm}$$

Observe that while motor speed has doubled, torque is reduced to one half but power is the same.

Series Motor

In this kind of motor, the field is series excited by the armature current as shown in Fig. 10.24. From speed Eq. (10.29)

$$n = K_N \left[\frac{V - I_a(R_a + R_{se})}{\Phi}\right] \tag{10.35}$$

Assuming linear magnetizaton

$$\Phi = K_{se} I_a \tag{10.36}$$

* The motor could be loaded to somewhat higher current because of enhanced cooling at higher speed.

From Eq. (10.28)

$$T = K_T K_{se} I_a^2 \tag{10.37}$$

Substituting for I_a from Eq. (10.37) in Eq. (10.35)

$$n = \frac{K_N}{K_{se}}\left[\frac{V\sqrt{K_T K_{se}}}{\sqrt{T}} - (R_a + R_{se})\right] \tag{10.38}$$

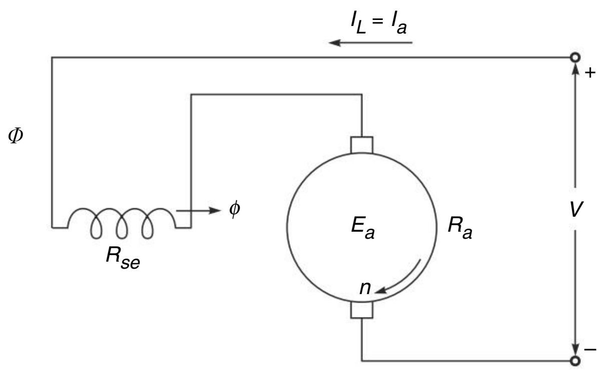


Fig. 10.24 DC series motor

As $(R_a + R_{sc})$ is very small and may be ignored. Thus

$$n \approx \frac{K_N V}{\sqrt{TK_{se}}} \tag{10.39}$$

It is inverse hyperpole type speed-torque characteristic as sketched in Fig. 10.25, with speed reducing as load torque increases. It is ideal for *traction-type* load. As the load torque is reduced, the motor speed rises sharply and will acquire dangerously high speed as the motor reaches no-load condition. *So the series motor must never be allowed to run at no-load even inadvertently.* This cannot happen in traction-type load as the load is always present.

Speed control in dc series motors is achieved as given below:

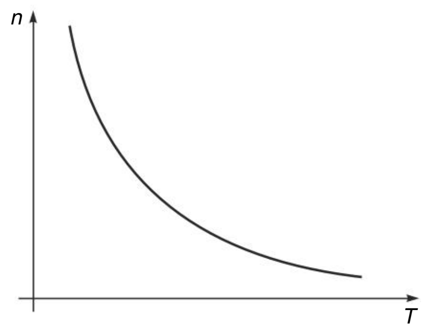


Fig. 10.25 Speed-torque characteristic of series motor

Field Control (K_{se} Varies as in Eq. (10.38)) This is achieved by two means:

– Tapped-field control—The connection diagram is illustrated in Fig. 10.26. As the field turns are reduced by means of tap changing gear, K_{se} (Eq. (10.36)) reduces causing the speed to go up for a given torque as shown in Fig. 10.27.

– Series–parallel field control—The field coils are divided in two equal halves and are connected in series/parallel as shown in Fig. 10.28. It is seen from

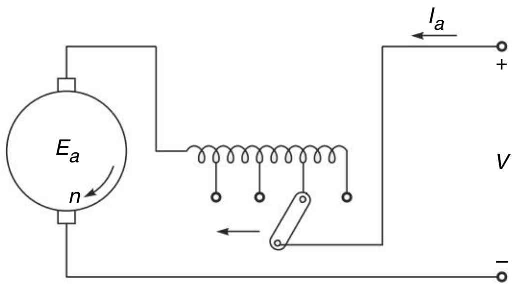


Fig. 10.26 Tapped field control (connection diagram)

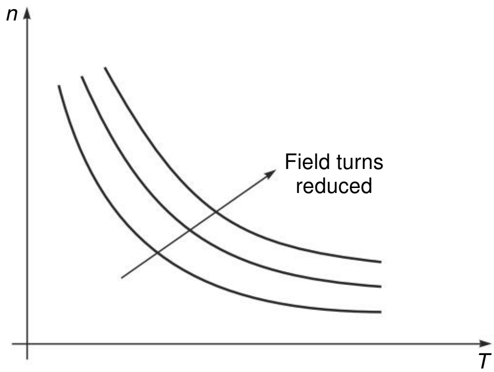


Fig. 10.27 Tapped field speed control of dc series motor

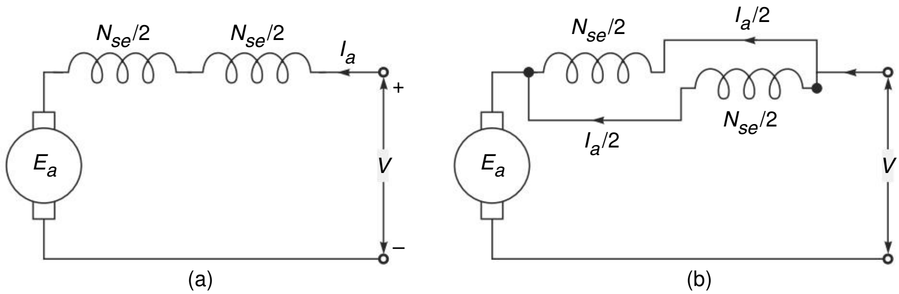


Fig. 10.28 Series–parallel field control of dc series motor

this figure that the series connection corresponds to N_{se} turns (low speed) and the parallel connection is equivalent to $N_{se}/2$ turns (high speed). Only two speeds are possible in this method which is commonly employed in traction.

Armature control This is obtained by series/parallel connection of two identical series motors (Fig. 10.29) which are mechanically coupled. The parallel connection with voltage V across each armature gives double the speed compared to the series connection with voltage $V/2$ across each armature.

Compound Motor

It has both shunt and series fields as shown in Fig. 10.30. If the series field is

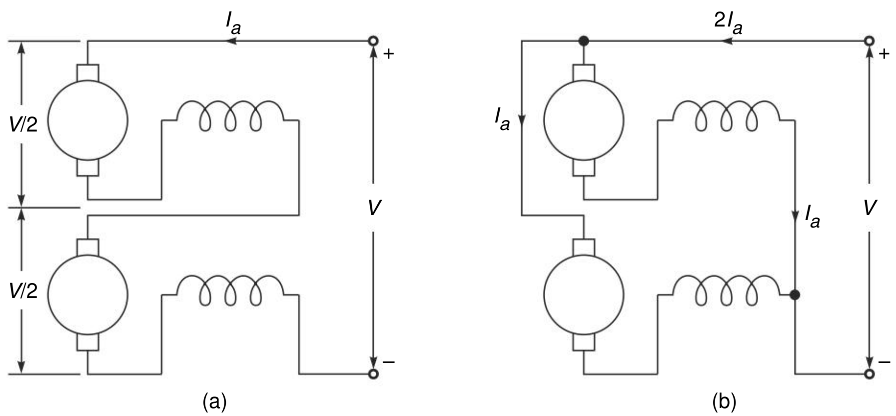


Fig. 10.29 Series–parallel speed control of series motors; case of constant load torque is illustrated

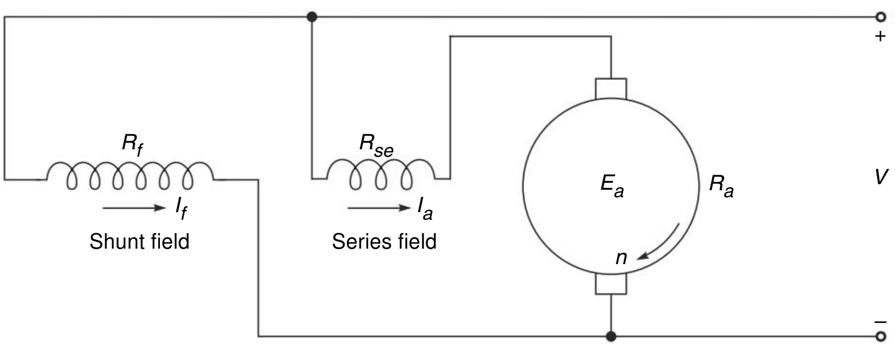


Fig. 10.30 Compound motor

connected to aid the shunt field, the motor is called *cumulative compound* and if it opposes the shunt field, it is called *differential compound*.

At no load, series field carries only a small no-load current and as a consequence the no-load speed is determined by the shunt field (Fig. 10.31). As the load increases, the series field current (= armature current) causes the flux/pole to increase in a cumulative compound motor so that its speed drops much more sharply than in a shunt motor as shown in Fig. 10.31. On the other hand, in a differential compound motor, the opposing series field reduces the flux/pole causing the motor speed to increase as shown in Fig. 10.31. It can be arranged that full load speed equals no-load speed (*called level compounding*). At heavy loads (overloads), the flux/pole decreases sharply and speed rises to dangerous values while the armature draws a very large current to meet the torque demand. Because of this serious drawback, a differential compound motor is not recommended for practical use.

A cumulative compound motor is preferred for rolling mills. Unlike a series motor, it has a finite no-load speed but speed drops sharply (similar to series motor) relieving the peak power drawn from the mains as the billet is passed through rolls.

Figure 10.31 is also illustrative of speed–torque characteristics of various types of dc motors.

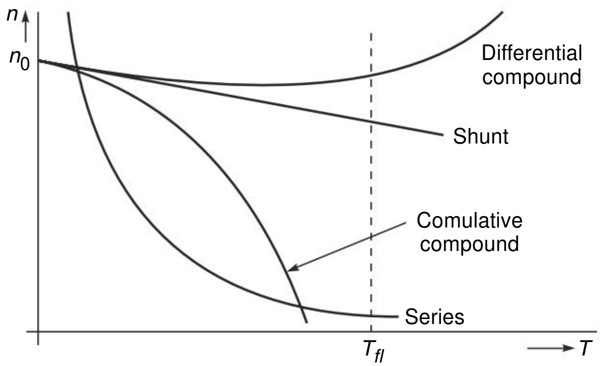


Fig. 10.31 Speed–torque characteristics of dc motors

Example 10.12 A dc series motor runs at 500 rpm drawing 40 A from 600 V mains. Determine the value of the resistance to be added in series with the armature for the motor to run at 450 rpm; the load torque is reduced in the square ratio of speed. Assume linear magnetization.

Given: $(R_a + R_{se}) = 0.5\ \Omega$

Solution Without additional resistance in armature circuit (n = 500 rpm)

$$E_a = 600 - 40 \times 0.5 = 580 \text{ V}$$

$$580 = K_a \Phi\omega_m = K'_a I_a \omega_m \text{ ; } \Phi \propto I_a \text{ in series motor} \quad \text{(i)}$$

or

$$K'_a = \frac{580}{40 \times \left(\dfrac{2\pi \times 500}{60}\right)} = 0.277$$

$$T = K_a \Phi I_a = K'_a I_2^2 \quad \text{(ii)}$$

$$= 0.277 \times (40)^2 = 443.2 \text{ Nm}$$

With additional resistance in armature circuit (n = 450 rpm)

$$T = 443.2 \times \left(\frac{450}{500}\right)^2 = 359 \text{ Nm}$$

$$I_a = \left(\frac{359}{0.277}\right)^{1/2} = 36 \text{ A}$$

$$E_a = 0.277 \times 36 \times \left(\frac{2\pi \times 450}{60}\right) = 470 \text{ V}$$

$$R \text{ (total)} = \left(\frac{600 - 470}{36}\right) = 3.6\ \Omega$$

$$R_{\text{ext}} = 3.6 - 0.5 = 3.1\ \Omega$$

Example 10.13 A 220 V dc series motor has an armature resistance of 1 Ω and series field winding resistance of 0.4 Ω. On a certain load the motor input current is 20 A. It is desired to reduce the speed of the motor by 30%. Calculate the resistance to be connected in series if the load torque varies as cube of the speed. Assume that the load torque includes the effect of mechanical loss of the motor. The magnetic circuit is unsaturated.

Solution

Motor speed n_1, $\quad I_a = 20$ A;

$$T = K_T \Phi I_a = K'_T I_a^2;\ \Phi_a \propto I_a$$

$$T_L = K_L n^3$$

$$K_L n_1^3 = K'_T I_{a1}^2 = 400\ K'_T \tag{i}$$

$$K_L (0.7\ n_1)^3 = I_{a2}^2 K'_T \tag{ii}$$

Dividing Eq. (ii) by (i)

$$(0.7)^3 = \frac{I_{a2}^2}{400}$$

or

$$I_{a2} = [400 \times (0.7)^3]^{1/2} = 11.7 \text{ A}$$

$$E_{a1} = 220 - 20\ (1 + 0.4) = 192 \text{ V}$$

$$E_a = K_E \Phi\ n = K'_E I_a n;\ \Phi\ \alpha\ I_a$$

$$192 = K'_E \times 20\ n_1 \tag{iii}$$

$$E_{a2} = K'_E \times 11.7 \times 0.7\ n_1 \tag{iv}$$

From Eqs. (iii) and (iv)

$$\frac{E_{a2}}{192} = \frac{11.7\ \ 0.7}{20}$$

or

$$E_{a2} = 78.6 \text{ V}$$

Additional resistance to be added in armature circuit

$$= \frac{220 - 78.6}{11.7} - 1.4 = 10.78\ \Omega$$

Example 10.14 A dc series motor runs at 1000 rpm drawing 25 A from a 250 V supply with its field halves connected in series. At what speed would it run if the field halves are reconnected in parallel? Also calculate the armature current. Assume (a) load torque to be proportional to square of speed and (b) voltage drop of armature resistance and field resistance to be negligible.

Solution Because of assumption (i) $E_a = V = 250$ whatever may be the armature current.

Field halves connected in series (Fig. 10.26(a))

$$E_a = K_a\ \Phi \omega_m$$

$$\Phi = K_f N_{se} I_a$$

$$\therefore \quad E_a = (K_a N_f I_{se})\ I_a\ \omega_m$$

or

$$250 = (K_a N_f N_{se}) \times 25 \times \frac{2\pi \times 1000}{160}$$

or $\quad K_a N_f N_{se} = 0.0955$

Now $\quad T\,(\text{developed}) = K_a \Phi I_a = (K_a K_f N_{se})\, I_a^2$

$$= 0.0955 \times (25)^2 = 59.7 \text{ Nm}$$

Field halves connected in parallel (Fig. 10.26(b)). We now have the relationship as

$$\Phi = \left(\frac{K_f N_{se}}{2}\right) I_a$$

$$E_a = \left(\frac{K_a K_f N_{se}}{2}\right) I_a \omega$$

$$T\,(\text{developed}) = \left(\frac{K_a K_f N_{se}}{2}\right) I_a^2$$

where I_a Φ, ω and T (developed) are new values

At new ω

But load torque $\quad T_L = 59.7 \times \left(\dfrac{\omega}{\dfrac{2\pi\times 1000}{60}}\right) = 59.7 \times \left(\dfrac{\omega}{104.7}\right)^2$

Now $\quad 250 = \dfrac{0.0955}{2} I_a\, \omega \qquad \text{(i)}$

Equating motor and load torques

$$59.7 \times \left(\frac{\omega}{104.7}\right)^2 = \left(\frac{0.0955}{2}\right) I_a^2 \qquad \text{(ii)}$$

or $\quad \omega = \left[\dfrac{0.0955\times(104.7)^2}{59.7\times 2}\right]^{1/2} I_a$

$$= 2.96\, I_a$$

Substituting in Eq. (i)

$$250 = \frac{0.0955}{2} \times 2.96 \times I_a^2$$

or $\quad I_a = 42 \text{ A}$

$$\omega = 2.96 \times 42 = 124.5 \text{ rad/s}$$

or $\quad n = \dfrac{124.5\times 60}{2} = 11.89 \text{ rpm}$

10.10 DC MOTOR STARTING

At the time of starting the motor, back emf being zero, the motor draws an armature current of

$$I_a\,(\text{start}) = \frac{V}{R_a} \text{ or } \frac{V}{R_a + R_{se}} \text{ (series motor)} \qquad (10.40)$$

Since the motor armature resistance may be as low as 0.01 pu (for large motors), it may draw 100 times its rated current at start. Such large currents in the motor can have serious consequences as follows:

- Heavy sparking and commutator damage;
- Sudden large starting torque will give a severe jolt to the motor shaft; and

- Such large current (even through for a short time) is not permitted to be drawn from the source of supply whose voltage would otherwise dip sharply.

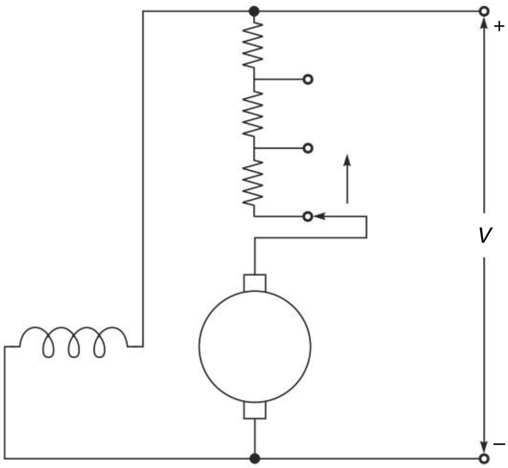


Fig. 10.32 Shunt motor starting

To limit the starting current to 1.5–2 times the rated current (this is necessary for quick acceleration), an external resistance has to be included in the armature circuit as shown in Fig. 10.32 for a shunt motor. This resistance is cut out in suitable time and resistance steps as the motor accelerates. These steps are so devised that peak current at each cut out remains limited to the prescribed value. To obtain high starting torque, full field current must be permitted to flow (no external resistance in field circuit) when the motor is started. Direct start when permissible does offer the advantage of quick acceleration and low ohmic loss per start–saving in energy and reduced temperature rise for frequent-starting situations.

Where a variable voltage dc source is available for speed control, low volt-age starting would be ideal and no starting resistance would then be necessary. Of course, the field at the time of starting must be excited with full voltage.

10.11 EFFICIENCY OF DC MOTORS

Figure 10.33 shows the flow of power in a dc motor. The various losses (as already categorized in Sec. 9.9) are as follows:

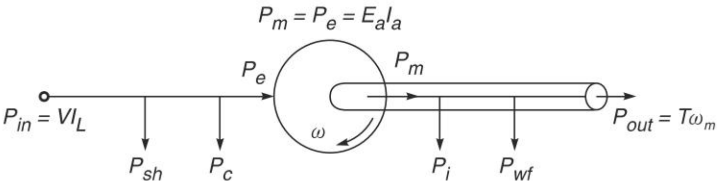


Fig. 10.33 Power flow in a dc motor

Constant Loss

P_i = core loss (including stray load core loss)

P_{wf} = windage and friction loss (at specified speed)

P_{sh} = shunt field copper loss (in a shunt machine)

$P_k = P_i + P_{wf} + P_{sh}$ = total constant loss

Variable Loss

$$P_c = I^2_a R_a$$

= Copper loss (inclusive of copper loss in series winding in a series motor, and also inclusive of stray-load copper loss)

Total motor loss, $P_L = P_k + P_c$

$$P_m = P_e = E_a I_a$$

The motor efficiency is given by

$$\eta = \frac{\text{Input} - \text{Losses}}{\text{Input}}$$

$$= \frac{VI_L - P_k - I^2_a R_a}{VI_L} \qquad (10.41)$$

Under load, $I_L \approx I_a$; I_f is small in a shunt motor.

Then

$$\eta = 1 - \frac{1}{V}\left(\frac{P_k}{I_a} + I_a R_a\right) \qquad (10.42)$$

The maximum motor efficiency occurs at

$$\frac{P_k}{I_a} = I_a R_a \qquad (10.43)$$

or $I^2_a R_a = P_k$

or Variable loss = constant loss

which is the same condition as was shown in a transformer (Eq. 7.66) (no wonder!).

Example 10.15 A 50 kW, 230 V dc shunt motor takes a current of 14.5 A when running light at 1640 rpm. The armature and field resistances are 0.15 Ω and 120 Ω respectively. Estimate the motor efficiency when the motor is drawing 215 A. What would be the maximum efficiency of the motor and the load current at which it would occur?

Solution Refer to Fig. 11.15.

$$I_f = \frac{230}{120} = 1.92 \text{ A}$$

$$P_{sh} = \frac{(230)^2}{120} = 441 \text{ W}$$

At no load

$$P_{in} = 14.5 \times 230 = 3335 \text{ W}$$

$$I_a = 14.5 - 1.92 = 12.6 \text{ A}$$

$$I^2_a R_a = (12.6)^2 \times 0.15 = 24 \text{ W (negligible)}$$

$$P_k = P_i + P_{wf} + P_{sh} = 3335 - 24 \approx 3311 \text{ W}$$

On load

$$I_a = 215 - 1.92 = 213 \text{ A}$$

$$I_a^2 R_a = (213)^2 \times 0.15 = 6805 \text{ W}$$
$$P_L = 6805 + 3311 = 10.116 \text{ kW}$$
$$P_{in} = 230 \times 215 = 49.45 \text{ kW}$$
$$\eta = \frac{49.45 - 10.116}{49.45} = 79.54\%$$

For maximum efficiency

$$0.15\, I_a^2 = 3311$$

or
$$I_a = 148.6 \text{ A}$$
$$I_L = 148.6 + 1.92 \approx 150 \text{ A}$$
$$P_L = 2 \times 3311 = 6622 \text{ W}$$
$$P_{in} = 230 \times 150 = 34.5 \text{ kW}$$
$$\eta = \frac{34.5 - 6.62}{34.5} = 80.8\%$$

Swinburne's Test A simple way of testing a dc shunt motor is to run it on load at rated speed and measure the no-load current I_0.

No-load power input

$$P_{i0} = P_i + P_{wf} + P_{sh} + I_a^2 R_a$$

As I_0 is very small $I_a^2 R_a$ may be ignored. Then

$$P_k \approx P_{i0}\text{ ; no-load power input}$$

The armature resistance Ra can be measured at rest by a battery test, $R_a = V/I$. R_a is then corrected for temperature rise.

At full-load current $I_a^2 R_a$, the copper loss, can be computed and motor efficiency then determined from Eq. (10.41).

The reader may note that the Swinburne's test is a *non-loading test*.

This is illustrated by the solved problem 10.16.

Example 10.16 A 60 kW, 250 V shunt motor takes 16 A when running on no load. The resistance of the armature and field are 0.2 Ω and 125 Ω respectively when hot (75°C).

(a) Estimate the motor efficiency when loaded to carry 152 A.

(b) Also estimate its efficiency when operating as a generator and delivering 152 A at 250 V.

Solution On no load:

$$I_f = 250/125 = 2 \text{ A}$$
$$I_{a0} = 16 - 2 = 14 \text{ A}$$

Constant losses $P_k = (P_{i0} + P_{wf} + P_{sh})$

$$P_{i0} + P_{wf} = 250 \times 14 - \underbrace{14^2 \times 0.2}_{\text{no - load coppper loss}} = 3500 - 39.2 = 3461 \text{ W}$$

$$P_{sh} = 250 \times 2 = 500 \text{ W}; \qquad P_k = 3461 + 500 = 3961 \text{ W}$$

Assumption: Stray load loss will be ignored.

(a) Machine Generating

$$I_a = 152 - 2 = 150 \text{ A}$$
$$P_L = I_a^2 R_a + P_k = (150)^2 \times 0.2 + 3961 = 8.46 \text{ kW}$$
$$P_{in} = 250 \times 152 = 38 \text{ kW}$$
$$\eta = \frac{38 - 8.46}{38} \times 100 = 77.73\%$$

(b) Machine Motoring

$$I_L = 152 \text{ A} \quad I_a = 152 + 2 = 154 \text{ A}$$
$$P_L = I_a^2 R_a + P_k = (154)^2 \times 0.2 + 3961 = 8.70 \text{ kW}$$
$$P_{\text{out}} = 250 \times 152 = 38 \text{ kW}$$
$$\eta = \frac{38 \times 100}{38 + 8.70} = 81.36\%$$

Note: Stray load can be accounted for by assuming it to be 1% of the output.

10.12 DC MOTOR APPLICATIONS

A DC motor in spite of its high cost as compared to an induction motor is still competitive in applications requiring control speed as its speed can be varied over a wide range by (i) field control, (ii) armature control or (iii) a combination of the two.

DC motors have high starting torque compared to induction motor; hence, there is faster load decleration.

Shunt Motor Medium starting torque; speed regulation 5-15% Application: centrifugal pumps, conveyors, machine tools, printing presses, etc.

Series motor High torque at low speed and low torque at high speed high starting torque (as much as 5 T (*fl*)), good speed control possible. Application: traction, hoists, crane, battery-powered vehicles, etc).

Compound motors Differentially compound motor is ideally suited for pulsating loads (needing flvwheel action).

Application Rolling mill, etc.

Example 10.17 A dc series motor operates at 800 rpm with a line current of 120 A from a 250 V mains. Determine the motor speed at a current of 60 A at 250 V assuming that the flux/pole at 60 A is 70% of its value at 120 A.

Given: $R_a = 0.15\ \Omega$, $R_{se} = 0.1\ \Omega$

Solution

$$R_a + R_{se} = 0.15 + 0.1 = 0.25\ \Omega$$

250 V, 120 A, 800 rpm operation:

$$E_{a1} = 250 - 120 \times 0.25 = 220 \text{ V}$$
$$E_{a1} \propto \phi_1 \times n$$

or $$220 \propto 800\, \phi_1 \quad \text{(i)}$$

250 V, 60 A, n_2 operation:

$$E_{a2} = 250 - 60 \times 0.25 = 235 \text{ V}$$

$$\phi_2 = 0.7\, \phi_1$$

$\therefore$ $$235 \propto n_2 \times 0.7\, \phi_1 \quad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i), we get

$$0.7\, n_2/800 = 235/220$$

or $$n_2 = 1221 \text{ rpm}$$

Example 10.18 A 220 V dc series motor yielded the following operational data:

Speed (rpm)	640	475	400
Current (A)	20	30	40

Find the speed at which the motor will run when connected to a 200 V mains with a series resistance of 2 Ω while drawing 35 A. Armature circuit resistance = 1.2 Ω.

Solution

$$E_a = 220 - I_a\,(R_a + R_{sc})$$

$$R_a + R_{sc} = 1.2\ \Omega \text{ gives}$$

Then

$$E_a = 220 - 1.2\, I_a$$

We prepare the table as below from the given data:

Speed (n)	640	475	400	rpm
Current (I_o)	20	30	40	A
1.2 I_a	24	36	48	V
E_a	196	134	172	V

As E_a–n relationship is non-linear, we find E_a and n at I_a = 35 A linear interpolation

$$E_a(35A) = 184 - (184 - 172) \times \frac{5}{2} = 178 \text{ V}$$

$$n(35A) = 475 - (475 - 400) \times \frac{5}{10} = 437.5 \text{ rpm}$$

At 200 Vwith added resistance of 2 Ω, 35 A

$$E_o = 200 - 35(1.2+2) = 88 \text{ V}$$

In a dc machine

$$E_a = K_a\, \Phi\, \omega_m = K_a\, \Phi\, n\,,\ \Phi \propto I_a$$

As I_a is same in both cases

$$E_a \propto n$$

Therefore

$$n\,(200V, 35A) = 437.5 \times \frac{88}{178} = 216.3 \text{ rpm.}$$

Example 10.19 A dc shunt motor rated 50 kW connected to a 250 V supply is loaded as to draw 200 A when running at a speed of 1250 rpm. Given: R_a = 0.22 Ω.

(a) Determine the load torque if the rotational loss (including iron loss) is 600 W.

(b) Determine the motor efficiency if the shunt field resistance is 125 Ω.

Solution

(a)
$$E_a = 250 - 200 \times 0.22 = 206 \text{ V}$$
$$E_a I_a = P_m \text{ (dev.)} = 206 \times 200 = 41.2 \text{ kW}$$
$$\text{Rotational loss} = 0.6 \text{ kW}$$
$$P_m \text{ (out)} = 41.2 - 0.6 = 40.6 \text{ kW}$$
$$\omega_m = (2\pi \times 1250)/60 = 130.9 \text{ rad/s}$$
$$T_L = \frac{40.6 \times 103}{130.9} = 310 \text{ Nm}$$

(b) Shunt field loss
$$= (250)^2/125 = 500 \text{ W}$$
$$P_e \text{ (in)} = 250 \times 200 + 500 = 50.5 \text{ kW}$$
$$\eta = [40.6/50.5] \times 100 = 80.3\%$$

ADDITIONAL SOLVED PROBLEMS

10.20 A 25 kW, 220V, 1600 rpm dc shunt generator with $R_a = 0.1\Omega$ has the OCC data at rated speed as given below:

I_f (A)	0.0	0.25	0.50	0.75	1.0	1.25	1.5
V_{oc} (V)	10	50	150	190	220	240	250

(a) What would be the field resistance for a no-load voltage of 230 V?

(b) At rated armature voltage and rated armature current, find the field current and field resistance.

(c) Find the value of the electromagnetic power and torque in part (b).

Solution

The OCC is drawn in Fig. 10.32, speed = 1600 rpm

(a) No-load voltage = 250 V

Connect the 250 V point on OCC to origin, which there is the R_f line corresponding

$$I_f = 1.5 \text{ A}$$
$$\therefore \quad R_f = \frac{250}{1.5} = 166.7 \ \Omega$$

(b) Rated voltage, $V = 220$ V ; speed = 1600 rpm

$$\text{Rated current, } I_a = \frac{250 \times 10^3}{220} = 113.6 \text{ A}$$
$$\text{Emf, } E_a = 220 - 113.6 \times 0.1 = 208.6 \text{ V}$$

From the OCC, the corresponding filed current is

$$I_f = 0.875 \text{ A}$$
$$R_f = \frac{220}{0.875} = 252.4 \ \Omega$$

(c) $P(\text{dev}) = E_a I_a = 208.6 \times 113.6 \times 10^{-3} = 23.7$ kW

$$T(\text{dev}) = \frac{23.7 \times 10^3}{\left(\frac{2\pi \times 1600}{60}\right)} = 141.4 \text{ Nm}$$

10.21 A dc machine is connected across 230 V supply and runs at 1200 rpm generating as emf f 210V with an armature current of 40 A.

(a) Is the machine motoring/generating?

(b) Calculate R_a.

(c) Calculate the electromanetic torque and power.

(d) If the load is thrown off, calculate the machine speed.

Solution

(a) V(terminal) = 230V, E_a = 210V

As $V > E_a$, the machine is motoring

(b) $$R_a = \frac{V - E_a}{I_a} = \frac{230 - 210}{40} = 0.5\ \Omega$$

(c) $$P\text{ (dev)} = E_a I_a = 210 \times 40 \times 10\text{–}3 = 8.4 \text{ kW}$$

$$n = 1200 \text{ rpm}, \omega_m = \frac{2\pi \times 1200}{60} 125.66 \text{ rad/s}$$

$$T\text{ (dev)} = \frac{8.4 \times 10^3}{125.66} = 66.85 \text{ Nm}$$

(d) Load thrown off, I_a is very small

$$E_a = V - I_{ao} R_a \approx V = 230 \text{ V}$$

Field current does not change, so

$$n \propto E_a$$

$$n\text{ (no-load)} = 1200 \times \frac{230}{210} = 1314 \text{ rpm}$$

10.22 A dc series motor run at 500 rpm drawing a current of 50 A from 400 V supply. Its armature circuit resistance is 0.5 Ω. What resistance must be included in the armature circuit to limit the starting current to 75 A at 400 V. What is the corresponding starting torque?

The motor has speeded upto 200 rpm, when part of the resistance is cut out limiting the current to 75 A. What is the value of the resistance in the circuit?

Assume the magnetization to be linear.

Solution

(i) 400 V, 50 A, 500 rpm, $R_a = 0.5\Omega$

$$E_a = 400 - 0.5 \times 50 = 375 \text{ V}$$

$$T(\text{dev}) = \frac{E_a I_a}{\omega_m} = \frac{375 \times 50}{\frac{2\pi \times 500}{60}} = 358.1 \text{ Nm}$$

For series motor

$$T\,(\text{dev}) \propto I_a^2,\; E_a \propto n\, I_a$$

(ii) At start

$$\frac{400}{0.5 + R_s} = 75 \text{ A} \Rightarrow R_s = 4.83$$

$$I_a\,(\text{start}) = 75 \text{ A}$$

$$T\,(\text{start}) = 358.1 \times \left(\frac{75}{50}\right)^2 = 805.7 \text{ Nm}$$

(iii) Motor speed = 200 rpm

$$I_a = 75 \text{ A (same), so } E_a\, \alpha\, n$$

$$E_a\,(\text{at 200 rpm}) = 375 \times \frac{200}{500} = 150 \text{ V}$$

Resistance in circuit

$$\frac{400 - 150}{0.5 + R_s} = 75$$

Or $$R_s = 2.83\ \Omega$$

10.23 The OCC of a generator at 1200 rpm is

I_f (A)	0	2	4	6	8
V_{oc} (V)	12	140	230	285	320

Estimate the critical field resistance at this speed. With resistance fixed at this value, the generator is shunt connected and its speed increased to 1600 rpm. What would its no-load voltage?

Solution

The OCC at 1200 rpm is plotted (reader should plot the data on graph sheet). A line from the origin is draw tangential to the OCC in its linear part which gives

$$R_f(\text{critical}) = \frac{260}{4} = 65\ \Omega$$

The speed is now increased to 1600 rpm. The value of V_{oc} at each field current increase by a factor of 1600/1200 = 4/3. The OCC (1600 rpm) is drawn in Fig. 10.33 The extended R_f(65 Ω) line interacts OCC to give

$$V\,(\text{no-load}) = 395 \text{ V}$$

10.24 The shunt generator of question 10.23 is run at 1600 rpm with $R_f = 65\ \Omega$. It is supplying a load of 500 A. Estimate the value of the terminal voltage. Given armature resistance R_a = 0.05 W; Brush voltage drop = 2 V(constant)

Solution

Armature voltage drop = 500 × 0.05 + 2 = 27

Terminal voltage

$$V_t = E_a(=V_{oc}) - 27 = V_{oc}\,(I_f) - 27 \qquad \text{(i)}$$

$$V_t = I_f R_f \tag{ii}$$

The solution of these two equations would yield V_t. Instead of translating V_{oc} down by 27 V, it is simpler to move up the R_f - line by 27 V. So in the OCC plot of Problem 10.23 draw a line parallel to R_f -line and 27 V above it. It intersects OCC at B and D from which we get

$$V_t = BC = 330 \text{ V} \quad \text{and } V_t = EF = 85 \text{ V}$$

The acceptable value is

$$V_t = 330 \text{ V}$$

10.24 A 3kW series motor has series field windings divided in two equal halves. The motor runs at 800 rpm on a 240V supply drawing 16A with field winding halves connected in series. Estimate the speed and current drawn by the motor with field winding halves connected in parallel. The load torque varies as square of speed. Addume magnetization to be linear and ignore losses.

Solution

$I_0 (R_a + R_{se})$ drop is ignored. Therefore $E_a \approx$ V = 240 V

- Field winding halves in series

$$\Phi \propto N_{se} I_a \;;\; N_{se} = \text{total turns of series field winding}$$

Therefore

$$E_a = K_E n I_a$$

$$T = K_T I_a I_a = K_T I_A^2 = K_L \text{n}^2$$

$$I_a = \left(\frac{K_L}{K_T}\right)^{1/2} n = K n \;; \qquad K = \left(\frac{K_L}{K_T}\right)^{1/2}$$

Substitiuting values

$$240 = K_E \times 800 \times 16 \;; \qquad K_E = \frac{3}{160}$$

$$16 = K \times 800 \;; \qquad K = \frac{16}{800} = \frac{1}{50}$$

- Field winding halves in parallel

$$\text{Field AT} = 2\left[\frac{N_{se} I_a}{4} + \frac{N_{se} I_a}{4}\right] = N_{se} = \left(\frac{I_a}{2}\right)$$

Thus effectively the field current reduces to $\left(\frac{I_a}{2}\right)$

Now

$$240 = K_E n. \frac{I_a}{2} = \left(\frac{3}{320}\right) n I_a \tag{i}$$

$$K_T I_a . \frac{I_a}{2} = K_L n^2$$

$$I_a = \sqrt{2} .K n = \left(\frac{\sqrt{2}}{50}\right) n \tag{ii}$$

Substituting I_a in Eq. (i)

$$240 = \left(\frac{3}{320}\right) \times \left(\frac{\sqrt{2}}{50}\right) n^2$$

Or $\quad n = 951.4$ rpm

and $$I_a = \left(\frac{\sqrt{2}}{50}\right) \times 951.4 = 26.9 \text{ A}$$

10.26 A 600V dc shunt motor drives a 60 kW load at 900 rpm. The shunt field resistance is 100 W and armature resistance is 0.16 W. If the motor efficiency at this load is 85%, determine (a) the speed at no-load and speed regulation (b) the rotational loss.

Solution

(a) $$P_{out} = 60 \text{ kW}, \eta = 0.85$$

$$P_{in} = \frac{60}{0.85} = 70.59 \text{ kW}$$

$$I_L = \frac{70 \times 10^3}{600} = 117.65 \text{ A}$$

$$I_f = \frac{600}{100} = 6 \text{ A}$$

$$\text{Then } I_a = 117.65 - 6 = 111.65 \text{ A}$$

$$E_a = 600 - 111.65 \times 0.16 = 582.14 \text{ A}$$

$$n = 900 \text{ rpm}$$

No load

$$E_a \approx V = 600 \text{ V}$$

$$n \propto E_a$$

$$n_o = 900 \times \frac{600}{582.14} = 927.6$$

$$\text{speed regulation} = \frac{927.6 - 900}{900} \times 100 = 3.1\%$$

(b) Total loss = 70.59 – 60 = 10.59 kW

$$\text{Copper loss} = I_a^2 R_a = (111.65)^2 \times 0.16 = 1.99 \text{ kW}$$

$$\text{Shunt loss} = (6)^2 \times 100 = 3.6 \text{ kW}$$

Hence

Rotational loss = 10.59 – 1.99 – 3.6 = 5 kW

It includes windage, friction and iron loss.

SUMMARY

- Constructional Features
  - Field poles on stator, always salient type with wide pole shoe about 70% of pole pitch; field poles d.c. excited
  - Armature is the rotor carries armature windings
  - Commutator on the rotor, copper segments with mica insulation between adjoining aegments. Functions—convert alternating armature current to d.c. which is collected by brushes (carbon) suitably placed around the commutator.

- Armature windings

  Lap winding

  Number of parallel paths, $A = P$, poles

  Number of brushes = $A = P$

  Conductor current, $I_c = I_a/A$

  where I_a is armature current.

  Wave winding

  Number of parallel paths, $A = 2$

  Number of brushes = 2 needed but P used in practice

  Conductor current, $I_c = I_a/2$

- EMF Equation

$$E_a = K_a\,\Phi\,\omega_m = \left(\frac{2\pi}{60}\right) K_a\,\Phi\,n \ \ \text{V}$$

ω_m = rad (mech)/s, n = rpm, Φ = flux /pole

$$K_a = \left(\frac{ZP}{2\pi A}\right), \text{ Z = number of armature conductors}$$

1. For linear magnetization, $\Phi \propto I_f$

$$E_a = K_a' I_f \omega_m,\ \omega_m = \left(\frac{2\pi}{60}\right)n$$

- Torque Equation

$$T = K_a\,\Phi\,I_a \text{ Nm} \quad ; K_a = \left(\frac{ZP}{2\pi A}\right) \text{ same as for emf}$$

2. For linear magnetization, $\Phi \propto I_f$

$$T = K_a' I_f I_a$$

- Power converted

$$T\,\omega_m = E_a I_a\ W$$

- Generating machine

  V(terminal) = $E_a - I_a R_a$; R_a = armature resistance, very small, order 0.01 pu

  $V_t < E_a$

  I_a flows out of the + terminal of machine

- Motoring machine

  V (terminal) = $E_a + I_a R_a$

  $V_t > E_a$

  I_a flows into the + terminal of machine, in opposition to E_a, called *back emf.*

- Machine axes

  d-axis, along the middle of field poles

  q-axis, along the magnetic netural

  d and *q*-axis are at 90° (elect) to each other

- Armature reaction

  AT_a at 90° (elect) to AT_f,cross-magnetizing

  Flux density deceases at one pole end and increases at the other. In linear magnetization no change in Φ, flux/pole. In sturation region of magnetization, Φ reduces slightly.

- Commutation

  As the armature coils move out of the influence of one pole pair to the next pole pair, the coil current must reverse, a process called commutation.

  Current reversal is oppsed by coil reactance emf ($L\, d_i/d_t$).

  If the current does not fully reverse, there is sparking at the brushes.

  Remedy – *Inter poles,* narrow poles placed in the *inter-polar* region to inject speed emf to aid current reversal in the commutating coils

- Excitation of the poles

  Shunt field winding – large number of turns to carry small current (winding resistance high) ; voltage excited, in shunt across voltage source.

  Series field winding – a few turns (small resistance), current excited exited in series with armature, so current flow is I_a.

- Field winding excitation

  Separately excited from an independent voltage source

  Shunt excited – in parallel with armature terminals

  Series excited – in series with armature, current I_a

  Compound excitation – both shunt and series excitations are employed, could be *cumulative differential*

  Shunt excitation can be controlled by a regulating resistance in series with shunt windings.

- OCC – open circuit characteristic. V_{oc} (= E_a) vs I_f at constant speed. The machine is run as separately excited generator with the armature open circuited and field current varied

  OCC is indeed the magnetization characteristic of the machine

- Critical field resistance (R_{fc}) for shunt generator. It is the maximum field resistance above which the generator fails to excite

  Critical speed – It is the minimum speed of a shunt generator with fixed field resistance below which the generator fails to excite

- Motor characteristics depend upon the type of excitation. Accodingly the motor types are

  Shunt motor, Series motor, Compound motor – cumulative compound and differential compound

- Shunt motor

  $n - T$ characteristic

  $$n = K_N \frac{E_{0.}}{\Phi} = K_N \frac{V - I_a R_a}{\Phi} \text{ ; no-load speed } n_0 = K_N \frac{V}{\Phi}$$

  At constant field current as load torque is increased, I_a increases and E_a reduces due to $I_a\, R_a$ drop and so the speed drops slightly from its no-load value (3-5% drop) This is the *shunt characteristic* where the speed is substantially constant.

- Speed Control

Shunt Motor

The speed in inversely proportional to excitation $\Phi(I_f)$ and directly proportional to $E_a \approx$ V (applied voltage).

Field Control

I_f is reduced by a regulating resistance the speed increases but torque reduces for rated current. This is constant – kW drive.

Armature Control

For fixed I_f, speed increases directly with applied armature voltage but torque in constant for rated current. This is the constant-torque drive

Combination of Armature and Field Control

Keeping field current at maximum, the armature voltage is increased to raise the speed till the voltage limit is reached (*constant-T drive*). After that, the armature voltage is kept at the limit and I_f is reduced to raise the speed till the armature current limit (rated value) is reached (*constant-kW drive*). The speed range can be as wide as 1:6.

This is the Ward-Leonard speed control

- Series Motor

The field winding is series connected and has very resistance ; $I_a (R_a - R_{se})$ drop can be ignored. Therefore

$$n \approx K_N \frac{V}{\Phi}, \Phi \propto I_a, T \propto I_a^2$$

As the load torque increases, I_a increases and so Φ increases and the speed reduces.

At light load, the speed can reach dangerous value. The motor should not be switched on at no load.

Ideal for traction type load

Speed Control

(i) tapped field control – field turns change

(ii) Series parallel field control – winding is in two halves which can be connected in series or parallels. Parallel connection is equivalent to half the total number of winding turns. Two speed operation.

(iii) Armature control – two identical motors are machanically coupled. The armatures can be connected in parallel (full voltage across each armature) or series (full voltage across each armature). Two speed operation.

(iv) Diverter control – a resistor is connected across the series field winding to reduce the winding current

- Compound Motor

Series field is provided along with the shunt field. It is comulative compound if series field and the shunt filed and it is differential compound if it opposes the shunt field.

Cumulative Compound

No load speed is controlled by the shunt field ; at heavily load the speed drops shortly due to the series excitation

Differential Compound

The series field by reducing flux keep the speed nearly constant till full-load.

Beyond that at heavy over load the series field causes the speed to increase to umacceptable levels. Not used in practice.

- Motor starting

  At start (n = 0) there is no back emf (E_a = 0) the starting current on direct start is unacceptably high (may be 100 times the rated current). So resistance in sries is connected when starting the motor to limit the starting current to about twice the rated value. As the motor speeds up the resistance is cut out in steps.

- Motor efficiency

  Constant loss (P_k) : core loss, windage and friction loss, shunt field loss (shunt motor)

  Variable loss : armature copper loss, $I_a^2 R_a$

  At η_{max}

  Variable loss = constant loss

$$I_a^2 R_a = P_k \text{ or } I_a = \sqrt{\frac{P_k}{R_a}}$$

Relationships To Remember (MUST)

$$n \propto \frac{E_a}{\Phi} \qquad T \propto \Phi I_a$$

If $\Phi \propto I_f$ (linear magnetization)

$$n \propto \frac{E_a}{I_f} \qquad T \propto I_f I_a, \qquad T \propto I_a^2 \text{ (series motor)}$$

$$\omega_m = \left(\frac{2\pi}{60}\right) n$$

Approximation

On no load $E_a \approx V$ (terminal)

This approximation may be used on load where less degrees of accuracy is acceptable.

REVIEW QUESTIONS

1. Why are the pole shoes made as large as 70% of pole pitch in a dc machine?
2. Comment on the statement “ The armature conductor current is alternating but non-sinusoidal”.
3. Discuss: Emf and torque of a dc machine depend on the flux/pole but are independent of the flux density distribution under the pole.

4. Write the expression for the induced emf and torque of a dc machine using standard symbols. What is the machine constant? What is the value of the constant relating ω_m and n?
5. Explain the meaning and significance of the critical field resistance of a shunt generator.
6. To how many coil ends is each commutator segment connected?
7. The two ends of coil in lap windings are connected to which commutator segment? What is the commutator pitch?
8. Why is the armature reaction in dc machine called cross-magnetizing? Can this affect the flux/pole?
9. What are inter-poles, their purpose, location and excitation? Explain each item.
10. Compare the number of parallel paths in the lap and wave windings.
11. State the condition which determines if a dc machine is generating or motor.
12. Write the expression relating the electrical power converted to the mechanical form in a dc motor. How are the electrical power input and mechanical power output different from these powers?
13. What is OCC and what information does it reveal about a dc machine? At what speed is it determined? What is thr air-gap line?
14. Write the basic proportionality relationships of a dc machine. What form do these take for linear magnetization?
15. State the types of dc motors. What is the basis of the classification?
16. Using emf and torque equation, explain how a dc motor has two powerful methods of speed control – through field excitation and armature voltage.
17. Sketch the speed-troque characteristic of a shunt motor at fixed field current. Explain the characteristic through relevant fundamental relationships of the machine
18. Sketch the speed-troque characteristic of a dc series motor and advance the underlying reasoning for the nature of the characteristic based on fundamental relationships of the dc machine.
19. Explain through sketch and derivations the speed-torque characteristic of a differentially compound dc motor.
20. Advance the methods of varying the shunt field and the series field excitation of a dc machine.
21. Discuss the method of speed control of a dc series motor.
22. How is a shunt motor started? Why should it not be started direct on line?
23. Why do we need a compensating winding and how is this winding excited and why?
24. Enumerate and classify the losses in a dc shunt motor.
25. How can you determine the load current of a dc shunt motor at which the motor efficiency is maximum?

PROBLEMS

10.1 The generator of Ex. 10.5 is now run at 1600 rpm.

(a) Find the no-load voltage and field current for a field resistance of 55 Ω.

(b) Find the value of critical field resistance and the critical speed.

(c) A load resistance of 0.8 Ω is connected across the generator terminals with the field resistance as in part (a). Find the generator terminal voltage and the load current.

Given: Generator armature resistance = 0.5 Ω.

Hint: Solve iteratively.

10.2 A shunt generator delivers 50 kW at 250 V and 400 rpm. The armature and field resistances are 0.02 Ω and 50 Ω respectively. Calculate the speed of the machine running as shunt motor and taking 50 kW input at 250 V.

10.3 In a 100 kW, 600 V, 1200 rpm shunt motor, the field resistance is 500 Ω and the armature and brush resistance is 0.13 Ω. The efficiency of the motor at rated output and speed is 90%. Find (a) the line current, (b) the field current, (c) induced emf, (d) mechanical power developed and (e) torque developed.

10.4 The resistance of the armature of a 250 V, 20 kW, 1200 rpm dc shunt motor is 0.252 Ω. When running light, the motor takes 6.32 A at rated voltage; the field current is 0.92 A while the speed is 1280 rpm. Determine (a) the speed when the motor takes 85 A with the field current remaining constant; (b) the speed when the current is 60 A.

10.5 A constant-torque load is being supplied by a 500 V dc shunt motor, having armature and field resistances of 0.8 Ω and 300 Ω respectively. The motor takes a current of 28 A from the mains when running on load at 750 rpm. Find the value of the field resistance which should be introduced in the field circuit to increase the speed to 1000 rpm. Find also the corresponding armature and line currents drawn by the motor. Assume linear magnetization.

10.6 A dc shunt motor operating at 300 V has a no-load speed of 1200 rpm. The motor is now reconnected to 600 V mains and draws an armature current of 150 A. Calculate the motor speed. Assume linear magnetization characteristic.

Given R_a = 0.2 Ω.

10.7 A 10 kW, 230 V dc shunt motor has armature resistance of 0.1 Ω. It runs at no load at a speed of 1500 rpm. When delivering a certain load, the motor draws an armature current of 200 A. Find the speed at which the motor will run at this load and the torque developed. Assume that the armature reaction on load causes a 4% reduction in the flux/pole compared to its no-load value.

10.8 A 250 V dc series motor has linear OCC with a slope of 12 V/field ampere at 1200 rpm. Find the speed at which the motor will run when developing a torque of 40 Nm. What current will it draw from the mains?

Given $R_a + R_{se} = 0.6$ Ω.

10.9 The armature and field resistances of a 25 kW, 250 V series motor are 0.12 Ω and 0.1 Ω respectively. The motor takes 85 A at a speed of 600 rpm. Find the motor speed when the motor takes (a) 100 A and (b) 40 A. (c) What will be current taken by the motor when it runs at 800 rpm? Assume straight line magnetization characteristic.

10.10 The armature and field resistances of a 60 kW, 600 V series railway motor are 0.215 Ω and 0.08 Ω respectively. At rated voltage and at a current of 80 A, the speed is 750 rpm. Find the speed and torque developed when the current is 95 A. Assume linear magnetization characteristic.

10.11 A 125 kW, 600 V dc series motor has $(R_a + R_{se}) = 0.15\ \Omega$. The full load current at rated voltage and speed is 220 A. The magnetization characteristic of the motor is assumed to be linear. It yielded an induced emf of 480 V at 600 rpm with a field current of 220 A.

(a) Calculate the motor speed at full-load current and rated voltage. Also calculate the full-load torque.

(b) The starting current is restricted to 300 A. Calculate the external resistance to be added in the motor circuit and the starting torque.

10.12 In Prob. 10.11, consider now that the motor has saturating type magnetization characteristic with two data points at 600 rpm given below:

EMF (V)	450	518
I_f (A)	220	300

Solve parts (a) and (b) once again.

10.13 Solve Ex. 10.14 assuming that the load torque remains constant as the field halves are reconnected in parallel.

10.14 A 500 V dc shunt motor when cold has a field resistance of 200 Ω and an armature resistance of 0.15 Ω. The motor runs at a no-load speed of 1000 rpm under cold conditions. After remaining continuously loaded at a load current of 70 A, field and armature temperatures rise to 40°C above the ambient temperature of 20°C. Calculate the motor speed under hot conditions. The temperature coefficient of copper = 1/234.5 per °C at 0°C.

10.15 A 220 V dc shunt motor has an armature resistance of 0.3 Ω and a field resistance of 200 Ω. The motor runs at 800 rpm with an armature current of 40 A. What resistance must be inserted in the field circuit to raise the motor speed to 1050 rpm, the load torque remaining constant? Assume linear magnetization characteristic.

10.16 A dc shunt motor having an armature resistance of 0.2 Ω takes 35 A from a 250 V supply and runs at 1500 rpm when driving (a) a load, the torque of which is proportional to the square of speed, and (b) a constant torque load. Calculate in each case the resistance to be added to the armature circuit to reduce the speed to 1200 rpm. Compare the loss in the external resistance in the two cases. Assume 100% mechanical efficiency.

10.17 A 115 V dc shunt motor draws an armature current of 25 A when running at 1500 rpm at full load torque. Motor armature circuit resistance is 0.3 Ω.

An additional resistance of 0.6 Ω is introduced in the armature circuit when the motor is operating at half full-load torque. Find the percentage change in the shunt field resistance which would cause the motor to have a speed of 1400 rpm.

10.18 In Ex. 10.14, what should be the armature resistance of the motor for it to have maximum efficiency while drawing 215 A? Calculate the value of the maximum efficiency.

10.19 A 15 kW, 220 V, 1200 rpm, 4-pole, dc shunt motor has 620 conductors connected as wave winding yielding an armature circuit resistance of 0.18 Ω. When delivering rated power at rated voltage and speed, it draws a line current of 79.8 A and a field current of 2.6 A. Calculate

(a) flux/pole,

(b) developed torque,

(c) rotational losses, and

(d) total losses and efficiency.

Chapter 11

SYNCHRONOUS MACHINE

MAIN GOALS AND OBJECTIVES

- *Excitation emf E_f of synchronous machine*
- *Concept of armature reactance equivalent of armature reaction, synchronous reactance (X_s)*
- *OCC and SC determination of X_s unsaturated and saturated*
- *Synchronizing synchronous machine to the mains*
- *Linear circuit model (equivalent circuit)*
- *Generating and motoring machine circuit equation, concept of power angle δ*
- *Performance characteristics*
  Fixed excitation with varying load
  Fixed load with varying excitation
  Synchronous Machine as a capacitor and as an inductor

11.1 INTRODUCTION

The constructional and operational features of a synchronous machine have been covered in fair detail in Chapter 9. The purpose of this chapter is to develop the circuit model of the synchronous machine and therefrom determine its operational qualities and features. It is one of the most important machines which is employed for large scale power generation in hundreds of megawatts. Its very special feature is the *spring-like* stator and rotor *synchronous link* with the rotor running at synchronous speed $n_s = 120\, f/P$. It is this link which enables the synchronous operation and power exchange between several generators located at vast distances through transmission lines. As a motor also, the synchronous machine serves the special purpose of speed constancy irrespective of load and a variable power factor operation.

Brush-up

Before we get into circuit modeling of the synchronous machine, it is necessary to brush-up the knowledge acquired in Chapter 9.

The stator of the synchronous machine carries the 3-phase *armature windings* which are the seat of the induced *emf* feeding current and exchanging power with the external circuit. The rotor carries the field poles which are *dc* excited through slip-rings from a small dc generator (exciter) carried as the machine shaft. The reader may refer to the cross-sectional view of a synchronous machine provided in Fig. 9.2. In this view, the rotor has *salient poles* (projecting poles) with concentrated field windings. The other constructional details of the machine are revealed by the half cross-sectional longitudinal view of Fig. 9.4. The cross-sectional view of the *non-salient* (cylindrical) pole machine is presented in Fig. 9. 6 with distributed field windings.

The air-gap B-wave is made nearly sinusoidal by chamfered salient poles and distributed windings for the non-salient pole machine.

To generate emf of frequency f Hz, the rotor must run at synchronous speed

$$n_s = \frac{120f}{P} \text{ rpm}$$

The 50 Hz machines are universal except in USA where the frequency is 60 Hz.

The armature (stator) has 3-phase distributed 2-layer winding with full pitch (or short-pitch) coils. The per phase emf induced in the armature phase by the rotating field vector $\vec{F}_f$

$$E_p \text{ (rms)} = \sqrt{2}\ \pi f N_{ph} \text{ (series) } \Phi \quad V;\ \sqrt{2}\ \pi = 4.44 \tag{11.1}$$

The flux/pole Φ is created by the sinusoidally distributed vector $\vec{F}_f$.

When the armature carries balanced 3-phase currents, it produces its own synchronously rotating field in the same direction as the rotor field. The two fields lock into each other and run at the synchronous speed (n_s). This is the *synchronous link.* The machine produces a torque

$$\begin{aligned} T &= F_1 F_2 \sin \lambda \\ &= F_r F_2 \sin \delta \end{aligned}$$

where F_1, F_2 are the peak values of the stator and rotor sinusoidally distributed (in space) field vectors and F_r is the resultant air-gap field.

λ = space angle between the axes of $\vec{F}_1$ and $\vec{F}_2$

δ = space angle between the axes of $\vec{F}_r$ and $\vec{F}_2$

- Generating action results when $\vec{F}_2$ leads $\vec{F}_r$ by angle δ
- Motoring action results when $\vec{F}_2$ lags $\vec{F}_r$ by angle δ

δ is know as the *torque* or *power angle.* If δ increases beyond 90°, the synchronous link is snapped, the machine operation is disrupted. The machine is said to *pull-out of stop* or *lose synchronism.*

Change in nomenclature

Stator field $\vec{\mathbf{F}}_2$ will be labeled as *armature reaction (AR),* $\vec{\mathbf{F}}_a$.

Rotor filed $\vec{\mathbf{F}}_2$ will be labeled as $\vec{\mathbf{F}}_f$.

Resultant field with continue to be labeled as $\vec{\mathbf{F}}_r$.

Resultant field, $\vec{\mathbf{F}}_r = \vec{\mathbf{F}}_f + \vec{\mathbf{F}}_a$. (11.2)

11.2 CIRCUIT MODEL (EQUIVALENT CIRCUIT)

Heuristic Treatment

Rather than finding the resultant field and therefrom determining the induced emf of the machine., we will proceed on the basis that the magenetic current is linear because of the predominant air-gap. It means that by the superposition theorem we can add the emf phasors corresponding to the individual fields to determine the resultant emf phasor. It implies that instead of the field Eq. (11.2), we use the phasor emf equation.

$$\overline{E}_r = \overline{E}_f + \overline{E}_a \tag{11.3}$$

We shall ignore at present the resistance and leakage reactance of the armature. Therefore, the terminal voltage

$$\overline{V}_t = \overline{E}_r \tag{11.4}$$

To model Eq. (11.3) as a circuit, we need to identify $\overline{E}_a$ as the voltage drop of a circuit element. We immendiately notice that $\overline{E}_a \propto \overline{I}_a$ as it is caused by $\vec{F}_a$. Further, we need to determine the effect on $\overline{E}_a$ as the phase angle of $\overline{I}_a$ with respect to $\overline{E}_f$ is varied over –90° to 0° to + 90°. We assume that $\overline{I}_a$ flows in the direction of $\overline{E}_f$ which means that the machine is *generating*. We proceed as per phase basis; phase 'a'

At *non-load* $\overline{I}_a = 0$ and so $\vec{F}_a = 0$. The rotor field $\vec{F}_f$ induces emf $\overline{E}_f$, which is called the *excitation* emf.

Case 1. $\overline{I}_a$ lags $\overline{E}_f$ by 90°

It means that the current maximum occurs 90° later than the emf maximum. Therefore, when the current of phase 'a' is maximum, its emf is zero and so $\vec{F}_f$ lies along the axis of phase 'a' coil as shown in Fig. 11.1 (a). The armature reaction field is also directed along the axis of phase 'a' coil but in opposition to $\vec{F}_f$. Thus the armature reaction is *demagnetizing* and the emf $\overline{E}_a$ is in phase opposition to $\overline{E}_f$. Thus $V_t = E_r < E_f$.

Case 2. $\overline{I}_a$ in-phase with $\overline{E}_f$

On the arguments as in Case 1, the 3-phase currents and the corresponding locations of $\vec{F}_f$ and $\vec{F}_a$ are shown in Fig. 11.1 (b). We find that $\vec{F}_a$ is *cross-magnetizing* and therefore, it will have only marginal effect on $\vec{F}_r$. So $V_t = E_r \approx E_f$.

Case 3. $\overline{I}_a$ *leading* $\overline{E}_f$ *by* 90°

This corresponds to the currents and pole positions as shown in Fig. 11.1 (c). $\vec{F}_a$ now directly aids $\vec{F}_f$, which means it is *magnetizing*. Therefore, $\overline{E}_a$ is in phase with $\overline{E}_f$ and so $V_t = E_r > E_f$ for the generating machine.

The above illustrated effect of the armature reaction field on the resultant induced emf can be simulated by the circuit model as shown in Fig. 11.2. The series inductive reactance X_a equivalently replaces the armature reaction induced emf $\overline{E}_a$. The generating circuit equation is

$$\overline{V}_t = \overline{E}_f + \overline{E}_a = \overline{E}_f - jX_a\overline{I}_a \tag{11.5}$$

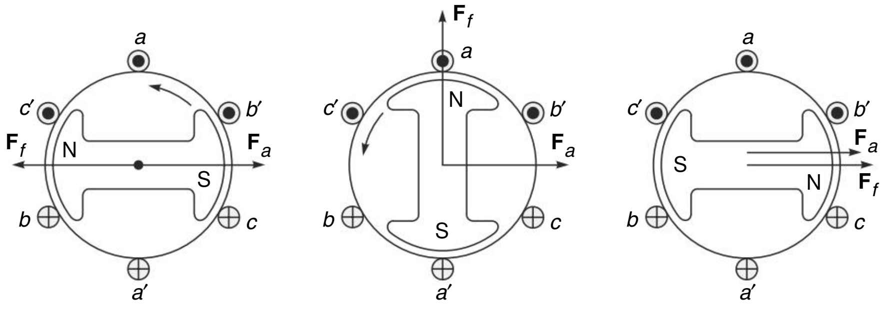


Fig. 11.1 Air-gap field picture for various armature current phase angles

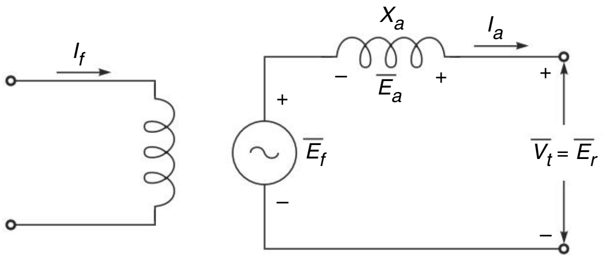


Fig. 11.2 Circuit model of synchronous machine, generating

where

$$\bar{E}_f = \text{excitation emf}$$

$$\bar{E}_a = \text{armature reaction emf} = -(-j\,X_a\,I_a)$$

The corresponding phasor diagram for $\bar{I}_a$, 90° lagging, in-phase and 90° leading are drawn respectively in Figs 11.3(a), (b) and (c). These phase diagrams confirm the conclusions drawn earlier in Cases 1. 2 and 3.

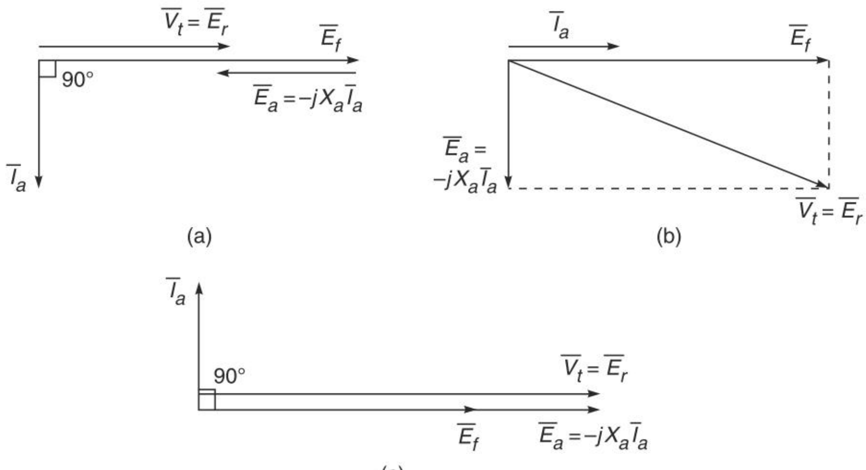


Fig. 11.3 Phasor diagram of synchronous machine generating

$$\bar{V}_t = \bar{E}_f + \bar{E}_a = \bar{E}_f - jX_a\bar{I}_a$$

$\bar{E}_f$ = excitation emf, $\bar{E}_a$ emf induced by armature reaction field

Circuit Model (Derivation via MMF Diagram)*

The magnetic circuit shall be assumed to be linear as the air-gap reluctance dominates. The superposition theorem therefore applies for the magnetic fields.

The rotor and stator mmfs are both sinusoidally distributed in space and rotate synchronously in space. Seen from any given point on the stator, these mmfs appear alternating (sinusoidally) in time and can be represented as phasors. The flux (linkages) phasor is in phase with the mmf phasor that causes it and its induced emf lags it by 90° (Fig. 9.12). It is intuitive to assume that the armature mmf phasor, called *armature reaction*, is in phase with armature current.

Let it be further assumed that the stator winding is devoid of any resistance and leakage reactance. It implies that the machine terminal voltage equals its resultant induced emf. The phasor diagram of the generating machine (positive current in the same direction as positive direction of induced emf) is drawn in Fig. 11.4. Here

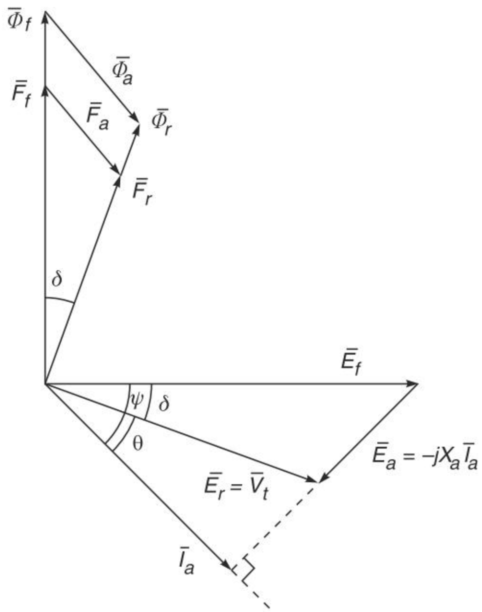


Fig. 11.4 Phasor diagram of synchronous machine (generating)

$\bar{F}_f, \bar{\Phi}_f$ = mmf and flux caused by dc rotor excitation
$\bar{F}_a, \bar{\Phi}_a$ = mmf and flux caused by stator current
$\bar{F}_r, \bar{\Phi}_r$ = resultant mmf and flux

Correspondingly

$\bar{E}_f$ = Phase emf induced by $\bar{\Phi}_f$ acting alone. It is called the *excitation emf.*
$\bar{E}_a$ = phase emf induced by armature reaction flux, $\bar{\Phi}_a$
$\bar{E}_r$ = phase emf induced by resultant flux, $\bar{\Phi}_r$
$\bar{I}_a$ = phase armature current

*This section may be skipped without loss of continuity if time does not permit

It is easily observed from the phasor diagram that $\bar{F}_f$ leads $\bar{F}_r$ by angle δ (a condition for generating machine). $\bar{E}_f$ leads $\bar{E}_r$ by the same angle δ.

In a motoring machine, the armature current is assumed to flow in opposition to induced emf (like in a dc machine, Sec. 10.4). $-\bar{F}_a$ is then in the direction of $-\bar{I}_a$, which is then the generating current. The reader should attempt this phasor diagram. Also the angle δ will become negative, i.e. $\bar{E}_f$ will lag $\bar{E}_r$.

From the phasor diagram of Fig. 11.4,

$$\bar{V}_t = \bar{E}_r = \bar{E}_f + \bar{E}_a \tag{11.6a}$$

But

$$\begin{aligned}\bar{E}_a &= -jK_e\ \bar{\Phi}_a;\ K_e = \text{machine emf constant}\\ &= -jK_e\ \mathcal{P}\bar{F}_a;\ \mathcal{P} = \text{per pole permeance of machine}\\ &= -jK_e\ \mathcal{P}K_a I_a;\ K_a = \text{armature winding constant}\end{aligned}$$

or

$$\bar{E}_a = -jX_a I_a \tag{11.6b}$$

where

$$\begin{aligned}X_a &= K_e\mathcal{P}K_a\\ &= \text{machine constant (inductive reactance equivalent of armature reaction); it will be higher for a machine with higher permeance, i.e. smaller air-gap}\end{aligned}$$

Substituting Eq. (11.6 (b)) in Eq. (11.6(a))

$$\bar{V}_t = \bar{E}_r = \bar{E}_f - jX_a\ \bar{I}_a \tag{11.7}$$

whose circuit representation is given in Fig. 11.2(a). It is easily seen from Eq. (11.7) that X_a has the nature of a reactance and it equivalently replaces the effect of the armature reaction field.

Circuit Model Inclusive of Armature Leakage Reactance

The circuit model of Fig. 11.5(a) or Fig. 11.2 modifies to that of Fig. 11.5(b) if the armature leakage reactance is accounted for. We define

$$\begin{aligned}X_s &= X_a + X_l;\ X_l = \text{leakage reactance} \qquad (11.8)\\ &= \textit{synchronous reactance (per phase)}\end{aligned}$$

In the simplified model of Fig. 11.5(c) the identity of E_r is not preserved as it is no longer necessary to do so. Observe that the circuit model is the same as in a transformer except that E_f is controlled by I_f.

In the ciruit model of a sychronous machine, the machine is *generating* when the armature current $\bar{I}_a$ is in the direction of $\bar{E}_f$ and so flows out of the positive terminal of the machine as shown in Fig. 11.5 (c). The machine is *motoring* when the armature current $\bar{I}_a$ flows in the positive terminal of the machine ans so in opposition to $\bar{E}_f$ as shown inf Fig. 11.6

The circuit equations for the two operations of the sychronous machine are:

$$\bar{V}_t = \bar{E}_f - jX_s\ \bar{I}_a\ \text{(generating)} \tag{11.9a}$$

$$= \bar{E}_f + jX_s\ \bar{I}_a\ \text{(motoring)} \tag{11.9b}$$

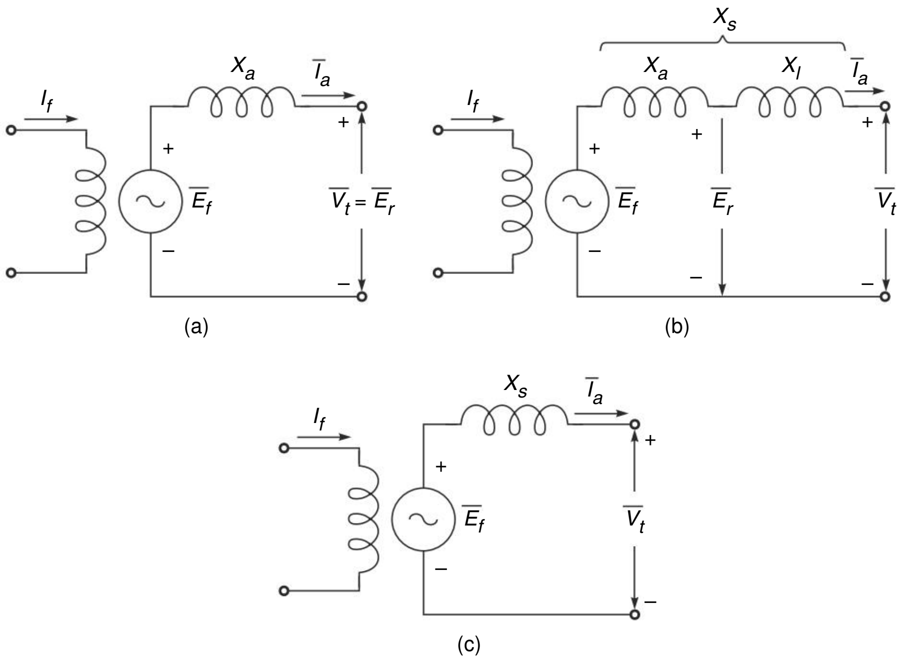


Fig. 11.5 Synchronous machine (generating)

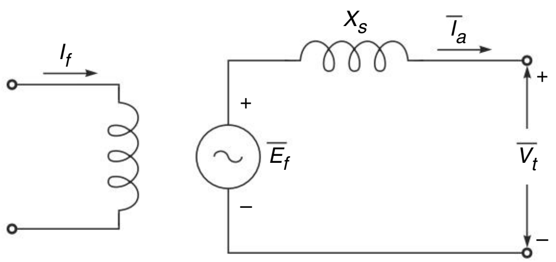


Fig. 11.6 Synchronous machine (motoring)

If the machine resistance is taken into account, the total impedance per phase would be

$$Z_s = [R_a^2 + X_a^2]^{1/2} = \textit{synchronous impedance}$$

The armature resistance in a synchronous machine is usually as low as 0.01 pu and can be ignored for all performance calculations except machine efficiency. The synchronous reactance is of the order of 0.5–1.0 pu (compare with transformer leakage reactance of 0.05 pu).

Determination of Synchronous Reactance

As in a transformer, X_s can be determined by the OC and SC tests as in the circuit diagram of Fig. 11.7. Under OC conditions, i.e. $I_a = 0$, it follows from Eq. (11.9a) that

$$E_f = \frac{Voc}{\sqrt{3}} \tag{11.10}$$

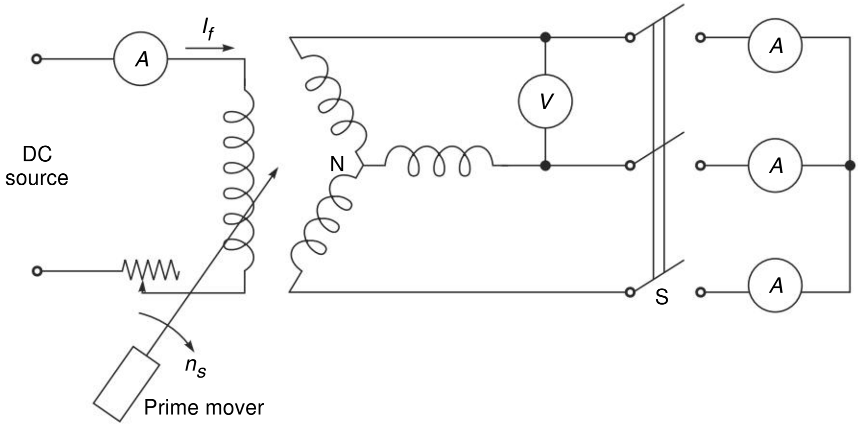


Fig. 11.7 Circuit diagram for *OC* and *SC* tests

where V_{OC} is the line-to-line open-circuit voltage. The open-circuit characteristic (*OCC*) is the plot of V_{OC} vs. I_f, which indeed is the magnetization characteristic of the machine and is drawn in Fig. 11.8. The linear part of the *OCC* is the *air-gap line*.

The *SC* test is carried out with field initially open and the armature switch S (Fig. 11.7) closed. The field current I_f is gradually raised till the armature current reaches I_a (rated). Because of armature short circuit, this would occur at very much reduced excitation (very low I_f). The short circuit characteristic (*SCC*) is the plot of I_{SC} vs I_f shown in Fig. 11.8. It is linear as the magnetic circuit is unsaturated (low I_f). Therefore, only one point on the *SCC* need to be determined.

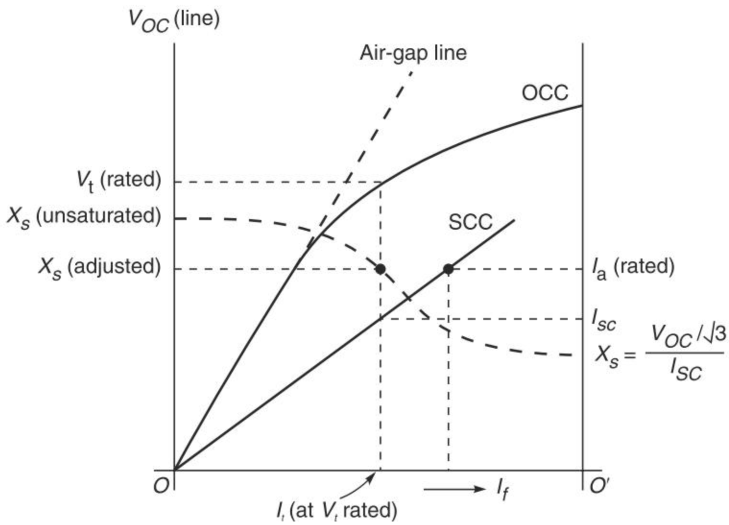


Fig. 11.8 *OCC*, *SCC* and determination of X_s

The *unsaturated synchronous reactance* is then found from the air-gap line and the SCC corresponding to the same value of I_f. It follows from Fig. 11.5 (c) and Eq. 11.9 (a) with $V_t = 0$ that

$$X_s \text{ (unsaturated)} = \left.\frac{V_{OC}/\sqrt{3}}{I_{SC}}\right|_{\text{at same } I_f} \tag{11.11}$$

X_s as obtained from various points corresponding to the OCC reduces sharply in the saturated region (dotted curve in Fig. 11.8). Under load conditions the machine is operated under a somewhat saturated magnetic condition when X_s will be less than that in the unsaturated region. It is therefore more realistic to use X_s (adjusted) obtained as below.

$$X_s \text{ (adjusted)} = \left.\frac{V_{oc}/\sqrt{3}}{I_{SC}}\right|_{\substack{I_f \text{ corresponding to} \\ V_t\text{(rated) on } OCC}} \tag{11.12}$$

This is shown in Fig. 11.8.

Voltage Regulation

It is the percentage change in the terminal voltage of a synchronous generator as full load at a specified power factor and rated voltage is thrown off. Thus

$$\text{Voltage Reg} = \left.\frac{V_t(\text{no load})\big|_{I_f \text{ same as at full load}} - V_t(\text{rated})}{V_t(\text{rated})}\right|_{\text{At specified } pf} \tag{11.13}$$

Obviously

$$V_t \text{ (no load)} = E_f \text{ (excitation emf)}$$

It is convenient to take all voltages in line-to-line.

As in a transformer, the voltage regulation can be positive, zero or negative depending upon the load *pf*. Unlike a transformer, the pu synchronous reactance of a synchronous machine is far larger in the range 0.5–0.8 pu or even 1.0. Voltage calculation must therefore proceed using Eq. (11.9a) and no approximation can be used.

Example 11.1 The *OC* and *SC* test data of a 3-phase 1 MVA, 6.6 kV, star-connected synchronous generator is given below:

I_f (A)	60	70	80	90	100	110
V_{oc} (line) (V)	4693	5500	6160	6600	6967	7260
SC (A)	98					

Find

(a) unsaturated synchronous reactance,

(b) adjusted synchronous reactance,

(c) excitation voltage needed to give rated voltage at full load, 0.8 pf lagging. Use adjusted synchronous reactance.

(d) also the voltage regulation for the load specified in part (c).

Solution The *OCC* and *SCC* are plotted in Fig. 11.9.

(a) Corresponding to $I_f = 60$ A (unsaturated region)

$$X_s \text{ (unsaturated)} = \frac{4700\sqrt{3}}{98}$$
$$= 27.7\ \Omega$$

(b) Corresponding to $V_{\text{rated}} = 6600$ V

$$X_s \text{ (adjusted)} = \frac{6600\sqrt{3}}{143}$$
$$= 26.65\ \Omega$$

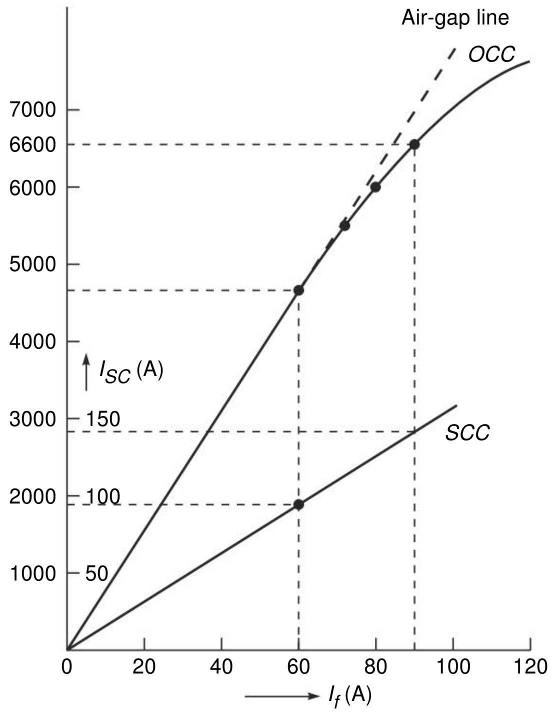


Fig. 11.9

(c) At full load, 0.8 *pf* lagging

$$I_a = \frac{100}{\sqrt{3} \times 6.6} = 87.48 \text{ A}$$

$$pf \text{ angle} = 39.9° \text{ lag}$$

$$\bar{I}_a = 87.48\,(0.8 - j\,0.6) = 70 - j\,52.5$$

From Eq. (11.9a)

$$\bar{E}_f = \bar{V}_t + j\,X_s\,\bar{I}_a$$

$$\bar{E}_f = 6600\ \angle 0° + j\,26.65\,(70 - j\,52.5)\,\sqrt{3};$$

(in line values)

$$= 6600 + 2423 + j\,3225 = 9023 + j\,3225$$

$$E_f = 9582 \text{ V (excitation voltage)}$$

(d) Voltage regulation $= \dfrac{9582 - 6600}{6600} \times 100$

$= 45.2\%$

Example 11.2 A 30 kW, 1000 rpm, 3-phase, 50 Hz, 440 V synchronous motor has a stator resistance of 0.2 Ω per phase, a field resistance of 35 Ω and synchronous reactance of 1.8 Ω per phase. Rotational losses are estimated to be 1500 W.

At stator input of 45 kVA, 0.8 pf leading and voltage, calculate the shaft power output, field current and efficiency.

Assume the magnetization curve to be linear with a slope of 85 V (line)/field ampere.

Solution

Referring to Fig. 11.6

$$\bar{I}_a = \frac{45 \quad 1000}{\sqrt{3} \quad 440} = 59 \text{ A}$$

$$pf = 0.8 \text{ leading}$$

$$\bar{I}_a = 59 \angle 36.9° \text{ A}$$

$$\bar{V}_t = \frac{440}{\sqrt{3}} \angle 0° = 254 \angle 0° \text{ V}$$

$$\bar{E}_f = 254 \angle 0° - 59 \angle 36.9° \times (0.2 + j\,1.8); \text{ Eq. 11.9(b)}$$

$$= 254 \angle 0° - 59 \times 1.81 \angle (36.9° + 83.7°)$$

$$= 308.4 - j\,92$$

$$E_f = 322 \text{ V}$$

$$\therefore \qquad I_f = \frac{322}{85} = 3.78 \text{ A}$$

Electrical power input to motor $= 45 \times 0.8 = 36$ kW

Field loss $= (3.78)^2 \times 35 = 0.5$ kW

Total power input to (ac + dc) $= 36.5$ kW

Stator copper loss $= 3 \times (59)^2 \times 0.2 = 2.088$ kW

Shaft power developed $= 36 - 2.088 = 33.912$ kW

Rotational losses $= 1.5$ kW

Net shaft power output $= 33.912 - 1.5 = 32.41$ kW

$$\eta = \frac{32.41}{36.5} = 88.8\%$$

Synchronizing to Mains For synchronizing to mains, the machine is run as a generator with terminals arranged to have the same phase sequence as the mains. Its speed and field current are adjusted such that

– the machine terminal voltage is nearly equal to that of the mains, and

– the machine frequency is nearly equal to that of the mains, i.e. its speed is close to synchronous.

The connection diagram is shown in Fig. 11.10. The two sets of 3-phase phasors rotate with respect to each other at the difference in their frequencies as shown in Fig. 11.11. The rms voltages V_{L1}, V_{L2} and V_{L3} respectively across lamps L_1, L_2 and L_3 oscillate at the difference frequency. At the instant of synchronization, when the two sets of phasors are co-phasal, $V_{L1} = 0$, $V_{L2} = V_{L3}$, i.e. lamp L_1 is dark and L_2, L_3 are equally bright. The machine is switched on the mains.

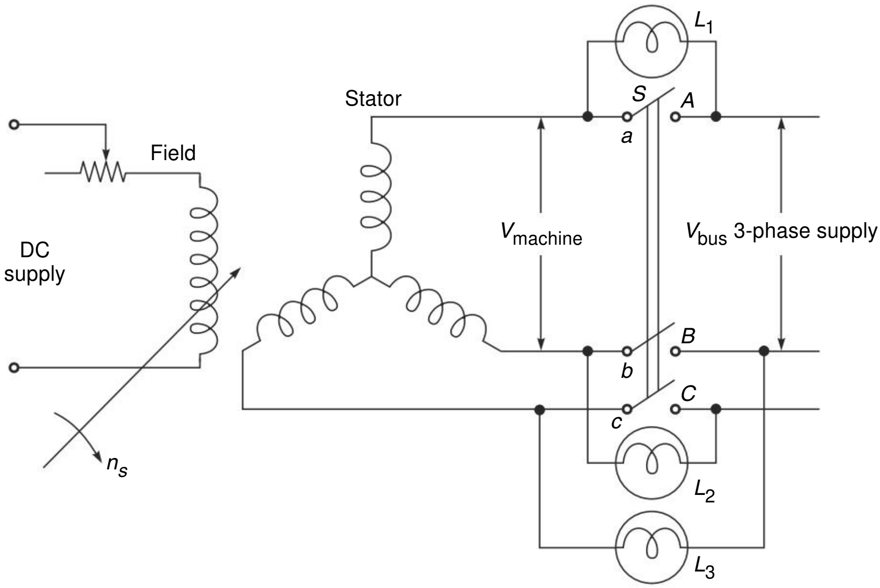


Fig. 11.10 Synchronizing to mains

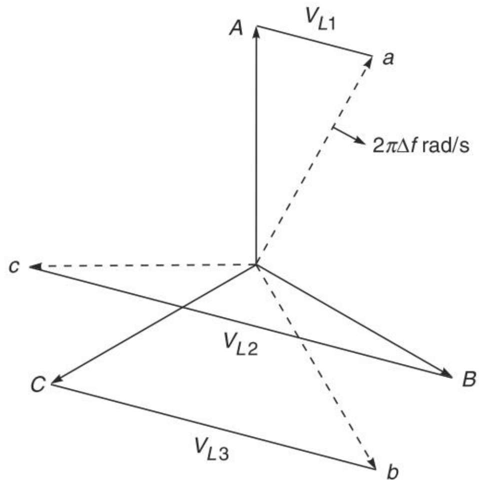


Fig. 11.11 Determining instant of synchronization

Acceptable phase difference in the two phasor sets is about 5°. For larger angular difference, the machine would get a current and torque jolt and may not synchronize (falls out of step). Instead of lamps, an instrument called *synchroscope* is employed in generating stations. Instrumentation schemes have been devised for complete autosynchronization.

The machine after synchronization would act as a generator or motor depending upon the mechanical conditions at its shaft.

It is immediately obvious from the above that to start a synchronous motor, a small pilot motor (induction type) must be coupled to it to bring it to the speed for synchronization.

Damper Winding Spring-like synchronous link along with rotor inertia results in oscillations, called *hunting*, initiated by disturbances of the electrical or mechanical sides of the machine. These oscillations are very undesirable electrically and would also fatigue the shaft. These are damped out by providing short-circuited copper bars, known as damper or *amortessur winding*, placed in the rotor pole faces. Damper winding because of induced currents when rotor oscillates wrt the rotating field produces the desired damping effect (damper torque always opposes the oscillatory movement).

Induced currents in the damper winding when the stator is switched on to the supply provide the starting torque (induction principle) for a synchronous motor. The field is switched on after the rotor reaches close to synchronous speed. Such an induction start (Chapter 12) synchronous motor is known as *synduction motor*.

11.3 OPERATING CHARACTERISTICS

Power–angle characteristic With reference to the circuit model of Fig. 11.12.

$$\bar{E}_f = \bar{V}_t + jX_s \bar{I}_a; \text{ generating mode} \tag{11.14a}$$

$$\bar{E}_f = \bar{V}_t - jX_s \bar{I}_a; \text{ motoring mode} \tag{11.14b}$$

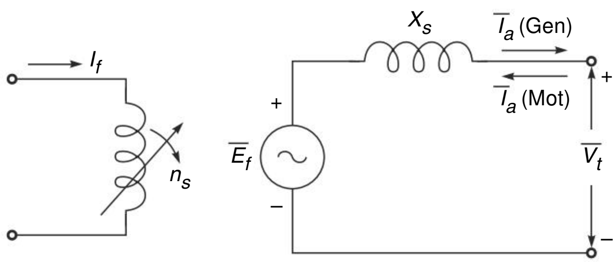


Fig. 11.12 Synchronous machine—generating/motoring modes

The phasor diagrams for the two modes as per Eqs (11.14a) and (11.14b) are drawn in Figs. 11.13(a) and (b). It is easily seen from these figures that

$\bar{E}_f$ leads $\bar{V}_t$ by angle δ in generating mode

$\bar{E}_f$ lags $\bar{V}_t$ by angle δ in motoring mode

These conclusions are parallel to those arrived at from field viewpoint in Sec. 9.8.

It easily follows from the geometry of the phasor diagrams of Figs. 11.13(a) and (b) that

$$PN = I_a X_s \cos \theta = E_f \sin \delta \tag{11.15}$$

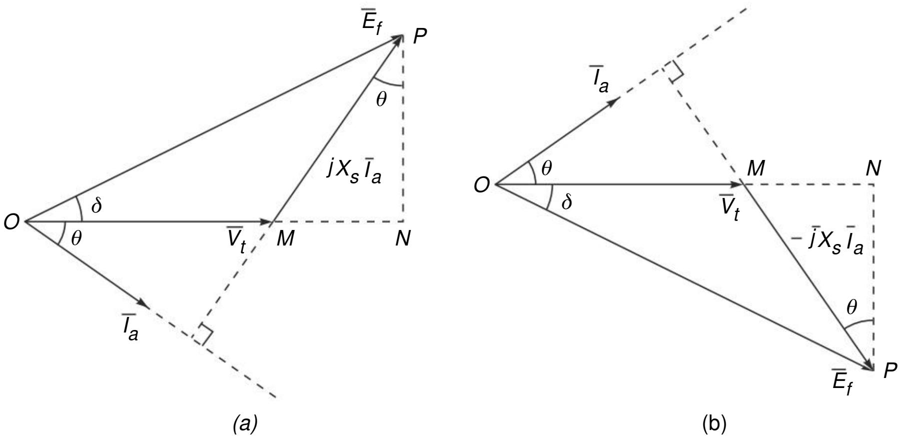


Fig. 11.13 Synchronous machine phasor diagram; constant excitation, variable load (a) Generating mode–lagging *pf,* (b) Motoring mode–leading *pf*

Multiplying Eq. (11.15) by V_t on both sides and manipulating

$$V_t I_a \cos \theta = \frac{V_t E_f}{X_s} \sin \delta \tag{11.16}$$

or
$$P_e = \frac{V_t E_f}{X_s} \sin \delta \tag{11.17}$$

where $P_e = V_t I_a \cos \theta$

= electrical power delivered to/drawn from mains

δ = angle by which E_f leads/lags V_t and is called the *power* (or *torque) angle*

This angle is not the same δ as in Fig. 9.24 as this takes into account the leakage reactance of the machine but notice the similarity in form.

The plot of Eq. (11.17) drawn in Fig. 11.14 is known as the *power–angle characteristic*. The motoring region of the machine has negative δ with the machine drawing electrical power and delivering mechanical power. For a given V_t and excitation emf E_f, δ is controlled by mechanical conditions at the shaft—positive δ when shaft power is input to the machine by a prime mover and negative δ when shaft power is drawn by a mechanical load. The machine loses synchronism for values of the electrical power demanded from a generator is more than $P_{e\,\max}$, or the mechanical load on a motor is more than $P_{e\,\max}$. In case of motors loss of synchronism is referred to as being "pulled out" of synchronism. For a machine operating at a value of δ even smaller than 90°, transient disturbances can cause it to lose synchronism. Steady value of δ is rarely allowed to exceed 30°.

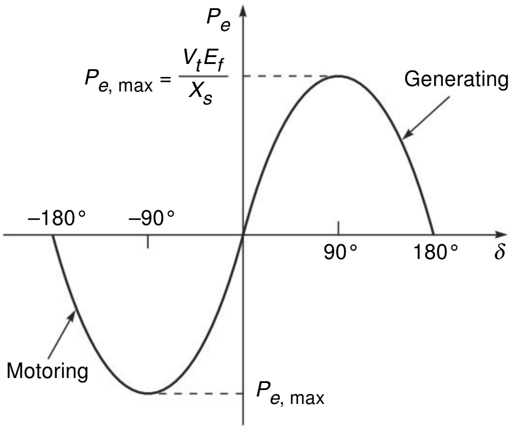


Fig. 11.14 Power–angle characteristic

Operation at constant load variable excitation For this type of operation, from Eqs (11.17) and (11.16)

$$E_f \sin \delta = \frac{P_e X_s}{V_t} = \text{constant} \tag{11.18a}$$

and

$$I_a \cos \theta = \frac{P_e}{V_t} = \text{constant} \tag{11.18b}$$

The motoring mode phasor diagram is drawn in Fig. 11.15 for unity power factor. The corresponding excitation is called *normal*. The loci of E_f and I_a are shown dotted in this figure. The cases of over excitation and under excitation are illustrated in Fig. 11.16 from which it can be concluded that an over excited motor draws leading current (acts like a capacitive load) but an under excited motor draws lagging current (acts like an inductive load).

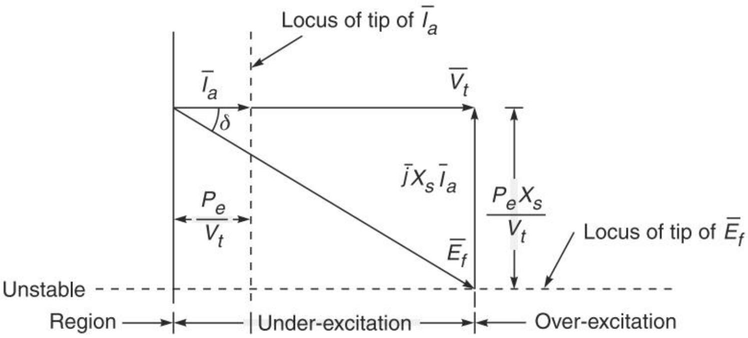


Fig. 11.15 Motoring machine—normal excitation

The case of variable generating machine excitation is just the reverse. Both these cases are summarized below:

Over-excitation: Leading *pf* in motoring mode; lagging *pf* in generating mode

Under-excitation: Lagging *pf* in motoring mode; leading *pf* in generating mode

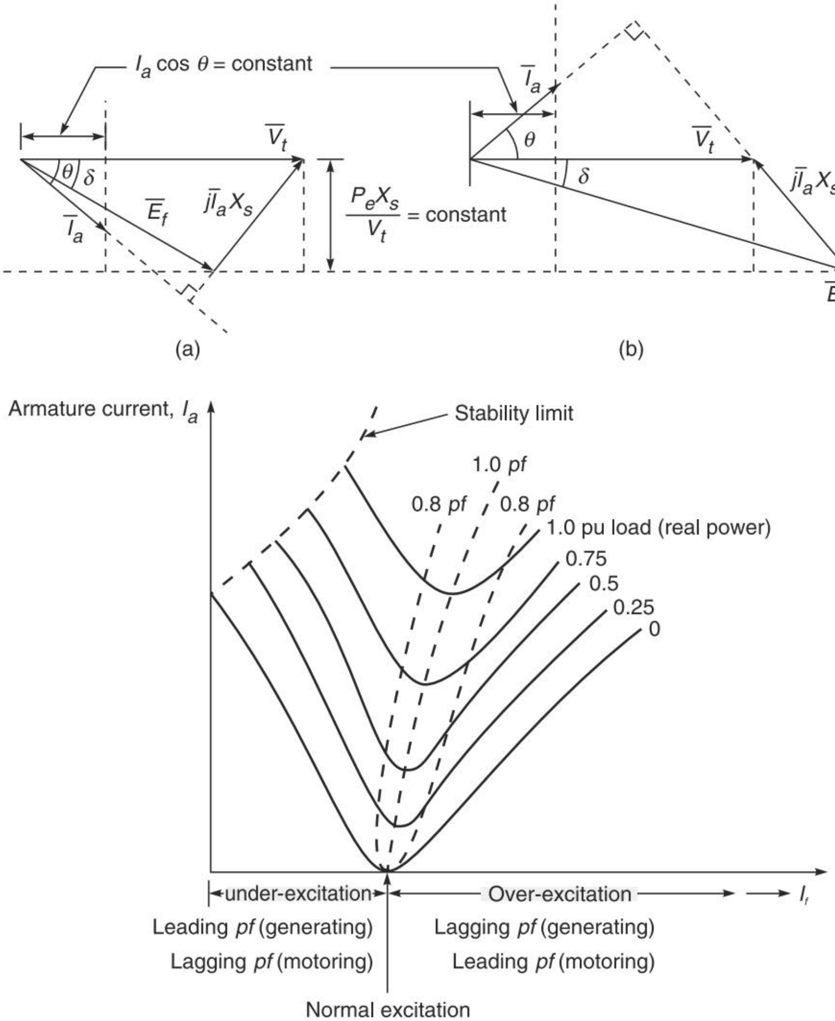


Fig. 11.16 (a) Under-excitation (lagging current); motoring
(b) Over-excitation (leading current); motoring machine
(c) *V*-curves of synchronous machine
(constant load (real power)), variable excitation

An unloaded synchronous motor may be used as a variable condenser or inductor by varying its excitation.

The plots of the variation of armature current of a synchronous machine for constant real power load but with variable excitation are shown in Fig. 12.16(c). These are known as *V-curves*.

Example 12.3 A 3-phase synchronous motor is synchronized to the mains at a terminal voltage of 12.5 kV. It has a synchronous reactance of 8.0 Ω.
Assuming the motor to be unloaded and neglecting rotational loss, draw the phasor

diagram and compute the current, active and reactive power drawn from the mains and power factor in the following two cases:

(a) The field current is raised to increase the machine excitation by 20% (over excitation)

(b) The field current is reduced to decrease the machine excitation by 20% (under-excitation).

Note: It is to be understood that at the time of synchronization, the machine excitation voltage is the same as the terminal voltage as the machine is just *floating* on the bus bars.

Solution

– Since motoring operation is considered, the direction of positive current is into the machine; (Fig. 12.6).

– Since the motor is on no load (also no rotational loss), power angle stays at zero; $\delta = 0$ and E_f and V_t are always in phase.

$$V_t = \frac{12.5}{\sqrt{3}} = 7.217 \text{ kV/phase}$$

At no load $\quad E_f = V_t = 7.217$ kV

(a) $\quad E_f = 7.217 \times 1.2 = 8.66$ kV/phase, $V_t = 7.217$ kV (mains)

The phasor diagram is drawn in Fig. 11.17. Observe that $\bar{I}_a$ must lead by 90°.

Now

$$I_a = \frac{8.66 - 7.217}{8} = 0.1804 \text{ kA}$$

Phase angle $\quad \theta = 90°$ lead

$$pf = 0 \text{ lead}$$

$$P_e = 0$$

$$Q_e = -\sqrt{3} \times 12.5 \times 0.1804$$

$$= -3.906 \text{ MVAR}$$

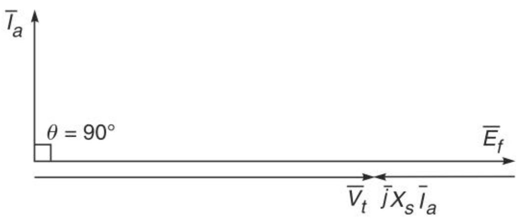


Fig. 11.17

(b) $\quad V_t = 7.217$ kV/phase

$$E_f = 7.217 \times 0.8 = 5.774 \text{ kV/phase}$$

The phasor diagram is drawn in Fig. 11.18. Observe that $\bar{I}_a$ must lag by 90°.

Now

$$I_a = \frac{7.217 - 5.774}{8} = 0.1804 \text{ kV}$$

$$\theta = 90° \text{ lag}$$

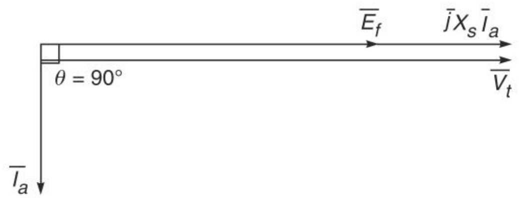


Fig. 11.18

$$pf = 0 \text{ lag}$$
$$P_e = 0$$
$$Q_e = +\sqrt{3} \times 12.5 \times 0.1804$$
$$= +3.906 \text{ MVAR}$$

Remark: When over-excited, the motor draws – 3.906 MVAR (or delivers + 3.906 MVAR to the mains) while when under-excited it draws + 3.906 MVA (or delivers – 3.906 MVA to the mains).

Example 11.4 The motor of Ex. 11.3 is run as a generator (it is now coupled to a prime mover) and it delivers 10 MW to the bus bars while it is over-excited by 20%. Find the current, pf and MVAR delivered.

Solution Since the generator is delivering active power to the bus bars (10 MW), the steam the valve of the turbine must have been opened. Now

$$P_e = 3\left(\frac{V_t E_f}{X_s}\right) \sin \delta$$

$$10 = 3 \times \left(\frac{7.217 \times 8.66}{8}\right) \sin \delta$$

or $$\delta = 25.3°$$

From Eq. (12.10a)

$$\bar{I}_a = \frac{\bar{E}_t - \bar{V}_t}{jX_s}\text{ ; } \bar{E}_f \text{ lead } \bar{V}_t \text{ by } 253.3°$$

$$= \frac{8.66\angle 25.3° - 7.217\angle 0°}{j8}$$

$$= 0.4626 - j\,0.0765 \text{ kA}$$

$$\bar{I}_a = 0.4689 \angle -9.4°$$

$$I_a = 0.4689 \text{ kA; } pf = 0.987 \text{ lag}$$

$$Q_e = +\sqrt{3} \times 12.5 \times 0.0765$$

$$= +1.656 \text{ MVAR}$$

Example 11.5 A 1000 kW, star-connected, 3.3 kV, 24-pole, 50 Hz synchronous motor has a synchronous reactance of 3.24 Ω; the resistance being negligible.

The motor is fed from infinite bus bars at 3.3 kV. It is drawing rated power at 0.9 *pf* leading from the bus bars. Calculate the maximum power and torque the motor can deliver while its excitation is maintained constant. Draw the phasor diagram under this condition. Find also the current, pf and reactive power drawn.

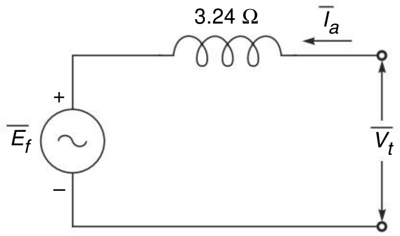


Fig. 11.19

Solution With reference to Fig. 11.19.

$$P_e\text{ (in)} = 1000 \text{ kW at } 0.9\ pf \text{ leading}$$

$$I_a = \frac{1000}{\sqrt{3} \times 3.33 \times 0.9} = 194.4 \text{ A}$$

$$\theta = \cos^{-1} 0.9 = 25.8°$$

or $$\bar{I}_a = 194.4 \angle 25.8°$$

$$V_t = \frac{3300}{\sqrt{3}} = 1905 \text{ V}$$

Now $$\bar{E}_f = 1905 \angle 0° - j\,194.4 \angle 25.8° \times 3.24$$

$$= 2179.5 - j\,566.9$$

$$E_f = 2252 \text{ V}$$

Under the condition when the motor is drawing maximum power from the mains, $\bar{V}_t$ leads $\bar{E}_f$ by 90°. The phasor diagram is drawn in Fig. 11.20.

$$P_{e,\ \max} = 3 \times \frac{1905 \times 2252}{3.24} = 3972 \text{ kW}$$

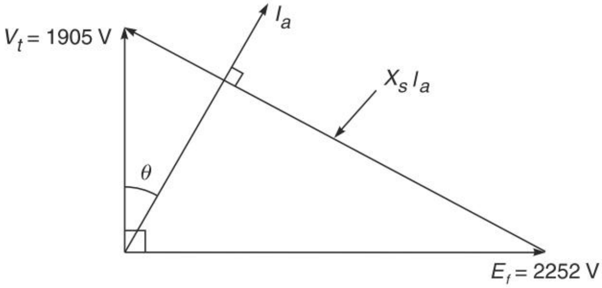


Fig. 11.20

From the phasor diagram

$$\bar{I}_a = \frac{V_t\angle 0° - E_f\angle -90°}{jX_s}$$

$$= \frac{1905 - 2252\angle -90°}{j3.24}$$

$$= 695.1 - j\,588 = 910 \angle -40.2° \text{ A}$$

$$I_a = 910 \text{ A};\ pf = 0.764 \text{ lag}$$

$$Q_e = \sqrt{3} \times 3.3 \times 588$$

$$= +\,3361 \text{ kVAR}$$

Example 11.6 A 3-phase, 25 kVA, 400 V, 4-pole, star-connected synchronous machine has X_S = 4.5 Ω/ph, the armature resistance being negligible. The machine is synchronized to a 400 V, 3-phase infinite bus. Neglect rotational loss.

(a) The machine is acting as a motor delivering 20 kW of mechanical power at 0.8 leading pf. Determine the excitation emf E and power angle.

(b) With the load remaining constant at 20 kW, the field current is gradually reduced. What is the minimum E and angle beyond which the machine will fall out of step? What is the value of the armature current and power factor under this operating condition? Also draw the phasor diagram.

Solution Refer to Fig. 11.41.

(a)
$$P_c = P_m = 20 \text{ kW (no losses)}$$
$$I_a = (20 \times 1000)/(\sqrt{3} \times 400 \times 0.8)$$
$$= 36.1 \text{ A} \therefore \bar{I}_a = 36.1 \angle 36.9° \text{ A}$$
$$\bar{V}_t = (400/\sqrt{3})\angle 0° = 231 \angle 0° \text{ V}$$
$$\bar{E}_f = 231 - j\,4.5 \times 36.1 \angle 36.9°$$
$$= 231 + j\,162.5 \angle -53.1°$$
$$= 375 \angle -28.7° \text{ V};$$
$$E_f \text{(line)} = 375\sqrt{3} = 650 \text{ V}, \delta = -28.7°$$

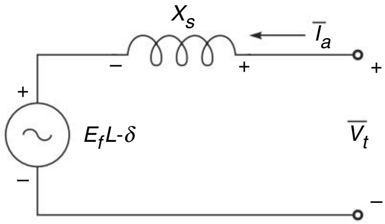


Fig. 11.21 (a)

(b) Stability limit is reached at δ = – 90°. Hence
$$P = 3\left\{\frac{E_f V_t}{X_s}\right\} \sin 90° = 3\left\{\frac{E_f V_t}{X_s}\right\}$$
$$20 \times 1000 = 3\,(E_f(\text{min})\,V_t)/X_s = 3\,(E_f(\text{min}) \times 231)/4.5$$
$$E_f(\text{min}) = 130 \text{ V} \quad \text{or} \quad 225 \text{ V (line)}, \delta = \angle -90°$$
$$\bar{I}_a = (231 + j\,130)/j\,4.5 = 28.9 - j\,51.3$$
$$= 58.9 \angle -60.6° \text{ A}$$
or
$$I_a = 58.9 \text{ A}, \text{ pf} = 0.49 \text{ lagging}$$

The phasor diagram is drawn in Fig. 11.21(b).

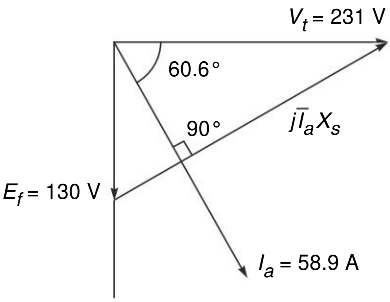


Fig. 11.21(b)

Remark The minimum excitation limit for stability is indicated by a dotted line in the V– curves of Fig. 12.16.

Example 11.7 A 3-phase hydroelectric synchronous generator is rated 110 MW, 0.8 *pf* lagging, 13.6 kV, Δ-connected, 50 Hz, 100 rpm. Determine the

(a) number of poles,
(b) kVA rating,
(c) prime mover rating if the full-load generator efficiency is 97.1% (leave out field loss) and
(d) output torque of the prime mover.

Solution

(a) $P = (120f)/n_s = (120 \times 50)/100 = 60$

(b) $(\text{kVA})_{\text{rating}} = 110/0.8 = 137.5$

(c) $(\text{kW})_{\text{turbine}} = 110/0.971 = 113.3$

(d) $T_{\text{PM}}\ (\text{output}) = (113.3 \times 1000 \times 60)/(2\pi \times 100)$

$= 10.82 \times 10^3$ Nm

Example 11.8 A 3-phase synchronous generator feeds into a 22 kV grid. It has a synchronous reactance of 8 Ω/phase and is delivering 12 MW and 6 MVAR into the system. Determine

(a) the phase angle of the current,
(b) the power (torque) angle and
(c) the generated emf.

Solution

(a) $S = 12 + j\,6 = 13.42 \angle 26.6°$ kVA

phase angle of current,

$\theta = 26.6°$ (lagging wrt grid voltage)

(b) $I_a = (13.42 \times 1000)/(\sqrt{3} \times 22) = 352.2$ A

$\bar{E}_f = (22/\sqrt{3}) \times 1000 + j\,8 \times 352.2 \angle -26.6°$

$= 13.96 + j\,2.52 = 14.18 \angle 10.2°$

Power angle $\delta = 10.2°$ ($\bar{E}_f$ leads $\bar{V}_t$)

(c) Generated emf $= 14.18\sqrt{3} = 24.56$ kV (line)

ADDITIONAL SOLVED PROBLEMS

11.9 A 1 MVA, 11 kV, star-connected synchronous machine has the following OCC and SCC data:

I_f	25	50	75	100	125	150
V (line) (kV)	6	10	13	16	17	18
I_{SC} (A)		60				

Calculate the synchronous reactance of the machine - unsaturated and saturated. The machine is now loaded as a generator to a line current of 50 A, 0.85 pf lagging. Calculate the excitation emf use saturated synchronous reactance. What is the value of the voltage regulation? What would be the required field current to reduce the OC voltage to the rated value?

Solution

The OCC from the given data is plotted in Fig. 11.22. Only one data point is given for SCC – I_{SC} = 60 A at I_f = 50 A. This corresponds to point *P* on the figure. A line drawn from the origin through *P* is the SCC, as it is linear. The air-gap line is also drawn on the figure.

Unsaturated Synchronous Reactance We can choose any point on the air-gap line. Let us choose the point P itself. The corresponding readings are

$$V_{oc}(\text{line}) = 15.3 \text{ kV} \qquad I_{sc} = 60 \text{ A}$$

$$\text{Then } x_s \text{ (unsaturated)} = \frac{15.3 \times 10^3 \sqrt{3}}{60} = 147\ \Omega$$

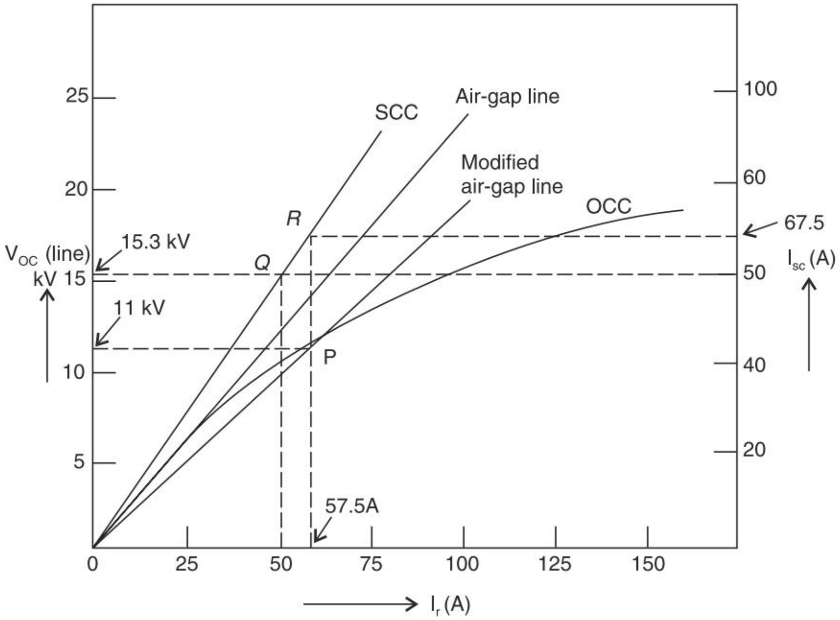


Fig. 11.22

Saturated Synchronous Reactance Corresponding to rated voltage 11 kV on the *OCC* at point *Q*, the I_{sc} is read against point *R* is 67.5 A. Therefore

$$X_s(\text{saturated}) = \frac{11 \times 10^3/\sqrt{3}}{67.5} = 94\ \Omega$$

Generator operation

$I_a = 50$ $\qquad pf = 0.85$ lag $\qquad V_t = 11$ kV; 6.35 kV (phase)

$$\theta = -31.8°$$

$$I_a = 50 \angle -31.8°$$

Excitation emf

$$\bar{E}_f = \bar{V}_L + j X_A \bar{I}_a$$
$$= 6.35 \angle 0° + j\, 94 \times 50 \angle -31.8°$$
$$= 6.35 + 4.7 \angle 58.2° = 9.68 \angle 24.4° \text{ kV}$$

$$\bar{E}_f(\text{line}) = 9.68\sqrt{3} = 16.77 \text{ kV}$$

Without changing the field current, the load is thrown off. Then

$$V_{OC} = E_f = 16.77 \text{ kV}$$

$$\text{Voltage regulation} = \frac{16.77 - 11}{11} \times 100 = 52.4\%$$

Field current to reduce V_{OC} to 11 kV is found from the *OCC*

$$I_f = 57.5 \text{ A}$$

11.10 A three-phase, 10 kVA, 400V, four-pole, 50 Hz star-connected synchronous machine has negligible armature resistance and a synchronous reactance of 16 Ω/phase. The machine is operating as a generator on 400 V bus bars (assumed infinite).

(a) Determine the excitation phase emf and torque (power) angle when the machine is delivering rated kVA at 0.8 lagging pf.

(b) While supplying the same real power as in part (a), the machine excitation is increased by 20%. Find the stator current, power factor and power angle.

(c) With field current fixed as in part (a), the load (real power) on the machine is increased (by putting in more power from the prime mover) till the steady state stability limit is reached (90°). Calculate the maximum power delivered by the machine, stator current and power factor. Draw the phasor diagram under these conditions.

Solution

Refer to Fig. 11.23.

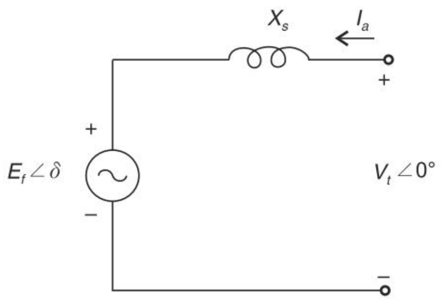


Fig. 11.23

(a)

$$\bar{I}_a = (10 \times 1000) / (\sqrt{3} \times 400) = 14.43 \text{ A}$$

$$\theta = \cos^{-1} 0.8 = 36.9° \text{ lag}$$

$$\bar{I}_a = 14.43 \angle -36.9°$$

$$\bar{V}_t = (400/\sqrt{3}) \angle 0° = 231 \angle 0° \text{ V}$$

$$\bar{E}_f = \bar{V}_t + jx_s \bar{I}_a$$

$$= 231 + j\, 16 \times 14.43 \angle -36.9° = 231 + 231 \angle 53.1°$$

$$= 413 \angle 26.4° \text{ V}$$

$$E_f = 413\ V;\ E_f \text{ (line)}$$

$$\delta = 26.5° \ (E_f \text{ leads } V_t)$$

(b) Excitation increased by 20% or $E_f = 1.2 \times 413 = 496$ V

P_e [same as in part (a)] = 10 × 0.8 = 8 kW

$$\left(\frac{E_f V_t}{x_s}\right) \sin \delta = P_e$$

or $$\left(\frac{496 \times 231}{16}\right) \sin \delta = (8\times1000/3) \text{ (per phase)}$$

or $$\delta = 21.8°$$

$$\bar{E}_f = 496 \angle 21.8°,\ \bar{V}_t = 231 \angle 0°$$

$$\bar{I}_a = (\bar{E}_f - \bar{V}_t) / (j\, x_s) = \frac{496\angle 8° - 231\angle 0°}{j16}$$

$$= 31 \angle -68° \text{A}$$

$$I_a = 31 \text{ A},\ pf = \cos 68° = 0.37 \text{ lagging}$$

$$\delta = 21.8°$$

(c) At steady state power limit, $\delta = 90°$

$$P_{e,\,max} = 3 \text{ (phases)} \times \frac{413 \times 231}{16} \sin (\delta = 90°)$$

$$= 17.9 \text{ kW}$$

$$\bar{E}_f = 413 \angle 90°; \text{ no change in field current}$$

$$\bar{I}_a = \frac{413\angle 90° - 231\angle 0°}{j16} = -\frac{231 - j413}{j16}$$

$$= 25.8 + j14.44 = 29.6 \angle 29.2° \text{ A}$$

$$\bar{I}_a = 29.6 \text{ A},\ pf = \cos 29.2° = 0.873 \text{ leading}$$

The phasor diagram is drawn in Fig. 11.24.

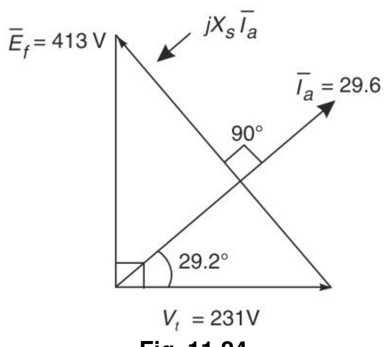


Fig. 11.24

11.11 The synchronous machine of 11.10 is acting as a motor.

(a) The mechanical load on the shaft is 8 kW and the rotational losses (mechanical and iron losses) equal 0.5 kW. The machine is excited to have an excitation emf of 750 V (line). Calculate the armature current, power factor and power angle. Also calculate the developed and shaft (load) torques. Ignore stator copper loss.

(b) The motor is on no-load and its losses can be ignored. Calculate the armature current and its power factor at an excitation emf of (i) 600 V and (ii) 300 V. Also calculate the kVAR drawn in each case.

(c) The motor is no-load (losses to be ignored). What should be its excitation for it to draw a leading kVAR of 6? Draw the phasor diagram.

Solution

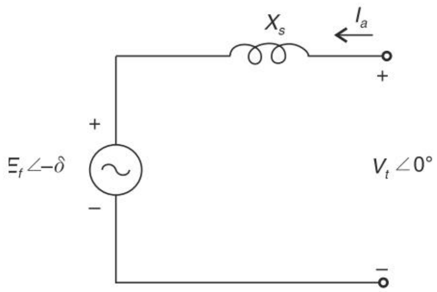


Fig. 11.25

(a) Gross mechanical load = 8 + 0.5 = 8.5 kW

$$P_e \text{ (in)} = 8.5 \text{ kW (copper loss ignored)}$$

$$P_e = \left(\frac{E_f V_t}{x_s}\right) \sin \delta$$

$$V_t = 231 \text{ V}, \qquad E_f = 750/\sqrt{3} = 433 \text{ V}$$

$$(8.5 \times 1000)/3 = [(433 \times 231)/16] \sin \delta$$

or

$$\delta = 27°, \quad (\bar{E}_f \text{ lags } \bar{V}_t)$$

$$\bar{I}_a = (\bar{V}_t - \bar{E}_f) / jx_s$$

$$\bar{I}_a = \left(\frac{230\angle 0° - 433\angle -27°}{j16}\right)$$

$$= 12.3 + j\,9.25 = 15.66 \angle 38.2°$$

$$\bar{I}_a = 15.4 \, A, \; pf = \cos 38.2° = 0.786 \text{ leading}$$

$$n_s = (120 \times 50)/4 = 1500 \text{ rpm or } 157.1 \text{ rad/s}$$

$$\text{T (dev)} = 8500/157.1 = 54.1 \text{ Nm}$$

$$\text{T (shaft)} = 8000/157.1 = 50.9 \text{ Nm}$$

(b) (i) $E_f = 600/\sqrt{3} = 346$ V

On no-load with no losses, power drawn from mains is zero and so $\delta = 0°$

Then $\bar{E}_f = 346\angle 0°\ V$ and $\bar{V}_t = 231 \angle 0°$V (i.e., these are in phase). Now

$$\bar{I}_a = (231 - 346)/j16 = j\,7.19 \text{ A} = 7.19 \angle 90° \text{ A}$$

$$\bar{I}_a = 7.19 \text{ A}, \; pf = 0, \text{ leading}$$

$$\text{kVAR (drawn)} = \sqrt{3} \times 400 \times 7.19 = 4.98 \text{ (leading)}$$

The motor acts like a capacitor with a per phase capacitance of

$$\bar{I}_a = 2\pi f C V_L$$

$$\therefore \quad C = (7.19/231)/(2\pi \times 50) = 99 \; \mu\text{F}$$

(ii) $\bar{E}_f = (300/\sqrt{3}) \angle 0° = 173.2\angle 0°\ V, \; \bar{V}_t = 231\angle 0°$

$$\bar{I}_a = (231 - 173.2)/j\,16 = -j\,3.6 \text{ A} = 3.6 \angle -90° \text{ A}$$

$$I_a = 3.6 \, A, \; pf = 0 \text{ (lagging)}$$

$$\text{kVAR (drawn)} = \sqrt{3} \times 400 \times 3.6 = 2.49 \text{ (lagging)}$$

The motor acts like an inductor with a per phase inductance of

$$V_t = 2\pi f L I_a$$

$$L = (231/3.6)/\,(2\pi \times 50) = 204 \text{ mH}$$

(c) kVAR (drawn) = 6, leading

$$6000 = \sqrt{3} \times 400 \times I_a$$

Or $\bar{I}_a = 8.66\ A$, zero leading pf

$$I_a = (230 - E_f)\,/\,j16 = j\,8.66$$

Or $E_f = 369.6$ V or 640 V (line)

The phasor diagram is drawn in Fig. 11.26.

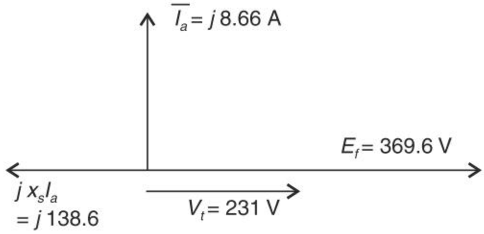


Fig. 11.26

11.12 A 1000 kVA, 6.6 kV, 3-phase star-connected synchronous generator has a synchronous reactance of 25 Ω per phase. It supplies full-load current at 0.8 lagging power factor at rated terminal voltage. Compute the terminal voltage for the same excitation when the generator supplies full-load current at 0.8 leading power factor.

Solution

$$I_a(\text{rated}) = \frac{1000}{\sqrt{3} \times 66} = 87.5 \text{ A}$$

$$V_t(\text{phase}) = \frac{6600}{\sqrt{3}} = 3810 \text{ V}$$

Operation at full-load 0.8 *pf* lag, rated voltage :

$$\bar{E}_f = \bar{V}_t + j\,x_s\,\bar{I}_a$$

$$= 3810\angle 0° + j\,25 \times 87.5\,(0.8 - j\,0.6)$$

$$= 5123 + j\,1750 = 5413\angle 18.8°$$

$$E_f = 5414 \text{ V}$$

Operation at full-load 0.8 *pf* lead, rated voltage, no change in excitation:

$$E_f = 5413\ V,\ I_a = 87.5 \text{ A}$$

I_a leads V_t by angle $\cos^{-1} 0.8 = 36.9°$

$$V_t \angle 0° = \bar{E}_f - j\,X_s\,\bar{I}_a$$

V_t and angle of E_f are unknown. It is easier to proceed from the phasor diagram drawn in Fig. 11.27.

For Δ *OAD*

$$OB = E_f = 5413 \text{ V}$$

$$AB = X_s I_a = 87.5 \times 25 = 2187.5 \text{ V}$$

$$\frac{5414}{\sin 531°} = \frac{2187.5}{\sin\delta}$$

or $\sin\delta = 0.323,\ \delta = 18.8°$

$\angle OBA = 180° - 53.1° - 18.8° = 108.1°$

$$\frac{V_t}{\sin 108.1°} = \frac{218.5}{0.323}$$

Or $V_t = 6437$ V (phase) or

11.15 kV(line)

Observation $V_t > E_f$ because of leading current

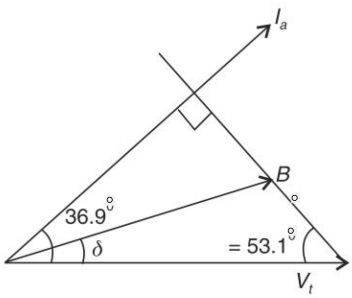


Fig. 11.27

11.13 A 40 kV, 600V, 3-phase, star-connected synchronous machine has a synchronous reactance of 8 Ω and negligible resistance. When excited to a 'per phase' emf of 600V and synchronized to 600 V main, it carries an armature current of 40 A. Find its power input/output and power factor when operating as (a) a generator and (b) motor. Also draw the phasor diagrams for both operations.

Solution

The circuit diagram as per phase basis is drawn in Fig. 11.28 (a).

Terminal voltage, $V_t = 600/\sqrt{3} = 346$ V (phase)

Excitation emf, $E_f = 600$ V (phase)

(a) *When operating as a generator*

$\bar{E}_f$ leads $\bar{V}_t$ by angle δ

$\bar{V}_t = 346\ \angle 0°,\ \bar{E}_f = 600\angle\delta$

Armature current

$$I_a = \left|\frac{600\angle\delta - 346}{j8}\right| = 40\text{ A} \qquad \text{(i)}$$

or $\left|\dfrac{\cos\delta + j\sin\delta - 346/600}{j}\right| = \dfrac{8\times 40}{600}$

or $|\sin\delta - j(\cos\delta - 0.577)| = 0.533$

or $\sin^2\delta + (\cos\delta - 0.577)^2 = (0.533)^2 = 0.284$

$1 - 2\times 0.577\cos\delta + 0.333 = 0.284$

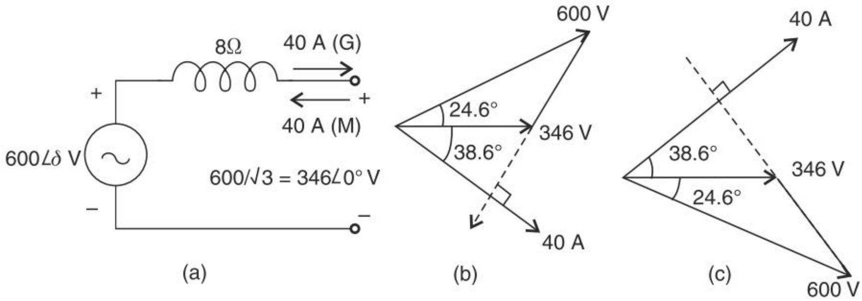


Fig. 11.28

From which we get

$$\cos\delta = 0.91,\ \delta = 24.6°$$

$$\angle\ \bar{I}_a = \theta = -\tan^{-1}\left[\frac{\cos\delta - 0.577}{\sin\delta}\right] = -38.6°$$

$$pf = \cos 38.6 = 0.78 \text{ lag}$$

$$P\text{ (out)} = \sqrt{3} \times 600 \times 0.78 \times 10^{-3} = 32.5 \text{ kW}$$

The phasor diagram is drawn in Fig. 11.28 (b).

(b) *When operating as a motor*

Same V_t, E_f and I_a (in)

$$I_a = \left[\frac{346\angle 0 - E_f\angle\delta}{j8}\right] = 40 \qquad \text{(ii)}$$

This is the same equation as Eq. (i) except for a minus sign within the absolute values, which makes no difference. So

$$\delta = 24.6°, \qquad \theta = +\ 38.6° \text{ (reversal in sign)}$$

$$\text{pf} = \cos 38.6° = 0.78 \text{ lead}$$

$$P\text{ (in)} = 32.5 \text{ kW}$$

The phasor diagram is drawn in Fig. 11.28 (c).

11.14 A synchronous motor is drawing 50 A from a 400 V, 3-phase supply at unity pf with a field current of 0.9 A. The synchronous reactance 1.3Ω.

(a) Find the power angle δ.

(b) Assuming no change in mechanical load, find the value of the field current which would result in a power factor of 0.8 leading. The magnetizing characteristic may be taken as linear.

Solution

$$V_t = \frac{400}{\sqrt{3}} = 231 \text{ V} \quad \text{Reference phasor}$$

$$I_a = 50 \text{ A} \ \ pf \text{ unity}, \theta = 0°$$

$\bar{I}_a$ is in phase with $\bar{V}_t$

(a) Excitation emf

$$\bar{E}_f = 231 - j\,1.3 \times 50 = 231 - j\,65$$

$$= 240\ \angle -15.7°$$

$$\bar{E}_f \text{ lags } \bar{V}_t \text{ by } \delta = 15.7° \text{ (motoring action)}$$

(b) Assuming no loss

$$P\text{ (mech)} = P\text{ (elect)} = \sqrt{3} \times 400 \times 50 \times 10^{-3} = 34.64 \text{ kW}$$

As there is no change in mechanical power output, so

P (elect) = $34.64 \times 10^3 = \sqrt{3} \times 400\, I_a \times 0.8$; $pf = 0.3$ leading $\phi = +\ 36.9°$

$$I_a = 62.5$$

$$\bar{I}_a = 62.5\ \angle\ 36.9°$$

Now

$$\bar{E}_f = 231 - j\,1.3 \times 62.5\ \angle 36.9°$$

$$= 231 - 1.3 \times 62.5\ \angle 126.9° = 231 - (-\,48.8 + j\,65)$$

$$= 279.8 - j\,65 = 28.3 \angle{-13°}$$

If increases proportionally. Thus

$$I_f = 0.9 \times \frac{287.3}{240} = 1.08 \text{ A}$$

11.15 A one MVA, 3 kV, 3-phase, 12-pole star-connected synchronous motor is connected to an infinite bus. The motor reactance of 0.8 pu. All losses are to be ignored. This is operating at 0.85 leading pf and is delivering 750 kW of mechanical power.

(a) Determine the value of E_f.

(b) With the value of E_f as found in part (a) renmaining fixed, determine the maximum power and torque the motor can deliver.

(c) While delivering the load of 750 kW, the excitation emf E_f is reduced by reduced by reducing the field current. Determine the minimum value of E_f when the motor will fall out of step. What is the corresponding armature current and the pf? Draw the phasor diagram.

Solution

$$X_s\,(\text{pu}) = 0.8 \times \frac{X_S(\Omega) \times (\text{MVA})_B}{(\text{kV})_B^2} = \frac{X_S(\Omega) \times 1}{(3)^2}$$

Or

$$X_s = 0.8 \times 9 = 7.2\Omega$$

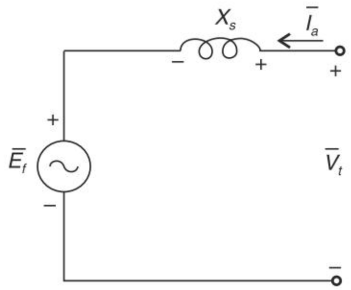


Fig. 11-29

(a) Refer to adjoining figure,

$$\bar{V}_t = \frac{3}{\sqrt{3}} = \sqrt{3} \times \angle 0° \text{ kV} = 1732 \angle 0° \text{ V}$$

$$P_e = P_m = 750 \text{ kW, no loss assumed}$$

$$pf = 0.85,\ \varphi = 31.8° \text{ lag}$$

$$I_a = \frac{750}{0.08\sqrt{3} \times 3} = 180 \text{ A},\ \bar{I}_a = 180.4 \angle{-318°}$$

From the figure

$$\bar{E}_f = \bar{V}_t - j\,x_s \bar{I}_a$$

$$= 1723 - j\,7.2 \times 180.4 \angle -31.8°$$

$$= 1048 - j\,1104$$

$$\bar{E}_f = 1522 \angle -46.5° \text{ (phase)}$$

$$E_f = 2636 \text{ V (line)}$$

(b) For maximum power output, $\delta = 90°$.

$$P\,(\max) = \frac{3V_tE_f}{X_S} = \frac{3\times1732\times1522}{7.2}\times10^{-3}$$
$$= 1098 \text{ kW}$$

$$n_s = \frac{120\times50}{12} = 500 \text{ rpm or } 52.36 \text{ rad (mech)/s}$$

$$T(\max) = \frac{1098\times10^3}{52.36} = 20015 \text{ Nm}$$

(c) For fixed power, E_f is minimum when $\delta = 90°$.

$$\frac{750}{3}\times10^3 = \frac{1732\times E_f}{7.2}$$

Or $E_f\,(\min) = 1039\text{V (phase)}$

$= 1.8 \text{ kV (line)}$

From the figure

$$\bar{I}_a = \frac{1732-(-j1039)}{j7.2} = \frac{1039-j1732}{7.2}$$
$$= (144.3 - j\,240.5) = 280.5\ \angle -59° \text{ A}$$

$pf = \cos 59° = 0.515$ lag

Phasor diagram:

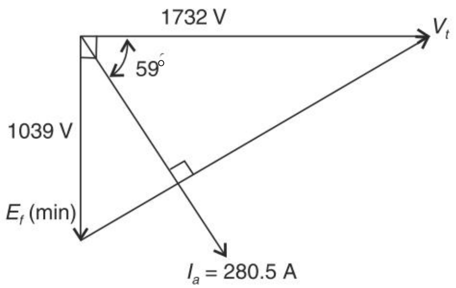


Fig. 11.30

SUMMARY

- Synchronous speed, $n_s = \frac{120f}{P}$ rpm, $\omega_s = \frac{4\pi f}{P}$ rad (mech)/s; in general $n_s = \frac{60}{2\pi}\omega$
- Excitation emf (E_f) is the emf induced in the armature winding due to field current (I_f) only. At no-load, $E_f = V_t$ (terminal voltage)
- Synchronous generator ($\bar{I}_a$ in the direction of $\bar{E}_f$)
  (i) When I_a lags E_f by 90°, the *armature reaction field* $\vec{F}_a$ is demagnetizing. It is in direct opposition to the rotor field $\vec{F}_f$.
  (ii) I_a in phase with E_f, $\vec{F}_a$ is cross-magnetizing, at 90° to $\vec{F}_f$.
  (iii) I_a leads E_f by 90°, $\vec{F}_a$ is magnetizing in the same direction as $\vec{F}_f$.
  In a synchronous motor, the direction of $\bar{I}_a$ reverses and so do the conclusions (i) and (ii) above interchange magnetizing and de-magenetizing.
- The armature reaction effects can be simulated by an armature reactance X_a

in series with $\bar{I}_f$. Further, X_a when combined with armature, leakage reactance is called the *synchronous reactance* $X_s = X_a + X_1$) of the amchine. Its value is of the order of 0.5 to 1 pu; far larger than in a transformer.

- OCC – Open Circuit Characteristic—plot of V_{OC} (line) vs I_f. It is the magnetizatic characteristic of the machine. A line from the origin tangential to the linear part of *OCC* is called the *air-gap line.*
- SCC – Short Circuit Characteristic—plot of short circuit current I_{SC} vs I_f; it is linear as I_f is very small.
- Determination of synchronous reactance x_s from *OCC* and *SCC*

$$X_s(\text{unsaturated}) = \left.\frac{V_{oc}/\sqrt{3}}{I_{sc}}\right| \text{at any point on air-gap line}$$

$$X_s\ (\text{adjusted}) = \left.\frac{V_{oc}/\sqrt{3}}{I_{sc}}\right| \text{If corresponding to } V_t\,(\text{rated}) \text{ on } OCC$$

- Voltage regulation

$$= \left.\frac{V_t(no\ load)\big|_{I_f\, same\ as\ at\ full\ load} - V_t(rated)}{V_t(\text{rated})}\right| \text{specified } pf$$

- Circuit model (equivalent circuit) of the synchronous machine

  — armature resistance negligible

  Governing equations

  Generating

$$\bar{V}_t = \bar{E}_f - j\,X_s\,I_a$$

  $\bar{E}_f$ lead $\bar{V}_t$ by angle δ

  Motoring

$$\bar{V}_t = \bar{E}_f + j\,X_s\,I_a$$

  $\bar{E}_f$ lead $\bar{V}_t$ by angle δ

- Starting—synchronous motor is non-self starting, started by an auxiliary motor (induction motor) and then *synchronized* to mains.

  Before loading, it is *floating* on the mains; drawing or delivering almost zero current.

- Operating characteristic:

$$P_e = \frac{V_t E_f}{X_s}\ .\ \sin\delta\ ;\ \text{power-angle characteristic}$$

  where $P_e = V_t I_a \cos\theta$; $\cos\theta = pf$

$$P_e\,(\max) = \frac{V_t E_f}{X_s}$$

  If $P_e > P_e$ (max), the machine loses synchronism or motor falls out of step

- Fixed load variable excitation:

  I_f controls the power factor

  Normal excitation—unity power factor

  Over-excitation—leading *pf* in motoring mode

lagging *pf* in generating mode
Under-excitation—lagging *pf* in motoring mode
leading *pf* in generating mode.

REVIEW QUESTIONS

1. Draw a suitable diagram indicating armature currents directions and field poles to show that for a motoring machine drawing 90° leading current, the armature reaction is demagnetizing.
2. In question 1, if the armature current is 45° leading, show that the armature reaction is both demagnetizing and cross-magnetizing.
3. A synchronous generator is supplying zero power factor (i) lagging and (ii) leading current. Show that the terminal voltage V_t and the excitation emf E_f are in phase.
4. In a synchronous motor drawing leading current at $pf = \cos\theta$, draw the phasor diagram. Find therefrom the phase and magnitude relationships between V_t and E_f. Here phase relationship means lag / lead and magnitude relationship means greater than / less than.
5. Show that in a generating synchronous machine, the phase relationship (lag/lead) between V_t and E_f is independent of the power factor (lag/lead). Draw the phasor diagrams to discover your answer.
6. In a generating synchronous machine connected to infinite bus, the mechanical power input is maintained constant, while its field current is increased from a low to a high value. Draw the phasor diagram to show how the magnitude of the armature current will change. Make a sketch of I_a vs I_f.
7. A synchronous motor with terminal voltage V_t is drawing zero *pf* current. Write the phasor expression for the excitation emf, E_f. Is E_f more or less than V_t magnitude-wise? What is the value of the power angle?
8. Explain what is meant by the armature reactance and synchronous reactance of a synchronous machine.
9. Expain what is the air-gap line.
10. Explain why the SCC is linear.
11. Distinguish between X_s (unsaturated) and X_x (saturated). Which one should be used for higher accuracy in predicting the voltage regulation of a synchronous generator?
12. Under what conditions does the voltage regulation of a synchronous generator become negative? Draw the phasor diagram in support of your answer.
13. Explain briefly the process of synchronizing a synchronous motor to the bus-bars. What conditions determine the instant of synchronization?
14. What is meant by the statement that a synchronous machine is 'floating' on the bus-bars?

15. Elaborate the statement that an unloaded synchronous motor can be made to act as a capacitor or as an inductor.

PROBLEMS

11.1 Draw the phasor diagram for a motoring synchronous machine.
(Hint: With reference to Fig. 11.1 $\mathbf{F}_a$ would be in phase with $-\mathbf{I}_a$ (motor)).

11.2 Redraw Figs. 11.12 and 11.13 for a generating synchronous machine.

11.3 A 62 kVA, 400 V, 50 Hz, 3-phase generator has the magnetization characteristic data as given below:

V_{OC} (line V)	160	320	400	454	520	580	612
I_f (A)	2	4	5	6	7.5	10	12

It has a synchronous reactance of 1.08 Ω/phase while the armature resistance is negligible.
The load in the generator is raised from no load to 125% full load, pf 0.8 lagging, with its terminal voltage held constant at rated value. Determine the range through which the field current will have to be adjusted in this operation.

11.4 A 400 V, 3-phase, star-connected synchronous motor has a synchronous reactance of 6.1 Ω/phase. For an armature current of 25 A, find the excitation emf for the pf to be (a) unity and (b) 0.8 leading. Calculate also the active and reactive power input in each case.

11.5 A 3-phase synchronous generator operates into a grid of 11 kV. The synchronous reactance of the generator is 4.2 Ω/phase. The generator is excited to have an emf equal in magnitude to the bus voltage. The machine is delivering 15 MW into the grid. Find
(a) the power angle δ,
(b) the line current and pf, and
(c) the reactive power delivered to the bus.

11.6 A star-connected, 3-phase, 4-pole, 50 Hz, 12.5 kV, synchronous motor has an armature resistance of 2 Ω/phase and a synchronous reactance of 48 Ω/phase. It is drawing electric power of 1050 kW from the supply. The field current is adjusted so that the motor draws a leading current of 60 A.
(a) At what pf is the motor operating?
(b) Calculate its excitation emf,
(c) mechanical torque developed, and
(d) the armature current and power factor if the load on the motor is thrown off with excitation remaining unchanged. Ignore armature resistance in this case.

11.7 A 3-phase, 4-pole, 50 Hz, star-connected synchronous motor has a synchronous reactance of 120 Ω/phase. The excitation is such as to give an open-circuit voltage of 13.2 kV. The motor is connected to 11.5 kV, 50 Hz supply. What maximum load can the motor supply before losing synchronism? What is the corresponding motor torque, line current and power factor?

11.8 A 22 kV, 3-phase, star-connected turbogenerator with a synchronous reactance of 1.5 Ω/phase is delivering 200 MW to the grid at 22 kV, unity pf. With the turbine power kept constant, the machine excitation is increased by 15%. Assuming linear magnetization characteristic, calculate the machine current and power factor. At the new excitation the turbine power is now increased gradually to 250 MW. Calculate once again the machine current and power factor.

11.9 A 3.3 kV, star-connected synchronous motor has a synchronous reactance of 5.5 Ω. It operates at rated terminal voltage and draws 750 kW from the supply at 0.8 leading pf. Calculate its pf when the motor shaft load is 1000 kW with the same excitation.

11.10 A 1000 kVA, 3-phase, 22 kV, star-connected synchronous motor has a synchronous reactance of 250 Ω/phase.

(a) Calculate the excitation emf and *pf* at rated current (leading), if power angle is 15°.

(b) What is the minimum excitation emf for the motor to deliver 800 kW without losing synchronism? What is the corresponding line current and power factor?

Assume supply voltage to be 22 kV.

Chapter 12

INDUCTION MOTOR

MAIN GOALS AND OBJECTIVES

- *Induction motor, the asynchronous machine, types and constructional features*
- *Understanding the basic relationships—resultant air-gap flux, the exciting current*
- *Standstill rotor emf, effective turn ratio, standstill rotor emf, the rotor circuit slip dependent*
- *Step-wise development of the circuit model (equivalent circuit), concept of load resistance*
- *Concept of power across air-gap, mechanical power developed*
- *Torque-slip characteristic, modes of operation—motor, generator, brake*
- *Test to determine circuit model parameters—no-load test, blocked-rotor test*
- *The starting problems, methods of starting squirrel-cage and slip-ring motors*
- *Problems encountered in speed control, comparison with dc motor.*

12.1 INDUCTION MOTOR*—AN INTRODUCTION

The induction machine is an important class of electric machines which finds wide applicability as a motor in industry and in its single-phase form in several domestic applications. More than 85% of industrial motors in use today are in fact induction motors. It is substantially a constant-speed motor with a shunt characteristic; a few per cent speed drop from no load to full load. It is a singly-fed motor (stator-fed), unlike the synchronous motor which requires as supply on the stator side and dc excitation on the rotor. The torque developed in this motor has its origin in current induction in the rotor, which is only possible at non-synchronous speed; hence, the name asynchronous machine. Torque in a synchronous machine, on the other hand, is developed only at synchronous speed when the 'locking' of the two fields takes place. Therefore, the induction motor is not plagued by the stability problem inherent in the synchronous motor. Since it is a singly-fed machine, it draw its excitation

* The reader is advised to brush-up Sec. 9.8.

current* from the mains to set up the rotating field in the air-gap which is essential for its operation. As a consequence, it inherently has a power factor less than unity, which usually must be corrected by means of shunt capacitors at motor terminals. There is no simple and inexpensive method of controlling the induction motor speed as is possible in a dc shunt motor and thus it finds stiff competition from the dc shunt motor in such applications.

The process of torque production in an induction machine was explained in Sec. 9.8 along with some constructional details. Here we shall give more constructional details and arrive at the circuit model of the machine.

12.2 CONSTRUCTION

Stator It is properly wound for three phases connected in star/delta. In some motors, all the six leads are brought out for changing the connection from star at the time of starting to delta during running. The pictorial view of a stator is shown in Fig. 12.1.

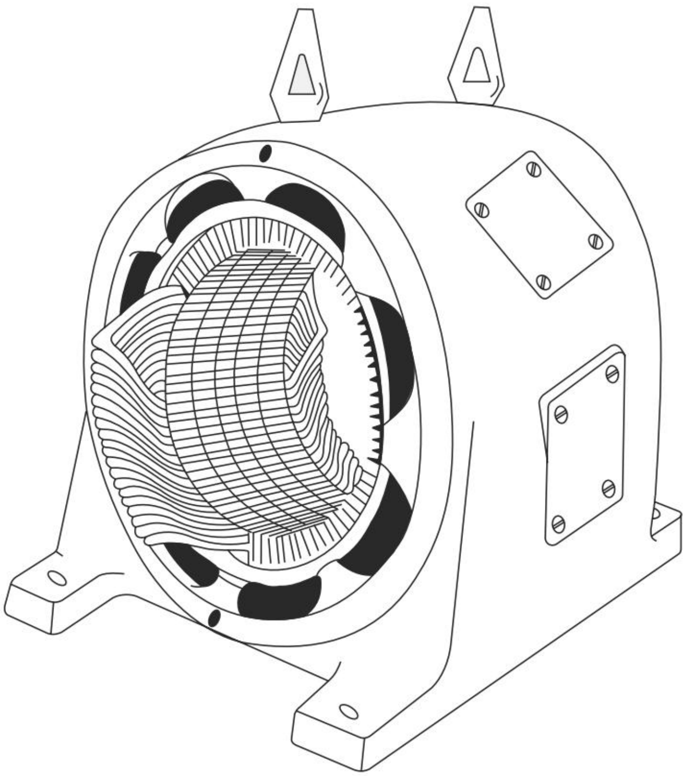
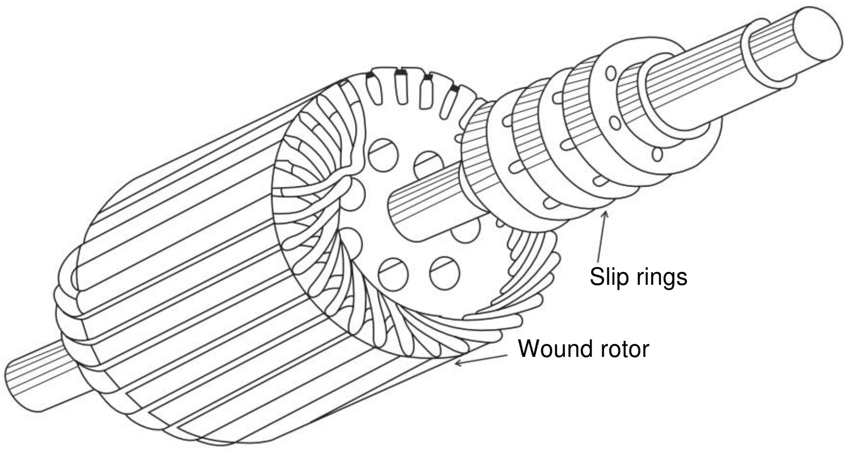


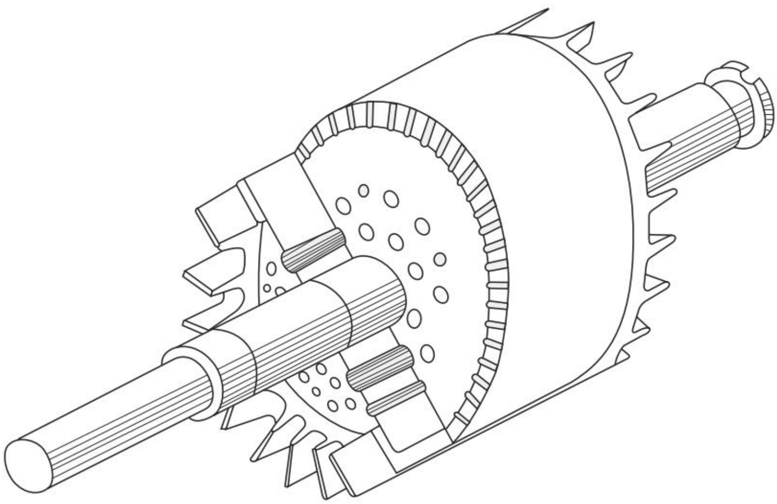
Fig. 12.1 Induction motor stator shown partially would double-layer winding

* Because of air-gap the excitation current is far larger in an induction motor than in a transformer for the same VA rating.

Rotor In one type of motor, rotor is properly wound for three-phases with three connecting leads brought out through slip rings as shown in Fig. 12.2(a). The slip rings are tapped by brushes. External resistances are connected in the rotor circuit at the time of starting but the slip rings are shorted out during running. Such a motor is called *wound-rotor* or *slip-ring* motor.



(a)



(b)

Fig. 12.22 (a) Wound rotor for induction motor
(b) Squirrel-cage rotor with cast aluminium bars and end rings

Figure 12.2(b) is that of a *squirrel-cage* rotor where conducting bars are placed in slots and are permanently shorted at each end. *Electrically it is equivalent to a*

3-phase winding which is shorted permanently such that no external resistance can be included in it.

Figure 12.3 shows the diagrammatic representation of rotor and stator winding connection for a slip-ring motor with delta-connected stator and star-connected rotor connected to slip rings to be shorted externally.

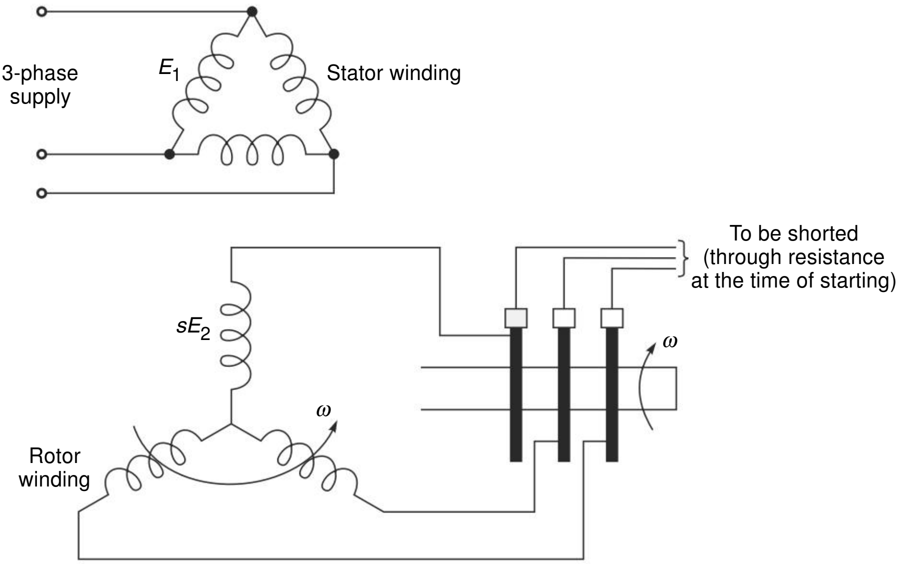


Fig. 12.3 3-phase, slip-ring induction motor—connection diagram

Rating

kW, Voltage, Speed (at full-load)

Frequency 50 Hz, (need not be specified)

kW is the rated output power called *shaft power*

12.3 CIRCUIT MODEL (EQUIVALENT CIRCUIT)

An induction motor is a generalized transformer in which rotor emf and frequency both depend upon the rotor speed and in the process, conversion of energy takes place. This is established step-by-step below from which would follow the circuit model.

– The resultant field in the air-gap must produce a flux Φ_r/pole which induces a counter emf in the stator winding to balance the applied voltage (per phase) minus the voltage drop in the stator impedance (resistance and leakage reactance). If this drop is neglected

$$V_1 \approx E_1 = \pi\sqrt{2}\, K_{w1}\, N_{\text{ph1}}\,(\text{series})\, f\, \Phi_r \tag{12.1}$$

where

$$V_1 = \text{applied voltage/phase}$$
$$K_{w1} = \text{stator winding factor}$$
$$N_{\text{ph1}} \text{ (series)} = \text{stator series turns/phase}$$
$$f = \text{stator frequency}$$

1. The stators must draw a 90° lagging magnetizing current component I_m to produce Φ_r. It will also be associated with a core loss component I_i. The net exciting current is

$$\bar{I}_0 = \bar{I}_m + \bar{I}_i \tag{12.2}$$

Magnitude-wise $I_m >> I_i$. The phasor diagram of the exciting current is drawn in

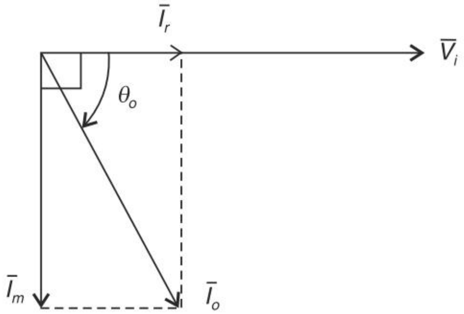


Fig. 12.4

the figure where θ_0 is only slightly less than 90°.

Because of air-gap in magnetic circuit of the induction motor (compared with transformer which has no air-gap being a static device), the net exciting current may be as large as 40% (magnitude-wise) of the full-load stator current.

2. Rotor induced emf at *standstill* (stator excited rotor block from rotation) is

$$E_2 = \pi \sqrt{2}\ K_{w2}\, N_{ph2} \text{ (series)} f\, \Phi_r \tag{12.3}$$

Thus

$$\frac{E_1}{E_2} = \frac{K_{w1} N_{ph1}}{K_w 2 N_{ph1}} = a \text{ (effective turn ratio)} \tag{12.4}$$

– Rotor frequency at any speed n (slip s) is

$$f_2 = sf \tag{12.5}$$

and the corresponding induced emf is sE_2. (12.6)

– Though the rotor carries currents of frequency sf, because of rotor speed, these interact magnetically with the stator field at speed n_s or to the stator these appear as currents of frequency f, the stator frequency.

– The stator apart from I_0, the exciting current, draws a current component $\bar{I}'_2$ to balance the ampere turns of the rotor current $\bar{I}_2$. Thus

$$\frac{\bar{I}'_2}{\bar{I}_2} = \frac{1}{a} \tag{12.7}$$

The net stator current is therefore

$$\bar{I}_1 = \bar{I}_0 + \bar{I}'_2 \tag{12.8}$$

Because of large lagging exciting current (Eq. (12.2)) the stator current has a pf less than unity (0.85 or less at full load). The pf will reduce as the load on motor is decreased ($\bar{I}'_2$ reduces while $\bar{I}_0$ remains unchanged in Eq. (12.8). Hence induction motor should not be operated at light load for long periods of time.

3. The rotor circuit impedance.

With the stator connected to the mains (V, f) and the rotor hold stationary (blocked from rotation) condition called *standstill,* the frequency of the rotor currents is f, the same as the stator. The rotor reactance under these conditions is X_2, the *standstill reactance.*

As the rotor runs at speed n (slip$s$), the frequency of rotor currents is $f_2 = sf$.

The rotor reactance at this frequency is

$$f_2\left(\frac{X_2}{f}\right) = sX_2 \tag{12.9a}$$

Thus the rotor circuit impedance at slip s (speed n) is

$$R_2 + js\,X_2$$

The power crossing the air-gap P_G is split into two parts-the mechanical poer output P_m and electrical loss in rotor resistance $3I^2{}_2R_2$.

4. With above argument, the induction motor is represented by the circuit model of Fig. 12.5 (a) in which the stator and rotor are coupled through an ideal generalized transformer. It has an effective turn ratio a: 1 for induced emfs and currents and also a frequency change of f to sf (rotor) and corresponding change in rotor unduced emf to sE_2.

In the next step, the transformer turn ratio is changed from a:1 to 1:1 like in a static transformer. The circuit model at this stage is drawn in Fig. 12.5 (b). It is to be observed this transformation is *power invariant* wherein P_G, the power crossing from the stator to the rotor and P_m, the mechanical power developed, are preserved as such.

In the rotor circuit of Fig. 12.5(b)

$$\bar{I}'_2 = \frac{s\bar{E}_1}{R'_2 + jsX'_2} \tag{12.10}$$

Dividing both numerator and denominator by s,

$$\bar{I}'_2 = \frac{\bar{E}_1}{R'_2/s + jX'_2} \tag{12.11}$$

The trick refers the rotor circuit to the stator frequency. This transformation is not power invariant and in the process the mechanical power developed per phase is now represented electrically as

$$\frac{P_m}{3} = \frac{I_2'^2\,R'_2}{s} - I_2'^2 R'_2$$

$$= I_2'^2\left(\frac{1}{s} - 1\right)R'_2 \tag{12.12}$$

The circuit model is now drawn in Fig. 12.5(c). The transformer here is an ordinary unit ratio transformer and can be dispensed with as in Fig. 12.5(d).

The final circuit model presented in Fig. 12.5(e) is approximate to the extent that the exciting shunt branch has been connected after the stator impedance. This approximation was also used in the transformer model but is a much stronger

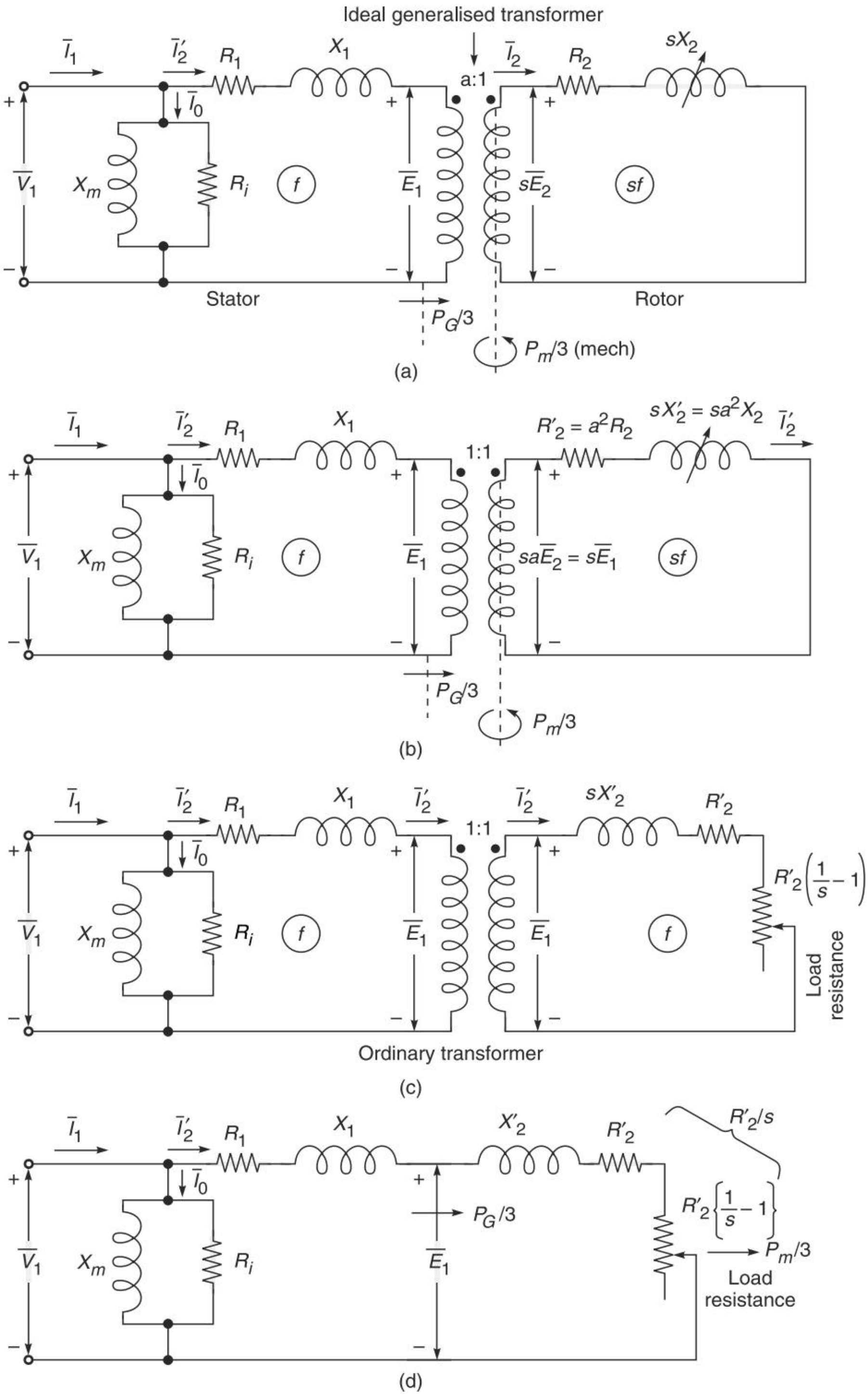
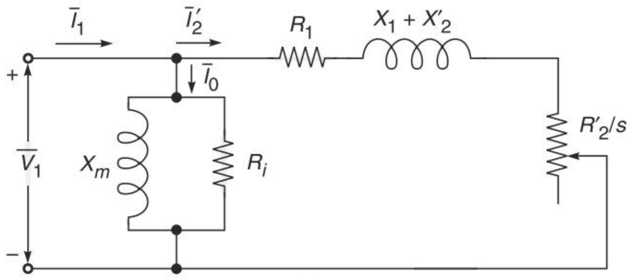

Ideal generalised transformer
a:1
Stator
Rotor
$P_G/3$
$P_m/3$ (mech)
(a)
$R'_2 = a^2R_2$
$sX'_2 = sa^2X_2$
1:1
$sa\bar{E}_2 = s\bar{E}_1$
$P_G/3$
$P_m/3$
(b)
$R'_2\left(\frac{1}{s}-1\right)$
Load resistance
Ordinary transformer
(c)
R'_2/s
$R'_2\left\{\frac{1}{s}-1\right\}$
$P_G/3$
$P_m/3$
Load resistance
(d)

Fig. 12.5



(e)

Fig. 12.5 Development of circuit model (equivalent circuit) of induction motor

approximation in induction motor as the exciting (or no-load) current I_0, because of the air-gap in the magnetic circuit, is much larger in the induction motor than it is in a transformer (2-5% of full-load current). I_0 in induction motor can be as large as 40% of the full-load current (in magnitude).

Power Across Air-Gap, Torque and Power Output With reference to Fig. 12.5(d), the power crossing the air-gap (after accounting for core loss and stator copper loss) is

$$P_G = \frac{3I_2'^2 R_2'}{s} \tag{12.13}$$

$$= \frac{3I_2^2 R_2}{s} \tag{12.14}$$

$$\therefore \text{Power across air-gap} = \frac{\text{rotor copper loss}}{\text{slip}}$$

It also follows that

Rotor copper loss $\quad 3I_2'^2 R_2' = s\,P_G$ (12.15)

Now the mechanical power output (gross) is

$$P_m = P_G - 3I_2'^2 R_2'$$

$$= 3I_2'^2 R_2' \left(\frac{1}{s} - 1\right) \tag{12.16}$$

$$= (1 - s)\,P_G \tag{12.17}$$

As per Eq. (12.16) the mechanical power output is the power absorbed by the load resistance $R'_2\,(1/s - 1)$ as shown in Fig. 12.4(d). The rotor speed is:

$$\omega = (1 - s)\omega_s \text{ rad (mech)/s}$$

The electromagnetic torque developed is given by

$$T = \frac{P_m}{\omega} = \frac{3I_2'^2 R_2' (1/s - 1)}{(1 - s)\omega_s}$$

$$= \frac{3I_2'^2 R_2'}{s\omega_s} = \frac{P_G}{\omega_s} \text{ Nm} \tag{12.18}$$

It immediately follows that

$$P_G = T\omega_s \tag{12.19}$$

i.e. power across the air-gap = torque × synchronous speed. Therefore P_G is also called torque in *synchronous watts*.

The net mechanical power output and torque are obtained by subtracting losses-windage, friction and stray-load loss.

Power Factor From the circuit model of Fig. 12.5 (e), the stator current (line current) 9s

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2'$$

The load component of the line current is given by

$$\bar{I}_2' = \frac{V}{(R_1 + R_2/s) + jX_2}$$

At low values of slip (2–5%) at full-load, R'/s predominates and $\bar{I}_2'$ has a small lagging angle (high pf). On the other hand, the exciting current has a large lagging phase angle (slightly less than 90°). Further, its magnitude is about 40% of the load current resulting the pf of the line current (I_1) of the order of 0.8 to 0.85 lagging. This fact is illustrated by the adjoining phasor diagram. Refer Example 12.7.

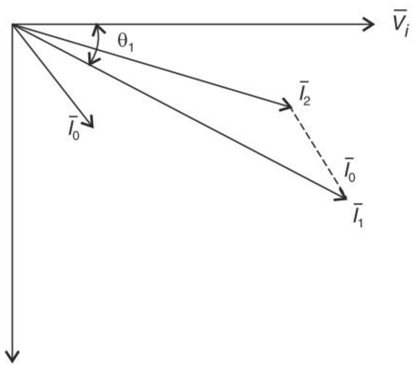


Fig. 12.6

If the motor load is lightened, $\bar{I}_2'$ reduces, while $\bar{I}_0$ remains constant. It can be easily visualised from the phasor diagram that θ_1 will increase and the pf will reduce. Therefore, an induction should not be run at light load for long periods of time. In choosing an induction motor, its rating should match the load. Over rated motor should not be used.

Example 12.1 The efficiency of a 400 V, 3-phase, 6-pole induction motor drawage a line current of 80 A at 0.75 pf at 4% slip is 85%. Calculate the shaft output and shaft torque.

Solution

$$\text{Input} = \sqrt{3} \times 400 \times 80 \times 0.75 \times 10^{-3}$$
$$= 41.57 \text{ kW}$$
$$\text{Shaft output} = 41.57 \times 0.85 = 35.33 \text{ kW}$$
$$n_s = 1000 \text{ rpm}$$
$$n = (1 - 0.04) \times 1000 = 960 \text{ rpm}$$
$$\omega = \frac{2\pi \times 960}{60} = 100.5 \text{ rad/s}$$
$$\text{Shaft torque} = \frac{35.33 \times 1000}{0.5} = 351.5 \text{ Nm}$$

Example 12.2 A 4-pole, 50 Hz, 3-phase induction motor when running on full load develops a useful torque of 100 Nm while the rotor emf is observed to make 120 cycles/

min. It is known that the torque lost on account of friction and core loss is 7 Nm. Calculate

(a) shaft power output,
(b) rotor copper loss,
(c) motor input, and
(d) motor efficiency.

The total core loss is given as 700 W.

Solution

$$f_2 = sf = \frac{120}{60} = 2 \text{ Hz}$$

$$s = \frac{2}{50} = 0.04$$

$$n_s = 1500 \text{ rpm}$$

$$n = (1 - 0.04) \times 1500 = 1440 \text{ rpm}$$

$$\omega = \frac{1440 \times 2\pi}{60} = 150.7 \text{ rad/s}$$

(a) Shaft power output $= T\omega = 100 \times 150.7 = 15.07$ kW

Mechanical power developed,

$$P_m = (100 + 7) \times 150.7 = 16.12 \text{ kW}$$

(b) From Eq. (12.16)

Roto copper loss $3I_2^2R_2 = P_m\left(\frac{s}{1-s}\right)$

$$= 16.12 \times \frac{0.04}{(1\text{–}0.04)} = 0.67 \text{ kW}$$

(c) Motor input $= 16.12 + 0.67 + 0.7 = 17.49$ kW

$$\eta = \frac{15.07}{17.49} = 86.16\%$$

12.4 TORQUE-SLIP CHARACTERISTIC

For the sake of simplicity of treatment, the stator impedance will be neglected.* The simplified circuit for obtaining torque expression is given in Fig. 12.7. The result given below follows immediately.

$$I'_2 = \frac{V_1}{\sqrt{(R'_2/s)^2 + X'^2_2}} = \frac{V_1/a}{\sqrt{(R_2/s)^2 + X_2^2}} \tag{12.20}$$

Substituting in Eq. (12.18)

$$T = \frac{P_G}{\omega_s} = \frac{3}{\omega_s}\frac{V_1^2(R_2'/s)}{(R_2'/s)^2 + X_2'^2} = \frac{3}{\omega_s}\frac{(V_1/a)^2(R_2/s)}{(R_2/s)^2 + X_2^2} \tag{12.21}$$

For maximum torque, $P_G/3$ in Fig. 12.7 must be maximum, which happens when R'_2/s matches with X'_2 (maximum power transfer theorem), i.e.

* This assumption causes a tolerable error in the region of low slip but unacceptable error for large slips. Yet it helps to get a feel of the complete T–s characteristic.

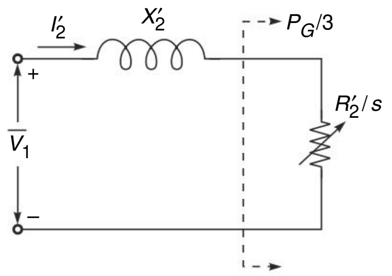


Fig. 12.7

$$\frac{R_2'}{s_{\max T}} = X'_2$$

or $$s_{\max\ T} = \frac{R_2'}{X_2'} = \frac{R_2}{X_2} = \frac{\text{rotor resistance}}{\text{standstill reactance}} \tag{12.22}$$

Substituting Eq. (12.22) in Eq. (12.21)

$$T_{\max} \text{ or } T_{\text{breakdown}} = \frac{3}{\omega_s}\left(\frac{V_1^2}{2X_2'}\right) \tag{12.23}$$

$$= \frac{3}{\omega_s}\left(\frac{(V_1/a)^2}{2X'_2}\right) \tag{12.24}$$

For load Torque $T_L > T_{\max}$, the motor stalls and so it is called breakdown torque.

Starting current and torque are given by substituting $s = 1$ in Eqs. (12.20) and (12.21). Thus

$$I'_2\,(\text{start}) = \frac{V_1}{\sqrt{R_2'^2 + X_2'^2}} = \frac{V_1/a}{\sqrt{R_2^2 + X_2^2}} \tag{12.25}$$

$$T_{\text{start}} = \frac{3}{\omega_s}\,\frac{V_1^2 R_2'}{(R_2')^2 + X_2'^2} = \frac{3}{\omega_s}\,\frac{(V_1/a)^2 R_2}{R_2^2 + X_2^2} \tag{12.26}$$

The starting torque increases by adding resistance in the rotor circuit. At the same time the starting current will reduce. This indeed is the advantage of the slip-ring induction motor in which a high starting torque is obtained at low starting current.

Observe that

$$T_{\text{start}}\,(\max) = T_{\text{breakdown}};\ R_2' = X_2'\ (\text{Eq. (12.22)}) \tag{12.27}$$

Further Approximations For low values of slip

$$\frac{R'_2}{s} >> X'_2$$

On ignoring X_2', it follows from Eqs. (12.20) and (12.21)

that $$I_2' = \frac{sV_1}{R_2'} = \frac{sV_1/a}{R_2} \tag{12.28}$$

and $$T = \frac{3}{\omega_s}\,\frac{sV_1^2}{R_2'} = \frac{3}{\omega_s}\,\frac{s(V_1/a)^2}{R_2};\ \text{(linear } T\text{–}s \text{ relationship)} \tag{12.29}$$

For large values of slip $X_2' > R_2'/s$.

Ignoring R_2'/s,

$$T = \frac{3}{\omega_s}\frac{V_1^2 R'_2}{sX'^2_2} = \frac{3}{\omega_s}\frac{(V_1/a)^2 R_2}{sX_2^2} \text{ (inverse law } T\text{–}s) \quad (12.30)$$

Stator Impedance Considered It is easy to see that if the stator impedance is accounted for the torque expression of Eq. (12.21) would become

$$T = \frac{3}{\omega_s}\frac{V_1^2(R'_2/s)}{(R_1 + R'_2/s)^2 + (X_1 + X'_2)^2} \quad (12.31)$$

Plot of Complete T–s Characteristic From the above results the complete *T–s* characteristic is plotted in Fig. 12.8. Its various operating modes are:

– *Motoring mode:* $0 \leq s \leq 1$; subsynchronous speed, motor runs in the direction of the rotating air-gap field;

– *Braking mode:* $s > 1$; motor runs in opposite direction to the rotating field; and

– *Generating mode*: $s < 0$; motor runs at supersynchronous speed in the direction of the rotating field. Negative *s* implies that mechanical power output (Eq. (12.16)) is negative (input) and so the electrical power is the output. Also from Eq. (12.13), power across air-gap change sign. Power transfers from rotor to stator across air-gap.

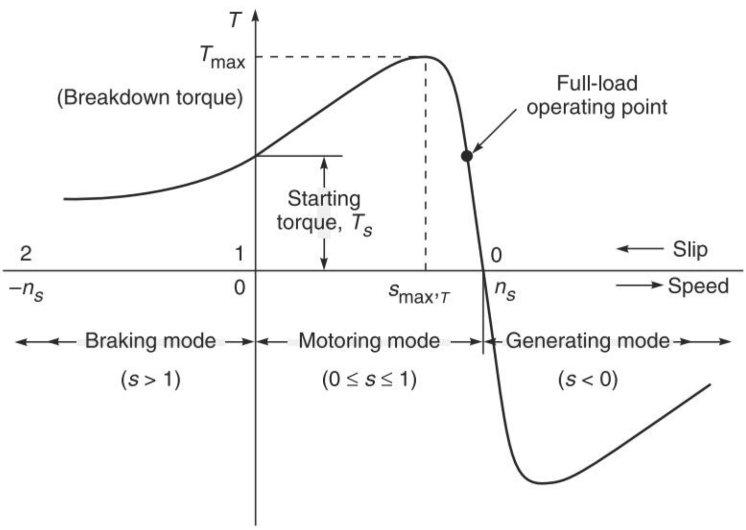


Fig. 12.8 Torque–slip characteristic of induction motor (at fixed terminal voltage)

External Resistance Added in Rotor Circuit This is only possible in a slip-ring induction motor. As resistance is added in the rotor circuit, we observe that

– breakdown torque remains unchanged (Eq. (12.24));

– slip at breakdown torque increases (Eq. (12.22)); and

– T_{start} becomes maximum (equal to $T_{breakdown}$) at $R_2 = X_2$.

The torque–slip characteristics of induction motor for increasing values of rotor resistance are plotted in Fig. 12.9.

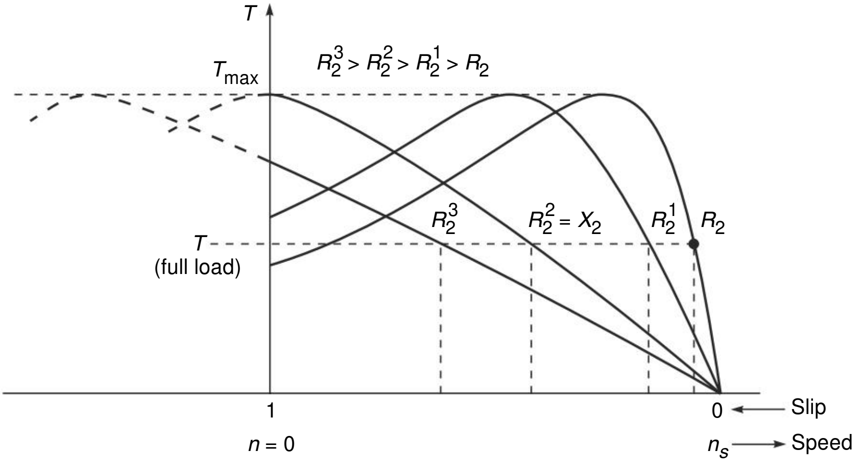


Fig. 12.9 Torque–slip characteristics of induction motor with increasing values of rotor resistance

Example 12.3 A 6.6 kV, 20-pole, 50 Hz, 3-phase star-connected induction motor has a rotor resistance of 0.12 Ω and a standstill reactance of 1.12 Ω. The motor has a speed of 292.5 rpm at full load. Calculate (a) slip at maximum torque and (b) the ratio of maximum to full-load torque. Neglect stator impedance.

Solution

$$n_s = \frac{120 \times 50}{20} = 300 \text{ rpm}$$

$$s_{fl} = \frac{300 - 292.5}{300} = 0.025$$

(a) From Eq. (12.36)

$$s_{\max T} = \frac{R_2}{X_2} = \frac{0.12}{1.12} = 0.107$$

(b) From Eq. (12.21)

$$T_{f1} = \frac{3}{\omega_s} \frac{(V/a)^2 \times 0.12/0.025}{(0.12/0.025)^2 + (1.12)^2} = \frac{3}{\omega_s}\left(\frac{V}{a}\right)^2 \times 0.1976 \qquad \text{(i)}$$

From Eq. (12.24)

$$T_{\max} = \frac{3}{\omega_s} \frac{0.5(V/a)^2}{1.12} = \frac{3}{\omega_s}\left(\frac{V}{a}\right)^2 \times 0.446 \qquad \text{(ii)}$$

Dividing Eq. (ii) by Eq. (i)

$$\frac{T_{\max}}{T_{fl}} = \frac{0.446}{0.1976} = 2.26$$

Example 12.4 Prove that in a 3-phase induction motor, the ratio of maximum to starting torque is $(1 + k^2)/2k$, where k is the ratio of rotor resistance to rotor reactance. Neglect stator impedance.

Solution From Eq. (12.21)

$$T = K_T \frac{sR_2}{R_2^2 + s^2 X_2^2}$$

$$s_{\max T} = \frac{R_2}{X_2} \quad \therefore \quad T_{\max} = \frac{K_T}{2X_2} \tag{i}$$

$$\text{Slip at starting} = 1$$

$$\therefore \qquad T_{\text{start}} = K_T \frac{R_2}{R_2^2 + X_2^2} \tag{ii}$$

Dividing Eq. (i) by Eq. (ii)

$$\frac{T_{\max}}{T_{\text{start}}} = \frac{R_2^2 + X_2^2}{2R_2X_2} = \frac{1 + (R_2/X_2)^2}{2R_2/X_2}$$

$$= \frac{1 + k^2}{2k} \tag{iii}$$

Example 12.5 An 8-pole, 50 Hz, 3-phase induction motor has rotor resistance and standstill reactance of 0.5 Ω and 5 Ω respectively. Calculate (a) the speed at which the torque is maximum, (b) the ratio of maximum torque to starting torque and (c) value of external resistance to be connected in series in the rotor circuit for the ratio in (b) to be 2. Neglect stator impedance.

Solution

(a)
$$n_s = 750 \text{ rpm}$$

$$s_{\max T} = \frac{R_2}{X_2} = \frac{0.5}{5} = 0.1$$

$$\therefore \qquad n = 750\,(1 - 0.1) = 675 \text{ rpm}$$

(b)
$$k = \frac{R_2}{X_2} = 0.1$$

Substituting in Eq. (iii) in Ex. 12.4

$$\frac{T_{\max}}{T_{\text{start}}} = \frac{1 + (0.1)^2}{2 \times 0.1} = 5.05$$

(c)
$$\frac{T_{\max}}{T_{\text{start}}} = \frac{1 + k^2}{2k} = 2$$

or $\quad k^2 - 4k + 1 = 0$

or $\quad k = 3.732 \quad \text{or} \quad 0.268$

Now $\quad R_2\,(\text{total}) = 5 \times 3.72 \quad \text{or} \quad 5 \times 0.268$

$$= 18.66 \quad \text{or} \quad 1.34\ \Omega$$

$$R_2\,(\text{ext}) = 18.66 - 0.5 = 18.16\ \Omega$$

or $\quad R_2\,(\text{ext}) = 1.34 - 0.5 = 0.84\ \Omega$

Figure 12.8 shows the torque–slip characteristics of the motor for the two values of R_2 (total). For R_2 (total) = 18.66 Ω the slip at maximum torque is $s_{\max T}$ = 18.66/5 = 3.732. As a result, the torque for slip less than unity in this case is less than the starting torque and hence the motor will speed up very slowly and that too only if T_{load} is much less than T_{start}. Hence this value of R_2 (ext) is

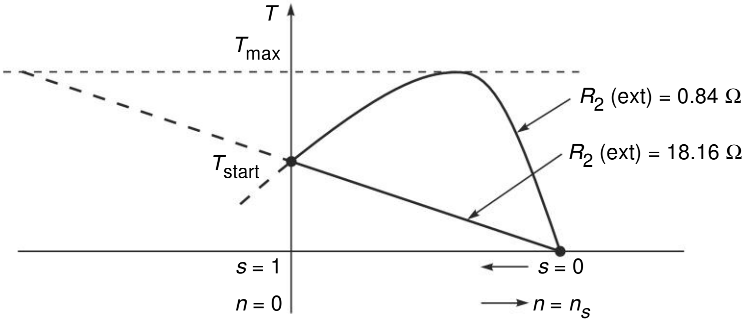


Fig. 12.10

rejected.

Thus

$$R_2 \text{ (ext) } = 0.84\ \Omega$$

Example 12.6 A 3-phase induction motor has a starting torque of 80% and a maximum torque of 200% of the full-load torque. Find (a) slip at maximum torque, (b) full-load slip, (c) rotor current at starting in percent-age of full-load rotor current and (d) percentage by which the rotor resistance must be increased to give a starting torque of 100% of the full-load torque.

Solution

(a) From Eqs (12.24) and (12.26)

$$\frac{T_{max}}{T_{start}} = \frac{0.5(R_2^2 + X_2^2)}{X_2 R_2} = \frac{0.5(R_2^2/X_2^2 + 1)}{R_2/X_2} \quad \text{(i)}$$

But $$R_2/X_2 = s_{max\ T} \quad \text{(ii)}$$

$$\frac{200}{80} = \frac{0.5(s^2_{max\ T} + 1)}{s^2_{max\ T}}$$

or $$s^2_{max\ T} - 5s_{max\ T} + 1 = 0 \quad \text{(iii)}$$

Solving $$s_{max\ T} = 0.21, 4.79 \text{ (rejected)}$$

(b) From Eqs (12.24) and (12.21)

$$\frac{T_{max}}{T_{fl}} = \frac{0.5[(R_2/s_{f1})^2 + X_2^2]}{R_2 X_2} \cdot s_{f1} \quad \text{(iv)}$$

But $$R_2/X_2 = s_{max\ T} = 0.21$$

Substituting values

$$2 = \frac{0.5[(0.21/s_{f1})^2 + 1]}{0.21/s_{f1}} \quad \text{(v)}$$

$$\left(\frac{0.21}{s_{f1}}\right)^2 - 4\left(\frac{0.21}{s_{f1}}\right) + 1 = 0 \quad \text{(vi)}$$

Solving

$$\frac{0.21}{s_{fl}} = 0.268, 3.73$$

or $$s_{fl} = 0.783 \text{ (rejected)}, 0.056$$

(c) From Eq. (12.21)

$$\frac{I_2(\text{start})}{I_2(\text{full load})} = \left[\frac{(R_2/s_{fl})^2 + X_2^2}{R_2^2 + X_2^2}\right]^{1/2} \quad \text{(vii)}$$

$$= \left[\frac{(s_{\max T}/s_{fl})^2 + 1}{s^2_{\max T} + 1}\right]^{1/2} \quad \text{(viii)}$$

$$= \left[\frac{(0.21/0.056) + 1}{(0.21)^2 + 1}\right]^{1/2}$$

$$= 3.8$$

Notice that the rotor current at starting is 3.8 times the full-load current; so would be the stator current. Measures have to be adopted to restrict the starting current.

(d) Let R_2 be increased to kR_2. From Eq. (12.21)

$$\frac{T_{\text{start}}}{T_{\text{full load}}} = \frac{(R_2/s_{f1})^2 + X_2^2}{k^2R_2^2 + X_2^2} \cdot s_{fl} \quad \text{(ix)}$$

$$1 = \left[\frac{(s_{\max T}/s_{fl})^2 + 1}{k^2 s^2_{\max T} + 1}\right] k\, s_{fl}$$

or $$1 = \frac{k[(0.21/0.056)^2 + 1]\times 0.056}{k^2 \times (0.21)^2 + 1}$$

or $$k^2 - 19.13\, k + 22.675 = 0$$

$$k = 1.27,\ 17.86 \text{ (rejected)}$$

Example 12.7 A 3-phase, Y-connected, 440 V, 7.5 kW, 50 Hz, 6-pole induction motor has the following circuit parameter constants.

$$R_1 = 1.06\ \Omega \qquad R'_2 = 0.576\ \Omega$$
$$X_1 = 1.68\ \Omega \qquad X'_2 = 0.75\ \Omega$$
$$X_m = 44.2\ \Omega$$

The total windage, friction and core losses may be assumed to be 415 W at any load. The motor runs at a speed of 975 rpm at rated voltage and frequency on a particular load. Calculate input torque and power, stator current, power factor and efficiency.

Solution

$$n_s = 1000 \text{ rpm},\ \omega_s = \frac{2\pi \times 1000}{60} = 104.7 \text{ rad/s}$$

$$n = 975 \text{ rpm}$$

$$s = \frac{1000 - 975}{1000} = 0.025$$

As the iron less resistance R_i is not proved in the circuit of Fig. 12.4 (e), it will be ignored and stator iron loss along with windage and friction loss will be subtracted from the gross mechanical power output. From this figure

$$\bar{Z} = (1.06 + 0.576/0.025) + j\,(1.68 + 0.75)$$
$$= 24.1 + j\,2.43 = 24.22\ \angle 5.8°$$

$$\bar{I}'_2 = \frac{440/\sqrt{3}}{24.22} \angle -5.8° = 10.5 \angle -5.8° \text{ A}$$

$$\bar{I}_0 \simeq \bar{I}_m = -j\,\frac{440/\sqrt{3}}{44.2} = -j\,5.75 \text{ A}$$

$$I_1 = \bar{I}_0 + \bar{I}'_2 = 10.5 \angle -5.8° - j\,5.75$$

$$= 12.45 \angle -33° \text{ A}$$

$$pf = \cos 33° = 0.84 \text{ lag}$$

$$\text{Power input} = \sqrt{3} \times 440 \times 12.45 \times 0.84$$

$$= 7.97 \text{ kW}$$

$$\text{Power output (gross)} = 3I'^2_2 R'_2 \left(\frac{1}{s} - 1\right)$$

$$= 3 \times (10.5)^2 \times 0.576 \left(\frac{1}{0.025} - 1\right)$$

$$= 7.43 \text{ kW}$$

$$\text{Power output (net)} = 7.43 - 0.415$$

$$= 7.015 \text{ kW}$$

$$\eta = \frac{7.015}{7.97} = 88\%$$

$$\text{Torque (net)} = \frac{7.015 \quad 1000}{104.7(1 - 0.025)}$$

$$= 68.72 \text{ Nm}$$

12.5 DETERMINATION OF CIRCUIT MODEL PARAMETERS

Since an induction machine is analogous to a transformer, similar non-loading tests would determine the circuit model parameters wherein the blocked rotor test now corresponds to the SC test of a transformer.

No Load Test

The machine is run as a motor on no load. It runs close to synchronous speed ($s \approx 0$). It cannot reach synchronous speed because of windage and friction loss. Under no-load condition, therefore, the circuit model of Fig. 12.24 (e) simplifies as Fig. 12.11 because

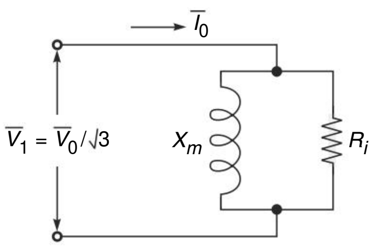


Fig. 12.11 Circuit model at no load (equivalent star basis)

$$\text{load resistance} = R'_2 \left(\frac{1}{s} - 1\right)\Big|_{s \simeq 1}$$

$$\approx \infty \text{ (open circuit)}$$

The data recorded during the no-load test are:

V_0 (line) rated value

I_0 = no-load current

P_o = no-load power (3 phase)

= stator core loss plus windage and friction loss

From these values X_m and R_i can be computed as illustrated in Example 12.8.

It is to be noted the R_i now includes the effect of windage and friction loss apart from the stator iron loss.

Blocked-Rotor Test

The stator is supplied with reduced voltage while the rotor is blocked, i.e. not allowed to rotate (s =1). Now

$$\text{load resistance } = R'_2\left(\frac{1}{s}-1\right)\bigg|_{s=1} \approx 0 \text{ (short circuit)}$$

because of which a much reduced voltage (about 25% of rated) has to be applied for circulating full-load current. At such low voltage, exciting current and stator core loss can be ignored, i.e. the shunt branch of the circuit model is disconnected reducing the circuit of Fig. 12.4(e) to that of Fig. 12.12.

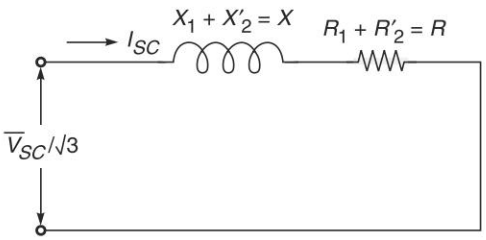


Fig. 12.12 Circuit model for blocked-rotor test

The data recorded during the test are:

V_{SC} (line), I_{SC} (line)

P_{SC} = full-load copper loss

From these values, we can compute

$R = R_1 + R_2$ and $X = X_1 + X_2$

as illustrated in Example 12.8. The stator phase resistance R_1 can be measured by dc test and corrected to ac value and R_1, R'_2 separated. For separating X_1, X'_2, we can use the approximation

$$\frac{X_1}{X_2} = \frac{R_1}{R'_2} \tag{12.32}$$

Voltage-Ratio (a) Test

It is possible to measure the voltage ratio ($E_1/E_2 = a$) only for a slip-ring motor with slip rings open-circuited (obviously rotor would be stationary).

Example 12.8 A 3-phase, 400 V, 25 kW, 50 Hz, 8-pole motor yielded the following data on testing:

No load	400 V	8.7 A	1210 Ω
Blocked rotor	200 V	47.8 A	6050 Ω

The stator phase resistance is 0.42 Ω (star basis).

(a) Estimate the parameters of the circuit model.

(b) The motor is run from 400 V, 50 Hz, 3-phase supply. Calculate

(i) Line current, power factor, torque developed and motor efficiency at 710 rpm,

(ii) Line current and torque developed at starting, and

(iii) Maximum torque and the slip at which it occurs.

Solution

(a) Motor parameters (equivalent star basis): Refer Fig. 12.4 (e).

No Load Test Shunt branch parameters

$$\frac{P_0}{3} = \frac{(V/\sqrt{3})^2}{R_i}$$

or $$R_i = \frac{(V/\sqrt{3})^2}{(P_0/3)} \tag{12.33}$$

Substituting values

$$R_i = (400/\sqrt{3})^2/(1210/3)$$
$$= 132.2\ \Omega \text{ (inclusive of windage and friction loss)}$$
$$P_0 = \sqrt{3}\ V I_0 \cos\theta_0$$

or

$$pf = \cos\theta_o = 1210/(400\sqrt{3} \times 8.7) = 0.2 \text{ lag}$$
$$\theta_0 = 87.5°$$
$$R_i = (400/\sqrt{3})^2/(1210/3)$$
$$= 132.2\ \Omega \text{ (inclusive of windage and friction loss)}$$
$$\cos\theta_0 = 1210/(\sqrt{3} \times 400 \times 8.7) = 0.2;\ \theta_0 = 78.5°$$
$$X_m = R_i/\tan\theta_0 = 27\ \Omega$$

Blocked Rotor Test

$$R = R_1 + R'_2 = \frac{6050}{3 \times (47.8)^2} = 0.883\ \Omega$$
$$R'_2 = 0.883 - 0.42 = 0.463\ \Omega$$
$$Z = \frac{200\sqrt{3}}{47.8} = 2.146\ \Omega$$
$$X = \sqrt{Z^2 - R^2}$$

Substituting values

$$X = X_1 + X'_2 = [(2.416)^2 - (0.883)^2]^{1/2}$$
$$= 2.25\ \Omega$$

(b) (i) $$n_s = 750 \text{ rpm}, \omega_s = 78.54 \text{ rad/s}$$

$$s = \frac{750 - 710}{750} = 0.053$$

With reference to Fig. 12.5(e)

$$\bar{Z} = (0.42 + 0.463/0.053) + j\,2.25$$
$$= 9.16 + j\,2.25 = 9.43\ \angle 13.8°\ \Omega$$
$$\bar{I}'_2 = \frac{400/\sqrt{3}\ \angle 0°}{9.43\ \angle 13.8°} = 24.5\ \angle -13.8° \text{ A}$$
$$\bar{I}_0 = 8.7\ \angle -78.5°$$
$$\bar{I}_1 = 8.7\ \angle -78.5° + 24.5\ \angle -13.8°$$
$$= 29.3\ \angle -29.4° \text{ A}$$

$$I_1 = 29.3 \text{ A}, pf = 0.87 \text{ lag}$$

Now
$$T = \frac{3I_2'^2 R_2'}{s\omega_s}; \text{ (Eq. (12.18))}$$
$$= \frac{3 \times (24.5)^2 \times 0.463}{0.053 \times 78.54}$$
$$= 200.3 \text{ Nm (net; because windage and friction loss is accounted for in the shunt branch)}$$

$$\text{Power output} = 3I_2^2 R'_2 \left(\frac{1}{s} - 1\right); \text{ (Eq. (12.30))}$$
$$= 3 \times (24.5)^2 \times 0.463 \left(\frac{1}{0.053} - 1\right)$$
$$= 14.897 \text{ kW (net; because of the same reason as advanced for torque)}$$

$$\text{Power input} = \sqrt{3} \times 400 \times 29.3 \times 0.87$$
$$= 17.66 \text{ kW}$$
$$\eta = \frac{14.897}{17.66} = 84.36\%$$

(ii)
$$s = 1$$
$$\bar{Z} = 0.883 + j\,2.25$$
$$= 2.42 \angle 68.6° \ \Omega$$
$$\bar{I}_2' = \frac{400/\sqrt{3} \angle 0°}{2.42 \angle 68.6°} = 95.43 \angle -68.6° \text{ A}$$
$$\bar{I}_1 = 8.7 \angle -78.5° + 95.43 \angle -68.6°$$
$$= 103.6 \angle -69.94° \text{ A}$$
$$T_{\text{start}} = \frac{3 \times (95.43)^2 \times 0.463}{78.54}$$
$$= 161.1 \text{ Nm}$$

(iii) For maximum torque P_G should be maximum. With reference to Fig. 12.5(e), this condition is fulfilled when (maximum power transfer theorem)

$$[R_1^2 + (X_1 + X'_2)^2]^{1/2} = \frac{R'_2}{s_{\max T}} \quad (12.34)$$

or
$$s_{\max T} = \frac{R'_2}{\left[R_1^2 + (X_1 + X_2')^2\right]^{1/2}} \quad (12.35)$$
$$= \frac{0.463}{\left[(0.42)^2 + (0.25)^2\right]^{1/2}} = 0.2$$

Now for
$$s = 0.2$$
$$\bar{Z} = \left(0.42 + \frac{0463}{0.2}\right)^2 + j\,2.75 \ \Omega$$
$$= 2.735 + j\,2.25 = 3.54 \angle 39.4°$$
$$I'_2 = \frac{400/\sqrt{3}}{3.54} = 65.24 \text{ A}$$
$$T_{\max} = \frac{3 \times (65.25)^2 \times 0.463/0.2}{78.54}$$
$$= 376.3 \text{ Nm}$$

12.6 STARTING

At starting (s = 1) short-circuit conditions prevail (load resistance = 0). The motor starting current can therefore be as large as four to six times the full-load current. Yet, at the same time, the starting torque is no higher than the full-load torque $\left(T_s = \left.\frac{I_s^2 R_2'}{s\omega_s}\right|_{s=1}\right)$. While such large currents, which flow for the short starting period, cannot damage the motor, these may not be permitted to be drawn from the power network depending upon its source impedance (low-impedance source can stand large current demands without undue voltage dips which are bothersome to other consumers). In general, however, motors above 5 kW should not be started direct-on-line (DOL) and also if permissible, these should be started on no load (motor decoupled from load) so that these can run to speed quickly.

Squirrel-Cage Motors

Reduced voltage is the only possible method of starting a squirrel-cage motor as the rotor cage is permanently shorted and cannot be tampered with. The starting torque reduces as voltage square (see Eq. 12.26) and therefore only no-load starting is possible. Various methods of starting squirrel-cage motors are discussed below.

Stator Impedance Starting Motor voltage is reduced by placing series resistance/reactance in the lines. The starting current reduces directly as the motor voltage but the torque reduces as square of voltage. This method can only be used for small fractional-kW motors.

Star–Delta Starting The motor is designed for delta running and is started in star as shown in Fig. 12.13. Let

$$Z_{SC} = \text{short-circuit phase impedance (delta connection)}$$
$$V = \text{line-to-line voltage}$$

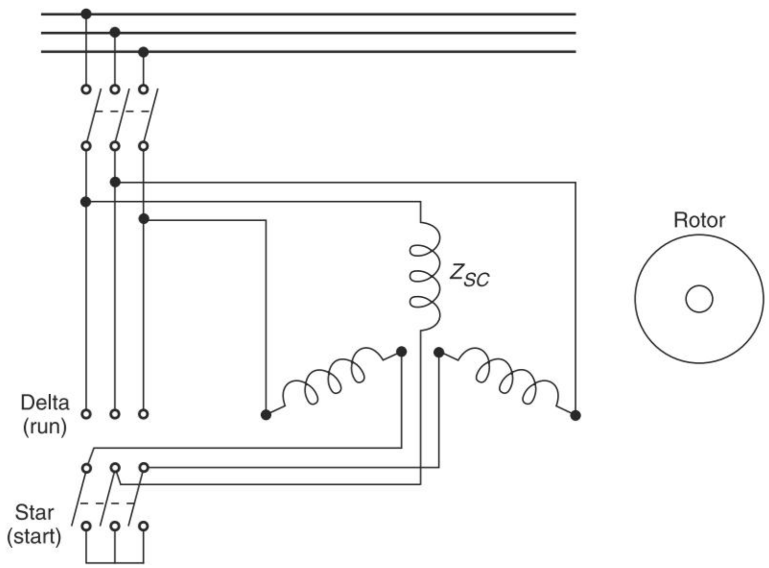


Fig. 12.13 Star–delta starting

DOL (direct on line) starting (in delta):

$$I_s\text{ (phase)} = \frac{V}{Z_{SC}} \tag{12.36}$$

$$I_s\text{ (line)} = \frac{\sqrt{3}\,V}{Z_{SC}} \tag{12.37}$$

$$T_s \propto I_s^2\text{ (phase)} = \frac{V^2}{Z_{SC}^2} \tag{12.38}$$

Starting in star:

$$I_s\text{ (line)} = I_s\text{ (phase)} = \frac{V^2}{\sqrt{3}\,Z_{sc}} \tag{12.39}$$

$$T_s \propto \frac{V^2}{3Z_{sc}^2} \tag{12.40}$$

From these results, we get

$$\frac{I_s(\text{line})(\Delta)}{I_s(\text{line})(\Delta)} = \frac{1}{3} \tag{12.41}$$

and

$$\frac{T_S(\Delta)}{T_S(\Delta)} = \frac{1}{3} \tag{12.42}$$

Observe that compared to DOL starting, torque reduces by the same factor (1/3) as the line current. Because of this advantage and simplicity of the method, it is popularly used.

Autotransformer Starting The connection diagram for this method of starting is drawn in Fig. 12.14. It is immediately observed that compared to DOL starting both torque and line current reduce by a factor of a^2 (a being the voltage ratio of autotransformer). It is also observed that star/delta starting corresponds to $a = 1/\sqrt{3}$ (fixed). The autotransformer can incidentally be used for small speed reduction, if desired. Because of the expense involved, this method is employed only for large squirrel-cage motors.

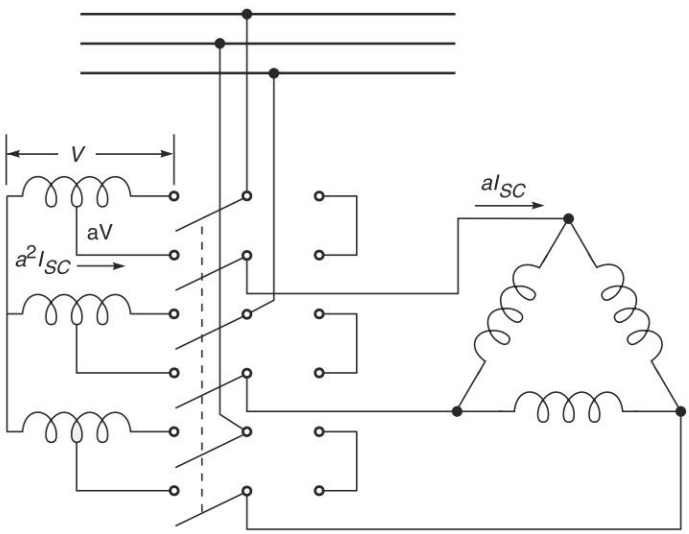


Fig. 12.14 Autotransformer starting

Slip-Ring Motors

Rotor resistance starting Resistances are included in the rotor circuit via slip rings as shown in Fig. 12.15 and are cut out in steps as the motor picks up speed. While the starting current reduces, the starting torque increases (unlike in squirrel-cage motor starting); the starting torque being maximum at R_2 (total) = X_2 (see Fig. 12.7). Thus the method is ideal for on-load starting. The slip-ring motor being more expensive than the squirrel-cage motor (because of wound rotor) is employed only where on-load starting is a necessity. High starting torque in the medium range can be obtained by the less expensive double-cage motor (described later).

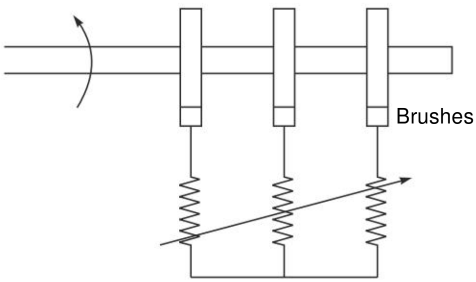


Fig. 12.15 Rotor resistance starting

Example 12.9 A squirrel-cage induction motor has a full-load slip of 4%. Its starting current is 5 times its full-load current. Calculate the starting torque in pu of the full-load torque. Neglect stator impedance and magnetizing current.

Solution

$$\text{Starting torque } T_s = \frac{3}{\omega_s} I_s^2 R'_2 \tag{i}$$

$$\text{Full-load torque } T_{fl} = \frac{3}{\omega_s} \frac{I_{f1}^2 R'_2}{s_{f1}} \tag{ii}$$

Dividing

$$\frac{T_S}{T_{f1}} = \left(\frac{I_S}{I_{f1}}\right)^2 s_{fl}$$

$$= 25 \times 0.04 = 1 \text{ pu}$$

Remark While the starting current is as high as five times full-load current, the starting torque just equals full-load torque. This corroborates the statement made earlier on starting.

Example 12.10 A 3-phase squirrel-cage induction motor has a ratio of maximum torque T_m to full-load torque T_{f1} as 3.5 : 1. Determine the ratio of starting torque T_s to full-load torque for (a) direct starting, (b) star–delta starting and (c) autotransformer starting with a tapping of 70%. The rotor resistance and standstill reactance per phase (delta) are 0.5 Ω and 5 Ω respectively. Neglect stator impedance.

Solution

$$T_m = K_T \frac{V^2}{2X_2}; \; V = \text{line voltage; Eq. (12.24)}$$

$$T_s = K_T \left(\frac{V^2 R_2}{R_2^2 + X_2^2}\right); \text{ Eq. (12.26)}$$

$$T_{fl} = \frac{T_m}{3.5} = \frac{K_T V^2}{7X_2} = \frac{K_T V^2}{35}$$

(a) Direct Start

$$\frac{T_S}{T_{f1}} = \frac{K_T V^2 R_2}{R_2^2 + X_2^2} \times \frac{35}{K_T V^2} = \frac{35 \times 0.5}{(0.5^2 + 5^2)} = 0.693$$

(b) Star–delta Starting

$$\frac{T_S}{T_{f1}} = \frac{K_T (V/\sqrt{3})^2 R_2}{R_2^2 + X_2^2} \times \frac{35}{K_T V^2} = \frac{35 \times 0.5}{3 \times (0.5^2 + 5^2)} = 0.231$$

(c) Autotransformer Starting

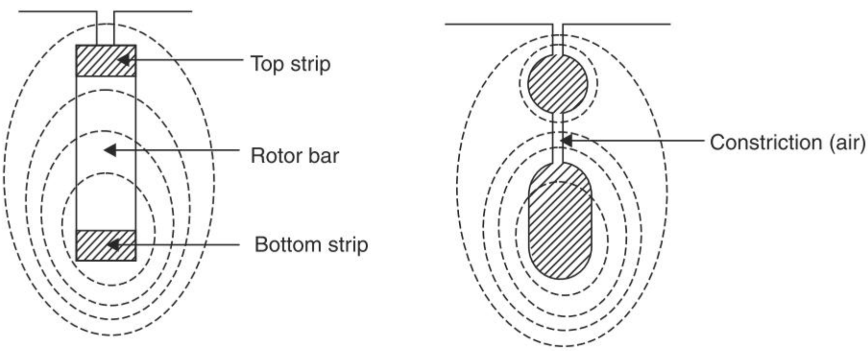
$$\frac{T_S}{T_{f1}} = \frac{K_T (0.7V)^2 R_2}{R_2^2 + X_2^2} \times \frac{35}{K_T V^2}$$

$$= 0.49 \times 0.693 = 0.4693$$

Deep-Bar / Double-Cage Induction Motor

To overcome the low starting torque drawback of the squirrel-case motor, the fact that the rotor frequency is high at starting, (f = 50 Hz) and low when running (sf = 2 Hz at is s = 4% say) is exploited in the deep-bar/double-cage type rotor as shown in the cross-sectional view of Fig. 12.16. In the deep-bar rotor, we can imagine the conductor to be made of strips, the upper and lower strips are shown in Fig. 12.16 (a). It can be seen that almost all the leakage flux (shown dotted) links the lower strip, while only a small part of it links the upper strip. The same holds to a smaller extent for the other strips. In the double-cage rotor bars of Fig. 12.16(b), this flux linkage difference is seen more clearly. Further, this flux linkage difference is accentuated by the fact that the self-flux of upper bar has to cross two air-gaps while that of the lower bar has to cross only one air-gap.

The above explained difference in the flux linkages of upper and lower conductor strips or conductor bars at starting causes the leakage inductance X_2 (lower part) to be much more than the X_2 (upper part). Therefore the rotor conductor current is unevenly divided, the upper part carries larger share of the total conductor current resulting



(a) Rotor with deep bars (b) Double cage rotor

Fig. 12.16

in effective reduction in the conductor cross-section and increase in its resistance. However, during running the rotor frequency (sf) being quite low sX_2 (upper) $\approx$ sX_2 (lower) and so the conductor current is almost evenly distributed resulting in lower conductor resistance. This means that at start the effective rotor resistance is higher than the actual values. As a result, the motor has high starting torque and low running resistance and good running performance; how slip and high efficiency.

The torque-slip (T-s) characteristic of a double-cage motor as the sum of the characteristics of the two cages is drawn in Fig. 12.17. Observe the high ratio of $T_{\text{start}} / T_{\text{Breakdown}}$ compared to a normal cage motor.

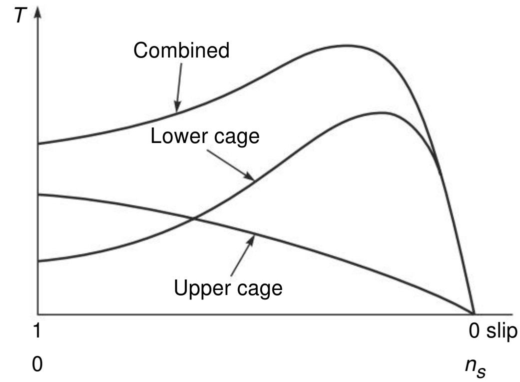


Fig. 12.17 *T–s* characteristic of double-cage rotor

Speed Control

The speed of an induction motor is given by

$$n = (1 - s)\, n_s$$
$$= (1 - s)\, \frac{120f}{P} \tag{12.43}$$

Equation (12.43) suggests two methods of speed control, namely

– Slip control
– Frequency control

These are discussed below.

Slip Control

Voltage Control Torque is proportional to the square of applied voltage (Eq. (12.21)). The *T–s* characteristics for various voltages are drawn in Fig. 12.18. Also shown in this figure is a fan-type load characteristic. From the intersection points it is easily seen that slip increases (speed drops) as the voltage is reduced. Because of the square law for fan torque, at lower voltages slip tends to increase disproportionately and the motor tends to draw a large current (Eq. (12.28)) causing inefficient operation and motor overheating. The method is obviously unsuitable for constant-torque loads. Even for fan-type loads it can only be used for small reduction in speed. It is used only in small motors where series resistance/reactances are employed for reducing the voltage applied to the motor.

Rotor Resistance Control Resistance is included in the rotor circuit via slip-rings as shown in Fig. 12.15. Obviously this type of control is only possible for a wound-rotor machine. Figure 12.19 shows the *T-s* characteristics for increasing

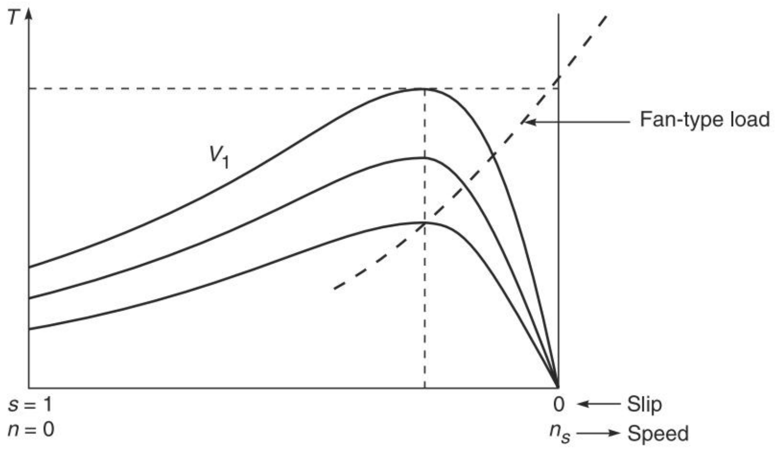


Fig. 12.18 Speed control by voltage control

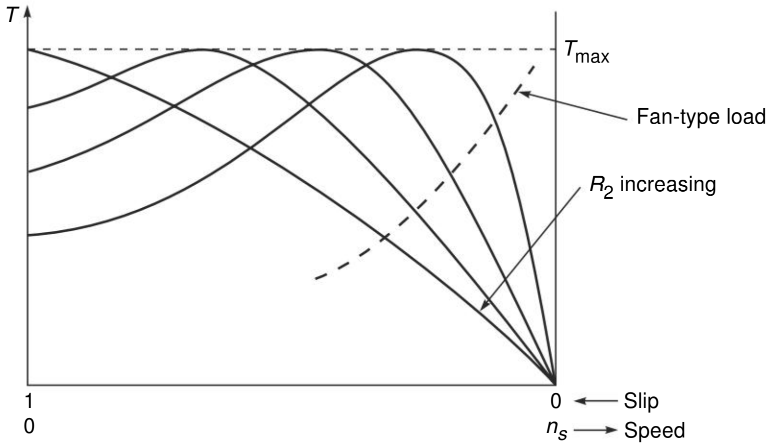


Fig. 12.19 Rotor resistance control

values of rotor resistance along with the fan-type load characteristic. It is easily seen that the slip increases as the rotor resistance is increased.

On approximate basis Eqs. (12.28) and (12.29) are reproduced below:

$$I_2 = \left(\frac{s}{R_2}\right)\left(\frac{V_1}{a}\right)$$

$$T = \frac{3}{\omega_s}\left(\frac{s}{R_2}\right)\left(\frac{V_1}{a}\right)^2$$

For constant load torque, as R_2 is increased, s increases proportionally and I_2 remains constant resulting in considerable increase in rotor resistance loss. At the same time the output $T\omega$ decreases. Thus the motor efficiency drops off sharply. The method is therefore useful only for small speed changes for short time periods in which case the starter resistance itself can be used for this purpose.

Frequency Control

In order to prevent abnormal rise in magnetizing current at below normal frequency, V/f must be kept constant (Eq. (12.1)) so that air-gap flux/pole remains fixed.

For constant (V/f), let us examine the effect on the breakdown torque. As per Eq. (12.23) reproduced below,

$$T_{BD} = \frac{3}{\omega_s}\frac{(V/a)^2}{2X_2}$$

$$\omega_s = \frac{120f}{P} \times \frac{2\pi}{60} = \left(\frac{4\pi}{P}\right)f$$

$$X_2 = 2\pi f L_2$$

Therefore

$$T_{BD} = \frac{3}{\left(\frac{4\pi}{P}\right)f}\frac{(V_1/a)^2}{2(2\pi\ f\ L_2)} = K_{BD}\left(\frac{V_1}{f}\right)$$

We conclude that with constant (V/f), the breakdown torque remains constant. Variable frequency and voltage supply is obtained from a thyristor inverter, which is itself fed from an ac/dc converter as shown in Fig. 12.20. Like in a dc motor a wide range of speed control becomes possible by this method. It is however expensive as full rated converter and inverter are needed.

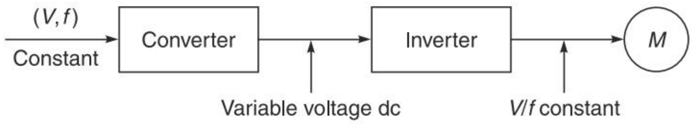


Fig. 12.20 *V/f* control of induction motor

12.7 INDUCTION GENERATOR

It can be observed from Fig. 12.8 that induction machine is in generating mode for $s < 0$ (negative slip). An induction generator is asynchronous in nature because of which it is commonly used as a windmill generator as a windmill runs at a nonfixed speed. These are used in remote areas to supplement power received from weak transmission links. A transmission line connected to an induction generator feeding a local load is drawn in Fig. 12.21. The primemover must be provided with automatic control to increase the generator speed when it is required to meet the increased load.

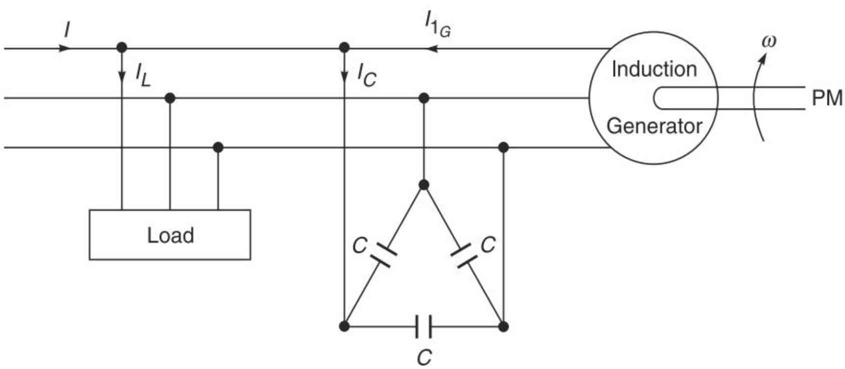


Fig. 12.21

The transmission line has to feed the lagging current component of the load as well as the magnetizing current of the induction generator. This places a severe lagging VARs load on the already weak links. The burden must be relieved by connecting balanced shunt capacitors (in delta) across the induction generator terminals. These feed the lagging magnetizing current of the generator.

An *isolated induction generator* feeding a load is shown in Fig. 12.22. The delta-connected capacitors across the generator terminals provide the magnetizing current necessary to excite the isolated generator.

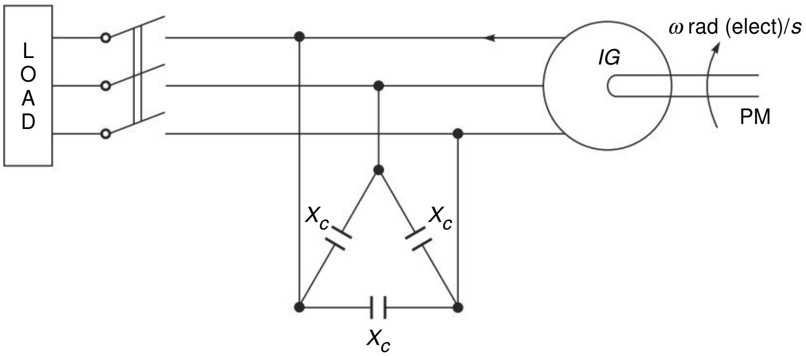


Fig. 12.22

12.8 HIGH EFFICIENCY INDUCTION MOTORS

With the ever increasing energy cost, the life time operating cost of an induction motor can be traded against a high efficiency and a high capital cost. With rising demand for high efficiency or energy efficient induction motors, designers and manufacturers are stepping up their production of such motors.

Example 12.11 The rotor of a 6-pole, 50 Hz, slip-ring induction motor has a resistance of 0.25 Ω/phase and runs at 960 rpm. Calculate the external resistance/phase to be added to lower the speed to 800 rpm, with the load torque reducing to 3/4 of the previous value. Assume stator impedance to be negligible.

Solution

For the range of slip considered,

$$R_2'/s >> X_2' \quad \text{(i)}$$

$$T = \left(\frac{3}{\omega_s}\right)\frac{sV_1}{R_2'} = k\,\frac{s}{R_2'} \quad \text{(ii)}$$

At 960 rpm: $s = (1000 - 960)/1000 = 0.04$

At 800 rpm: $s = (1000 - 800)/1000 = 0.2$

$$T = k\,(0.04/0.25) \quad \text{(iii)}$$

$$(3/4)\,T = k\,[0.2/(0.25 + R_2\,(\text{ext}))]$$

Dividing Eq. (iii) by Eq. (iv), we get

$$4/3 = \frac{0.25 + R_2(\text{ext})}{0.2 \times 0.25} \times 0.04/0.25$$

Solving, we get R_2 (ext)= 1.42 Ω

Example 12.12 A 6-pole, 50 Hz slip-ring induction motor has a rotor resistance of 0.25 Ω and a maximum torque of 180 Nm, while it runs at 860 rpm. Calculate (a) the torque at 4.5% slip and (b) the resistance to be added in the rotor circuit to obtain the maximum torque at starting.

Solution

$$s_{\max T} = (1000 - 860)/1000 = 0.14$$

$$R_2 = 0.14\, X_2 \text{ (for max. torque)}$$

$$\therefore \quad X_2 = 0.25/0.14 = 1.79\ \Omega$$

$$T_{\max} = \frac{3}{\omega_s}\left(\frac{0.5(V_1/a)^2}{X_2}\right) = k/X_2$$

$$180 = k/1.79 \text{ or } k = 322.2$$

(a) Now at slip $s = 0.045$

$$T = \left(\frac{3}{\omega_s}\right)\left(\frac{(V_1/a)^2}{(R_2/s)^2 + X_2^2}\right)$$

$$R'_2/s = \frac{2 \times 100.6}{(0.25/0.045)^2 + (1.79)^2} \times \frac{0.25}{0.045}$$

$$= 105 \text{ Nm}$$

(b) For maximum torque at starting,

$$R_2 + R_2 \text{ (ext)} = X_2 = 1.79\ \Omega$$

or $$R_2 \text{ (ext)} = 1.79 - 0.25 = 1.54\ \Omega$$

Example 12.13 A 400 V, 5 kW, 50 Hz induction motor runs at 1445 rpm at full load. The rotational losses are 285 W. If the maximum torque occurs at 900 rpm, calculate its value.

Solution

$$\text{Mechanical output} = 5000 \text{ W}$$

$$\text{Rotational loss} = 285 \text{ W}$$

Mechanical power developed,

$$P = 5285 \text{ W}$$

$$\text{slip } s = (1500 - 1445)/1500 = 0.0367$$

$$P_m = 3\left(\frac{1}{s} - 1\right) I'^2_2 R'_2 \quad \text{(i)}$$

Ignoring stator impedance

$$I'_2 = V / [R'_2/s + X'^2_2]^{0.5} \quad \text{(ii)}$$

Substituting I'_2 from Eq. (ii) in Eq. (i), we get

$$P_m = 3\left(\frac{1}{s} - 1\right)\frac{V_1^2 R'_2}{(R'_2/s) + X'^2_2} \quad \text{(iii)}$$

Substituting values

$$5285 = 3\,(1/0.0367 - 1) \times \frac{(400/\sqrt{3})R'_2}{(R'_2/0.0367)^2 + X'^2_2} \quad \text{(iv)}$$

or $$0.94\, R'^2_2 + 0.00126\, X'^2_2 = R'_2 \quad \text{(v)}$$

At maximum torque,

$$s = (1500 - 900)/1500 = 0.4$$

$$R'_2 = 0.4\, X'_2 \quad \text{(vi)}$$

Solving Eqs. (v) and (vi), we get

$$X'_2 = 2.65\ \Omega,\ R'_2 = 1.06\ \Omega$$

Now,

$$T_{max} = \frac{3}{\omega_s}\frac{0.5\, V_1^2}{X_2'} \quad \text{(vii)}$$

$$\omega_s = (2\pi \times 1500)/60 = 50\,\pi \text{ rad/s}$$

$$T_{max} = (3/50\,\pi)\left\{\frac{0.5(400/\sqrt{3})^2}{2.65}\right\}$$

$$= 192.2 \text{ Nm}$$

ADDITIONAL SOLVED PROBLEMS

12.14 A 3-phase, 20 kW, 4-pole, 50 Hz, 1440 rpm induction motor delivers full-load mechanical power. The windage and friction loss is 1.5 kW. Determine the

(a) mechanical power developed,

(b) power across air-gap, and

(c) rotor copper loss

Solution

(a) Net mechanical power output = 20 kW

Windage and friction loss = 1.5 kW

Mechanical power developed = 20 + 1.5 = 21.5 kW

(b) $$n_s = 1500 \text{ rpm}\ ;\ n = 1440 \text{ rpm}$$

$$s = (1500 - 1440) / 1500 = 0.04$$

$$P_m = (I - s)\, P_G$$

or $$P_G = \frac{21.5}{1 - 0.04} = 22.4 \text{ kW} = \text{power across air-gap}$$

(c) $$\text{Copper loss} = s\, P_G$$

$$= 0.04 \times 22.4 = 900 \text{ W.}$$

12.15 The rotor of a slip-ring inductor motor has resistance of 0.25 Ω / phase. The motor is running at a speed of 60 rpm. Calculate the resistance / phase to be added to the rotor to reduce the speed to 800 rpm at a load torque of 3/4th of the previous value. Neglect stator resistance.

Solution

For $n = 960$ rpm; $n_s = 1000$ rpm

At $n = 960$ rpm, $s = (1000 - 960)/1000 = 0.04$

At $n = 800$ rpm, $s = (1000 - 800)/1000 = 0.2$

For this range of slip, it is fair to assume

$$\frac{R_2'}{s} >> X_2$$

With this assumption, torque developed

$$T = \left(\frac{3}{\omega_s}\right)\frac{sV_1^2}{R_2} = K\frac{s}{R_2}$$

The two torque equations are

$$T = K\,\frac{0.04}{0.25}\ ;\ R_2 = 0.25\ \Omega$$

and

$$\frac{3}{4}T = K\,\frac{0.2}{\left(0.25 + R_2(\text{ext})\right)}$$

Dividing out, we get

$$\frac{4}{3} = \frac{0.25 + R_2(\text{ext})}{0.25}\ \ \frac{0.04}{0.2}$$

Solving, we find

$$R_s\,(\text{ext}) = 1.42\ \Omega$$

12.15 A 3-phase, 400 V, 5 kW, 50 Hz, 6 pole induction motor with delta connected stator gave the following test results:

No-load	400 V	3.5 A	445 W
Blocked rotor	200 V	16.7 A	2220 W

The dc resistance measured between two stator terminals (raised to75°C) = 2.28 Ω, ac resistance is 10% more than the dc value.

Find the equivalent circuit parameters on delta phase basis. Calculate

(a) the shaft torque, line current and efficiency at a speed of 935 rpm,

(b) the starting torque when started with phases connected in star.

Solution

Note: Calculations will be made on delta phase basis.

Stator resistance (see Fig. 12.23)

$$2.28 = \frac{R_1 \times 2R_1}{R_1 + 2R_1} \text{ or } R_1 = 3.42\ \Omega$$

$$R_1\,(\text{ac}) = 1.1 \times 3.76\ \Omega$$

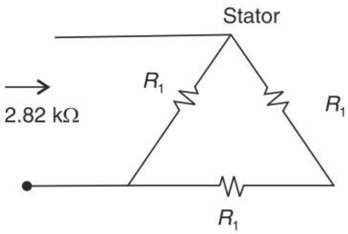


Fig. 12.23

No-load test

$$Y_0 = \frac{3.5\sqrt{3}}{400} = 5.052 \times 10^{-3}$$

$$G_i = \frac{445/-3}{(400)^2} = 0.927\times10^{-3}$$

$$B_m = [(5.052)^2 - (0.927)^2]^{1/2} \times 10^{-3} = 4.966 \times 10^{-3}$$

or

$$R_i = \frac{1}{0.927\times10^{-3}} = 1078\ \Omega \text{ (includes windage and friction loss)}$$

$$X_m = \frac{1}{4.966\times10^{-3}} = 201\ \Omega$$

Blocked-rotor test

$$Z = \frac{200}{16.7/\sqrt{3}} = 20.74\ \Omega$$

$$R = \frac{2220/3}{(16.7/\sqrt{3})^2} = 7.97\ \Omega = R_1 + R_2'$$

$$R = [(20.74)^2 - (7.97)^2]^{1/2} = 19.15\Omega = X_1 + X_2'$$

$$R_1 = 3.76\ \Omega,\ R_2' = 7.96 - 3.76 = 4.20\ \Omega$$

The circuit model is drawn in Fig. 12.24.

(a) $$n = 935 \text{ rpm},\ n_s = 1000$$

$$s = \frac{1000-935}{1000} = 0.065$$

$$\frac{R_2'}{s} = \frac{4.2}{0.065} = 64.6\ \Omega$$

$$\bar{Z} = (3.76 + 64.6) + j19.15 = 71- \angle 15.6°\text{A}$$

$$\bar{I}_2' = \frac{400\angle 0°}{71\angle 15.6°} = 5.63\angle -15.6°\text{A}$$

$$= 5.42 - j\,1.51 \text{ A}$$

$$\bar{I}_i = \frac{400}{1078} - j\frac{400}{201} = 0.37 - j1.99 \text{ A}$$

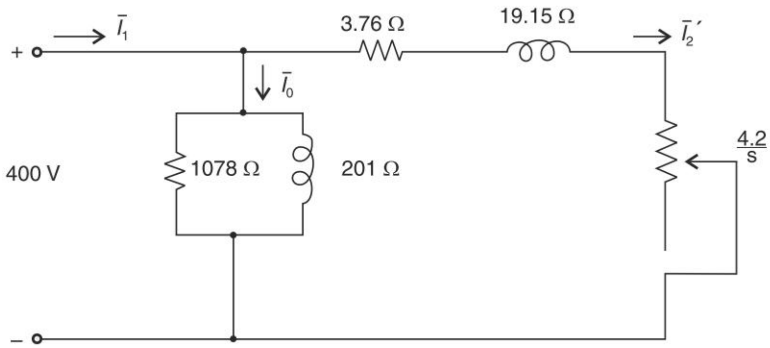


Fig. 12.24

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2' = 5.79 - j\,3.5 = 6.77 \angle -31.2° \text{ A}$$

Line current = $6.77\sqrt{3}$ = 11.73 A pf = cos 31.2° = 0.855 lagging

Synchronous speed, $\omega_s = \dfrac{2\pi \times 1000}{60}$ = 104.7 rad (mech)/s

Shaft torque, $T = 3\left(\dfrac{I_2'^2 R_2'/s}{\omega_s}\right) = 3 \times \dfrac{(5.63)^2 \times 4.02/0.065}{104.7}$

= 58.7 Nm

Efficiency

Mechanical power output

$$P_m = 3I_2^2 R_2\left(\frac{1}{s} - 1\right)$$

$$= 3 \times (5.63)^2 \times 4.2 \times \left(\frac{1}{0.065} - 1\right)$$

$$= 5745 \text{ W}$$

Note : This is the net mechanical power output (shaft power) as the windage and friction has been accounted for in R_i (no-load test) power input, $P_i = \sqrt{3} \times 400 \times 11.73 \times 0.855 = 6948$ W

$$\eta = \frac{5745}{6948} \times 100 = 82.7\%$$

(b) Phases connected in star, voltage applied = 400 V (line)

At start, s = 1

$$\bar{Z} = (3.76 + 4.2) + j19.15 = 20.74 \angle 67.4° \ \Omega$$

Magnetizing branch can be ignored

$$I_2'^2 = \frac{400\sqrt{3}}{20.74} = 11.74 \text{ A}$$

$$T\text{ (start)} = 3\times \frac{I_2^2 R_2'}{\omega_s} = \frac{3}{104.7} \times (11.74)^2 \times 4.2$$

$$= 16.6 \text{ Nm}$$

Compare with full-load torque of 58.7 Nm even though rated voltage is applied.

12.16 A 5 kW, 400 V, 50 Hz induction motor runs at a speed of 1445 rpm at full-load. The rational losses are 285 W. The maximum torque occurs at a speed of 900 rpm Calculate its value.

Solution

P_m (out) = 500 W

Rotational loss = 285 W

P_m (developed) = 5000 + 285 = 5285 W (i)

P_m (dev) = $(1 - s)\, P_G = 3\left(\dfrac{1}{s} - 1\right) I_2'^2 R_2'$

Ignoring stator impedance (for convenience we are dropping super dash)

$$I_2 = \frac{V_1}{\sqrt{\left(\frac{R_2}{s}\right)^2 + (X_2)}} \qquad \text{(ii)}$$

Substituting I_2 from Eq. (ii) in Eq. (i)

$$P_m\,(\text{dev}) = \left(\frac{1}{s} - 1\right)\frac{3V_1^2 R_2}{\left(\frac{R_2}{s}\right)^2 + X^2} \qquad \text{(iii)}$$

$$s = \frac{1500 - 1445}{1500} = 0.0367$$

Substituting known values in Eq. (iii)

$$5285 = 3(26.25) \times \frac{(400/\sqrt{3})^2 R_2}{742 R_2^2 + X_2^2}$$

$$1.258 \times 10^{-3}\,(742\,R_2^2 + X_2^2) = R_2$$

$$0.933\,R_2'^2 + 1.258 \times 10^{-3}\,X_2^2 = R_2$$

As R_2, X_2, are two unknowns we need another equation. At maximum torque

$$R_2 = s_{\max\,T}\,X_2$$

$$s_{\max\,T} = \frac{1500 - 900}{1500} = 0.4$$

$$\therefore \qquad R_2 = 0.4\,X_2 \text{ or } X_2 = 2.5\,R_2$$

Substituting in Eq. (iii)

$0.933\,R_2^2 + 1.258 \times 10^{-3} \times (2.5)^2\,R_2^2 = R_2$

which yields

$$R_2 = 1.06\ \Omega \quad \text{so} \quad X_2 = 2.5\,R_2 = 2.65\ \Omega$$

We then find

$$T_{\max} = \left(\frac{3}{\omega_s}\right)\left(\frac{V_1^2}{2X'_2}\right)$$

$$\omega_s = \frac{2\pi \times 1500}{60} = 50\pi \text{ rad (mech)/s}$$

Then

$$T_{\max} = \left(\frac{3}{50\pi}\right)\left(\frac{(4000/\sqrt{3})^2}{2 \times 2.65}\right)$$

$$= 19.22 \text{ Nm}$$

12.17 A 50 Hz, 4-pole induction motor draws 25 A from a 440 V, 3 phase supply at 0.85 pf lagging. Various losses are

Stator copper loss = 950 W

Rotor copper loss = 450 W

Core loss = 750 W

Windage and friction loss = 250 W

Calculate (a) power across air-gap, P_G (b) the mechanical power developed, P_m (c) the net mechanical power output (d) the efficiency (e) the motor operating speed and (f) the developed and output torques.

Solution

Input power, $P_i = \sqrt{3} \times 440 \times 225 \times 0.85 = 16{,}194$ W = 16.194 kW

(a) $P_G = P_i$ – core loss – stator copper loss

$= 16{,}194 - 750 - 950 = 14494$ W

(b) Mechanical power developed

$$P_m = P_G - \text{rotor copper loss}$$
$$= 14044$$

(c) Net mechanical power output = P_m – windage and friction loss

$$P\,(\text{out}) = 14044 - 250 = 13794 \text{ W}$$

(d) $$\eta = \frac{13794}{16.194}. \times 100 = 85.2\%$$

(e) Rotor copper loss $= s\,P_G$

$$450 = s \times 14494 \text{ or } s = 0.031$$

Motor speed, $n = (1 - 0.031) \times 1500 = 1453.5$ rpm

(f) Motor speed, $\omega = \dfrac{2\pi \times 1453.5}{60} = 152.2$ rad/s

$$T\,(\text{developed}) = \frac{P_m}{\omega} = \frac{14044}{152.2} = 92.27 \text{ Nm}$$

$$T\,(\text{net}) = \frac{P(\text{out})}{\omega} = \frac{13794}{152.2} = 90.63 \text{ Nm}$$

SUMMARY

- Induction motors are of two types:

  Squirrel- cage Induction Motor—copper/aluminium bars in rotor slots shorted by end rings. Aluminium is commonly used being low cost.

  Wound-rotor or slip-ring Induction Motor—The rotor has 3-phase windings with connections brought out through three slip-rings, the winding is shorted externally, also external resistance can be included at the time of starting; expensive used only where high-starting torque is must.
- Results and statements made here are on per phase basis, powers—active and reactive on 3-phase basis. Winding connection will be assumed star (or equivalent star) except where specified otherwise.
- Stator resistance and leakage reactance ignored

  $$V_1 \approx E_1 = \sqrt{2}\,\pi\, K_{\omega 1}\, N_{ph1}\,(\text{series})\, f\, \Phi_r \text{ V}$$

  f = stator frequency (frequency of V_1), Φ_r = resultant air-gap flux
- Exciting current

  $$\bar{I}_0 = \bar{I}_m + \bar{I}_i$$

  I_m = magnetizing current; 90° lagging

  I_i = core loss current; in-phase

  Magnitude-wise $I_m >> I_i$

  PF of I_0 is very low, phase angle slightly less than 90°
- Rotor standstill emf E_2, frequency f

- At speed n (slips)

  Rotor induced emf $= sE_2$

  Rotor frequency $f_2 = s_f$

- $\frac{I_2'}{I_2} = \frac{I}{a}$; I_2' stator current to counter I_2

- The net stator current

$$\bar{I}_1 = \bar{I}_0 + \bar{I}_2' = \bar{I}_0 + \left(\frac{I}{a}\right) \bar{I}_2$$

  I_0 is almost 40% of I_1 (full-load)

- Power Factor: Because of large I_0 with phase angle slightly less than 90°, the pf of the line current is of the order of 0.8 to 0.85. At light load I_2' reduces and so does the power factor. Therefore induction motor should not be run at light load for long period of time.

- Rotor standstill reactance $= X_2$

  Rotor circuit impedance

$$\bar{Z}_2 = R_2 + j\, s\, X_2$$

- Power across air-gap

$$P_G = \text{Gross mechanical power output + rotor copper loss}$$
$$= P_m + 3\, I_2^2\, R_2$$

- Rotor resistance equivalent of mechanical power output

$$\left(\frac{1}{s} - 1\right) R_2$$

  which means

$$\frac{P_m}{3} = \left(\frac{1}{s} - 1\right) I_2^2\, R_2$$

- Circuit model

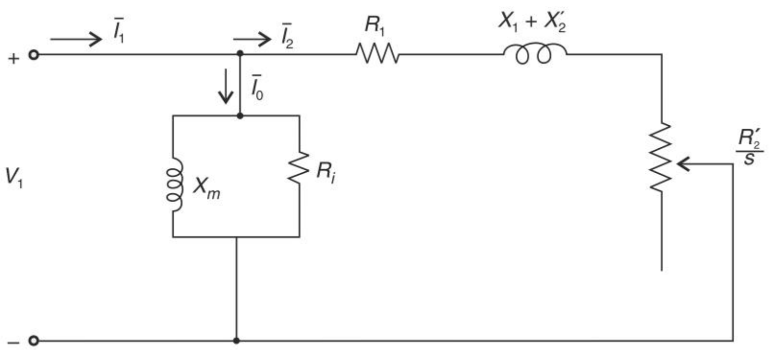


Fig. 12.25

- $$P_G = 3\, \frac{I_2 R_2}{s} = 3\, \frac{I_2^2 R_2}{s}$$

$$P_m = (1 - s)\, P_G$$

  $T = \frac{P_G}{\omega_w}$, ω_s = synchronous speed in rad (mech)/s

  P_G is known as torque in *synchronous watts*

- $T = \frac{3}{\omega_s} \cdot \frac{V_1^2 (R_2'/s)}{(R_2'/s)^2 + X_2^2} = \frac{3}{\omega_s} \cdot \frac{(V_1/a)^2 (R_2/s)}{(R_2/s)^2 + X_2^2}$

Stator impedance ignored

- Torque-slip characteristic; see Fig. 12.8

  Motoring $0 \le s \le 1$

  Generating $s < 0$

  Breaking $s > 1$

- $T_{\max} = T_{\text{breadown}}\ (T_{\text{BD}})$

  For $T > T_{BD}$; motor stalls

- Resistance added in rotor circuit—slip-ring induction motor only. T_{BD} no change, slip at given torque increase, motor speed reduces, T(start) increases

- At $T_{\max}$

$$R_2 = s_{\max T} X_2$$

$$R_2 = s_{\max T} X_2$$

$$T_{\max} = T_{BD} = \frac{3}{\omega_s}\left(\frac{V_1^2}{2X_2'}\right) = \frac{3}{\omega_s}\left(\frac{(V_1/a)^2}{2X_2}\right)$$

Stator impedance ignored

- Starting—stator impedance ignored

$$I_2\,(\text{start}) = \frac{V_1}{\sqrt{R_2^2 + X'^2_2}} = \frac{V_1/a}{\sqrt{R_2^2 + X_2^2}}$$

$$T\,(\text{start}) = \frac{3}{\omega_s}\left[\frac{V_{12}^2\ R_2'}{\sqrt{R_2'^2 + X_2'^2}}\right] = \frac{3}{\omega_s}\left[\frac{(V_1/a)^2 R_2}{R_2^2 + X_2^2}\right]$$

- Determination of circuit model parameters

  Quantities measured in test: voltage, current and power

  No-load test

  Conducted at rated voltage

  Determines

  Sum of core loss and windage and friction loss and X_m, R_i

  Blocked rotor test

  Conducted at reduced voltage, full-load current

  Determines

  Full-load copper loss

  R_2 , X_2

  Important Note:

  In the computation of performance based the circuit model as determined by the above two tests, the windage and friction loss is accounted for in R_i. Therefore the mechanical power out-put $P_m = 3\left(\frac{1}{s} - 1\right) I'^2_2 R_2$ is the net mechanical power output called the *shaft power*.

- Methods of starting

  Squirrel-cage motor

  DOL (direct-on-line) starting not permitted for motors 5 kW and above. As the motor current is 5 to 6 times full-load current, the power supply companies do not allow such heavy short-time currents to be drawn.

—Reduced voltage start

—Series resistance starting—can be used for fractional—kW motor only

—Star / delta starting

Start in star, run in delta

Starting current and torque both reduce by a factor of 1/3.

—Auto transformer starting

Expensive, used for very large motor

Both starting current and torque reduce by a factor of a^2, a = voltage reduction factor

- Speed control—slip control, frequency control

—slip control—reduced voltage for very small motors, inefficient

— rotor resistance control for slip-ring induction motor, reduce efficiency drastically. Not suitable

—frequency control—in varying frequency (V/f) must be maintained constant for constant air-gap flux

Requires expensive full rated thyristor convertor/inverter Equipment

REVIEW QUESTIONS

1. Give a brief account of squirrel-cage induction motor. Explain qualitatively as to how it develops torque and the nature of its torque-slip characteristic. Why is it called asynchronous motor?
2. What is the effective turn-ratio of an induction motor?
3. What is standstill rotor emf and what is its frequency? How does the emf magnitude and frequency vary with speed?
4. Explain what is meant by standstill reactance of induction motor. How does it vary with speed?
5. The stator of a slip-ring induction motor with slip-ring terminals open-circuited has at stator excited from 3-phase source. The rotor is run by a prime mover. What will be the frequency of rotor induced emf at the following speeds?
   (a) half synchronous speed in the same direction as the air-gap field (AGF)
   (b) half synchronous speed in opposite to AGF
   (c) at synchronous speed in opposite direction to AGF
6. What is meant by the excitation current of an induction motor? Draw its phasor diagram with applied voltages as the reference phasor showing its components. Which is the larger component and why?
7. What is the difference between excitation current and no-load current?
8. Draw the phasor diagram of an induction motor showing applied voltage, magnetizing, coreloss, load current and the line current. Label each component.

9. Write the expression for the resistance in the circuit model, the loss in which is equivalent to the mechanical power developed.
10. What is meant by the torque in synchronous watts? Write its expression in terms of circuit model quantities then find the torque developed.
11. Show that the maximum torque occurs at a slip $s = \frac{X_2}{R_2}$ and further show that T_{max} is independent of s.
12. Draw the T-s characteristic of an induction motor. Indicate the region where the characteristic is nearly linear.
13. Show that the motor can operate stably at $s \angle s_{\text{max T}}$. Use perturbation technique.
14. Neglect the stator impedance and show that the maximum power output (developed power) occurs at slip $s = \frac{R_2 + \sqrt{R_2^2 + X_2^2}}{R_2}$.

    Hint: In the circuit model of Fig. 12.5 (e), use maximum power transfer theorem. The magnitude of fixed impedance should match $\left(\frac{1}{s} - 1\right) R_2$
15. Show that at super synchronous speed the induction machine acts as a generator. Write the expression for P_G. In which direction does it flow? How can you find the net mechanical power input and net electrical power output?
16. Neglecting stator impedance, derive the expression for the starting torque of an induction motor. Show that it increases with rotor resistance. At what resistance value it reaches the maximum? Resistance added to the rotor of a slip-ring induction motor.
17. Show that in star/delta starting of squirrel-cage induction motor, do the starting current and torque get reduced by a factor of 1/3 compared to DOL starting.
18. Elaborate the statement "rotor resistance starting of slip-ring induction motor reduces starting current and increases starting torque".
19. The power input on no-load running of induction motor is consumed in what losses?
20. The power input in blocked rotor test at reduced voltage is consumed in what losses?
21. No-load test determines what parameters of the circuit model of induction motor.
22. Which parameters of the circuit model of induction motor are determined by the blocked-rotor test.
23. Why is DOL starting current very high but the starting torque is still low? Why is DOL not permitted in starting even though the short duration current cannot harm the motor?
24. What methods are used in starting squirrel-cage induction motor? Which method is used in what size of motor? Which is the most common method and what is it superiority?

25. Compare the speed control features of induction motor with dc shunt motor.
26. From no-load to full-load, what is the type of speed-load characteristic of induction motor?
27. Compare and contrast the squirrel-cage and slip-ring induction motors.
28. Upon reducing the load on an induction motor, why does its pf come down?

PROBLEMS

12.1 A 40 kW, 440 V, 3-phase, 50 Hz, 8-pole squirrel-cage induction motor has a slip of 0.03 when operated at rated voltage and frequency. It has full-load line current of 68.9 A and an efficiency of 89.6%.
Find (a) the shaft torque delivered to the load and (b) the power factor at which the motor operates.

12.2 A 7.5 kW, 440 V, 3-phase, 50 Hz, 6-pole squirrel-cage induction motor operates at a full-load slip of 4.0% when rated voltage and frequency are impressed.
Assume that the torque–slip characteristic is linear (Eq. (12.43)). What would be the motor speed if the load torque is increased to 125%, while the impressed voltage is reduced to 80% of the rated value.

12.3 The no-load test data yielded the following parameters (on star basis) for the shunt branch of the circuit model of a 400 V, 50 Hz, 4-pole induction motor.

$$X_m = 31.25\ \Omega, \quad R_i = 242\ \Omega$$

The motor developes a torque of 95.6 Nm at a slip of 5%. Calculate
(a) mechanical output,
(b) rotor copper loss, and
(c) motor efficiency.

12.4 Assuming for an induction motor the stator iron loss to be negligible, derive an expression for efficiency as a function of s in terms of stator and rotor circuit parameters.
A 3-phase, 6-pole induction motor rated 7.5 kW, 400 V, 50 Hz has the following impedance parameters referred to the stator:

$$R_1 = 0.975\ \Omega \quad R_2' = 0.496\ \Omega$$
$$X_1 + X_2' = 2.38\ \Omega$$

Calculate the motor efficiency at slips of 0.04, 0.1 and 0.5.

12.5 A 3-phase, 400 V, 5 kW, 50 Hz, 6-pole induction motor with star-connected stator gave the following test results:

No load	400 V	3.5 A	444.5 W
Blocked rotor	200 V	16.7 A	2220 W

Stator phase resistance = 1.25 Ω
Calculate the line current, power factor and efficiency at a speed of 935 rpm.
Calculate also the maximum torque and the speed at which it occurs.

12.6 The motor of Ex. 12.13 is to drive at a constant torque with a load of 250 Nm. Given effective turn ratio, stator/rotor = 2.45/1.
(a) Calculate the minimum rotor resistance to be added in the rotor circuit for the machine to start up. Ignore variations in mechanical loss due to change in speed.
(b) At what speed will the motor run if the added resistance is (i) left in and (ii) subsequently shorted out?

(c) To what value the applied voltage must be reduced if the speed in part (b) (i) above is to be achieved with normal rotor resistance?

(d) Compare also the motor efficiency for these two methods of speed control.

12.7 A 3-phase, 6-pole induction motor rated 7.5 kW, 400 V, 50 Hz has the following impedance parameters referred to the stator:

$$R_1 = 0.975\ \Omega \quad R_2' = 0.496\ \Omega$$
$$X_1 + X_2' = 2.38\ \Omega$$

The motor is driving a load requiring torque of

$$T_L = 75\ (n/n_s)^2\ \text{Nm}$$

(a) Determine the speed at which the motor would run at rated terminal voltage.

(b) What would be the motor speed if the terminal voltage is reduced by 15%?

12.8 A 3-phase squirrel-cage induction motor is started by reducing the applied voltage by a factor of 1/2. The exciting current can be ignored. Find the factor by which

(a) the starting current is reduced and

(b) the starting torque is reduced.

12.9 A 25 kW, 400 V, 3-phase, 50 Hz, 6-pole, delta-connected squirrel-cage motor has the following circuit model parameters:

$$R_1 = 0.45\ \Omega, \quad R_2' = 0.36\ \Omega \text{ per phase}$$
$$X_1 + X_2' = 4.5\ \Omega \text{ per phase}$$

(a) Estimate the starting line-current and torque for DOL starting.

(b) Repeat part (a) if the motor is reconnected in star.

Chapter 13

FRACTIONAL-kW MOTORS

13.1 INTRODUCTION

In Chapter. 12, 3-phase, ac motors, which are used for high-power rating applications, have been discussed. For reasons of economy, most homes, offices and also rural areas are supplied with single-phase ac, as the power requirements of individual load items are rather small. Even though input point power to homes or offices may be 3-phase, the inside wiring is single-phase, 220–230 V for reasons of safety. This has led to the availability of a wide variety of small-size motors of fractional kilowatt ratings. These motors are employed in fans, refrigerators, mixers, vacuum cleaners, washing machines, other kitchen equipment, tools, small farming appliances, etc. Individual room air conditioners (single-phase) for home and office (where there is no central air conditioning) are being increasingly used.

Obviously the total number of such small motors in use far exceeds the number of integral kW motors in industrial use. Though these motors are simpler in construction as compared to their 3-phase counterparts, their analysis happens to be more complex and requires certain concepts which have not been developed so far. Also the design of such motors is carried out by trial and error till the desired prototype is achieved. Because of the vast numbers in which these motors are produced, even a fractional efficiency increase or a marginal cost saving is extremely important. Nowadays, as in other fields, computers are employed for more accurate and optimum paper designs.

The treatment of the fractional-kW motors as presented in this chapter is concerned mainly with their method of operation, classification and characteristics of various types, and their typical applications. The analysis of the performance of such motors is beyond the scope of this book.*

13.2 SINGLE-PHASE INDUCTION MOTORS

A single-phase induction motor comprises a single-phase winding on the stator and a

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* However, another text by the same authors' namely 'Electric Machines' 2nd edn. Ch. 10, may be consulted for a detailed analysis of fractional-kW motors

squirrel-cage rotor as shown in Fig. 13.1. The stator winding is connected to a single-phase source. The winding mmf is

$$\mathcal{F} = F_m \cos \omega t \cos \theta; \text{ (see Eq. (10.32))} \tag{13.1}$$

where $F_m = N I_m$

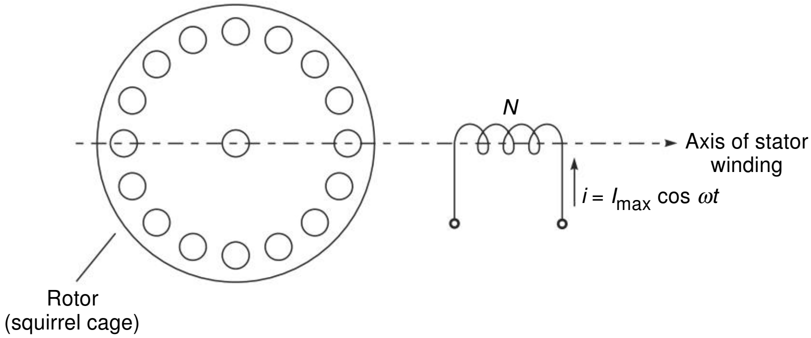


Fig. 13.1 Single-phase induction motor

As per Sec. 10.6, this pulsating space-distributed field can be split into time rotating fields as

$$\mathcal{F} = \frac{1}{2} F_m (\omega t + \theta) + \frac{1}{2} F_m \cos (\omega t - \theta) \tag{13.2}$$

$$= \mathcal{F}_f + \mathcal{F}_b$$

or vectorially

$$\mathbf{F} = \mathbf{F}_f + \mathbf{F}_b \tag{13.3}$$

where F_f and F_b are respectively forward and backward rotating fields rotating at synchronous speed ($\omega = 2\pi f$ rad (elect/s)). Each field has the same peak mmf equal to 1/2 F_m.

Rotor Slip with Respect to Two Rotating Fields

Let the rotor be assumed to run at a speed n in the direction of the forward field as shown in Fig. 13.2. It easily follows that

Rotor slip wrt forward field, $s_f = \dfrac{n_s - n}{n_s} = s$ (13.4)

Rotor slip wrt backward field, $s_b = \dfrac{n_s - (-n)}{n_s} = \dfrac{2n_s - (n_s - n)}{n_s}$

$$= (2 - s) \tag{13.5}$$

At $s = 0.05$ say

$$\frac{s_f}{s_b} = \frac{0.05}{1.45} = \frac{1}{39}$$

At $s = 1$ (standstill rotor)

$$s_f = s_b = 1$$

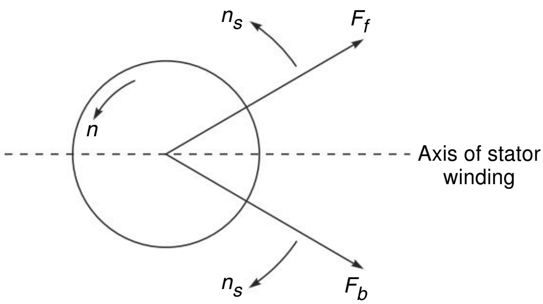


Fig. 13.2

Torque–Speed Characteristic

At standstill, the rotor slip is $s = 1$ wrt both the rotating fields. The two fields are therefore equal in strength inducing equal currents in the rotor. As a result, these produce equal but opposite torques with net zero torque. The single-winding, single-phase motor (Fig. 12.1) is therefore *non-self starting*. The two rotating fields induce stator emfs which together balance the applied voltage (if low impedance stator is assumed).

If now the rotor is made to run at speed n in the direction of the forward field, the rotor slips wrt the two fields are now vastly different, i.e. $(2 - s) >> s$. The forward field (low rotor slip) induces low, high pf currents in the rotor while the backward field (high rotor slip $(2 - s)$) induces high, low pf currents in the rotor. As a consequence, the backward field gets highly attenuated in strength while the strength of the forward field enhances in comparison. The forward torque therefore becomes several times the backward torque (torque being nearly proportional to square of field strength). The single-phase induction motor in this region of slip has T–s characteristic similar to that of a 3-phase motor but has a low efficiency because of the rotor loss caused by the backward field.

The T–s characteristic of a single-winding single-phase induction motor as sum of forward and backward field T–s characteristics is shown in Fig. 13.3 from which it is obvious that the motor has no starting torque.

The problem posed now is how to create a starting torque. This will be tackled by strengthening the forward field and weakening the backward field at $s = 1$.

Two-Phase Motor

Figure 13.4 shows a 2-phase motor. Let

$$\sqrt{2}\,N_m I_m = \sqrt{2}\,N_a I_a = F_m$$

The windings are displaced 90° (elect) in space phase and carry currents with 90° time phase difference. The resultant mmf distribution is

$$\begin{aligned} \mathcal{F} &= F_m \cos \omega t \cos \theta + F_m \cos (\omega t - 90°) \cos (\theta - 90°) \\ &= F_m \cos (\omega t - \theta) \qquad (13.6) \\ &= \mathcal{F}_f \end{aligned}$$

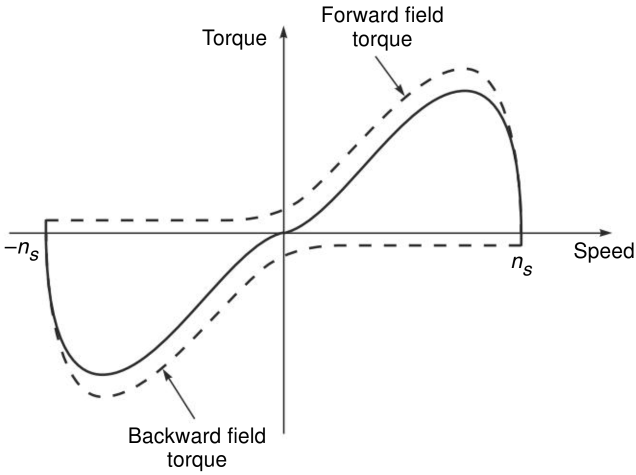


Fig. 13.3 *T–s* characteristic of a single-winding, single-phase induction motor

A single rotating field is thus established rotating at synchronous speed. But the problem now is that to create these currents on the 2-phase windings a 2-phase supply would be necessary, which is not practicable.

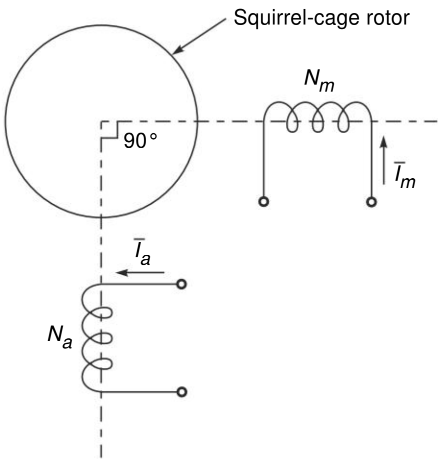


Fig. 13.4 Two-phase motor

Split-Phase Motor

It is a 2–winding, single–phase motor in which the two windings are placed at 90° (elect) but are fed from single phase. The time phase difference in winding currents is obtained by placing suitable impedance in series with one of the windings called the *auxiliary* winding *a* while the other winding is called the *main* winding *m*. The current I_a in the higher impedance auxiliary winding is less than the current I_m in the main winding. The auxiliary winding has fewer turns of thinner wire. Unbalanced 2-phase field conditions are thus created at the start and as a result, the forward rotating field, becomes sufficiently stronger than the backward field resulting in production of starting torque. The auxiliary winding may or may not be left in circuit after the motor starts. For opening the auxiliary winding after motor starts, a centrifugal switch is employed. After starting the motor runs only on the main winding.

Depending on the method of *phase-splitting* (causing time phase difference in the currents of the two windings), there are two types of single-phase motors.

Resistance Split-Phase Motor The schematic diagram of the resistance split-phase motor is shown in Fig. 13.5. Here high *R*/*X* ratio is used for the auxiliary wind-

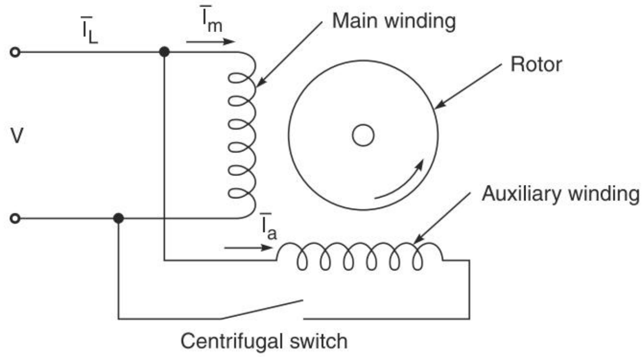


Fig. 13.5 Resistance split-phase motor

ings. A phase difference of about 30° is achievable as shown in Fig. 13.6(a) while Fig. 13.6(b) gives a typical *T–s* characteristic.

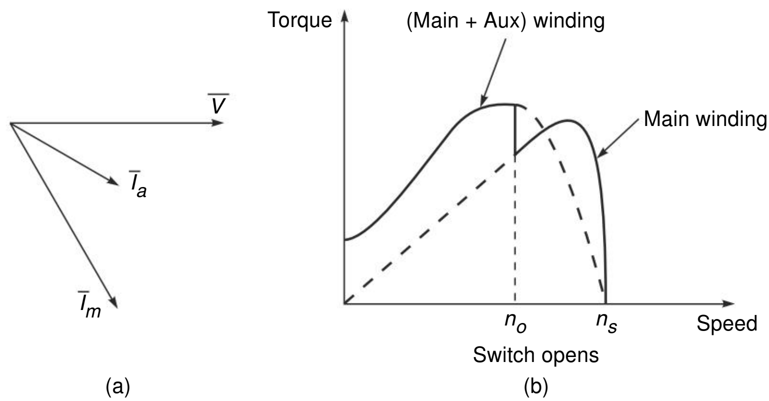


Fig. 13.6 (a) Phasor diagram at start (b) *T–s* characteristic

It is a low efficiency, low pf motor and is available in sizes of 1/20 – 1/2 kW.

Capacitor split-phase motor For phase splitting a capacitor is placed in series with the auxiliary winding as shown in Fig. 13.7 along with phasor diagram at start. While the main winding draws a lagging current, the current in the auxiliary winding is leading and it is possible to make the phase difference between them as 90° at start. During running the auxiliary winding is cut out so that capacitor is only short-time rated. Such a motor is known as capacitor-start motor. It has a far larger starting torque compared to a resistance-start motor. It has wide applications in machine tools, refrigeration, air-conditioning, etc. and is available in up to 5 kW size.

Two-Value Capacitor Motor

The connection diagram is given in Fig. 13.8. A larger capacitance (*C* (run) and *C* (start) in parallel) is employed to yield best starting conditions. The phase separation is adjusted to more than 90° (Fig. 13.9(a)). The *C* (start) is cut out at a certain speed leaving *C* (run) in circuit to give best running performance; phasor diagram of Fig.

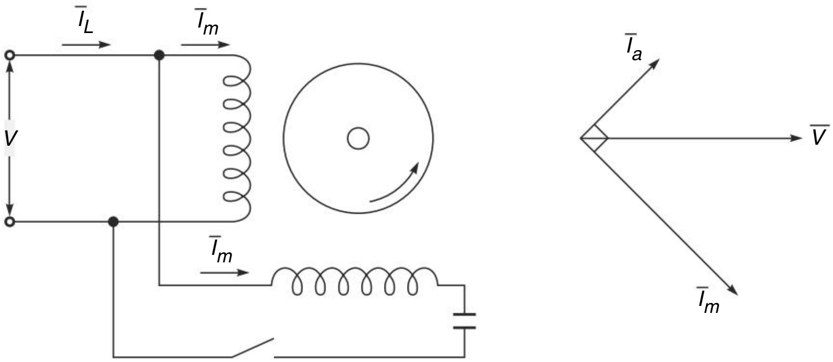


Fig. 13.7 Capacitor-start motor

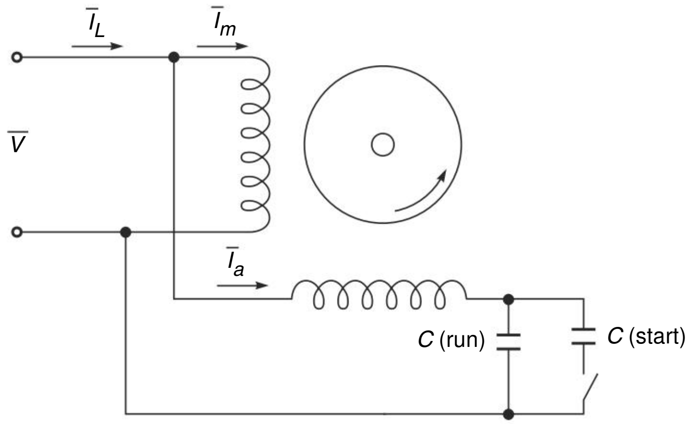
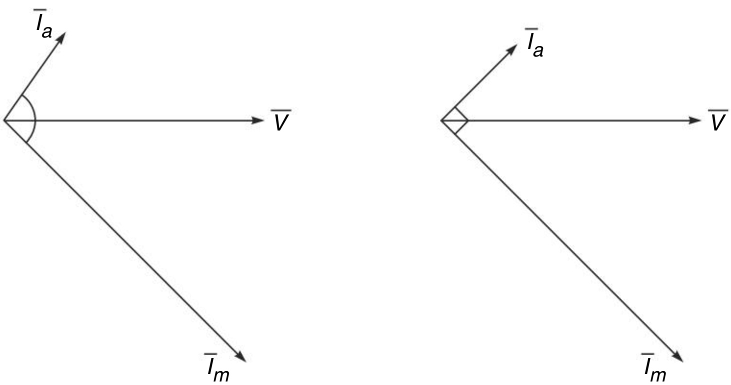


Fig. 13.8 Two-value capacitor motor



(a) (b)

Fig. 13.9 Phasor diagram two-value capacitor motor
(a) At start (b) During running

13.9(b). *C* (run) also helps to improve the overall pf of the motor. While *C* (run) is continuous rated, *C* (start) need only the short-time rated. The composite *T–s* characteristic is shown in Fig. 13.10. This motor is employed for hard to start loads.

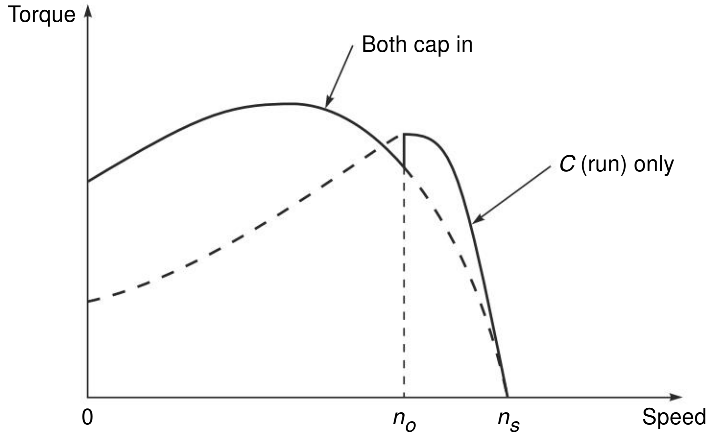


Fig. 13.10 *T–s* characteristic two-value capacitor motor

Shaded-Pole Motor

Figure 13.11 shows a shaded-pole motor. It has a projecting pole stator excited from single-phase ac while part of the poles is enclosed by short-circuited shading coils (sometimes a single shading ring is employed). The shading coil has a large lagging current induced in it which produces a lagging flux. As a consequence, the resultant flux passing down the shaded portion of each pole lags behind the flux passing down the remaining portion. This difference in phase angle of the two portions of flux causes the production of starting torque.

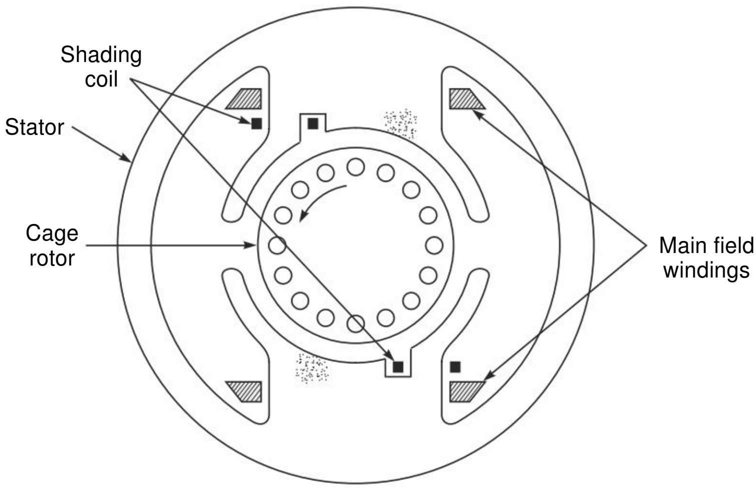


Fig. 13.11 Shaded pole motor

The direction of rotation of the rotor is from the leading flux portion of the pole to the lagging flux portion of the pole, i.e. from the unshaded to the shaded part of the pole. It is as if the flux glides part the rotor surface from the leading to the lagging part of the pole. Reversal of direction of rotation is possible only by providing shading coils at both the pole ends and open circuiting one of these.

Shaded-pole motor inherently has low pf and is available in sizes up to 1/20 kW. It finds application in small fans, convectors, vending machines, photocopying machines, advertising displays, etc.

13.3 SINGLE-PHASE SYNCHRONOUS MOTORS

Reluctance Motor

In single-phase reluctance motor, the rotating field is produced by main and auxiliary winding (split phase) placed in stator slots. The rotor is made of stampings with a projecting structure as shown in Fig. 13.12. Short-circuited bars are placed in the projected stator portions. The motor starts like a single-phase induction motor. Near synchronous speed, the rotor gets locked with the rotating magnetic field in the position of minimum reluctance and thereafter runs at synchronous speed.

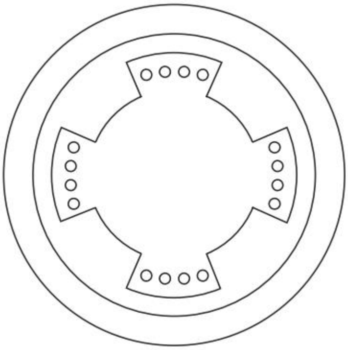


Fig. 13.12 4-pole single-phase reluctance motor

Hysteresis Motor

A split-phase stator and a solid iron rotor with high hysteresis loss is employed in a hysteresis motor. The motor starts by induction action and is pulled into synchronism near synchronous speed. Because of hysteresis, the rotor field lags the stator field by an angle δ with consequent torque production as shown in Fig. 13.13. Because of smooth rotor, it has a very low noise figure and is used in phonographic appliances.

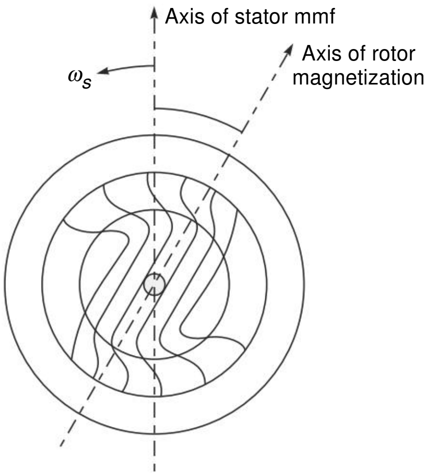


Fig. 13.13 Torque production in hysteresis motor

13.4 AC SERIES MOTOR–UNIVERSAL MOTOR

The torque in a dc series motor is given by the expression

$$T = K_T i_a^2$$

If this motor is ac excited, the torque would be unidirectional with an average component and a second harmonic oscillating component as shown in Fig. 13.14.

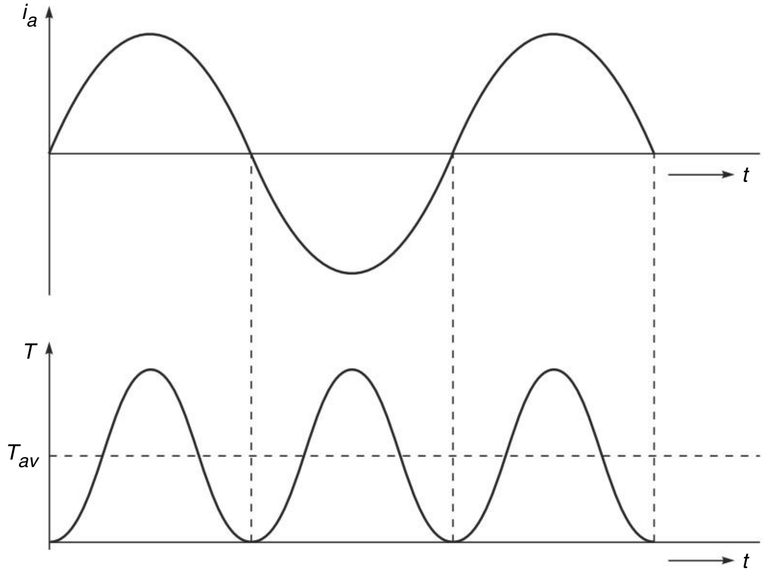


Fig. 13.14 Torque in ac-excited series motor

The average torque can be obtained from

$$T_{av}w = E_aI_a$$

where $\overline{E}_a$ and $\overline{I}_a$ must be in phase in a series excited dc armature.

There are certain consequences of ac excitation of the dc series motor.

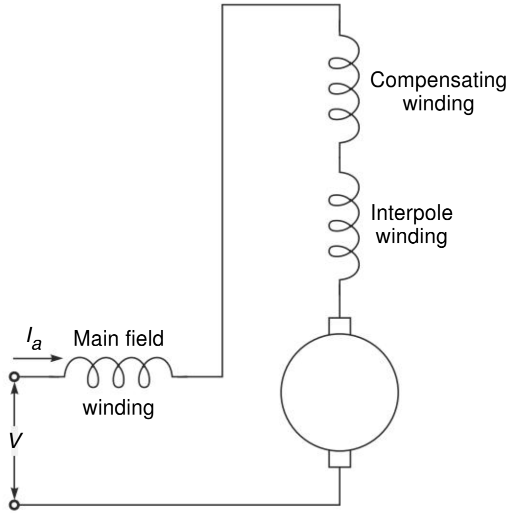


Fig. 13.15 Connection diagram of ac-excited series motor

– The field and yoke carry alternating flux and must therefore be laminated.
– Apart from speed emf E_a, the armature coils have transformer emf induced in them by the alternating field flux. This reactance emf seriously impairs the commutation qualities of the machine. Interpoles must therefore be provided.
– Because of alternating armature flux (armature reaction), the armature offers a high reactance causing the motor to have a very poor pf. The armature reaction must therefore be cancelled out by providing compensating winding placed in pole faces.

The schematic connection diagram of a series motor is shown in Fig. 13.15.

The no-load speed of a universal motor may be as high as 20,000 rpm unlike that of other motors. Therefore, it has a smaller physical size for a given power capacity. It finds applications where light weight is important and high operating speeds are desired, as in vacuum cleaners and portable tools.

SUMMARY

1. Fractional kW power motors, are geneally single phase motors such as fans, refrigerators, vacuum cleaners, etc.
2. Single phase induction motor is not self-starting. It can be made as self-starting motor by using two winding (starting, main) which is used only at the time of starting.
3. Both reluctance and hysteresis motor are synchronous speed motor (constant speed motors).
4. A universal motor will run with both ac and dc supply. It is a light weight and a high speed motor (nearly 20,000 rpm).

REVIEW QUESTIONS

1. Why is single phase induction motor not self-starting?
2. How can the starting torque in a single phase induction motor be improved?
3. What is the working principle of reluctance motor?
4. Why is the power factor of the universal motor slow?
5. How does the universal motor run with both AC and DC? Discuss the features of universal motor.
6. What is hystersis effect and how is it used in the hysteresis motor?
7. Discuss the advantages of two value capacitor motor briefly.

Chapter 14

MEASUREMENT TECHNIQUES AND ELECTRIC AND ELECTRONIC INSTRUMENTATION

14.1 INTRODUCTION

Measurement normally involves an instrument as a physical means of determining a variable or quantity. An instrument is defined as a device for finding out the value or magnitude of a variable or quantity. Measurement is a means to achieve the final goal, i.e. instrumentation. The electronic instrument depends on electrical or electronic principles for its measurement function.

The measurement of a given quantity is nothing but the result of a comparison between the quantity and a predefined standard (direct method). In engineering applications, indirect methods are normally preferred. It consists of a transducer which converts the quantity to be measured in an analogous form. This analog signal is then processed by some intermediate means and is fed to the final device which finally gives the measurement result.

14.2 ELECTRICAL AND ELECTRONIC INSTRUMENTS

The elements used in such instruments are

(i) a detector,

(ii) an intermediate transfer device, and

(iii) an indicator, a recorder or a storage device.

Mechanical instruments, due to their high inertia and noise, are hardly used nowadays. Their application is restricted to measurement of a slowly varying pressure.

Electrical instruments depend on the mechanical movement of an indicating device having some inertia and thus have a limited time response (0.5–24s). Nowadays electronic instruments are used for fast responses required for most scientific and industrial measurements. They are used for the detection of electromagnetically produced signals such as radio, video and microwaves, space applications and computers.

Some important terms pertaining to 'measurement' are defined now.

Instrument: A device for finding the value or magnitude of a quantity or variable.

Accuracy: It tells us about the nearness of the measured value towards the true value, i.e. the measure of conformity to the true value.

Precision: It refers to the degree of agreement within a group of measurements or instruments i.e. the measure of reproductivity. Precision has two characteristics: conformity, and the number of significant figures to which measurements may be made.

Resolution: It is defined as the smallest change in input that can be detected by an instrument.

Sensitivity: It is the ratio of output signal or response of the instrument to a change of input or measured variable.

True Value: (A_t) It is the average of the infinite number of measurements, when the average deviation tends to become zero.

Error: An error is a deviation from the true value of the measured variable.

Errors

No measurement can be made with perfect accuracy. There are three types of errors: gross, systematic and random. *Gross errors* are mainly human errors like misreading of instruments, incorrect adjustment, improper application of instruments and computational mistakes. *Systematic errors* have the same magnitude and sign for a given set of conditions. These errors accumulate at the end of the measurement. *Random errors* are caused due to random variations in the parameter or the system of measurement. These errors result in the deviation of magnitude of the variable measured by the instrument from the true value of the variable. The difference in the measured value and the true measured value gives rise to static errors.

Absolute static error $\delta A = A_m - A_t$ (14.1)

where, A_m is the measured value and

A_t is the true value.

Relative static error $\varepsilon_r = \delta A/A_t = (A_m - A_t)/A_t$ (14.2)

Static error correction $\delta C = A_t - A_m = -\delta A$ (14.3)

Limiting Error In most indicating instruments, accuracy is guaranteed to a certain percentage of full-scale reading. The limits of these deviations from the specified value are known as limiting errors.

$$\delta A = A_a - A_s \quad (14.4)$$

where A_a is the actual measurement, and

A_s is the specified (nominal) value.

Guarantee error (ε_r)

$$\varepsilon_r = \delta A/A_s \quad (14.5)$$

Instrument Efficiency

1. Ammeter

$$\eta_A = I_{\text{fsd}} / \text{power consumed} = 1/V_{\text{fsd}} \tag{14.6}$$

2. Voltmeter

$$\eta_v = V_{\text{fsd}} / \text{power consumed} = 1/I_{\text{fsd}} \tag{14.7}$$

where I_{fsd} is the full scale current (A), and

V_{fsd} is the full scale voltage (V).

Example 14.1 A 60 mV/120 mV dual range millivoltmeter, when used to measure the voltage across two points in a dc circuit, gives a reading of 27.5–30 mV when 60 mV and 120 mV ranges, respectively, are employed. Assuming that the meter has been correctly calibrated, estimate the true value of the voltage existing across the two points in the dc circuit. It is known that the millivoltmeter has a sensitivity of 10 kΩ/volt.

Solution Let R_x be the resistance between the two points and I be the current through R_x.

For 60 mV range,

Resistance of millivoltmeter (R_v)= 60 mV × 10 kΩ/V = 600 Ω

$$\therefore \quad 27.5 \text{ mV} = I \times 600\, R_x/(R_x + 600) \tag{i}$$

For 120 mV range,

Resistance of millivoltmeter (R_v) = 120 mV × 10 kΩ/V = 1200 Ω

$$\therefore \quad 30 \text{ mV} = I \times 1200\, R_x/(R_x + 1200) \tag{ii}$$

Dividing Eq. (i) by (ii), we get

$$27.5/30 = (1200 + R_x)/2\,(600 + R_x)$$

Solving $\quad R_x = 120\ \Omega;\ I = 0.275$ mA

$$\therefore \quad \text{Actual voltage} = R_x I = 120 \times 0.275 = 33 \text{ mV}$$

14.3 CLASSIFICATION OF INSTRUMENTS

Instruments are broadly divided into two classes:

1. Absolute instruments: These give the quantity to be measured in terms of an instrument constant and its deflection, e.g. tangent galvanometer.
2. Secondary instruments: These directly give the magnitude of the electrical quantity to be measured, e.g. ammeter, voltmeter. The principle of working of all electrical measuring instruments depends on the various effects of electric current or voltage. The effects utilized in the manufacturing of electrical instruments are magnetic, heating, chemical and electromagnetic in nature. Indicating instruments consist essentially of a pointer moving over a calibrated scale attached to the moving system pivoted on jewelled bearings.

Basic Requirements for Measurement

For a satisfactory working of indicating instruments, the following three types of torques are required:

1. Deflection (Operating) Torque: It is necessary to make the moving system of the instrument (pointer) move from its zero position.

$$T_{\mathrm{dc}} = N\,Bi\,ld = Gi \tag{14.8}$$

$$\therefore \qquad T_{\mathrm{dc}} \propto \text{measurand}$$

where

N = number of turns of coil

B = flux density in the air-gap at the coil position

l = length of vertical side of coil

d = length of horizontal side of coil

i = current through the coil

G = $NBld$ displacement constant of the galvanometer

2. Controlling (Restoring) Torque: Some controlling force is employed either by a spring or gravity to limit the movement.

(a) A spring is generally used for controlling torque (phosphor bronze is the most suitable material).

$$T_c = k\theta \tag{14.9}$$

where θ is the deflection of the pointer and k is the controlling torque constant.

(b) Gravity method as shown in Fig. 14.1 is also sometimes used for controlling torque.

$$T_c \propto \sin\theta$$

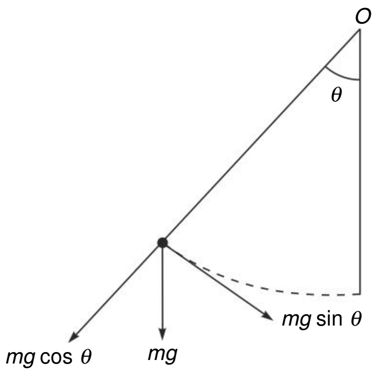


Fig. 14.1 Controlling torque due to gravity

3. Damping torque: This torque is necessary to avoid oscillation of the moving system about its final deflected position owing to the inertia of the moving parts and to bring the moving system to rest in its deflected position quickly.

The various methods of obtaining damping are air friction, fluid friction and eddy current. Figure 14.2 shows the possible cases of damping of an instrument.

$$\text{Damping torque } T_{dm} = D\,\mathrm{d}\theta/\mathrm{d}t \tag{14.10}$$

where D is called the damping constant.

The restarting torque due to inertia of the moving system is

$$T_i = J\,(d^2\theta/\mathrm{d}t^2) \tag{14.11}$$

where J is called the inertia constant.

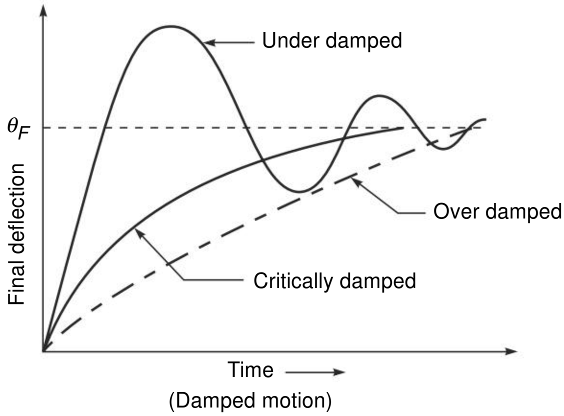


Fig. 14.2 Different cases of damping of an instrument

Equation of Damping Motion The general differential equation of damping motion is

$$J(d^2\theta/dt^2) + D\,(d\theta/dt) + k\theta = Gi \tag{14.12}$$

Hence, the auxiliary is

$$Jm^2 + Dm + k = 0$$

$$\therefore \quad m = [-D \pm j\sqrt{(4\,\text{kJ} - D^2)}]/2\,\text{J}$$

Thus, Deflection $\theta = A\,e^{m1t} + B\,e^{m2t}$

Under steady state,

$$\frac{d_2\theta}{dt^2} = 0, \quad d\theta/dt = 0,\ \theta = \theta_f$$

$$\therefore \quad \theta_f = Gi/k = \text{final steady state deflection}$$

Case 1. If $D^2 < 4$ kJ then the response is underdamped

Case 2. If $D^2 = 4$ kJ then the response is critically damped

Case 3. If $D^2 > 4$ kJ then the response is overdamped

The angular frequency of damped oscillation is

$$\omega_d = \sqrt{4\,\text{kJ} - D^2}\,/2J \tag{14.13}$$

$$\therefore \quad R = G^2/2\sqrt{(\text{kJ})} \tag{14.14}$$

where, R is the series resistance for critical damping.

Electromagnetic Damping

Electromagnetic damping is produced by the induced effects when the coil moves in the magnetic field and a closed path is provided for the currents to flow. Electromagnetic damping is because of

1. eddy currents produced in the metal core and
2. current circulated in the coil circuit by emf generated in the coil when it rotates.

14.4 TYPES OF INDICATING INSTRUMENTS

Indicating instruments can be divided into different types according to their working principles.

D'Arsonval Movement

The basic permanent magnet moving coil (PMMC) mechanism is often called the d'Arsonval Movement, named after its inventor. PMMC instruments are accurate and suitable for dc measurements only.

When the current I passes through a coil, a deflecting torque is produced on the coil.

$$T_{dc} = K_1 I$$

where K_1 is a constant $= NBA$

N = number of turns in a coil

B = flux density

A = area of the coil

The deflecting torque causes a restoring torque in the spring attached to the pointer, which is given by

$$T_c = K_2 \theta$$

where θ is the angular deflection of the pointer and k_2 is the controlling torque constant.

For final steady deflection

$$T_{\text{dc}} = T_c$$

$$K_1 I = K_2 \theta$$

$$\therefore \quad \theta = K_1 I/K_2 \tag{14.15}$$

These instruments require very low power consumption and current for full-scale deflection.

Galvanometer Sensitivity The sensitivity of a galvanometer can be specified in terms of current sensitivity and voltage sensitivity.

$$\text{Current sensitivity } s_I = d/I \text{ mm/}\mu\text{A} \tag{14.16}$$

where d is the deflection of the glavanometer in scale divisions in mm and I is the galvanometer current in μA.

$$\text{Voltage sensitivity } S_v = d/V \text{ mm/mV} \tag{14.17}$$

where V is the voltage applied to the galvanometer in mV.

DC Ammeter The coil winding of the PMMC movement is small and light and it can carry only small currents. For large current measurement, major part of the current is bypassed through a shunt resistance, R_{sh} (Fig. 14.3)

Clearly, $\quad I_{sh} R_{sh} = I_m R_m$

$$\therefore \quad R_{sh} = I_m R_m/(I - I_m)$$

$$\therefore \quad I_{sh} = I - I_m$$

$$= R_m/(m - 1) \tag{14.18}$$

where

$m = I/I_m$ = multiplying factor

R_m = internal resistance of the meter

I_{sh} = shunt current > I_m

I = full-scale current of the ammeter including the shunt

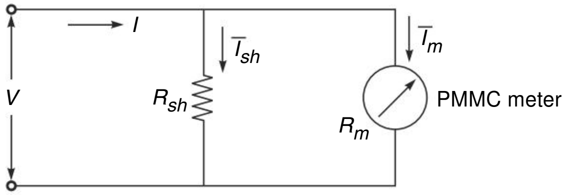


Fig. 14.3 DC ammeter

Multirange Ammeter The current range of the dc ammeter may be further extended by a number of shunts, selected by a range switch. Such a meter is called a multirange ammeter. The meter shown in Fig. 14.4 is called a universal shunt.

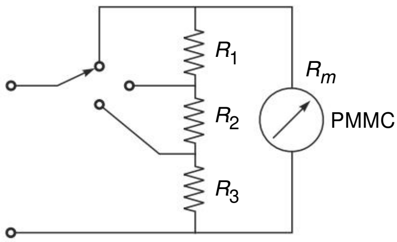


Fig. 14.4 Multirange ammeter

DC Voltmeter The basic d'Arsonval movement can be converted into a dc voltmeter with the addition of a series resistor or multiplier, as shown in Fig. 14.5. The multiplier limits the current through the movement so as not to exceed the value of the full-scale deflection current (I_{fsd}).

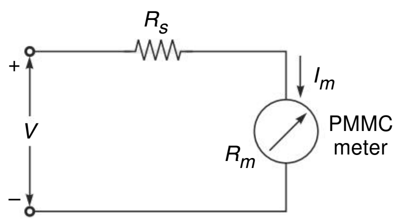


Fig. 14.5 DC voltmeter

A dc voltmeter is connected across a source of emf or a circuit component. The terminals are generally marked positive and negative since polarity must be observed.

The value of a multiplier required to extend the voltage is calculated as follows:

$$V = I_m (R_s + R_m) \text{ or } R_s = V/I_m - R_m \tag{14.19}$$

Multiplying factor $m = V/V_m = I_m (R_m + R_s)/I_m R_m$

$$m = 1 + R_s/R_m$$

$$\therefore \quad R_s = (m - 1) R_m \tag{14.20}$$

where

I_m = deflection current of the meter

R_s = multiplier resistance

V = full-scale voltage of the instrument

V_m = Voltage across movement

Multirange Voltmeter A multirange voltmeter can be obtained by the addition of a number of multipliers together with a range switch. There are two types of multi-range voltmeters.

1. The meter shown in Fig. 14.6(a) has only one resistance connected to the PMMC meter
2. The meter shown in Fig. 14.6(b) has all multipliers connected in series with the PMMC meter.

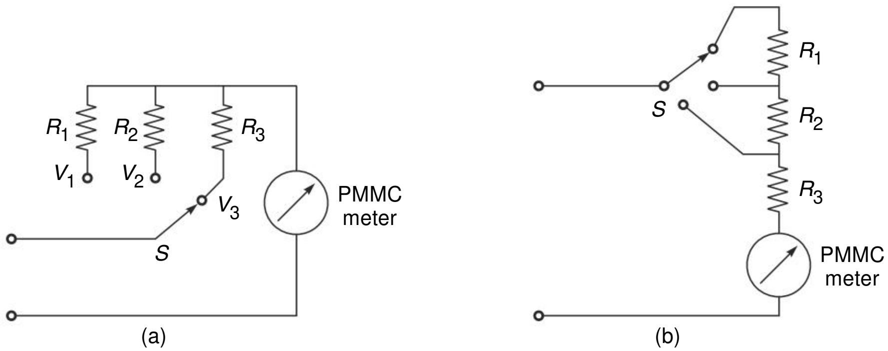


Fig. 14.6 Multirange voltmeters

Voltmeter sensitivity (ohm/V rating)

$$S = 1/I_{\text{fsd}}\ \Omega/\text{v} \tag{14.21}$$

The total resistance of the voltmeter will be

$$R_{\text{total}} = S \times V,\ R_s = (S \times V) - R_m \tag{14.22}$$

Loading Effect A low sensitivity meter may give a correct reading when measuring voltages in low resistance circuits. A voltmeter when connected across two points in a highly resistive circuit, acts as a shunt for that portion of the circuit and thus reduces the equivalent resistance in that portion of the circuit. The meter will give a lower indication of the voltage drop that actually existed before the meter was connected. This effect is called the *loading effect* of an instrument.

Example 14.2 A moving coil instrument gives full-scale deflection with 25 mA. The resistance of the coil is 5 Ω. It is required to convert this meter into an ammeter to read up to 5A. Find (a) the resistance of the shunt to be connected in parallel with the meter and (b) the value of series resistance for the above meter to read up to a voltage of 20 V.

Solution

(a) Full scale $\quad I_{fsd} = 25$ mA

$\therefore$ Current through shunt resistance $(R_x) = 4.975$ A

$$25 \text{ mA} \times 5\ \Omega = R_x \times 4.975 \text{ A} \qquad \therefore R_x = 0.025\ \Omega$$

(b) For meter to read 20 V

$$20 \text{ V} = (R_x + 5) \times 25 \text{ mA} \qquad \therefore R_x = 795\ \Omega$$

Example 14.3 If the instrument of Example 14.2 is to be converted into a multirange voltmeter to read up to 40 V and 60 V, find the additional resistance to be connected in series with the instrument.

Solution Let R_1 be the resistance in series (Fig. 14.7):

$$40 = (R_1 + 5) \times 0.025 \qquad \therefore R_1 = 1595\ \Omega$$

Let R_2 be the other resistance in series than

$$60 = (R_1 + R_2 + 5) \times 0.025$$

$$R_2 = 800\ \Omega$$

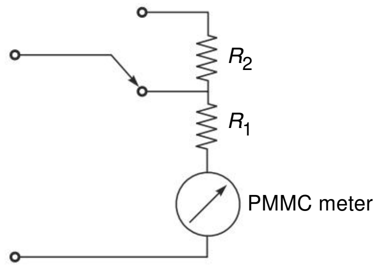


Fig. 14.7 Multirange voltmeter

Ohmmeter

This instrument is used to measure resistance. It is mainly of two types.

1. *Series Type Ohmmeter:* It essentially consists of a d'Arsonval movement connected in series with a resistance and a battery to a pair of terminals to which the unknown resistance is connected, as shown in Fig. 14.8. The current through the movement thus depends on the magnitude of the unknown resistor R_x and the meter indication is proportional to the value of R_x

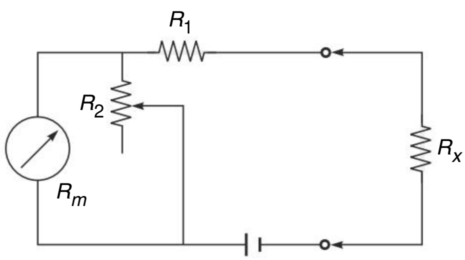


Fig. 14.8 Series type ohmmeter

When R_x is zero, maximum current flows in the circuit and shunt resistance R_2 is adjusted until the movement indicates full-scale current. The full-scale current position of the pointer is marked 0 Ω on the scale. When R_x is infinity, the current drops to zero and movement indicates zero current, which is marked as '∞' on the scale.

A convenient quantity to use in the design of a series type ohmmeter is the value of R_x (= R_h) which causes half-scale deflection of the meter.

$$R_h = R_1 + (R_2 \,\|\, R_m)$$

Then, total resistance for the battery is 2 R_h.

$$\therefore \quad I_t = 2\, I_h = E/R_h,\ I_2 = I_t - I_{\text{fsd}}$$

where

I_t = current through R_x for producing full-scale meter deflection

I_h = current for producing half-scale meter deflection

I_{fsd} = current through movement causing full-scale deflection

I_2 = current through R_2.

Also, voltage across shunt (R_2) is equal to voltage across movement,

$$(R_m),\ \text{i.e}\ E_{\text{sh}} = E_m$$

or
$$I_2\, R_2 = I_{\text{fsd}}\, R_m \tag{14.23}$$

$$\therefore \quad R_2 = I_{\text{fsd}}\, R_m\, R_h/(E - I_{\text{fsd}}\, R_h)$$

$$R_1 = R_h - I_{\text{fsd}}\, R_m\, R_h/E \tag{14.24}$$

2. *Shunt Type Ohmmeter (Fig. 14.9):* When R_x is zero, the meter current is zero. If R_x is infinity, a current finds a path only through the meter and by appropriate selection of the value of R_1, the pointer can be made to read full scale. This ohmmeter has a zero mark at the right hand side of the scale. It is particularly used for low resistance.

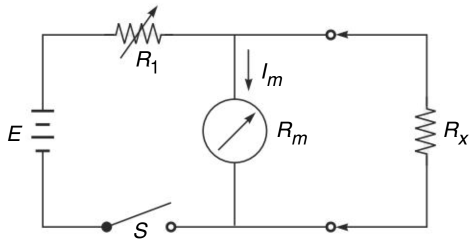


Fig. 14.9 Shunt type ohmmeter

When R_x is zero, the full-scale meter current will be

$$I_{\text{fsd}} = V/(R_1 + R_m)$$

For any value of R_x,

$$I_m = E\, R_x/[R_1\, R_m + R_x\,(R_1 + R_m)]$$

Then

$$S = I_m/I_{\text{fsd}}$$

$$\therefore \quad S = R_x/[R_x + R_1 R_m/(R_1 + R_m)] = R_x/(R_x + R_p) \tag{14.25}$$

where

$$R_p = R_1 \,\|\, R_m$$

AC Indicating Instruments

PMMC meters cannot be used for measuring ac signals because the meter will show the average value of ac, i.e. zero deflection.

Electrodynamometer It can be used for ac as well as dc measurements. It contains two types of coils, a fixed coil and a moving coil. The field is produced by a fixed coil. This coil is divided into two sections to give a more uniform field.

$$\text{Deflecting torque } T_d = BANI \tag{14.26}$$

where

B = flux density of the magnetic field in which the coil moves

A = area of cross-section of coil

I = current through the coil

N = number of turns of coil

But $\quad B \propto I$

$\therefore \quad T_d \propto I^2$

$$\text{Instantaneous deflecting torque } T_i = i_1\, i_2 \frac{\mathrm{d}M}{\mathrm{d}\theta} \tag{14.27}$$

where

i_1 = instantaneous current in the fixed coil

i_2 = instantaneous current in the moving coil

M = mutual inductance of coils

θ = deflection of pointer

Operation with dc:

$$T_d = I_1\, I_2 \frac{\mathrm{d}M}{\mathrm{d}\theta} \tag{14.28}$$

where I_1 and I_2 are dc currents through fixed and moving coils, respectively.

This deflecting torque deflects the moving coil to such a position where the controlling torque ($k\theta$) of the spring is equal to the deflecting torque.

$$\therefore \quad k\theta = I_1\, I_2 \frac{\mathrm{d}M}{\mathrm{d}\theta}$$

$$\therefore \quad \theta = (I_1\, I_2/k) \frac{\mathrm{d}M}{\mathrm{d}\theta} \tag{14.29}$$

ac currents: The meter will show the average value of the deflecting torque.

$$\therefore \quad T_{\text{av}} = \frac{\mathrm{d}M}{\mathrm{d}\theta} \frac{1}{T} \int i_1 i_2\, dt \tag{14.30}$$

Sinusoidal currents:

Let $i_1 = I_{m1} \sin \omega t$

and $\quad i_2 = I_{m2} \sin (\omega t - \phi)$

The average torque $(T_{\text{av}}) = (\mathrm{d}M/\mathrm{d}\theta)\ (1/T)$

$$\int I_{m1}\, I_{m2} \sin \omega t \sin (\omega t - \phi)\, dt$$

$$= (I_{m1}\, I_{m2}/2) \cos \phi \; \mathrm{d}M/\mathrm{d}\theta$$

$$\therefore \qquad T_{av} = I_1\, I_2 \cos \phi \, \frac{\mathrm{d}M}{\mathrm{d}\theta} \qquad (14.31)$$

where I_1 and I_2 are the rms values of i_1 and i_2, respectively. At steady state,

$$k\theta = I_1\, I_2 \cos \phi \, \frac{\mathrm{d}M}{\mathrm{d}\theta}$$

$$\therefore \qquad \theta = \frac{I_1 I_2}{k} \cos \phi \, \frac{\mathrm{d}M}{\mathrm{d}\theta} \qquad (14.32)$$

Electrodynamometer in Current and Voltage Measurement In an electrodynamometer, current under measurement itself produces a magnetic field flux in which the movable coil rotates.

A fixed coil (FC) splits into equal halves and provides the magnetic field in which the movable coil (MC) rotates. The two coil halves are connected in series and are fed by the current under measurement as shown in Fig. 14.11

Meter deflection $\propto \sqrt{\ }$(average i^2)

The meter therefore reads the rms or effective value of ac.

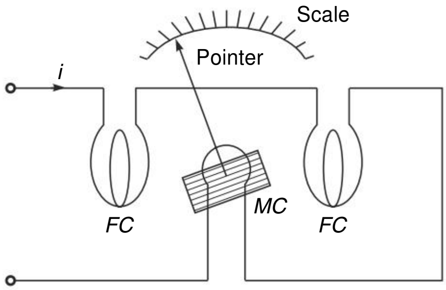


Fig. 14.11 Electrodynamometer type ammeter

In ammeters, fixed and movable coils are connected in series and, therefore, carry the same current. Hence

$$I_1 = I_2 = I, \phi = 0$$

$$\therefore \qquad \theta = (I^2/k) \, \frac{\mathrm{d}M}{\mathrm{d}\theta} \qquad (14.33)$$

In the voltmeter, the fixed and movable coils are connected in series with a high noninductive resistance.

Hence,

$$I_1 = I_2 = V/Z, \phi = 0$$

$$\therefore \qquad \theta = (V^2/Z^2 k) \; \mathrm{d}M/\mathrm{d}\theta \qquad (14.34)$$

The appropriate selection of the shunt value converts the electrodynamometer into the desired range of the ammeter and the addition of the series resistance converts the meter into a voltmeter exactly like the dc ammeter and voltmeter discussed earlier.

These types of ammeters and voltmeters can measure either ac or dc quantities.

Electrodynamometers in Power Measurement The electrodynameter movement is used extensively in measuring power. It may be used to indicate both dc and ac power for any wave form of voltage and current and is not restricted to a sinusoidal waveform.

The fixed or field (current) coil, shown in Fig. 14.11 as two separate elements, is connected in series and carry the total line current (i_c). The movable (potential) coil located in the magnetic field of fixed coils is connected in series with a current-limiting resistor (R_p) across the power line and carries the small current (i_p). The instantaneous value of current in the movable coil is

$$i_p = e/R_p$$

where e is the instantaneous voltage across the power line and, R_p is the total resistance of the movable coil and its series resistor.

The deflection of movable coil is proportional to the product of the two currents i_c and i_p.

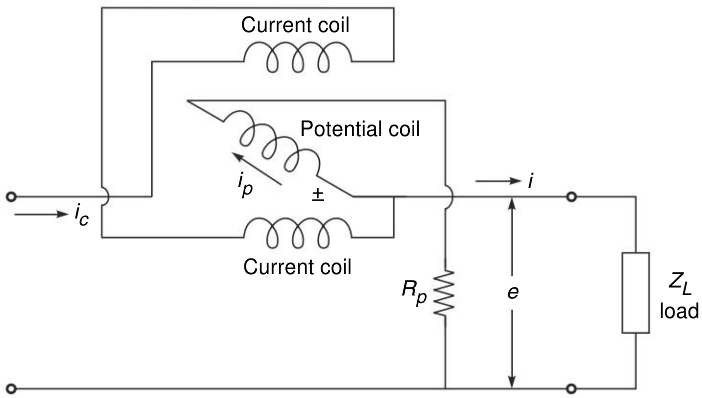


Fig. 14.11 An electrodynamometer in power measurement

Average deflection over one period

$$\theta_{av} = (k/T) \int i_c\, i_p\, dt$$

Average power in circuit

$$P_{av} = 1/T \int ei\, dt \tag{14.35}$$

$$\therefore \quad P_{av} \propto \theta_{av}$$

which indicates that the electrodynamometer movement of Fig. 14.12 has a deflection proportional to the average power.

Electrodynamometer Wattmeters An electrodynamometer wattmeter, shown in Fig. 14.12, has a current coil and a pressure coil.

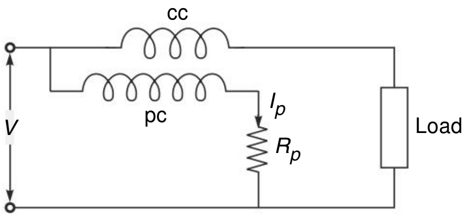


Fig. 14.12 Electrodynamometer wattmeter

The instantaneous torque is

$$T_i = i_1\, i_2 \frac{\mathrm{d}M}{\mathrm{d}\theta}$$

where i_1 and i_2 are the instantaneous currents in the two coils and M is the mutual inductance of coils. Therefore, the average deflecting torque is

$$T_i = (V\, I_c/R_p) \cos \phi \, \frac{\mathrm{d}M}{\mathrm{d}\theta}$$

where ϕ is the logging phase angle of current in the current coil, I_c is the current coil current and, R_p is the series resistance of pressure coil circuit.

$$\theta = (V\, I_c/kR_p) \cos \phi \, \frac{\mathrm{d}M}{\mathrm{d}\theta} \tag{14.36}$$

If the pressure coil has some inductance, then

$$W = (V\, I_c/kR_p) \cos (\phi - \alpha) \cos \alpha \, \frac{\mathrm{d}M}{\mathrm{d}\theta} \tag{14.37}$$

where α is the lagging phase angle between current in pressure coil and voltage supply. But the true value of power is

$$W_{\text{true}} = (VI_c/kR_p) \cos \phi \, \frac{\mathrm{d}M}{\mathrm{d}\theta}$$

$$\therefore \quad W/W_{\text{true}} = \cos \alpha\, [\cos \alpha + (\sin \alpha)\, (\tan \phi)] \tag{14.38}$$

Three-Phase Power Measurement

There are several ways in which 3-phase power can be measured. Most important and common of these is the one called the *two wattmeter method*, especially used when the load is unbalanced. Two wattmeters are used in a 3-wire system (Fig. 14.13) with delta or star-connected load. The total instantaneous power consumed by the load is $v_1\, i_1 + v_2\, i_2 + v_3\, i_3$

Instantaneous reading of $W_1 = i_1\, (v_1 - v_3)$

Instantaneous sending of $W_2 = i_2\, (v_2 - v_3)$

$\therefore$ Total instantaneous power in the load $= v_1\, i_1 + v_1\, i_2 - v_3\, (i_1 + i_2)$

From Kirchhoff's law, $i_3 = -\,(i_1 + i_2)$

$\therefore$ Total instantaneous power $= v_1\, i_1 + v_2\, i_2 + v_3\, i_3$

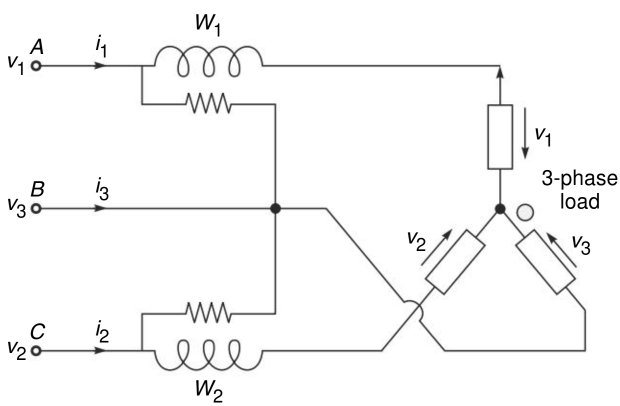


Fig. 14.13 Two-wattmeter method

Thus, the sum of the two wattmeter readings is equal to the power consumed by the load (balanced or unbalanced). Figure 14.14 shows the phasor diagram for a balanced star-connected load of Fig. 14.13.

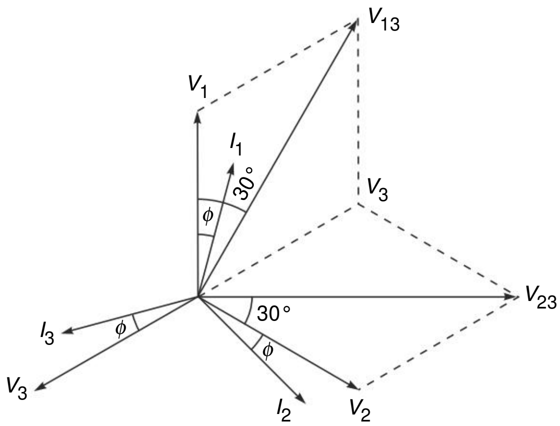


Fig. 14.14 Phase diagram for balanced star-connected load of Fig. 14.14

Consider a balanced load for the sake of simplicity.

Let $\quad I_1 = I_2 = I_3 = I;\ V_1 = V_2 = V_3 = V$ (say) rms values

Line voltages $\quad V_{13} = V_{23} = V_{12} = \sqrt{3}\,V$

The phase currents lag the corresponding phasor voltages by an angle ϕ. The current I_1 flows through W_1 and voltage across its pressure coil is V_{13}. I_1 leads V_{13} by an angle $(30°–\phi)$.

$$\therefore \quad \text{Reading of } W_1 = V_{13}\, I_1 \cos (30° - \phi) = \sqrt{3}\, VI \cos (30–\phi) \tag{14.39}$$

The current through wattmeter W_2 is I_2 and voltage across its pressure coil is V_{23}. I_2 lags V_{23} by an angle $(30° + \phi)$.

$$\therefore \text{ Reading of wattmeter } W_2 = V_{23}\, I_2 \cos (30°+\phi)$$
$$= \sqrt{3}\; VI \cos (30°+\phi) \tag{14.40}$$

$$\therefore \text{Sum of the wattmeter readings} = W_1 + W_2$$
$$= \sqrt{3}\, VI\, [\cos (30° - \phi) + \cos (30°+\phi)] = 3\, VI \cos \phi \tag{14.41}$$
$$= \text{3-phase power in the load}$$

Difference of readings of two wattmeters is

$$W_1 - W_2 = \sqrt{3}\; VI\, [\cos (30°- \phi) - \cos (30° + \phi)]$$
$$= \sqrt{3}\; VI \sin \phi \tag{14.42}$$

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{\sqrt{3}\, VI \sin\phi}{3VI\cos\phi} = \frac{1}{\sqrt{3}} \tan \phi$$

or
$$\phi = \tan^{-1} \sqrt{3}\; \frac{W_1 - W_2}{W_1 + W_2} \tag{14.43}$$

From Eq. (14.43), power factor (cos ϕ) may be found.

Some points worth noting are as follows:

(i) At unity power factor, the readings of the two wattmeters are equal.

(ii) When the power factor is 0.5, one of the wattmeters reads zero and the other reads total power.

(iii) With zero power factor, the readings of the two wattmeters are equal, but of opposite sign.

(iv) It should be noted that when the power factor is below 0.5, one of the wattmeters will give negative reading. Thus to read the wattmeter, we must either reverse the current coil or the pressure coil connections. The wattmeter will then give a positive reading but this must be taken as negative for calculating the total power.

Example 14.4 Two wattmeters are connected to measure power in a 3-phase circuit. One of the wattmeters reads 500 W and the other points out in reverse direction. After reversing the voltage coil terminals, the reading of this wattmeter is found to be 200 W. Determine the power factor of the load and the total 3-phase power of the circuit.

Solution

$$W_1 = 500 \text{ W} \quad W_2 = -200 \text{ W}$$

$$\therefore \quad \text{Total power} = W_1 + W_2 = 300 \text{ W}$$

$$\tan\phi = \frac{500-(-200)}{500+(-200)} = \frac{700}{300} = 7/3 \qquad \therefore \phi = 66.8°$$

$$\therefore \qquad \text{pf} = \cos\phi = 0.39$$

Induction Type Instruments

Induction type of instruments depend upon magnetic induction for operation and are used for ac measurement only.

$$\text{Deflecting torque } (T_d) \propto \phi_1\, \phi_2 \tag{14.44}$$

$\therefore$ Fluxes ϕ_1 and ϕ_2 both are proportional to some current I.

$$\therefore \qquad T_d \propto I^2$$

The induction type ammeter and voltmeter employ coil spring control and electromagnetic damping. The induction type instrument is primarily used as a Watthour meter or energy meter.

Watthour Meter or Energy Meter It is used for commercial measurement of electrical energy. Figure 14.15 shows the elements of a single-phase watthour meter.

The current coil is connected in series with the line, and the voltage coil is connected across the line. Both coils are wound on a metal of special design providing two magnetic circuits. A light aluminium disk is suspended in the air-gap of the current coil field which causes edddy currents to flow in the disk. The reaction of the eddy currents and the field of voltage coil creates a torque on the disk, causing it to rotate. The number of rotations of the disk is proportional to the energy consumed by the load in a certain time interval.

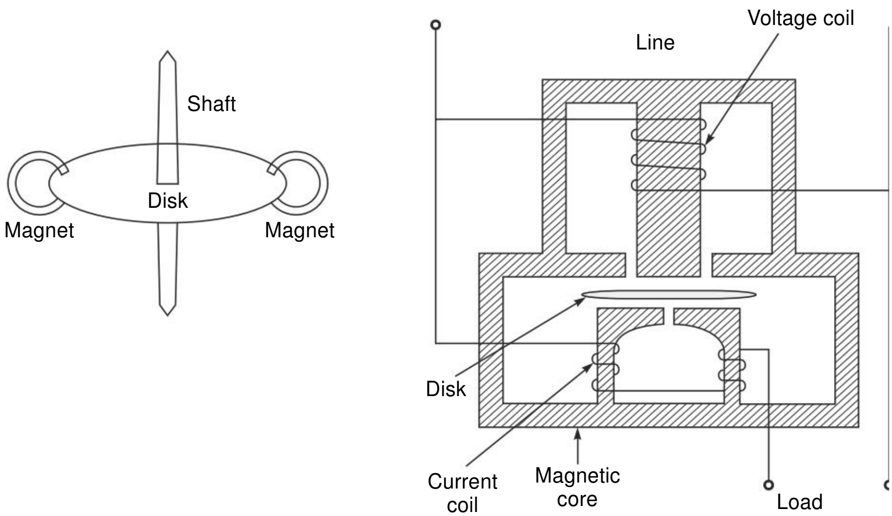


Fig. 14.15 Watthour meter or energy meter

At very light loads, the voltage component of the field produces zero torque that is not directly proportional to the load. Compensation for error is provided by inserting a shading coil with the meter operating at 10% of rated load. Two holes are drilled in the disk of the energy meter on the opposite side of the spindle to eliminate creeping on no load.

Moving Iron Instrument In this type of instrument, a plate of soft iron is the moving element of the system. This iron moves in a magnetic field produced by a stationary coil. The coil is excited by a current in voltage under measurement.

Thus are two types of moving iron instruments:

1. Attraction type: In this type, a sheet of soft iron is attracted towards a solenoid.
2. Repulsive type: In this type, two parallel strips of soft iron magnetized inside a solenoid repel each other.

The deflection of the pointer, $\theta = (I^2/2k)\ \mathrm{d}L/\mathrm{d}\theta$ (14.45)

where k is the control spring constant, L is the instrument inductance and I is the initial current.

$$\therefore \quad \theta \propto I^2_{\text{rms}} \text{ for ac}$$
$$\propto I^2 \text{ for dc}$$

Thermocouple Instrument

In this instrument, the heat at the junction is produced by the electrical current flowing in the heater element while the thermocouple produces an emf at its output terminals which are connected to a PMMC meter, as shown in Fig. 14.16. The emf produced is proportional to the temperature and hence to the rms value of the current. Therefore, the scale of PMMC instrument can be calibrated to read the current through the meter.

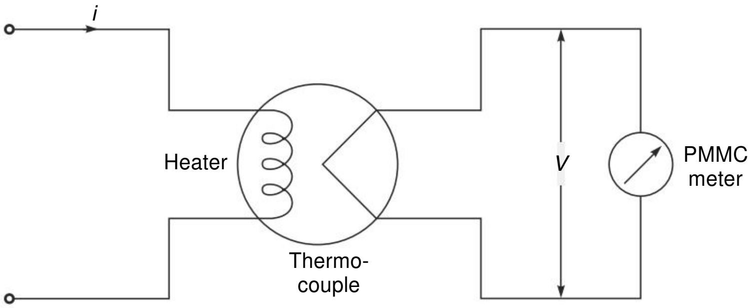


Fig. 14.16 Thermocouple instrument

Electrostatic Instruments

In this type of instrument, the deflecting torque is produced by the action of electric field on charged conductors. Such instruments are essentially voltmeters, but they can be used to measure current and power with the help of external components. In these instruments, there are two oppositely charged electrodes. One of them is fixed and the other is movable. This electrode can be moved by the attraction force of opposite charges.

Power Factor Meter

The most common type is the crossed coil power factor meter. This instrument is basically an electrodynamometer movement where the moving element consists of two coils, mounted on the same shaft but at right angles to each other.

The moving coil rotates in the magnetic field provided by the field coil that carries the line current. The connections for this meter in a single-phase circuit are shown in Fig. 14.17.

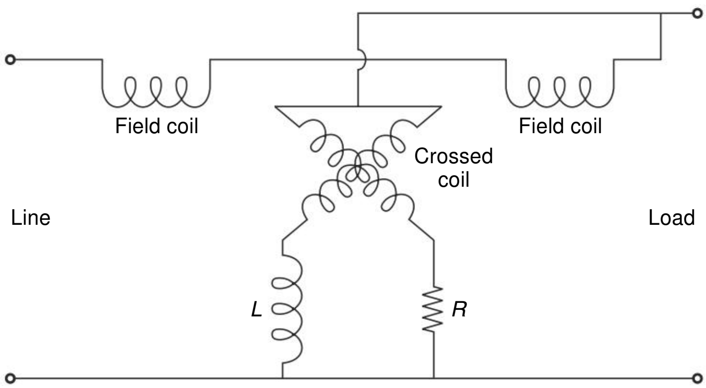


Fig. 14.17 Power factor meter

The field coil is connected as usual in series with the line and carries the line current. One coil of the movable element is connected in series with an inductor across the lines. Since no control springs are used, the balance position of the movable element depends on the resulting torque developed by the two crossed coils.

The torque developed is proportional to the mutual inductance between the crossed coil and the stationary field coil. When the movable element is at balance, it can be easily shown that its angular displacement is a function of the phase angle between the line current (field coil) and line voltage (crossed coil). The indication of the pointer, which is connected to the movable element, is calibrated directly in terms of the phase angle or power factor.

Frequency Meter

Figures 14.18 shows an electrodynamometer type frequency meter. Here the field coil forms the part of two repeated resonant circuits. The resonant circuit comprising L_1, C_1 and FC_1 is tuned to a frequency slightly below the low end of the instrument scale whereas L_2, C_2 and FC_2 form another resonant circuit tuned to a frequency slightly higher than the high end of the scale. The torque on the movable element is proportional to the current through the moving coil.

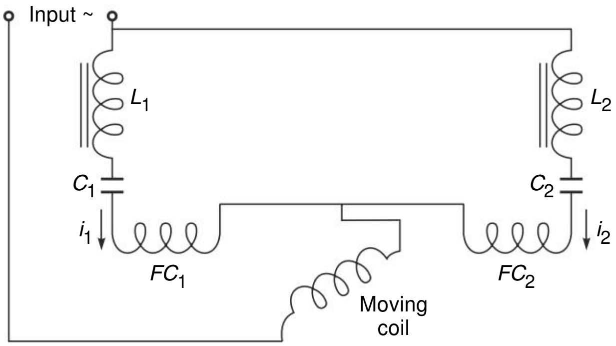


Fig. 14.18 Frequency meter

Megger

Megger is an insulation testing instrument. It is used to measure very high resistances of the order of megaohms. This instrument works on the principle of an ohmmeter. The required deflecting torque is produced by both the system voltage and the current. Because of interaction between the magnetic fields produced by the voltage and the current, the deflecting torque is produced.

14.5 INSTRUMENT TRANSFORMER

Transformers are used in ac systems for the measurement of current, voltage, power and energy. Instrument transformers are classified according to their uses as current transformer (CT) and potential transformer (PT)

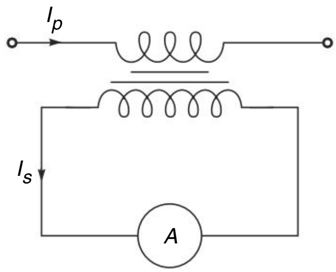


Fig. 14.19 Current transformer

Current Transformer

It has a primary winding with a small number of turns, connected in series with a line carrying the current to be measured. The secondary winding has a larger number of turns and is connected to a current meter or a relay coil as shown in Fig. 14.19.

The transformer ratio = primary current/secondary current

$$R \approx N + I_0/I \sin(\delta + \alpha) \quad (14.46)$$

where

N = turns ratio = no. of turns of secondary winding/no. of turns of primary winding

I_0 = exciting current

I = secondary current

δ = angle between the secondary induced emf and secondary current, and

α = angle between I_0 and working flux

Phase angle θ is the angle by which I, when reversed, differs in phase from the primary current.

$$\theta = 180/\pi(I_0/NI_s) \cos(\delta + \alpha)] \text{ degree} \quad (14.47)$$

Ratio error = (nominal ratio – actual ratio) × 100/actual ratio

$$= (k_n - R) \times 100/R \quad (14.48)$$

where

k_n = nominal ratio = rated primary current/rated secondary current

Potential Transformer

Potential transformer is used to operate voltmeters, potential coils of wattmeters and relays from high voltage lines. The primary winding of the transformer is connected across the lines carrying the voltage to be measured and the voltage circuit is connected across the secondary winding (Fig. 14.20).

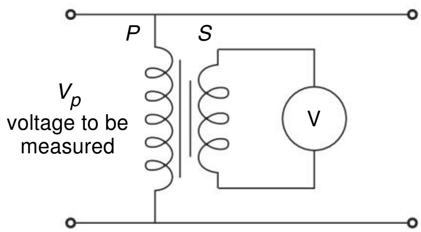


Fig. 14.20 Potential transformer

The transformer ratio R is

$$R = N + [NI_s (R_2 \cos\beta + x_2 \sin\beta) + I_c r_1 + I_m x_1]/V_s \quad (14.49)$$

where

N = turns ratio

I_s = secondary winding current

I_c = exciting current

I_m = magnetizing component of no-load (exciting) current

β = phase angle of secondary load circuit

V_s = secondary winding terminal voltage

r_1, x_1 = resistance and reactance of primary windings

R_2, X_2 = equivalent resistance and reactance of a transformer referred to secondary side.

The phase angle θ is

$$\theta = I_s (X_2 \cos\beta - R_2 \sin\beta)/V_s + (I_c x_1 - I_m r_1)/N V_s \text{ rad} \qquad (14.50)$$

Range Extension Instrument transformers extend the range of the ac measuring instruments.

Advantages

1. Moderate size instrument transformers are used for metering.
2. Cheaper single range instrument can be used to cover a large current or voltage range.
3. The metering circuit consumers lower power and is isolated from high power circuits.
4. Several instruments can be operated from a single instrument transformer.

14.6 BRIDGE MEASUREMENTS OR TRANSFORM

Bridge measurements help in measuring the values of components like resistance, inductance, capacitance, admittance, conductance and any other impedance parameters. Bridges are considered better than meters for measuring impedance parameters because accuracy of meter depends on the quality of internal resistance and the meter movement.

Wheatstone Bridge

This is the simplest bridge to measure resistance. Figure 14.21 shows the schematic of a Wheatstone bridge. The bridge has four resistive arms together with a source of emf and detector, usually a galvanometer or other sensitive current meter.

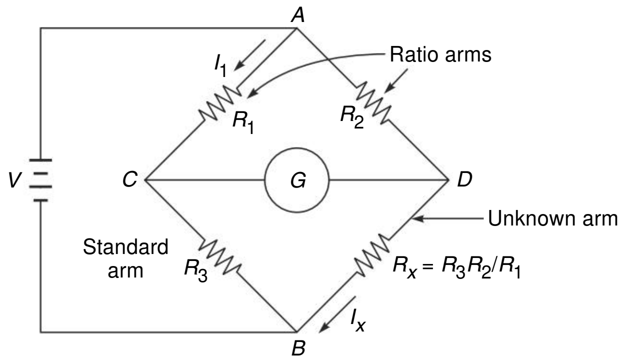


Fig. 14.21 Wheatstone bridge

If the sensitivity of the galvanometer is not sufficient to indicate the balance position to the required degree of precision, then the measured value of R_x can have some error.

Thevenin Equivalent Circuit of Wheatstone Bridge To find whether or not the galvanometer has the required sensitivity to detect an inbalance condition, it is necessary to calculate the galvanometer current. The Thevanin equivalent circuit is determined by looking into the galvanometer terminals 'c' and 'd' as shown in Fig. 14.22.

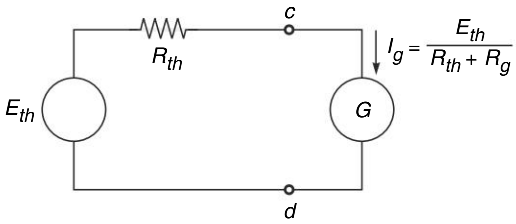


Fig. 14.22 Thevenin equivalent of a Wheatstone bridge

Assuming $R_{\text{battery}} = 0$, we have

$$R_{th} = (R_1 \parallel R_3) + (R_2 \parallel R_4) \parallel R_4 = R_x \tag{14.51}$$

$$E_{th} = \parallel E = V\,E\,[R_2/(R_2 + R_4) - R_1/(R_1 + R_3)] \tag{14.52}$$

$\therefore$ Current through galvanometer is

$$I_g = E_{th}/(R_{th} + R_g) \tag{14.53}$$

The Kelvin bridge is more accurate than the Wheatstone bridge and is used for measuring very low resistances.

AC Bridge

The ac bridge is a natural extension of the dc bridge and consists of four bridge arms, a source of excitation and a null detector (e.g. a pair of headphones).

The general equation for bridge balance is

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{or} \quad Y_1 Y_4 = Y_2 Y_3$$

where Z and Y are the impedance and admittance of the arm of the bridge.

Maxwell Bridge

It measures an unknown inductance in terms of a known capacitance. Here (Fig. 14.23),

$$Z_x = Z_2 Z_3 Y_1$$

$$R_x = R_3 R_2/R_1,\ L_x = R_2 R_3 C_1$$

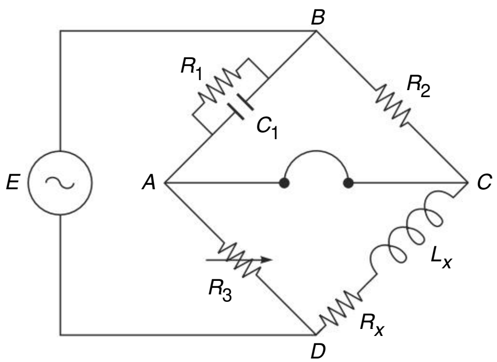


Fig. 14.23 Maxwell bridge

Hay Bridge

It is used for the measurement of high Q inductors having a value more than 10.

$$R_x = \omega^2 C_1^2 R_1 R_2 R_3/(1 + \omega^2 C_1^2 R_1^2)$$
$$L_x = R_2 R_3 C_1/(1 + \omega^2 C_1^2 R_1^2)$$

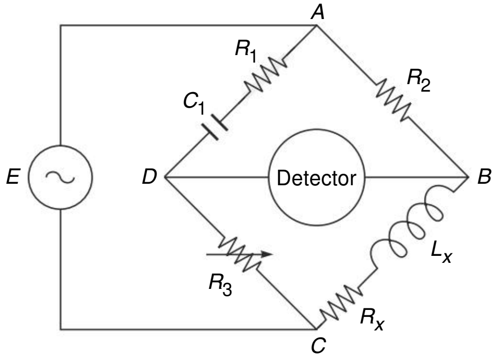


Fig. 14.24 Hay bridge

L_x and R_x both depend on the frequency of signal. Therefore, ω should be accurately known.

The Q-factor of the coil is

$$Q = \omega L_x/R_x = \frac{1}{\omega C_1 R_1}$$

$$\therefore \quad R_x = \omega^2 C_1 R_1 R_2 R_3/[1 + (1/Q)^2]$$
$$L_x = R_2 R_3 C_1/[1 + (1/Q)^2]$$

For large Q, $1/Q^2$ can be neglected. Thus $L_x = R_2 R_3 C_1$, which is the same as the Maxwell bridge. For smaller values of Q (< 10), the term $1/Q^2$ becomes important and cannot be neglected.

Schering Bridge

It is used extensively for measurement of capacitance.

$$Z_x = Z_2 Z_3 Y_1$$
$$R_x = R_2 C_1/C_3$$
$$C_x = C_3 R_1/R_2$$
$$\text{pf} = R_x/X_x = \omega C_x R_x$$

The dissipation factor of a series RC circuit is defined as the co-tangent of the phase angle and, therefore, by definition, the dissipation factor is

$$D = R_x/X_x = \omega C_x R_x$$

Substituting the values of C_x

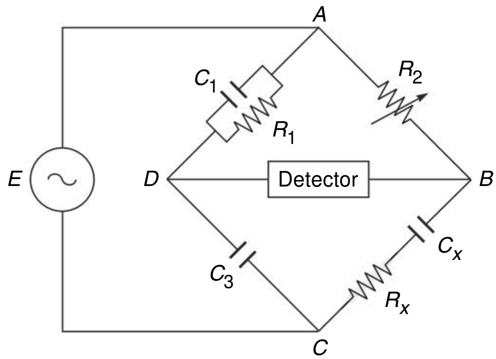


Fig. 14.25 Schering bridge

and R_x, we get

$$D = \omega\, C_1\, R_1$$

If R_1 is fixed, then dial of C_1 may be calibrated directly in dissipation factor D.

Owen's Bridge

It is used for the measurement of inductance.

$$R_x = R_3\, C_4/C_2$$

$$L_x = R_2\, R_3\, C_4$$

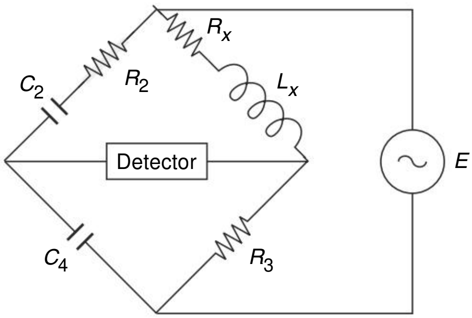


Fig. 14.26 Owen's bridge

Anderson's Bridge

It is also used for the measurement of inductance.

$$R_x = R_2\, R_3/R_4$$

$$L_x = C\,(R_3/R_4)\,[R(R_2 + R_4) + R_2R_4]$$

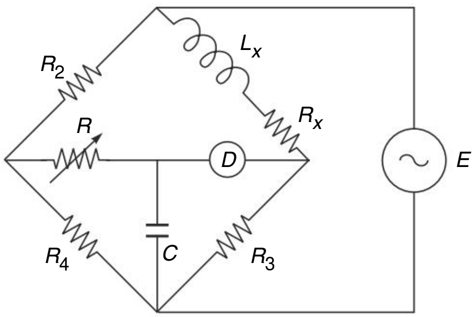


Fig. 14.27 Anderson's bridge

Wien's Bridge

It is used for frequency measurement and also for various other applications such as harmonic distortion analyzer (as a notch filter), audio and HF oscillators used as frequency determining elements.

Frequency of applied voltage, $f = 1/2\pi\sqrt{C_1\, C_3\, R_1\, R_3}$

From Fig. 14.28, $R_2/R_4 = R_1/R_3 + C_3/C_1$

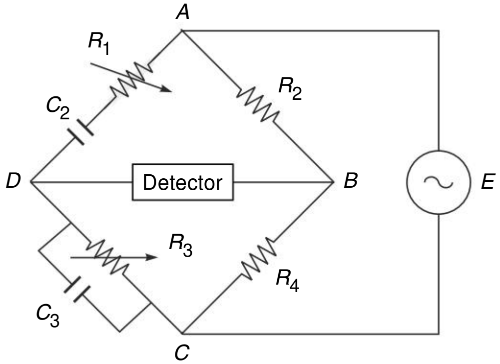


Fig. 14.28 Frequency measurement with the Wien bridge

Normally, for a Wien bridge $R_1 = R_3$, $C_1 = C_3$

$$\therefore \qquad R_2/R_4 = 2, f = 1/(2\pi RC)$$

Example 14.5 Figure 14.29 shows a Wheatstone bridge. The battery voltage is 4 V and the internal resistance is ignored. The galvanometer has a current sensitivity of 8 mm/μA and an internal resistance of 80 Ω. Calculate the deflection of the galvanometer caused by the 5 Ω imbalance in arm BC.

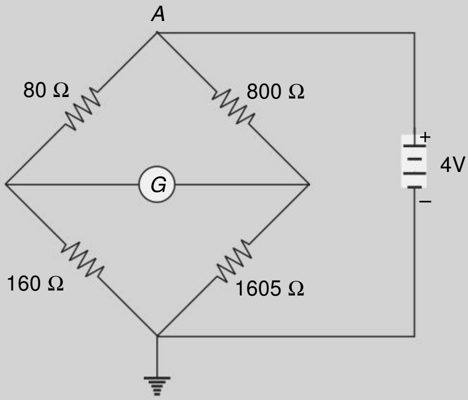


Fig. 14.29 Wheatstone bridge

Solution Because of the unbalanced resistor, there will be some current in the galvanometer. Hence,

$$E_{th} = E\,[R_2/(R_2 + R_4) - R_1/(R_1 + R_3)]$$
$$= 4\,[80/240 - 800/2405] = 0.00277 \text{ V}$$
$$R_{th} = R_1R_3/(R_1 + R_3) + R_2R_4/(R_2 + R_4)$$
$$= 800 \times 1605/2405 + 80 \times 160/240 = 587.22\ \Omega$$
$$\therefore \qquad I_g = E_{th}/(R_{th} + R_g) = 0.00277/667.22 = 4.15\ \mu\text{A}$$
$$\text{Deflection} = \text{sensitivity} \times I_g = 8 \times 4.15 = 33.2 \text{ mm}$$

Example 14.6 A balanced ac bridge has the following constants: arm *AB*, $R = 2000\ \Omega$ in parallel with $C = 0.045\ \mu F$; arm *BC*, $R = 1\ k\Omega$ is series with $C = 0.45\ \mu F$; arm *CD* unknown; arm *DA*, $C = 0.4\ \mu F$. The oscillator frequency is 1 kHz. Find the constants of *CD*.

Solution

$$\omega = 2\pi f = 6283.2 \text{ rad}$$

From Fig. 14.30, we see that

$$Y_1 = 2000 \parallel 0.045 = 5.744 \times 10^{-4} \angle 29.5°$$

$$Z_2 = 1000 + 1/(j\omega\ 0.45) = 1060.7 \angle -19.47°$$

$$Z_3 = 1/(j\omega\ 0.4) = 398 \angle -90°$$

$$Z_4 = Z_x$$

$$Z_1 Z_4 = Z_3 Z_2$$

$$Z_x = Z_3 Z_2 Y_1$$

$$= 1060.7 \angle -19.47° \times 5.744 \times 10^{-4} \angle 29.5° \times 398 \angle -90°$$

$$= 242 \angle -80°$$

$$\therefore \quad Z_x = 42 - j\,238.3$$

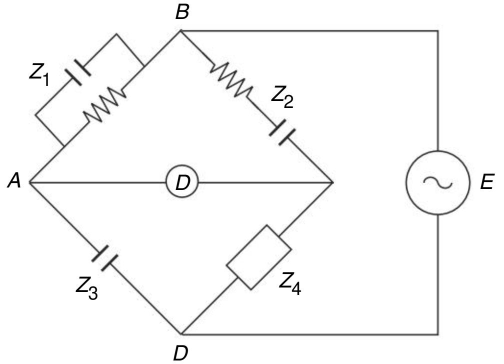


Fig. 14.30

14.7 ELECTRONIC VOLTMETER (EVMS)

So far, we have described the measuring instruments which employed the movement of an electromagnetic meter to measure electrical quantities such as *V*, *I* and *R*. Now we shall describe electronic instrumentation for the measurement of small signals.

Special features:

1. Sensitivity of EVMs is of the order of 10 microvolts (full scale).
2. In EVM we also put some impedance matching stages (Z_{in} is very high but Z_{out} is low), i.e. a buffer.
3. EVMs also possess some intelligent functions such as autozeroing and autoranging.
4. EVMs generally have internal protection against overloading.
5. EVMs have wide frequency operating range (even a GHz range).

DC Electronic Voltmeter

In this meter, the amount of power drawn from a circuit under test is decreased by increasing the input impedance using an amplifier with unity gain. Then this amplification of input source is applied to the MPMC meter as shown in Fig. 14.31.

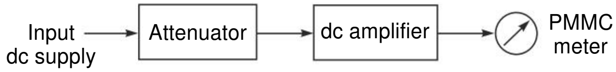


Fig. 14.31 dc EVM

AC Electronic Voltmeter

AC electronic voltmeters are basically identical to dc electronic voltmeters except that the ac input voltage must be rectified before it can be applied to the dc meter (PMMC) circuit.

There are five types of electronic ac voltmeters.

(i) Average responding EVM

(ii) Peak-responding ac EVM;

(iii) True rms responding EVM

(iv) Tuned EVM

(v) Heterodyne type EVM.

1. Average Responding EVM This is the most common type of meter: with the meter scale calibrated in terms of the rms value of a sine wave. Figure 14.32 shows the block diagram of an average responding EVM.

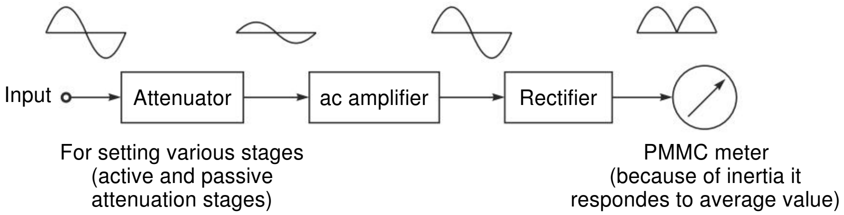


Fig. 14.32 Block diagram of an average responding EVM

The rms value of a voltage wave that has equal positive and negative excursion is related to the average value by the form factor. The form factor, as the ratio of the rms value to the average value of this sinusoidal waveform can be expressed as

$$k = \sqrt{}\left[(1/T)\int e^2 dt\right] \Big/ \left[(2/T)\int e^2 dt\right] \tag{14.54a}$$

Value : $\left(\frac{2}{T}\right)\int e.\mathrm{d}t$

Let $\quad e = E_m \sin \omega t$. Then

$$k = E_m\, 0.707/(E_m\, 0.636) = 1.11 \tag{14.54b}$$

Therefore, an average responding voltmeter has scale markings corresponding to the rms value of the applied sinusoid input waveform, whose markings are actually corrected by a factor 1.11 from the true (average) value of the applied voltage.

A non-sinusoidal waveform, when applied to this voltmeter, will cause the meter to read either high or low, depending on the form factor of the waveform.

Example 14.7 The symmetrical square wave voltage of Fig. 14.33 is applied to an average responding voltmeter with a scale calibrated in terms of the rms value of a sine wave. Calculate the error in the meter indication.

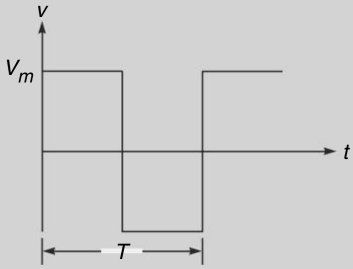


Fig. 14.33

Solution First we find the value of the form factor k.

$$V_{\text{rms}} = \left[(1/T)\int v^2 dt\right] = V_m$$

$$V_{\text{av}} = 2/T \int v\text{d}t = V_m$$

$$\therefore \quad k = 1 = V_{\text{rms}}/V_{\text{av}}$$

Now the meter scale is calibrated in terms of the rms value of a sine-wave voltage, where $V_m = k\, V_{\text{av}}$

$$\therefore \quad V_{\text{rms}} = 1.11\, V_{\text{av}}$$

But for square-wave voltage, $V_{\text{rms}} = V_{\text{av}}$ since $k = 1$. Thus the meter indication for the square-wave voltage is higher by a factor of

$$k_{\text{sine wave}}/k_{\text{square wave}} = 1.11/1 = 1.11$$

$$\therefore \quad \%\ \text{error} = (1.11 - 1) \times 100/1 = 11\%$$

This problem shows that any deviation from a sinusoidal waveform may cause an appreciable error in the result.

2. Peak-Responding ac EVM In this *EVM*, the ac input is fed to a peak detector which gives out only the peak value of supply as output. Figure 14.34 shows the

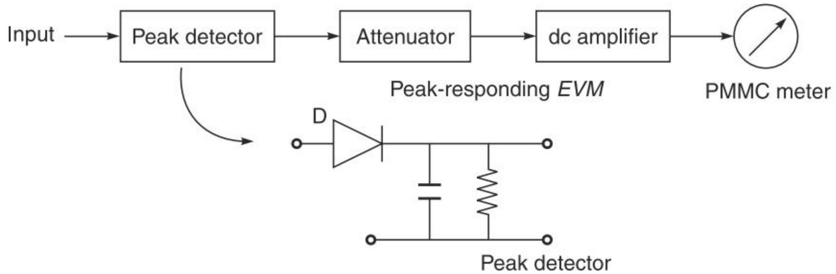


Fig. 14.34 Peak–responding ac *EVM*

block diagram of a peak-responding ac EVM.

Advantage: The range of operating frequency is increased (100 Hz–100 MHz).

Disadvantage: It uses a costly dc amplifier and possesses lower sensitivity.

3. True rms-Responding EVM Complex waveforms are most accurately measured with the runs-responding voltmeter. This instrument produces a meter indication by sensing the waveform heating power, which is proportional to the square of the rms value of the voltage.

4. Tuned EVM This instrument can be used to measure the distortion factor (δ), which is given by

$$\delta = \sqrt{(V_2^2 + V_3^2 + \ldots)/V_1^2} \tag{14.55}$$

where V_1 is the rms fundamental component and $V_2 \ldots V_n$ are the rms values of harmonics. It consists of a tunable amplifier which responds only to that frequency to which it has been tuned. Thus the PMMC meter measures the rms value of the selected frequency component as shown in Fig. 14.35

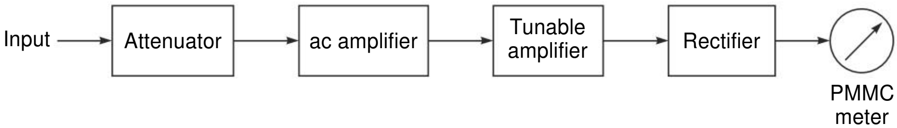


Fig. 14.35 Block diagram of a tuned *EVM*

5. Heterodyne Type EVM (Fig. 14.36) Hetrodyne is the mixing of the two high and close frequencies to generate the beat frequency ($f - f_0$). Heterodyne type EVM's sensitivity can be as good as 1 μV full-scale, because of tuning of the noise being very small and the sensitivity being very high. The operating frequency range is much wider than in the tuned EVM (100 kHz–1000 MHz).

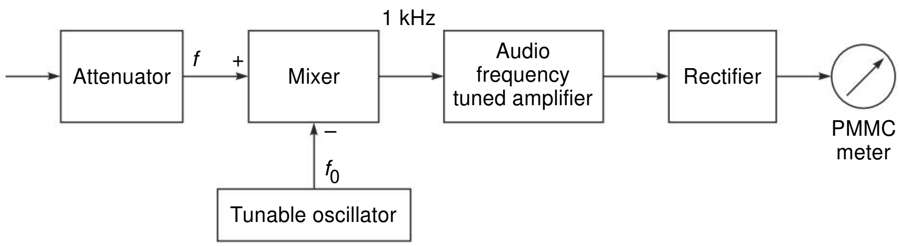


Fig. 14.36 Block diagram of hetrodyne type *EVM*

14.8 ELECTRONIC MULTIMETER (EMM)

An EMM is capable of measuring dc and ac voltages, current and resistance. The three main characteristics of EMMS are

1. The input impedance is very high, varying from 1 to 10 MΩ.
2. EMMs can respond to a high frequency of 3MHz.
3. The voltage placed across the components during a resistance measurement can be lower than the voltage from a nonelectronic multimeter. Because of their high input impedance, field effect transistors (FETs) are generally used. That is why

EMMs are also called FET VOM (volt-ohm-milliameter).

Scheme of dc Voltage Measurement

Figure 14.37 shows the block diagram for dc voltage measurement.

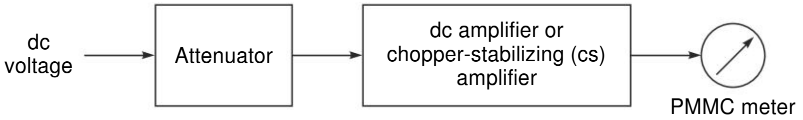


Fig. 14.37 dc voltage measurement

The chopper-stabilizing amplifier is a technique for amplifying direct currents and relatively low-frequency alternating currents. This circuit eliminates the effects of dc offset current and the drift of other dc parameters by using an ac-coupled amplifier for the necessary gain. In this technique dc, input signal is first converted into ac signal. After high gain amplification, the signal is again converted into a dc signal.

Scheme of ac Voltage Measurement

A schematic block diagram of ac voltage measurement is shown in Fig. 14.38(a). The circuit for RC attenuator is also shown in Fig. 14.38(b). This serves the purpose of an attenuator for both ac and dc measurements correctly.

Scheme of dc and ac Current Measurement

Current is converted into voltage using shunt arrangement. The variable *R* is used for multiranging facility, as shown in Fig. 14.39.

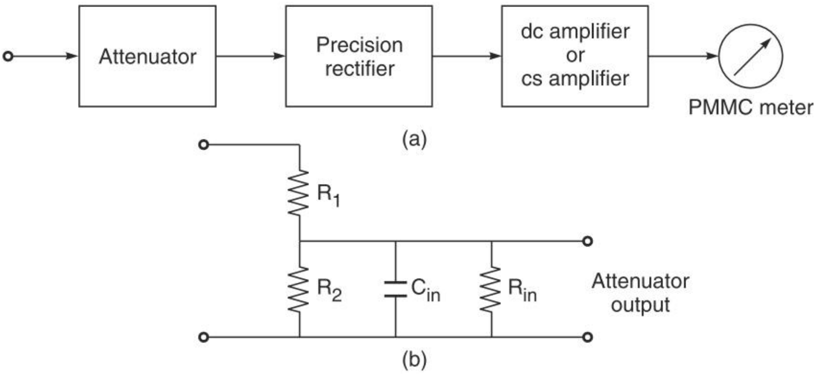


Fig. 14.38 ac voltage measurement

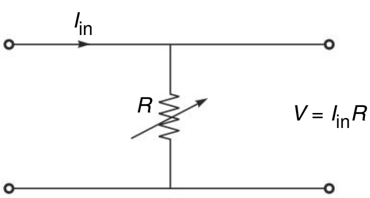


Fig. 14.39 Current measurement

Resistance Measurement (Medium Resistance Measurement)

Figure 14.40 shows the different techniques for resistance measurement.

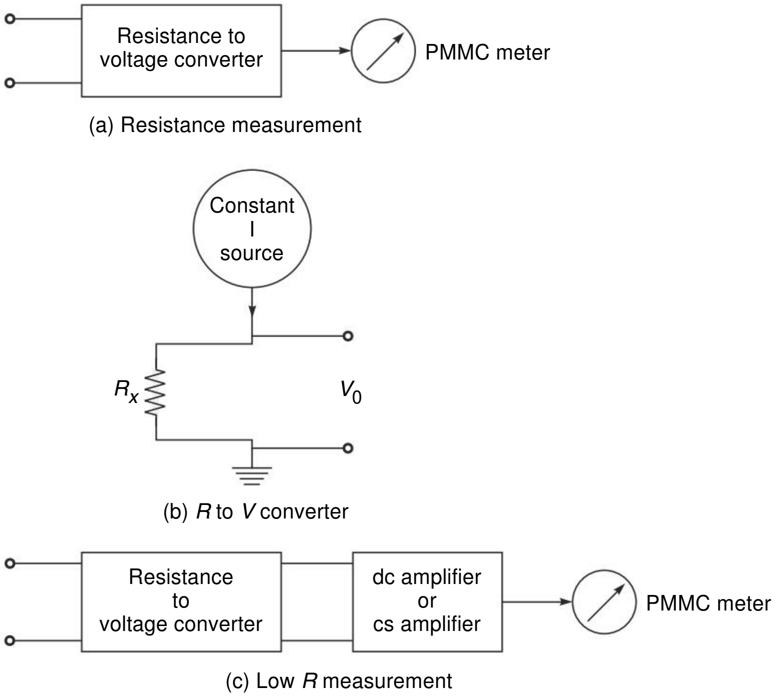


Fig. 14.40 Electronic multimeter

When an unknown R_x resistance is connected to the terminals of the multimeter, the internal battery supplies current through one of the range resistors and the unknown resistance to the ground. Voltage drop V_x is applied to the input of the bridge amplifier and causes a deflection on the meter.

Since the voltage drop across R_x is directly proportional to its resistance, the meter scale can be calibrated in terms of resistance.

14.9 MEASUREMENT OF ELECTRONIC COMPONENTS

The methods of performing electronic inductance and capacitance measurements are described below.

1. Small Inductance and Capacitance Measurement Using Active Resonant Circuit: Figure 14.41(a) shows the circuit for capacitance measurement. Steps for measuring capacitance are as follows:

(a) S open: adjust value of $C = C_1$ so that again value of 'f' is 'f_0'.

(b) S close: again adjust value of $C = C_2$ so that again value of 'f' is 'f_0'. Since the values of L and f_0 are same for both the measurements, we have

$$C_1 + C_s = C_2 + C_s + C_x, \quad \text{total capacitance in both the cases will be same}$$

$$\therefore \qquad C_x = C_1 - C_2$$

Figure 14.41(b) shows the circuit for measurement of inductance. For measurement take four readings.

(i) For $f = f_1$., (a) S open adjust $C = C_1$

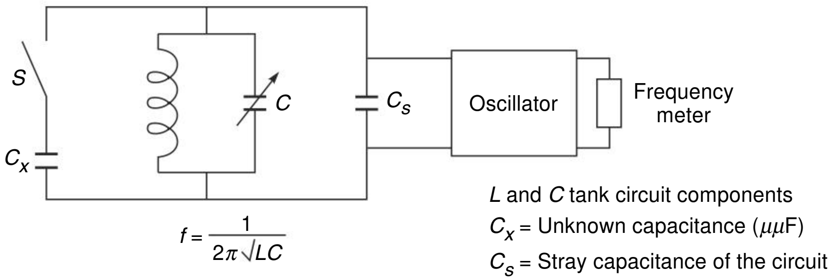
(b) S close adjust $C = C_2$

(ii) For $f = f_2$, (a) S open adjust $C = C_3$

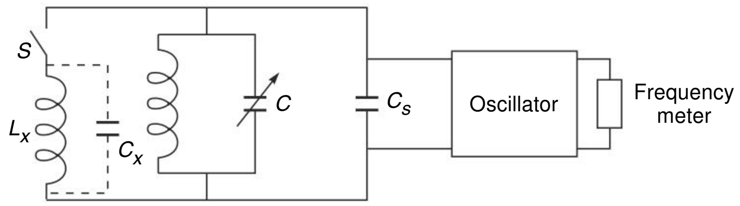
(b) S close adjust $C = C_4$

After solving the equations for each reading, we get

$$L_x = \{1/[(2\pi)^2 (C_4 - C_3 - C_2 + C_1)]\} \times (1/f_1^2 - 1/f_2^2) \tag{14.56}$$



(a) Capacitance measurement



(b) Small inductance measurement

Fig. 14.41

2. Using Voltage Divider Circuit. In this method the current through the capacitor is sampled across a known resistance and the resultant voltage is amplified and measured as shown in Fig. 14.42.

Amplification provides the necessary gain to the current through the capacitor. This gain can be quite small and within practicality.

The voltage across the resistor can be expressed as

$$V = RV_{in} / \sqrt{R^2 + \{1/(2\pi fC)\}^2}$$

3. Phase Shift Method

(a) Capacitance measurement: In the circuit shown in Fig. 14.48(a), the phase angle

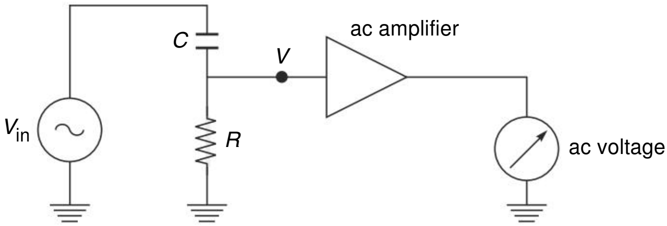


Fig. 14.42 *Voltage divider circuit*

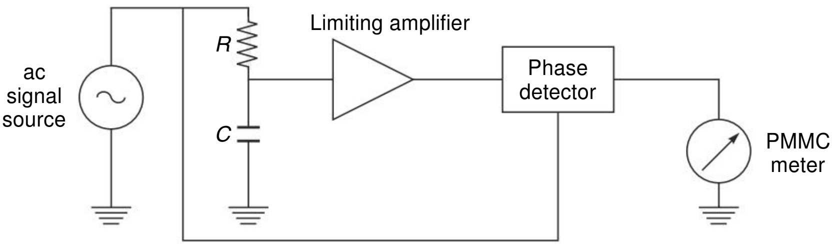
between the applied voltage and the voltage across the capacitance is measured.

$$\theta = \tan^{-1}(R/X_c) = \tan^{-1}(2\pi fRC)$$

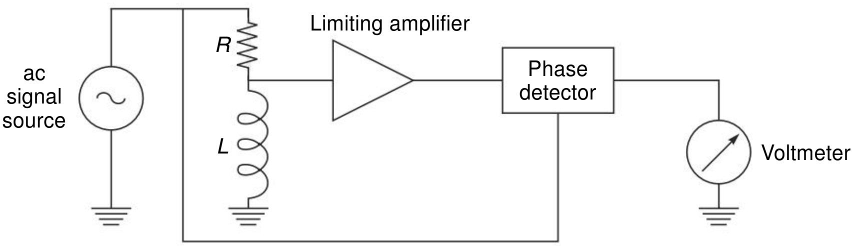
$$\theta \approx 2\pi RC \text{ for small values of } \theta \text{ in radians}$$

(b) Inductance measurement: In the circuit shown in Fig. 14.43(b), the phase shift is

$$\theta = \tan^{-1}(x_L/R) = \tan^{-1}(2\pi fL/R)$$



(a) Capacitance measuring meter using phase-shift characterstics of *RC* circuit



(b) Inductance measuring meter

Fig. 14.43

Example 14.8 For measuring a small value of capacitance, a 50 MHz signal source is to be used in a capacitance meter. What value of series resistance is required if the phase shift is to remain below 5° for a full-scale capacitance reading of 1.10 and 100 pF?

Solution

$$\tan\theta = X_c/R$$

Now $\theta = 5° = 0.0875$ rad

For $\theta < 0.0875$ or $1/(2\pi fRC) < 0.0875$

$$R > 1/(0.0875 \times 2\pi fC)$$

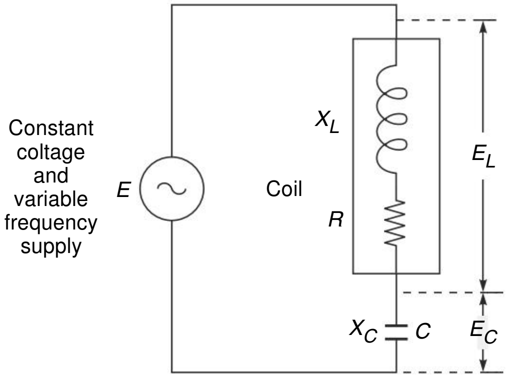


Fig. 14.44 Q-factor measurement

(a) $C = 1$ pF $\quad \therefore \quad R = 36.4$ kΩ
(b) $C = 10$ pF $\quad R = 3.64$ kΩ
(c) $C = 100$ pF $\quad R = 0.364$ kΩ

14.10 Q-METER

It is a device to measure electrical properties of coils and capacitors. The measurement of Q-factor is

$$Q = X_L/R = \text{quality factor of a coil}$$

Under resonance condition,

$$|X_L| = |X_C|$$

Hence the total impedance of the circuit is

$$Z_{\text{total}} = R$$

$$\therefore \quad I = E/Z = E/R$$

$$Q = I\,X_C/IR = E_c/E$$

If E is a constant voltage source, then

$$Q \propto E_c \tag{14.57}$$

Measurement Methods

There are three different methods for connecting unknown components to the test terminals of a Q-meter: direct, series and parallel. The method of connection is governed by the type of component and its size.

1. Direct Connection Generally coils can be connected directly across the test terminals, exactly as shown in Fig. 14.45.

The circuit is resonated by adjusting either the oscillator frequency or the then resonating capacitor. If E is constant, then

$$Q = E_c/E \Rightarrow E_c = QE \quad \text{or} \quad E_c\ \alpha\ Q$$

2. Series Connection The connection shown in Fig. 14.46 is used for low-value

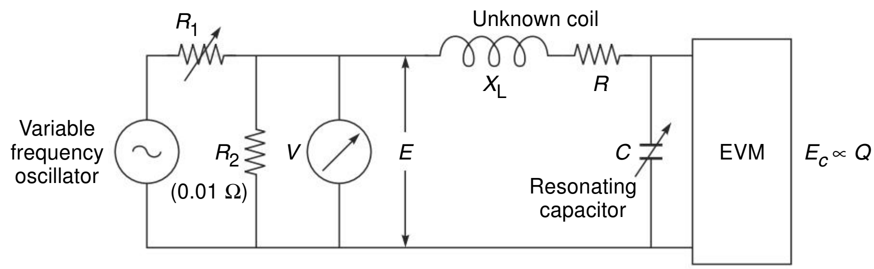


Fig. 14.45 Direct connection

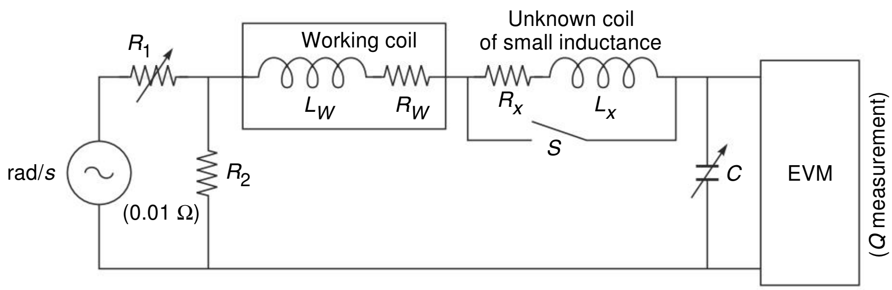


Fig. 14.46 Series connection

resistance, small coils and large capacitors. Here, the component to be measured is placed in series with a working coil across the test terminals.

Two measurements are made:

(a) Only working coil (S closed): Reference conditions are established. The value of tuning capacitance (C_1) and indicated $Q(Q_1)$ are noted.

(b) In the second measurement (S open), i.e. measurement with an unknown coil, new values of tuning capacitance (C_2) and indicated $Q(Q_2)$ are noted. Then

$$X_x = (C_1 - C_2)/(\omega C_1 C_2),\ R_x = (C_1Q_1 - C_2Q_2)/(\omega C_1 C_2 Q_1 Q_2)$$

$$Q_x = (C_1 - C_2)\, Q_1 Q_2/(C_1 Q_1 - C_2 Q_2)$$

• If the unknown is purely resistive, then

$$C_1 = C_2$$

Hence $$R_x = (Q_1 - Q_2)/(\omega C_1 Q_1 Q_2)$$

• If the unknown is a small inductor, then

$$L_x = (C_1 - C_2)/(\omega^2 C_1 C_2)$$

• If the unknown is a large capacitor, then

$$C_x = C_1 C_2/(C_2 - C_1)$$

3. Parallel connection The connection shown in Fig. 14.47 is used for high impedance components, such as high-value resistance, certain inductors and small capacitors. Two measurements as made:

(a) Before connecting the unknown impedance, the circuit is resonated to establish reference values of Q and C (Q_1 and C_1), respectively.

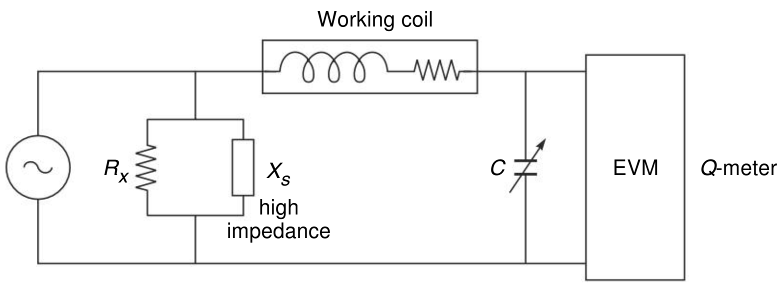


Fig. 14.47 Parallel connection

(b) Then the unknown impedance is connected, the capacitor is again adjusted for resonance and new values of $Q(Q_2)$ and $C(C_2)$ are noted. So,

$$X_x = 1/\omega(C_1 - C_2) \text{ and } R_x = Q_1\, Q_2/[\omega\, C_1\,(Q_1 - Q_2)]$$

$$\therefore \quad Q_x = (C_1 - C_2)\, Q_1\, Q_2/C_1\,(Q_1 - Q_2)$$

- If the unknown is an inductor, then

$$L_x = 1/\omega^2\,(C_1 - C_2)$$

- If the unknown is a capacitor, then

$$C_x = C_1 - C_2$$

Sources of Error

The most important factor affecting measurement accuracy is the distributed capacitance or self-capacitance of the measuring circuit as shown in Fig. 14.48.

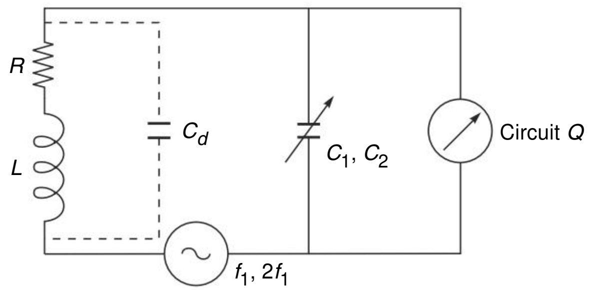


Fig. 14.48 Circuit showing distributed capacitance

The method of finding the distributed capacitance (C_d) of a coil requires two measurements at different frequencies. The coil under test is connected directly to the test terminals of the Q-meter. The tuning capacitor is set to a high value and the circuit is resonated by varying the oscillator frequency. The values of the tuning capacitor (C_1) and the oscillator frequency (f_1) are used. The frequency is then doubled ($f_2 = 2f_1$) and the circuit is then tuned by tuning the resonance capacitor (C_2). The resonant frequency of an LC circuit is given by

$$f = 1/2\pi\sqrt{LC}$$

$$\therefore \quad f_1 = 1/2\pi/\sqrt{\;}[L(C_1 + C_d)]$$

$$f_2 = 1/2\pi/[L\,(C_2 + C_d)]$$

Since $f_2 = 2f_1$, we have

$$C_d = (C_1 - 4\,C_2)/3$$

The effective Q of the coil (Q_e) with distributed capacitance is less than the true Q by a factor that depends on the value of the self-capacitance and the resonating capacitor (C).

$$Q_{\text{true}} = Q_e\,(C + C_d)\;IC \qquad (14.58)$$

Example 14.9 A coil with a resistance of 5 Ω is connected to the terminals of the Q-meter. Resonance occurs at an oscillator frequency of 8 MHz and resonating capacitance of 150 pF. Calculate the percentage error introduced by the insertion resistance of 0.1 Ω.

Solution

$$Q = 1/(\omega CR)$$

$$\therefore \quad Q_1 = 1/2\pi\,(8 \times 10^6)\,(150 \times 10^{-12}) \times 5 = 26.53\ \Omega$$

Insertion resistance = 0.1 Ω

Thus, $Q_2 = 1/\omega C\,(R + 0.1) = 26.01\ \Omega$

$$\%\text{ error} = (26.53 - 26.01) \times 100/26.53 = 2\%$$

Example 14.10 A circuit consisting of a coil, a resistance and a variable capacitor connected in series is tuned to a resonance using a Q-meter. For the frequency of 100 kHz and capacitance of 400 pF, calculate the effective inductance and resistance of the coil if the Q-meter indicates 100.

Solution

$$Q = X_c/R \quad \therefore\ R = X_c/Q = 39.8\ \Omega$$

$$\therefore \quad Q = X_c/R = X_L/R$$

So, $X_c = X_L \quad L = 1/C\omega^2 = 6.33 \times 10^{-3}$ H

14.11 FREQUENCY MEASUREMENT

1. CRO Method

(a) From the calibrated time base Display the wave and see the time interval for a cycle and calculate the frequency.

$$\text{Frequency } f = (1/T)\text{ Hz} \qquad (14.59)$$

where T is the time period in seconds.

(b) Dual trace method (Fig. 14.49) Display a known frequency wave on one channel to an unknown wave on the second channel. Count the number of cycles of both the waves.

$$f_{\text{unknown}} = (n/N)\,f_{\text{known}} \qquad (14.60)$$

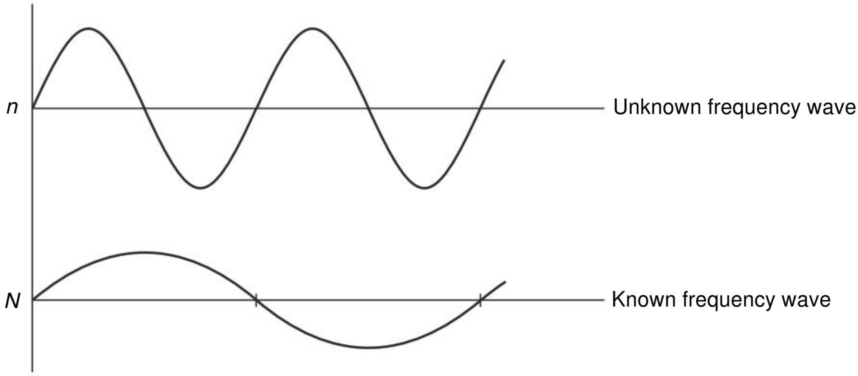


Fig. 14.49 Dual trace method

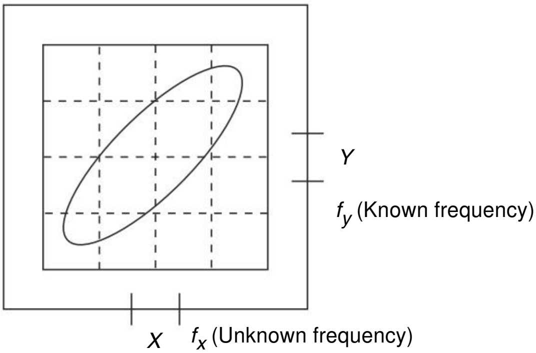


Fig. 14.50 Lissajous pattern method

(c) Lissajous Pattern Method An unknown frequency wave is applied to the X-plates and a known frequency wave is applied to the Y-plates of the CRO of Fig. 14.50. Adjust the known frequency wave until an elliptical loop appears on the screen as shown in Fig. 14.50. At this point the two frequencies will be equal.

If f_x and f_y are equal, then the lissajous pattern could take one of the three shapes depending on the phase difference between the two signals as shown in Fig. 14.51.

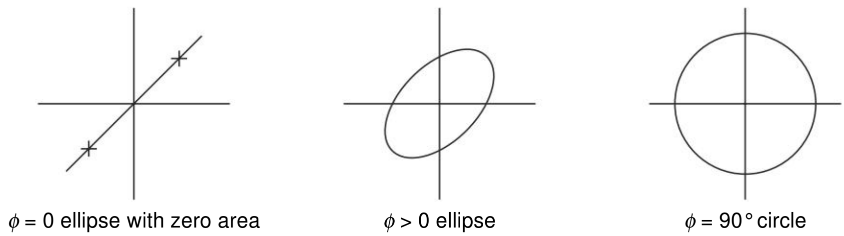


Fig. 14.51

(d) Z modulation method This method is specially useful for high frequency measurement. It could be used up to a known to unknown frequency ratio of 1:50.

2. Wein Bridge Frequency Meter (Fig. 14.52) Here resistance in arms *AB* and *BC* are equal and gauged together.

An unknown signal is applied to excite the circuit. Then for a balanced bridge condition

$$\omega = 1/RC$$

$$\therefore \quad f = \frac{1}{2\pi RC}$$

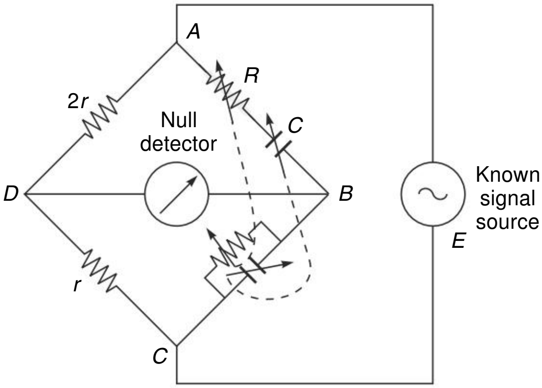


Fig. 14.52 Wein bridge frequency meter

4.12 PHASE MEASUREMENT

CRO Method The two signals for which the phase difference is to be measured are applied to the X-plates and Y-plates to obtain a lissajous pattern, as shown in Fig. 14.53. Then

$$\sin\phi = B/A = D/C$$

$\therefore$ Phase difference $\phi = \sin^{-1}(B/A) = \sin^{-1}(D/C)$

Direct reading Analogue Phase Meter (Fig. 14.54)

$V_{av} = E_0\, t/T = E_0\, \phi°/360°$

Since E_0 and 360° are constant, $V_{av} \propto \phi°$

A multivibrator is being set at the edge of signal V_1 and get, reset at the edge of signae V_2, as shown in Fig. 14.55. Reading of the PMMC meter is calibrated in terms of the phase difference angle.

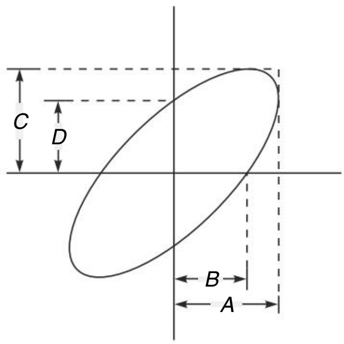


Fig. 14.53 CRO method

14.13 DIGITAL INSTRUMENTS

Analogue instruments display the quantity to be measured in terms of the deflection of a pointer. Digital instruments indicate the value of the measured in the form of a decimal number. The digital meters work on the principle of quantization.

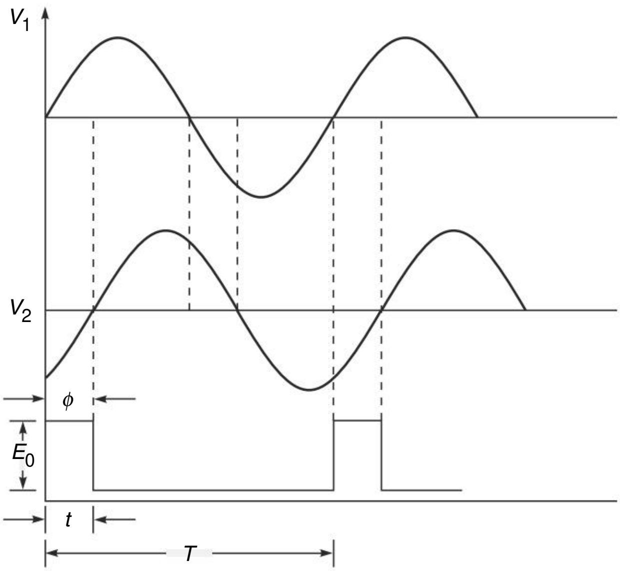


Fig. 14.54 Direct reading analogue phase meter

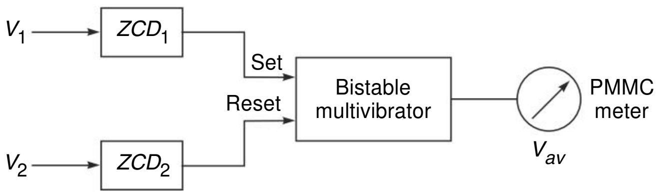


Fig. 14.55 Block diagram

The advantages of digital instruments are as follows:

1. The readings are indicated directly in decimal numbers and therefore errors on account of human factors, such as errors due to parallax and approximation, are eliminated.
2. The readings may be carried to any significant figure by merely positioning the decimal point, i.e there is higher accuracy.
3. As compared to analog meters, digital instruments have got a very high resolution.
4. Since output is in digital form, it may be directly fed into memory devices like tape recorders, printers, and digital computers etc, for storage and future computations.

Resolution in Digital Meters

The number of digits used in a digital meter determines the resolution. Thus a 3-digit display voltmeter (DVM) for a 0–1 V range will be able to indicate values from zero to 999 mV, with the smallest increment or resolution of 1 mV.

Half Digit In practice a fourth digit, usually capable of indicating 0 or 1 only is placed to the left of active digits. This permits going above 999–1999 to give an overlap between ranges for convenience. This is called 'over ranging'. This type of display is known as a 3 half digit.

The resolution of a digital meter, however, is determined by the number of active or full digits used. If n is the number of full digits, then resolution is $1/10^n$.

For an 8-digit display, the resolution is 1 in 10^8, while for analog meters in general it is only 1 in 500.

Sensitivity of Digital Meters

Sensitivity is defined as the smallest change in the input which a digital meter is able to detect.

$$\text{Sensitivity } S = (fs)_m \times R \tag{14.61}$$

where $(fs)_m$ is the lowest full-scale value of meter and, R is the resolution expressed as decimal.

Example 14.11 A 4½ digit voltmeter is used for voltage measurement.

(a) Find its resolution.

(b) How would 14.76 V be displayed on the 10 V range?

(c) How would 0.5434 be displayed on the 1 V range?

(d) How would 0.5434 be displayed on the 10 V range?

Solution

(a) Resolution = $1/10^4$ = 0.0001 or 0.01% V

(b) There are 5 digit places in a 4 digit display

∴ 14.76 V would be displayed as 14.760 V on its 10 V scale.

(c) Resolution on the 1 V range = 1 × 0.0001 = 0.0001 V

∴ on the 1 V range, any reading can be shown to the fourth decimal place.

Thus, 0.5434 V would be displayed as 0.5434 V on the 1 V range.

(d) Resolution on the 10 V range = 10 × 0.0001 = 0.001 V

Hence on a 10 V range readings can be displayed only upto the third decimal place.

∴ 0.5434 V will be shown as 00.543 V on a 10 V range. The digit 4 in the fourth decimal place will be lost. However, by employing a suitable range, e.g. 1 V, digit 4 can be retained.

Digital Voltmeters

The schematic block diagram of a DVM is shown in Fig. 14.56.

Analog to Digital Converter (ADC)

This is the most critical block of a DVM. It decides the accuracy, resolution etc.

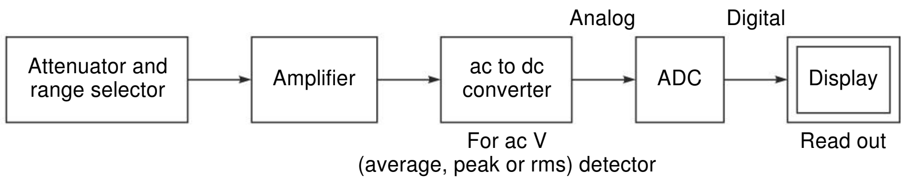
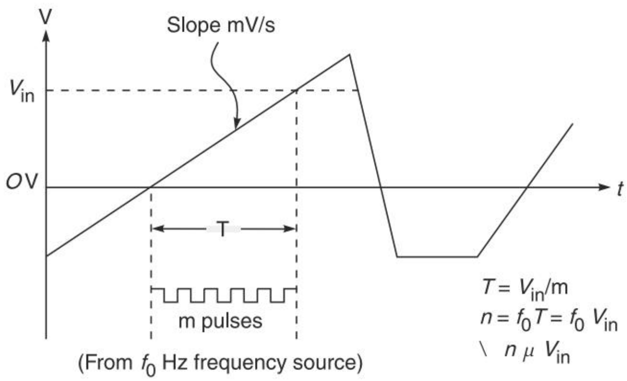


Fig. 14.56 A digital voltmeter

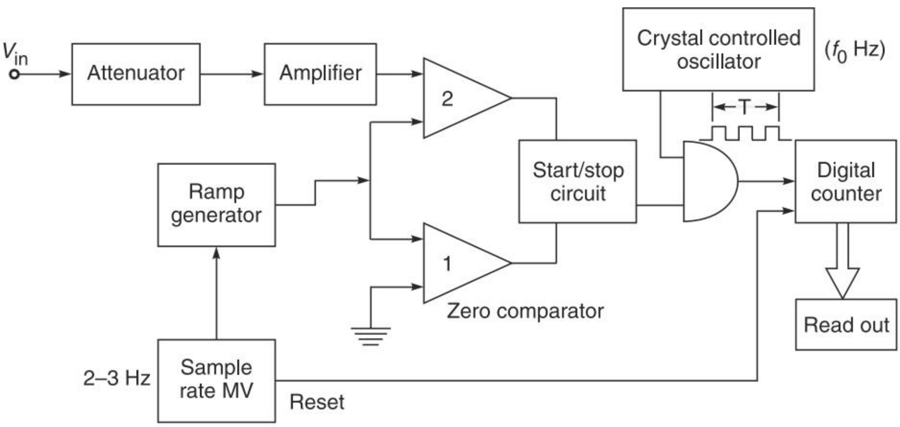
The types of DVM (according to the ADC principle) are as follows:

1. Ramp Type DVM: The principle behind ramp-type DVM is based on the measurement of the time it takes for a linear ramp voltage to rise from zero to the level of the input voltage, or to decrease from the level of the input voltage to zero.

Conversion from a voltage to a time interval is illustrated by the waveform diagram of Fig. 14.57 (a). At the start of the measurement cycle a ramp voltage is initiated, which is continuously compared with the unknown input voltage. At the instant ramp voltage is zero, comparator 1 generates a pulse; another pulse is generated by comparator 2, when continuously increasing ramp voltage equals the unknown voltage. The start/stop circuit gives a pulse of width T using the two pulses.



(a) Waveforms for ramp type DVM



(b) Block diagram of a ramp type DVM

Fig. 14.57

The output of oscillator (f_0 Hz) is ANDED with the pulse of width T. Gate output goes to number of decade counting units (DCUs), which totalize the number of pulses passed through GATE. The decimal number displayed by the readout is the input voltage.

2. Dual slope type DVM: We change the capacitor for fixed time T_0 by V_{in}; then, it is discharged to fixed voltage V_{ref} and discharge time T is measured. (see Fig. 14.58).

Charging: $nV_p = -(1/RC) \int V_{in}\, dt$

where

V_p = voltage across the capacitor after charging for time T_0

R = charging resistance (in series with the capacitor)

Discharging: $V_p = -(1/RC) \int V_{ref}\, dt$

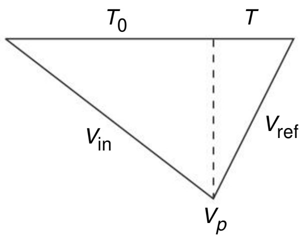


Fig. 14.58

$$V_p = -(1/RC) \int V_{in}\, dt = -(1/RC) \int V_{ref}\, dt$$

$$T = V_{in}\, T_0/V_{ref} \quad \text{or} \quad T \propto V_{in}$$

Both T_0 and T times are measured by pulses on the same frequency (f_0) with the same counter.

T_0: count by number of pulses

T: N count: pulses passed discharging

n: $V_{in}\, N_0/V_{ref}$ or $n \propto V_{in}$

Figure 14.59 shows a complete dual slope A/D converter. Electronic switches, usually FET switches, are used to switch the input of the integrator alternatively between the reference voltage and the unknown. Another pair of switches apply the integrator output to the automatic zero capacitor and ground the input for the automatic zero function. The switch timing and the counting of the clock pulses to determine the unknown voltage are under control of the control logic. The output is made available to the external electronics after the conversion is complete.

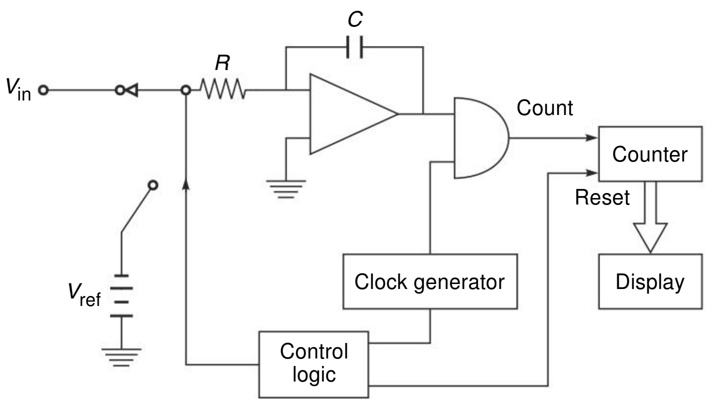


Fig. 14.59

Example 14.12 A dual slope integrating type of A/D converter has an integrating capacitor of 0.22 μF and a resistance of 100 kΩ. If the reference voltage is 5 V and the output of the integrator is to remain below 10V, find the maximum time the reference voltage can be integrated.

Solution Let T be the maximum time for which the reference voltage can be integrated. Then, the voltage across the capacitor after charging for time T is

$$V_p = (1/RC) \int V_{\text{ref}}\, dt$$

$$V_p = 10 \text{ V},\ V_{\text{ref}} = 5 \text{ V}$$

$$\therefore \quad 10 = (1/10^5 \times 0.22 \times 10^{-6}) \int 5\, dt$$

Thus, $$T = 44 \text{ ms}$$

3. Successive Approximation Type DVM: This is the fastest as compared to any other type of DVM. It is an electronic implementation of a technique called *binary regression*. This converter compares the analogue input to a DAC reference voltage which is repeatedly divided in half. The process is shown in Fig. 14.60, where a 3-digit binary number, representing the full voltage E_r, is divided in half (binary number 100), to the corresponding voltage $E_r/2$. A comparison of this reference voltage $E_r/2$ and the analogue voltage is made. If the result of this comparison shows that this first approximation is too small (i.e. $E_r/2$ is smaller than the analogue input),

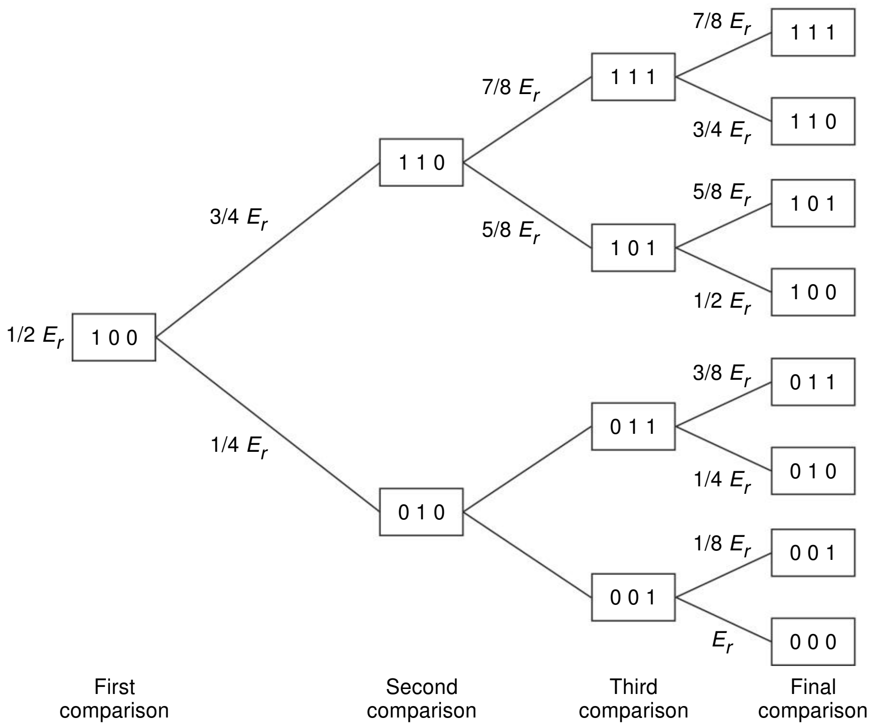


Fig. 14.60 Operation of successive approximation A/D converter

then the next comparison is made against $E_r/4$ (binary number 010). After four successive approximations the digital number is resolved.

At the start of the conversion cycle, both the control resistor and the distribution register are set with a 1 in MSB and 0 in all bits of less significance. Thus, the distribution registor shows 1000 and this causes the output voltage at the D/A convertor section to be one half of reference supply. At the same time, a pulse enters the time delay circuitry. By the time the D/A converter and the comparator have settled, this delayed pulse is gated with the comparator output.

When the next MSB is set in control register by the action of the timing circuit, the MSB remains in state 1 or it is reset to 0, depending on the comparator output. The single 1 in the distribution registor is shifted to the next position and keeps track of the comparison mode.

The procedure repeats itself (see Fig. 14.61) until the final approximation has been made and the distribution register indicates the end of the conversion.

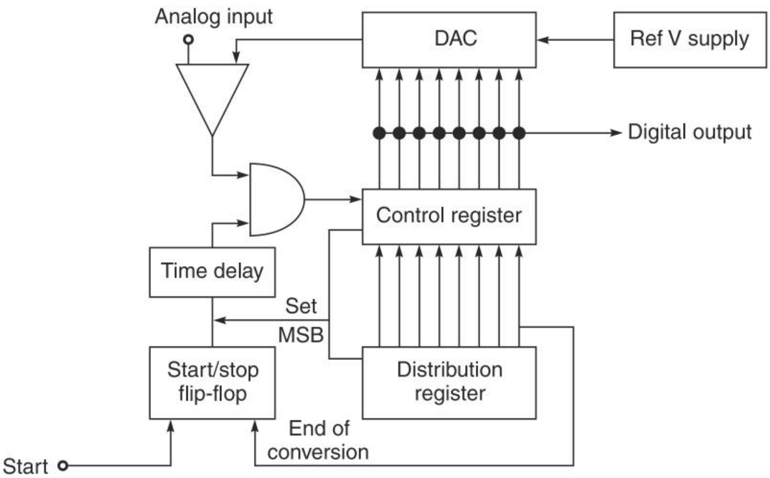


Fig. 14.61 Block diagram of the successive approximation A/D converter

Example 14.13 Find the successive approximation A/D output for a 4-bit converter to a 3.217 V input if the reference is 5 V.

Solution For a 4-bit converter only 4 digits can be shown. Input 3.217 V is approximately equal to 3.25 V. The output can be shown in tabular form as follows:

Pulse number	Value represented	Output (binary number)
1	2.5	1
2	1.25	0
3	0.625	1
4	0.3125	0

$\therefore$ 4 bit representation is 1010.

Digital Frequency Meter and Timer

This multifunction instrument can perform the following functions:

1. Pulse Counting or Totalizing As the name implies, totalizers count and provide a readout of the total number of pulses received by the DCA with no specific gate time used, as shown in Fig. 14.62.

2. Time Interval Measurement (Fig. 14.63) This measurement is very useful in determining the pulse width of a certain waveform.

Select the clock frequency and select the unit of the time interval measurement (microsec, millisec or sec) depending upon the range selector position, which decides the clock frequency. The gate control circuit enables the gate for the time interval to be measured. The display gives the time interval in the sealed unit.

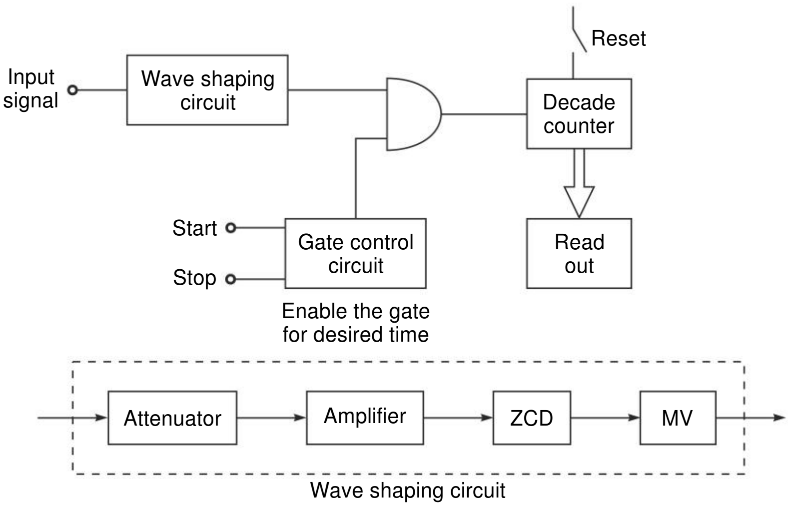


Fig. 14.62 Pulse counting

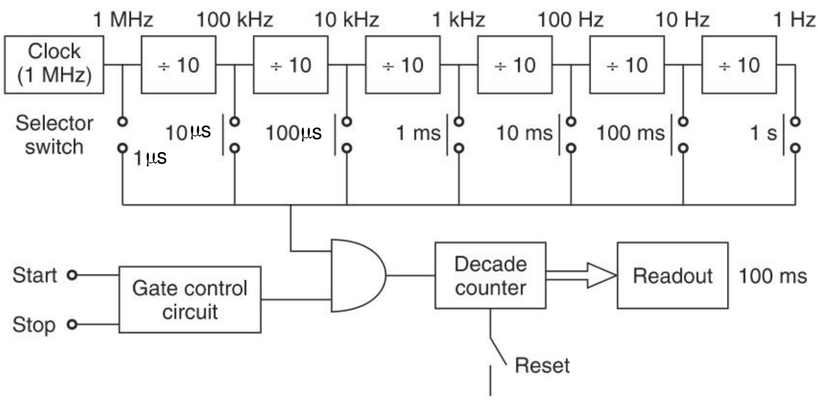


Fig. 14.63 Time interval measurement

3. Frequency Measurement (Fig. 14.64) The gate for the predecided time and a number of pulses are passed, counted by a decade counter. The readout displays the frequency (scale calibrated used depends upon the selector switch position).

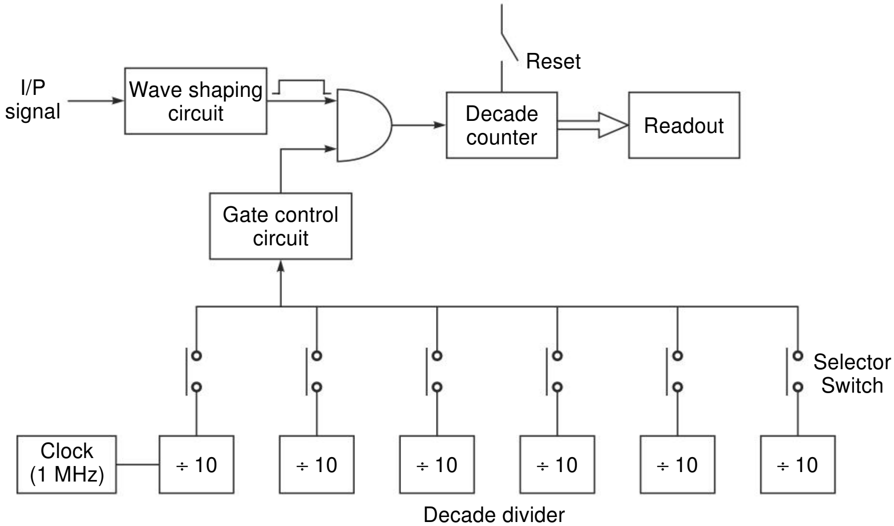


Fig. 14.64 Frequency measurement

4. Time Period Measurement (Fig. 14.65) The gate is enabled only for one time period of the signal by the gate control circuit. The number of pulses passed gives the measure of the signal's time period.

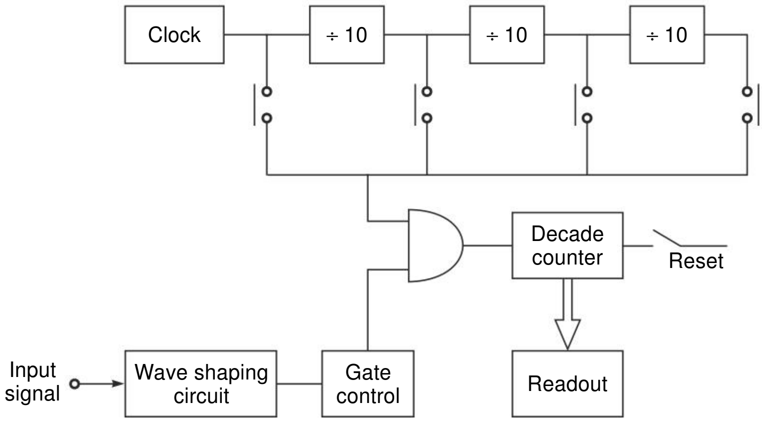


Fig. 14.65 Time period measurement

5. Frequency Ratio Measurement (Fig. 14.66) The gate is enabled for one or more number of time periods of low frequency signal and we count the number of pulses passed of high frequency signal within this interval. The pulses passed provide the measure of the frequency ratio (f_1/f_2).

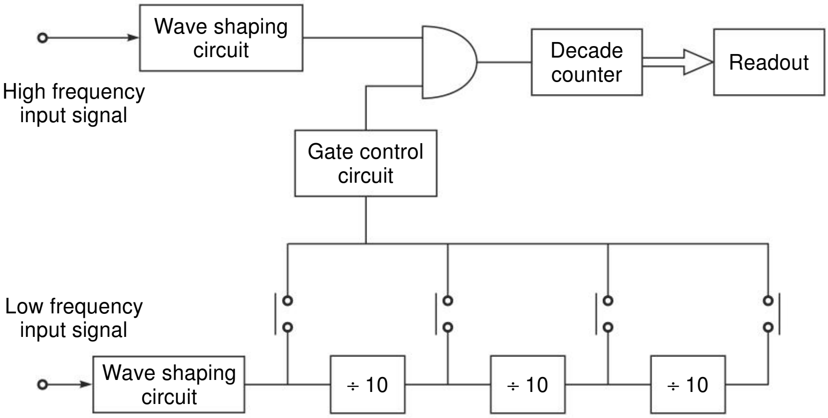


Fig. 14.66 Frequency ratio measurement

Example 14.14 A digital timer with 8-digit readout is said to have an accuracy of 0.005% of reading ± 1 in the final digit. The readout is in seconds, milliseconds and microseconds. Assuming that the instrument meets its specification, what are the maximum likely errors when the reading is

(a) 05 00 0000 μs

(b) 00 00 0500 s

What is the maximum nominal accuracy in time units with which reading (b) could be made?

Solution

(a) The reading is 05 000 000 μs or 5 s.

$$0.005\% \text{ of reading } = 250\ \mu s$$

Error due to final digit= $\pm$ 1 μs = $\pm$ 251 μs

(b) The reading is 0 000 500 s

$$0.005\% \text{ of reading } = 0.025 s$$

Error due to final digit= $\pm$ 1s

$\therefore$ Maximum error = $\pm$ 0.025 s $\pm$ 1s = $\pm$ 1.025 s

14.14 TRANSDUCERS

An instrumentation system consists of a number of components used to perform a measurement and record the result. Such a system generally consists of three main components: an input device, a signal conditioning or processing device, and an output device. The input device receives the quantity under measurement and delivers a proportional electrical signal to the signal-conditioning device. Here the signal is amplified, filtered or suitably changed to a format acceptable to the output device. The output device may be a simple indicating meter, an oscilloscope, or a chart recorder for visual display. A magnetic tape recorder may be used for storing the input data or a computer may be employed for data manipulation or process control.

The input for most instrumentation systems is nonelectrical. Normally this is converted into an electrical signal by a device called a *transducer*.

The mechanical transducers are distinguished from electrical transducers since their output signals are also mechanical by nature. Electrical transducers respond to nonelectrical quantities but develop output signals which are electrical by nature. Electrical transducers may be classified as passive or active type. *Passive transducers* are those that need to be excited by electrical supply in order to recognize their response to the measurand. Electrical circuit elements constitute the group of passive transducers. The strain gauge is another example of a passive transducer.

Active transducers are those which function as energy converters. A thermocouple is an active transducer. If itself acts as a source of emf. A closed-loop system can act as *feedback transducers*. These are described in detail in Ref [IV 6].

14.15 OSCILLOSCOPE

The cathode ray oscilloscope (CRO) is probably the most versatile tool for the development of electronic circuits and systems. The CRO allows the amplitude of electrical signals (e.g. V, I or P) to be displayed as a function of time. The CRO depends on the movement of an electron beam, which is bombarded (impinged) on a screen coated with a fluorescent material, to produce a visible spot. If the electron beam is deflected on both the conventional axes (X and Y axes), a two-dimensional display is produced. Typically, the X-axis of the oscilloscope is deflected at a constant rate, relative to time, and the vertical or Y-axis is deflected in response to an input stimulus such as voltage. This produces the time-dependent variation of the input voltage, which is very important to the design and development of electronic circuits.

The oscilloscope is basically an electron beam voltmeter. The electron beam follows rapid variations in signal voltage and traces a visible path on the CRT (Cathode Ray Tube) screen which is the heart of the oscilloscope. Thus, rapid variations, pulsations or transients are reproduced and the analyst can observe the waveform as well as measure amplitude at any instant of time.

The oscilloscope can reproduce HF waves which are too fast for electromechanical devices to follow. Thus it is a kind of recorder which used an electron beam instead of a pen. The oscilloscope is capable of displaying events that take place over periods of microseconds and nanoseconds.

A storage CRT can retain the display much longer, upto several hours after the image was first written on the phosphor. The retention feature will be useful while displaying the waveform of a very low frequency signal. A better method of trace storage is the digital storage oscilloscope. In this technique, the waveform to be stored is digitized, stored in a digital memory and retrieved for display on the storage oscilloscope. One very important feature of a digital storage oscilloscope is its ability to provide 'pretrigger view'. This means that the oscilloscope can display what happened before a trigger input is applied. This is useful when a failure takes place. To find the reason of the failure, it would be necessary to see different waveforms before the failure. The reader may refer to references [IV 1,3] for a detailed account on *Oscilloscopes*.

By a combining a special fibre optic CRT with an oscillograph type paper drive (which panes the paper over the CRT face where it is exposed by light from the CRT phosphor), a recording oscilloscope with useful characteristics is obtained.

14.16 SIGNAL GENERATION, SIGNAL ANALYSIS AND FIBRE OPTICS MEASUREMENTS

Because of the importance of the sine function, the sinewave generator represents the largest single category of signal generators. This instrument covers the frequency range from a few hertz to many gigahertz. The simple sine-wave generator consists of two basic blocks, an oscillator and an attenuator. The frequency accuracy and stability and freedom from distortion depend on the design of the oscillator, while the amplitude accuracy depends on the design of the attenuator.

An oscillator is one of the most basic and useful instrument among the electrical and electronic measuring instruments. Oscillators provide a convenient source of power or test voltage for all practical measurements. They generate a sine-wave signal of known frequency and amplitude. The Wien bridge oscillator is one of the most popular types of generators used in the audio and subaudio frequency ranges because of its simplicity, low distortion, good amplitude stability and the relative case of frequency variation.

It can be shown mathematically that any complex waveform consists of a fundamental and its harmonics. It is often desired to measure the amplitude of each harmonic or fundamental individually. This can be performed by instruments called *wave analyzers*. This is the simplest form of analysis in the frequency domain, and can be performed with a set of tuned filters and a voltmeter. Wave analyzers are also called frequency selective voltmers or carrier frequency voltmeters. The instrument is tuned to the frequency of one component whose amplitude is measured. Harmonic distortion analyzers measure the total harmonic content in the waveforms.

The introduction of fibre optics into the main stream of communications electronics has given rise to a new meaning to measurements. Fibre optics communications use light energy not visible to the human eye in the infrared region of the spectrum. The light source used is either a light-emitting diode (LED) or a laser diode. Ref [IV 1] describes a typical fibre optic power meter.

14.17 DATA ACQUISITION SYSTEMS

A typical data acquisition system consists of individual sensors with the necessary signal conditioning, data conversion, data processing, multiplexing, data handling and associated transmission, storage and display systems.

Data may be transmitted over long distances or short distances. The data may be displayed on a digital panel or on a CRT. The same may be stored temporarily (for immediate use) or permanently for future reference. Data acquisition normally relates to the process of collecting the input data in digital form as rapidly, accurately and economically as required. The basic instrumentation used may be a digital panel meter (DPM) with digital outputs, a shift digitiser or a modern high speed resolution device. To match the input requirement of the converter with the output available from the sensor, some sort of scaling and off-selting is necessary and is performed with an amplifier/attenuator. For converting analog information from more than one source, either additional converters or a multiplexer may be required; to increase the speed with which information is to be accurately converted, a sample and hold circuit may be required.

A setematic block diagram of a general Data Acquisition System (DAS) is given in Fig. 14.67.

Data acquisition systems are used in a large and even increasing number of applications in a variety of industrial and scientific areas, such as power systems, biomedical, aerospace and telemetry industries.

The type of DAS, whether analog or digital, depends mainly on the intended use of the recorded input data. Normally, analog data systems are used when wide bandwidth is required and high accuracy is not required. Digital systems are employed when the physical process being monitored is slowly varying (narrow bandwidth) and high accuracy and low per channel cost in desired.

The advent of real-time computation and control by means of digital computers is primarily responsible for the development of telemetry systems. Telemetry (telemetering) is useful in presentation of measured values at a location remote from the site of measurement. The subject of telemetry plays vital role in the centralized control and supervision of power generation and distribution systems.

The important factors that decide the configuration and subsystems of the DAS are given below.

(i) Accuracy and resolution

(ii) Number of channels to be monitored

(iii) Snalog or digital signal

(iv) Single/multichannel

(v) Sampling rate per channel

(vi) Signal conditioning requirements of each channel

(viii) Cost

DAS must be able to collect, summarise and store data for diagonosis of operation and record purpose. It must be able to compute unit performance indices using on-line, real-time data.

ADDITIONAL SOLVED PROBLEMS

14.15 A series type ohm-meter is designed to operate with a 6.0 V battery with a circuit diagram as shown in Fig. 14.68. The meter movement has an internal resistance of 2 kΩ and requires a current of 100 μA for full-scale deflection. The value of R_1 = 49 kΩ.

(a) Assuming the battery voltage has fallen to 5.9 V, calculate the value of R_2 required

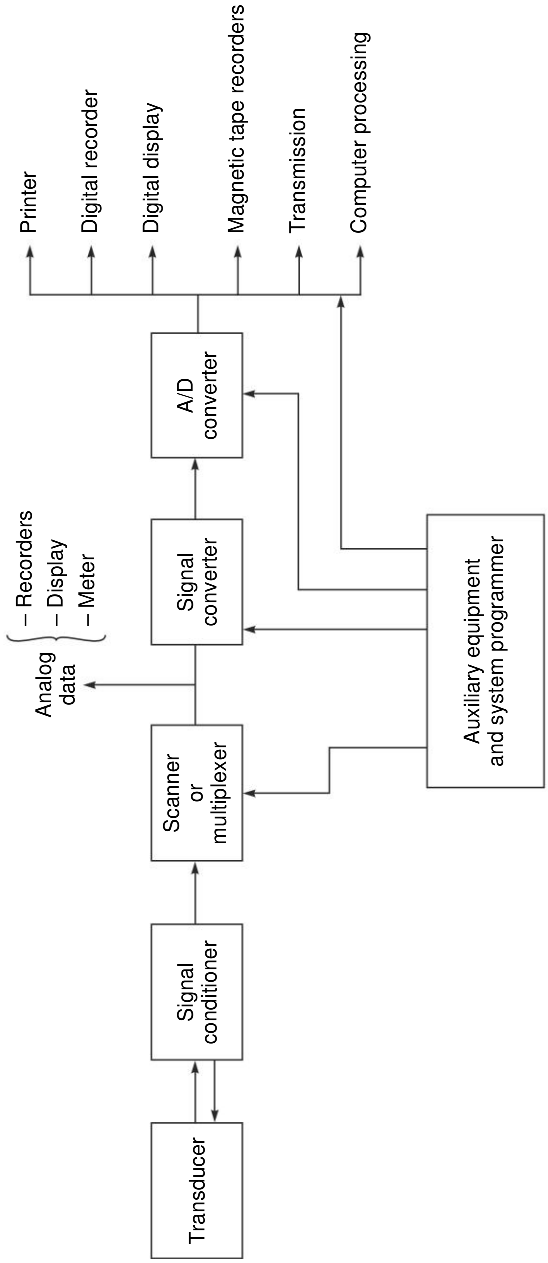


Fig. 14.67 A data-acquisition system

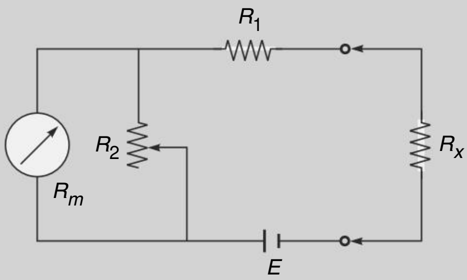


Fig. 14.68

to obtain zero reading in the meter.

(b) Under the condition mentioned in part (a), an unknown resistor is connected to the meter causing 60% meter deflection. Calculate the value of the unknown resistance.

Solution

(a) $R_x = 0$ for zero meter reading, $R_m = 2\ \text{k}\Omega$, $R_1 = 49\ \text{k}\Omega$,

$I_{\text{fsd}} = 100\ \mu\text{A}$.

Voltage across movement = 2000 × 100 μA = 0.2 V

Then $IR_1 + 0.2 = 5.9$

$I = 116.32\ \mu\text{A}$

$I_{\text{sh}} = 116.32\ \mu\text{A} - I_{\text{fsd}} = 16.32\ \mu\text{A}$

$R_2 = 0.2\ \text{V}/16.32\ \mu\text{A} = 12.25\ \text{k}\Omega$

(b) for 60% deflection

$I = 60 \times 116.32\ \mu\text{A}/100 = 69.8\ \mu\text{A}$

R_{eq} (for meter) $= 5.9/69.8\ \mu\text{A} = 84.53\ \text{k}\Omega$

$R_x = 84{,}530 - 49000 - 2000 \times 12.25/14.25$

$= 33.81\ \text{k}\Omega$

14.16 The resistnace of the pressure coil branch of the wattmeter W in the circuit of Fig. 14.69 is $R_p\ \Omega$. In position 2 of the switch an inductive reactance of $R_p\ \Omega$ is connected in series with the pressure coil branch. If the reading of the wattmeter in switch positions 1 and 2 are W_1 and W_2 respectively, determine the reactive power taken by the load in terms of W_1 and W_2. Neglect current coil impedance and pressure coil reactance.

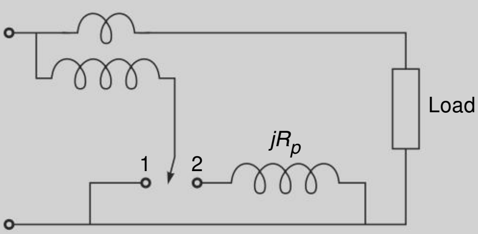


Fig. 14.69

Solution

The wattmeter reading at position 1 is

$$W_1 = (VI_c/k\,R_P) \cos\phi \text{ d} M/\text{d}\theta \quad \text{(i)}$$

The wattmeter reading at position 2 is

$$W_2 = (VI_c/kR_p) \cos(\phi - \alpha) \cos\alpha \frac{dM}{d\theta} \quad \text{(ii)}$$

$$\text{where } \alpha = \tan^{-1}(\omega L/R_p)$$

$$\omega L = R_p, \text{ reactance in series with pressure coil}$$

$$\alpha = 45°$$

Dividing Eq. (ii) by Eq. (i), we get

$$W_2/W_1 = \cos 45° \cos(\phi - 45°)/\cos\phi$$

$$= (1 + \tan\phi)/2$$

$$\therefore \quad \tan\phi = (2\,W_2 - W_1)/W_1$$

Now reactive power $(P_r) = W_1 \sin\phi$

Thus, $P_r = W_1 (2W_2 - W_1)/\sqrt{4W_2^2 + 2W_1^2 - 4W_1W_2}$

14.17 Hay's ac bridge is used to measure the effective resistance and self-inductance of an iron-cored coil. Its four arms are arranged as follows:

Arm AB:Unknown coil (L_x, R_x)

Arm BC:Nonreactive resistor (1 kΩ)

Arm CD:Nonreactive resistor (833 Ω) in series with a standard capacitor of 0.38 μF,

Arm DA:Nonreactive resistor (16.8 kΩ)

Calculate the value of effective resistance and self-inductance. The bridge is supplied from a 50 Hz frequency source.

Solution

From Fig. 14.70, we see that

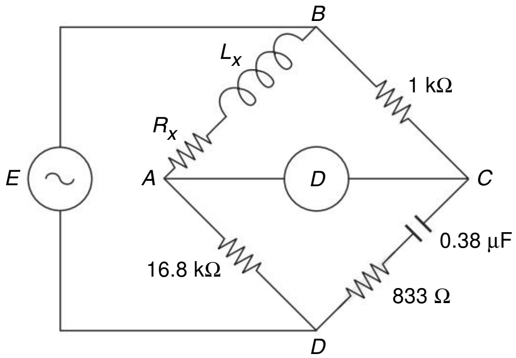


Fig. 14.70

$$Z_1 = Z_x = R_x + j\omega L_x$$

$$Z_2 = 1000\ \Omega$$

$$Z_3 = 16800\ \Omega$$

$$Z_4 = 833 - j/(0.38\,\omega) \times 10^{-6}$$
$$Z_1 Z_4 = Z_3 Z_2$$
$$(R_x + j\omega L_x)\,[833 - j/(0.38\omega)\,10^{-6}] = 1.68 \times 10^7$$
$$R_x + j\omega L_x = 1995.75 \angle 84.32°$$
$$= 197.52 + j\,1985.95$$
$$R_x = 197.52\ \Omega;\ L_x = 6.32\ \text{H}$$

14.18 Evaluate the readings of the following instruments under the stated conditions. Assume that the instruments have been correctly calibrated and the readings use below the full-scale mark in each case.

(a) A dc voltmeter of very high resistance connected as shown in Fig. 14.72(a) containing a sinusoidal source, an ideal diode and ideal capacitor.

(b) A conventional PMMC ammeter connected to a 10 V, 50 Hz sinusoidal voltage source in series with a pure resistance of 5 Ω and an ideal diode.

Solution

(a) First the capacitor will be charged to a maximum value of supply V_0. Then the capacitor will act as an open switch. Also, the capacitor cannot discharge due to the diode. Therefore voltmeter will read V_0 only.

(b) The PMMC ammeter will read the average value of the current in the circuit.

The circuit is shown in Fig. 14.71(b). Here the circuit acts as a half-rectifier for the ac voltage source. Hence,

$$I_{av} = 1T \int 2 \sin \omega t\, dt$$
$$= \omega/2\pi \int 2 \sin \omega t\, dt$$
$$I_{av} = 2/\pi$$

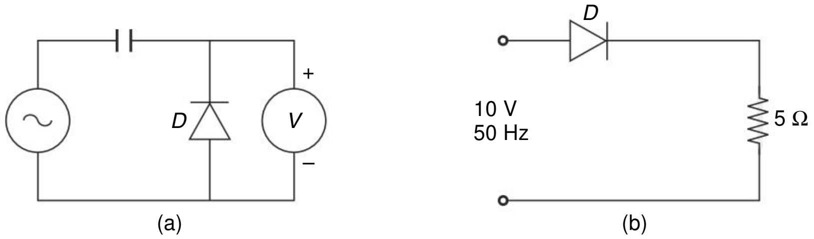


Fig. 4.71

SUMMARY

- Moving Iron (MI) Instrument is used to measure both ac and dc, but it gives rms value of ac and average value of dc.
- The two types of MI meters are

  1 Attraction type 2. Repulsion type.

- Megger is an insulation testing instrument. It is used to measure very high resistance of the order of mega ohms.
- Instrument transformers are used in ac systems for the measurement of current, voltage, power and energy.
  Types : Current transformer, Potential transformer.
- Wheatstone bridge is used to measure resistance.
  Maxwell bridge is used to measure inductance.
  Hay bridge is used to measure high inductance.
  Schering bridge is used to measure high inductance.
  Oven's bridge is used to measure inductance.
  Anderson's bridge is used to measure inductance
  Wien's bridge is used to measure frequency.
- The cause of error in measuring instrument is distributed capacitance and self-capacitance.
- Sensitivity of digital meters:
  It is defined as the smallest change in the input which a digital meter is able to detect.
  Sensitivity $S = (Fs)\ m \times R$
  $(Fs)\ m \longrightarrow$ lowest fuel scale value of meter.
  R - resolution expressed in decimal.
- Electrical transducer is a device which converts non-electrical input into electrical output.

REVIEW QUESTION

1. What are the basic elements of a measuring instrument?
2. How can we get true values from measurements?
3. Why is the controlling torque needed in a measuring instrument?
4. How can the range of ammeter / voltmeter be extended?
5. How can a electro dynamo meter be converted into an ammeter/voltmeter?
6. Can we measure the 3Φ power with single watt meter and how?
7. In three-phase measurement using two watt meter method, if the power factor is zero, what are the two watt readings?
8. Thermocouple instrument measure average value or RMs value? Why?
9. How can a instrument transformers extend the range of the ac measuring instrument?
10. Which will give accurate reading, bridge or meter measurement?

PROBLEMS

14.1 A 0-150 V voltmeter has a guaranteed accuracy of 1% full-scale reading the voltage measured by this instrument is 85 V. Calculate the limiting error in percentage.

14.2 The coil of a moving coil galvanometer has 250 turns and a resistance of 150 Ω. The coil dimensions are 2 cm × 2.5 cm. The strength of uniform magnetic field in 0.12 Wb/m^2. The inertia constant of the moving system in 1.6×10^{-7} kg m^2 and the control torque constant is 2.4 × 10– 6 Nm per radian. Assuming that the damping in entirely electromagnetic, determine the value of the resistance to be connected across the galvanometer terminals to obtain critical damping of the moving system.

14.3 For the instrument mentioned in Example 14.2, find the values of shunt resistances to convert the interament into a multirange ammeter reading up to 10 A, 15 A.

14.4 The power input to a 3-phase balanced load is measured by the two-wattmeter method. The ranges and accuracies of the instruments used and the readings obtained are tabulated as follows:

Wattmeter	W_1	W_2
Full scale range	1500 W	1500 W
Max error as a % of full scale	0.5%	0.5%
Reading obtained	– 500 W	1000 W

Estimate the maximum uncertainly in the load pF computed from the two wattmeter readings.

14.5 An ac bridge has the following constants: arm *AB*, R = 1 kΩ in parallel with C = 0.16 μF; arm *DA*, R = 1 kΩ arm *CD*, R = 0.5 kΩ, arm *BC*, C = 0.65 μF in series with an unknown resistance (R_x).

Find the frequency for which this bridge is in balance and determine the value of the resistance in arm BC to achieve this balance.

14.6 A Schering bridge as shown in Fig. 14.72 balances under the following condition: R_1 = 1 kΩ, C_1 = 100 pF and C_3 = 500 pF. The driving frequency is 1 kHz. Find the values of R_x, C_x and dissipation factor D. Also convert these values to parallel equivalent values.

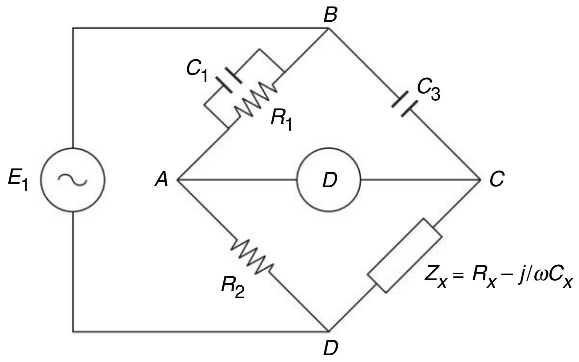


Fig. 14.72

14.7 Figure 14.73 shows a bridge for measuring the resistance and inductance of a choke.

(a) Write down the condition for bridge balance and obtain expressions for *R* and *L*.

(b) If the resistances R_1, R_2 and R_3 have a variance of ± 0.2% and C a variation of ± 0.1% from their nominal values, estimate the percent error in the evaluation of *R* and *L*.

14.8 A 30 mA full-scale current meter with an internal resistance of 150 Ω is available for constructing an AC voltmeter with a voltage range of 200 V_{rms}. Using four diodes in bridge arrangement, where each diode has a forward resistance of 300 Ω and infinite

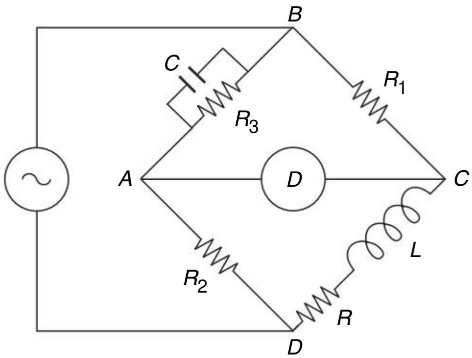


Fig. 14.73

reverse resistance, calculate the necessary series limiting resistance for the 200 V_{rms} range.

14.9 Figure 14.74 displays the lissajous patterns for cases where voltage of the same frequency out of different phases are connected to the the *Y* and *X* plates of the oscilloscope. Find the phase difference in each case. The spot generating the patterns moves in a clockwise direction.

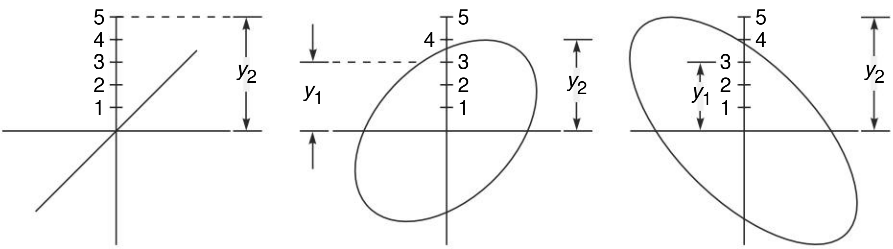


Fig. 14.74

14.10 Repeat Example 14.7 if the voltage applied to the meter consists of a sawtooth waveform with a peak value of 200 V and a period of 4 s as shown in Fig. 14.75.

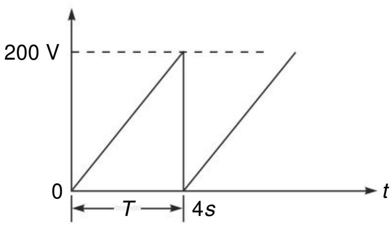


Fig. 14.75 Sawtooth waveform

14.11 What would a true rms reading meter indicate if a pulse waveform of 10 V peak and a 25% duty cycle were applied? What would the meter indicate if 10 V dc input was applied?

14.12 Meter '*M*' in Fig. 14.76 is a rectifier type 200 V full-scale voltmeter having a sensitivity of 10 kΩ/V. What will be the reading in the meter if the source voltage V_s is a symmetrical square wave of 800 V peak-to-peak?

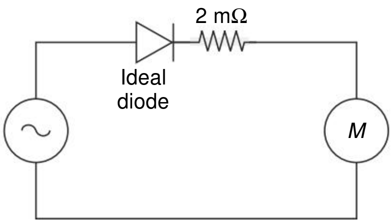


Fig. 14.76

14.13 The self-capacitance of the coil is to be measured by a Q-meter. The circuit is set into resonance at 2 MHz and the tuning capacitor is set at 450 pF. The frequency is now adjusted to 4 MHz and resonance conditions are obtained by tuning capacitor at 100 pF. Calculate the value of distributed capacitance of the coil.

14.14 Compute the value of self-capacitance of a coil when the following measurements are taken. At frequency f_1 = 2 MHz, the tuning capacitor is set at 500 pF. When the frequency is increased to 6 MHz, the tuning capacitor is tuned at 60 pF.

14.15 A coil with a resistance of 12 Ω is connected in the 'direct connection' mode. Resonance occurs when the oscillator frequency is 1.0 MHz and the resonating capacitor is set at 70 pF. Calculate the % error introduced in the calculated value of Q by the 0.2 Ω infection resistance:

14.16 The lowest range of 4.5 digit voltmeter is 10 mV full scale. Find the sensitivity of the meter.

14.17 A certain 3 digit DVM has an accuracy specification of ! 0.5% of reading $\pm$ 2 digits.

(a) What is the possible error in volts, when the instrument is reading 6.00 V on its 10 V range?

(b) What is the possible error in volts, when the instrument is reading 0.20 V on the 10 V range?

(c) What percentage of the reading is the possible error in the case of (b)?

Chapter 15

POWER SYSTEMS

15.1 INTRODUCTION

Electric energy is most convenient and efficient for production of light and rotational mechanical motion. It can be transported easily and efficiently over long distance from production site to myriad points of use (compare this with transporting coal). Electric energy must be generated centrally and instantly transported to vast geographical regions within and beyond national boundaries. It cannot be stored in large quantities, except in batteries for limited use.

Because of these qualities of electric energy, its use has been growing fast in all sectors: industrial, commercial and domestic. With increasing industrial production, new products and new home appliances, the demand for electric energy has been rising very fast. This is particularly so in developing countries, which are trying to catch up with the developed countries, and in a chain reaction, the undeveloped countries are following suit. Apart from industrial conditioning, comfort conditioning of commercial and home areas is adding newer and fast growth electric loads. With limited resources of the globe, this may not go on for ever. Intensive research efforts are already on to (i) make electric energy products more efficient, (ii) look for renewable energy sources, (iii) make industrial drives, processes and appliances more efficient, (iv) conserve energy and avoid its wastefull use and (v) limit growth to a sustainable level. This sustainable level has so far eluded every nation.

Usage of electricity in the world has been growing at the rate of about 7%, implying doubling of demand every 10 years. This growth is presented in Fig. 15.1. alongwith the Indian picture. In India, the means (i.e. generation of electric energy) to meet this burgeoning demand have considerably lagged behind all along and more so in the past two decades when the effects of compounded growth rate have shown up. Of course there are many other factors, like inefficient generation, maintenance, insufficient outlay of transmission lines and above all management, which have contributed to the acuteness of the problem. Radical solutions are being groped but have to be applied fast.

The readers interested in reading ‘Power Systems’ in greater and fuller details, may refer to the following text books on Power Systems.

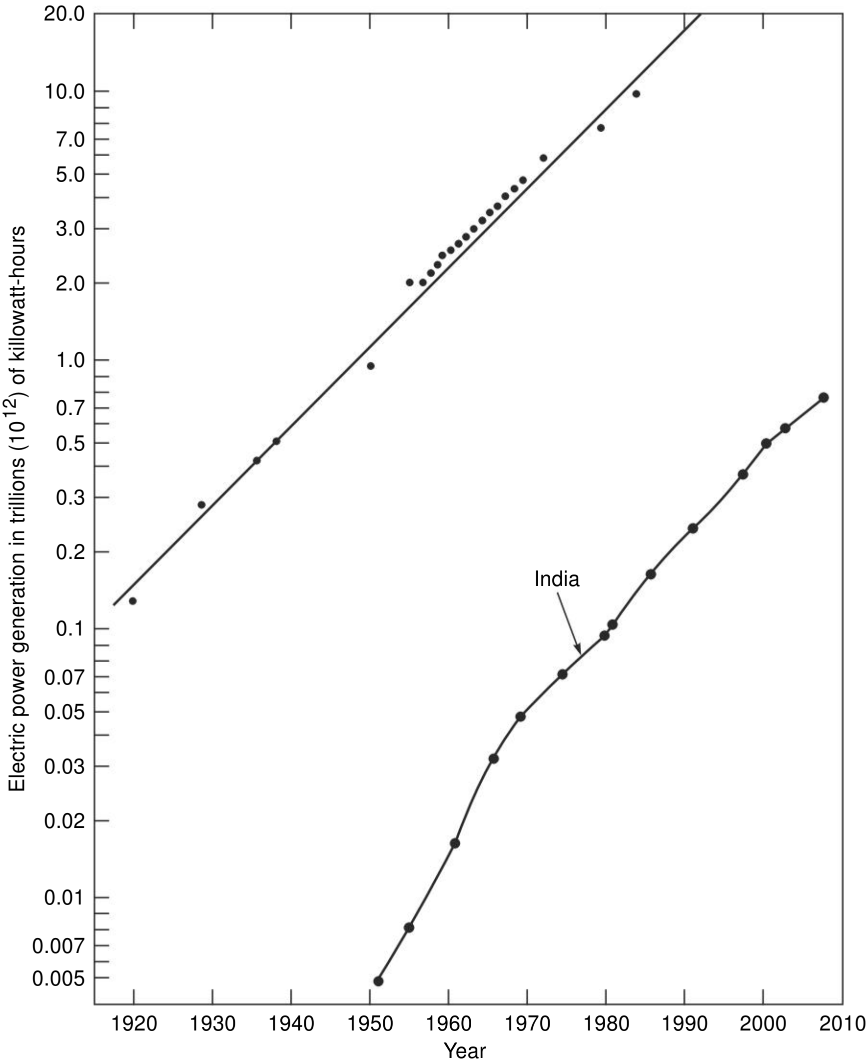


Fig. 15.1 World growth rate for electricity

1. I.J. Nagrath and D.P. Kothari, 'Power System Engineering', Tata McGrow Hill, 1994.
2. D.P. Kothari and I.J. Nagrath, 'Modern Power System Analysis', 3rd edn. TMH, 2003.

Factors Influencing Generation and Transmission

Four main factors that influence electricity supply are as follows:

1. Typical hourly load curves for summer and winter months in a North Indian metropolis are shown in Fig. 15.2. The generation and transmission system must meet this *fluctuation* load, which is not under the control of generating station engineers. The study of these curves show that there is a steady

component of the load called the base load plus peaks depending on the season and time of the day.

2. Demand is continuously rising over the years as seen from Fig. 15.1. This requires planning, addition of generation and also adding high voltage and medium voltage transmission lines. Planning of expansion and construction work must start years ahead. In India generation and transmission are both heavily lagging behind the rising load. The result is load shedding, both scheduled and unscheduled, particularly in summer.

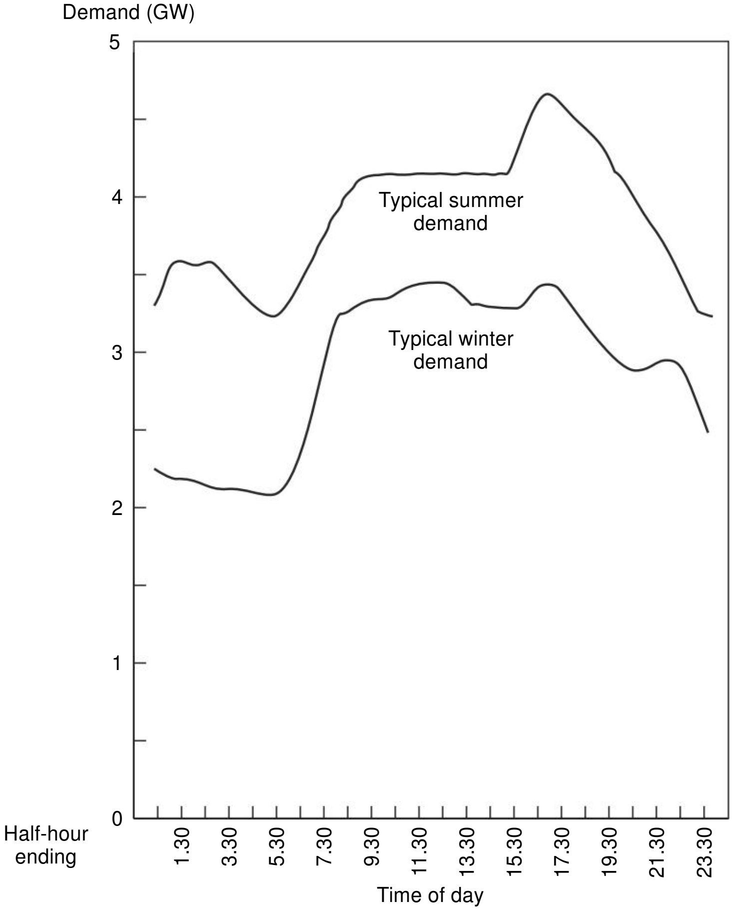


Fig. 15.2 Typical hourly load demand curves in giga weth (G.W)

3. Where coal is the fuel (this is largely so in India), the generating stations have to be sited near the coal mines. The electric power then has to be transmitted over long distances by HV (high voltage) transmission lines. In India, northern and western regions are devoid of coal.

4. Nuclear stations can be sited reasonably close to load centres. Also with gas grid continuously expanding, peaking load can be met by gas generating stations.

A picture of the intensity with which load demand is galloping in India, the following statistics, based on Electric Power Survey and prepared by Central Electricity Authority (CEA), are presented in Table 15.1.

Table 15.1

Year	Energy demand (billion kWh)	Peaking demand (MW)	Annual load factor*
1998–99	469.06	78,936	67.83
2001–2002	569.65	95,757	67.91
2002–2007	781.86	1,30,944	68.16

*This will be defined in Sec. 15.13.

15.2 ENERGY CONVERSION

Electric energy is obtained by conversion from other forms of energy stored in naturally occurring materials or from energy being continuously received from the sun in its primary form or in its secondary manifestations, for example, rain, snow at high altitude, wind, plants, etc.

Energy is stored in natural materials (coal, oil and gas and in the atom) in a chemical form. Coal, oil and gas (methane) were formed by natural processes over enormous periods of time aeons ago. These are available near the earth surface or underground, mostly at great depths, particularly, oil and gas. These are limited resources provided by nature and are nonreplenishable. Their extraction leaves gorges, which, when near earth surface, render vast tracts of land unfit for use. It is not clearly known what effect is caused by voids deep under the earth caused by oil extraction.

Energy from the atom (nuclear energy) can be obtained from certain materials with a high atomic number like uranium/thorium. Their resources are also limited, though they contain a great amount of energy.

Energy directly received from the sun (during day time) can be used directly but the surface density of solar energy is quite low and is variable during day, cloudy weather and different seasons. It is the solar energy which is responsible for rain/snow and winds. Rain collected at high altitude has potential energy and winds possess kinetic energy. Further, trees, plants and vegetation absorb solar energy, which is stored there in a chemical form. Pull of the moon on earth imparts energy to sea water in the form of tidal waves. High winds cause energy to be imparted to sea waves. Energy in the forms enumerated here are replenishable (*renewable*) and further they are nonpolluting, when used for conversion to the electric energy form.

The aim of this section is to describe various means and processes for converting the above listed energy forms to electric energy. With certain exceptions, this conversion requires the first step of conversion to rotational mechanical energy, which then is used to run a generator for conversion to electric form.

A panoramic view of energy conversion to electric form is presented in Fig. 15.3, which at a glance brings into focus all aspects of electric energy including conservation. Energy is converted to a mechanical rotational form by means of the following turbines.

- Steam turbine
- Gas turbine
- Hydraulic turbine

Steam is raised in a boiler by heat released by combustion of coal/oil or by atomic fission in a suitable vessel called the reactor. Combustion and steam raising is combined into a single boiler unit for coal-oil-based operation. However, in fission process, steam is raised by heat exchange processes from the reactor to boiler (or directly in the reactor). A gas turbine directly extracts energy from the products of combustion. In a hydraulic turbine, water's potential energy is directly converted to a rotational form.

15.3 ELECTRIC SUPPLY SYSTEMS

Electric power systems serve two basic functions.

1. Generation of continuous electrical energy at minimum economic and ecological cost.
2. Transporting or transmitting this energy to consumers with maximum efficiency and high reliability and quality (at almost constant voltage and fixed frequency which is 50 Hz universally, except in USA, where it is 60 Hz.)

The electric energy can be the in ac (alternating current) from or dc (direct current) from. It is now universal to generate ac electric power. Electric energy is integral of power over a period of time. As the size of a power equipment is concerned with power, we shall mostly use the term power but sometimes the term energy could also be used. Its transmission is also predominantly ac. However, in part of the power system (modern power system is a complex network) transmission could be in dc. Dc for this purpose is obtained by converters, composed of solid-state switching devices, which interconvert ac to dc back to ac. This makes the dc transmission line(s) part of the rest of the system which is ac. Dc power wherever needed for consumer use is obtained from ac line through converters.

Three phase System: Generation and transmission of electric power is universally ac three phase 50 Hz. At utilization end of the system, single phase loads are fed from each phase.

Components of a power are broadly classified as :

Prime mover - generator : The prime mover is normally a turbine which through mechanical coupling runs a synchronous generator. This arrangement converts mechanical power to electrical from which is 3-phase ac at usually 11/25 kV.

Transformers : These step-up the generated voltage to much higher level for transmitting large amounts of power. Transformers are then employed to reduce the voltage to several lower levels till the consumer level of 440V/231 is reached (this is the lowest voltage for electric use).

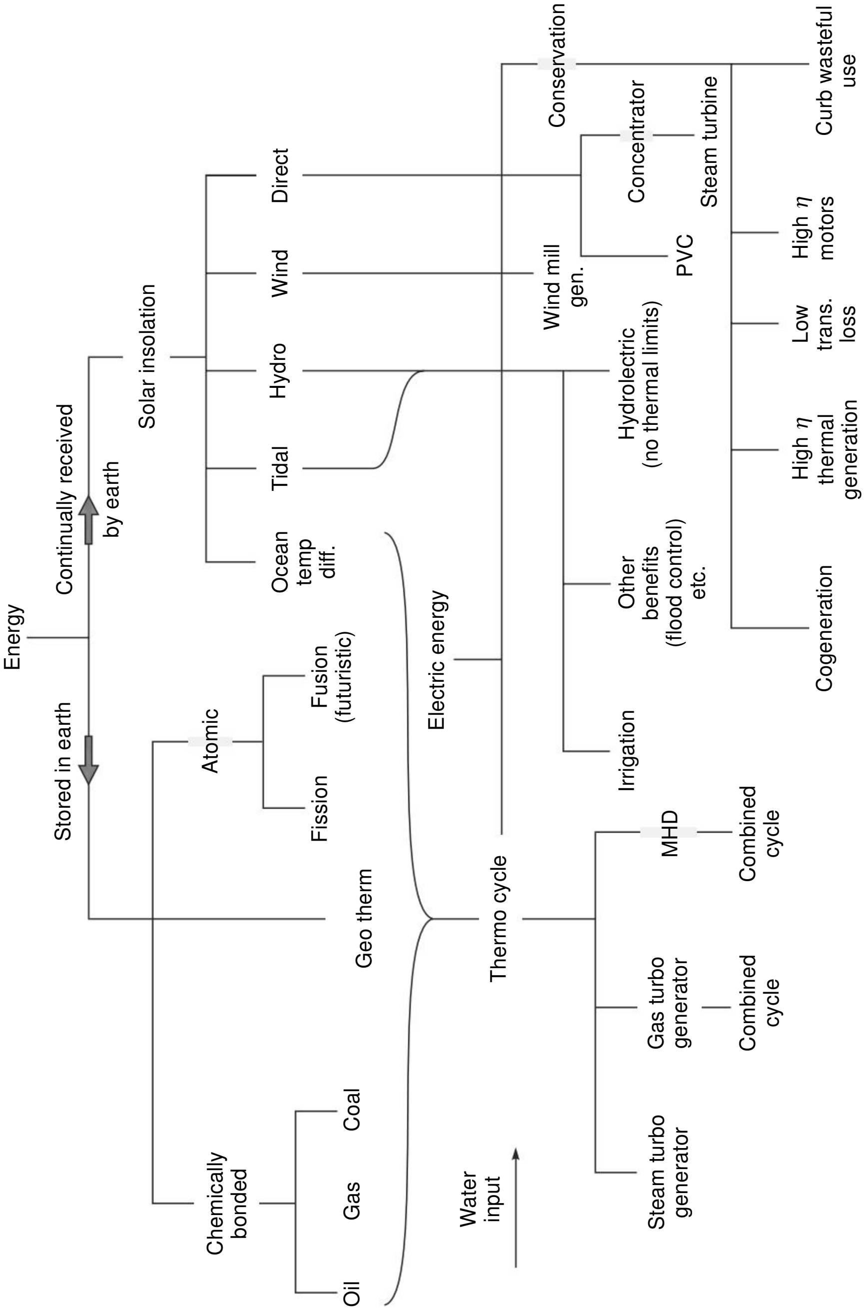

Energy
Stored in earth
Continually received by earth
Solar insolation
Chemically bonded
Oil
Gas
Coal
Atomic
Fission
Fusion (futuristic)
Geo therm
Ocean temp diff.
Tidal
Hydro
Wind
Direct
Water input
Thermo cycle
Electric energy
Wind mill gen.
Conservation
Steam turbo generator
Gas turbo generator
Combined cycle
MHD
Combined cycle
Irrigation
Other benefits (flood control) etc.
Hydroelectric (no thermal limits)
PVC
Concentrator
Steam turbine
Cogeneration
High η thermal generation
Low trans. loss
High η motors
Curb wasteful use

Fig. 15.3

Transmission Lines These connect the generator-transformer units to various load centres or to a large system. Power can be transmitted to a large system over a line either way depending on the need.

Power Station A power station or a power plant is a complex where electrical power is generated starting from the primary energy source (coal, gas, hydro, nuclear) for transmission over line. Present day power plants are super thermal or mega power plants with large unit sizes to cater to the ever increasing demand for electric power. In India such plants, to avoid the high cost of transporting coal, are being build at pit head (near coal mines).

In thermal ultra mega power plant, the (UMPP) unit system is adopted where a boiler, turbo generator and transformer (step up) form an independent unit, the unit sizes are in the range of 500 -750 MW while 1000 MW unit is being completed.

Hydro plants are dependent on source of water and nuclear plants must be located at remote places away from populated areas due to safety-considerations. This necessitates that power (in hundreds of megawatts) must be evacuated from these plants and transported over EHV (extra high voltage) transmission line covering distances of several hundred kilometers. EHV are voltage in the range is above 220 kV and UHV (ultra high voltage) means voltage greater than 700 kV.

Power Grid A modern power system is a large complex network feeding several load centres from where the consumers are fed. The power system spreads over vast geographical areas wherein transmission lines may be several hundred kilometers long, transporting power from one area to another. Such power flows are directionally controllable.

This kind of interconnection in power system parlance is known as power grid (or grid in short). The advantage and economics of grid formation shall be taken up later in this chapter. In India, we have a state-owned company Power Grid Corporation of India to manage power flows as per inter-area contracts for sale/purchase of power.

15.4 PASSIVE ELECTRICAL ELEMENTS

Electrical equipment are composed of (or can be modeled as) active and passive elements. Active elements are sources or sinks of power which can supply or receive unlimited amount of energy. There are electric generators or motors in electrical power systems. The passive – elements are dissipative (nonconservative) and conservative. The conservative elements are inductor and capacitor which can store electric energy or the same can be retrieved from the element without any loss of energy. The dissipative element is the resistor which only absorbs electric energy and converts it to heat form (the process is irreversible).

Electric power system components are modeled as sources, with resistors (R), inductors (L) and capacitors (C) elements. Their electrical terminal behaviour is governed by linear laws and has been discussed at length in Section 1.6. The resistor element has temperature dependency which has been pointed out there but law of temperature dependence has not been presented. This is of importance in power systems as its components exhibit a rise in temperature caused by power loss in resistance element (s), whose resistance value then indergoes a change (in fact a rise). This aspect of a resistor is now taken up here.

Electrical resistance is the property of material by virtue of which it opposes the electrons flow through it. The unit of resistance is "ohm" (W). It is obtained form the ohm's low as discussed in Chapter 1. (pp 10 - 11). Some of the relationships are reproduced below for clarity and completeness.

$$v = Ri \sqrt{V} \tag{15.1}$$

Power dissipated by the resistance (Fig. 1.9) is

$$p = vi = i^2 R = V^2/R \tag{15.2}$$

The energy lost in resistance in the form of heat can be expressed as

$$\text{Energy} = \int_0^t p \, dt = \int_0^t i^2 R \, dt = \int_0^t \frac{V^2}{R} \, dt \tag{15.3}$$

Effect of Temperature on Resistance: Normally, the resistance of a material changes with change in temperature. The effect of temperature in resistance varies with type of material.

(i) The resistance of pure metals (eg. Cu, Al ,etc) increases with increase of temperature. The graph of temperature versus resistance is almost a straight line for normal range of temperature (see Fig. 15.4) It means that a metal has positive temperature co-efficient of resistance.

(ii) The resistance of insulators (e.g. glass, mica, etc) and semiconductors (e. g. germanium, silicon, etc) decreases with the increase in temperature thus having a negative temperature co-efficient of resistance.

(iii) The resistance of alloys increases with temperature by a very amount and in a an irregular fashion. In fact in some cases like manganin and constantan, there is hardly any change over a wide range of temperature.

Temperature coefficient of resistance : From Fig. 15.4. for a temperature rise of t° C, the change in resistance is

$$R_t - R_0 \propto R_0$$

$$R_t - R_0 \propto t$$

so it can be expressed as

$$(R_t - R_0) \propto R_0 t$$

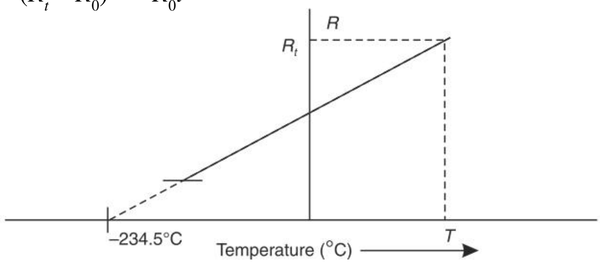


Fig. 15.4 Variation of resistance of metal with temperature

$$R_t - R_0 = \propto R_0 t$$

$$R_t = R_0(1+ \alpha\, t) \tag{15.4}$$

Here $\propto$ is called temperature coefficient of resistance and is given by

$$\alpha = \frac{(R_t - R_0)}{R_0 t} \tag{15.5}$$

Note: The reader is advised to refer Sec. 8.5 for Electromagnetic Induction and Force,

Sec. 8.6 for Self and Mutual Inductance,
Sec. 8.7 for Energy Stored in Magnetic Systems, and
Sec. 1.6 for Capacitance.

15.5 CONCEPT OF POWER TRANSMISSION

Consider an elementary system composed of an ac generator (coupled to primemover), *LV* - *HV* transformer, a transmission line and *HV* - *LV* transformer feeding the load as shown in Fig 15.5. a. In a power system diagram, the prime mover is normally not indicated. This is the representation of *one phase of a 3-phase* system. A 3-phase transmission line has only three lines as the common neutral line is not needed to carry any current.

We shall develop the circuit model of this system item-wise.

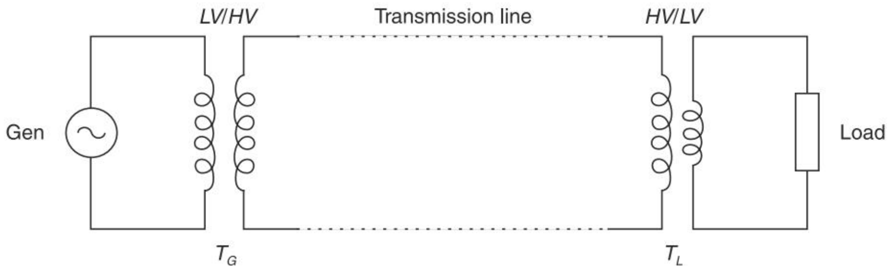


Fig. 15.5(a)

Circuit Model

Generator It has an induced emf and a series inductive reactance (synchronous reactance).

Transformer It has a series inductive reactance but negligible resistance.

Transmission Line It has resistance and inductance in series and capacitance between lines which is in shunt. These parameters (R, L, C) are proportional to line length.

Load It can be represented by resistances and inductances in series or in parallel as the overall load is normally inductive. Load comprises lighting, fans (small motors), household appliances driven by small motors, cooking ranges (heating elements), industrial motors (predominantly induction motors), and welding appliances, etc.

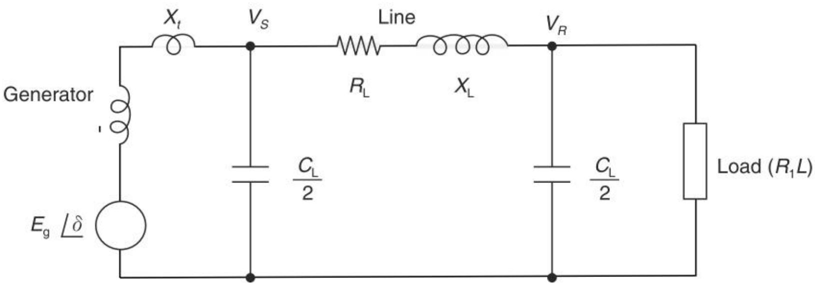


Fig. 15.5 (b)

The circuit model of the system of Fig. 15.5 (a) is drawn in Fig. 15.5 (b). In load I^2R is converted to useful form at the consumer end, while I^2R in the transmission line is lost (in form of heat to environment). This loss determines the power efficiency of the system, so it should be kept low.

The line capacitance is very small and can be ignored except for long lines. In our study, we shall consider the line to have only resistance and inductance.

In the system of Fig. 15.5 (a) the power flows form the generator to load. It can be shown and should be noted that power always flows from terminal whose voltage leads the other terminal.

Classification of Lines

Direct current (DC) Systems

(a) DC 2-wire, 1 wire earthed
(b) DC 1-wire, earth return
(c) DC 2-wire, mid-point earthed
(d) DC 3-wire

Alternating Current (AC) systems

(a) Single-phase, 2-wire, 1-wire earthed
(b) Single-phase, 1-wire, earth return
(c) Single-phase, 2-wire, mid-point earthed
(d) Single-phase, 3-wire
(e) Two-phase, 3-wire
(f) Two-phase, 4-wire
(g) Three-phase, 3-wire
(h) Three-phase, 4-wire, one is neutral which is earthed

All these systems can be used in practice. Each has its own field of application. Direct current 2 wire, mid-point earthed system is used for transmitting large amounts of power over long distances. Three-phase 3-wire AC system is used for transmission and primary distribution. For secondary distribution, three-phase, 4-wire system is normally employed and it supplies industrial and other large consumers. The domestic and other small loads are supplied with single-phase whose one wire is the phase of 3-phase, of 4-wire and the second wire is the neutral.

The lines are predominantly overhead. Underground cables are used where it is a must as in a congested area. For connection across small sea distance, the only choice is an underwater cable. Cables are several times more expensive than overhead lines.

15.6 SYSTEM VOLTAGE AND TRANSMISSION EFFICIENTY

Consider a 3-phase system

Let P = Power to be transmitted per phase in MW

V = Voltage/phase in kV

I = Current/phase in A

L = Line length in km

A = Conductor cross - sectional area in m^2

ρ = Specific resistance of the conductor material in ohm-m

R = Resistance of each conductor in ohms

J = Permissible current density in A/m^2

$\cos\phi$ = Power factor of the load

Active (Real) power per phase is given by

$$P = VI\cos\phi$$

$$\therefore \quad I = P/V\cos\phi \tag{15.6}$$

But current density $J = \dfrac{I}{A} \;\therefore\; A = \dfrac{P}{JV\cos\phi}$ (15.7)

Resistance of conductor is $R = \dfrac{\rho L}{A} = \dfrac{\rho LJV\cos\phi}{P}$ (15.8)

Real Power Loss The power loss in the line per phase is

$$P_L = I^2R = \left(\frac{P}{V\cos\phi}\right)^2 \frac{\rho LJV\cos\phi}{P}$$

or

$$P_L = \frac{J\rho LP}{V\cos\phi} \tag{15.9}$$

Then it is easy to see that *transmission line power loss is inversely proportional to both the system voltage and the power factor*. Lower power loss implies high system efficiency as is shown by derivation given below.

Transmission efficiency The efficiency of transmission (η_T) is given by

$$\eta_T = \frac{\text{Line output}}{\text{Line output} + \text{Line loss}}$$

$$\eta_T = \frac{P}{P + \dfrac{J\rho LP}{V\cos\phi}} = \frac{1}{1 + \dfrac{J\rho L}{V\cos\phi}}$$

$$= \left[1 + \frac{J\rho L}{V\cos\phi}\right]^{-1} \tag{15.10}$$

Expanding using Binomial theorem and neglecting higher order terms, we get

$$\eta_T \cong 1 - \frac{J\rho L}{V\cos\phi} \tag{15.11}$$

It is clear from Eq (15.11) that the *efficiency of transmission increases with increase of supply voltage and power factor.* It is obvious from Eq (15.5) the cross-sectional area of conductor decreases by increasing line voltage and by increasing power factor of the load. Consequently the conductor cost comes down.

It immediately follows from the above results that for large power transmission, there is no choice but to go for higher line voltage such as 440 kV (line to line) or 750 kV (line to line).

Kelvin's Law

The most economical size of the copper conductor for the transmission of electrical energy will be formed by comparing the annual interest of money value of the conductor copper with the money value of the energy lost annually due to current flow in conductor. The most economical conductor area is given by the Kelvin's law

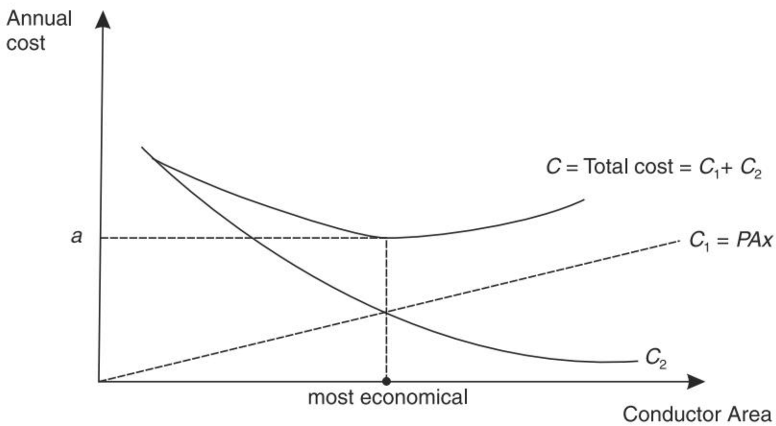


Fig. 15.6

presently graphically in Fig. 15.6, in which

$C_1 = PAx;$ P – Constant
A – Area of the conductor
x – Annual interest and depreciation
$C_2 = \frac{Q}{A}$; Q – Constant
A – Area of the conductor

The annual cost of conductor is proportional to conductor area, so it is a straight line. The annual cost of energy dissipated is inversely proportional to conductor area so it is a rectangular hyperbola shaped in the graph. It is also clear that the most economical conductor area is given by the point where the two graphs intersect each other.

Modified Kelvin's Law The actual Kelvin's law is based on the assumption that cost of towers and their foundation, insulators and their erection etc., are all independent of the conductor area but practically it is not true. The increase in conductor size results in increased mechanical stress to the towers and so the need heavier insulators and towers. Therefore $C_1 = R_s\ (PA + K)\ x$ where P and K are constants and x is the rate of annual interest and depreciation.

15.7 COMPARISON OF CONDUCTOR COSTS OF TRANSMISSION SYSTEMS

The conductor cost forms the main expenditure of the line, therefore it is necessary to compare it in various systems. It is assumed that power transmitted, length of line, and maximum voltage to earth are same in all cases.

(i) DC 3-wire system: (Fig 15.7) In this system, there are three conductors namely, two outers and one middle conductor. The middle conductor is earthed at supply end. When the load is balanced, the current in the middle (neutral) conductor is zero. Therefore,

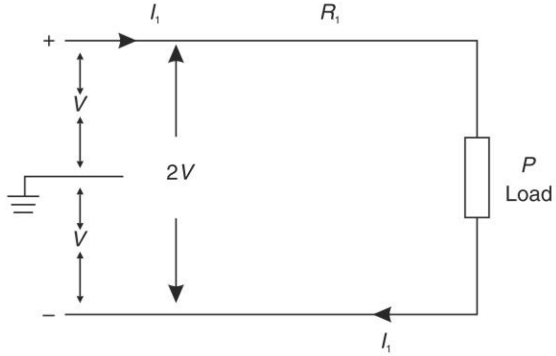


Fig. 15.7

Current $$I_1 = \frac{P}{2V}$$

$$\text{Line losses} = 2I^2 . R_1 = 2\frac{P^2}{4V^2} R_1 = \frac{P^2 R_1}{2V^2} \tag{15.12}$$

(ii) Single-phase 2-wire system with 1-wire earthed (Fig. 15.8)

Let V be the maximum voltage to earth.

∴ the rms voltage to be earth is

$$V_{\text{rms}} = V/\sqrt{2}$$

Let I_2 be the load current and cos ϕ be the power factor of the load.

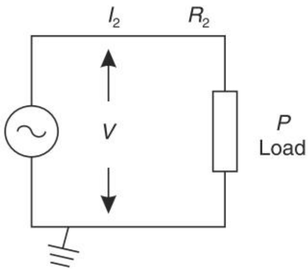


Fig. 15.8

$$\therefore \qquad I_2 = \frac{P}{V_{\text{rms}} \cos\phi} = \frac{P}{\frac{V}{\sqrt{2}} \cos\phi}$$

$$\therefore \qquad \text{line losses} = 2\, I_2^2\, R_2$$

$$= \frac{2 \times 2P^2 R_2}{V^2 \cos^2 \phi} = \frac{4P^2 R_2}{V^2 \cos^2 \phi} \tag{16.13}$$

(iii) Three-phase, 3-wire, star connected system : (Neutral point at earth potential) The system is shown in Fig (15.9).

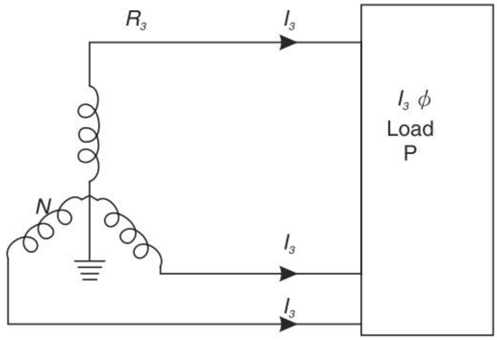


Fig. 15.9

RMS phase voltage $= V/2$

Line voltage $= 3\ V/\sqrt{2}$

$$\text{Current } = I_3 = \frac{P}{\sqrt{3}\frac{\sqrt{3}}{2}V\cos\phi} = \frac{\sqrt{2}\,P}{3V\cos\phi}$$

$$\text{Line losses} = 3\ I_3^2\ R_3 = \frac{3 \times 2P^2 R_3}{9V^2 \cos^2 \phi} = \frac{2P^2 R_3}{3V^2 \cos^2 \phi}\,. \tag{15.14}$$

Comparison We assume equal line losses and therefore equal efficiency of transmission.

Equating equations (15.2) and (15.14) we get

$$\frac{P^2 R_1}{2V_2} = \frac{4P^2 R_2}{V^2 \cos^2 \phi} = \frac{2P^2 R_3}{3V^2 \cos^2 \phi}$$

$$\frac{R_1}{2} = \frac{4R_2}{\cos^2 \phi} = \frac{2R_3}{3\cos^2 \phi}, \text{ Let us compute the ratios 01}$$

Resistances i.e. Area

$$\frac{R_2}{R_1} = \frac{\cos^2 \phi}{8}$$

$$\therefore \qquad \frac{A_1}{A_2} = \frac{\cos^2 \phi}{8}; \quad \frac{R_3}{R_1} = \frac{3\cos^2 \phi}{4}$$

$$\frac{A_1}{A_3} = \frac{3\cos^2 \phi}{4}$$

$$\left[A_1 : A_2 : A_3 = 1 : \frac{8}{\cos^2 \phi} : \frac{4}{3\cos^2 \phi}\right] \tag{15.15}$$

In cases (i) and (ii), there are two conductors, but in case (iii) there are 3 conductors.

Hence the volume of copper or aluminium is

$$V_1 : V_2 : V_3 = 2 : \frac{16}{\cos^2\phi} : \frac{12}{3\cos^2\phi}$$

$$= 1 : \frac{8}{\cos^2\phi} : \frac{2}{\cos^2\phi} \tag{15.16}$$

Thus it is concluded that DC 2-phase, 2 wire mid point earthed system is the cheapest on the basis of conductor cost alone and AC 3-phase, 3-wire star-connected system is far more economical than single-phase system.

Even though 3-wire DC transmission is cheapest on copper volume and on line loss basis, dc voltage levels cannot be changed as easily and as cheaply as the ac voltages with transformer. Consequently dc has come into use recently for transmission of large chunk of power at high voltage. As such, it is interconnected to the remaining part of the system which is high voltage ac.

15.8 POWER FACTOR IMPROVEMENT

This has already been discussed briefly in Chapter 4. At low power factor, the load draws a large current with consequent increase in voltage drop, losses resulting in lower efficiency and higher voltage regulation. Larger current requires higher current rated lines, switch gear, transformers, etc. with consequent increase of equipment cost. Thus both the capital and running costs are increased due to low *pf* (lagging or leading) and this is uneconomical from the supplier's point of view. The usual reason for low power factor is the presence of large number of inductive loads like induction motors particularly if these are lightly loaded. Further, arc lamps, electric discharge lamps, industrial are furnaces and welding equipment operate at low lagging power factors.

As low load *pf* is uneconomical to the electric supply company, it charges a two part tarrif; one based on electric energy consumed and the other based on maximum KVA demand. As low *pf* for a given real power load means larger current and so higher KVA, the consumer is thereby forced to install power factor improvement at its installation.

Concept of VARs - positive and negative (refer Section 4.4)

This concept is helpful in understanding the method of *pf* improvement or in general *pf* correction. Consider an inductance and capacitance current as in Figs. 15.10. (a) and (b) respectively.

Inductance : Its current I lags the voltage V across it by 90°. The inductance draws volts - amps of

$$VAR = VI = V\frac{V}{X_L} = \frac{V^2}{X_L}$$

$$= (I X_L)\, I = I^2 X_L$$

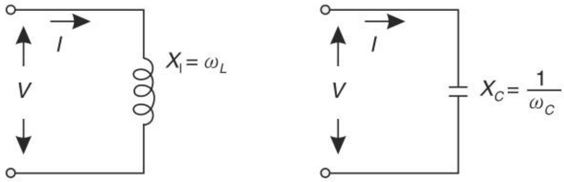


Fig. 15.10 VARs concept

The VARs drawn by inductance (or inductive reactance) are labeled as positive VARs. It can be alternatively considered that inductive reactance supplies negative VARs.

Capacitance Its current leads the voltage V across it by 90°. The capacitance draws volt amps of

$$VAR = VI = V^2/X_C = I^2 X_C$$

The capacitance (capacitive reactance) draws negative VARs or alternatively supplies positive VARs.

The basic principal of pf improvement is thus to supply positive VARs (from a capacitor across load) to compensate the VARs drawn by the inductive (low pf) load.

The power factor of any load can be improved by the following methods.

(i) By using static compensator (static capacitors)

(ii) By using dynamic compensator (synchronous motors)

Power factor correction by static capacitors

Consider RL load supplied at voltage V as given in Fig 15.11 (a). The phasor diagram of this circuit is given in Fig. 15.11 (b).

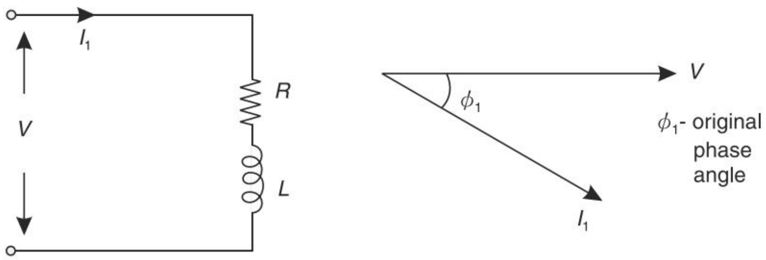


Fig. 15.11

Let a capacitor C be placed in parallel with the load (Fig 15.12 (a)). It will draw I_C (leading) form the supply. The resultant current will now be $\bar{I}_2 = \bar{I}_1 + \bar{I}_C$.

The phasor diagram is drawn in Fig. 15.12 (b) wherein we find $\phi_2 < \phi_1$. The real power P in given as

The phasor diagram is drawn in Fig. 15.11(b).

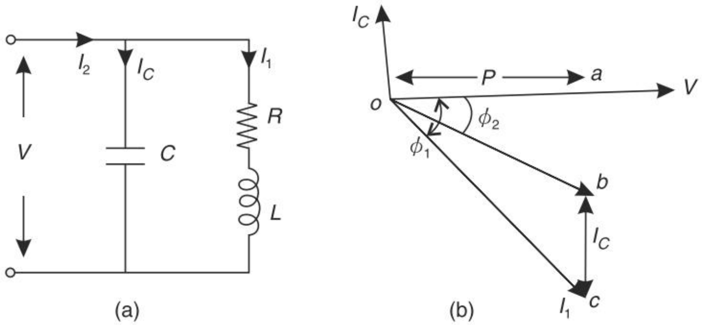


Fig. 15.12

$$Oa = I_1 \cos \phi_1 = I_2 \cos \phi_2, \cos \phi_2 > \cos \phi_1. \qquad \therefore I_2 < I_1$$

It means that the current I_2 drawn form the supply is less than the load current I_1 thereby reducing system losses and improving efficiency as well.

Further since $VI_2 \cos \phi_2 = VI_1, \cos \phi_1$, real power taken from the supply remains the same.

Computation of Capacitor Rating From Fig. (15.12 (a))

$$I_C = I_1, \sin \phi_1 - I_2 \sin \phi_2$$

or $$VI_C = VI_1 \sin \phi_1 - VI_2 \sin \phi_2$$

Using the fact that load power

$$P = VI \cos \phi_1 = VI_2 \cos \phi_2$$

We can write $$Q_C = P (\tan \phi_1 - \tan \phi_2) \tag{15.17 a}$$

where $$Q_C = VI_C = \text{capacitor volt-amp}$$

The capacitor current is given as

$$I_C = I_1 \cos \phi_1 (\tan \phi_1 - \tan \phi_2) \tag{15.17 b}$$

Also. $$I_C = \frac{V}{X_C} = V\omega_C; Q_C = VI_C = V^2\omega_C \tag{15.17c}$$

which gives the capacitor value of $C = \dfrac{Q_C}{\omega V^2}$. (15.18)

For a given power P, $\cos \phi_1$ load *pf* and required *pf* $\cos \phi_2$ are known from which we can find the capacitor needed from Eqs. (15.17 a) and (15.18). It is seen that compensating capacitor needed for pf improvement is inversely proportional to V^2.

For achieving unity *pf* ($\phi_2 = 0$)

$$Q_C = P \tan \phi_1 \tag{15.19}$$

Apart from the consumer improving the pf of its load to reduce the electricity bill as per two part tariff, there is need to install VAR compensators (or generators) at certain important system buses to improve the system voltage and to increase the real power transfer over a high tension (high voltage) transmission line. Under heavy load condition, the system voltage tends to sag which is remedied by injecting positive VARs into the system by the compensator. Under light load conditions, the voltage tends to rise and so negative VARs are injected into the system.

Advantages and drawbacks of Static Capacitors Capacitors are static, robus, easy to install, occupy less space and not require any special foundation. They can be used in modules; easy maintenance and loss free are other advantages.

When negative VARs are needed, inductor banks are switched on.

Capacitor banks can be switched on in steps. However, stepless (smooth) VAR control can now be achieved using SCR (Silicon Controlled Rectifier) circuitry. The effectiveness of capacitors becomes less as the voltage sags under full load conditions. (see Eq. (15.17)c). If the system voltage contains appreciable harmonics, the capacitors may be overloaded considerably. $I_C = V\omega C$, at high ω, I_C is high causing overload. Also capacitors act as short circuit when switched in and further there is a possibility of series resonance with line inductance particularly at harmonic frequencies.

Power Factor correction by Synchronous motors

When synchronous motor runs at no load with adjustable excitation over a wide range, pf can be improved. It can generate or absorb VARs by varying the excitation of its field winding. It can be made to feed positive VARs into the line under overexcited conditions and feed negative VARs when underexcited. A machine thus running is called a *Synchronous condenser* or a *dynamic compensator*. They can provide both positive and negative VARs which are continuously adjustable. VAR injection at a given excitation is less sensitive to change in bus voltage. But these are costly as installation, maintenance and remote-controlled operations are not easy.

Economics of Power Factor Improvement As KVA demand is reduced with improvement in pf, it results in annual saving over the maximum KVA demand charges. However, there is capital investment on the power factor correction equipment. Thus there is expenditure every year in the form of interest and depreciation etc. On the initial cost of pf improvement equipment. The net annual saving will be equal to the annual saving on the maximum KVA demand charges minus annual expenditure on the equipment. The most economical pf will be that pf at which net annual saving is maximum.

15.9 THE ONE - LINE (SINGLE - LINE) DIAGRAM

A one - line diagram of a power system shows the main connections and arrangement of components. Any particular component may or may not be shown depending on the information required in a system study, e.g. circuit breakers need not be shown in a load flow study but are a must for a protection study. Power system networks are represented by one-line diagrams using suitable symbols for generators, motors transformers and loads. It is a convenient practical way of network representation rather than drawing the actual three-phase diagram which may indeed be quite cumbersome and confusing for a practical size network. Generator and transformer connections - star, delta and neutral grounding are indicated by symbols drawn by the side of the representation of these elements. Circuit breakers are represented as rectangular blocks. Figure 15.13. shows the one-line diagram of a simple power system.

A balanced three-phase system is studied on a per-phase basis. A three-phase balanced system is effectively and concisely represented by a single-line diagram. A single line diagram cannot be used for all types of studies. For example, it fails to depict the conditions during unbalanced operation of a power system.

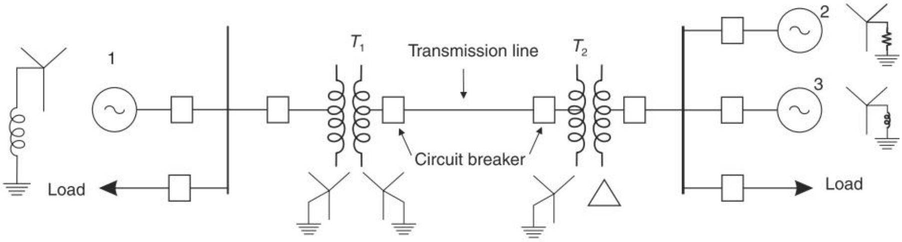


Fig. 15.13 One-line representation of a simple power system

15.10 TRANSMISSION LINE PERFORMANCE

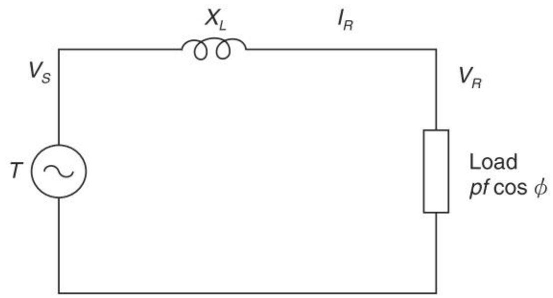


Fig. 15.14

One phase diagram of a transmission line feeding load is drawn in Fig. 15.14. Here line resistance is ignored. In this diagram, V_S = sending end voltage, V_R = receiving end voltage, X_L = Line reactance. The phasor diagram for this system is drawn in Fig. 15.15. wherein reactance voltage drop in line = $I_R X_L$. It leads line current I_R by 90°

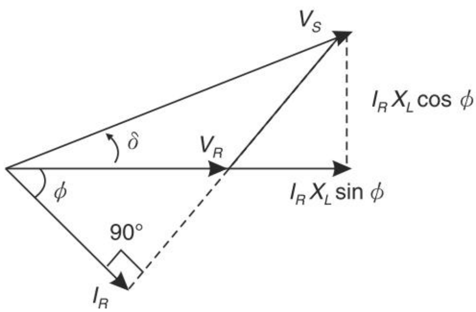


Fig. 15.15

From the phasor diagram

$$V_S = [(V_R + I_R \times X_L \sin\phi)^2 + (I_R \times X_L \cos\phi)^2]^{1/2} - \quad (15.20)$$

$$= [V_R^2 + 2IR\, X_L \sin\phi.\ VR + I_R^2 \times X_L^2]^{1/2}$$

$$V_S = V_R\left[1 + \frac{2I_R X_L}{V_R}\sin\phi + \frac{I_R^2 X_L^2}{V_R^2}\right]^{1/2} \quad (15.21)$$

For short lines

$$I_R X_L << V_R\text{ ; so }\frac{I_R^2 X_L^2}{V_R^2} \simeq 0$$

It is sufficiently accurate to consider only the first term in Taylor series expansion of Eq. (15.21). Thus

$$V_S = V_R + I_R X_L \sin\phi \quad (15.22)$$

Voltage regulation of the line is the line voltage drop as a percentage of the receiving and voltage at full load.

Thus

$$\text{Voltage regulation} = \frac{V_S - V_R}{V_R}\text{ ; under full load at specified pf}$$

$$= \frac{I_R X_L \sin\phi}{V_R} \quad (15.23\text{ a})$$

(15.22 a) get modified as

$$V_S = V_R + I_R X_L \sin\phi\ I_R R_L \cos\phi \quad (15.22\text{ b})$$

$$\text{and voltage regulation} = \frac{I_R X_L \sin\phi + I_R R_L \cos\phi}{V_R} \quad (15.23\text{ b})$$

Active (Real) and Reactive Power Flows

Active (real) power received at receiving or fed to load.

$$P = V_R I_R \cos\phi;\ \cos\phi = \text{pf} \quad (15.24)$$

Reactive power received at receiving end or fed to load :

$$Q = V_R I_R \sin\phi \quad (15.15)$$

We will now find the expressions for P and Q in terms of V_S and V_R. For this voltage are expressed in phasor form as

$$V_S = V_S \angle\delta,\quad V_R = V_R \angle 0°$$

δ = angle of V_S with respect to V_R ; this is called Power angle.

Then

$$\bar{I}_R = \frac{V_S\angle\delta - V_R\angle 0}{jX_L} \quad 16.26$$

or

$$\bar{I}_R = \frac{V_S}{X_L}\angle\ \delta - 90° - \frac{V_R}{X_L}\ \angle -90°$$

Expanding in polar form we get

$$\bar{I}_R = \frac{1}{X_L}\ [V_S \cos(90°-\delta) + jV_S \sin(90°-\delta) - V_R \cos(-90°) - jV_R \sin(-90°)]$$

or $$\bar{I}_R = \frac{V_S}{X_L}\sin\delta - j\frac{V_S}{X_L}\cos\delta + j\frac{V_R}{X_L}$$

$$\bar{I}_R\cos\phi_R - jI_R\sin\phi_R = \frac{V_S}{X_L}\sin\delta - j\frac{(V_S\cos\delta - V_R)}{X_L}$$

Equating real and imaginary parts

$$I_R\cos\phi_R = \frac{V_S}{X_L}\sin\delta$$

$$I_R\sin\phi_R = (V_S\cos\delta - V_R)/X_L$$

Multiplying both the above equations by V_R. We get

$$P = \frac{V_S V_R}{X_L}\sin\delta \tag{15.27}$$

$$Q = \frac{V_S V_R}{X_L}\cos\delta - \frac{V_R^2}{X_L} \tag{15.28}$$

Eqs (15.27) and (15.28) can also be easily obtained from the phasor diagram of Fig. 15.15. Active and reactive powers flowing from sending – end to receiving – end are sinusoidal and cosinusoidal function respectively of angle δ (the angle by which V_S leads V_R). Real power flow is maximum for $\delta = 90°$. Actual operation must be at well below this value. The real power flow would reverse if V_S were to behind V_R. It means that real flows from the terminal (bus) whose voltage leads the voltage of the terminal (bus) where real power is received. This is general statement.

15.11 TRANSMISSION AND DISTRIBUTION SYSTEMS

Large power plants are located close to regions where fossil fuels are mined, hydraulic dams can be buit near foothills or where there is sufficient change in river level, even nuclear power plant are located away from populated areas. This means that electrical power has to be transported to load centres over transmission lines and suitably distributed.

Electricity supply system are invariably three phase and they are so designed and loaded that the operation is balanced three phase. This is assured by allotting nearly equal domestic load on each phase; industrial loads are usually three phase and balanced.

Symbols used in one line diagram of a power system are depicted in Fig. 15.16. A typical line diagram of part of a power system is drawn in Fig. 15.17. for illustration. The generator is star connected with neutral grounded through resistance. The purpose of grounding is that if a line or any phase of any part of the system faults to ground, sufficient current will flow from fault point to neutral via the ground connection. This is sensed by protective devices which disconnect the faulty part of the system so that no equipment damage occurs.

The generator voltage is raised to transmission line level by the two (independent) line transformers. The voltage is brought down by the transformers at the end of the line to the values required for feeding the load. Circuit breakers are located on each side of the transformer to disconnect transformer and line from the generator and load in the event of a fault on the line. Two lines in parallel build security into the

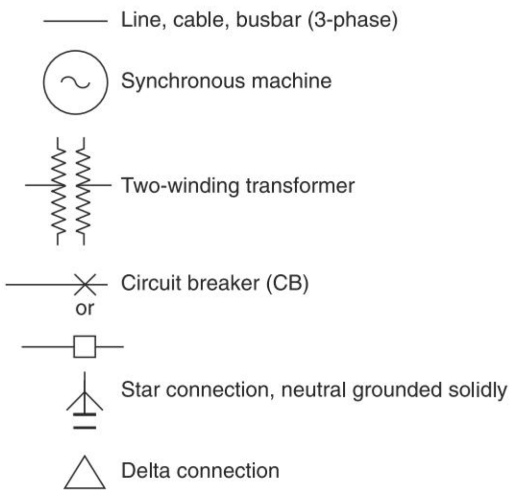


Fig. 15.16 Symbols for representing the components of a 3-phase power system.

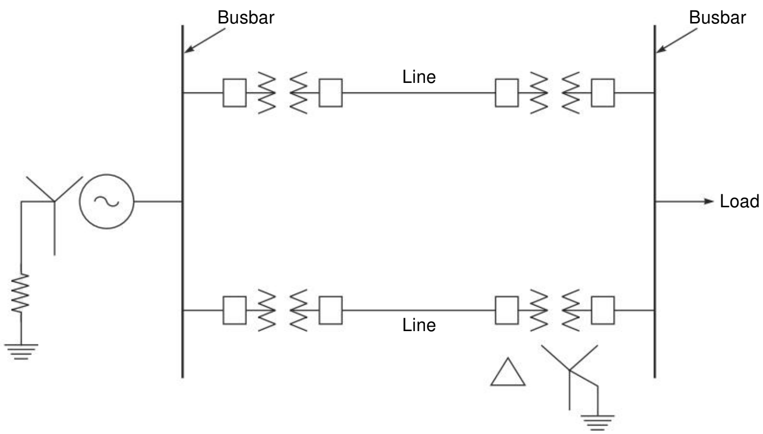


Fig. 15.17 Line diagram of part of a power system

system as in the event one is faulted, the other can feed the load, though not the full load.

Before proceeding further, the terminology used in power systems is listed below, with explanation where necessary.

Systems The complete electrical networks: prime movers, generators, transformers, lines and loads.

Busbar It is a solid electrical connection (of zero impedance) made of aluminium or copper bars connecting various power system components like generators, transformers, lines, loads etc. Busbars are shown in Fig. 15.17.

Load It is device or devices which draw electrical power from the busbar to do useful work for the consumers: drive motors and other processes in industry; domestic load is lighting, refrigeration, comfort conditioning, small electrical appliance, etc.

Earthing Earth connecting entails burying deep into ground large assemblies of (grounding) conducting rods embedded in moist coal (powdered) and lime. Conductors of large cross-section are used to bring the earth connection to the ground level. This is used to connect the frames of electrical devices, neutral of generators and ground conductor of the lines, sheathing of cables, etc. This prevents the voltage of any of these devices rising above the ground voltage in the event of fault to earth on any part of the system.

Outage Removal of a circuit either deliberately (in case of fault or for maintenances) or inadvertently.

Security of supply Provisions made to ensure continuity of supply to consumers in the event of some outage or loss of generator. Only certain combination of events can be covered, while other combinations are considered rare event (very low probability of occurrence).

In India, with long hours of load shedding being a daily routine, it is not meaningful to talk of security at present.

Transmission

Transmission part of the system implies bulk transfer of power by high voltage links between generation and load centres. Distribution network conveys the power form the bulk load centres to consumers by lower voltage network, which then is reduced to consumer voltage level at the distribution substations.

Generator voltage is usually in the range of 11 to 25 kV, which is raised by transformers to the main transmission voltage. Large amount of power are transmitted at ac voltage of 400 kV, 500 kV and even 750 kV (not in India yet). Network formed by these high voltage lines is referred as supergrid. Subtransmission voltage levels are 132 kV, 115 kV and distribution voltage level are 33 kV, 11 kV. The consumer feeders are at 400/230 V 3-phase/1-phase.

High voltage dc (HVDC) transmission is presented in Section 15.12. Higher voltage levels imply that large chunk of power can be transmitted for same current level, but it has to be economically balaced against cost of higher transmission towers and higher cost of insulators, stronger conductor reinforcement becauese of longer span between towers. To make it economical, towers may be designed to carry two three phase lines, on both the sides.

A small portion of a large transmission network is shown in Fig. 15.18 in one-line. The figure shows the high voltage grid, grid power supply (GPS) and interconnection to the rest of the system and one distribution bus with embedded generators, local generators.

It is seen from above that the power system is made up of interconnected networks(grids), at various voltage levels and also fed by power plants of large, medium and small size, This is illustrated in the conceptual diagram if Fig. 15.19.

Why Interconnection Interconnection or grid formation in a power system provides the following facilities and advantages:

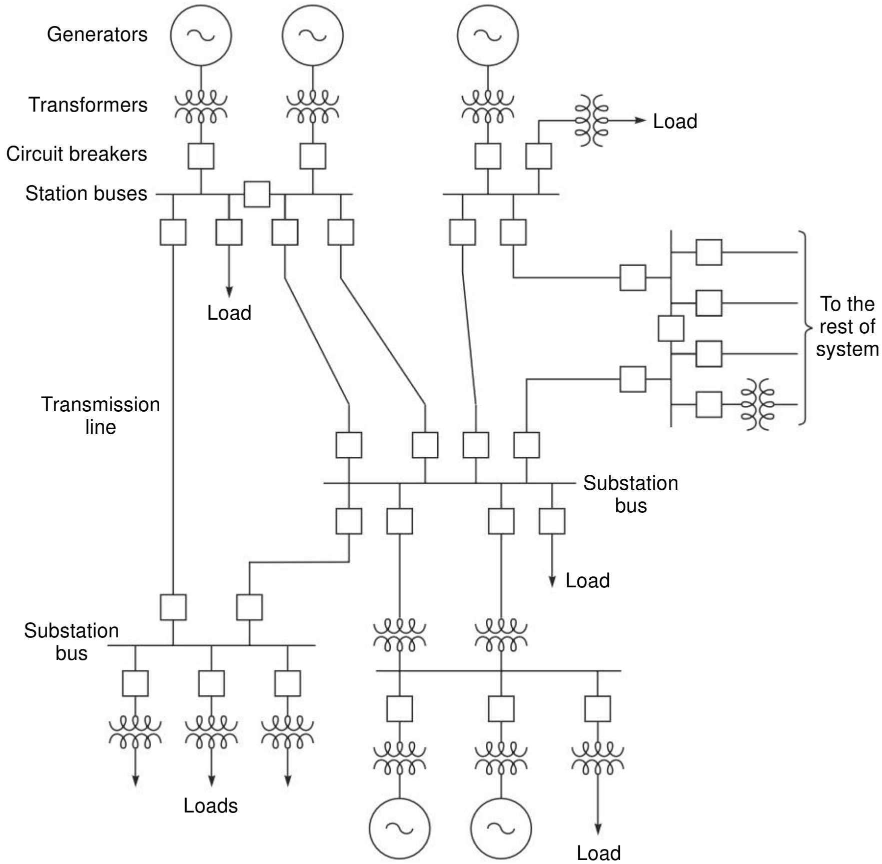


Fig. 15.18 One-line diagram, power system grid

- Evacuating power from surplus areas and transporting it to power deficient areas. This situation can also arise in case of generation outage.
- Load can still be met in case of a line outage: also see Fig. 15.17.
- Power plants of various kinds, efficiencies and ages can be scheduled to meet the loads on the grid in a economical manner by scheduling a particular mix of generation on hourly basis. The main base load is always fed by high efficiency generators, nuclear plant, etc.

Distribution System

Here the voltage is stepped down o 132 kV or 11 kV/400 V (230 V single phase). This part of the system feeds industrial, commercial and domestic consumers. Usually there are no interconnections, but isolators are used to interconnect two sections, when there is an outage on one side. A typical radial distribution system along with its link to subtransmission and transmission is shown in Fig 15.20.

Ring Mains: The radical distribution is simple and econimic. But the reliability of the system is poor and it leads to interruption of energy supply if there is fault in the line. To overcome this, ring main type distribution is used.

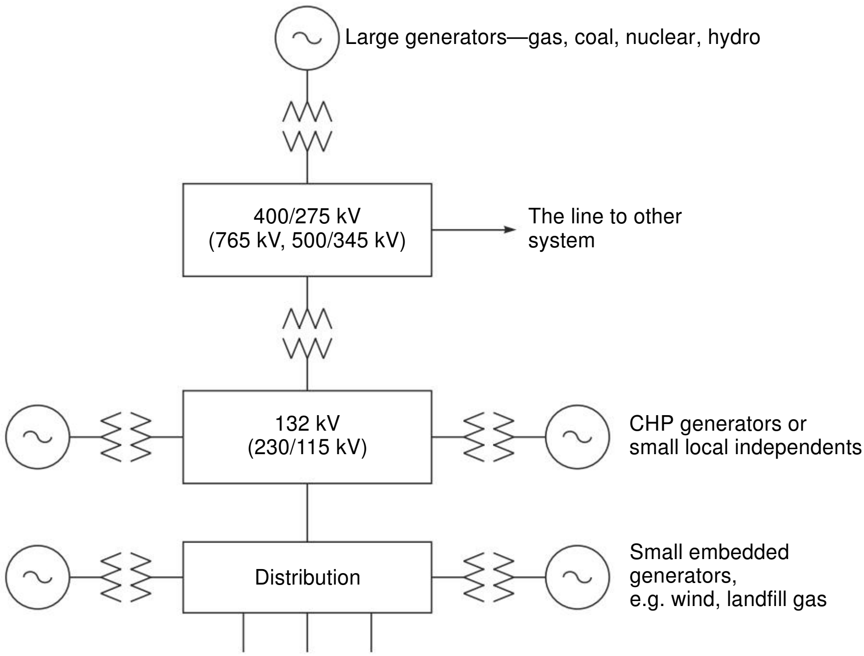


Fig. 15.19 Schematic diagram of the constituent networks of a power supply system

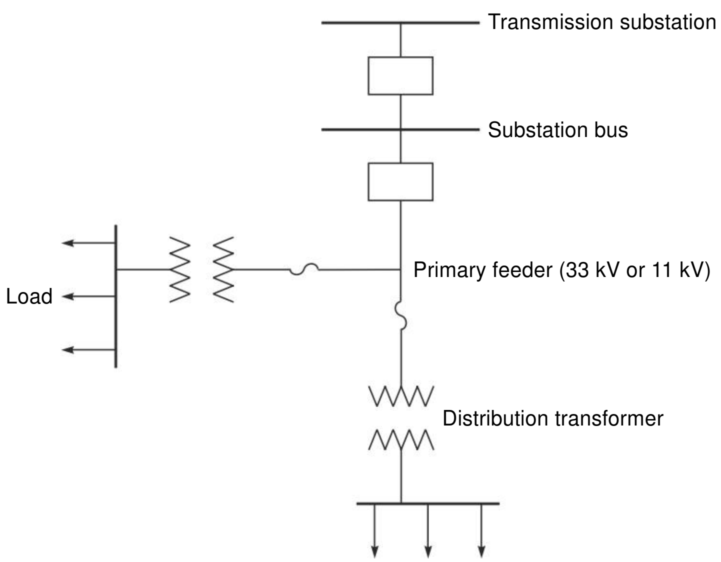


Fig. 15.20 Simple form of radial distribution system

The ring main distributors are of two types:

(a) Single ring: Here the distribution is carried out by forming a closed circle i.e., two ends are connected together as shown in Fig. 15.21.

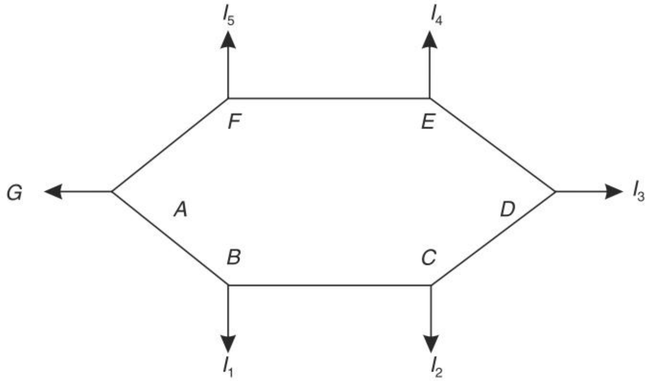


Fig. 15.21

(b) Double ring distributor: It looks like interconnected distribution system. To minimize the voltage drop at the far end, the inter connector is used as shown in Fig. 15.22.

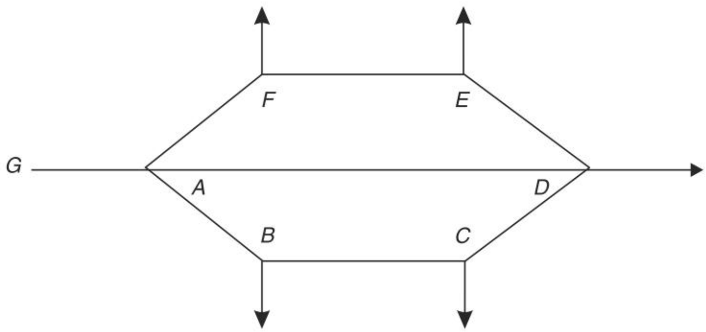


Fig. 15.22

Line Loss In transmission and distribution network, there are I^2R losses in lines and transformers. These average at about 20% of the power input. It is also to be remarked here that only 1/5 of the heat energy put in the furnace reaches the user. All this should make us realize that electric energy must be utilized in a conservative manner.

15.12 HIGH VOLTAGE DC (HVDC) TRANSMISSION

It has been shown in Sec. 15.7 that dc transmission requires much less conductor than ac 3-phase transmission. However, dc transmission was not possible as high voltage needed high power transmission which has not been possible since large power could not be generated. Further it is not been possible to transfer level of dc volts. However, dc transmission became operational as early as 1954 with the development of power switching devices. After the advent of high current, high voltage solid- state switching devices (thyristor), several HVDC lines have been constructed in the world for transmitting dc power in thousands of MWs. Such HVDC lines are embedded in an overall ac transmission and are converted to the rest of the system for power exchanging through transformer and converters.

Apart from 3 back-to-back HVDC stations already in operation in India, HVDC lines from Chandrapur to Padghe have been commissioned in 1999. The rating of this system is 1500 MW, ± 500 kV bipolar with a length of 754 km.

Principal of AC/DC Conversion

HVDC transmission consists of two converter which are connected to each other by a DC cable or an overhead DC line. A typical arrangement of main components of an HVDC transmission is shown in Fig. 15.23.

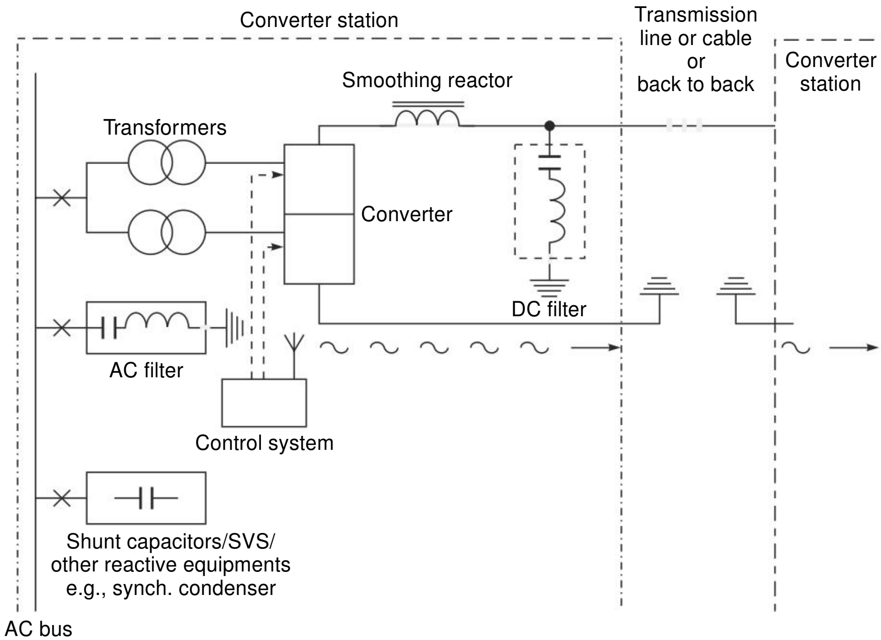


Fig. 15.23 Main components of a HVDC transmission—a typical arrangement

Two series connected 6-pluse converters (12-pulse bridge) consisting of thyristor valves and converter transformers are used. The valves convert AC to DC, and the transformer provides a suitable voltage ratio to achieve the desired direct voltage and galvanic separation of the AC and DC systems. A smoothing reactor in the DC circuit reduces the harmonic currents in the DC line, and the possible transient overcurrents.

Filters are used to take care of harmonics generated at the conversion. Thus we see that in an HVDC transmission, power is taken from one point in an AC network, where it is converted to DC in a converter station (rectifier), transmitted to another converter station (inverter) via line or a cable and injected into an AC system.

Economics of DC Transmission The cost of terminal equipment is much more in case of DC (converting stations) than in case of AC (transformer/substations). If we plot the variation of cost of power as a function of transmission distance, it will be as shown in Fig. 15.24. The slope gives the cost per unit length of the line and other accessories. The point of intersection *P* is called a breakeven point which shows that

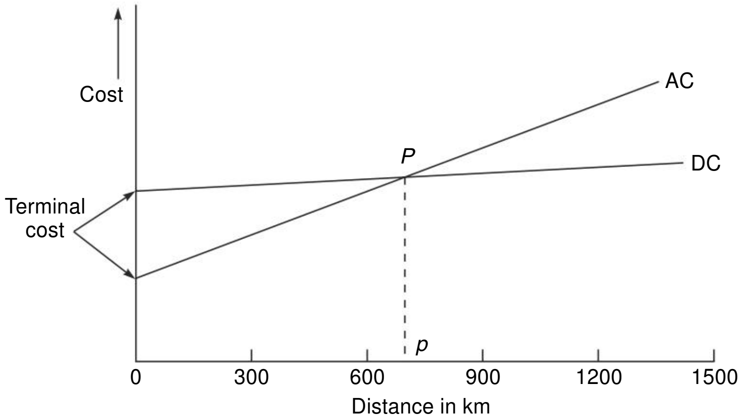


Fig. 15.24

if the transmission distance is more than 0*p*, it is preferable to use DC; otherwise AC should be used.

There is hardly any scope to reduce the cost of AC terminal equipment. But a lot of progress has been made in the development of converting devices, and the breakeven distances are reducing with further development of these devices.

Present day breakeven distance in favour of DC transmission is 700 km for overhead lines. However, the breakeven distance varies with each individual project and should always be checked.

Advantages of HVDC Systems The advantages of the HVDC systems are as follows:

1. These systems are economical for long distance bulk power transmission by overhead lines.
2. There is greater power per conductor and simpler line construction.
3. Ground return is possible
4. There is no charging current.
5. The voltage regulation problem is must less serious for DC, since only the IR drop is involved ($I_X = 0$).
6. There is easy reversibility and controllability of power flow through a DC link.
7. There is considerable insulation economy. The peak voltage of the 400 kV AC line is $2 \times 400 = 564$ kV. So the AC line requires more insulation between the tower and conductors, as well as greater clearance above the earth as compared to corresponding 400 kV DC line.
8. Smaller amount of right of way is required. The distance between two outside conductors of a 400 kV AC line is normally 20 m, whereas the same between a corresponding DC line is roughly half, i.e. 10 m only.
9. Line losses are smaller.

Disadvantages of HVDC Systems

1. The systems are costly since installation of complicated converters and DC switchgear is expensive.
2. Harmonics are generated which require filters.
3. Converters do not have overload capability.
4. Lack of HVDC circuit breakers hampers network operation. There is no DC device which can perform excellent switching operations and ensure protection.
5. There is nothing like DC transformer which can change the voltage level in a simple way. Voltage transformation has to be provided on the AC sides of the system.

Further, power system would include a transmission mix of AC and DC. Future controllers would be more and more microprocessor based, which can be modified or upgraded without requiring hardware changes, and without bringing the entire system down. While one controller is in action, the duplicate controller is there as a 'hot standby' in case of a sudden need

It is by now clear that HVDC transmission is already a reliable, efficient and cost-effective alternative to HVAC for many applications. Currently a great deal of effort is being devoted to further research and development in solid state technology.

Example 15.1 In DC two-wire system, a feeder is working on 250 V to supply a constant load. If the supply voltage is increased to 480 V with power transmitted remaining the same, compute the percentage saving in conductor material.

Solution

For 250 V supply

$$\text{Line current } I_1 = \frac{P}{V_1} = \frac{P}{250}$$

Cross-sectional area of conductor required

$$A_1 = \frac{I_1}{J} = \frac{P}{250J}.$$

where J = permissible current density

Volume of conductor material

$$\text{Vol 1} = 2l\,A_1 = \frac{2lP}{250J} = \frac{lP}{125J};\ l = \text{line length} \qquad \text{(i)}$$

For 480 supply

$$I_2 = \frac{P}{V_2} = \frac{P}{480}$$

$$\text{Cross-sectional area } A_2 = \frac{I_2}{J} = \frac{P}{480j}$$

Volume of conductor material required

$$\text{Vol 2} = 2l\,\text{A}_2 = \frac{2lP}{480J} = \frac{lP}{240J} \qquad \text{(ii)}$$

$$\therefore \qquad \frac{\text{Vol 2}}{\text{Vol 1}} = \frac{lP}{240J} \times \frac{125J}{LP} = \frac{125}{240} = 0.52$$

$\therefore$ Percentage saving in conductor material $= \dfrac{\text{Vol 1} - \text{Vol 2}}{\text{Vol 1}} \times 100$

$$= \left(1 - \frac{\text{Vol 2}}{\text{Vol 1}}\right) \times 100$$

$$= (1 - 0.52) \times 100 = 48\%$$

Example 15.2 A 50 km long transmission line supplies a load of 5MVA at 0.85 power factor lagging at 33 kV. The efficiency of transmission is 90%. Calculate the volume of aluminium conductor required for the line for (a) single-phase 2-wire system and (b) three-phase, 3-wire system. Given $\rho_{Al} = 3 \times 10^{-8}$ Ωm.

Solution

(a) Power transmitted = 5 × 0.85 = 4.25 MW

Line loss = 10% of power transmitted = 0.425 MW (a) Single-phase system

$$MVA = VI \times 10^{-3}$$

$$5 = 33 \times I \times 10^{-3}$$

$$\therefore \qquad I = \frac{5 \times 10^3}{33} = 151.5 \text{ A}$$

Let A_1 be the area of cross-section of each conductor.

Line loss, $\quad P_L = 2\, I^2R = 2I^2 \dfrac{Pl}{A_1}$

$$A_1 \therefore \qquad \frac{2I^2Pl}{P_L} = \frac{2 \times (151.5)^2 \times 3 \times 10^{-8} \times 50 \times 10^3}{0.425 \times 106}$$

$$= 1.62 \times 10^{-4} \text{ m}^2$$

Volume of conductor required

$$\text{Vol}_1 = 21A_1 = 2 \times 50 \times 10^3 \times 1.62 \times 10^{-4} = 16.2 \text{ m}^3$$

(b) Three-phase, three-wire AC system :

$$MVA = \sqrt{3}\, V_L I_L \times 10^{-3}$$

$$S = \sqrt{3} \times 33 \times I_L \times 10^{-3}$$

$$I_L = \frac{5}{\sqrt{3} \times 33 \times 10^{-3}} = 87.48 \text{ A}$$

Let the area of each phase conductor be A_2

Total line loss $= 3I_L^2 R_2 = 3\, I^2_L \dfrac{el}{A_2}$.

$$A_2 = \frac{3I_L^2 \rho l}{P_L} = \frac{3 \times (87.48)^2 \times 3 \times 10^{-8} \times 50 \times 10^3}{0.425 \times 10^6}$$

$$= 0.81 \times 10^{-4} \text{ m}^2$$

$$\text{Vol}_2 = 3l\, A_2 = 3 \times 50 \times 10^3 \times 0.81 \times 10^{-4} = 12.15 \text{ m}^3$$

Example 15.3 An electric (specify the type lamp with real less pf) draws a current of 0.8A when connected across a 220 V. 50 Hz supply. The power consumed by the lamp is 75 W. Find the value of the capacitance to be connected in parallel with the lamp to improve the power factor to (a) 0.9 lagging and (b) unity.

Solution:

$$I = 0.8 \text{ A } V = 220 \text{ } V, P = 75 \text{ w}$$

$$P = VI \cos \phi$$

$$\cos \phi = \frac{75}{220 \times 0.8} = 0.426. \therefore \phi = 64.79°$$

(a) $\cos \phi_{\text{new}} = 0.9 \therefore \phi_{\text{new}} = 25.84 \therefore \tan \phi_{\text{new}} = 0.484$

$$I_c = I \cos \phi (\tan \phi - \tan \phi_{\text{new}}) = 0.8 \times 0.426 (2.12 - 0.484)$$

$$= 0.557 \text{ A.}$$

$$C = \frac{I_c}{V_\omega} = \frac{0.557}{220 \times 2\pi \times 50} = 8\mu\text{F.}$$

(b) $\cos \phi_{\text{new}} = 1; \tan \phi_{\text{new}} = 0$

$$I_C = 0.8 \times 0.426 \times 2.12 = 0.7224.$$

$$C = \frac{0.7224}{220 \times 100 \times \pi} = 10.45 \text{ } \mu\text{F.}$$

Example 15.4 A synchronous condenser costs Rs. 100 per KVA. If a consumer is charged at Rs. 60 per annum per KVA of maximum demand and the interest and depreciation charges are 10% on the capital invested, determine the most economical power factor. The power factor of the consumer load is 0.8 lagging. Assume that there are no losses in the synchronous condenser.

Solution Let P_{kw} be the load of the consumer. Then KVA supplied to consumer is $p/\cos \phi_1$. Let the improved pf be $\cos\phi$ due to installation of the synchronous condenser. New KVA maximum demand of the consumer

$$= P\cos \phi_2$$

Saving due to decrease in maximum demand

$$= \text{Rs } 60 \left(\frac{P}{\cos \phi_1} - \frac{P}{\cos \phi_2}\right)$$

KVAR supplied by the synchronous condenser

$$= P \tan \phi_1 - P \tan \phi_2$$

Capital cost of condenser

$$= \text{Rs } 100 \times P (\tan \phi_1 - \tan \phi_2)$$

Annual interest and depreciation charge @ 10%

$$= \text{Rs } 10 \text{ } P (\tan \phi_1 - \tan \phi_2)$$

Net saving of the consumer :

$$S = \text{Rs } 60 \text{ } P \left(\frac{1}{\cos \phi_1} - \frac{1}{\cos \phi_2}\right) - 10 \text{ } P (\tan \phi_1 - \tan \phi_2)$$

For maximum saving $\frac{dS}{d\phi_2} = 0.$

$$\frac{dS}{d\phi_2} = 60P\left(\frac{\rho \sin\phi_2}{\cos^2\phi_2}\right) + 10\,P\sec^2\phi_2 = 0 \quad \text{or}$$

$$\frac{60\sin\phi_2}{\cos^2\phi_2} = 10\sec^2\phi_2 = \frac{10}{\cos^2\phi}$$

$\therefore$ $\sin\phi_2 = 1/6$ $\therefore \phi_2 = 9.6$ $\therefore$ $\cos\phi_2 = 0.986$ lagging

$\therefore$ most economical *pf* of the consumer is 0.986 lagging.

Example 15.5 What will be the rating of a phase advancing plant in KVA if it improves the *pf* from 0.8 log to 0.9 log? The consumer load is 400 kW and the current taken by the phase advancer leads the supply voltage at a pf of 0.25.

Solution: Let ϕ_2 be the pf angle of supply current after the phase advancing plant is used and ϕ_3 be the pf angle of phase advancing plant.

From the phasor diagram of Fig. 15.25, we have

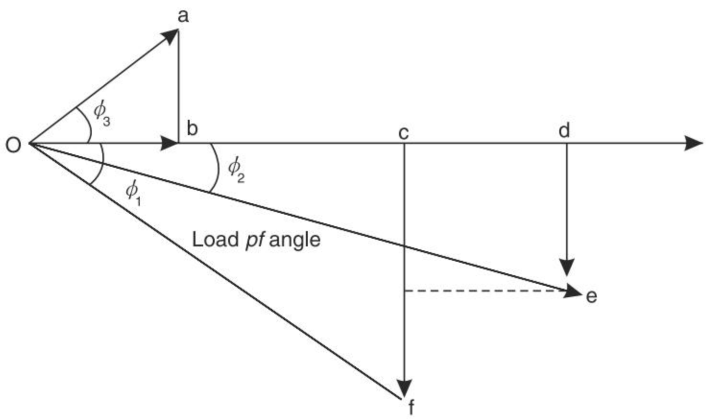


Fig. 15.25

Oc = power taken by load

Ob = power taken by the phase advancing plant

Od = *Oc* + *Ob* = total power taken by load and phase advancing plant.

cf = reactive KVAR (logging) of the load

ab = reactive KVAR (leading) of the phase advancing plant

dc = reactive KVAR (logging) taken by load and phase advancing plant = *cf* – *ab*

Oc = 400 kW $\cos\phi_1 = 0.8$; $\sin\phi_1 = 0.6$

$$\therefore\ ab = \frac{400}{\cos\phi_1} \times \sin\phi_1 = \frac{400 \times 0.6}{0.8} = 300 \text{ kW}$$

$$\cos\phi_3 = 0.25;\ \sin\phi_3 = 0.968$$

Let the rating of the phase advancing plant be γ KVA

kW taken by the plant $ob = \gamma\cos\phi_3 = 0.25\,\gamma = cd$

$$ab = \gamma \sin\phi_3 = 0.968\,\gamma$$

$$od = (400 + 0.25\gamma)\text{ kW } cd = (300 - 0.968\gamma)\text{ KVAR}$$

$$\cos \phi_2 = ed/od = 0.484 = \frac{300 - 0968\gamma}{400 + 0.25\gamma}.$$

Solving $\gamma = 97.7$

$\therefore$ Rating of the phase advancing plant = 97.7 KVA

Example 15.6 A 3-phase overhead line has a reactance of 10 Ω, while its resistance can be ignored. The receiving-end load is 2 MW at 0.8 logging pf. The receiving end voltage is required to be maintained at 11 kV. What should be the sending end voltage? What is the voltage regulation of the line?

If the sending end voltage is also 11 kV, what should be the pf of 2 MW receiving-end load? In order to achieve this pf what will be the Q of the shunt capacitor connected in shunt to load as above i.e. 2 MW, 0.8 pf.

Solution

Refer to Fig. 15.14

Line current

$$P\,(3\text{ phase}) = \sqrt{3}\; V_R I_R \cos \phi$$

$$2 \times 10^6 = \sqrt{3}\; (11 \times 10^3)\, I_R \times 0.8$$

or $$I_R = \frac{2 \times 10^6}{\sqrt{3} \times 11 \times 10^3 \times 0.8} = 131.2 \text{ A}$$

Sending end voltage (eq. (16.22))

$$V_S = V_R + I_R X_L \sin \phi$$

$$V_R\,(\text{phase}) = 11/\sqrt{3} = 6.35 \text{ kV or } 6350 \text{ V}$$

Then

$$V_S = 6350 + 131.2 \times 10 \times 0.6 = 7137 \text{ V or } 7.137 \text{ kV}$$

$$V_S = (\text{line-to-line}) = \sqrt{3} \times 7.137 = 12.36 \text{ kV}$$

$$\text{Voltage regulation} = \frac{12.36 - 11}{11} = 0.1236 \text{ or } 12.36\%$$

Now it is required that sending end voltage should also be 11 kV or

$$V_S\,(\text{phase}) = 11\sqrt{3} = 6.35 \text{ kV}$$

As $V_S = V_R$, voltage drop (magnitude) in zero or from Eq. (16.22)

$$I_R X_L \sin \phi = 0$$

As I_R is not zero,

$$\sin \phi = 0 \text{ or } \phi = 0°$$

$$\therefore \quad \text{pf} = \cos \phi = 1$$

It means Q = 0 for load plus capacitor

Load is 2 MW, pf = 0.8 $\therefore \phi = 36.7°$

$$\tan \phi = \frac{Q}{P} \text{ or } Q = P \tan \phi$$

$$Q = 2 \tan 36.7° = 1.5 \text{ MVAR}$$

As Q = 0, capacitor will provide

$$Q_c = 1.5 \text{ MVAR}$$

To find the value of the capacitor, from Eq. (16.17c)

$$Q_c = V_R^2\, \omega C$$

$$\frac{1.5}{3} \times 10^6 = \left(\frac{11}{\sqrt{3}} \times 10^3\right)^2 \times (2\Omega \times 50)\, C;\ f = 50 \text{ H}_z$$

or $C = 3.946 \times 10^{-5}$ $F = 39.46$ μF.

Example. 15.7 A 3-phase load of 24 MW, 0.8 pf in fed from a transmission line with a receiving end voltage of 33 kV. The line reactance of the line is 20W and its resistance is 4 Ω. What should be the sending end voltage and line current?

Shunt capacitor is to be installed at this load end so that this sending end voltage is neme as receiving end voltage. Calculate also the value of capacitance per phase.

Solution

Converting data to per phase

$$V_R = \frac{33}{\sqrt{3}} = 19.05 \text{ kV}$$

$$P = \frac{24}{8} = 8 \text{ mw } 0.8\ pf,\ \cos\phi = 0.8,\ \sin\phi = 0.6$$

$$\text{Line current } I_a = \frac{8 \times 10^6}{19.05 \times 10^3 \times 0.8} = 525 \text{ A. or } 0.525 \text{ kf}$$

$$\text{Given, } R_L = 4\Omega,\ X_L = 20\ \Omega$$

As per eqn. (15.22b)

$$V_S \simeq V_R + I_R\,(X_L \sin\phi + R_L \cos\phi)$$

$$= 19.05 + 0.525\,(20 \times 0.6 + 4 \times 0.8)$$

$$= 27.03 \text{ kV}$$

$$V_S \text{ (line)} = \sqrt{3} \times 27.03 = 46.8 \text{ kV}$$

$$\text{Voltage regulation} = \frac{27.03 - 19.05}{19.05} = 41.9\ \%$$

It is noted that voltage regulation is unexpectedly high. So capacitor compensation is needed.

$$\text{For } V_R = V_S$$

$$X_L \sin\phi_1 + R_L \cos\phi_1 = 0.$$

$$20 \sin\phi_1 + 4 \cos\phi_1 = 0 \text{ or } \tan\phi_1 = -0.2 \text{ or } \phi = -11.3°$$

Load plus shunt capacitor has pf of $\cos\phi_1 = 0.981$ leading. Real power load remains the same i.e. $P = 25/3$ MW. current drawn by load is

$$\bar{I}_a = 0.525\,(0.8 - j\,0.6) = 0.42 - j\,0.315 \text{ A.}$$

Current drawn by load plus capacitor

$$\bar{I}_{a1} = \frac{8 \times 10^6}{19.05 \times 10^3 \times 0.981} = 428 \text{ A} = 0.428 \text{ kA}$$

Observation: Capacitor compensation reduces the line current from 0.525 kA to 0.428 kA.

$$\cos\phi_1 = 0.981 : \quad \sin\phi_1 = 0.196.$$

$$\bar{I}_{a1} = 0.428\,(0.981 + j\,0.196);\ \text{leading pf.}$$

$$= 0.42 + j\,0.257$$

One line diagram of capacitor compensation B drawn in Fig. 15.26. from which

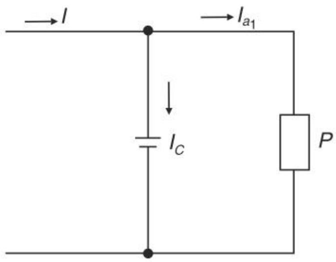


Fig. 15.26

$$\bar{I}_C = \bar{I}_{a1} - \bar{I}_a$$

$$= j\,0.257$$

$$+ j\,0.315$$

$$= j\,0.572 \text{ kA}$$

The capacitor current

$$I_C = 0.572 \text{ kA}$$

$$I_C = V_R C\omega$$

$$C = \frac{I_c}{\omega V_R} = \frac{0.572 \times 10^3 \times 10^6}{2\pi \times 50 \times 19.05 \times 10^3}$$

$$= 95.58\ \mu\text{F}.$$

Efficiency comparison

$$I_a = 0.525 \text{ kA}$$

$$\text{Line loss} = (0.525)^2 \times 4 = 1.1025 \text{ MW/phase}$$

Compensated case : $I_{a1} = 0.428$ kA

$$\text{Line loss} = (0.428)^2 \times 4 = 0.737 \text{ MW/phase}$$

$$\eta = \frac{8}{8 + 0.737} = 91.6\%$$

Observation: Compensation increases the transmission efficiency from 78.9% to 91.6%.

SUMMARY

- High voltage is essential for transmission of large power in order to reduce the conductor cost and bring the efficiency to acceptable levels. Of, *all possible power transmission methods the most economical is dc 2-wire followed by ac 3-phase, 3-wire.*
- Ac transmission is universal because of generating power at high voltage and the facility of step-up are step-down of voltage levels. This is not possible in dc transmission.
- Econonmy of HVDC transmission becomes visible for transmitting large power over long distances (700 km and above). Devoted lines are used for HVDC embedded in large interconnected ac power network.
- For improving the efficiency of power transmission lines, the receiving-end load power factor must be high.
- Most power loads have lagging power factor, which can be improved by

shunt capacitors which supply positive VARs.

- The value of capacitor bank to provide Q_C VARs is

$$C = \frac{Q_C}{\omega V^2} \text{ Per phase basic}$$

- At light load the receiving-end voltage becomes high. The corrective measure is to switch in inductor bank to feed negative VARs.
- In place of capacitor and inductor banks synchronous condenser (synchronous motor at low-load) can be employed. When over-excited, it feeds positive VARs to the line and upon under-exciting, it feeds negative VARs. Installation is expensive needs regular maintenance.
- To provide security and reliability of power supply and for power sharing, large power networks are interconnected through transmission forming a grid.

REVIEW QUESTIONS

1. List out the merits and demerits of having high transmission line voltage.
2. Tabulate the current, voltage and line losses in various transmission system types.
3. Study the effects of poor power factor and compare the power factor correction methods.
4. Brief the effect of reactive power flow on transmission line voltage.

PROBLEMS

15.1 A single-phase 50 Hz generator, supplies an inductive load of 5000 kW at a pf of 0.707 lagging by means of an overhead transmission line 20 km long. The line inductance is 0.63 mH/km. The voltage at the receiving-end is required to be kept constant at 10 kV. Find (a) the sending-end voltage and voltage regulation of the line (b) the value of the capacitors to be placed in parallel with the load such that the regulation is 50% of that obtained in part (a), and (c). Compare the transmission efficiencies in part (a) and (b).

15.2 What is the percentage saving in feeder copper if the line voltage in a 2-wire DC systems is raised from 100 V to 200 V for the same power transmitted over the same power distance and having the same power loss?

(**Ans :** 75%)

15.3 A single-phase AC system supplies a certain power, and this system is converted to three-phase AC system by running a third similar copper conductor. Calculate the percentage additional load that can be supplied for the same voltage between the conductors, and the same percentage loss.

(**Ans :** 100%)

15.4 A sample of copper wire has a resistance of 50 ohm at 10° C. What must be the maximum operating temperature of the wire if its resistance is to increase by 10%? Take the temperature coefficient at 10° C to be $\alpha = 0.004\ C^{-1}$.

15.5 A load of three impedances each (8 + j12) ohm is supplied through a line having an impedance of (2 + j4) ohm. The supply voltage is 400 V, 50 Hz. Determine the power input and output when the load is (i) star connected and, (ii) delta connected.

15.6 A 3-phase load of 9 MW at 0.707 lagging pf is supplied from a transmission line having a resistance of 0.4 Ω and a reactance of 4 Ω. The load voltage is required to be 12 kV.

(a) Find the sending-end voltage and voltage regulation.

(b) The voltage regulation is required to be reduced to half of that found in part (a). Find the size of the capacitor to be connected in shunt across load to achieve this.

(**Ans:** 131 μF)

15.7 A 3-phase 33 kV line has reactance of 20 Ω per phase, its resistance can be ignored. The line is feeding a balanced 3-phase load of 15 MW with voltage of 33 kV at both receiving and sending ends (zero voltage regulation). What is the reactive power drawn by the load and power angle?

What is the maximum power that can be supplied by the line end? What is the corresponding reactive power of the load?

(**Ans:** 0.708 MVAR, 15.99°, 0)

Chapter 16

DOMESTIC WIRING

MAIN GOALS AND OBJECTIVES

- *Electrical domestic wiring*
- *Types of wiring*
  – Suitability of a particular wiring system for a given installation
- *Corridor and staircase lighting*
- *Necessity of earthing*
  – Different types of earthing
- *Fuses and different types of fuses*

Electrical wiring done in residential and commercial buildings to provide power for lights, fans, pumps and other domestic appliances is known as domestic wiring. There are several wiring systems in practice such as tree system and distribution systems.

16.1 TYPES OF WIRING

- Cleat wiring
- CTS wiring or TRS wiring or batten wiring
- Metal sheathed wiring or lead sheathed wiring
- Casing and capping
- Conduit wiring

1. Cleat Wiring In this type of wiring, insulated conductors (usually VIR, Vulcanized Indian Rubber) are supported on porcelain or wooden cleats. The cleats have two halves: one base and the other cap. The cables are placed int the grooves provided in the base and then the cap is placed. Both are fixed securely on the walls by 40 mm long screws. The cleats are easy to erect and are fixed 4.5–15 cms apart. This wiring is suitable for temporary installations where cost is the main criteria but not the appearance.

2. CTS (Cable Tyre Sheathed)/ TRS (Tough Rubber Sheathed)/Batten Wiring In this wiring system, wires sheathed in tough rubber are used which are quite flexible. They are clipped on wooden battens with brass clips (link or joint) and fixed on to the walls or ceilings by flat head screws. These cables are moisture and chemical proof. They are suitable for damp climate but not suitable for outdoor use in sunlight. TRS wiring is suitable for lighting in low voltage installations.

3. Metal Sheathed or Lead Sheathed Wiring The wiring is similar to that of CTS but the conductors (two or three) are individually insulated and covered with a common outer lead-aluminium alloy sheath. The sheath protects the cable against dampness, atmospheric extremities and mechanical damages. The sheath is earthed at every junction to provide a path to ground for the leakage current. They are fixed by means of metal clips on wooden battens. The wiring system is very expensive. It is suitable for low voltage installations.

4. Casing and Capping It consists of insulated conductors laid inside rectangular, teakwood or PVC boxes having grooves inside it. A rectangular strip of wood called capping having same width as that of casing is fixed over it. Both the casing and the capping are screwed together at every 15 cms. Casing is attached to the wall. Two or more wires of same polarity are drawn through different grooves. The system is suitable for indoor and domestic installations.

5. Conduit Wiring: In this system, PVC (polyvinyl chloride) or VIR cables are run through metallic or PVC pipes providing good protection against mechanical injury and fire due to short circuit. They are either embedded inside the walls or supported over the walls, and are known as concealed wiring or surface conduit wiring (open conduit) respectively. The conduits are buried inside the walls on wooden gutties and the wires are drawn through them with fish (steel) wires. The system is best suited for public buildings, industries and workshops.

16.2. SPECIFICATION OF WIRES

The conductor material, insulation, size and the number of cores, specifies the electrical wires. These are important parameters as they determine the current and voltage handling capability of the wires. The conductors are usually of either copper or aluminium. Various insulating materials like PVC, TRC, and VIR are used. The wires may be of single strand or multistrand. Wires with combination of different diameters and the number of cores or strands are available.

For example: The VIR conductors are specified as 1/20/,3/22,.....7/20 ....

The numerator indicates the number of strands while the denominator corresponds to the diameter of the wire in SWG (Standard Wire Gauge). SWG 20 corresponds to a wire of diameter 0.914 mm, while SWG 22 corresponds to a wire of diameter 0.737 mm.

Wiring may be the simplest wiring like controlling the appliances from single point. Or two-way and three-way control is also possible with reasonable complexity.

16.3 EARTHING

Earthing is to connect any electrical equipment to earth with a very low resistance wire, making it to attain earth's potential. The wire is usually connected to a copper plate placed at a depth of 2.5 to 3 meters from the ground level. The potential of the earth is considered to be at zero for all practical purposes as the generator (supply) neutral is always earthed. The body of any electrical equipment is connected to the earth by means of a wire of negligible resistance to safely discharge electric energy, which may be due to failure of the insulation, or line coming in contact with the casing, etc. Earthing brings the potential of the body of the equipment to ZERO i.e. to the earth's potential, thus protecting the operating personnel against electrical shock. The body of the electrical equipment is not connected to the supply neutral due to long transmission lines and intermediate substations, the same neutral wire of the generator will not be available at the load end. Even if the same neutral wire is running, it will have a self-resistance, which is higher than the human body resistance. Hence, the body of the electrical equipment is connected to earth only.

Necessity of Earthing:

1. The protect the operating personnel from danger of shock in case they come in contact with the charged frame due to defective insulation.
2. To maintain the line voltage constant under unbalanced load condition.
3. To protect the equipments.
4. T protect large buildings and all machines fed from overhead lines against lightning.

16.4 METHODS OF EARTHING

The important methods of earthing are the plate earthing and the pipe earthing. The earth resistance for copper wire is 1 ohm and that of *GI* wire less than 3 ohms. The earth resistance should be kept as low as possible so that the neutral of any electrical system, which is earthed, is maintained almost at the earth potential. The typical value of the earth resistance at powerhouse is 0.5 ohm and that at substation is 1 ohm.

Plate Earthing In this method, a copper plate of 60 cm × 60 cm × 3.18 cm or a *GI* plate of the size 60 cm × 60 cm × 6. 35 cm is used for earthing. The plate is placed vertically down inside the ground at a depth of 3 m and is embedded in alternate layers of coal and salt for a thickness of 15 cm. In addition, water is poured for keeping the earth electrode resistance value well below a maximum of 5 ohms. The earth wire is securely bolted to the earth plate. A cement masonry chamber is built with a cast iron cover for easy regular maintenance.

Pipe Earthing Earth electrode made of a *GI* (galvanized) iron pipe with 38mm diameter and length of 2 m (depending on the current) with 12 mm holes on the

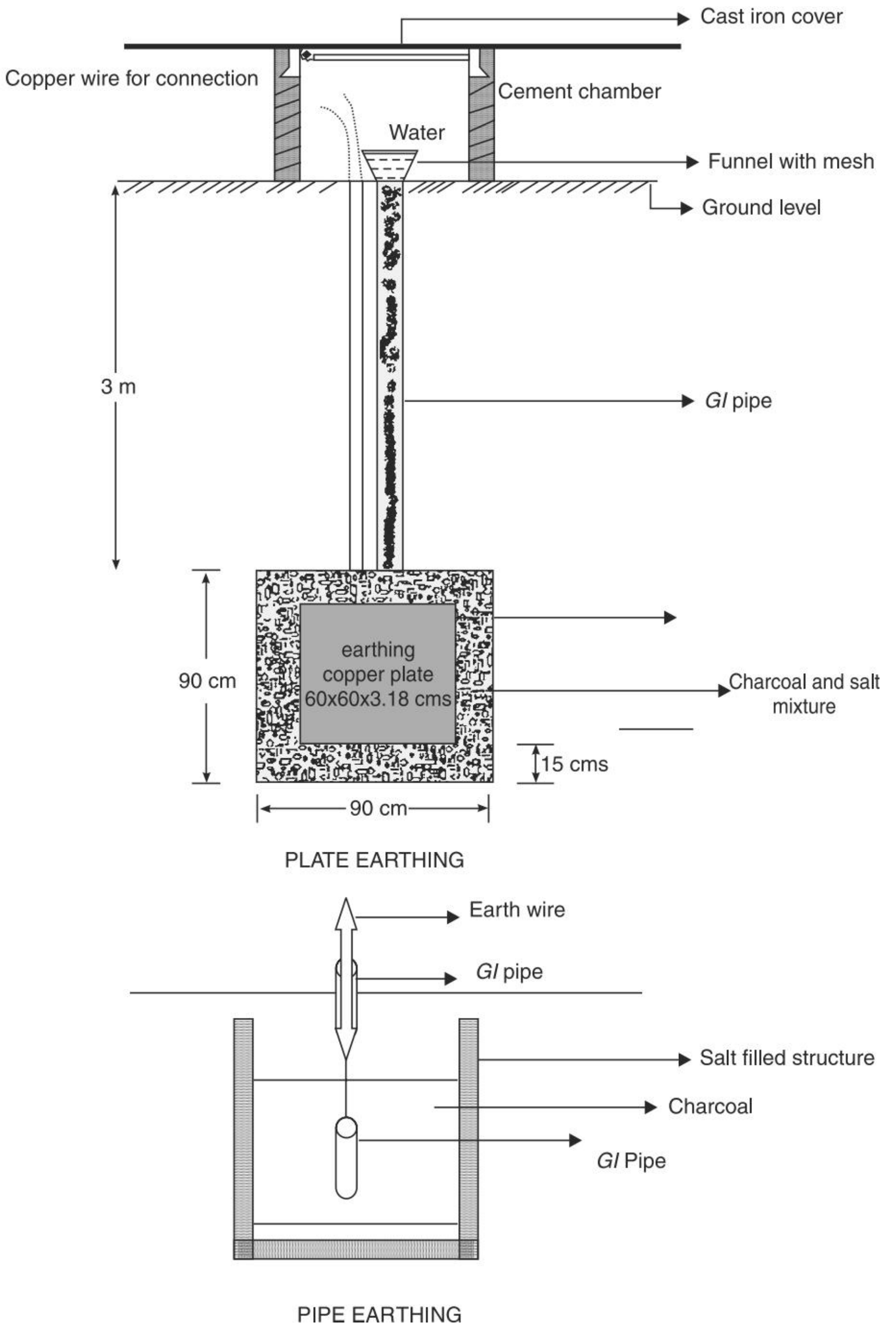


Fig. 16.1 Pipe earthing

surface is placed upright at a depth of 4.75 m in a permanently wet ground. To keep the value of the earth resistance at the desired level, the area (15 cm) surrounding the *GI* pipe is filled with a mixture of salt and coal. The efficiency of the earthing system is improved by pouring water through the funnel periodically. The *GI* earth wires of sufficient cross-sectional area are run through a 12.7 mm diameter pipe (at 60 cms below) from the 19 mm diameter pipe and secured tightly at the top as shown in the following figure.

When compared to the plate earth system, the pipe earth system can carry larger leakage currents as a much larger surface area is in contact with the soil for a given electrode size. The system also enables easy maintenance as the earth wire connection is housed at the ground level.

16.5 PROTECTIVE DEVICES

Protection for electrical installation must be provided in the event of faults such as short circuit, overload and earth faults. The protective circuit or device must be fast acting and isolate the faulty part of the circuit immediately. It also helps in isolating only required part of the circuit without affecting the remaining circuit during maintenance. The following devices are usually used to provide the necessary protection:

- Fuses
- Relays
- Miniature circuit breakers (MCB)
- Earth leakage circuit breakers (ELCB)

The simplest form of protection element is fuse. The remaining three devices are advanced and are placed in the electrical circuit to protect the circuit and other equipment from the faulty part. The electrical equipments are designed to carry a particular rated value of current under normal circumstances. Under abnormal conditions such as short circuit, overload or any fault, the current raises above this value, damaging the equipment and sometimes resulting in fire hazard. Fuses are pressed into operation under such situations. Fuse is a safety device used in any electrical installation, which forms the weakest link between the supply and the load. It is a short length of wire made of lead / tin / alloy of lead and tin/ zinc having a low melting point and low ohmic losses. Under normal operating conditions it is designed to carry the full load current. If the current increases beyond this designed value due any of the reasons mentioned above, the fuse melts (said to be blown) isolating the power supply from the load.

REVIEW QUESTIONS

1. Analyze the advantages and drawbacks of different types of wiring.
2. What are the deciding factors of selection of wires?
3. Discuss the importance of earthing?
4. What are basic requirements of fuses?

Appendix A

GRAPH THEORY

In Chapter 1 certain definitions were advanced concerning networks—*node*, *branch*, *path*, *loop* and *mesh*. In drawing the graph (*linear graph*) of a network each element is represented by a line segment which are then connected as in the *topology* of the network. The linear graph of the network of Fig. A.1(a) is drawn in Fig. A.1(b). Linear graph is a general concept but here we have defined it in specific relation to electric networks.

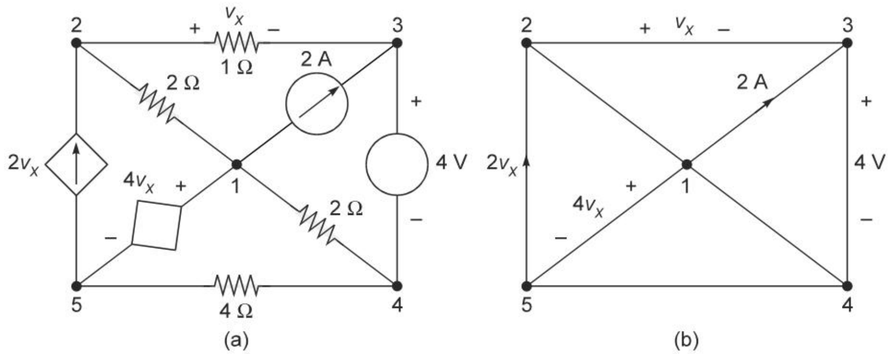


Fig. A.1 (a) Network (b) Linear graph of the network of Fig. A.1(a)

CERTAIN DEFINITIONS

Connected Graph A graph is said to be connected if a path can be found between any two nodes of it. Figure A.1(b) is a connected graph.

Tree It is the subgraph of a connected graph containing all the nodes of the graph, and is itself a connected graph and contains no loops. Many trees can be found for a graph, i.e. a tree of a graph is *nonunique*. Figure A.2 gives one of the trees for the graph of Fig. A.1(b)) (thick lines). The branches of the tree are called *tree branches*.

$$N = \text{number of nodes}$$
$$\text{Number of tree branches} = N - 1$$

Cotree The branches of a graph not included in the tree are called *links*. The set of links of a tree constitutes the cotree. The cotree of the graph of Fig. A.1(b) is also shown in Fig. A.2 (thin lines).

$$\text{Number of links} = B - (N - 1) = B - N + 1$$

where B is the number of branches of the graph.

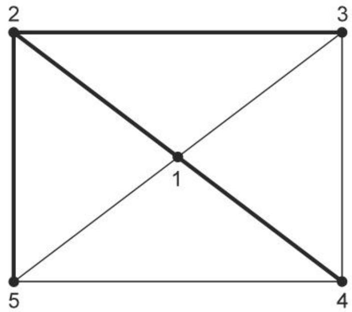


Fig. A.2 Tree and cotree of the graph of Fig. A.1(b)
Tree branches: Thick lines; cotree branches: thin lines

It is easily observed form Fig. A.2 that as one link is brought in at a time, it causes the creation of a single mesh. Following this procedure, as many meshes as the number of links $(B - N + 1)$ can be formed.

NODAL ANALYSIS

Sufficiency of Tree Branch Voltages

Reconsider the graph of Fig. A.2 with tree and cotree indicated. This is redrawn in Fig. A.3 with all the tree branch voltages shown. It easily follows that these are sufficient to determine the voltage of all the links. For example going round the mesh involving link 1 and applying KVL, we have

$$v_{t1} + v_{l1} - v_{t4} - v_{t2} = 0$$

or
$$v_{l1} = -v_{t1} + v_{t2} + v_{t4}$$

Similarly other link voltages can be found.

Independence of Nodal Equations

With one of the nodes as reference, KCL equations can be written at the remaining $(N-1)$ nodes for $(N - 1)$ unknown tree branch voltages. Every time a KCL equation is written for a new node, a new tree branch connecting that node to the remaining tree is encountered and, as a consequence, the $(N - 1)$ nodal equations form an independent set.

Consider the graph and the tree shown in Fig. A.3. Let node 5 be the reference node. As we write down KCL equations for nodes 4, 1, 3, 2 in that order each time a new tree branch voltage (v_{t4}, v_{t2}, v_{t1}, v_{t3}) is encountered in the nodal equations. So no equations can be obtained from a linear combination of the remaining three equations, and hence the set of four nodal equations is an independent set.

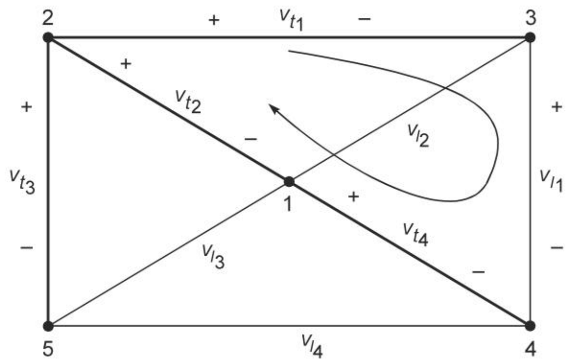


Fig. A.3

Rules for Selection Tree

1. Place all voltage sources (independent/dependent) in the tree.
2. Place all current sources (independent/dependent) in the cotree.
3. Place all control voltage branches of a voltage-controlled source in the tree, if possible.
4. Place all control current branches of a current-controlled source in the cotree, if possible.

As per the above rules, tree/cotree for the network of Fig. A.1(a) are drawn in Fig. A.4.

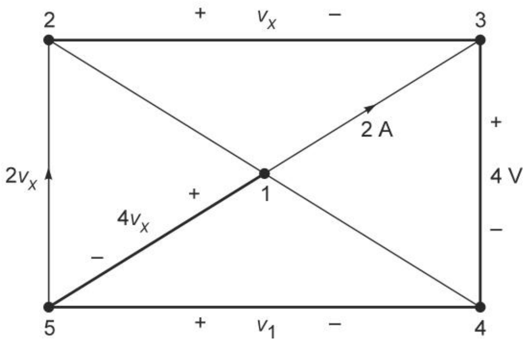


Fig. A.4 Tree and cotree of the network of Fig. A.1(a)

Nodal Equations

Before writing down nodal equations the tree branches corresponding to voltage sources (independent and dependent) and their nodes are coalesced into a single supernode as these tree branches pertain to known voltages or are specified as scaled values of some other branch voltages (tree or cotree). By carrying out this step the graph of Fig. A.4 is now redrawn in Fig. A.5 (the independent voltage source (4V) and its nodes 3 and 4 constitute the supernode (sn)1 and the dependent voltage source with its nodes 1 and 5 constitute the supernode (sn)2 which is also the reference node for this example.

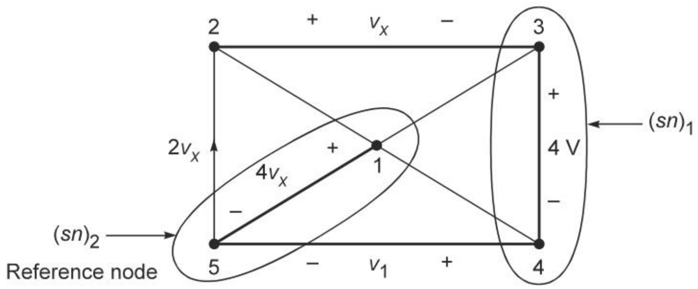


Fig. A.5 Voltage sources represented as supernodes

Nodal equations can now be written as below.

Node 2

$$\frac{v_x}{1} - 2v_x + \frac{v_x + 4 + v_1 - 4v_x}{2} = 0$$

$$-2.5v_x + 0.5v_1 = -2 \qquad \text{(i)}$$

Supernode $(sn)_1$

$$-\frac{v_x}{1} - 2 + \frac{v_1 - 4v_x}{2} + \frac{v_1}{4} = 0 \qquad = 0$$

$$-3v_x + 0.75v_1 = 2 \qquad \text{(ii)}$$

Solving Eqs (i) and (ii), we get

$$v_x = 6.67 \text{ V}, \; v_1 = \; 29.33 \text{ V}$$

Voltages and currents in all the branches can now be found.

Current through 4V source (i_{34})

Apply KCL at node 3

$$-\frac{6.67}{1} - 2 + i_{34} = 0$$

or $$i_{34} = 8.67 \text{ A}$$

Voltage of 2A source

Applying KVL round the loop 1, 5, 4, 3, 1

$$v_{13} + 4 + v_1 - 4v_x = 0$$

or $$v_{13} + 4 + 29.33 - 4 \times 6.67 = 0$$

or $$v_{13} = -2.65 \text{ V}$$

MESH ANALYSIS

Sufficiency and independence It has been seen above at the end of the definition of cotree that bringing in one link at a time creates one mesh, i.e. as many meshes as the number of link currents. The KVL equation for each mesh has a new link current which establishes the independence of the mesh equations ($B - N + 1$).

The set of independent mesh currents is shown in Fig. A.6 for the graph of Fig. A.3. It easily follows from Fig. A.6 that all branch currents can be obtained from the mesh link currents. For example for the branch 23

$$i_{23} = i_{l1} + i_{l4}$$

Also $\quad i_{12} = i_{l1} - i_{l2} - i_{l3} - i_{l4}$

Reconsider the example solved earlier by the method of nodal analysis. The graph with tree and cotree is redrawn in Fig. A.7. The four meshes corresponding to the four link currents (i_1, i_2, 2A, $2v_x$) are shown in the figure. Out of these there are only two independent link currents (i_1, i_2). The corresponding KVL mesh equations are written below:

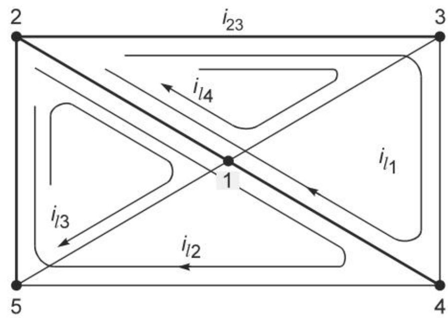


Fig. A.6

Mesh i_1 $\quad 2i_1 + 4(i_1 + i_2 + 2 + 2v_x) - 4v_x = 0 \quad$ (i)

Mesh i_2 $\quad 2i_1 + v_x + 4 + 4(i_1 + i_2 + 2 + 2v_x) - 4v_x = 0 \quad$ (ii)

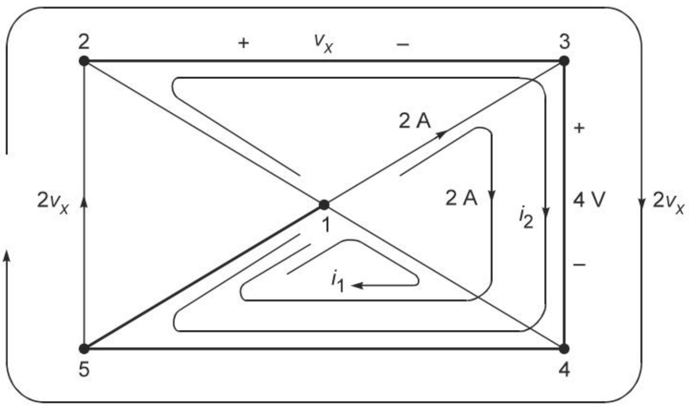


Fig. A.7

Also for branch 23

$$v_x = (i_2 + 2v_x) \times 1$$

or $\quad v_x = -\, i_2 \quad$ (iii)

Substituting Eq. (iii) Eqs. (i) and (ii), we get

$$i_1 = -\frac{4}{3}\text{ A}$$

$$4i_1 - i_2 = -12$$

or $\quad i_2 = -\dfrac{4}{3} = -6.67$ A

$\therefore \quad v_x = -\, i_2 = 6.67$ V (same as before)

PROBLEMS

A.1 For the circuit of Fig. P.A.1 draw the graph and define a suitable tree. Carry out nodal analysis and find the current through the 10 V source and voltage of the 2 A source.

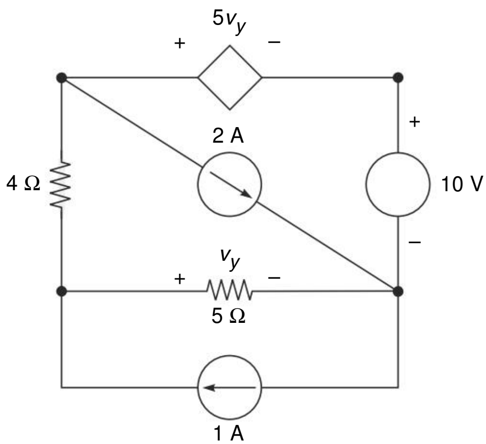


Fig. A.8

A.2 For the circuit of Fig. A.9 draw the graph and define a suitable tree. Carry out mesh analysis and find the current through the 12 V source and voltage of the 5 A source.

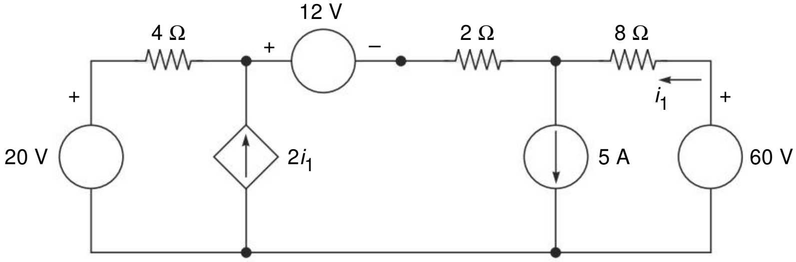


Fig. A.9

A.3 For the circuit of Fig. A.10 draw the graph and identify the tree. Carry out nodal as well as mesh analysis and therefrom find the voltage v25 and current i34.

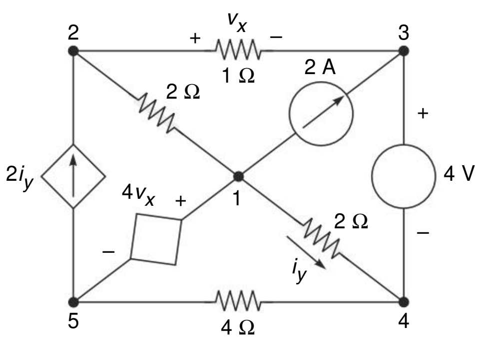


Fig. A.10

ANSWERS

A.1 –0.75 A, 3.75 V
A.2 4 A, 52 V
A.3 –13.33 V, –2A

Appendix B

RESISTANCE

Conduction in Metals

As has been explained in the text in Section 1.5 the conduction in metal is caused by the drift velocity (v_d) of charge carriers (electrons) under influence of electric field E.

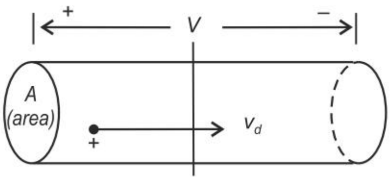


Fig. B.1

Though the charge carriers are electrons with charge $-q$ (1.6×10^{-19}C) we shall consider equivalently positive charge carriers, charge $+q$, as the conventional current is in the direction of flow of positive charges.

For simplicity we consider a long straight conduction with voltage V applied across it as shown in Fig. B.1. The charge carriers acquire a drift velocity.

$$v_d = k_d E \text{ m/s} \tag{B.1}$$

where

$E = \dfrac{V}{L}$, electric field in V/m

k_d = drift constant (mobility factor)

Let the concentration of charge carriers be n/m^3. Then the rate of charge flow across any cross-section of the conductor in time dt is

$$\mathrm{d}Q = q\,(nAv_d\,\mathrm{d}t)$$

The current flow is then given by

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t} = qnAv_d = qnAk_dE \tag{B.2}$$

or $$I = qn\,k_dA\,\frac{V}{L} \tag{B.3}$$

or $$I = \frac{V}{R} \tag{B.4}$$

where R = conductor *resistance* and Eq. (B.4) is the *Ohm's law.* From Eq. (B.3)

$$R = \rho\,\frac{L}{A}\ \Omega, \quad \rho = \frac{1}{qnk_d} \tag{B.5}$$

where ρ = *resistivity* if the conductor in Ω.m

The inverse of resistivity is

conductivity σ in units $(\Omega\text{–m})^{-1}$ or ℧/m

We can write Eq (B.2) as

$$J = \frac{I}{A} = qnk_d EA/\text{m}^2$$

where J = current density we can write

$$J = \frac{E}{\rho} = \sigma E \qquad \text{(B.6)}$$

where ρ = resistivity, It is the basic form of Ohm's law.

The resistivity of conductors, semiconductors and insulators is given in Table B.1.

Table B1

	Substance	ρ (Ω.*m*)	Substance	ρ (Ω.*m*)
Conductors	*Semiconductors*			
Metals:	Silver	1.47×10^{-8}	Pure carbon (graphite)	3.5×10^{-5}
	Copper	1.72×10^{-8}	Pure germanium	0.60
	Gold	2.44×10^{-8}	Pure silicon	2300
	Aluminum	2.75×10^{-8}	**Insulators**	
	Tungsten	5.25×10^{-8}	Amber	5×10^{14}
	Steel	20×10^{-8}	Glass	$10^{10} – 10^{14}$
	Lead	22×10^{-8}	Lucite	$>10^{13}$
	Mercury	95×10^{-8}	Mica	$10^{11} – 10^{15}$
Alloys:	Manganin (Cu 84%, Mn 12%, Ni 4%)	44×10^{-8}	Quartz (fused)	75×10^{16}
	Constantan (Cu 60%, Ni 40%)	49×10^{-8}	Sulphur	10^{15}
	Nichrome	100×10^{-8}	Teflon	$>10^{13}$
			Wood	$10^{8} – 10^{11}$

Resistivity and Temperature

It has been qualitatively explained in Section 1.5 that the resistivity of conductors increase with temperature. In not too large a range of temperature linear law applies

$$\rho(T) = \rho_0[1 + \alpha\,(T - T_0)] \qquad \text{(B.7)}$$

where

α = *temperature coefficient of resistivity at temperature* T_0,

T_0 is usually taken as 0°C as 20°C

The law of Eq. (B.7) will apply to the resistance of any particular conductor (replace ρ by R)

The temperature coefficients of resistivity for various materials is given in Table B.2.

Table B.2 Temperature Coefficients of Resistivity (approximate Values Near Room Temperature, 20°C)

Material	$\alpha[(°C)^{-1}]$	Material	$\alpha[(°C)^{-1}]$
Aluminium	0.0039	Lead	0.0043
Brass	0.0020	Manganin	0.00000
Carbon (Graphite)	–0.0005	Mercury	0.00088
Constantan	0.00001	Nichrome	0.0004
Copper	0.00393	Silver	0.0038
Iron	0.0050	Tungsten	0.0045

Example B.1 An 18-gauge[1] copper wire has a nominal diameter of 1.02 mm. It is carrying a constant current of 1.56 A. The density of free electrons in 8.5×10^{28} electrons/m^3. Find the value of current density and also the drift velocity.

Solution

$$A = \frac{\pi d^2}{4} = \frac{\pi(1.02 \times 10^{-3})^2}{4} = 8.17 \times 10^{-7}\ \text{m}^2$$

Current density

$$J = \frac{I}{A} = \frac{1.56}{8.17 \times 10^{-7}} = 1.906 \times 10^6\ \text{A/m}^2$$

From Eq. (B.2)

$$v_d = \frac{I}{A}\frac{1}{nq} = \frac{J}{nq}$$

Substituting values

$$v_d = \frac{1.906 \times 10^6}{(8 \cdot 5 \times 10^{28})(1 \cdot 6 \times 10^{-19})} = 1.4 \times 10^{-4}\ \text{m/s}$$

Example B.2 The 18–gauge copper wires of example B.1 has a diameter of 1.02 mm and so a cross-sectional area of $A = 8.17 \times 10^{-7}$ m^2. The wire is carrying a constant current of 1.56A.

Find (a) the magnitude of electrical field, (b) the potential difference between two points, if the wire 50 m apart, and (c) the resistance of 50 m length of wire.

Solution

From Table B.1 the resistivity of copper is 1.72×10^{-8} Ωm

(a) From Eq. (B.6)

$$E = \rho J = \rho \frac{I}{A}$$

$$\text{or } E = \frac{1.72 \times 10^{-8} \times 1.56}{8.02 \times 10^{-7}} = 0.0327\ \text{V/m}$$

(b) The potential difference is

$$V = EL = 0.0327 \times 50 = 1.635\ \text{V}$$

(c) Resistance of 50 m length of wire

$$R = \frac{V}{I} = \frac{1.635}{1.56} = 1.05\ \Omega$$

or directly

$$R = \rho \frac{L}{A} = \frac{(1.72 \times 10^{-8}) \times 50}{8.20 \times 10^{-7}} = 1.05\ \Omega$$

Example B.3 The resistance of the wire in the above two examples is found to be 1.05 Ω at 20°C. Find its resistance at 0°C and at 100°C.

Solution

$$R = R_o[1 + \alpha\,(T - T_o)]$$

From Table B.2 for copper $\alpha = 0.00393\ (\mathrm{C}°)^{-1}$ at 20°C

At, $T = 0°\mathrm{C}$

$$R = 1.05\ [1 + 0.00393\ (20–0)] = 0.97\ \Omega$$

At $T = 100°\mathrm{C}$

$$R = 1.05\ [1 + 0.00393\ (100–20)] = 1.38\ \Omega$$

Observation

It is found from above that increasing the temperature from 0°C to 100°C increases the wire resistance by a factor of

$$\frac{1.38}{0.97} = 1.42 \text{ or an increase of } 42\%$$

which is significant and so must always be accounted for copper.

Copper conductor, change of resistance with temperature.

It is convenient to use α(at 0°C). For copper conductor

$$\alpha\ (0°\mathrm{C}) = \frac{1}{234.5} = 0.00426$$

We can find the copper conductor resistance at another temperature if it is known at one temperature by the relationship.

$$\frac{R\,(T_2)}{R\,(T_1)} = \frac{234.5 + T_2}{234.5 + T_1} \tag{B.8}$$

Change of Resistance with Frequency

AC and DC resistances of copper conductor are related as

$$\frac{R_{AC}}{R_{DC}} = \rho(f) = \text{function of frequency} \tag{B.9}$$

When ac is flowing through a conductor, the current is non-uniformly distributed over the cross-section. This effect becomes more pronounced as frequency is increased. This phenomenon is called *skin effect*. It causes larger power loss for a given rms ac than the loss when the same value of dc is flowing through the conductor. Consequently, the effective conductor resistance is more for ac than for dc.

Apart from skin effect, non-uniformity of current distribution is also caused by *proximity effect*. Thus the ratio of R_{AC} / R_{DC} is a function of frequency. R_{AC} / R_{DC} versus frequency plot for copper is shown in Fig. B.2.

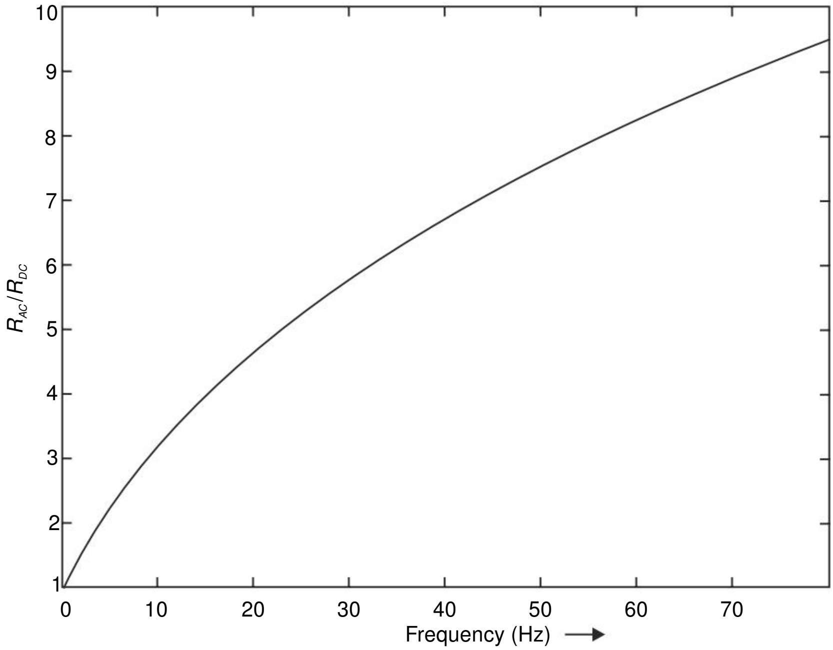


Fig. B.2

Appendix C

COMPLEX NUMBERS

MAIN GOALS AND OBJECTIVES

- *Definition of complex numbers*
- *Complex number arithmetic*
- *Euler's identify*
- *Exponential and polar forms of complex number.*

We shall begin by introducing the concept of complex number.

C-1. THE COMPLEX NUMBER

When we encounter algebraic equations of the kind $x^2 = -5$, no real number solution can be found. To deal with such as equation, the concept of the *imaginary operator* designated by symbol j is introduced.

By definition $j^2 = -1$ and thus $j = \sqrt{-1}$, $j^3 = -j$ and $j^4 = 1$ and so forth. A real number multiplied by the imaginary operator is called *imaginary number* like $j3$. The sum of a real number and an imaginary number is known as *complex number*. Thus $\overline{A} = a + jb$ is a complex number when a and b are real numbers. This can also be expressed as

$$R_e = [\overline{A}] = 0 \text{ and } Im\,[\overline{A}] = b$$

where R_e means real part of and Im means imaginary part of $\overline{A}$. By definition the imaginary component is a real number.

The Complex Plane

To graphically represent a complex number we form a complex plane with orthogonal coordinates as the real axis and imaginary axis as depicted in Fig. C.1.

Two complex numbers $\overline{M} = 5 + j_2$ and $\overline{N} = 3 - j_2$ are represented on the complex plane. Representation of complex numbers on complex plane is not essential but visually helpful.

A complex number expressed as sum of a real and an imaginary number like $\overline{A} = a + jb$ is said to be in *rectangular* or *Cartesian form.* Other forms of expressing a complex number will be studied later in the appendix.

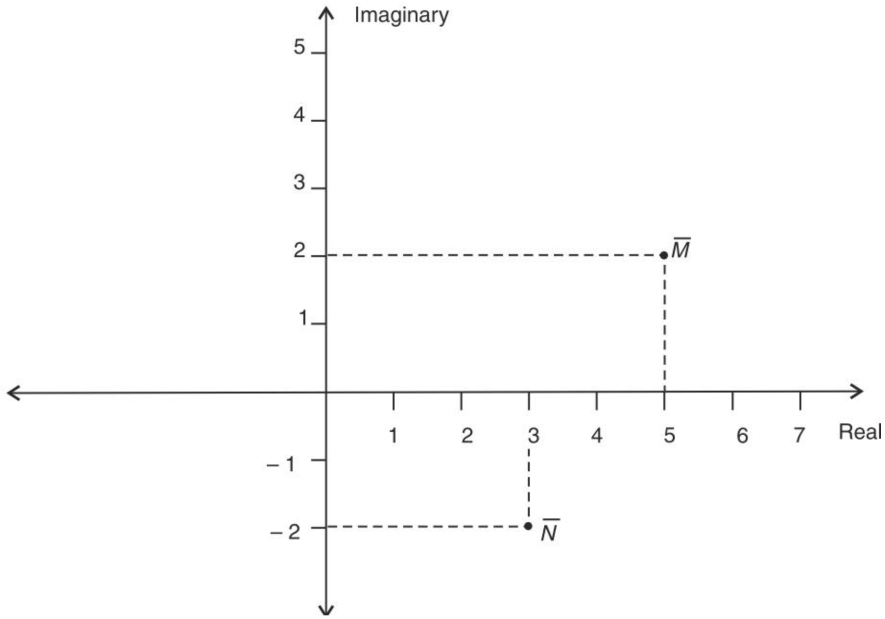


Fig. C.1

Equality: Two complex numbers are equal only if their real parts are equal and complex parts are equal. Consider two complex numbers

$$\overline{A} = a + jb, \quad \overline{B} = c + jd$$

Then if $$\overline{A} = \overline{B}$$

it is necessary that

$$a = b \text{ and } c = d$$

Addition / Subtraction

$$(a + jb) + (c + jd) = (a + c) + j(c + d)$$

For example

$$(2 + j3) +(5 - j7) = 7 - j4$$

$$(2 + j3) - (5 - j7) = -3 + j10$$

Addition can be carried out graphically on the complex plane. For this each complex number is represented by a directed line (vector) from the origin to the number point. Consider two complex numbers

$$\overline{M} = 3 + j1$$

$$\overline{N} = 2 - j3$$

These are drawn as vectors in Fig. C.2.

Their addition is carried by a - parallelogram or by drawing the vector $\overline{N}$ from the tip of vector $\overline{M}$. The tip of the resultant vector gives

$$\overline{M} + \overline{N} = 5 - j2$$

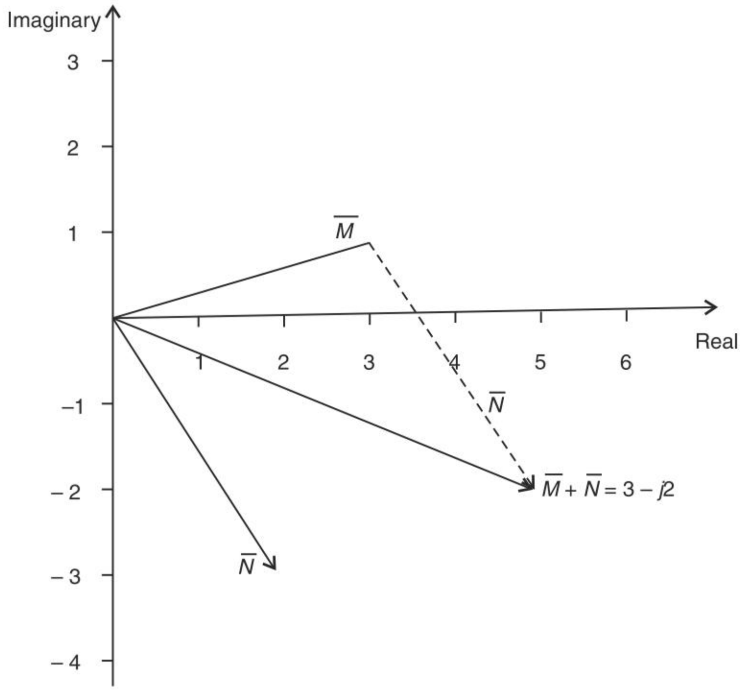


Fig. C.2

Multiplication

It is carried out by direct multiplication.

Accounting for j and collecting real numbers and imaginary numbers.

$(a + jb)\ (c + jd) = (ac + jad) + (jbc - bd) = (ac - bd) + j(bc + ad)$

As an example

$(3 + j4)\ (4 - j3) = (12 - j9) + (j16 + 12) = 24 + j7$

Conjugate of a complex number is defined as

$\overline{A} = a + jb$

Conjugate $\overline{A}^* = a - j\,b$; changing the sign of

$(\overline{A}^*)^* = \overline{A}$

$\overline{A} + \overline{A}^* = 2\ Re\ [\overline{A}]$

$\overline{A} + \overline{A}^* = 2\ j\ Im\ [\overline{A}]$

$\overline{A} \cdot \overline{A}^* =$ real number

$(a + j\,b)(a - j\,b) = a^2 - (j\,b)^2 = a^2 + b^2$

Quotient of two complex numbers

$$\frac{\overline{A}}{\overline{B}} = \frac{\overline{A} \cdot \overline{B}^*}{\overline{B} \cdot \overline{B}^*}$$

and thus

$$\frac{a+jb}{c+jd} = \frac{(a+jb)(c-jd)}{(c+jd)(c-jd)} = \frac{(ac+cd)+j(bc-ad)}{c^2+d^2}$$

The process is called *rationalizing the denominator.*

As an example

$$\frac{5+j3}{3-j2} = \frac{(5+j3)(3+j2)}{(3-j2)(3+j2)} = \frac{9+j19}{9+4} = 0.692 + j1.462$$

Multiplication and division by these methods become cumbersome when many complex numbers operations are involved. A faster way is to converting each complex number to exponential or polar form before multiplication or division. These forms will be provided Sections A.4.

C.2. DIFFERENTIATION AND INTEGRATION

Functions of time which contain complex numbers are differentiated and integrated with respect to the real variable and in the same way as with real constants. Consider a complex function of time

$$\bar{f}(t) = a \cos ct + jb \sin ct$$

then

$$\frac{d\,\bar{f}(t)}{\mathrm{d}t} = -\,a\,c \sin ct + jb\,c \cos ct$$

and

$$\int \bar{f}(t)\,\mathrm{d}t = \frac{a}{c} \sin ct - \frac{jb}{c} \cos ct + C$$

where C = constant of integration, a complex number in general.

C.3. EULER'S IDENTITY

The level for which this book is meant the Euler's Identity is presented below without proof.

$$e^{j\theta} = \cos\theta + j \sin\theta \qquad (1)$$

And $$e^{-j\theta} = \cos\theta - j \sin\theta \qquad (2)$$

This is a very useful result as it gives the equivalent of a complex exponential in Cartesian or rectangular form. Adding and subtracting Eqs. (1) and (2) we get its inverse form

$$\cos\theta = \frac{1}{2}\,[e^{j\theta} + e^{-j\theta}] \qquad (3)$$

$$\sin\theta = \frac{1}{2j}\,[e^{j\theta} - e^{-j\theta}] \qquad (4)$$

Examples:

1. $e^{-j1} = \cos 1 - j \sin 1$, 1 is rad

$= 0.540 - j\,0.841$

2. $\cos(-j1) = \frac{1}{2}[e^1 + e^{-1}] = 1.543$

3. $e^{1-j1} = e^1 \cdot e^{-j1} = e^1(0.540 - j0.841)$

$= 1.468 - j2.286$

4. $\sin(-j\,1) = \frac{1}{2j}[e^1 - e^{-1}] = -j\,.175$

C.4. THE EXPONENTIAL FORM

Let us again write down the Euler's identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

Multiply both sides by constant C

$$Ce^{j\theta} = C\cos\theta + jC\sin\theta \tag{5}$$

$$= a + j\,b \tag{6}$$

The right side of Eq. (6) is a complex number in rectangular form, our purpose it to convert it to exponential form as on the left side. From Eqs. (5) and (6).

$$a = C\cos\theta, \qquad b = C\sin\theta \tag{7}$$

Squaring and adding

$$a^2 + b^2 = C^2$$

or

$$C = +\sqrt{a^2 + b^2} \tag{8}$$

This gives the magnitude of exponential form. Observe that it is always a positive quantity.

From Eq. (7) by dividing we get

$$\tan\theta = \frac{b}{a}$$

or $$\theta = \tan^{-1}\frac{b}{a} \tag{9}$$

We thus have the exponential form

$Ce^{j\theta}$ of rectangular form $(a + j\,b)$.

Representation of a complex number by a point in the complex plane is shown in Fig. C.3. It is seen that a complex number is represented as $(a + j\,b)$ or $Ce^{j\theta}$. From this right angle triangle we can convert from one form to the other. The value of angle θ depends on the signs of a and b and not on the combined sign of $\frac{b}{a}$.

A simple method of finding the angle is to locate the complex number graphically in the complex plane, which determines the quadrant in which the point lies.

Consider for example a complex number

$$\bar{A} = 4 - j3$$

It is located in the complex plane in Fig. C.4. where

$$C = \sqrt{(4)^2 + (-3)^2} = 5$$

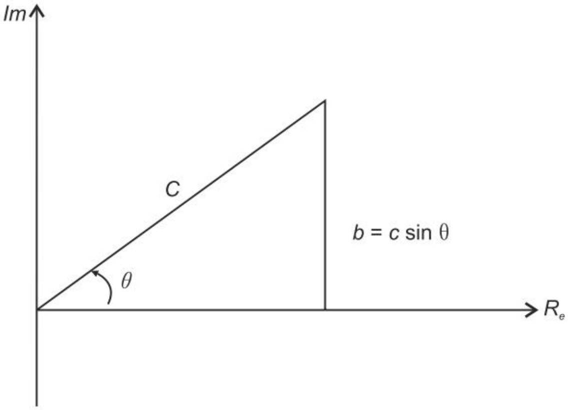


Fig. C.3

$$\theta = -\tan^{-1}\frac{3}{4} = -36.9°$$

Or $\theta = -36.9° + 360° = 323.1°$

Making a sketch on the complex plane to determine the value of θ is not necessary, it can even be mentally visualized.

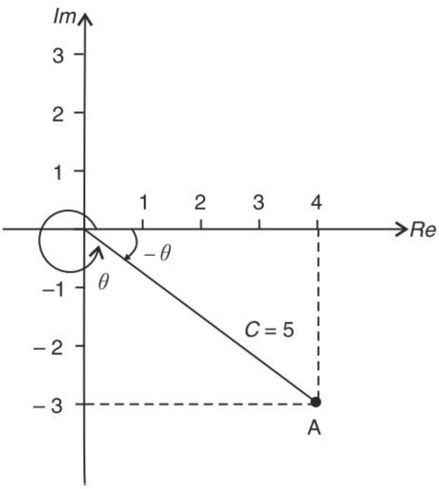


Fig. C.4

Examples: Express the following complex numbers in exponential forms with the angles lying in range

$$-180° < \theta \leq 180°.$$

1. $\bar{A} = -21.5 - j29.1°$

$$-(21.5 + j29.1) = -Ce^{j\theta}$$

$$C = \sqrt{(21.5)^2 + (29.1)} = 31.2$$

$$\theta = \tan^{-1}\frac{29.1}{21.5} = 53.5°$$

We have then

$$\bar{A} = -31.2\ e^{j53.5}$$

The minus sign corresponds to j^2 , each j means turning by 90°. Therefore minus sign can be removed by adding an angle of 180° to θ i.e., 53.5°+180°= 233.5°. Thus

$$\bar{A} = 31.2\ e^{j233.5°}$$

Instead we could subtract an angle of 180° i.e, 53.5° – 180° = –126.5°. So

$$\bar{A} = 31.2\ e^{j(-126.5°)}$$

A quick sketch is drawn in Fig. C.5.

2. $\bar{A} = -21.6 + j12.4$

$$C = \sqrt{(21.6)^2 + (12.4)^2} = 24.9$$

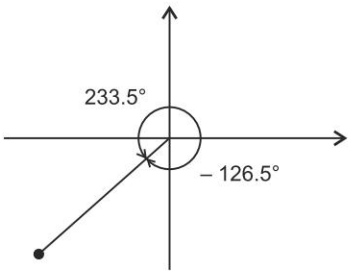


Fig. C-5

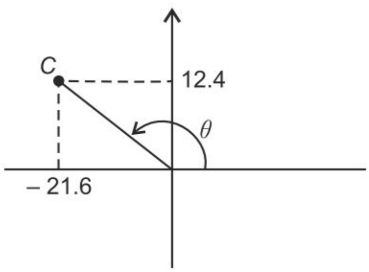


Fig. C-6

$$\theta = 180° - \tan^{-1}\frac{12.4}{21.6} = 180° - 29.86°$$

$$= 150.14°$$

Thus $\bar{A} = 24.9\ e^{j150.14°}$, it is sketched in Fig. C-6.

C.5. THE POLAR FORM

It is essentially the same as the exponential form except for the difference in symbolism. The exponential angle $e^{j\theta}$ is symbolized as $\angle\theta$. Thus

$$C\ e^{j\theta} \text{ is written as } C\ \angle\theta$$

It employs the polar coordinates, magnitude C at angle θ.

Consider the complex number

$$\bar{A} = -3 + j5$$

In exponential form it is written as

$$\bar{A} = 5.8\ e^{j120°}$$

In polar form it is

$$\bar{A} = 5.8\ \angle 120°$$

Multiplication and Division

Let $\bar{A} = 6\ e^{j52.1°}$, $\bar{B} = 1.5\ e^{-j30.5°}$

Then

$$\bar{A}\cdot\bar{B} = 6\ e^{j52.1°} \times 1.5\ e^{-j\,30.5°}$$

$$= (6 \times 1.5)\ e^{j52.1°} \cdot e^{-j\,30.5°}$$

$$= 9\ e^{j(52.1°-30.5)}$$

$$= 9\ e^{j21.6°}$$

In polar form

$$\bar{A} = 6\angle 52.1°, \quad \bar{B} = 15\angle -30.5$$

Then

$$\bar{A}\cdot\bar{B} = (6 \times 5)\angle 52.1° - 30.5$$

$= 90\angle 21.6°$

Rule: Multiply magnitudes and algebrically add angles consider division

$$\frac{\overline{A}}{\overline{B}} = \frac{6e^{j52.1^\circ}}{15e^{-j30.5^\circ}} = \frac{6}{15}\cdot e^{j52.1^\circ}\cdot e^{j30.5}$$

$$= 0.4\ e^{j(52.1^\circ + 30.5^\circ)}$$

$$= 0.4\ e^{j82.6^\circ}$$

In polar form

$$\frac{\overline{A}}{\overline{B}} = \frac{6\angle 52.1^\circ}{15\angle -30.5^\circ} = 0.4\angle 52.1^\circ - (-30.5^\circ)$$

$$= 0.4\angle 82.6°$$

Remark Multiplication and division are accomplished easily in exponential/polar form, while addition and subtraction are easily performed in rectangular form. A complex number operations may therefore require conversion from one form to another more than once.

The inter conversions are easily performed by most scientific calculators.

Examples

1. $\dfrac{40}{3.75\angle 81.6^\circ} + 6.15\angle 63.4°$; result in polar form

$= 10.67\angle -81.6° + 6.15\angle 63.4°$

$= (1.56 - j10.56) + (2.75 + j5.50)$

$= 4.31 - j5.05 = 6.04\angle -49.5°$

2. Find $\overline{Z}$ in rectangular form

$$\overline{Z} + j2 = \frac{3}{\overline{Z}}$$

$$\overline{Z}^2 + j\,2\,\overline{Z} - 3 = 0$$

Solving the quadratic in $\overline{Z}$

$$\overline{Z} = \frac{-j2 \pm \sqrt{(j2)^2 + 12}}{2}$$

$$= -j1 \pm \sqrt{2} = 1.414 - j1,\ -1.414 - j1$$

3. Find $\overline{Z}$ in rectangular form

$$\overline{Z} = 2\ln(2 - j3)$$

$(2 - j3) = 3.6\ e^{-j56.3^\circ}$; $56.3° \longrightarrow 0.983$ rad

$= 3.6\ e^{-j0.983}$

$\overline{Z} = 2\ \ln[3.6\ e^{-j0.983}] = 2\{\ln 3.6 - j\ 0.983\} = 2.56 - j1.966$

Appendix D

ANSWERS TO PROBLEMS

CHAPTER 1

1.1 (a) 529 Ω (b) 0.435 A (c) 800 Wh

1.2 (i) (b) 4 H (c) 4 F (c) 2 W

(ii) (b) 50 J (c) 50 J (d) 50 W

(iii) 100 J

1.3 (a) 8.88 sin 314t (b) i leads v by 90°

(c) instantaneous power oscillates at double frequency (628 rad/s) with zero average value

1.4 (a) –1.8 cos 314t (b) i lags v by 90°

(c) same as P.13 part (c)

1.5 (a) $\sqrt{2}/5 \sin 314t$ (b) i is in phase with v

(c) $P_{av} = 40$ W, oscillates at double frequency

1.6 $\frac{1}{2} LI_0^2$

1.7 (c) $V_1 = 32/15$ V, $V_2 = 1.6$ V

1.8 (a) 3 A (b) 2A, 1A and 2V, 1V (c) 3 V

1.9 (b) Node 1

$$\frac{V_s - V_1}{R_1} = \frac{V_1 - V_2}{R_2} + \frac{V_1 - V_3}{R_4}$$

Node 2

$$\frac{V_1 - V_2}{R_2} + I_s = \frac{V_2}{R_s} + \frac{V_2 - V_3}{R_3}$$

Node 3

$$\frac{V_1 - V_3}{R_4} + \frac{V_2 - V_3}{R_3} = \frac{V_3}{R_6}$$

1.10 $8e^{-2t}$

1.11 9 cos 2t – sin 2t

1.12 1 A

1.13 10 V, 24 V

1.14 8 A, 4 A; 64 W, 128 W

1.15 1.59 sin $(314t + 32.8°)$

1.16 (a) 2 A (b) – 1/2 A

1.17 4.1 V

CHAPTER 2

2.1 $V_1 = 0.75$ V
$I_2 = 0.035$ A

2.2 1 V

2.3 4.5 Ω

2.4 (i) $3.99 \approx 4\ \Omega$ (ii) 2.44 Ω

2.5 22.2 V, 17.1 V, 2.55 A

2.6 7 V, 5 V, 6.5 V

2.7 10.72 V

2.8 0.0055 A

2.9 I(1 Ω) = 5/3 A, i(2 Ω left) = 10/3 A, i(2 Ω right) = – 3A, i(3 Ω) = – 2 A

2.10 (a) $V_1 = 5$ V, $V_2 = 3$ V, $i_{12}(5\ \Omega) = 0$
(b) power output; current source = 5 W, voltage source = 2.5 W

2.11 0.162 A, – 0.307 A, 0.469 A; 10.14 W (0.5 A source), 17.58 W (0.6 A source)

2.12 0.6 V

2.13 1.665 A

2.14 (i) –272.7 (ii) 2.479×10^5

2.15 (i) $\dfrac{h_{21}h_{11}/R_L}{h_{11}h_{21} + 2h_{12}h_{21} + h_{11}/R_L}$ (ii) $-\left(\dfrac{h_{21}}{h_{11}h_{21} + h_{12}h_{21} + h_{11}/R_L}\right)$

(iii) $\dfrac{h_{11}(h_{11}h_{22} - h_{12}h_{21} + h_{11}/R_L)}{h_{11}h_{22} - 2h_{12}h_{21} + h_{11}/R_L}$

2.16 (i) 2 A (ii) 128 V

2.17 56.93 V, 25.36 A

2.18 0.0159 A

2.19 1.5 A

2.20 10 Ω, 4.83 W

2.21 $V_{OC} = 10$ V, $R_0 = 1\ \Omega$

2.22 $\dfrac{2}{3}$ A

2.23 $V_{OC} = 12$ V, $R_0 = 2\ \Omega$

CHAPTER 3

3.1 8 V, – 1 A, – 4 V/s

3.2 $8e^{-4t}$, $t > 0$

3.3 $(2 - e^{-12t})$; $t > 0$

3.4 $v_e(t)\ 2\ (1 - e^{-t})\ u(t)$; $v_R(t) = \left(-\left(\frac{1}{2}\right)e^{-t} + 2\right)\ u(t)$

3.5 $\frac{1}{6}e^{-20t} + \frac{2}{3}$; $t > 0$

3.6 $-1 \pm j2$; $\omega_0 = \sqrt{5}$; $\omega_d = 2$

3.7 $\frac{1}{2}(1 - e^{-2t})$; $t > 0$

3.8 $\frac{1}{2}(1 + e^{-5/2t})$ A, $3\left(1 - \frac{3}{4}e^{-5/2t}\right)$ V; $t > 0$

3.9 $2 - e^{-2t}$; $t > 0$

3.10 $6.7\,e^{-4t}\cos(2t + 63.4°)$

3.11 1 A, 3 V

3.12 (a) $\ddot{v}_C + 6\dot{v}_C + 10v_C = 20$ (b) 10 V, – 40 A/s
(c) $17.89e^{-3t}\cos(t + 63.43°)$

3.13 (a) $\ddot{v} + 6\dot{v} + 2v = 2$ (b) – 2, – 1
(c) $i_L(0^+) = 0$, $v(0^+) = 0$, $v(0^+) = 0$
(d) $1 + e^{-2t} - 2e^{-t}$; $t > 0$

3.14 $\ddot{v} + 3\dot{v} + 2v = 0$; $v(0^+) = 0$, $v(0^+) = -2$
$v(t) = 2e^{-2t} - e^{-t}$; $t > 0$

3.15 $i(0^+) = 0$; $\frac{di(0^+)}{dt} = 3$ A/s

3.16 $8e^{-2t}$; $t > 0$

CHAPTER 4

4.1 (a) 74.7 mH (b) 500 W
(c) 0.208 lag

4.2 (a) 31.4 Ω (b) 19.9 Ω
(c) 14 Ω, 16.43 A, 0.57 lag (d) 515.9 V, 327 V

4.3 (a) 2.64 Ω, 39.9 Ω, 40 Ω (b) 66 W
(c) 0.066 lag

4.4 0.714 Ω, 0.517 Ω, 0.073 mH, 2.475 kHz, 31 V

4.5 63.6 μF

4.6 1.21 Ω, 0.638 lag

4.7 214.4 V, 0.74 lag

4.8 (a) $0.5\angle -45°$ A (b) 0.0625 F

4.9 (a) $\frac{4 + j3\omega}{4 + j5\omega - \omega^2}$ (b) $\sin(2t - 33.7°)$

4.10 (a) $\bar{I}(1\,\Omega) = 1\angle -90°$, $\bar{V}(1\,\Omega) = -j1$, $\bar{V}(1\text{ F}) = \bar{V}(½\,\Omega)$
$= \sqrt{2}\,(-45°)$, $\bar{I}(1\text{ F}) = \sqrt{8}\,\angle 45°$, $I(\text{ W}) = \angle -45°$
(b) $5.83\cos(2t - 14°)$
(c) 5 W, – 3 VAR
(d) P(1 Ω) = 1 Ω, $P(½\,\Omega) = 4$ W

4.11 $\sqrt{10}\cos(2t + 18.4°)$

4.12 $1.11\sin(2t + 78.7°)$ V, 0.153 W

4.13 $\bar{V}_1 = 1.91\angle -65.3°$ V, $_2 = -0.894\angle -26.6°$

4.14 (b) 14 W
4.15 (a) 275.7 V
(b) 320 V, 0.67 lag, 38.82 $\angle 48°$ kVA
4.16 (a) 268.8 $\angle 4.4°$ V, 251.9 –2.4° V
(b) 0.812 lag, 7.14 $\angle 35.7°$ kVA
4.17 0.322 V
4.18 $v_2(t) = 0.201\sqrt{2}\cos(2t + 13.5°)$
4.19 $\bar{V}_1 = 7.8\angle -40.9°$, $\bar{V}_2 = 8.21\angle -81.9°$ V
4.20 (a) 228.9 $\angle 29.7°$ V (b) – 1.7 kW
(c) 15.65 kW
4.21 0.644 Ω, 0.786 Ω, 39.6 Ω; 0.786 Ω, 39.14 W
4.22 2.81 $\angle 56.3°$ VA

CHAPTER 5

5.1 (a) $\dfrac{j2\omega + 1}{j10\omega + 1}$ (b) $0.5 \times \dfrac{j\omega + 1}{j0.5\omega + 1}$

5.2 $\dfrac{1}{2} \times \dfrac{(j\omega + 1)(j2\omega + 1)}{\left(j\dfrac{\omega}{0.22} + 1\right)\left(j\dfrac{\omega}{2.28} + 1\right)}$

5.3 44 Ω 0.0633 mH, 9.04
5.4 657 Ω, 3.72 μF, 2.73 H
5.5 100.37, 99.52 Hz
5.6 cos 400t, 50 sin 400t, –50 sin 400t
5.7 (a) 0.001 V (b) 0.0068 V
5.8 0.796 MHz, 50, 2 V
5.9 0.318 MHz, 37.7, 333 kHz
5.10 4.51 μF, 0.0238 A

5.11 $v(t) = \dfrac{V_m}{\pi} + \dfrac{V_m}{2}\cos\omega t + \dfrac{2V_m}{3\pi}\cos 2\,\omega t - \dfrac{2V_m}{15\pi}\cos 4\,\omega t + \dfrac{2V_m}{35\pi}\cos 6\,\omega t \ldots$

5.12 $v(t) = \dfrac{V}{2} + \dfrac{2V}{\pi}\cos\omega t - \dfrac{2V}{3\pi}\cos 3\,\omega t - \dfrac{2V}{5\pi}\cos 5\,\omega t + \ldots$

5.13 $v_0(t) = \dfrac{V}{2} + 0.667\,V\cos(\omega t - 19.2°) - 0.223\,V\cos(3\omega t - 83.2°) + 0.059\cos(5\omega t - 133°)$

5.14 $a_0 = \dfrac{1}{2} I_m$, $a_m = -\left(\dfrac{4}{m^2\pi^2}\right) I_m$, m odd, = 0, m even

5.15 $v_0(t) = 10 - 3\cos(2000\pi t - 68.3°) - 0.117\cos(16000\pi t - 82.6°)$
$- 0.025\cos(1000\pi t - 85.4°)$

5.16 $v_0(t) = 63.7 + 6.66\cos(628t - 81°)$
$- 0.672\cos(1256t - 85.4°) + \ldots$

5.17 $\dfrac{2(j\omega)^2}{\left(1 + j\dfrac{\omega}{3.186}\right)\left(1 + j\dfrac{\omega}{0.314}\right)}$

CHAPTER 6

6.1 13.28 A, 0.8 lag, 4.232 kW,
3.174 kVAR, 5.29 kVA

6.2 16 Ω, 308 μF

6.3 19.9 A, 0.8 lag, 6.34 kW, 4.70 kVAR, 7.93 kVA

6.4 52 A, 36 kW

6.5 30 A, 0.978 leading
36 kVAR

6.6 (a) 17.32 A, –37° (b) 5.27 kW
(c) 17.6 Ω

CHAPTER 7

7.1 0.175 A

7.2 2.3 A

7.3 2mWb

7.4 1.675 A

7.5 1690 AT

7.6 0.587 A, 0.94 m Wb

7.7 7609

7.8 (a) 1.046 J (b) 1.11 sin 314 t
(c) 144.7 mH, 161 mH

CHAPTER 8

8.1 (a) 125 A, 41.67 A (b) (i) 14.4 Ω, (ii) 1.6 Ω
(c) 0.045 Wb
(d) 720 V (e) 0.056 Wb

8.2 (a) $R_1 = 0.139\ \Omega$, $X_1 = 0.735\ \Omega$; $R_2 = 0.0154\ \Omega$, $X_2 = 0.0817\ \Omega$;
$R(\text{pu}) = 0.097$, $X(\text{pu}) = 0.51$
(b) 5.2%

8.3 (a) 36.9 A, 110.6 A (b) 35.6 A, 106.8 A (c) 3.4%

8.4 97.4%

8.5 $\bar{Z}(\text{HV}) = 0.25 + j125$; $\bar{Z}(\text{LV}) = 0.01 + j0.05$
$\bar{Z}(\text{pu}) = 0.01 + j0.052$

8.6 (a) 1142.9 V, 3.9% (b) 1074.5 V, –2.32%

8.7 97.2%

8.8 892 A

8.9 (a) 213.1 V (b) 225.1 V

8.10 (a) $(0.1 - j0.2)$ A (b) 13.3 V, 112.5 W, 0.34 lag

8.11 (a) 96.14%, 13.2 kW (b) 0.76 leading

8.12 150 kVA; 125 A, 25 A

8.13 (a) 1 (b) 1/9

8.14 20 kVA, 244 kVA, 99.84%

8.15 Rating of each unit: 170 kVA, 22/199.2 kV

Y-side: 0.853 $\angle -11.3°$, 0.853 $\angle -131.3°$, 0.853 $\angle -251.3°$ kA (line)

$\overline{V}_{AB} = 345 \angle 30°$, $\overline{V}_{BC} = 345 \angle 90°$,

$\overline{V}_{CA} = 345 \angle -210°$ kV

D-side: $\overline{V}_{ab} = 22 \angle 0°$, $\overline{V}_{bc} = 22 \angle -120°$, $\overline{V}_{ca} = 22 \angle -240°$ kV

$\overline{I}_a = 13.376 \angle -41.3°$, $\overline{I}_a = 13.376 \angle -161.3°$,

$\overline{I}_c = 13.376 \angle -281.3°$ kA (line)

Reference phasor: $\overline{V}_{AN} = 199.2 \angle 0°$

8.16 (a) 484 Ω (b) 78.75 A (c) 454.5 A, 136.4 A

8.17 *I*(load leg) = 78.75 A, *I*(secondary, phase/line) = 136.4 A

I(primary, phase) = 454.6 A, *I*(primary, line) = 787.4 A

CHAPTER 9

9.1 0.96, 0.985

9.2 50 Hz, 4430 V

9.3 2338 V, 81 kVA

9.4 (a) 41.77 kV, 417.7 kVA (b) 20.95 kV, 628.5 kVA

9.5 (a) 7478 V, 59.8 kVA (b) 7458 V, 59.8 kVA
(c) 6497 V, 90 kVA

9.6 $$\mathcal{P} = \left(\frac{2Dl}{P} \times \frac{\mu_0}{g}\right) \text{ Wb/AT}$$

9.7 16

9.8 (a) 6 (b) 1%, 5%
(c) 10 rpm, 0 rpm, 1000 rpm
50 rpm, 0 rpm, 1000 rpm

9.9 (a) 4% (b) 6 (c) 920 rpm

CHAPTER 10

10.1 (a) 220 V, 4 A (b) 65 Ω, 1354 rpm
(c) 191.4 V, 23.75 A

10.2 387.4 rpm

10.3 (a) 185.2 A (b) 1.2 A
(c) 576 V (d) 106 kW
(e) 843.5 Nm

10.4 (a) 1178 rpm (b) 1209 rpm

10.5 106.5 Ω, 35.67 A, 36.9 A

10.6 2280 rpm

10.7 1427 rpm, 281 Nm

10.8 1159 rpm, 20.5 A

10.9 (a) 503 rpm (b) 1330 rpm (c) 65.6 A

10.10 625 rpm, 830.3 Nm

10.11 (a) 709 rpm, 1681 Nm (b) 1.85 Ω, 3132 Nm

10.12 (a) 756 rpm, 1576 Nm (b) 1.85 Ω, 2473 Nm

10.13 1413 rpm

10.14 1068.6 rpm

10.15 68 Ω

10.16 (a) 2.25 Ω, 1129 W (b) 1.37 Ω, 1678 W

10.17 3.5% reduction

10.18 0.071 Ω, 86.65%

10.19 (a) 8.31 mWb (b) 126.6 Nm
(c) 903 W (d) 2548 W, 85.5%

CHAPTER 11

11.3 5 to 8.83 A

11.4 (i) 479.4 V (line); 17.32 kW, 0 kVAR
(ii) 579 V (line); 13.86 kW, + 10.39 kVAR

11.5 (a) 31.4° (b) 819 A, 0.96 leading
(c) +4222 kVAR

11.6 (a) 0.81 leading (b) 15.9 kV (line)
(c) 6650 Nm (d) 40.8 A, 0 leading

11.7 1.264 MW, 8051 Nm, 0.656 lag

11.8 5.527 A, 0.95 lag; 6627 A, 0.99 lag

11.9 0.9 leading

11.10 (a) 31.1 kV (line), 0.708 leading
(b) 9.09 kV (line) 54.98 A, 0.382 lagging

CHAPTER 12

12.1 (a) 525 Nm (b) 0.85 lag

12.2 (a) 922 rpm

12.3 (a) 14.26 kW (b) 751 W (c) 91.1%

12.4 83%, 75.2%, 25.2%

12.5 11.73 A, 0.856 lag, 82.7%, 98.7 Nm, 785 rpm

12.6 (a) 0.032 Ω (b) (i) 686.25 rpm, 705 rpm
(c) 377.3 V (d) 86.2%, 86.74%

12.7 (a) 974 rpm (b) 964 rpm

12.8 (a) $\frac{1}{2}$ (b) $\frac{1}{4}$

12.9 (a) 87.48 A, 78.9 Nm (b) 50.5 A, 26.3 Nm

CHAPTER 14

14.1 1.765%

14.2 181.546 Ω

14.3 0.0042 Ω, 0.0083 Ω

14.4 0.0087

14.5 376.92 Ω, 803.857 Hz

14.6 200 Ω, 5×10^{-9} F, 6.28×10^{-3}

14.7 $R = R_1 R_2/R_3$, $L = CR_1 R_2$, ± 0.6%, ± 0.5%

14.8 8676.67 Ω

14.9 0°, 36.87°, – 36.87°

14.10 –3.896%

14.11 5V, 10 V

14.12 200 V

14.13 50/3 pF

14.14 260/3 pF

14.15 1.65

14.16 10^{-6} V

14.17 ± 00.05 V, ± 00.021 V, 20.5%

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