Basic Electrical Engineering

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To our Spouses

PREFACE

A well established, huge, and an exciting field, Electrical Engineering broadly involves working with various electronic devices ranging from pocket calculators to super computers. A growing presence of electronic devices and instrumentation in all aspects of engineering design and analysis greatly emphasizes the importance of the study of Electrical Engineering.

Basic Electrical Engineering is a compulsory course in all Engineering Colleges, Institutes and Universities. The book, targeted at the undergraduate level, presents the fundamental concepts of the subject in a comprehensive manner. Offering a lucid explanation of theory, supported by illustrations and large number of worked out examples; the text has been structured in a logical sequence as per syllabi of Engineering Colleges and Universities.

Key Features

The text not only lays strong emphasis on basic principles, but also incorporates topics and contents with advanced concepts. A careful selection of the text material and worked out examples (using a 'simple to more complex topic' approach) enable the students to gradually master the course of Basic Electrical Engineering. Examples throughout the text have been solved in detail so that the reader can follow each example thoroughly. Exercises, including a number of problems, are given at the end of each chapter, to evaluate the understanding of topics. All the problems have answers, wherein some of them contain hints to assist the reader in solving the problem.

The book has been written utilising the long experience of the authors in teaching Electrical Engineering. In relevant portions of the text, additional discussions and illustrative examples have been presented to make the pursuit of the study of Electrical Engineering a stimulating experience for the students.

The major highlights of this book can be summarized as

- The subject has been presented in a graded manner, and contains a detailed exposition of the fundamentals.

- Clearly illustrated examples demonstrate relevant applications of electrical engineering.
- Step-by-step and simple problem solving methodology enables the students to sharpen their problem solving skills.
- The figures and diagrams support the concepts and aid in the understanding of the subject.

Chapter Organisation

This book contains fifteen chapters, out of which the first thirteen chapters have been developed on the important topics, to include fundamental laws, concepts in electrical circuits, basic principles on electrical machines, and electrical measurements and measuring instruments. Chapter 14 on Review Problems contains a substantial number of well chosen, worked out examples. Another unique feature is Chapter 15 on Multiple Choice (objective type) Questions, covering the whole syllabus of Basic Electrical Engineering.

Web Supplements

To complement the contents of the book, an exhaustive online learning centre has been designed. Students can access additional Solved Problems and Links for further reference, enabling them to explore practical engineering applications of the devices and systems, in greater detail. The website also features Model Question Papers. A collection of resources available to teachers include detailed Solutions Manual and PowerPoint Slides that serve as valuable teaching tools. The site will be updated regularly and any suggestion towards this end is welcome.

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Constructive criticism and/or comments about the book are welcome.

Authors

CONTENTS

Preface *xv*

1. Fundamentals **1**

1.1 System of Units *1*
1.2 Fundamental and Derived Units in SI system *1*
1.3 Definitions of Fundamental Units and Derived Units *2*
1.4 Concept of Per Unit (p.u.) System *4*
1.5 Concept of Current Flow *11*
1.6 Concept of Source and Load *11*
1.7 Sign Notation for Voltages *12*
1.8 Concept of Positive and Negative Currents and Voltages *12*
1.9 Ohm's Law and Concept of Resistance *13*
1.10 Concept of Resistivity *13*
1.11 Expressions of Power and Energy in Resistive Circuits *14*
1.12 Temperature Coefficient of Resistance *17*
Additional Examples *22*
Exercises *30*

2. Electrostatics **36**

2.1 Introduction *36*
2.2 Coulomb's Law and Concept of Permittivity *36*
2.3 Permittivity *37*
2.4 Electric Potential and Potential Difference *38*
2.5 Expression for Potential at a Point within an Electric Field *38*
2.6 Electric Field Intensity *39*
2.7 Electric Field Intensity and Potential of Isolated Point Charge ($+q$) *40*
2.8 Electric Field Intensity and Potential Gradient *41*
2.9 Electric Potential Energy *42*
2.10 Relation between Electric Field Strength and Potential *43*
2.11 Electric Field Inside a Conductor *43*

2.12 Gauss' Law and its Derivation *44*
2.13 Electric Dipole *45*
2.14 Electric Field and Potential due to a Dipole at an Axial Point *46*
2.15 Electric Field and Potential due to Dipole on Equatorial Line *46*
2.16 Capacitor and Capacitance *54*
2.17 Series and Parallel Connection of Capacitors *55*
2.18 Concept of Dielectric Strength *56*
2.19 Types of Capacitors Commonly Used *56*
2.20 Capacitance of a Parallel-plate Capacitor *57*
2.21 Capacitance of a Multi-plate Capacitor *58*
2.22 Capacitance of a Parallel-plate Capacitor with Composite Dielectrics *58*
2.23 Capacitance of an Isolated Sphere *60*
2.24 Capacitance of Concentric Spheres *60*
2.25 Capacitance of a Parallel Plate Capacitor when an Uncharged Metal Slab is Introduced between Plates *62*
2.26 Expression of Instantaneous Current and Voltage in a Capacitor *74*
2.27 Charging and Discharging of Capacitance *75*
2.28 Energy Store in Capacitor *82*
Additional Examples *84*
Exercises *98*

3. Electromagnetism and Magnetic Circuits **109**
3.1 Introduction *109*
3.2 Magnetic Field around a Current-carrying Conductor *112*
3.3 Force on a Current-carrying Conductor in a Magnetic Field *113*
3.4 Force between Two Parallel Current-carrying Conductors *113*
3.5 Biot Savart's (or Laplace's) Law *117*
3.6 Application of Biot-Savart Laws *118*
3.7 Electromagnetic Induction *123*
3.8 Concept of Self-inductance and Mutual Inductance *125*
3.9 Concept of Magnetic Coupling *126*
3.10 Calculation of Self-inductance *129*
3.11 Energy Stored in Inductor *129*
3.12 Magnetic Energy Density (U_M) *130*
3.13 Combination of Inductances *131*
3.14 Lifting Power of a Magnet *138*
3.15 Concept of Magnetic Circuit *139*
3.16 Concept of Magneto-motive Force (MMF) *139*
3.17 Magnetic Field Intensity *140*
3.18 Concept of Reluctance *140*
3.19 Permeability and Relative Permeability *140*
3.20 Definition of Ampere *144*
3.21 B-H Characteristics *145*
3.22 Ferromagnetic Materials *146*
3.23 Types of Magnetic Materials *147*

3.24 Magnetic Circuit Laws *147*
3.25 Comparison between Electric and Magnetic Circuits *148*
3.26 Distinction between Magnetic and Electric Circuits *150*
3.27 Leakage Flux in Magnetic Circuit and Fringing and Staking *155*
3.28 Magnetic Hysteresis *156*
3.29 Hysteresis Loss 158
3.30 Eddy Currents (or Foucault's Currents) and Eddy Current Loss *159*
3.31 Rise and Decay of Current in Inductive Circuit *160*
3.32 Ampere's Circuital Law *167*
Additional Examples 167
Exercises 175

4. DC Network Analysis 180
4.1 Introduction *180*
4.2 Characteristics of Network Elements *180*
4.3 Series Resistive Circuits *181*
4.4 Parallel Resistance Circuits *187*
4.5 Series–Parallel Circuits *191*
4.6 Kirchhoff's Laws *196*
4.7 Nodal Analysis *201*
4.8 Mesh Analysis (or Loop Analysis) *207*
4.9 Star Delta Conversion *217*
4.10 Voltage Sources and Current Sources *225*
4.11 Superposition Theorem *232*
4.12 Thevenin's Theorem *242*
4.13 Norton's Theorem *256*
4.14 Equivalence of Thevenin's and Norton's Theorems *266*
4.15 Maximum Power Transfer Theorem *268*
Additional Examples 274
Exercises 325

5. Steady State Analysis of AC Circuit 336
5.1 Generation of Alternating emf *336*
5.2 Definitions Relating to Alternating Quantity *337*
5.3 Phasor Representation of an Alternating Quantity *343*
5.4 AC Voltage as Applied to Pure Resistance, Pure Inductance and Pure Capacitance *346*
5.5 Series RL Circuit *348*
5.6 Series RC Circuit *349*
5.7 Series RLC Circuit *350*
5.8 Impedances in Series *352*
5.9 Parallel AC Circuit *357*
5.10 Admittance, Conductance and Susceptance of AC Circuit *358*
5.11 Average Power in ac Circuits *359*
5.12 Complex Notation Applied to ac Circuits *376*
5.13 Series Parallel ac Circuits *377*

5.14 Series Resonance *382*
5.15 *Q* factor in Series Resonance *383*
5.16 Different Aspects of Resonance *384*
5.17 Resonance in Parallel Circuit *391*
5.18 Properties of Parallel Resonant Circuits *392*
5.19 *Q* Factor in Parallel Circuit *393*
5.20 Parallel Resonance in RLC Circuit *393*
5.21 Parallel Resonance in RC-RL Circuit *394*
Additional Problems 397
Exercises 439

6. AC Network Analysis **444**
6.1 Superposition Theorem (AC Application) *444*
6.2 Thevenin's Theorem (AC Application) *448*
6.3 Norton's Theorem (AC Application) *453*
6.4 Maximum Power Transfer Theorem (AC Application) *457*
Additional Problems 461
Exercises 482

7. Three-phase Circuits **485**
7.1 Three-phase System *485*
7.2 Advantages of a Three-phase System *485*
7.3 Generation of a Three-phase Supply *485*
7.4 Interconnection of Phases *486*
7.5 One Line Equivalent Circuit for Balanced Loads *491*
7.6 Measurement of Power in a Three-phase Three-wire System *496*
7.7 Measurement of Power for a Three-phase System using Two Wattmeters (Assuming Balanced Load and Sinusoidal Voltages and Currents) *498*
7.8 Unbalanced Four-Wire Star *499*
7.9 Unbalanced Delta Connected Load *499*
7.10 Unbalanced Three-Wire Star Connected Load *499*
Additional Problems 504
Exercises 527

8. Transformers **530**
8.1 Definition *530*
8.2 Principle of Operation *530*
8.3 EMF Equation *531*
8.4 Construction of Single-phase Transformers *531*
8.5 Transformation Ratio (or Turns Ratio) *532*
8.6 Impedance Transformation *533*
8.7 No Load Operation of a Transformer *537*
8.8 Working of a Transformer on Load *538*
8.9 Equivalent Circuit of Transformer *541*
8.10 Regulation of a Transformer *544*
8.11 Condition for Zero (minimum) Regulation *546*
8.12 Losses and Efficiency of Transformer *552*

8.13 Condition for Maximum Efficiency *554*
8.14 Expression for Load at which Efficiency is Maximum *554*
8.15 Testing of Transformers *560*
8.16 Parallel Operation of Two-single Phase Transformers *566*
8.17 Single-phase Auto Transformer *568*
8.18 Transformer Cooling *569*
8.19 Conservator and Breather *570*
8.20 Distribution Transformers and Power Transformers *570*
8.21 Name Plate and Ratings *570*
8.22 All Day Efficiency *571*
8.23 Three-phase Transformer *571*
Additional Problems 578
Exercises 592

9. DC Machines **598**
9.1 Introduction *598*
9.2 Principal Parts of a DC Machine *598*
9.3 Magnetic Flux Path in a DC Generator *599*
9.4 Equivalent Circuit of a DC Machine *599*
9.5 Different Types of Excitations in DC Machine *600*
9.6 Process of Voltage Build up in Self-excited Generator *602*
9.7 EMF Equation of a DC Machine *603*
9.8 Types of Windings *603*
9.9 Armature Reaction *605*
9.10 Commutation *606*
9.11 Characteristics of DC Generators *607*
9.12 Principle of Operation of a DC Motor *611*
9.13 Back EMF *611*
9.14 Torque Equation of a DC Motor *611*
9.15 Speed Equation of a DC Motor *616*
9.16 Speed Regulation of DC Motor *617*
9.17 Speed vs. Armature Current Characteristic of DC Motor (N/I_a Characteristics) *617*
9.18 Speed Torque Characteristic of DC Motors *619*
9.19 Speed Control of DC Motors *622*
9.20 Losses in a DC Machine *629*
9.21 Need for Starter in a DC Motor *638*
9.22 Reversal of Rotation of DC Motor *639*
9.23 DC Machine Applications *639*
Additional Examples 640
Exercises 655

10. Three-Phase Induction Motors **660**
10.1 Introduction *660*
10.2 Construction of Induction Machines *661*
10.3 Comparison of Squirrel Cage and Wound Rotors *663*
10.4 Advantages and Disadvantages of a Three-phase Induction Motor *663*

10.5 Principle of Operation *663*
10.6 Concept of Production of Rotating Field *664*
10.7 The Concept of Slip *666*
10.8 Frequency of Rotor Voltages and Currents *667*
10.9 Torque Expression of an Induction Motor *670*
10.10 Torque Slip Characteristics of a Three-phase Induction Motor *678*
10.11 Equivalent Circuit of Induction Motor *680*
10.12 Losses and Efficiency *681*
10.13 Determination of Motor Efficiency *686*
10.14 Starting of Three-phase Induction Motors *695*
10.15 Comparison Among Direct on Line Starter, Star Delta starter and Auto-transformer Starter *699*
10.16 Speed Control of a Three-phase Induction Motor *700*
10.17 Reversal of Rotation *701*
Additional Examples 703
Exercises *719*

11. Synchronous Machines **728**
11.1 Introduction *728*
11.2 Operating Principle *729*
11.3 Types of Rotors *729*
11.4 Stator *730*
11.5 Field and Armature Configurations *730*
11.6 Advantages of Rotating Field *730*
11.7 Expression of Frequency *731*
11.8 Distributed Armature Winding *731*
11.9 Short-pitch Armature Winding *731*
11.10 Single and Double Layer Windings *732*
11.11 Expression for Induced EMF in a Synchronous Machine *732*
11.12 Armature Reaction and Armature Winding Reactance *737*
11.13 Armature Leakage Reactance and Synchronous Impedance *738*
11.14 Equivalent Circuit and Phasor Diagram *738*
11.15 Voltage Regulation of a Synchronous generator *739*
11.16 Phasor Diagram for a Salient Pole Alternator: Two-reaction Theory *743*
11.17 Steady State Parameters *745*
11.18 Power Angle Characteristic of Salient Pole and Cylindrical Pole Machines *745*
11.19 Synchronous Motor 749
11.20 Principle of Operation *749*
11.21 Effect of Change of Excitation of a Synchronous Motor (V-curves) for the Motor Driving a Constant Load *750*
11.22 Starting of Synchronous Motors *751*
11.23 Application of Synchronous Motor *752*
11.24 Comparison between Synchronous and Induction Motor *752*
11.25 Hunting of Synchronous Machine *753*

11.26 Equivalent Circuit and Phasor Diagram of Synchronous Motor *753*
11.27 Power and Torque Developed in a Cylindrical Rotor Motor *755*
11.28 Power and Torque Developed in Salient Pole Motor *756*
Additional Examples 760
Exercises 765

12. Single-phase Induction Motors 770
12.1 Introduction *770*
12.2 Production of Torque *770*
12.3 Equivalent Circuit of a Single-Phase Induction Motor *772*
12.4 Determination of Parameters of Equivalent Circuit *774*
12.5 Starting of Single-phase Induction Motors *777*
12.6 Split Phase Induction Motors *778*
12.7 Capacitor Start Motor *779*
12.8 Capacitor Start Capacitor-Run Motor *781*
12.9 Shaded Pole Motors *784*
Exercises 786

13. Electrical Measuring Instruments 788
13.1 Introduction *788*
13.2 Types of Instruments *788*
13.3 Working Principle of Electrical Instruments *789*
13.4 Different Torques in Indicating Instruments *789*
13.5 Damping Torque (T_d) *792*
13.6 Types of Indicating Instruments *794*
13.7 Measurement of Resistance *809*
13.8 Rectifier Type Instruments *810*
Exercises 812

14. Review Problems 813

15. Multiple Choice Questions 923
15.1 Circuit Elements, Kirchhoff's Laws, and Network Theorems *923*
15.2 Electromagnetic Induction and Inductance *925*
15.3 Fundamentals of AC Circuits *927*
15.4 DC Machine *930*
15.5 Transformer *935*
15.6 Induction Motor *941*
15.7 Synchronous Machine *947*
15.8 Basic Circuit; Capacitance; AC Circuit *949*
15.9 Magnetic Circuits *950*
15.10 Electrical Machines *951*

Index 956

1 FUNDAMENTALS

1.1 SYSTEM OF UNITS

In our daily life as well as in commerce, science and engineering, the system of units have been devised to designate the standards of reference of physical quantities and used to quantify (or measure) them in that standard frame of reference. In fact, everything we see, feel, buy, sell, use is measured and compared by means of units. The units also bear a direct numerical relationship to each other, usually expressed as a whole number. In the English system of units (*FPS system*), they are related to each other by multiples of 3, 12 and 36 while in the Metric system (*CGS system*), the units are related to each other by multiples of 10. In conversion, this system (also known as the *decimal system*) is thus more advantageous than the English system.

Nowadays the International system of units (abbreviated as *SI*) is a commonly acceptable metric system. It possesses the following remarkable features:

(a) It is a decimal system.
(b) It is versatile and diversified (e.g., it can measure weights as kilogram, power as kilowatt, potential as volt, current as ampere, length as metre, time as second, temperature as Kelvin, etc.).
(c) It is used by common people as well as by specialists and is very simple.

1.2 FUNDAMENTAL AND DERIVED UNITS IN SI SYSTEM

Table 1.1 represents the fundamental units in SI system.

From these seven fundamental units in SI system, we can derive any number of derived units to express physical quantities such as area, force, flux, potential, charge, pressure, etc.

Table 1.2 represents a list of a few electrical derived units in SI system which are commonly in use.

Table 1.1 *Fundamental units in SI system*

Physical Quantity	*Unit*	*Symbol*
Mass	Kilogram	kg
Length	Metre	m
Time	Second	s (or sec.)
Temperature	Kelvin	K
Electric Current	Ampere	A (or Amp.)
Luminous Intensity	Candela	Cd
Amount of substance	Mole	mol

Table 1.2 *Derived electrical units in SI system*

Capacitance (C)	Farad (F)
Inductance (L)	Henry (H)
Conductance (G)	Siemens (S) [also in use Mho]
Resistance (R)	Ohm (Ω)
Susceptance (B)	Mho (℧) or Siemens (S)
Impedance (Z)	Ohm (Ω)
Admittance (Y)	Mho (℧) or Siemens (S)
Potential (V)	Volt (V)
Energy	Joule (J)
Magnetic flux (ϕ)	Weber (Wb)
Flux density (B)	Tesla (T)
Power (P)	Watt (W)
Frequency (f)	Hertz (Hz)
Phase Angle (δ)	Radian (rad)

1.3 DEFINITIONS OF FUNDAMENTAL UNITS AND DERIVED UNITS

A few commonly used fundamental units are defined as under:

(a) *Metre* (m): Length of the path travelled by a light ray in space during a time interval of 1/299792458 of a second.

(b) *Kilogram* (kg): Unit of mass that is equal to the mass of the international prototype of the kilogram. (It is a particular cylinder of platinum-iridium alloy that is preserved at France by the International Bureau of Weights and Measures.)

(c) *Second* (s): Unit of time defined as the duration of 9192631770 periods of radiation corresponding to the transition between the two hyperfine levels of the ground state of "Cesium-133" atom.

(d) *Kelvin* (K): Unit of temperature expressed as the fraction (1/273.16) of the thermodynamic temperature of the "triple point" of water [triple point is equal to 0.01 degree Celsius (°C)]. Thus a temperature of 0°C is equal to 273.16 Kelvin.

(e) *Ampere* (A): It is the constant current which, if passed through two straight parallel conductors of infinite length (of negligible cross sectional area), and if placed in vaccum at 1 metre apart, would produce a force of 2×10^{-7} Newton/metre length of conductors and between these two conductors.

(f) *Candela* (Cd): It is the luminous intensity of a source in a given direction emitting monochromatic light of frequency 540×10^{12} Hz and it has an intensity of 1/683 watts per steredian in that direction.

(g) *Mole* (mol): It is the amount of substance of a system that has as many elementary entities as there are atoms in 0.012 kg of "Carbon-12" element.

A few commonly used derived units are defined as under:

(a) *Coulomb* (Q): It is defined as the quantity of electric charge passed in one second by a current of strength one ampere

$$1 \text{ coulomb} = 1 \text{ A/sec.}$$

(b) *Henry* (H): It is the inductance of a closed circuit in which an emf of one volt is produced when the circuit current varies at a uniform rate of 1 A/sec.

$$1 \text{ Henry} = 1 \text{ V. sec./A}$$

(c) *Ohm* (Ω): It is the electrical resistance offered by a passive current carrying element across its two terminal points when a constant potential difference of one volt is maintained across their two terminal points and the current passing through this conductor is one ampere.

$$1 \text{ ohm} = 1 \text{ V/A}$$

(d) *Siemens* (S): It is the unit of electric conductance, equal to reciprocal of ohm. Siemen is also represented by mho.

(e) *Farad* (F): It is the capacitance between the plates of a capacitor having a p.d. of one volt across them and when it is equal by one coulomb of electricity.

$$1 \text{ F} = 1 \text{ coulomb/volt}$$

(f) *Hertz* (Hz): Frequency of a periodic wave; when the time period of the periodic wave is one sec., frequency is one Hz.

$$f = \frac{1}{T}$$

(g) *Tesla* (T): It is defined as the flux density in a medium equal to one weber per square metre.

$$1 \text{ T} = 1 \text{ Wb/m}^2$$

(h) *Volt* (V): It is the electric potential difference between two points of a conductor when a constant current of one ampere passes through it with a power dissipation of one watt.

$$1 \text{ V} = 1 \text{ W/A.}$$

(i) *Watt* (W): It is the unit of electrical power that produces energy at the rate of one Joule in one second.

$$1 \text{ W} = 1 \text{ joule/sec}$$

(j) *Radian* (rad): It is the unit of phase angle that measures a plane angle with its vertex at the centre of a circle and subtended by an arc equal to the length of the radius.

$$1 \text{ rad} = \frac{\pi^\circ}{180}$$

1.4 CONCEPT OF PER UNIT (P.U.) SYSTEM

The SI system is enough to specify the magnitude of any physical quantity. However, we can get a better idea by comparing the physical quantity in question to the size (or quantity) of "something" very similar to it. This in fact is another concept of creating a new unit and specify the size (or quantity) of similar quantities compared to this new unit. This unit gives rise to the concept of *per unit (p.u.)* system of expressing the magnitude of a physical quantity. It is extensively expressed in electrical engineering.

In representing p.u. quantities, it is important to select a "base" unit against which the physical quantities would be expressed. Though this method is applicable in all systems, in the present text we will explain the application of this concept to electrical engineering only.

Let us say we have three bulbs of 25 W, 100 W and 200 W. We arbitrarily select a base power 100 W. We can then represent the corresponding bulb power ratings as 0.25 p.u., 1 p.u. and 2 p.u. respectively. If we select another base, say 50 W, the corresponding powers become 0.5 p.u., 2 p.u., and 4 p.u. respectively. It is thus important to first decide the base unit in the p.u. system. If we do not know its magnitude, the actual values cannot be calculated.

An illustration

Let us assume that the base value of impedance for the elements, represented in Fig. 1.1(a), be 100 ohm. Figure 1.1(b) then represents the same network configuration but with a p.u. represenation.

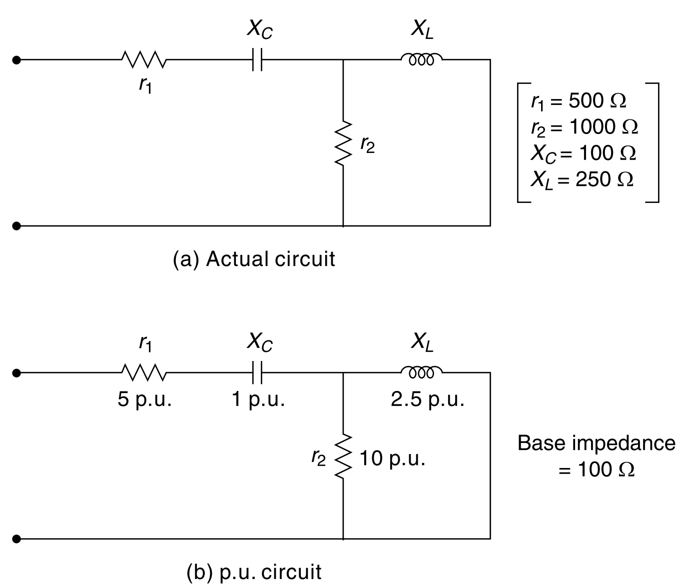


Fig. 1.1 ***(a) Actual circuit (b) p.u. circuit***

In order to maintain the consistancy in the p.u. system, two base quantities have been chosen in electrical engineering—voltage (V) and voltamperes (VA).

The base voltage is normally the nominal (i.e. rated) system voltage and the voltampere base may be selected as some integral multiple of one equipment rating. (In power system engineering, the largest rotating machine rating is normally chosen as the base *VA* power).

If the selected voltage base is (V_B) and the selected *VA* base is $(VA)_B$, then the remaining base quantities may be derived as:

$$I_B = \frac{(VA)_B}{V_B}. \tag{1.1}$$

$$Z_B = \frac{V_B}{I_B} \quad \text{or} \quad \frac{V_B^2}{(VA)_B}. \tag{1.2}$$

(The three-phase systems are generally represented as single-phase quantities, in which case the base quantities are all phase values.)

If an equipment in a system has a different base value than that of the system, we can represent the equipment p.u. impedance $Z_{e(\text{p.u.})}$ in terms of its own base $[Z_{e(\text{base})}]$

$$Z_{e(\text{p.u.})} = \frac{Z}{Z_{e\,(\text{base})}}$$

i.e.

$$Z = Z_{e(\text{p.u.})} \times Z_{e(\text{base})} \tag{1.3}$$

Equipment p.u. impedance on system base $(Z_{s(\text{base})})$ is represented as

$$Z_{s(\text{p.u.})} = \frac{Z}{Z_{s\,(\text{base})}} = Z_{e(\text{p.u.})} \times \frac{Z_{e\,(\text{base})}}{Z_{s\,(\text{base})}}$$

i.e.,

$$Z_{s(\text{p.u.})} = Z_{e(\text{p.u.})} \times \left(\frac{V_e}{V_s}\right)^2 \times \left(\frac{VA_s}{VA_e}\right) \tag{1.4a}$$

These formulae can be used to convert the p.u.. (Z) obtained in one base to p.u. (Z) of another base. Here

$$Z_{\text{new(p.u.)}} = Z_{\text{old(p.u.)}} \times \left(\frac{V_{\text{old base}}}{V_{\text{new base}}}\right)^2 \times \left(\frac{VA_{\text{new base}}}{VA_{\text{old base}}}\right) \tag{1.4b}$$

In order to clarify the concept of per unit representation, we include two more illustrations as given below.

Illustration 1

Let us assume that in an electrical system the base voltage is 3 KV while the base power is 300 kVA. The base current would be

$$I_B = \frac{\text{Base power}}{\text{Base voltage}} = \frac{300 \times 10^3}{3 \times 10^3} = 100 \text{ A}$$

The base impedance is

$$Z_B = \frac{V_B}{I_B} = \frac{3\times10^3}{100} = 300\ \Omega$$

Once we have selected the base quantities, we can find the p.u. resistance, p.u. current, p.u. voltage drop and the p.u. power dissipated across a 500 Ω resistor (say) carrying a current of 50 A.

We can write:

$$\text{p.u. } (R) = \frac{R}{Z_B \text{ (or } R_{\text{base}})} = \frac{500}{300} = 1.67$$

$$\text{p.u. } (I) = \frac{I}{I_B} = \frac{50}{100} = 0.5$$

$$\begin{aligned}\text{p.u. } (V_{\text{drop}}) \text{ across } (R) &= I_{(\text{p.u.})} \times R_{(\text{p.u.})} \\ &= 0.5 \times 1.67 = 0.835 \\ \text{p.u. power} &= V(\text{pu.}) \times I(\text{p.u.}) \\ &= 0.835 \times 0.5 = 0.4175\end{aligned}$$

Illustration 2

Let us suppose that the voltage drop in the referred resistor of the previous illustration is 0.835 p.u. (given) while the power dissipated is 0.4175 p.u. (given). We would like to know the actual values of the voltage drop and power dissipated using the base values. We can do it as follows:

$$V = V_B \times V_{(\text{p.u.})} \qquad \left[\because V_{(\text{p.u.})} = \frac{V}{V_B}\right]$$

$$= 3 \times 10^3 \times 0.835 = 2.505 \times 10^3 \text{ V } (= 2.505 \text{ kV})$$

Here (V) represents the actual drop across the resistor.

Actual power dissipated (S) is obtained as:

$S = S_{(\text{p.u.})} \times S_B$, where S stands for VA power

Here, $S = 0.4175 \times 300 \times 10^3$

$[\because$ VA(p.u.) = 0.475, given VA(base) = 300 kVA$]$

$= 125.25 \times 10^3$ VA = 125.25 kVA.

1.1 A 6 kV energy source delivers power to a 100 Ω resistor and a 250 kVA electric heater. Find the p.u. currents to the 100 Ω resistor and the electric heater. Also find the total p.u. current from the source. What is the p.u. power of the resistor and the heater? Also determine the actual power loss in the resistance and find the actual line current. Assume base voltage is 3 kV and base power is 100 kVA.

Solution

Let us first draw the circuit diagram (Fig. 1.2).

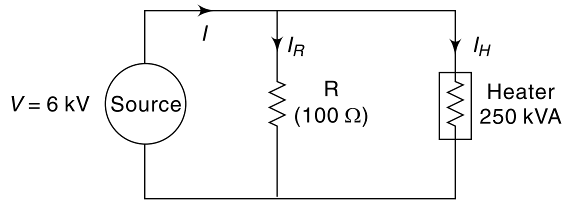


Fig. 1.2 ***Circuit of Ex. 1.1***

From the given data,

$$V_{(\text{p.u.})} = \frac{V}{V_B} = \frac{6 \times 10^3}{3 \times 10^3} = 2 \text{ p.u.}$$

$$R(\text{p.u.}) = \frac{R}{Z_B} = \frac{100}{90} = 1.111$$

$$\left[\because Z_B = \frac{V_B}{I_B} = \frac{V_B}{(VA)_B / V_B} = \frac{V_B^2}{(VA)_B} = \frac{(3 \times 10^3)^2}{100 \times 10^3} = 90\,\Omega \right]$$

Heater $\quad (VA)_{\text{p.u.}} = S(\text{p.u.}) = \dfrac{S}{S_B} = \dfrac{250 \times 10^3}{100 \times 10^3} = 2.5;$

p.u. current of (R) is given by

$$I_R(\text{p.u.}) = \frac{V_{(\text{p.u.})}}{R_{(\text{p.u.})}} = \frac{2.000}{1.111} = 1.8$$

$$I_H(\text{p.u.}) = \frac{\text{Heater power (p.u.)}}{V_{(\text{p.u.})}} = \frac{S_{(\text{p.u.})}}{V_{(\text{p.u.})}} = \frac{2.5}{2.0} = 1.25$$

$\therefore$ p.u. currents of the 100 ohm resistor is 1.8 p.u. while that of the heater is 1.25 p.u.

Total p.u. current $I_{(\text{p.u.})}$ drawn from the supply is 1.8 + 1.25 = 2.05

Per unit power of (R) is obtained as S(p.u.) of resistor = $V_{(\text{p.u.})} \times I_{R(\text{p.u.})} = 2 \times 1.8 = 3.6$, while the p.u. power of heater is $S_{(\text{p.u.})}$ [of heater] = $V_{(\text{p.u.})} \times I_{H(\text{p.u.})} = 2 \times 1.25 = 2.5$

Actual power loss in the resistor is obtained as

$$S_H = S_B \times S_H(\text{p.u.})$$

$$= 100 \text{ kVA} \times 2.5 = 250 \text{ kVA } (= 250 \text{ kW})$$

[Also $S_H = (I_H(\text{p.u.}) \times I_b)^2 \times R_H(\text{p.u.}) \times Z_B$ $\qquad (\because \text{ loss} = I^2R)$

$$= \left(1.25 \times \frac{(VA)_B}{V_B}\right)^2 \frac{V\,(\text{p.u.})}{I_H\,(\text{p.u.})} \times R_B \qquad (\because Z_B \equiv R_B)$$

$$= \left(1.25 \times \frac{100 \times 10^3}{3 \times 10^3}\right)^2 \times \frac{2}{1.25} \times 90$$

= 250 kVA (= 250 kW) (for a resistive circuit)]

Actual line current is

$$I = I(\text{p.u.}) \times I_B$$

$$= 2.05 \times \frac{(VA)_B}{V_B} = 2.05 \times \frac{100\times10^3}{3\times10^3}$$

$$= 68.333 \text{ A.}$$

1.2 A sample 3-phase power system is displayed in Fig. 1.3. The component ratings are as shown in Table 1.3.

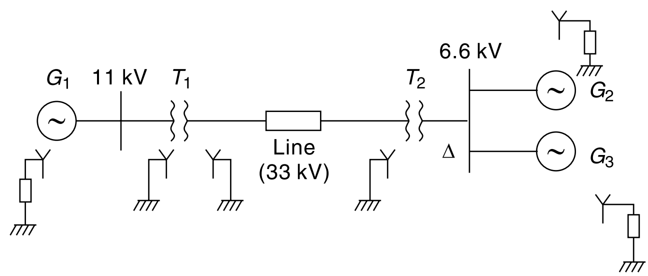


Fig. 1.3 *A sample power system*

Table 1.3 *Component rating*

Item	*Power*	*Voltage (L-L)*	*Reactance*
Generator no. 1 (G_1)	30 MVA	10.5 kV	2 Ω
Generator no. 2 (G_2)	20 MVA	6.6 kV	1.8 Ω
Generator no. 3 (G_3)	10 MVA	6.6 kV	1.2 Ω
Transformer no. 1 (T_1)	20 MVA	11 kV/33 kV	15 Ω on HT side
Transformer no. 2 (T_2)	20 MVA	6.6 kV/33 kV	18 Ω on HT side
Line	—	—	20 Ω/phase

Assuming 25 MVA is the base power and 33 KV the base voltage, find the p.u. reactances of the sample system components on a common base and hence draw the reactance diagram.

Solution

The voltage base of G_1 is 11 kV while the voltage base of G_2 and G_3 is 6.6 kV.

$\therefore$ for G_1, $Xg_1 = 2\ \Omega$ (given), $MVA_B = 25\ MVA$, and $V_B = 11$ kV.

$$\therefore \qquad I_B = \frac{MVA_B \times 10^6}{KV_B \times 10^3} = \frac{25\times10^6}{11\times10^3} = 2272.73 \text{ A}$$

$$\text{Thus,} \qquad X_{\text{base}(G_1)} = \frac{V_B}{I_B} = \frac{11\times10^3}{2272.73} = 4.84\ \Omega$$

$$\text{while} \qquad X(\text{p.u.})_{G_1} = \frac{X_{g1}}{X_{\text{base}(G_1)}} = \frac{2}{4.84} = 0.413.$$

Next, for G_2 and G_3,

$$I_B = \frac{MVA_B \times 10^6}{KV_B \times 10^3} = \frac{25 \times 10^6}{6.6 \times 10^3} = 3.788 \times 10^3 \text{ A}$$

$$X_{\text{base}(G_2)} = \frac{V_B}{I_B} = \frac{6.6 \times 10^3}{3.788 \times 10^3} = 1.742\ \Omega$$

$$\therefore \qquad X_{(\text{p.u.})G_2} = \frac{X_{g_2}}{X_{\text{base}(G_2)}} = \frac{1.8}{1.792} = 1.034$$

and

$$X_{(\text{p.u.})G_3} = \frac{X_{g_3}}{X_{\text{base}(G_3)}} = \frac{1.2}{1.792} = 0.689.$$

For T_1 and T_2,

$$V_B = 33 \text{ KV}, \quad \text{MVA}_B = 25 \text{ MVA}$$

$$\therefore \qquad I_B = \frac{MVA_B}{V_B} = \frac{25 \times 10^6}{33 \times 10^3} = 0.758 \times 10^3 \text{ A}$$

$$X_{\text{base}(T_1)} = X_{\text{base}(T_2)} = \frac{V_B}{I_B} = \frac{33 \times 10^3}{0.758 \times 10^3}$$

$$= 43.54\ \Omega.$$

For $\quad T_1,\ X_{T_{1\,(\text{p.u.})}} = \dfrac{X_{T_1}}{X_{\text{base}(T_1)}} = \dfrac{15}{43.54} = 0.345.$

For $\quad T_2,\ X_{T_{2\,(\text{p.u.})}} = \dfrac{X_{T_2}}{X_{\text{base}(T_2)}} = \dfrac{18}{43.54} = 0.413.$

For transmission line,

$$V_B = 33 \text{ KV; MVA}_B = 25 \text{ MVA}$$

$$\therefore \qquad I_B = \frac{25 \times 10^6}{33 \times 10^3} = 0.758 \times 10^3 \text{ A}$$

$$X_{\text{base(line)}} = \frac{V_B}{I_B} = \frac{33 \times 10^3}{0.758 \times 10^3} = 43.54\ \Omega$$

Thus,

$$X_{\text{line(p.u.)}} = \frac{X_L}{X_{\text{base(line)}}} = \frac{20}{43.54}$$

$$= 0.459.$$

The reactance diagram is shown in Fig. 1.4. (Figures are in p.u. reactances)

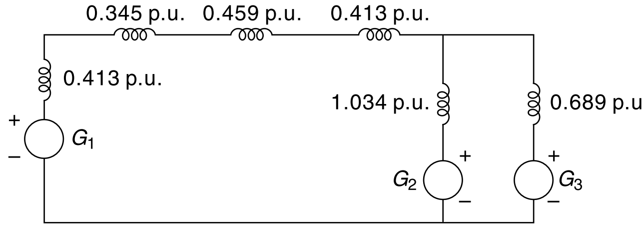


Fig. 1.4 ***Reactance diagram***

1.3 Express the p.u. impedance ($Z_{p.u.}$) and the p.u. admittance ($Y_{p.u.}$) of an electric power network in terms of the base voltage (V_B) and base voltamperes $(VA)_B$.

Solution

$$Z_B = \frac{V_B}{I_B} = \frac{V_B}{\left(\dfrac{(VA)_B}{V_B}\right)} = \frac{V_B^2}{(VA)_B}$$

Also $$Z_{\text{p.u.}} = \frac{Z_{(\text{actual})}}{Z_{\text{base}}}\left(= \frac{Z}{Z_B}\right) = \frac{Z\,(VA)_B}{V_B^2}$$

$$Y_{\text{p.u.}} = \frac{1}{Z_{\text{p.u.}}} = \frac{YV_B^2}{(VA)_B}$$

1.4 A 11 kV overhead line has a series impedance of 25 Ω and shunt admittance of 0.02 mho. Assuming the base power to be 25 MVA and line voltage as base voltage, find the p.u. impedance and p.u. admittance of the line.

Solution

$$Z_{\text{p.u.}} = \frac{Z}{Z_B}$$

but $$Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{(VA)_B}$$

Here, $$Z_B = \frac{(11 \times 10^3)}{25 \times 10^6} = 4.84\ \Omega$$

∴ $$Z(\text{p.u.}) = \frac{25\,\Omega}{4.84\,\Omega} = 5.165$$

$$Y(\text{p.u.}) = \frac{Y}{Y_B} = Y \times Z_B = 0.02 \times 4.84$$
$$= 0.097 \text{ mho.}$$

1.5 A 5 kVA, 220 V alternator has a reactance of 10 Ω. Using the rated kVA and voltage as base values, obtain the p.u. reactance. Then refer this p.u. value to a 230 V, 7.5 kVA base.

Solution

For the first base (5 kVA, 220 V),

$$V_B = 220 \text{ V} = 1 \text{ p.u.}; \; S_B = (VA)_B = 5 \text{ kVA}$$

$$\therefore \quad I_B = \frac{S_B}{V_B} = \frac{5 \times 10^3}{220} = 22.73 \text{ A} = 1 \text{ p.u.}$$

$$X_B(\text{base reactance}) = \frac{V_B}{I_B} = \frac{220}{22.73} = 9.679\ \Omega;$$

$$\text{p.u. reactance} = X(\Omega)/X_B = 10/9.679 = 1.033.$$

For 230 V, 7.5 kVA base, we obtain

$$X_{\text{p.u.}}(\text{new base}) = X_{\text{old(p.u.)}} \times \left(\frac{V_{\text{old}}}{V_{\text{new}}}\right)^2 \left(\frac{(VA)_{B\,(\text{new})}}{(VA)_{B\,(\text{old})}}\right)$$

$$= 1.033 \times (220/230)^2 \times (7500/5000) = 1.418.$$

1.5 CONCEPT OF CURRENT FLOW

In an energy source (say, a battery) there is a potential difference (p.d.) between the positive ("+ve" or "+") and negative ("–ve" or "–") terminals measured in volts. When there is no external connection between these two terminals, this potential difference is the "emf" of the cell. The potential difference is due to an excess of electrons at the negative terminal with respect to the positive terminal.

As soon as these two terminals (+ve and –ve) are connected by a conductor, the p.d. causes an electric current to flow in the circuit. This current is composed of a steady stream of electron coming out of the negative terminal and passing through the wire and re-entering the battery through its positive terminal. Though the direction of flow of the electrons in the circuit is from –ve to +ve terminal, the conventional direction of current flow is just taken as in the direction opposite to electron flow. Thus the flow of current in the circuit is always assumed to be from the +ve to the –ve polarity.

1.6 CONCEPT OF SOURCE AND LOAD

A 'source' delivers electric power (or energy) while a 'load' absorbs it. Usually a resistor, an inductor, a motor, etc. can be treated as loads while a generator, battery, etc. can be treated as source. However, it is interesting to note that a number of devices like a motor, a capacitor, and inductor and some other electric devices can act both as source and load (only exception is a reistor which can act as a load only; similarly a photocell can act as a source only). Take the example of a capacitor; when it is charged, it acts as a load, but during discharging it acts as a source. During charging current enters into it through a particular terminal which develops +ve polarity. During discharging current comes out from the +ve polarity and it than acts as a source. Similar things happens for a battery. During charging, current enters through the positive polarity and the battery becomes a load while during discharging, current comes out of this positive polarity and the battery acts as a source. Thus, in a general sense we can say:

- in a load current enters the device through the positive terminal and
- in a source current comes out of the device from positive terminal.

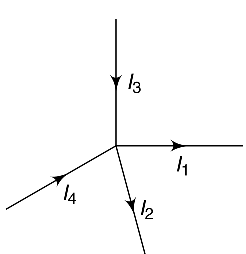
In both the cases, the '+ve' polarity remains the same.

1.7 SIGN NOTATION FOR VOLTAGES

Usually we represent the positive terminal by (+) and having the higher potential while the negative terminal by (–) having the lower potential. If we use subscript, then it represents the potential of a particular point in the circuit (i.e., the node); i.e., we denote the voltage at point '*A*' (say) in a circuit as (V_A) or (E_A). If $V_A = 10$ V it means point *A* has potential of 10 V (+ve). If (V_A) = –10 V, it means that polarity of point *A* is –ve and the magnitude of the potential is still 10 V. If we use double subscript we then represent the potential of that point with respect to another point; e.g., (V_{AB}) = 10 V means the potential difference between points *A* and *B* in a circuit is 10 V while point *A* is at a higher potential than that at point *B*. This means potential at *A* is higher and (+10) V with respect to the lower potential point *B*. If we say (V_{AB}) = –10 V, it means, the voltage between *A* and *B* is still 10 V but point *A* is negative with respect to point *B* (i.e., point *B* has higher potential than point *A*). It may be remembered that a voltage (V_{AB}) can always be represented by the voltage ($-V_{BA}$).

1.8 CONCEPT OF POSITIVE AND NEGATIVE CURRENTS AND VOLTAGES

A current flow is said to be +*ve* when the current comes out from the +ve polarity of the device while it is said to be –*ve* when it enters the device through the +ve polarity. Thus in a junction point in a circuit, a current is positive when it comes out of the junction and is negative when it approaches the junction, the voltage at the junction being +ve. In Fig. 1.5(a), (I_1) and (I_2) are +ve currents while (I_3) and (I_4) are –ve currents.



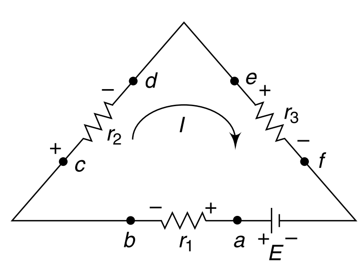


Fig. 1.5(b)

Fig. 1.5(a) ***Illustration of +ve and –ve currents and voltage drops***

We next focus our attention to a closed loop circuit as shown in Fig. 1.5(b). A battery of emf E circulates a current I though a circuit having series resistances r_1, r_2 and r_3. The drops (Ir_1), (Ir_2) and (Ir_3) are taken as the +ve voltages as current through the resistors r_1, r_2 and r_3 passes from the +ve to –ve polarity of each of the elements (since current passes from higher potential to lower potential, points a, c and e are higher potentials than their corresponding points b, d and f respectively). Thus, the voltage drop in a load is +ve provided current flows within the device through its +ve to –ve polarity. On the other hand, for the battery, current flows through –ve to +ve terminal within the battery and thus this voltage (E) is –ve. These conventions are important in understanding and dealing with Kirchhoff's current and voltage laws described in the chapter 'DC Network Analysis' later in this book.

1.9 OHM'S LAW AND CONCEPT OF RESISTANCE

Ohm's law states that electric current flowing through a conductor is directly proportional to the potential difference between its two ends when the temperature and other physical parameters of the conductor remains unchanged.

Assuming (V_{AB}) be the p.d. between the two ends A and B of a conductor, with terminal A at higher potential, current (I) being flowing through the conductor, we can write as per Ohm's law

$$V_{AB} \propto I$$

or,
$$V_{AB} = R.I \tag{1.5}$$

(R) being the *resistance* of the conductor (the constant in the equation of proportionality).

Thus, $R = V/I$; (V_{AB}) being generalised as (V), i.e., resistance = volt/ampere

Ohm is the unit of resistance, the symbol being Ω (Greek capital letter "omega"). If 1 ampere flows through a conductor, the voltage difference being 1 V across its two ends, the resistance of the conductor is then said to be 1 Ω.

[The reciprocal of resistance is called *conductance* (G). The resistance of the conductor being the property of it which opposes the current flow through it, conductance is the property that assists current flow through it. Obviously,

$$G = \frac{1}{R}.$$

The unit of conductance is mho or Siemens and following Ohm's law,

$$V = IR \quad \text{or,} \quad I = \frac{V}{R} = VG \tag{1.6}$$

1.10 CONCEPT OF RESISTIVITY

Let us consider a conductor of length 'l' m and area of cross-section 'a' m^2 having the resistance (R) ohm. It can be written,

$$R \propto l, \text{ when '}a\text{' is constant}$$

$$R \propto \frac{1}{a}, \text{ when '}l\text{' is constant.}$$

Combining the two equations of proportionality,

$$R \propto \frac{l}{a} \text{ when both '}l\text{' and '}a\text{' are not constant (i.e. varying)}$$

$$\therefore \qquad R = \rho \frac{l}{a}. \tag{1.7}$$

(ρ) being the constant of proportionality; ρ (rho) is known as *resistivity* of the material.

If $l = 1$ m; $a = 1$ m^2, $R = \rho$.

Also, $\rho = \dfrac{R \cdot a}{l} = \dfrac{\Omega \times \text{m}^2}{\text{m}} = \Omega$-m (unit of resistivity is thus Ω-m). We also introduce another term here, known as *conductivity* (σ) (reciprocal of resistivity).

$$\sigma = \frac{1}{\rho}$$

The unit of conductivity is Siemens (or mho)

Also, $$\sigma = \frac{1}{\rho} = \frac{1}{\frac{Ra}{l}} = \frac{lG}{a} \tag{1.8}$$

Unit of conductivity is Siemens/m.

1.11 EXPRESSIONS OF POWER AND ENERGY IN RESISTIVE CIRCUITS

If I be the strength of current in a resistive circuit, (V) being the potential difference across the terminals of the conductor having resistance (R) in the circuit, the power absorbed by the resistor is given by

$$P = VI = (IR)\, I = I^2R = \frac{V^2}{R} \text{ W} \tag{1.9}$$

and the energy disipated in the resistance (in form of heat) is then obtained as

$$W = \int_0^t P.dt = Pt = I^2R \times t = \left(\frac{V^2}{R} \times t\right) \text{Joule} \tag{1.10}$$

Joule is the basic unit of electrical energy but commercially we frequently use a bigger unit kWhr; it is a practical unit of electrical energy and is called *Board of Trade Unit* (BOT), 1 kWhr = BOT. Also, 1 kWhr = 1 kW $\times$ 1 Hr = 1000 $\times$ 60 $\times$ 60 W.sec = 36 $\times$ 10^5 W.sec = 36 $\times$ 10^5 J.

[It may be noted here that if the power absorbed by an element (or device) is found to be –ve, we can say that the element is delivering power to the circuit without absorbing power in real sense. Power is absorbed when current enters through the positive polarity of the device while power is delivered when the current comes out of the device from its positive terminal.]

1.6 A copper conductor of circular cross-section has length 10 m and diameter 2 mm. Calculate its resistance if the resistivity of copper is given as 1.72×10^{-8} Ω.m.

Solution

$$R = \frac{\rho l}{a} = \frac{1.72 \times 10^{-8} \times 10}{\frac{\pi \times (2 \times 10^{-3})^2}{4}} = \frac{6.88 \times 10^{-7}}{12.56 \times 10^{-6}} = 0.0548\ \Omega$$

1.7 A cube has resistivity of its material as 1.1×10^{-6} Ω.m. If it has all sides of length 2 cm each, determine the resistance of the cube between any two faces.

Solution

$$R = \frac{\rho l}{a} = \frac{1.1 \times 10^{-6} \times 2 \times 10^{-2}}{(2 \times 10^{-2})^2} = \frac{1.1 \times 10^{-6}}{2 \times 10^{-2}}$$

$$= 0.55 \times 10^{-4}\ \Omega = 55\ \mu\Omega.$$

1.8 A conductor is 50 m long. It has a cross-sectional area of 2 mm^2 while it offers a resistance of 10 Ω. Find the conductivity of the material.

Solution

$$\sigma = \frac{1}{\rho} = \frac{l}{R.a} = \frac{50}{10 \times 2 \times (10^{-3})^2}$$

$$= 2.5 \times 10^6 \text{ Siemens/m}$$

$$= 2.5 \text{ MS/m} \qquad [\because 10^6 = 1 \text{ Mega} = 1 \text{ M}]$$

1.9 Among two cubes, the first one has a length of l m while the second one has a length of $2l$ m. Find the ratio of conductivities of the materials of the cubes so that the resistance between any two faces of one cube is the same as that of the other cube.

Solution

We have seen in the text that.

$$\sigma \text{ (conductivity)} = \frac{1}{\rho}$$

$$\therefore \qquad \sigma = \frac{l}{R \cdot a} \quad \text{or} \quad R = \frac{l}{\sigma \cdot a}.$$

Here, for the cubes,

$$R_1 = \frac{l}{\sigma_1 l^2} = \frac{1}{l\sigma_1}$$

and

$$R_2 = \frac{2l}{\sigma_2 (2l)^2} = \frac{1}{2l\sigma_2}.$$

$$\therefore \qquad \frac{R_1}{R_2} = \frac{2\sigma_2}{\sigma_1} = 1 \qquad [\because \text{ as per given question } R_1 = R_2]$$

$$\therefore \qquad \frac{\sigma_1}{\sigma_2} = 2.$$

1.10 A rectangular bus bar is made of aluminium and is 90 cm long, 10 cm wide and 1 cm thick. If the bus bar current flows along its length, find the bus bar conductance provided the bus bar conductivity is 3.6×10^8 S/m.

Solution

$$R = \frac{l}{\sigma \cdot a} = \frac{0.9}{(3.6 \times 10^8)(0.1 \times 1 \times 10^{-2})} = 2.5 \times 10^{-6}\ \Omega$$

$$\therefore \qquad G(\text{Conductance}) = \frac{1}{R} = 0.4 \times 10^6 \text{ S.}$$

1.11 How many electrons pass a given point in a conductor in 10 sec if the current strength is 18 A? Assume charge of electron as 1.6×10^{-19} C.

Solution

Charge (Q) = $I \times t$ (I = Current strength; t = time in sec)
Here $Q = 18 \times 10 = 180$ C.
$\because$ 1.6×10^{-19} C corresponds to charge of one electron, 180 C of charge would have $\left(\frac{1}{1.6 \times 10^{-19}} \times 180\right)$, i.e. 112.5×10^{19}.

1.12 A charge of 400 C passes through a conductor in 40 sec. What is the corresponding current in amperes?

Solution

$$Q = I \times t$$

$$\therefore \qquad I = \frac{Q}{t} = \frac{400}{40} = 10 \text{ A.}$$

1.13 What is the p.d. across a resistance dissipating 50 W of power while the strength of current is 5 A? What is the ohmic value of the resistance?

Solution

$$V = \frac{\text{Watt}}{\text{Current}} = \frac{50}{5} = 10 \text{ V}$$

Watt (power dissipated) = I^2R

$$\therefore \qquad R = \frac{W}{I^2} = \frac{50}{5^2} = 2\ \Omega$$

1.14 A current of 10 A flows in a resistor for 10 sec. How many coulombs of charge pass within this time?

Solution

$$Q = I \times t = 10 \times 10 = 100 \text{ C}$$

1.15 An electric heater consumes 1 kWhr of energy in 30 min at 220 V. What is the current rating of the heating element?

Solution

Current, $I = \dfrac{\text{Power}}{\text{Votage}} = \dfrac{\text{Energy/Time}}{\text{Voltage}}$

$$= \frac{1\times 10^3/0.5}{220} \qquad [\because \text{ time} = 30 \text{ min} = \tfrac{1}{2} \text{ hr}]$$

$= 9.09$ A.

1.16 An energy source suppliers 500 J of energy at 100 V for a certain period of time. Determine the quantity of charge passed through.

Solution

$$Q = I \times t = \frac{\text{Power}}{\text{Voltage}} \times \text{Time} = \frac{\text{Energy}}{\text{Voltage}} = \frac{500}{100} = 5 \text{ C}$$

1.12 TEMPERATURE COEFFICIENT OF RESISTANCE

The resistance of almost all electricity conducting materials changes with the variation in temperature. This variation of resistance with change in temperature is governed by a property of a material called *temperature coefficient of resistance* (α). *The temperature coefficient of resistance can be defined as the change in resistance per degree change in temperature and expressed as a fraction of the resistance at the reference temperature considered.*

If suffix 1 indicates the initial condition and suffix 2 the final condition, we can write, from the definition of the temperature coefficient of resistance,

$$\alpha_1 = \frac{\text{Change in resistance/Change in temperature}}{\text{resistance at the reference temperature}}$$

$$= \frac{(R_2 - R_1)/(t_2 - t_1)}{R_1} \tag{1.11}$$

while $\alpha_1$is the *temperature coefficient* of the material at t_1 °C.

Similarly, at temperature t_2 °C, the temperature coefficient of resistance being expressed as α_2, we can write

$$\alpha_2 = \frac{(R_2 - R_1)/(t_2 - t_1)}{R_2} \tag{1.12}$$

It is usual to specify the reference temperature as 0 °C while the temperature coefficient of resistance is α_0. If t °C is the rise in temperature and the resistance changes from R_0(at 0 °C) to R_t and t °C, we can write

$$\alpha_o = \frac{(R_t - R_0)/(t-0)}{R_0} = \frac{R_t - R_0}{R_0 t}$$

or, $(R_t - R_0) = R_0 \alpha_0 t$

$\therefore$ $R_t = R_0 + \alpha_0 R_0 t$

$= R_0(1 + \alpha_0 t)$ (1.13)

Again from equations (1.11) and (1.12) we find

$$\frac{\alpha_2}{\alpha_1} = \frac{R_1}{R_2} = \frac{R_1}{R_1 [1+\alpha_1 (t_2 - t_1)]}$$

$$\therefore \quad \alpha_2 = \frac{\alpha_1}{1+\alpha_1 (t_2 - t_1)} = \frac{1}{\frac{1}{\alpha_1} + (t_2 - t_1)} \tag{1.14}$$

In general, if the reference temperature is 0 °C, for t °C rise in temperature we can write

$$\alpha_t = 1/\frac{1}{\alpha_0} + (t - 0) = 1/\frac{1}{\alpha_0} + t \tag{1.15}$$

The unit of α is Ω/Ω °C, i.e. per °C; e.g the temperature coefficient of resistance of copper is

$$\alpha_0 = \frac{1}{234.5} /°\text{C}$$

[It may be noted here that though the temperature coefficient of resistance has a +ve value for all metallic conductors, some of the non-metallic conducors (e.g carbon) has –ve temperature coefficient of resistance; α is –ve for these elements.]

Effect of Temperature on Resistance

The temperature coefficient of resistance (α) being given by

$$\alpha_0 = \frac{R_t - R_0}{R_0 t}$$

t will be positive for rise in temperature of the resistance from 0 °C to t °C. If $R_t > R_0$, α_0 is +ve and the profile of R_t versus t can be plotted as show in Fig. 1.6.

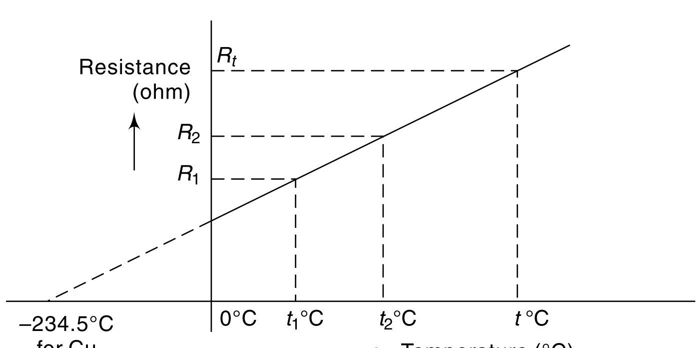


Fig 1.6 ***Resistance-temperature graph***

The profile of R_t vs t is a straight line. If we assume $\alpha_0 = 0$, $(R_t - R_0)$ tends to zero and this given $R_t = R_0$, then we can say if the temperature coefficient of resistance for a conductor is negligible, the resistance of that conductor remains same for any variation of temperature. All metallic conductors have a positive temperature coefficient of resistance while non-metals, electrolytes and liquid conductors have a negative temperature coefficient of resistance. Semiconductors and insulators also have negative temperature coefficients. A few alloys like constantan, nickel chromium, manganin and eureka have almost zero temperature coefficient of resistance. The resistance of those conductors increase with increase in temperature whose temperature coefficient of resistance is +ve while the resistance of those conductors decrease with increase in temperatures whose temperature coefficient of resistance is –ve.

1.17 A piece of coppper wire has a resistance of 25 Ω at 10 °C. What is the maximum operating temperature if the resistance of the wire is to be increased by 20%? Assume α at 10 °C = 0.0041 °C^{-1}.

Solution

$$R_1 = 50\ \Omega;\ t_1 = 10\ °\text{C};\ \alpha_1 = 0.0041/\ °\text{C}$$
$$R_2 = 50 + 0.2 \times 50 = 60\ \Omega$$
t_2 = the unknown temp. at which R will be 60 Ω.

Since $\quad R_{t_2} = R_{t_1}\ [1 + \alpha_{t_1}(t_2 - t_1)],$

here we have

$$R_2 = R_1\ [1 + \alpha_{t_1}(t_2 - t_1)]$$

∴ $\quad 60 = 50\ [1 + 0.0041 \times (t_2 - 10)]$

or $\quad t_2 - 10 = \left(\frac{60}{50} - 1\right) \times \frac{1}{0.0041}$

∴ $\quad t_2 = 58.78\ °\text{C}.$

1.18 A particular metal filament has a resistance of 10 Ω at 0 °C. At 20 °C the resistance become 12 Ω. Calculate the temperature coefficient of resistance of that filament at 20 °C. What is the temperature coefficient at 0 °C?

Solution

In the text we have seen

$$\alpha_t = \frac{R_t - R_0}{R_t \cdot t}$$

On simplification we may write

$$R_0 = R_t\ (1 - \alpha_t \cdot t)$$

Here, $\quad R_0 = 10\ \Omega;\quad R_t = 12\ \Omega$

$$t = 20°\text{C}\ [=(t_2 - t_1) = (20 - 0)°\text{C})]$$

Hence $\quad 10 = 12\ (1 - \alpha_t \cdot 20)$

or $\quad 10 = 12 - 12 \times 20 \times \alpha_t$

∴ $\quad \alpha_t = \frac{12 - 10}{12 \times 20} = 0.00833/°\text{C},$

i.e. $\alpha_{20} = 0.00833/°C$.

Also, $R_t = R_0(1 + \alpha_0 t)$ [as derived in the text.]

Here, $12 = 10(1 + \alpha_0 \times 20)$

$\therefore$ $\alpha_o = \left(\frac{12}{10} - 1\right) \times \frac{1}{20} = 0.01\ °C^{-1}$

i.e. temperature coefficient at 0 °C is 0.01/°C.

1.19 The temperature coefficient of carbon at 0°C is –0.000515 $°C^{-1}$ and that of platinum is 0.00357 $°C^{-1}$ at 40 °C. A carbon filament has a resistance of 10 Ω while the platinum foil has a resistance of 8 Ω at 0 °C. At what temperature will the two elements have the same resistance?

Solution

$$\alpha_0 = \frac{R_t - R_0}{R_0 t}$$

$$\alpha_0 R_0 t = R_t - R_0$$

or $$R_t = R_0(1 + \alpha_0 t) \tag{1}$$

and $$\alpha_t = \frac{R_t - R_0}{R_t \cdot t}$$

$\therefore$ $$R_t \cdot \alpha_t \cdot t = R_t - R_0$$

or $$R_0 = R_t(1 - \alpha_t\, t) \tag{2}$$

We can write, using equation (1) in (2)

$$R_0 = R_0(1 + \alpha_0 t)\,(1 - \alpha_t\, t)$$

or $$\alpha_o - \alpha_t - \alpha_0\, \alpha_t\, t = 0 \tag{3}$$

or $$\alpha_0 = \alpha_t(1 + \alpha_0 t) \tag{4}$$

$\therefore$ $$\alpha_t = \frac{\alpha_0}{1 + \alpha_0 t}.$$

Also, from (3),

$$\alpha_0(1 - \alpha_t \cdot t) = \alpha_t$$

$\therefore$ $$\alpha_0 = \frac{\alpha_t}{1 - \alpha_t \cdot t}. \tag{5}$$

In this problem the (α) value at 0° is given for carbon but the α value of platinum is provided for 40°C. Thus (α) of platinum at 40°C is to be converted to (α) at 0°C for platinum.

$\therefore$ For platinum,

$$\alpha_0 = \frac{0.00357}{1 - 40 \times 0.00357} = 0.00416\ °C^{-1}.$$

Let the resistances of the carbon and platinum filaments be equal at t °C.

$\because$ $R_t = R_0\,(1 + \alpha_0 t)$, we can write

$$8(1 + 0.00416\ t) = 10(1 - 0.000515 \cdot t).$$

[Note that α_0 for platinum is 0.00416/ °C as obtained and that of carbon is given as (–0.000515/ °C)]

or $$8 + 0.03328t = 10 - 0.00515\ t$$

or $$0.03843t = 2 \quad \therefore\ t = 52.043\ °C.$$

Then at 52.043 °C, the resistances of both the filaments are equal.

1.20 The power consumed by a heating element made of copper wire is 250 W at 220 V and at 30 °C. Obtain the power consumed by the same element at 220 V and 100 °C. The temperature coefficient of the element at 30 °C is 0.004 °C^{-1}.

Solution

Power loss at 30 °C(P_{30}) = $\dfrac{V^2}{R_{30}}$

$$\therefore \qquad R_{30} = \frac{V^2}{P_{30}} = \frac{220^2}{250} = 193.6\Omega.$$

We know,

$$R_{100\,°C} = R_{30\,°C}[1 + \alpha_{30}(100\,°C - 30\,°C)].$$

From the given data

$$R_{100\,°C} = 193.6(1 + 0.004 \times 70) = 247.81\ \Omega.$$

∴ Power loss at 100 °C is obtained as

$$P_{100\,°C} = \frac{220^2}{247.81} \quad \left(\because \frac{V^2}{R} \text{ is power loss}\right)$$
$$= 195.31 \text{ W}.$$

[Note that with increase of temperature, voltage applied being the same, power dissipated is reduced though the resistance term in (I^2R) is increased. Since current has predominant effect in power loss than resistance hence with increase of temperature when resistance increases, current decreases and hence power loss decreases.]

1.21 A resistor is made up of Alloy 1 dissipating 50 W of electrical power at 110 V at 20°C. Another resistance of Alloy 2 is made having the same resistance as the first resistor but consuming double amount the power of the first one. What is the current flowing through Alloy 2 resistor? Assume temperature remain constant during the entire process.

Solution

$$R_{\text{alloy 1}} = \frac{V^2}{P} = \frac{110^2}{50} = 242\ \Omega$$

As per question, $R_{\text{alloy 2}} = R_{\text{alloy 1}}$

Let I be the current flowing through the Alloy 2 resistor.

∴ $I^2R_2 = 2 \times 50$. [∵ P_{loss} in the second one is twice than that of alloy-1.]

$$\therefore \qquad I^2 = \frac{100}{242}.$$

and $I = 0.643$ A.

1.22 A heat-dissipating wire dissipates 100 W at 50°C when subjected to applied voltage of 220 V. If the wire diameter is 0.01 mm and resistivity is $2 \times 10^{-8}\ \Omega \cdot \text{m}$, find the length of the wire assuming the temperature coefficient of resistance at 20°C as 0.005°C^{-1}.

Solution

At 50°C, $R_{50} = \dfrac{V^2}{P} = \dfrac{220^2}{100} = 484\ \Omega.$

Also $R_{50} = R_{20}[1 + \alpha_{20}(50 - 20)]$

or $$484 = R_{20}[1 + 0.005 \times 30].$$

$\therefore$ $$R_{20} = \frac{484}{3.42} = 141.52\ \Omega.$$

But $$R_{20} = \frac{\rho l}{a} = \frac{2\times 10^{-8} \times l}{\frac{\pi}{4}[(0.01)^2 \times 10^{-6}]}$$

$[\because\ d = 0.01 \text{ mm} = 0.01 \times 10^{-3} \text{ m.}]$.

$\therefore$ $$l = \frac{141.52 \times \frac{\pi}{4}[(0.01)^2 \times 10^{-6}]}{2 \times 10^{-8}}$$

$$= 0.5555 \text{ m.}$$

Thus, the required length is 0.5555 m.

1.23 A non-metallic resistor has temperature coefficient of resistance of –0.0005/ °C at 20 °C. It dissipates 50 W power while drawing 1 A current at 20 °C. It is now connected to a 230 V source at 100 °C. What will be the power dissipation?

Solution

At 20°C, $$P_{20} = I^2 R_{20}$$

$\therefore$ $$R_{20} = \frac{50}{1.0^2} = 50\ \Omega$$

At 100°C, $$R_{100} = R_{20}[1 + \alpha_{20} \cdot t]$$

Here $$R_{100} = 50[1 - 0.0005\ (100 - 20)]$$
$$= 48 \text{ ohm.}$$

$\therefore$ $$P \text{ (Power loss at 100°C)} = \frac{V^2}{R_{100}} = \frac{(230)^2}{48} \simeq 1102 \text{ W.}$$

ADDITIONAL EXAMPLES

1.24 A heater is operated at 220 V and has an efficiency of 99%. The energy consumed is 1.5 kWhr in one hour. If it is required to boil a liquid that requires 100 kJ of energy, find the time needed to boil it. What is the resistance of the heater?

Solution

Let I be the input current to the heater.

$$I = \frac{\text{Power}}{\text{Voltage}} = \frac{\text{(Energy/time)}}{\text{Voltage}} = \frac{1.5 \times 10^3 / 1}{220} = 6.82 \text{ A}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100$$

Here $$\frac{99}{100} = \frac{\text{Output}}{\text{Input}}.$$

To have an output energy of 100 kJ, the input is (Output ÷ 0.99), i.e. 100/0.99 = 101.01 kJ.

But Energy = Power × Time

∴ Time = Energy ÷ Power

$$= \frac{101.01 \times 10^3}{220 \times 6.82} \quad [\because P = V \times I = 220 \times 6.82]$$

$$= 67.32 \text{ sec.}$$

Thus it will take 67.32 sec to boil the liquid. The heater resistance is obtained as

$$R = \frac{V}{I} = \frac{220}{6.82} = 32.26\ \Omega.$$

1.25 The resistivity of a material at 0°C is $8 \times 10^{-8}\ \Omega \cdot \text{m}$, while it is $10 \times 10^{-8}\ \Omega \cdot \text{m}$ at 30 °C. Assuming the resistivity versus temperature profile of the material to linear, find its resistivity at 10 °C.

Solution

Since the resistivity-temperature curve of the given material is linear, hence the slope (m) can be expressed as

$$m = \frac{\rho_{30} - \rho_0}{t_{30} - t_0} = \frac{(10-8)10^{-8}}{30-0} = 0.067 \times 10^{-8}$$

Also, from this equation $m = \dfrac{\rho_{30} - \rho_0}{t_{30} - t_0}$, we get

$$\rho_{30} = \rho_0 + m(t_{30} - t_0).$$

Generalising this for temperature t_2 and t_1 when $t_2 > t_1$, we can write

$$\rho_{t_2} = \rho_{t_1} + m\,(t_2 - t_1)$$

where $$m = \frac{\rho_{t_2} - \rho_{t_1}}{t_2 - t_1}.$$

Then for 10°C,

$$\begin{aligned}\rho_{10} &= \rho_{20} + m(10 - 20)\\ &= 10 \times 10^{-8} + 0.067 \times 10^{-8} \times (-10)\\ &= 9.33 \times 10^{-8}\ \Omega \cdot \text{m}.\end{aligned}$$

Thus the resistivity of the material at 10°C is $9.33 \times 10^{-8}\ \Omega \cdot \text{m}$.

1.26 A given conductor has a resistor of 5 Ω. What is the resistance of another conductor of the same material, which has one-half the diameter and five times the length of the given conductor?

Solution

$$R_1 = \rho\frac{l_1}{a_1}; \quad R_2 = \rho\frac{l_2}{a_2}$$

$$\therefore \quad \frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{a_1}{a_2}.$$

Here $a_1 = \pi \dfrac{d_1^2}{4}$; $a_2 = \pi \dfrac{d_2^2}{4}$

Also $d_2 = \dfrac{d_1}{2}$ (given)

$$\therefore \quad \frac{a_1}{a_2} = \frac{d_1^2}{d_2^2} = \left(\frac{d_1}{d_2}\right)^2 = \left(\frac{d_1}{d_{1/2}}\right)^2 = 4$$

Also $l_2 = 5l_1$ or $\dfrac{l_2}{l_1} = 5$

$$\therefore \quad \frac{R_2}{R_1} = \frac{l_2}{l_1} \times \frac{a_1}{a_2} = 5 \times 4 = 20.$$

$\because$ $R_1 = 5\ \Omega$ $\therefore R_2 = R_1 \times 20 = 100\ \Omega$.

1.27 Obtain the ratio of powers lost in two resistors when

(a) each being connected across the same voltage

(b) each resistor is to carry the same current.

Assume same length and material for each resistor but the first one having a diameter twice that of the other. The cross-section of each of the resistor wire is circular.

Solution

(a) Voltage applied remains same for each resistor:

$$P_1 = \frac{V^2}{R_1} = \frac{V^2}{\rho l / a_1} = \frac{V^2 a_1}{\rho l} = \frac{\pi}{4} \frac{V^2 d_1^2}{\rho l}.$$

Similarly, $P_2 = \dfrac{V^2}{R_2} = \dfrac{\pi}{4} \dfrac{V^2 d_2^2}{\rho l}$

Since $d_1 = 2d_2$,

$$\frac{P_1}{P_2} = \frac{\frac{\pi}{4}\left(\frac{V^2 d_1^2}{\rho l}\right)}{\frac{\pi}{4}\left(\frac{V^2 d_2^2}{\rho l}\right)} = \frac{d_1^2}{d_2^2} = \frac{4 d_2^2}{d_2^2} = 4.$$

(b) Each resistor carries same current:

$$P_1 = I^2 R_1 = I^2 \times \rho \times \frac{l}{a_1} = \frac{4}{\pi} \cdot I^2 \cdot \frac{\rho l}{d_1^2}$$

Similarly, $P_2 = I^2 R_2 = \dfrac{4}{\pi} \cdot I^2 \cdot \dfrac{\rho l}{d_2^2}$

$\because$ $d_1 = 2d_2$,

$$\frac{P_1}{P_2} = \frac{\frac{4}{\pi} \cdot I^2 \cdot \rho l / d_1^2}{\frac{4}{\pi} \cdot I^2 \cdot \rho l / d_2^2} = \frac{d_2^2}{d_1^2} = \frac{d_2^2}{4 d_2^2} = \frac{1}{4}.$$

1.28 A copper wire having a cross-sectional area of 0.5 mm^2 and a length of 0.1 m is initially at 25 °C and is thermally insulated from the surroundings. If a current of 10 A is set up in this wire,

(a) Find the time in which the wire will start melting.

(b) What will this time be, if the length of the wire is doubled?

Assume the resistance of the wire remains unaffected with variation in temperature. Density of copper is 9×10^3 K/m^3; specific heat of copper = 9×10^{-2} cal/(kg ×°C) and melting point of copper is 1075 °C. Specific resistance of copper is 1.6×10^{-8} Ωm.

Solution

(a) Mass of copper = Volume × Density

$$= 0.5 \times 10^{-6} \times 0.1 \times 9 \times 10^3$$
$$= 45 \times 10^{-5} \text{ kg.}$$

Rise in temperature (0 °C) is (1075–25)°C, i.e. 1050 °C.

Heat required (H) to melt the copper wire is

$$H = \frac{I^2 Rt}{4.2}$$

[I = current in the wire having resistance R and t is the time in sec for which current flows]

However, from the concept of calorimetry,

$$H = ms\theta$$

[m = mass of wire, s = specific heat, θ = temp. rise].

Also, $$R = \rho\frac{l}{a} = 1.6 \times 10^{-8} \times \frac{0.1}{0.5 \times 10^{-6}}$$
$$= 3.2 \times 10^{-3}\ \Omega.$$

Since $\dfrac{I^2 Rt}{4.2} = ms\theta$, we have

$$\frac{10^2 \times 3.2 \times 10^{-3} \times t}{4.2} = 45 \times 10^{-5} \times 9 \times 10^{-2} \times 1050$$

or $$t = 558 \text{ sec.}$$

(b) Electric energy = I^2Rt. When the length of the wire is doubled, R is also doubled. However the mass remaining constant, H is also doubled. Hence the wire will melt in the same time.

1.29 A heating element is made of some material having the temperature coefficient of resistance an 0.00065 °C^{-1} at 0 °C. What will be the ratio of its resistance at temperatures 50 °C and 30 °C?

Solution

Ler R_0 be the resistance at 0 °C for the element. At temperature θ_1°; we can write

$$R_{\theta_1} = R_0[1 + \alpha_0(\theta_1° - 0°)]$$

while at temperature θ_2° we can write

$$R_{\theta_2} = R_0[1 + \alpha_0(\theta_2° - 0°)]$$

$$\therefore \quad \frac{R_{\theta_1}}{R_{\theta_2}} = \frac{1+\alpha_0\theta_1°}{1+\alpha_0\theta_2°} (1 + \alpha_0\theta_1°)(1 + \alpha_0\theta_2°)^{-1}$$

$$= (1 + \alpha_0\theta_1°)[1 - \alpha_0\theta_2° + (\alpha_0\theta_2°)^2 - \ldots] \quad (1)$$

Since α_0 is small, we can neglect higher power of α_0. Then we can write from eqn (1)

$$\frac{R_{\theta_1}}{R_{\theta_2}} \simeq 1 + \alpha_0(\theta_1^\circ - \theta_2^\circ).$$

Here $$\frac{R_{\theta_1}}{R_{\theta_2}} = 1 + 0.00065\ (50^\circ - 80^\circ) = 0.9805, \text{ where}$$

$$\theta_1 = 50°\text{C}; \quad \theta_2 = 80°\text{C}.$$

1.30 A rectangular metal strip has dimensions x = 100 cm; y = 1 cm and z = 0.5 cm. Obtain the ratio of resistances R_x, R_y and R_z between respective pairs of opposite faces.

Solution

In Fig. 1.7, R_x = resistance along the length

$$= \rho \frac{x}{y \times z} \quad \left[\because R = \rho \frac{l}{a}\right]$$

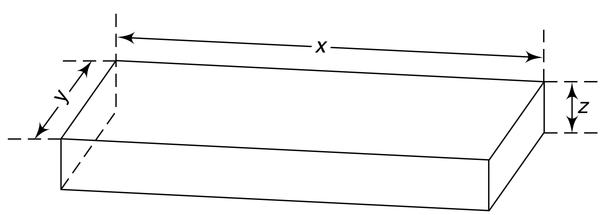


Fig. 1.7 ***A rectangular metal strip***

$$R_y = \rho \frac{y}{zx} \qquad [R_y = \text{resistance along the breadth}]$$

and $$R_z = \frac{z}{x \times y} \qquad [R_z = \text{resistance along the thickness}]$$

$$\therefore \quad R_x : R_y : R_z = \frac{\rho x}{yz} : \frac{\rho y}{zx} : \frac{\rho z}{xy} = \frac{x}{yz} : \frac{y}{zx} : \frac{z}{xy}$$

$$= \frac{100}{1 \times 0.5} : \frac{1}{0.5 \times 100} : \frac{0.5}{100 \times 1}$$

$$= 200 : \frac{1}{50} : \frac{1}{200}$$

$$= 40{,}000 : 4 : 1.$$

1.31 Temperature of a heating element is governed by the relation θ °C = 20t, t being expended in seconds. The temperature coefficient of the material at 0°C is 0.0005 °C^{-1}. If the initial resistance of the heating element is 5 Ω, find the energy dissipated in the heating element over 10 sec period of time. The applied voltage is 220 V (dc).

Solution

At $t = 0$, $\theta\,°C = 20 \times 0 = 0°C$ [$\because\ \theta = 20t$, given]
Resistance at any temperature θ_x°C is given by

$$R_{\theta_x} = R_0[1 + \alpha_0(\theta_x° - 0°)]$$

i.e., $$R_{\theta_x} = R_0[1 + \alpha_0\theta_x°] = R(t), \text{ in this case.}$$

Given $R_0 = 5\Omega$, $\alpha_0 = 0.005°C^{-1}$, $\theta_x = 20\,t$

$\therefore$ $R(t) = 5(1 + 0.005 \times 20t) = (5 + 0.5t)\ \Omega$.

However, $$\text{Energy} = \int_0^t P \cdot dt = \int_0^t \frac{V^2}{R(t)} \cdot dt$$

$$= \int_0^{10} \frac{220^2}{(5+0.5\,t)} \cdot dt$$

$$= 220^2 \int_0^{10} \frac{dt}{(5+0.5\,t)} = \frac{220^2}{0.5}\left[\ln(5+0.5\,t)\right]_0^{10}$$

$$= 96800[\ln(5 + 5) - \ln(5 + 0)] \simeq 67.1 \text{ kJ}.$$

1.32 In a current carrying conductor the current density is expressed in δ A/mm^2 while its resistivity is $\rho \times 10^{-6}\ \Omega$ cm. If the specific gravity is (S), find the power loss expressed in W/kg.

Solution

$$P_{\text{loss}} = RI^2 = \rho\frac{l}{a} \cdot I^2 \times 10^{-6} = \rho\frac{l}{a} \cdot a^2\left(\frac{I}{a}\right)^2 \times 10^{-6}$$

[$\therefore$ current density = current/area; area is expressed by a cm^2.]

or $$P_{\text{loss}} = \rho\left(\frac{I}{a}\right)^2 \times la \times 10^{-6}$$

However, ($l \times a$) represents the volume of the conductor.

$\therefore$ $$P_{\text{loss}} = \rho\left(\frac{I}{a}\right)^2 \times 10^{-6} \text{ W/cm}^3.$$

However, mass of 1 cm^3 of material = S gm

$\therefore$ $$\text{Loss in watts for } S \text{ gm} = \rho\left(\frac{I}{a}\right)^2 \times 10^{-6}$$

i.e., loss in watts for one gm = $\rho\left(\frac{I}{a}\right)^2 \times \frac{1}{5} \times 10^{-6}$

or $$\text{loss in watts per kg} = \frac{\rho \times 10^{-6} \times (\sigma \times 10^2)^2 \times 10^3}{S}$$

$$\left[\because \sigma = \frac{I}{a} \text{ A/cm}^2 = \frac{I}{a} \times 10^2 \text{ A/mm}^2\right]$$

$$= \frac{10\,\rho\sigma^2}{S}$$

1.33 For a particular element the temperature coefficient of resistance is 0.005 °C^{-1} at 0 °C. The temperature varies linearly with time and is given by the relation θ °C = (10 + 10t), where t is expressed in seconds. If the initial resistance of the element is 1 ohm, find its resistance after 30 sec. The element does not melt up to 500 °C.

Solution

At $t = 0$; $\theta_0 = 10$ °C, as obtained from the relation θ °C = (10 + 10t). Also, $R_{10} = 1\ \Omega$

At $t = 30$ sec, θ °C = 10 + 10 × 30 = 310 °C

$$\therefore \quad R_{310} = R_{10}[1 + \alpha_{10}(310 - 10)] \tag{1}$$

But
$$\alpha_{10} = \frac{\alpha_0}{1 + \alpha_0(\theta_0 - 0)}$$

[We derived this relation earlier in Ex. 1.26]

Here
$$\alpha_{10} = \frac{0.005}{1 + 0.005 \times 10} = 0.0048/°\text{C}.$$

Thus from (1),

$$R_{310} = 1[1 + 0.0048 \times 300]$$
$$= 2.44\ \Omega.$$

Thus, after 30 sec, the temperature of the element would be 2.44 Ω.

1.34 A heating element is subjected to a voltage of 220 V. The resistance of the element is a function of time when voltage is applied across it and follows the relation $R(t) = 5e^{2t}\ \Omega$ (t being expressed in seconds). How much heat energy would it generate after 10 seconds?

Solution

$$\text{Energy} = \int_0^t P \cdot dt = \int_0^t \frac{V^2}{R(t)} \cdot dt$$

$$= \int_0^t \frac{220^2}{5e^{2t}} \cdot dt = \frac{(220^2)}{5} \int_0^t (e^{-2t}) \cdot dt$$

$$= 9680\ (1 - e^{-2t}) \text{ Joule.}$$

After 10 seconds the energy generation is

$$9680\ (1 - e^{-10 \times 2}) \simeq 9680 \text{ Joule.}$$

1.35 Find the current flowing through a 60 W bulb at the instant of switching across a 220 V dc supply if the incandescent bulb filament temperature is 2020 °C and the temperature coefficient at 20 °C is 0.005. Also assume the room temperature is 20 °C.

Solution

Just at the moment of switching the filament temperature is 20°C and we are to find current at this temperature. However from the given data, utilising the formula in the text,

$$R_{2020} = R_{20}[1 + \alpha_{20}(t_2 - t_1)]$$
$$= R_{20}[1 + 0.005\ (2020 - 20)]$$
$$= R_{20} \times 11.$$

$$\therefore \quad R_{20} = \frac{R_{2020}}{11}$$

However, $$R_{2020} = \frac{V}{I\,(\text{at steady state})} = \frac{\text{V}}{(\text{W/V})}$$

$$= \frac{220^2}{60} \simeq 807\ \Omega$$

$$\therefore \quad R_{20} = \frac{807}{11} = 73.36\ \Omega.$$

Thus current at the moment of switching, i.e. at 20 °C would be

$$I_{sw} = \frac{V}{R_{20}} = \frac{220}{73.36} \simeq 3\ \text{A}.$$

1.36 Two materials, A and B, have resistance temperature coefficient of 0.004 and 0.0004 respectively at a particular temperature. In what proportion must A and B be joined in series to produce a circuit having a temperature coefficient of 0.001?

Solution

Let us assume that at the lower temperature let the resistance of A be 1 Ω and that of B be r ohm. As the temperature is raised by t °C, the resistance of the combination becomes $[(1 + r)\{1 + 0.001 \times t\}]$.

However, individual resistance of element A at t°C is $[1(1 + 0.004t)]$ and that of B is $r(1 + 0.0004t)]$.

Since both the elements A and B are in series.

$$[1(1 + 0.004t)] + [r(1 + 0.0004t)] = [(1 + r)\ \{1 + 0.001 \times t\}]$$

or $$1 + 0.004\ t + r + 0.0004t \times r = 1 + 0.001\ t + r + 0.001t \times r$$

or $$0.004 + 0.0004r = 0.001 + 0.001r$$

$$\therefore \quad r = 5.$$

Thus the required proportion A and B that must be joined in series is A : B = 1 : 5.

1.37 In Fig. 1.8(a), assume all the resistances are equal to r. A battery with $V = 2$ volts and internal resistance of 0.1 Ω is connected across the given circuit. Calculate the value of r for which the heat generated is maximum.

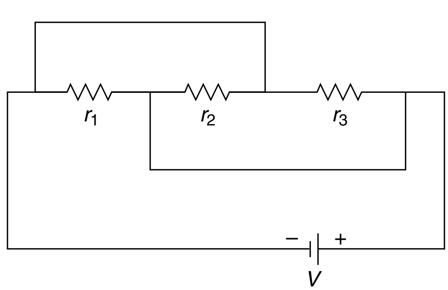


Fig. 1.8(a) *Circuit of Ex. 1.37*

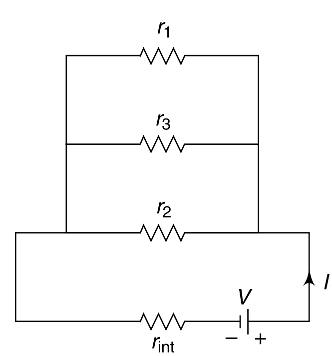


Fig. 1.8(b) *Simplified circuit of Fig. 1.8.1*

Solution

Let us simplify the circuit as shown in Fig. 1.8(b)

$\therefore \quad R_{eq}$ = parallel of r_1, r_2 and r_3

Since $r_1 = r_2 = r_3 = r$

$R_{eq} = r/3$

Current from the battery is $I = \dfrac{V}{R_{eq} + r_{int}} = \dfrac{V}{\dfrac{r}{3} + r_{int}} = \dfrac{3V}{r + 3r_{int}}$.

The rate at which heat is generated in the circuit is given by

$$W = I^2 R_{eq} = \left(\frac{3V}{r + 3r_{int}}\right)^2 \times \frac{r}{3} = \frac{3V^2 r}{(r + 3r_{int})^2}.$$

Heat will be maximum when

$$\frac{dW}{dr} = 0.$$

i.e.,
$$\frac{dW}{dr} = \frac{3V^2}{(r + 3r_{int})^2} - \frac{6V^2 r}{(r + 3r_{int})^3} = 0$$

or
$$\frac{3V^2}{(r + 3r_{int})^2}\left[1 - \frac{2r}{r + 3r_{int}}\right] = 0$$

∴ $2r = r + 3\,r_{int}$

or $r = 3\,r_{int} = 3 \times 0.1 = 0.3\ \Omega$

Thus, at $r_1 = r_2 = r_3 = r = 0.3\ \Omega$, the heat generated will be maximum.

EXERCISES

1. Name the fundamental units in SI system. Why is the SI system preferred?
2. What are derived units in SI system? Name a few used in electrical engineering.
3. What is per unit (p.u.) system of measurement? What is its convenience? How do you convert a p.u. quantity from one base to another base?
4. How does electric current flow? How do you use sign notation for voltages?
5. How do you distinguish between source and load in electrical engineering?
6. State Ohm's law and define resistance. How do you express Ohm's law in terms of conductance? Can we apply Ohm's law in any electrical circuit?
7. What is resistivity? How is it expressed? What is its unit in SI system?
8. Write the expression of electric power and energy in terms of current and resistance. How is voltage related to electric power expression? What do you mean by negative power?
9. (a) Define temperature coefficient of resistance. If α_t be the temperature coefficient of resistance at t °C, α_0 be its value at 0 °C for any metal, show that we can represent α_t as

$$\alpha_t = \frac{1}{\dfrac{1}{\alpha_0} + t}$$

(b) When can temperature coefficient of resistance be negative?

10. Find the power consumed by the combination of resistances when connected to a voltage source of 100 V at terminals *P-Q* (Fig. 1.9).

(*Ans:* 3.125 kW)

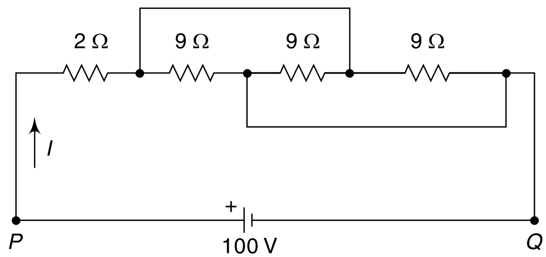


Fig. 1.9

[*Hint:* Fig. 1.9 can be reduced as shown below:

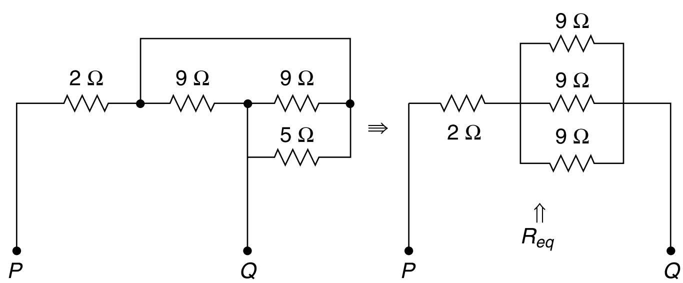


Fig. 1.10

$$R_{eq} = 1 + \frac{9}{3} = 5\ \Omega$$

$\therefore \quad I = \dfrac{100}{5} = 25$ A; $P_{loss} = I^2R = 25^2 \times 5 = 3.125$ kW.]

11. A circular wire of resistance 5 Ω is stretched to twice its original length. What will be the resistance of the wire now?

(*Ans:* 20 Ω)

[*Hint:* Suffix 1 represents the parameters of the original wire while suffix 2 represents the parameters in the stretched condition. The mass (m) of the wire remains the same.

$\therefore \quad m = \pi r_1^2 l_1 \times d = \pi r_2^2 l_2 \times d \quad [\because m = \text{volume} \times \text{density}]$

or $\quad r_1^2 l_1 = r_2^2 l_2$

Since $\quad l_2 = 2l_1,\ r_2^2 = r_1^2/2.$

If R_1 is the resistance of the original wire, R_2 that of the stretched wire, we have

$$R_1 = \rho \frac{l_1}{\pi r_1^2}\ ; \quad R_2 = \rho \frac{l_2}{\pi r_2^2}$$

$$\therefore \qquad \frac{R_2}{R_1} = \frac{l_2\, r_1^2}{l_1\, r_2^2} = 2 \times 2 = 4.$$

Then $\qquad R_1 = 4R_1$; Here $R_2 = 4 \times 5 = 20\ \Omega$]

12. A heater wire length 50 cm and 1 mm^2 in cross-section carries a current of 2A when connected across a 2 V battery. What is the resistivity of the wire? (*Ans:* $2 \times 10^{-10}\ \Omega$m)

[*Hint:* $\rho = R\frac{a}{l} = \frac{V}{I} \times \frac{a}{l} = \frac{2}{2} \times \frac{1 \times 10^{-6}}{50 \times 10^{-2}} = 2 \times 10^{-10}\ \Omega\text{m}$]

13. A copper rod of length 20 cm and cross-sectional area 2 mm^2 is joined with an identical aluminium rod, as shown in Fig. 1.11. Find the resistance of the combination along its length and between the ends. Assume $\rho_{cu} = 1.7 \times 10^{-8}\ \Omega$ m; $\rho_{al} = 2.6 \times 10^{-8}\ \Omega \cdot$ m. (*Ans:* 1.0 mΩ)

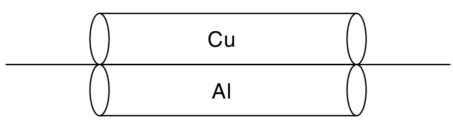


Fig. 1.11

[*Hint:* $\qquad R_{cu} = \rho_{cu}\frac{l}{a} = \frac{1.7 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}} = 1.7 \times 10^{-3}\ \Omega$

$$R_{al} = \rho_{al}\frac{l}{a} = \frac{2.6 \times 10^{-8} \times 20 \times 10^{-2}}{2 \times 10^{-6}} = 2.6 \times 10^{-3}\ \Omega$$

The rods are in parallel and hence

$$R = \frac{R_{cu} \times R_{al}}{R_{cu} + R_{al}} = \frac{1.7 \times 10^{-3} \times 2.6 \times 10^{-3}}{1.7 \times 10^{-3} + 2.6 \times 10^{-3}}$$

$= 1.0$ m Ω.]

14. A wire of resistance 20 Ω is bend to form a complete round loop. Find the amount of heat it will generate if a voltage of 10 V is applied at diametrical opposite points $m - n$ in the loop (Fig. 1.12). (*Ans:* 20 W)
[*Hint:* Let us assume two points p, q at the top and bottom of the circle and diametrically opposite. It is obvious that across 10 V source, current will enter at m and pass to n through two parallel paths *mpn* and *mqn*. Thus the equivalent

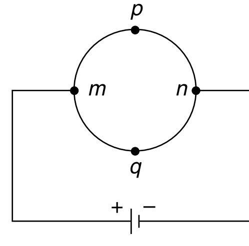


Fig. 1.12

resistance across *mn* is given by $R_{mn} = \dfrac{R_1 \times R_2}{R_1 + R_2}$; R_1 = resistance of *mpn* and R_2 = resistance of *mqn.*

$$= \frac{10 \times 10}{10 + 10} = 5\ \Omega.$$

$$\therefore \qquad P_{\text{loss}} = \frac{V^2}{R_{\text{mn}}} = \frac{100}{5} = 20\ \text{W}].$$

15. Find the equivalent resistance between points *a* and *b* of the infinite ladder shown in Fig. 1.13. (*Ans:* $R_{\text{eq}(a-b)} = \dfrac{1+\sqrt{5}}{2} \cdot R$)

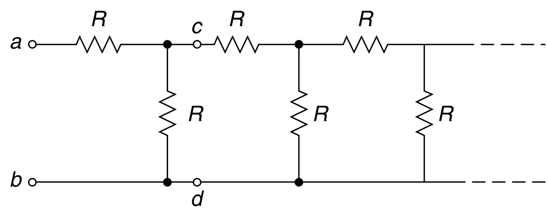


Fig. 1.13

[*Hint:* Let $R_{\text{eq}(a-b)}$ be the equivalent resistance between $a - b$. As the ladder is infinite, R_{eq} is also the equivalent resistance of the ladder to the right of *c* and *d* point. Thus, we can replace the part of right of *cd* by a resistance R_{eq} and reduce the given ladder network as shown in Fig. 1.14.

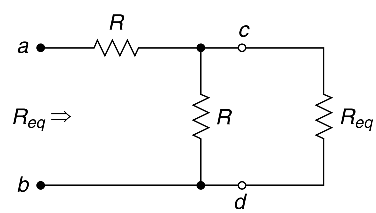


Fig. 1.14

$$R_{\text{eq}} = R + \frac{R_{eq} \times R}{R_{eq} + R}$$

or $$R_{\text{eq}}^2 - R\,R_{\text{eq}} - R^2 = 0$$

$$\therefore \qquad R_{\text{eq}} = \frac{R + \sqrt{R^2 + 4R^2}}{2} = \frac{1+\sqrt{5}}{2} \cdot R\ ]$$

16. A copper wire of resistivity 1.7×10^{-8} ohm · m and density 8900 kg/m^3 and an aluminium wire of resistivity 2.8×10^{-8} ohm · m and density 2700 kg/m^3 here the same mass per unit length. What would be the ratio of resistances of aluminium and copper wire per unit length?

(*Ans:* $\dfrac{R_{\text{al}}/\text{length}}{R_{\text{cu}}/\text{length}} = \dfrac{1}{2}$)

[*Hint:* Let ξ_1 and ξ_2 be the resistance per unit length of copper and aluminium.

$$\frac{\xi_1}{\xi_2} = \frac{R_1/l_1}{R_2/l_2} = \frac{\rho_1/a_1}{\rho_1/a_2} = \frac{\rho_1}{\rho_2} \times \frac{A_2}{A_1}$$

Also, mass = volume × density. Here,

$$m_1 = (a_1 l_1)d_1;\ m_2 = (a_2 l_2)d_2$$

But as per question, $\frac{m_1}{l_1} = \frac{m_2}{l_2}$

$$\therefore \qquad a_1 d_1 = a_2 \mathrm{d}_2 \quad \text{or} \quad \frac{a_2}{a_1} = \frac{d_1}{d_2}$$

Hence $$\frac{\xi_1}{\xi_2} = \frac{\rho_1}{\rho_2} \times \frac{d_1}{d_2} = \frac{1.7 \times 10^{-8}}{2.8 \times 10^{-8}} \times \frac{8900}{2700} = 2$$

Hence $$\frac{\xi_2}{\xi_1} = \frac{\text{resistance/length of aluminium}}{\text{resistance/length of copper}} = \frac{1}{2}$$

17. A 12 V, 24 W filament bulb is to be used against a battery of N number of cells. Each cell has emf of 1.5 V and internal resistance of 0.25 Ω. How many cells are to be used so that bulb runs at rated power?
(*Ans:* $N = 12$)
[*Hint:* $W = VI = 24$; $V = 12$ V; $I = 2A$. As the bulb is connected to the battery in series, for each cell $V = E - Ir$; r being the internal resistance of each cell.
Or, $V = 1.5 - 2 \times 0.25 = 1$ V, assuming the cells are in series.
Thus 12 cells are required so that the voltage across the bulb is 12 V.]
18. Two wires made of the same materials and same cross-sectional area but the second wire has twice the mass than the first wire. If same current I flows through both the wires, what is the ratio of heat produced in the two wires? (*Ans:* In series $H_1 : H_2 = 1 : 2$, in parallel, $H_1 : H_2 = 2 : 1$)
[*Hint:* Let the first wire have mass m while the second wire have mass ($2m$). Using the relation, $m = (a\,l)d$ ($\because$ mass = volume × density) we find

$$\frac{m}{2m} = \frac{a l_1 d}{a l_2 d} \quad \text{or,} \quad \frac{l_1}{l_2} = \frac{1}{2}.$$

But $$R = \rho\frac{l}{a}, \qquad \therefore \frac{R_1}{R_2} = \frac{l_1}{l_2} = \frac{1}{2}.$$

If they are connected in series, in the first one heat produced is $H_1 = I^2R_1$, while in the second one heat produced is $H_2 = I^2R_2$.

$$\therefore \qquad \frac{H_1}{H_2} = \frac{R_1}{R_2} = \frac{1}{2}$$

On the other hand, when they are in parallel, V across each of them remain same. This time $H_1 = \dfrac{V^2}{R_1}$, while $H_2 = \dfrac{V^2}{R_2}$.

$$\therefore \qquad \frac{H_1}{H_2} = \frac{R_2}{R_1} = \frac{2}{1}\,]$$

19. Current flows through a copper bar of length 5 m long and 10 cm^2 cross-sectional area. If the resistivity of the bar at 0 °C is 1.6×10^{-8} Ωm, find the resistance of the bar at 0°C. If the strength of current is 5 kA, find the potential drop across the bar. If the bar is now stretched to form a thinner bar of 5 cm^2 cross-section, find the new resistance of the bar. If the same current is passed through the bar in original condition as well as in stretched condition, what is the ratio of heat produced in two conditions?

(*Ans:* $R_0 = 8 \times 10^{-5}$ Ω; $V = 0.4$ Volt; $R_{stretched} = 3.2 \times 10^{-4}$ Ω; Heat loss increases 4 times on stretching)

[*Hint:* (i) $R_0 = \rho \dfrac{l}{a} = \dfrac{1.6 \times 10^{-8} \times 5}{10 \times 10^{-4}} = 8 \times 10^{-5}\ \Omega$

(ii) $V_{drop} = I \times R_0 = 5000 \times 8 \times 10^{-5} = 0.4$ V

(iii) After the bar is stretched,

$$l = \frac{\text{Volume}}{\text{New area}} = \frac{(5 \times 10 \times 10^{-4})}{5 \times 10^{-4}}$$

($\because$ volume remains same before and after stretching)

$= 10$ m.

$$R_{new} = \rho \times \frac{l}{a} = \frac{1.6 \times 10^{-8} \times 10}{5 \times 10^{-4}} = 3.2 \times 10^{-4}\ \Omega.$$

(**Note:** resistance increases under stretched condition.) If H_1 and H_2 be the heat produced in two conditions,

$$H_1 = I^2 R_0 t\ ;\ H_2 = I^2 R_{new} \times t$$

$\therefore \quad H_1 : H_2 = R_0 : R_{new} = 0.25.$]

20. A battery of emf 6 V and internal resistance of 0.1 Ω is charged by a dc charger (Fig. 1.15) with a constant current of 5A. What is the p.d. across the terminals x-y? (*Ans:* 6.5 V)

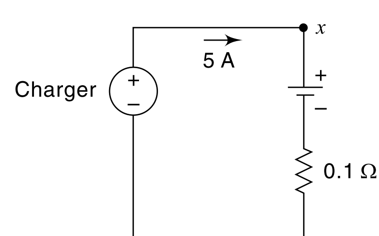


Fig. 1.15

[*Hint:* During charging current enters the battery through +ve terminal i.e., current passes from $x - y$. The battery is absorbing energy.

Here $V_x - V_y$ is p.d. across the battery. Since the battery emf is 6 V, we can write

$$V_x - 6 - \text{drop in 0.1 Ω internal resistance} - V_y = 0$$

i.e. $\quad V_x - 6 - 5 \times 0.1 - V_y = 0$

$\therefore \quad V_x - V_y = 6.5$ V.]

2 ELECTROSTATICS

2.1 INTRODUCTION

In *electrostatics* we deal with static electricity (i.e, when charges are at rest). The knowledge of electrostatics is important in electrical engineering as we frequently come across the design process of electrical insulations and performance of various equipment, cables and overhead lines when subjected to electric stress. Natural phenomenon like lightning is very much related to electrostatic processes and laws. Electrostatics finds extensive applications in extra high voltage systems (ehv) in transmission engineering. *Capacitors* play a vital role in different spheres of electrical engineering as well as electronics engineering. The performance of a capacitor can be best analysed and it can be properly designed with knowledge in electrostatics.

2.2 COULOMB'S LAW AND CONCEPT OF PERMITTIVITY

Coulomb's first law states that like charges repel each other while opposite charges attract each other. *Coulomb's second law* states that the force of attraction between two opposite charges or force of repulsion between two like charges is

(a) directly proportional to the product of the charges, the distance between them being same;
(b) inversely proportional to the square of the straight distance between them, magnitude of the charges being constant.

If we assume the charges to be of magnitude (Q) and (q) separated by a straight distance x, then from Coulomb's second law we can write

$F \propto Q \cdot q/x^2$, (F) being the force of repulsion if both charges are alike or force of attraction if the charges have opposite polarity. The force (F) is measured in Newtons when the magnitude of the charges are expressed in Coulombs and the distance in meters.

$$\therefore \qquad F = K\frac{Q \cdot q}{x^2} \tag{2.1}$$

(K) being constant of variation and in SI unit (K) in vacuum is given by $1/4\pi\varepsilon_o$; otherwise $K = 1/4\pi\varepsilon$, when the charges are placed in any other medium other than vacuum or space.

$$\therefore \qquad F = \frac{Q \cdot q}{4\pi\varepsilon x^2} \tag{2.2}$$

In electrical engineering we term this ε as permittivity of the medium in which charges are placed. It is known as absolute permittivity and is represented as

$$\varepsilon = \varepsilon_o \times \varepsilon_r \tag{2.3}$$

where (ε_o) is the *permittivity* of space while (ε_r) is the *relative permittivity* of the medium where the charges are placed.

In SI unit,

$$\varepsilon_o = 8.854 \times 10^{-12} \text{ Farad/metre}$$

$$\therefore \qquad \frac{1}{4\pi\varepsilon_o} = \frac{1}{4\pi \times 8.854 \times 10^{-12}} = 9 \times 10^9$$

Thus, Coulomb's law can be written as

$$F = 9 \times 10^9 \frac{Q \times q}{\varepsilon_r x^2}. \tag{2.4a}$$

When the charges are placed in vacuum (or space),

$$F = 9 \times 10^9 \frac{Q \cdot q}{x^2}. \tag{2.4b}$$

If $Q = q = 1$ Coulomb and $x = 1$ metre, from Coulombs law, $F = 9 \times 10^9$ Newtons (in space). This gives rise to the definition of *unit charge* (i.e. 1 Coulomb) which means that it is such a charge which when placed at one metre apart from another similar charge experiences a force of 9×10^9 Newton in vacuum.

2.3 PERMITTIVITY

Permittivity of a medium is basically that property of the medium which permits electric flux to pass through it. If the permittivity is more it means that the medium allows more flux to pass through it and hence this medium is more susceptible to the electric field.

Absolute permittivity (ε) is the ratio of electric flux density in a dielectric medium to the corresponding electric field strength and is expressed as Farad/meter.

$$\text{i.e} \qquad \varepsilon = \frac{\delta}{E} \text{ F/m} \tag{2.5}$$

where δ is electric flux density and E is the strength of the field.

Also, $\varepsilon = \varepsilon_o \times \varepsilon_r$, where ε_o is the permittivity of free space (8.854×10^{-12} F/m) and (ε_r) is the relative permittivity of a dielectric medium. It is defined as the ratio of flux densities of the dielectric medium to that in vacuum produced by the same electric field strength.

$$[\varepsilon_r = \frac{\delta}{\delta_o} = \frac{(\varepsilon E)}{(\varepsilon_o E)} = \frac{\varepsilon}{\varepsilon_o}$$

$$\therefore \qquad \varepsilon = \varepsilon_r \times \varepsilon_o].$$

Relative permittivity of space is 1 while that of air is 1.0006. In practice, we assume ε_r of vacuum and air as 1. Commonly used dielectric medium have permittivity between 2 and 10.

2.4 ELECTRIC POTENTIAL AND POTENTIAL DIFFERENCE

The *electric potential* at any point in an electric field is defined as the work done in joules in moving a unit positive charge from infinity (i.e., from zero potential) to that point against the electric field.

$$\therefore \qquad \text{Electric potential} = \frac{\text{Work done}}{\text{Electric charge}}$$

or

$$V = \frac{W}{Q} \tag{2.6}$$

When W is expressed in joules, Q in coulombs, V is expressed in volts. Then the electric potential at a particular point in an electric field is one volt provided one joule of work is done in moving a unit positive charge from zero potential to that point against the field.

In electrical engineering we are more interested in measuring the potential difference between two points in a field than to know the absolute value of the electric potential at any point s in the field. The *potential difference (p.d.)* is the work done in joule in moving a unit positive charge from the point of lower potential to higher potential within the field.

The potential difference is obviously measured in volts and the p.d. of one volt means one joule of work is done in bringing a unit positive charge from the point of lower potential to the point of higher potential within the electric field.

2.5 EXPRESSION FOR POTENTIAL AT A POINT WITHIN AN ELECTRIC FIELD

Let us consider two positive charges, the first one having a charge of Q coulombs while the second one is a unit positive charge. Both the charges are assumed to be placed in space at a straight distance x metres between them. From Coulomb's law we can express the force of repulsion between these two charges as

$$F = \frac{Q \times 1}{(4\pi\varepsilon_o)x^2} \text{ ; } \varepsilon_o \text{ being the permittivity of space.}$$

The work done dW in moving the unit charge towards the charge Q for a small distance dx metre will be given as

$$dW = \left[\frac{Q}{(4\pi\varepsilon_o)\,x^2}(-dx) \right] \text{ joule}$$

Work done is negative as the charge is moved against a repulsive force and against the direction of the field.

In order to find the total work done in moving the unit positive charge from infinity to any point d metres away from the charge Q against the field, we will integrate the expression of dW obtained within the limit of integral ∞ to d.

$$\therefore \qquad W = \int_{\infty}^{d} \frac{+Q(-dx)}{(4\pi\varepsilon_o)\,x^2} = \frac{-Q}{4\pi\varepsilon_o}\left[-\frac{1}{x} \right]_{\infty}^{d} = \frac{Q}{(4\pi\varepsilon_o)\cdot d} \text{ joules.}$$

Thus, from definition we can write the potential at a point d metres away from the charge Q is simply $\left(\dfrac{Q}{4\pi\varepsilon_o \cdot d} \right)$ volts.

$$\therefore \qquad V = \frac{Q}{(4\pi\varepsilon_o)\cdot d} \text{ volts} \tag{2.7a}$$

If the analysis be performed assuming the surrounding medium as a dielectric of negative permittivity ε_r, we can modify the expression for potential at distance d away from Q as

$$V = \frac{Q}{4\pi\varepsilon_o\,\varepsilon_r \cdot d} \text{ volts} \tag{2.7b}$$

If we consider an isolated sphere of radius R placed in space and having +ve charge Q coulombs uniformly distributed over its surface, the potential at the surface of the sphere would be

$$V = \frac{Q}{4\pi\varepsilon_o R} \tag{2.8}$$

[the charge on the sphere would act as a concentrated charge at the centre O of the sphere.]

The potential will remain constant for the space between O and R in the sphere and will be same as the potential V at the surface of the sphere. The surface of the sphere would be termed as equipotential surface and electric lines of force always cross such a equipotential surface normally.

2.6 ELECTRIC FIELD INTENSITY

The *intensity of the electric field* at a point is defined as the mechanical force per unit charge placed at that point. The direction of the intensity is same as direction of the force exerted on a positive charge.

Thus, if F be the force experienced by a test charge q placed at a point in an electric field, the intensity E at that point is given by

$$E = \frac{F}{q}. \tag{2.9}$$

E is expressed in newton per coulomb or in volt/metre (V/m). Frequently the term *electric field strength* is also used instead of the term electric field intensity.

Let us assume a positive point charge $+Q$ is placed at a point M and a test charge $+q$ is placed at point N, as shown in Fig. 2.1.

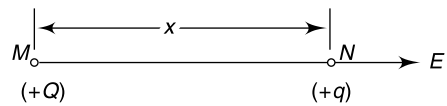


Fig. 2.1 ***A (+ve) charge (+Q) placed in an electric field***

The force F experienced by q is given by

$$F = \frac{Qq}{4\pi\varepsilon_o x^2}.$$

Since intensity E is given by F/q, we can write

$$E = \frac{Q}{4\pi\varepsilon_o x^2}. \tag{2.10}$$

The direction of E is towards the point charge or away from it according as the charge is negative or positive.

[In vector form the intensity can be expressed as

$$\vec{E} = \frac{1}{4\pi\varepsilon_o} \times \frac{Q}{r^2} . \hat{r}$$

or
$$\vec{E} = \frac{1}{4\pi\varepsilon_o} \times \frac{Q}{r^3} . \vec{r} \tag{2.11}$$

where x is the distance between the charges Q and q; $\hat{r}$ is unit vector along $\vec{r}$ and is given by

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}$$

2.7 ELECTRIC FIELD INTENSITY AND POTENTIAL OF ISOLATED POINT CHARGE (+q)

Figure 2.2 represents an isolated point charge $+q$ placed in space. We are to find the field intensity and potential at point P.

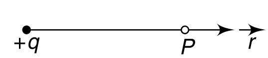


Fig. 2.2 ***An isolated point charge (+q) placed in space***

To find intensity:
By definition of electric field intensity, the intensity at a point along the direction $\vec{r}$ is given by (in vector form)

$$\vec{E} = \frac{1}{4\pi\varepsilon_o}\frac{q}{r^2}\hat{r} \tag{2.12a}$$

where q = positive point charge
r = distance between $+q$ and point P and
$\hat{r}$ = unit vector along $\vec{r}$.

$$\therefore \quad \vec{E} = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r^3}\right)\vec{r} \qquad \left(\because \hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{\vec{r}}{r}\right) \tag{2.12b}$$

From Eq. (2.10), we can also write field intensity as

$$E = \frac{1}{4\pi\varepsilon_o} \times \frac{Q}{x^2} \tag{2.12c}$$

To find potential:
By definition of potental at a point P,

$$V = -\int_{\infty}^{r} |\vec{E}|\, dr = -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r^2}\right) dr = -\left[-\frac{1}{4\pi\varepsilon_o}\left(\frac{q}{r}\right) - 0\right] = \frac{q}{4\pi\varepsilon_o r}$$

$$\therefore \quad V = \frac{q}{4\pi\varepsilon_o r}. \tag{2.12d}$$

2.8 ELECTRIC FIELD INTENSITY AND POTENTIAL GRADIENT

Electric field strength E due to a point charge at any point in the vicinity of the charge is defined as the force experienced by a unit positive charge placed at that point within the field. It is expressed in Newton/Coulomb (or volt/meter). If this force is stronger, the electric field strength is more. We also can state that the work done in moving a unit positive charge through a small distance dx meters in the direction of the field is given by

(dW) = force × displacement of the charge
$= E \times dx$ joules, where E is expressed in Newton/Coulomb.

Obviously this work done would be equal to the "drop" or reduction in potential as this time the unit positive charge is moved along the direction of the field and the work done would consequently be positive.

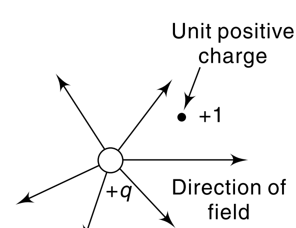


Fig. 2.3 ***A unit (+ve) charge in a field***

Then we can write $dV = E \times dx$

or $$E = \frac{dV}{dx} \tag{2.13}$$

(dV/dx) is known as the *potential gradient* and is thus the drop in potential per meter in the direction of the field. It is expressed as volt/meter. We thus find that the electric field strength and potential gradient being same, both are expressed in volt/meter.

2.9 ELECTRIC POTENTIAL ENERGY

Let us consider a system of two charges Q_1 and Q_2. Suppose Q_1 is a fixed charge at a point M while the charge Q_2 is taken from a point N to a point P along the line MNP (Fig. 2.4).

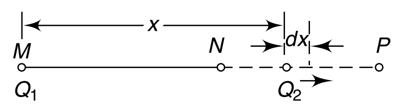


Fig. 2.4 *A system of two charges*

Let distance $MN = x_1$, while distance $MP = x_2$. We consider a small displacement of charge Q_2. Its distance from M then changes to $(x + dx)$. The electric force on Q_2 is given by

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_o x^2}, \text{ in direction } M \text{ to } N.$$

[We assume the medium in which the charges are placed in space and hence $\varepsilon_r = 1$.]

The work done by the force in making small displacement dx by the charge is

$$dW = \frac{Q_1 Q_2}{4\pi\varepsilon_o x^2} \cdot dr$$

[$\because$ Work done = force $\times$ displacement]

The total work done as Q_2 moves from N to P is thus

$$W = \int_{x_1}^{x_2} \frac{Q_1 Q_2}{4\pi\varepsilon_o x^2} \cdot dx$$

$$= \frac{Q_1 Q_2}{4\pi\varepsilon_o}\left[\frac{1}{x_1} - \frac{1}{x_2}\right] \text{ joule} \tag{2.14}$$

[Charges are expressed in Coulomb and distance in metres]

[It may be noted here that no work is done by the electric force on the charge Q_1 as it remains fixed.]

The change in potential energy is thus

$$u(x_2) - u(x_1) = -W = -\frac{Q_1 Q_2}{4\pi\varepsilon_o}\left[\frac{1}{x_1} - \frac{1}{x_2}\right]$$

$$= \frac{Q_1 Q_2}{4\pi\varepsilon_o}\left[\frac{1}{x_2} - \frac{1}{x_1}\right] \tag{2.15a}$$

[We define *change in electric potential energy* of the system as negative of work done by the electric force.]

If one charge is placed at infinity, its potential is zero and consequently $u(\infty)$ = 0.

The potential energy, when the separation is x, can be obtained as

$$U(x) = u(x) - u(\infty)$$

$$= \frac{Q_1 Q_2}{4\pi\varepsilon_o}\left(\frac{1}{x} - \frac{1}{\infty}\right) = \frac{Q_1 Q_2}{4\pi\varepsilon_o x} \tag{2.15b}$$

The equations derived here assume that one of the charges is fixed and the other is moving. However, the potential energy depends essentially on the separation between charges and is independent of the spatial location of the charged particles.

2.10 RELATION BETWEEN ELECTRIC FIELD STRENGTH AND POTENTIAL

Let us suppose the electric field at a point n due to a charge distribution is E, while the electric potential at the same point is V. Let us assume the point charge of strength q is moved slightly from the point x to $(x + dx)$. The force on the charge is $F = q \cdot E$, while the work done is

$$dW = F(-dx) = qE(-dx)$$

[In article 2.8, we have assumed dx in the direction of the field, i.e. from +q towards infinity. If any charge is moved against the field, dx becomes –ve.] The change in potential energy due to this displacement is

$$du = +dW = -q \cdot E \cdot dx.$$

The change in potential is $dV = \dfrac{du}{q}$ i.e. $dV = -E\,dx$.

[If the test charge is moved along the field, $dV = E \times dx$ (as shown in article 2.8).]

Integrating between x_1 and x_2, we get

$V_2 - V_1 = -\int_{x_1}^{x_2} E dx$, where V_2 and V_1 are the potentials at x_2 and x_1 respectively. If we select point x_1 as reference having zero potential, we can write $V(r) = -\int_{\infty}^{x} E \cdot dx$, where x is distance equal to x_2.

2.11 ELECTRIC FIELD INSIDE A CONDUCTOR

When there is no electric field around a conductor the conduction electrons are almost uniformly distributed within the conductor. In any small volume of the conductor the number of electrons is equal to the number of proton in the nuclei of each atom of the conductor. The net charge in the volume is then zero. Next we suppose that an electric field E is created in the direction left to right across the conductor. This field will exert a force on the free electrons in the atoms of the conductor from right to left. The free electron then move towards the left and consequently the number of electrons in the left will increase while the number of electrons in the right decreases. The left side of the conductor then becomes negatively charged while the right side is positively charged. The electron continue to drift towards the left. The result is the creation of an electric field of strength E' within the conductor in the direction opposite to the applied field. With passage of time a situation comes when the field E' inside the conductor is equal to the magnitude of the external field E. The net electric field inside the conductor is zero. Then a steady state is reached when some positive and negative charges appear at the surface of the conductor while there is no electric field inside the plate. Thus there is no electric field inside the conductor when it is subjected to an external electric field. The redistribution of electrons take place in such a way that charges remain at the surface of the conductor only.

It may be recalled here from the basic concepts of physics that in conductors there is always existence of free electrons while in insulators all atomic electrons are tightly bound to their respective nuclei. When insulators are placed in an electric field they may slightly shift their parent position but cannot drift from their parent atoms and hence cannot move long distance. These materials are then said to act as dielectrics. If the external field is strengthened further, a time will come when the bonding of the electron with their nuclei may break causing them to drift apart. We call this phenomenon as breakdown of dielectric medium.

2.12 GAUSS' LAW AND ITS DERIVATION

Statement of Gauss' Law: The flux of the net electric field through a closed surface is equal to the net charge enclosed by the surface divided by ε_o.

i.e.,
$$\oint E\, ds = \frac{q_{in}}{\varepsilon_o} \tag{2.16}$$

where $\oint E\, ds$ represents the flux ϕ through a closed surface and q_{in} is the net charge enclosed by the surface through which the flux passes.

Derivation of Gauss' Law from Coulomb's Law

Let us suppose that a charge q is placed at a point O inside a closed surface (Fig. 2.5). We assume a point P on the surface and consider a small area Δs on the surface around P.

Let OP = x.

The electric field at point P due to the charge q is given by, $E = q/4\pi\varepsilon_o \cdot x^2$, directed along the line OP.

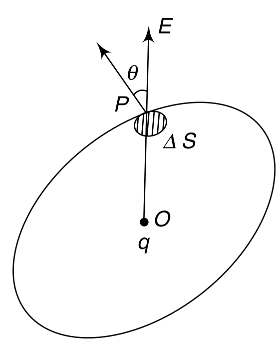


Fig. 2.5 *A charge (q) placed inside a closed surface*

Let us suppose this line OP makes an angle (θ) with the outward normal to the surface Δs. The flux of the electric field through Δs is given by

$$\Delta\phi = E\,\Delta s \cos\theta$$

$$= \frac{q}{4\pi\varepsilon_o x^2} \cdot \Delta s \cos\theta$$

$$= \frac{q}{4\pi\varepsilon_o} \cdot \Delta\sigma$$

where $\Delta\sigma = \dfrac{\Delta s \cdot \cos\theta}{x^2}$ [Actually ($\Delta\sigma$) is the solid angle subtended by (Δs) at O]

$$\therefore \qquad \phi = \sum \frac{q}{4\pi\varepsilon_o} \cdot \Delta\sigma = \frac{q}{4\pi\varepsilon_o} \sum \Delta\sigma$$

We can see that [$\Sigma(\Delta\sigma)$] represents the sum that is actually the total solid angles subtended by a closed surface at O. Obviously this total solid angle is 4π.

$\therefore$ The total flux of the electric field due to the internal charge q through the closed surface is

$$\phi = \frac{q}{4\pi\varepsilon_o} \cdot 4\pi = \frac{q}{\varepsilon_o}$$

$$\because \qquad \phi \equiv \oint E\,ds \text{, hence we have}$$

$$\oint E\,ds = \frac{q_{\text{in}}}{\varepsilon_o} \text{ (proof of Gauss' law)}$$

where, $\dfrac{q_{\text{in}}}{\varepsilon_o} = \sum \dfrac{q_i}{\varepsilon_o}$ (i.e. the sum of all charges $q_1, q_2, \ldots, q_i, \ldots q_n$ located in the said closed surface. We do not consider external charges as the solid angle ($\Delta\sigma$) subtended by a closed surface at any external point is zero, then ϕ becomes zero.

2.13 ELECTRIC DIPOLE

Definition: Two equal and opposite charges separated by a finite distance (Fig. 2.6) is said to constitute an *electric dipole*. It is characterised by dipole moment vector $\vec{P}$.

Dipole moment vector $\vec{P}$ is defined as

$$\vec{P} = q\vec{l}$$

and is aligned along the same line that join the two equal and opposite charges.

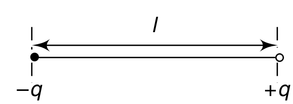


Fig. 2.6 ***Two equal and opposite charges separated by a finite distance***

2.14 ELECTRIC FIELD AND POTENTIAL DUE TO A DIPOLE AT AN AXIAL POINT

Let the charges ($-q$) and ($+q$) be kept at ($-a$, o) and (a, o) (Fig. 2.7). The electric field at $P(x, o)$ then

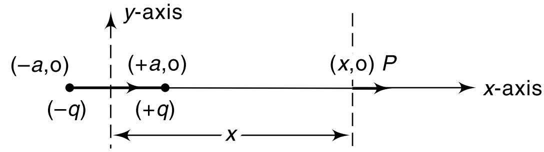


Fig. 2.7 ***Charges (+q) and (–q) placed at (+a, 0) and (–a, 0)***

$$\vec{E}_{\text{axial}} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \frac{kq}{(x-a)^2}\hat{i} - \frac{kq}{(x+a)^2}\hat{i}; \left(k = \frac{1}{4\pi\varepsilon_o}\right)$$

$$= \frac{k(q \cdot 2a)2x}{(x^2-a^2)^2}\hat{i}$$

$$= \frac{2K\vec{P}x}{(x^2-a^2)^2} \quad [\vec{P} = \text{dipole moment vector } (2aq)\hat{i}.$$

Assuming $x >> a$,

$$\vec{E}_{\text{axial}} = \frac{2K\vec{P}}{x^3} = \frac{\vec{P}}{2\pi\varepsilon_o x^3}. \tag{2.17}$$

Also, potential = V_{axial} $= -\int_{\infty}^{x} |\vec{E}_{\text{axial}}|\, dx$

$$= \int_{\infty}^{x} \frac{P}{2\pi\varepsilon_o x^3}\, dx$$

$$\therefore \qquad V_{\text{axial}} = \frac{P}{4\pi\varepsilon_o x^2}. \tag{2.18}$$

2.15 ELECTRIC FIELD AND POTENTIAL DUE TO DIPOLE ON EQUATORIAL LINE

At P (Fig. 2.8),

$$\vec{E}_{eq} = \vec{E}_{+q} + \vec{E}_{-q}$$

$$= \left(\frac{Kq}{y^2 + a^2}\right)(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$+ \left(\frac{Kq}{y^2 + a^2}\right)(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$$

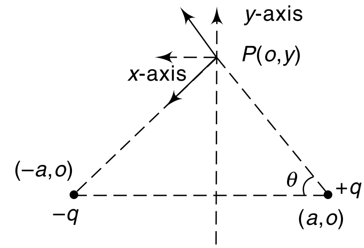


Fig. 2.8 ***Field and potential on equitorial line***

$$= \frac{-2Kq}{y^2 + a^2}\cos\theta\,\hat{i} = \frac{-K\vec{P}}{(y^2 + a^2)^{3/2}}$$

$$= \frac{-\vec{P}}{4\pi\varepsilon_o (y^2 + a^2)^{3/2}} \qquad \left[\because \vec{P} = 2aq\hat{i} \text{ and } \cos\theta = \frac{a}{(a^2 + y^2)^{1/2}}\right]$$

Assuming $y >> a$, $\quad \vec{E}_{eq} = -\frac{K\vec{P}}{y^3} = -\frac{\vec{P}}{4\pi\varepsilon_o y^3}$. (2.19)

Potential V_{eq} = (Potential at P due to $-q$) + (Potential at P due to $+q$)

$$= \frac{-Kq}{r} + \frac{Kq}{r}$$

$$= O, \text{ where } K = \frac{1}{4\pi\varepsilon_o} \quad \text{and} \quad r = \sqrt{a^2 + y^2}\,.$$

2.1 Three equal charges, each of magnitude 3.0×10^{-6}C, are placed at three corners of a right-angled triangle of sides 3 cm, 4 cm and 5 cm. Find the force on the charge at the right-angle corner.

Solution

Force on A due to B (Fig. 2.9)

$$(= F_1) = \frac{(3.0 \times 10^{-6})(3.0 \times 10^{-6})}{4\pi\varepsilon_o (4 \times 10^{-2})^2}$$

$$= 9 \times 10^9 \times 9.0 \times 10^{-12} \times \frac{1}{16 \times 10^{-4}}$$

$$= 5.0625 \times 10^1 = 50.625 \text{ N.}$$

This force acts along BA. Similarly, force on A due to C is, F_2 = 90 N in direction CA.

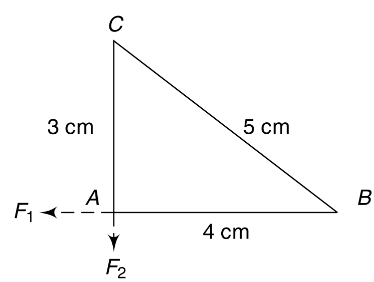


Fig. 2.9 ***Right angle triangle of Ex. 2.1***

$$\therefore \text{ Net electric force} = F = \sqrt{F_1^2 + F_2^2}$$

$$= \sqrt{(50.625)^2 + (90)^2}$$

$$= 103.261 \text{ N.}$$

The resultant makes an angle of θ with BA where $\tan\theta = \dfrac{90}{50.625} = 1.778$.

2.2 A charge Q is divided between two point charges. What should be the values of the charges on the objects so that the force between them is maximum?

Solution

Let charge on the objects be q and $(Q - q)$.

$$\therefore \text{ force between them } (= F) = \frac{q(Q-q)}{4\pi\varepsilon_o d^2} \qquad \text{(i)}$$

where d is the distance between them.

For maximum F, numerator of (i) is maximum. Let $q(Q - q) = y$.

$\therefore$ y should be maximum.

Differentiating y w.r.t. (q) we get

$$\frac{dy}{dq} = Q - 2q.$$

Equating to zero (to get the maxima of y), $Q - 2q = 0$ or $q = Q/2$.

Thus, the charge should be equally distributed between the objects.

2.3 Three concentric thin spherical shells A, B, C of radii r_1, r_2, r_3 are kept as shown in Fig. (2.10i). Shells A and C are given charges q and $-q$ respectively, shell B is earthed. Find charges appearing on the surfaces of B and C.

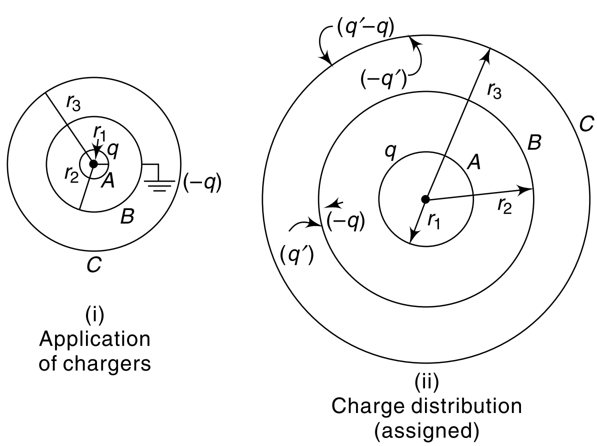


Fig. 2.10 ***Three concentric spherical shells (Ex. 2.3)***

Solution

Inner surface of B (by Gauss's law) must have charge $-q$. Let the outer surface of B have charge q'. The Inner surface of C must have change $-q'$ from Gauss's law. As net charge on C must be $-q$, its outer surface should have a charge $(q' - q)$. The charge distribution is shown in Fig. (2.10ii)

$$\text{Potential at } B \text{ due to charge } q \text{ on } A = \frac{q}{4\pi\varepsilon_o r_2}$$

due to charge $-q$ on the inner surface of $B = \dfrac{-q}{4\pi\varepsilon_o r_2}$

due to charge q' on outer surface of $B = \dfrac{q'}{4\pi\varepsilon_o r_2}$

due to charge $-q'$, on inner surface of $C = \dfrac{-q'}{4\pi\varepsilon_o r_3}$

and due to charge $(q' - q)$ on outer surface of $C = \dfrac{q' - q}{4\pi\varepsilon_o r_3}$.

Net potential on B is obtained adding all the potentials at B. We then obtain

$$V_B = \frac{q'}{4\pi\varepsilon_o r_2} - \frac{q}{4\pi\varepsilon_o r_3}.$$

But $V_B = 0$ as B is earthed.

$$\therefore \qquad q' = \frac{r_2}{r_3} q.$$

The final charge distribution is shown in Fig. 2.11.

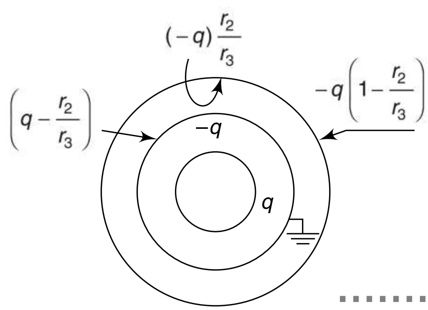


Fig. 2.11 ***Final charge distribution***

2.4 A charge 8×10^{-8} C is distributed uniformly on the surface of a sphere of radius 1 cm. It is covered by a concentric, hollow conducting sphere of radius 5 cm.

(a) Find the electric field at a point 2 cm away from the centre.

(b) A charge 6×10^{-8} C is placed on the hollow sphere. Find surface charge density on the outer surface of the hollow sphere.

Solution

To find the field at P of Fig. 2.12(i). Let us consider a Gaussian surface through P.

$$\therefore \text{ Flux through surface } = \oint \vec{E} \cdot d\vec{s}$$

$$= E \oint ds = 4\pi x^2 E$$

where $x = 2 \times 10^{-2}$ m.

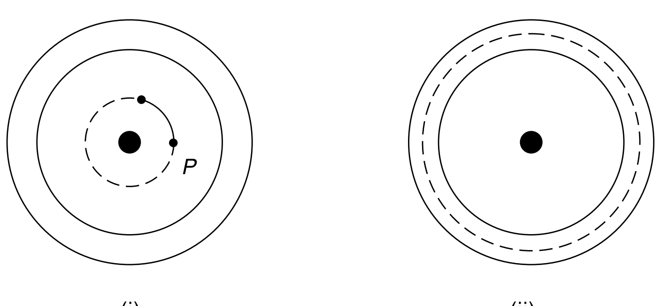


Fig. 2.12 ***Concentric spheres (Ex. 2.4)***

From Gauss' law,

$$4\pi x^2 E = q/\varepsilon_o$$

or
$$E = \frac{q}{4\pi x^2 t_o} = 9 \times 10^9 \times \frac{8\times10^{-8}}{4\times10^{-4}} = 18 \times 10^5 \text{ N/C.}$$

In Fig. (2.12ii) we take a Gaussian surface through the material of the hollow sphere.

As electric field in a conducting material is zero,

$\therefore \oint \vec{E}\cdot d\vec{s} = 0$ (through this Gaussian surface).

Using Gauss' law, the total charge enclosed must be zero.

Hence charge on inner sphere of the hollow sphere is (-8×10^{-8} C). But the total charge given to this hollow sphere is (6×10^{-8} C).

$\therefore$ Charge on the outer surface will be (2×10^{-8} C).

2.5 There are two thin wire rings, each of radius R, whose axes coincide. The charges of the rings are ($+Q$) and ($-Q$). Find the potential difference between the centres of the rings separated by a distance a.

Solution

The arrangement of the rings are shown in Fig. 2.13. The potential at point 1 is given by V_1 = potential at 1 due to ring 1 + potential at 1 due to the ring 2;

i.e., $$V_1 = \frac{Q}{4\pi\varepsilon_o R} + \frac{-Q}{4\pi\varepsilon_o (R^2 + a^2)^{1/2}}.$$

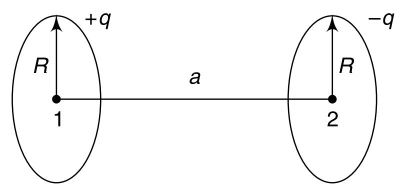


Fig. 2.13 ***Arrangement of rings in Ex. 2.5***

Similarly, the potential at point 2 is

$$V_2 = \frac{-Q}{4\pi\varepsilon_o R} + \frac{Q}{4\pi\varepsilon_o (R^2 + a^2)^{1/2}}$$

$$\therefore \quad V = V_1 - V_2 = \Delta V = 2\left(\frac{Q}{4\pi\varepsilon_o R} + \frac{-Q}{4\pi\varepsilon_o (R^2 + a^2)^{1/2}}\right)$$

$$= \frac{Q}{2\pi\varepsilon_o R}\left[1 - \frac{1}{\sqrt{1 + (a/R)^2}}\right]$$

2.6 Three point charges q, $2q$ and $8q$ are to be placed on a 9 cm long straight line. Find the position where the charges should be placed such that the potential energy of this system is minimum. In this situation, what is the electric field at the position of the charge q due to the other two charges?

Solution

Let charges q, $2q$ and $8q$ be placed along a straight line of length 9 cm or 0.09 m with distance between the charges q and $2q$ being x metres. Then distance between $2q$ and $8q$ would be $(0.09 - x)$m. Thus the potential energy u of the system is given by

$$u = \frac{1}{4\pi\varepsilon_o}\left[\frac{q\cdot 2q}{x} + \frac{2q\cdot 8q}{(0.09 - x)} + \frac{q\cdot 8q}{0.09}\right] = 9 \times 10^9 \times 2q^2\left[\frac{1}{x} + \frac{8}{0.09 - x} + \frac{4}{0.09}\right]$$

u to be minimum,

$$\frac{du}{dx} = 0, \text{ i.e } 0 = -\frac{1}{x^2} + \frac{8}{(0.09 - x^2)}.$$

This, gives $$x^2 = \frac{(0.09 - x^2)}{8}$$

or $$2\sqrt{2}\, x = \pm(0.09 - x)$$

or $$2\sqrt{2}\, x \pm x = \pm 0.09$$

$\therefore$ $$x(\text{minimum}) = \frac{0.09}{2\sqrt{2}+1} = 0.0235 \text{ m}.$$

Again with E_1 and E_2 as the electric fields at the position of charge q due to charge $2q$ and $8q$ respectively,

$$E_1 = \frac{1}{4\pi\varepsilon_o} \times \frac{2q}{x^2} \quad \text{and} \quad E_2 = \frac{1}{4\pi\varepsilon_o} \cdot \frac{8q}{(0.09)^2}$$

The electric field at q due to the other two charges is $(E_1 + E_2)$

$\therefore$ $$E_1 + E_2 = \frac{1}{4\pi\varepsilon_o}\left[\frac{2q}{(0.0235)^2} + \frac{8q}{(0.09)^2}\right]$$

$$= 9 \times 10^9 \times 2q \left[\frac{1}{(0.0235)^2} + \frac{4}{(0.09)^2}\right]$$

$$= 4.15 \times 10^{13}\, q \text{ N/C}.$$

2.7 An infinite number of charges each equal to Q coulomb are placed along the x-axis at $x = 1$, $x = 2$, $x = 8$,… and so on. Find the potental and the electric field at the point ($x = 0$) due to these charges. What will be the potential and electric field if, in the above setup, the consecutive charges have opposite signs?

Solution

Refering to Fig. 2.14, the potential at $x = 0$ due to this set of charges is given by

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{1} + \frac{q}{2} + \frac{q}{4} + \frac{q}{8} + \ldots\right)$$

$$= \frac{q}{4\pi\varepsilon_o}\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots\right)$$

$$= \frac{q}{4\pi\varepsilon_o} \times \frac{1}{1 - 1/2} = \frac{2q}{4\pi\varepsilon_o}$$

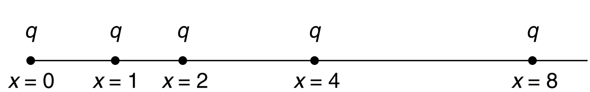


Fig. 2.14 ***Infinite number of charges placed along x-axis (Ex. 2.7)***

Since the point charges are along the same straight line, the intensities at $x = 0$ are also along the x-axis.

$$E = \frac{1}{4\pi\varepsilon_o}\left[\frac{q}{1^2}+\frac{q}{2^2}+\frac{q}{4^2}+\frac{q}{8^2}+\ldots\right]$$

$$= \frac{q}{4\pi\varepsilon_o}\left[1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+\ldots\right]$$

$$= \frac{1}{4\pi\varepsilon_o}\left\{\frac{q}{1-1/4}\right\} = \frac{1}{4\pi\varepsilon_o}\times\frac{4q}{3} = \frac{q}{3\pi\varepsilon_o}$$

If the consecutive charges are of opposite sign, the potential at $x = 0$ is

$$V = \frac{1}{4\pi\varepsilon_o}\left(\frac{q}{1}-\frac{q}{2}+\frac{q}{4}-\frac{q}{8}+\frac{q}{16}-\frac{q}{32}+\ldots\right)$$

$$= \frac{q}{4\pi\varepsilon_o}\left\{\left(1+\frac{1}{4}+\frac{1}{16}+\ldots\right)-\left(\frac{1}{2}+\frac{1}{8}+\frac{1}{32}+\ldots\right)\right\}$$

$$= \frac{q}{4\pi\varepsilon_o}\left\{\frac{1}{1-1/4}-\frac{1}{2}\times\frac{1}{1-1/4}\right\} = \frac{q}{4\pi\varepsilon_o}\left[\frac{4}{3}-\frac{2}{3}\right]$$

$$= \frac{q}{6\pi\varepsilon_o}$$

$$\therefore \qquad V = \frac{1}{4\pi\varepsilon_o}\left(\frac{2q}{3}\right)$$

The electric field intensity at $x = 0$ is

$$E = \frac{1}{4\pi\varepsilon_o}\left\{\frac{q}{(1)^2}-\frac{q}{(2)^2}+\frac{q}{(4)^2}-\frac{q}{(16)^2}\ldots\right\}$$

$$= \frac{q}{4\pi\varepsilon_o}\left\{\left(1+\frac{1}{16}+\frac{1}{256}+\ldots\right)-\left(\frac{1}{4}+\frac{1}{64}+\frac{1}{1024}+\ldots\right)\right\}$$

$$= \frac{q}{4\pi\varepsilon_o}\left\{\frac{1}{1-1/16}-\frac{1}{4}\times\frac{1}{1-1/16}\right\}$$

$$= \frac{q}{4\pi\varepsilon_o}\left\{\frac{16}{15}-\frac{1}{4}\times\frac{16}{15}\right\}$$

$$= \frac{1}{4\pi\varepsilon_o}\left(\frac{4q}{5}\right)$$

2.8 Some equipotential surfaces are shown in Fig. 2.15a and 2.15b. What are the magnitudes and directions of the electric field intensity for these two figures?

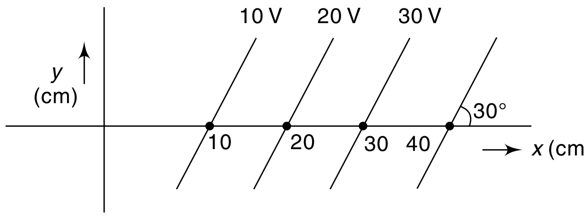


Fig. 2.15a *Equipotential surfaces (linear)*

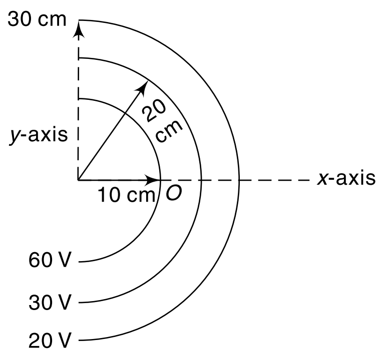


Fig. 2.15b *Equipotential surfaces (circular)*

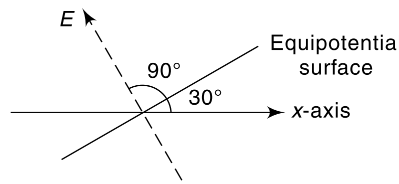


Fig. 2.15c *Direction of field of equipotential surfaces shown in Fig. 2.15a*

Solution

We know electric field is normal to the equipotential surface in the direction of the decreasing potential.

Thus for the equipotential surfaces of Fig. 2.15a, the field will be at an angle making an angle 120° to the x-axis (Fig. 2.15c)

Magnitude of the electric field in this case is

$$E \cos 120° = -\frac{(20-10)}{(20-10)10^{-2}} \left[\because E = -\frac{dv}{dx}\right]$$

or $$E \times \left(-\frac{1}{2}\right) = -\frac{10}{0.10}$$

$\therefore$ $$E = 200 \text{ V/m}.$$

In Fig. 2.15b,

direction of electric field will be radially outward, similar to a point charge kept at centre,

i.e. $V = \dfrac{Kq}{r}$, (r) being the radius.

When $$V = 60 \text{ V},$$

$$60 = \frac{Kq}{(0.1)}$$

$\therefore$ $$Kq = 6.$$

Then, potential at any distance from the centre is

$$V(r) = \frac{6}{r}\left[\because V = \frac{Kq}{r}\right]$$

Hence $$E = -\frac{dv}{dr} = \left(\frac{6}{r^2}\right) \text{ V/m}$$

2.9 A square frame of edge 20 cm is placed with its positive normal making an angle of 60° with a uniform electric field of 10 V/m. Find the flux of the electric field through the surface bounded by the frame.

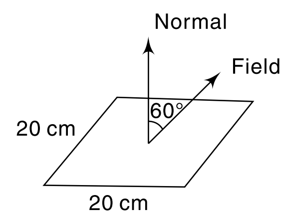


Fig. 2.16 ***Situation of the square of Ex. 2.9***

Solution

The situation is displayed in Fig. 2.16. The surface considered is plane and the electric field is uniform. The flux is

$$\begin{aligned}\Delta\phi &= \bar{E} \times \Delta S \\ &= E\,\Delta S \cos 60° \\ &= (10 \text{ V/m})\left(20 \times 20 \times \frac{1}{2} \times 10^{-4} \text{ m}^2\right) \\ &= 0.2 \text{ V.m.}\end{aligned}$$

2.10 A charge q is placed at the centre of a sphere (Fig. 2.17). Taking the outward normal as positive, find the flux of the electric field through the surface of the sphere due to the enclosed charge.

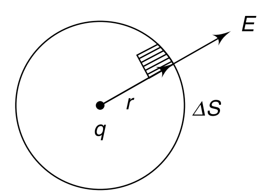


Fig. 2.17 ***(+q) charge is placed at the centre of a sphere (Ex. 2.10)***

Solution

The electric field here is radially outward and has the magnitude $q/4\pi\varepsilon_o r^2$, (r) being the radius of the sphere. As the positive normal is outward, $Q = 0$ and the flux through this part is

$$\Delta\phi = \bar{E}\,\Delta S = \frac{q}{4\pi\varepsilon_o r^2} \times \Delta S.$$

Summing over all the parts of the spherical surface,

$$\begin{aligned}\phi &= \sum \Delta\phi = \frac{q}{4\pi\varepsilon_o r^2} \sum \Delta S = \frac{q}{4\pi\varepsilon_o r^2} \cdot 4\pi r^2 \\ &= \frac{q}{\varepsilon_o}.\end{aligned}$$

2.16 CAPACITOR AND CAPACITANCE

A combination of two conductors placed close to each other and separated by a dielectric medium forms a *capacitor*. One of the conductors is given a positive charge (+Q) while the other one is charged by the same amount of negative charge (–Q). The conductor with (+Q) charge is called the *positive plate* while that with (–Q) charge is known as the *negative plate*. The charge stored in the positive or in the negative plate is the charge on the capacitor [note that the total charge on the capacitor is (+Q + (–Q)) zero]. The potential difference (V) between the plates is called the *potential of the capacitor*. If the positive plate has a potential V(+) while the negative plate has a potential V(–), then (V) = V(+) – V(–).

For any given capacitor, the charge Q on the capacitor is proportional to the p.d. (V) between the plates-

i.e. $Q \alpha V$

or $Q = CV$. (2.20)

The constant of proportionality being C, it is called *capacitance* of the capacitor. It depends on the shape, size and geometrical spacing of the conductors as well as the medium between them.

In *SI* system capacitance is expressed in coulomb/volt and is termed as *Farad*. Since Farad is a large unit by magnitude, in electrical engineering frequently *microfarad* (10^{-6} F) or μF is used.

If $Q = 1$, $V = 1$, then $C = 1$F, i.e. the capacitor is one Farad if it requires a charge of one coulomb when the potential difference is one volt across its plates. It may be noted here that when the capacitor is fully 'charged' i.e., if it is full to its capacity of containing charges across a voltage source, then the p.d. across its plates is always equal to the magnitude of the voltage source.

2.17 SERIES AND PARALLEL CONNECTION OF CAPACITORS

(a) Capacitors in Series Let us assume three capacitors of capacitances (C_1), (C_2) and (C_3) are connected in series across a dc supply of potential difference (V) through a switch K(Fig. 2.18). On closing the switch, the capacitors get charged and at steady state the p.d. across (C_1), (C_2) and (C_3) are (V_1), (V_2) and (V_3) respectively while the charge in each capacitor is (Q) (since the capacitors are connected in series, same charging current would flow resulting in accumulation of charge (Q) in each capacitor).

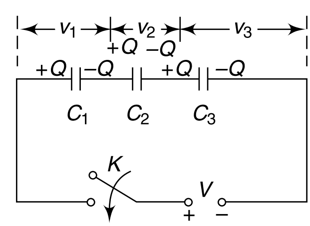


Fig. 2.18 ***Capacitors in series connection***

Obvviously,

$$V_1 = \frac{Q}{C_1}; \quad V_2 = \frac{Q}{C_2}; \quad V_3 = \frac{Q}{C_3}$$

Since $V = V_1 + V_2 + V_3$, we can write

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

or $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$ [(C) being the hypothetical capacitance equivalent to three capacitances (C_1), (C_2) and (C_3) in series].

∴ For n number of capacitances in series,

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots + \frac{1}{C_n} \tag{2.21}$$

Thus we can conclude that for series connection of capacitance across a voltage source, the charge on each capacitor being same while the voltages vary. Also, the sum of individual voltage drops across each capacitor gives the total supply voltage. *The reciprocal of the equivalent capacitance of the series combination is equal to the sum of the reciprocals of the capacitances of the individual capacitors.*

(b) Capacitors in Parallel In this arrangement (Fig. 2.19) on closing K, charges Q_1, Q_2 and Q_3 would accumalate in capacitances C_1, C_2 and C_3 during steady state while the voltage will remain V across each capacitors in the parallel combination. Obviously,

$$Q = Q_1 + Q_2 + Q_3,$$

where Q is the total charge drained from the source.

or $$CV = C_1V + C_2V + C_3V$$

[(C) is assumed to be the equivalent hypothetical capacitance of this parallel combination of capacitance]

i.e $$C = C_1 + C_2 + C_3 \qquad (2.22)$$

Generalising for n number of capacitances

Thus, in case of parallel combination of capacitances (where voltage across each capacitance remain the same but the capacitors share the charge depending on the value of their capacitance), *the equivalent capacitance is equal to the sum of their individual capacitance.*

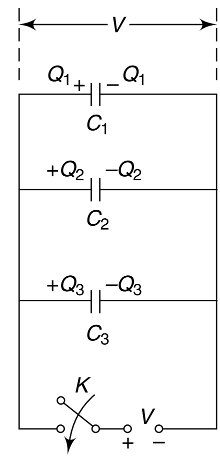


Fig. 2.19 ***Capacitors in parallel connection***

2.18 CONCEPT OF DIELECTRIC STRENGTH

Dielectric strength is the potential gradient required to cause breakdown of a dielectric medium. It is usually expressed in megavolts/millimetre (MV/mm). Dielectric strength depends on the moisture content, carbon content or presence of other impurity and thickness of the medium. With pressure of moisture and other impurities, dielectric strength drops while with increase of thickness the dielectric strength increases.

2.19 TYPES OF CAPACITORS COMMONLY USED

Depending or the nature of dielectric medium the following types of capacitors are usually available:

(a) Air Capacitors These have two sets of metal foils (aluminium or brass) and the inbetween medium is ordinary air. These capacitors are used in voltage ranges 100 V to 3000 V and the capacity varies up to 500 μF.

(b) Paper Capacitors These have a pair of elongated foil of metal (aluminium or copper or tin) interrelated with oil impregnated paper. Multiple layers of foils with paper is available. They can be used in the range of 100 V to 100 KV and is applicable for both AC and DC circuits. The capacitances are small and is usually in the range of pF.

(c) **Mica Capacitors** These consist of a series of aluminuim or tin foils separated by very thin layers of mica sheets. Usually multiple sheets are used and alternate plates are connected to each other. These capacitors are used in the range of 100–500 V and the capacitances in the range of pF to μF. These capacitors can be used in AC circuits.

(d) **Ceramic Capacitors** These capacitors are made of discs of ceramic material and the parallel facing surfaces are coated with silver. They have application in the range of a few volts to 3000 volts and the capacitances are from low values of pF to low values of μF. They are extensively used in AC and DC circuit.

(e) **Electrolytic Capacitors** Usually aluminium foils or cylinders are used as electrodes while electrolytes like porous paper, plastic, aluminium oxide, tantalum powder, etc. are used as dielectric. These capacitors are used in DC circuits and applicable in the range of 1 V to 1 kV. The range of capacitances are usually from 1pF to even Farad.

2.20 CAPACITANCE OF A PARALLEL-PLATE CAPACITOR

Let us consider two identical plates A and B are kept in close proximity and parallel to each other and separated by a dielectric medium of thickness (x) metre and relative permittivity (ε_r) (Fig. 2.20). Let us connect the parallel plates with a potential difference (V) volts and we assume (Q) coulombs of charge is accumulated by the parallel plate combination acting on a parallel plate capacitor. The electric flux is ψ between the plates having charge of (Q) coulombs and the area of each plate is considered to be (A) square metres.

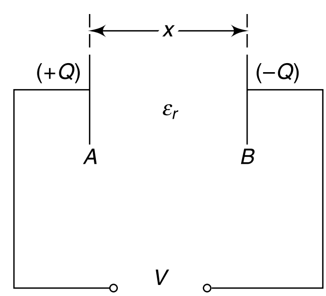


Fig. 2.20 ***Parallel plate capacitor***

Since the charge Q is distributed uniformly over each plate, the electric field between the plates is nearly uniform. Let δ represent the electric flux density while E the intensity (or the potential gradient) and C the capacitance in Farads for this parallel plate capacitor.

Here

$$\delta = \frac{\psi}{A} = \frac{Q}{A} \text{ coulomb/square metre.}$$

But $$E = \frac{V}{x}, \ V \text{ being the potential;}$$

also, $$\frac{\delta}{E} = \varepsilon \text{ [see equation (2.5)]}$$

or $$\frac{Q/A}{V/x} = \varepsilon = \varepsilon_o \times \varepsilon_r$$

$$\therefore \qquad \frac{Q}{V} = C = \frac{\varepsilon_o \varepsilon_r \times A}{x} \text{ Farad.} \tag{2.23a}$$

If the dielectric medium of the capacitance is vacuum, $\varepsilon_r = 1$ and hence

$$C = \frac{\varepsilon_o A}{x} \tag{2.23b}$$

Hence we find capacitance C of a parallel plate capacitor becomes

(i) proportional to the area of the plate,
(ii) inversely proportional to the distance of separation (x) between plates, and
(iii) directly proportional to the relative permittivity of the medium of separation of plates.

2.21 CAPACITANCE OF A MULTI-PLATE CAPACITOR

We just obtained the capacitance of a parallel plate capacitor having only two plates held in parallel. If there are n number of parallel plates, each being identical to the other and alternate plates being connected to the same polarity of the supply potential (Fig. 2.21). We can say that there are (n – 1) space between n number of parallel plates. Thus the capacitor is equivalent to (n – 1) number of parallel plate capacitor consisting of two parallel plates.

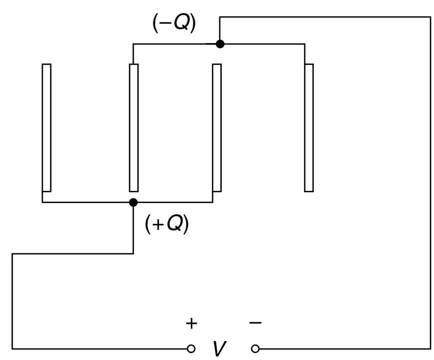


Fig. 2.21 ***Alternate plates of parallel plate capacitor being connected to same polarity***

$\therefore$ Total capacitance C of multiple parallel plate capacitor (containing n number of plates)

$$= (n - 1) \times \text{capacitance of one pair of plates}$$

$$= (n - 1) \times \frac{\varepsilon_o \varepsilon_r A}{x} \text{ Farad} \tag{2.24}$$

where ε_o = absolute permittivity of space,
ε_r = relative permittivity of dielectric medium,
x = thickness of dielectric medium between any two parallel plates in metres, and
A = area of each plate in m^2.

2.22 CAPACITANCE OF A PARALLEL-PLATE CAPACITOR WITH COMPOSITE DIELECTRICS

Let us assume a parallel plate capacitor with two different dielectrics having relative permittivities ε_{r_1} and ε_{r_2}. The separation of plates are x_1 and x_2 metres, as shown in Fig. 2.22. The plates are of identical cross-sectional area (A) square metre and the charge accumulated in the capacitor is Q coulombs when a p.d. of (V) volts is applied across the capacitor terminals.

Electric flux density is given by

$$\delta = \frac{\psi}{A} = \frac{Q}{A} \text{ coulomb/m}^2.$$

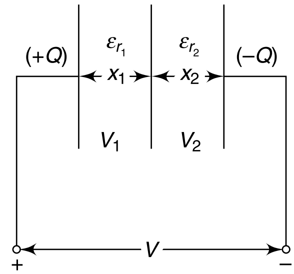


Fig. 2.22 ***Parallel plate capacitor with composite dielectric***

Since $\varepsilon = \dfrac{\delta}{E}$, E being the electric field intensity, we can write

$$\varepsilon_1 = \frac{\delta}{E_1} \text{ and } \varepsilon_2 = \frac{\delta}{E_2}$$

i.e.
$$E_1 = \frac{\delta}{\varepsilon_1} = \frac{\delta}{\varepsilon_o \varepsilon_{r_1}}$$

and
$$E_2 = \frac{\delta}{\varepsilon_2} = \frac{\delta}{\varepsilon_o \varepsilon_{r_2}}.$$

If V_1 and V_2 be the p.d. across the respective dielectrics, we can write

$$V = V_1 + V_2$$

$$= E_1 x_1 + E_2 x_2 \quad \left[\because \text{Intensity } E = \frac{\text{Potential }(V)}{\text{Distance }(x)}\right]$$

$$= \frac{\delta}{\varepsilon_o \varepsilon_{r_1}} \cdot x_1 + \frac{\delta}{\varepsilon_o \varepsilon_{r_2}} \cdot x_2$$

$$= \frac{\delta}{\varepsilon_o}\left[\frac{x_1}{\varepsilon_{r_1}} + \frac{x_2}{\varepsilon_{r_2}}\right]$$

$$= \frac{Q}{\varepsilon_o A}\left[\frac{x_1}{\varepsilon_{r_1}} + \frac{x_2}{\varepsilon_{r_2}}\right]$$

$$\therefore \text{ Capacitance }(C) = \frac{Q}{V} = \frac{Q}{\dfrac{Q}{\varepsilon_o A}\left[\dfrac{x_1}{\varepsilon_{r_1}} + \dfrac{x_2}{\varepsilon_{r_2}}\right]}$$

or
$$C = \frac{\varepsilon_o A}{\left[\dfrac{x_1}{\varepsilon_{r_1}} + \dfrac{x_2}{\varepsilon_{r_2}}\right]} \text{ Farad} \qquad (2.25a)$$

i.e. $$C = \frac{\varepsilon_o A}{\sum \frac{x}{\varepsilon_r}}, \text{ for more number of dielectrics.} \qquad (2.25b)$$

2.23 CAPACITANCE OF AN ISOLATED SPHERE

We have seen earlier that in case of an isolated sphere, charged with Q coulombs of electricity, the potential at the surface is given by

$$V = \frac{Q}{4\pi\varepsilon_o R},$$

R being the radius of the sphere. V being expressed in volts, we can find the capacitance of this sphere as

$$C = \frac{Q}{V} = 4\pi\varepsilon_o \cdot R \text{ Farad.}$$

If the medium within the sphere is filled up with a dielectric medium of relative permittivity ε_r, we can modify this expression of capacitance as

$$C = 4\pi\, \varepsilon_0\, \varepsilon_r \cdot R \text{ Farad.} \qquad (2.26)$$

2.24 CAPACITANCE OF CONCENTRIC SPHERES

A pair of concentric sphere S_1 and S_2 of radii r_1 and r_2 metres, separated by a dielectric medium of permittivity ε_r forms a spherical capacitance. We will consider two cases of this spherical capacitor.

Case-A

S_2 (the outer sphere) is earthed:

Let the inner sphere S_1 be charged by ($+Q$) coulomb of charge. It will induce ($-Q$) coulomb charge at the inner surface of S_2 and ($+Q$) coulomb charge at the outer surface of S_2. But as the outer surface of S_2 is earthed, this ($+Q$) charge at the outer surface of S_2 will escape (ε_1) to the earth (Fig. 2.23).

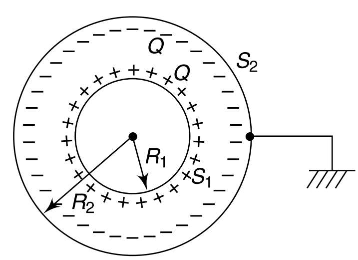


Fig. 2.23 *Charge distribution of concentric spheres (outer sphere earthed)*

$\therefore$ Surface potential of S_1 is given by

$$V_{S_1} = \frac{+Q}{4\pi\varepsilon_o\varepsilon_r R_1},$$

while surface potential at the inner surface of S_2 is given by $V_{S_2} = \dfrac{-Q}{4\pi\varepsilon_o\varepsilon_r R_2}$.

$\therefore$ Potential difference between S_1 and S_2 is

$$V = V_{S_1} - V_{S_2}$$

$$= \frac{Q}{4\pi\varepsilon_o\varepsilon_r}\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \frac{Q}{4\pi\varepsilon_o\varepsilon_r} \times \frac{R_2 - R_1}{R_1 R_2}$$

Then, $$C = \frac{Q}{V} = \frac{Q}{\dfrac{Q}{4\pi\varepsilon_o \varepsilon_r} \times \dfrac{R_2 - R_1}{R_2 R_1}}$$

$$= 4\ \pi\varepsilon_o\varepsilon_r \left[\frac{R_2 - R_1}{R_2 R_1}\right] \text{ Farad} \tag{2.27}$$

Case-B

S_1(the inner sphere) earthed: This time the outer sphere S_2 is given a charge of ($+Q$) coulomb. This charge is uniformly distributed in the outer and inner surface of S_2; we assume ($+Q_2$) charge remain at the outer surface while ($+Q_1$) at the inner surface of S_2. The charge ($+Q_1$) at the inner surface of S_2 would induce a charge of ($-Q_1$) coulomb on the outer surface of S_1; ($+Q_1$) charge induced in the inner surface of S_1 would pass to the earth as the inner surface of S_1 is earthed (Fig. 2.24).

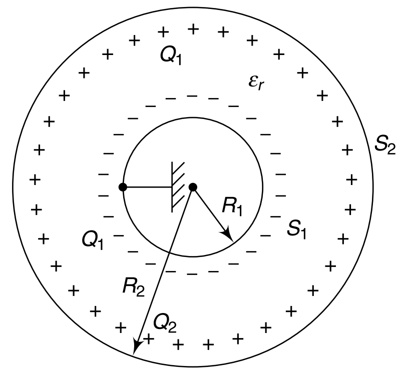


Fig. 2.24 ***Charge distribution of concentric spheres (inner sphere earthed)***

The system is now composed of two subsystems of capacitors as described below:

(i) Capacitor formed by inner surface of S_2 and outer surface of S_1 and is similar to the case we described in case A

$$\therefore \text{ Its capacitance, } C_1 = 4\pi\varepsilon_o\varepsilon_r\left[\frac{1}{R_2} - \frac{1}{R_1}\right]$$

$$= 4\pi\varepsilon_o\varepsilon_r \left[\frac{R_1 - R_2}{R_2 R_1}\right]$$

(ii) Capacitor formed by the outer surface of outer sphere S_2 and earth with air as dielectric.

$\therefore$ Its capacitance, $C_2 = 4\pi\varepsilon_o\varepsilon_r\, R_2$

Since these two subsystems of capacitors are electrically parallel, we can find the total capacitance (C) as

$$C = C_1 + C_2$$

$$= 4\pi\varepsilon_o\varepsilon_r\left[\frac{R_1 - R_2}{R_1 + R_2}\right] + 4\pi\varepsilon_o\, R_2$$

$$= 4\pi\varepsilon_o \left[\varepsilon_r \cdot \frac{R_1 - R_2}{R_1 + R_2} + R_2\right] \text{ Farad} \tag{2.28}$$

2.25 CAPACITANCE OF A PARALLEL PLATE CAPACITOR WHEN AN UNCHARGED METAL SLAB IS INTRODUCED BETWEEN PLATES

Let us consider each parallel plate has area of A m^2 and the distance between them is x m, the dielectric medium being air. If the charge retained by the capacitor is Q coulombs, the charge density δ is given by

$$\delta = \frac{Q}{A} \text{ C/m}^2.$$

Also $$C = \frac{\varepsilon_o A}{x} \text{ Farad}$$

And, $$\varepsilon = \frac{\delta}{E}, \ (E) \text{ being the field intensity}$$

$\therefore$ $$E = \frac{\delta}{\varepsilon_o} = \frac{Q}{\varepsilon_o A}; \ \varepsilon_o \text{ being the permittivity of air medium}$$

When an uncharged metal plate of thickness a ($a < b$) is inserted between the plates, equal and opposite charges are induced on the slab (Fig. 2.25) and the net charge on the slab is equal to zero. Thus the electric field inside the slab is zero. Then electric field would act in the distance $(x - a)$ m.

However, p.d. $(V) = E \times$ Distance

In our case, $V = E \times (x - a) = \dfrac{Q}{\varepsilon_o A} \times (x - a)$

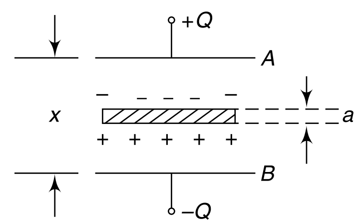


Fig. 2.25 ***Parallel plate capacitor with uncharged metal plate in between***

Thus, the new capacitance C' is given by

$$C' = \frac{Q}{V} = \frac{\varepsilon_o A}{(x - a)} \text{ Farad} \tag{2.29}$$

2.11 Find the equivalent capacitance of the network shown in Fig. 2.26 connected across terminals a and b.

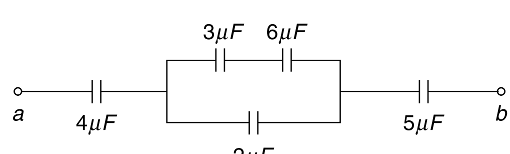


Fig. 2.26 ***Capacitance configuration of Ex. 2.11***

Solution

The capacitors of 3 μF and 6 μF are connected in series. Hence their equivalent capacitance

$$C_1 = \frac{1}{\frac{1}{3} + \frac{1}{6}} = \frac{6}{2+1} = 2 \ \mu\text{F}$$

However C_1 and 2 μF are in parallel, therefore their equivalent capacitance $C_2 = C_1 + 2 = 2 + 2 = 4$ μF.

Thus the network of capacitors reduces to that as shown in Fig. 2.26a.

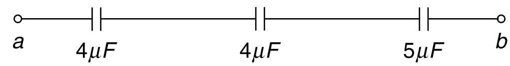


Fig. 2.26a ***Reduced network of capacitors***

If C_{eq} be the equivalent capacitance of the new network configuration then,

$$\frac{1}{C_{eq}} = \frac{1}{4}+\frac{1}{4}+\frac{1}{5}=\frac{5+5+4}{20}=\frac{14}{20}\ \mu\text{F}$$

Hence $$C_{eq} = \frac{20}{14}\ \mu\text{F} = 1.43\ \mu\text{F}.$$

2.12 Find the equivalent capacitance of the system of capacitances shown in Fig. 2.27.

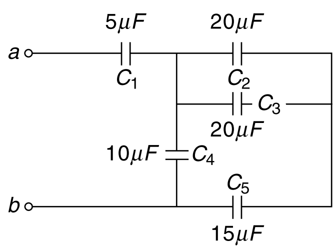


Fig. 2.27 ***Capacitor configuration of Ex. 2.12***

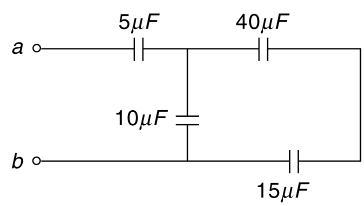


Fig. 2.27a ***Reduced network***

Solution

The capacitors C_2 and C_3 (20 μF each) are conected in parallel. Hence their equivalent capacitance is 20 + 20 = 40 μF. The network is shown in Fig. 2.27a.

40 μF and 15 μF are connected in series. Hence their equivalent capacitance is $\frac{40 \times 15}{40+15}$ i.e 10.91 μF.

Thus, 10 μF and 10.91 μF are connected in parallel. Their equivalent capacitance is 10 + 10.91 = 20.91 μF. The corresponding network is shown in Fig. 2.27b.

Hence the equivalent capacitance of the system is

$$C_{eq} = \frac{5 \times 20.91}{5+20.91}\ \mu\text{F, i.e } 4.035\ \mu\text{F}$$

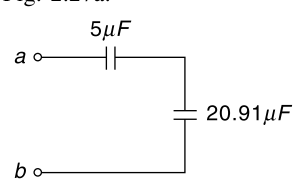


Fig. 2.27b ***Equivalent network of capacitances of Ex. 2.12***

2.13 A 50 μF capacitor is initially charged to accumulate 100 μ coulomb of charge. One uncharged capacitor of 200 μF is connected across it in parallel. How much charge will be transferred?

Solution

Let V be the voltage across the capacitors connected in parallel. We know that $V = Q/C$, where Q is the charge in coulomb and C is the capacitance in Farad.

Hence $Q_1/C_1 = Q_2/C_2$, where Q_1 is the charge of capacitor C_1 and Q_2 is the charge of capacitor C_2. Voltage across the parallel combination of C_1 and C_2 remain the same.

Therefore, $$\frac{Q_1}{Q_2} = \frac{C_1}{C_2} = \frac{50}{200} = \frac{1}{4} \qquad \text{(i)}$$

Again, Total charge = Initial charge accumulated by C only = 100 μ Coulumb.

Hence $$Q_1 + Q_2 = 100 \qquad \text{(ii)}$$

Solving equations (i) and (ii)

$$Q_1 + 4Q_1 = 100\ \mu C$$

or $$Q_1 = 20\ \mu C \text{ and } Q_2 = 80\ \mu C$$

Therefore 80 μC charge will be transferred from C_1 to C_2.

2.14 Find the equivalent capacitance across terminals x-y in Fig. 2.28. Also find the time to charge the capacitances by a direct current of 10A.

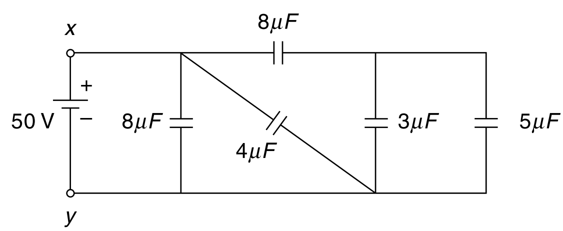


Fig. 2.28 *Network of capacitances of Ex. 2.14*

Solution

Equivalent capacitance of 3 μF and 5 μF is (3 + 5)μF, i.e 8 μF

The equivalent capacitance of two 8 μF capacitors in series is 8 × 8/8 + 8 μF, i.e 4 μF. The equivalent capacitance of two 4 μF capacitors in parallel is (4 + 4) μF, i.e 8 μF. Here we find two 8 μF capacitors are in parallel with the voltage source.

Hence the equivalent capacitance of the circuit across terminals $(x - y)$ is (8 + 8) μF, i.e 16 μF.

Now, charge $= 50 \times 16 \times 10^{-6}$ coulomb

$= 800 \times 10^{-6}$ coulomb

If t be the charging time and i be the current then

$$i \times t = 800 \times 10^{-6} \quad [\because Q = i \times t]$$

or $$t = \frac{800 \times 10^{-6}}{10}\text{s} = 80\mu \text{ second.}$$

2.15 A voltage of 90V d.c is applied across two capacitors in series having capacitances of 50 μF and 25 μF. Find the voltage drop across each capacitor. What is the charge in coulomb in each capacitor?

Solution

Since the capacitors are in series, same charge Q is flowing across each of them.

Hence $Q = C_1V_1 = C_2V_2$, where C_1 and V_1 are the capacitance and voltage across one capacitor and C_2 and V_2 are the capacitance and voltage across the other.

Therefore, $$50V_1 = 25V_2 \text{ or, } V_2 = 2V_1 \qquad \text{(i)}$$

Again $V_1 + V_2 = 90$ (ii)

Solving equations (i) and (ii), $V_1 + 2V_1 = 90$

or $V_1 = 30$ and $V_2 = 60$.

Hence voltage drop across the capacitors are 30 V and 60 V.

Since both the capacitors are in series

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \quad \text{or} \quad C = \frac{C_1 C_2}{C_1 + C_2} = \frac{50\times10^{-6}\times25\times10^{-6}}{50\times10^{-6}+25\times10^{-6}}$$

$$= 16.67\ \mu\text{F}.$$

The charge supplied by the dc source is

$$Q = CV = 16.67 \times 10^{-6} \times 90 \simeq 1500\ \mu\text{C}.$$

In series combination, each capacitor has equal charge and this charge equals the charge supplied by the dc source.

$\therefore$ each capacitor would retain a 1500 μC charge in fully charged condition.

2.16 Calculate the capacitance of a parallel plate capacitor having 20 cm × 20 cm square plates separated by a distance of 1.0 mm. Assume the dielectric medium to be air with permittivity of 8.85×10^{-12} F/m.

Solution

$$C = \frac{\varepsilon_o A}{x}, \text{ for parallel plate capacitor}$$

$$= \frac{8.85\times10^{-12}\times400\times10^{-4}}{1\times10^{-3}}$$

$$= 3.54 \times 10^{-10}\ \text{F} = 354\ \text{pF}.$$

2.17 In Fig. 2.29, a voltage source is connected across a combination of capacitances at terminals $(x - y)$. Find the current supplied by the battery to charge this combination if the time taken to charge is 50 m sec.

$[C_1 = C_2 = C_3 = 10\ \mu\text{F}$

$V = 100\ \text{V}]$

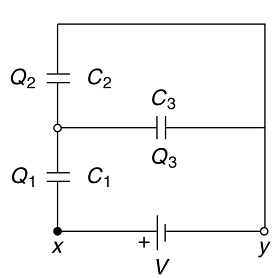


Fig. 2.29 ***Capacitance configuration (Ex. 2.17)***

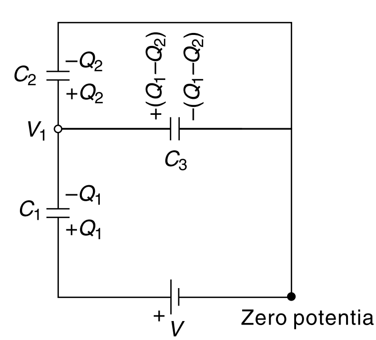


Fig. 2.29a ***Voltage-charge distributiion***

Solution

Let us redraw the diagram with voltage and charge distribution (Fig. 2.29a)

Here

$$Q_1 = C_1(V - V_1) \quad \text{(i)}$$

$$Q_2 = C_2V_1 \quad \text{(ii)}$$

$$(Q_1 - Q_2) = C_3V_1 \quad \text{(iii)}$$

From equations (ii) and (iii)

$$Q_1 = (C_2 + C_3)V_1$$

i.e.

$$V_1 = \frac{Q_1}{C_2 + C_3} \quad \text{(iv)}$$

From (i), $\dfrac{Q_1}{C_1} = V - V_1$ (v)

Adding (iv) and (v)

$$V = \frac{Q_1}{C_2 + C_3} + \frac{Q_1}{C_1}$$

or

$$V = \frac{(C_1 + C_2 + C_3)Q_1}{C_1(C_2 + C_3)}$$

$$\therefore \quad C \text{ (equivalent capacitance)} = \frac{Q_1}{V} = \frac{C_1(C_2 + C_3)}{C_1 + C_2 + C_3}$$

$$= \frac{10 \times 10^{-6}(10 + 10)10^{-6}}{(10 + 10 + 10)10^{-6}}$$

$$= 6.67 \times 10^{-6} \ \mu\text{F}$$

Hence, Q = charge drawn from source
$= CV = 6.67 \times 10^{-6} \times 100 = 6.67 \times 10^{-4}$ coulombs

Also, Charge = Current × Time

$$\therefore \text{ Current } (I) = \frac{\text{Charge}}{\text{Time}} = \frac{6.67 \times 10^{-4}}{50 \times 10^{-3}} = 13.34 \text{ mA.}$$

2.18 What is the capacitance across AD in Fig. 2.30?

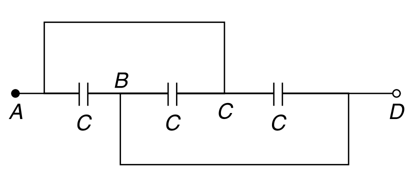


Fig. 2.30 ***Capacitance configuration (Ex. 2.18)***

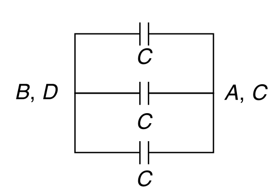


Fig. 2.30a ***Reduced network (Ex. 2.18)***

Solution

Observation reveals that B and D are electrically same points while A and C are electrically same points. The given figure then reduces as shown in Fig. 2.30a. Thus the equivalent capacitance of this parallel combination becomes $3C$.

2.19 If capacitance between adjacent parallel plates be C, find the total capacitance in the system shown in Fig. 2.31.

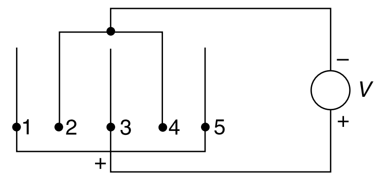


Fig. 2.31 *Circuit of Ex. 2.19*

Solution

Let us redraw the circuit in a conventional form (Fig. 2.31a).

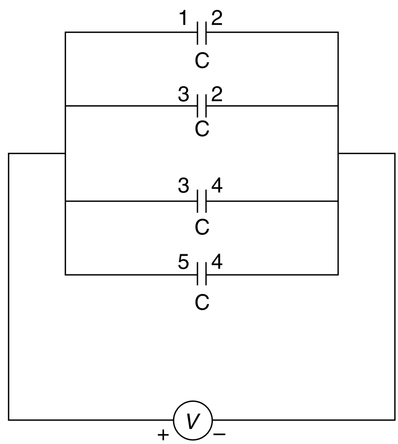


Fig. 2.31a *Equivalent circuit of Fig. 2.31*

Hence we find that the plate pairs are in parallel and hence the net capacitance is 4C.

2.20 Find the value of the capacitance C if the equivalent capacitance between points X and Y is to be 1 μF. All capacitances are in μF.

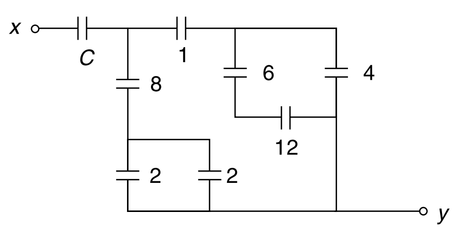


Fig. 2.32 *Capacitance configuration (Ex. 2.20)*

Solution

The series combination of capacitances 6 and 12 is 6 × 12/ 6 + 12 i.e. 4 μF. The parallel combination of 2 and 2 is (2 + 2), i.e. 4 μF. Figure 2.32 is reduced to Fig. 2.32a.
The parallel combination of 4 and 4 is 8 μF in Fig. 2.32a, while the series combination of 8 and 4 is 8 × 4/8 + 4, i.e. 8/3 μF. We can reduce Fig. 2.32a further to Fig. 2.32b.

The series combination of 1 and 8 yield 1 × 8/1 + 8, i.e. 8/9 μF and this 8/9 μF is in parallel to 8/3 μF in Fig. 2.32b. The equivalent capacitance is then (8/9 + 8/3) i.e., 32/9 μF. Thus finally we reduce the network of Fig. 2.32b to Fig. 2.32c, where C is in series with 32/9 μF. By the given question,

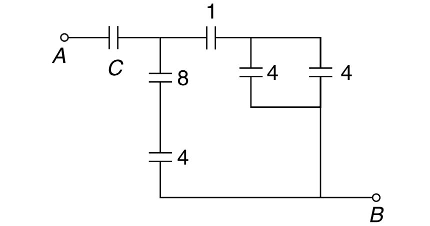


Fig. 2.32a *Partly reduced network of Fig. 2.32*

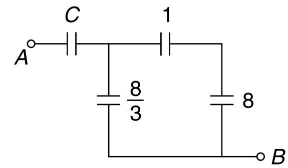


Fig. 2.32b *Further network reduction of Fig. 2.32a*

$$1 = \frac{1}{C} + \frac{9}{32}$$

$$\therefore \qquad C = \frac{32}{23} = 1.39\ \mu\text{F}.$$

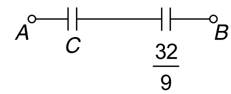


Fig. 2.32c *Finally reduced network of Fig. 2.32b*

2.21 In Fig. 2.33, C = 9F; C_1 = 6F. Find the equivalent capacitance across $(a - b)$.

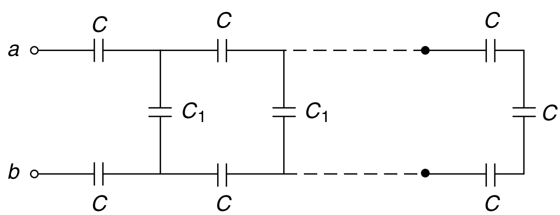


Fig. 2.33 *Capacitance configuration (Ex. 2.21)*

Solution

We may note that the last three capacitors are all C, i.e. all are 9F each. Since they are in series, the net capacitance of these three capacitors is 3F. This 3F equivalent capacitor is in parallel to C_1 of the previous loop (Fig. 2.33a). Thus parallel combination of C_1(6F) and C_q(3F) gives C_{q1} = 9F.

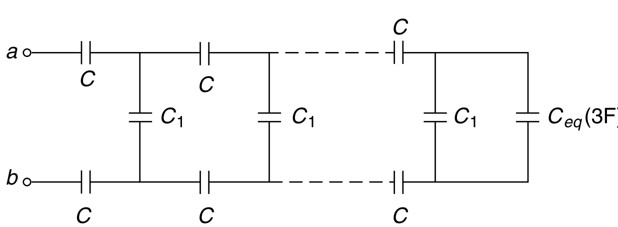


Fig. 2.33a *Reduced network of Fig. 2.33*

Thus in this loop, there are again two capacitors of C Farad (9F) each in series with C_{q1}. The net capacitance of this loop again becomes 3F. This process continues and finally we come to the first loop while the same result is obtained.

$\therefore$ Equivalent capacitance across ab becomes 3F (Fig. 2.33b).

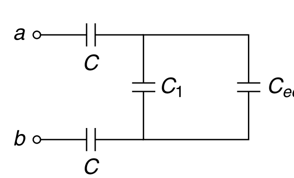


Fig. 2.33b *Finally reduced network of Ex. 2.21*

2.22 Find the equivalent capacitance across XY (Fig. 2.34).

Solution

Each vertical column is having equivalent capacitance of

$$C_1 = 1\text{F}$$

$$C_2 = \frac{1}{2}\text{F}$$

$$C_3 = \frac{1}{4}\text{F}$$

$$C_4 = \frac{1}{8}\text{F}$$

and so on.

It may be noticed that all these capacitors C_1, C_2, … are in parallel. (Fig. 2.34a)

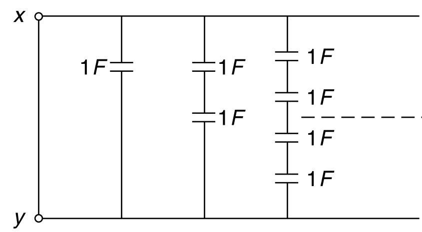


Fig. 2.34 *Capacitance network of Ex. 2.22*

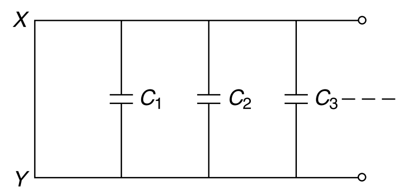


Fig. 2.34a *Equivalent network of Fig. 2.34*

$\therefore \quad C = C_1 + C_2 + C_3 + \ldots$

$= 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots$

$= 2$ [$\because$ it is a geometric series whose sum is 2]

$\therefore \quad C = 2$ F.

2.23 In the network of Fig. 2.35, find the capacitance between points x and y. Also find the charges on the three capacitors. Assuming the potential of y to be zero, find the potential at z.

Solution

Equivalent capacitance of 12 μF and 6 μF capacitors (being joined in series) is 12 × 6/12 + 6 = 4 μF (C_x), across XY.

This equivalent capacitance C_x is in parallel to the 2 μF capacitor. The final equivalent capacitance C is then

$$C = C_x + 2 = 6 \text{ μF}.$$

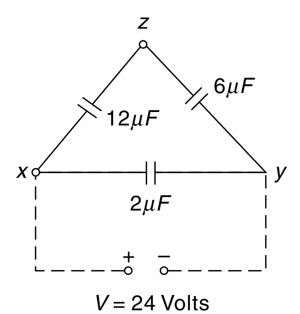


Fig. 2.35 *Circuit of Ex. 2.23*

C is the net capacitance between x and y points. The charge supplied by the battery is then

$$Q = CV = 6 \text{ μF} \times 24 \text{ V}$$
$$= 144 \text{ μC}$$

[$\because$ Voltage across the equivalent capacitance C is 24 V]

Since the p.d. across the 2 μF capacitor is 24 V hence charge on the 2 μF capacitor is

$$2 \text{ μF} \times 24 \text{ V} = 48 \text{ μC}.$$

The charge on each of the 12 and 6 μF capacitors is then (144 μC – 48 μC), i.e. 96 μC. $\therefore$ Drop across the 6 μF capacitor is obtained as 96 μC/6 μF = 16 Volts. Observation reveals that this 16 V drop is actually the potential V_{zy} (i.e. $V_z - V_y$). Since V_y is zero, hence $V_{zy} = V_z = 16$ V. Then potential at z with respect to y is 16 V.

2.24 The plates of a parallel plate air capacitor of a capacitance C consists of two circular plates, each of 10 cm radius and placed 0.2 cm apart. The capacitor is charged to 100 V and connected across an electrostatic voltmeter. The space between the plates is then filled up by a dielectric medium so that the capacitance of the parallel plate capacitor becomes $4.5C$ and the voltmeter now reads 25 V. What is the capacitance of the electrostatic voltmeter?

Solution

Let V be the p.d. across the combination (condenser C in parallel to capacitance C' of voltmeter). Since C and C' are in parallel (Fig. 2.36),

$$Q = CV + C'V = (C + C')V$$

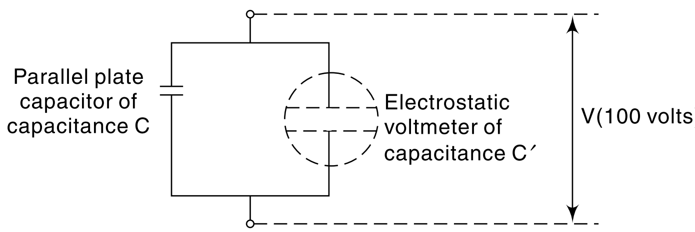


Fig. 2.36 ***Fig. of Ex. 2.24***

Let us now replace the air medium of C and fill it by a dielectric medium such that the new capacitance is $4.5C$.

Total charge remaining the same we can now write

$$4.5C\,V_1 + C'V_1 = Q = (C + C')\,V$$

[V_1 is the new voltage across the capacitor]

or $$(4.5C + C')\,V_1 = (C + C')\,V.$$

$\therefore$ $$\frac{4.5C + C'}{C + C'} = \frac{V}{V_1} = \frac{100}{25} = 4$$

$\therefore$ $$C' = \frac{C}{6} \text{ (on simplification).}$$

Now $$C = \frac{\varepsilon_o A}{x}, \text{ where } A = \pi r^2$$

Here, $$C = \frac{8.85 \times 10^{-12} \times \pi \times (10 \times 10^{-2})^2}{0.2 \times 10^{-2}}$$

$$= \frac{8.85 \times \pi \times 10^{-14}}{0.2 \times 10^{-2}}$$

$$= 1.39 \times 10^{-10} \text{ F}$$

Hence $C' = \dfrac{C}{6} = 2.32 \times 10^{-11}$ F.

2.25 In Fig. 2.37, find the p.d. between (x-y) and (x-z) in steady state. Figures shown against capacitances are in μF.

Solution

At steady state all the capacitors are fully charged and no current passes through the circuit. Thus points y and z are at some potential as points a and b respectively.

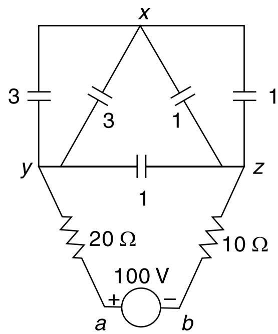


Fig. 2.37 *Network of Ex. 2.25*

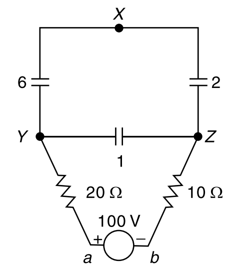


Fig. 2.37a *Reduced network of Fig. 2.37*

Figure 2.37a shows the reduced network where both 3 μF capacitors at the left hand side are replaced by their equivalent capacitance as well as two 1 μF capacitors at the right hand side of Fig. 2.37 are also replaced by their equivalent capacitance.

Further reduction of the network (shown in Fig. 2.37a) is possible (Fig. 2.37b).

We thus find the 3/2 μF equivalent capacitor is placed in parallel to the 1 μF capacitor. The charge retained by each of them will be different. We find that charge retained by the 3/2 μF equivalent capacitance is

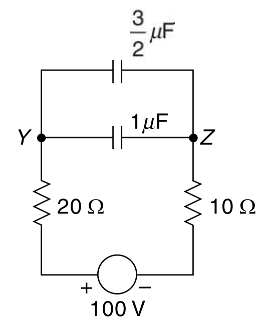


Fig. 2.37b *Final reduced network of Fig. 2.37a*

$$Q = CV = \frac{3}{2} \times 10^{-6} \times 100 = 150\ \mu\text{C}.$$

If we go back to the capacitor configuration of Fig. 2.37a, we find this 150 μC charge will be retained by capacitors 6 μF and 2 μF. Thus, the p.d. between x and y is actually the drop across the 6 μF capacitor.

$$\therefore \qquad V_{x\text{-}y} = \frac{Q}{C} = \frac{150 \times 10^{-6}}{6 \times 10^{-6}} = 25\ \text{V}.$$

Similarly, p.d. at $x - z$ will be the drop across the 2 μF capacitor

$$\text{i.e.} \qquad V_{x\text{-}z} = \frac{150 \times 10^{-6}}{2 \times 10^{-6}} = 75\ \text{V}.$$

[Check that, $V_{x\text{-}y} + V_{x\text{-}z} = V$.]

2.26 Show that if a dielectric of thickness t and with the same area as the plates of parallel plate capacitor is introduced, the capacitor would then have the capacitance

$$C = \frac{\varepsilon_o A}{\left[d - t + \dfrac{t}{\varepsilon_r}\right]}$$

Solution

Let us suppose that we have a parallel plate capacitor with air as the dielectric medium and capacitance C. Obviously, $C = \varepsilon_0 A/d$, (d) being the separation between the plates.

Next we imagine that the capacitor is filled up by another dielectric of dielectric strength ε_r replacing the air medium (Fig. 2.38). Let C' be the new capacitance.

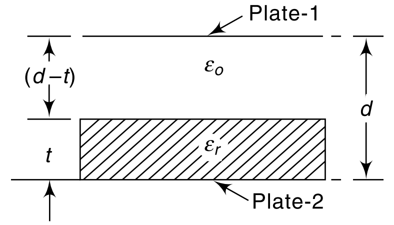


Fig. 2.38 ***Parallel plate capacitor with two dielectric medium***

$$C' = \frac{\varepsilon_o \varepsilon_r A}{d} = \frac{\varepsilon_o A}{d/\varepsilon_r}$$

Now, if the two capacitances are supposed to be equal then we find that d/ε_r replaces d in the original expression when air was the dielectric medium.

Thus d distance between the plates with air as the dielectric medium is equivalent to distance d/ε_r in air medium.

Therefore, if a dielectric of thickness t is introduced then it being equivalent to t/ε_r of air medium, the effective air distance between the plate is $(d - t + t/\varepsilon_r)$.

$$\therefore \qquad C = \frac{\varepsilon_o A}{d - t + (t/\varepsilon_r)}.$$

[Also, the problem may be solved in another way:
C_1 is the capacitance with ε_o.

$$\therefore \qquad C_1 = \frac{\varepsilon_o A}{d - t}$$

C_2 is the capacitance with ε_r.

$$\therefore \qquad C_2 = \frac{\varepsilon_r \varepsilon_o A}{t}.$$

Since C_1 and C_2 are in series

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{\varepsilon_o A/d - t \times \varepsilon_r \varepsilon_o A/t}{\varepsilon_o A/d - t + \varepsilon_r \varepsilon_o A/t}$$

$$= \frac{\varepsilon_o^2 \varepsilon_r A^2}{\varepsilon_o A(t + d\varepsilon_r - t\varepsilon_r)} = \frac{\varepsilon_o A}{\left(d - t + \dfrac{t}{\varepsilon_r}\right)}.$$

We have obtained same result using the previous method.]

2.27 Three plates are held parallel to a common plate (Fig. 2.39). A is the area in m^2 for each of the three parallel plates, while d metre is the distance between each pair of plates. What is the equivalent capacitance?

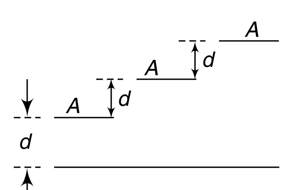


Fig. 2.39 ***Three plates parallel to a common plate***

Solution

Each of the three plates form a parallel plate capacitance with the common plate (the bottom most plate). Also by virtue of their placement and configuration, these capacitances are parallel.

∴ Equivalent capacitance

$$C = C_1 + C_2 + C_3$$

$$= \frac{\varepsilon_o A}{d} + \frac{\varepsilon_o A}{2d} + \frac{\varepsilon_o A}{3d}$$

$$= \frac{\varepsilon_o A}{d}\left[1 + \frac{1}{2} + \frac{1}{3}\right] = \frac{11}{6} \times \frac{\varepsilon_o A}{d}$$

Hence, equivalent capacitance is $\frac{11}{6}\frac{\varepsilon_o A}{d}$.

2.28 Figure 2.40 shows a set of capacitor configurations. Find charges on the three capacitors.

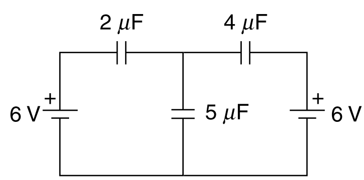


Fig. 2.40 ***Capacitor configuration in Ex. 2.28***

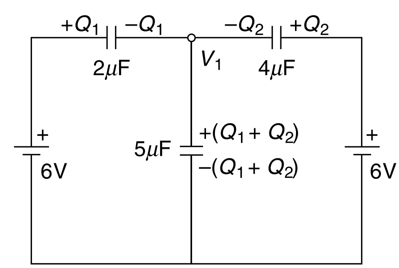


Fig. 2.40a ***Charge distribution (Ex. 2.28)***

Solution

Let us redraw the given figure with charge distribution diagram (Fig. 2.40a)
Since $Q = CV$, we can write

$$Q_1 = 2 \times 10^{-6}\,(6 - V_1) \quad \text{(i)}$$

$$Q_2 = 4 \times 10^{-6}\,(6 - V_1) \quad \text{(ii)}$$

$$(Q_1 + Q_2) = 5 \times 10^{-6}\,(V_1 - 0) \quad \text{(iii)}$$

$$Q_2 = 2Q_1 \quad \text{(iv)}$$

Solving for (ii) and (iii) we get

$$5Q_2 + 4(Q_1 + Q_2) = 20 \times 10^{-6} \times 6$$

$$4Q_1 + 9Q_2 = 120 \times 10^{-6}$$

or $$4Q_1 + 18Q_1 = 120 \times 10^{-6}$$

∴ $$Q_1 = 5.45\ \mu\text{C}$$

Thus $$Q_2 = 10.9\ \mu\text{C}.$$

2.29 Figure 2.41 represents a capacitive ladder network. Obtain equivalent capacitance across $(x - y)$.

Solution

As the capacitive ladder network is infinitely long, the capacitance of the ladder to the right of points M and N is the same as that of the ladder to the right of the points $(x - y)$.

Let the equivalent capacitance of the network to the right of $M - N$ be C_1. We must draw the reduced network (Fig. 2.41a).

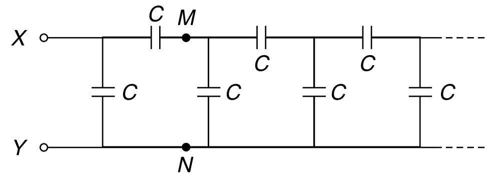


Fig. 2.41 *A capacitive ladder network*

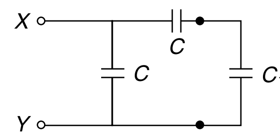


Fig. 2.41a *Reduced network of Fig. 2.41*

The equivalent capacitance between $x - y$ is

$$C_{x-y} = C + \frac{CC_1}{C + C_1}$$

However, the ladder is symmetric and hence all the loops are identical to the adjacent loop. Hence the equivalent capacitance of the ladder is also C_1, i.e. $C_{x-y} = C_1$.

$$\therefore \qquad C_1 = C + \frac{CC_1}{C + C_1}$$

or $$C_1^2 - CC_1 - C^2 = 0$$

$$\therefore \qquad C_1 = \frac{C + \sqrt{C^2 + 4C^2}}{2} = \frac{1+\sqrt{5}}{2}C$$ [negative sign is neglected]

2.30 The space between two plates of a parallel plate capacitor C is filled up with three different dielectric slabs of identical size as shown in Fig. 2.42. If the dielectric constants of the three slabs be ε_1, ε_2, and ε_3, find the new value of the capacitance. The plate cross-sectional area is A and the separation is d.

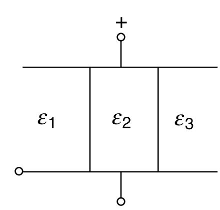


Fig. 2.42 *Capacitor of Ex. 2.30*

Solution

Let us consider each 1/3 assembly as separate capacitors C_1, C_2 and C_3.

$$C_1 = \frac{\varepsilon_1 (A/3)}{d}; \quad C_2 = \frac{\varepsilon_2 (A/3)}{d}; \quad C_3 = \frac{\varepsilon_3 (A/3)}{d}$$

As the three capacitors are in parallel (the +ve plates are joined together for all capacitors as well as the negative plates are also connected together),

$$C_{eq} = C_1 + C_2 + C_3 = \frac{A}{3d}(\varepsilon_1 + \varepsilon_2 + \varepsilon_3).$$

2.26 EXPRESSION OF INSTANTANEOUS CURRENT AND VOLTAGE IN A CAPACITOR

The instantaneous current in a capacitor is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv) = C\frac{dv}{dt}. \tag{2.30a}$$

Thus, the voltage across the capacitor being constant, current through it is zero. This means, on application of dc voltage across the capacitor and with no initial charge the capacitor first acts as short circuit but as soon as it accumulates full charge, it behaves like an open circuit.
Also, from above

$$dv = \frac{1}{C} \cdot i \cdot dt$$

or $$\int_{v_o}^{v_f} dv = \frac{1}{C}\int_o^t i \cdot dt$$ [v_o = initial voltage in the capacitor, if any and v_f = the final voltage in the capacitor]

or $$v_f - v_o = \frac{1}{C}\int_o^t i \cdot dt$$

$\therefore$ $$v_f = \frac{1}{C}\int_o^t i \cdot dt + v_o \tag{2.30b}$$

[Normally, $v_o = 0$ and hence $v_f = v_C = \frac{1}{C}\int_o^t i\, dt$]

2.27 CHARGING AND DISCHARGING OF CAPACITANCE

(a) Charging

Let a dc voltage V be applied (at $t = 0$) by closing a switch S in a series RC circuit (Fig. 2.43). The capacitor being charged, at $t > 0$, the charging current becomes i. We can write

$$Ri + \frac{1}{C}\int_o^t i\, dt = V \tag{2.31}$$

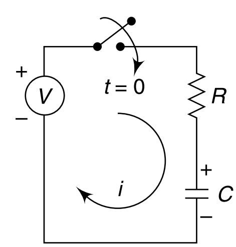


Fig. 2.43 *Charging and discharging of capacitor*

[$\because$ drop across the resistor = Ri and the drop across (C) is obtained from the instantaneous current (i) given by

$$i = C\frac{dv}{dt}$$

i.e. $$v = \frac{1}{C}\int i\, dt$$]

It may be noted here that as the charging gets started, upper plate of (C) will start accumulating +ve charges while the lower plate accumulates –ve charge.
Differentiation of equation 2.31 results

$$R\frac{di}{dt} + \frac{i}{C} = 0 \tag{2.31a}$$

or $$\frac{di}{i} = -\frac{1}{RC}dt$$

Integrating both sides

$$\log_e i = -\frac{t}{RC} + K_1, \text{ where } K_1 \text{ is a constant}$$

or $$\log_e \frac{i}{K_2} = \log_e e^{-t/RC} \quad (\text{where } K_1 = \log_e K_2)$$

or $$i = K_2\ e^{-t/RC} \qquad (2.32)$$

With application of voltage and assuming no initial charge across the capacitor, the capacitor will not produce any voltage across it but acts as a short circuit causing the circuit current to be (V/R).

i.e. at $$t = 0^+,\ i(0^+) = \frac{V}{R}$$

Hence from equation (2.37) at $t = 0^+$

$$\frac{V}{R} = K$$

Finally we then obtain, $i = \dfrac{V}{R} e^{-t/RC}$ (2.33)

It may be observed that the charging current is a decaying function, the plot being shown in Fig. 2.44a. As the capacitor is getting charged, the charging current dies out.

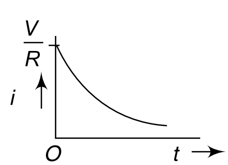


Fig. 2.44a ***Profile of current in RC charging circuit***

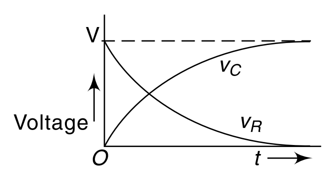


Fig. 2.44b ***Profiles of v_R and v_C in RC charging circuit***

The corresponding voltage drops across the resistor and capacitor can be obtained as follows:

$$v_R = iR = V\ e^{-t/RC} \qquad (2.34)$$

and $$v_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int_o^t \frac{V}{R}\, e^{-\frac{t}{RC}}\,dt$$

$$= V\,(1 - e^{-t/RC}) \qquad (2.35)$$

Observing equations (2.34) and (2.35) it reveals that (v_R) is a decaying function while (v_C) is an exponentially rising function [profiles of (v_R) and (v_C) are shown in Fig. 2.44b]. The steady state voltage across capacitor is V volts.

The constant is obtained by substituting $t = RC$ which gives $v_C = V(1 - 0.368) = 0.632V$, i.e the item by which the capacitor attains 63.2% of steady state voltage. The instantaneous powers are given by

$$p_R = iv_R = \frac{V^2}{R} e^{-2t/RC}$$

and $$p_C = iv_C = \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)$$

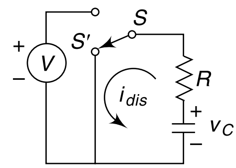


Fig. 2.45 ***Discharging in RC series circuit***

(b) Discharging

Let us now study the discharging case when the switch S is thrown to a contact S' such that the R-C circuit is shorted and the voltage source is withdrawn (Fig. 2.45). Here we can write

$$Ri + \frac{1}{C}\int i\,dt = 0 \tag{2.36}$$

Differentiating equation (2.36) we get

$$R\frac{di}{dt} + \frac{i}{C} = 0 \tag{2.37}$$

Solution of equation (2.37) is

$$i = K'e^{-t/RC} \tag{2.38}$$

where K' is a constant.

However at $t = 0^+$, the voltage across the capacitor will start discharging current through the resistor in opposite direction to the original current (shown by i_{dis} in Fig. 2.45). Hence the direction of i during discharge is negative and its magnitude is given by (V/R).

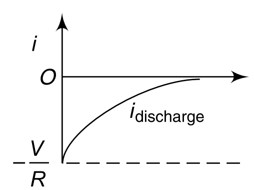


Fig. 2.46 ***Current decay transient in RC discharging circuit***

Hence from equation (2.38) we get

$$-\frac{V}{R} = K'(\text{at } t = 0^+)$$

The complete solution is then

$$i = -\frac{V}{R} e^{-t/RC} \tag{2.39}$$

The decay transient is plotted in Fig. 2.46.

The corresponding transient voltages are given by

$$v_R \text{ (voltage drop across } R) = iR = -Ve^{-t/RC} \tag{2.40a}$$

and $$v_C \text{ (voltage drop across } C) = \frac{1}{C}\int i\,dt = Ve^{-t/RC} \tag{2.40b}$$

Obviously, $v_R + v_C = 0$

Figure 2.47 represents the profiles of v_R and v_C with t. In the discharging circuit, the time constant is given by the product of R and C such that $v_C = Ve^{-1} =$

$0.369V \simeq 0.37V$, i.e. the time by which the capacitor discharges to 37% of its initial voltage.

The instantaneous powers are given by

$$p_R = v_R\, i = \frac{V^2}{R} e^{-2t/RC} \qquad (2.41a)$$

and

$$p_C = v_C\, i = -\frac{V^2}{R} e^{-2t/RC} \qquad (2.41b)$$

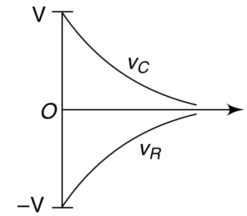


Fig. 2.47 ***vR and vC in RC discharging circuit***

[The charge stored in the capacitor during charging is given by $q = Cv_C = CV\left(1 - e^{-t/RC}\right)$ or $q = Q\left(1 - e^{-t/RC}\right)$ while that during discharging is given by $q = Cv_C = CVe^{-t/RC}$ coulombs or, $q = Qe^{-t/RC}$].

2.31 Calculate the time taken by the capacitor of 1 μF and in series with a 1 μΩ. resistance to be charged upto 80% of the final value.

Solution

The time constant T is given by

$$T = RC = 1 \times 10^6 \times 10^{-6} = 1 \text{ sec.}$$

The charging of capacitor is expressed by the following equation

$$q = Q_o\left(1 - e^{-t/RC}\right).$$

Here$q = 0.8\ Q_o$; $R = 1$ sec.

$\therefore \qquad 0.8 = 1 - e^{-t} \quad$ or, $e^{-t} = 0.2$

Hence $t = 1.61$ sec.

2.32 A dc constant voltage source feeds a resistance of 2000 kΩ in series with a 5 μF capacitor. Find the time taken for the capacitor when the charge retained will be decayed to 50% of the initial value, the voltage source being short circuited.

Solution

Time constant $T = RC = 2 \times 10^6 \times 5 \times 10^{-6} = 10$ sec.

The decaying condition is represented by the following expression

$$q = Q_o e^{-t/T}$$

However, $\qquad q = 0.5\ Q_o,$

$\therefore \qquad 0.5\ Q_o = Q_o e^{-t/T}$

or $\qquad 0.5 = e^{-t/T} = e^{-t/T}$

or $\qquad -t/10 = \log_e (0.5)$

or $\qquad t = 6.938$

2.33 In Fig. 2.48 the switch K is closed. Find the time when the current from the battery reaches to 500 mA.

$[R_x = 50\ \Omega;\ R_y = 70\Omega;\ C = 100\ \mu F]$

Solution

Let current through R_x be I_x and through C be I_y after switch K is closed.

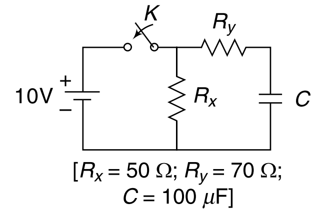


Fig. 2.48 ***Circuit of Ex. 2.33***

$$I_x = \frac{10}{50} = 0.2A = 200 \text{ mA}$$

However $\quad I = I_x + I_y$

[I being the current from the supply]

or $\quad 500 = 200 + I_y \quad [\because \text{ supply current is 500 mA}]$

$$I_y = 300 \text{ mA}$$

But $\quad I_y = \dfrac{V}{R_y} e^{-t/T} \quad [T = RC = 70 \times 100 \times 10^{-6} = 0.007 \text{ sec}]$

or $\quad 0.3 = \dfrac{10}{70} e^{-t/0.007}$

or $\quad -\dfrac{t}{0.007} = \log_e (2.1)$

$$t = 5.2 \text{ m-sec.}$$

This is the time required when the d.c. source current flow will be 500 mA.

2.34 A 10 μF capacitor is initially charged to 100 volts dc. It is then discharged through a resistance of (R) ohms for 20 seconds when the p.d. across the capacitor is 50 V. Calculate the value of (R).

Solution

In the discharging condition of the capacitor,

$$q = Q_0 e^{-t/RC} \quad \text{or} \quad v = V_0 e^{-t/RL}$$

As per the question capacitor p.d. gets discharged to 50 V from the initial p.d. of 100 V.

$\therefore \quad v = 0.5V_0$

Hence we obtain, $0.5 = e^{-t/R \times 10 \times 10^{-6}}$

or $\quad \log_e 0.5 = -\dfrac{t}{R \times 10^{-5}} = -\dfrac{20}{R \times 10^{-5}}$

or $\quad -0.7 = -\dfrac{20}{R \times 10^{-5}}$

$\therefore \quad R = 28.86 \times 10^5\ \Omega = 2.86\ M\Omega.$

2.35 A resistance R and 5μF capacitor are connected in series across a 100 V d.c. supply. Calculate the value of R such that the voltage across the capacitor becomes 50 V in 5 sec after the circuit is switched on.

Solution

In case of charging

$$q = Q_0\left(1 - e^{-t/T}\right) \quad [T = RC = (5 \times 10^{-6}\, R) \text{ sec.}]$$

or $$v = V_0\left(1 - e^{-t/T}\right)$$

As per the question, the p.d. across the capacitor is 50 V within 5 sec.

$\therefore$ $v = 0.5\ V_0$ $[V_0$ = final p.d. in steady state = 100 V]

and $t = 5$ sec.

$\therefore$ $$0.5 = 1 - e^{(-5)/(5\times 10^{-6})R} \text{ or, } -0.5 = -e^{-10^6/R}$$

$$R = 1.45 \text{ M}\Omega$$

2.36 A 5 μF capacitor is initially charged with 500 μC. At $t = 0$, the switch K is closed (Fig. 2.49). Determine the voltage drop across the resistor at $t < T$ and $t = \infty$.

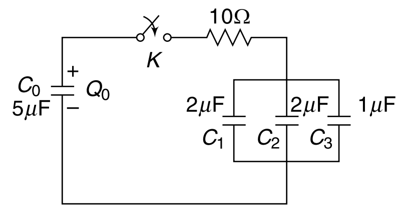


Fig. 2.49 *Circuit of Ex. 2.36*

Solution

The equivalent capacitance of bank of parallel capacitances is 5μF. As soon as K is closed, the equivalent 5μF capacitor is in series with C_0 and the net capacitance becomes 2.5 μF.

$\therefore$ T(time constant) = $R\ C_{net} = 10 \times 2.5 \times 10^{-6} = 25\mu$ sec.

The initial voltage V_0 across capacitor C_o is given by

$$V_0 = \frac{Q_0}{C_0} = \frac{500\,\mu C}{5\times 10^{-6}} = 100 \text{ V}$$

With closing of K, capacitor C_0 will start discharging, however at $t = 0^+$, there will be no voltage across C_1, C_2 or C_3.

Thus the entire voltage drop will be across R only (v_R) at $t = 0^+$ time.

i.e. $$v_R = V_0 \text{ (decaying)}$$

$$= V_0\, e^{-t/RC} = 100\, e^{-t/25\times 10^{-6}}$$

$$= 100\, e^{-4\times 10^4\, t} \text{ V}$$

At $t = \infty$, v_R becomes zero.

[It is also evident that in steady state ($t = \infty$), the charge of C_0 will be distributed through C_1, C_2 and C_3 and no current will flow through the circuit. Hence $i = 0$, $v_R = i_R = 0$].

2.37 In Fig. 2.50, a capacitor of capacitance C is charged to a voltage V_0 (dc) and is allowed to discharge through a resistance R while charging another capacitor of capacitance αC. Determine the final voltage at terminals $(a - b)$ under steady state condition.

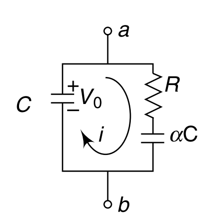


Fig. 2.50 *Circuit of Ex. 2.37*

Solution

Let V_f be the final voltage appearing across $(a - b)$ after discharging of C charging αC through R.

Equating the charge of the two capacitors

$$V_f(\alpha C) = V_0 C - V_f C$$

or $$V_f(1 + \alpha) = V_0$$

$$\therefore \qquad V_f = \frac{V_0}{1+\alpha}$$

[It may be noted that the final voltage across ($a - b$) is independent of R].

2.38 In Example 2.37 what fraction of the energy originally stored is lost?

Solution

Initial energy = $\frac{1}{2}CV_o^2$ [the proof is furnished in Article 2.28]

$$\text{Final energy} = \frac{1}{2}(\alpha C)V_f^2 + \frac{1}{2}CV_f^2 = \frac{1}{2}CV_f^2(1+\alpha)$$

$$= \frac{1}{2}(1+\alpha)C\left(\frac{V_o}{1+\alpha}\right)^2 = \frac{1}{2}\frac{V_o^2 C}{(1+\alpha)} \quad \left[\because\ V_f = \frac{V_o}{1+\alpha}, \text{ as described in Ex. 2.37}\right]$$

Hence, loss in energy = initial energy – final energy

$$= \frac{1}{2}CV_o^2 - \frac{1}{2}CV_o^2 \cdot \frac{1}{\alpha+1}$$

$$= \frac{1}{2}CV_o^2\left[1 - \frac{1}{\alpha+1}\right] = \frac{1}{2}CV_o^2 \cdot \frac{\alpha}{\alpha+1}$$

[It may be noted that this loss of energy is due to presence of resistance R. However, the energy loss expression is independent of R].

2.39 The 10 μF capacitor in RC circuit of Fig. 2.51 has initial charge of 100 μC with polarities as shown in Fig. 2.51. At $t = 0$, the switch being closed, a dc voltage of 100 V is applied. Find the expression for the current.

Solution

In the charging case

$$100 = 500 \times i + \frac{1}{10 \times 10^{-6}}\int i\, dt$$

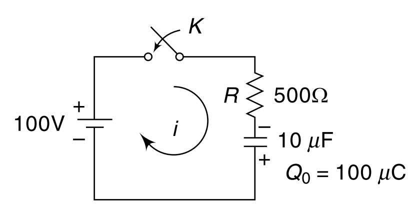


Fig. 2.51 ***Circuit of Ex. 2.39***

or $$0 = 500\frac{di}{dt} + \frac{i}{C}$$

or $$\frac{di}{i} = -\frac{1}{500\,C}dt$$

or $i = Ke^{-200\,t}$, where K is constant.

However, due to initial charge of 100 μC in the polarity shown, the equivalent voltage becomes

$$V_o = \frac{q_o}{C} = \frac{100 \times 10^{-6}}{10 \times 10^{-6}} = 10 \text{ V}$$

This 10 V also sends current in the direction of i.

Hence at $t = 0$

$$i_0 = (V + V_o)/R = \frac{110}{500} = 0.22 \text{ A}$$

Thus at $t = 0$, $0.22 = Ke^{-200 \times 0}$

or $K = 0.22$

Thus the expression for current becomes

$$i = 0.22\, e^{-200t} \text{ A.}$$

2.28 ENERGY STORED IN CAPACITOR

A capacitor never dissipates energy and only stores it when the capacitor is assumed to be ideal. It can store finite amount of energy, even if the steady state current through it is zero. A capacitor discharges its energy when connected in a circuit having resistances.

The power absorbed by the capacitor is given by

$$p = v \cdot i = v \cdot C \cdot \frac{dv}{dt}$$

$\therefore$ Energy stored by the capacitor is

$$W = \int_o^t p dt = \int_o^t v \cdot C \cdot \frac{dv}{dt} \cdot dt$$

$$= \frac{1}{2} C v^2 \tag{2.39}$$

The energy stored by the capacitor is then $1/2Cv^2$ Joules.

$$\left[\text{Also,} \quad W = \frac{1}{2} C v^2 = \frac{1}{2} C \cdot \frac{Q^2}{C^2} = \frac{Q^2}{2C}\right] \tag{2.40}$$

W is always expressed in joules.

2.40 A 10F capacitance is charged to 5 V and is isolated. It is then connected in parallel to a 40 F capacitor. What is the decrease in total energy of the system?

Solution

$$Q_1 = C_1 V_1 = 10 \times 5 = 50 \text{ C}$$

$$W_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2} \times 10 \times (5)^2 = 125 \text{ J}$$

Next, with parallel combination of 10 F and 40 F, the equivalent capacitance of the system becomes 50 F.

$\therefore$ W_2 (final energy when both the capacitors are connected in parallel)

$$= \frac{1}{2} C V^2 = \frac{1}{2} \cdot \frac{Q^2}{C} = \frac{1}{2} \times (50)^2 \times \frac{1}{50} = 25 \text{ J}$$

Thus, the decrease in total energy of the system is (125 J – 25 J), i.e. 100 J.

2.41 A parallel plate capacitor of plate area A and plate separation d is charged to a potential difference V and then the battery is disconnected. A slab of dielectric constant ε is then inserted between the plates so as to fill the space between the plates. If Q, E and W denote respectively, the magnitude of charge on each plate, the electric field between the plates (after the slab is inserted), and the work done on the system, show that in the process of inserting the slab the work done is given by

$$W = \frac{\varepsilon_o AV^2}{2d}\left(1 - \frac{1}{\varepsilon}\right)$$

Solution

Let us assume that the capacitor retain charge Q when charged to voltage V at the initial condition. This charge will remain same even when the slab is inserted; however, the electric field intensity will reduce by a factor ε.

$$\therefore \qquad Q = CV = \varepsilon_o \frac{AV}{d}\ ;\ E \text{ (field intensity)} = \frac{V}{\varepsilon d}$$

Energy of the system before dielectric ε is inserted,

$$W_1 = \frac{1}{2}\frac{Q^2}{C} = \frac{\varepsilon_o^2 A^2 V^2}{2d^2} \times \frac{d}{\varepsilon_o A} \qquad \left[\because \text{from above } C = \frac{\varepsilon_o A}{d}\right]$$

$$= \frac{\varepsilon_o AV^2}{2d}$$

After insertion of dielectric

$$W_2 = \frac{1}{2}\frac{Q^2}{C}, \quad \text{where } C_1 = \varepsilon_o \frac{\varepsilon A}{d}$$

$$= \frac{1}{2}\frac{\varepsilon_o^2 A^2 V^2}{2d^2} \times \frac{d}{\varepsilon_o \varepsilon A} = \frac{\varepsilon_o AV^2}{2\varepsilon d}$$

$$\therefore \qquad W_1 - W_2 = \frac{\varepsilon_o AV^2}{2d}\left(1 - \frac{1}{\varepsilon}\right).$$

2.42 A capacitor of capacitance C is fully charged by a 220 V supply. It is then discharged through a small resistance embedded in a thermally insulated block of specific heat 2.5×10^2 J kg^{-1} K^{-1} and of mass 0.2 kg. If the temperature of the block rises by 1°K, find the value of C.

Solution

Energy stored in the capacitor is

$$W = \frac{1}{2}CV^2 = \frac{1}{2} \times C \times (220)^2 \text{ Joule}$$

Energy supplied as heat in the block is obtained as

$H = m\ s\ t$

where H = heat,

m = mass,

s = specific heat, and

t = temperature rise.

Here $H = 0.2 \times 2.5 \times 10^2 \times 1$ Joule = 50 Joule

In a thermally insulated system

$$W = H$$

$$\therefore \quad \frac{1}{2} \times C \times (220)^2 = 50$$

$$\therefore \quad C = \frac{50}{(220)^2} \times 2 = 2066\ \mu\text{F}.$$

2.43 An uncharged capacitor is connected to a battery. Show that half the energy supplied by the battery is lost as heat while charging the capacitor.

Solution

The charge required by the capacitor is $Q = CV$ while the work done by the battery is

$$W = Q \cdot V$$

The capacitor would store energy of $1/2CV^2$.

However, $\frac{1}{2}CV^2 \equiv \frac{1}{2}QV$.

Then the remaining energy is $\left(QV - \frac{1}{2}QV\right)$

i.e. $1/2QV$ is lost as heat. Then half the energy supplied by the battery is lost as heat.

ADDITIONAL EXAMPLES

2.44 A ring of radius r contains a charge Q distributed uniformly over its length. Find the electric field at a point on the axis of the ring at a distance x from the centre.

Solution

Let us consider a small element of the ring at point A having a charge dQ. The field at P due to this element is

$$dE = \frac{dQ}{4\pi\varepsilon_o (AP)^2}$$

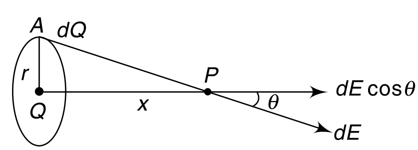


Fig. 2.52 ***A ring of radius r containing a charge Q***

The component of dE along x-axis is

$$dE \cos\theta = \frac{dQ}{4\pi\varepsilon_o (AP)^2} \times \frac{QP}{AP}$$

$$= \frac{x \cdot dQ}{4\pi\varepsilon_o (r^2 + x^2)^{3/2}}$$

$\therefore$ The net field at P is

$$E = \int dE \cos\theta = \int \frac{x\, dQ}{4\pi\varepsilon_o (r^2 + x^2)^{3/2}}$$

$$= \frac{x}{4\pi\varepsilon_o (r^2 + x^2)^{3/2}} \int dQ = \frac{xQ}{4\pi\varepsilon_o (r^2 + x^2)^{3/2}}\ \text{V/m}$$

2.45 Three charges, each equal to q, are placed at three corners of a square of side s. Find the electric field at the fourth corner.

Solution

Let the charges be placed along A, B, C (Fig. 2.53).

Now, the electric field at D, $\vec{E}_D = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

[where $\vec{E}_1$ = field at D due to A,

$\vec{E}_2$ = field at D due to C, and

$\vec{E}_3$ = field at D due to B]

$$= \frac{kq}{s^2}\hat{i} + \frac{kq}{s^2}\hat{j} + \frac{kq}{(\sqrt{2}s)^2}(\hat{i} + \hat{j})$$

$$= \frac{kq}{s^2}\left(1+\frac{1}{2}\right)\hat{i} + \frac{kq}{s^2}\left(1+\frac{1}{2}\right)\hat{j}$$

$$= \frac{3kq}{2s^2}(\hat{i} + \hat{j})$$

$$\therefore \quad |\vec{E}_D| = \frac{3}{2}\frac{kq}{s^2}\sqrt{1^2+1^2} = \frac{3\sqrt{2}}{2}\frac{kq}{s^2}, \text{ where } K = \frac{1}{4\pi\varepsilon_o}$$

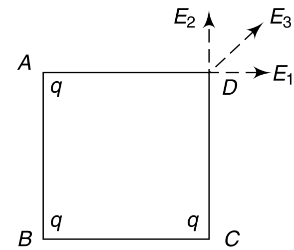


Fig. 2.53 ***A square of side s having charges q at three corners***

2.46 Six similar charges, each of value q are placed at the corners of a regular hexagonal pyramid of side l. Find the resultant electric field strength at the apex due to charges if the diagonal of the base equals slant edge of the pyramid.

Solution

With reference to Fig. 2.54, side of base = l; slant edge = $2l$.

As charge is same for all points and slant edge is also same hence magnitude of electric field due to each charge is equal to $kq/(2l)^2$ though the directions are different. Solving them into components, we find horizontal components will cancel among themselves.

Resultant electric field = 6 E cos θ

$$= 6 \times \frac{1}{4\pi\varepsilon_o} \times \frac{q}{(2l)^2} \times \frac{\sqrt{3}}{2} \quad [\because \theta = 30°]$$

$$= \frac{1}{4\pi\varepsilon_o} \times \frac{3\sqrt{3}}{4} \times \frac{q}{l^2}$$

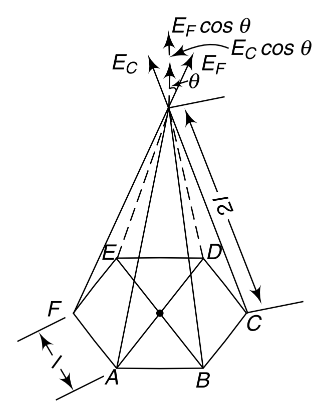


Fig. 2.54 ***Charges placed at the corners of a hexagonal pyramid***

2.47 A 1.0 μF parallel plate capacitor with air in dielectric medium is charged to 200 V at steady state. Assuming the distance between the two parallel plates to be 1.0 cm, find

(i) the electric stress on dielectric

(ii) the electric stress on plate surface and electric flux density

(iii) the charges on the plate.
(iv) if the dielectric medium of air is replaced by another dielectric medium of permittivity 4, recalculate the answers for (i) , (ii) and (iii).

Solution

(i) $E = \dfrac{V}{x}$ V/m [E = electric stress on dielectric i.e. field intensity]

$$= \frac{200}{1 \times 10^{-2}} = 20 \text{ kV/m}$$

(ii) Electric stress on plate surface will also be 20 kV/m while the flux density is given by

$$\delta = \varepsilon_o \times E = 8.854 \times 10^{-12} \times 20 \times 10^3$$
$$= 1.771 \times 10^{-7} \text{ C/sq. m.}$$

(iii) $Q = CV$; Q is the charge on the plate
$= 1 \times 10^{-6} \times 200 = 200$ μC.

(iv) If the air medium is replaced by another medium of permittivity 4, we can obtain the new values of E, δ and Q as follows:

$$C_{\text{new}} = \frac{\varepsilon_o\, \varepsilon_r\, A}{x}$$

$$= \varepsilon_r \times \frac{\varepsilon_o\, A}{x} = \varepsilon_r \times 1 \times 10^{-6}$$

[∵ original capacitance of the given capacitor with air on dielectric medium is given as 1 μF.]

∴ $C_{\text{new}} = 4 \times 1 \times 10^{-6} = 4$ μF

Also, $E_{\text{new}} = \dfrac{V}{x} = \dfrac{200}{1 \times 10^{-2}} = 20$ kV/m [distance x remains same]

$$\delta_{\text{new}} = \varepsilon_o\, \varepsilon_r\, E = 8.854 \times 10^{-12} \times 4 \times 20 \times 10^3$$
$$= 7.1 \times 10^{-7} \text{ C/m}^2$$

The new charge accumulation is

$$Q_{\text{new}} = C_{\text{new}} \times V = 4 \times 10^{-6} \times 200$$
$$= 8 \times 10^{-4} \text{ Coulomb.}$$

2.48 Two capacitors C_1 and C_2 are placed in (i) Series and (ii) Parallel. If $C_1 = 100$ μF; $C_2 = 50$ μF, find the maximum energy stored when a 220 dc supply is applied across the combination.

Solution

If C is the equivalent capacitance, for series connection of C_1 and C_2,

$$C = \frac{C_1 C_2}{C_1 + C_2} = \frac{100 \times 50}{100 + 50} = 33.33 \text{ μF.}$$

∴ Maximum energy stored is $\frac{1}{2} CV^2$, i.e. $\frac{1}{2} \times 33.333 \times 10^{-6} \times (220)^2$ i.e. 0.807 J.

When the capacitors are in parallel, $C = C_1 + C_2 = 150$ μF

$\therefore$ Maximum energy stored is

$$\frac{1}{2} \times 150 \times 10^{-6} \times (220)^2 \text{ i.e. } 3.63 \text{ J.}$$

[It may be observed here that capacitor conserves maximum energy when they are in parallel configuration.]

2.49 Find the equivalent capacitance between A and B in Fig. 2.55. Assume the capacitances are equal to each other and having a value of 2 μF each.

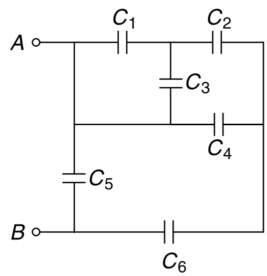


Fig. 2.55 ***Capacitance configuration of Ex. 2.49***

Solution

We can reduce the given circuit as shown in Fig. 2.55(a).

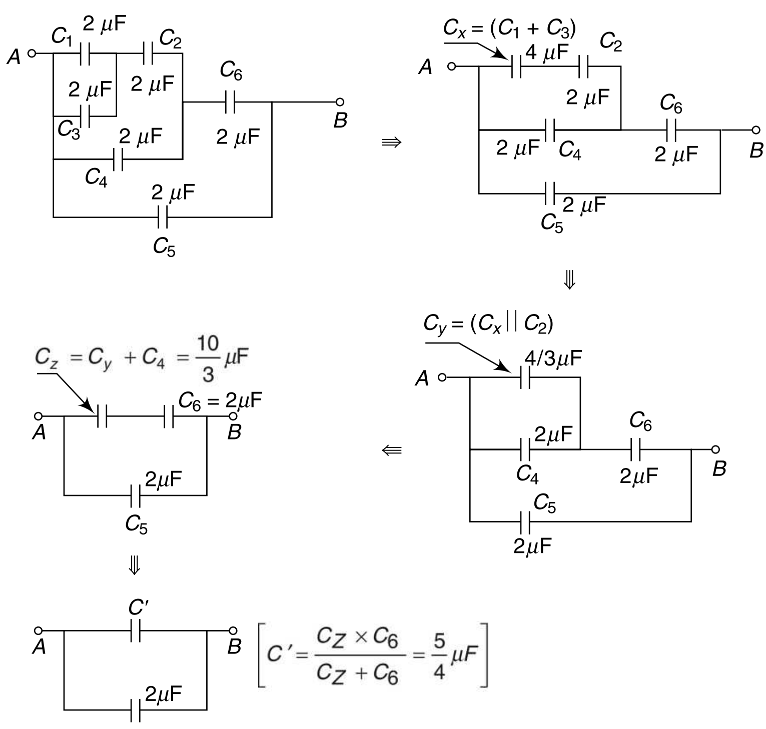


Fig. 2.55a ***Circuit reduction for Ex. 2.49***

Finally we obtain

$$C = C' \| C_5 = C' + C_5 = \frac{5}{4} + 2 = \frac{13}{4}\ \mu\text{F}$$

2.50 In Fig. 2.56, if $C_1 = C_2 = C_3 = C_4 = \ldots = C_{12} = 1$ F, and $V = 10$ V, find the charge supplied by the battery. If the charging current drawn from the battery is 10 A, how much time would the battery take to charge the capacitor cube? What is the energy stored in the capacitances of the cube?

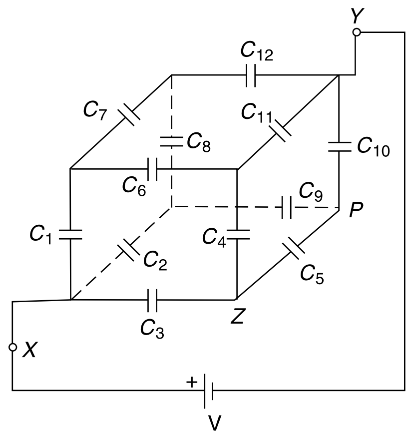


Fig. 2.56 *Cube of Ex. 2.50*

Solution

Let us assume Q be the charge entering terminal x from the battery to the cube. Obviously, capacitance C_1, C_2 and C_3 would store charges $Q/3$, $Q/3$ and $Q/3$. Since charge (Q) leaves out terminal y hence charge on capacitors C_{10}, C_{11} and C_{12} must also be $\left(\frac{Q}{3}\right)$ each. On the other hand since C_3 is connected to C_4 and C_5 at terminal z, hence C_4 and C_5 should have a total charge equal to that stored in C_3. Hence we can say since C_3 stores $\left(\frac{Q}{3}\right)$, hence C_4 and C_5 would individually have $\left(\frac{Q}{6}\right)$ each.

Following charging current path in the cube from x to y we find

$$V_{xy} = V_x - V_y = (V_x - V_z) + (V_z - V_p) + (V_p - V_y)$$

$$= V_{xz} + V_{zp} + V_{py} = \frac{Q/3}{C} + \frac{Q/6}{C} + \frac{Q/3}{C}$$

$$= \frac{5Q}{6C} \quad [\because C_1 = C_2 = C_3 = \ldots = C_{12} = C]$$

or
$$\frac{Q}{C_{\text{eq}}} = \frac{5Q}{6C}$$

$$C_{\text{eq}} = \frac{6}{5}C = \frac{6}{5}\text{F} \quad [\because C = 1\text{ F}]$$

Hence charge stored in the cube is

$$Q = C_{\text{eq}} \cdot V = \frac{6}{5} \times 10 = 12 \text{ coulomb.}$$

The battery supplies 12 coulomb of electricity. Also Q = Charging current × Time.

$$\therefore \quad \text{Time} = \frac{Q}{I_{\text{ch}}} = \frac{12}{10} = 1.2 \text{ sec.}$$

Energy stored in the capacitor cube

$$= \frac{1}{2}\frac{Q^2}{C_{\text{eq}}} = \frac{(12)^2}{2 \times 6/5} = 60 \text{ J.}$$

2.51 Find the amount of heat generated in the circuit shown in Fig. 2.57 after the switch is shifted from position 1 to position 2.

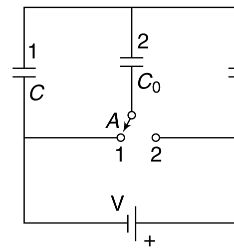


Fig. 2.57 ***Circuit of Ex. 2.51***

Solution

When the switch is in position 1, the combination has C and C_0 in parallel and C in series for which the equivalent capacitance is

$$C_{eq} = \frac{(C_0 + C)C}{C_0 + 2C}$$

The total charge on the combination is

$$Q = V \times C_{eq} = \frac{VC(C + C_0)}{2C + C_0}$$

The total charge in the three capacitors can be obtained as

$$Q_3 = VC_{eq} = \frac{VC(C + C_0)}{2C + C_0}$$

$$Q_2 = \frac{VC(C + C_0)C_0}{(2C + C_0)(C + C_0)} = \frac{VC\,C_0}{2C + C_0}$$

$$Q_1 = \frac{VC(C + C_0)C}{(2C + C_0)(C + C_0)} = \frac{VC^2}{2C + C_0}$$

When the switch is in position 2, the charge distinguish on the three capacitors is

$$Q_3' = \frac{VC^2}{2C + C_0};\ Q_2' = Q_2 \text{ and } Q_1' = \frac{VC(C + C_0)}{2C + C_0}$$

Heat produced = loss in stored electrical energy + extra energy drawn from the battery. Since the equivalent capacitance C_{eq} remain unchanged in both the positions of the key, the loss in stored energy is zero.

Hence,

$$\begin{aligned}\text{Heat produced} &= \text{Energy drawn from the battery}\\ &= V(Q_1' - Q_1)\\ &= V(Q_3 - Q_3')\\ &= V\left[\frac{VC(C + C_0)}{2C + C_0} - \frac{VC^2}{2C + C_0}\right]\\ &= \frac{V^2 C\,C_0}{2C + C_0}.\end{aligned}$$

2.52 A parallel plate capacitor has plate area of 0.1 m^2 and plate separation 0.015 cm. The dielectric medium between the plates has relative permittivity 3. The capacitors retain a charge of 1.0 μC when placed across a dc voltage source. Find the flux density, electric field strength and voltage across the plates. Assume ε_o, the permittivity space as 8.854×10^{-12} F/m.

Solution

Given:

$$A = 0.1 \text{ m}^2;\ x = 0.015 \text{ cm} = 0.015 \times 10^{-2} \text{ m}$$
$$\varepsilon_r = 3;\ Q = 1.0\ \mu\text{C}.$$
$$\varepsilon_o = 8.854 \times 10^{-12} \text{ F/m}$$

$\therefore$ for the given parallel plate capacitor,

$$C = \frac{\varepsilon_o \varepsilon A}{x} = \frac{8.854 \times 10^{-12} \times 3 \times 0.1}{0.015 \times 10^{-2}}$$
$$= 0.01771\ \mu\text{F}.$$

Flux density is obtained identical to charge density.

$$\therefore \qquad \delta = \frac{Q}{A} = \frac{1.0 \times 10^{-6}}{0.1} = 10\ \mu\text{C/m}^2.$$

Field strength (E) is obtained as

$$E = \frac{\delta}{\varepsilon_o \varepsilon_r} = \frac{10 \times 10^{-6}}{8.854 \times 10^{-12} \times 3}$$
$$= 37.65 \times 10^4 \text{ V/m}.$$

The p.d. across plates is formed as

$$V = \frac{Q}{C} = \frac{1 \times 10^{-6}}{0.01771 \times 10^{-6}} = 56.47 \text{ V}.$$

2.53 Find the charge that will flow through the battery B when switch S is closed (Fig. 2.58).

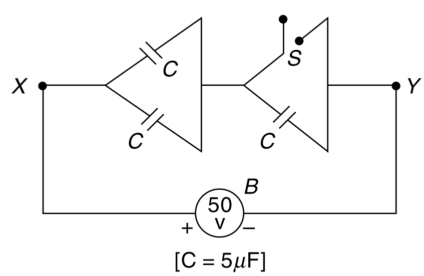


Fig. 2.58 ***Circuit of Ex. 2.53***

Solution

First we consider S is open. Equivalent capacitance across $X - Y$ is

$$C_{X-Y} = \frac{C \times 2C}{C + 2C} = \frac{2}{3}C$$

Hence Q_1 (charge retained by C_{X-Y} when S is open)

$$= C_{X-Y} \cdot V = \frac{2}{3} C \cdot V \text{ coulomb}$$

Next we consider S is closed. Equivalent capacitance C'_{X-Y} across XY is

$$C'_{X-Y} = 2C$$

$\therefore Q_2$ (charge retained by C'_{X-Y} when S is closed)

$$= C'_{X-Y} \times V = 2CV.$$

Hence we can find charge flowing through battery as $|Q_1 - Q_2|$

This becomes $|Q_1 - Q_2| = \frac{2C}{3} \cdot V - 2CV$

$$= \frac{4}{3}CV = \frac{4}{3} \times 5 \times 10^{-6} \times 50 = 333.33\ \mu C.$$

Thus 333.33 μC of charge would pass through the battery upon switching S.

2.54 A 20μF capacitor is charged to 100 V and then discharged through a resistor of 10 kΩ. Find (i) initial value of current, (ii) value of current when t = time constant, and (iii) rate at which current begins to decrease.

Solution

(i) As soon as the capacitor is switched to the discharging charging circuit having a series resistance 10 kΩ, the initial value of discharging current would be

$$I = \frac{100}{10^4} = 0.01 \text{ A}$$

(ii) While discharging

$$i = Ie^{-t/RC};$$

at t = time constant (RC),

$$i = Ie^{-t/t} = I \times \frac{1}{e}.$$

Here, $i = 0.01 \times \frac{1}{e} = 0.00368$ A

(iii) Normally the time constant is RC in the discharging circuit and hence $RC = 10^4 \times 20 \times 10^{-6} = 0.2$ sec.

$$i = I \times e^{-t/RC} = 0.01 \times e^{-\frac{t}{0.2}}$$
$$= 0.01\ e^{-5t}$$

$$\therefore \quad \frac{di}{dt} \text{ (= rate of discharging)}$$
$$= 0.01(-5)e^{-5t} = -0.05e^{-5t} \text{ A/s.}$$

2.55 Two metal plates form a parallel plate capacitor with an in-between metal plate of the same material. There are two dielectric medium K_1 and K_2 having relative permittivity ε_{r_1} and ε_{r_2} respectively as shown in figure (Fig. 2.59). If the metal plate is removed find the work done in slowly removing the plate when a p.d. of (V) volts is applied across the capacitors.

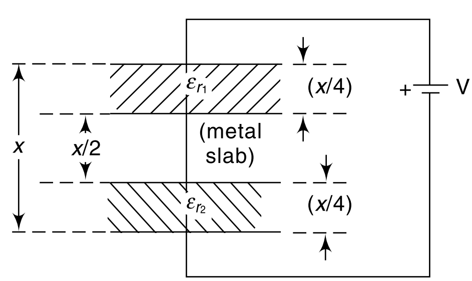


Fig. 2.59 ***A parallel plate capacitor (Ex. 2.55)***

Solution

From the given (Fig. 2.59) it is evident that the capacitor consists of two series capacitors C_1 and C_2 when C_1 is formed with ε_{r_1} while C_2 is formed with ε_{r_2}.

$\therefore \qquad C_1 = \dfrac{\varepsilon_o \varepsilon_{r_1} A}{(x/4)}; \quad C_2 = \dfrac{\varepsilon_o \varepsilon_{r_2} A}{(x/4)}$

and $\qquad C_{eq} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2}\right)^{-1} = \left(\dfrac{x}{4\varepsilon_o \varepsilon_{r_1} A} + \dfrac{x}{4\varepsilon_o \varepsilon_{r_2} A}\right)^{-1}$

$$= \frac{4\varepsilon_o A}{x}\left[\frac{\varepsilon_{r_1} \times \varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$$

$\therefore$ Energy stored $(E_1) = \dfrac{1}{2} C_{eq} V^2 = \dfrac{1}{2} \times \dfrac{4\varepsilon_o A}{x}\left[\dfrac{\varepsilon_{r_1} \times \varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right] V^2$

When the metal slab is removed, there are now three capacitors formed, the first one is C_1 as it was, the second one is with dielectric medium air C_A (as the metal slab is removed, the space between ε_{r_1} and ε_{r_2} is now air) and the third one is C_2 as it was and now, $C_A = \dfrac{\varepsilon_o A}{x/2} = \dfrac{2\varepsilon_o A}{x}$.

$\therefore \qquad C'_{eq} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_A} + \dfrac{1}{C_2}\right)^{-1}$ i.e., $\left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_A}\right)^{-1}$

$$= \left(\frac{x}{4\varepsilon_o \varepsilon_{r_1} A} + \frac{x}{4\varepsilon_o \varepsilon_{r_2} A} + \frac{x}{2\varepsilon_o A}\right)^{-1}$$

$$= \frac{4\varepsilon_o A}{x}\left[\frac{1}{\varepsilon_{r_1}} + \frac{1}{\varepsilon_{r_2}} + 2\right]^{-1}$$

$$= \frac{4\varepsilon_o A}{x}\left[\frac{\varepsilon_{r_1} + \varepsilon_{r_2} + 2\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1}\varepsilon_{r_2}}\right]^{-1}$$

$$= \frac{4\varepsilon_o A}{x}\left[\frac{\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2} + 2\varepsilon_{r_1}\varepsilon_{r_2}}\right]^{-1}$$

$\therefore$ Energy stored $(E_2) = \dfrac{1}{2} C'_{eq} V^2$

$$= \frac{1}{2} \times \frac{4\varepsilon_o A}{x}\left[\frac{\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2} + 2\varepsilon_{r_1}\varepsilon_{r_2}}\right] \cdot V^2.$$

Since work done (ΔE) is given by ($\Delta E = E_1 - E_2$), it represents the work done to remove the metal slab.

$\therefore \qquad \Delta E = E_1 - E_2 = \dfrac{1}{2} V^2 \times \dfrac{4\varepsilon_o A}{x}\left[\dfrac{\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}} - \dfrac{\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2} + 2\varepsilon_{r_1}\varepsilon_{r_2}}\right]$

$$= V^2 \times \frac{2\varepsilon_o A \varepsilon_{r_1}\varepsilon_{r_2}}{x}\left[\frac{1}{\varepsilon_{r_1} + \varepsilon_{r_2}} - \frac{1}{\varepsilon_{r_1} + \varepsilon_{r_2} + 2\varepsilon_{r_1}\varepsilon_{r_2}}\right]$$

$$= V^2 \times \frac{2\varepsilon_o A\varepsilon_{r_1}\varepsilon_{r_2}}{x}\left[\frac{2\varepsilon_{r_1}\varepsilon_{r_2}}{(\varepsilon_{r_1}+\varepsilon_{r_2})(\varepsilon_{r_1}+\varepsilon_{r_2}+2\varepsilon_{r_1}\varepsilon_{r_2})}\right]$$

$$= \frac{4\varepsilon_o AV^2\varepsilon_{r_1}^2\varepsilon_{r_2}^2}{x(\varepsilon_{r_1}+\varepsilon_{r_2})(\varepsilon_{r_1}+\varepsilon_{r_2}+2\varepsilon_{r_1}\varepsilon_{r_2})} \text{ Joules.}$$

This is the work done in removing the metal slab from the capacitor.

2.56 Three delta connected capacitors are set to form a unit as shown in Fig. 2.60. It is required to transform the delta unit to equivalent star. Find these star capacitances for equal capacitances between similar terminals in both the connections.

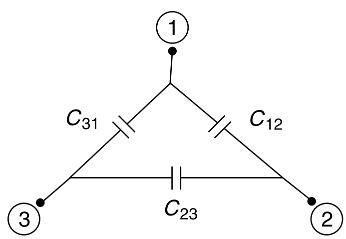


Fig. 2.60 ***Delta connected capacitors (Ex. 2.56)***

⇒

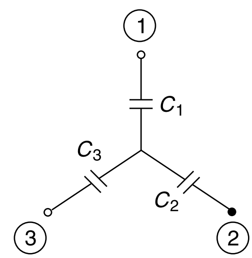


Fig. 2.60a ***Equivalent star connection of delta connected capacitors in Ex. 2.56***

Solution

Figure 2.60a represent the equivalent star capacitances provided net capacitances between terminals ① – ②, ② – ③, ③ – ① remain same for both the configurations. Capacitances between ① and ② in star connection is $\left(\frac{C_1 C_2}{C_1 + C_2}\right)$ while that between ① and ② in delta connection is $\left(C_{12} + \frac{C_{23} C_{31}}{C_{23} + C_{31}}\right)$.

For equal capacitance between similar terminals,

$$\frac{C_1 C_2}{C_1 + C_2} = \frac{C_{23} C_{31}}{C_{23} + C_{31}} + C_{12} = \frac{C_{23} C_{31} + C_{12} C_{23} + C_{31} C_{12}}{C_{23} + C_{31}}$$

or
$$\frac{C_1 + C_2}{C_1 C_2} = \frac{C_{23} + C_{31}}{C_{23} C_{31} + C_{12} C_{23} + C_{31} C_{12}} = \frac{C_{23} + C_{31}}{\Delta} \qquad \text{(i)}$$

where $\Delta = C_{12}\, C_{31} + C_{23}\, C_{31} + C_{12}\, C_{23}$

Similarly, capacitance between ② and ③ in star connection is $\left(\frac{C_2 C_3}{C_2 + C_3}\right)$ while that in delta connection is

$$\frac{C_{31} C_{12}}{C_{31} + C_{12}} + C_{23}$$

By the same reasoning,

$$\frac{C_2 C_3}{C_2 + C_3} = C_{23} + \frac{C_{31} C_{12}}{C_{31} + C_{12}}$$

or $$\frac{C_2 + C_3}{C_2 C_3} = \frac{C_{31} + C_{12}}{\Delta} \quad \text{(ii)}$$

Also, capacitance between ③ and ① in star connection is $\dfrac{C_3 C_1}{C_3 + C_1}$ and that between ③ and ① in delta connection is $\dfrac{C_{12} C_{23}}{C_{12} + C_{23}} + C_{31}$.

$\because$ $$\frac{C_3 C_1}{C_3 + C_1} = \frac{C_{12} C_{23}}{C_{12} + C_{23}} + C_{31}\text{, we can write}$$

$$\frac{C_3 + C_1}{C_3 C_1} = \frac{C_{12} + C_{23}}{\Delta} \quad \text{(iii)}$$

Adding (i) and (ii) we have

$$\frac{C_1 + C_2}{C_1 C_2} + \frac{C_2 + C_3}{C_2 C_3} = \frac{C_{23} + C_{31}}{\Delta} + \frac{C_{31} + C_{12}}{\Delta}$$

or, $$\frac{C_1 C_3 + C_2 C_3 + C_1 C_2 + C_1 C_3}{C_1 C_2 C_3} = \frac{C_{23} + C_{31} + C_{31} + C_{12}}{\Delta} \quad \text{(iv)}$$

Subtracting (iii) from (iv) we have

$$\frac{C_1 C_3 + C_2 C_3 + C_1 C_2 + C_1 C_3 - C_2 C_3 - C_2 C_1}{C_1 C_2 C_3}$$

$$= \frac{C_{23} + C_{31} + C_{31} + C_{12} - C_{12} - C_{23}}{\Delta}$$

or $$\frac{2 C_1 C_3}{C_1 C_2 C_3} = \frac{2 C_{31}}{\Delta}$$

or $$\frac{1}{C_2} = \frac{C_{31}}{\Delta}, \quad C_2 = \frac{\Delta}{C_{31}}$$

$\therefore$ $$C_2 = \frac{C_{12} C_{31} + C_{23} C_{12} + C_{31} C_{23}}{C_{31}} \quad \text{(iva)}$$

$$= C_{12} + C_{23} + \frac{C_{23} C_{12}}{C_{31}} \quad \text{(ivb)}$$

Similarly,

$$C_1 = C_{31} + C_{12} + \frac{C_{31} C_{12}}{C_{23}} \quad \text{(ivc)}$$

and $$C_3 = C_{23} + C_{31} + \frac{C_{23} C_{31}}{C_{12}} \quad \text{(ivd)}$$

2.57 Three star connected capacitances C_1, C_2 and C_3 are to be transformed to delta. Show that for equal capacitances between similar terminals in both connections,

$$C_{12} = \frac{C_1 C_2}{C_1 + C_2 + C_3}$$

$$C_{23} = \frac{C_2 C_3}{C_1 + C_2 + C_3}$$

$$C_{31} = \frac{C_3 C_1}{C_1 + C_2 + C_3}$$

Solution

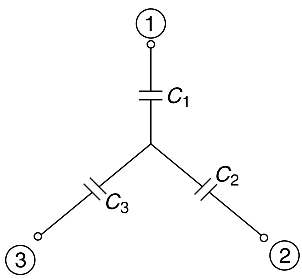


Fig. 2.61 ***Star connected capacitors (Ex. 2.57)***

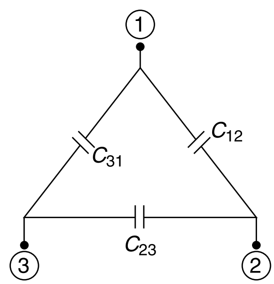


Fig. 2.61a ***Equivalent delta connected capacitors of Ex. 2.57***

With reference to Fig. 2.61 and Fig. 2.61a we have found out in the previous example that for capacitance between identical terminals being same for both star and delta, delta capacitances can be successfully converted to star where

$$C_1 = C_{12} + C_{31} + \frac{C_{12}C_{31}}{C_{23}} = \frac{C_{12}C_{23} + C_{23}C_{31} + C_{31}C_{12}}{C_{23}}$$

$$= \frac{\Delta}{C_{23}} \qquad \text{(i)}$$

$$C_2 = C_{23} + C_{12} + \frac{C_{23}C_{12}}{C_{31}} = \frac{\Delta}{C_{31}} \qquad \text{(ii)}$$

$$C_3 = C_{31} + C_{23} + \frac{C_{31}C_{23}}{C_{12}} = \frac{\Delta}{C_{12}} \qquad \text{(iii)}$$

Multiplying equations (i) and (ii), (iii) and (iv) and (v) and (vi), we have

$$C_1\, C_2 = \frac{\Delta^2}{C_{23}C_{31}} \qquad \text{(iv)}$$

$$C_2\, C_3 = \frac{\Delta^2}{C_{31}C_{12}} \qquad \text{(v)}$$

$$C_3\, C_1 = \frac{\Delta^2}{C_{12}C_{23}} \qquad \text{(vi)}$$

Inverting and adding equations (iv), (v) and (vi),

$$\frac{1}{C_1 C_2} + \frac{1}{C_2 C_3} + \frac{1}{C_3 C_1} = \frac{C_{12} C_{23} + C_{31} C_{12} + C_{23} C_{31}}{\Delta^2}$$

or $$\frac{C_3 + C_1 + C_2}{C_1 C_2 C_3} = \frac{\Delta}{\Delta^2} = \frac{1}{\Delta}$$

or $$\frac{C_1 C_2 C_3}{C_1 + C_2 + C_3} = \Delta$$

or $$\frac{\Delta}{C_3} = \frac{C_1 C_2}{C_1 + C_2 + C_3} \qquad \text{(vii)}$$

However, we have proved earlier in the preceding example

$$C_3 = C_{23} + C_{31} + \frac{C_{23} C_{31}}{C_{12}} = \frac{C_{23} C_{12} + C_{31} C_{12} + C_{23} C_{31}}{C_{12}}$$

$$= \frac{\Delta}{C_{12}}$$

∴ From equation (iii), using $C_3 = \frac{\Delta}{C_{12}}$, we can write

$$\frac{\Delta}{C_3} = C_{12} = \frac{C_1 C_2}{C_1 + C_2 + C_3}$$

Similarly, $$C_{23} = \frac{C_2 C_3}{C_1 + C_2 + C_3}$$

and $$C_{31} = \frac{C_3 C_1}{C_1 + C_2 + C_3}$$

2.58 In the circuit of Fig. 2.62, switch S is switched on at $t = 0$ and kept at on position for long time. At steady state, the switch is suddenly opened. Instantaneously, the dielectric mediums of the capacitors are replaced by another dielectric medium having dielectric constant of 0.5. Find the ratio of total electrostatic energy in both the capacitors before and after opening of the switch at steady state.

Solution

Condition 1: Switch closed, steady state prevails. Both the capacitors C_1 and C_2 are at same potential and the total electrostatic energy of both C_1 and C_2 are given by

$$W_1 = \frac{1}{2}CV^2 + \frac{1}{2}CV^2 = CV^2.$$

Condition 2: Switch opened and dielectric medium replaced.

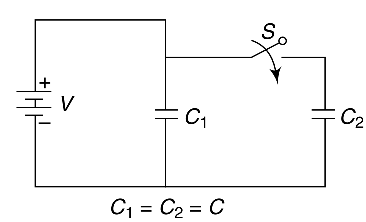


Fig. 2.62 ***Circuit of Ex. 2.58***

This time the capacitance of each of the capacitors becomes 0.5C. Let v be the new potential of C_2 and C_2 is isolated as the switch is already opened. However it will retain the same charge before and after switching and hence we can write

$$CV \text{ (charge before } S \text{ is opened)}$$
$$= 0.5\, CV' \text{(charge after } S \text{ is opened)}$$

i.e., $V' = 2V$

The new electrostatic energy of the system is now the summation of electrostatic energies of each of the capacitors.

$$\therefore \quad W' = \frac{1}{2} \cdot (0.5C) \cdot V^2 + \frac{1}{2} \cdot (0.5C) \cdot V'^2$$
$$= \frac{1}{2} \times 0.5C \times V^2 + \frac{1}{2} \times 0.5C \times (2V)^2$$
$$= 1.25\; CV^2$$

$$\therefore \quad \frac{W}{W'} = \frac{CV^2}{1.25\, CV^2} = \frac{1}{1.25}$$

Hence the required ratio is 1 : 1.25.

2.59 A 8 μF capacitor is connected in series with a 0.5 MΩ resistor across a 200 V dc supply. Calculate (i) the time constant during charging of the capacitor, (ii) the initial charging current, (iii) the time taken for the p.d. across the capacitor to grow to 160 V and (iv) the current and the p.d. across the capacitor in 4 sec after it is connected to the supply.

Solution

(i) Time constant = $RC = 0.5 \times 10^6 \times 8 \times 10^{-6} = 4.0$ sec.

(ii) $I = \dfrac{V}{R} = \dfrac{200}{0.5 \times 10^6} = 0.4$ mA

[capacitor acts as short circuit as soon as voltage is applied and hence initial charging current is (V/R)]

(iii) $\because \quad v_C = V\left(1 - e^{-t/RC}\right)$, [refer text]

Here, $\quad 160 = 200\,(1 - e^{-t/4})$

i.e. $\quad \dfrac{160}{200} = 1 - e^{-t/4}$ or, $0.2 = e^{-t/4}$

or, $\quad \log_{10} 0.2 = -\dfrac{t}{4} \cdot \log_{10} e$

$\therefore \quad t = 6.42$ sec.

(iv)
$$i = Ie^{-t/RC}$$
$$= (0.4 \times 10^{-3})e^{-t/4}$$
$$= (0.4 \times 10^{-3})2.718$$
$$= 0.147 \text{ mA}$$
$$v = V(1 - e^{-t/RC})$$
$$= 200(1 - e^{-4/4})$$
$$= 126.40 \text{ V.}$$

EXERCISES

1. State and explain Coulomb's law in Electrostatics and hence define "Coulomb", the unit for electric charge.
2. What is permittivity? What do you mean by relative permittivity of a medium? Why it does not have any unit?
3. Define electric potential and potential difference with their units. Find an expression for potential at a point within an electric field. What is equipotential surface?
4. What is meant by electric field intensity? Discuss the various factors upon which it depends.
5. Find the expression of electric field intensity and electric potential of an isolated point charge in vector form.
6. Define potential gradient. What is its unit? Why do we say electric field intensity and potential gradient both are expressed in volt/m?
7. Derive an expression of potential energy in an electric field.
8. Find a relationship between electric field strength and electric potential.
9. State Gauss' Law and derive from Coulomb's law.
10. What do you mean by electric dipole? Obtain an expression of electric field and potential due to a dipole at an axial point.
11. (a) Define electric capacitance and derive an expression for the capacitance of a parallel plate capacitor.
    (b) Discuss the various factors upon which the value of capacitance of parallel plate capacitor depends.
12. State the factors on which the capacitance of a condenser would depend.
13. Derive expression for the equivalent capacitance for a number of capacitors connected in (i) series (ii) parallel.
14. Derive the expression of capacitance of a parallel plate capacitor with (i) uniform dielectric medium (ii) compound dielectric medium.
15. How would you find capacitance of multiplate capacitors?
16. Find the capacitors of an isolated sphere.
17. Derive the expression to find capacitance of concentric spheres.
18. How do you find the capacitance of a parallel plate capacitor if a metal plate (uncharged) is introduced within the parallel plates?
19. Derive an expression for the energy stored in a condenser charged to a potential.
20. Explain charging and discharging of a capacitor alongwith necessary derivation when the capacitor is connected across a dc voltage source and when discharging from steady state with resistance in series in both the cases.
21. What is the potential at $x = 0$ due to these charges? x = distance in m.

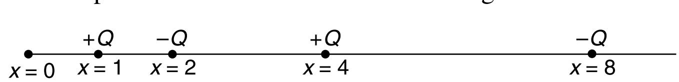


Fig. 2.63

$$\left(Ans: V = \frac{Q}{6\pi\varepsilon_o} \right)$$

[Hint: Potential V_1 at $x = 0$ due to +ve charges is given by

$$V_1 = \frac{Q}{4\pi\varepsilon_o(1)} + \frac{Q}{4\pi\varepsilon_o(4)} + \frac{Q}{4\pi\varepsilon_o(16)} + \ldots$$

$$= \frac{Q}{4\pi\varepsilon_o}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right) = \frac{Q}{4\pi\varepsilon_o}\left[\frac{1}{1-1/4}\right]$$

$$= \frac{Q}{3\pi\varepsilon_o}$$

Potential V_2 at $x = 0$ due to –ve charges is given by

$$V_2 = \frac{-Q}{4\pi\varepsilon_o(2)} + \frac{-Q}{4\pi\varepsilon_o(8)} + \frac{-Q}{4\pi\varepsilon_o(32)} + \ldots$$

$$= \frac{-Q}{4\pi\varepsilon_o}\left[1 + \frac{1}{4} + \frac{1}{16} + \ldots\right] = \frac{-Q}{6\pi\varepsilon_o}$$

$\therefore$ $$V = V_1 - V_2 = \frac{Q}{6\pi\varepsilon_o}$$]

22. Find the potential and field intensity at $x = 0$ due to these set of charges; x represents the distance from origin in x-axis. Q is magnitude of charge.

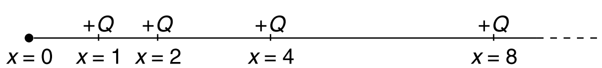


Fig. 2.64

$$\left(Ans: V = \frac{Q}{2\pi\varepsilon_o}; \frac{Q}{3\pi\varepsilon_o}\right)$$

[Hint:

$$V = \frac{Q}{4\pi\varepsilon_o}\left[1 + \frac{1}{2} + \frac{1}{4} + \ldots\right] = \frac{Q}{4\pi\varepsilon_o} \times 2 = \frac{Q}{2\pi\varepsilon_o}$$

$$E = \frac{Q}{4\pi\varepsilon_o x^2} = \frac{Q}{4\pi\varepsilon_o}\left[1 + \frac{1}{2^2} + \frac{1}{4^2} + \ldots\right]$$

$$= \frac{Q}{4\pi\varepsilon_o}\left(1 + \frac{1}{4} + \frac{1}{16} + \ldots\right) = \frac{4Q}{3} \times \frac{1}{4\pi\varepsilon_o} = \frac{Q}{3\pi\varepsilon_o}$$]

23. Three point charges $4q$, Q and q are placed in a straight line of length l at point of distance 0, $l/2$ and l from origin respectively. The net force on charge q is zero. What is the value of Q?

[*Ans:* $Q = -q$]

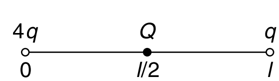


Fig. 2.65

[Hint: With ref. to Fig. 2.65, the net force on Q is zero when

$$\frac{4q \times q}{4\pi\varepsilon_o \times (l)^2} + \frac{Q \times q}{4\pi\varepsilon_o (l/2)^2} = 0$$

i.e. $4q^2 + 4Qq = 0$

i.e. $Q = -q$]

24. Eight charged drops of a fluid carry a charge of 10^{-4} μC each. Each drop has a diameter of 2 mm. If they merge together to form a single drop, find the potential of the merged big drop. (*Ans:* 3.6 kV)

[Hint: Total charge of 8 drops = 8×10^{-4} μC (=q)

Potential due to a charge 'q' is given by.

$V = \frac{1}{4\pi\varepsilon_o} \times \frac{q}{R}$, where R is the radius of the big drop and q is its charge.

Since the single big drop is equivalent to eight drops, assuming all drops to be perfect spheres,

$$8 \times \frac{4}{3}\pi r^3 \times \rho = \frac{4}{3}\pi R^3 \times \rho,$$

ρ being the density of the fluid and $\frac{4}{3}\pi r^3$ is the volume of each small drop.

$\therefore$ We have $8r^3 = R^3$

or $R = 2r$.

$$\therefore \qquad V = \frac{1}{4\pi \times 8.854 \times 10^{-12}} \times \frac{8 \times 10^{-4} \times 10^{-6}}{2 \times 10^{-3}} = 3600 \text{ V}$$]

25. Show that when a dielectric slab of thickness t and permittivity ε_r is inserted between two fixed charges Q_1 and Q_2, the force of repulsion between them is given by

$$F = \frac{1}{4\pi\varepsilon_o} \cdot \frac{Q_1 Q_2}{(d - t + \sqrt{\varepsilon_r}\, t)^2}$$

and hence find the dielectric constant of the slab, if on interposing another slab of same material of thickness (d/2) the force of repulsion reduceds in the ratio (9/4). Assume the distance between the fixed charges to be d.

(*Ans:* $\varepsilon_r = 4$)

[Hint: $F = \frac{1}{4\pi\varepsilon_o \varepsilon_r} \times \frac{Q_1 Q_2}{d^2}$

Let us assume that when the same two charges are placed d' apart in air, same force of repulsion arises between them.

$$\therefore \qquad F = \frac{1}{4\pi\varepsilon_o} \times \frac{Q_1 Q_2}{d'^2}$$

Then, $\varepsilon_r d^2 = d'^2$ i.e., $d' = \sqrt{\varepsilon_r} \cdot d$

Then, d of medium is equivalent to d' of air

i.e., $\sqrt{\varepsilon_r} \cdot d$ in air.

$\therefore$ Effective air separation is $(d - t + \sqrt{\varepsilon_r} \cdot t)$

$$F = \frac{1}{4\pi\varepsilon_o} \times \frac{Q_1 Q_2}{(d - t + \sqrt{\varepsilon_r} \cdot t)^2}$$

Substituting $t = d/2$, $F' = \dfrac{1}{4\pi\varepsilon_o} \times \dfrac{Q_1 Q_2}{\left(d - \dfrac{d}{2} + \sqrt{\varepsilon_r} \cdot \dfrac{d}{2}\right)^2}$

$$= \frac{1}{4\pi\varepsilon_o} \times \frac{4 Q_1 Q_2}{(1 + \sqrt{\varepsilon_r})^2 \, d^2}$$

Force in air is $F = \dfrac{1}{4\pi\varepsilon_o} \times \dfrac{Q_1 Q_2}{d^2}$

$\therefore$ $F'/F = \dfrac{4}{(1 + \sqrt{\varepsilon_r})^2}$. But it is given $F'/F = 4/9$

$\therefore$ $\dfrac{4}{(1 + \sqrt{\varepsilon_r})^2} = \dfrac{4}{9} \therefore \sqrt{\varepsilon_r} = 2$]

26. If the capacitance between two successive plates is 0.5C (Fig. 2.66), find the capacitance of the equivalent system between points P and Q.

(*Ans:* 1.5 C)

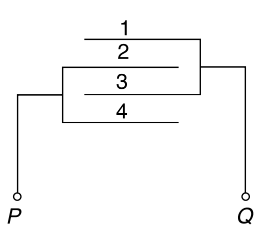


Fig. 2.66

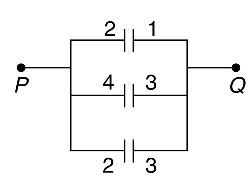


Fig. 2.66a

[Hint: The four plates form three capacitors (Fig. 2.66a)

$\therefore$ $C_{eq} = 0.5C + 0.5C + 0.5C = 1.5C$]

27. Two capacitors are once connected in parallel and then in series. If the equivalent capacitance in the two cases be 10F and 5F respectively, find the capacitance of each of the capacitors.

[Hint: $C_1 + C_2 = 10$; $\dfrac{C_1 C_2}{C_1 + C_2} = 2.1$

Solving, $C_1 = 7\text{F}$, $C_2 = 3$ F]

28. Two dielectrics of equal size are introduced inside a parallel plate capacitor as shown in Fig. 2.67. How does the effective capacitance charge?

(*Ans:* Increase by $\frac{\varepsilon_1 + \varepsilon_2}{2}$)

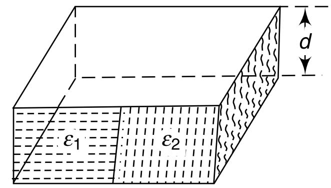


Fig. 2.67

[Hint: With introduction of two dielectrics two parallel capacitors are formed.

$$C_{eq} = \frac{\varepsilon_o \varepsilon_1 (A/2)}{d} + \frac{\varepsilon_o \varepsilon_2 (A/2)}{d} = \frac{\varepsilon_o A}{d}\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right)$$

while without these dielectrics it was $\frac{\varepsilon_o A}{d}$, assuming the area of plates to be A.

$$\therefore \quad \text{Ratio} = \frac{\frac{\varepsilon_o A}{d}\left(\frac{\varepsilon_1 + \varepsilon_2}{2}\right)}{\frac{\varepsilon_o A}{d}} = \frac{\frac{\varepsilon_1 + \varepsilon_2}{2}}{1}$$

$\because$ ε_1 and ε_2 are both more than 1, hence the new capacitance is increased by $\frac{\varepsilon_1 + \varepsilon_2}{2}$ times].

29. Find the charge drawn from the battery at steady state when K is closed. Assume $C_1 = C_2 = C$. (Fig. 2.68) $\left(Ans: \frac{1}{2} CV\right)$

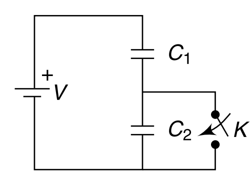


Fig. 2.68

[Hint: K open:

$$Q = C_{eq} \cdot V = \frac{C}{2} \cdot V$$

Next K closed; C_2 gets shorted. Charge is now drawn by C_1 only.

$$Q' = \text{CV}$$

$$Q' - Q = \frac{1}{2} CV$$

This is the amount of charge drawn with K closed].

30. If each of the capacitances is C, find the equivalent capacitance between A and B (Fig. 2.69).

(*Ans:* $2C$)

[Hint: The given circuit can be redrawn as shown below:

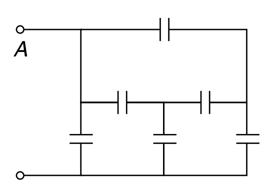


Fig. 2.69

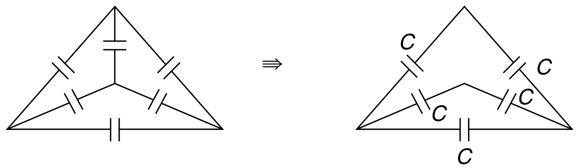


Fig. 2.69a

One capacitor may be deleted from star as no charge will flow through it since p.d. will be same across it. This simplifies the figure and we find net capacitance 2C. The problem can also be solved using ($Y - \Delta$) conversion].

31. A capacitor is formed by two parallel metal plates of area 5 cm^2 separated by a distance of 1.5 cm in air medium. It is then connected to a 1000 V dc supply. A flat sheet of a dielectric material is now introduced in the capacitor and pasted to the +ve plate. The sheet has a permittivity of 3 and thickness of 0.5 cm. What is the new capacitance? What is the electric stress developed in air? (*Ans:* 37.84×10^{-14} F; 85.5 V/m)
[Hint: Ref. Fig. 2.70]

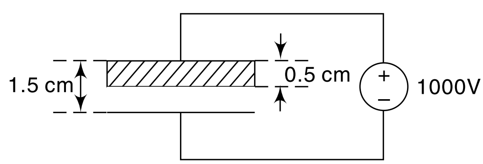


Fig. 2.70

$$C_{\text{new}} = \frac{\varepsilon_o A}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2}} \text{ Farad}$$

$$= \frac{\varepsilon_o \times 5 \times 10^{-4}}{\dfrac{0.5 \times 10^{-2}}{3} + \dfrac{1 \times 10^{-2}}{1}}$$

$$= \varepsilon_o \times 4.274 \times 10^{-2}$$

$$= 37.84 \times 10^{-14} \text{ F}$$

$\because$ $Q = CV$, here

$$Q = 37.84 \times 10^{-14} \times 1000 = 37.84 \times 10^{-11} \text{ C}$$

$$\delta = \frac{Q}{A} = \frac{37.84 \times 10^{-11}}{5 \times 10^{-4}} = 7.57 \times 10^{-7} \text{ C/m}^2$$

$\therefore$ $$E = \frac{\delta}{\varepsilon_o} = \frac{7.57 \times 10^{-7}}{8.854 \times 10^{-12}} = 85.5 \text{ V/m}].$$

32. Two plates of a capacitor of capacitance C are given charges q_1 and q_2. The capacitor is connected across a resistance r (Fig. 2.71). Find the charges on the plate after time t.

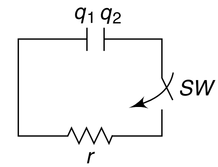


Fig. 2.71

$$\left(Ans: Q\,(\text{left plate}) = \frac{q_1 + q_2}{2} + \left(\frac{q_1 - q_2}{2}\right) \cdot e^{-t/RC} \right.$$

$$\left. Q\,(\text{right plate}) = \frac{q_1 + q_2}{2} - \left(\frac{q_1 - q_2}{2}\right) \cdot e^{-t/RC} \right)$$

[Hint: Ref. Fig. 2.71a for charge distribution diagram before closing of switch.

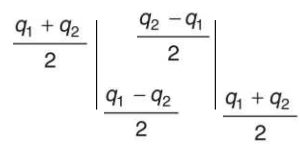


Fig. 2.71a

Initially the p.d. is $\frac{q_1 - q_2}{2C}$. During discharging, the charges on the outer surfaces would not change as the p.d. is independent of charges in the outer system. Charge on the inner surface decreases as $Q = \left(\frac{q_1 - q_2}{2}\right) \cdot e^{-t/RC}$

$$\therefore \quad Q\ (\text{left plate}) = C \cdot \frac{q_1 - q_2}{2C} + \frac{q_1 - q_2}{2} \cdot e^{-t/RC}$$

$$= \frac{q_1 - q_2}{2} + \frac{q_1 - q_2}{2} \cdot e^{-t/RC}$$

Similarly for right plate it is $\frac{q_1 + q_2}{2} - \frac{q_1 - q_2}{2} \cdot e^{-t/RC}$]

33. A dc voltage of 100 V is connected to a capacitor of 0.1F. The source is removed when steady state is reached and the charged capacitor is connected to a second uncharged capacitor. If the charge is equally distributed on these two capacitors, find the total energy stored in these two capacitors. Find the ratio of final to initial energy. (*Ans:* 250 J; 1 : 2)

[Hint: Initial stored energy $E = \frac{1}{2} \times 0.1 \times 100^2 = 500$ J. Removing the voltage source this capacitor is connected to another 0.01F and charge distribution is same for both. Hence final voltage is 100/2, i.e. 50 V.

$$\text{Final stored energy} = \frac{1}{2} \times 0.1 \times 50^2 + \frac{1}{2} \times 0.1 \times 50^2$$

$$= 250 \text{ J.}$$

$\therefore$ Ratio of energy = 1 : 2.]

34. A 6 μF capacitor is charged to 100 V and another 10 μF capacitor is charged to 200 V. They are then connected across each other. Find the p.d. across them and calculate the heat produced. (*Ans:* 162.5 V; 0.0188 J)

[Hint: $q_1 = C_1V_1 = 6 \times 10^{-6} \times 100 = 600\ \mu C$

$q_2 = C_2V_2 = 10 \times 10^{-6} \times 200 = 2000\ \mu C$

After connecting one across the other, the common p.d. is V across them.

$\therefore\ V = \dfrac{q_1 + q_2}{C_1 + C_2} = \dfrac{q_1' + q_2'}{C_1 + C_2} = \dfrac{q_1'}{C_1} = \dfrac{q_2'}{C_2}$; q_1' and q_2' are new charge distributions

i.e. $$V = \frac{600 + 2000}{6 + 10} = 162.5\ \text{V}$$

$$W_1 = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 = \frac{1}{2} \times 6 \times 10^{-6} \times 100^2 + \frac{1}{2} \times 10 \times 10^{-6} \times 200^2 = 23 \times 10^{-2}\ \text{J}$$

$$W_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}(16) \times 10^{-6} \times 162.5^2 = 21.12 \times 10^{-2}\ \text{J}]$$

35. A parallel plate capacitor has 30 plates each of 0.1 m^2 area, the separation between them being 0.2 cm. If the dielectric constant is 3, find the capacitance and the capacitive energy stored if the capacitance stores 10 μC.

(*Ans:* 385.15×10^{-10}F; 0.1298 J)

[Hint: $$C = \frac{(n-1)\,\varepsilon_o\,\varepsilon_r\,A}{d}$$

$$= \frac{(30-1) \times 8.854 \times 10^{-12} \times 0.1 \times 3}{0.2 \times 10^{-2}} = 385.15 \times 10^{-10}\ \text{F}$$

$$\text{Energy stored} = \frac{Q^2}{2C} = \frac{(10 \times 10^{-6})^2}{2 \times 385.15 \times 10^{-10}} = 0.1298\ \text{J}$$

36. A 1μF and a 2μF capacitors are connected in series across a 1200 V supply.
   (i) find the charge on each capacitor and voltage across each capacitor,
   (ii) the charged capacitors are disconnected from the supply and are now connected in parallel with terminals of like sign together. Find the final charge on each capacitor and the volatage across each.

(*Ans:* 800 μC; 800 V across 1 μF and 400 V across 2 μF; In parallel, charges are $\dfrac{1600}{3}$ μC across 1 μF and $\dfrac{3200}{3}$ μC across 2 μF. Final voltage = (1600/3) Volts)

[Hint: $Q = \dfrac{2}{2+1} \times 1200 \times 10^{-6} = 800\ \mu C$

V (across 1 μF) $= \dfrac{800}{1} = 800$ V

$$V \text{ (across 2 μF)} = \frac{800}{2} = 400 \text{ V}$$

$$\text{When in parallel, } V = \frac{\text{net charge}}{\text{net capacitance}} = \frac{1600 \times 10^{-6}}{3 \times 10^{-6}} = \frac{1600}{3} \text{ V}$$

$$\therefore \; Q_f \text{ (across 1 μF)} = 1 \text{ μF} \times \frac{1600}{3} = \frac{1600}{3} \text{ μC}$$

$$Q_f \text{ (across 2 μF)} = 2 \text{ μF} \times \frac{1600}{3} = \frac{3200}{3} \text{ μC]}$$

37. A 100 μF capacitor is charged to a p.d. of 100 V. It is then connected to an uncharged capacitor of 20 μF. What will be the new p.d. across the 100 μF capacitor? (*Ans:* 83.33 V)

[Hint: $Q = CV = 100 \times 10^{-6} \times 100 = 10^{-2}$ C

$$V' = \frac{Q}{C_{eq}} = \frac{10^{-2}}{(100+20)10^{-6}} = 83.33 \text{ V]}$$

38. A circuit is displayed in Fig. 2.72. Find
   (i) the charge across each capacitor,
   (ii) the potential difference across each capacitor,
   (iii) the stored energy of each capacitor.

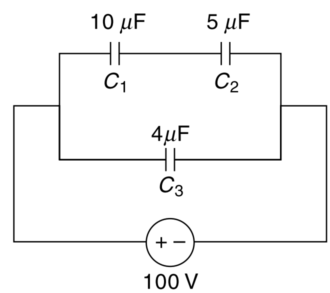


Fig. 2.72

$(Ans: Q_{C_1} = 333.33 \text{ μC} = Q_{C_2},$
$Q_{C_3} = 400 \text{ μC}$

$V_{C_1} = 33.33$ V; $V_{C_2} = 66.66$ V; $V_{C_3} = 100$ V

$W_{C_1} = 5.5 \times 10^{-3}$ J; $W_{C_2} = 10.9 \times 10^{-3}$ J

$W_{C_3} = 2 \times 10^{-2}$ J).

[Hint: $$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} + C_3 = \frac{22}{3} \text{ μF}$$

$$Q_{C_1} = \frac{C_1 C_2}{C_1 + C_2} \times 100 = \frac{1000}{3} \text{ μC}$$

$$Q_{C_2} = \frac{C_1 C_2}{C_1 + C_2} \times 100 = \frac{1000}{3} \text{ μC}$$

$$Q_{C_3} = 4 \times 100 = 400 \text{ μC}$$

(all capacitances are expressed in μF)

$$V_{C_3} = 100 \text{ V}; \; V_{C_1} = \frac{1000/3}{10} = 33.33 \text{ V}$$

$$V_{C_2} = \frac{1000/3}{5} = 66.66 \text{ V}$$

$$W_{C_1} = \frac{1}{2}C_1 \times \left(\frac{100}{3}\right)^2 = 5.5 \times 10^{-3} \text{ J}$$

$$W_{C_2} = \frac{1}{2}C_2 \times (66.66)^2 = 10.9 \times 10^{-3} \text{ J}$$

$$W_{C_3} = \frac{1}{2} \times C_3 \times 100^2 = 2 \times 10^{-2} \text{ J}$$

39. A 40 μF capacitor is charged to a potential difference of 400 V and then discharged through a 100 kΩ resistor. Derive an expression representing the discharge current. How much energy will be dissipated in the resistor over a period of 4s following the initiation of discharge. [*Ans:* 2.767 J]

[Hint: $\lambda = RC = 100 \times 10^3 \times 40 \times 10^{-6} = 4$

$$v_c = 400\ e^{-t/4}$$

$$i_c = C\frac{dv_c}{dt} = 40 \times 10^{-6} \times 400\left(-\frac{1}{4}\right)e^{-t/4}$$
$$= 0.004\ e^{-t/4}$$

Initial stored energy $= \frac{1}{2} \times 40 \times 10^{-6} \times (400)^2$ J = 3.2 J

After 4s $v_c = 400\ e^{-4/4} = 147.15$ V

Stored energy $= \frac{1}{2} \times 40 \times 10^{-6}(147.15)^2$ J = 0.433 J

Energy dissipated = 3.2 – 0.433 = 2.767 J]

40. A capacitor is constructed from two square metal plates each of side 120 mm. The plates are separated by a dielectric of thickness 2 mm and relative permittivity 5. Calculate the capacitance. If the electric field strength in the dielectric is 12.5 kV/mm calculate the total charge on each plate.

[*Ans:* C = 318.7 μF Q = 7.968 C]

[Hint: $$C = \frac{\varepsilon_o \varepsilon_r A}{d} = \frac{8.854 \times 10^{-12} \times 5 \times 0.12 \times 0.12}{2 \times 10^{-3}} \text{ F}$$

$$= 0.3187 \times 10^{-9} \text{ F} = 318.7\ \mu\text{F}$$

$$Q = CV = CEd = 318.7 \times 10^{-6} \times 12.5 \times 10^3 \times 2$$
$$= 7.9675 \text{ C]}$$

41. A parallel plate capacitor is formed using three dielectric substances having permittivities ε_1, ε_2 and ε_3 (Fig. 2.73). If the plate area is 'a', find the equivalent capacitance between A and B.

$$\left(Ans: \frac{2\varepsilon_o \varepsilon_3 A}{d}\left(\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 + \varepsilon_2 + 2\varepsilon_3}\right)\right)$$

[Hint: Ref. equivalent circuit Fig. 2.73.

$$C_3 = \frac{\varepsilon_o \varepsilon_3 A}{d/2}\ ;\ C_2 = \frac{\varepsilon_o \varepsilon_2 (A/2)}{d/2} = \frac{\varepsilon_o \varepsilon_2 A}{d}$$

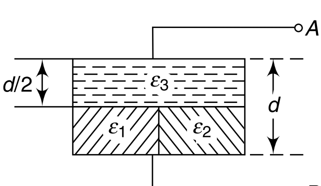


Fig. 2.73

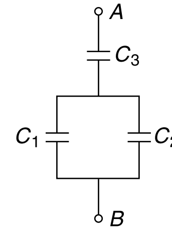


Fig. 2.73a

Similarly $C_1 = \dfrac{\varepsilon_o \varepsilon_1 A}{d}$.

Since C_1 and C_2 are in parallel contribution and this is in series with C_3 we can write

$$C = \frac{(C_1 + C_2) \times C_3}{C_1 + C_2 + C_3} = \frac{\dfrac{\varepsilon_o A}{d}(\varepsilon_1 + \varepsilon_2) \times \dfrac{2\varepsilon_o \varepsilon_3 A}{d}}{\dfrac{\varepsilon_o A}{d}(\varepsilon_1 + \varepsilon_2 + 2\varepsilon_3)}$$

$$= \frac{2\varepsilon_o \varepsilon_3 A}{d}\left[\frac{\varepsilon_1 + \varepsilon_2}{\varepsilon_1 + \varepsilon_2 + 2\varepsilon_3}\right].$$

42. Find the capacitance of an isolated spherical conductor of radius r_1 surrounded by a concentric layer of dielectric of outer radius r_2 and relative permittivity ε_r.

$$\left(Ans: \frac{4\pi\varepsilon_o \varepsilon_r r_1 r_2}{r_2 + (\varepsilon_r - 1) r_1}\right)$$

$$\left[\text{Hint:} \quad C_1 = \frac{4\pi\varepsilon_o \varepsilon_r r_1 r_2}{r_1 - r_2}; \; C_2 = 4\pi\varepsilon_o r_2\right.$$

$$\left. C = \frac{C_1 C_2}{C_1 + C_2} = \frac{4\pi\varepsilon_o \varepsilon_r r_1 r_2}{r_2 + (\varepsilon_r - 1) r_1}\right]$$

3 ELECTRO-MAGNETISM AND MAGNETIC CIRCUITS

3.1 INTRODUCTION

The fascinating properties of magnets have been known since ancient times. The word ‘magnet’ comes from the ancient Greek city of Magnesia (The modern town Manisa in Western Turkey), where the natural magnets called lodestones were found.

The fundamental nature of magnetism is the interaction of moving electric charges. Unlike electric forces which act on electric charges whether they are moving or not, magnetic forces act only on moving charges and current-carrying conductors.

The power of a magnet by which it attracts certain substances is called *magnetism* and the materials which are attracted by a magnet are called *magnetic materials*.

Few important characteristics of magnets are:

(a) Magnets can exist only in dipole and the pole strength of its two poles is the same.
(b) Magnets always attract iron and its alloys.
(c) A magnetic field is established by a permanent magnet, by an electric current or by other moving charges.
(d) Between magnets, like poles repel and unlike poles attract (Coulomb’s first law of magnetism).
(e) A magnetic substance becomes a magnet, when it is placed near a magnet. This phenomenon is known as *magnetic induction*.
(f) Magnetic field, in turn, exerts forces on other moving charges and current carrying conductors.
(g) The two poles of a magnet cannot be isolated (i.e., separated out) Magnetics monopole does not exist.
(h) It can be magnetically saturated.
(i) It can be demagnetized by beating, mechanical jerks, heating and with lapse of time.

(j) It produces magnetism in other materials by induction.
(k) On bending a magnet its pole strength remains unchanged but its magnetic moment changes.
(l) The magnetism of materials is mainly due to the spin motion of its electrons.

Electromagnetism was discovered by H.C. Oersted in 1820. He found that electric current in a conductor can produce a magnetic field around it. This discovery by Oersted provided the interaction between electricity and magnetism. The strength of this magnetic field can be increased by increasing the current in the conductor. The strength can also be increased by forming the wire into a coil of many turns as only by providing an iron core. The magnetic force has been conventionally assumed to act along a curved line from the N-pole to the S-pole.

Properties of Lines of Force

(a) They are always in the closed curves existing from a N-pole and terminating on a S-pole.
(b) They never cross one another
(c) Parallel lines of forces acting in the same direction repel one another.
(d) They always take the path of least opposition.
(e) They never have an origin nor an end.

Coulomb's Law

The mechanical force produced between two magnetic poles is produced to the product of their pole strengths, and inversely proportional to the square of the distance between them.

$$F \propto \frac{m_1 m_2}{d^2}$$

In SI system, the law is given by

$$F = \frac{m_1 m_2}{\mu_o \cdot \mu_r \cdot d^2} \tag{3.1}$$

where F is the force between the poles (in Newtons), m_1 and m_2 are pole strengths, d is the distance between the poles in meters, μ_r is the relative permeability of the medium in which the poles are situated, and μ_o is the permeability of free space (in air). μ = Absolute permeability of air (or vacuum) × relative permeability = $\mu_o \cdot \mu_r = 4\pi \times 10^{-7} \times \mu_r$ wb/AT.

Gauss's Law of Magnetism

If there were such a thing as a single magnetic charge (magnetic monopole), the total magnetic flux through a closed surface would be proportional to the total magnetic charge enclosed. But as no magnetic monopole has ever been observed, we can conclude that the total magnetic flux through a closed surface is zero.

$$\oint \vec{B} \cdot d\vec{s} = 0 \tag{3.2}$$

i.e. the total magnetic flux entering a closed surface equals the total magnetic flux leaving.

Intensity of Magnetism

When a material is placed in a magnetic field, it acquires magnetic moment M. The intensity of magnetization is defined as the magnetic moment per unit volume. It's unit is Ampere/meter. The magnetic susceptibility is defined as the intensity of magnetisation per unit magnetizing field. It has no unit and is dimensionless.

Lorentz Force

If a charge q moves in a region where both electric field $\vec{E}$ and a magnetic field $\vec{B}$ are present the resultant force acting on it is

$$\vec{F} = q(\vec{E} + v \times \vec{B}) \tag{3.3}$$

This is called the *Lorentz force*.

The phenomenon of magnetizing an unmagnetized substance by the process magnetic induction is called *magnetization*.

The process of protecting any apparatus from the effect of earth's magnetic field is known as *magnetic shielding*.

The phenomenon of decreasing or spoiling magnetic strength of a material is known as *demagnetization*.

The state of a material after which the increase in its magnetic strength stops is known as *magnetic saturation*.

Curie Law

The magnetic susceptibility of paramagnetic substances is inversely proportional to its absolute temperature, i.e. magnetic susceptivity (x) $\propto 1/T$

$$x = \frac{C}{T}$$

where C = Curies constant and T = absolute temperature.

On increasing the temperature, the magnetic susceptibility of paramagnet materials decreases and vice versa. The magnetic susceptibility of ferro-magnetic substances does not change according to Curie law.

Curie Temperature

The temperature above which a ferromagnetic materials behaves like a paramagnetic material is defined as the Curie temperature.

For Ni, T_c = 358°C
For Fe, T_c = 770°C
For Co, T_c = 1120°C

At this temperature the ferro magnetism of the substance suddenly vanishes.

Curie-Weiss Law

At temperatures above the Curie temperature the magnetic susceptibility (x) of ferromagnetic material is inversely proportional to ($T - T_c$), i.e.

$$x \propto \frac{1}{T - T_C}, \quad x = \frac{C}{T - T_C} \tag{3.4}$$

where T_c = curie-temperature, C = variation constant (Curie constant).
$x - T$ curve is shown in Fig. 3.1.

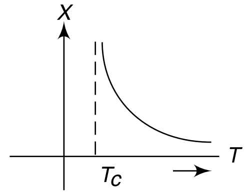
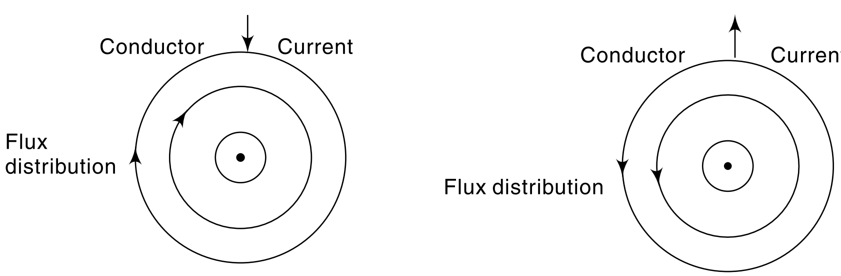


Fig. 3.1 *x – T characteristic*

3.2 MAGNETIC FIELD AROUND A CURRENT-CARRYING CONDUCTOR

If electric current passes through a conductor, a magnetic field immediately builds up due to the motion of electrons. When a magnetic field is applied to a conductor, the electrons come in motion. This is the converse phenomenon of the previous one.

When a conductor carries current downwards, i.e. away from the observer, the flux distribution is shown in Fig. 3.2(a) when it carried current upwords, i.e. towards observer, flux distribution is shown in Fig. 3.2(b).



(a) Current away from the observer (b) Current towards observer

Fig. 3.2 *Magnetic field distribution around a current carrying conductor*

The dot and cross symbol may be thought of a viewing an arrow in the direction of current flow and in the direction away from the current flow. Direction of the field (flux) can be determined by using Ampere's right-hand rule for a conductor, which states that when the conductor is being held by the right hand in such a way that the thumb outstretched parallel to the conductor and pointing the direction of current flow, the closed fingers then give the direction of flux around the conductor. (It can be obtained from right hand *cork screw rule*.)

3.2.1 Fleming's Left hand Rule

This rule is used to determine the direction of force acting on a current carrying conductor placed in a magnetic field. According to this rule, if the middle finger, forefinger and the thumb of the left hand are at right angles to one another and if

* Remember: Flux is a scalar quantity. It's SI unit is Weber (Wb), CGS unit is Maxwell, 1 Wb = 10^8 Maxwell.

the middle finger and forefinger represent the direction of current and magnetic field respectively, then the thumb will indicate the direction of force acting on the conductor (Fig. 3.3) This rule is used to determine the direction of motion of a conductor (rotor) in the magnetic field produced by the stator for a motor.

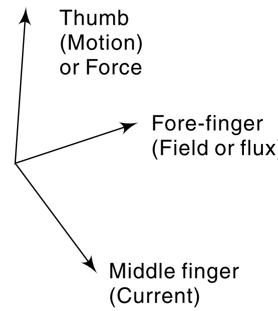


Fig. 3.3 ***Diagramatic explanation of Fleming's left hand rule***

3.3 FORCE ON A CURRENT-CARRYING CONDUCTOR IN A MAGNETIC FIELD

If has been found that when a current carrying conductor is placed in a right angles to the direction of magnetic field, the conductor experiences a mechanical force, which is right angles to both the direction of magnetic field as well as flow of current. This force experienced is directly proportional to:

(a) Current (I) flowing through the conductor
(b) Flux density (B) and
(c) The length (l) of the conductor.

∴ Force acting on conductor,

$$F = BIl \text{ N (Newtons).} \tag{3.5a}$$

Now, say the conductor is not placed at the right angles to the field, but instead placed at an angle (θ), then from natural reasoning,

$$F = BIl \sin\theta \text{ N} \tag{3.5b}$$

The direction of the mechanical force (F) is found by Fleming's left hand rule (described in previous section).

3.4 FORCE BETWEEN TWO PARALLEL CURRENT-CARRYING CONDUCTORS

Figure 3.4 shows two conductors, kept in parallel, and carrying currents I_1 and I_2, each has a length of l meters and placed at distance d meters from each other in air. When these two parallel conductors are carrying currents in the same direction [Fig. 3.5a], lines of force in circle each other in the same direction and as a result, resultant field tends to attract the conductor together towards each other. On the other hand, when two parallel conductors are carrying currents in the opposite direction [Fig. 3.5b] then lines of force in the same direction are crowded

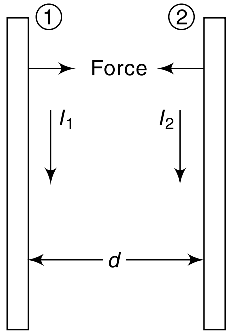


Fig. 3.4 ***Force between two parallel current-carrying conductors***

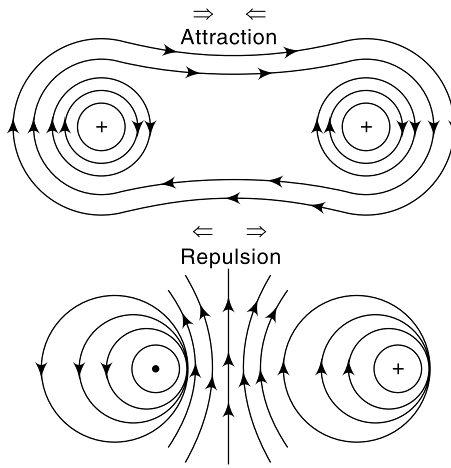


Fig. 3.5 ***Direction of lines of force between two parallel current carrying conductors***

between the two conductors, and thereby experience a mutual repulsive force. In fact, a mechanical force is developed between these two conductors.

In order to determine the magnitude of force between two parallel current-carrying conductors, one of the two conductors (say conductor 1) is considered placed in a magnetic field due to the other (i.e. due to conductor 2).

Magnetic field due to conductor 2 is given by

$$H = \frac{I_2}{2\pi d} \tag{3.6}$$

$\therefore$ Force acting on conductor 1 is $F = BI_1 l$, where B is the *flux density* of the field due to conductor 2.

$$\therefore \quad F = \mu_o \mu_r H I_1 l \tag{3.7a}$$

where $\quad B = \mu_o \mu_r H$

[μ_r is the *relative permeability* of the medium in which both the conductors are placed]

Substituting the value of (H) in equation (3.7a), we have

$$F = \frac{\mu_o \, \mu_r \, I_1 \, I_2 \, l}{2\pi d} \cdot \text{N}$$

$$= \frac{\mu_o \, I_1 \, I_2 \, l}{2\pi d} \text{ N (in air } \mu_r = 1]$$

$$= \frac{4\pi \times 10^{-7} \times I_1 \, I_2 \, l}{2\pi d} \text{ N}$$

$$= \frac{2 I_1 \, I_2 \, l \times 10^{-7}}{d} \text{ N} \tag{3.7b}$$

In differential form, we can write

$$\frac{dF}{dl} = \frac{2 \times 10^{-7} \times I_1 I_2}{d} \text{ N} \tag{3.8}$$

From the above expression it can be concluded that *the larger the currents carried by the conductor, and less is the distance between the conductors, greater is the force between them.*

3.1 In uniform field of 1 Wb/m^2, a direct current of 70 A is passed through a straight wire of 1.5 m placed perpendicular to the field. Calculate:

(a) The magnitude of the mechanical force produced in the wire.
(b) The prime mover power (i.e. the mechanical power) in watts to displace the conductor against the force at a uniform velocity of 5m/sec.
(c) The emf generated in the current carrying wire.
How do you show that the electrical power produced is same as the mechanical power in creating the motion?

Solution

(a) Force = $BIl \sin \theta$
$= 1.0 \times 70 \times 1.5 \sin 90°$ [here $\theta = 90°$]
$= 105$ N

(b) Prime-mover, i.e. mechanical power
$= F \times v$
$= 105 \times 5 = 525$ W

(c) Emf generated = Blv
$= 1 \times 1.5 \times 5 = 7.5$ V.
The electrical power = $eI = 7.5 \times 70 = 525$ W. (= mechanical power)

3.2 Calculate the force developed per meter length between two current-carrying conductors 10 cm apart and carrying 1000 A and 1500 A currents respectively.

Solution

Given $I_1 = 1000$ A, $I_2 = 1500$ A, $d = 10$ cm = 0.10 m

$$\text{Force per meter length} = \frac{2 \times 10^{-7}\, I_1 I_2}{d}$$

$$= \frac{2 \times 10^{-7} \times 1000 \times 1500}{0.10}$$

$$= 3 \text{ N}$$

3.3 Two long straight conductors each carrying an electric current of 5.0 A, are kept parallel to each other at a separation of 2.5 cm. Calculate the magnitude of magnetic force experienced by 10 cm of a conductor.

Solution

The field at the side of one conductor due to the other is

$$B = \frac{\mu_o \cdot I}{2\pi d} = \frac{2 \times 10^{-7} \times 5}{2.5 \times 10^{-2}} = 4.0 \times 10^{-5}\ T$$

∴ The force experienced by 10 cm of the conductor due to the other is

$$F = IlB$$
$$= 5.0 \times 10 \times 10^{-2} \times 4.0 \times 10^{-5}$$
$$= 2 \times 10^{-5} \text{ N.}$$

3.4 Two long straight parallel wires situated in air at a distance of 2.5 cm having a current of 100 A in each wire in the same direction. Determine the magnetic force on each wire. Justify whether the force is of the repulsion or attraction type.

Solution

Given, $d = 2.5 \text{ cm} = 2.5 \times 10^{-2}$ m

$I_1 = I_2 = 100$ A.

∴ Force on each wire per meter length

$$F = \frac{4 \times 10^{-7} \times I_1 \cdot I_2}{2d} = \frac{2 \times 10^{-7} \times 100 \times 100}{2.5 \times 10^{-2}} = 0.08 \text{ N}$$

Force is attraction type since currents in both the wires are flowing in same direction.

3.5 Two infinite parallel conductors carrying parallel currents of 50 A each. Find the magnitude and direction of the force between the conductors per meter length if the gap between them is 25 cm.

Solution

We know
$$F = \mu_0 \cdot \frac{I_1 I_2 d}{2\pi d}$$
$$= \frac{2 \times 10^{-7} \times 50 \times 50 \times 1}{25 \times 10^{-2}} \text{ N}$$
$$= 10^2 \times 10^2 \times 10^{-7} \times 2$$
$$= 2 \times 10^{-3} \text{ N.}$$

The direction of force will depend on whether the two currents are flowing in the same direction or not. For same direction, it will be force of attraction and for opposite direction it will be force of repulsion.

3.6 A wire of 1 m length long is bend to form a square. The plane of the square is right angled to a uniform field having a flux density of 2 m Wb/mm^2. Determine the work done if the wire carries a current of 20 Amps through it, is changed to a circular shape.

Solution

Side of the square $= \frac{1}{4}$ m $= 0.25$ m

A_1 = area of square = $(0.25)^2 = 0.0625$ m^2

Circumference of a circle is 1 m = $2\pi r$

$$\therefore \quad r = \frac{1}{2\pi} = 0.15909 \text{ m}$$

$$\therefore \quad A_2 = \text{area of the circle} = \pi r^2$$
$$= \frac{22}{7} \times (0.15909)^2$$
$$= 0.0795 \text{ m}^2$$

$\therefore$ Work done = difference between the torques = $BIA_2 - BIA_1$

$$= 2 \times 10^{-3} \times 20\ (0.0795 - 0.0625)$$
$$= 0.00068 \text{ J}$$

3.5 BIOT SAVART'S (OR LAPLACE'S) LAW

If a current carrying conductor is placed at an angle (θ) to the direction of a magnetic field, the effective length being ($l \sin \theta$), the force on the conductor (Fig. 3.6) will be $F = BIl \sin \theta$. (The effective length is the length of the conductor lying within the magnetic field).

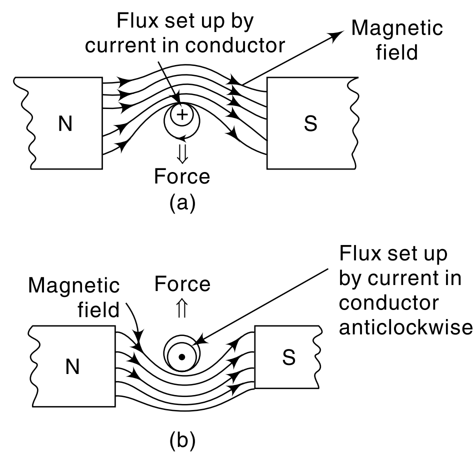


Fig. 3.6 (a, b) ***Force on a current-carrying conductor lying in a magnetic field***

In magnetics, there are basically two methods of calculating magnetic field at some point. One is Bio-Savart Law (or Laplace's Law) which gives the magentic field due to an infinitesimally small current-carrying conductor (wire) at some point and another is Ampere's law.

Let dH be the magnetic field at a point P associated with length element dl carrying a steady current of I amperes. Imagine a unit N-pole is present at point P (Fig. 3.6(c). Then flux density B due to the pole strength of 1 wb is given by

$$B = \frac{1}{4\pi r^2} \text{ wb/m}^2$$

$\therefore$ Mechanical force acting at the element (dl)

$$dF = BI(dl) \sin \theta = \frac{I \cdot dl \cdot \sin \theta}{4\pi r^2} \text{ N} \qquad (3.9)$$

(Since action and reaction are equal and opposite)

Now, as per Newton's third law the magnetic field produced by the current element (dl) on the unit pole at point P, is given by

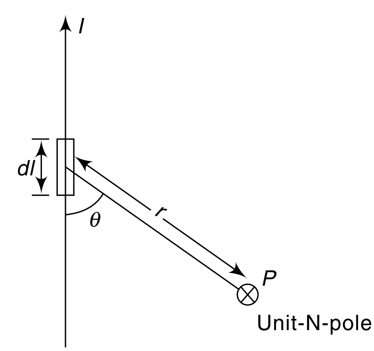


Fig. 3.6(c) ***A current carrying conductor in a magnetic field***

$$dH = \frac{I \cdot dl \cdot \sin \theta}{4\pi r^2} \text{ AT/m} \qquad (3.10)$$

with the direction of (dH) and perpendicular to both (dl) and the unit vector r directed from (dl) to P.

This equation is known as Biot-Savart's Law (or Laplaces' law).

The following point is worth noting regarding the Biot-Savart law.

The magnitude of dH is given by

$$|dH| = \frac{I \cdot dl \cdot \sin\theta}{4\pi r^2}$$

$|dH|$ is zero at $\theta = 0°$ or $180°$
and maximum at $\theta = 90°$.

This law is employed for calculating the field strength near any system of conductors.

3.6 APPLICATION OF BIOT-SAVART LAWS

3.6.1 Determination of Magnetic Field Surrounding a Straight Long Conductor of Finite Size

Consider XY (Fig. 3.7) be a straight long finite conductor carrying a current I in the direction for X to Y, and P be a point at which the magnetic field H is required. Let us draw a perpendicular from P which meets the conductor at Q. Let $PQ = R$.

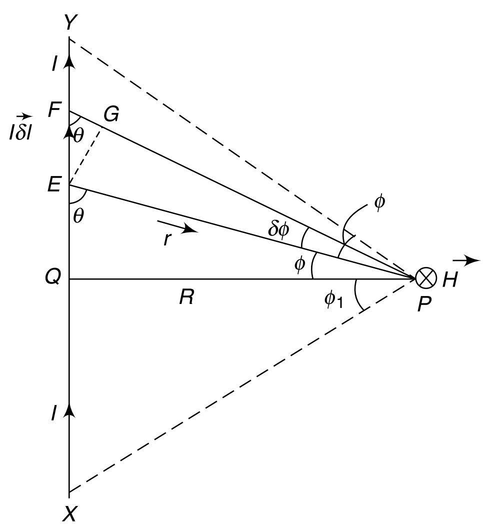


Fig. 3.7 *Application of Biot-Savarts Law for a straight conductor*

Let us consider the conductor to be divided into many small current elements and let $(i \cdot d\vec{l})$ be one such (vector) element, where length EF (= δl). Let ($\vec{r}$) be the position vector from the element to the point P. Let $\angle EPQ = \phi$, $\angle FPE = \delta\phi$ and $\angle EFP = \theta$. Since the length EF (= δl) is very small, $\angle QEP = \theta$. Let EG be a perpendicular from E to FP.

By Biot-Savart law, the magnetic field at P due to the current-element $(i \cdot d\vec{l})$ is given by

$$\delta\vec{H} = \frac{\mu_o}{4\pi} \cdot \frac{i\delta\vec{l} \times \vec{r}}{r^3}$$

The angle between vectors $(i\,d\vec{l}\,)$ and $(\,\vec{r}\,)$ is $(180 - \theta)$ and so the magnitude of the field at P is

$$\delta H = \frac{\mu_o}{4\pi} \cdot \frac{i\delta l \sin(180-\theta)}{r^2} = \frac{\mu_o}{4\pi}\frac{i\delta l \sin\theta}{r^2} \tag{3.11}$$

By right-hand screw rule of vector product, the field $(\delta\vec{H}\,)$ at point P is perpendicular to the page directed 'downwards'.

Now from the similar triangles ΔEFG and ΔPEG, we have

$$EG = EF \sin\theta = \delta l \sin\theta$$

and $$EG = EP \sin\delta\phi = r \sin\delta\phi = r\cdot\delta\phi \qquad (\delta\phi \text{ being very small})$$

$$\therefore \qquad \delta l \sin\theta = r\cdot\delta\phi \tag{3.12}$$

Making this substitution in equation (3.11) we have

$$\delta H = \frac{\mu_o}{4\pi}\frac{i\cdot\delta\phi}{r}$$

From right-angled triangle EQP, we have

$$r = \frac{R}{\cos\phi}$$

$$\therefore \qquad \delta H = \frac{\mu_o}{4\pi}\cdot\frac{i\cos\phi\,\delta\phi}{R} \tag{3.13}$$

Let us join PX and PY. Let $\angle QPX = \phi_1$ (anticlockwise) and $\angle QPY = \phi_2$ (clockwise). Then the magnitude of the magnetic field $\vec{H}$ at point P due to the whole conductor is

$$H = \int_{-\phi_1}^{\phi_2} \frac{\mu_o}{4\pi}\cdot\frac{i}{R}\cos\phi\, d\phi = \frac{\mu_o}{4\pi}\cdot\frac{i}{R}[\sin\phi]_{-\phi_1}^{\phi_2}$$

$$= \frac{\mu_o\cdot I}{4\pi R}[\sin\phi_2 - \sin(-\phi_1)]$$

$$= \frac{\mu_o\cdot I}{4\pi R}[\sin\phi_1 + \sin\phi_2] \tag{3.14}$$

For a conductor of infinite length, we have $\phi_1 = \phi_2 = 90°$

$$H = \frac{\mu_o}{4\pi}\cdot\frac{I}{R}(\sin 90° + \sin 90°) = \frac{\mu_o\cdot 2\cdot I}{4\pi\cdot R}$$

$$= \frac{\mu_o}{2\pi}\cdot\frac{I}{R} \tag{3.15a}$$

For a semi-infinite conductor the field is

$$H = \frac{\mu_o\cdot I}{4\pi r}(1 + \sin\phi) \tag{3.15b}$$

3.6.2 Field Strength Due to Circular Loop

Consider a circular coil of length l meters and radius r meters having N turns, and carrying a current of I amperes as shown in Fig. 3.8. Let a unit N-pole be placed on the axis of the coil at P at a distance x meters from the centre of the coil; then force dH experienced on unit N-pole, due to small are length dl, is given by (from Biot-Savart's law)

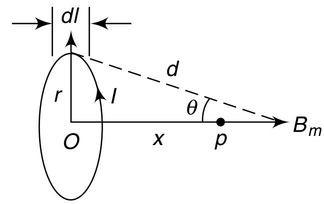


Fig. 3.8 ***A circular coil***

$$\delta H = \frac{NIdl}{4\pi d^2}$$

Axial component of (δH) is thus obtained as $\left(\frac{NIdl}{4\pi d^2}\sin\theta\right)$

$\therefore$ Total force experienced on unit N-pole placed at P due to entire coil is given by

$$H = \frac{NI\sin\theta}{4\pi d^2}\int_0^l dl = \frac{NI\sin\theta}{4\pi d^2}\cdot l$$

$$= \frac{NI\sin\theta}{4\pi d^2}\times 2\pi r = \frac{NIr\sin\theta}{2d^2}$$

$$= \frac{NIr^2}{2(r^2+x^2)^{3/2}} \quad \left[\because \frac{r}{\sin\theta} = (r^2+x^2)^{1/2}\right] \tag{3.16}$$

Now if we want to calculate the field strength at the centre of the loop, x becomes zero.

H(Force experienced on a unit N-pole placed at that point) is given by = $\frac{NI}{2r}$ AT/m.

In the above expression if we want to calculate the field strength far away from the centre of the circular loop, i.e. if $x \gg r$, $r^2 + x^2 \approx x^2$,

then, $$H = \frac{1}{4\pi}\cdot\frac{2M}{x^3}$$

where M = magnetic moment of the loop = $NIA = NI\pi r^2$.

3.6.3 Field in a Solenoid

A solenoid is a helical coil and is a very effective method of producing a magnetic field. Figure 3.9 shows a solenoid of length l carrying a steady current of I amperes. Let the number of turns be N and the radius of the coil be R. Since the spacing between the turns is very small, the wire (conductor) can be considered as current sheet of width $\left(\frac{l}{N}\right)$ and negligible spacing between turns. Then the current in the solenoid causes a cylindrical current sheet having a linear current density K.

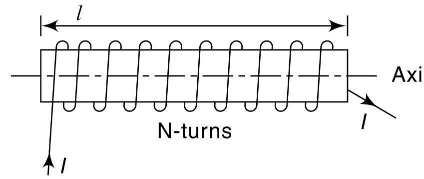


Fig. 3.9 ***Solenoid of length l***

i.e., $K = \frac{NI}{l}$ ampere turns/meter.

For an element (dx), shown in Fig. 3.10,

The current density = $Kdx = NI\left(\frac{dx}{l}\right)$

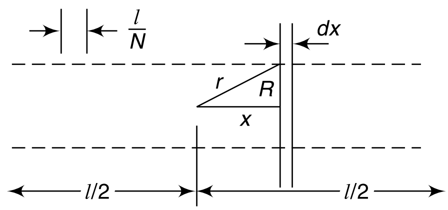


Fig. 3.10 ***Field in an elemental portion in the solenoid***

Using the equation

$H = \frac{NIR^2}{2(R^2+x^2)^{3/2}}$, the flux density ($dB$) at the centre of the solenoid, due to element (dx) is

$$dB = \frac{\mu NIR^2}{2l(R^2+x^2)^{3/2}}dx \tag{3.17}$$

The total flux density can be formed by integrating (dB) over the length of the solenoid, i.e. from $x = -\frac{l}{2}$ to $x = +\frac{l}{2}$. Thus

$$B = \frac{\mu NIR^2}{2l}\int_{-l/2}^{l/2}\frac{dx}{(R^2+x^2)^{3/2}}$$

$$= \frac{\mu NI}{4(R^2+l^2)^{1/2}} \tag{3.18}$$

If the length is much larger than the radius, i.e. if $l \gg R$, we have

$$B = \frac{\mu NI}{l} = \mu \cdot K$$

The obtained equations give B at the centre of the solenoid. If the limit of integration is changed from 0 to l, we get B at one end of the solenoid. This value is

$$B = \frac{\mu NI}{2(R^2+l^2)^{1/2}} \tag{3.19}$$

If $l \gg R$, this flux density at one end is

$$B = \frac{\mu NI}{2l} = 0.5\ \mu K \quad \left[\because K = \frac{NI}{l}\right]$$

A comparison of the above equations shows that flux density at one end of the coil is half of that at the centre.

3.7 A circular current-carrying coil has a radius R. Show that the distance from the centre of the coil, on the axis, where B will be $\left(\frac{1}{8}\right)$ of its value at the centre of the coil is $\left(\sqrt{3}r\right)$.

Solution

$$B_{\text{axis}} = \frac{1}{8} B_{\text{centre}} \quad \text{(given)}$$

$$\therefore \quad \frac{\mu_o \cdot NIr^2}{2(r^2 + x^2)^{3/2}} = \frac{1}{8}\left(\frac{\mu_o \cdot NI}{2 \cdot r}\right)$$

or $$8r^3 = (r^2 + x^2)^{3/2}$$

or $$(2r)^3 = \left\{\left(\sqrt{r^2 + x^2}\right)\right\}^3$$

or $$\sqrt{r^2 + x^2} = 2r$$

or $$r^2 + x^2 = 4r^2$$

or $$x = \sqrt{3}\, r.$$

3.8 A current of 10 A is flowing in a flexible conductor of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform field of 2 T. Calculate the angle between the magnetic field and the direction of the current.

Solution

We know $$F = BIl \sin \theta$$

$$15 = 2 \times 10\ (1.5) \sin \theta$$

$$\therefore \quad \sin \theta = \frac{15}{30} = \frac{1}{2} = \sin^{-1}(30°)$$

$$\therefore \quad \theta = 30°$$

3.9 A straight conductor is carrying 2000 A and a point P is situated in such that ϕ_1 = 60° and ϕ_2 = 30°. Calculate the field strength at P if its perpendicular distance from the conductor is 0.2 meter.

Solution

$$H = \frac{\mu_o \cdot I}{4\pi r} (\sin \phi_1 + \sin \phi_2)$$

Given $\phi_1 = 60°$, $\phi_2 = 30°$, $r = 0.2$ meter

$I = 2000$ Amps and $\mu_o = 1$ [in air]

$$\therefore \quad H = \frac{2000}{4 \times \pi \times 0.2} [\sin 60° + \sin 30°]$$

$$= 1087.05 \text{ A/m.}$$

3.10 A solenoid of 400 turns is wound on a continuous ring of iron, the mean diameter of the ring being 10 cm. The relative permeability is 1250. What current is required in order that the flux density (B) in the iron shall be 12,000 maxwells/cm^2.

Solution

given l = mean circumference of ring = $10 \times \pi$ cm

$= 0.10\ \pi$ meter

and B = 12,000 maxwells/cm^2

$= 12{,}000 \times 10^{-8} \times 10^4 = 1.2$ wb/m^2

We know $B = \mu_r \cdot \mu_o \dfrac{NI}{l}$

$$\therefore \quad I = \frac{B \cdot l}{\mu_r\, \mu_o \cdot N} = \frac{1.2 \times 0.10\, \pi}{1.25 \times 10^3 \times 4\pi \times 10^{-7} \times 400}$$

$$= \frac{0.12}{2000 \times 10^{-4}} = \frac{0.12}{2 \times 10^{-1}} = \frac{1.2}{2} = 0.6 \text{ A}$$

3.11 Prove that the magnetic field due to a current-carrying coil on the axis at a large distance x from the centre of the coil varies approximately as x^{-3}.

Solution

We know $$B_{\text{axis}} = \frac{\mu_o \cdot NIr^2}{2\,(r^2 + x^2)^{3/2}}.$$

For $x \gg r$

$$B_{\text{axis}} \cong \frac{\mu_o \cdot NIr^2}{2 \cdot x^3}$$

$\therefore$ $B_{\text{axis}} \propto \dfrac{1}{x^3}$ and hence proved.

3.7 ELECTROMAGNETIC INDUCTION

Whenever magnetic flux is linked with a circuit changes, an emf is induced in the circuit. If the circuit is closed, a current is also induced in it. The emf and current so produced lasts as long as the flux linked with the circuit changes (in direction and or in magnitude). This phenomenon is called *electromagnetic induction (EMI).* The magnetic force can be considered as magnetic induction and its SI unit is tesla (T).

3.7.1 Laws of Electromagnetic Induction

In 1831, Michael Faraday, after performing a number of experiments, summarised a phenomenon of electromagnetic induction, which are stated as follows:

Faraday's first law states that "*whenever magnetic flux linked with a close coil changes, an induced emf is set up in the coil and the induced emf lasts as long as the change in magnetic flux continues.*"

Faraday's second law states that *the magnitude of the induced emf is proportional to the rate of change of magnetic lines of force.*

If ϕ is the magnetic flux linked with the current [coil of (N) turns] at any instant t, then the induced emf's expression in differential form is

$$|e| = \frac{d(N\phi)}{dt} = N \cdot \frac{d\phi}{dt} \text{ V}.$$

Since the induced emf (e) sets up a current in a direction that opposes the very cause of producing magnetic field, so a minus sign is given to the induced emf.

Therefore, $$e = -N\frac{d\phi}{dt} \text{ V} \tag{3.20}$$

3.7.2 Lenz's Law

Even though Faraday's laws give no idea regarding the direction of induced emf, the direction of induced emf is, however, given by Lenz's law which is based on the law of conservation of energy and it states that "*The direction of the induced current (or emf) is such that it opposes the very cause producing this current (or emf), i.e. it opposes the change in magnetic flux.*

In view of Lenz's law, the induced emf equation takes the form $e = -N \cdot \frac{d\phi}{dt}$.

In 1834, Heinrich Lenz, a German Physicist, enunciated a simple rule, now known as Lenz's law. In fact, this law basically interpreted the law of conservation of energy which can be justified as follows.

When the north pole of a magnet is moved towards the coil, the induced current flows in a direction so as to oppose the motion of the magnet towards the coil. This is only possible when the nearer face of the coil acts as a magnetic north pole which makes an anticlockwise current to flow in the coil. Then the repulsion between the two similar poles opposes the motion of the magnet towards the coil.

Similarly, when the magnet is moved away from the coil, the direction of induced current is such as to make the nearer face of the coil as a south pole which makes a clockwise induced current to flow in the coil. Then the attraction between the opposite poles opposes the motion of the magnet away from the coil. In either case, therefore, work has to be done in moving the magnet. It is this mechanical work which appears as electrical energy in the coil. Hence the production of induced emf or induced current in the coil as in accordance with the law of conservation of energy.

3.7.3 Fleming's* Right Hand Rule

The direction of the induced emf (and hence the induced current) is given by Fleming's Right Hand Rule which states that "stretch the fore finger, middle finger and the thumb of right hand in such a way that all three are mutually perpendicular to each other. If fore finger points in the direction of field, thumb points in the direction of motion of conductor, then middle finger will point along the direction of induced coventional current", as shown in Fig. 3.11.

* John Ambrose Fleming (1849-1945) was professor of Electrical Engineering at University College, London.

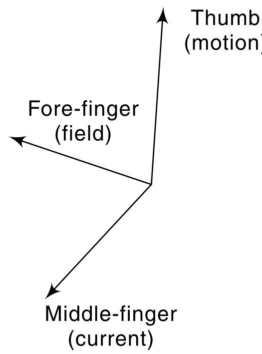


Fig. 3.11 ***Fleming's right hand rule.***

3.8 CONCEPT OF SELF-INDUCTANCE AND MUTUAL INDUCTANCE

Self-Inductance

When a coil carries a current it establishes a magnetic flux (Fig. 3.12). When the current in the coil changes, the magnetic flux linking with the coil also changes. It is observed that this change in the value of current or flux in the coil is opposed by the instantaneous induction of opposing emf. This property of the coil by which it opposes the change in value of current or flux through it due to the production of self induced emf is called *self-inductance*. It is measured in terms of co-efficient of self-inductance L. It obeys Faraday's law of electromagnetic induction like any other induced emf.

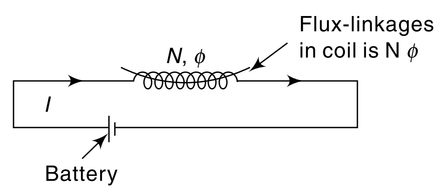


Fig. 3.12 ***Diagramatic explanation of self-inductance***

For a given coil (provided no magnetic material such as iron is nearby) the magnetic flux linked with it will be proportional to the current, i.e.

$$\phi \propto I \text{ or } \phi = LI \tag{3.21a}$$

where L is called the self-inductance (or simply inductance) of the coil.
The induced emf is given by

$$E = -\frac{d\phi}{dt} = -L\frac{dI}{dt} \tag{3.21b}$$

The S.I. unit of inductance is henry (symbol H). Henry is a big unit of inductance. Smaller units millihenry (mH) and microhenry (μH) are used

$$1 \text{ mH} = 10^{-3} \text{ H and } 1\ \mu\text{H} = 10^{-6} \text{ H}$$

Thus, the self-inductance of a coil is 1 H if an induced emf of 1 volt is set up when the current in the coil changes at the rate of one ampere per second. Also, 1 H = 1 wb A^{-1}. This is also termed as *co-efficient of self-induction* or simply *self-induction*.

The role of self-inductance in an electrical circuit is the same as that of the inertia in mechanical motion. Thus the self-inductance of a coil is a measure of its ability to oppose the change in current through it and hence is also called *electrical inertia*.

Mutual Inductance

Whenever a change in current occurs in a coil, an induced emf is set up in the neighbouring coil. This process is called *mutual induction*. The coil in which the emf is induced is called the *secondary coil*. If a current I_1 flows in the primary coil, the magnetic flux linked with the secondary coil (Fig 3.13) will be

$$\phi_2 = M \cdot I_1 \tag{3.22a}$$

where M is called the mutual inductance between the two coils or circuits.

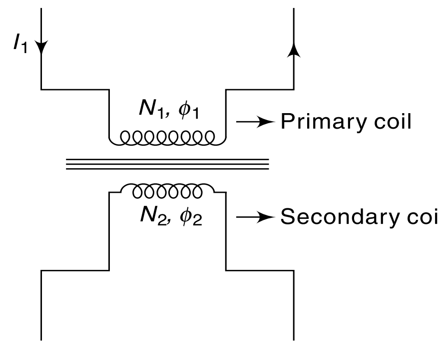


Fig. 3.13 *Explanation of mutual inductance*

The emf induced in the secondary coil is given by

$$E_2 = \frac{d\phi_2}{dt} = -M\frac{dI_1}{dt} \tag{3.22b}$$

Thus the mutual inductance of a pair of circuits is 1 H if a rate of change of current of one ampere per second induces an emf of 1 V in the other circuit.

3.9 CONCEPT OF MAGNETIC COUPLING

The coils are said to be *magnetically coupled* if either or part of the magnetic flux produced by one links that of the other. If L_1 = self-inductance of coil 1 and L_2 be the self-inductance of coil 2 and M be the mutual inductance of two coils, then $M = K\sqrt{L_1 L_2}$ where K is co-efficient of coupling. If the total flux produced by coil 1 links with the flux produced by coil 2, then $K = 1$ and $M = \sqrt{L_1 L_2}$.

On the other hand, if there is no common flux between the two coils, then they are said to be magnetically isolated. Therefore, co-efficient of coupling K between the coils = $\dfrac{\text{'actual' mutual inductance}}{\text{maximum possible value}}$

When the two coils are closely coupled magnetically through an iron core, K is close to unity. On the other hand, when the two coils are loosely coupled magnetically, K is equal to 0.5 or even less. In the magnetically isolated case, $K = 0$, i.e. $M = 0$.

3.12 A magnetic flux of 400 μ Wb passing through a coil of 1500 turns is reversed in 0.1 s. Determine the average value of the emf induced in the coil.

Solution

The magnetic flux has to decrease from 400 μ Wb to zero and then increase to 400 μ Wb in the reverse direction, hence the increase of flux in the original direction is 800 μ Wb.

We know, average emf induced in the coil is

$$e = \frac{N\phi}{t}$$

$$= \frac{1500 \times (800 \times 10^{-6})}{0.1}$$

$$= 12 \text{ V.}$$

3.13 A coil has a self-inductance of 40 milli-henry. Determine the emf in the coil when the current in the coil

(a) increase at the rate of 300 A

(b) raises from 0 to 10 A in 0.05 sec.

Solution

Given (a) Self-inductance $L = 40 \times 10^{-3}$ H

$$E = L \cdot \frac{dI}{dt} \quad \text{(only magnitude)}$$

$$= 40 \times 10^{-3} \times 300 = 12 \text{ V}$$

(b) $L = 40 \times 10^{-3}$ H

$dI = 10 - 0 = 10$ A

$dt = 0.05$ sec

$$\therefore \qquad e = L\frac{dI}{dt}$$

$$= \frac{40 \times 10^{-3} \times 10}{0.05} = 8 \text{ V}$$

3.14 Determine the emf induced in a coil of 4.19×10^{-4} Henry when a current of 5 A is reversed in 60 milliseconds.

Solution

Given $L = 4.19 \times 10^{-4}$ H

Also given $dI = 5 - (-5) = 10$ A

$dt = 60$ milliseconds $= 60 \times 10^{-3}$ sec.

$$\therefore \qquad \text{emf induced} = L\frac{di}{dt} \quad \text{(only magnitude)}$$

$$= \frac{4.19 \times 10^{-4}}{60 \times 10^{-3}} \times 10$$

$$= \frac{4.19}{60}$$

$$= 0.0698 \text{ V.}$$

3.15 Two identical coils X and Y each having 1000 turns lie in parallel planes such that 60% of the flux produced by one coil links with the other. A current of 10 A in coil X produces in it a flux of 10^{-4} wb. If the current in the coil X changes from +15 A to –15 A in 0.03 seconds, what would be the magnitude of the emf induced in the coil Y?

Solution

Given $N_1 = N_2 = 1000$ turns

$I_x = 10$ A

$\phi_x = 10^{-4}$ Wb.

The amount of flux linking with second coil = 0.6×10^{-4} Wb.

$$\therefore \quad \frac{dI_x}{dt} = \frac{[15-(-15)]}{0.03} = \frac{30}{0.03} = 1000 \text{ A/sec.}$$

We have

$$e_{My} = M \cdot \frac{dI_1}{dt} \text{ volts (ignoring –ve sign)}$$

where
$$M = \frac{N_2 \text{ (Amount of } \phi_1 \text{ linking with the coil } Y)}{I_1}$$

$$= \frac{1000 \times (0.6 \times 10^{-4})}{10}$$

$$= 0.6 \times 10^{-2} \text{ H}$$

$$\therefore \quad e_{My} = 0.6 \times 10^{-2} \times 1000$$

$$= 6 \text{ volts.}$$

3.16 A square coil of 10 cm side and with 120 turns in rotated at a uniform speed of 1000 rpm about and axis at right angles to a uniform magnetic field having a flux density of 0.5 Wb/m^2. Determine the instantaneous value of the electromotive force have the plane of the coil

(a) at right angles to the field
(b) at 30° to the field
(c) in the plane of the field.

Solution

We know

$$\phi = B.A = 0.5 \times (10 \times 10^{-2})^2 = 5 \text{ milli-Wb}$$

$$\psi = \phi \cdot N = 5 \times 10^{-3} \times 120 = 0.6$$

$$\omega = \frac{2\pi n}{60} = \frac{2 \times \pi \times 1000}{60} = 104.719 \text{ radian/sec}$$

$$\therefore \quad \psi = \psi_m \cos \omega t = 0.6 \cos 104.719t$$

$$\therefore \quad e = -\frac{d\psi}{dt} = +0.6 \times 104.719 \sin 104.719t$$

$$= 62.83 \sin 104.719t \text{ volt.}$$

(a) When $\theta = 0°$

$e = 62.83 \sin 0° = 0$ V

(b) When $\theta = 90° - 30° = 60°$

$$e = 62.83 \sin 60° = 62.83 \times \frac{\sqrt{3}}{2} = 54.412 \text{ V.}$$

(c) When $\theta = 90°$

$e = 62.83$ V.

3.10 CALCULATION OF SELF-INDUCTANCE

3.10.1 For a Circular Coil

Consider a circular coil of radius r and number of turns N. If current I passes in the coil, then magnitude field at centre of coil

$$B = \frac{\mu_o Nl}{2r}$$

the effective magnetic flux linked with this coil,

$$\phi = NBA = \frac{N(\mu_o Nl)A}{2r}.$$

Since, by definition, $L = \dfrac{\phi}{I}$

$$\therefore \qquad L = \frac{\mu_o N^2 A}{2r} = \frac{\mu_o N^2 \pi r^2}{2r} \quad [\because A = \pi r^2 \text{ for a circular coil}]$$

$$\text{or,} \qquad L = \frac{\mu_o N^2 \pi r}{2}. \tag{3.23}$$

3.10.2 For a Solenoid

Consider a solenoid with n number of turns per meter. Let current I flow in the windings of solenoid, then the magnetic field inside solenoid is given by

$$B = \mu_0 n I$$

the magnetic flux linked with its length l is $\phi = NBA$, where N is the total number of turns in length l of solenoid.

$$\phi = (nl)BA = (nl)(\mu_0 \cdot n \cdot I) A \quad (\because N = nl)$$

$$\text{Since,} \qquad L = \frac{\phi}{I}$$

$$\therefore \qquad L = \mu_0 n^2 A I \tag{3.24}$$

$$\text{Since} \qquad n = \frac{N}{l}$$

$$\therefore \text{ Self-inductance, } L = \frac{\mu_o N^2 A}{l} \tag{3.25}$$

3.11 ENERGY STORED IN INDUCTOR

When the current in a circuit of a coil of inductance L henry increases from zero to its maximum steady value of I amperes, work has to be done against the opposing induced emf.

Let dw be the infinitesimal work done in time dt, then $dw = VI\,dt$, where V is voltage across an inductor.

Since $$V = L\frac{di}{dt}$$

$\therefore$ $$dw = \left(L\frac{di}{dt}\right) \cdot I \cdot dt = LI\,dI$$

or $$w = \int_0^I LI\,dI = \frac{1}{2}LI^2 \text{ J.} \qquad (3.26)$$

This work done is stored in the form of energy of the magnetic field in an inductor.

Also we can write $L = \dfrac{2w}{I^2}$ if $I = 1$ Amp, $L = 2w,$ using equation (3.26)

Thus the self-inductance of a circuit is numerically equal to twice the work done against the inductance emf in establishing a circuit of 1 A in the coil. Again, from the definition of self-inductance (Ref. equation number 3.25)

$$L = \frac{\mu_o\,\mu_r\,A \cdot N^2}{l}$$

And stored energy $= \dfrac{1}{2}LI^2 = \dfrac{1}{2}\dfrac{\mu_o \cdot \mu_r \cdot A \cdot N^2 \cdot I^2}{l}$ J

But magnetic field intensity, $H = \dfrac{NI}{l}$

$\therefore$ Stored energy $= \dfrac{1}{2}\mu_0 \cdot \mu_r \cdot A.l \cdot \dfrac{N^2 I^2}{l^2} = \dfrac{1}{2}\mu_o\mu_r\,A.l \cdot H^2$ J

Now, Al = volume of the magnetic field in m^3

$$\text{Energy stored/m}^3 = \frac{1}{2}\mu_o \cdot \mu_r \cdot H^2 \text{ J}$$

$$= \frac{1}{2}BH\,J = \frac{1}{2} \cdot B \cdot \frac{B}{\mu_o \cdot \mu_r} \text{ J } [\because\ B = \mu_o\,\mu_r\,H]$$

$$= \frac{B^2}{2\mu_o} \quad [\because\ \mu_r = 1 \text{ in air}]. \qquad (3.27)$$

3.12 MAGNETIC ENERGY DENSITY (U_M)

Since, $$W = \frac{1}{2}LI^2, \quad \text{for a solenoid}$$

$$L = \mu_o n^2 Al \text{ and } B = \mu_o nI$$

$$\therefore \qquad W = \frac{1}{2}(\mu_o \cdot n^2 Al)\left(\frac{B}{\mu_o n}\right)^2$$

$$= \frac{B^2}{2\mu_o}(Al) \qquad (3.28)$$

Again we know $\frac{W}{Al} = \frac{\text{Energy}}{\text{Volume}}$ = Magnetic energy density (say U_M)

$$\therefore \qquad U_M = \frac{B^2}{2\mu_o} \text{ J.} \qquad (3.29)$$

Thus we can conclude that the energy stored in magnetic field in terms of per unit volume of the magnetic material used is called the *magnetic energy density*.

3.13 COMBINATION OF INDUCTANCES

3.13.1 Dot Covention

The emf induced due to mutual inductance may either aid or oppose the emf induced due to self-inductance in a magnetic coupling circuit. It depends on the relative direction of currents, the relative modes of windings of the coils as well as the physical location, i.e. either far away or very close with respect to the other. Figure 3.14 clearly explains the sign of mutually induced emf.

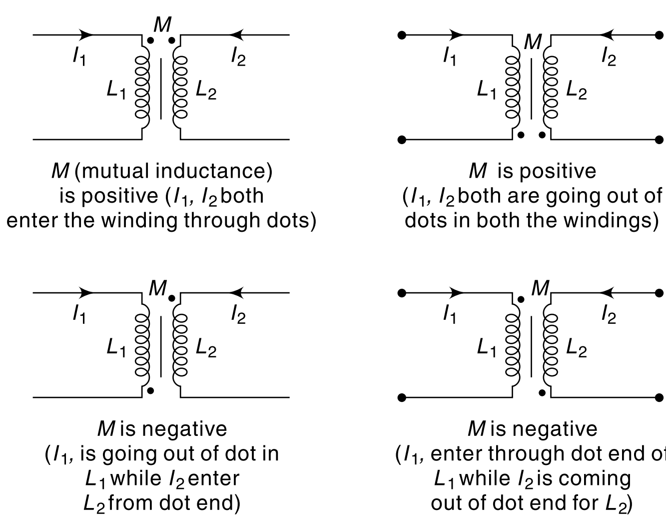


Fig. 3.14 ***Dot convention to determine sign of M (A dot represents the +ve polarity of the winding at any instant).***

3.13.2 Inductances in Series and Parallel

Series Connection

When two inductors are coupled in series, a mutual inductance exists between them.

Figure 3.15(a) shows the connection of two inductive coils in series aiding. The flux produced by the two coils are additive in nature as per dot convention. Let L_{12} be is the self-inductance of the coil 1 and L_2 is the self-inductance of coil 2 and M is the mutual inductance between coil 1 and coil 2.

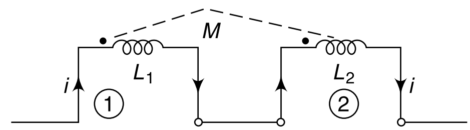


Fig. 3.15(a) *Inductance in series (Cumulative coupling) (flux-aiding)*

For coil 1, the self-induced emf $e_1 = -L_1 \dfrac{di}{dt}$ and the mutual induced emf = $-M \dfrac{di}{dt}$ due to change of current in coil 2.

For coil 2, self-induced emf $e_2 = -L_2 \dfrac{di}{dt}$ and mutually induced emf = $-M \dfrac{di}{dt}$ due to change of current in coil 1.

Therefore the total induced emf of the above connection can be written as

$$e = -L_1 \frac{di}{dt} - L_2 \frac{di}{dt} - 2M \frac{di}{dt}$$

$$= -(L_1 + L_2 + 2M) \frac{di}{dt}.$$

If (L_s) is the equivalent inductance of the coil in series then it can be expressed as

$$e = -L_s \frac{di}{dt}.$$

Now comparing the above two equations, we have

$$\therefore \qquad -L_s \frac{di}{dt} = -(L_1 + L_2 + 2M) \frac{di}{dt}$$

$$\therefore \qquad L_s = L_1 + L_2 + 2M \quad \left[\because \frac{di}{dt} \neq 0\right]. \tag{3.30a}$$

Similarly, for the series opposing, from the Fig. 3.15(b) we can write

$$L_s = L_1 + L_2 - 2M \tag{3.30b}$$

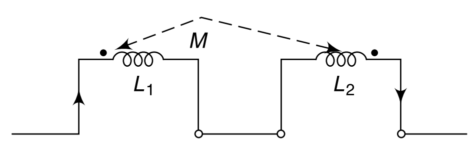


Fig. 3.15(b) *Inductance in series (flux opposing) (Differential coupling)*

If the two coils of self-inductances L_1 and L_2 having mutual inductance M are in series and are far away from each other, so that the mutual inductance between them is negligible, then the net self-inductance is

$$L_s = L_1 + L_2 \quad [\because M = 0].$$

Parallel Connection

When two coils are coupled in parallel and mutual inductance M exists, the equivalent inductance can be calculated as follows:

For Parallel Aiding The Fig. 3.16 shows the connection of parallel connection of two coils where the flux is additive as per dot convention. Using Kirchoff's voltage law, we have

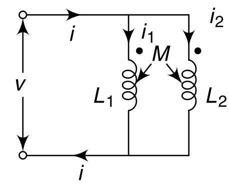


Fig. 3.16 ***Inductances in parallel flux aiding***

$$V = L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} \qquad (3.31a)$$

also,
$$V = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt} \qquad (3.31b)$$

From the above two equations, we can write

$$L_1 \frac{di_1}{dt} + M\frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M\frac{di_1}{dt}$$

Here $\quad i = i_1 + i_2 \quad \therefore \quad i_2 = i - i_1.$

Thus,
$$L_1 \frac{di_1}{dt} + M\frac{d}{dt}(i - i_1) = L_2 \frac{d}{dt}(i - i_1) + M\frac{di_1}{dt}$$

or
$$L_1 \frac{di_1}{dt} + M\frac{di}{dt} - M\frac{di_1}{dt} = L_2\frac{di}{dt} - L_2\frac{di_1}{dt} + M\frac{di_1}{dt}$$

i.e.
$$(L_1 + L_2 - 2M)\frac{di_1}{dt} = (L_2 - M)\frac{di}{dt}$$

$$\therefore \qquad \frac{di_1}{dt} = \frac{L_2 - M}{L_1 + L_2 - 2M}\cdot\frac{di}{dt}. \qquad (3.32a)$$

Similarly
$$\frac{di_2}{dt} = \frac{L_1 - M}{L_1 + L_2 - 2M}\cdot\frac{di}{dt}. \qquad (3.32b)$$

Using equations (3.32a) and (3.32b), we can write,

$$V = L_1\left(\frac{L_2 - M}{L_1 + L_2 - 2M}\right)\frac{di}{dt} + M\left(\frac{L_1 - M}{L_1 + L_2 - 2M}\right)\frac{di}{dt}$$

$$= \frac{L_1 L_2 - L_1 M + L_1 M - M^2}{L_1 + L_2 - 2M}\cdot\frac{di}{dt}$$

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}\frac{di}{dt}. \qquad (3.32c)$$

If L_P be the equivalent inductance of the parallel combination, it can be written as

$$V = L_P\frac{di}{dt} \qquad (3.32d)$$

Comparing equation (3.32c) and (3.32d) we have

$$L_P\frac{di}{dt} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}\left(\frac{di}{dt}\right)$$

$$\therefore \qquad L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} \qquad (3.33a)$$

Equation (3.33a) gives the required equivalent inductance for parallel connections when inductances are in the flux aiding mode.

Similarly, for inductances in parallel opposing, we can write

$$L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} \tag{3.33b}$$

This is the required equivalent inductance in parallel for the flux opposing mode.

Again, if the two coils are far away from each other, $M \cong 0$ and hence

$$L_P = \frac{L_1 L_2}{L_1 + L_2} \tag{3.33c}$$

Therefore by combining parallel aiding and parallel oppositing, the final expression for the *equivalent inductance* is

$$L_P = \frac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M} \tag{3.33d}$$

3.17 Two coils are connected in parallel as shown in Fig. 3.17. Calculate the net inductance of the connection.

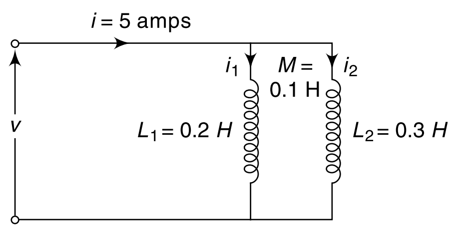


Fig. 3.17

Solution

The net inductance in the given circuit

$$= \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

$$= \frac{0.2 \times 0.3 - (0.1)^2}{0.2 + 0.3 + 2 \times 0.1}$$

$$= \frac{0.06 - .01}{0.7} = \frac{0.05}{0.7} = 0.0714 \text{ H.}$$

3.18 The combined inductance of the two coils connected in series is 0.60 H and 0.40 H, depending on the relative directions of currents in the coils. If one of the coils, when isolated, has a self-inductance of 0.15 H, then find: (a) the mutual inductance, and (b) the co-efficient of coupling K.

Solution

$$L_{\text{additive}} = L_1 + L_2 + 2M$$

$$0.60 = 0.15 + L_2 + 2M \tag{i}$$

$$L_{\text{subtractive}} = L_1 + L_2 - 2M$$

$$0.40 = 0.15 + L_2 - 2M \tag{ii}$$

adding equations (i) and (ii), we have

$$1.0 = 0.\ 3 + 2L_2.$$

$$\therefore \quad L_2 = \frac{(1.0 - 0.3)}{2} = 0.35 \text{ H}$$

Substituting this value of L_2 in equation (i),

$$0.60 = 0.15 + 0.35 + 2M$$

or $\quad M = 0.05$ H

(b) Co-efficient of coupling, $K = \dfrac{M}{\sqrt{L_1 L_2}}$

$$= \frac{0.05}{\sqrt{0.15 \times 0.35}} = 0.218 = 0.22.$$

3.19 Pure inductors each of inductance 3 H are connected as shown in Fig. 3.18. Find the equivalent inductance of the circuit.

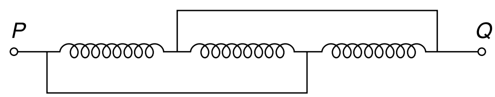


Fig. 3.18 ***The equivalent inductance of the circuit***

Solution

Since all three are in parallel. Hence the equivalent inductance is $L/3 = 3/3 = 1$ H.

3.20 A current of 10 A when flowing through a coil of 2000 turns establishes a flux of 0.6 milliwebers. Calculate the inductance L of the coil.

Solution

Given
$$I = 10 \text{ A}$$
$$N = 2000 \text{ turns}$$
$$\phi = 0.6 \times 10^{-3} \text{ wb}$$
$$L = \text{to be calculated.}$$

We have
$$L = \frac{N\phi}{I} = \frac{2000 \times 0.6 \times 10^{-3}}{10}$$
$$= 0.12 \text{ H.}$$

3.21 Determine the inductance L of a coil of 500 turns wound on an air cored torroidal ring having a mean diameter of 300 mm. The ring has a circular cross section of diameter 50 mm.

Solution

Given $N = 500$ turns

Mean diameter, $D = 300 \text{ mm} = 300 \times 10^{-3}$ m

$l = \pi D = \pi \times 300 \times 10^{-3}$ m

$= 0.942$ m

Cross-sectional diameter $d = 50$ mm

$= 50 \times 10^{-3}$ m

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (50 \times 10^{-3})^2}{4}$$
$$= 1.963 \times 10^{-3} \text{ m}^2.$$

For air cored torroidal ring, $\mu_r = 1$ and $L =$ is to be calculated.

We have, inductance $L = \dfrac{N^2}{\text{Reluctance}}$

$$\left[\because \quad L = \frac{\mu_o \mu_r A N^2}{l}\right.$$

$$= \frac{N^2}{l/\mu_o \mu_r A} = \frac{N^2}{\text{Reluctance}}$$

$$\left.\text{where Reluctance} = \frac{l}{\mu_o \cdot \mu_r \cdot A}.\right]$$

(The concept of reluctance is explained in article 3.18 and 3.24)

Here

$$\text{Reluctance} = \frac{\pi \times 300 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 1.963 \times 10^{-4}}$$

$$= 3.818 \times 10^8 \text{ AT/Wb}$$

$$\therefore \quad L = \frac{N^2}{\text{Reluctance}} = \frac{500 \times 500}{3.818 \times 10^8}$$

$$= 0.000654 \text{ H}$$

$$= 6.54 \times 10^{-4} \text{ H.}$$

3.22 Two coils having 80 and 350 turns respectively are wound side by side on a closed iron circuit of mean length 2.5 m with a cross-sectional area of 200 cm^2. Calculate the mutual inductance between the coils. Consider relative permeability of iron as 2700.

Solution

Given

N_1 = 80 turns
N_2 = 350 turns
l = 2.5 m
A = 200 cm^2 = 200 × 10^{-4} m^2
μ_r = 2700
μ_0 = 4π × 10^{-7} H/m
M = to be calculated.

We have

$$M = \frac{N_1 \cdot N_2}{\text{Reluctance}} \qquad \text{[for two coils of turns } N_1 \text{ and } N_2]$$

$$\text{where reluctance} = \frac{l}{\mu_o \cdot \mu_r \cdot A} \qquad \text{(Ref. article 3.18)}$$

$$= \frac{2.5}{4\pi \times 10^{-7} \times 2700 \times 200 \times 10^{-4}}$$

$$= 36860 \text{ AT/Wb}$$

$$\therefore \text{ Mutual inductance } (M) = \frac{80 \times 350}{36860}$$

$$= 0.760 \text{ H.}$$

3.23 A solenoid 60 cm long and 24 cm in radius is wound with 1500 turns. Calculate:

(a) the inductance

(b) the energy stored in the magnetic field when a current of 5 A flows in the solenoid.

Solution

Given: $l = 60$ cm $= 0.6$ m, $N = 1500$ turns, $A = \pi\,(0.24)^2$ m^2

$\mu = \mu_o\mu_r = 4\pi \times 10^{-7} \times 1$, $I = 5$ A.

(a) *Inductance:*

We know $$L = \frac{\mu \cdot N^2 \cdot A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times (1500)^2 \times \pi\,(0.24)^2}{0.6}$$

$$= 0.8534 \text{ H}$$

(b) *Energy stored:*

We have $$W = \frac{1}{2} LI^2$$

$$= \frac{1}{2}(0.8534)\,(5)^2 = 10.67 \text{ J.}$$

3.24 Two coils of inductance 8 H and 10 H are connected in parallel. If their mutual inductance is 4 H, determine the equivalent inductance of the combination if (a) mutual inductance assists the self-inductance, (b) mutual inductance opposes the self-inductance.

Solution

It is given that

$$L_1 = 8 \text{ H}, L_2 = 10 \text{ H}, M = 4 \text{ H}$$

(a) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M} = \frac{8 \times 10 - 4^2}{8 + 10 - 2 \times 4} = \frac{80 - 16}{18 - 8} = 6.4 \text{ H.}$$

(b) $$L = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M} = \frac{8 \times 10 - 4^2}{8 + 10 + 2 \times 4} = \frac{80 - 16}{26} = 2.46 \text{ H.}$$

3.25 Three coils are connected in series. Their self-inductances are L_1, L_2 and L_3. Each coil has a mutual inductance M with respect to the other coil. Determine the equivalent inductance of the connection. If $L_1 = L_2 = L_3 = 0.3$ H and $M = 0.1$ H, calculate the equivalent inductance. Consider that the fluxes of the coil are additive in nature.

Solution

Let the current i and v_1, v_2, v_3 be the voltage across the three coils.

$$\therefore \quad v_1 = L_1\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v_2 = L_2\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v_3 = L_3\frac{di}{dt} + M\frac{di}{dt} + M\frac{di}{dt}$$

$$v = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3 + 6M)\frac{di}{dt}$$

$\because$ Equivalent inductance = $L_1 + L_2 + L_3 + 6M$.
Now putting the values of L_1, L_2, L_3 and M, we have

$$\begin{aligned}\text{Equivalent inductance} &= 0.3 + 0.3 + 0.3 + 6 \times 0.1 \\ &= 0.9 + 0.6 \\ &= 1.5 \text{ H.}\end{aligned}$$

3.26 In a telephone receiver, the cross-section of the two poles is 10 cm × 0.2 cm. The flux between the poles and the diaphragm is 6×10^{-6} wb. With what force is the diaphragm attracted to the poles? Assume $\mu_0 = 4\pi \times 10^{-7}$.

Solution

Here $\phi = 6 \times 10^{-6}$ wb

$A = 1.0 \text{ cm} \times 0.2 \text{ cm} = 0.2 \text{ cm}^2 = 2 \times 10^{-5} \text{ m}^2$

$$B = \frac{\phi}{A} = \frac{6 \times 10^{-6}}{2 \times 10^{-5}} = 0.3 \text{ wb/m}^2.$$

$\therefore$ Force acting on the diaphragm

$$F = \frac{B^2 A}{2\mu_o} \text{ N}$$

$$= \frac{(0.3)^2 \times 2 \times 10^{-5}}{2 \times 4\pi \times 10^{-7}} = 0.7159 \text{ N.}$$

3.27 Determine the force in kg necessary to separate two surfaces with 200 cm^2 of contact area, when the flux density perpendicular to the surfaces is 1.2 wb/m^2.

Solution

Given $A = 200 \text{ cm}^2 = 2 \times 10^{-2} \text{ m}^2$

and $B = 1.2 \text{ wb/m}^2$

$$\therefore \quad F = \frac{B^2 A}{2\mu_o \times 9.81} \text{ kg} = \frac{(1.2)^2 \times 2 \times 10^{-2}}{2 \times 4 \times \pi \times 10^{-7} \times 9.81}$$

$$= 1167.64 \text{ kg.}$$

3.14 LIFTING POWER OF A MAGNET

Magnetic force is utilised in lifting magnets, operation of brakes, relays, circuit breakers, etc. All such devices have electromagnets and a steel armature. An air gap exists between the forces of the electromagnet and the armature and energy is stored in the air gap.

Let us consider two poles, north and south, as shown in Fig. 3.19.

Here, A = Area of cross-section (in m^2) of each pole

F = Force in Newtons between the poles.

Suppose one of the poles (say N-pole) is pulled apart against this attractive force through a small distance of dx meters. Then, work done in moving the S-pole against the force of attraction is $F\,dx$ joules. However in moving the N-pole through a small distance dx, the volume of the path is increased to $(A\,dx)$m^3.

Also, energy stored E in the magnetic field (air) is given by $E = \dfrac{B^2}{2\mu_o}$ joules/m^3

i.e., Increase in stored energy $= \dfrac{B^2}{2\mu_o} \cdot A \cdot dx$ joules.

As we know, the increase in stored energy = work done, hence we can write $\dfrac{B^2}{2\mu_o} A \cdot dx = F \cdot dx$, when F is the force of attraction or lifting power of the magnet.

$$\therefore \qquad F = \frac{B^2}{2\mu_o} \text{ Newtons} = \frac{B^2 A}{2\mu_o \times 9.81} \text{ kg}$$

and lifting power per unit area $= \dfrac{B^2}{2\mu_o}$ Newton/meter^2

where B is the flux density in wb/m^2.

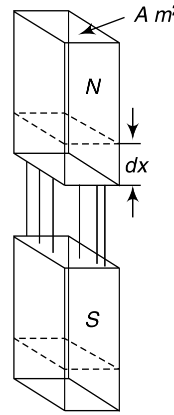


Fig. 3.19 ***Lifting power of magnet***

3.15 CONCEPT OF MAGNETIC CIRCUIT

The path of a magnetic flux is known as magnetic circuit. Similarly, the flow of magnetic flux is almost analogus to the flow of electric current in an electric circuit. In fact, the laws of magnetic circuit are almost similar (but not exactly same) to those electric circuits. It is known to us that to carry electric current in an electric circuit, usually aluminium or copper wires are used because the resistance of these materials is comparatively much lower than other materials. Similarly, to carry magnetic flux, iron or soft steel circuits are used as "opposition" of these materials to flux is low in comparison with other materials.

The study of magnetic circuit concepts is essential in the design analysis and application of electromagnetic devices like transformers, electromagnetic relays, electrical machines, etc.

3.16 CONCEPT OF MAGNETO-MOTIVE FORCE (MMF)

It is the amount of work done (in joules) required to carry a unit magnetic pole once through the entire magnetic field. In fact it is a kind of magnetic flux through a magnetic circuit and is called the *magneto motive force*. It's unit being ampere-turns (AT), it is actually measured by the product of number of turns N in the coil of a magnetic circuit, and the current I in amperes required to produce the magneto motive force.

Thus, $\qquad$ F(MMF) = NI AT/m

Where N = number of turns in the coil and I = current through the coil (in Amps). It should be noted that through the unit of MMF is ampere turn (AT), it's dimension is taken as ampere since N is dimensionless.

3.17 MAGNETIC FIELD INTENSITY

The MMF for unit length (along the path of magnetic flux) is defined as the *magnetic field intensity* and it is designated by the symbol H. The magnetic field intensity thus can be expressed as

$$H = \frac{\text{Magneto motive force}}{\text{Mean length of the magnetic path}}$$

i.e.
$$H = \frac{F}{l} = \frac{NI}{l} \text{ AT/m.} \tag{3.34}$$

where l is the *mean length* of the magnetic circuit in meters. Magnetic field intensity is also termed as *magnetising force* or magnetic *field strength.*

3.18 CONCEPT OF RELUCTANCE

It is designated by the symbol "*S*" and is analogus to resistance of an electric circuit. Flux in a magnetic circuit is limited by *reluctance*. Thus, reluctance S is a measure of the opposition offered by a magnetic circuit to the establishment of magnetic flux.

It is directly proportional to the length and inversely proportional to the area of cross section of the magnetic path.

$$S \propto \frac{l}{a}$$

$$\therefore \quad S = \frac{l}{\mu_o \cdot \mu_r \cdot a} \tag{3.35}$$

The unit of reluctance is AT/Wb.
For air, vaccum and non-magnetic materials $\mu_r = 1$

$$\therefore \quad S = \frac{l}{\mu_o \cdot a} \tag{3.36}$$

the reciprocal of reluctance is called the "permeance". It is designated by the symbol A.

$$\therefore \quad \text{Permeance } (= A) = \frac{1}{S} \text{ wb/A [or henry (H)]} \tag{3.37}$$

Concept of magnetic reluctance is based on the following assumptions:

(a) B-H curve of the magnetic core is linear
(b) Leakage flux is of negligible order.

3.19 PERMEABILITY AND RELATIVE PERMEABILITY

Permeability of a material μ is defined as *its conducting power for magnetic lines of force.* It is the ratio of the flux density B produced in a material to the magnetic field strength H, i.e. $\mu = \dfrac{B}{H}$.

Permeability of free space or vaccum is minimum and its value in SI units is $4\pi \times 10^{-7}$ henry/meter.

It may be noted here that flux density (B) is usually expressed as flux (Wb) per unit area (sq. m) and its unit in SI system is Tesla (T).

$\therefore \qquad 1 \text{ T} = 1 \text{ Wb/m}^2$

Relative Permeability

When the magneto motive force is applied to a ferromagnetic material, the flux produced is very large compared with that in air, free space (vaccum) or a non-magnetic material. *The ratio of the flux density produced in material to the flux density produced in air or a non-negative material by the same magnetic field intensity is called the relative permeability of that material.* It is designated by the symbol μ_r. The relative permeability of vaccum is taken as unity.

Thus, permeability of any medium,

$$\mu = \text{Absolute permeability of air} \times \text{Relative permeability}$$
$$= \mu_o \cdot \mu_r = 4\pi \times 10^{-7} \times \mu_r \text{ Wb/AT.}$$

(with special nickel-iron alloys the value of μ_r may be as high as 2×10^5 whereas for most commonly used magnetic materials the values of μ_r is much smaller).

3.28 A coil of 600 turns and of resistance of 20 Ω is wound uniformly over a steel ring of mean circumference 30 cm and cross-sectional area 9 cm^2. It is connected to a supply of 20 V (DC). If the relative permeability of the ring is 1,600 find (a) the reluctance, (b) the magnetic field intensity, (c) the mmf, and (d) the flux.

Solution

Here N = 600 turns, resistance of the coil is 20 Ω, l = 30 cm = 0.3 m, A = 9 cm^2 = 9×10^{-4} m^2, relative permeability μ_r = 1600 and $\mu_o = 4\pi \times 10^{-7}$.

(a) Reluctance $S = \dfrac{l}{\mu_o \cdot \mu_r \cdot a} = \dfrac{0.3}{4\pi \times 10^{-7} \times 1600 \times 9 \times 10^{-4}}$

$$= 1.657 \times 10^5 \text{ At/wb}$$

(b) The magnetic field intensity

$$H = \left(\frac{NI}{l}\right) = \frac{600 \times 1}{0.3} = 2000 \text{ AT}$$

$$\left(\text{where} \quad I = \frac{V}{\text{resistance of the coil}} = \frac{20}{20} = 1 \text{ Amp}\right)$$

(c) MMF = (NI) = 600 × 1 = 600 AT

(d) $\text{Flux}(\phi) = \dfrac{\text{mmf}}{\text{reluctance}} = \dfrac{NI}{S} = \dfrac{600 \times 1}{1.657 \times 10^5} = \dfrac{600}{1.65 \times 10^5}$

$$= 3.62 \times 10^{-3} \text{ wb} = 3.62 \text{ m Wb.}$$

3.29 What is the value of the net mmf acting in the magnetic circuit shown in Fig. 3.20.

Solution

Net mmf acting in the magnetic circuit = mmf of all the three coils = $N_1I_1 + N_2I_2 + N_3I_3$ = (10 × 2 + 50 × 1 + 300 × 1)AT = 370 AT.

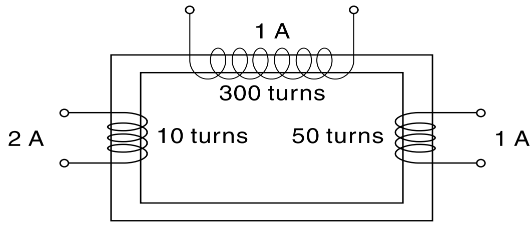


Fig. 3.20

[It may be observed here that since the flux in all the coils are additive within the core, hence AT's are additive.]

3.30 A mild-steel ring having a cross-sectional area of 400 mm^2 and a mean circumference of 400 mm has a coil of 200 turns wound uniformly around it. [given μ_r = 300]. Determine:

(a) the reluctance S of the ring,
(b) the current required to produce a flux of 800 μ Wb in the ring.

Solution

(a) Flux density B in the ring is $\dfrac{800 \times 10^{-6}}{400 \times 10^{-6}}$ = 2 Wb/m^2.

$\therefore$ The reluctance S of the ring is $\dfrac{0.4}{300 \times 4\pi \times 10^{-7} \times 0.4 \times 10^{-3}}$ = 2.65 × 10^6 A/Wb

Again we know $\phi = \dfrac{\text{mmf}}{\text{Reluctance}}$

$$800 \times 10^{-6} = \frac{\text{mmf}}{\text{Reluctance}}$$

$$\therefore \quad \text{mmf} = 800 \times 10^{-6} \times 2.65 \times 10^{6}$$
$$= 2.122 \times 10^{3} \text{ AT}$$

and magnetizing current is $\dfrac{\text{mmf}}{\text{No. of turns}} = \dfrac{2.122 \times 10^3}{200}$

$$= 10.6 \text{ A.}$$

3.31 A magnetic circuit having 150 turns coils and the cross-sectional area and length of the magnetic circuit are 5 × 10^{-4} m^2 and 25 × 10^{-2} m respectively. Determine H and the relative permeability μ_r of the core when the current is 2 A and the total flux is 0.3 × 10^{-3} Wb.

Solution

When I = 2 A

$$\text{mmf} = NI = 150 \times 2 = 300 \text{ A/T}$$

$$\therefore \quad H = \frac{NI}{l} = \frac{300}{25 \times 10^{-2}} = 1200 \text{ A/m}$$

$$B = \frac{\text{flux } (\phi)}{\text{area } (a)} = \frac{0.3 \times 10^{-3}}{5 \times 10^{-4}} = 0.6 \text{ T}$$

$$\mu = \frac{B}{H} = \frac{0.6}{1200} = 500.00 \times 10^{-6} \text{ H/m}$$

$$\mu_r = \frac{\mu}{\mu_o} = \frac{500.00 \times 10^{-6}}{4\pi \times 10^{-7}} = 3.9788 \times 10^2 = 397.88.$$

3.32 An air cored coil has 500 turns. The mean length of magnetic flux path is 50 cm and the area of cross-section is 5×10^{-4} m^2. If the exciting current is 5 A, determine (a) H (b) the flux density and (c) the flux (ϕ).

Solution

$$\text{mmf} = NI = 500 \times 5 = 2500 \text{ A}$$

Given,

$$l = 50 \text{ cm} = 0.5 \text{ m}$$

$$a = 5 \times 10^{-4} \text{ m}^2$$

(a) $\therefore\ H = \dfrac{NI}{l} = \dfrac{2500}{0.5} = 5000.00$ A/m.

(b) B = flux density = $\mu \cdot H = \mu_r \cdot \mu_0 \cdot H$

$= \mu_o \cdot H \quad [\because\ \mu_r = 1]$

$= 4\pi \times 10^{-7} \times 5000.00$

$= 6.283 \times 10^{-3}$ T.

(c) Flux (ϕ) = $B \times a = 6.283 \times 10^{-3} \times 5 \times 10^{-4}$

$= 3.1415 \times 10^{-6}$ Wb.

3.33 Two identical co-axial circular loops carry a current I each circulating in the same direction. If the loops approach each other, the current in each decreases. Justify the statement.

Solution

When the loops approach each other, the field becomes strong, which should not be allowed in accordance with Lenz's law. So, the current in both should be in such a way that the field decreases and hence I decreases.

3.34 An iron ring 10 cm mean circumference is made from a round iron of cross-section 10^{-3} m^2. It's relative permeability is 500. If it is wound with 250 turns, what current will be required to produce a flux of 2×10^{-3} Wb?

Solution

The lines of magnetic flux follow the circular path of the iron so that

$$l = 100 \text{ cm} = 1 \text{ m}$$

$$a(\text{area}) = 10^{-3} \text{ m}^2$$

$$\therefore \quad \text{Reluctance } S = \frac{1}{\mu_r \mu_o a} = \frac{1}{(500 \times 4\pi \times 10^{-7} \times 10^{-3})}$$

$$= 1.59 \times 10^6 \text{ A/Wb.}$$

Given $\quad$ Flux (ϕ) = 2×10^{-3} Wb

$$\therefore \quad H = \phi \cdot S = 2 \times 10^{-3} \times 1.59 \times 10^6$$

$$= 3.1847 \times 10^3 \text{ AT.}$$

As we know $H = NI$

$$\therefore \quad I = \frac{H}{N} = \frac{3.1847 \times 10^3}{250}$$

$$= 12.738 \text{ A.}$$

3.35 An air gap 1.1 mm long and 40 sq. cm in cross-section exists in a magnetic circuit. Determine (a) Reluctance S of the air-gap, and (b) mmf required to create a flux of 10×10^{-4} Wb in the air gap.

Solution

(a) $$\text{Reluctance } (S) = \frac{l}{\mu_r \cdot \mu_o a} = \frac{l}{m_o \cdot a} \quad [\because \mu_r = 1]$$

$$= \frac{1.1 \times 10^{-3}}{4\pi \times 10^{-7} \times 40 \times 10^{-4}} = 2.1885 \times 10^5 \text{ A/m}$$

(b) Since $\phi = 10 \times 10^{-4}$ Wb (given)

$$\therefore \quad \text{mmf} = \text{flux} \times \text{reluctance}$$
$$= 10 \times 10^{-4} \times 2.1885 \times 10^5$$
$$= 218.85 \text{ AT.}$$

3.36 An iron ring of mean length 110 cm with an air gap of 1.5 mm has a winding of 600 turns. The relative permeability of iron is 600. When a current of 4 A flows in the winding, calculate the flux density B. Do not consider fringing.

Solution

Given that $l_i = 110 \text{ cm} - 0.15 \text{ cm}$
$= 109.85$ cm
$= 1.0985$ m
$l_g = 1.5 \text{ mm} = 1.5 \times 10^{-3}$ m.
$N = 600$ turns, $\mu_r = 600$ (given)
$I = 4$ A

$$\therefore \text{ Flux density } B = \frac{\mu_o \cdot NI \cdot \mu_r}{l_i} + \frac{m_o\, NI \cdot 1}{l_g}$$

$$= \mu_o NI \left(\frac{\mu_r}{l_i} + \frac{1}{l_g} \right)$$

$$= 4\pi \times 10^{-7} \times 600 \times 4 \times \left[\frac{600}{1.0985} + \frac{1}{1.5 \times 10^{-3}} \right] \text{Wb/m}^2$$

$$= 3.0159 \times 10^{-3} \times (5.46 \times 10^2 + 6.666 \times 10^2)$$
$$= 3.657 \text{ Wb/m}^2.$$

3.20 DEFINITION OF AMPERE

Consider two parallel wires separated by 1 m in space carrying a current of 1 A each, then $I_1 = I_2 = 1$ A and $d = 1$ m. From the expression of magneto motive

force developed between two parallel conductors, we have $dF/dl = 2 \times 10^{-7}$ Newton per meter.
This is used to formally define the unit 'ampere' of electric current.

Therefore, ampere is defined as the current, which when flowing in each of the two infinitely long parallel wires (conductors) of negligible cross-section and placed 1 meter apart in vaccum produces on each wire a force of 2×10^{-7} Newton per meter length.

3.21 B-H CHARACTERISTICS

The graph between the flux density B and field intensity H of a magentic material is called B-H or familiar magnetization curve. Figure 3.21 shows a typical B-H curve for an iron specimen. As seen from the curve it can be divided into four distinct regions OA, AB_1, B_1C and the region beyond C.

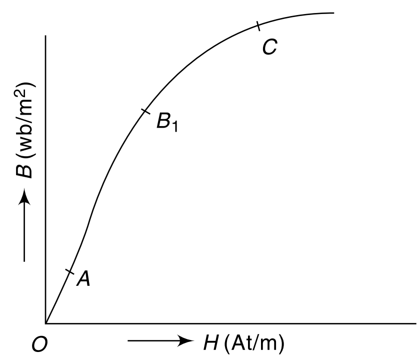


Fig. 3.21 ***B-H Curve for an iron sample***

The slope of the B-H curve for iron (Fig. 3.21) can be explained as follows:

(a) In the region OA (in step region), magnetic field strength H (another name is magnetizing force) is too weak to cause any appreciable alignment of domains (or elementary magnets). Consequently, the increase in flux density B is small. In the neighbourhood of the origin the graph is a straight line through the origin, and the slope gives the initial permeability.

(b) In the region AB_1, more and more domains get aligned as H increases, consequently, B increases almost linearly with H.

(c) In the region B_1C only a fewer domains are left unaligned, consequently, the increase in B with H is very small.

(d) Beyond C, i.e. beyond the knee zone, no more domains are left unaligned and the iron material is said to be magnetically saturated. This upper portion of the curve is represented with fair accuracy by Frohlich's equation

$$B_S = B - \mu_o H = \frac{H}{a + bH}$$

$$\therefore \qquad \frac{1}{B_S} = \frac{a + bH}{H} = \frac{a}{H} + b$$

where (a) is a hardness constant while $\left(b = \dfrac{1}{B_S}\right)$. Here B_S is the saturation flux density.

A magnetic material is said to be magnetically saturated when it fails to attain a still higher degree of magnetization, even when the magnetizing power is increased enormously. The slope of the curve beyond C shows that the increase in

B with increase in H is nominal. This point C is called the *point of saturation* for the material.

The slope of the B-H curve at any point E is given by $\tan\theta = B/H$, again $B = \mu_o$

$$\mu_r H;\ \mu_r = 1/\mu_o \cdot \tan\theta \left[\text{since } \frac{B}{H} = \tan\theta\right]$$

Thus, (μ_r) is proportional to the slope of the B-H curve at any point. Starting from as definite value at the origin, the slope increases as B increases until it becomes a maximum. It then gradually decreases as B increases further. The slope becomes almost zero in the saturation region when the curve becomes almost horizontal. The B-H curve shows that the permeability μ_r of a magnetic material changes with the flux density B. It is important to note that the lesser the impurities like carbon, sulphur and phosphorus in iron (the magnetic material) higher is its permeability.

Importance of B-H curve

(a) It helps in resulting the magnetic material for a specific application.
(b) It helps in making practical magnetic calculations in the design or analysis of the magnetic circuits.

3.22 FERROMAGNETIC MATERIALS

Ferromagnetism, characterised by the strong attraction or repulsion of one magnetised body by another. In fact, in some materials, the permanent atomic magnetic moments have a strong tendency to align themselves even without any external field. These materials are called *ferromagnetic materials* and permanent magnets are made from them. The force between the neighbouring atoms, responsible for their alignment, and it can only be explained on the basis of Bohr's theory of atomic structure.

According to this theory, the electrons revolve around the nucleus in fixed orbits. Since these electrons are in motion they constitute electric currents. These currents produce magnetic fields called *orbital magnetic* fields. In addition to this, each electron spins on is axis as it revolves in an orbit. (the spin of an electron is similar to the spin of the earth). A spinning electron has a charge in motion and, therefore, constitutes an electric current. This current produces a magnetic field called the *spin magnetic* field.

For an individual atom these fields are very weak. Strong magnetic fields can be produced if the atoms are grouped in a material in such a way that their orbital and spin fields reinforce one another. In ferromagnetic materials there is an appreciable interaction between neighbouring atoms. Atoms do not act singly but in groups called *domains*. Each domain contains between 10^9 and 10^{20} atoms. If a magnetic field is applied, the domains which are aligned along the direction of the field grow in size and those opposite to it get reduced. Also, domains may orient themselves in favour of the applied field. Consequently the resultant magnetic flux density inside the material is much greater than the flux density of the applied field. Examples of ferromagnetic materials are iron, cobalt and nickel but there many ferromagnetic alloys, some of which do not even possess iron as one of their components. The ferromagnetic materials are all solids.

3.23 TYPES OF MAGNETIC MATERIALS

From an engineering point of view, magnetic materials can be broadly classified into two groups, namely:

(a) Soft Magnetic Materials i.e. soft iron (3.5 to 4.5% silicon content) and are used in transformers, electric machines and taperecorder tapes. Their magnetization can be changed rapidly. Its susceptibility, permeability and retentivity are greater while coercivity and hysteresis loss per cycle are smaller than those of steel. These possess a uniform structure (i.e. well-aligned crystal grains).

(b) Hard Magnetic Materials like steel or alloy alnico (Al + Ni + Co) are used for permanent magnets, coercivity and curie temperature for these materials are high and their retentivity is low. Their hysteresis loops are generally characterized by a broad hysteresis loop of large area compared to soft magnetic materials. Its demagnetization takes place with difficulty.

3.24 MAGNETIC CIRCUIT LAWS

The path of the magnetic flux is called the magnetic circuit. Just as the flow of electric current in an electric circuit necessiates the presence of an emf, so the establishment of a magnetic flux requires the presence of a mmf. In fact there is a close mathematical analogy between magnetic and d.c. resistive circuits. For a d.c. resistive circuits, Ohm's law relationship is

$$I = \frac{\text{e.m.f}}{\text{resistance}} = \frac{V}{R}, \quad \text{also} \quad R = \rho \cdot \frac{l}{a}\,;\, \rho \cdot \frac{1}{\sigma}$$

where σ is the conductivity.

For a magnetic circuit, magentic flux ϕ is equal to mmf divided by reluctance,

$$\text{i.e. } \phi = \frac{\text{mmf}}{\text{reluctance}} = \frac{F}{S} \text{ (in symbols)} \tag{3.38}$$

Again we know for electric circuits $R = \rho \cdot \dfrac{l}{a}$ and for magnetic circuits

$$S = \frac{1}{\mu_o \cdot \mu_r} \cdot \frac{l}{a}$$

Therefore the definition of reluctance S is obtained from the above analogy. Reluctivity is *specific reluctance* and is comparable to resistivity ρ which is *specific resistance*.

Again ϕ can be written as from the equation 3.38

$$\text{Flux} = \phi = \frac{\text{mmf}}{\text{reluctance}} = \frac{\text{AT}}{\dfrac{1}{\mu_o \cdot \mu_r} \cdot \dfrac{l}{a}}$$

$$\therefore \qquad \text{AT} = \frac{\phi}{a} \cdot \frac{l}{\mu_o \cdot \mu_r} = \frac{B \cdot l}{\mu_o \cdot \mu_r} = H \times l \tag{3.39}$$

= field strength of the particular magnetic path × length of this particular magnetic path.

Therefore by analogy, σ the conductivity of the conductor is equal to the $(\mu_o \cdot \mu_r)$ in the magnetic circuit.

No name is in common use for the unit of reluctance; it is evidently measured in ampere-turns per weber of magnetic flux in the circuit.

By analogy, the laws of resistance in series and parallel also hold good for reluctances in a composite magnetic circuit.

In case of a composite electric circuit we have for series connected conductors

$$R_{\text{Total}} = R_1 + R_2 + R_3 + \cdots + R_n$$

Similarly, with a composite magnetic circuit, we have to substitute S for R.

i.e. $$S_{\text{Total}} = S_1 + S_2 + S_3 + \cdots + S_n \tag{3.40}$$

Again in case of parallel magnetic circuit, the same mmf is applied to each of the parallel paths and the total flux divides between paths in inverse proportion to their reluctances.

$$\phi = \phi_1 + \phi_2 + \phi_3 + \cdots + \phi_n$$

The total reluctance S_{Total} of a number of reluctancs in parallel is given by

$$\frac{1}{S_{\text{Total}}} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3} + \cdots + \frac{1}{S_n} \tag{3.41}$$

Similarly, *permeances* (reciprocal of reluctances) in series and parallel obey the same rules as electrical conductances.

3.25 COMPARISON BETWEEN ELECTRIC AND MAGNETIC CIRCUITS

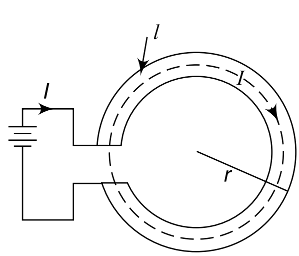
<table>
<tr><td>1.</td><td>Basic Model</td></tr>
<tr><td>The toroidal copper ring (Fig. 3.22) is assumed open by an infinitesmal amount with the ends connected to a battery.


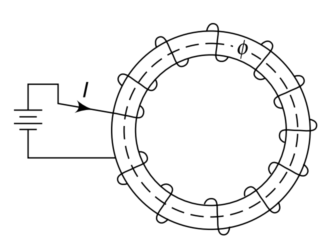
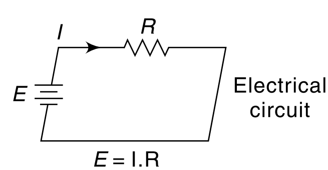
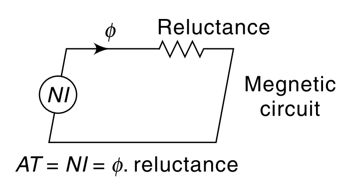
Fig. 3.22 (a)</td><td>The toroidal iron ring is assumed wound with N turns of wire with a current i flowing through it. The magneto motive force creates the flux ϕ.


Fig. 3.22 (b)</td></tr>
<tr><td>2.</td><td>Driving Forces</td></tr>
<tr><td>Applied battery voltage is E or V</td><td>Applied Ampere turns is AT or NI</td></tr>
</table>

3.	**Response**
Current $= \dfrac{\text{Driving force}}{\text{Eletric resistance}}$ $\therefore \quad I = \dfrac{E}{R}$	Flux $= \dfrac{\text{Driving force}}{\text{Magnetic resistance}}$ $= \dfrac{AT}{R}$ where $R = \dfrac{l}{a \cdot \mu_o \cdot \mu_r}$ AT/Wb where a = area of magnetic path
4.	**Impedance**
Resistance $R = \rho \times \dfrac{l}{a}$, here $l = 2\pi r$ = mean length of turn of the toroid and a is the toroidal cross-section	Reluctance (= R) $= \dfrac{l}{\mu_o \mu_r a} = \dfrac{l}{\mu \cdot a}$ where $l = 2\pi r$ = mean length of turn of the toroid and a is the toroidal cross-sectional area.
5.	**Equivalent Circuit**
Here electromotive force E is the driving force and electric current flows through the circuit.  **Fig. 3.23 (a)**	Here electromoive force is the driving force and magnetic flux does not flows and it only links with the coil.  **Fig. 3.23 (b)**
6.	**Field Intensity**
Electric field intensity: with the application of the voltage E to the homogeneous copper toroid, an electric potential gradient ε produced within the material and is given by $\varepsilon = \dfrac{E}{l} = \dfrac{E}{2\pi n}$ V/m This electric field must occur in a closed loop path if it is to be maintained. If then follows the closed line integral of ε is equal to the battery voltage E. Thus $\oint \varepsilon \times dl = E$	Magnetic field intensity: when a magnetomotive force is applied to the homogeneous iron toroid, there is produced within the material a magnetic potential gradient given by $H = \dfrac{NI}{l} = \dfrac{NI}{2\pi r}$ AT/m As already pointed out in connection with Ampere's circuited law, the closed line integral of H equals the enclosed magnetomotive force. Thus $\oint H \times dl = NI$

7. Voltage drop	mmf drop
Voltage drop is given $V = I \times R$ where R is the resistance of the copper toroid between the two points.	NI = ϕ. Reluctance where R is the reluctance of the iron toroid between the points.
8. Current density	**Flux density**
Current density is the amount of ampere per unit area = $\frac{\text{Amp.}}{m^2} = \frac{I}{a}$	Flux density is expressed as webers per unit area i.e. $\frac{\text{Wb}}{\text{m}^2} = \frac{\phi}{a}$.

3.26 DISTINCTION BETWEEN MAGNETIC AND ELECTRIC CIRCUITS

1. In an electric circuit current does not flow in air, unless the dielectric strength between the curent-carrying conductor and nearest earth fails but flux in a magnetic circuit can flow in air.
2. Current flows (actual flow of electrons) in an electric circuit but flux does not flow because magnetic flux lines are imaginary.
3. Based on property, some materials act as insulators and some as conductors but there is no such material as 'insulators' to magnetic circuits.
4. The resistance in electric circuit changes with temperature but at constant ampere turns (AT), reluctance does not change with temperature.
5. The electric current can be confined to flow in an accurately defined path but there is no good magnetic insulator to confine all the magnetic flux to one prescribed path in a magnetic circuit. There is always some leakage flux.

3.37 An iron ring has a mean circumference of 80 cm and having cross-sectional area of 5 cm^2 and having coil of 150 turns. Using the following data, calculate the existing current for a flux of 6.4×10^{-4} Wb. Also calculate the relative permeability (μ_r).

B (Wb/m^2)	:	0.9	1.1	1.2	1.3
H (A/m)	:	260	450	600	820

Solution

It is given that $\phi = 6.4 \times 10^{-4}$ Wb

$$\therefore \text{ flux density } B = \frac{\phi}{\text{area}} = \frac{6.4 \times 10^{-4}}{5 \times 10^{-4}} = 1.28 \text{ Wb/m}^2$$

Assuming B-H curve to be linear in the range from 1.2 to 1.3 Wb/m^2,

$$H = 600 + \frac{820 - 600}{1.3 - 1.2} \times (1.28 - 1.2)$$

$$= 776 \text{ A/m.}$$

$$\therefore \qquad \text{mmf} = 776 \times \frac{80}{100} = 620.8 \text{ A}$$

$$\therefore \qquad I = \frac{620.8}{150.0} = 4.138 \text{ A}$$

We know $\qquad B = \mu \cdot H = \mu_o \cdot \mu_r \cdot H$

$$\therefore \qquad \mu_r = \frac{B}{\mu_o \cdot H} = \frac{1.28}{4\pi \times 10^{-7} \times 776} = 1312.089$$

3.38 In the magnetic circuit shown in Fig. 3.24, the cross-sectional area of limbs Q and R are 0.01 m^2 and 0.02 m^2 respectively and the lengths of air-gap are 1.1 mm and 2.1 mm respectively, are cut in the limbs Q and R. If the magnetic medium can be assumed to have infinite permeability and the flux in the limb is 1.5 Wb, calculate the flux in the limb P.

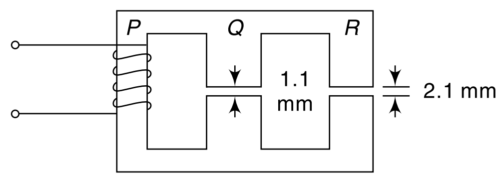


Fig. 3.24 *Magnetic circuit of Ex. 3.38*

Solution

It is given that

Area of cross-section of limb Q = 0.01 m^2

Area of cross-section of limb R = 0.02 m^2

Length of air-gap = 1.1 mm for limb Q and 2.1 mm for limb R

Flux in limb Q = 1.5 Wb

As because the permeability of the magnetic medium is infinity, reluctance of the given iron path is zero. The electrical equivalent is shown in Fig. 3.25.

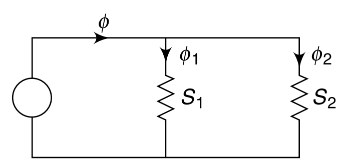


Fig. 3.25 *Electrical equivalent*

Now if we assume S_1 is the reluctance of air-gap of limb Q and S_2 is the reluctance of the air-gap of limb R respectively. Let ϕ_1 is the flux across the air-gap of limb Q and ϕ_2 is the flux across the air-gap of limb R.

$$\therefore \qquad S_1 \times \phi_1 = S_2 \times \phi_2$$

$$\frac{l_1}{\mu_o \times a_1} \times \phi_1 = \frac{l_2}{\mu_o \times a_2} \times \phi_2$$

$$\therefore \qquad \phi_2 = \frac{a_2}{a_1} \times \frac{l_1}{l_2} \times \phi_1$$

$$= \frac{0.02}{0.01} \times \frac{1.1}{2.1} \times 1.5$$

$$= 1.5714 \text{ Wb.}$$

$\therefore$ Flux in the limb $P = \phi_1 + \phi_2$

$$= 1.5 + 1.5714 = 3.0714 \text{ Wb.}$$

3.39 A ring, made of steel has a rectangular cross-sectional area. The outer diameter of the ring is 25 cm while the inner diameter is 20 cm, the thickness being 2 cm. The ring has a winding of 500 turns and when carrying a current of 3A, produces a flux density of 1.2 T in the air gap produced when the ring is cut to have an air gap of 1 mm length (Fig. 3.26). Find (a) the magnetic field intensity of the steel ring and in the air gap,

(b) relative permeability of the magnetic material, (c) total reluctance of the magnetic circuit, (d) inductance of the coil and (e) emf induced in the coil when the coil carries a current of $i_{(ac)} = 5 \sin 314\,t$.

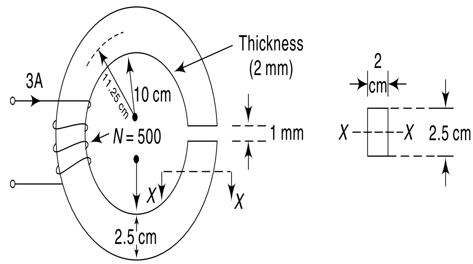


Fig. 3.26

Solution

$$NI = 500 \times 3 = 1500 \text{ AT}$$
$$B_{steel} = B_{gap} = 1.2 \text{ T}$$

(a) $\therefore \quad H_{gap} = \dfrac{1.2}{4\pi \times 10^{-7}} = 9.55 \times 10^5$ AT/m

Since $\quad NI = H_{gap} \times l_g + H_{core} \times l_{core}$,

where l_{gap}(mean length of gap) = 1 mm = 1×10^{-3} m,

$$l_{core} = 2\pi \times \left(10 + \frac{2.5}{2}\right) = (2\pi \times 11.25) \text{ cm}$$
$$= 2\pi \times 11.25 \times 10^{-2} \text{ m.}$$

We can write,

$$1500 = 9.55 \times 10^5 \times 10^{-3} + H_{core} \times 2\pi \times 11.25 \times 10^{-2}$$

i.e. $\quad H_{core} = 771.20$ AT/m.

(b) Also, $H_{core} = \dfrac{B_{core}}{\mu_o \mu_r}$; $\quad \therefore \quad \mu_r = \dfrac{1.2}{4\pi \times 10^{-7} \times 771.20} = 1238.2$

(c) $S = S_1 + S_2 = \dfrac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 2 \times 2.5 \times 10^{-4}}$

$$+ \frac{2\pi \times 11.25 \times 10^{-2}}{4\pi \times 10^{-7} \times 1238.2 \times 2 \times 2.5 \times 10^{-4}}$$
$$= 2.5 \times 10^6 \text{ AT/Wb}$$

(d) $L = \dfrac{N\phi}{I} = \dfrac{500 \times (B \times a)}{3} = \dfrac{500 \times (1.2 \times 2.5 \times 2 \times 10^{-4})}{3} = 0.1$ H

(e) $E = L\dfrac{di}{dt} = 0.1 \times \dfrac{d}{dt}(5 \sin 314\,t) = 0.5\dfrac{d}{dt} \sin(314\,t)$

$$= 157 \cos 314\,t.$$

3.40 An iron ring of circular cross-section of 5×10^{-4} m^2 has a mean circumference of 2 m. It has a saw-cut of 2×10^{-3} m length and is wound with 800 turns of wire. Determine the exciting current when the flux in the air gap is 0.5×10^{-3} Wb. [given: μ_r of iron = 600 and leakage factor is 1.2] Assume areas of air gap and iron are same.

Solution

The flux linking with the iron ring is

$$\phi_{iron} = \phi_{air\text{-}gap} \times \text{Leakage factor}$$
$$= 0.5 \times 10^{-3} \times 1.2 \qquad \text{[As the leakage factor is given as 1.2]}$$
$$= 0.6 \times 10^{-3} \text{ Wb.}$$

Again we know,

Ampere turns required = NI

$$= \left[\frac{\phi_{\text{iron}} \times l_{\text{iron}}}{\mu_r \times \mu_o \text{ of iron} \times \text{area}} + \frac{\phi_{\text{air-gap}} \times l_{\text{air}}}{\mu_o \text{ of air} \times \text{area}}\right] \qquad [\because \mu_r \text{ for air} = 1]$$

$$\therefore \qquad I = \frac{1}{800}\left[\frac{0.6 \times 10^{-3} \times 2}{4 \times \pi \times 10^{-7} \times 600 \times 5 \times 10^{-4}} + \frac{0.5 \times 10^{-3} \times 2 \times 10^{-3}}{4\pi \times 10^{-7} \times 5 \times 10^{-4}}\right]$$

$= 5.95$ A.

3.41 An iron ring of 100 cm mean circumference is made from round iron of cross-section 10 cm^2. It's relative permeability is 500. Now a saw-cut of 2 mm wide has been made on it. It is wound with 200 turns. Determine the new current required to produce a flux of 0.12×10^{-2} Wb in the air-gap, given that the leakage factor x is 1.24, and that the relative permeability of the iron under the new condition is 350.

Solution

Given:

$$\phi_{\text{air-gap}} = 1.2 \times 10^{-3} \text{ Wb}$$
$$l_{\text{air-gap}} = 0.2 \times 10^{-2} = 2 \times 10^{-3} \text{ m}$$
$$a_{\text{air-gap}} = 10^{-3} \text{ m}^2$$
$$\mu_{\text{air}} = 1.$$

$\therefore$ Reluctance in the air-gap is

$$\frac{2 \times 10^{-3}}{(4\pi \times 10^{-7} \times 1 \times 1.2 \times 10^{-3})} = 1.325 \times 10^6 \text{ AT/Wb}$$

$$\phi_{\text{iron-path}} = 1.25 \times 1.2 \times 10^{-3} \text{ Wb}$$
$$= 1.5 \times 10^{-3} \text{ Wb}$$
$$l_{\text{iron-path}} = 0.998 \text{ m}$$
$$a_{\text{iron-path}} = 10^{-3} \text{ m}^2$$
$$\mu_{\text{iron-path}} \text{ (new)} = 350.$$

$$\therefore \text{ Reluctance in the iron-path} = \frac{998 \times 10^{-3}}{(350 \times 4\pi \times 10^{-7} \times 1.2 \times 10^{-3})}$$

$$= 1890912.3 \text{ AT/Wb} = 1.89 \times 10^6 \text{ AT/Wb}.$$

As we cannot add up the values of air-gap reluctance and iron-path reluctance to get the total reluctance, we therefore calculate in this way.

$$H = H_{\text{air-gap}} + H_{\text{iron-path}}$$
$$= (\text{Reluctance of air-gap} \times \text{Flux in this path})$$
$$+ (\text{Reluctance of iron-path} \times \text{Flux in this path})$$
$$= (1.325 \times 10^6 \times 1.2 \times 10^{-3}) + (1.89 \times 10^6 \times 1.5 \times 10^{-3})$$
$$= 4.425 \times 10^3 = 4425$$

$$\therefore \qquad \text{Current } (= I) = \frac{4,425}{200} = 22.125 \text{ A}.$$

3.42 A magnetic circuit shown in Fig. 3.27 is constructed of wrought iron:

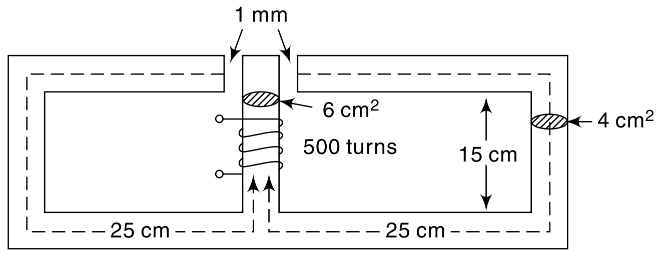


Fig. 3.27 ***Magnetic circuit of Ex. 3.42***

The cross-section of the central limb is 6 cm^2, and each outer limb is 4 cm^2. If the coil is wound with 500 turns, determine the exciting current required to set up a flux of 1.0 m Wb in the central limb.

B-H curve of wrought iron are:

B (Wb/m^2)	1.25	1.67
H (AT/m)	600	2,100

Solution

Given that Flux (ϕ_1) in the central limb = 1.0×10^{-3} Wb
Area (a_1) of the central limb = 6×10^{-4} m^2
l_1 = 15 cm = 0.15 m

$$B_1 = \frac{\phi_1}{a_1} = \frac{1.0 \times 10^{-3}}{6.0 \times 10^{-4}} = 1.667 \text{ Wb/m}^2$$

$\therefore$ AT required = $H_1 l_1$ = 2100 × 0.15
= 315 AT

For outer limb flux (ϕ_2) = $\frac{1}{2} \times 1.0 \times 10^{-3}$ Wb
Area (a_2) = 4×10^{-4} m^2,
Length l_2 = 25 cm = 0.25 cm

$$\therefore \quad B_2 = \frac{1/2 \times 1.0 \times 10^{-3}}{4 \times 10^{-4}} = 1.25 \text{ Wb/m}^2$$

From B-H curve, H_2 = 600 AT/m

$\therefore$ AT required = $H_2 l_2$ = 600 × 0.25
= 150 AT

Air-gap $A_g = B_g$ = 1.25 Wb/m^2
$l_g = 1 \times 10^{-3}$ m

$$\therefore \quad \text{AT required} = \frac{B_g \cdot l_g}{\mu_o} = \frac{1.25 \times 1 \times 10^{-3}}{4\pi \times 10^{-3}} = 994.45 \text{ AT}$$

$\therefore$ Total AT required
= 315 + 150 + 994.45
= 1,459.45 AT.

$\therefore$ The exciting current I

$$= \frac{NI}{N} = \frac{1459.45}{500} = 2.92 \text{ A}$$

3.43 An iron ring made up of three parts, $l_1 = 12$ cm, $a_1 = 6$ cm^2; $l_2 = 10$ cm, $a_2 = 5$ cm^2, $l_3 = 8$ cm and $a_3 = 4$ cm^2. It is surrounded by a coil of 200 turns. Determine the exciting current required to create a flux of 0.5 m wb in the iron ring. [Given $\mu_1 = 2670$, $\mu_2 = 1055$, $\mu_3 = 680$.]

Solution

Total reluctance $S = S_1 + S_2 + S_3$

$$= \sum_{\mu_r \frac{l}{a}=1}^{3} \frac{l}{\mu_o \mu_r a} = \frac{l_1}{\mu_o \mu_r a_1} + \frac{l_2}{\mu_o \mu_r a_2} + \frac{l_3}{\mu_o \mu_r a_3}$$

$$= \frac{1}{4\pi \times 10^{-7}} \left[\frac{0.12}{2670 \times 6 \times 10^{-4}} + \frac{0.1}{1055 \times 5 \times 10^{-4}} + \frac{0.08}{680 \times 4 \times 10^{-4}} \right]$$

$$= \frac{1}{4\pi \times 10^{-7}} [0.074906 + .189573 + 0.294117]$$

$= 4.445 \times 10^5$ AT/Wb.

$$\therefore \qquad \text{Flux } (\phi) = \frac{\text{mmf}}{\text{reluctance}} = \frac{NI}{4.445 \times 10^5}$$

$$\therefore \qquad I = \frac{\text{flux} \times 4.45 \times 10^5}{N}$$

$$= \frac{0.5 \times 10^{-3} \times 4.45 \times 10^5}{200}$$

$= 1.11125$ A

$= 1111.25 \times 10^{-3}$ Amps $= 1111.25$ mA.

3.27 LEAKAGE FLUX IN MAGNETIC CIRCUIT AND FRINGING AND STAKING

In a magnetic circuit, it is never possible to confine all the fluxes in the direction of designated path, since a portion of the total flux will follow different paths from the intended path (generally through air). The shapes of these paths, and the amount of flux in them, depend on the geometry of the magnetic circuit and also in the value of the relative permeability μ_r. Therefore the part of the magnetic flux that has its path within the magnetic circuit is known as the useful flux or main flux and that taking other paths is called *leakage flux*. This phenomenon of wastage of some flux is called *magnetic leakage*. Sum of the two parts is called the *total flux produced*.

The ratio of the total flux produced by the magnet to the main flux is called *leakage co-efficient* or *leakage factor*.

Mathematically, leakage factor = $\dfrac{\phi_T}{\phi_m} = \dfrac{\text{Total useful flux}}{\text{Main flux}} = \dfrac{\phi_l + \phi_m}{\phi_m}$

where ϕ_l = leakage flux
ϕ_m = main flux and
ϕ_T = Total flux.

This leakage co-efficient is generally designated by λ and its value ranges from 1.12 to 1.25, i.e. is always greater than unity.

Magnetic leakage in magnets is undesirable since it increases their weight as well as cost of manufacturer.

Fringing

Figure 3.28 shows a ring provided with an air gap. The flux lines crossing this air-gap tend to repel each other and therefore buldge out across the edges of the air gap. This phenomenon is known as *fringing*. Due to this fringing, the effective gap area is larger than that of the ring. Longer the air-gap, greater is the fringing. Generally, the increase in cross-sectional area of air-gap due to fringing is assumed to be about 9 to 10%.

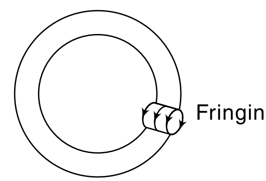


Fig. 3.28 *Concept of fringing*

Stacking

Magnetic circuits are generally laminated to reduce eddy current loss. These laminations are coated with insulating varnish. Therefore, a small space is present between the successive laminations. So the effective magnetic cross-sectional area is less than the overall area of the stack. *Stacking factor* is defined as the ratio of the effective area to the total area. This factor plays an important role during calculation of flux densities in magnetic parts. This factor is usually less than unity.

3.28 MAGNETIC HYSTERESIS

When a bar of ferromagnet material is magnetised by a varying magnetic field and the intensity of magnetizaton B is measured for different values of magnetizing field H, the graph of B versus H is shown in Fig. 3.29 and it is called B-H curve or magnetization curve. From graph, it is observed that

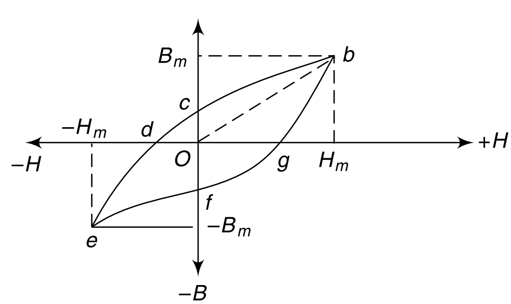


Fig. 3.29 *Magnetic hysteresis*

(a) When the magnetizing field is increased from 0, the intensity of magnetization (H) increases and becomes maximum. This maximum value is called the saturation value.

The state of magnetic material in which the value of H becomes maximum and does not increase further on increasing the value of H is called the *state of magnetic saturation.*

(b) When H is reduced, B reduces but is not zero when $H = 0$. The remainder value OC of magnetization, when $H = 0$, is called the *residual magnetism* or *retentivity*. The property by virtue of which the magnetism B remains in material even on the removal of magnetizing field is called *retentivity* or *residual magnetism* or *ramnant magnetism*.

(c) When magnetic field H is reversed, the magnetization decreases and for a particular value of H, denoted by H_C, it becomes zero, i.e. $H_C = od$ when $I = 0$. This value of H is called *coercivity*.

So, the process of demagnetizing a material completely by applying magnetizing field in a negative direction is defined as coercivity. Coercivity assesses the softness or hardness of a magnetic material. If the coercivity of a magnetic material is low then it is magnetically soft and when its value is high then the material is magnetically hard.

(d) When the field H is further increased in reverse direction the intensity of magnetization attains saturation value in reverse direction (i.e., point e).

(e) When H is decreased to zero and changed direction in steps, we get the part *efgb*.

Thus complete cycle of magentization and demagnetization is represented by *bcdefgb*. In the complete cycle the intensity of magnetization H is lagging behind the applied magnetizing field. This is called *hysteresis* and the closed loop *bcdefgb* is called *hysteresis cycle*.

The energy loss in magnetizing and demagnetizing a specimen is proportional to the area of hysteresis loop.

The selection of a material for a specific purpose depends on its hysteresis loop. When the magnet is to operate on ac voltage it undergoes a large number of reversals every second. The material for such application should have a low hysteresis loss and therefore, the hysteresis loop should enclose a small area. Soft iron is one such example.

In recent times some development has been made in Ni-Fe alloys. They are called *square loop* materials produced by maintaining the alloy for a time in a magnetic field at a temperature of 400°C to 590°C. The ultimate of this development is to make the knee point of magnetizing curve very sharp, the coercive force becomes small and the permeability is very high. The hysteresis curve showing variation of B with H of in "square loop" material is shown in Fig. 3.30. In fact these properties are essential in devices like magnetic storage of information like in computers.

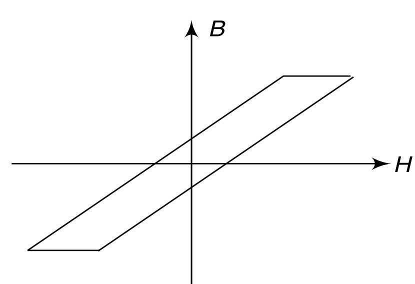


Fig. 3.30 ***B-H curve for square loop material***

3.29 HYSTERESIS LOSS

This loss occurs due to the *B-H* magnetization curve which swings to positive and negative maximum B_m before returning to zero. Ideally the energy absorbed during the positive swing should be returned to the source during reversal of the magnetizing cycle. But in actuality there is only a partial return to source, the rest being dissipated as heat.

Figure 3.31 represents the hysteresis loop obtained of a steel ring of mean circumference (l) meters and cross-sectional area (a) square meters. Let (N) be the number of turns on the magnetizing coil.

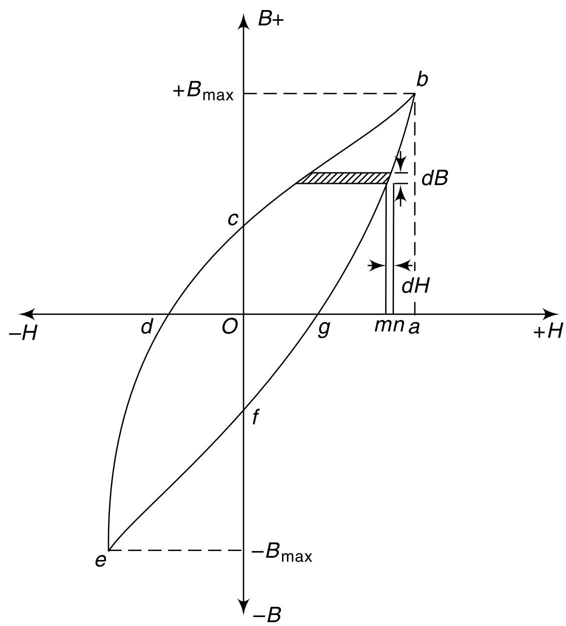


Fig. 3.31 ***Hysteresis loop for steel ring***

Let (dB) = increase of flux density when the magnetic field intensity is increased by a very small amount dH (say) in dt seconds, and i = current in amperes corresponding to om, i.e., $om = \dfrac{Ni}{l}$.

Instantaneous emf induced in the winding is $a \times dB \times \dfrac{N}{dt}$ V.

and component of applied voltage to neutralize this emf equals

$$\left(an \times \frac{dB}{dt}\right) \text{V}.$$

Therefore instantaneous power supplied to the magnetic field is

$$\left(i \times an \times \frac{dB}{dt}\right) \text{W}.$$

and energy supplied to the magnetic field in time dt second is

$$(i \times an \times dB) \text{ J}$$

Since, $$om = \frac{Ni}{l}$$

$$i = l \times \frac{om}{N}$$

hence energy supplied to magnetic field in time dt is

$$l \times \frac{om}{N} \times an \times dB \text{ J} = (om \times dB \times lA) \text{ J}$$

$$= \text{area of shaded strip, J/m}^3.$$

Thus energy supplied to the magnetic field when H is increased from zero to oa is equal to area $f\ g\ b\ B_m\ f$ J/m^3. Similarly, energy returned from the magnetic field when H is reduced from oa to zero is area $b\ B_m cb$ J/m^3. Then net energy absorbed by the magnetic field is

Area $f\ g\ b\ B_m f$ J/m^3.

Hence, hysteresis loss for a complete cycle is

area of $e\ f\ b\ c\ e$ jouled per m^3.

If we define hysteresis loss as P_n,

$$\therefore \qquad P_n = v.f.\ (K_h \cdot B_m^n) \text{ W} \qquad (3.42)$$

where v is volume of core material and f is the frequency of variation of H in Hz.

The value of n is between 1.5 and 2.5 ($1.5 \leq n \leq 2.5$) but mostly it is 1.6.

B_m is the maximum flux density in Tesla and K_H is a constant and n is the exponent which depends on the material.

The constant K_H (depending on the chemical properties of the material and the heat treatment and mechanical treatment the material has been subjected to) may have value as low as 5×10^{-7} for permalloys and as high as 6×10^{-5} for cast iron. However K_H for electrical sheet steel is generally 4×10^{-5}.

The exponent n has been found by Steinmetz as 1.6 and does not have any theoretical basis. This value suits most material at flux densities generally not exceeding 1 wb/m^2. However, for higher value of flux densities, the value may be as great as 2.5.

3.30 EDDY CURRENTS (OR FOUCAULT'S CURRENTS) AND EDDY CURRENT LOSS

When a metallic body is moved in a magnetic field in such a way that the flux through it changes or is placed in a changing magnetic field, induced currents circulate throughout the volume of the body. These are called *eddy currents*. If the resistance of the said conductor is small, then the magnitudes of the eddy currents are large and the metal gets heated up. This heating effect is a source of power loss in iron-cored devices such as dynamos, motors and transformers,

The eddy current loss is given by

$$P_e = K_e \cdot f \cdot B_m^2\, t^2\, v^2 \text{ W} \qquad (3.43)$$

where K_e is a constant, f is frequency, B_m is the maximum flux density, t is the thickness of the core material and v is the total volume of the core material.

If the core is made of laminations insulated from one another, the eddy currents are confined to their respective sheets, the eddy current loss is thereby reduced. Thus, if the core is split up into five laminations, the emf per lamination is only a fifth of that generated in the solid core. Also, the cross-sectional area per path is reduced to about a fifth, so that the resistance per path is roughly five times that of the solid core. Consequently the current per path is about one-twenty-fifth of that in the solid core. Hence:

$$\frac{I^2 R \text{ loss per lamination}}{I^2 R \text{ loss in solid core}} = \left(\frac{1}{25}\right)^2 \times 5 = \frac{1}{125} \qquad \text{(approx.)}$$

Since there are five laminations,

$$\frac{\text{Total eddy current loss per laminated core}}{\text{Total eddy current loss in solid core}} = \frac{1}{125}\left(\frac{1}{5}\right)^2$$

It follows that the eddy current loss is approximately proportional to the square of the thickness of the laminations.

Hence the eddy current loss can be reduced to any desired value, but if the thickness of the laminations is made less than about 0.4 mm, the reduction in the loss does not justify the extra cost of construction.

Since the emfs induced in the core are proportional to the frequency and the flux, therefore the eddy current loss is proportional to (frequency $\times$ flux)^2.

Eddy current loss can also be reduced considerably by the use of silicon-iron alloy and employing conducting material of high resistivity.

The hysteresis and eddy current losses are together known as *core losses* or *iron losses*. For any particular material B_m and f are also nearly constant and does not vary with current. Therefore the core losses are also known as *constant losses* and is independent of the load current.

3.31 RISE AND DECAY OF CURRENT IN INDUCTIVE CIRCUIT

Let us consider a circuit (Fig. 3.32) consisting of a battery of emf E, a coil of self-inductance L and a resistor R. The resistor R may be a separate circuit element or it may be the resistance of the inductor windings. Growth of current by closing switch S_1, we connect R and L in series with constant emf E. Let i be the current at some time t after switch S_1 is closed and $\left(\frac{di}{dt}\right)$ be its rate of change at that time. Applying Kirchoff's law starting at the negative terminal and proceeding counter clockwise around the loop,

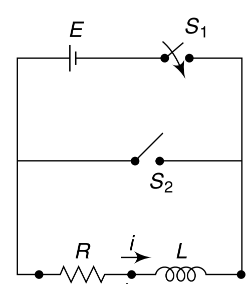


Fig. 3.32 ***Charging and discharging in inductive circuit***

$$E - V_{ab} - V_{bc} = 0$$

$\therefore$
$$E - iR - L\frac{di}{dt} = 0$$

or
$$E - iR = L\frac{di}{dt}$$

or
$$\int_0^t \frac{dt}{L} = \int_0^i \frac{di}{E - iR}$$

or
$$I = \frac{E}{R}\left(1 - e^{-\frac{R}{L}t}\right) \tag{3.44a}$$

By letting $\frac{E}{R} = i_0$ and $\frac{L}{R} = \Upsilon$, the above expression reduces to

$$i = i_0\left(1 - e^{\frac{t}{\Upsilon}}\right) \tag{3.44b}$$

and $\Upsilon = L/R$ is called the *time constant* of the L-R circuit.

If $t \to \infty$, then the current $i = i_0 = E/R$. It is also called the *steady state current* or the *maximum current* in the circuit.

At a time equal to one time constant the current has risen to $(1 - e^{-1})$ or about 63% of its final value i_0.

The *i-t* graph is as shown in Fig. 3.33.

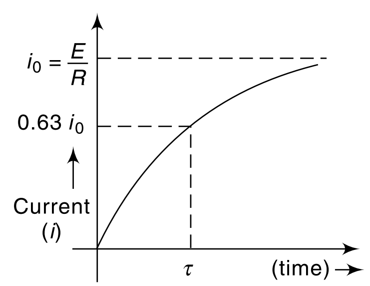


Fig. 3.33 ***i-t charging characteristic***

Note that the final current i_0 does not depend on the inductance L, it is the same as it would be if the resistance R alone were connected to the source with emf E. Let us have an insight into the behaviour of an L-R circuit from energy considerations.

The instantaneous rate at which the source delivers energy to the current $P = Ei$ is equal to the instantaneous rate at which energy is dissipated in the resistor $(=i^2R)$ plus the rate at which energy is stored in the inductor

$\because$
$$\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} \tag{3.45}$$

Thus
$$E \cdot i = i^2R + Li\frac{di}{dt} \tag{3.46}$$

Decay of Current

Now, let us suppose switch S_1 in the circuit shown in Fig. 3.34 has been closed for a long time and that the current has reached its steady state value i_0. Resetting our stopwatch to reduce the initial value the close switch S_2 at time $t = 0$ and at the same time we should open the switch S_1 to by-pass the battery. The current through L and R does not instantaneously go to zero but decays exponentially. Applying Kirchhoff's law to find current in the circuit (Fig. 3.34) at time t, we can write.

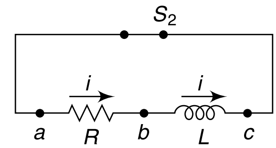


Fig. 3.34 ***L-R circuit***

$$(V_a - V_b) + (V_b - V_c) = 0$$

or
$$i \cdot R + L\left(\frac{di}{dt}\right) = 0 \qquad [\text{as } V_a = V_c]$$

$$\frac{di}{i} = -\frac{R}{L}\,dt$$

or
$$\int_0^i \frac{di}{i} = -\frac{R}{L}\int_0^t dt$$

or
$$i = i_o e^{-\frac{t}{\Upsilon}} \tag{3.47}$$

where $\Upsilon \left(=\frac{L}{R}\right)$ is the time for current to decrease to $\left(\frac{1}{e}\right)$ or about 37% of its original value.

The current (i)—time (t) graph for the decaying condition is as shown in Fig. 3.35

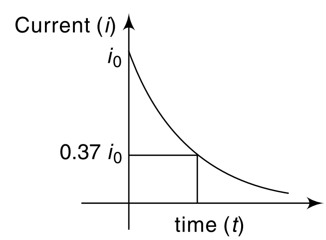


Fig. 3.35 ***Current decay in an inductive circuit***

The energy that is needed to maintain the current during this decay is provided by the energy stored in the magnetic field. Thus, the rate at which energy is dissipated in the resistor is equal to the rate at which the stored energy decreases in the magnetic field of the inductor.

$\therefore$
$$i^2R = -\frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\left(-\frac{di}{dt}\right)$$

or
$$i^2R + Li\frac{di}{dt} = 0$$

or $iR + L\dfrac{di}{dt} = 0$, which confirms when R-L circuit is short-circuited the current does not cease to flow immediately (i.e. at $t = 0$) but reduced to zero gradually.

3.44 The hysteresis loop of a specimen of 5×10^{-4} m^3 of iron is 12 cm^2. The scale is 1 cm = 0.4 Wb/m^2 and 1 cm = 400 AT/m. Find out the hysteresis loss, when subjected to an alternating flux density of 50 c/sec.

Solution

Hysteresis loss = 12 cm^2 × (400 AT/m) cm^{-1} × (0.4 Wb/m^2) cm^{-1}
= 1.92 × 10^3 Wb/m^3
= 1.92 × 10^3 × 50 J/m^3/sec
= 9.6 × 10^4 J/m^3/sec.

It given that volume of the specimen is 5 × 10^{-4} m^3.

∴ Hysteresis loss = 9.6 × 10^4 × 5 × 10^{-4} J/sec.
= 48 J/s = 48 W.

3.45 The flux in a magnetic core is varying sinusoidally at a frequency of 600 c/s. The maximum flux density $B_{\max}$ is 0.6 Wb/m^2. The eddy current loss then is 16 W. Find the eddy current loss in this core, when the frequency is 800 c/sec, and the flux density is 0.5 Wb/m^2 (Tesla).

Solution

We know, eddy current loss $\propto B_{\max}^2 \times f$

$$\text{at 600 c/sec: } P_{e_1} \propto (0.6)^2 \times 600 \quad \text{(i)}$$

$$\text{at 800 c/sec: } P_{e_2}\text{(say)} \propto (0.5)^2 \times 800 \quad \text{(ii)}$$

Dividing equation (ii) by equation (i) gives:

$$\frac{P_{e_2}}{16} = \frac{(0.5)^2 \times 800}{(0.6)^2 \times 600} \quad [\because P_{e_1} \text{ is 16 W}]$$

$$= 9.259 \times 10^{-1}$$

$$\therefore \quad P_{e_2} = 16 \times 9.259 \times 10^{-1} \text{ W}$$

$$= 14.8148 \text{ W.}$$

3.46 A coil having a resistance of 10 Ω and inductance of 15 H is connected across a d.c. voltage of 150 V. Calculate: (i) The value of current at 0.4 sec after switching on the supply. (ii) With the current having reached the final value the time it would take for the current to reach a value of 9 A after switching off the supply.

Solution

It is given that

$V(d.c) = 150$ V
$R = 10\ \Omega$
$L = 15$ H

(i) ∴ The value of the current

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}t}\right) = \frac{150}{10}\left(1 - e^{-\frac{10}{15}\times 0.4}\right)$$

$$= 15\left(1 - e^{-\frac{4}{15}}\right)$$

$$= 3.51 \text{ A}$$

(ii) Let us assume that at $t = t_1$, $i = 9$ A

$$\therefore \quad 9 = 15 \times e^{-\frac{t_1}{1.5}} \qquad \text{(for decaying)}$$

$$e^{-\frac{t_1}{1.5}} = \frac{9}{15}$$

taking $\log_e$ in both sides,

$$-\frac{t_1}{1.5} = \log_e \frac{9}{15}$$

$$\therefore \quad t_1 = 0.7662 \text{ sec.}$$

3.47 For the network shown in Fig. 3.36

(a) Find the mathematical expression for the variation of the current in the inductor following the closure of the switch at $t = 0$ on to position '*a*';

(b) The switch is closed on to position '*b*' when $t = 100$ m/sec, calculate the new expression for the inductor current and also for the voltage across R;

(c) Plot the current waveforms for $t = 0$ to $t = 200$ m/sec.

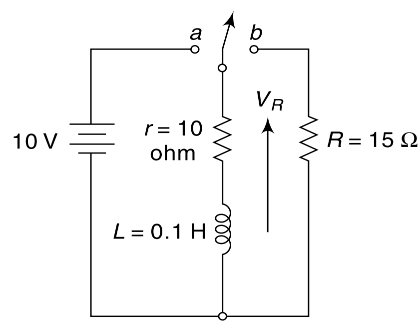


Fig. 3.36 ***Network of Ex. 3.47***

Solution

(a) For the switch in position '*a*', the time constant is

$$\Upsilon_a = \frac{L}{r} = \frac{0.1}{10} = 10 \text{ milli-sec(ms)}$$

$$\therefore \quad i_a = \frac{V}{r}\left(1 - e^{-\frac{t}{\Upsilon_a}}\right) = \frac{10}{10}\left(1 - e^{-\frac{t}{10 \times 10^{-3}}}\right)$$

$$= \left(1 - e^{-\frac{t}{10^{-2}}}\right) \text{A.}$$

(b) For the switch in position '*b*' the time constant

$$\Upsilon_b = \frac{L}{R + r} = \frac{0.1}{15 + 10} = 4 \text{ ms}$$

$$\therefore \quad i_b = \frac{V}{R} e^{-\frac{t}{\Upsilon_b}} \qquad \text{(for decaying)}$$

$$= \frac{10}{10} e^{-\frac{t}{4 \times 10^{-3}}} = e^{-\frac{t}{4 \times 10^{-3}}} \text{ A.}$$

The current continues to flow in the same direction as before, therefore the voltage drop across R is negative to the direction of the arrow shown in Fig. 3.36. $v_R = i_b \cdot R = -15 \times e^{-t/4 \times 10^{-3}}$ V.

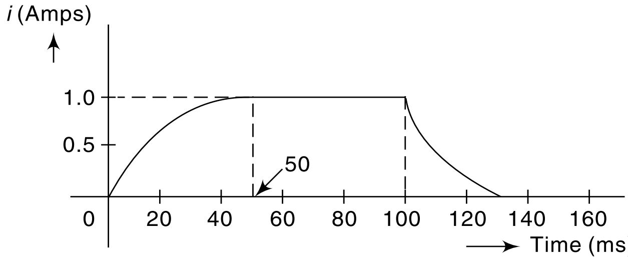


Fig. 3.37 ***Current profile***

It will be noted that in the first switched period, five times the time constant is 50 m/sec. The transient has virtually finished at the end of this time and it would not have mattered whether the second switching took place then or later. During the second period the transient took only 25 m/sec.

(c) The profile current waveform has been plotted in Fig. 3.37.

3.48 For the network shown in Fig. 3.36 (Ex. No. 3.47) the switch is closed on the position '*a*'. Next, it is closed on to position '*b*' when τ = 10 ms. Again, find the expression of current and hence draw the current wave forms.

Solution

For the switch in position '*a*', the time constant ϒ is 10 m/sec as in Ex. No. 3.47, and the current expression as is before. However, the switch is moved to position '*b*' while the transient is proceeding. When t = 10 m/sec.

$$i = \left(1 - e^{-\frac{t}{10\times10^{-3}}}\right) = \left(1.0 - e^{-\frac{10\times10^{-3}}{10\times10^{-3}}}\right)$$

$= (1 - e^{-1}) = 0.632$ A.

i.e., the second transient commences with an initial current in R of 0.632 A.

$\therefore$ The current decay is, $i_b = 0.632 \times \frac{t}{4\times10^{-3}}$ A. which is shown in Fig. 3.38.

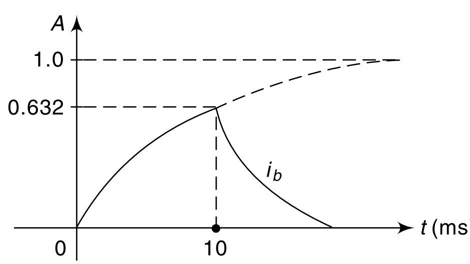


Fig. 3.38

3.49 A d.c. voltage of 150 V is applied to a coil whose resistance is 10 Ω and inductance is 15 H. Find: (i) the value of the current 0.3 sec after switching on the supply; (ii) with the current having reached the final value, how much time it would take for the current to reach a value of 6 A after switching off the supply.

Solution

(a) It is given that

V = 150 V, R = 10 Ω, L = 15 H

$\therefore$ The value of the current 0.3 sec after switching on is

$$i = \frac{150}{10}\left(1 - e^{-\frac{10}{15}\times0.3}\right) = 2.72 \text{ A}.$$

(b) After switching off the supply, the current will be decaying and is given by

$$i = \frac{V}{R} e^{-\frac{R}{L}t} \quad \therefore \quad 6 = \frac{150}{10} e^{-\frac{10}{15}\times t}$$

$\therefore$ $t = 1\ .375$ sec.

3.50 A coil of resistance 24 Ω and having inductor 36 H is suddenly connected to a d.c. of 60 V supply. Determine

(a) the initial rate of change of current $\left(\frac{di}{dt}\right)$

(b) the time-constant

(c) the current after 3 sec.

(d) the enrgy stored in the magnetic field at $t = 3$ sec.

(e) the energy lost as heat energy at $t = 3$ sec.

Solution

It is given that: $V = 60$ V, $R = 24\ \Omega$, $L = 36$ H

(a) Initial rate of change of current:

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right)$$

$$\therefore \quad \frac{di}{dt} = -\frac{V}{R}\cdot\left(-\frac{R}{L}\right)\cdot e^{-\frac{R}{L}\cdot t}$$

$$= \frac{V}{L} e^{-\frac{R}{L}\cdot t}:$$

When $t = 0$,

$$\frac{di}{dt} = \frac{V}{L}\cdot e^{o} = \frac{V}{L} = \frac{60}{36} = 1.67 \text{ A/sec:}$$

(b) Time constant (ϒ):

$$\Upsilon = \frac{L}{R} = \frac{36}{24} = 1.5 \text{ sec.}$$

(c) Current; the current at $t = 3$ sec is

$$i = \frac{V}{R}\left(1 - e^{-\frac{R}{L}\cdot t}\right) = \frac{60}{24}\left(1 - e^{-\frac{24}{36}\times 3}\right) = 2.16 \text{ A.}$$

(d) Energy stored:

at $t = 3$ sec, the energy stored in the magnetic field is $\frac{1}{2}Li^2$

$$= \frac{1}{2} \times 36 \times (2.16)^2 = 84 \text{ J.}$$

(e) Energy lost as heat energy:

at $t = 3$ sec, the energy lost as heat energy is

$$i^2 \times R = (2.16)^2 \times 24 \cong 112 \text{ J.}$$

3.32 AMPERE'S CIRCUITAL LAW

We know the integral of static (time independent electric field around a closed path is zero but what about the integral of the magnetic field around a closed path? Actually, the quantity (*Hdl*) does not represent some physical quantity, and certainly not work. Although the static magnetic force does no work on a moving charge, we cannot conclude that the path integral of the magnetic field around a closed path is zero.

The line integral $\left(\oint B \cdot dl\right)$ of the resultant magnetic field along a closed plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant.

Thus $$\oint B \cdot dl = \mu_0 \cdot I$$

This is known as *Ampere's circuital law*. The above equation can be simplified as $B \cdot l = \mu_0 \cdot I$

But this equation can be used only under the following condition.

(a) At every point of the closed path B is parallel to dl.

(b) Magnetic field has the same magnitude B at all placed on the closed path.

If this is not the case, then the above equation can be written as

$$B_1 dl_1 \cos\theta_1 + B_2 dl_2 \cos\theta_2 + \cdots = \mu_0 \cdot I$$

ADDITIONAL EXAMPLES

3.51 If the vertical component of the earth's magnetic field be 4.0×10^{-5} Wb m^{-2}, then what will be the induced potential difference produced between the rails of a meter-gauge running north-south when a train is running on them with a speed of 36 km h^{-1}?

Solution

When a train is on the rails, it cuts the magnetic flux lines of the vertical component of the earth's magnetic field. Hence, a potential dfifference is induced between the ends of its axle.

Distance between the rails = 1 m; speed of train (v) = 36 km/hr = 10 m/sec. Magnetic field $B_V = 4.0 \times 10^{-5}$ Wb/m. $\therefore$ The induced potential difference in $e = Bvl = (4.0 \times 10^{-5}) \times 10 \times 1 = 4.0 \times 10^{-9}$ V.

3.52 The current in the coil of a large electromagnet falls from 6 A to 2 A in 10 ms. The induced emf across the coil is 100 V. Find the self-inductance of the coil.

Solution

The self-induced emf is given by

$$e = -L\frac{di}{dt}$$

Here $di = 2 - 6 = -4$ A

$dt = 10$ ms $= 10^{-2}$ sec

and $e = 100$ V

$$\therefore \quad L = -e\frac{dt}{di} = -100 \times \frac{10^{-2}}{-4} = 0.25 \text{ H.}$$

3.53 The current (in ampere) in an inductor is given by $i = 5 + 16t$, where t is in seconds. The self-induced emf in it is 10 mV. Find (a) the self-inductance, and (b) the energy stored in the inductor and the power supplied to it at $t = 1$.

Solution

The induced emf in the inductor due to current change is

$$|e| = L\frac{di}{dt}$$

$$\therefore \qquad L = \frac{|e|}{di/dt}$$

Hence $\quad i = 5 + 16t$, from this, we have

$$\frac{di}{dt} = 0 + 16 = 16 \text{ A sec}^{-1}, \text{ and } e = 10 \text{ mV} = 10 \times 10^{-3} \text{ V}$$

$$\therefore \qquad L = \frac{10 \times 10^{-3}\, V}{15\, A \sec^{-1}} = 0.666 \times 10^{-3} \text{ H} = 0.666 \text{ mH}$$

(b) The current at $t = 1$ sec is

$$i = 5 + 16t = 5 + 16 \times 1$$
$$= 21 \text{ A}$$

$\therefore$ Energy stored in the inductor is

$$\frac{1}{2}Li^2 = \frac{1}{2} \times (0.666 \times 10^{-3}) \times (21)^2$$
$$= 137.8 \times 10^{-3} = 137.8 \text{ mJ}$$

Power supplied to the inductor at $t = 1$ sec is

$$P = li = (10 \times 10^{-3} \text{ V}) \times 21 = 0.21 \text{ W}.$$

3.54 Calculate the self-inductance of an air-cored solenoid, 40 cm long, having an area of cross-section 20 cm^2 and 800 turns.

Hints: $\quad L = \dfrac{\mu_0 \cdot N^2 \cdot A}{l}$

[here we assume $\mu_0 = 4\pi \times 10^{-7}$ Hm^{-1}].

$$\therefore \qquad L = \frac{4\pi \times 10^{-7} \times 800^2 \times 20 \times 10^{-4}}{40 \times 10^{-2}} = 4.022 \times 10^{-3} \text{ H}$$

3.55 A solenoid of inductance L and resistance R is connected to a battery. Prove that the time taken for the magnetic energy to reach 1/4 of its maximum value is $L/R \log_e(2)$.

Solution

The growth of current in an LR circuit is given by

$$I = I_0\left(1 - e^{-\frac{R}{L}\cdot t}\right) \qquad \text{(i)}$$

where I_0 is the maximum current. The energy stored at time t is

$$u = \frac{1}{2}LI^2$$

We are required to find the time at which the energy stored is 1/4 the maximum value, i.e., when $u = \frac{u_o}{4}$ where $u_o = \frac{1}{2}LI_0^2$.

i.e., $$\frac{1}{2}LI^2 = \frac{1}{4}\left(\frac{1}{2}LI_0^2\right) \quad \text{or} \quad I = \frac{I_0}{2}$$

∴ Using the equation 1, we have

$$\frac{I_0}{2} = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

$$\frac{1}{2} = 1 - e^{-\frac{R}{L}\cdot t}$$

or $$e^{-\frac{R}{L}t} = \frac{1}{2}$$

or $$-\frac{R}{L}t = \log_e\left(\frac{1}{2}\right) - \log_e(2)$$

∴ $$t = \frac{L}{R}\log_e(2)$$

3.56 An aeroplane with a 20 m wingspread is flying at 250 m/s parallel to the earth's surface at a plane where the horizontal component of the earth's magnetic field is 2×10^{-5} Tesla and angle of dip 60°. Calculate the magnitude of the induced emf between the tips of the wings.

Solution

As the aeroplane is flying horizontally parallel to the earth's surface the flux linked with it will be due to the vertical component B_V on the earth's field.

∴ $$B_V = B_H \tan\theta = 2 \times 10^{-5} \times \tan 60°$$

$$= 2\sqrt{3} \times 10^{-5} \text{ Wb/m}^2.$$

∴ Induced emf is $|e| = B_V lv \sin\theta$

$$= 2\sqrt{3} \times 10^{-5} \times 20 \times 250 \times \sin 90°$$

or $$l = \frac{\sqrt{3}}{10} \text{ V} = 0.173 \text{ V}.$$

3.57 A rectangular loop of sides 25 cm × 10 cm carries a current of 15 A. It is placed with its longer side parallel to a long straight conductor 2.0 cm apart and carrying a current at 25 A. Find the net force on the loop. What will be the difference in force if the current in the loop be reversed?

Solution

Let $ABCD$ be the loop (length l, width b and current i_1), with its longer side AB placed parallel and at a distance d from a long conductor XY, carrying current i_2 as shown in Fig. 3.39.

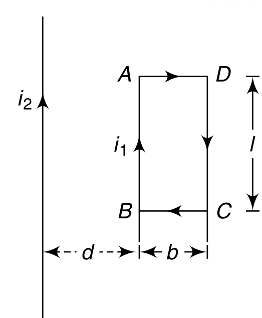


Fig. 3.39 ***Rectangular loop***

The attractive force on the side AB of the loop, due to current i_2, is

$$F_1 = \frac{\mu_o}{2\pi} \cdot \frac{i_1 \cdot i_2 \cdot l}{d} \text{ towards } XY$$

Similarly, the (repulsive) force in the side CD of the loop is

$$F_2 = \frac{\mu_o}{2\pi} \cdot \frac{i_1 \cdot i_2 \cdot l}{(d+b)} \text{ away from } XY$$

$\therefore$ The forces on the sides AD and BC of the loop; being equal, opposite and collinear, cancel each other.

$\therefore$ Net force on the loop is

$$F_1 - F_2 = \frac{\mu_o}{2\pi} i_1 \cdot i_2 \cdot l \left[\frac{1}{d} - \frac{1}{(d+b)} \right]$$

$$= \frac{\mu_o}{2\pi} \cdot \frac{i_1 \cdot i_2 \cdot l \cdot b}{d(d+b)} \text{ towards } XY.$$

Substituting the values:

$$i_1 = 15 \text{ A}, \; i_2 = 25 \text{ A}$$

$$l = 0.25, \; b = 0.10 \text{ m}$$

$$d = 0.02 \text{ m}, \; d + b = 0.12 \text{ m}$$

and $\quad \dfrac{\mu_o}{2\pi} = 2 \times 10^{-7}$ NA^{-2}, we get

$$F_1 - F_2 = (2 \times 10^{-7}) \times \frac{15 \times 25 \times 0.25 \times 0.10}{0.02 \times 0.12}$$

$$= 7.8 \times 10^{-4} \text{ N.}$$

The net force is directed towards the long conductor. If the current in the loop, or in the long conductor, be reversed, the net force will remain same in magnitude but will then be directed away from the long conductor.

3.58 Show that the time for attaining half the value of the final steady current in an L-R series circuit is $0.6931L/R$.

Solution

The instantaneous current during its growth in an L-R series circuit is given by $i = i_0\left(1 - e^{-\frac{R}{L}\cdot t}\right)$

where i_0 = final steady current for $i/i_0 = 1/2$, we have

$$\frac{1}{2} = 1 - e^{-\frac{R}{L}\cdot t}$$

$$\therefore \quad e^{-\frac{R}{L}\cdot t} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \quad e^{\frac{R}{L}\cdot t} = 2$$

$$\frac{R}{L} \cdot t = \log_e 2 = 0.6931$$

$$\therefore \qquad t = 0.6931 \times \frac{L}{R}.$$

3.59 The resistors of 100 Ω and 200 Ω and an ideal inductance of 10 H are connected to a 3-V battery through a key *K*, as shown in the Fig. 3.40.

If *K* is closed at $t = 0$, calculate

(a) the initial current drawn from the battery
(b) the initial potential drop across the inductance
(c) the final current drawn from the battery
(d) the final current through 100 Ω resistance.

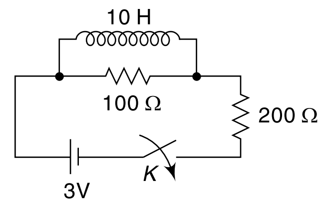


Fig. 3.40 ***Circuit of Ex. 3.59***

Solution

(a) 'Immediately' after closing *K*, there is almost no current in the inductance due to self-induction, the current is only in resistance. Thus, the initial current is

$$i = \frac{3v}{(100+200)\,\Omega} = 0.01 \text{ A}$$

(b) The p.d. across the inductor is same as across the 100 Ω resistance, that is $0.01 \times 100 = 1$ V.

(c) When the current has become steady the opposing emf in the inductance is zero and it short circuits the 100 Ω resistance. The resistance of the circuit is now only 200 Ω and so the current drawn from the cell is $3V/200\ \Omega = 0.015$ A. This is the final current in the 200 Ω resistance.

(d) The final current through the 100 Ω is zero.

3.60 A horizontal power-line carries a current of 90 A from east to west. Compute the magnetic field generation at a point 1.5 meter below the line.

Solution

The magnitude of the magnetic field $\vec{B}$ due to a long current carrying conductor at a distance *R* is given by

$$B = \frac{\mu_o}{2\pi} \cdot \frac{i}{R}$$

where $\frac{\mu_o}{2\pi} = 2 \times 10^{-7}$ Tm A^{-1}.

Putting, $i = 90$ A, $R = 1.5$ m, we have

$$B = (2 \times 10^{-7}) \times \frac{90}{1.5} = 1.2 \times 10^{-5} \text{ T}.$$

Applying Right Hand Rule, we find $\vec{B}$ is directed towards the south.

3.61 Two coils with terminals T_1, T_2 and T_3, T_4 respectively are placed side by side. Measured separately, the inductance of the first is 1200 μH and that of the second coil is 800 μH. With T_2 joined with T_3 (Fig. 3.41), the total inductance between the two coils is

2500 μH. What is the mutual inductance? If T_2 is joined with T_4 instead of T_3, what would be the value of equivalent inductance of the two coils?

Solution

Given L_1 = 1200 μH, L_2 = 800 μH, T_{14} = 2500 μH.

Let the mutual inductance between the two coils be M, then total inductance $L_1 + L_2 + 2M$. In the first case (refer (Fig. 3.41)

$$T_{14} = L_1 + L_2 + 2M$$

$$2500 = 1200 + 800 + 2M$$

$$\therefore \quad M = \frac{500}{2} = 250 \text{ μH.}$$

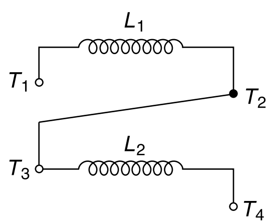


Fig. 3.41 ***Connection of two coils, 1st case***

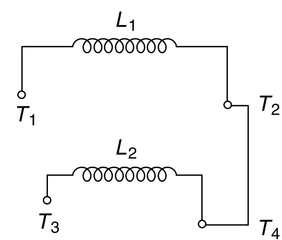


Fig. 3.42 ***Connection of two coils, 2nd case***

If T_{13} is the total inductance in the second case, then

$$T_{13} = L_1 + L_2 - 2M \qquad \text{(See Fig. 3.42)}$$
$$= 1200 + 800 - 2 \times 250$$
$$= 1500 \; \mu\text{H.}$$

3.62 A coil has a resistance of 5 Ω and an inductance of 1 H. At $t = 0$ it is connected to a 2 V battery. Find (a) the rate of rise of current at $t = 0$; (b) the rate of rise of current when $i = 0.2$ Amps and (c) the stored energy when $i = 0$ and $i = 0.3$ A.

Solution

$$\Upsilon = \frac{L}{R} = \frac{1}{5} = 0.2 \text{ sec.}$$

(a) $$\frac{di}{dt} = \frac{E}{L} e^{-\frac{t}{\Upsilon}} = 2e^{-5t}$$

at $t = 0$, $$\frac{di}{dt} = 2 \text{ A/sec.}$$

(b) $$i = \frac{E}{R}(1 - e^{-5t})$$
$$= 0.4(1 - e^{-5t})$$

time t_1 when $i = 0.2$ A is

$$0.2 = 0.4\,(1 - e^{-5\,t_1})$$

or $$t_1 = 0.1386 \text{ sec.}$$

$$\frac{di}{dt} = 2e^{-5(0.1386)}$$
$$= 1 \text{ A/sec}$$

(c) At $i = 0$, stored energy = 0
when $i = 0.3$ A, stored energy

$$= \frac{1}{2} Li^2 = \frac{1}{2} \times 1 \times (0.3)^2 = 0.045 \text{ J.}$$

3.63 A small flat coil of area 2.0×10^{-4} m^2 with 25 closely wound turns is placed with its plane perpendicular to a magnetic field. When the coil is suddenly withdrawn from the field, a charge of 7.5 mc flows through the coil. The resistance of the coil is 0.50 Ω. Estimate the strength of the magnetic field.

Solution

The magnetic flux passing through each turn of coil of area A, perpendicular to a magnetic field B is given by

$$\phi_B = BA$$

when the coil is withdrawn from the field, the flux through it vanishes. Therefore, the change in flux is $d\phi_B = 0 - BA = -BA$.
By Faraday's law, the emf induced in the coil is

$$e = -N \cdot \frac{d\phi_B}{dt} = \frac{NBA}{dt}$$

where dt is the time taken in withdrawal. The induced current in the coil of resistance (say R) is

$$i = \frac{e}{R}.$$

This current persists only during the time interval dt. Hence the charge flowed through the coil is $q = i \times dt = \dfrac{e}{R} \cdot dt = \dfrac{NBA}{R}$.

$$\therefore \qquad B = \frac{qR}{NA}.$$

Substituting the given values, we have

$$B = \frac{(7.5 \times 10^{-3}) \times 0.50}{25 \times (2.0 \times 10^{-4})} = 0.75 \text{ Wb/m}^2.$$

3.64 A copper rod of length 1.0 m is revolving with a frequency of 50 rev/sec. around one of its ends, perpendicular to a uniform magnetic field of 1.0 Wb/m^2. Find the emf developed between the two ends of the revolving rod.

Solution

The magnetic flux linked with an area of perpendicular to a uniform magnetic field of magnitude B is given by $\phi_B = BA$.

Suppose the copper rod of length l (say) revolving about its one end O is completing n revolutions per unit time is shown in Fig. 3.43, then, the rate of change of magnetic flux linked with the revolving rod is given by

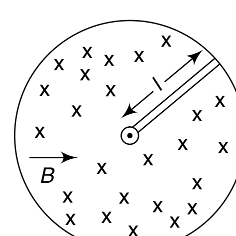


Fig. 3.43 ***Revolving copper rod***

$$\frac{d\phi_B}{dt} = B \cdot \frac{dA}{dt} = B \times (\text{area swept by the rod per unit time})$$

$$= B \times (\pi l^2) \cdot n$$

By Faraday's law of electromagnetic induction, the emf induced across the rod is given by

$$|e| = \frac{d\phi_B}{dt} = B\,\pi\, l^2 \cdot n$$

Substituting the given values, we have

$$|e| = 1.0 \times 3.14 \times (1.0)^2 \times 50$$
$$= 157 \text{ Wb/sec} = 157 \text{ V}$$

3.65 A rectangular iron-core is shown below in Fig. 3.44. It has a mean length of magnetic path of 100 cm, cross-section of (2 cm × 2 cm), relative permeability of 1400 and an air-gap of 5 mm is cut in the core. The three coils carried by the core have number of turns $N_1 = 335$, $N_2 = 600$ and $N_3 = 600$, and the respective currents are $I_1 = 1.6$ A, $I_2 = 4.0$ A and $I_3 = 3.0$ A. The directions of the currents are as shown. Calculate the flux in the air-gap.

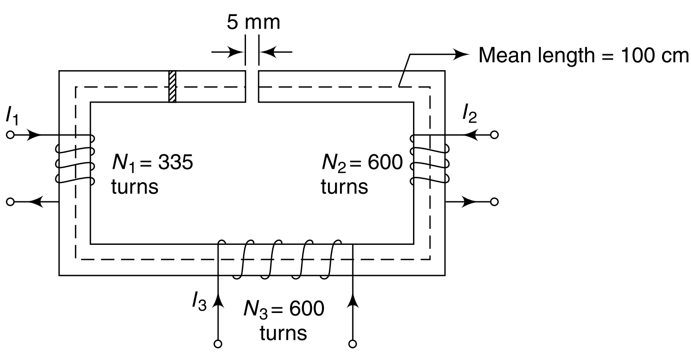


Fig. 3.44 ***A rectangular iron-cone (Ex. 3.65)***

Solution

The mmf acting in the magnetic circuit (current considering in the clockwise direction)

$$= \Sigma NI = -335 \times 1.6 + 600 \times 4 - 600 \times 3 = 64 \text{ AT}$$

$$\therefore \quad 64 \text{ AT} = \frac{\phi}{\mu_o A}\left[\frac{l_i}{\mu_r} + l_g\right],$$

where l_i = mean length, μ_r = relative permeability and

l_g = air-gap cut length.

$$\therefore \quad 64 = \frac{\phi}{4\pi \times 10^{-7} \times (2 \times 2) \times 10^{-4}}\left[\frac{1}{1400} + 5 \times 10^{-3}\right]$$

$$= 1.136 \times 10^7\, \phi$$

$$\therefore \quad \text{Flux, } \phi = \frac{64}{1.136 \times 10^7} = 5.63 \times 10^{-6} \text{ Wb}$$

$$= 5.63\ \mu\text{ Wb}$$

EXERCISES

1. Define mmf, reluctance and permeability. Deduce an expression for the force between two parallel conductors. Explain the significance of reluctance in a magnetic circuit.
2. Compare magnetic and electric circuits.
3. What are the different types of magnetic losses? What is the eddy-current loss? What are the considerable effects of eddy currents? How can they be minimised? Mention some application of eddy currents. How is the different types of losses to be minimised?
4. Explain the terms: magnetic leakage and flux fringing. Derive an expression for the weight which can be lifted by a horse-shoe magnet.
5. Why do magnetic circuits usually have air-gaps? How does the presence of air-gaps affect the magnetic circuit calculations which has higher reluctance an air-gap or an iron path? And why? Prove that $B = \mu H$.
6. Draw and explain the *B-H* curves for air and a magnetic material.
7. Explain with the aid of a typical *B-H* curve the meaning of the following terms:
   Relative permeability, coercivity, and remanence.
   What information can be derived from the *B-H* loop?
   What is meant by magnetic hysteresis?
8. Explain briefly under what conditions it is advantageous to use in a magnetic circuit:
   (a) a granulated iron core
   (b) a laminated iron core.
9. An air-cooled solenoid has a diameter of 30 cm and a length of 5070 cm and is wound with 3000 turns. If a current of 6 A flow in the solenoid find the energy stored in its magnetic field.

   [Hints: Calculate $A = \dfrac{\pi D^2}{4} = \dfrac{\pi \times 9 \times 10^{-4}}{9}$

   and $L = \dfrac{\mu_o \cdot \mu_r \cdot N^2 A}{l}$,

   have $\mu_o = 4\pi \times 10^{-7}\ \mu_r = 1$

   $\therefore$ Energy stored $= E = \dfrac{1}{2} LI^2$ J.] [*Ans:* 2875 J]

10. An air cored torodial coil has 450 turns and a mean diameter of 300 mm and a cross-sectional area of 300 mm^2. Determine the self-inductance of the coil and the average voltage induced in it when a current of 2 A is reversed in 40 m/sec. [*Ans:* 8.1 mH; 8.1 mV]

    [Hint: $L = \dfrac{\mu_o \mu_r AN^2}{l} = \dfrac{4\pi \times 10^{-7} \times 1 \times 300 \times 10^{-6} \times (450)^2}{\pi \times 300 \times 10^{-3}}$ H

    $= 0.81 \times 10^{-4}$ H.

    $$\text{emf} = L\frac{di}{dt} = 0.81 \times 10^{-4} \times \frac{2+2}{40 \times 10^{-3}} = 0.0081 \text{ V.]}$$

11. Two identical coils, having 1000 turns each, lie in parallel planes such that 90% flux produced by one coil links with the other. If a current of 4 A flowing in one coil produces a flux of 0.05 m Wb in it, find the magnitude of mutual inductance between the two coils. [*Ans:* 9 mH]
12. A solenoid of length 1 meter, and diameter 10 cm has 5000 turns. Calculate: (i) the approximate inductance, and (ii) the energy stored in a magnetic field when a current of 2 A flows in the solenoid.
[*Ans:* L = 0.247 H; Energy stored = 0.494 J]
13. The coils having 150 and 200 turns respectively are wound side by side on a closed magnetic circuit of cross-section 1.5×10^{-2} m^3 and mean length 3 m. The relative pearmeability of the magnetic circuit is 2000. Calculate (a) the mutual inductance between the coils; (b) the voltage induced in the second coil if the current changes from 0 to 10 A in the first coils is 20 m/sec.

[*Hints:* $N_1 = 150,\ N_2 = 200$
$a = 1.5 \times 10^{-2},\ l = 3$ m, $\mu_r = 2000$

(a) $M = \mu_o \cdot \mu_r \cdot N_1 \cdot N_2 \cdot \dfrac{a}{l} = 0.377$ H

(b) $di_1 = 10 - 0 = 10$ A.
$dt = 20$ ms $= 20 \times 10^{-3}$ sec.

$$\therefore \qquad e_2 = M \cdot \frac{di_1}{dt} = 188.5 \text{ V}]$$

14. Two coils of negligible resistance and of self-inductances 0.2 H and 0.1 H, are connected in series. If the mutual inductance is 0.1 H, calculate the effective inductance of the combination. [*Ans:* 0.5 H or 0.1 H]
15. Two coils A and B, each with 100 turns, are mounted so that part of the flux set up by one links the other. When the current through coil A is changed from +2A to –2A in 0.5 second, an emf of 8 mV is induced in coil B, calculate: (i) the mutual inductance between the coils, and (ii) the flux produced in coil B to 2A in coil A. [*Ans:* M = 1 mH; ϕ = 200 m wb]

[*Hints:* $N_A = N_B = 100$ turns
$I_A = 2A,\ dI_A = 2 - (-2) = 4$ A
$dt = 0.5$ sec, $e_M = 8 \times 10^{-3}$ V

(i) Now $e_M = M \cdot \dfrac{di_A}{dt}$, $M = 1 \times 10^{-3}$ H

(ii) Flux induced in $B = \phi_B = \dfrac{MI_A}{N_B} = 2 \times 10^{-5}$ Wb.

16. A coil has 100 turns of wire, and a flux of 5 m Wb linkage with this coil changes to zero in 0.05 second. Determine the self-induced emf in the coil. [*Ans:* 10 V]
17. Two long single-layer solenoids have the same length and the same number of turns but are placed coaxially one within the other. The diameter of

the inner coil is 60 mm and that of the outer coil is 75 mm. Determine the co-efficient of coupling between the coils. [*Ans:* 0.8]

18. A coil has a self-inductance of 1 H. If a current of 25 mA is reduced to zero in a time of 12 m/sec, find the average value of the induced voltage across the terminals of the coil. [*Ans:* 25 V]

19. A conductor of length 300 cm moves at an angle of 30° to the direction of uniform magnetic field of strength 2.0 Wb/m^2 with a velocity of 100 m/sec. Calculate the emf induced. What will be the emf induced if the conductor moves at right angles to the field?
[*Hints:* (i) $e = Blv \sin 30° = 300$ V;
(ii) $e = Blv \sin 90° = 600$ V.]

20. A conductor having a length of 80 cm is placed in a uniform magnetic field of 2 Wb/m^2 (Tesla). If the conductor moves with a velocity of 50 m/sec, find the induced emf when it is (i) at right angle (ii) at an angle of 30° and (iii) parallel to the magnetic field. [*Ans:* 80 V, 40 V and 0 V]
[*Hint:* B = 2 Wb/m^2, l = 0.8 m, v = 50 m/sec.
(i) $e = Blv = 2 \times 0.8 \times 50 = 80$ V [$\because \sin\theta = \sin 90° = 1$]
(ii) $e = 80 \sin 30° = 40$ V;
(iii) $e = 80 \sin 0° = 0$.]

21. Calculate the mmf required to produce a flux of 0.01 Wb across an air-gap of 2 mm. of length having an effective area of 200 cm^2 of a wrought iron ring of mean iron path of 0.5 m and cross-sectional area of 125 cm^2. Assume a leakage co-efficient of 1.25. The magnetization curve of the wrought iron is given below:

B(Wb/m^2):	0.6	0.8	1	1.2	1.4
H(AT/m):	75	125	250	500	1000

[*Ans:* 921.18 AT]

[*Hint:* $\phi = 0.01$ Wb

Air-gap: $$H = \frac{B}{\mu_o} = \frac{\phi}{A \cdot \mu_o} = \frac{0.01}{200 \times 10^{-4} \times 4\pi \times 10^{-7}} \text{ AT/m}$$
$$= 3.98 \times 10^5 \text{ AT/m.}$$

Total AT required = $3.98 \times 10^5 \times 2 \times 10^{-3} = 796.178$

Iron path: $$B = \frac{0.01 \times 1.25}{125 \times 10^{-4}} \text{ Wb/m}^2 = 1 \text{ Wb/m}^2$$

$\therefore$ $H = 250$ AT/m

Total AT required = $250 \times 0.5 = 125$

Total mmf = $796.178 + 125 = 921.178$]

22. (a) An iron ring, having a mean diameter of 75 cm and a cross-sectional area of 5 cm^2 is wound with a magnetizing coil of 120 turns. Using the following data, calculate the current required to set-up a magnetic flux of 630 μ Wb in the ring.

Flux density (T)	0.9	1.1	1.2	1.3
A/m	260	450	600	820

(b) The air gap in a magnetic circuit is 1.1 m long and 20 cm^2 in cross-section. Calculate (i) the reluctance of the air-gap and (ii) the ampere turns required to send a flux of 700 μ Wb across the air-gap.

[*Ans:* a = 4.5 A. (b) 4.375 × 10^5 A/Wb, 306 A]

23. A mild steel having a cross-sectional area of 10 cm^2 and a mean circumference of 60 cm has a coil of 300 turns wound around it. Determine
   (i) reluctance of the steel ring,
   (ii) current required to produce a flux of 1 mole in the ring. Relative permeability of the given steel is 400 at the flux density developed in the core.
   (iii) if a slit of 1 mm. is cut in the ring, what will be the new value of current? Assume no fringing effect.

[*Ans:* 119.43 × 10^4 A/Wb; 3.98 A; 6.635 A]

[*Hint:* (i) Reluctance $= \dfrac{l}{\mu_o \mu_r a}$

$$= \frac{0.6}{4\pi \times 10^{-7} \times 400 \times 10 \times 10^{-4}} \text{ A/Wb}$$

$$= 119.426 \times 10^4 \text{ A/Wb}$$

(ii) AT = 1 × 10^{-3} × 119.426 × 10^4 = 1194.26

$$I = \frac{1194.26}{300} \text{ A} = 3.98 \text{ A}$$

(iii) Reluctance of iron path $= \dfrac{1\times 10^{-3}}{4\pi \times 10^{-7} \times 10 \times 10^{-4}}$ A/Wb

$$= 79.62 \times 10^4 \text{ A/Wb}$$

AT for air gap = 79.62 × 100 × 1 × 10^{-3} = 796.2.

$$\therefore \quad I = \frac{1194.26 + 796.2}{300} = 6.635 \text{ A.]}$$

24. A magnetic circuit has an area of 10 cm^2 and length of 0.2 m. If μ_r = 2000, find the reluctance. [*Ans:* 0.79 × 10^5 A/Wb]

25. Find out the inductance and energy stored in the magnetic field of an air-cored solenoid of 100 cm long, 10 cm in diameter and some wound with 900 turns and a current of 7.5 A is passing through the solenoid.

[*Ans:* L = 8 mH; E = 0.225 J]

[*Hints:* Inductance of solenoid

$$L = \frac{N^2 \cdot A \cdot \mu_o \cdot \mu_r}{l} = 0.008\ H$$

and energy stored = $E = \dfrac{1}{2} LI^2$ = 0.225 J]

26. A circuit has 2000 turns enclosing a magnetic circuit of 30 cm^2 sectional area. When with 5A, the flux density is 1.2 Wb/m^2 and 10 A it is

1.7 Wb/m^2, find the mean value of the inductance between these current limits and the induced emf if the current falls from 10 A to 4 A in 0.08 sec.

[*Hints:* $L = N \cdot \dfrac{d\phi}{dt} = N \cdot \dfrac{d(BA)}{dt} = NA\dfrac{dB}{dt}$

here $dB = 1.7 - 1.2$ and $dt = 10 - 5$

$\therefore$ $L = 0.6$ H

again $dI = 10 - 4 = 6$ A and $dt = 0.08$

$\therefore$ $e_L = L \cdot \dfrac{dI}{dt} = 45$ V]

27. An iron ring of mean length 200 cm and circular cross-section of area 15 cm^2 has an air gap of 6 mm and a winding of 200 turns. Calculate the inductance of the coil. μ_r for iron is given as 800. [*Ans:* 200 V]
28. A conductor 1.5 m long carries a current of 50 A at right angles to a magnetic field of density 1.2 T. Calculate the force on the conductor. [*Ans:* F = 90 N]

[*Hints:* $F = BIl \sin\theta$]

29. A horseshoe electromagnet is required to lift a 200 kg weight. Find the exciting current requried if the electromagnet is wound with 500 turns. The magnetic length of the electromagnet is 60 cm and is of permeability 500. The reluctance of the load can be neglected. The pole face has a cross-section 20 sq. cm. [*Ans:* 2.35 A]

[*Hint:* $F = 2 \times \dfrac{B^2 A}{2\mu_o} = \dfrac{B^2 A}{\mu_o}$

$\therefore$ $B^2 = \dfrac{\mu_o F}{A} = \dfrac{4\pi \times 10^{-7} \times 200 \times 9.81}{20 \times 10^{-4}}$

or $B = 1.232$ Wb/m^2

$$H = \frac{B}{\mu_o \mu_r} = \frac{1.232}{4\pi \times 10^{-7} \times 500} = 1961.78 \text{ AT/m}$$

Total AT $= 1961.78 \times 0.6 = 1177$

$$I = \frac{1177}{500} = 2.35 \text{ A.}]$$

4 DC NETWORK ANALYSIS

4.1 INTRODUCTION

When a number of network elements* are connected together to form a system that consists of set(s) of interconnected elements performing specific or assigned functions, it is called a “network”. An electrically closed network is a “circuit”. An electrical network is a combination of numerous electric elements (e.g., resistance R, inductance L, capacitance C, etc.).

Some important definitions related to an electrical network are as follows:

Node: It is the junction in a circuit where two or more network elements are connected together.

Branch: It is that part of the circuit which lies between two junctions in a circuit.

Loop: It is a closed path in a circuit in which no element or node is encountered more than once.

Mesh: It is such a loop that contains no other loop within it.

4.2 CHARACTERISTICS OF NETWORK ELEMENTS

4.2.1 Linear and Non-linear Elements

A *linear element* shows linear characteristics of voltage vs current. Thus the parameters of linear elements remain constant (i.e., the parameters do not change with voltage or current applied to that element). Resistors, inductors and capacitors are linear elements.

On the other hand, for a *non-linear element*, the current passing through it does not change linearly with the linear change in applied voltage across it, at a particular temperature and frequency. In a non-linear element the parameters change with applied voltage and current changes. Semiconductor devices like diodes, transistors, thyristors, etc. are typical examples of non-linear elements. Ohm’s law is not valid for non-linear elements.

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*A network element is a component of a circuit having different characteristics like linear, non-linear, active or passive etc. and will be defined shortly.

4.2.2 Active and Passive Elements

If a circuit element has the capability of enhancing the energy level of an electric signal passing through it, it is called an *active element*, viz., a battery, a transformer, semiconductor devices, etc. Otherwise the element that simply allows the passage of the signal through it without enhancement is called *passive element* (viz., resistors, inductors, thermistors and capacitors). Passive elements do not have any intrinsic property of boosting an electric signal.

4.2.3 Unilateral and Bilateral Elements

If the magnitude of the current passing through an element is affected due to change in polarity of the applied voltage, the element is called a *unilateral element*. On the other hand if the current magnitude remains the same even if the polarity of the applied voltage is reversed, it is called a *bilateral element*. Unilateral elements offer varying impedances with variation in the magnitude or direction of flow of the current while bilateral elements offer same impedance irrespective of the magnitude or direction of flow of current. A resistor, an inductance and a capacitor, all are bilateral elements while diodes, transistors, etc. are unilateral elements.

4.3 SERIES RESISTIVE CIRCUITS

Resistors are said to be in *series* when they are connected in such a way that there is only one path through which current can flow. Therefore the current in a series circuit is the same at all parts in the circuit. The voltage drop across each component in a series circuit depends on the current levels and the component resistance (or impedance).

4.3.1 Currents and Voltages in a Series Circuits

The circuit diagram for three series connected resistors and a d.c. voltage source is shown in Fig. 4.1.

The total resistance connected across the voltage source is $R = R_1 + R_2 + R_3$. (R is called the *equivalent resistance* in ohms for the given circuit.)

For a series circuit with n resistors, the equivalent resistance R is thus

$$R = R_1 + R_2 + R_3 + \cdots + R_n \qquad (4.1)$$

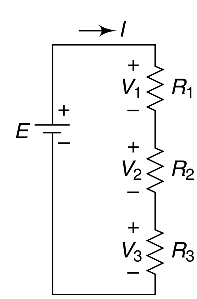

Fig. 4.1 ***A simple series resistive circuit***

The equivalent circuit for the series resistance circuit is shown in Fig. 4.2.

The equivalent circuit consists of the voltage source E and the equivalent resistance R. The current I flows from the positive terminal of the voltage source. Using Ohm's law the current through the series circuit in ampere is obtained as

$$I = \frac{E}{R} = \frac{E}{R_1 + R_2 + R_3 + \cdots + R_n}. \qquad (4.2)$$

There is only one path for current flow in a series circuit.

The current flow causes a voltage drop V or *potential difference* across each resistor in the circuit of Fig. 4.1. Using Ohm's law, the voltage drops across each resistor in volts are obtained as

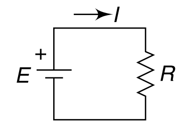

Fig. 4.2 *Equivalent of a simple series resistive circuit*

$$V_1 = IR_1, \; V_2 = IR_2 \quad \text{and} \quad V_3 = IR_3.$$

Since the sum of the resistive voltage drops is equal to the applied emf, for any series circuit,

$$E = V_1 + V_2 + V_3 + \cdots + V_n$$

or

$$E = I(R_1 + R_2 + R_3 + \cdots + R_n). \tag{4.3}$$

Next we consider series connection of voltage sources instead of series connection of resistors.

If three voltage sources are connected in series as shown in Fig. 4.3, the resultant voltage in volt is

$$E = E_1 + E_2 + E_3.$$

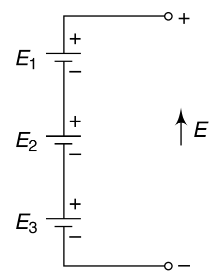

Fig. 4.3 *Series connection of three-voltage sources*

In Fig. 4.4 the lowermost voltage source E_3 has its negative terminal connected to the negative terminal of the middle cell. The resultant voltage in this case is

$$E = E_1 + E_2 - E_3$$

In Fig. 4.3 the voltage sources assist one another to produce the circuit current, so they are said to be in "series aiding". In Fig. 4.4 the bottommost voltage source will attempt to produce current in the opposite direction to that formed by the other two. Therefore this bottommost source is connected in "series opposing" with the top two cells.

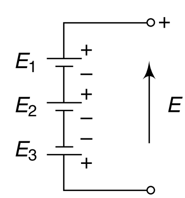

Fig. 4.4 *Series connections of three voltage sources with the polarity of one source reversed*

4.3.2 Voltage Divider

In Fig. 4.5 two series connected resistors are used as a *voltage divider* or *potential divider*.

Here,

$$V_1 = IR_1 = \frac{E}{R_1 + R_2} \cdot R_1$$

$$\left[\because I = \frac{E}{R_2 + R_2}\right]$$

Also

$$V_2 = I_2R_2 = \frac{E}{R_1 + R_2} \cdot R_2$$

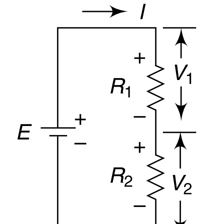

Fig. 4.5 *Voltage divider (or potential divider) circuit*

If $R_1 = R_2$ then $V_1 = V_2 = \dfrac{E}{2}$.

When n number of resistors are connected in series then voltage drop (V_i) across any resistance (R_i) is given by

$$V_i = E \times \frac{R_i}{R_1 + R_2 + R_3 + \cdots + R_n} \tag{4.4}$$

V_i and E are expressed in volt and resistors are given in ohms.

Voltage Divider Theorem

In a series circuit, the portion of applied emf developed across each resistor is the ratio of that resistor's value to the total series resistance in the circuit.

4.3.3 Potentiometer

The circuit diagram of a variable resistor employed as a *potentiometer* is shown in Fig. 4.6. *The potentiometer is essentially a single resistor with terminals at each end and a movable contact that can be set to any point on the resistor.* Terminals A and B are the end terminals and terminals C is the adjustable contact (Fig. 4.6).

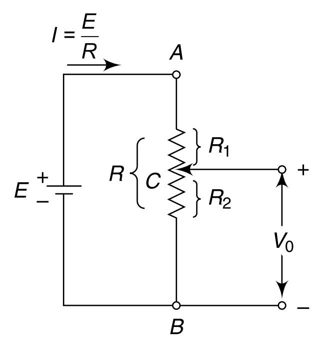

Fig. 4.6 ***A simple potentiometric circuit***

The output voltage (V_o), in volt, is given as

$$V_o = E \times \frac{R_2}{R_1 + R_2} \tag{4.5}$$

If the moving contact is half way between the two end terminals then

$$R_1 = R_2 = \frac{R}{2}$$

or $\quad V_o = E \times \dfrac{1}{2}$

When $\quad R_2 = R,\ V_o = E$

and when $\quad R_2 = 0,\ V_o = 0$

Thus it is seen that the potentiometer can be adjusted to give an output voltage ranging linearly from 0 to E.

4.3.4 Power in a Series Circuit

In Fig. 4.5, the power (VA) dissipated in R_1 is given by

$$P_1 = V_1 I = \frac{V_1^2}{R_1} = I^2 R_1$$

$\therefore$ For any series circuit containing n number of resistors the power dissipated is

$$\begin{aligned} P &= P_1 + P_2 + P_3 + \cdots + P_n \\ &= V_1 I + V_2 I + V_3 I + \cdots + V_n I \\ &= I(V_1 + V_2 + V_3 + \cdots + V_n) \\ &= IE \end{aligned}$$

$$\therefore \qquad P = \frac{E^2}{R_1 + R_2 + R_3 + \cdots + R_n} \tag{4.6}$$

In dc circuit volt-ampere power (VA) is same as power expressed in watts. Thus P is usually expressed in watts in dc circuits.

4.3.5 Current-limiting Resistor

Sometimes a resistor is included in series with an electrical circuit or electronic device to drop the supply voltage down to a desired level. This resistor can be treated as a *current-limiting resistor.*

In Fig. 4.7, R_s provides a voltage drop to the series connected lamps L_1 and L_2. The lamps operate to a voltage level lower than the source voltage even in series connection. Also the resistor R_s limits the current I to the level required by the lamps.

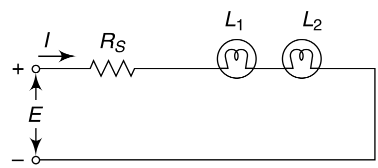

Fig. 4.7 ***Application of current-limiting resistor R_S***

Here circuit current in ampere is obtained as

$$I = \frac{E}{R_S + (\text{sum of resistances of lamps})}$$

or $$R_s = (E/I) - (\text{sum of resistances of lamps}) \tag{4.7}$$

4.3.6 Open Circuits and Short Circuits in a Series Circuit

An *open circuit* occurs in a series resistance circuit when one of the resistors (or any series network element) becomes disconnected from the adjacent one. Open circuit can also occur when one of the resistors (or an element) has been destroyed by excessive power dissipation.

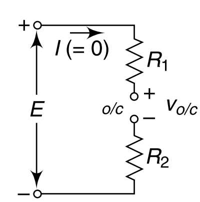

Fig. 4.8 ***An open circuit***

In the circuit shown in Fig. 4.8, the open circuit can be thought of another resistance in series with value "infinity". Therefore the current, $I = \dfrac{E}{R_1 + R_2 + \infty} = 0$.

The voltage drop across the open circuit ($V_{O/C}$) in volts is obtained as

$$V_{O/C} = E - IR_1 = E - 0 = E$$

Figure 4.9 shows a series resistance R_3 short circuited in the series circuit. Here the resistance between the terminals of R_3 becomes zero after short circuit. Therefore, the circuit current I in ampere is given by

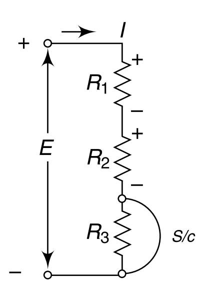

Fig. 4.9 ***Deactivation of a resistance (R_3) in a series circuit by shorting terminals of R_3***

$$I = \frac{E}{R_1 + R_2 + 0} = \frac{E}{R_1 + R_2}$$

4.1 Find the current that flows through the resistors 10 Ω, 20 Ω, and 30 Ω connected in series across a 240 V supply.

Solution

$$\text{Current } I = \frac{V}{10+20+30}\,A = \frac{240}{10+20+30} = 4A$$

4.2 Determine the voltage drops across each resistor of the circuit shown in Fig. 4.10.

Solution

The current flowing through each resistor is given as

$$I = \frac{100}{5+2+3}\text{ A} = 10\text{ A}$$

Voltage drop across the 5 Ω resistor = 10 × 5 = 50 V
Voltage drop across the 2 Ω resistor = 10 × 2 = 20 V
Voltage drop across the 3 Ω resistor = 10 × 3 = 30 V

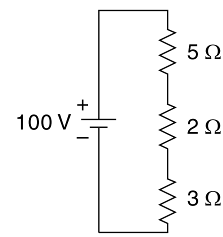

Fig. 4.10 *Circuit of Ex. 4.2*

Polarities are marked in Fig. 4.10(a). Check that total voltage drop is 100 V, same as the supply voltage.

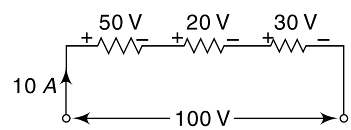

Fig. 4.10(a) *Voltage drops for the series circuit shown in Fig. 4.10*

4.3 In the circuit shown in Fig. 4.11, if E_1 = 10 V and E_2 = 7 V, find the current through the resistors.

Solution

The current through the resistors

$$I = \frac{E_1 + E_2}{R_1 + R_2 + R_3} = \frac{10+7}{2+7+3}\text{ A} = 1.4167\text{ A}$$

(Note that E_1 and E_2 are in series aiding connection.)

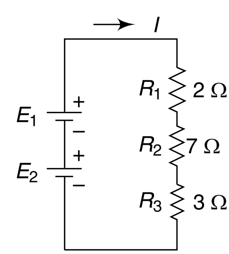

Fig. 4.11 *Circuit of Ex. 4.3*

4.4 Determine the current through the resistors in the circuit shown in Fig. 4.11 when the polarity of E_2 is reversed.

Solution

$$\text{Current } I = \frac{E_1 - E_2}{R_1 + R_2 + R_3} = \frac{10-7}{2+7+3}\text{ A} = 0.25\text{ A}$$

(this time E_1 and E_2 are in series opposition).

4.5 Calculate the minimum and maximum values of V_o that can be obtained from the circuit shown in Fig. 4.12. *P* is the moving contact and can slide linearly along a 300 Ω resistor.

Solution

By inspection it is evident that, if P is at the bottommost point of 300 Ω resistor, V_o is minimum.

$$\therefore \quad V_{o(\min)} = 240 \times \frac{800}{800+300+500} = 120 \text{ V.}$$

On the other hand, if P is at the topmost point of 300 Ω resistor, V_o is maximum.

$$V_{o(\max)} = 240 \times \frac{800+300}{800+300+500} = 165 \text{ V.}$$

It is possible to obtain values of (V_o) between 120 V and 165 V by sliding P suitably across the 300 Ω resistor.

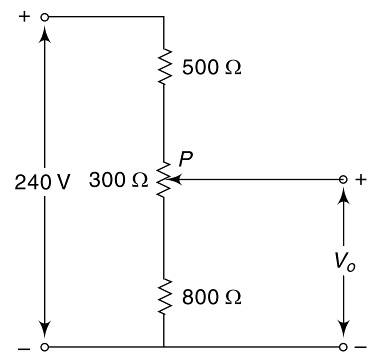

Fig. 4.12 *Circuit of Ex. 4.5*

4.6 Determine the power dissipated in each resistor of Fig. 4.11 and also find the total power.

Solution

Power dissipated across the 2 Ω resistor in Fig. 4.11 is $I^2 \times 2$, i.e., $(1.4167)^2 \times 2$, i.e., 4.014 W. (The value of circuit current has been obtained in Ex 4.3 as 1.4167 A.)

Power dissipated across the 7 Ω resistor is $I^2 \times 7$, i.e., $(1.4167)^2 \times 7$ or 14.05 W.

Similarly, power dissipated in the 3 Ω resistor is $(1.4167)^2 \times 3$, i.e., 6.02 W.

Total power is ($E \times I$), i.e., $(E_1 + E_2) \times I$.

This gives (10 + 7) × 1.4167, i.e., 24.084 W.

[Check: Total power is $I^2(2 + 7 + 3)$, i.e., $(1.4167)^2 \times 12$ or 24.085 W.]

4.7 In the circuit shown in Fig. 4.13 find the value of the resistor R so that the lamps L_1 and L_2 operate at rated conditions. The rating of each of the lamps is 12 V, 9 W. If L_2 becomes short circuited find the current through the circuit and the power dissipated in each of the lamps.

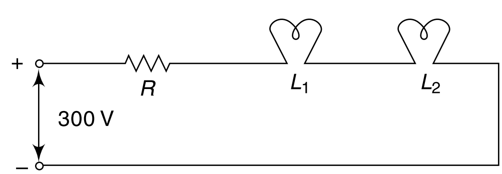

Fig. 4.13 *Circuit of Ex. 4.7*

Solution

Voltage rating of the lamps is 12 V each, while power rating of each of the lamps is 9 W.

If I be the rated current through the lamps then

$$VI = P$$

or
$$I = \frac{P}{V} = \frac{9}{12} = 0.75 \text{ A}$$

If R_L be the resistance of each lamp,

$$I^2 R_L = 9 \text{ or, } R_L = \frac{9}{(0.75)^2}\ \Omega$$

$$= 16\ \Omega.$$

Supply voltage = 300 V (given)

$\therefore$ Voltage across resistor (R) is (300 – 2 × 12) i.e., 276 V

Also, current through R is 0.75 A

$$\therefore \quad R = \frac{276}{0.75}\,\Omega = 368\ \Omega.$$

If L becomes short circuited, resistance across terminals of L_2 is 0.

If the current now is I', we can write

$$300 = I'(R + R_L) = I'(368 + 16)$$

or $$I' = 0.78 \text{ A}.$$

Power dissipated in L_1 is now $(0.78)^2 \times 16 = 9.73$ W (L_1 will glow brighter)

Power dissipated in L_2 is obviously 0.

4.4 PARALLEL RESISTANCE CIRCUITS

Resistors are said to be connected in parallel when equal voltages appear across each resistor (or network element). The total current taken from the supply is the sum of all the individual resistor or network elements' currents.

4.4.1 Currents and Voltages in Parallel Circuits

Resistors are said to be connected in parallel when the circuit has two terminals which are common to each resistor. Figure 4.14 represents a circuit having three resistors connected in parallel.

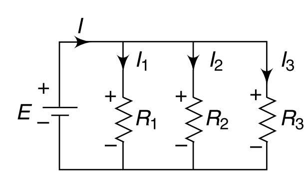

Fig. 4.14 *The resistors in parallel connection*

The voltage across each resistor is E volts and the current through R_1 is I_1, through R_2 is I_2 and through R_3 is I_3.

$$\therefore \quad I_1 = \frac{E}{R_1},\ I_2 = \frac{E}{R_2} \quad \text{and} \quad I_3 = \frac{E}{R_3}$$

The current supplied by the battery in ampere is $(I) = I_1 + I_2 + I_3$

If R be the equivalent resistance (in ohms) of the circuit in Fig. 4.14,

$$I = \frac{E}{R} = \frac{E}{R_1} + \frac{E}{R_2} + \frac{E}{R_3}$$

or $$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

or $$R = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}. \qquad (4.8)$$

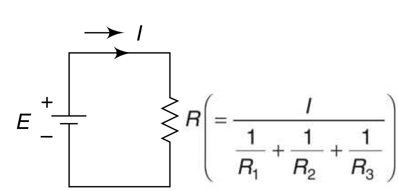

Fig. 4.15 *Equivalent circuit of three resistances in a parallel circuit*

The equivalent circuit is shown in Fig. 4.15.

If n resistors are connected in parallel then we have

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \qquad (4.9)$$

where R is the *equivalent resistance*.

Therefore, the reciprocal of the equivalent resistance of resistors in parallel connection is equal to the sum of the reciprocals of the individual resistances.

4.4.2 Conductances in Parallel

In dc circuits *conductance* is the reciprocal of resistance and its unit is "Siemens" (S) in SI units or "mho" in cgs units. If $G_1, G_2, G_3, \ldots, G_n$ be the conductances of the resistors connected in parallel then the equivalent conductance (G) in Siemens is given by

$$G = G_1 + G_2 + G_3 + \cdots + G_n \tag{4.10}$$

According to Ohm's law, $I = \dfrac{V}{R} = VG$, where V is the applied voltage, G is the equivalent conductance of a parallel circuit, and I is the source current.

4.4.3 Current Divider

Parallel resistance circuits are often called *current divider circuits* because the supply current is divided among the parallel branches.

The circuit in Fig. 4.16 can be called as a current divider circuit. Here

$$I_1 = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{E}{R_2}$$

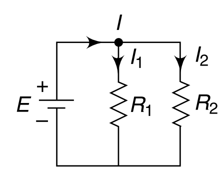

Fig. 4.16 ***Concept of current division***

Again, $$I = I_1 + I_2$$

$$= \frac{E}{R_1} + \frac{E}{R_2} = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

or $$I = E\,\frac{R_1 + R_2}{R_1 \times R_2}$$

or $$E = I \times \frac{R_1 R_2}{R_1 + R_2} = IR.$$

$$\left[\text{If } (R) \text{ be the equivalent resistance then, } R = \frac{R_1 R_2}{R_1 + R_2}.\right]$$

Now, $$I_1 = \frac{E}{R_1} = \frac{1}{R_1}(I) \times \frac{R_1 R_2}{R_1 + R_2} = I\,\frac{R_2}{R_1 + R_2}. \tag{4.11}$$

Similarly $$I_2 = I\,\frac{R_1}{R_1 + R_2}. \tag{4.12}$$

I_1 and I_2 are the currents in the branches of this current divider circuit in amperes.

These two equations (4.11 and 4.12) can be used to determine how a known supply current is divided into two individual currents through parallely connected resistors or network elements.

If G_1 and G_2 be the conductances of the resistors R_1 and R_2,

$$I_1 = I\frac{\frac{1}{G_2}}{\frac{1}{G_1}+\frac{1}{G_2}} = \frac{G_1}{G_1+G_2}\cdot I \tag{4.13}$$

and
$$I_2 = I\frac{\frac{1}{G_1}}{\frac{1}{G_1}+\frac{1}{G_2}} = \frac{G_2}{G_1+G_2}\cdot I\,. \tag{4.14}$$

If there are n number of resistors with conductances G_1, G_2 ..., G_n connected in parallel across a voltage source then current in any resistor with conductance G_i is

$$I_i = \frac{G_i}{G_1+G_2+G_3+\cdots+G_n}\cdot I \tag{4.15}$$

[I being the supply current in ampere while I_i is the currrent through G_i].

4.4.4 Power in Parallel Circuits

For the circuit in Fig. 4.14, the power (in VA) across resistor R_1 is given by

$$P_1 = EI_1 = \frac{E^2}{R_1} = I_1^2 R_1$$

Total power $P = E(I_1 + I_2 + I_3)$

$$= E^2\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) = I_1^2 R_1 + I_2^2 R_2 + I_3^2 R_3$$

$$= P_1 + P_2$$

When n resistors are connected in parallel

$$P = P_1 + P_2 + P_3 + \cdots + P_n \tag{4.16}$$

(P) can be expressed in VA or in Watts in dc circuits.

4.4.5 Open Circuits and Short Circuits in Parallel Circuits

When one of the components in a parallel resistive circuit is open circuited, as shown in Fig. 4.17, no current flows through that branch. The other branch currents are not affected by the open circuit as they still have the normal supply voltage applied across each of them. In Fig. 4.17, $I_1 = 0$. Supply current $I = I_2 + I_3$. All currents are expressed in amperes.

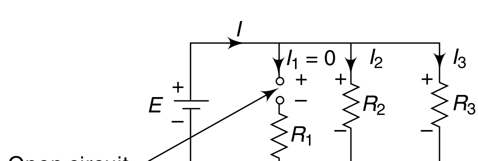

Fig. 4.17 ***Open circuit in a branch in a parallel resistive circuit***

Figure 4.18 shows a short circuit across resistor (R_3).

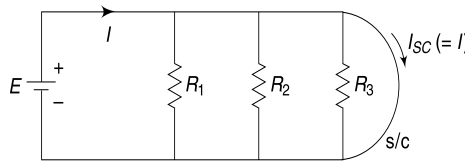

Fig. 4.18 ***Short circuit in a branch in a parallel circuit***

As there is a short circuited path across R_3, i.e., across one of the resistors in the parallel circuit, no current will flow through resistors R_1, R_2 and R_3. Total current will flow from the battery through the short circuited path and the current ($I_{S/C}$) = $I = E/0 = \infty$. However, in practice this current is limited by the internal resistance of the battery and lead resistances of the wires. If the internal resistance of the battery be taken only and is equal to R_i, then current $I = E/R_i$ which is also very high (as the internal resistance of a battery is very small).

4.8 Calculate the total current supplied by the battery in Fig. 4.19.

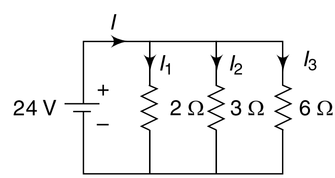

Fig. 4.19 ***Circuit of Ex. 4.8***

Solution

$$I_1 = \frac{24}{2}\text{ A} = 12\text{ A},\ I_2 = \frac{24}{3}\text{ A} = 8\text{ A and}$$

$$I_3 = \frac{24}{6}\text{ A} = 4\text{ A}$$

$\therefore$ The total current $I = I_1 + I_2 + I_3 = (12 + 8 + 4)$ A = 24 A

4.9 Determine the equivalent resistance of the four resistances connected in parallel across a 240 V supply. Also find the total current. The resistances are of 10 Ω, 15 Ω, 25 Ω and 40 Ω.

Solution

The equivalent resistance

$$R = \frac{1}{\frac{1}{10}+\frac{1}{15}+\frac{1}{25}+\frac{1}{40}} = \frac{1}{0.1+0.067+0.04+0.025}$$

$$= 4.31\ \Omega$$

Total current $$I = \frac{E}{R} = \frac{240}{4.31}\text{ A} = 55.68\text{ A}.$$

4.10 Three resistors of conductances 0.1 Siemens, 0.2 Siemens and 0.5 Siemens are connected in parallel. Calculate the equivalent resistance of the circuit.

Solution

Equivalent conductance (G) = $G_1 + G_2 + G_3$

$$= 0.1 + 0.2 + 0.5 = 0.8\text{ Siemens}$$

Equivalent resistance $$R = \frac{1}{G} = \frac{1}{0.8}\ \Omega = 1.25\ \Omega.$$

4.11 Using the current divider rule find the current in the resistors R_1 and R_2 connected in parallel across a voltage source. The supply current is 50 A, R_1 = 10 Ω and R_2 = 20 Ω.

Solution

Total current I = 50 A

$$\text{Current through resistor } R_1 \text{ is } (I_1) = I \times \frac{R_2}{R_1 + R_2}$$

$$= 50 \times \frac{20}{20+10}$$

$$= 33.33 \text{ A}$$

$$\text{Current through resistor } R_2 \text{ is } (I_2) = I \times \frac{R_1}{R_1 + R_2}$$

$$= 50 \times \frac{10}{10+20}$$

$$= 16.67 \text{ A}$$

4.12 In the circuit shown in Fig. 4.19 find the power dissipated across each resistor and the total power.

Solution

$$\text{Power dissipated across 2 Ω resistor } (P_1) = I_1^2 \times 2 = (12)^2 \times 2 = 288 \text{ W}.$$

$$\text{Power dissipated across 3 Ω resistor } (P_2) = I_2^2 \times 3 = (8)^2 \times 3 = 192 \text{ W}.$$

$$\text{Power dissipated across 6 Ω resistor } (P_3) = I_3^2 \times 6 = (4)^2 \times 6 = 96 \text{ W}.$$

$$\text{Total power } (P) = P_1 + P_2 + P_3 = 288 + 192 + 96 = 576 \text{ W}.$$

[The values of I_1, I_2, and I_3 have been obtained as 12 A, 8 A and 4 A in Ex 4.8]. Also, (P) = EI = 24 V × 24A = 576 W (check).

4.5 SERIES-PARALLEL CIRCUITS

Series-parallel resistive circuits consist of combinations of series connected and parallel connected resistors (or other passive network elements). Figure 4.20 represents a simple series-parallel resistive circuit. In this circuit R_2 and R_3 are connected in parallel. The parallel combination of R_2 and R_3 is $R_2R_3/(R_2 + R_3)$ (= R_{eq}).

The equivalent circuit is shown in Fig. 4.21.

Since R_1 and R_{eq} are connected in series, therefore the equivalent resistance of the whole circuit is [(R_1 + R_{eq}) Ω].

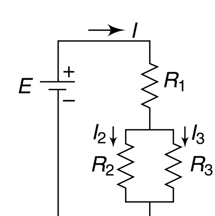

Fig. 4.20 ***A series parallel circuit***

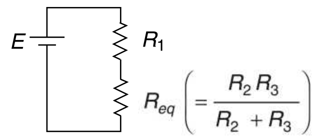

Fig. 4.21 *Equivalent of series-parallel circuit*

4.5.1 Currents and Voltages in Series-parallel Circuits

In Fig. 4.20 the supply current I flows through resistance R_1. Then I splits into I_2 and I_3 flowing through R_2 and R_3 respectively.

Obviously, $I = I_2 + I_3$, I being expressed in amps

The currents I_2 and I_3 can easily be calculated using the current divider rule.

The voltage across resistor R_1 is given by $V_1 = IR_1$

The voltage across resistors R_2 and R_3 are equal as they are connected in parallel. Here

$$V_2 = V_3 = I_2R_2 = I_3R_3$$

Also, $E = V_1 + V_2 = V_1 + V_3$.

Once the branch currents are known, the voltages across each resistor can easily be calculated.

4.5.2 Open Circuits and Short Circuits in a Series Parallel Circuit

The effect of open circuit in a series-parallel circuit is shown in Fig. 4.22(a) and Fig. 4.22(b).

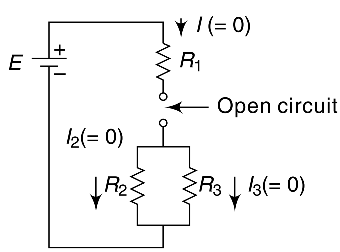

Fig. 4.22(a) *Open circuit in series-parallel circuit*

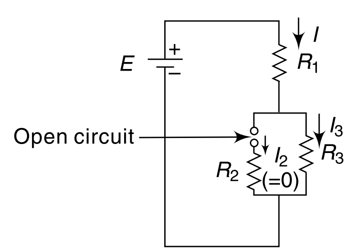

Fig. 4.22(b) *Open circuit in a branch of series-parallel circuit*

In Fig. 4.22(a), open circuit occurs at one terminal of R_1. This has the same effect as an open circuit in the supply line, so that the main current flowing in any part of the circuit is zero. Also as the main current is zero there is no voltage drop across the resistors and the supply voltage E appears across the open circuit.

When open circuit occurs at one end of one of the parallel resistors, as shown in Fig. 4.2(b), the current through that resistor only is zero. Here, $I_2 = 0$.

Also, R_1 and R_3 can be assumed to be connected in series.

Hence $$I = I_3 = \frac{E}{R_1 + R_3}.$$

As there is no current through R_2 so there is no voltage drop across it and the voltage across the open circuit is equal to the voltage across R_3, i.e. V_3.

When short circuit occurs across the terminals of R_1 as shown in Fig. 4.23(a), the resistance across the terminals of R_1 is 0.

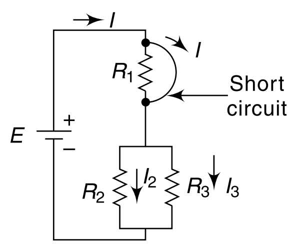

Fig. 4.23(a) ***Short circuit in series part of series-parallel circuit***

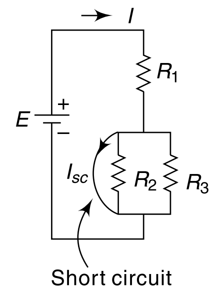

Fig. 4.23(b) ***Short circuit in a parallel branch of a series-parallel circuit***

The total current is obtained as, $I = \dfrac{E}{R_2 \parallel R_3} = \dfrac{E}{R_2 R_3/(R_2 + R_3)}$

$$I_2 = I \cdot \frac{R_3}{R_2 + R_3} \quad \text{and} \quad I_3 = I \cdot \frac{R_2}{R_2 + R_3}$$

When short circuit occurs across the terminals of R_2, as shown in Fig. 4.23(b), the resistance across the terminals of R_2 is 0. Therefore no current will pass through R_3 as there is a short circuited path in parallel with it.

Hence $\qquad I = \dfrac{E}{R_1}$;

also, $\qquad I_3 = 0 = I_2$

If $I_{s/c}$ be the current in amps through the short circuited path then $(I_{s/c})$ $(= I)$ = $\dfrac{E}{R_1}$.

4.5.3 Analysis of a Series Parallel Circuit

The following are the steps for solving series-parallel circuits.

1. Draw a circuit diagram identifying all components by number and showing all currents and resistor voltage drops.
2. Convert all series branches of two or more resistors into a single equivalent resistance.
3. Convert all parallel combinations of two or more resistors into a single equivalent resistance.
4. Repeat procedures 2 and 3 until the desired level of simplification is achieved.

The final circuit should be simple series or parallel circuit. Once the current through each equivalent resistance or the voltage across it is known, the original circuit can be used to determine individual resistor currents and voltages.

4.13 Find the supply current and the currents in the parallel branches in the circuit shown in Fig. 4.24.

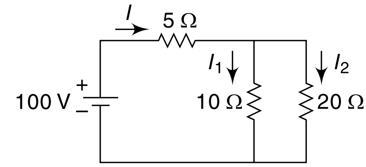

Fig. 4.24 *Circuit of Ex. 4.13*

Solution

In the circuit shown in Fig. 4.24, 10 Ω and 20 Ω are in parallel. The equivalent resistance of the parallel combination is $\dfrac{10 \times 20}{10+20} = \dfrac{200}{30}\ \Omega$ = 6.67 Ω. 5 Ω and 6.67 Ω are now in series as shown in Fig. 4.24(a)

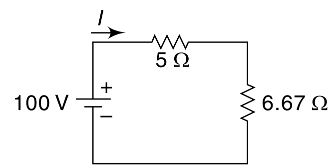

Fig. 4.24(a) *Equivalent circuit of the series-parallel circuit of Fig. 4.24*

The supply circuit is $I = \dfrac{100}{5+6.67}$ A = 8.57 A

From Fig. 4.24,

$$I_1 = 8.57 \times \frac{20}{10+20} = 5.71 \text{ A}$$

$$I_2 = 8.57 \times \frac{10}{10+20} = 2.857 \text{ A}$$

4.14 Find all resistor currents and voltages in the circuit shown in Fig. 4.25.

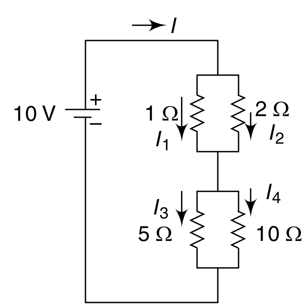

Fig. 4.25 *Circuit of Ex. 4.14*

Solution

The parallel combination of 1 Ω and 2 Ω is $\dfrac{1\times 2}{1+2} = \dfrac{2}{3}\ \Omega$.

The parallel combination of 5 Ω and 10 Ω is $\dfrac{5\times 10}{5+10}$ $= \dfrac{50}{15}\ \Omega = \dfrac{10}{3}\ \Omega$.

Also, $\dfrac{2}{3}\ \Omega$ and $\dfrac{10}{3}\ \Omega$ are in series as shown in Fig. 4.25(a)

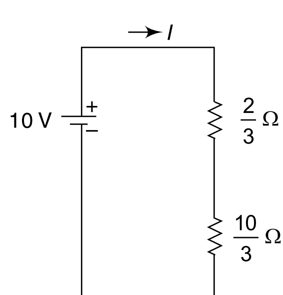

Fig. 4.25(a) *Simplified equivalent of circuit of Fig. 4.25*

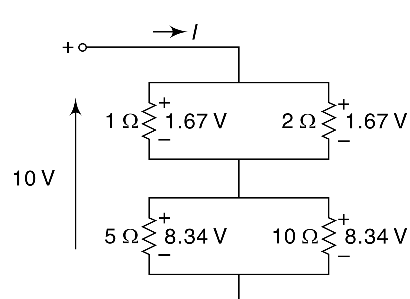

Fig. 4.25(b) *Polarity of voltage drops of the circuit of Fig. 4.25*

Figure 4.25(a) represents a simple series circuit. The supply current is given by

$$I = \frac{10}{\frac{2}{3}+\frac{10}{3}}\text{ A} = \frac{10\times 3}{12}\text{ A} = 2.5\text{ A}$$

From Fig. 4.25,

Current through 1 Ω resistor $I_1 = 2.5 \times \frac{2}{1+2}\text{ A} = 1.67\text{ A}$

Current through 2 Ω resistor $I_2 = 2.5 \times \frac{1}{1+2}\text{ A} = 0.83\text{ A}$

Current through 5 Ω resistor $I_3 = 2.5 \times \frac{10}{10+5}\text{ A} = 1.676\text{ A}$

Current through 10 Ω resistor $I_4 = 2.5 \times \frac{5}{10+5}\text{ A} = 0.834\text{ A}$

Therefore

Voltage across 1 Ω resistor is $1.67 \times 1 = 1.67$ V

Voltage across 2 Ω resistor is $0.833 \times 2 \simeq 1.67$ V

Voltage across 5 Ω resistor is $1.676 \times 5 = 8.34$ V

Voltage across 10 Ω resistor is $0.834 \times 10 = 8.34$ V

[Polarities of voltage drops are shown in Fig. 4.25(b)].

4.15 Find the current through 5 Ω resistor in Fig. 4.26 when the terminals across a 10 Ω resistor is (i) open circuited and (ii) short circuited. Also find the current through the short circuited path.

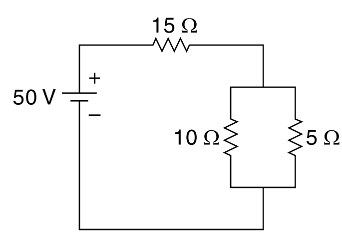

Fig. 4.26 *Circuit of Ex. 4.15*

Solution

(i) When terminals across 10 Ω resistor is open circuited as shown in Fig. 4.26(a), 15 Ω and 5 Ω are in series. Hence current I flows through both 15 Ω and 5 Ω. The current through the 10 Ω resistor is obtained as

$$I = \frac{50}{15+5}\text{ A} = \frac{50}{20}\text{ A} = 2.5\text{ A}$$

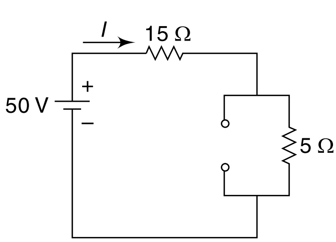

Fig. 4.26(a) *One resistor in circuit of Fig. 4.26 open*

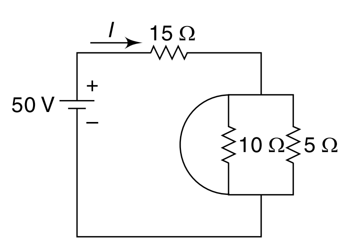

Fig. 4.26(b) ***10 Ω** resistor is shorted in circuit of Fig. 4.26*

(ii) When terminals across 10 Ω resistor is short circuited as shown in Fig. 4.26(b), no current will pass through the 5 Ω resistor as there is a short-circuited path in parallel with it. Therefore current through the 5 Ω resistor is 0.

The supply current I will pass through 15 Ω and through the short circuited path. Hence the current through the short circuited path is 50/15 A = 10/3 A = 3.33 A.

4.6 KIRCHHOFF'S LAWS

A German physicist Gustav Kirchhoff developed two laws enabling easier analysis of circuits containing interconnected impedances. The first law deals with flow of current and is popularly known as *Kirchhoff's current law* (KCL) while the second one deals with voltage drop in a closed circuit and is known as *Kirchhoff's voltage laws* (KVL).

4.6.1 Kirchhoff's Current Law (KCL)

It states that in any electrical network the algebraic sum of currents meeting at any node of a circuit is zero.

In Fig. 4.27, i_1 and i_2 are the inward currents towards the junction 0 and are assumed as negative currents. Currents i_3, i_4 and i_5 are outward currents and taken as positive. As per KCL,

$$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

i.e., $$i_1 + i_2 = i_3 + i_4 + i_5 \qquad (4.18)$$

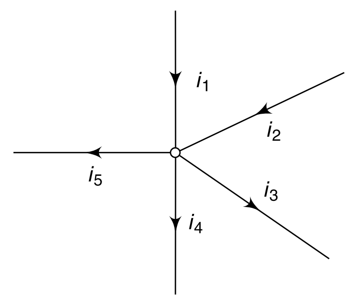

Fig. 4.27 *Explanation of KCL*

i.e., the algebraic sum of currents entering a node must be equal to the algebraic sum of currents leaving that node.

4.6.2 Kirchhoff's Voltage Law (KVL)

It states that the algebraic sum of voltages (or voltage drops) in any closed path, in a network, traversed in a single direction is zero.

In Fig. 4.28, if we travel clockwise in the network along the direction of the current, application of KVL yields

$$-V_1 + iR_1 + V_2 + iR_2 + iR_3 = 0$$

or $$V_1 = i(R_1 + R_2 + R_3) + V_2 \qquad (4.19)$$

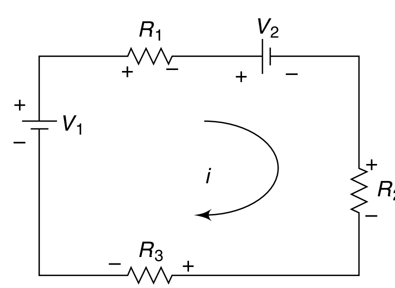

Fig. 4.28 *Explanation of KVL*

[We can also write equation (4.19) as follows:

$$V_1 - V_2 = i(R_1 + R_2 + R_3)$$

or $$i = \frac{V_1 - V_2}{R_1 + R_2 + R_3}. \qquad (4.20)]$$

We consider the voltage drop as positive when current flows from positive to negative potential. Hence V_1 is negative while V_2 is positive in the first step of equation (4.19).

4.6.3 Network Analysis Procedure using Kirchhoff's Laws

1. Convert all current sources to voltage sources.
2. Letter or number all junctions on the network as A, B, C or 1, 2, 3 etc.
3. Identify current directions and voltage polarities and number them according to the resistor involved.
4. Identify each current path according to the lettered junctions and applying Kirchhoff's voltage law, write the voltage equations for the paths.
5. Applying Kirchhoff's current law, write the equations for the currents entering and leaving all junctions where more than one current is involved.
6. Solve the equations by substitution to find the unknown currents and or voltages.

4.16 Find the magnitude and direction of the unknown currents in Fig. 4.29. Given $i_1 = 20\ A$, $i_2 = 12\ A$ and $i_5 = 8\ A$.

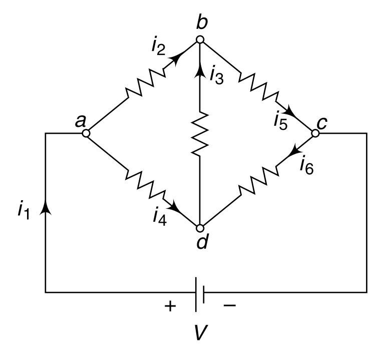

Fig. 4.29 *Circuit of Ex. 4.16*

Solution

Applying KCL at node '*a*'

$$-i_1 + i_2 + i_4 = 0 \cdots (x)$$

or $$i_4 = i_1 - i_2 = 20 - 12 = 8 \text{ A}$$

Applying KCL at node '*b*'

$$-i_2 - i_3 + i_5 = 0 \cdots (y)$$

or $$i_3 = i_5 - i_2 = 8 - 12 = -4 \text{ A}$$

Applying KCL at node '*d*'

$$-i_4 + i_3 - i_6 = 0 \cdots (z)$$

or $$i_6 = i_3 - i_4 = -4 - 8 = -12 \text{ A}$$

The actual currents are now marked in Fig. 4.29(a).

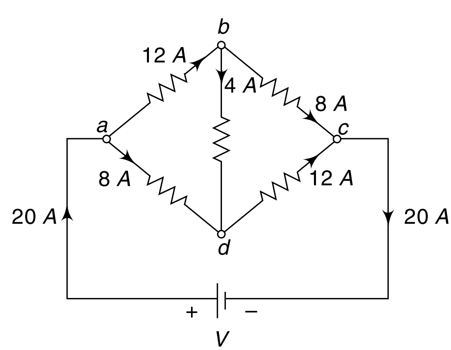

Fig. 4.29(a) *Actual current flows in circuit of Fig. 4.29*

We can interpret as follows:

$$i_3 = -4\ A(\text{from } d \text{ to } b)$$

i.e. $$i_3 = 4\ A(\text{from } b \text{ to } d)$$

$$i_4 = 8\ A(\text{from } a \text{ to } d)$$

$$i_6 = -12\ A(\text{from } c \text{ to } d)$$

or $$i_6 = 12\ A(\text{from } d \text{ to } c)$$

4.17 In Fig. 4.30, find v. Also find the magnitudes and direction of the unknown currents through 10 Ω, 2 Ω and 5 Ω resistors.

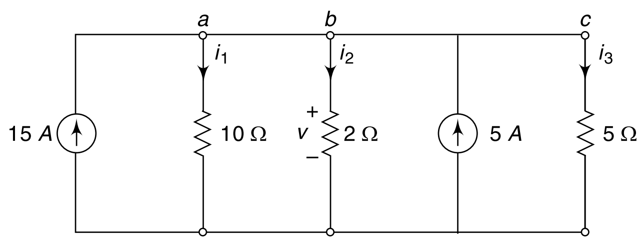

Fig. 4.30 *Circuit of Ex. 4.17*

Solution

Applying KCL at node 'a', (Fig. 4.30),

$$-15 + i_1 + i_2 - 5 + i_3 = 0$$

or $$i_1 + i_2 + i_3 = 20 \tag{i}$$

From Ohm's Law, $i_2 = \dfrac{v}{2}$; $i_1 = \dfrac{v}{10}$ and $i_3 = \dfrac{v}{5}$.

Then from equation (i), we have

$$\frac{v}{10} + \frac{v}{2} + \frac{v}{5} = 20$$

or $$v + 5v + 2v = 200$$

∴ $$v = 25 \text{ V.}$$

Hence $$i_1 = \frac{v}{10} = \frac{25}{10} = 2.5 \text{ A}$$

$$i_2 = \frac{25}{2} = 12.5 \text{ A}$$

$$i_3 = \frac{25}{5} = 5 \text{ A.}$$

4.18 In the part of the electrical network, shown in Fig. 4.31, find v_1. Assume $i_2 = (10e^{-3t})A$, $i_4 = 6(\sin t)\ A$ and $v_3 = (8e^{-3t})\ V$.

Solution

Applying KCl at the node '0' in Fig. 4.31,

$$-i_1 - i_2 - i_3 + i_4 = 0$$

or $$i_1 + 10e^{-3t} + c\frac{dv_3}{dt} - 6\sin t = 0$$

or $$i_1 + 10e^{-3t} + 2\times\frac{d}{dt}(8e^{-3t}) - 6\sin t = 0$$

or $$i_1 + 10e^{-3t} - 48e^{-3t} - 6\sin t = 0$$

$$\therefore \quad i_1 = 38e^{-3t} + 6\sin t$$

Now, $$v_1 = L\frac{di}{dt} = 4\times\frac{d}{dt}(38e^{-3t} + 6\sin t)$$

$$= 4\{-114e^{-3t} + 6\cos t\}$$

$$\therefore \quad v_1 = (24\cos t - 456\,e^{-3t})\ V$$

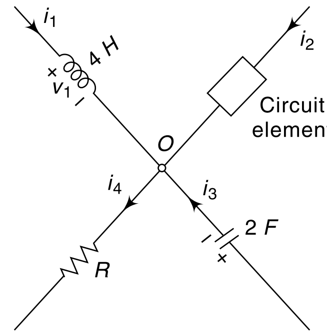

Fig. 4.31 ***Circuit of Ex. 4.18***

4.19 Find branch currents in the bridge circuit shown in Fig. 4.32.

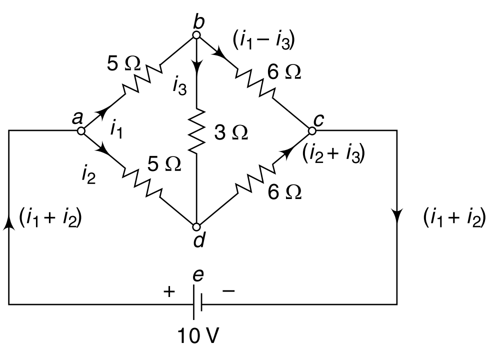

Fig. 4.32 ***Circuit of Ex. 4.19***

Solution

We assume currents i_1, i_2 and i_3 in the directions as shown in Fig. 4.32.

Applying KVL in loop '*abda*', we find

$$5i_1 + 3i_3 - 5i_2 = 0. \qquad \text{(i)}$$

Applying KVL in loop '*bcdb*', we find

$$6(i_1 - i_3) - 6(i_2 + i_3) - 3i_3 = 0$$

or $$6i_1 - 6i_2 - 15i_3 = 0. \qquad \text{(ii)}$$

Applying KVL in loop '*adcea*', we find

$$5i_2 + 6(i_2 + i_3) - 10 = 0$$

or $$11i_2 + 6i_3 - 10 = 0. \qquad \text{(iii)}$$

Solving equations (i), (ii) and (iii) we get

$$i_1 = i_2 = 0.91\text{ A};\ i_3 = 0.$$

4.20 In the network of Fig. 4.33, find v_1 and v_2 using KVL.

Solution

In loop '*abca*', from KVL we can write,

$$1 + v_2 - v_1 = 0$$

or $$v_1 - v_2 = 1. \qquad \text{(i)}$$

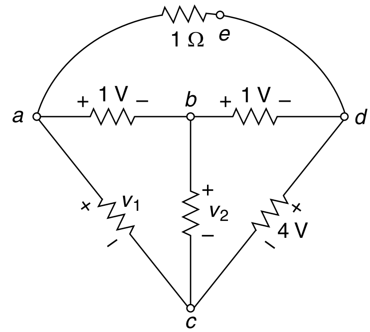

Fig. 4.33 ***Circuit of Ex. 4.20***

In loop '*bcdb*', using KVL we find,

$$-v_2 + 1 + 4 = 0$$

or $$v_2 = 5 \text{ V}.$$

Substituting the value of v_2 in equation (i) we get

$$v_1 = 6 \text{ V}.$$

4.21 Find current i in the circuit shown in Fig. 4.34.

Solution

The assumed and given currents in various branches of the circuit shown in Fig. 4.34 are drawn in Fig. 4.34(a).

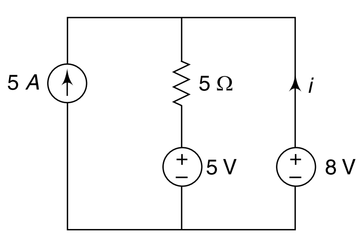

Fig. 4.34 ***Circuit of Ex. 4.21***

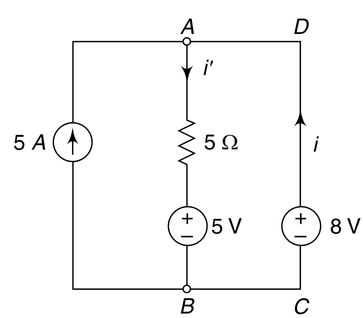

Fig. 4.34(a) ***Circuit of Ex. 4.21***

Using KCL at node '*A*',

$$-5 + i' - i = 0$$

or $$i' - i = 5. \quad \text{(i)}$$

Applying KVL in loop '*ABCDA*',

$$5i' + 5 - 8 = 0$$

i.e. $$i' = \frac{3}{5} = 0.6 \text{ A}.$$

Substituting the value of i' in (i), we have $i = 0.6 - 5 = -4.4$ A.

Thus i(4.4 *A*) flows from node *D* to node *C* in the actual circuit.

4.7 NODAL ANALYSIS

Nodal analysis is based on Kirchhoff's current law. This method has the advantage that a minimum number of equations are needed to determine the unknown quantities. Moreover, it is particularly suited for networks having many parallel branches and also when there are current sources in the network.

For the application of this method one of the nodes in the network is regarded as the *reference* or *datum* node or *zero potential* node. The number of simultaneous equations to be solved becomes $(n - 1)$, where n is the number of independent nodes.

Illustration

Referring Fig. 4.35, we find that nodes 'A' and 'B' are independent nodes. Let node 'B' be considered as reference node and the voltages at nodes 'A' and 'B' be (V_A) and (V_B) respectively. Obviously, $V_B = 0$.

Using Ohm's Law,

$$I_1 = \frac{E_1 - V_A}{R_1};$$

$$I_2 = \frac{E_2 - V_A}{R_2}$$

$$I_3 = \frac{V_A - V_B}{R_3} = \frac{V_A}{R_3}.$$

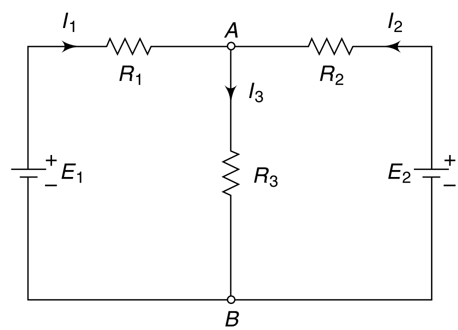

Fig. 4.35 ***Illustration for nodal method***

Applying KCL at node A,

$$-I_1 - I_2 + I_3 = 0$$

i.e.,
$$-\frac{E_1 - V_A}{R_1} - \frac{E_2 - V_A}{R_2} + \frac{V_A}{R_3} = 0. \tag{4.20}$$

This equation represents the *nodal* form of KCL. *In nodal analysis we usually assume inward currents as negative while outward currents as positive.*

4.7.1 Nodal Analysis Procedure

1. Convert all voltage sources to current sources and redraw the circuit diagram.
2. Identify all nodes and choose a reference node. (Usually, the common node is the reference node.)
3. Write the equation for the currents flowing into and out of each node, with the exception of the reference node.
4. Solve the equation to determine the node voltage and the required branch currents.

4.22 Find the voltage v in the circuit shown in Fig. 4.36.

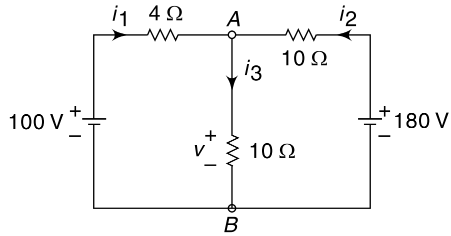

Fig. 4.36 *Circuit of Ex. 4.22*

Solution

Considering 'B' as reference node, $V_B = 0$. Let V_A be the potential at node 'A'.

Obviously, $V_A - V_B = v$, i.e., $V_A = v$.

Using nodal analysis at node 'A', we get

$$\frac{V_A - 100}{4} + \frac{V_A}{10} + \frac{V_A - 180}{10} = 0$$

or $$\frac{v - 100}{4} + \frac{v}{10} + \frac{v - 180}{10} = 0$$

or $$v\left(\frac{1}{10} + \frac{1}{10} + \frac{1}{4}\right) = 43$$

$\therefore$ $$v = \frac{43 \times 10}{4.5} = 95.55 \text{ V}.$$

4.23 Find the currents in different branches of the network shown in Fig. 4.37 using nodal analysis.

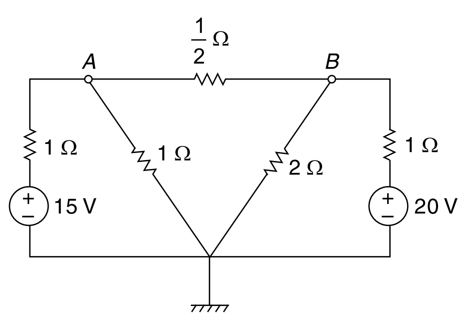

Fig. 4.37 *Circuit of Ex. 4.23*

Let V_A and V_B be the nodal voltages at nodes 'A' and 'B' in the given figure. The ground node is the reference node. Using nodal analysis, at node 'A' we can write

$$\frac{V_A - 15}{1} + \frac{V_A}{1} + \frac{V_A - V_B}{1/2} = 0$$

or $$4V_A - 2V_B = 15. \qquad \text{(i)}$$

At node '*B*' we can write

$$\frac{V_B - V_A}{1/2} + \frac{V_B}{2} + \frac{V_B - 20}{1} = 0$$

or $$3.5\ V_B - 2\ V_A = 20 \qquad \text{(ii)}$$

Solving equations (i) and (ii), we get

$$V_B = 11\text{ V};\ V_A = 9.25\text{ V}.$$

Hence, current through the respective resistors can be calculated as follows:

$$I_1 = \frac{V_A - 15}{1} = -5.75\text{ A}$$

$$I_2 = \frac{V_A}{1} = 9.25\text{ A}$$

$$I_3 = \frac{V_A - V_B}{1/2} = -3.5\text{ A}$$

$$I_4 = \frac{V_B}{2} = 5.5\text{ A}$$

$$I_5 = \frac{V_B - 20}{1} = -9\text{ A}.$$

Figure 4.37(a) represents the circuit alongwith associated currents.

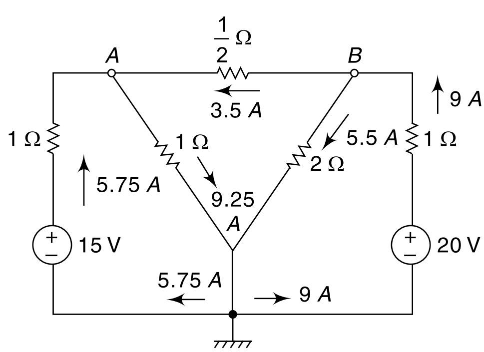

Fig. 4.37(a) ***Current values in branches of Fig. 4.37***

4.24 Find the node voltages (V_x) and (V_y) using nodal analysis (Fig. 4.38).

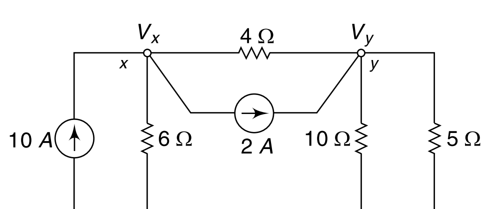

Fig. 4.38 ***Circuit of Ex. 4.24***

Solution

At node 'x', we have

$$-10 + \frac{V_x}{6} + 2 + \frac{V_x - V_y}{4} = 0$$

or $$V_x\left(\frac{1}{4}+\frac{1}{6}\right)-\frac{V_y}{4} = 8$$

or $$5V_x - 3V_y = 96. \qquad \text{(i)}$$

Applying nodal analysis at 'y', we get

$$-2 + \frac{V_y - V_x}{4} + \frac{V_y}{10} + \frac{V_y}{5} = 0$$

or $$-\frac{V_x}{4} + V_y\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{10}\right) = 2$$

or $$5V_x - 11V_y = -40. \qquad \text{(ii)}$$

Solving equations (i) and (ii), we get

$$V_x = 29.4 \text{ V}; \; V_y = 17 \text{ V}.$$

4.25 Find current in the 15 Ω resistor using nodal method (Fig. 4.39).

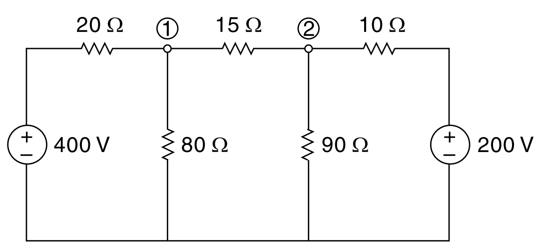

Fig. 4.39 ***Circuit of Ex. 4.25***

Solution

Let us first designate the nodes '1' and '2' in Fig. 4.39 and assume nodal voltages to be (V_1) and (V_2) respectively

At node '1',

$$\frac{V_1 - 400}{20} + \frac{V_1}{80} + \frac{V_1 - V_2}{15} = 0$$

or $$V_1\left(\frac{1}{20}+\frac{1}{80}+\frac{1}{15}\right)-\frac{V_2}{15} = 20$$

or $$\frac{31}{240}\cdot V_1 - \frac{1}{15}\cdot V_2 = 20 \qquad \text{(i)}$$

Similarly, using nodal analysis at node '2',

$$\frac{V_2 - 200}{10} + \frac{V_2}{90} + \frac{V_2 - V_1}{15} = 0$$

or $$-\frac{V_1}{15}+V_2\left(\frac{1}{10}+\frac{1}{15}+\frac{1}{90}\right)V_2=20$$

or $$-\frac{1}{15}\cdot V_1+\frac{16}{90}\cdot V_2 = 20 \tag{ii}$$

Soling equations (i) and (ii) we get

$$V_1 = 264.88 \text{ V}; \quad V_2 = 211.33 \text{ V}$$

Hence, current in the 15 Ω resistor is obtained as

$$I_{15} = \frac{V_1 - V_2}{15} = \frac{264.88 - 211.33}{15} = 3.57 \text{ A}$$

This current is directed from node '1' and node '2'.

4.26 In Fig. 4.40, find "v" in the given circuit using nodal analysis.

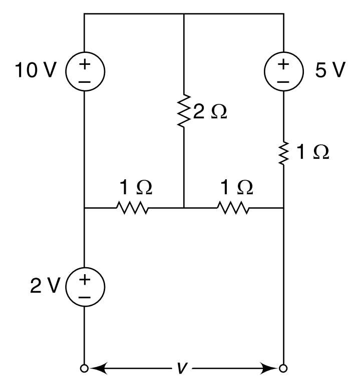

Fig. 4.40 ***Circuit of Ex. 4.26***

Solution

Let us mark the junction of two resistors (1 Ω and 2 Ω) as node 'A' and assume the voltage at this node to be (V_A). Applying nodal analysis at 'A' we get

$$\frac{V_A}{2}+\frac{V_A+5}{1+1}+\frac{V_A+10}{1} = 0$$

or $$V_A\left(\frac{1}{2}+\frac{1}{2}+1\right)+\frac{5}{2} + 10 = 0$$

or $$V_A = -6.25 \text{ V}.$$

We now find the currents passing through both side resistors of the node 'A'. We redraw Fig. 4.40 as Fig. 4.40(a) and mark the corresponding resistors as r_1 and r_2. The current through r_1 is given by $i_{r_1} = V_A + 10/1$ = 3.75 A, directed outwards of node 'A'. Similarly, current through r_2 is given by $i_{r_2} = \frac{V_A+5}{1+1}$ = –0.625 A, directed towards the node 'A'.

∴ Voltage drop across r_1 is (3.75 × 1) i.e., 3.75 V while that across r_2 is (–0.625 × 1) i.e., –0.625 V. The corresponding polarities have been marked in Fig. 4.40(a).

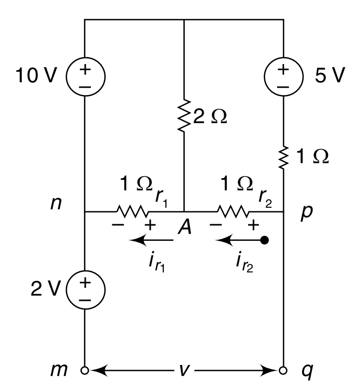

Fig. 4.40(a) ***Figure 4.40, redrawn, for analysis***

Finally, in loop '$mnApq$' we can write, from KVL,

$$-v - 2 - 3.75 + 0.625 = 0$$

i.e. $$v = -5.125 \text{ V}.$$

(It means, polarity of 'm' is actually negative while polarity of 'q' is actually positive in Fig. 4.40(a)).

4.27 Obtain the current through the 1 Ω resistor using node voltage method for the circuit shown in Fig. 4.41.

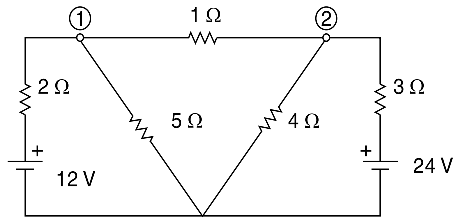

Fig. 4.41 *Circuit of Ex. 4.27*

Solution

Let us first mark the nodes '1' and '2' in Fig. 4.41 and assume corresponding nodal voltages to be V_1 and V_2.

At node '1', we have

$$\frac{V_1 - 12}{2} + \frac{V_1}{5} + \frac{V_1 - V_2}{1} = 0$$

or $$V_1\left(\frac{1}{5} + \frac{1}{2} + 1\right) - V_2 = 6$$

or $$17\ V_1 - 10\ V_2 = 60. \quad (1)$$

At node '2', we have

$$\frac{V_2 - 24}{3} + \frac{V_2}{4} + \frac{V_2 - V_1}{1} = 0$$

or $$-V_1 + V_2\left(\frac{1}{3} + \frac{1}{4} + 1\right) = 8$$

or $$-V_1 + \frac{19}{12}V_2 = 8$$

or $$-12V_1 + 19V_2 = 96. \quad (ii)$$

Solving (i) and (ii) we get, $V_1 = 10.35$ V; $V_2 = 11.6$ V

Hence the current through 1 Ω resistor is

$$I_1 = \frac{V_2 - V_1}{1} = \frac{11.6 - 10.35}{1}.$$

$= 1.25$ V, directed from node '2' and to node '1'.

4.28

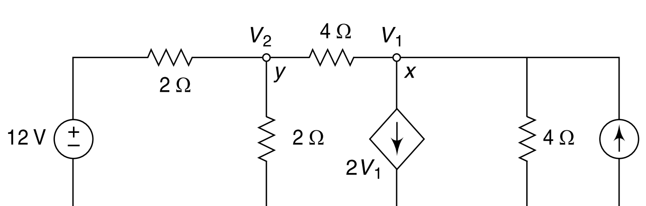

Fig. 4.42 *Circuit of Ex. 4.28*

In Fig. 4.42, find I so that $V_1 = 0$.

Solution

At node 'x' we can write, using nodal analysis,

$$\frac{V_1 - V_2}{4} + 2V_1 + \frac{V_1}{4} - I = 0$$

or $$2.5\ V_1 - 0.25\ V_2 = I \tag{i}$$

At node 'y', using nodal analysis, we write

$$\frac{V_2 - V_1}{4} + \frac{V_2}{2} + \frac{V_2 - 12}{2} = 0$$

or $$1.25\ V_2 - 0.25\ V_1 = 6 \tag{ii}$$

But as per question, $V_1 = 0$.

$\therefore$ from (ii), $V_2 = \dfrac{6}{1.25} = 4.8$ V.

Also, from (i), $I = -4.8 \times 0.25 = -1.2$ A

Thus the current source I pushing current in the reverse direction and of magnitude 1.2 A will make $V_1 = 0$.

4.29 Obtain the value of V_R in the network shown in Fig. 4.43.

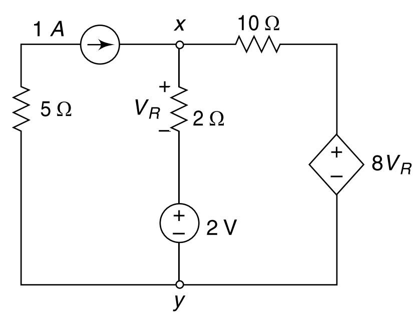

Fig. 4.43 ***Circuit of Ex. 4.29***

Solution

In the network of Fig. 4.43 let us assume that the node voltage at node 'x' be 'V_x'. Thus at node 'x' we can write,

$$-1 + \frac{V_x - 2}{2} + \frac{V_x - 8V_R}{10} = 0$$

or $$\frac{V_x}{2} + \frac{V_x}{10} - 1 - \frac{4}{5}V_R = 1$$

or $$V_R = \left(\frac{3}{4}V_x - \frac{5}{2}\right) \text{V} \tag{i}$$

Also, in branch 'xy', $V_x - 2 = V_R$

i.e. $$V_x = (V_R + 2) \text{ V} \tag{ii}$$

Substituting the value of V_x from equation (ii) in equation (i), we get

$$V_R = \frac{3}{4}(V_R + 2) - \frac{5}{2} = \frac{3}{4}V_R - 1$$

$\therefore$ $$V_R = -4 \text{ V}.$$

4.8 MESH ANALYSIS (OR LOOP ANALYSIS)

The *mesh* or *loop analysis* is based on Kirchhoff's voltage law. Here the currents in different meshes are assigned continuous paths so that they do not split at a junction into branch currents. In this method loop voltage equations are written by KVL in terms of unknown loop currents. Circuits with voltage sources are comparatively easier to be solved by this method.

Illustration

Figure 4.44 shows that two batteries having emf E_1 and E_2 are connected in a network containing five resistors. There are two loops and the respective loop currents are I_1 and I_2. Applying KVL in loop 1, we have

$$-E_1 + I_1R_1 + (I_1 - I_2)R_2 + I_1R_4 = 0$$

or $E_1 = I_1(R_1 + R_2 + R_4) - I_2R_2$ (i)

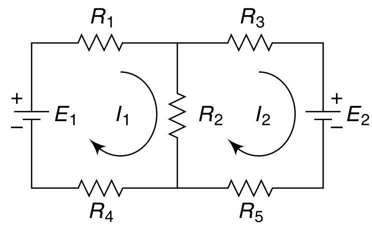

Fig. 4.44 ***Illustration of mesh analysis***

Applying KVL in loop 2, we get

$$E_2 + I_2R_3 + (I_2 - I_1)\, R_2 + I_2R_5 = 0$$

or $E_2 = I_1R_2 - (R_2 + R_3 + R_5)I_2.$ (ii)

Solving equations (x) and (y), we can find the values of I_1 and I_2 and subsequently branch currents can be evaluated.

4.8.1 DC Circuit Analysis Procedure using Loop Equations

1. Convert all current sources to voltage sources.
2. Draw all loop currents in a clockwise direction and identify them.
3. Identify all resistor voltage drops as + to – in the direction of the loop current and assume these drops to be positive.
4. Identify all voltage sources according to their correct polarity.
5. Write the equations for the voltage drops around each loop in turn, by equating the sum of the voltage drops to zero.
6. Solve the equations to find the unknown currents and/or voltage drops.

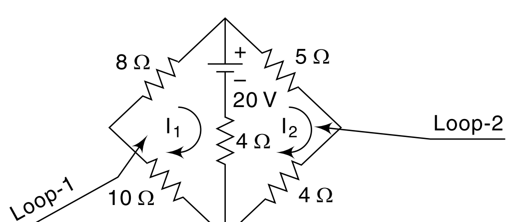

Fig. 4.45 ***Circuit of Ex. 4.30***

4.30 Calculate the current supplied by the battery in Fig. 4.45 using loop current method.

Applying KVL in loop-1

$$8I_1 + 20 + (I_1 - I_2)\,4 + 10I_1 = 0$$

or $\quad 22I_1 - 4I_2 = -20$

or $\quad 11I_1 - 2I_2 = -10$

or $$I_2 = \frac{11I_1 + 10}{2} \quad \text{(i)}$$

Applying KVL in loop-2,

$$5I_2 + 4I_2 + (I_2 - I_1)\,4 - 20 = 0$$

or $$-4I_1 + 13I_2 = 20 \quad \text{(ii)}$$

Substituting the value of I_2 from equation (i) in equation (ii), we get

$$-4I_1 + 13 \times \frac{10 + 11I_1}{2} - 20 = 0$$

or $$I_1 = -0.667 \text{ A}$$

Also, $$I_2 = \frac{10 + 11(-0.667)}{2} = 1.33 \text{ A.}$$

Hence the current supplied by the battery is obtained as $(I_2 - I_1)$, i.e, 1.33 + 0.667 = 1.997 A.

4.31 Find the currents in 2 Ω, 3 Ω, 4 Ω, 5 Ω and 10 Ω resistances in the circuit shown in Fig. 4.46 using loop method.

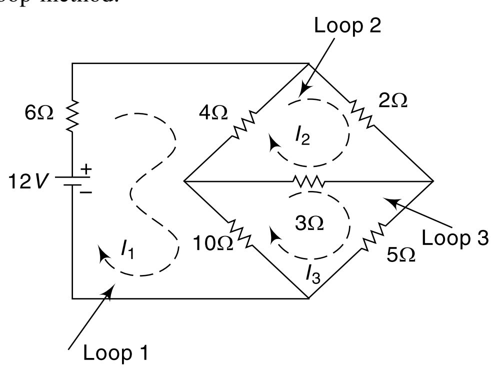

Fig. 4.46 ***Circuit of Ex. 4.31***

Solution

Let us first mark the loop current in Fig. 4.46 as shown by dotted arrows.

For loop 1 we can write,

$$-12 + 6I_1 + (I_1 - I_2)\,4 + (I_1 - I_3)\,10 = 0$$

or $$10I_1 - 2I_2 - 5I_3 = 6. \quad \text{(i)}$$

Applying mesh method in loop 2, we have

$$2I_2 + (I_2 - I_3)\,3 + (I_2 - I_1)\,4 = 0$$

or $$4I_1 - 9I_2 + 3I_3 = 0. \quad \text{(ii)}$$

Applying mesh method in loop 3, we have

$$5I_3 + (I_3 - I_1)\,10 + (I_3 - I_2)\,3 = 0$$

or $$10I_1 + 3I_2 - 18I_3 = 0. \quad \text{(iii)}$$

Comparing equations (i) and (iii), we get

$$5I_2 - 13I_3 = -6$$

∴ $$I_2 = \frac{13I_3 - 6}{5} \quad \text{(iv)}$$

Again, from equation (ii) we can write

$$I_1 = \frac{9I_2 - 3I_3}{4}.$$

Substituting this value of I_1 in equation (i), we get

$$10 \times \frac{9I_2 - 3I_3}{4} - 2I_2 - 5I_3 = 6.$$

Simplifying,

$$I_2 = \left(6 + \frac{50}{4} I_3\right)\frac{4}{82}. \quad \text{(v)}$$

From equations (iv) and (v), we have

$$\frac{13I_3 - 6}{5} = \frac{4}{82}\left(6 + \frac{50}{4} I_3\right)$$

or $82(13I_3 - 6) = 20\left(6 + \frac{50}{4} \times I_3\right)$

or $$I_3 = \frac{612}{816} = 0.75 \text{ A}.$$

∴ From equation (iv) we now can write

$$I_2 = \frac{13 \times 0.75 - 6}{5} = 0.75 \text{ A}$$

Also from equation (i), we can write

$$10I_1 = 2 \times 0.75 + 5 \times 0.75 + 6 = 11.25$$

∴ $$I_1 = 1.125 \text{ A}.$$

Thus current in the 2 Ω and 5 Ω resistors is 0.75 A each;

current in the 4 Ω resistor is $(I_1 - I_2)$ i.e., 0.375 A,

current in the 10 Ω resistor is $(I_1 - I_3)$ i.e., 0.375 A,

and current in the 3 Ω resistor is $(I_2 - I_3)$ i.e., 0 A.

4.32 From the mesh analysis find the current flow through a 50 V source in Fig. 4.47.

Solution

Let us designate the loop currents by dotted arrows in the network of Fig. 4.47.

In loop 1 we have

$$-50 + 5i_1 + (i_1 - i_2)\, 2 + 20 = 0$$

or $$7i_1 - 2i_2 - 30 = 0 \quad \text{(i)}$$

In loop 2 we can write,

$$3i_2 + 10 - 20 + (i_2 - i_1)\, 2 = 0$$

or $$i_1 = \frac{5}{2} i_2 - 5 \quad \text{(ii)}$$

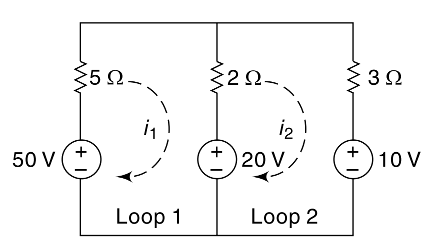

Fig. 4.47 ***Circuit of Ex. 4.32***

Substituting the value of i_1 from equation (ii) to equation (i), we get

$$7\left(\frac{5}{2} i_2 - 5\right) - 2i_2 - 30 = 0$$

Simplification yields,

$$i_2 = 4.19 \text{ A}$$

Thus from equation (ii) we get

$$i_1 = \frac{5}{2} \times 4.19 - 5 = 5.475 \text{ A.}$$

The current through the 50 V source is thus 5.475 A.

4.33 Find the voltage drop between terminals (*y*) and (*d*) in the network of Fig. 4.48.

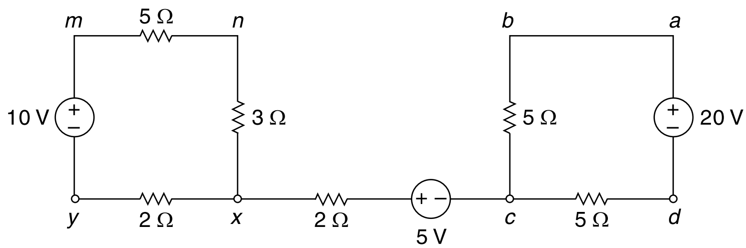

Fig. 4.48 *Circuit of Ex. 4.33*

Solution

The current supplied by the 10 V source in the loop-*mnxy* is obtained as

$$i_1 = \frac{10}{5+3+2} = 1 \text{ A.}$$

The current supplied by the 20 V source in the loop-*badc* is given by

$$i_2 = \frac{20}{5+5} = 2 \text{ A.}$$

The corresponding drops with polarities are shown in Fig. 4.48(a).

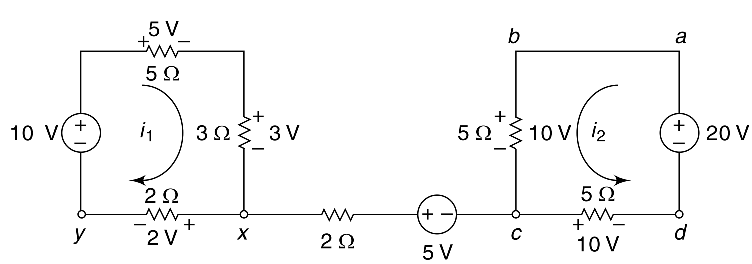

Fig. 4.48(a) *Currents and voltages for the circuit of Ex. 4.33*

$$\therefore \qquad V_{yd} = -V_{yx} + 5V + V_{cd} = -2 + 5 + 10 = 13 \text{ V.}$$

The drop across terminal (*y*) and (*d*) is 13 V.

4.34 Find the current through the resistors using mesh method for the network shown in Fig. 4.49.

Solution

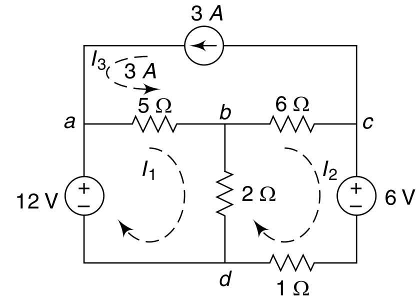

Fig. 4.49 ***Circuit of Ex. 4.34***

Let us first draw the loop currents in the network of Fig. 4.49. The loop currents are shown by dotted arrows. It may be noted that due to presence of current source of 3 A, the corresponding loop current I_3 is 3 A.

In the loop containing 12 V source, we have

$$-12 + (I_1 + 3)5 + (I_1 - I_2)2 = 0$$

or $\quad 7I_1 - 2I_2 + 3 = 0. \qquad$ (i)

Applying mesh analysis in the loop containing 6*V* source, we get

$$I_2\,1 + (I_2 - I_1)\,2 + (I_2 + 3)\,6 + 6 = 0$$

or $\quad I_1 = \frac{9}{2} \cdot I_2 + 12. \qquad$ (ii)

Substituting the value of I_1 from equation (ii) in (i), we get

$$7\left(\frac{9}{2}I_2 + 12\right) - 2I_2 + 3 = 0$$

or $\quad I_2 = -2.95$ A

Thus from equation (ii), we get

$$I_1 = \frac{9}{2} \times (-2.95) + 12 = -1.275 \text{ A}$$

We now can find currents in respective resistors:

Current through 5 Ω resistor (= $I_1 + I_3$) = −1.275 + 3 = 1.725 A.

[It may be noted that the current obtained through the 5 Ω resistor is directed from *a* to *b*].

Current through the 6 Ω resistor (= $I_2 + I_3$) = −2.95 + 3 = 0.05 A.

[This current is directed from *b* to *c*].

Current through the 2 Ω resistor (= $I_1 + I_2$) = −1.275 + 2.95 = 1.675 A.

[The current through the 2 Ω resistor is directed from *b* to *d*].

Finally, the current through the 1 Ω resistor (I_2) is (−2.95 *A*) and is directed from *d* to *c*.

4.35 In the bridge network shown in Fig. 4.50 find the current through the galvanometer having 20 Ω internal resistance. Use mesh analysis.

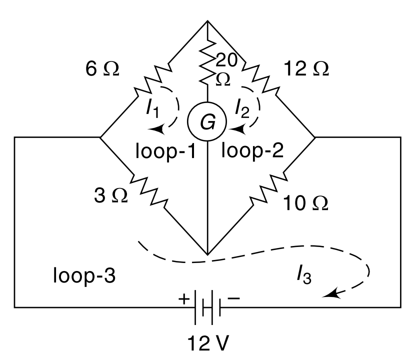

Fig. 4.50 ***Circuit of Ex. 4.35***

Solution

We first assign loops as *loop 1*, *loop 2* and *loop 3* with circulating currents I_1, I_2 and I_3 through these loops (Fig. 4.50).

In loop 1 we have

$$6\,I_1 + (I_1 - I_2)\,20 + (I_1 - I_3)\,3 = 0$$

or $\quad 29\,I_1 - 20\,I_2 - 3\,I_3 = 0. \qquad$ (i)

In loop 2 we have

$$12\,I_2 + (I_2 - I_3)\,10 + (I_2 - I_1)\,20 = 0$$

or $\quad -20\,I_1 + 42\,I_2 - 10I_3 = 0. \qquad$ (ii)

Similarly, in loop 3 we can write

$$-12 + (I_3 - I_1)\,3 + (I_3 - I_2)\,10 = 0$$

or $$-3\,I_1 - 10\,I_2 + 13\,I_3 - 12 = 0. \qquad \text{(iii)}$$

Let us now solve these three simultaneous equations.

From equation (i), $I_1 = \dfrac{20\,I_2 + 3\,I_3}{29}$ (iv)

and from equation (ii), $I_1 = \dfrac{42\,I_2 - 10\,I_3}{20}$ (v)

Comparing equation (iv) and (v) we get

$$\frac{20\,I_2 + 3\,I_3}{29} = \frac{42\,I_2 - 10\,I_3}{20}$$

or $$818\,I_2 - 350\,I_3 = 0 \qquad \text{(vi)}$$

Again, from equation (iii) we find

$$I_1 = \frac{-10\,I_2 + 13\,I_3 - 12}{3} \qquad \text{(vii)}$$

Comparing equation (v) with equation (vii) we get

$$\frac{-10\,I_2 + 13\,I_3 - 12}{3} = \frac{42\,I_2 - 10\,I_3}{20}$$

or $$326\,I_2 - 290\,I_3 = -240 \qquad \text{(viii)}$$

From equation (vi) we find $I_2 = (350\,I_3/818)$; substitution of value of I_2 in equation (viii) yields

$$326 \times \frac{350}{818} \cdot I_3 - 290\,I_3 = -240$$

or $$I_3 = 1.59 \text{ A}.$$

From (vi), I_2 can be found as $I_2 = (350/818) \times 1.59$

i.e., $I_2 = 0.68\ A$

From (vii) we can find the value of I_1;

$$I_1 = \frac{-10 \times 0.68 + 13 \times 1.59 - 12}{3} = 0.633 \text{ A}.$$

The current $(I_2 - I_1)$ through the galvanometer is then obtained. Obviously,

$$(I_2 - I_1) = I_G = 0.68 - 0.633 = 0.047 \text{ A (directed upwards).}$$

4.36 Find current in all branches of the network shown in Fig. 4.51.

Solution

Let the current in the arm *AF* be *I* amps, as shown by dotted arrow. Using the concept of KCL, the currents at each of the branches have been identified in Fig. 4.51 in terms of the assumed current *I*. Next we apply the mesh analysis at the hexagonal network *AFEDCBA*. We have

$$0.02(I) + 0.01(I - 60) + 0.03(I) + 0.01(I - 120)$$
$$+ 0.01(I - 50) + 0.02(I - 80) = 0.$$

Solving for *I*, we get $I = 39$ A.

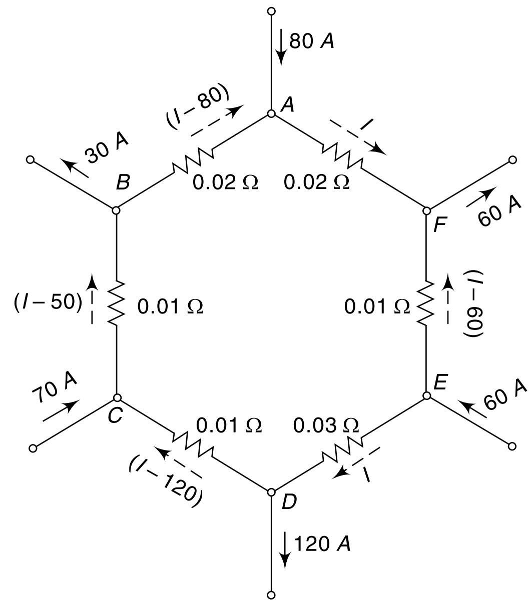

Fig. 4.51 *Circuit of Ex. 4.36*

Thus we can identify the branch currents as

current in $AF = I_{AF} = 39\ A\ (=I)$

current in $FE = (I - 60) = -21$ A

current in $ED = I_{ED} = I = 39$ A

current in $DC = I_{DC} = (I - 120) = -81$ A

current in $CB = I_{CB} = (I - 50) = -11$ A

current in $BA = I_{BA} = (I - 80) = -41$ A.

4.8.2 Mesh Analysis Using Matrix Form

Let us consider the network shown in Fig. 4.52; it contains three meshes. The three mesh currents are I_1, I_2 and I_3 and they are assumed to flow in a clockwise direction.

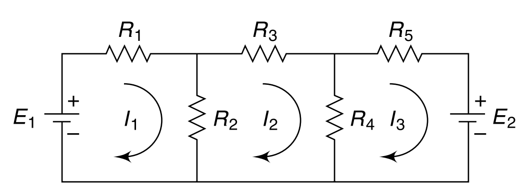

Fig. 4.52 *Concept of mesh analysis in matrix form*

Applying KVL to mesh 1

$$-E_1 + (I_1 - I_2)R_2 + I_1R_1 = 0$$

or $$I_1R_1 + (I_1 - I_2)R_2 = E_1$$

or $$I_1(R_1 + R_2) - I_2R_2 = E_1$$

or $$I_1(R_1 + R_2) + I_2(-R_2) = E_1. \quad (4.21)$$

Applying KVL to mesh 2

$$(I_2 - I_1)R_2 + I_2R_3 + (I_2 - I_3)R_4 = 0$$

or $$-I_1R_2 + I_2(R_2 + R_3 + R_4) - I_3R_4 = 0$$

or $$I_1(-R_2) + I_2(R_2 + R_3 + R_4) + I_3(-R_4) = 0 \tag{4.22}$$

Applying KVL to mesh 3

$$E_2 + I_3R_5 + (I_3 - I_2)R_4 = 0$$

or $$-I_2R_4 + I_3(R_4 + R_5) = -E_2. \tag{4.23}$$

It should be noted that the signs of resistances in the above equations have been so arranged as to make the items containing self-resistances positive. The matrix equivalent of the above three equations is

$$\begin{bmatrix} R_1 + R_2 & -R_2 & 0 \\ -R_2 & R_2 + R_3 + R_4 & -R_4 \\ 0 & -R_4 & R_4 + R_5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \\ -E_2 \end{bmatrix}$$

In general the resistance matrix $[R]$ can be written as

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

where R_{11} = *self-resistance* of mesh 1 = $R_1 + R_2$

R_{22} = *self-resistance* of mesh 2 = $R_2 + R_3 + R_4$

R_{33} = *self-resistance* of mesh 3 = $R_4 + R_5$

$R_{12} = R_{21}$

= – [sum of all the resistances common to meshes 1 and 2]

$= -R_2$

$R_{23} = R_{32}$

= – [sum of all the resistances common to meshes 2 and 3]

$= -R_4$

$R_{13} = R_{31}$

= – [sum of all the resistances common to meshes 3 and 1]

= 0 (here).

[R_{11}, R_{22}, R_{33} … are called *diagonal elements* of the resistance matrix while R_{12}, R_{13}, R_{21}, R_{23}, … are called *off-diagonal elements*.]

4.37 Find the mesh currents in Fig. 4.53 using mesh current method.

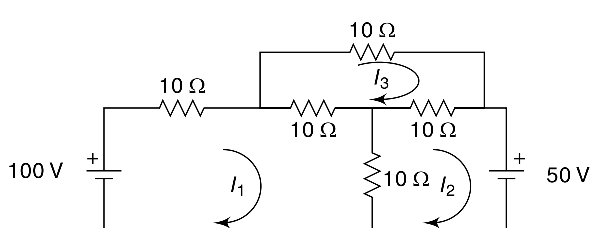

Fig. 4.53 ***Circuit of Ex. 4.37***

Solution

Applying KVL in loop 1

$$-100 + 10(I_1 - I_2) + 10(I_1 - I_3) + 10I_1 = 0$$

or $$I_1(10 + 10 + 10) - 10I_2 - 10I_3 = 100$$

Applying KVL in loop 2

$$50 + 10(I_2 - I_3) + 10(I_2 - I_1) = 0$$

or $$-10I_1 + I_2(10 + 10) - 10I_3 = -50$$

Applying KVL in loop 3

$$10I_3 + 10(I_3 - I_1) + 10(I_3 - I_2) = 0$$

or $$-10I_1 - 10I_2 + I_3(10 + 10 + 10) = 0$$

The above equations in matrix form can be written as

$$\begin{bmatrix} 30 & -10 & -10 \\ -10 & 20 & -10 \\ -10 & -10 & 30 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 100 \\ -50 \\ 0 \end{bmatrix}$$

Hence $$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 30 & -10 & -10 \\ -10 & 20 & -10 \\ -10 & -10 & 30 \end{bmatrix}^{-1}\begin{bmatrix} 100 \\ -50 \\ 0 \end{bmatrix}$$

or $$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \frac{1}{8000}\begin{bmatrix} 500 & 400 & 300 \\ 400 & 800 & 400 \\ 300 & 400 & 500 \end{bmatrix}\begin{bmatrix} 100 \\ -50 \\ 0 \end{bmatrix}$$

Therefore, $$I_1 = \frac{500 \times 100 - 400 \times 50}{8000} A = 3.75 \text{ A}$$

$$I_2 = \frac{400 \times 100 - 50 \times 800}{8000} A = 0 \text{ A}$$

and $$I_3 = \frac{300 \times 100 - 400 \times 50}{8000} A = 1.25 \text{ A}$$

4.38 Find the ammeter current in Fig. 4.54 using mesh analysis.

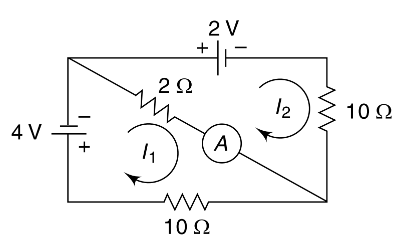

Fig. 4.54 ***Circuit of Ex. 4.38***

Solution

Applying mesh method in mesh 1

$$4 + 10I_1 + 2(I_1 - I_2\ ) = 0$$

or $$12I_1 - 2I_2 = -4.$$

Applying mesh method in mesh 2

$$2 + 2(I_2 - I_1) + 10I_2 = 0$$

$$-2I_1 + 12I_2 = -2.$$

In the matrix form the above equations can be written as

$$\begin{bmatrix} 12 & -2 \\ -2 & 12 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

or $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 12 & -2 \\ -2 & 12 \end{bmatrix}^{-1}\begin{bmatrix} -4 \\ -2 \end{bmatrix} = \frac{1}{144 - 4}\begin{bmatrix} 12 & 2 \\ 2 & 12 \end{bmatrix}\begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

or $$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{140}\begin{bmatrix} -52 \\ -32 \end{bmatrix}$$

Hence $I_1 = \dfrac{-52}{140}$ A and $I_2 = -\dfrac{32}{140}$ A.

Therefore, the current through ammeter is

$I_2 - I_1 = \dfrac{52}{140} - \dfrac{32}{140} = \dfrac{1}{7}$ A [in the direction of (I_2) as shown in Fig. 4.54].

4.9 STAR DELTA CONVERSION

Like series and parallel connections the resistances may be connected in *star* (Y) or *delta* (Δ) connection as shown in Fig. 4.55(a) and Fig. 4.55(b).

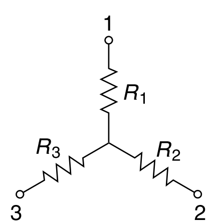

Fig. 4.55(a) *A star (or* T*) connection*

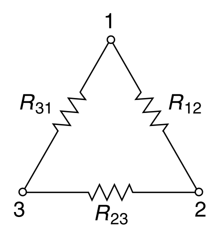

Fig. 4.55(b) *A delta (or mesh) connection*

Circuits shown in Fig. 4.55(a) and Fig. 4.55(b) are identical provided their respective resistances from terminals (12), (23) and (32) are equal.

In star connection,

Resistance between terminals 1 & 2 is $(R_1 + R_2)$

Resistance between terminals 2 & 3 is $(R_2 + R_3)$

Resistance between terminals 3 & 1 is $(R_3 + R_1)$.

Similarly in delta connection,

Resistance between terminals 1 & 2 is $[R_{12} || (R_{23} + R_{31})]$

$$= \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminals 2 & 3 is $[R_{23} || (R_{31} + R_{12})]$

$$= \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}$$

Resistance between terminals 3 & 1 is $[R_{31} || (R_{12} + R_{23})]$

$$= \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}}.$$

Now, we equate the resistances in star and delta across appropriate terminals.

i.e. $$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \tag{4.24}$$

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \qquad (4.25)$$

$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \qquad (4.26)$$

Subtracting equation (4.25) from equation (4.24) we get

$$R_1 - R_3 = \frac{R_{12}(R_{23} + R_{31}) - R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}}$$

$$= \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \qquad (4.27)$$

Adding equations (4.26) and (4.27)

$$2R_1 = \frac{2 R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

or $$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}}$$

In a similar way, $R_2 = \dfrac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}}$ and

$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}}$$

Thus we see that if the resistances in delta connected resistance network are known, we can find the equivalent star network where

$$R_1 = \frac{R_{31} R_{12}}{R_{12} + R_{23} + R_{31}} \qquad 4.28(a)$$

$$R_2 = \frac{R_{12} R_{23}}{R_{12} + R_{23} + R_{31}} \qquad 4.28(b)$$

$$R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \qquad 4.28(c)$$

R_1, R_2 and R_3 being equivalent resistances in the star network and R_{12}, R_{23} and R_{31} the resistances in the delta network.

Next, multiplying each equation [4.28(a), 4.28(b) and 4.28(c)] with another and adding

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{31} R_{12}^2 R_{23} + R_{12} R_{23}^2 R_{31} + R_{23} R_{31}^2 R_{12}}{(R_{12} + R_{23} + R_{31})^2}\,. \qquad (4.29)$$

Dividing equation (4.29) by (R_1), we get

$$R_2 + \frac{R_2 R_3}{R_1} + R_3 = \frac{R_{12} R_{23} R_{31}(R_{12} + R_{23} + R_{31})}{R_1 (R_{12} + R_{23} + R_{31})^2}$$

$$= \frac{R_{12} R_{23} R_{31}}{R_1 (R_{12} + R_{23} + R_{31})} \tag{4.30}$$

Substituting the value of R_1 from equation (4.28(a)) in equation (4.30) we get

$$R_2 + R_3 + \frac{R_2 R_3}{R_1} = \frac{R_{12} R_{23} R_{31}(R_{12} + R_{23} + R_{31})}{R_{31} R_{12} (R_{12} + R_{23} + R_{31})}$$

$$= \frac{R_{12} R_{23} R_{31}}{R_{31} R_{12}} = R_{23}$$

i.e.,
$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}.$$

Similarly, dividing equation (4.29) by R_2 we get

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

and dividing equation (4.29) by (R_3) we get

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}.$$

Thus we find R_{12}, R_{23} and R_{31}, i.e. the equivalent delta network provided R_1, R_2 and R_3 of the star network are given.
The equations are

$$R_{12} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3} \qquad [4.31(a)]$$

$$R_{23} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \qquad [4.31(b)]$$

$$R_{31} = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \qquad [4.31(c)]$$

4.9.1 Delta-Star (Δ-Y) and Star-Delta (Y-Δ) Transformation Procedures

1. When starting with a Δ network, draw a *Y* network; when starting with a *Y* network, draw a Δ network.
2. Identify the three corresponding terminals on each network as 1, 2 and 3.
3. Identify the resistors on the Δ network as follows:
   Resistor between terminals 1 and 2 as (R_{12})
   Resistor between terminals 1 and 3 as (R_{13})
   Resistor between terminals 2 and 3 as (R_{23}).

4. Identify the resistors on the Y network as follows:
   Resistor connected to terminal 1 as (R_1)
   Resistor connected to terminal 2 as (R_2)
   Resistor connected to terminal 3 as (R_3).
5. For Δ to Y transformation, substitute the Δ network resistor values into equations 4.28(a), 4.28(b) and 4.28(c) to obtain the Y network resistor values.
6. For Y to Δ transformation, substitute the Y network resistor values into equations 4.31(a), 4.31(b) and 4.31(c) to obtain the Δ network resistor values.
   [A Y network is also called as T(Te) network while a Δ network may be called as a mesh or π(pi) network].

4.39 Convert the π network shown in Fig. 4.56 into equivalent T network.

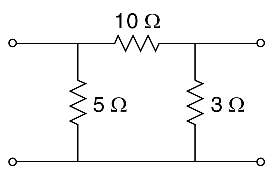

Fig. 4.56 ***Circuit of Ex. 4.39***

Solution

The network in Fig. 4.56 can be redrawn as shown in Fig. 4.56(a), Here R_1, R_2 and R_3 in star combination represent the equivalent of the given delta network.

$$R_1 = \frac{R_{12} \times R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{10 \times 5}{10 + 3 + 5} = 2.78\ \Omega$$

$$R_2 = \frac{R_{23} \times R_{12}}{R_{12} + R_{23} + R_{31}} = \frac{3 \times 10}{10 + 3 + 5} = 1.67\ \Omega$$

$$R_3 = \frac{R_{23} \times R_{31}}{R_{12} + R_{23} + R_{31}} = \frac{3 \times 5}{10 + 3 + 5} = 0.83\ \Omega$$

Thus we have obtained the equivalent star (or T) resistances given by

$R_1 = 2.78\ \Omega$; $R_2 = 1.67\ \Omega$; $R_3 = 0.83\ \Omega$.

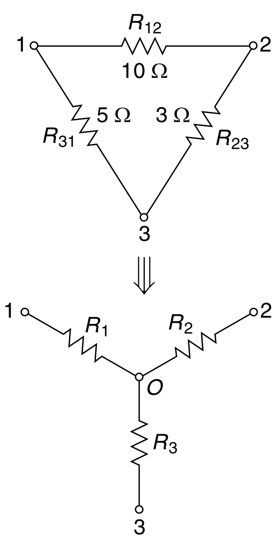

Fig. 4.56(a) ***Δ-Y conversion of the given network***

4.40 Find the input resistance (R) of the network shown in Fig. 4.57.

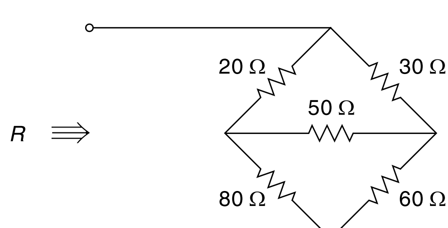

Fig. 4.57 ***Circuit of Ex. 4.40***

Solution

Converting the upper delta network of Fig. 4.57 into a star network [Fig. 4.57(a)] we obtain the arm impedances of the equivalent star network as

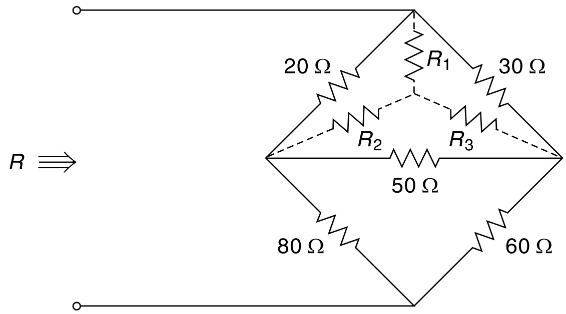

Fig. 4.57(a) ***Conversion of upper delta network to equivalent star for the network of Ex. 4.40***

$$R_1 = \frac{20 \times 30}{20 + 30 + 50} = 6\ \Omega$$

$$R_2 = \frac{20 \times 50}{20 + 30 + 50} = 10\ \Omega$$

$$R_3 = \frac{30 \times 50}{20 + 30 + 50} = 15\ \Omega.$$

Next we further reorient the network as shown in Fig. 4.57(b).

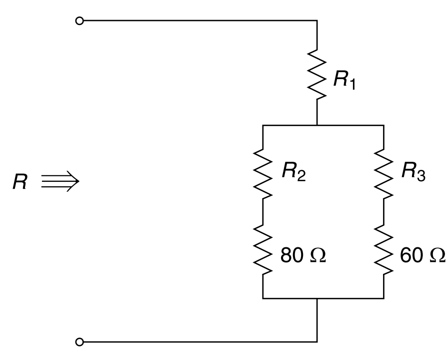

Fig. 4.57(b) ***Simplified equivalent of network of Fig. 4.57(a)***

Here, $$R = 6 + \frac{(10 + 80) \times (15 + 60)}{(10 + 80) + (15 + 60)}$$

$$= 6 + \frac{90 \times 75}{90 + 75}$$

$$= 46.9\ \Omega.$$

Thus, the equivalent resistance of the network given in Fig. 4.57 is 46.9 Ω.

4.41 Using star-delta conversion, find the equivalent resistance between terminals A and B in the network shown in Fig. 4.58.

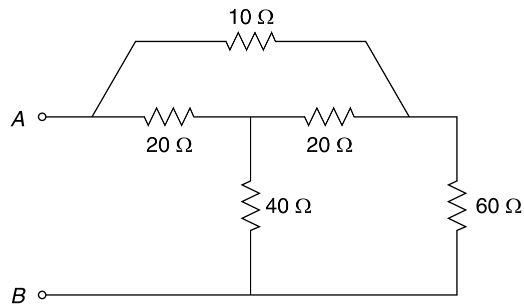

Fig. 4.58 *Circuit of Ex. 4.41*

Solution

Let us first convert the star connected network using 20 Ω, 20 Ω and 40 Ω resistors to an equivalent delta network [Ref. Fig. 4.58(a)].

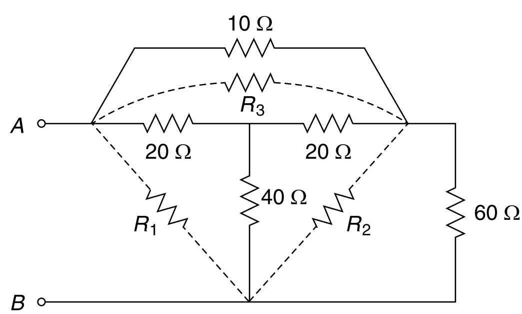

Fig. 4.58(a) *Equivalent Δ network of a part of circuit of Fig. 4.58*

Here,
$$R_1 = \frac{20 \times 40 + 20 \times 40 + 20 \times 20}{20} = 100\ \Omega$$
$$R_2 = \frac{20 \times 40 + 20 \times 40 + 20 \times 20}{20} = 100\ \Omega$$
$$R_3 = \frac{20 \times 40 + 20 \times 40 + 20 \times 20}{40} = 50\ \Omega.$$

Figure 4.58(b) is the final form of the given network as reduced in Fig. 4.58(a).

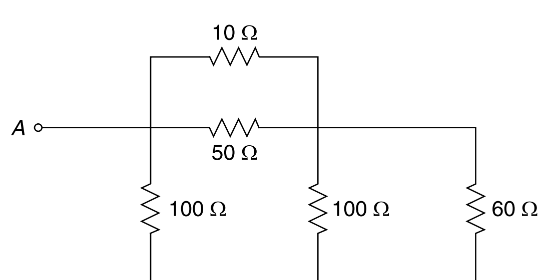

Fig. 4.58(b) *Simplified equivalent of network of Ex. 4.41*

Thus the equivalent resistance between terminals A and B is obtained as

$$R_{eq} = 100 \parallel \left[\frac{50\times 10}{50+10} + \frac{100\times 60}{100+60}\right]$$

$$= 100 \parallel \left(\frac{50}{6} + \frac{600}{16}\right)$$

$$= 100 \parallel 45.83$$

$$= \frac{100\times 45.83}{100+45.83} = 31.43\ \Omega.$$

The equivalent resistance across terminals A and B is thus 31.43 Ω.

4.42 Find the resistance across terminals AB for the circuit shown in Fig. 4.59.

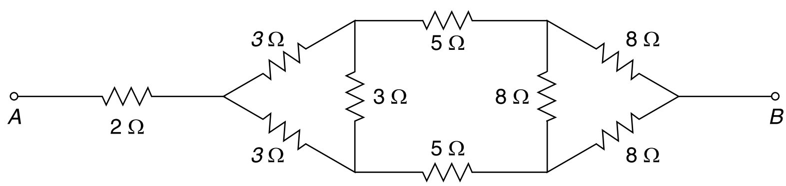

Fig. 4.59 *Circuit of Ex. 4.42*

Solution

We convert the two delta networks formed in the given circuit to equivalent star networks as shown in Fig. 4.59(a).

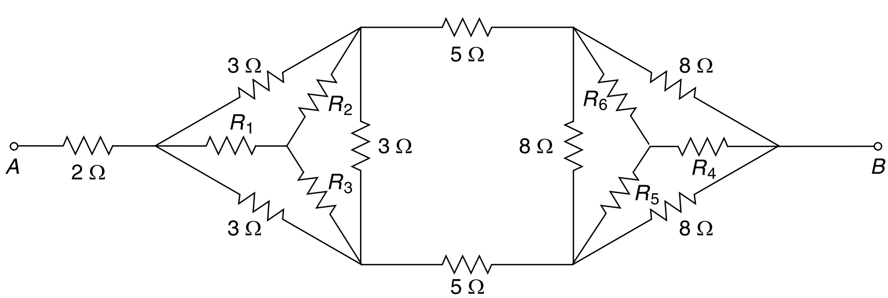

Fig. 4.59(a) *Formation of equivalent stars for the given network in Ex. 4.42*

We find, $$R_1 = R_2 = R_3 = \frac{3\times 3}{3+3+3} = 1\ \Omega$$

and $$R_4 = R_5 = R_6 = \frac{8\times 8}{8+8+8} = 2.67\ \Omega.$$

The equivalent resistance between terminals CD (Fig. 4.59(b)) can be obtained by redrawing Fig. 4.59(a) as Fig. 4.59(b).

$$R_{CD} = (R_2 + 5 + R_6) \parallel (R_3 + 5 + R_5)$$
$$= (1 + 5 + 2.67) \parallel (1 + 5 + 2.67) = 4.335\ \Omega$$

Hence the resistance between terminals AB of the given network is

$$R = 2 + R_1 + 4.335 + R_4 = 2 + 1 + 4.335 + 2.67$$
$$= 10\ \Omega.$$

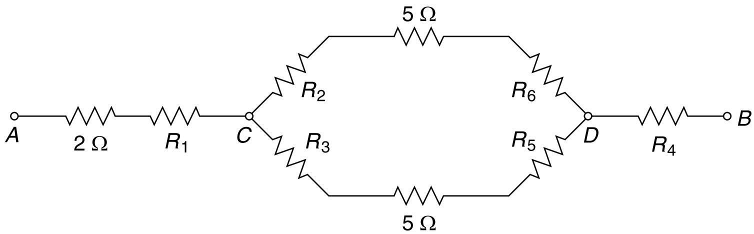

Fig. 4.59(b) *Equivalent network of the circuit shown in Fig. 4.59(a)*

4.43 Determine the resistance between points A and B in the network shown in Fig. 4.60.

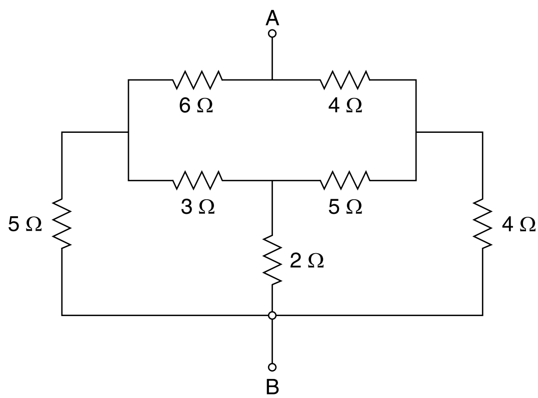

Fig. 4.60 *Circuit of Ex. 4.43*

Solution

Figure 4.60(a) is drawn to represents a star equivalent to the part of the given network containing resistaces 3 Ω, 2 Ω and 5 Ω.

In Fig. 4.60(a),

$$R_1 = \frac{3 \times 2}{3+2+5} = 0.6\ \Omega$$

$$R_2 = \frac{2 \times 5}{5+2+3} = 1\ \Omega$$

$$R_3 = \frac{3 \times 5}{3+5+2} = 1.5\ \Omega.$$

We redraw Fig. 4.60(a) as Fig. 4.60(b) and the circuit is further reduced to Fig. 4.60(c). We draw a star equivalent for the delta connected resistances 5.6 Ω, 4 Ω and 1 Ω in Fig. 4.60(c).

Here, $$R_4 = \frac{1 \times 4}{1+4+5.6} = 0.377\ \Omega$$

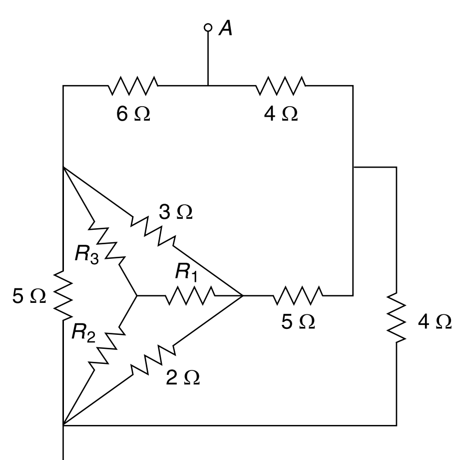

Fig. 4.60(a) *Formation of star equivalent for a portion of network shown in Fig. 4.60*

$$R_5 = \frac{1 \times 5.6}{1 + 4 + 5.6} = 0.528\ \Omega$$

$$R_6 = \frac{4 \times 5.6}{1 + 4 + 5.6} = 2.11\ \Omega.$$

The final configuration of the given network is shown in Fig. 4.60(d). The resistance between terminals AB is then obtained as

$$R_{AB} = \{(7.5 + 0.528) \parallel (4 + 2.11)\} + 0.377$$

$$= \frac{8.028 \times 6.11}{8.028 + 6.11} + 0.377 = 3.84\ \Omega.$$

The equivalent resistance across AB is then 3.84 Ω.

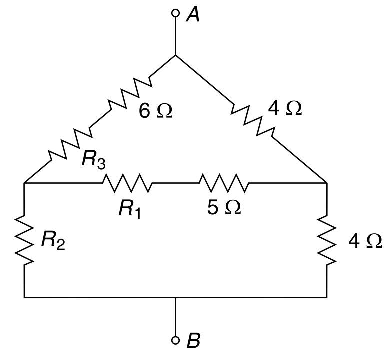

Fig. 4.60(b) *Reduction of circuit shown in Fig. 4.60(a)*

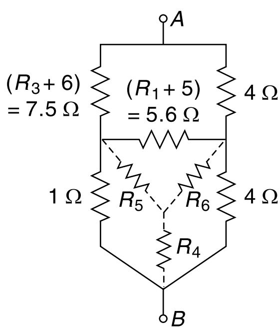

Fig. 4.60(c) *Further simplification of circuit shown in Fig. 4.60(c)*

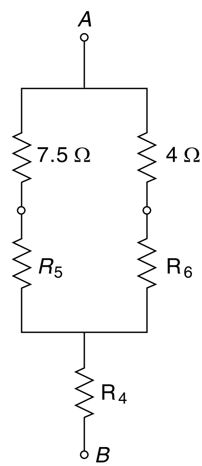

Fig. 4.60(d) *Final simplified equivalent circuit of Ex. 4.43*

4.10 VOLTAGE SOURCES AND CURRENT SOURCES

A network can sometimes be simplified by converting *voltage sources* to *current sources* and vice versa.

Voltage sources can be represented by an ideal voltage cell in series with the internal resistance of the cell or battery. The ideal cell is assumed to be a constant voltage source and the output current produces a voltage drop across the internal resistance. Figure 4.61 shows such a constant voltage source, with voltage E, source resistance R_S and an external load resistance R_L. Using the voltage divider rule in Art. 4.3.2 the output voltage developed across R_L can be determined.

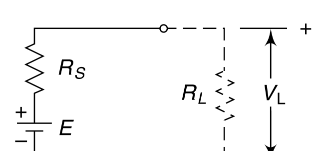

Fig. 4.61 *A constant voltage source with a load resistor*

$$V_L = E\frac{R_L}{R_S + R_L} \tag{4.32}$$

If $R_S \ll R_L$, then $V_L \cong E$

When the load resistance is very much larger than the source resistance, the constant voltage source is assumed to have zero source resistance and all of the source voltage is assumed to be applied to the load.

Certain electronic devices can produce a current that tends to remain constant regardless of how the load resistance varies. Hence it is possible to have a constant current source. The circuit of constant current source is shown in Fig. 4.62 with its source resistance R_S and a load resistance R_L.

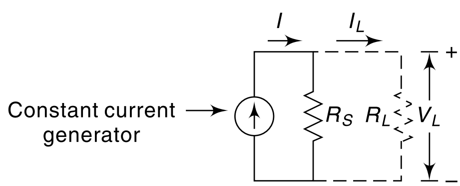

Fig. 4.62 ***A constant current source with a load resistor***

Here R_S is in parallel with the current source. Hence some current flows through R_S and remaining through R_L. Using the current divider rule as shown in Art 4.4.3, the output current (or load current) from a constant current source can be determined in terms of R_L and R_S:

$$I_L = I\frac{R_S}{R_S + R_L} \tag{4.33}$$

If $R_S \gg R_L$ then $I_L \cong I$.

Figure 4.63(a) and Fig. 4.63(b) show how a voltage source can be converted into an equivalent current source that will produce the same current level in a given load resistor.

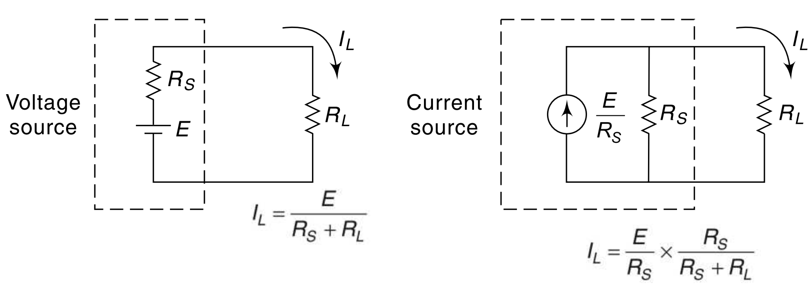

Fig. 4.63(a) & (b) ***Conversion of a constant voltage source to an equivalent constant current source***

When the load resistance is very much smaller than the source resistance, a constant current source is assumed to have an infinite source resistance, and all of the source current is assumed to flow through the load.

4.10.1 Source Conversion

According to source conversion technique a given voltage source with a series resistance can be converted into an equivalent current source with a parallel resistance (as explained in Art. 4.10). Similarly, a current source with a parallel resistance can be converted into a voltage source with a series resistance.

Here we explain again the conversion of the constant voltage source shown in Fig. 4.64 into an equivalent constant current source.

The current supplied by the constant voltage source when a short circuit is placed across terminals 1 & 2 is $I = V/R$.

A constant current source supplying this current I and having the same resistance R connected in parallel with it represents the equivalent current source as shown in Fig. 4.65.

Similarly, a constant current source of I and a parallel resistance R can be converted into a constant voltage source of voltage V (= IR) and a resistance R in series with it.

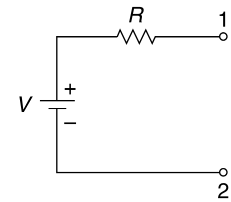

Fig. 4.64 *A constant voltage source with series resistor*

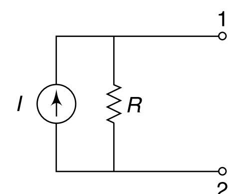

Fig. 4.65 *Equivalent constant current source of the voltage source (V)*

4.44 Convert the constant voltage source shown in Fig. 4.66 into equivalent current source.

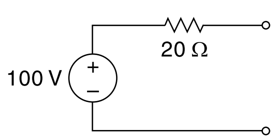

Fig. 4.66 *Circuit of Ex. 4.44*

Solution

The current supplied by the 100 V source when a short circuit is placed across the output terminals is I = 100/20 = 5 A. So the value of the equivalent constant current source is 5 A; the equivalent circuit with current source is shown in Fig. 4.67.

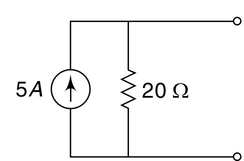

Fig. 4.67 *Equivalent current source of the 100 V voltage source*

4.45 Convert the constant current source of Fig. 4.68 into equivalent voltage source.

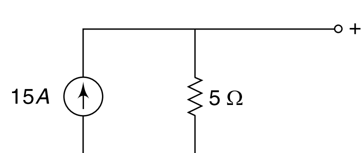

Fig. 4.68 *Circuit of Ex. 4.45*

Solution

The value of the equivalent constant voltage source is given as V (= IR) = 15 × 5 = 75 V. The equivalent network with the voltage source is shown in Fig. 4.68(a).

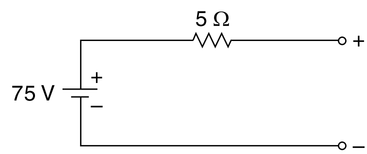

Fig. 4.68(a) ***Equivalent voltage source of 15 A current source***

4.46 Use source transformation technique to find the current through the 2 Ω resistor in Fig. 4.69.

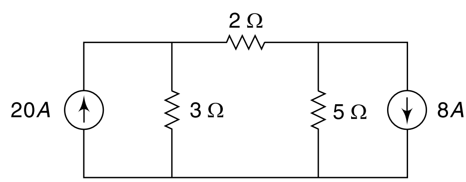

Fig. 4.69 ***Circuit of Ex. 4.46***

Solution

Converting the two sources into equivalent voltage source, the network shown in Fig. 4.70 is obtained.

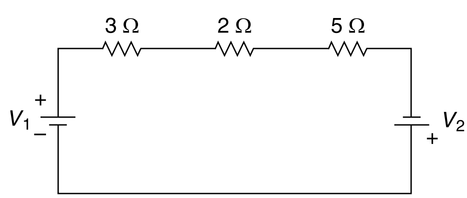

Fig. 4.70 ***Current sources of Fig. 4.69 converted to voltage sources***

The values of V_1 and V_2 are 20 × 3 = 60 V and 8 × 5 = 40 V respectively.

As the voltage sources are series connected hence they deliver current in the same direction. Hence the current through 2 Ω resistor is

$$\frac{60+40}{3+2+5} = 10 \text{ A.}$$

4.47 By using source conversion technique find the value of voltage across R_L where R_L = 4 Ω in Fig. 4.71.

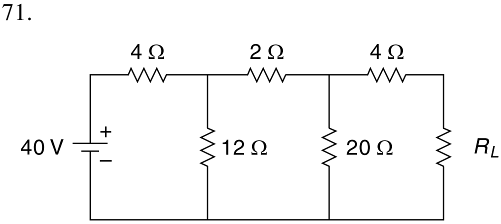

Fig. 4.71 ***Circuit of Ex. 4.47***

Solution

Converting 40 V source into equivalent current source, in Fig. 4.71(a) the value of the current source is 40/4 = 10 A.

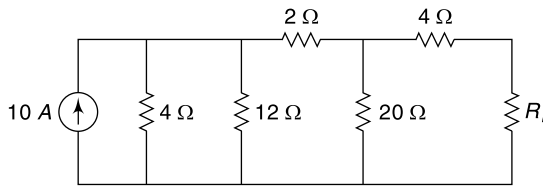

Fig. 4.71(a) ***Conversion of 40 V voltage source to equivalent current source***

The combination of the parallel resistances of 4 Ω and 12 Ω is (12 × 4)/(12 + 4) = 3 Ω (the network is shown in Fig. 4.71(b)).

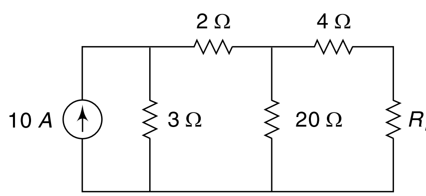

Fig. 4.71(b) ***Reduction of network shown in Fig. 4.71(a)***

Converting 10 A current source into equivalent voltage source, the value of the voltage source is 10 × 3 = 30 V, the equivalent circuit is shown in Fig. 4.71(c).

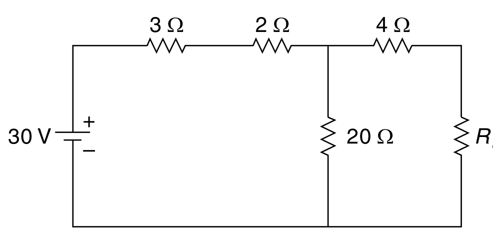

Fig. 4.71(c) ***Conversion of 10 A current source to equivalent voltage source***

Again converting the voltage source into current source, the network in Fig. 4.71(d) is obtained where the parallel combination of 5 Ω and 20 Ω is $\dfrac{5\times 20}{5+20}$ = 4 Ω.

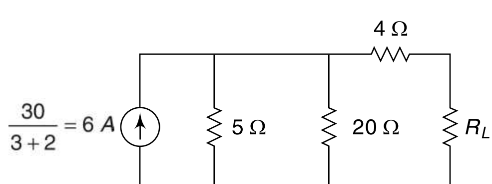

Fig. 4.71(d) ***Conversion of 30 V voltage source to an equivalent current source***

Further converting the current source into voltage source (Fig. 4.71 (e)) we get current through R_L as $\dfrac{24}{4+4+4}$ = 2 *A* and the voltage across R_L is 4 × 2 = 8 V.

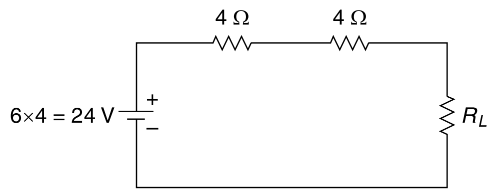

Fig. 4.71(e) *Conversion of 6 A current source to equivalent voltage source*

4.48 Using source conversion technique find the current *I* in Fig. 4.72.

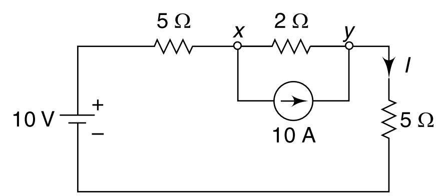

Fig. 4.72 *Circuit of Ex. 4.48*

Solution

The current source is connected in parallel with a 2 Ω resistor; so the value of the equivalent voltage source is obtained as, *V* = 10 × 2 = 20 V (as shown in Fig. 4.73)

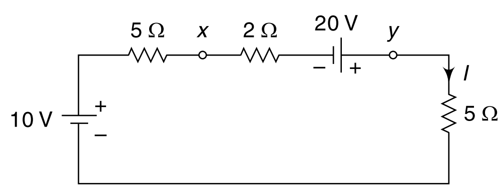

Fig. 4.73 *Conversion of 10 A current source to equivalent voltage source*

The current delivered by a 10 A source would flow from *y* to *x* in 20 Ω resistor. The polarity of the 20 V source is shown in Fig. 4.73.

Therefore, $I = \dfrac{20+10}{5+2+5}$ = 2.5 A.

4.49 Convert the circuit of Fig. 4.74 into a single voltage source in series with a single resistor.

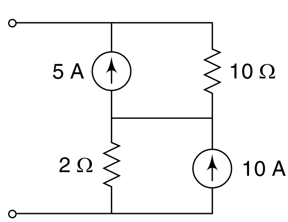

Fig. 4.74 *Circuit of Ex. 4.49*

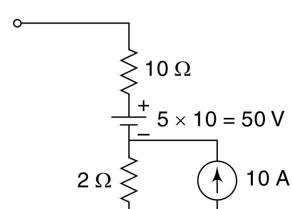

Fig. 4.74(a) *Conversion of 5 A current source to equivalent voltage source*

Solution

Figure 4.74(a) represents the conversion of 5 A source into equivalent voltage source.

Fig. 4.74(b) represents conversion of 10 A current source into equivalent voltage source.

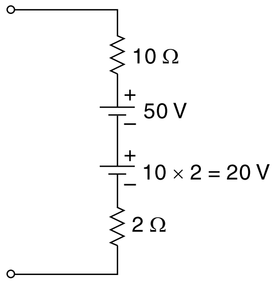

Fig. 4.74(b) ***Conversion of 10 A current source to equivalent voltage source***

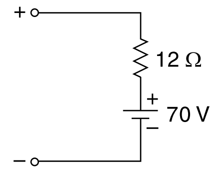

Fig. 4.74(c) ***Equivalent circuit of network shown in Fig. 4.74***

The net voltage of the single voltage source is thus (50 + 20) V = 70 V and the net resistance is (10 + 2) Ω = 12 Ω.

The equivalent circuit is shown in Fig. 4.74(c).

4.10.2 Independent and Dependent Sources

The voltage or current sources which do not depend on any other quantity in the circuit (i.e the strength of voltage or current in the sources), and do not change for any change in the connected network, are called *independent sources*. Independent sources are represented by circles. An independent voltage source and an independent current source is shown in Fig. 4.74.1(a) and 4.74.1(b)

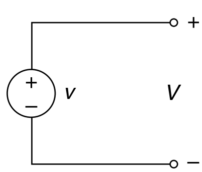

Fig. 4.74-1(a) ***Independent voltage source***

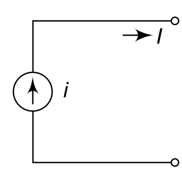

Fig. 4.74-1(b) ***Independent current source***

A *dependent voltage or current source* is one which depend on some other quantity in the circuit (may be either voltage or current) i.e the strength of voltage or current changes in the source for any change in the connected network. Dependent sources are represented by diamond-shaped symbol. There are four possible dependent sources:

Voltage dependent voltage source, as shown in Fig. 4.74.1(c).

Voltage dependent current source, as shown in Fig. 4.74.1(d).

Current dependent current source, as shown in Fig. 4.74.1(e).

Current dependent voltage source as shown in Fig. 4.74.1(f).

In the above figures a, b, c and d are the constants of proportionality a and c has no units, unit of b is siemens and unit of d is ohms.

Some examples of independent sources are battery, dc (or ac) generator. Dependent sources are parts of models which are used to represent electrical properties of electronic devices such as operational amplifiers and transistors etc.

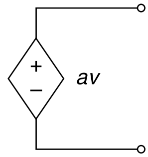

Fig. 4.74-1(c) ***Voltage dependent voltage source***

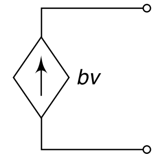

Fig. 4.74-1(d) ***Voltage dependent current source***

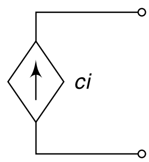

Fig. 4.74-1(e) ***Current dependent current source***

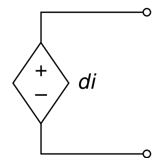

Fig. 4.74-1(f) ***Current dependent voltage source***

4.11 SUPERPOSITION THEOREM

Statement: In a linear bilateral network containing several sources, the current through or voltage across any branch in the network equals the algebraic sum of the currents or voltages of each individual source considered separately with all other sources made inoperative, i.e., replaced by resistances equal to their internal resistances.

It may be noted here that while removing the voltage source it should be replaced by its internal resistance (if any) or by a short circuit and while removing the current source it should be replaced by an open circuit. Superposition theorem is applicable only to linear networks (both ac and dc) where current is linearly related to voltage as per Ohm's law.

Illustration

Let us find the current I as shown in Fig. 4.75 applying superposition theorem.

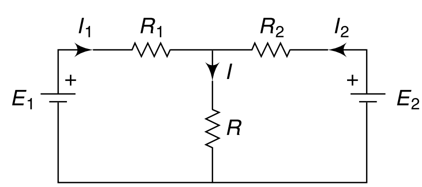

Fig. 4.75 ***A simple resistive network with two voltage sources***

Considering the voltage source E_1 acting alone and removing the other voltage source E_2 after replacing it by its internal resistance (if any) otherwise short circuiting the source, the current through R_1 is [Fig. 4.75(a)]

$$I_1' = \frac{E_1}{R_1 + \dfrac{RR_2}{R + R_2}}. \tag{4.34}$$

Hence current through R is

$$I' = I_1' \times \frac{R_2}{R + R_2} = \frac{E_1 R_2}{RR_1 + R_1 R_2 + R_2 R}. \tag{4.35}$$

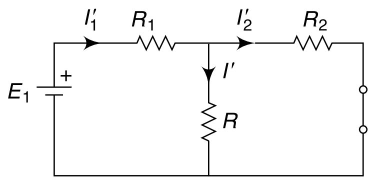

Fig. 4.75(a) ***Source E_1 retained, E_2 deactivated***

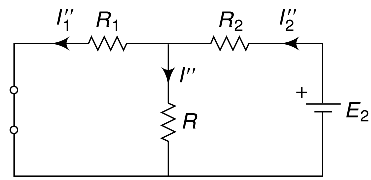

Fig. 4.75(b) ***Source E_2 retained, E_1 deactivated***

Similarly considering the voltage source E_2 acting alone removing the source E_1 and replacing it by a short circuit [Fig. 4.75(b)], the current through R_1, R_2 and R being I_1'', I_2'' and I'' repectively, we find for E_2 acting alone

$$I_2'' = \frac{E_2}{R_2 + \dfrac{RR_1}{R + R_1}} \tag{4.36}$$

and
$$I'' = I_2'' \times \frac{R_1}{R + R_1} = \frac{E_2 R_1}{RR_2 + R_2 R_1 + RR_1} \tag{4.37}$$

Therefore, if there are two sources connected through a network, the resultant current flowing through R is

$$I = I' + I'' = \frac{E_1 R_2 + E_2 R_1}{RR_2 + R_2 R_1 + RR_1} \tag{4.38}$$

4.11.1 Procedure for Applying Superposition Theorem

1. Select one source and replace all other sources by their internal impedances.
2. Determine the level and direction of the current that flows through the desired branch as a result of the single source acting alone.
3. Repeat steps 1 and 2 using each source in turn until the branch current components have been calculated for all sources.
4. Algebrically sum the component currents to obtain the actual branch current(s).

4.50 Compute the current in the 10 Ω resistor as shown in Fig. 4.76 using Superposition theorem.

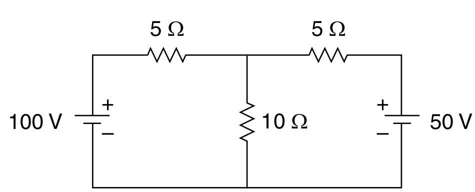

Fig. 4.76 ***Circuit of Ex. 4.50***

Solution

Considering the 100 V source acting alone, the direction of currents supplied by the source has been shown in Fig. 4.76(a).

Here $$I_1 = \frac{100}{5+\dfrac{10\times 5}{10+5}} = \frac{1500}{125}\ \text{A}$$

Hence current through 10 Ω resistor $I' = I_1 \times \dfrac{5}{5+10} = 4$ A

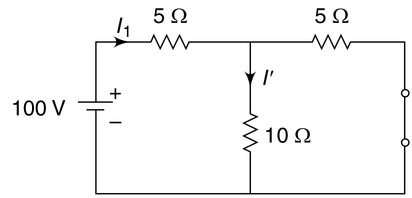

Fig. 4.76(a) ***Source 100 V only considered***

Considering a 50 V source acting alone the direction of currents supplied by the source are shown in Fig. 4.76(b).

Here, $$I_2 = \frac{50}{5+\dfrac{10\times 5}{5+10}} = \frac{750}{125}\ \text{A}$$

Hence current through the 10 Ω resistor is

$$I'' = I_2 \times \frac{5}{5+10} = 2\ \text{A}$$

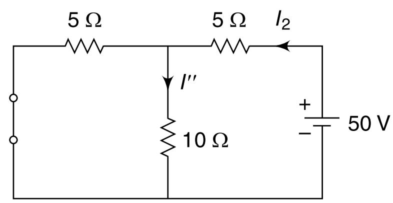

Fig. 4.76(b) ***Source 50 V only considered***

When both the sources are acting simultaneouly, the current through 10 Ω resistor (according to Superposition theorem) is given by $(I' + I'')$ i.e., $(4\,A + 2\,A = 6\,A)$.

4.51 Find the current in the 50 Ω resistor in Fig. 4.77 using Superposition theorem.

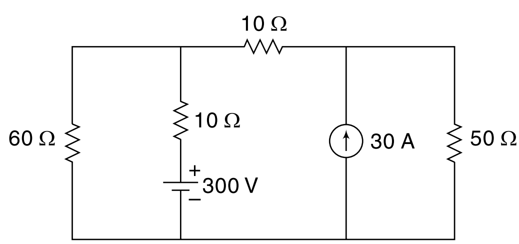

Fig. 4.77 ***Circuit of Ex. 4.51***

Solution

Considering the voltage source acting alone and removing the current source (the corresponding figure being shown in Fig. 4.77(a)) the total current supplied by the voltage source is

$$I_1 = \frac{300}{10+\dfrac{60\,(10+50)}{60+(10+50)}} = \frac{15}{2}\ \text{A}$$

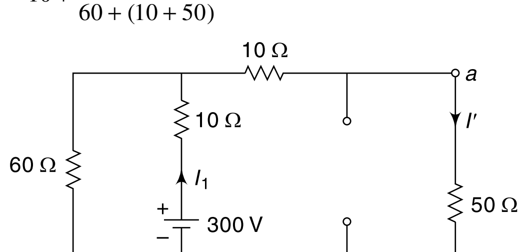

Fig. 4.77(a) ***Voltage source is acting only***

Hence the current through the 50 Ω resistor due to the voltage source acting alone is

$$I' = \frac{15}{2} \times \frac{60}{60+10+50} = \frac{15}{4} \text{ A} = 3.75 \text{ A (from } a \text{ to } b)$$

Next, removing the voltage source and considering the current source acting alone (the corresponding networks being shown in Fig. 4.77(b) and Fig. 4.77(c)), the current through the 50 Ω resistor is

$$I'' = 30 \times \frac{10+60/7}{50+10+60/7} = 8.124 \text{ A (from } a \text{ to } b)$$

[The combined resistance of the 60 Ω and 10 Ω in parallel is $\frac{60 \times 10}{60+10} = \frac{60}{7}$ Ω]

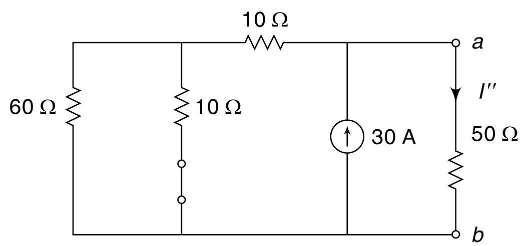

Fig. 4.77(b) *Current source is acting only*

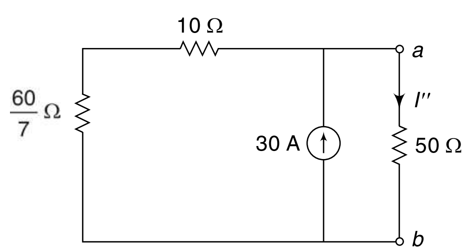

Fig. 4.77(c) *Simplified circuit of network shown in Fig. 4.77(b)*

According to the Superposition theorem when both the sources are acting simultaneously, the current through the 50 Ω resistor is

$$I' + I'' = (3.75 + 8.124) \text{ A} = 11.874 \text{ A (from } a \text{ to } b)$$

4.52 Obtain I using the Superposition theorem for the network shown in Fig. 4.78.

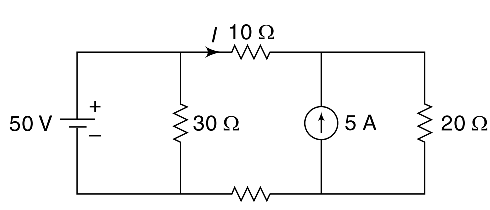

Fig. 4.78 *Circuit of Ex. 4.52*

Solution

Considering the voltage source acting alone [Fig. 4.78(a)] the current supplied by the source is

$$I_1 = \frac{50}{\dfrac{30(10+20+20)}{30+(10+20+20)}} = \frac{8}{3}\ \text{A}$$

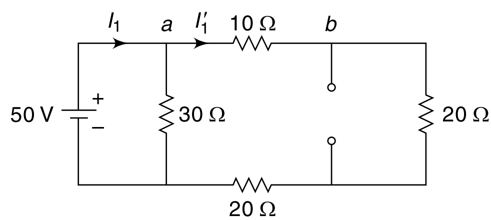

Fig. 4.78(a) ***Voltage source is acting alone***

Hence the current through the 10 Ω resistor is

$$I_1' = \frac{8}{3} \times \frac{30}{30+10+20+20} = 1\ \text{A (from } a \text{ to } b)$$

Removing the voltage source and considering the current source acting alone [Fig. 4.78(b)] the current through the 30 Ω resistor is zero as there is a short circuit path in parallel with it. Hence the network of Fig. 4.78(b) reduces to that in Fig. 4.78(c). The current through the 10 Ω resistor is then given by

$$I'' = 5 \times \frac{20}{20+20+10} = 2\ \text{A} \quad \text{(from } b \text{ to } a\text{) (or } -2\text{ A from } a \text{ to } b)$$

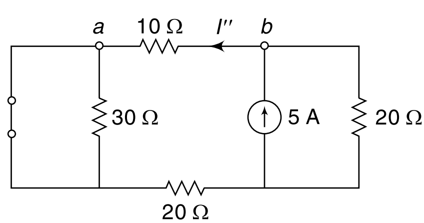

Fig. 4.78(b) ***Current source is acting alone***

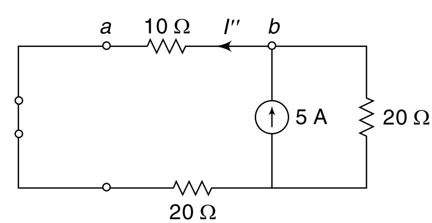

Fig. 4.78(c) ***Simplified circuit of Fig. 4.78(b)***

Therefore according to the Superposition theorem when both the sources are acting simultaneously the current

$$I = I' + I'' = 1 - 2 = -1\ \text{A (from } a \text{ to } b)$$

4.53 Find the voltage across 20 Ω resistor using the Superposition theorem in Fig. 4.79.

Solution

When 1 A current source is acting alone (the corresponding figure being shown in Fig. 4.79(a), the current through the 20 Ω resistor under this condition is obtained as 1 ×

$$\frac{4+5}{20+4+5} = \frac{9}{29}\ \text{A (from } a \text{ to } b)$$

Hence voltage across 20 Ω resistor is $\dfrac{9}{29} \times 20 = \dfrac{180}{29}$ V $(= V_{ab}')$

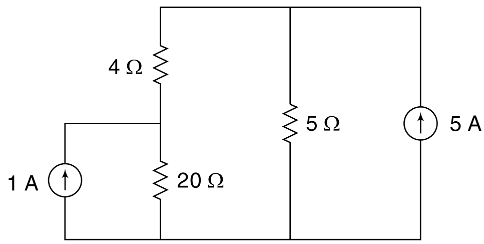

Fig. 4.79 *Circuit of Ex. 4.53*

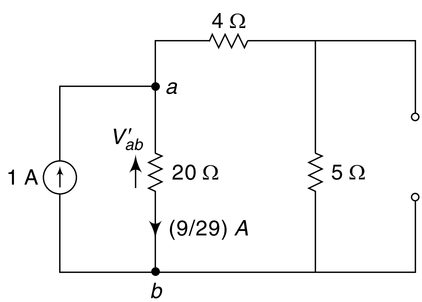

Fig. 4.79(a) *Current source (1 A) acting alone*

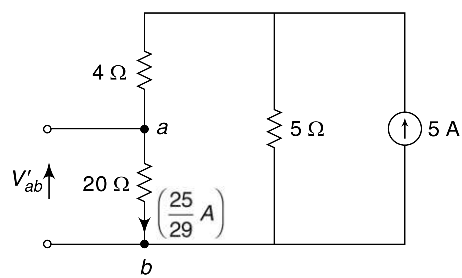

Fig. 4.79(b) *Current source (5 A) is acting alone*

Fig. 4.79(b) shows the network when 1 A source is deactivated and 5 A source acts alone.

The current through 20 Ω resistor under this condition is

$$5 \times \frac{5}{5+4+20}\ \text{A} = \frac{5 \times 5}{29}\ \text{A} = \frac{25}{29}\ \text{A (from } a \text{ to } b)$$

The voltage across the 20 Ω resistor is then $V''_{ab} = \dfrac{25}{29} \times 20\ \text{V} = \dfrac{500}{29}\ \text{V}$

According to the Superposition theorem the voltage across 20 Ω resistor (V_{ab}) when both sources are acting simultaneously is

$$V_{ab} = V'_{ab} + V''_{ab} = \frac{180}{29} + \frac{500}{29} = \frac{680}{29}\ \text{V} = 23.45\ \text{V}$$

4.54 Find the current through 40 Ω resistor using Superposition theorem in Fig. 4.80.

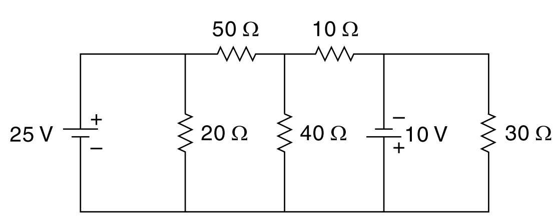

Fig. 4.80 *Circuit of Ex. 4.54*

Solution

Let us consider that the 25 V source is acting alone and the other source is deactivated. The corresponding figures are shown in Fig. 4.80(a) and Fig. 4.80(b).

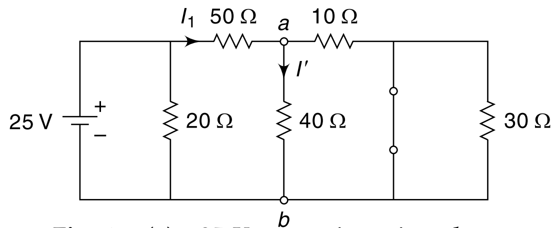

Fig. 4.80(a) *25 V source is acting alone*

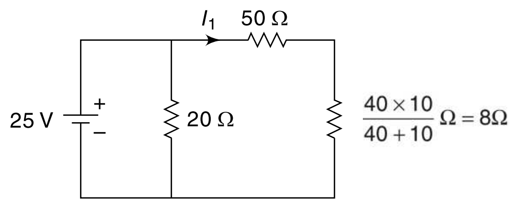

Fig. 4.80(b) *Simplified circuit of Fig. 4.80(a)*

The current through 50 Ω resistor is

$$I_1 = \frac{25}{\dfrac{20\times 58}{20+58}} \times \frac{20}{20+50+8} = 0.431 \text{ A}$$

Hence the current through 40 Ω resistor due to 25 V source alone is

$$I' = 0.431 \times \frac{10}{40+10} \text{ A} = 0.0862 \text{ A} \qquad \text{(from } a \text{ to } b \text{ in Fig. 4.80(a))}$$

Next consider the 10 V source acting alone deactivating the 25 V source.
The current through the 10 Ω resistor [Fig. 4.80(c) and Fig. 4.80(d)] is

$$I_2 = \frac{10}{\dfrac{30\times(200/9+10)}{30+200/9+10}} \times \frac{30}{30+200/9+10} = 0.31 \text{ A}$$

Hence the current through the 40 Ω resistor is

$$I'' = 0.31 \times \frac{50}{50+40} = 0.172 \text{ A (from } b \text{ to } a)$$

Using the Superposition theorem the current through the 40 Ω resistor is

$$I'' - I' = 0.172 - 0.086 = 0.086 \text{ A (from } b \text{ to } a)$$

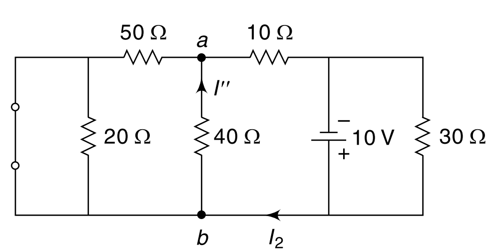

Fig. 4.80(c) *10 V source is acting alone*

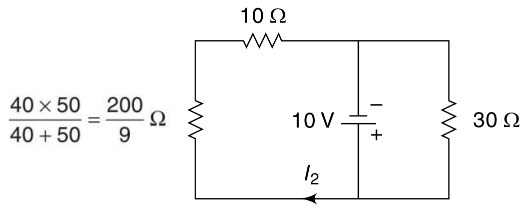

Fig. 4.80(d) ***Simplified circuit of Fig. 4.80(c)***

4.55 Utilising the Superposition theorem find the current through the 20 Ω resistor for the network shown in Fig. 4.81.

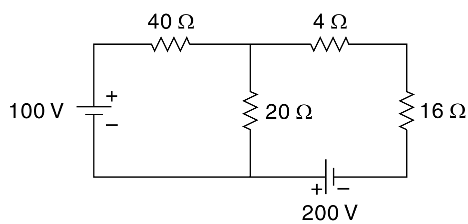

Fig. 4.81 ***Circuit of Ex. 4.55***

Solution

Considering the 10 *V* source acting alone [Fig. 4.81(a)] the current through the 20 Ω resistor is

$$I_1 = \frac{100}{40 + \dfrac{20 \times (4+16)}{20+4+16}} \times \frac{4+16}{4+16+20} \text{ A}$$

$$= 1 \text{ A (from } a \text{ to } b)$$

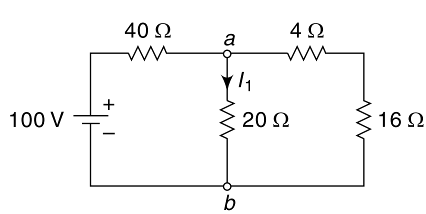

Fig. 4.81(a) ***100 V source is acting alone***

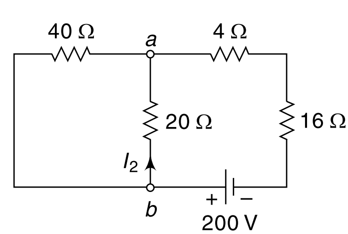

Fig. 4.81(b) ***200 V source is acting alone***

Considering the 200 V source acting alone [Fig. 4.81(b)] the current through the 20 Ω resistor is

$$I_2 = \frac{200}{16+4+(40 \times 20)/(40+20)} \times \frac{40}{40+20} = 4 \text{ A (from } b \text{ to } a)$$

Then, according to the Superposition theorem, the current through 20 Ω resistor is $(I_2 - I_1) = 4 - 1 = 3$ A (from *b* to *a*).

4.56 Find the current through 1 Ω resistor applying the Superposition theorem in Fig. 4.82.

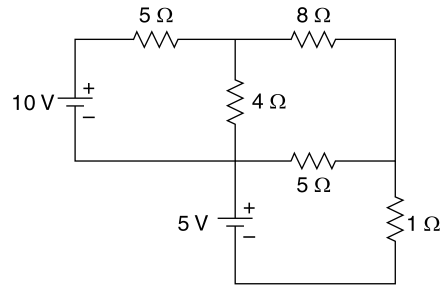

Fig. 4.82 *Circuit of Ex. 4.56*

Solution

Consider 10 V source acting alone (the corresponding figures are shown in Fig. 4.82(a) and Fig. 4.82(b)).

$$I_1 = \frac{10}{5+\dfrac{4\times(8+(5/6))}{4+8+(5/6)}} \times \frac{4}{4+8+(5/6)}$$

$= 0.4$ A

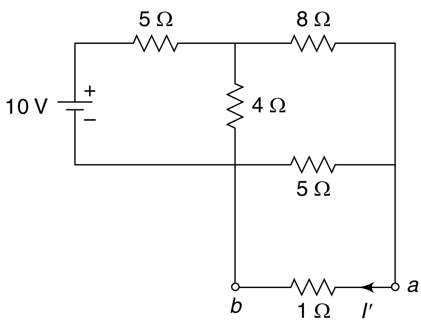

Fig. 4.82(a) *10 V source is acting alone*

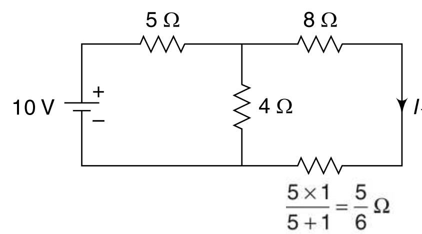

Fig. 4.82(b) *Simplified circuit of Fig. 4.82(a)*

Therefore current through the 1 Ω resistor is

$$I' = 0.4 \times \frac{5}{5+1} = 0.33 \text{ (from } a \text{ to } b\text{)}$$

Next considering the 5 V source acting alone (corresponding figures are shown in Fig 4.82(c) and Fig. 4.82(d)

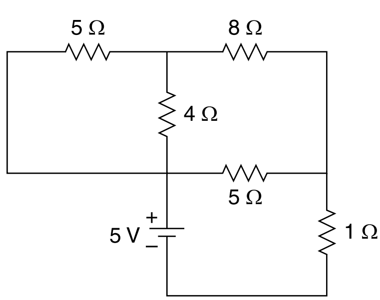

Fig. 4.82(c) *5 V source is acting alone*

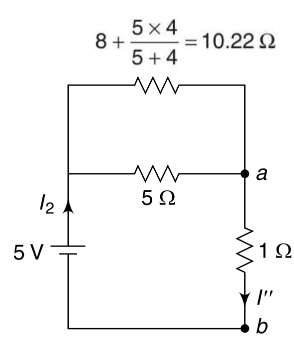

Fig. 4.82(d) *Simplified circuit of Fig. 4.82(c)*

The current supplied by 5 V source is

$$I_2 = \frac{5}{1+\dfrac{5\times 10.22}{5+10.22}} \text{ A} = 1.147 \text{ A}$$

The current through the 1 Ω resistor due to the 5 V source acting alone is then

$$I'' = I_2 = 1.147 \text{ (from } a \text{ to } b)$$

Hence according to Superposition theorem the current through the 1 Ω resistor is obtained as

$$I' + I'' = 0.333 + 1.147 = 1.48 \text{ A}$$

4.57 Find the current through resistance (R_L) for the network shown in Fig. 4.83 using the Superposition theorem

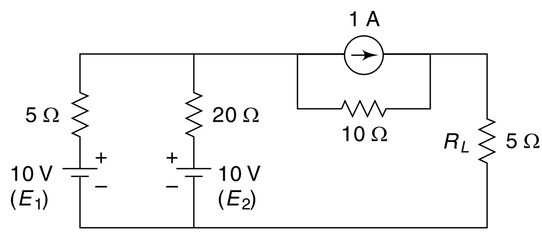

Fig. 4.83 ***Circuit of Ex. 4.57***

Solution

Considering the 10 V source (E_1) acting alone the current through R_L [Fig. 4.83(a)] is

$$I_1 = \frac{10}{5 + \frac{20 \times 15}{20 + 15}} \times \frac{20}{20 + 15} = 0.42 \text{ A (from } a \text{ to } b)$$

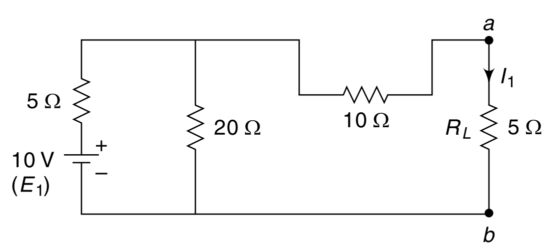

Fig. 4.83(a) ***10 V source (E_1) is acting alone***

Next, considering the other 10 V source (E_2) acting alone the current through R_L [Fig. 4.83(b)] is

$$I_2 = \frac{10}{20 + \frac{5 \times 15}{5 + 15}} \times \frac{5}{5 + 15} = 0.105\, A \text{ (from } a \text{ to } b)$$

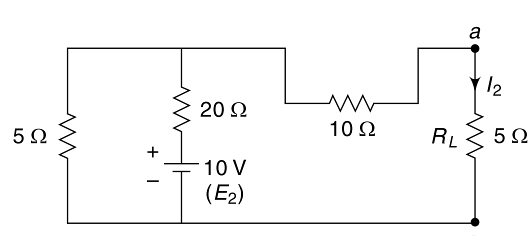

Fig. 4.83(b) ***Another 10 V source (E_2) is acting alone***

Considering the current source (1A) acting alone the current through R_L [Fig 4.83(c) and Fig. 4.83(d)] is

$$I_3 = 1 \times \frac{10}{10+5+4} = \frac{10}{19} \text{ A} = 0.5263 \text{ A (from } a \text{ to } b).$$

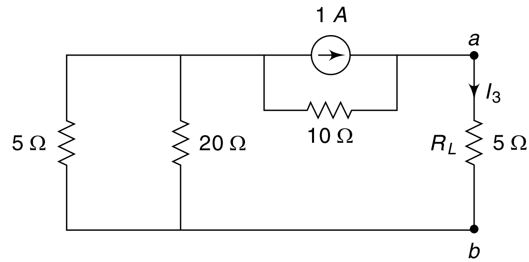

Fig. 4.83(c) ***1 A current source is acting alone***

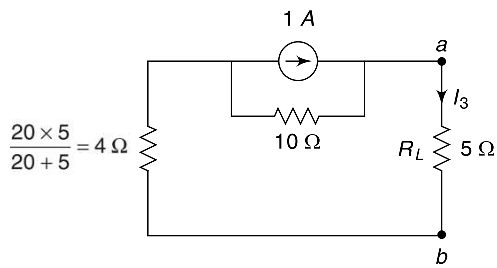

Fig. 4.83(d) ***Simplified circuit of Fig. 4.83(c)***

Hence, according to the Superposition theorem the current through R_L, when all the sources are acting simultaneously, is obtained as $I_1 + I_2 + I_3 = 1.0513$ A.

4.12 THEVENIN'S THEOREM

Statement: The current flowing through a load resistance R_L connected across any two terminals *A* and *B* of a linear, bilateral network is given by $\dfrac{V_{oc}}{R_i + R_L}$, where V_{oc} is the open circuit voltage (i.e voltage across terminals *AB* when R_L is removed) and R_i is the internal resistance of the network as viewed back into the open circuited network from terminals *AB* deactivating all the independent sources. The following are the limitations of this theorem:

(i) Thevenin's theorem can not be applied to a network which contains non-linear impedances.

(ii) This theorem can not calculate the power consumed internally in the circuit or the efficiency of the circuit.

Thevenin's theorem can be explained with the help of the following simple example. The steps are as follows:

Step I

R_L is to be removed from the circuit terminals *a* and *b* for the network shown in Fig. 4.84.

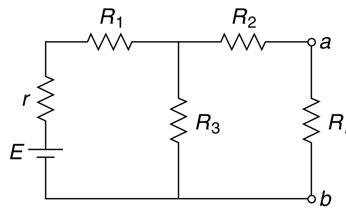

Fig. 4.84 ***Circuit to explain Thevenin's theorem***

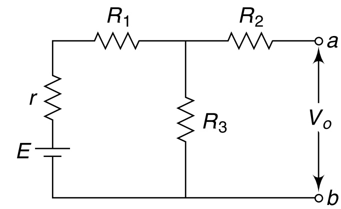

Fig. 4.84(a) ***R_L removed from circuit of Fig. 4.84***

Step II

The open circuit voltage (V_{oc}) which appears across terminals a and b in Fig. 4.84(a) is calculated as

$$V_{oc} \text{ (= Voltage across } R_3) = \frac{E}{r + R_1 + R_3} \times R_3$$

$V_{o/c}$ is called the "Thevenin's voltage" (V_{Th})

Hence,
$$V_{\text{Th}} = \frac{ER_3}{r + R_1 + R_3} \tag{4.39}$$

Step III

Removing the battery from the circuit leaving the internal resistance (r) of the battery behind it [Fig. 4.84(b)] when viewed from terminals a and b, the internal resistance of the circuit is given by

$$R_i = R_2 + \frac{R_3\,(R_1 + r)}{R_3 + R_1 + r}$$

This resistance R_i is called *Thevenin's equivalent resistance* (R_{Th})

$\therefore$
$$R_i = R_{\text{Th}} = R_2 + \frac{R_3\,(R_1 + r)}{R_3 + R_1 + r} \tag{4.40}$$

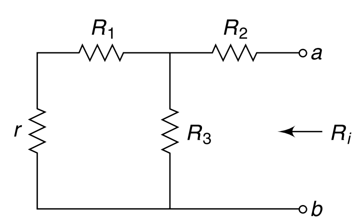

Fig. 4.84(b) ***Internal resistance R_i of the given network***

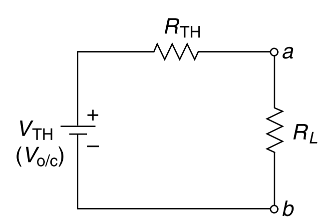

Fig. 4.84(c) ***Thevenin's equivalent circuit***

Step IV

Thevenin's equivalent circuit is drawn as shown in Fig. 4.84(c) and R_L is reconnected acrosss terminals a and b. The current through R_L is

$$I_{\text{Th}} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L}$$

Different methods of finding R_{Th}

(a) *For independent sources:* Deactivate the sources, i.e for independent current source deactivate it by open circuiting its terminals and for voltage source deactivate it by shorting it. Then find the internal resistance of the network looking through the load terminals kept open circuited. In case these independent sources are non-ideal, the internal resistance will remain connected across the deactivated source terminals.

(b) *For dependent sources in addition or in absence of independent source:*

First Method

(i) Find open circuit voltage V_{oc} across the open circuited load terminals. Next short circuit the load terminals and find the short circuit current (I_{sc}) through the shorted terminals.
The Thevenin's equivalent resistance is then obtained as

$$R_{\text{Th}} = \frac{V_{oc}}{I_{sc}}$$

Second Method

(ii) Remove the load resistance and apply a dc voltage V_{dc} at the open circuited load terminals. keep the other independent sources deactivated. A dc current I_{dc} will flow in the circuit from the load terminals. The Thevenin's equivalent resistance is then

$$R_{\text{Th}} = \frac{V_{dc}}{I_{dc}}$$

4.12.1 Thevenizing Procedure

1. Calculate the open circuit voltage (V_{Th}) across the network terminals.
2. Redraw the network with each independent source replaced by its internal resistance. This is called "deactivation of the sources".
3. Calculate the resistance (R_{Th}) of the redrawn network as seen from the output terminals.

4.58 Using Thevenin's theorem find the current through the 15 Ω resistor in Fig. 4.85.

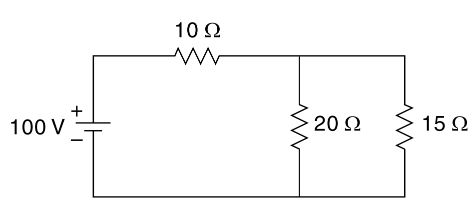

Fig. 4.85 ***Circuit of Ex. 4.58***

Solution

Removing the 15 Ω resistor the open circuit voltage across a and b [Fig. 4.85(a)] is $V_{oc} = \frac{100}{10+20} \times 20 \text{ V} = \frac{200}{3} \text{ V}$.

The Thevenin's equivalent voltage is

$$V_{\text{Th}}\ (= V_{oc}) = \frac{200}{3}\ \text{V}$$

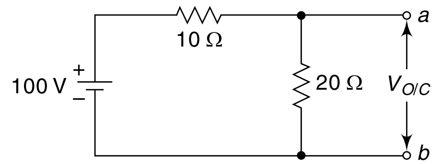

Fig. 4.85(a) *Finding ($V_{o/c}$)*

Next removing the source, the internal resistance of the network as viewed from the open circuited terminals [Fig. 4.85(b)] is $R_i = \dfrac{10 \times 20}{20 + 10}$ i.e. Thevenin's equivalent resistance is $R_{\text{Th}} = R_i = (20/3)\ \Omega$. Thevenin's equivalent circuit is shown in Fig. 4.85 (c). The current through the 15 Ω resistor (according to Thevenin's theorem) is then given by

$$I_{15} = \frac{(200/3)}{(20/3) + 15}\ A = \frac{200}{65}\ \text{A} = 3.077\ \text{A}$$

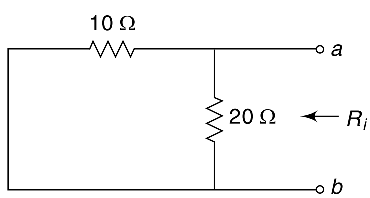

Fig. 4.85(b) *Finding of R_i (R_{th})*

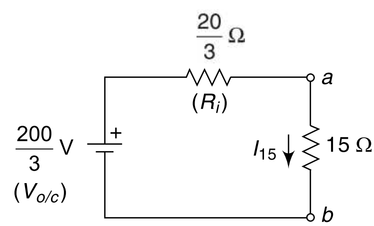

Fig. 4.85(c) *Thevenin's equivalent circuit (Ex. 4.58)*

4.59 Find the current through the 2 Ω resistor using Thevenin's theorem [Fig. 4.86].

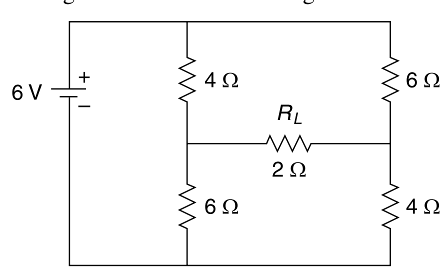

Fig. 4.86 *Circuit of Ex. 4.59*

Solution

The circuit is redrawn in Fig. 4.86(a) with terminals of R_L open circuited.

Thevenin's equivalent voltage is

$$V_{\text{Th}} = V_{bd} = V_{cd} - V_{ab}$$

$$= 6 \times \frac{6}{6+4} - 6 \times \frac{4}{6+4}$$

$$= 3.6 - 2.4$$

$$= 1.2\ \text{V}\ [V_b \text{ is higher potential}].$$

Deactivating the voltage source, Thevenin's equivalent resistance is shown in Fig. 4.86(b) and Fig. 4.86(c).

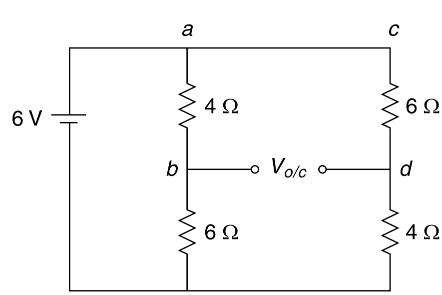

Fig. 4.86(a) *Finding of ($V_{o/c}$)*

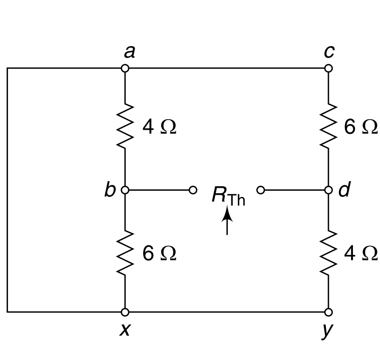

Fig. 4.86(b) *Finding of (R_{th})*

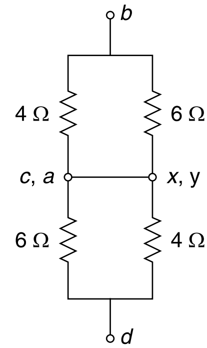

Fig. 4.86(c) *Reduced equivalent network to find R_{th}*

$$R_{Th} = (4||6) + (6||4) = 2 \times \frac{4 \times 6}{4+6}\ \Omega = 4.8\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 4.86(d)

The current through (R_L) is then $I_L = \dfrac{1.2}{4.8+2}\ A$

$$= \frac{1.2}{6.8} = 0.176A$$

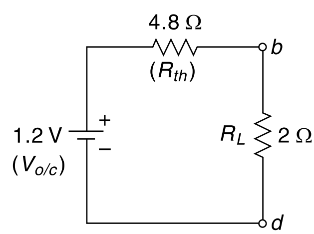

Fig. 4.86(d) *Thevenin's equivalent circuit of Ex. 4.59*

4.60 Find the current through 10 Ω resistor in Fig. 4.87 using Thevenin's theorem.

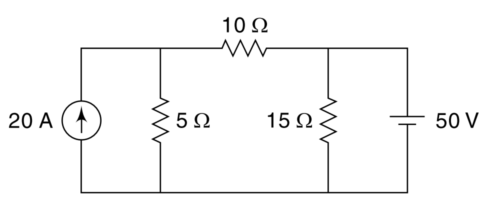

Fig. 4.87 *Circuit of Ex. 4.60*

Solution

Removing the load resistance of 10 Ω from its terminals, the open circuit voltage across terminals *a* and *b* (as shown in Fig. 4.87(a)) can be found out.

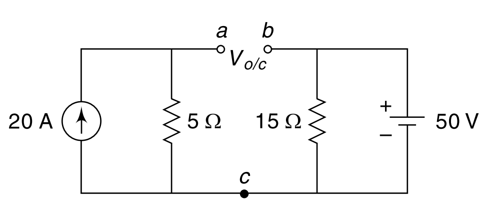

Fig. 4.87(a) *Finding of $V_{o/c}$*

The voltage across the 15 Ω resistor is due to the current supplied by the voltage source only.

$\therefore$ Voltage across the 15 Ω resistor is 50 V.

Hence $\qquad V_{bc} = 50$ V; Also, $V_{ac} = 20 \text{ A} \times 5\ \Omega = 100$ V.

Therefore voltage across open circuit terminals a and b is

$$V_{o/c} = V_{ab} = V_{ac} - V_{bc} = (100 - 50) \text{ V} = 50 \text{ V}$$

i.e $\qquad V_{\text{Th}} = 50 \text{ V } (= V_{o/c}).$

Deactivating all the sources as shown in Fig. 4.87(b), the internal resistance of the network as viewed from the open circuited terminals is,

$$R_{\text{Th}} = 5\ \Omega \text{ (as 15 Ω resistor is short circuited)}$$

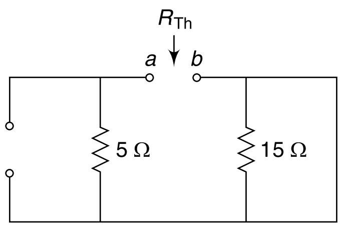

Fig. 4.87(b) *Finding of R_{Th}*

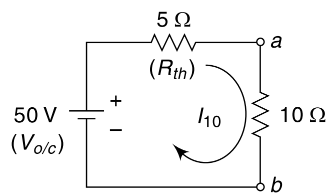

Fig. 4.87(c) *Thevenin's equivalent circuit of Ex. 4.60*

Thevenin's equivalent circuit is shown in Fig. 4.87(c). The current through 10 Ω resistor is $I_{10} = \dfrac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \dfrac{50}{5+10}$ A = 3.33 A.

4.61 Find the current through 15 Ω resistor for the network shown in Fig. 4.88 using Thevenin's theorem.

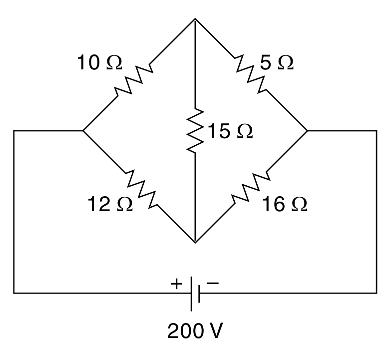

Fig. 4.88 *Circuit of Ex. 4.61*

Solution

Removing 15 Ω resistor the open circuit voltage across its terminals is found out in the network of Fig. 4.88(a).

The current through 10 Ω resistor is obtained as $\dfrac{200}{10+5}\text{ A} = \dfrac{200}{15}\text{ A}$.

Voltage across the 10 Ω resistor is given by $V_{xa} = 10 \times \dfrac{200}{15} = \dfrac{2000}{15}$.

Current through the 12 Ω resistor is found as $\dfrac{200}{12+16}\text{ A} = \dfrac{200}{28}\text{ A}$.

Voltage across the 12 Ω resistor is obtained as $V_{xb} = 12 \times \dfrac{200}{28}\text{ V} = \dfrac{2400}{28}\text{ V}$.

$$V_{ab} = V_{xb} - V_{xa}$$

$$= \frac{2400}{28} - \frac{2000}{15} = 85.71 \text{ V} - 133.33 \text{ V}.$$

$$= -47.62 \text{ V}.$$

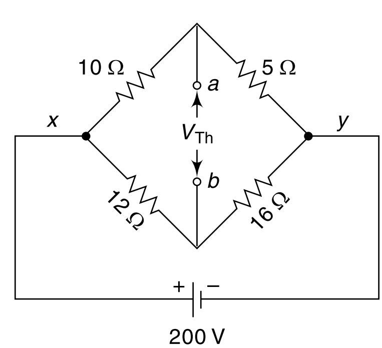

Fig. 4.88(a) *Finding of V_{Th}*

Hence b is at higher potential with respect to a. Therefore $V_{\text{Th}} = V_{ba} = 47.62$ V.

Deactivating the voltage source, Thevenin's equivalent resistance can be obtained as shown in Fig. 4.88(b) and Fig. 4.88(c).

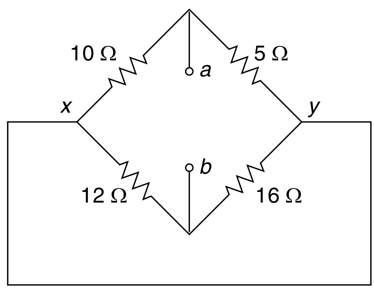

Fig. 4.88(b) *Finding of* **R**$_{\mathbf{i}}$

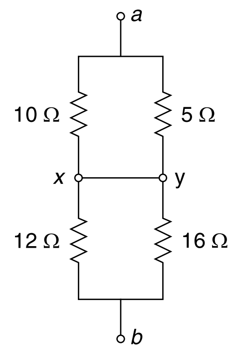

Fig. 4.88(c) ***Reduced network to find R$_i$***

The resistance between a and b is then found as

$$R_{\text{Th}} = (10\|5) + (12\|16)$$

$$= \frac{10\times 5}{10+5} + \frac{12\times 16}{12+16}$$

$$= 10.19\ \Omega\ (= R_i)$$

$\therefore$ Current through 15 Ω resistor [Fig. 4.58d] is

$$I_{15} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{47.62}{10.19+15} = 1.89\ \text{A}$$

[flowing from terminal b to terminal a]

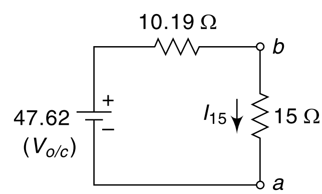

Fig. 4.88(d) ***Thevenin's equivalent circuit of Ex. 4.61***

4.62 Find Thevenin's equivalent circuit of the network (shown in Fig. 4.89) across terminals x-y.

Solution

The voltage across the open circuited terminals is same as the voltage across the 6 Ω resistor.

$\therefore$ V_{Th} = Voltage across the 6 Ω resistor

$$= \frac{50}{5+6}\times 6 = \frac{300}{11} = 27.27\ \text{V}.$$

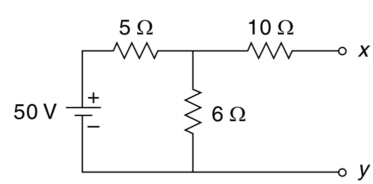

Fig. 4.89 ***Circuit of Ex. 4.62***

Removing the source, Thevenin's equivalent resistance R_{Th} (Fig. 4.89(a)) is $10 + \dfrac{5\times 6}{5+6} = 10 + \dfrac{30}{11} = 12.72\ \Omega$.

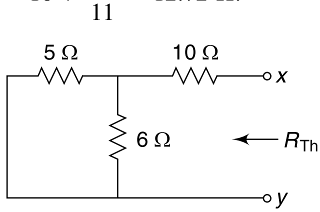

Fig. 4.89(a) *Finding of* ***(R_{Th})***

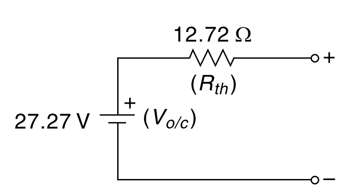

Fig. 4.89(b) ***Thevenin's equivalent circuit of Ex. 4.62***

Thevenin's equivalent circuit is shown in Fig. 4.89(b).

4.63 Find Thevenin's equivalent circuit of the network shown in Fig. 4.90 across terminals *a-b*.

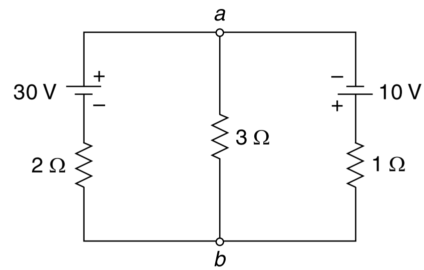

Fig. 4.90 *Circuit of Ex. 4.63*

Solution

Removing the 3 Ω resistor, the circuit is redrawn as shown in Fig. 4.90(a). From Fig. 4.90(a) the circulating current is,

$$I = \frac{30+10}{2+1} \text{ A} = 13.33 \text{ A}.$$

Applying KVL in loop *abyx*,

$$V_{ab} = -13.33 \times 2 + 30 = 3.34 \text{ V}$$

$\therefore \quad V_{o/c}\ (= V_{\text{Th}}) = 3.34$ V.

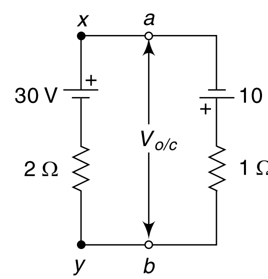

Fig. 4.90(a) *Finding of ($V_{o/c}$)*

Next deactivating the sources, Thevenin's equivalent resistance [Fig. 4.90(b)] is given by

$$R_{\text{Th}} = \frac{2 \times 1}{2+1} = 0.667 \ \Omega.$$

Thevenin's equivalent circuit is then drawn in Fig. 4.90(c).

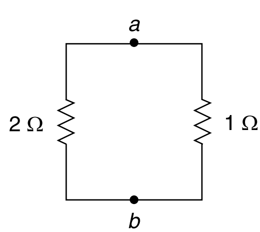

Fig. 4.90(b) *Finding of R_{Th}*

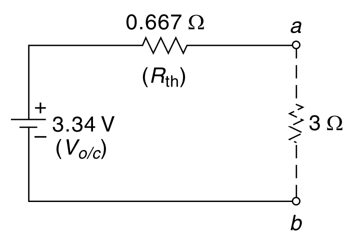

Fig. 4.90(c) *Thevenin's equivalent circuit of Ex. 4.63*

4.64 Find the current through the 5 Ω resistor using Thevenin's theorem in Fig. 4.91.

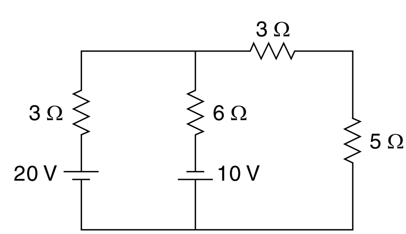

Fig. 4.91 *Circuit of Ex. 4.64*

Solution

Removing the 5 Ω resistor [Fig. 4.91(a)] the current circulating in the loop *abyx* is

$$I = \frac{20+10}{3+6} \text{ A} = \frac{10}{3} \text{ A}$$

(in the clockwise direction)

Let V_{Th} = voltage across branch *xy* = voltage across branch *ab*

Here, $\quad V_{xy} = -10 + 6 \times \frac{10}{3} = 10\ V = V_{\text{Th}}.$

$$\left[\text{Otherwise} \qquad V_{ab} = 20 - 3 \times \frac{10}{3} = 10\ V = V_{\text{Th}}. \right]$$

Deactivating all the sources, $\quad R_{\text{Th}} = 3 + (6||3) = 3 + \frac{6 \times 3}{6+3} = 5\ \Omega.$

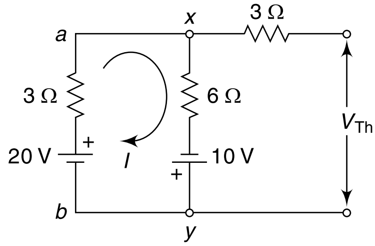

Fig. 4.91(a) *Finding of (V_{Th})*

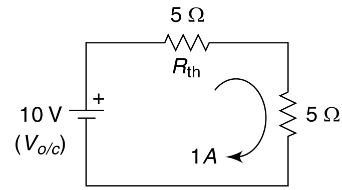

Fig. 4.91(b) *Thevenin's equivalent circuit of Ex. 4.64*

From Fig. 4.91(b) the current through 5 Ω resistor is $\dfrac{10}{5+5}$ = 1 A

4.65 Find the Thevenin's equivalent circuit of Fig. 4.92, across R_L.

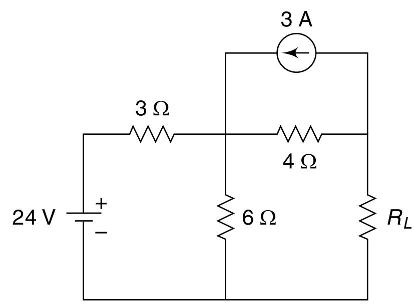

Fig. 4.92 *Circuit of Ex. 4.65*

Solution

R_L is removed and the terminals are open circuited as shown in Fig. 4.92(a). The current supplied by the 24 V source circulates through the 3 Ω and 6 Ω resistor only while the current due to the current source circulated through the 4 Ω only when the circuit is open circuited at *a* and *b*.

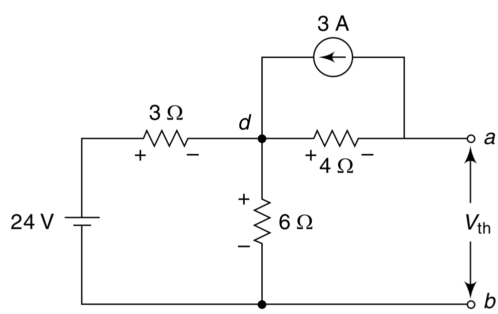

Fig. 4.92(a) *Finding of (V_{Th})*

Voltage across *dc* is

$$V_{dc} = \frac{24 \times 6}{3+6}\ V = 16\ \text{V}$$

Voltage across *da* is V_{da} = 3 × 4 V = 12 V.

Applying KVL in the loop *abcd*

$$V_{ab} = 16 - 12 = 4\ \text{V i.e } V_{\text{Th}} = 4\ \text{V}$$

Next, all the sources in the network is deactivated [Fig. 4.92(b)].

$$\therefore \qquad R_{\text{Th}} = 4 + \frac{3 \times 6}{3+6} = 6\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 4.92c.

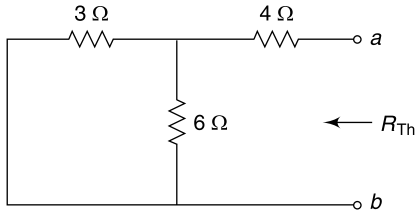

Fig. 4.92(b) *Finding of (R_{Th})*

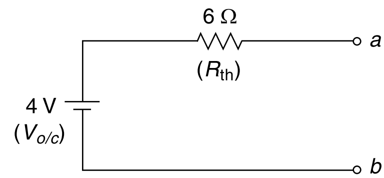

Fig. 4.92(c) *Thevenin's equivalent circuit of Ex. 4.65*

4.66 Find the current in the ammeter of the 2 Ω resistance as shown in Fig. 4.93 using Thevenin's theorem.

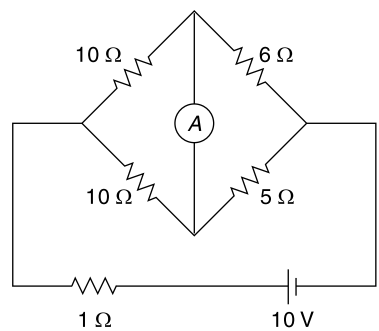

Fig. 4.93 *Circuit of Ex. 4.66*

Solution

The ammeter is removed and the circuit is shown in Fig. 4.93(a).

The total current delivered by 10 *V* source is

$$I = \frac{10}{1+\dfrac{(10+6)\times(10+5)}{(10+6)+(10+5)}} = \frac{10}{1+\dfrac{16\times 15}{31}}$$

$$= 1.144 \text{ A}$$

$$\therefore \qquad I_1 = 1.144 \times \frac{10+5}{15+16} = 0.55 \text{ A}$$

and $$I_2 = 1.144 \times \frac{10+6}{15+16} = 0.59 \text{ A.}$$

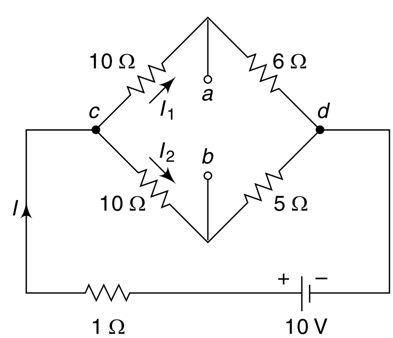

Fig. 4.93(a) *Finding of (V_{Th})*

Voltage across open circuited terminals *a* and *b* is

$$V_{Th} = V_{ab} = V_{cb} - V_{ca}$$
$$= 0.59 \times 10 - 0.55 \times 10 = 0.4 \text{ V.}$$

Deactivating the voltage source, the corresponding figure is drawn in Fig. 4.93(b). In this figure using delta star conversion values of R_1, R_2 and R_3 can be found out.

$$R_1 = \frac{10\times 6}{10+6+1}\,\Omega = \frac{60}{17}\,\Omega$$

$$R_2 = \frac{6\times 1}{10+6+1}\,\Omega = \frac{6}{17}\,\Omega$$

$$R_3 = \frac{1\times 10}{10+6+1} = \frac{10}{17}\,\Omega\,.$$

Resistance between *a* and *b* as shown in Fig. 4.93(c) is given by

$$R_{Th} = \frac{60}{17} + \left[\left(\frac{10}{17}+10\right)\middle\|\left(\frac{6}{17}+5\right)\right]$$

$$= 3.53 + \frac{10.59\times 5.35}{10.59+5.35}$$

$$= 7.084\ \Omega$$

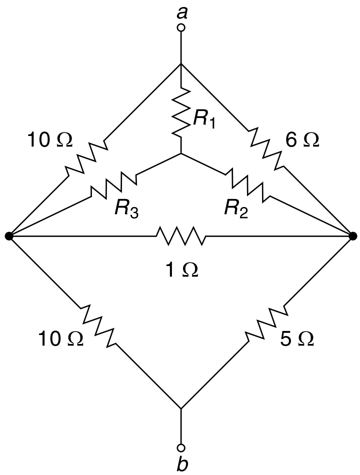

Fig. 4.93(b) ***Network reduction to find (R_{Th})***

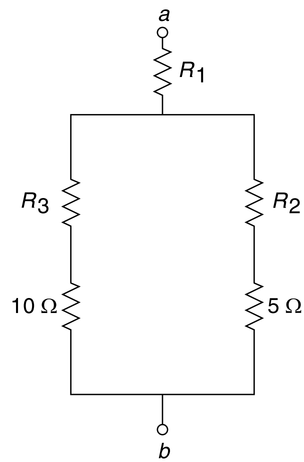

Fig. 4.93(c) ***Final network reduction to find (R_{Th})***

The current through the 2 Ω resistor is

$$I_{2\Omega} = \frac{V_{\text{Th}}}{R_{\text{Th}} + 2} = \frac{0.4}{7.0841 + 2} = 0.044 \text{ A (directed from } a \text{ to } b)$$

4.67 Find the current through the 5 Ω resistor in the network of Fig. 4.94 using Thevenin's theorem.

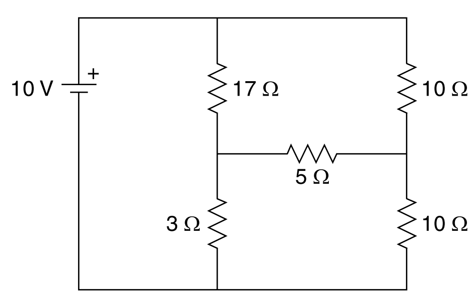

Fig. 4.94 ***Circuit of Ex. 4.67***

Solution

The 5 Ω resistor is first removed. The circuit configuration is shown in Fig. 4.94(a).

The current through 17 Ω resistor is $\frac{10}{17+3}$ A = 0.5 A.

The current through pair of 10 Ω resistors is $\frac{10}{10+10}$ A = 0.5 A.

Voltage across 17 Ω resistor is $V_{ca} = 17 \times 0.5 = 8.5$ V

Voltage across 10 Ω resistor is $V_{cb} = 10 \times 0.5 = 5$ V

Hence $V_{ab} = V_{cb} - V_{ca} = 5 - 8.5 = -3.5$ V

or $V_{ba} = 3.5$ V (i.e b is positive terminal)

i.e $V_{\text{Th}} = 3.5$ V

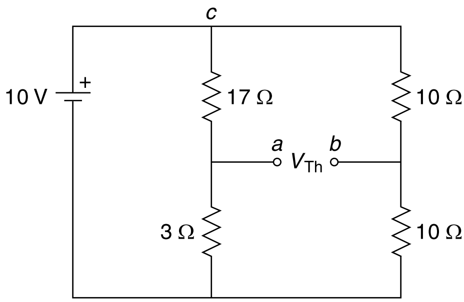

Fig. 4.94(a) *Finding of V_{Th}*

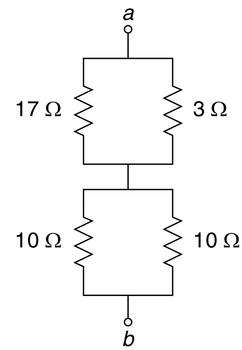

Fig. 4.94(b) *Finding of R_{Th}*

For finding R_{Th}, the circuit is redrawn in Fig. 4.94(b) deactivating the source;

$$R_{Th} = \frac{17 \times 3}{17+3} + \frac{10 \times 10}{10+10} = 7.55\ \Omega$$

Current through the 5 Ω resistor [Fig. 4.94(c)] is obtained as

$$I_{5\Omega} = \frac{V_{Th}}{R_{Th}+5} = \frac{3.5}{7.55+5}\ \text{A} = 0.279\ \text{A}$$

(flowing from b to a).

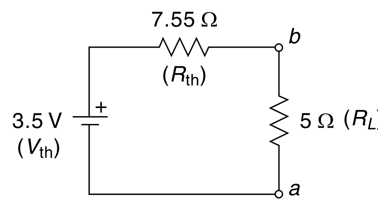

Fig. 4.94(c)

4.68 Find the current in the 5 Ω resistor (using Thevenin's theorem) in Fig. 4.95.

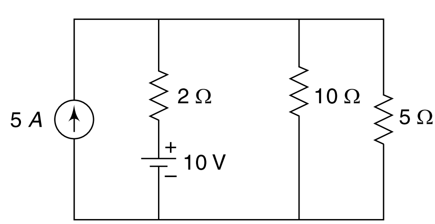

Fig. 4.95 *Circuit of Ex. 4.68*

Solution

Let us first remove the 5 Ω resistor. The circuit configuration is shown in Fig. 4.95(a).

Applying the Superposition theorem, we consider one source at a time. Considering 5*A* source alone and removing the voltage source, the current through the 10 Ω resistor is

$$5 \times \frac{2}{2+10} = \frac{10}{12}\ \text{A}.$$

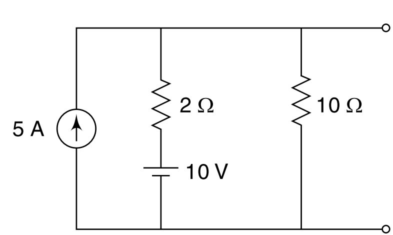

Fig. 4.95(a) *Finding of V_{Th} $V_{O/C}$*

Considering the voltage source acting alone and removing the current source, the current through the 10 Ω resistor is

$$\frac{10}{2+10} = \frac{10}{12}\ \text{A}.$$

Both the currents are directed in the same direction through 10 Ω resistor. So net current through 10 Ω resistor is $\frac{10}{12}+\frac{10}{12}=\frac{20}{12}$ A and voltage across 10 Ω resistor is $\frac{20}{12}$ × 10 = $\frac{200}{12}$ V.

As 10 Ω is connected in parallel with the open circuited terminals hence,

$$V_{\text{Th}} = \text{Voltage acros 10 } \Omega \text{ resistor}$$

$$= \frac{200}{12} = \frac{50}{3} = 16.67 \text{ V.}$$

Removing all the sources, R_{Th} is found out and is shown in Fig. 4.95(b).

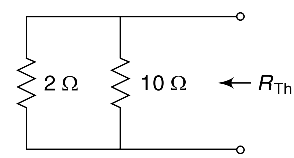

Fig. 4.95(b) *Finding of R_{Th}*

$$R_{\text{Th}} = \frac{10 \times 2}{10+2} = \frac{20}{12} = \frac{5}{3} = 1.67 \ \Omega$$

So the current through the 5 Ω resistor is obtained as

$$I_{5\Omega} = \frac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \frac{16.67}{1.67+5} = 2.5 \text{ A}$$

4.69 Find the power loss in 10 Ω resistor using Thevenin's theorem (Fig. 4.96).

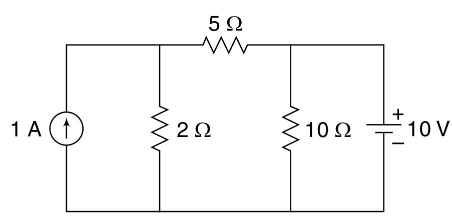

Fig. 4.96 *Circuit of Ex. 4.69*

Solution

Removing 10 Ω resistor the circuit configuration is shown in Fig. 4.96(a).

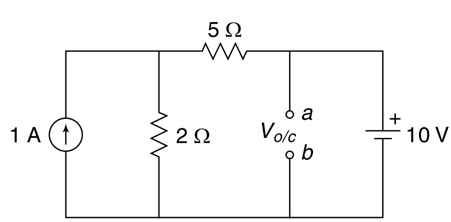

Fig. 4.96(a) *Finding of $V_{O/C}$*

As the open circuit voltage ($V_{o/c}$) across terminals *a* and *b* is in parallel with the 10 *V* source hence the open circuit voltage is becoming 10 *V* (or $V_{o/c}$ = V_{Th} = 10 *V*)

Removing all the sources R_{Th} is found out from Fig. 4.96(b). However there is a short circuit path across *ab*, so R_{Th} = 0 Ω.

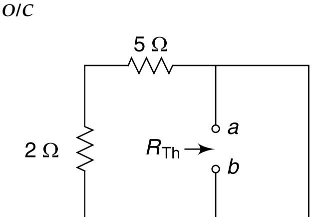

Fig. 4.96(b) *Finding of R_{Th}*

Current through the 10 Ω resistor according to Thevenin's theorem is $\dfrac{V_{\text{Th}}}{R_{\text{Th}} + R_L} = \dfrac{10}{0+10}$ A = 1 A

Therefore power loss in 10 Ω resistor is

$$I^2R = 1^2 \times 10 \text{ W}$$
$$= 10 \text{ W.}$$

4.70 Find Thevenin's equivalent circuit of the network across R_L in Fig. 4.97.

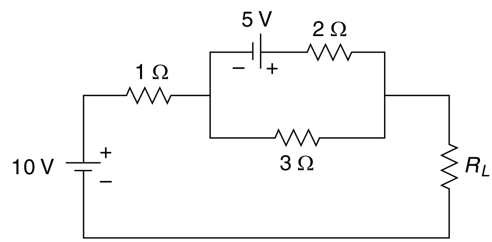

Fig. 4.97 *Circuit of Ex. 4.70*

Solution

R_L is removed first and the corresponding figure is shown in Fig. 4.97(a). Under this condition 10 *V* source can not deliver any current. Current due to 5 *V* source circulates through 2 Ω and 3 Ω resistor

$$\therefore \qquad I = \frac{5}{2+3} \text{ A} = 1 \text{ A}$$

Voltage across 3 Ω resistor is 3 × 1 V = 3 V

Applying KVL in loop *a b c d e f a* of Fig. 4.97(a)

$$V_{ab} = 10 + 3 = 13 \text{ V}$$

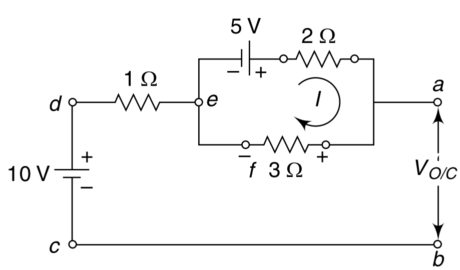

Fig. 4.97(a) *Finding of* $V_{O/C}$

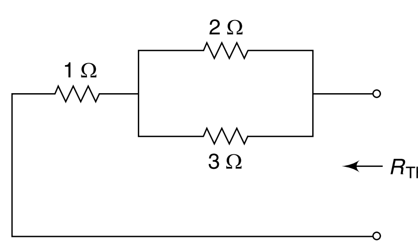

Fig. 4.97(b) *Finding of* R_{Th}

Deactivating the sources R_{Th} is found out [Fig. 4.97(b)].

$$\therefore \qquad R_{\text{Th}} = 1 + \frac{3 \times 2}{3+2}$$
$$= 2.2 \ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 4.97(c).

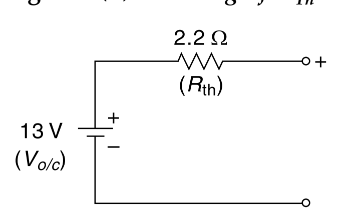

Fig. 4.97(c) *Thevenin's equivalent circuit of Ex. 4.70*

4.13 NORTON'S THEOREM

According to this theorem, *any two-terminal active network containing voltage sources and resistances when viewed from its output terminals is equivalent to a constant current source and an internal (parallel) resistance. The constant current source (known as Norton's equivalent current source) is of the magnitude of the short circuit current at the terminals. The internal resistance is the equivalent resistance of the network looking back into the terminals with all the sources replaced by their internal resistances.*

A network is shown in Fig. 4.98 to explain Norton's theorem. Let us find out the current through R_L using Norton's theorem.

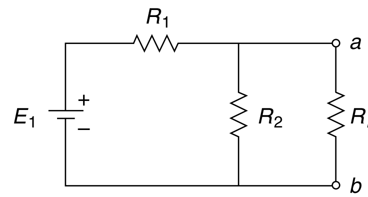

Fig. 4.98 *Circuit to explain Norton's theorem*

The steps are as follows:

Step I

Remove (R_L) and short circuit the terminals *a* and *b* [Fig. 4.98(a)]. The current through the short circuited path is $I_{sc} = E/R_1$ ($= I_N$), where I_N is the Norton's equivalent current.

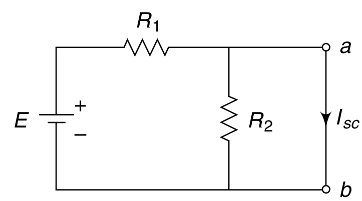

Fig. 4.98(a) *Developing Norton's current source*

Step II

For finding internal resistance R_i of the network, terminals *a* and *b* is open circuited and the source is deactivated [Fig. 4.98(b)].

$$R_i = \frac{R_1 R_2}{R_1 + R_2} \quad (= R_N), \text{ where } R_N \text{ is called}$$

the Norton's equivalent resistance.

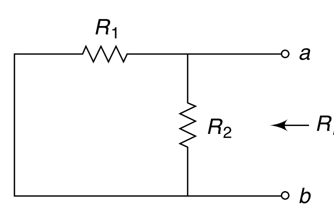

Fig. 4.98(b) *Finding of internal resistance*

Step III

Norton's equivalent circuit is shown in Fig. 4.98(c). It contains Norton's current source I_N and a parallel resistance equal to internal resistance of the circuit R_i.

Step IV

Connect R_L across terminals *a* and *b* and find current I_L through R_L

$$I_L = I_N \times \frac{R_N}{R_N + R_L}. \qquad (4.41)$$

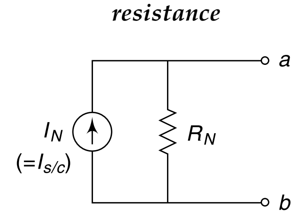

Fig. 4.98(c) *Norton's equivalent circuit*

4.13.1 Nortonizing Procedure

1. Calculate the short-circut current I_N at the network terminals.
2. Redraw the network with each source replaced by its internal resistance.

3. Calculate the resistance R_N of the redrawn network as seen from the output terminals.
4. Draw Norton's equivalent circuit.

4.71 Find the current through R_L in Fig. 4.99 using Norton's theorem.

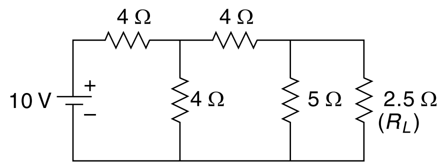

Fig. 4.99 *Circuit of Ex. 4.71*

Solution

R_L is removed and the terminals are short circuited as shown in Fig. 4.99(a). The short circuit current is

$$I_{sc} = \frac{10}{4 + \dfrac{4 \times 4}{4 + 4}} = \frac{10}{4 + 2} = 1.67 \text{ A}$$

∴ Norton's equivalent current $I_N = 1.67$ A

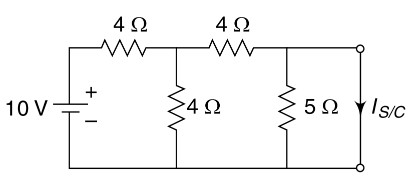

Fig. 4.99(a) *Finding of $I_{S/C}$*

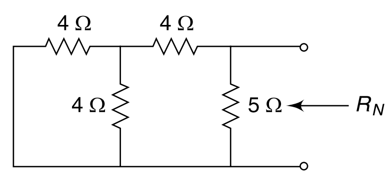

Fig. 4.99(b) *Finding of internal resistance R_N*

Norton's equivalent resistance R_N is found from Fig. 4.99(b).

$$R_N = \left(\frac{4 \times 4}{4 + 4} + 4\right) \Bigg\| 5 = \frac{6 \times 5}{6 + 5} = 2.73 \ \Omega.$$

Norton's equivalent circuit is drawn in Fig. 4.99(c).

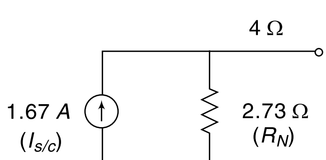

Fig. 4.99(c) *Norton's equivalent circuit*

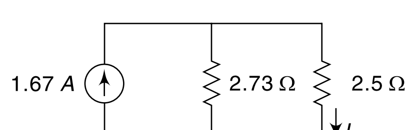

Fig. 4.99(d) *Finding of I_L*

Replacing R_L across the open circuited terminals as shown in Fig. 4.99(d), the current through R_L is

$$I_L = 1.67 \times \frac{2.73}{2.73 + 2.5} = 0.87 \text{ A}$$

4.72 Find the current through 10 Ω resistor in Fig. 4.100 using Norton's theorem.

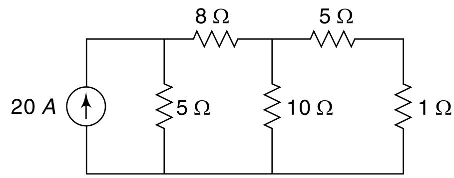

Fig. 4.100 *Circuit of Ex. 4.100*

Solution

The 10 Ω resistor is removed and the terminals are short circuited as shown in Fig. 4.100(a).

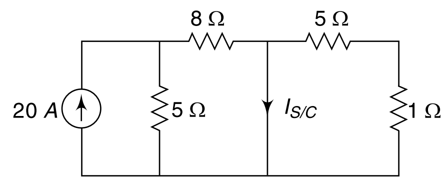

Fig. 4.100(a) *Finding of $I_{S/C}$*

The current through the short circuited path is

$$I_{sc} = 20 \times \frac{5}{5+8} \text{ A} = 7.69 \text{ A}$$

Hence Norton's equivalent current I_N = 7.69 A

Norton's equivalent resistance as seen from the open circuited terminals of the network (Fig. 4.100(b)), is obtained as

$$R_N = (8 + 5)||(5 + 1) = \frac{13 \times 6}{13+6} \ \Omega = 4.1 \ \Omega$$

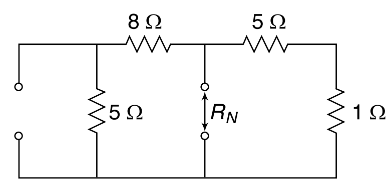

Fig. 4.100(b) *Finding of R_N*

Norton's equivalent circuit is shown in Fig. 4.100(c).

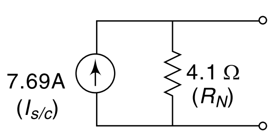

Fig. 4.100(c) *Norton's equivalent circuit of Ex. 4.72*

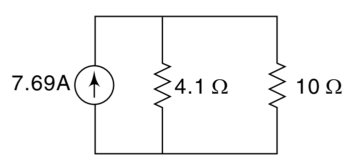

Fig. 4.100(d) *Current I_L through the 10 Ω resistor*

Therefore the current through the 10 Ω resistor is [Fig. 4.100(d)]

$$I_L = 7.69 \times \frac{4.1}{4.1+10} \text{ A} = 2.236 \text{ A}$$

4.73 Find current in 6 Ω resistor using Norton's theorem for the network shown in Fig. 4.101.

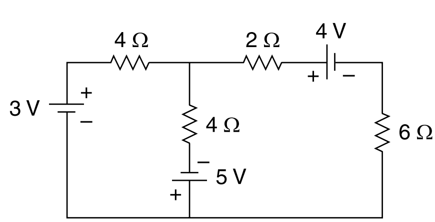

Fig. 4.101 *Circuit of Ex. 4.73*

Solution

The load resistance 6 Ω is short-circuited as shown in Fig. 4.101(a).

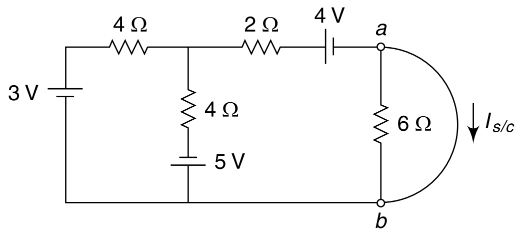

Fig. 4.101(a) ***Determination of $I_{S/C}$***

The current through the short circuited path *ab* due to the 3 V source acting alone is

$$I_{sc_1} = \frac{3}{4+\dfrac{2\times 4}{2+4}} \times \frac{4}{4+2} = 0.375 \text{ A (from } a \text{ to } b).$$

The current through the short-circuited path *ab* due to the 5 V source acting alone is

$$I_{sc_2} = \frac{5}{4+\dfrac{4\times 2}{4+2}} \times \frac{4}{4+2} = \frac{5\times 4}{24+8} = 0.625 \text{ A (from } b \text{ to } a).$$

The current through the short-circuited path *ab* due to the 4 *V* source acting alone is

$$I_{sc_3} = \frac{4}{2+\dfrac{4\times 4}{4+4}} = 1 \text{ A (from } b \text{ to } a).$$

Applying the superposition theorem when all the sources are acting simultaneously the short circuit current is obtained as

$$I_{sc} = (1 + 0.625 - 0.375) \text{ A} = 1.25 \text{ A (from } b \text{ to } a)$$

Hence Norton's equivalent current is $I_N = 1.25$ A.

Norton's equivalent resistance [Fig. 4.101(b)] is obtained as

$$R_N = 2 + \frac{4\times 4}{4+4} = 4\ \Omega$$

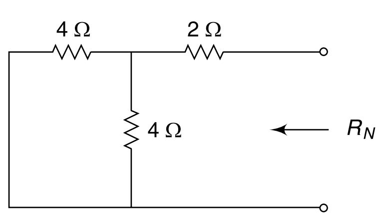

Fig. 4.101(b) ***Finding of R_N***

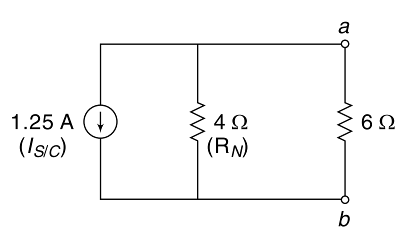

Fig. 4.101(c) ***Norton's equivalent circuit***

The current through 6 Ω resistor is $I = 1.25 \times \dfrac{4}{4+6} = 0.5$ A (from *b* to *a*).

4.74 Find current through 5 Ω resistor in the circuit of Fig 4.102 using Norton's theorem.

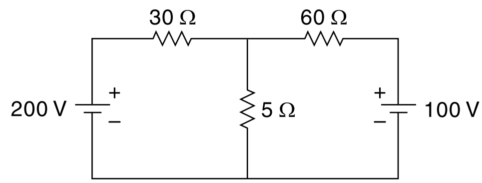

Fig. 4.102 *Circuit of Ex. 4.74*

Solution

5 Ω resistor is short circuited as shown in Fig. 4.102(a). The current through the short-circuited path $I_{sc} = \dfrac{200}{30} + \dfrac{100}{60}$ = 8.33 A from *a* to *b*.

Hence $\quad I_N = 8.33$ A

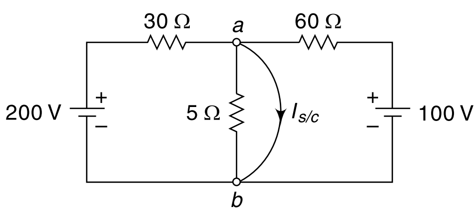

Fig. 4.102(a) *Determination of ($I_{S/C}$)*

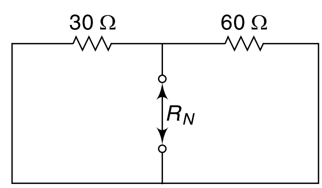

Fig. 4.102(b) *Determination of (R_N)*

The Norton's equivalent resistance is obtained by removing all sources and looking from open circuited terminals *a* and *b* in Fig. 4.102(b) as

$$R_N = \frac{30 \times 60}{30 + 60} = 20\ \Omega$$

Therefore, current through the 5 Ω resistor [Fig. 4.102(c)] is $I = 8.33 \times \dfrac{20}{20+5}$ = 6.664 A from *a* to *b*.

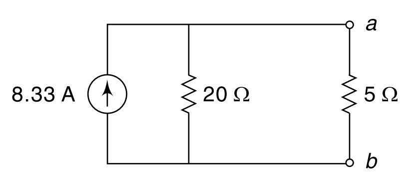

Fig. 4.102(c) *Norton's equivalent circuit of Ex. 4.74*

4.75 Find the current through R_L in Fig. 4.103 using Norton's theorem.

Solution

Removing R_L and short circuiting its terminals the network is redrawn in Fig. 4.103(a).

The current through the short circuited path is obtained as

$$I_{sc} = \frac{1}{2} - \frac{2}{1} - \frac{1}{1} = 0.5 - 2 - 1$$

$$= -2.5 \text{ A from } a \text{ to } b \text{ or } 2.5 \text{ A from } b \text{ to } a$$

i.e. $\quad I_N = 2.5$ A.

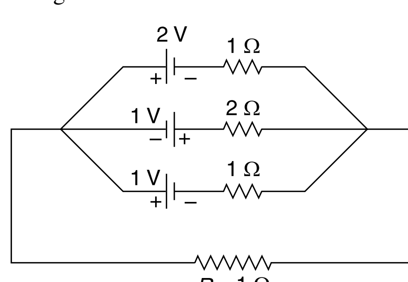

Fig. 4.103 *Circuit of Ex. 4.75*

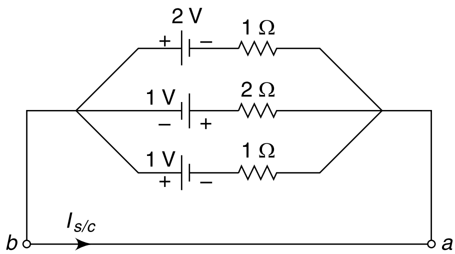

Fig. 4.103(a) ***Determination of $I_{S/C}$***

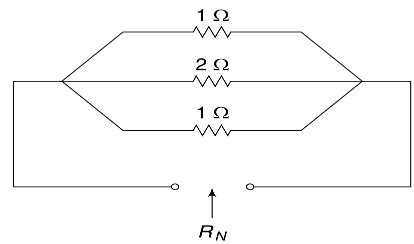

Fig. 4.103(b) ***Determination of R_N***

Removing the sources and open circuiting the short circuited path [as shown in Fig. 4.103(b)], we get

$$R_N = \frac{1 \times 2 \times 1}{1 \times 2 + 2 \times 1 + 1 \times 1} \ \Omega = 0.4 \ \Omega$$

The current through R_L [Fig 4.103(c)] is (I)

$$= 2.5 \times \frac{0.4}{1 + 0.4} \text{ A} = 0.714 \text{ A from } b \text{ to } a.$$

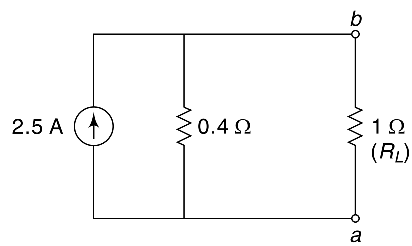

Fig. 4.103(c) ***Norton's equivalent circuit of Ex. 4.75***

4.76 Find current through the 20 Ω resistor in Fig. 4.104 using Norton's theorem.

Solution

Short circuiting 20 Ω resistor [Fig. 4.104(a)] the current through the short circuited path due to 20 A source acting alone is I_{sc_1} = 20 A from a to b.

Considering the 40 V source acting alone the current through the short circuited path is $I_{sc_2} = \frac{40}{10}$ A = 4 A (from a to b). Considering the 80 V source acting alone the current through the short circuited path $I_{sc_3} = \frac{80}{10}$ A = 8 A (from b to a).

Applying the Supperposition theorem the net current through the short circuited path I_{sc} = (20 + 4 – 8) A = 16 A (from a to b).

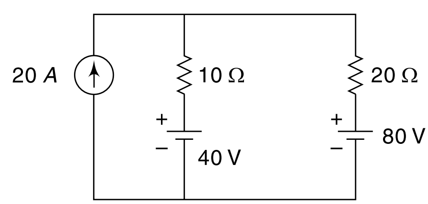

Fig. 4.104 ***Circuit of Ex. 4.76***

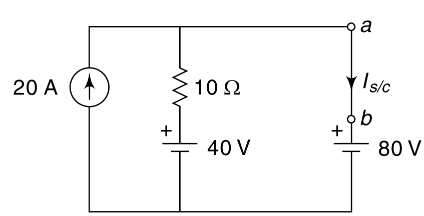

Fig. 4.104(a) ***Determination of ($I_{S/C}$)***

Thus, Norton's equivalent current is I_N = 16 A. Next, removing all the sources, R_N is found out from Fig. 4.104(b) as R_N = 10 Ω.

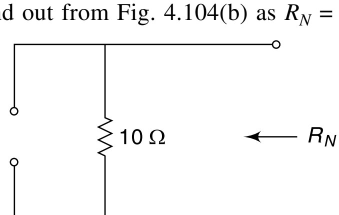

Fig. 4.104(b) ***Determination of (R_N)***

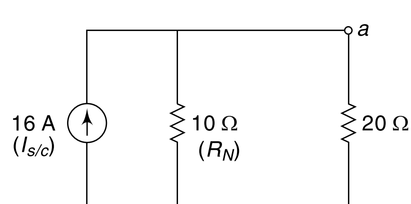

Fig. 4.104(c) ***Norton's equivalent circuit of Ex. 4.76***

From Fig. 4.104(c) the current through the 20 Ω resistor can be found out as

$$I = 16 \times \frac{10}{10+20} \text{ A} = 5.33 \text{ A (from } a \text{ to } b\text{).}$$

4.77 Find the current through the 2 Ω resistor in Fig. 4.105 using Norton's theorem.

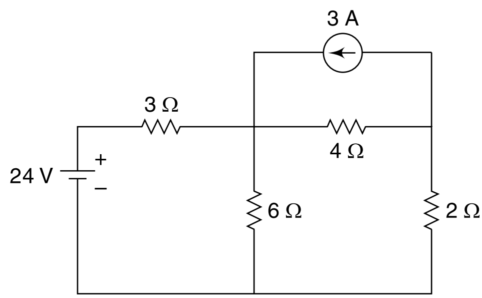

Fig. 4.105 *Circuit of Ex. 4.77*

Solution

Short circuiting the 2 Ω resistor [as shown in Fig. 4.105(a)], and with 25 V source acting alone, the short circuit current through *ab* is $I_{sc_1} = \dfrac{24}{3+\dfrac{6\times 4}{6+4}} \times \dfrac{6}{6+4}$ = 2.67 A from *a* to *b*.

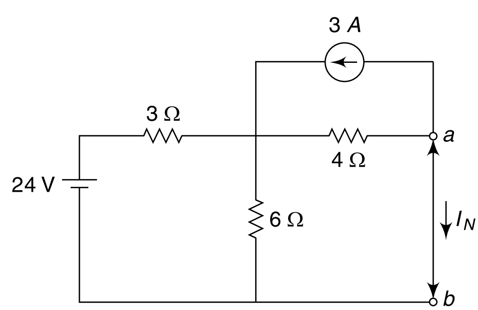

Fig. 4.105(a) *Determination of I_N*

Next with 3 A source acting alone, the current through *ab* is $I_{sc_2} = 3 \times \dfrac{4}{4+\dfrac{3\times 6}{3+6}} = 2$ A from *b* to *a*.

∴ the current through *ab* is

$$I_N = I_{sc_1} - I_{sc_2} = 2.67 \text{ A} - 2 \text{ A} = 0.67 \text{ A from } a \text{ to } b.$$

Norton's equivalent resistance [Fig. 4.105(b)] is

$$R_N = 4 + \frac{3\times 6}{3+6} = 6\ \Omega$$

The current through the 2 Ω resistor [Fig. 4.105(c)] is $I = 0.67 \times \dfrac{6}{6+2}$ = 0.5 A from *a* to *b*.

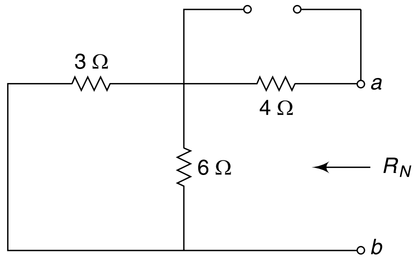

Fig. 4.105(b) *Determination of R_N*

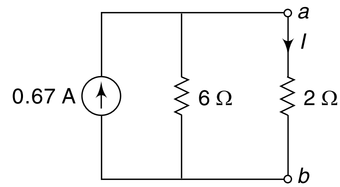

Fig. 4.105(c) *Norton's equivalent circuit*

4.78 Find Norton's equivalent circuit for the network shown in Fig. 4.106.

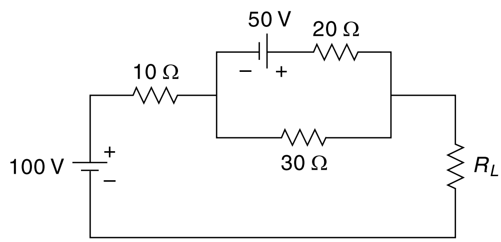

Fig. 4.106 *Circuit of Ex. 4.78*

Solution

Remove R_L and short circuit the terminals [as shown in Fig. 4.106(a)].

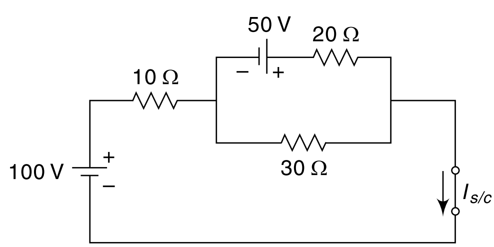

Fig. 4.106(a) *Determination of $I_{S/C}$*

The short circuit current is

$$I_{sc} = \frac{100}{10+\dfrac{20\times 30}{20+30}} + \frac{50}{20+\dfrac{30\times 10}{30+10}} \times \frac{30}{30+10}$$

$$= \frac{100}{10+12} + \frac{50\times 30}{800+300}$$

= 5.9 A from a to b i.e I_N = 5.9 A.

Norton's equivalent resistance looking back from the open circuited terminals [Fig. 4.106(b)] is $(R_N) = 10 + \dfrac{20\times 30}{20+30} = 22\ \Omega$.

Norton's equivalent circuit is shown in Fig. 4.106(c).

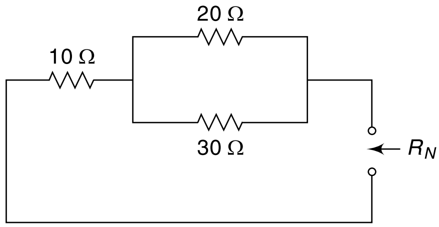

Fig. 4.106(b) *Finding of R_N*

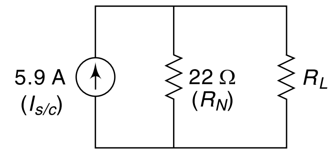

Fig. 4.106(c) *Norton's equivalent circuit of Ex. 4.78*

4.79 Find the current through the 1 Ω resistor in the network shown in Fig. 4.107 using Norton's theorem.

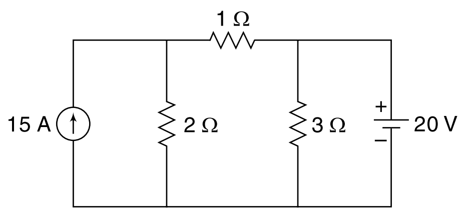

Fig. 4.107 *Circuit of Ex. 4.79*

Solution

1 Ω resistor is removed and terminals are short circuited as shown in Fig. 4.107(a). The current through the short circuited path is

$$I_{sc} = 15 - \frac{20}{\frac{2\times 3}{2+3}} \times \frac{3}{3+2}$$

$$= 15 - \frac{20\times 3}{6}$$

$$= 5 \text{ A (from } a \text{ to } b)$$

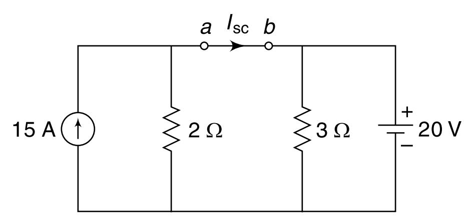

Fig. 4.107(a) *Determination of $I_{S/C}$*

Removing all the sources and open circuiting terminals a and b [Fig. 4.107(b)], $R_N = 2\ \Omega$

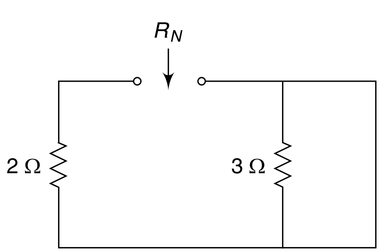

Fig. 4.107(b) *Determination of R_N*

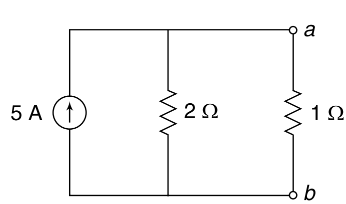

Fig. 4.107(c) *Norton's equivalent circuit*

Thus the current through 1 Ω resistor [Fig. 4.107(c)] is

$$I = 5 \times \frac{2}{2+1} \text{ A} = 3.33 \text{ A}$$

4.80 Find the current through 8 Ω resistor using Norton's theorem in the network of Fig 4.108.

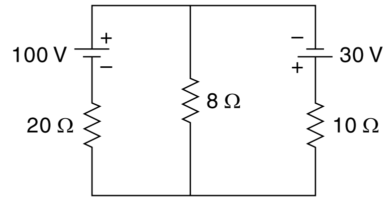

Fig. 4.108 *Circuit of Ex. 4.80*

Solution

Short circuiting the 8 Ω resistor as shown in Fig. 4.108(a), the current through the short circuted path is $I_N = \dfrac{100}{20} - \dfrac{30}{10}$ = 2 A (from *a* to *b*).

Open circuting *ab* and removing the sources the Norton's equivalent resistance [Fig. 4.108(b)] is

$$R_N = \frac{20 \times 10}{20 + 10}\ \Omega = 6.67\ \Omega$$

The current through the 8 Ω resistor [from Fig. 4.108(c)] is $(I) = 2 \times \dfrac{6.67}{6.67 + 8}$ = 0.9 A.

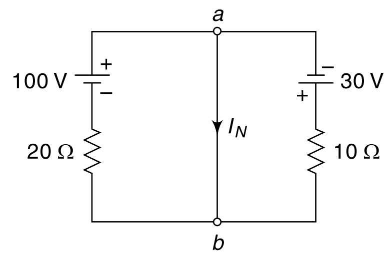

Fig. 4.108(a) *Determination of I_N*

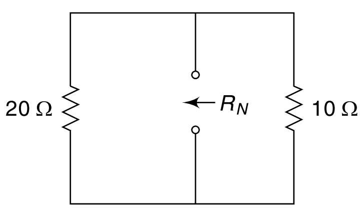

Fig. 4.108(b) *Determination of R_N*

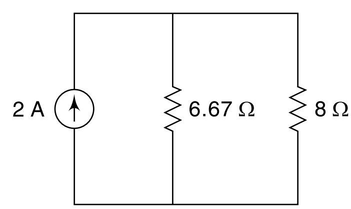

Fig. 4.108(c) *Norton's equivalent circuit of Ex. 4.80*

4.81 Find the current through the 20 Ω resistor in Fig. 4.109 using Norton's theorem.

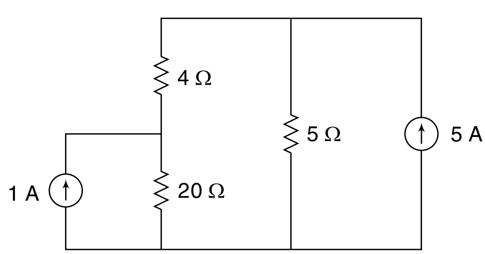

Fig. 4.109 *Circuit of Ex. 4.81*

Solution

The 20 Ω resistor is short circuited and the circuit is redrawn in Fig. 4.109(a). The current through the short-circuited path due to 1 A current source only is I_{sc_1} = 1 A (from *a* to *b*).

The current through the short-circuited path due to 5 A source only is $I_{sc_2} = 5 \times \dfrac{5}{5+4}$ A $= \dfrac{25}{9}$ A (from *a* to *b*). Norton's equivalent current is then

$$I_N\ (= I_{sc}) = I_{sc_1} + I_{sc_2} = 1 + \frac{25}{9} = 3.78\ \text{A}.$$

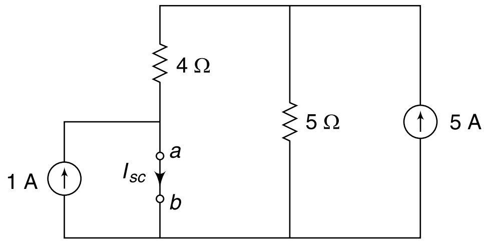

Fig. 4.109(a) *Determination of $I_{S/C}$*

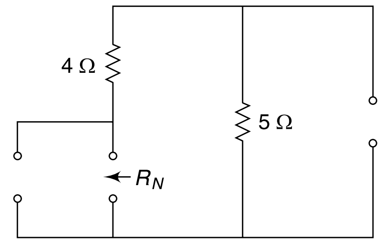

Fig. 4.109(b) *Finding of R_N*

Next, removing the sources and open-circuiting terminals *a* and *b* [as shown in Fig. 4.109 (b)] R_N is obtained as $(R_N) = 4 + 5 = 9\ \Omega$.

The current *I* through the 20 Ω resistor is obtained from Fig. 4.109(c), where

$$I = 3.78 \times \frac{9}{9+20} = 1.173 \text{ A (from } a \text{ to } b)$$

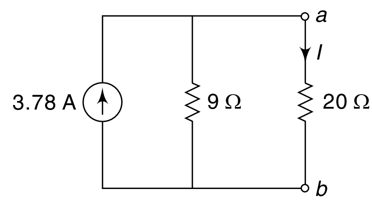

Fig. 4.109(c) *Norton's equivalent circuit of Ex. 4.81*

4.14 EQUIVALENCE OF THEVENIN'S AND NORTON'S THEOREMS

Figure 4.110 shows the equivalency of Thevenin's and Norton's theorems. It can be proved that the equivalent circuits given by Thevenin's and Norton's theorem yield exactly the same current and same voltage in the load impedance and they are effectively identical to one another. In any particular problem, either theorem can therefore be used. In most cases Thevenin's theorem is the easier to apply, although when the network impedance is high compared with the load impedance, the Norton's theorem concept may simplify calculations.

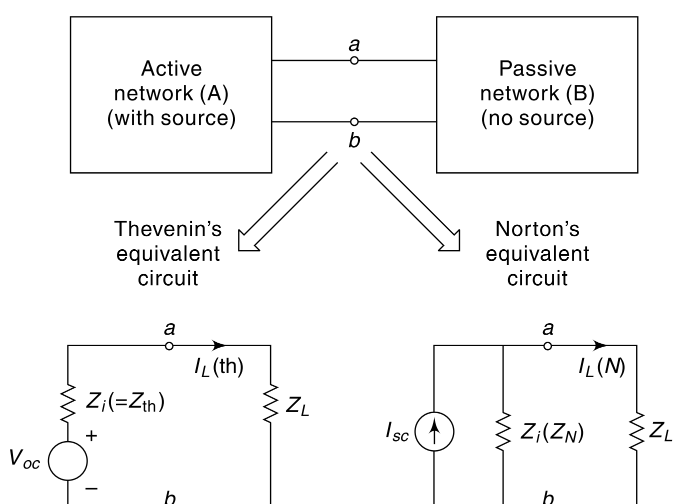

Fig. 4.110 *Equivalence of Thevenin's and Norton's circuits*

From Fig. 4.110 by applying Thevenin's theorem the load current is given by

$$I_{L(\text{Th})} = \frac{V_{oc}}{Z_i + Z_L} \tag{4.42}$$

where V_{oc} = Open circuit voltage (Thevenin's equivalent voltage source)
Z_i = Thevenin's equivalent impedance (or resistance for dc circuit), and
Z_L = Load impedance of the load network.

On short circuiting the terminals a and b of the Thevenin's equivalent,

$$I_{sc} = \frac{V_{oc}}{Z_i} \tag{4.43}$$

or

$$V_{oc} = I_{sc} \times Z_i \tag{4.44}$$

However from Norton's equivalent circuit [Fig. 4.110(b)], the load current is given by

$$I_{L(N)} = \frac{I_{sc} \times Z_i}{Z_i + Z_L} \tag{4.45}$$

Substituting the equation (4.44) in equation (4.45),

$$I_{L(N)} = \frac{V_{oc}}{Z_i + Z_L} \tag{4.46}$$

Comparing equation (4.42) and equation (4.46)

$$I_{L(\text{Th})} \equiv I_{L(N)} \tag{4.47}$$

Thus for any passive network, being connected to an active network, one can have equivalent representation of Norton's equivalent or Thevenin's equivalent circuit (i.e. both the theorems are equivalent to each other). For easy understanding, a simple example is shown in the circuit of Fig. 4.111(a).

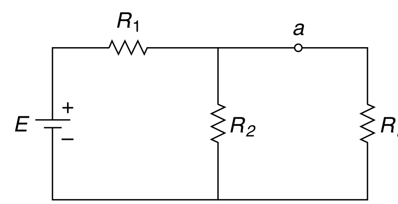

Fig. 4.111(a) ***Circuit for illustrating equivalence of Thevenin's and Norton's theorems***

From Fig. 4.111(b) the load current is

$$I_{L(\text{Th})} = \frac{\dfrac{ER_2}{R_1 + R_2}}{\dfrac{R_1 R_2}{R_1 + R_2} + R_L} = \frac{ER_2}{R_1 R_2 + R_1 R_L + R_2 R_L} \tag{4.48}$$

[$\because$ In Fig. 4.111(a), removing R_L the equivalent resistance R_i looking back to the network from $a - b$, is $\left\{\dfrac{R_1 R_2}{R_1 + R_2}\right\}$ and V_{oc} is then $\left\{\left(\dfrac{E}{R_1 + R_2}\right) \times R_2\right\}$].

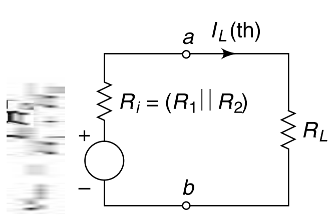

Fig. 4.111(b) ***Thevenin's equivalent circuit***

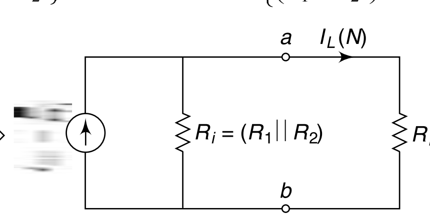

Fig. 4.111(c) ***Norton's equivalent circuit***

On the other hand, from Fig. [4.111(c)] the load current is given by

$$I_{L(N)} = \frac{\dfrac{E}{R_1} \times \dfrac{R_1 R_2}{R_1 + R_2}}{\dfrac{R_1 R_2}{R_1 + R_2} + R_L} = \frac{\dfrac{ER_2}{R_1 + R_2}}{\dfrac{R_1 R_2 + R_1 R_L + R_2 R_L}{R_1 + R_2}}$$

$$= \frac{ER_2}{R_1 R_2 + R_1 R_L + R_2 R_L}$$

[$\because$ Removing R_2 from a–b terminal and applying short circuit at $a - b$, current through the terminals a–b is (I_{sc}) i.e. $\left(\dfrac{E}{R_1}\right)$ while the internal resistance of the network is $\left\{R_i = \dfrac{R_1 R_2}{R_1 + R_2}\right\}$]

$$\therefore \qquad I_{L(\text{Th})} = I_{L(N)} = \frac{ER_2}{R_1 R_2 + R_1 R_L + R_2 R_L} \tag{4.49}$$

4.15 MAXIMUM POWER TRANSFER THEOREM

As applied to dc networks this theorem may be stated as follows: *A resistive load abstracts maximum power from a network when the load resistance equals the internal resistance of the network as viewed from the output terminals, with all energy sources removed, leaving behind their internal resistances.*

This theorem is aplicable to all branches of electrical engineering including analysis of communication networks. However the overall efficency of a network supplying maximum power to any branch is only 50%; hence application of this theorem to power transmission and distribution networks is limited because in that case, the final target is high efficiency and not maximum power transfer. But in electronics and communication network as the purpose is to receive or transmit maximum power, even at low efficiency, the problem of maximum power transfer is of crucial importance in the operation of communication lines and antennas.

Illustration

Figure 4.112 shows a simple resistive network in which a load resistance R_L is connected across terminals a and b of the network. The network consists of a generator emf(E) and internal resistance r along with a series resistance R. The internal resistance of the network as viewed from the terminals a and b is (R_i) = $r + R$.

According to maximum power transfer theorem R_L will abstract maximum power from the network when

$$R_i = R_L \text{ or } R_L = (r + R).$$

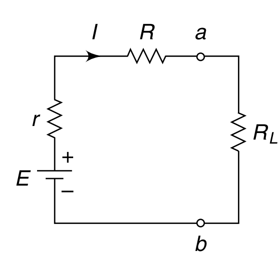

Fig. 4.112 ***Circuit for illustrating maximum power transfer theorem***

4.15.1 Proof of Maximum Power Transfer Theorem

Let us assume that current I flows through R_L in the circuit shown in Fig. 4.112. Obviously, $I = \dfrac{E}{R_i + R_L}$.

Power across the load $(P_L) = I^2 R_L = \dfrac{E^2}{(R_i + R_L)^2} \cdot R_L = \dfrac{E^2 R_L}{(R_i + R_L)^2}$ (4.50)

For P_L to be maximum,

$$\frac{dP_L}{dR_L} = 0$$

Differentiating Eq. (4.50),

$$\frac{dP_L}{dR_L} = E^2 \left[\frac{(R_i + R_L)^2 - 2R_L (R_i + R_L)}{(R_i + R_L)^4} \right] = 0$$

or $\quad R_i + R_L = 2R_L$

or $\quad R_L = R_i = r + R$

Thus for maximum power transfer, $R_L = R_i$.

The maximum power is $(P_{L\max}) = I^2 R_L = \dfrac{E^2}{(R_L + R_L)^2} \times R_L = \dfrac{E^2}{4R_L}$

The power delivered by the source is $(EI) = \dfrac{E^2}{(R_L + R_L)} = \dfrac{E^2}{2R_L}$.

So the effeciency under maximum power transfer condition is $\dfrac{E^2/4R_L}{E^2/2R_L} = \dfrac{1}{2}$ (or 50%).

4.82 Calculate the value of R_L which will abstract maximum power from the circuit shown in Fig. 4.113 Also find the maximum power.

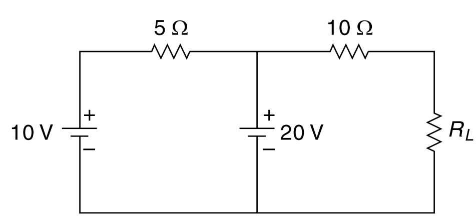

Fig. 4.113 *Circuit of Ex. 4.82*

Solution

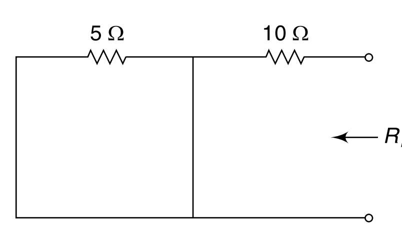

Fig. 4.113(a) *Determination of (R_i)*

Removing all the sources and open circuiting the terminals of R_L [Fig 4.113(a)] the internal resistance R_i of the network is found out as 10 Ω.

i.e., $\quad R_i = 10\ \Omega$

∴ For maximum power transfer

$$R_L = R_i = 10\ \Omega$$

Again for $R_L = 10\ \Omega$, the total current through R_L due to both sources is given by

$$I = \frac{20}{\dfrac{5\times(10+10)}{5+10+10}} \times \frac{5}{5+10+10} = 1\ \text{A}$$

[The current due to 10 V source circulates through 5 Ω resistor and 20 V source only]
The maximum power across load is

$$I^2R_L = (1)^2 \times 10 = 10\ \text{W}$$

4.83 Calculate the value of R_L which will absorb maximum power from the circuit shown in Fig. 4.114. Also calculate the value of this maximum power.

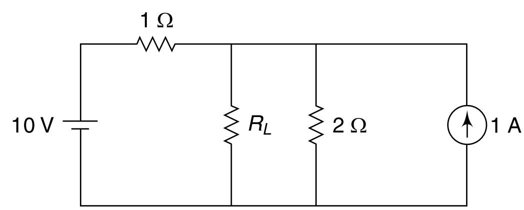

Fig. 4.114 *Circuit of Ex. 4.83*

Solution

Let R be removed and internal resistance of the network is calculated looking from the open circuited terminals after removing all the sources as shown in Fig. 4.114(a).

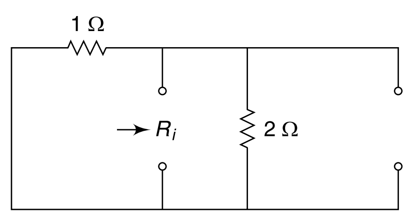

Fig. 4.114(a) *Determination of (R_i)*

Here $$R_i = \frac{1\times 2}{1+2}\ \Omega = \frac{2}{3}\ \Omega$$

i.e. $$R = R_i = \frac{2}{3}\ \Omega = 0.667\ \Omega \text{ [for maximum power transfer]}$$

The current through R due to both the sources acting simultaneously is given by

$$I = \frac{10}{1+\dfrac{0.667\times 2}{0.667+2}} \times \frac{2}{2+0.667} + 1\times \frac{\dfrac{2\times 1}{2+1}}{0.667+\dfrac{2\times 1}{2+1}}$$

$$= 4.998 + 0.5 = 5.5\ \text{A}$$

The value of the maximum power is $(5.5)^2 \times 0.667$ W = 20 W.

4.84 Obtain the maximum power transferred to R_L in the circuit of Fig. 4.115 and also the value of R_L.

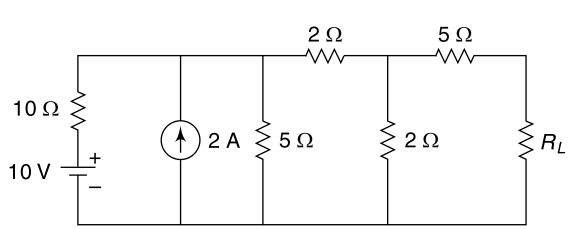

Fig. 4.115 *Circuit of Ex. 4.84*

Solution

R_L is removed and its terminals are open circuited. Deactivating the sources the internal resistance R_i of the network can be found out from Fig. 4.115(a).

$$R_i = \left[\left(\frac{10\times 5}{10+5}+2\right) \| 2\right] + 5 = 6.45\ \Omega$$

Thus, according to the maximum power transfer theorem the value of R_L is 6.45 Ω for maximum power transfer.

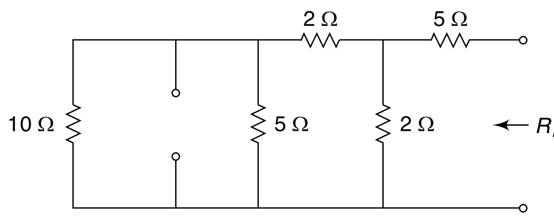

Fig. 4.115(a) ***Finding of (R_i)***

Next, considering the 10 V source acting alone in the network the total current supplied by the 10 V source [Fig. 4.115(b)] is

$$I = \frac{10}{10+\dfrac{5\times 3.7}{5+3.7}} = 0.82\ \text{A}.$$

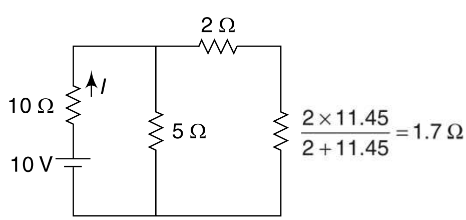

Fig. 4.115(b) ***Current (I) for 10 V source only***

∴ Current through R_L due to the 10 V source only is

$$I_1 = 0.82 \times \frac{5}{5+2+1.7} \times \frac{2}{2+5+6.45} = 0.07\ \text{A}.$$

Again considering the 2 A source acting alone, the current through the 2 Ω resistor is

$$I' = 2 \times \frac{10}{10+\dfrac{5\times 3.7}{5+3.7}} \times \frac{5}{5+2+1.7}\ \text{A} = 0.4739\ \text{A}$$

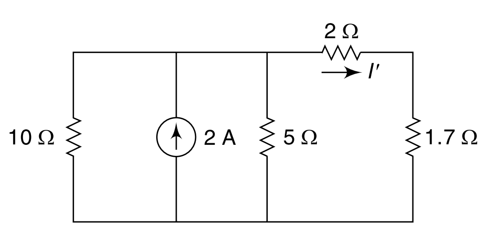

Fig. 4.115(c) ***Determination of current through 2 Ω resistor for 2 A source only***

Hence the current due to the 2 A current source through R_L is

$$I_2 = 0.4739 \times \frac{2}{2+5+6.45} \text{ A} = 0.07 \text{ A}$$

Applying the superposition theorem current through R_L (when both the sources are acting simultaneously) is

$$I = I_1 + I_2 = 0.07 + 0.07 = 0.14 \text{ A}$$

∴ Maximum power transferred across R_L is

$$I^2R_L = (0.14)^2 \times 6.45 = 0.126 \text{ W}.$$

4.85 Find the value of R in the circuit of Fig. 4.116 such that maximum power transfer takes place. What is the amount of this power?

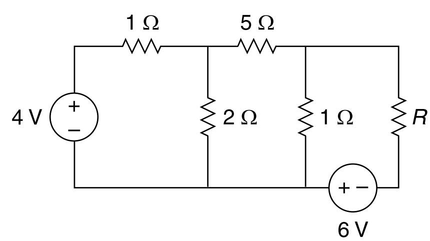

Fig. 4.116 ***Circuit of Ex. 4.85***

Solution

Deactivating all the sources, internal resistance R_i of the network is found out as shown in Fig. 4.116(a).

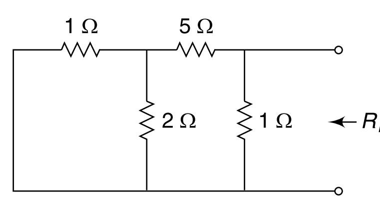

Fig. 4.116(a) ***Determination of (R_i)***

$$R_i = \left(\frac{2\times 1}{2+1}+5\right) \parallel 1$$

$$= \frac{5.67\times 1}{5.67+1} = 0.85\Omega$$

According to the maximum power transfer theorem the maximum power takes place across R when $R = R_i = 0.85\ \Omega$.

The current through R due to the 4 V source acting alone is

$$I_1 = \frac{4}{\left[\left(\frac{0.85\times 1}{0.85+1}+5\right)\parallel 2\right]+1} \times \frac{2}{2+5+\frac{1\times 0.85}{1+0.85}} \times \frac{1}{1+0.85}$$

$$= \frac{4}{\frac{5.46\times 2}{5.46+2}+1} \times \frac{2}{13.8} = 0.235 \text{ A}$$

The current through (R) due to 6 V source acting alone is

$$I_2 = \frac{6}{\left[\left(\frac{1\times 2}{1+2}+5\right)\parallel 1\right]+0.85} = \frac{6}{\frac{5.67\times 1}{5.67+1}+0.85} = 3.53 \text{ A}$$

According to superposition, the current through R when both the sources are acting simultaneously is

$$I = I_1 + I_2 = 0.235 + 3.53 = 3.765 \text{ A}.$$

Thus the maximum power is $I^2R = (3.765)^2 \times 0.85 = 12$ W.

4.86 Assuming maximum power transfer from the source to R find the value of this power in the circuit of Fig. 4.117.

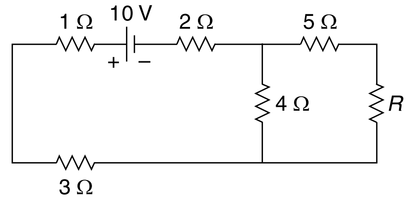

Fig. 4.117 *Circuit of Ex. 4.86*

Solution

Deactivating the source the internal resistance R_i of the network is found from Fig. 4.117(a);

$$R_i = 5 + \frac{4(1+2+3)}{4+(1+2+3)}$$

$$= 5 + \frac{4 \times 6}{10} = 5 + 2.4 = 7.4\ \Omega$$

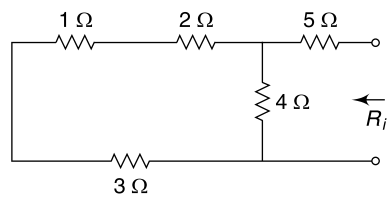

Fig. 4.117(a) *Determination of (R_i)*

According to the maximum power transfer theorem, the maximum power transfer from the source to R occurs when

$$R = R_i = 7.4\ \Omega.$$

The current through R due to 10 V source is

$$I = \frac{10}{1+2+3+\dfrac{4\times(5+7.4)}{4+(5+7.4)}}\ \text{A} = 1.108\ \text{A}$$

Hence the maximum power transfer from source to R is

$$I^2R = (1.108)^2 \times 7.4 = 9.08\ \text{W}.$$

4.87 Find the value of R_L for which the power transfer across R_L is maximum and find the value of this maximum power [Fig. 4.118].

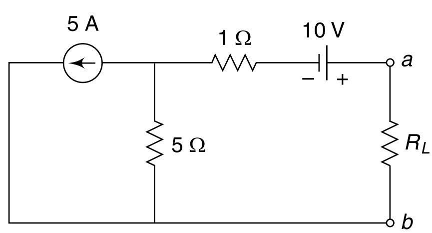

Fig. 4.118 *Circuit of Ex. 4.87*

Solution

Deactivating the sources the internal resistance of the network is found out looking back from the open circuited terminals of R_L [as shown Fig. 4.118(a)].

$\therefore \qquad R_i = 1 + 5 = 6\ \Omega.$

Power transfer across R_L is maximum when

$$R_i = R_L = 6\ \Omega.$$

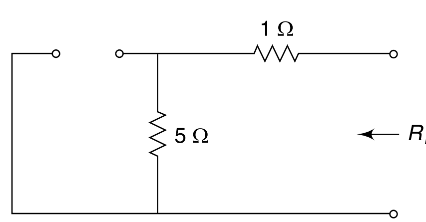

Fig. 4.118(a) *Determination of (R_i)*

The current through (R_L) is

$$I = \frac{10}{6+5+1} - 5 \times \frac{5}{5+1+6}\ \text{(from } a \text{ to } b\text{)}$$

$$= -1.25\ \text{A (from } a \text{ to } b\text{)} \quad \text{or} \quad 1.25\ \text{A from (} b \text{ to } a\text{)}$$

The value of the maixmum power is obtained as

$$I^2R_L = (1.25)^2 \times 6 = 9.375\ \text{W}.$$

ADDITIONAL EXAMPLES

4.88 In the network shown in Fig. 4.119 determine all branch currents and the voltage across the 5 Ω resistor by loop current analysis.

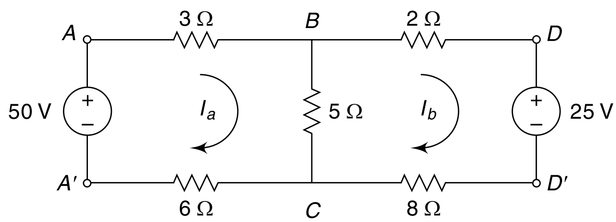

Fig. 4.119 *Circuit of Ex. 4.88*

Solution

Let I_a and I_b be the loop currents. Applying KVL to loop *ABCA′A*

$$3I_a + 5\ (I_a - I_b) + 6I_a = 50$$

or $\quad 14I_a - 5I_b = 50.$ (i)

Applying KVL to loop *BDD′CB*

$$2I_b + 25 + 8I_b + 5(I_b - I_a) = 0$$

or $\quad -5I_a + 15I_b = -25.$ (ii)

Solution of equations (i) and (ii) yields

$$I_a = 3.3784 \text{ A and } I_b = -0.541 \text{ A}$$

The current through 3 Ω and 6 Ω resistors is thus 3.3784 A from *A* to *B* and *C* to A′ respectively. The current through 2 Ω and 8 Ω resistors is 0.541 A from *D* to *B* and *C* to *D′* respectively, while the curent through 5 Ω resistor is

$$I_a - I_b = 3.9194 \text{ A from } B \text{ to } C.$$

Voltage across 5 Ω resistor is 5 × 3.9194 = 19.597 V.

4.89 In the circuit shown in Fig. 4.120 find current I_a.

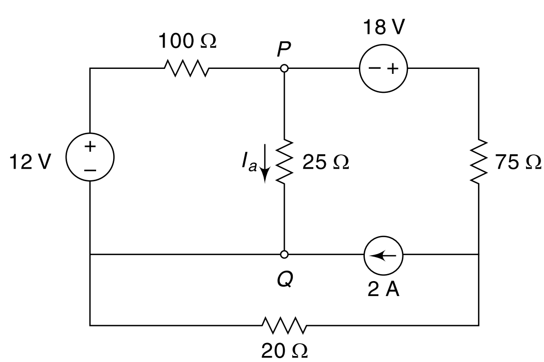

Fig. 4.120 *Circuit of Ex. 4.89*

Solution

The circuit is redrawn as shown in Fig. 4.120(a).

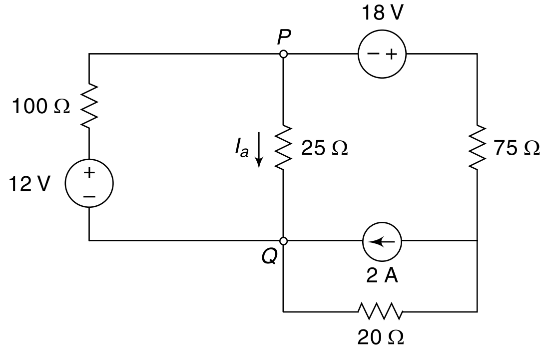

Fig. 4.120(a) *Circuit of Ex. 4.89 redrawn*

The 2 A current source can be replaced by an equivalent voltage source of $20 \times 2 = 40$ V in series with a 20 Ω resistance and the modified circuit is shown in Fig. 4.120(b).

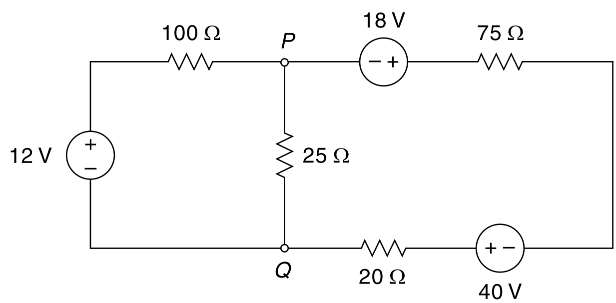

Fig. 4.120(b) *Modified circuit*

The two-voltage sources in series can be combined into a single source as shown in Fig. 4.120(c).

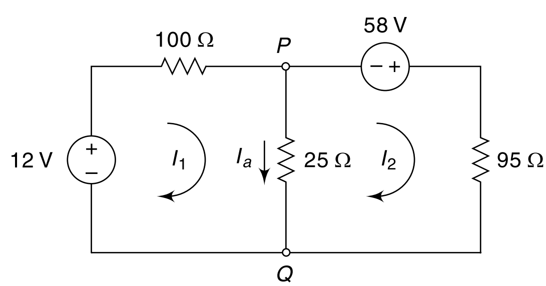

Fig. 4.120(c) *Finally reduced circuit of Ex. 4.89*

Let I_1 and I_2 be the loop currents applying KVL in these two loops

$$100\ I_1 + 25(I_1 - I_2) = 12 \tag{i}$$

and $$95I_2 + 25(I_2 - I_1) = 58 \tag{ii}$$

Solving the two equations (i) and (ii).

$$I_1 = 0.201 \text{ A and } I_2 = 0.525 \text{ A}$$

∴ $$I_a\ (= I_1 - I_2) = -0.3242 \text{ A (from } P \text{ to } Q)$$

or, $$I_a = 0.3242 \text{ A (from } Q \text{ to } P).$$

4.90 From the circuit shown in Fig. 4.121, use loop analysis to determine the loop currents I_1, I_2, I_3.

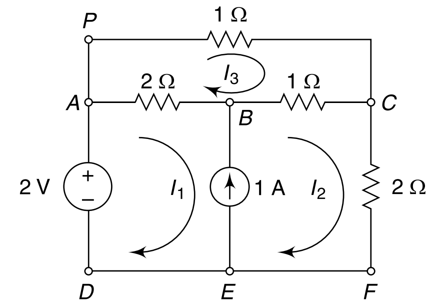

Fig. 4.121 ***Circuit of Ex. 4.90***

Solution

From Fig. 4.121 the current source of 1 A is equivalent to $(I_2 - I_1)$, i.e, $I_2 - I_1 = 1$ A. (i)

From loop A*BCPA*

$$2(I_3 - I_1) + 1(I_3 - I_2) + I_3 \times 1 = 0$$

or $-2I_1 - I_2 + 4I_3 = 0$ (ii)

From loop A*BCFEDA*

$$2(I_1 - I_3) + 1(I_2 - I_3) + 2I_2 - 2 = 0$$

or $2I_1 + 3I_2 - 3I_3 = 2$ (iii)

Solving the three equations (i), (ii), (iii) we obtain

$$I_1 = -\frac{1}{11} \text{ A}, \quad I_2 = \frac{10}{11} \text{ A} \quad \text{and} \quad I_3 = \frac{2}{11} \text{ A.}$$

4.91 In the circuit shown in Fig. 4.122 determine the voltages at nodes 1 and 2 with respect to the reference point.

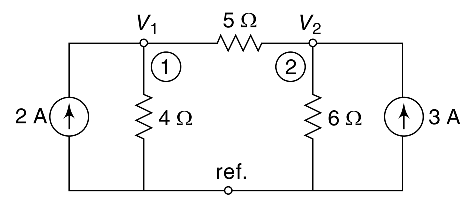

Fig. 4.122 ***Circuit of Ex. 4.91***

Solution

Applying nodal analysis at node (1),

$$\frac{V_1}{4} + \frac{V_1 - V_2}{5} - 2 = 0$$

or $9V_1 - 4V_2 = 40$ (i)

Applying nodal analysis at node (2),

$$\frac{V_2 - V_1}{5} + \frac{V_2}{6} - 3 = 0$$

or $-6V_1 + 11V_2 = 90$ (ii)

Solving equations (i) and (ii)

$$V_1 = 10.667\ V \text{ and } V_2 = 14 \text{ V.}$$

4.92 In the circuit shown in Fig. 4.123 find voltage at node A.

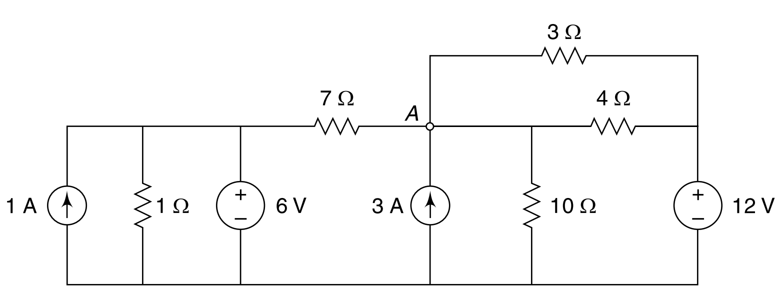

Fig. 4.123 ***Circuit of Ex. 4.92***

Solution

Since the 3 A current source is in parallel with the 10 Ω resistor, hence converting the current source into the equivalent voltage source and replacing the parallel combination of 3 Ω and 4 Ω by a single resistance [Fig. 4.123(a)] we can write nodal equation at node A as

$$\frac{V_A - 30}{10} + \frac{V_A - 6}{7} + \frac{V_A - 12}{\frac{12}{7}} = 0$$

or $$V_A\left(\frac{1}{10} + \frac{1}{7} + \frac{7}{12}\right) - 3 - \frac{6}{7} - 7 = 0$$

or $$V_A = 13.14 \text{ V}$$

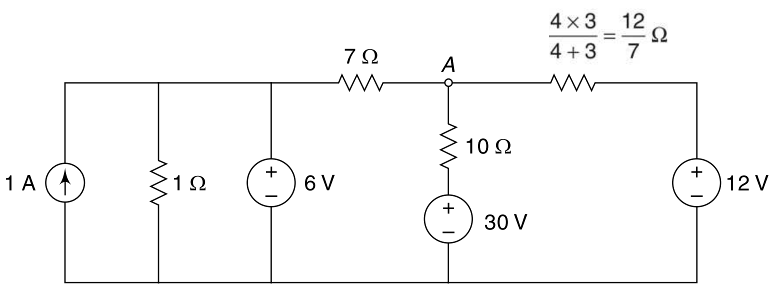

Fig. 4.123(a) ***Network reduction of the circuit shown in Fig. 4.123***

4.93 Using mesh analysis obtain the values of all mesh currents of the network shown in Fig. 4.124.

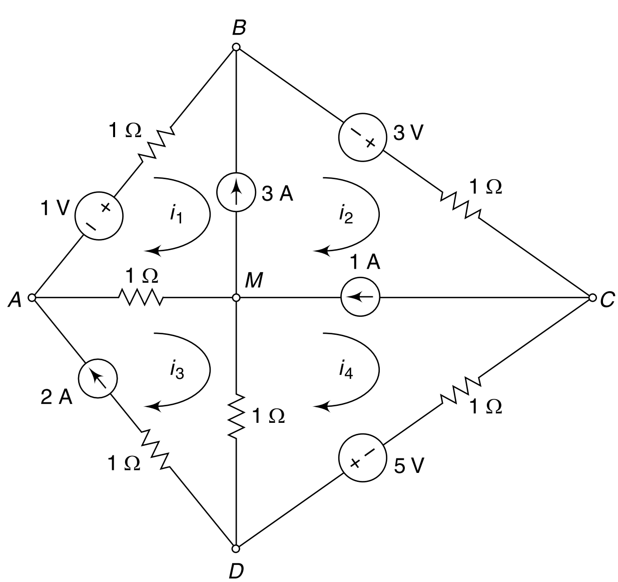

Fig. 4.124 ***Circuit of Ex. 4.93***

Solution

From Fig. 4.124 it can easily be observed that

$$i_3 = 2 \text{ A}$$

$$i_2 - i_1 = 3 \text{ A}$$

and $$i_2 - i_4 = 1 \text{ A}$$

By applying KVL to a closed loop which does not have any current source (loop A*BCDMA*) we obtain

$$-1 + i_1 - 3 + i_2 + i_4 - 5 + (i_4 - i_3) + (i_1 - i_3) = 0$$

or $$2i_1 + i_2 + 2i_4 = 13$$

As $i_1 = (i_2 - 3)$ and $i_4 = (i_2 - 1)$, hence from above we can write,

$$2(i_2 - 3) + i_2 + 2(i_2 - 1) = 13$$

or $$i_2 = 4.2 \text{ A}$$

Hence, $i_1 = 4.2 - 3 = 1.2$ A and $i_4 = 4.2 - 1 = 3.2$ A.

$\therefore$ $i_1 = 1.2$ A; $i_2 = 4.2$ A; $i_3 = 2$ A; $i_4 = 3.2$ A.

4.94 Determine the current in the conductor of 2 Siemens of the network shown in Fig. 4.125 using node voltage analysis.

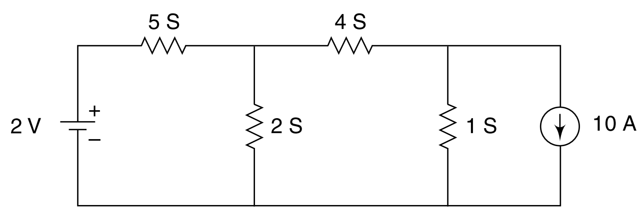

Fig. 4.125 ***Circuit of Ex. 4.93***

Solution

Replacing the current source by an equivalent voltage source the new transformed circuit is shown in Fig. 4.125(a). Here there are two nodes (1) and (2). Let node (2) be taken as the reference node and let *V* be the potential at node (1).

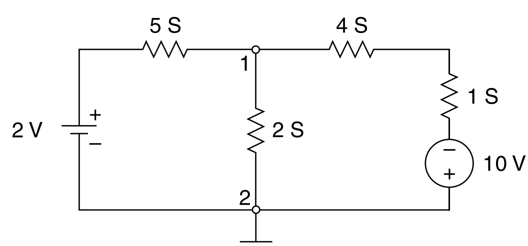

Fig. 4.125(a) ***Transformed circuit of Fig. 4.125***

Hence $$(V - 2)5 + V \times 2 + (V + 10)(4 + 1) = 0$$

or $$V = -\frac{40}{12} = -3.333.$$

Hence current through conductance of 2 Siemens is $-3.333 \times 2 = -6.67$ A. This current is directed from (2) to (1).

4.95 Using loop equations obtain the current in the 12 Ω resistor of the network shown in Fig. 4.126.

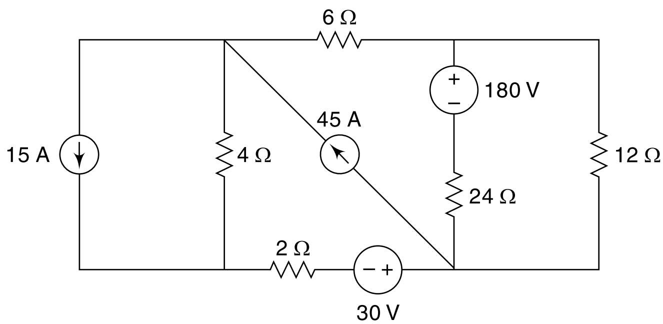

Fig. 4.126 *Circuit of Ex. 4.95*

Solution

Let us first replace 15 A current source by an equivalent voltage source; the corresponding figure is shown in Fig. 4.126(a).

Here $I_2 - I_1 = 45$ (i)

Applying KVL in loop A*BCDA*

$$6I_2 + 180 + 24(I_2 - I_3) + 30 + 6I_1 + 60 = 0$$

or $6I_1 + 30I_2 - 24I_3 = -270$

or $I_1 + 5I_2 - 4I_3 = -45$ (ii)

Applying KVL in loop *BEFCB*

$$12I_3 + 24(I_3 - I_2) - 180 = 0$$

or $-24I_2 + 36I_3 = 180$

or $-2I_2 + 3I_3 = 15$ (iii)

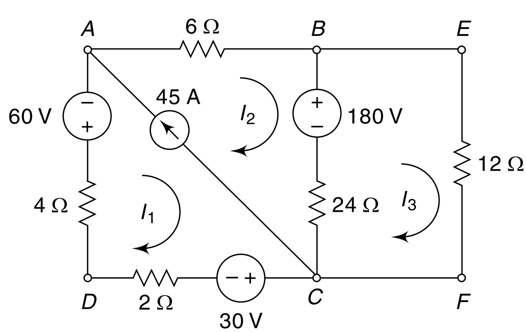

Fig. 4.126(a) *Transformed circuit of Fig. 4.126*

Solving equations (i), (ii) and (iii)

$$I_2 = 6 \text{ A}, I_3 = 9 \text{ A and } I_1 = -39 \text{ A}$$

∴ Current in the 12 Ω resistor is $I_3 = 9$ A.

4.96 Use node voltage analysis determine the power in the 2 Ω and 4 Ω resistor in the network of Fig. 4.127.

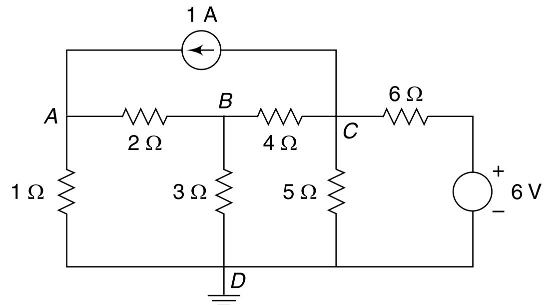

Fig. 4.127 ***Circuit of Ex. 4.96***

Solution

There are four nodes in the network of which D is considered as the reference node.

At node A

$$\frac{V_A - V_B}{2} + V_A - 1 = 0$$

or $\quad 3V_A - V_B = 2 \quad$ (i)

At node B

$$-\frac{V_A - V_B}{2} + \frac{V_B - V_C}{4} + \frac{V_B}{3} = 0$$

or $\quad 6V_B - 6V_A + 3V_B - 3V_C + 4V_B = 0$

or $\quad -6V_A + 13V_B - 3V_C = 0 \quad$ (ii)

At node C

$$\frac{V_C - V_B}{4} + \frac{V_C}{5} + \frac{V_C - 6}{6} + 1 = 0$$

or $\quad 15V_C - 15V_B + 12V_C + 10V_C - 60 + 60 = 0$

or $\quad -15V_B + 37V_C = 0 \quad$ (iii)

Solving equations (i), (ii) and (iii)

$$V_A = 0.8031 \text{ V}, V_B = 0.4088 \text{ V and } V_C = 0.1658 \text{ V}$$

$$\therefore \text{ Power in the 2 Ω resistor} = \frac{(V_A - V_B)^2}{2} = \frac{(0.8031 - 0.4088)^2}{2} = 0.078 \text{ W}$$

$$\text{Power in 4 Ω resistor} = \frac{(V_B - V_C)^2}{4}\left(= \frac{(0.4088 - 0.1658)^2}{4}\right) = 0.01476 \text{ W.}$$

4.97 Determine the Thevenin's equivalent circuit with respect to terminals A, B for the network shown in Fig. 4.128.

Solution

From Fig. 4.128 it is evident that open circuit voltage V_{oc} between A and B is the voltage across the 4 Ω resistor. The current through the 4 Ω resistor due to the 12 A

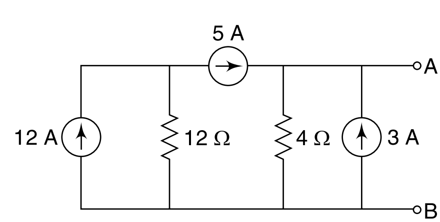

Fig. 4.128 ***Circuit of Ex. 4.97***

source is zero due to the presence of 5 A current source. The other two current sources deliver 5 A and 3 A current in the same direction through the 4 Ω resistor. So voltage across the 4 Ω resistor V_{oc} is [4 × (5 + 3) = 32 V]. Next, removing all the sources Thevenin's equivalent resistance can be obtained as shown in Fig. 4.128(a).

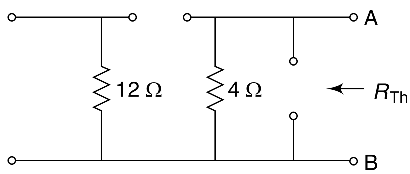

Fig. 4.128(a) ***Determination of Thevenin's equivalent resistance***

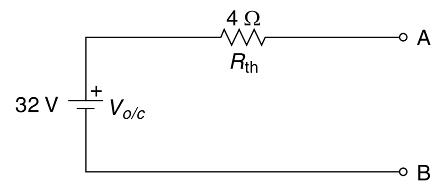

Fig. 4.128(b) ***Thevenin's equivalent network of Ex. 4.97***

Here R_{Th} = 4 Ω

The Thevenin's equivalent circuit is shown in Fig. 4.128(b).

4.98 Find the Thevenin's equivalent circuit at terminals A*B* for the network shown in Fig. 4.129 and hence determine the power dissipated in a 5 Ω resistor connected between A and *B*.

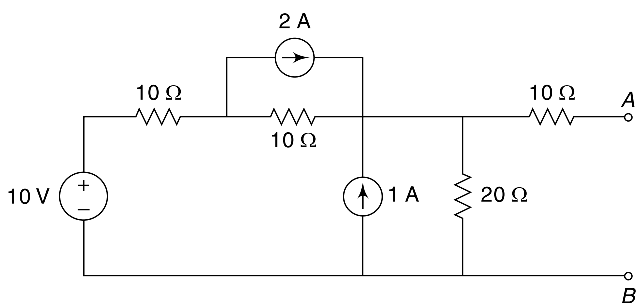

Fig. 4.129 ***Circuit of Ex. 4.98***

Solution

Converting the current sources into equivalent voltage sources [Fig. 4.129(a)], current *I* through 20 Ω resistor is given as

$$I = \frac{20 + 10 - 20}{10 + 10 + 20} = 0.25 \text{ A}$$

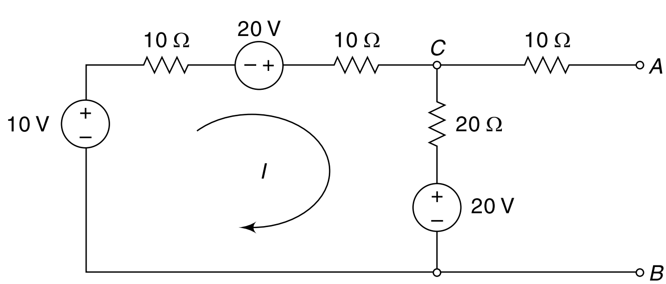

Fig. 4.129(a) ***Modified circuit***

$\therefore$ V_{Th} = Voltage across CD
$= 20 + 20 \times 0.25 = 25$ V

To find R_{Th}, deactivating all sources [Fig. 4.129(b)] we get

$$R_{Th} = 10 + \frac{20 \times (10 + 10)}{20 + (10 + 10)} = 20\ \Omega$$

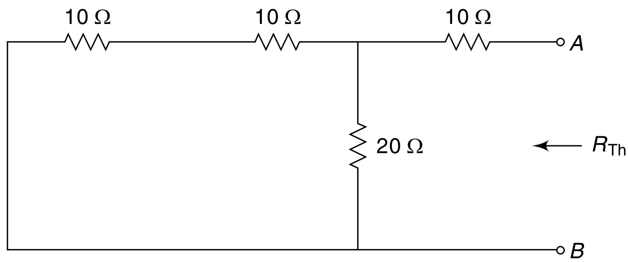

Fig. 4.129(b) ***Determination of R_{TH}***

Thevenin's equivalent circuit is shown in Fig. 4.129(c).

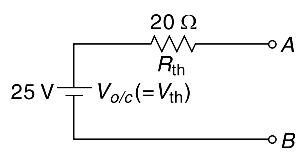

Fig. 4.129(c) ***Thevenin's equivalent circuit of Ex. 4.98***

4.99 Obtain Thevenin's equivalent circuit with respect to terminals A and *B* of the network shown in Fig. 4.130.

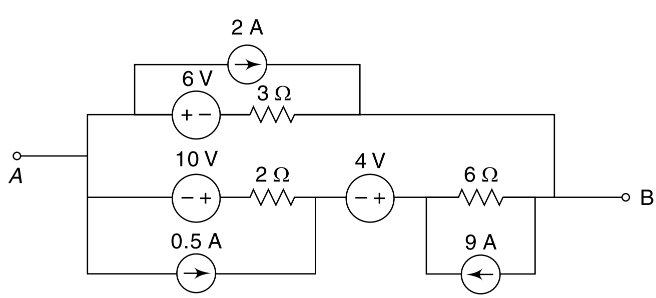

Fig. 4.130 ***Circuit of Ex. 4.99***

Solution

Let us first convert 6 V and 10 V voltage sources into corresponding current sources and 9 A current source into voltage source [Fig. 4.130(a)]. Next, Fig. 4.130(a) is reduced to Fig. 4.130(b).

Next we convert 5.5 A current source into equivalent voltage source as shown in Fig. 4.130(c). Figure 4.130(d) shows further network reduction.

The current I through the loop in Fig. 4.130(d) is

$$I = \frac{39}{8+3}\text{ A} = 3.55\text{ A}$$

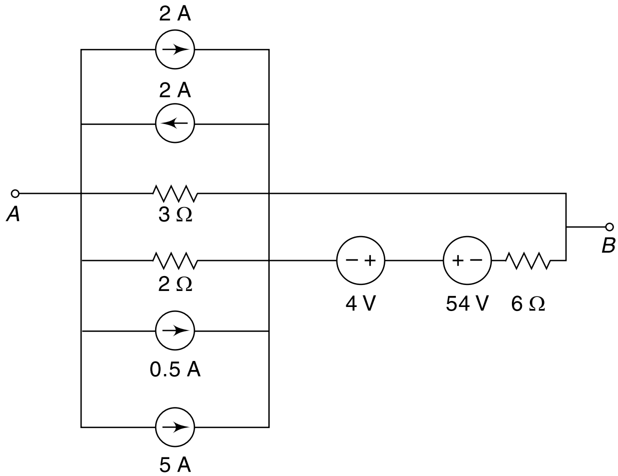

Fig. 4.130(a) *Conversion of sources*

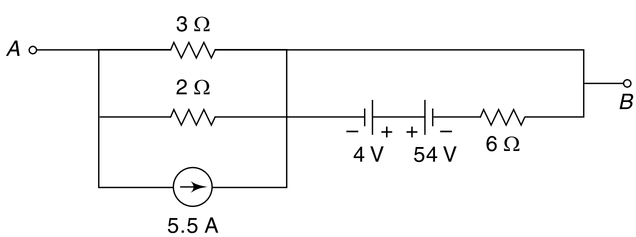

Fig. 4.130(b) *Network reduction*

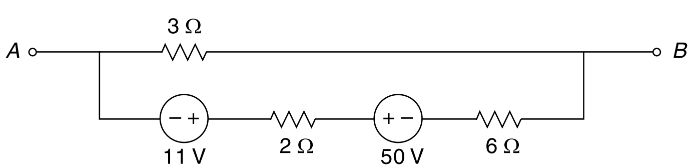

Fig. 4.130(c) *Reduced network*

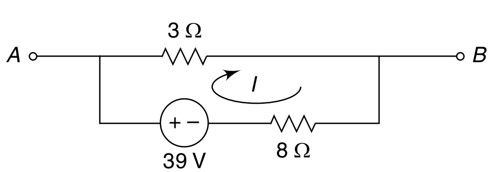

Fig. 4.130(d) *Finally reduced network*

∴ V_{Th} = Voltage across the 3 Ω resistor = 3 × 3.55 V = 10.65 V.

Thevenin's equivalent resistance $R_{\text{Th}} = \dfrac{8 \times 3}{3+8}\ \Omega$

= 2.18 Ω.

Thevenin's equivalent circuit is shown in Fig. 4.130(e).

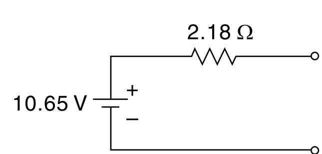

Fig. 4.130(e) ***Thevenin's equivalent network of Ex. 4.99***

4.100 Determine the Thevenin's equivalent of the bridge network shown in Fig. 4.131 as seen from the galvanometer terminals B and D and hence determine the galvanometer current when $R_G = 50\ \Omega$.

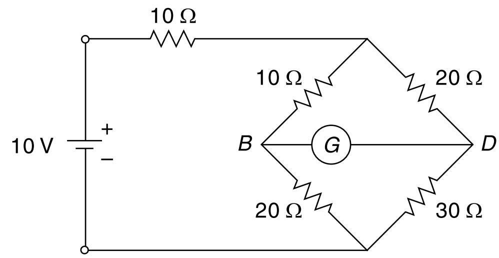

Fig. 4.131 *Circuit of Ex. 4.100*

Solution

To find the Thevenin's equivalent voltage across BD, the galvanometer is open-circuited and the corresponding figure is shown in Fig. 4.131(a). The circuit of Fig. 4.131(a) can then be reduced to that shown in Fig. 4.131(b).

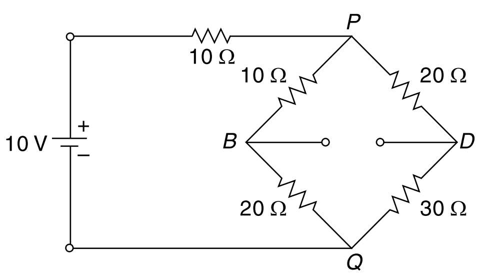

Fig. 4.131(a) *Circuit configuration with galvanometer removed*

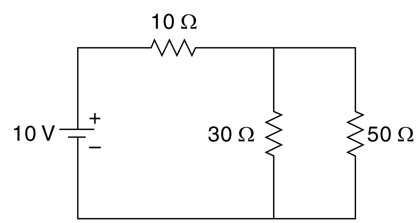

Fig. 4.131(b) *Reduced network*

Current through the 30 Ω resistor = $\dfrac{10}{10+\dfrac{30\times 50}{30+50}} \times \dfrac{50}{50+30}$ = 0.217 A.

Current through the 50 Ω resistor = $\dfrac{10\times 30}{2300}$A = 0.13 A.

∴ Currents through PB and PD in Fig. 4.131(a) are 0.217 A and 0.13 A respectively.

$$V_{\text{Th}} = V_{BD} = V_{PD} - V_{PB} = 20 \times 0.13 - 0.217 \times 10 = 0.43 \text{ V}$$

To find Thevenin's equivalent resistance the voltage source is short circuited as shown in Fig. 4.131(c)

Converting delta network into equivalent star network Fig. 4.131(d) is obtained.

$$R_1 = \frac{10\times 10}{10+20+10} = 2.5\ \Omega$$

$$R_2 = \frac{10\times 20}{40} = 5\ \Omega$$

$$R_3 = \frac{20\times 10}{40} = 5\ \Omega$$

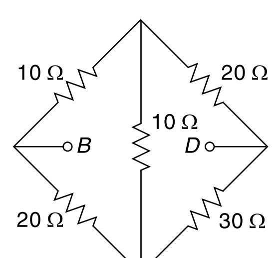

Fig. 4.131(c) *Finding of R_{Th}*

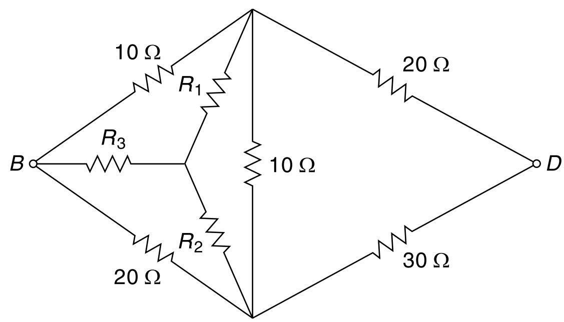

Fig. 4.131(d) ***Network reduction for network shown in Fig. 4.131(c)***

The equivalent resistance across terminal *BD* can be found out from Fig. 4.31(e) as

$$R_{\text{Th}} = 5 + \frac{22.5 \times 35}{22.5 + 35} = 18.696\ \Omega.$$

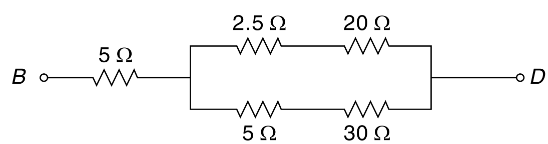

Fig. 4.131(e) ***Finally reduced network***

Thevenin's equivalent of the bridge network is shown in Fig. 4.131(f).

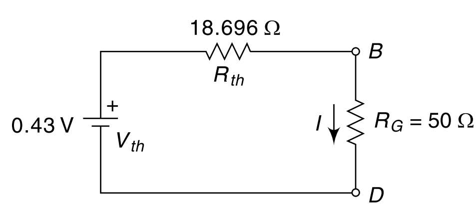

Fig. 4.131(f) ***Thevenin's equivalent of Ex. 4.100***

The galvanometer current is given by

$$I = \frac{0.43}{18.696 + 50}\ \text{A} = 0.0063\ \text{A} = 6.3\ \text{mA}$$

4.101 Find Norton's equivalent circuit at terminals *A* and *B* for the network shown in Fig. 4.132 and hence determine the power dissipated in a 5 Ω resistor to be connected between terminals *A* and *B*.

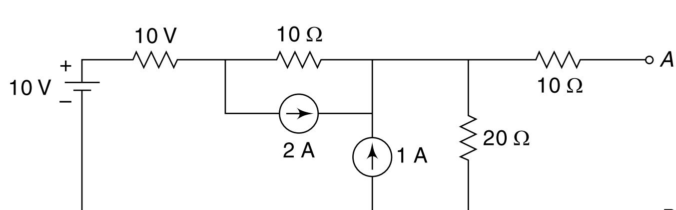

Fig. 4.132 ***Circuit of Ex. 4.101***

Solution

First we convert the current sources into equivalent voltage sources and short circuit terminals A*B* [Fig. 4.132(a)].

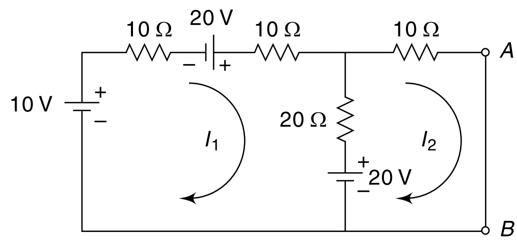

Fig. 4.132(a) *Conversion of sources*

If I_1 and I_2 be the loop currents then

$$-10 + (10 + 10)\,I_1 + 20(I_1 - I_2) = 0 \quad \text{(i)}$$

and $$-20 + 20(I_2 - I_1) + 10I_2 = 0 \quad \text{(ii)}$$

Solving Eqs (i) and (ii) we get

$$I_2 = 1.25 \text{ A}$$

Now, Norton's equivalent current i.e., the current through short-circuited path A*B* is given by

$$\therefore \quad I_N = 1.25 \text{ A}$$

To find Norton's equivalent resistance, A*B* is open circuited and the sources are removed as shown in Fig. 4.132(b).

$$R_N = 10 + \frac{20 \times 20}{20 + 20} = 20\ \Omega$$

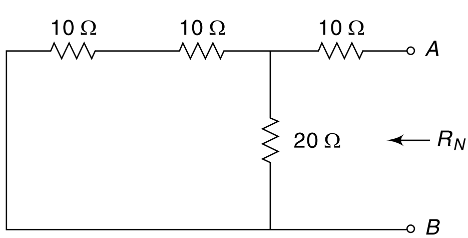

Fig. 4.132(b) *Finding of R_N*

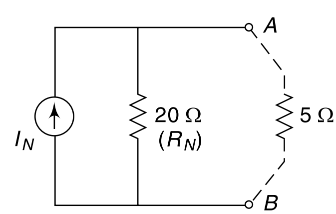

Fig. 4.132(c) *Norton's equivalent circuit of Ex. 4.101*

Norton's equivalent circuit is shown in Fig. 4.132(c).

So, the current *I* through 5 Ω resistor connected between terminals A and *B* is

$$I = 1.25 \times \frac{20}{20 + 5} = 1 \text{ A}$$

Hence power dissipated through 5 Ω resistor = $1^2 \times 5 = 5$ W.

4.102 In Fig. 4.133 the galvanometer *G* has a conductance of 10 S. Determine the current through the galvanomneter using Thevenin's theorem.

Solution

Let us first see open-circuiting terminals A*B* [Fig. 4.133(a)]

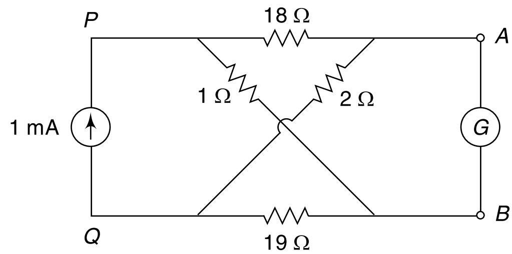

Fig. 4.133 *Circuit of Ex. 4.102*

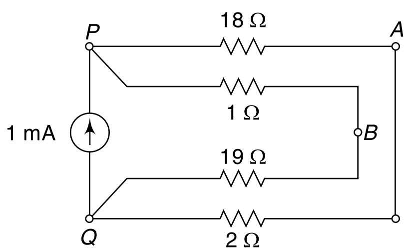

Fig. 4.133(a) *Circuit with galvanometer removed*

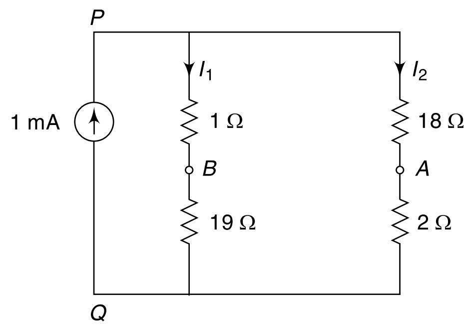

Fig. 4.133(b) *Modified circuit of Fig. 4.133(a)*

Figure 4.133(a) is redrawn as shown in Fig. 4.133(b).

From Fig. 4.133(b) current through the 1 Ω resistor is

$$I_1 = 1 \times \frac{20}{20+20} = 0.5 \text{ mA}$$

and current through 18 Ω resistor is also

$$I_2 = 1 \times \frac{20}{20+20} = 0.5 \text{ mA}.$$

Now $\quad V_{AB} = V_{PB} - V_{PA}$

$\quad = 1 \times 0.5 - 18 \times 0.5 = -8.5$ mV

$\therefore \quad V_{\text{Th}} = V_{BA} = 8.5$ mV

[terminal *B* is at higher potential].

To find Thevenin's equivalent resistance current source is open-circuited and the network of Fig. 4.133(c) is obtained.

Hence $\quad R_{\text{Th}} = \dfrac{(18+1)(19+2)}{(18+1)+(19+2)} = 9.975\ \Omega$

From Fig. 4.133(d) current through the galvanometer of 10 S, i.e. 1/10 Ω resistance is

$$\frac{8.5 \times 10^{-3}}{9.975 + \frac{1}{10}} \text{ A} = 0.844 \times 10^{-3} \text{ A}$$

$$= 0.844 \text{ mA}$$

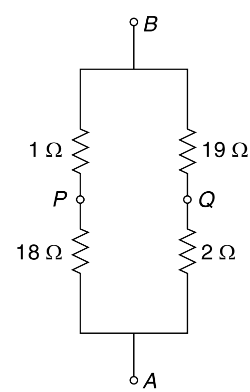

Fig. 4.133(c) *Determination of R_{TH}*

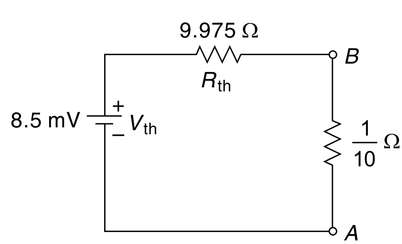

Fig. 4.133(d) *Thevenin's equivalent of Ex. 4.102*

4.103 Determine the current through the 1 Ω resistor connected across *A*, *B* of the network shown in Fig. 4.134 using Norton's theorem.

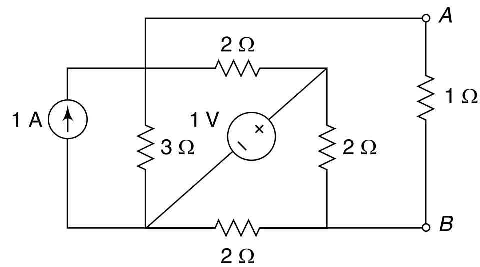

Fig. 4.134 *Circuit of Ex. 4.134*

Solution

Removing the 1 Ω resistor and short-circuiting the terminals *AB* the circuit is redrawn as shown in Fig. 4.134(a). The 1 A current source has been transformed into voltage source. Applying KVL to the three loops we get the following three equations:

$$3I_1 + 2(I_1 - I_3) + 1 - 3 = 0$$

or $$5I_1 - 2I_3 = 2 \qquad \text{(i)}$$

$$2I_2 - 1 + 2(I_2 - I_3) = 0$$

or $$4I_2 - 2I_3 = 1 \qquad \text{(ii)}$$

and $$2(I_3 - I_2) + 2(I_3 - I_1) = 0$$

or $$-2I_1 - 2I_2 + 4I_3 = 0 \qquad \text{(iii)}$$

Solving the three equations (i), (ii) and (iii) we get I_3 = 0.59 A. Hence the current through the short circuited path *AB* is I_3 = 0.59 A, i.e. I_N = 0.59 A.

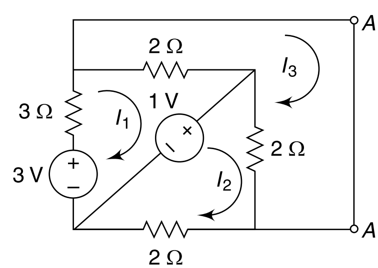

Fig. 4.134(a) *Determination of (I_N)*

To find R_N, all the sources are deactivated and open circuiting terminals *AB* [Fig. 4.134(b)], we get

$$R_N = \frac{3 \times 2}{3 + 2} + \frac{2 \times 2}{2 + 2} = 2.2\ \Omega$$

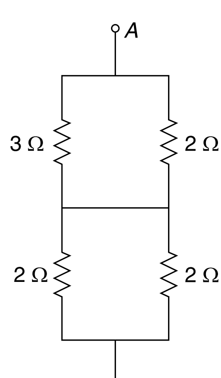

Fig. 4.134(b) *Determination of (R_N)*

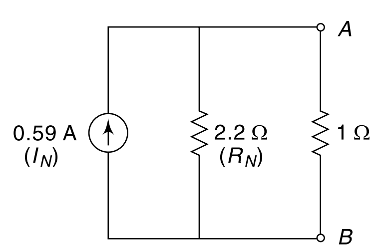

Fig. 4.134(c) *Norton's equivalent circuit of Ex. 4.103*

From Fig. 4.134(c) the current through the 1 Ω resistor is

$$0.59 \times \frac{2.2}{2.2 + 1}\ \text{A} = 0.4056\ \text{A}.$$

4.104 Solve the above problem (Example 4.103) using the superposition theorem.

Solution

Considering a 1 A current source acting alone, the circuit shown in Fig. 4.134, transforms into the circuit shown in Fig. 4.135.

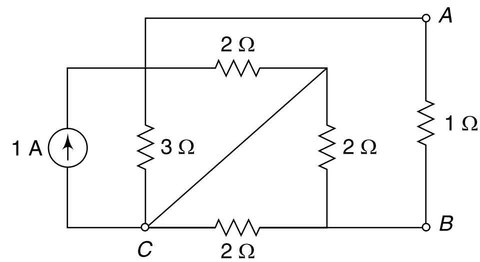

Fig. 4.135 ***1 A source is acting alone in circuit of Fig. 4.134***

The circuit further reduces as shown in Fig. 4.135(a).

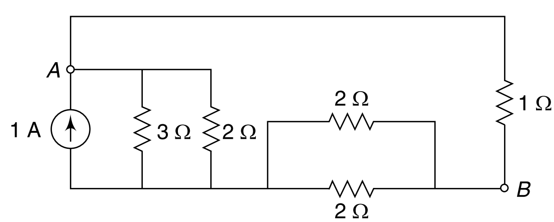

Fig. 4.135(a) ***Reduced circuit***

Next, Fig. 4.135(a) is simplified into Fig. 4.135(b) and then into Fig. 4.135(c).

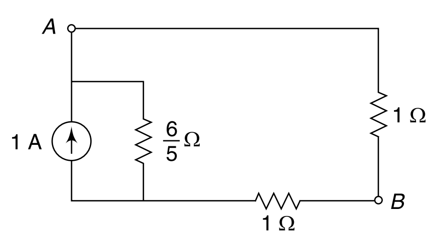

Fig. 4.135(b) ***Network reduction***

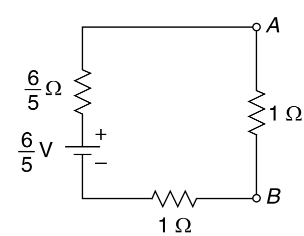

Fig. 4.135(c) ***Finally reduced circuit with 1 A source acting only***

The current through the 1 Ω resistor when the current source acts alone is given by

$$\frac{\frac{6}{5}}{\frac{6}{5}+1+1}\ \text{A} = 0.375\ \text{A (from } A \text{ to } B\text{)}$$

Next, considering the voltage source acting alone, the network in Fig. 4.134 transforms into Fig. 4.135(d).

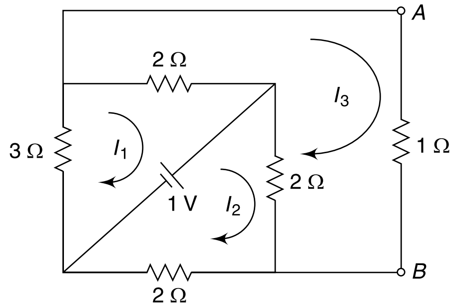

Fig. 4.135(d) ***Voltage source acting alone in circuit of Fig. 4.134***

Applying KVL in the three loops the following three equations are obtained:

$2(I_1 - I_3) + 1 + 3I_1 = 0$ (i)

$2(I_2 - I_3) + 2I_2 - 1 = 0$ (ii)

and $I_3 + 2(I_3 - I_2) + 2(I_3 - I_1) = 0$ (iii)

Solving these three equations, I_3 = 0.03125 A (from A to *B*).

Applying superposition theorem current through the 1 Ω resistor (when both the sources are acting simultaneously) is 0.375 + 0.03125 = 0.40625 A (from *A* to *B*).

4.105 Using the superposition theorem find the voltage across the 20 Ω resistor of the circuit shown in Fig. 4.136.

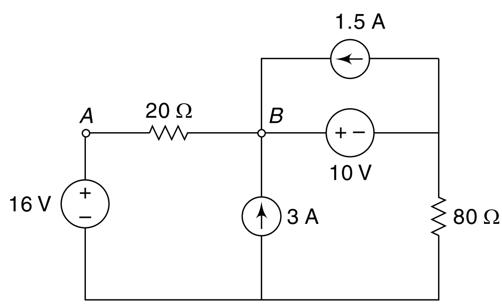

Fig. 4.136 ***Circuit of Ex. 4.105***

Solution

Let us consider that the 16 V source acts alone; removing the other sources the circuit configuration is shown in Fig. 4.136(a). The current through the 20 Ω resistor is

$$I_1 = \frac{16}{20+80}\text{ A } = 0.16 \text{ A from A to } B$$

Considering 10 V source acting alone the circuit is redrawn as shown in Fig. 4.136(b).

Current through the 20 Ω is $I_2 = \dfrac{10}{20+80}$ A

= 0.1 from *B* to *A*.

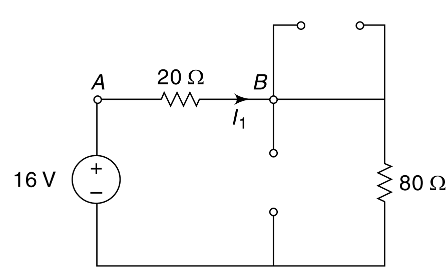

Fig. 4.136(a) ***16 V source acting alone***

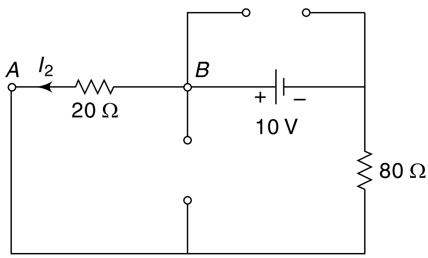

Fig. 4.136(b) ***10 V source acting alone***

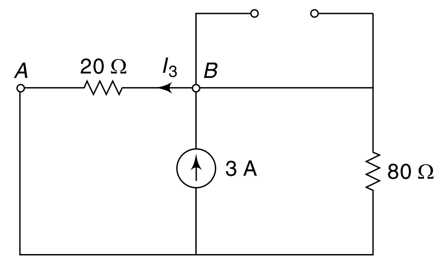

Fig. 4.136(c) ***3 A source is acting alone***

Next, considering 3 A source acting alone the corresponding circuit is shown in Fig. 4.136(c).

Current in the 20 Ω resistor is $I_3 = 3 \times \dfrac{80}{20+80}$ A $= 2.4$ A from B to A.

Considering the 1.5 A source acting alone the correspoding circuit is shown in Fig. 4.136(d).

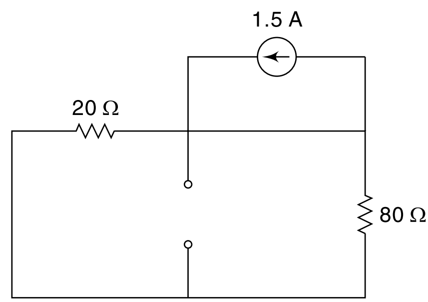

Fig. 4.136(d) ***1.5 A source acting alone***

As there is a short circuit path in parallel with 1.5 A current source, hence no current flows through 20 Ω resistor due to this source.

Applying superposition theorem, when all the sources are acting simultaneously the current through the 20 Ω resistor is $(I_2 + I_3 - I_1) = (0.1 + 2.4 - 0.16) = 2.34$ A from B to A. or voltage across the 20 Ω resistor is $2.34 \times 20 = 46.8$ V.

4.106 Determine R_L in Fig. 4.137 for maximum power transfer to the load.

Solution

The two-delta networks, one formed by 3 numbers of 6 Ω resistors and another by 3 numbers of 21 Ω resistors, are first converted into equivalent star network.

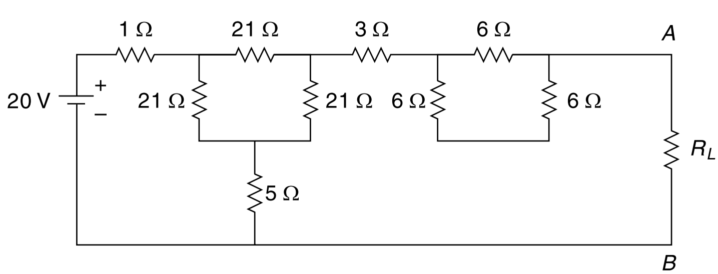

Fig. 4.137 ***Circuit of Ex. 4.106***

Here

$$R_1 = \frac{21 \times 21}{21+21+21}\,\Omega = 7\,\Omega$$

$$R_2 = \frac{6 \times 6}{6+6+6}\,\Omega = 2\,\Omega$$

The corresponding network is shown in Fig. 4.137(a).

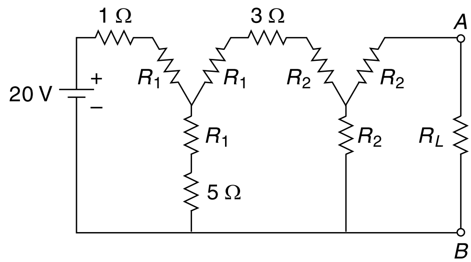

Fig. 4.137(a) *Circuit reduction*

The network shown in Fig. 4.137(a) can further be reduced to Fig. 4.137(b).

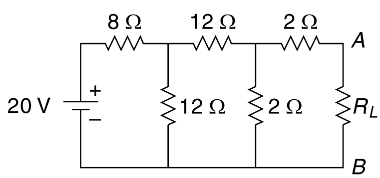

Fig. 4.137(b) ***Finally reduced circuit***

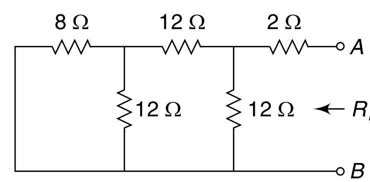

Fig. 4.137(c) ***Finding of (R_i)***

For maximum power transfer to the load R_L the value R_L should be equal to R_i which is equal to the internal resistance of the network. R_i can be found from Fig. 4.137(c) removing the source and open circuiting terminals A*B* (Fig. 4.137 (c)).

$$R_L = R_i = \left\{\left(\frac{8\times 12}{8+12}+12\right) \| 12\right\}+2$$

$$= \frac{16.8\times 12}{16.8+12}+2 \ = 9\ \Omega$$

4.107 Using the superposition theorem, find the current through R_L in the circuit shown in Fig. 4.138.

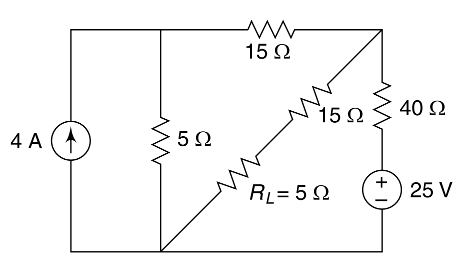

Fig. 4.138 ***Circuit of Ex. 4.107***

Solution

Converting the current source into equivalent voltage source the transformed network is shown in Fig. 4.138(a)

Considering the 20 V source acting alone, the circuit is shown in Fig. 4.138(b).

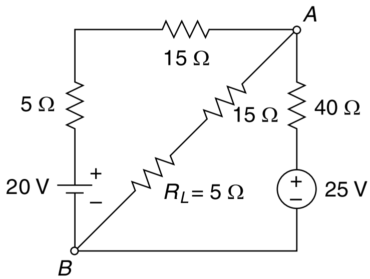

Fig. 4.138(a) *Conversion of source*

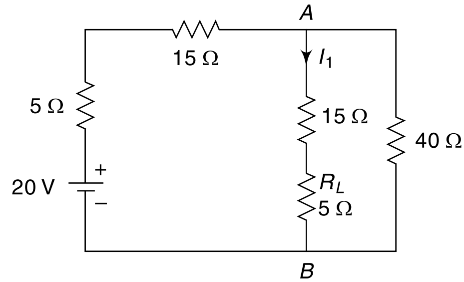

Fig. 4.138(b) *20 V source acting alone*

The current through R_L is

$$I_1 = \frac{20}{20 + \frac{20 \times 40}{20 + 40}} \times \frac{40}{40 + 20}$$

$$= \frac{20 \times 40}{20 \times 60 + 20 \times 40} = \frac{800}{1200 + 800} = \frac{8}{20} = 0.4 \text{ A (from } A \text{ to } B)$$

Considering the 25 V source acting alone from the circuit, is shown in Fig. 4.138(c), the current through (R_L) is

$$I_2 = \frac{25}{40 + \frac{20}{2}} \times \frac{1}{2} = \frac{25}{2 \times 50} = 0.25 \text{ A (from } A \text{ to } B).$$

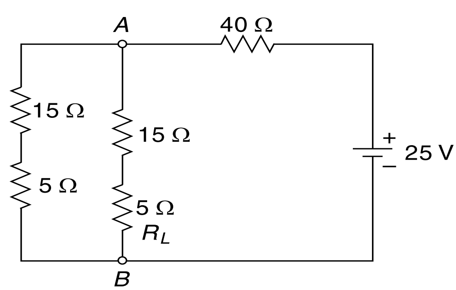

Fig. 4.138(c) *25 V source acting alone*

Applying the superposition theorem when both the sources are acting simultaneously the current through R_L is

$$I_1 + I_2 = 0.4 + 0.25 = 0.65 \text{ A (from } A \text{ to } B)$$

4.108 Find the current through the 2 Ω resistor as shown in Fig. 4.139 using the Superposition theorem.

Solution

Considering the 2 A source acting alone, the corresponding circuit is shown in Fig. 4.139(a).

Fig. 4.139(a) is redrawn in Fig. 4.139(b).

Now the current through the 2 Ω resistor is

$$I_1 = 2 \times \frac{1}{1 + 2 + \frac{4 \times 3}{4 + 3}} = \frac{2}{4.71} = 0.424 \text{ A (from } P \text{ to } A)$$

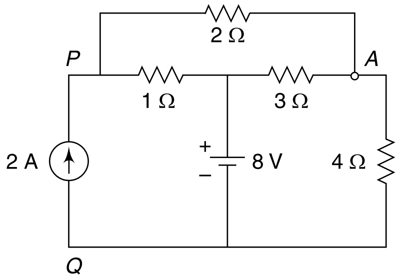

Fig. 4.139 *Circuit of Ex. 4.108*

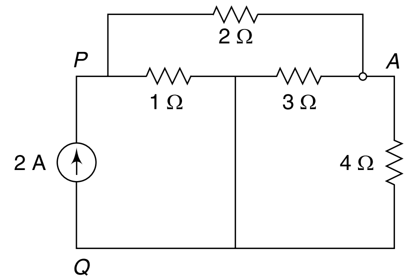

Fig. 4.139(a) *2 A source acting alone*

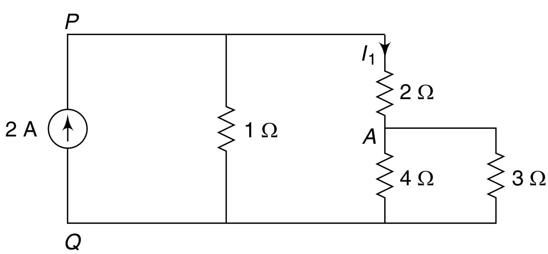

Fig. 4.139(b) *Simplified circuit*

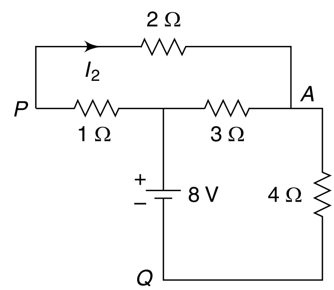

Fig. 4.139(c) *8 V Source acting alone*

Considering the 8 V source acting alone, the corresponding circuit is shown in Fig. 4.139(c).
Current through the 2 Ω resistor is

$$I_2 = \frac{8}{4+\frac{3}{2}} \times \frac{1}{2} = \frac{4}{5.5} = 0.727 \text{ A (from } P \text{ to } A)$$

Using superposition theorem, net current through 2 Ω resistor is $I_1 + I_2 = 0.424 + 0.727 =$ 1.151A.

4.109 Find the current through the 2 Ω resistor of Fig. 4.139 using Norton's theorem.

Solution

Let us short-circuit the terminals PA after removing the 2 Ω resistor. Now we consider the 2 A source acting alone the (corresponding circuit being shown in Fig. 4.140).

Figure 4.140 can be further reduced to the circuit shown in Fig. 4.140(a).
The short circuit current due to the 2 A source acting alone is

$$I_{sc1} = 2 \times \frac{1}{1+\frac{4\times 3}{4+3}} = \frac{2\times 7}{19} = \frac{14}{19} \text{ A}$$

(from P to A).

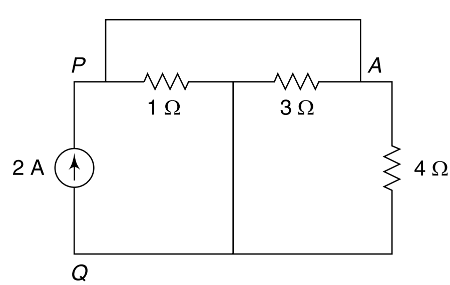

Fig. 4.140 *Circuit of Ex. 4.109*

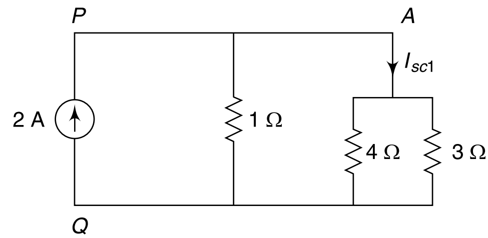

Fig. 4.140(a) *Reduced network with 2 A source acting alone*

Considering the 8 V source acting alone, the current through the short circuited path can be found from Fig. 4.140(b). Current through short circuited path due to the 8 V source acting alone is

$$I_{sc2} = \frac{8}{4 + \frac{3 \times 1}{3+1}} \times \frac{3}{3+1} = \frac{24}{19} \text{ (from } P \text{ to } A).$$

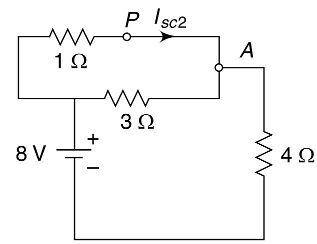

Fig. 4.140(b) *8 V source acting alone*

Applying the superposition theorem the current through the short circuited path when both the sources are acting simultaneously is

$$I_{sc} = \left(\frac{14}{19} + \frac{24}{19}\right) = 2 \text{ A}$$

Hence, Norton's equivalent current $I_N = 2$ A.

Now to find Norton's equivalent resistance R_N, all the sources are deactivated and open circuiting terminals *PA* the circuit configuration shown in Fig. 4.140(c) is obtained.

$$R_N = 1 + \frac{3 \times 4}{3+4} = 1 + \frac{12}{7} = \frac{19}{7} \,\Omega.$$

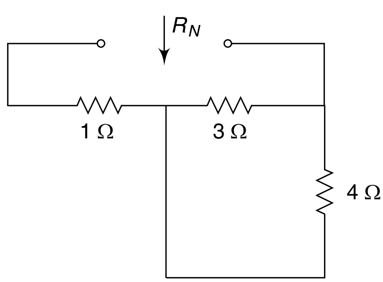

Fig. 4.140(c) *Determination of (R_N)*

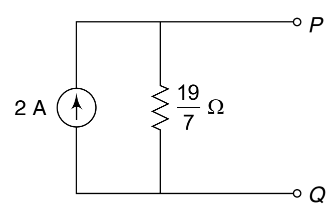

Fig. 4.140(d) *Norton's equivalent circuit of Ex. 4.109*

Norton's equivalent circuit is shown in Fig. 4.140(d).

The current through the 2 Ω resistor connected between terminals *P&A* using Norton's theorem is $2 \times \dfrac{\frac{19}{7}}{2 + \frac{19}{7}} = \dfrac{2 \times 19}{33}$ A $= 1.151$ A.

4.110 Find the value of V_R in the circuit shown in Fig. 4.141.

Solution

Let V_a be the voltage at node a. Applying KCL at node a

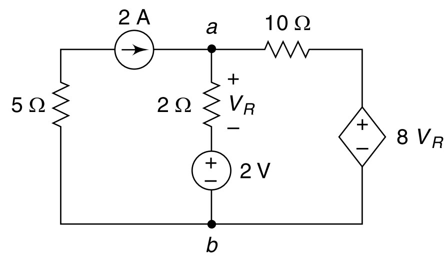

Fig. 4.141 ***Circuit of Ex. 4.110***

$$\frac{V_a - 2}{2} + \frac{V_a - 8V_R}{10} - 2 = 0$$

or $\quad 5V_a - 10 + V_a - 8V_R = 20$

or $\quad 6V_a - 8V_R = 30$

Again $\quad V_R = V_a - 2$ or, $V_a = V_R + 2$

Hence, $\quad 6(V_R + 2) - 8V_R = 30$

or $\quad -2V_R = 18$

or $\quad V_R = -9$ V [this means node "a" is of negative polarity.]

4.111 Applying kirchhoff's voltage law find the values of current i and the voltages v_1 and v_2 in the circuit shown in Fig. 4.142.

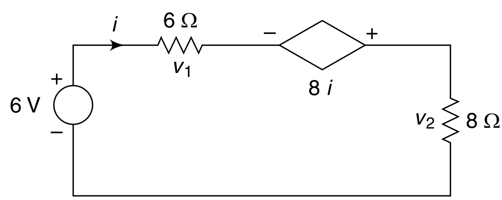

Fig. 4.142 ***Circuit of Ex. 4.111***

Solution

Applying Kirchhoff's voltage law in Fig. 4.142

$$6 - v_1 + 8i - v_2 = 0$$

or $\quad v_1 + v_2 = 6 + 8i$

Now $\quad v_1 = 6i \quad$ and $\quad v_2 = 8i$

Hence $\quad 6i + 8i = 6 + 8i$

or $\quad i = 1$

Therefore $\quad v_1 = 6 \times 1 = 6$ Volts and $v_2 = 8 \times 1 = 8$ V.

4.112 Applying KCL find the value of current i in the circuit shown in Fig. 4.143.

Solution

Applying KCL at node (x),

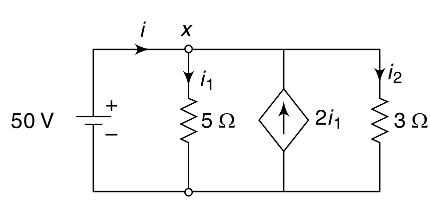

Fig. 4.143 ***Circuit of Ex. 4.112***

$$i - i_1 + 2i_1 - i_2 = 0$$

or $\quad i + i_1 - i_2 = 0$

$\because \quad i_1 = \dfrac{50}{5} = 10$ A and $i_2 = \dfrac{50}{3}$ A,

$$i + 10 - \frac{50}{3} = 0$$

or, $\quad i = \dfrac{50}{3} - 10 = \dfrac{20}{3} = 6.67$ A

4.113 Find i_1, i_2 and i_3 in Fig. 4.144.

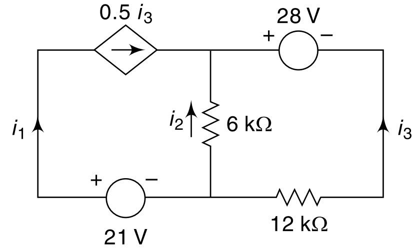

Fig. 4.144 *Circuit of Ex. 4.113*

Solution

Let us consider mesh currents i_x and i_y in the two meshes as shown in Fig. 4.144(a).

Applying loop equations in the two meshes

$$6 \times 10^3(i_x - i_y) - 21 = 0$$

and $$6 \times 10^3(i_y - i_x) + 12 \times 10^3 i_y + 28 = 0$$

or $$i_x = i_y + \frac{21}{6 \times 10^3} \quad \text{(i)}$$

and $$18 \times 10^3 i_y - 6 \times 10^3 i_x + 28 = 0 \quad \text{(ii)}$$

Solving these two equations

$$i_y = -0.583 \text{ mA} \quad \text{and} \quad i_x = 2.917 \text{ mA}$$

From Fig. 4.144(a) it is evident that

$$0.5 i_3 = 2.917 \text{ mA}$$

or $$i_3 = 5.834 \text{ mA.}$$

Applying KCL at node a

$$0.5 i_3 + i_2 + i_3 = 0$$

or $$i_2 + 1.5 i_3 = 0$$

or $$i_2 = -1.5 \times 5.834 = -8.751 \text{ mA}$$

Also, $$i_1 = 0.5 i_3 = 0.5 \times 5.834 = 2.917 \text{ mA.}$$

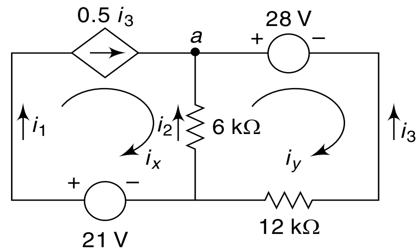

Fig. 4.144(a) *Network of Fig. 4.144 with mesh currents*

4.114 Find the power dissipated in the 100 Ω resistor and find the voltage rating of the dependent source in Fig. 4.145.

Solution

Applying KVL in the given figure,

$$6 - 500 i_o + 2 - 100 i_o = 0$$

or $$i_o = \frac{8}{600} = 13.33 \text{ mA.}$$

Power dissipated in the 100 Ω resistor = $(100) \times (0.0133)^2 = 17.7$ m Watts. Hence voltage rating of the dependent source is $500 \times i_o = 500 \times 0.0133 = 6.65$ V.

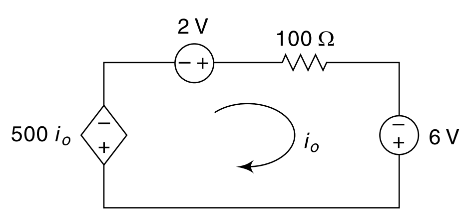

Fig. 4.145 *Circuit of Ex. 4.113*

4.115 Using node analysis find the value of α for the circuit shown in Fig. 4.146 when the power loss in the 1 Ω resistor is 9 W.

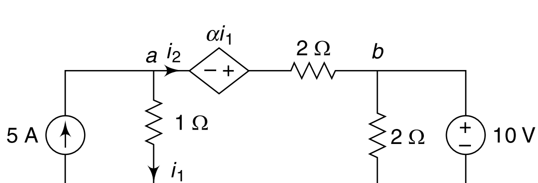

Fig. 4.146 *Circuit of Ex. 4.115*

Solution

Power loss in the 1 Ω resistor is

$$i_1^2 \times 1 = 9$$

or $$i_1 = 3 \text{ A}$$

and $$v_a = 3 \times 1 = 3 \text{ V}.$$

Applying KCL at node a

$$i_1 + i_2 = 5$$

or $$i_2 = 5 - i_1 = 5 - 3 = 2 \text{ A}$$

Also $$v_a + \alpha i_1 - 2i_2 = v_b$$

or $$3 + 3\alpha - 4 = v_b$$

Since $$v_b = 10 \text{ V}$$

hence $$3\alpha = 10 + 1 = 11$$

or $$\alpha = 3.67.$$

4.116 Find i_1 and i_2 in the circuit shown in Fig. 4.147 using superposition theorem.

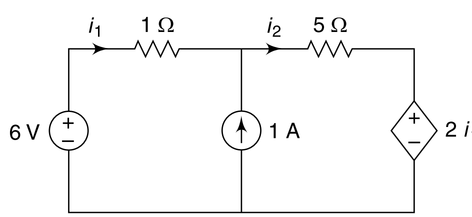

Fig. 4.147 ***Circuit of Ex. 4.116***

Solution

Considering 6 *V* source acting alone and removing the current source (as shown in Fig. 4.147(a)), we get

$$6 - 2i_1 = (1 + 5)i_1 \quad \text{or} \quad i_1 = \frac{6}{8} = \frac{3}{4} \text{ A}$$

Also $$i_2 = \frac{3}{4} \text{ A}$$

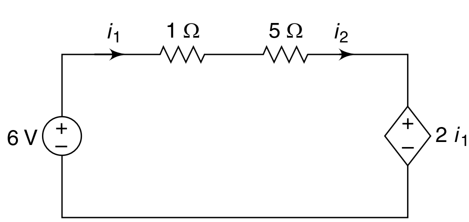

Fig. 4.147(a) ***6 V source acting alone***

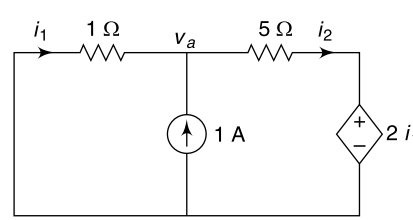

Fig. 4.147(b) ***1 A source acting alone***

Now, considering the 1 A current source acting alone and removing the others, from the corresponding circuit (shown in Fig. 4.147(b)), we have

$$1 + i_1 - \frac{v_a - 2i_1}{5} = 0$$

or $\quad 5 + 5i_1 - v_a + 2i_1 = 0$

or $\quad 7i_1 - v_a + 5 = 0$

$\because \quad \dfrac{v_a}{1} = -i_1,$

hence $\quad 7i_1 + i_1 + 5 = 0$

or $\quad i_1 = -\dfrac{5}{8}$ A

and $\quad i_2 = \dfrac{v_a - 2i_1}{5} = \dfrac{-i_1 - 2i_1}{5} = \dfrac{3}{5} \times \dfrac{5}{8} = \dfrac{3}{8}$ A

Applying superposition theorem when both the sources are acting simultaneously

$$i_1 = \frac{3}{4} - \frac{5}{8} = \frac{6-5}{8} = \frac{1}{8} = 0.125 \text{ A}$$

and $$i_2 = \frac{3}{4} + \frac{3}{8} = \frac{6+3}{8} = \frac{9}{8} = 1.125 \text{ A}$$

4.117 Find v in the circuit shown in Fig. 4.148 using superposition theorem.

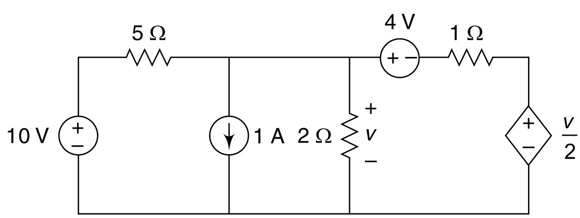

Fig. 4.148 *Circuit of Ex. 4.117*

Solution

Let us consider the 10 V source only removing the 1 A and 4 V source. The corresponding circuit is shown in Fig. 4.148(a) At node a,

$$\frac{v-10}{5} + \frac{v - \frac{v}{2}}{1} + \frac{v}{2} = 0$$

or $\quad 2v - 20 + 10v - 5v + 5v = 0$

or $\quad 12v = 20$ or, $v = 1.67$ V.

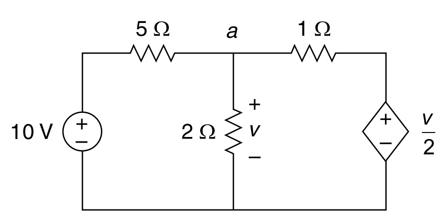

Fig. 4.148(a) *10 V source considered only*

Now, let us consider 1 A source acting alone. The corresponding figure is shown in Fig. 4.148(b).

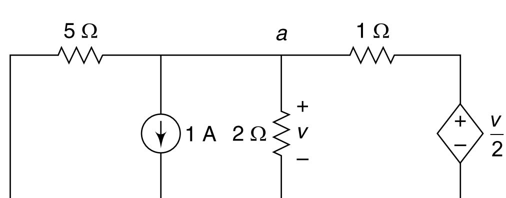

Fig. 4.148(b) *1 A source acting alone*

At node a,

$$\frac{v}{2}+\frac{v}{5}+1+\frac{v-\frac{v}{2}}{1} = 0$$

or $$v + 5 + 5v = 0$$

or $$v = -\frac{5}{6} = -0.833 \text{ V}$$

Finally let us consider 4 V source acting alone [as shown in Fig. 4.148(c)].

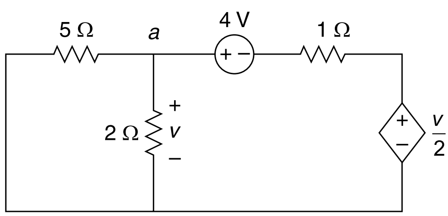

Fig. 4.148(c) *4 V source acting alone*

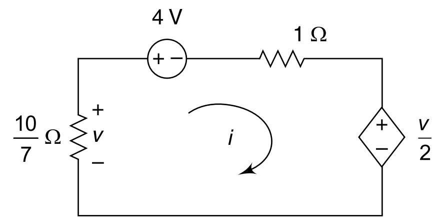

Fig. 4.148(d) *Simplified network with 4 V source*

Here 5 Ω and 2 Ω are in parallel. The transformed network is shown in Fig. 4.148(d).

In the circuit of Fig. 4.148(d),

$$v - 4 - 1 \times i - \frac{v}{2} = 0$$

or $$\frac{v}{2} - i = 4$$

Again, $$v = -\frac{10}{7} \times i$$

Hence $$-\frac{10i}{2\times 7} - i = 4$$

or $$i = -2.33$$

∴ $$v = \frac{10}{7} \times 2.33 = 3.33 \text{ V}$$

Using superposition theorem, when all the sources are acting simultaneously we have

$$v = 1.67 - 0.833 + 3.33 = 4.17 \text{ V}.$$

4.118 Find power loss in the 2 Ω resistor shown in Fig. 4.149 using superposition theorem.

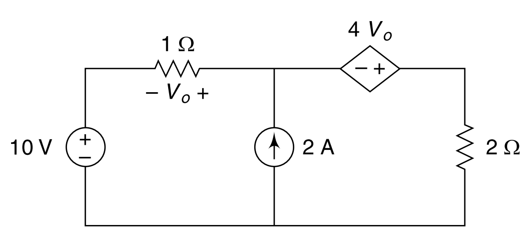

Fig. 4.149 *Circuit of Ex. 4.118*

Solution

Considering the 10 V source acting alone in the circuit [Fig. 4.149(a)] the loop equation

$$10 + V_o + 4V_o - 2.i = 0$$

or $\quad 5V_o - 2.i + 10 = 0$

Now $\quad 1 \times i = -V_o$

Hence $\quad 5(-i) - 2i + 10 = 0$

or $\quad -7i + 10 = 0$, i.e. $\quad i = \dfrac{10}{7}\text{ A} = 1.43\text{ A}$

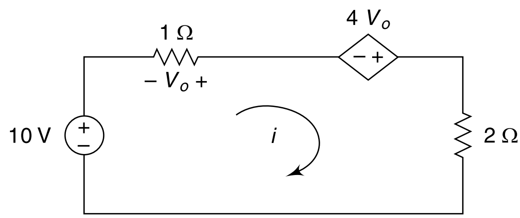

Fig. 4.149(a) ***10 V source acting alone***

Considering 2 A source acting alone [Fig. 4.149(b)] and applying KCL at node *a* we have

$$2 - \frac{v_a}{1} - \frac{v_a + 4V_o}{2} = 0$$

or $\quad 4 - 2\,v_a - v_a - 4\,V_o = 0$

or $\quad 4 - 3\,v_a - 4\,v_o = 0$

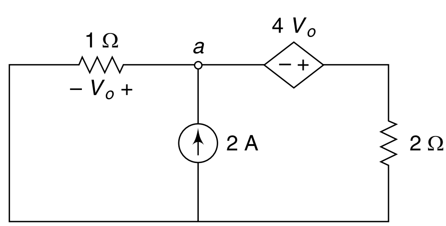

Fig. 4.149(b)   2 A source acting alone

Now, from the given figure, $v_a = V_o$

Hence from (i) $\quad 4 - 7\,V_o = 0$ i.e. $V_o = \dfrac{4}{7} = 0.57$ V

Current through 2 Ω resistor is

$$\frac{v_a + 4V_o}{2} = \frac{V_o + 4V_o}{2} = \frac{5}{2} \times 0.57 = 1.425\text{ A}$$

Applying superposition theorem the current through 2 Ω resistor is thus (1.43 + 1.425) A i.e 2.855 A.

Hence power loss in 2 Ω resistor is $(2.855)^2 \times 2 = 16.31$ W.

4.119 Solve Example 4.118 using Thevenin's theorem.

Solution

Removing 2 Ω resistor as shown in Fig. 4.150, the open circuit voltage is obtained and

$$V_{oc} = 10 + V_o + 4V_o = 10 + 5V_o$$

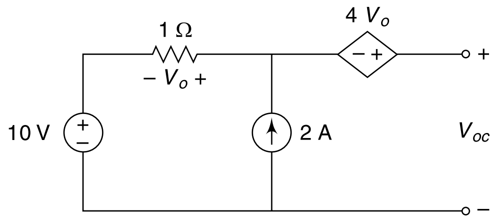

Fig. 4.150 ***Determination of V_{OC}***

However, V_o = voltage across the 1 Ω resistor
= 1 × current through the 1 Ω resistor
= 1 × 2 = 2 V

Hence V_{oc} = 10 + 5 × 2 = 20 V.

To find out R_{Th}, let us first short-circuit the output terminals as shown in Fig. 4.150(a).

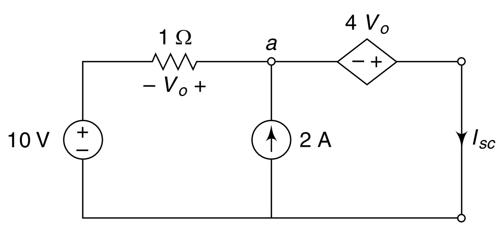

Fig. 4.150(a) ***Determination of I_{SC} and R_{Th}***

Applying KVL in the circuit,

$$10 + V_o + 4V_o = 0$$

or $$V_o = -\frac{10}{5} = -2$$

Applying nodal analysis at node *a*,

$$2 - \frac{V_o}{1} - I_{sc} = 0$$

or $$I_{sc} = 2 - \frac{-2}{1} = 4 \text{ A}$$

Hence $$R_{\text{Th}} = \frac{V_{oc}}{I_{sc}} = 5\ \Omega.$$

The Thevenin's equivalent circuit is shown in Fig. 4.150(b)

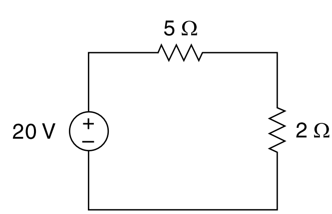

Fig. 4.150(b) ***Thevenin's equivalent circuit***

The current through 2 Ω resistor = $\frac{20}{7}$ A = 2.857 A.

Hence the power loss in the 2 Ω resistor is $(2.857)^2 \times 2 = 16.31$ W

4.120 Obtain Thevenin's equivalent circuit across terminals *a*–*b* in the Fig. 4.151.

Solution

The current through 1 kΩ resistor is $\left(\frac{10}{1000} + \frac{V_o}{2000}\right) A$

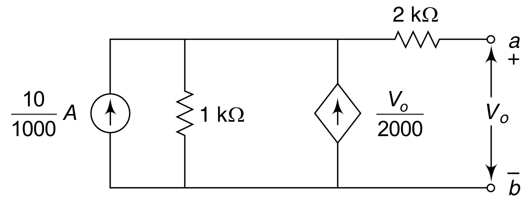

Fig. 4.151 *Circuit of Ex. 4.120*

Open circuit voltage across *a–b* is the voltage across the 1 kΩ resistor

Hence $$V_o = \left(\frac{10}{1000} + \frac{V_o}{2000}\right) \times 1000 = 10 + 0.5\ V_o$$

i.e . $V_o = 20$ V

To find out Thevenin's equivalent resistance (R_{Th}) let us short circuit terminals *ab* as shown in Fig. 4.151(a).

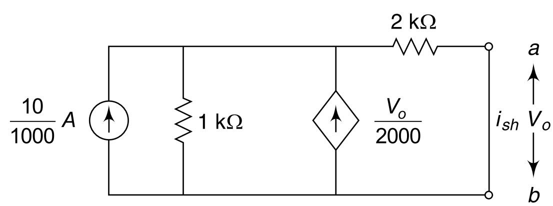

Fig. 4.151(a) *Determination of R_{Th}*

As *ab* is short-circuited V_o is zero. The network reduces to that shown in Fig. 4.151(b).

Hence $$i_{sh} = \frac{10}{1000} \times \frac{1}{1+2} = \frac{10}{3000}\ \text{A}$$

Therefore, $$R_{\text{Th}} = \frac{V_o}{i_{sh}} = \frac{20}{10} \times 3000 = 6000\ \Omega$$

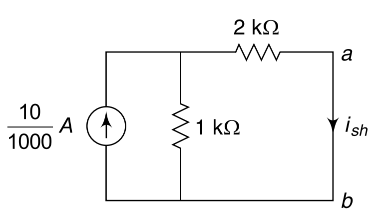

Fig. 4.151(b) *Reduced network of the circuit shown in Fig. 4.151(a)*

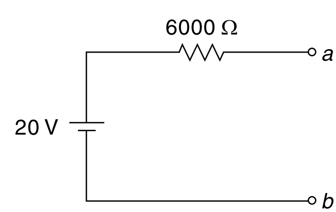

Fig. 4.151(c) *Thevenin's equivalent circuit of Ex. 4.120*

Thevenin's equivalent circuit is shown in Fig. 4.151(c).

4.121 Find the current in the 2 Ω resistor using Thevenin's theorem in the circuit shown in Fig. 4.152.

Solution

Let us remove the 2 Ω resistor. The corresponding figure is shown in Fig. 4.152(a).

Obviously the current supplied by the dependent current source $2i$ is zero.

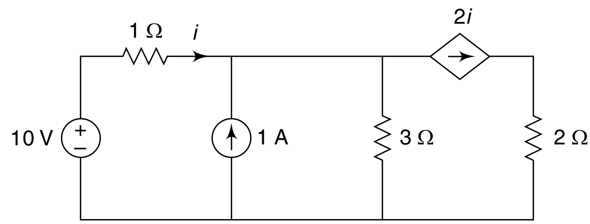

Fig. 4.152 *Circuit of Ex. 4.121*

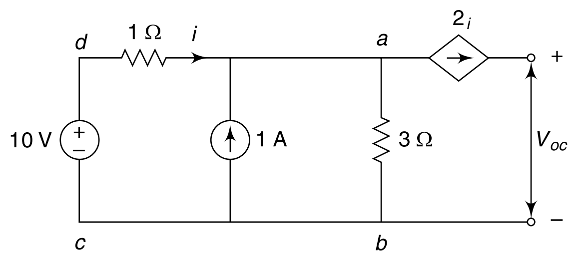

Fig. 4.152(a) ***Determination of V_{OC}***

Applying nodal method at node (a),

$$\frac{V_{oc}}{3} - 1 - i = 0$$

or $\quad V_{oc} = 3(i + 1) \quad$ (i)

Applying KVL in loop *abcd*

$$10 - i - V_{oc} = 0 \quad \{V_{oc} = \text{voltage across the } 3\ \Omega \text{ resistor}\}$$

or $\quad V_{oc} = 10 - i. \quad$ (ii)

Solving the two equations (i) and (ii),

$$3i + 3 = 10 - i$$

or $\quad 4i = 7$ or, $i = 1.75$ A

and $\quad V_{oc} = 10 - i = 8.25$ A.

To find out R_{Th}, terminals across the 2 Ω resistor are shorted as shown in Fig. 4.152(b).

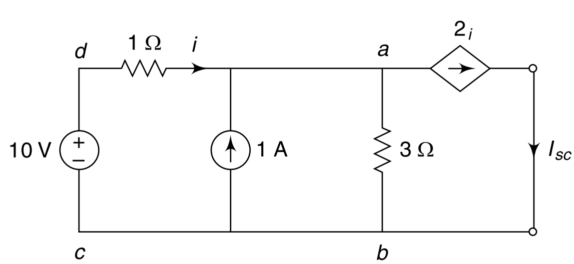

Fig. 4.152(b) ***Determination of R_{Th}***

$$I_{sc} = 2i$$

Applying the nodal method at node a,

$$i + 1 = \frac{v_a}{3} + 2i$$

where v_a is the potential at node (a) w.r.t node (b)

Hence $3i + 3 = v_a + 6i$

or $3i = 3 - v_a$ (iii)

Applying KVL in loop *abcd*

$$10 - i - v_a = 0$$

i.e. $v_a = 10 - i.$ (iv)

Solving the two equations (iii) and (iv),

$$3i = 3 - 10 + i$$

i.e. $2i = -7$ or, $i = -3.5$ A

Hence $I_{sc} = 2i = -7$

and $$R_{Th} = \left|\frac{V_{oc}}{I_{sc}}\right| = \frac{8.25}{7} = 1.18\ \Omega.$$

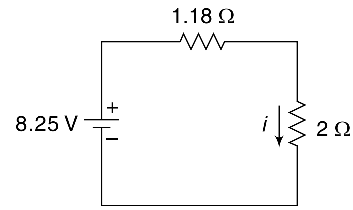

Fig. 4.152(c) ***Thevenin's equivalent circuit of Ex. 4.121***

The Thevenin's equivalent circuit is shown in Fig. 4.152(c).

Hence current in the 2 Ω resistor is $\dfrac{8.25}{2+1.18} = 2.59$ A.

4.122 Find the Norton's equivalent circuit for the transistor amplifier circuit shown in Fig. 4.153.

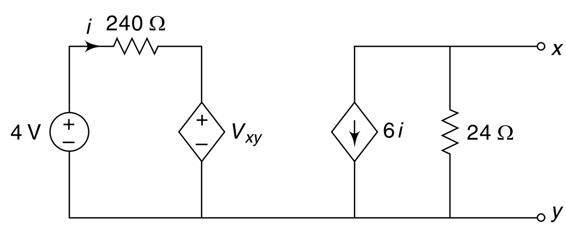

Fig. 4.153 ***Circuit of Ex. 4.122***

Solution

To find the Norton's equivalent current source (i_N) let us short circuit *xy*. The corresponding figure is shown in Fig. 4.153(a).

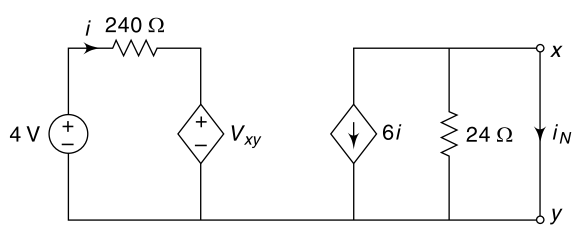

Fig. 4.153(a) ***x-y terminals shorted in the circuit of Fig. 4.153***

The voltage across short-circuited terminals *xy* is zero, i.e. $V_{xy} = 0$.

Hence $i = \dfrac{4}{240}$ A = 0.0167 A

and $i_N = -6i = -6 \times 0.0167 = -0.1002$ A (from *x* to *y*)

or $i_N = 0.1002$ (from *y* to *x*)

To find the Norton's equivalent resistance R_N let us find the open circuit voltage V_{xy} from Fig. 4.153.

$$V_{xy} = \text{Voltage drop across the 24 } \Omega \text{ resistor}$$
$$= -6i \times 24 = -144i$$

or
$$i = -\frac{V_{xy}}{144}$$

Again aplying KVL equation we find

$$4 - 240i - V_{xy} = 0$$

or
$$4 - 240\left(-\frac{V_{xy}}{144}\right) - V_{xy} = 0$$

or
$$V_{xy} = -6$$

Hence
$$R_N = \frac{V_{xy}}{i_N} = \frac{6}{0.1002} \simeq 60$$

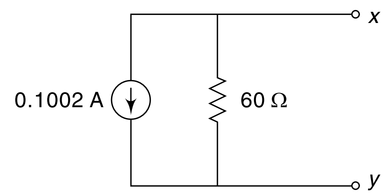

Fig. 4.153(b) ***Norton's equivalent circuit of Ex. 4.122***

The Norton's equivalent circuit is shown in Fig. 4.153(b).

4.123 Find the current through R_L in the circuit shown in Fig. 4.154 using Norton's theorem.

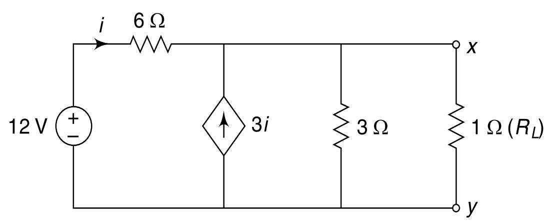

Fig. 4.154 ***Circuit of Ex. 4.123***

Solution

Let us short-circuit the terminals *xy* to find out the Norton's equivalent current (Fig. 4.154(a)).

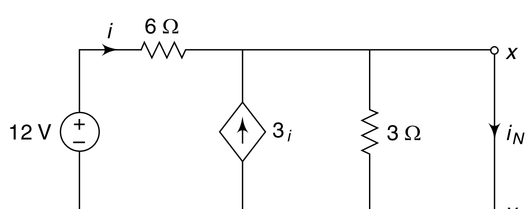

Fig. 4.154(a) ***Determination of i_N***

$$i_N = i + 3i = 4i$$

Now,
$$i = \frac{12}{6} A = 2 \text{ A}$$

Hence
$$i_N = 4 \times 2 = 8 \text{ A}$$

To find Norton's equivalent resistance R_N let us open circuit terminals *xy*. The corresponding circuit is shown in Fig. 4.154(b).

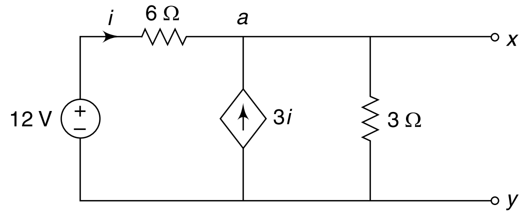

Fig. 4.154(b) *Determination of V_{xy}*

At node a,

$$i + 3i - \frac{V_{xy}}{3} = 0$$

$$V_{xy} = 12i$$

But, $$i = \frac{12 - V_{xy}}{6}$$

Hence $$V_{xy} = 12\,\frac{12 - V_{xy}}{6} = 24 - 2V_{xy}$$

$$3V_{xy} = 24 \text{ i.e. } V_{xy} = 8 \text{ V.}$$

Therefore $$R_N = \frac{8}{8} = 1\ \Omega.$$

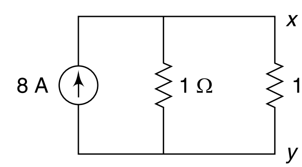

Fig. 4.154(c) *Norton's equivalent circuit of Ex. 4.123*

Norton's equivalent circuit is shown in Fig. 4.154(c).

Hence current through 1 Ω resistor $= 8 \times \dfrac{1}{1+1} = 4$ A.

4.124 Using maximum power transfer theorem find the value of the load resistance R_L so that the maximum power is transfered across R_L in the circuit shown in Fig. 4.155.

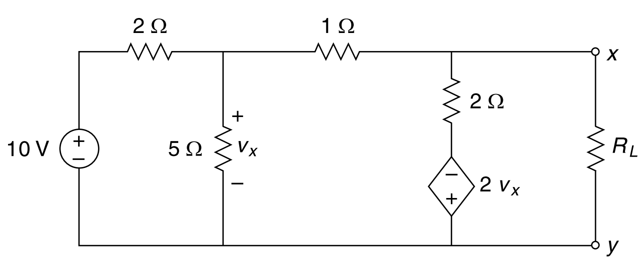

Fig. 4.155 *Circuit of Ex. 4.124*

Solution

Let us remove R_L and open circuit terminals xy to find out the internal resistance R_i of the circuit. According to maximum power transfer theorem the maximum power will be transfered through R_L when

$$R_L = R_i$$

From Fig. 4.155(a) applying KVL,

$$10 - 2i_1 - 5(i_1 - i_2) = 0$$

i.e. $$7i_1 - 5i_2 = 10 \qquad \text{(i)}$$

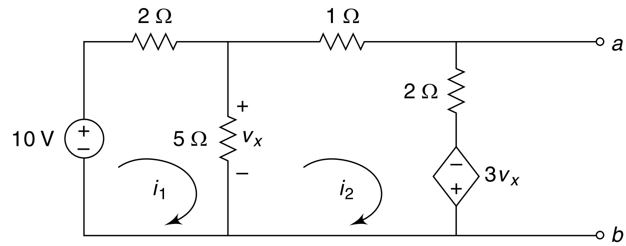

Fig. 4.155(a) *Determination of V_{ab}*

and $\quad 3v_x - 5(i_2 - i_1) - i_2 \times 1 - 2i_2 = 0$

i.e. $\quad 3 \times 5(i_1 - i_2) - 5(i_2 - i_1) - 3i_2 = 0 \quad [\because v_x = 5(i_1 - i_2)]$

or $\quad 20i_1 - 23i_2 = 0$ i.e., $i_1 = \dfrac{23}{20} i_2$ (ii)

Using equation (ii) in equation (i) we get

Hence $\quad 7 \times \dfrac{23}{20} i_2 - 5i_2 = 10$

or $\quad i_2 = 3.28$ A

and $\quad i_1 = 3.772$ A

$\therefore \quad v_x = 5(3.772 - 3.28) = 2.46$ V.

Now,
$$\begin{aligned} v_{ab} &= -3v_x + 2i_2 \\ &= -3 \times 2.46 + 2 \times 3.28 \\ &= -0.82 \text{ V.} \end{aligned}$$

Let us now short circuit the terminals *xy* as shown in Fig. 4.155(b).

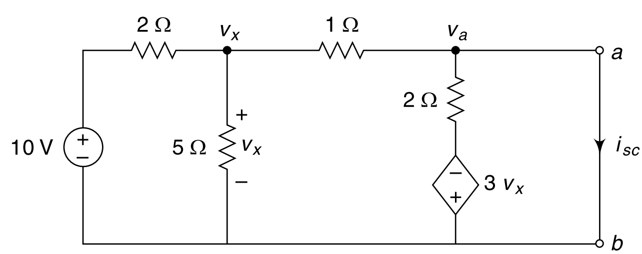

Fig. 4.155(b) *Determination of i_{SC}*

At node *a*,

$$\frac{v_a - v_x}{1} + \frac{v_a + 3v_x}{2} + i_{sc} = 0$$

As $\quad a - b$ are shorted $v_a = 0$

Hence $\quad -v_x + 1.5v_x + i_{sc} = 0$

i.e. $\quad i_{sc} = -0.5v_x$

Now, v_x = Voltage across 5 Ω resistor. Current through 5 Ω resistor

$$I_{5\Omega} = \frac{10}{2 + \dfrac{5 \times 1}{5 + 1}} \times \frac{1}{5+1} = \frac{60}{12+5} \times \frac{1}{6} = 0.588 \text{ A}$$

Hence $\quad v_x = 5 \times 0.588 = 2.94$ V

and $\quad i_{sc} = -0.5 \times 2.94 = -1.47$ A

Therefore, $\quad R_L = R_i = \dfrac{-0.82}{-1.47} = 0.558\ \Omega.$

4.125 Determine the resistance connected across terminals *a–b* which will transfer maxmum power across it in the circuit shown in Fig. 4.156.

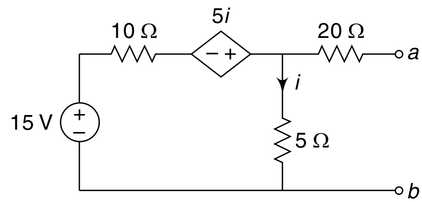

Fig. 4.156 ***Circuit of Ex. 4.125***

Solution

Applying KVL in the closed loop (Fig. 4.156), we have

$$15 - 10i + 5i - 5i = 0$$

or $$i = \frac{15}{10} = 1.5 \text{ A}$$

Hence $V_{ab} = 1.5 \times 5 = 7.5$ V

For finding out the internal resistance R_i of the circuit let us short circuit the path *ab* as shown in Fig. 4.156(a).

The mesh equations in the two loops are

$$-15 + 10i_1 - 5i + 5(i_1 - i_2) = 0$$

i.e., $-15i_1 + 5i_2 + 5i + 15 = 0$

and $20i_2 + 5(i_2 - i_1) = 0$

i.e. $5i_1 - 25i_2 = 0$

$\therefore$ $i_1 = 5i_2$

Also $i = i_1 - i_2$

Hence $-15(5i_2) + 5i_2 + 5(i_1 - i_2) + 15 = 0$

With $i_1 = 5i_2$ we get

$i_2 = 0.3$ A $(= i_{sh})$

$$\therefore \quad R_i = \frac{v_{ab}}{i_2} = \frac{7.5}{0.3} = 25\ \Omega$$

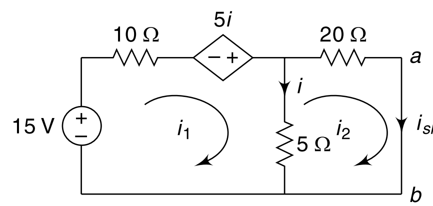

Fig. 4.156(a) ***Determination of R_i***

According to the maximum power transfer theorem maximum power will be transferred across *ab* when the resistance connected across *ab* is equal to R_i i.e., 25 Ω.

4.126

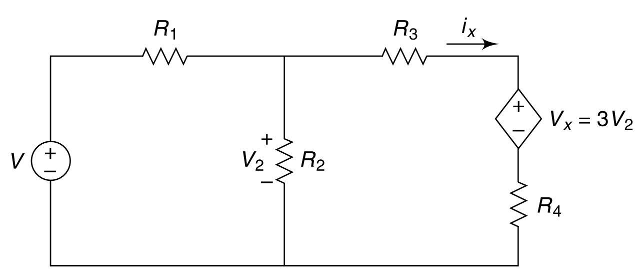

Fig. 4.157 ***Circuit of Ex. 4.126***

[*Given:* $R_1 = 4\ \Omega$, $R_2 = 6\ \Omega$; $R_3 = 2\ \Omega$; $R_4 = 10\ \Omega$, $V = 9$ V.]

In the circuit shown in Fig. 4.157, find i_x.

Solution

Let us first simplify the circuit by clubbing R_3 with R_4 to get $R(=R_3 + R_4)$ in the right loop (loop 2) in the given circuit. The simplified circuit is shown in Fig. 4.157(a).

In loop 2, $V_2 + V_x = i_xR = 12i_x$

or $V_2 + 3V_2 = 12i_x$

$\therefore$ $V_2 = 3i_x$ (1)

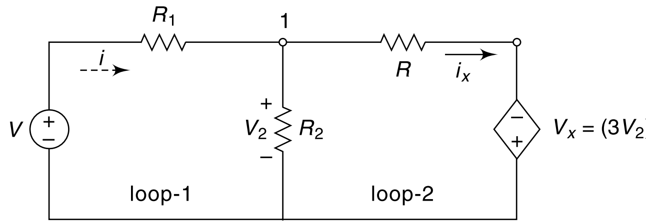

Fig. 4.157(a) ***Simplified circuit***

Again at loop 1,

$$V - iR_1 = V_2 \quad \text{[assuming current '}i\text{' passing through } R_1]$$

Using (1)

$$V - 4i = 3i_x \tag{2}$$

But $\quad i = i_x + \dfrac{V_2}{6}$ [applying KCL at node 1]

$$= i_x + \frac{3i_x}{6} \quad \text{[using (1)]}$$

$$= 1.5i_x$$

From (2) we then get

$$V - 4 \times 1.5i_x = 3i_x$$

or $\quad V = 9i_x$

$\therefore \quad i_x = \dfrac{V}{9} = \dfrac{9}{9} = 1 \text{ A.}$

4.127 In the circuit shown in Fig. 4.158, find i_o assuming $\beta = 8$. Use the superposition principle.

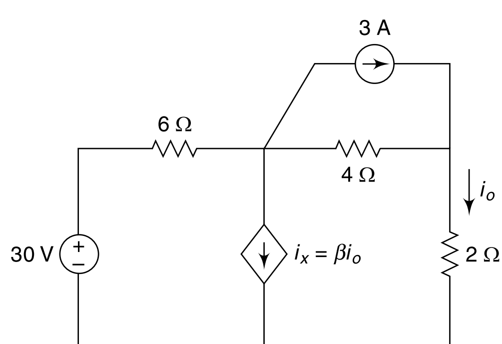

Fig. 4.158 ***Circuit of Ex. 4.127***

Solution

First we take the 30 V source [Ref. Fig. 4.158(a)]

Here, $\quad i_1 = i_x + i_{01} = 9i_{01}$ (1)

In loop *abcde*, $\quad -30 + 6 \times 9i_{01} + (4 + 2)\, i_{01} = 0$

$\therefore \quad i_{01} = \dfrac{1}{2} \text{ A} = 0.5 \text{ A}$

Next we consider the 3 A constant current source. [Ref. 4.158(b)].

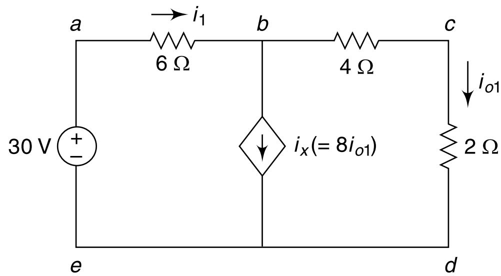

Fig. 4.158(a) *30 V source acting alone*

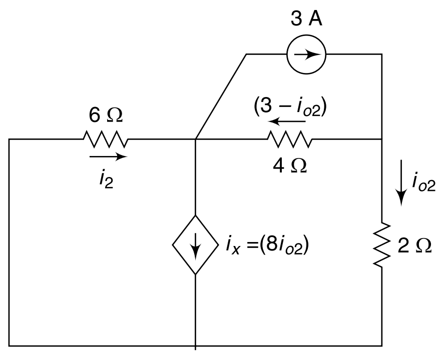

Fig. 4.158(b) *3 A source acting alone*

Current through the 4 Ω resistor is $(3 - i_{02})$ A, and current through the 6 Ω resistor is $i_2 = 9\ i_{02}$.

∴ We can write from loop equation,

$$i_2 \times 6 - (3 - i_{02}) \times 4 + i_{02} \times 2 = 0$$

as $\qquad 6 \times 9i_{02} - 12 + 6i_{02} = 0 \quad \therefore \quad i_{02} = \text{(since } i_2 = 9i_{02}\text{) A.}$

Then using superposition principle

$$i_0 = i_{01} + i_{02} = 0.5 + 0.2 = 0.7 \text{ A}$$

4.128 In the circuit shown in Fig. 4.159, find I.

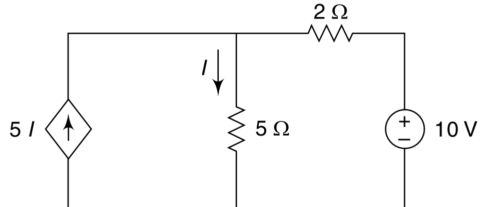

Fig. 4.159 *Circuit of Ex. 4.128*

Solution

We redraw the circuit with arbitrary current distribution and node number [Fig. 4.159(a)]

At node x,

$$5I = I + I_2$$

$$= \frac{V}{5} + \frac{V-10}{2}$$

[Assuming voltage at node x to be (V)]

$$\therefore \quad 5I = \frac{7V-50}{10}$$

or $$50I = 7V - 50 \quad (1)$$

But $$I = \frac{V}{5}.$$

$\therefore$ From (1), $10\ V = 7\ V - 50$

or $$V = -\frac{50}{3} = -16.67\ \text{V}$$

Thus $$I = \frac{V}{5} = -\frac{16.67}{5} = -3.34\ \text{A}$$

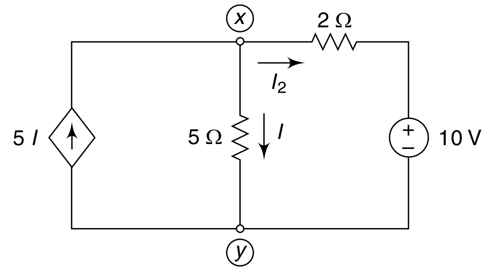

Fig. 4.159(a) *Circuit of Ex. 4.128 redrawn with currents and nodes designated*

The actual current I is directed upwards (i.e., towards x from y) in Fig. 4.159(a).

4.129 In the network shown in Fig. 4.160, find the value of the dependent source using (i) nodal method and (ii) superposition theorem.

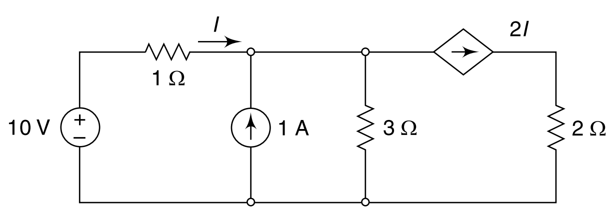

Fig. 4.160 *Circuit of Ex. 4.129*

Solution

Using nodal method: We first redraw the figure and assign a node x for application of nodal method (Fig. 4.160(a))

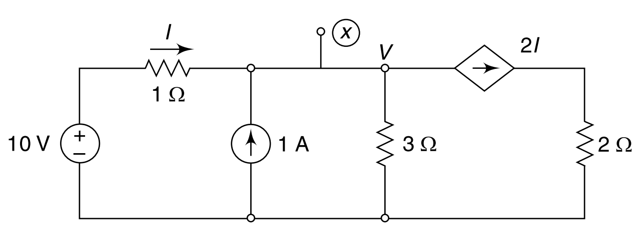

Fig. 4.160(a) *Solution by nodal method*

At the node x, we have, $\dfrac{10-V}{1} + 1 = \dfrac{V}{3} + 2I$

[(V) being the node voltage at x]

Since $I = \dfrac{10-V}{1}$, we can write, $\dfrac{10-V}{1}+1=\dfrac{V}{3}+2\left(\dfrac{10-V}{1}\right)$

Simplification yields V = 13.5 V.

Thus, $I = \dfrac{10-13.5}{1} = -3.5$ and the dependent source would have value $2I$, i.e., $2 \times (-3.5)$ i.e., –7 A.

It may be noted here that the actual direction of the currents I and $2I$ would be just the reverse than given in the question.

Using Superposition Theorem

Let us first assume the 10 V source only (Fig. 4.160(b)).

At node a, we find

$$\frac{10-V}{1} = \frac{V}{3}+2\left(\frac{10-V}{1}\right)$$

Simplifying,

$$V = 15 \text{ Volts (at node } a)$$

$$\therefore \quad I_1 = \frac{10-V}{1} = \frac{10-15}{1} = -5 \text{ A}$$

while $\quad 2I_1 = -10$ A

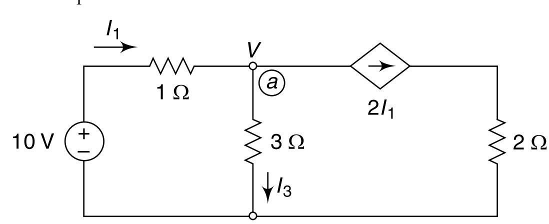

Fig. 4.160(b) ***Solution by superposition method (10 V source acting only)***

Next we consider the constant current source 1 A only (Fig. 4.160(c)).

We select node b where we find

$$1 + I_2 = \frac{V}{3} + 2I_2$$

or $\quad \dfrac{V}{3} + I_2 = 1 \qquad$ (i)

But $\quad I_2 = \dfrac{-V}{1} \qquad$ (ii)

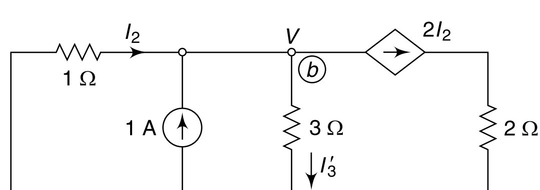

Fig. 4.160(c) ***1 A source acting only***

Using (ii) in (i), we get,

$$\frac{V}{3} - V = 1, \quad \text{i.e.,} \quad V = -1.5 \text{ V}.$$

$\therefore$ $$I_2 = 1.5 \text{ A}; \ 2I_2 = 3 \text{ A}.$$

Finally, using the principle of superposition, we get

$$2I = 2I_1 + 2I_2 = -10 + 3 = -7 \text{ A}$$

(the same result that we obtained earlier).

4.130 Find Thevenin's equivalent for the given circuit in Fig. 4.161.

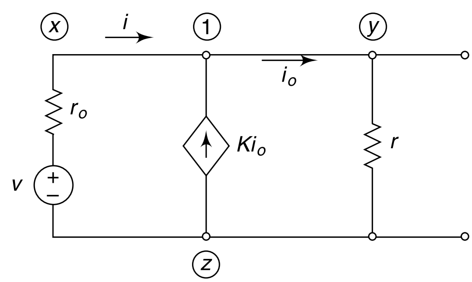

Fig. 4.161 ***Circuit of Ex. 4.130***

Solution

At node $\frac{60}{7}$W, we can write,

$$i + Ki_o = i_o$$

$$i = i_o(1 - K)$$

Also, in loop $x - 1 - y - z$ we can write

$$-v + ir_o + i_o r = 0$$

or $$v = r_o i_o (1 - K) + i_o r \quad [\text{using } i = i_o(1 - K) \text{ for } i]$$

$$= i_o[r_o(1 - K) + r]$$

$\therefore$ $$i_o = \frac{v}{r_o(1-K)+r}$$

Hence $$V_{o/c} = i_o \times r = \frac{v \times r}{r_o(1-K)+r}$$

Let us now short the output terminals. Resistance r is bypassed [Fig. 4.161(a)]

Here, $$-v + ir_o = 0$$

or $$i = \frac{v}{r_o}.$$

However, $$i = i_{s/c} - Ki_{s/c}, \text{ at node } \tfrac{60}{7}\text{W}$$

or $$\frac{v}{r_o} = i_{s/c}(1 - K)$$

$\therefore$ $$i_{s/c} = \frac{v}{r_o(1-K)}.$$

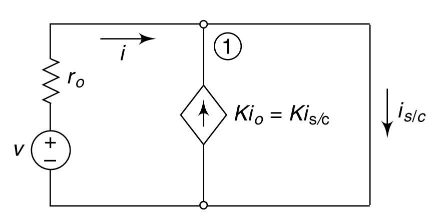

Fig. 4.161(a) ***Determination of i_{SC}***

$\therefore$ z_{Th} (Thevenin equivalent impedance) $= \dfrac{V_{o/c}}{i_{s/c}}$

$$= \frac{v \times r}{r_o(1-K)+r} \Bigg/ \frac{v}{r_o(1-K)}$$

$$= \frac{r \times r_o(1-K)}{r+r_o(1-K)}.$$

Thus, for the given circuit,

$$V_{o/c} = \frac{v \times r}{r_o(1-K)+r}; Z_{th} = \frac{r \times r_o\,(1-K)}{r+r_o\,(1-K)}.$$

4.131 Obtain the values of I_1, I_2 and I_3 in the circuit shown in Fig. 4.162.

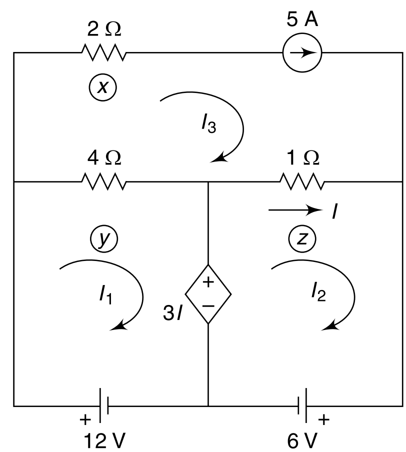

Fig. 4.162 *Circuit of Ex. 4.131*

Solution

In loop (x), the circulating current will be driven by 5 A constant current source and hence $I_3 = 5$ A.

In loop (y), we write

$$(I_1 - I_3)\,4 + 3I - 12 = 0 \qquad \text{(i)}$$

But $\quad I = I_2 - I_3$ (in loop (z))

∴ From (i), $\quad (I_1 - I_3)\,4 + 3(I_2 - I_3) - 12 = 0$

or $\quad 4I_1 + 3I_2 - 7I_3 = 12 \qquad$ (ia)

Since $I_3 = 5$ A, equation (1a) gives

$$4I_1 + 3I_2 - 7 \times 5 = 12$$

or $\quad 4I_1 + 3I_2 = 47 \qquad$ (ii)

For loop-(z), we write

$$1 \times I + 6 - 3I = 0 \quad \text{or,} \quad I = 3 \text{ A}$$

Using $I = 3$ A, $I = I_2 - I_3$, we get

$$I_2 - I_3 = 3 \quad \text{or,} \qquad I_2 = I_3 + 3 = 8 \text{ A} \quad (\because I_3 = 5 \text{ A})$$

Thus from (ii), $\quad 4I_1 = 47 - 3I_2 = 23$ A

or $\quad I_1 = 5.75$ A

Thus finally, $\quad I_1 = 5.75$ A, $\quad I_2 = 8$ A, $\quad I_3 = 5$ A.

4.132 Find v_a and v_b using the principle of superposition in Fig. 4.163.

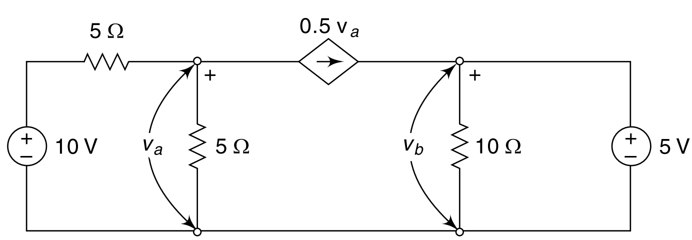

Fig. 4.163 *Circuit of Ex. 4.132*

Solution

Let us first take 10 V source (Fig. 4.163(a)). It may be observed that the 10 Ω resistor is shorted due to deactivation of the 5 V source.

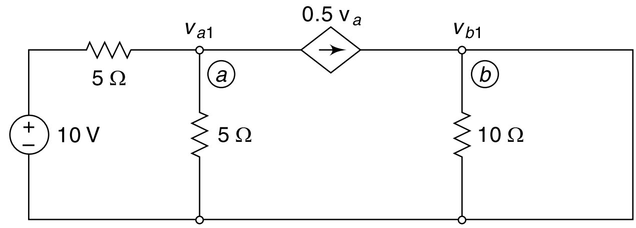

Fig. 4.163(a) ***10 V source acting alone***

At node a, we have

$$\frac{10 - v_{a_1}}{5} = \frac{v_{a_1}}{5} + 0.5\, v_{a_1}$$

or $$2 - \frac{v_{a_1}}{5} = \frac{v_{a_1}}{5} + 0.5\, v_{a_1}$$

or $$0.9\, v_{a_1} = 2$$

$\therefore$ $$v_{a_1} = \frac{2}{0.9} = \frac{20}{9} \text{ V}$$

Obviously $$v_{b_1} = 0.$$

Next, we deactivate the 10 V source and activate the 5 V source. (Fig. 4.163(b))

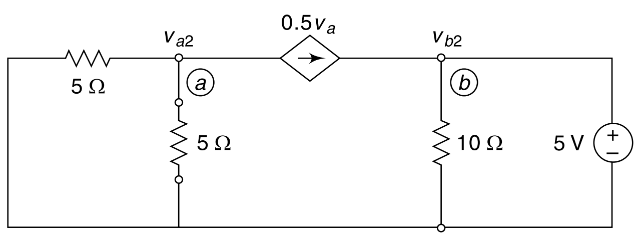

Fig. 4.163(b) *5 V source acting alone*

At node a, we have

$$\frac{v_{a_2}}{5} + \frac{v_{a_2}}{5} + 0.5\, v_{a_2} = 0$$

$\therefore$ $$v_{a_2} = 0.$$

v_{b_2} is then 5 V.

Using superposition principle,

$$v_a = v_{a_1} + v_{a_2} = \frac{20}{9} = 2.22 \text{ V}$$

$$v_b = v_{b_1} + v_{b_2} = 5 \text{ V.}$$

4.133 Obtain Thevenin equivalent across x-y in the network shown in Fig. 4.164.

Solution

$$V_{o/c} = V_{xy} = 10(v - v_o) = v$$

$\therefore$ $$9\, v = 10\, v_o \quad \text{or} \quad v = \frac{10}{9} v_o \qquad \text{(i)}$$

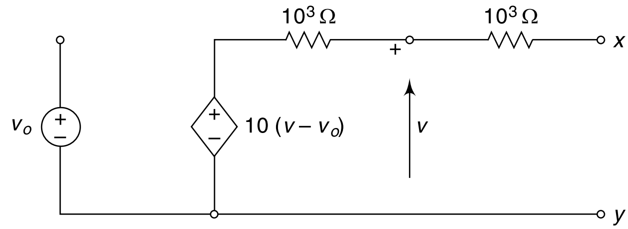

Fig. 4.164 *Circuit of Ex. 4.133*

Thus, $$V_{o/c} = V_{xy} = \frac{10}{9} v_o$$

Next we short terminals *x-y*. (Fig. 4.164(a)).

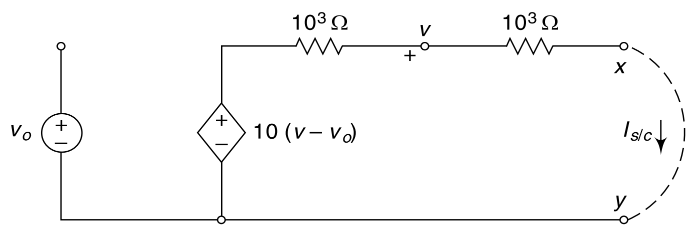

Fig. 4.164(a) *Network of Fig. 4.164 with (x-y) shorted*

Here,

$$10(v - v_o) = 10^3 \times 2 \times I_{s/c}$$

$$\therefore \quad I_{s/c} = \frac{10\,(v - v_o)}{2 \times 10^3} = \frac{v - v_o}{200} \qquad \text{(ii)}$$

Also, $$v = 10^3 \times I_{s/c} = 1000\, I_{s/c} \qquad \text{(iii)}$$

Using (iii) in (ii) we get

$$I_{s/c} = \frac{1000\, I_{s/c} - v_o}{200} = 5\, I_{s/c} - \frac{v_o}{200}$$

$$\therefore \quad 4I_{s/c} = \frac{v_o}{200}, \quad \text{i.e.,} \quad I_{s/c} = \frac{v_o}{200} \times \frac{1}{4}$$

We now obtain the internal resistance of the circuit as

$$R_{\text{int}} = \frac{V_{o/c}}{I_{s/c}} = \frac{\frac{10}{9} v_o}{\frac{v_o}{800}} = \frac{10}{9} \times 800 = 888.89\ \Omega$$

The Thevenin's equivalent circuit is thus obtained as

$$V_{o/c} = \frac{10}{9} v_o\,;\ R_{\text{int}} = 888.89\ \Omega.$$

4.134 In the network shown in Fig. 4.165 find *I* using Thevenin theorem and verify the result using Norton's theorem.

Solution

Let us first remove 1 Ω resistor from the given network. With reference to Fig. 5.165(a), we can write, $-10 + V_{o/c} + V_1 = 0$

$$\therefore \quad V_{o/c} = 10 - V_1$$

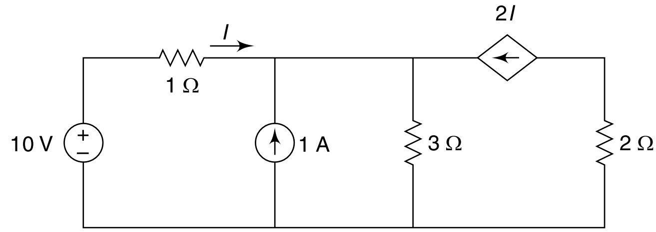

Fig. 4.165 *Circuit of Ex. 4.165*

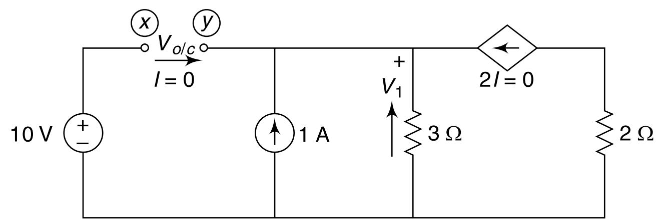

Fig. 4.165(a) *Determination of $V_{O/C}$*

By inspection, $V_1 = 1\text{ A} \times 3\ \Omega = 3\text{ V}.$

$\therefore$ $V_{o/c} = 7\text{ V}.$

Next, we short terminals (x)-(y) [Ref Fig. 4.165(b)]

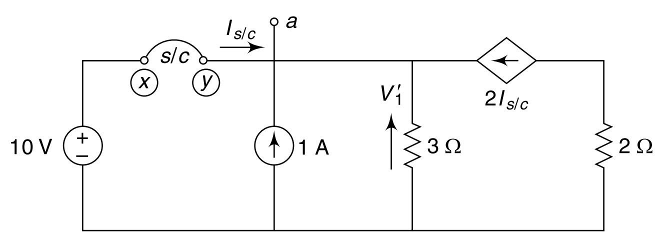

Fig. 4.165(b) *Determination of $I_{S/C}$*

By inspection, $V_1' = 10\text{ V}.$

At node a, we can write

$$I_{s/c} + 1 = \frac{10}{3} - 2\,I_{s/c} \text{ i.e., } I_{s/c} = \frac{7}{9}\text{ A}$$

Thus, the internal resistance of the circuit is

$$R_{\text{int}} = \frac{V_{o/c}}{I_{s/c}} = \frac{7}{(7/9)} = 9\ \Omega.$$

We now draw both Thevenin's and Norton's equivalent circuits. (Fig. 4.165(c) and 4.165(d) respectively).

In Fig. 4.165(c),

$$I = \frac{V_{o/c}}{R_{\text{int}} + 1} = \frac{7}{9+1} = 0.7\text{ A}$$

Thus we have obtained current in 1 Ω resistor by Thevenin's theorem which is 0.7 A.

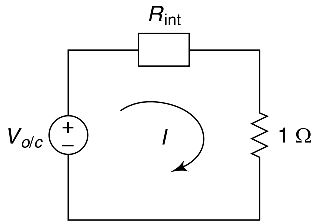

Fig. 4.165(c) ***Thevenin's equivalent network***

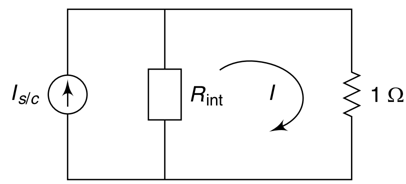

Fig. 4.165(d) ***Norton's equivalent network***

If we apply Norton's theorem, from equivalent circuit of Fig. 4.165(d), we get

$$I = I_{s/c} \times \frac{R_{int}}{R_{int}+1} = \frac{7}{9} \times \frac{9}{9+1}$$
$$= 0.7 \text{ A.}$$

Thus the current obtained in the 1 Ω resistor using Thevenin's theorem has been verified by Norton's theorem.

4.135 Find i and i' the circuit shown in Fig. 4.166. Use node analysis.

Solution

Let nodal voltage at nodes x and x' be V_x and V_x'.

At node x we can write

$$\frac{V_x}{2} + \frac{V_x - V_x'}{1} + \frac{V_x - 8}{2} + 1 = 0$$

[Please note here that electrically x, y, z nodes are at the same potential and hence treated as one common node x; node 0 is taken as references node].

or $\quad 2V_x - V_x' = 3 \quad$ (1)

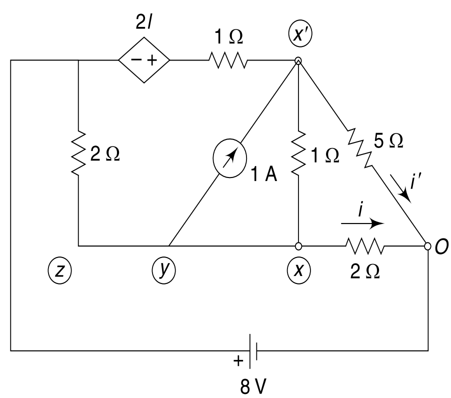

Fig. 4.166 ***Circuit of Ex. 4.135***

At node x' we can write

$$\frac{V_x' - V_x}{1} + \frac{V_x' - 2i - 8}{1} - 1 + \frac{V_x'}{5} = 0$$

or $\quad 2.2V_x' - V_x - 2i - 9 = 0 \quad$ (ii)

But $\quad i = \dfrac{V_x}{2}$

∴ We can write from (ii)

$$2.2V_x' - V_x - 2 \times \frac{V_x}{2} - 9 = 0$$

or $\quad -2V_x + 2.2V_x' = 9 \quad$ (iia)

Solving for (i) and (iia), $V_x = 10$ V and $V_x' = 17$ V.

∴ $\quad i = \dfrac{V_x}{2} = 5$ A ; $\quad i' = \dfrac{V_x'}{5} = 3.4$ A

4.136 In the network shown in Fig. 4.167, verify, using Thevenin theorem

$$V_{o/c} = \frac{Vr_1}{r_1 + r_2 + r_3}$$

$$I_{s/c} = \frac{V}{r_2 - r_3\,(\alpha - 1)}$$

$$z_{\text{th}} = \frac{r_1[r_2 - r_3(\alpha - 1)]}{r_1 + r_2 + r_2}$$

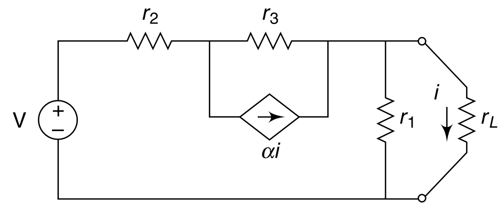

Fig. 4.167 *Circuit of Ex. 4.136*

Solution

With (r_L) opened, $i = 0$. This makes source $\alpha i = 0$.

$\therefore$ $V_{o/c}$ (i.e., voltage across open circuited output after removing r_L) is given by

$$V_{o/c} = \frac{r_1 \times V}{r_1 + r_2 + r_3}$$

Next we apply short across the output so that (r_L) and r_1 are shorted [Fig. 4.167(a)]. $\therefore$ Current through r_2 is $i_{s/c}$; current through r_3 is ($i_{sc} - \alpha i_{sc}$). Use of KVL in this loop gives

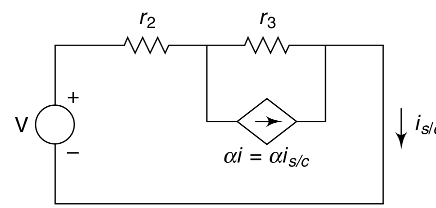

Fig. 4.167(a) *Determination of ($i_{S/C}$) and (Z_{Th})*

$$-V + i_{s/c}\, r_2 + (i_{s/c} - \alpha i_{s/c})r_3 = 0$$

or $$V = i_{s/c}\,(r_2 + r_3 - \alpha r_3)$$

$\therefore$ $$i_{s/c} = \frac{V}{r_2 + r_3 - \alpha r_3} = \frac{V}{r_2 - r_3\,(\alpha - 1)}$$

and $$z_{\text{Th}} = \frac{V_{o/c}}{i_{s/c}} = \frac{r_1 \times V}{r_1 + r_2 + r_3} \Bigg/ \frac{V}{r_2 - r_3(\alpha - 1)}$$

$$= \frac{r_1\,[r_2 - r_3(\alpha - 1)]}{(r_1 + r_2 + r_3)}$$

4.137 In the network shown in Fig. 4.168, find the current through R_L. Assume $V = 5$ V.

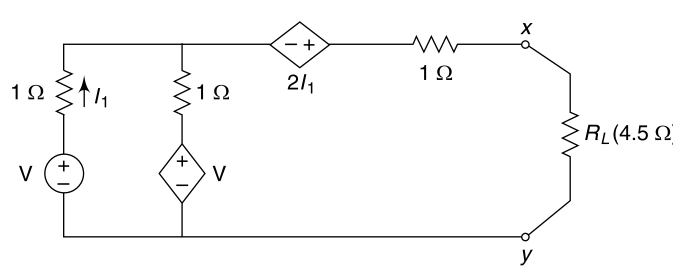

Fig. 4.168 *Circuit of Ex. 4.137*

Solution

Let us remove R_L from *x-y* terminals. In Fig. 4.168(a), in loop *abcd*,

$$-V + I_1 \times 1 + I_1 \times 1 + V = 0$$

or $$I_1 = 0.$$

Also, $$2I_1 = 0.$$

$\therefore$ $$V = V_{o/c}$$

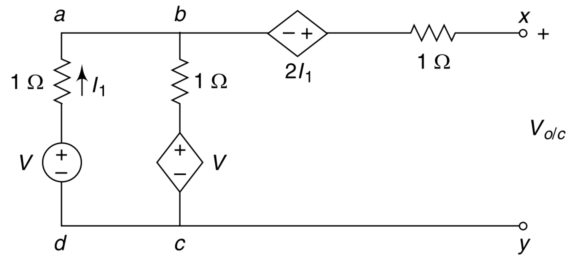

Fig. 4.168(a) *Determination of* $(V_{o/c})$

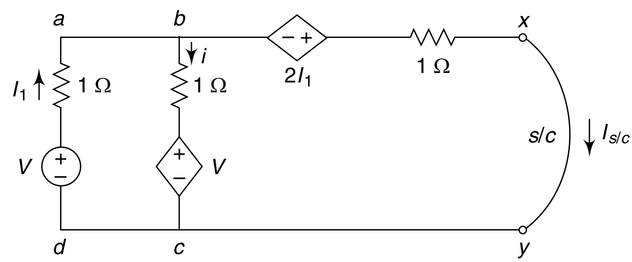

Fig. 4.168(b) *Determination of* $I_{s/c}$

Next, we short terminal *x-y* (Fig. 4.168(b))

At node *b*,

$$I_1 = i + I_{s/c}$$

$$i = I_1 - I_{s/c}$$

Again in loop *ab xy cda* we have

$$-V + 1 \times I_1 - 2I_1 + 1 \times I_{s/c} = 0$$

$\therefore$ $I_{s/c} = I_1 + V.$

Also, in loop *abcd*

$$-V + 1 \times I_1 + 1 \times i + V = 0$$

or $I_1 + (I_1 - I_{s/c}) = 0$

or $2I_1 - I_{s/c} = 0$

$\therefore$ $I_{s/c} = 2I_1 = 2(I_{s/c} - V) \quad [\because I_{s/c} = I_1 + V]$

i.e., $I_{s/c} = 2$ V.

Next, we find the internal resistance as

$$R_{\text{int}} = \frac{V_{o/c}}{I_{s/c}} = \frac{V}{2V} = \frac{1}{2}\,\Omega \ \ (= 0.5\ \Omega)$$

The Thevenin's equivalent circuit can now be drawn as shown in Fig. 4.168(c).

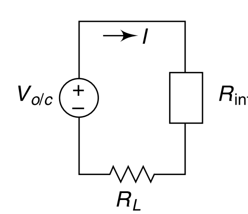

Fig. 4.168(c) *Determination of* $(V_{o/c})$

Here $$I = \frac{V_{o/c}}{R_{\text{int}} + R_L} = \frac{5}{4.5 + 0.5}$$

$$= 1 \text{ A.}$$

Thus, the current through 4.5 Ω resistor in the given circuit is 1 A.

4.138 Find (I) using Thevenin's theorem and find the value of R_L to have maximum power transfer from source (Fig. 4.169). Also find the maximum value of power transfer.

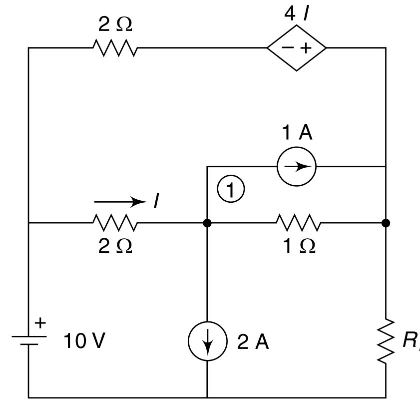

Fig. 4.169 *Circuit of Ex. 4.138*

Solution

Let V_x be the voltage at node $\frac{60}{7}$W when we remove R_L from the given circuit. The circuit is redrawn in Fig. 4.169 (a). Using node analysis at node $\frac{60}{7}$W we get

$$\frac{V_x - 10}{2} + 2 + 1 + \frac{V_x - V_{o/c}}{1} = 0$$

or $\quad 1.5\ V_x - V_{o/c} = 2 \qquad (1)$

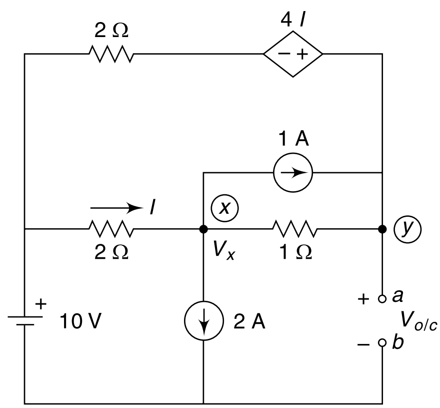

Fig. 4.169(a) *Determination of $V_{o/c}$*

Again, using nodal analysis at node (y) we get

$$\frac{V_{o/c} - V_x}{1} + \frac{V_{o/c} - 4I - 10}{2} - 1 = 0$$

or $\quad 1.5\ V_{o/c} - V_x - 2I = 6;$

$$\text{But } I = \frac{10 - V_x}{2}.$$

Hence we can write

$$1.5\ V_{o/c} - V_x - 2 \cdot \frac{10 - V_x}{2} = 6$$

or $\quad 1.5\ V_{o/c} - V_x - 10 + V_x = 6$

or $\quad V_{o/c} = 10.67$ V. and $V_x = 8.44$ V.

Next we short the terminals a-b in Fig. 4.169(b). Let the voltage at node $\frac{60}{7}$W be V. Using nodel method at node $\frac{60}{7}$W we can write,

$$\frac{V - 10}{2} + 1 + \frac{V}{1} + 2 = 0.$$

or $\quad 1.5\ V = 2$ or, $V = 1.33$ V.

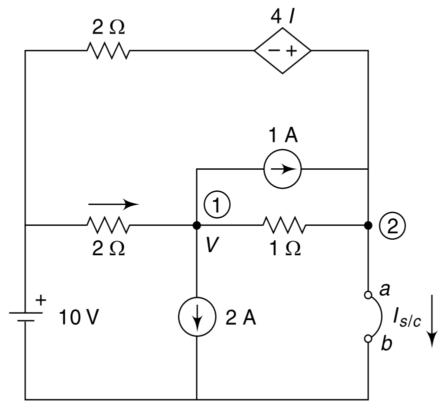

Fig. 4.169(b) *Determination of ($I_{S/C}$)*

Next we use the nodal method at shorted node (2). Here we get

$$\frac{0-V}{1} + I_{s/c} + \frac{0-4I-10}{2} - 1 = 0$$

or $$-V + I_{s/c} - 2I - 5 - 1 = 0$$

∴ $$I_{s/c} = 6 + 2I + V$$

But $$I = \frac{10-V}{2}$$

∴ $$I_{s/c} = 6 + 2 \cdot \frac{10-V}{2} + V = 16 \text{ A}$$

Hence $$R_{\text{Th}} = \frac{V_{o/c}}{I_{s/c}} = \frac{10.67}{16}$$

$$= 0.67\ \Omega.$$

It is obvious that maximum power will flow to R_L provided $R_L = R_{\text{Th}}$. Thus R_L is to 0.67 Ω to have maximum power flow for source to R_L.

Also, $$P_{\max} = \frac{V_{o/c}^2}{4R} = \frac{(10.67)^2}{4 \times 0.67} = 42.48 \text{ W}.$$

4.139 Determine the current I in the network of Fig. 4.170 using superposition theorem.

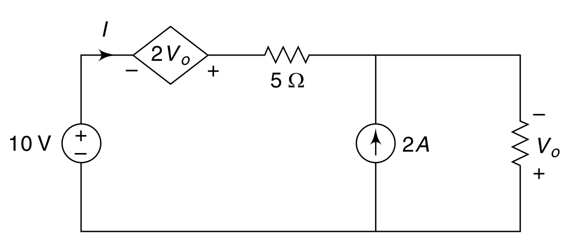

Fig. 4.170 *Circuit of Ex. 4.139*

Solution

Let us first assume the 10 V source only removing 2 A source as shown in Fig. 4.170(a).

Here we have

$$-10 - 2\,V_o + 5I' - V_o = 0 \quad \text{(i)}$$

and $$V_o = -2I' \quad \text{(ii)}$$

Using (ii) in (i), we get

$$-10 - 3\,(-2I') + 5\,I' = 0$$

$\therefore$ $$I' = 0.91 \text{ A.}$$

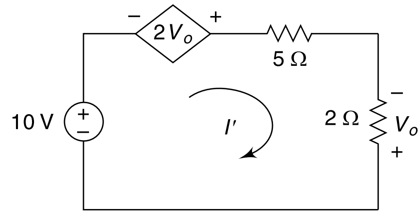

Fig. 4.170(a) ***10 V source acting alone***

Next, removing the 10 V source and connecting the 2 A current source as shown in Fig. 4.170(b), at node p,

$$I'' + (-2) + I_{2\Omega} = 0$$

or $$\frac{-V_o - 2V_o}{5} - 2 - \frac{V_o}{2} = 0$$

or $$V_o = -\frac{20}{11} \text{ V}$$

$\therefore$ $$I'' = \frac{-3V_o}{5} = 1.09 \text{ A}$$

Hence, $$I = I' - I'' = -0.18 \text{ A}$$

(the current flows opposite to given direction of (I) given in Fig. 4.170).

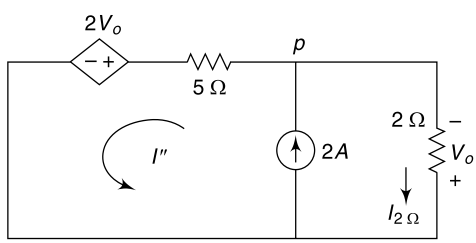

Fig. 4.170(b) ***2 A current source acting alone***

4.140 Find the current in the 14 Ω resistor using Thevnein's theorem in Fig. 4.171.

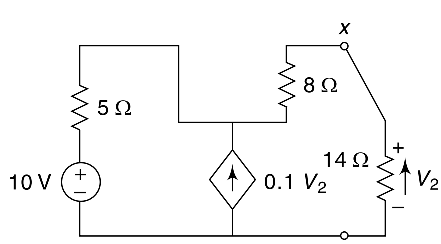

Fig. 4.171 ***Circuit of Ex. 4.140***

Solution

Let us first open circuit x-y. See Fig. 4.171(a).

In loop $axyb$,

$$-10 - 5 \times 0.1\,V_2 + V_{o/c} = 0$$

$\therefore$ $$V_{o/c} = 10 + 0.5\,V_2$$

$$= 10 + 0.5\,V_{o/c} \quad [\because\ V_{o/c} \equiv V_2]$$

$\therefore$ $$V_{o/c} = 20 \text{ V.}$$

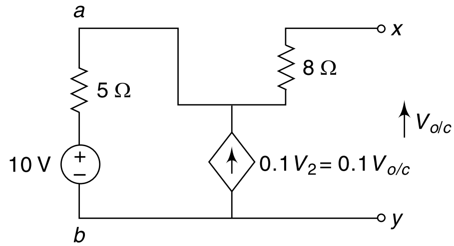

Fig. 4.171(a) ***Determination of $V_{O/C}$***

Next, when we short terminals *x-y* (Fig. 4.171(b), we find $V_2 = 0$ due to short circuit, thus $0.1\ V_2 = 0$.

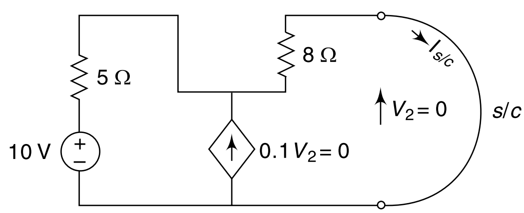

Fig. 4.171(b) ***Determination of $I_{S/C}$***

$$\therefore \qquad I_{s/c} = \frac{10}{5+8} = \frac{10}{13}\ A$$

$$\therefore \qquad R_{\text{int}} = \frac{V_{o/c}}{I_{s/c}}$$

$$= \frac{20}{\frac{10}{13}} = 26\ \Omega.$$

From Thevenin's equivalent circuit, we can then write

$$I = \frac{V_{o/c}}{R_{\text{int}} + 14} = \frac{20}{26+14} = 0.5\ \text{A}$$

$\therefore$ Current through 14 ohm resistor is 0.5 A.

EXERCISES

1. State and explain Thevenin's theorem. What are the limitations of this theorem?
2. State and prove maximum power transfer theorem.
3. State and explain Kirchhoff's voltage law and current law.
4. Distinguish between dependent and independent sources. How do you transform a voltage source into a current source?
5. Distinguish between
   (a) Linear and non-linear elements
   (b) Active and passive elements
   (c) Unilateral and bilateral elements.

6. State superposition theorem and explain it.
7. State Norton's theorem and explain it.
8. Prove that under maximum power transfer condition the power transfer efficiency of the circuit is only 50%.
9. Find the equivalent resistance for the circuit shown in Fig. 4.172. [*Ans:* 10 Ω]

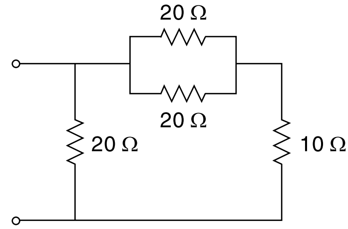

Fig. 4.172

10. Find R_1 and R_2 for the potential divider in Fig. 4.173 so that current I is limited to 1 A when V_o = 20 V. [*Ans:* R_2 = 20 Ω, R_1 = 80 Ω]

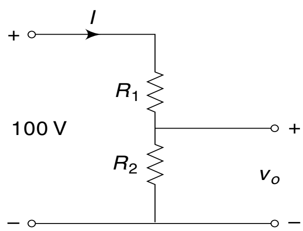

Fig. 4.173

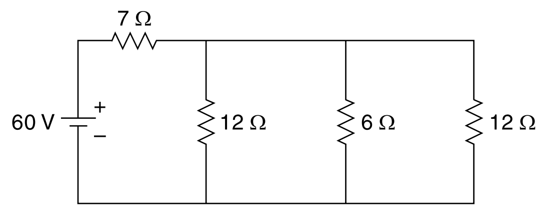

Fig. 4.174

11. Use branch currents in the network shown in Fig. 4.174 to find the current supplied by a 60 V source. [*Ans:* 6 A]
12. Solve problem no. 11 by mesh current method.
13. Two ammeters x and y are connected in series and a current of 20 A flows through them. The potential difference across the ammeters are 0.2 V and 0.3 V respectively. Find how the same current will divide between x and y when they are connected in parallel. [*Ans:* 12 A and 8 A]
14. Obtain the source current I and the power delivered to the circuit in Fig. 4.175. [*Ans:* 6 A, 228 W]

   [Hint: $I_{2\Omega}$ = 4 A; $V_{\text{drop}(2\Omega)}$ = 4 × 2 = 8 V;

   Hence $I_{4\Omega} = \dfrac{8}{4} = 2$ A.

   $\therefore I = I_{2\Omega} + I_{4\Omega} = 6$ A;

   $P = 6^2 \times 5 + 2^2 \times 4 + 4^2 \times 2 = 228$ W]

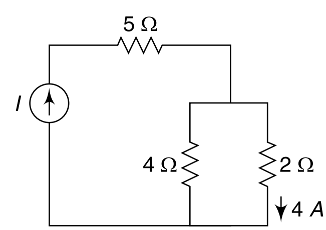

Fig. 4.175

15. For the circuit shown in Fig. 4.176 find the potential difference between x and y. [*Ans:* –2.85 V]

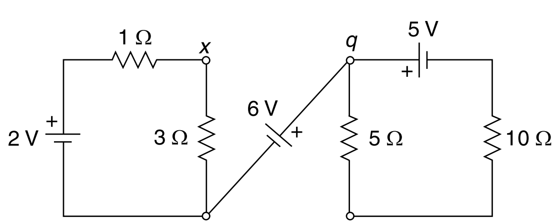

Fig. 4.176

[Hint: In left loop, $I = \dfrac{2}{1+3}$ = 0.5 A; in right loop, $I = \dfrac{5}{15}$ = 0.33 A.

$V_{x\text{-}y} = V_{xp} + (-6) + V_{qy} = 0.5 \times 3 - 6 + 0.33 \times 5 = -2.85$ V]

16. Reduce the circuit in Fig. 4.177 to a voltage source in series with a resistance between terminals A and B.

$$\left[Ans\text{: } V = \frac{90}{34}\text{ V and } R = \frac{15}{23}\,\Omega \right]$$

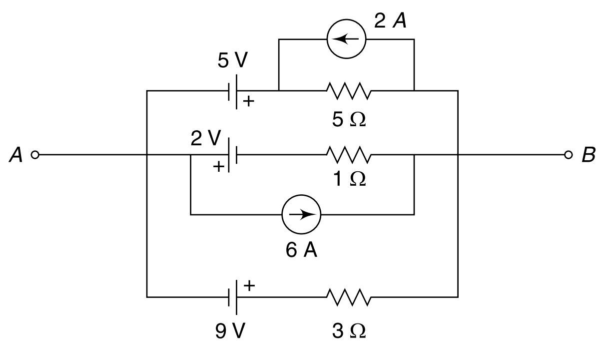

Fig. 4.177

17. For the network shown in Fig. 4.178 find V which makes I = 7.5 mA. [*Ans:* 1.02 V]

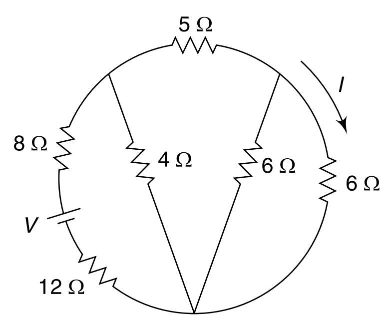

Fig. 4.178

$\Big[I$ = 7.5 mA; $I_{6\Omega} = \dfrac{7.5\text{ mA} \times 6\,\Omega}{6\,\Omega}$ = 7.5 mA

$\therefore$ $I_{5\Omega}$ = 15 mA. Drop in 5 Ω = 15 mA × 5 Ω = 75 mV.

Then drop across 4 Ω is 75 mV + 7.5 mA × 6 Ω = 120 mV.

$\therefore$ $I_{4\Omega} = \dfrac{120}{4}$ = 30 mA. Current from battery is then (30 + 15) i.e.,45 mA. Hence voltage drop in 8 Ω is 45 mA × 8 = 360 mV. Drop in 12 Ω is 45 mA × 12 = 540 mV.

$\therefore$ V = 360 + 75 + 45 + 540 = 1020 mV. i.e., 1.02 V $\Big]$

18. In the network shown in Fig. 4.179 find the resistance between (i) A and a (ii) C and A.

$$\left[Ans\text{: (i) } 1\,\Omega \text{ (ii) } \frac{7}{12}\,\Omega \right]$$

[Hint: Convert delta (abc) to star first and proceed]

19. For the ladder network shown in Fig. 4.180, find the applied voltage V. [*Ans:* 800 V]

[Hint: Find current through the 40 Ω resistor and then proceed as shown in Problem no. 17]

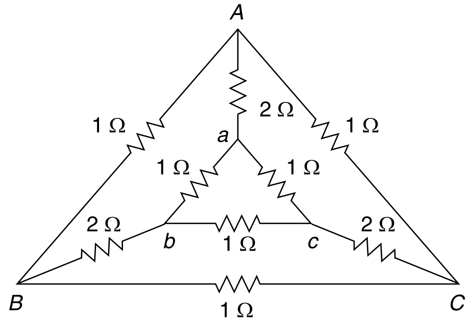

Fig. 4.179

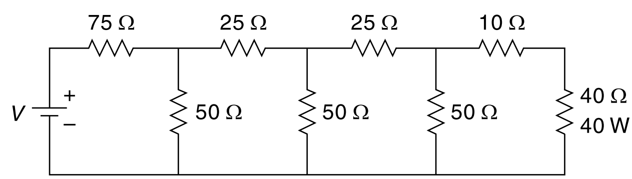

Fig. 4.180

20. Find the current in the 10 Ω resistance in the network shown in Fig. 4.181 using Thevenin's theorem. [*Ans:* 4 A]

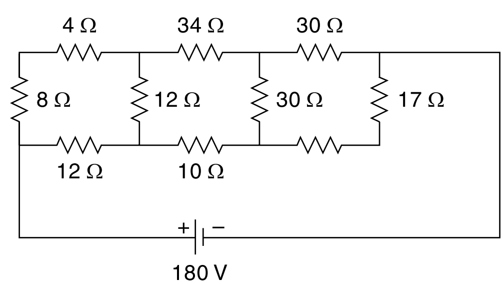

Fig. 4.181

21. Using Norton's theorem find the current through 64 Ω resistance in the circuit shown in Fig. 4.182. [*Ans:* 0.3 A from *A* to *B*]

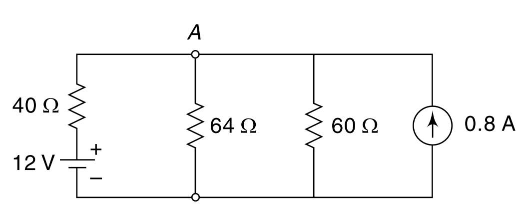

Fig. 4.182

22. In the circuit shown in Fig. 4.183 find the value of R_L so that it abstracts maximum power and also calculate that power. What percentage of power delivered by the battery reached R_L? [*Ans:* 25 Ω, 900 W, 35.7%]

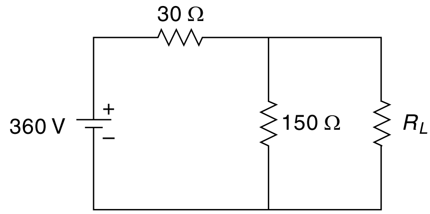

Fig. 4.183

23. In the network shown in Fig. 4.184, find Thevenin's equivalent network across $x - y$ terminals. [*Ans:* $V_{o/c} = V_{x-y} = 25$ V; $R_{Th} = 17\ \Omega$]

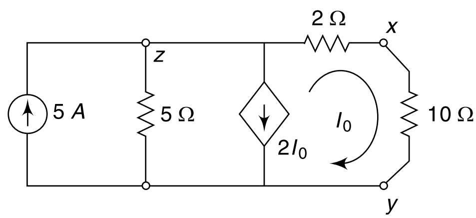

Fig. 4.184

[Hint: 10 Ω resistor is removed. $V_{o/c} = 5\text{ A} \times 5\ \Omega = 25$ V. Next $x - y$ is shorted. At node z, we can write

$5 = \dfrac{V}{5} + 2I_{s/c} + I_{s/c}$; But $I_{s/c} = \dfrac{V}{2}$ (V being the voltage at node z).

$\therefore\ I_{s/c} = 1.47$ A and $R_{Th} = \dfrac{V_{o/c}}{I_{s/c}} = 17\ \Omega$]

24. In the circuit shown in Fig. 4.185 use loop analysis to determine the loop currents i_1, i_2 and i_3.

$$\left[Ans\text{: } i_1 = -\frac{1}{11}\text{ A},\ i_2 = \frac{10}{11}\text{ A},\ i_3 = \frac{2}{11}\text{ A} \right]$$

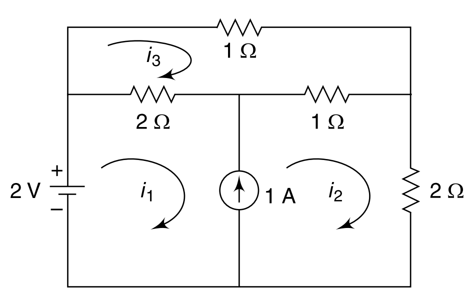

Fig. 4.185

25. Find the Thevenin's equivalent circuit at terminals A, B for the network shown in Fig. 4.186. [*Ans:* $V_{Th} = 25$ V, $R_{Th} = 20\ \Omega$]
26. Find V_1 and V_2 in Fig. 4.187 using nodal voltages analysis method. [*Ans:* $V_1 = 2.468$ V and $V_2 = 1.156$ V]

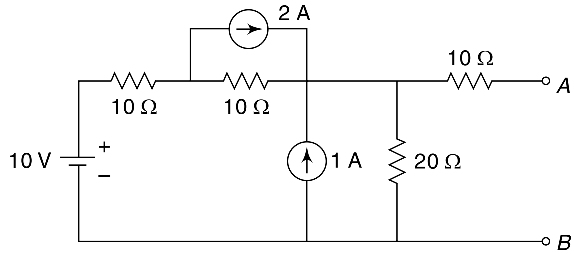

Fig. 4.186

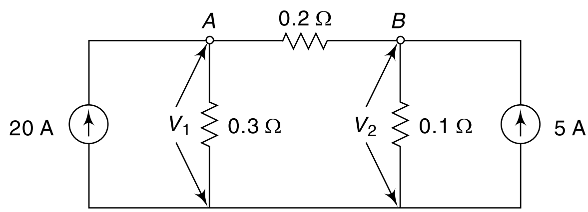

Fig. 4.187

[Hint: At node A, $20 = \dfrac{V_1}{0.3} + \dfrac{V_1 - V_2}{0.2}$; At node B, $5 = \dfrac{V_2}{0.1} + \dfrac{V_2 - V_1}{0.2}$].

27. Find the current through the resistance R in Fig. 4.188 by nodal voltage analysis. [*Ans:* 0 A]

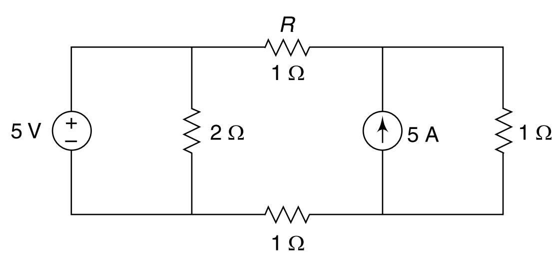

Fig. 4.188

28. In the network shown in Fig. 4.189 show that the internal impedance of the network when looked into it through terminals 1-2 is

$$z_{\text{int}} = \frac{r_1 r_2 (1-m)}{r_2 + r_1 (1-m)}$$

Apply Thevenin's theorem.

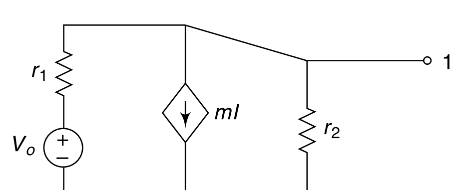

Fig. 4.189

29. Obtain Thevenin's equivalent with respect to terminals *A* and *B* of the network shown in Fig. 4.190. [*Ans:* V_{Th} = 10.637 V, R_{Th} = 2.182 Ω]

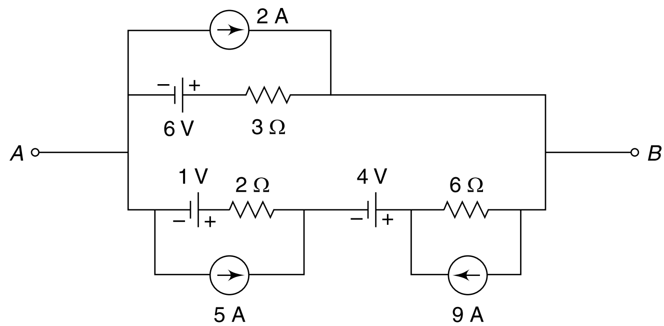

Fig. 4.190

30. Determine the value of R_L for maximum power transfer to the load and determine the load power in the circuit shown in Fig. 4.191.

[*Ans:* 9 Ω, 0.694 W]

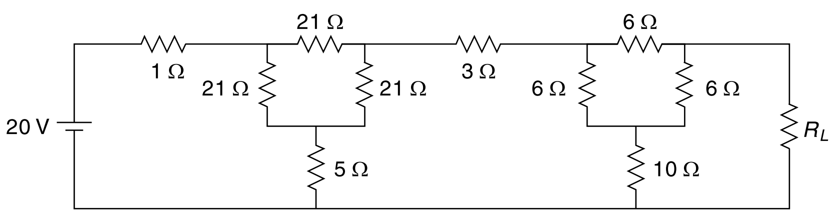

Fig. 4.191

31. Using superposition theorem find the value of V_x in the circuit shown in Fig. 4.192. [*Ans:* −46.8 V]

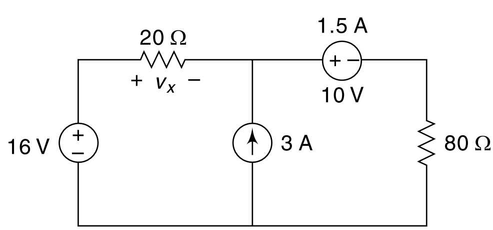

Fig. 4.192

32. Determine Thevenin's equivalent circuit as viewed from the open circuit terminals *a* and *b* of the network shown in Fig. 4.193. [*Ans:* 3 V, 5 Ω]

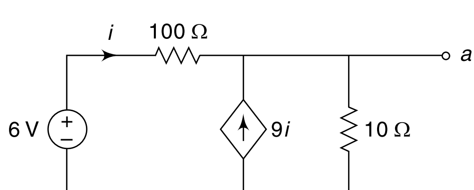

Fig. 4.193

33. Find i_0, i_2 and the value of the dependent source for the network shown in Fig. 4.194. [*Ans:* 2 A, –4 A ; 4 A]

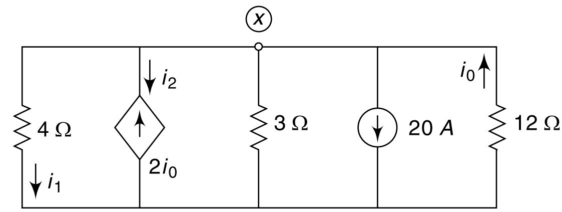

Fig. 4.194

$\Bigg[$ Hint: At node x, assuming node voltage to be v, we have, $\frac{v}{4}+\frac{v}{3}+\frac{v}{12}$ + $20 = 2i_0$

However, $-i_0 = \frac{v}{12}$ A.

$\therefore$ $v = -24$ V and $i_1 = \frac{v}{4} = -6$ A; $i_0 = -\frac{v}{12} = 2$ A.

Value of dependent source is 4 A.

$i_2 = -2i_0 = -4$ A. $\Bigg]$

34. Find the current in the 6 Ω resistor of Fig. 4.195 using Thevenin's theorem. [*Ans:* 1 A]

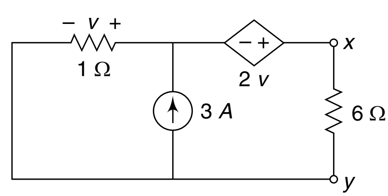

Fig. 4.195

35. Find the loop currents i_1, i_2 and i_3 in the network shown in Fig. 4.196 by mesh method.

$$\left[Ans: i_1 = \frac{29}{17}\text{ A}, i_2 = \frac{11}{17}\text{ A}, i_3 = \frac{57}{17}\text{ A}\right]$$

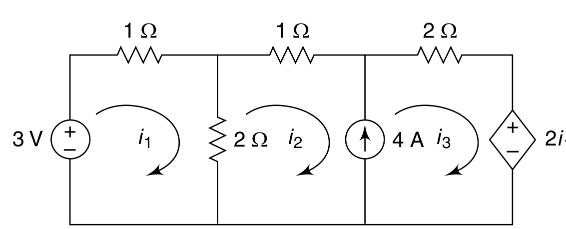

Fig. 4.196

36. What is the power supplied by the dependent source in the circuit of Fig. 4.197. [*Ans:* –84 W]

[Hint: In the right loop, $-5 - V_0 + 2i + 2V_0 = 0 \quad \therefore V_0 + 2i = 5.$

But $V_0 = -(i + 1) \times 1 = -i - 1$

Solving, $i = 6$ A and $V_0 = -7$ V

$\therefore$ Power supplied by the dependent source is $2\ V_0 \times i = -84$ W]

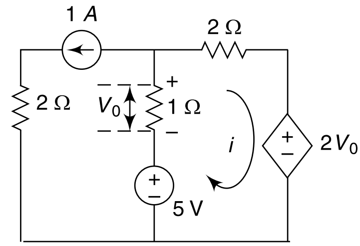

Fig. 4.197

37. Find Norton's equivalent circuit of the network shown in Fig. 4.198. [*Ans:* 1.17 A, 6 Ω]

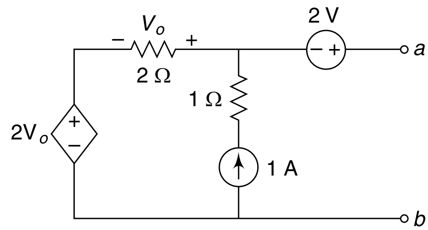

Fig. 4.198

38. Using Norton's theorem find the current in the 5 Ω resistor in the network shown in Fig. 4.199. [*Ans:* 4.166 A]

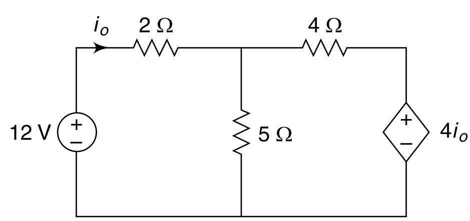

Fig. 4.199

39. In the circuit of Fig. 4.200, if $r = 5\ \Omega$, $R_L = 10\ \Omega$, $v_o = 10$ V, $i_o = 2$ A, find the current through R_L using Thevenin's theorem. [*Ans:* 1.33 A]

[Hint: R_L is removed.

$$V_{o/c} = i_o \times r + v_o = 20 \text{ V}$$

$$R_{\text{Th}} = 5\ \Omega\ (= r)$$

$$\therefore \qquad I_{R_L} = \frac{V_{o/c}}{R_{\text{Th}} + R_L} = 1.33 \text{ A} \Big]$$

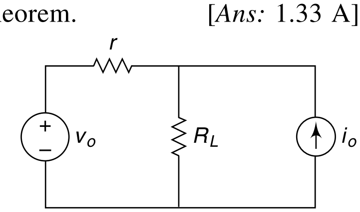

Fig. 4.200

40. Find v by superposition theorem (Fig. 4.201). [*Ans:* $v = 23.37$ V]

[Hint: With 10 V source only,

$$v_1 = 10 \times i = 10 \times \frac{10}{5+10} = 6.67 \text{ V}$$

With 5 A source only,

$$i_{10} = 5 \times \frac{5}{5+10} = 1.67 \text{ A}$$

$\therefore$ $v_2 = 1.67 \times 10 = 16.7$ V

Thus, $v = v_1 + v_2 = 23.37$ V]

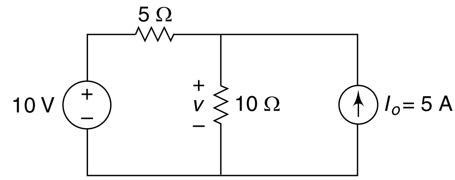

Fig. 4.201

41. The galvanometer in Fig. 4.202 has a resistance of 5 Ω. Find the current through the galvanometer using Thevenin's theorem. [*Ans:* 15.9 *mA*]

[Hint: Open circuiting *BD*, current through 10 Ω resistor

$$I_1 = \frac{10}{10+15} \text{ A} = 0.4 \text{ A}.$$

Current through the 12 Ω resistor

$$I_2 = \frac{10}{12+16} \text{ A} = 0.357 \text{ A}.$$

$V_{Th} = V_{BD} = V_{AD} - V_{AB} = 12 \times 0.357 - 10 \times 0.4 = 0.284$ V

$$R_{Th} = \frac{10\times15}{10+15} + \frac{12\times16}{12+16} = 12.857 \ \Omega.$$

Currrent through galvanometer = $\frac{0.284}{12.857+5}$ A = 0.0159 A [*B* to *D*]]

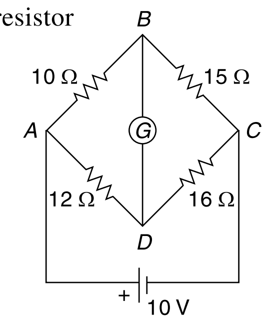

Fig. 4.202

42. For the electrical network shown in Fig. 4.203 find the value of load resistance R_L for which source will supply maximum power to the load. Find also the maximum power. [*Ans:* 8 W]

[Hint: $R_L = \frac{6\times3}{6+3} \ \Omega = \frac{18}{9} \ \Omega = 2 \ \Omega$

(= R_{int})

$$\therefore \quad I_L = \frac{12}{3+\frac{6\times2}{6+2}} \times \frac{6}{6+2} = 2 \text{ A}$$

$$P_{max} = (2)^2 \times 2 \text{ W} = 8 \text{ W}$$]

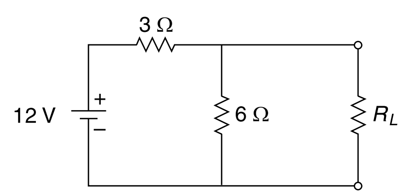

Fig. 4.203

43. Determine the current passing through the 20 Ω (*BD*) resistor of the network as shown in Fig. 4.204 with the help of Thevenin's theorem.

[*Ans:* *I*(*B* to *D*) = –7.79 A]

[Hint: Removing the 20 Ω resistor the open circuit voltage

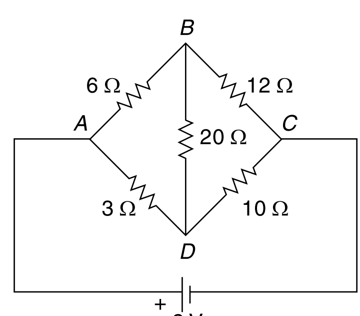

Fig. 4.204

$$V_{BD} = V_{\text{Th}} = V_{AD} - V_{AB} = \frac{2}{3+10} \times 3 - \frac{2}{6+12} \times 6 = -0.205 \text{ V}$$

$$R_{\text{Th}} = \frac{6\times 12}{6+12} + \frac{3\times 10}{3+10} = 6.307 \ \Omega.$$

Current through the 20 Ω resistor = $\dfrac{-0.205}{20+6.307}$ A from *B* to *D* or 7.79 mA from *D* to *B*]

44. Find the current in each branch of the network shown in Fig. 4.205 using Kirchhoff's law.

[*Ans:* $I_{1\Omega} \simeq 1.978$ A; $I_{2\Omega}$ = 1.12 A (*AD*)
$I_{4\Omega}$ = 0.066 A (*BD*); $I_{2\Omega}$ = 1.912 A (*BC*)
$I_{3\Omega}$ = 1.186 A (*DC*);
Current through battery = 3.098 A]

[Hint: Taking 3 mesh currents I_1, I_2 and I_3 in loops *ABDA*, *BCDB* and *ADC* (12 V) A,

$I_1 + (I_1 - I_2)4 + (I_1 - I_3)2 = 0$
$2I_2 + 3(I_2 - I_3) + 4(I_2 - I_1) = 0$
$2I_3 + 2(I_3 - I_1) + 3(I_3 - I_2) = 12$

Solving I_1 = 1.978 A; I_2 = 1.912 A; I_3 = 3.098 A currents in all branches can be found out from I_1, I_2 and I_3]

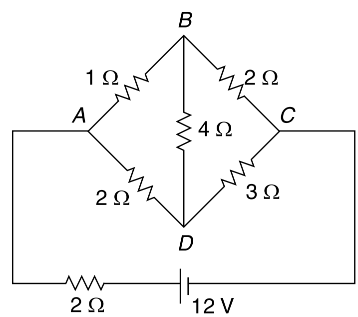

Fig. 4.205

STEADY STATE ANALYSIS OF AC CIRCUIT

5.1 GENERATION OF ALTERNATING EMF

Let us consider a rectangular coil (Fig. 5.1), having N number of turns and A m^2 cross-sectional area, which is rotating in a uniform magnetic field with an angular velocity ω radian/s. If in t seconds the coil rotates through an angle $\theta = \omega t$ from the X-axis, the component of the flux perpendicular to the plane of the coil is $\phi = \phi_m \cos \omega t$ (where ϕ_m = maximum flux density perpendicular to the axis of rotation, when the plane of the coil coincides with the X-axis).

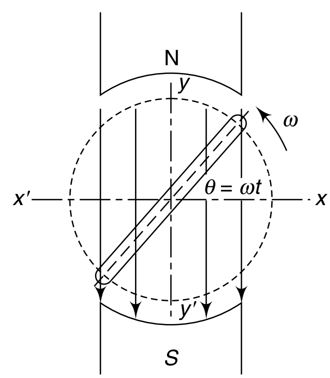

Fig 5.1 *Generation of alternating emf*

We know from Faraday's laws of electromagnetic induction that, "the induced emf in the coil is equal to the rate of change of flux linkages of the coil". Again, Lenz's law states that, "when a circuit and a magnetic field move relatively to each other the electric current induced in the circuit will have a magnetic field opposing the motion". Combining these two laws, the instantaneous induced emf at time t is given by

$$e = -N\frac{d\phi}{dt} = -N\frac{d}{dt}(\phi_m \cos \omega t) \quad [\because \quad \phi = \phi_m \text{ cost } \omega t]$$

$$= \omega N \phi_m \sin \omega t = (\omega N \phi_m \sin \theta) \text{ V}$$

when $\theta = 90°$, $e = \omega N \phi_m = E_m$ (say) where (E_m) is the maximum value of the instantaneous induced emf.

Now, if f be the frequency of rotation of coil in Hertz and B_m the maximum flux density in $\omega b/\text{m}^2$,

$$e = E_m \sin \theta = (2\pi f N B_m A) \sin \theta \text{ V} \qquad [\because \quad B_m \cdot A = \phi_m]$$

Let i be the instantaneous value of the current in the coil. Therefore, $i = I_m \sin \omega t$, where I_m is the maximum value of the current.

As both the induced emf and induced current varies sinusoidally hence the emf or current can be plotted against (time). A sinusoidal curve is obtained as shown in Fig. 5.2

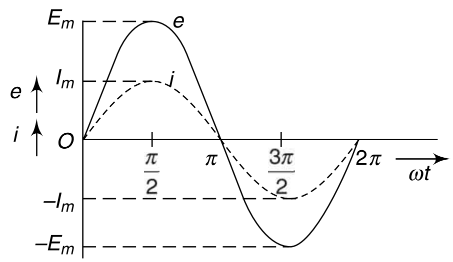

Fig. 5.2 *AC Sinusoidal wave form*

5.2 DEFINITIONS RELATING TO ALTERNATING QUANTITY

1. Amplitude (Peak Value) It is the maximum value, positive or negative of an alternating quantity.

2. Instantaneous Value It is the value of the alternating quantity at any instant.

3. Cycle One complete set of positive and negative values of an alternating quantity is known as cycle.

4. Time Period It is the time required by an alternating quantity to complete 1 cycle; so for a 50 Hz a.c the time period is 1/50 second.

5. Frequency The number of cycles per second is called the frequency of the alternating quantity. Its unit is Hertz (Hz).

6. Phase Phase of an alternating quantity is fraction of the time period that has elapsed since the quantity last passed through the chosen zero position of reference.

7. Phase Angle It is the equivalent of phase in radians or degrees. So phase angle is $\left(2\pi \frac{t}{T}\right)$, where t is the instantaneous time and T is the time period.

8. Phase Difference Phase difference between two alternating quantities is the fractional part of a period by which one has advanced over or lags behind the other. To measure phase difference the frequency of the alternating quantities should be same.

(a) The alternating quantities are in phase when each pass through their zero value, maximum and minimum values at the same instant of time.

(b) A leading alternating quantity is one which reaches its maximum, minimum or zero value earlier than the other quantity. A lagging quantity is one which reaches the maximum, minimum and zero values later than the other quantity.

In Fig. 5.3 alternating quantity e_B is leading with respect to e_C and is lagging with respect to e_A. If we consider e_B as reference then

$$e_B = E_m \sin \omega t,$$

where E_m is the amplitude and ω is the angular frequency of (e_B).

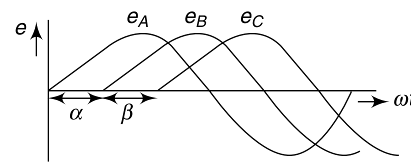

Fig. 5.3 *Lagging and leading alternating quantities*

Therefore $\quad e_A = E_m \sin(\omega t + \alpha)$ and $e_C = E_m \sin(\omega t - \beta)$

where α is the phase difference between e_A and e_B and (β) is the phase difference between (e_B) and (e_C).

9. Roots Mean Square (RMS Value) The rms value of the alternating current is that steady current, i.e., d.c current which if passed through a circuit produces the same amount of heat as produced by the alternating current flowing through the same circuit for the same period of time. The heat produced by direct current I or its equivalent rms value of the alternating quantity i is proportional to i^2. So the area under the curve i^2 vs. 2π is the total heat produced by an alternating current [Fig. 5.4(a) and Fig. 5.4(b)].

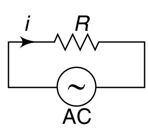

Fig. 5.4(a) *AC through pure resistance*

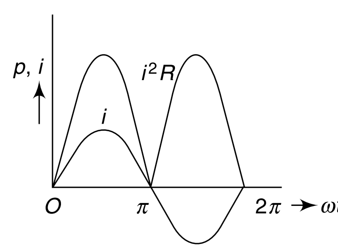

Fig. 5.4(b) *RMS value of alternating quantity*

Rms value is given by

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} i^2\, dt}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} I_m^2 \sin^2\theta\, d\theta} \qquad \text{[substituting } i = I_m \sin\theta]$$

$$= \sqrt{\frac{I_m^2}{4\pi}\int_0^{2\pi} 2\sin^2\theta\, d\theta}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}\int_0^{2\pi} (1-\cos 2\theta)\, d\theta}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi}}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}(2\pi)}$$

$$= \frac{I_m}{\sqrt{2}} = 0.707\ I_m.$$

Hence, the rms value of an alternating quantity = 0.707 × maximum value of that alternating quantity.

10. Average (or Mean) Value The average value of an alternating current is that steady or d.c. current which transfers across any circuit the same amount of charge as transferred by that alternating current during the same period of time.

The average value of an alternating current is given by

$$I_{\text{av}} = \frac{1}{\pi}\int_0^{\pi} i\, d\theta$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta\, d\theta$$

$$= \frac{I_m}{\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{2I_m}{\pi} = 0.637\ I_m.$$

Thus, the average value of an alternating quantity = 0.637 × maximum value of that alternating quantity.

11. Crest or Amplitude or Peak Factor K_a It is the ratio of the peak or maximum value to the rms value of an alternating quantity. For a sinusoidal wave,

$$K_a = \frac{I_m}{I_{\text{rms}}} = \frac{I_m}{0.707\ I_m} = 1.414$$

The knowledge of crest factor is important for measuring iron losses, as iron loss depends on the value of maximum flux. Also in dielectric insulation testing the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage.

12. Form factor K_f It is the ratio of the rms value to the average value of an alternating quantity. For a sinusoidal wave

$$K_f = \frac{I_{\text{rms}}}{I_{\text{av}}} = \frac{0.707\ I_m}{0.637\ I_m} = 1.11.$$

5.1 An alternating emf of frequency 50 Hz, has an amplitude of 100 V. Write down the equation for the instantaneous value. Also find the instantaneous value of the emf after 1/600 second.

Solution

The instantaneous equation for the emf is

$$e = 100 \sin 2\pi ft = 100 \sin 2\pi \times 50t = 100 \sin 100\pi t$$

At $t = \frac{1}{600}$ sec,

$$e = 100 \sin 100\pi \times \frac{1}{600} = 100 \sin \frac{100 \times 180°}{600}$$

$$= 100 \sin 30° = 50 \text{ A.}$$

5.2 An alternating current has rms value of 50 A and frequency 60 Hz. Find the time taken to reach 50 A for the first time.

Solution

$\because$ rms value = 50 A i.e $I_{rms} = 50$ A

So $I_m = 50\sqrt{2} = 70.71$ A.

The instantaneous equation of the current is

$$i = I_m \sin 2\pi ft = 70.71 \sin 2\pi \times 60t = 70.71 \sin 120\,\pi t$$

when $i = 50$ A

$$50 = 70.71 \sin 120\,\pi t$$

$$\therefore \quad \sin 120\,\pi t = \frac{50}{70.71} = \frac{1}{\sqrt{2}} = \sin\frac{\pi}{4}$$

Hence $t = \frac{1}{120 \times 4} = 0.0021$ sec.

5.3 An alternating sinusoidally varying voltage with angular frequency of 314 radian/second has an average value of 127.4 V. Find the instantaneous value of the emf (a) $\frac{1}{300}$ sec and (b) $\frac{1}{75}$ sec after passing through a positive maximum value.

Solution

$$E_{av} = 127.4 \text{ V}$$

$$\therefore \quad E_m = \frac{E_{av}}{0.637} = 200 \text{ V.}$$

Reckoning the time from the instant when the voltage waveform has maximum value, the equation of the sinusoidal voltage wave is $e = E_m \cos \omega t = 200 \cos 314\, t$.

(a) When $t = \frac{1}{300}$ sec

$$e = 200 \cos 314\, t = 200 \cos 100\,\pi \times \frac{1}{300}$$

$$= 200 \cos \frac{\pi}{3} = 100 \text{ V.}$$

(b) When $t = \frac{1}{75}$ sec

$$e = 200 \cos 314\, t = 200 \cos 100\, \pi t$$

$$= 200 \cos 100\, \pi \times \frac{1}{75} = 200 \cos \frac{4\pi}{3} = -100 \text{ V}$$

5.4 An alternating voltage is given by the equation $v = 282.84 \sin\left(377\, t + \frac{\pi}{6}\right)$. Find the (a) rms value, (b) frequency, and (c) the time period.

Solution

(a) $$V_m = 282.84 \text{ V}$$

$$V_{\text{rms}} = \frac{282.84}{\sqrt{2}} = 200 \text{ V}$$

(b) $$\omega = 377 \text{ rad/s}$$

$$f = \frac{377}{2\pi} = \frac{377}{3.14 \times 2} = 60 \text{ Hz}$$

(c) $$T = \frac{1}{f} = \frac{1}{60} = 0.0167 \text{ sec.}$$

5.5 If the form factor of a current wave form is 2 and the amplitude factor is 2.5, find the average value of the current if the maximum value of the current is 500 A.

Solution

$$K_f = 2 \text{ and } K_a = 2.5,\ I_m = 500 \text{ (Given)}$$

Therefore $$K_f = 2 = \frac{I_{\text{rms}}}{I_{\text{av}}} \text{ and } K_a = 2.5 = \frac{I_m}{I_{\text{rms}}} = \frac{500}{I_{\text{rms}}}$$

So $$I_{\text{rms}} = \frac{500}{2.5} = 200 \text{ A and } I_{av} = \frac{I_{\text{rms}}}{2} = \frac{200}{2} = 100 \text{ A.}$$

5.6 Find the average and rms value of the wave form shown in Fig. 5.5.

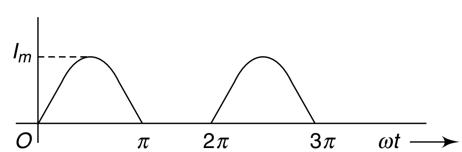

Fig. 5.5 ***Waveform of Ex. 5.6***

Solution

$$I_{\text{av}} = \frac{1}{2\pi}\int_0^{\pi} I_m \sin \omega t\, d(\omega t)$$

$$= \frac{I_m}{2\pi}\left[-\cos \omega t\right]_0^{\pi} = \frac{I_m}{\pi} = 0.318\, I_m$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_0^{\pi} I_m^2 \sin^2 \omega t\, d(\omega t)} = \sqrt{\frac{I_m^2}{4\pi}\int_0^{\pi}(1 - \cos 2\,\omega t)\, d(\omega t)}$$

$$= \frac{I_m}{2}\sqrt{\frac{1}{\pi}\left[\omega t - \frac{\sin^2 \omega t}{2}\right]_0^{\pi}} = \frac{I_m}{2}\sqrt{\frac{1}{\pi}(\pi)} = 0.5 I_m.$$

5.7 Find the rms and average value of the waveform shown in Fig. 5.6

Solution

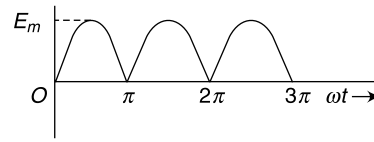

Fig. 5.6 ***Waveform of Ex. 5.7***

$$E_{\text{rms}} = \sqrt{\frac{1}{\pi}\int_0^{\pi} E_m^2 \sin^2 \omega t \, d(\omega t)}$$

$$= \sqrt{\frac{E_m^2}{2\pi}\int_0^{\pi} (1 - \cos 2\,\omega t)\, d(\omega t)}$$

$$= \frac{E_m}{\sqrt{2\pi}}\sqrt{\left[\omega t - \frac{\sin 2\,\omega t}{2}\right]_0^{\pi}} = \frac{E_m}{\sqrt{2\pi}}\sqrt{\pi} = \frac{E_m}{\sqrt{2}}$$

$$= 0.707\ E_m$$

$$E_{\text{av}} = \frac{1}{\pi}\int_0^{\pi} E_m \sin \omega t \, d(\omega t) = \frac{E_m}{\pi}\left[-\cos \omega t\right]_0^{\pi} = \frac{E_m}{\pi} \times 2$$

$$= 0.637\ E_m.$$

5.8 Find the average and rms of the wave form shown in Fig. 5.7.

Solution

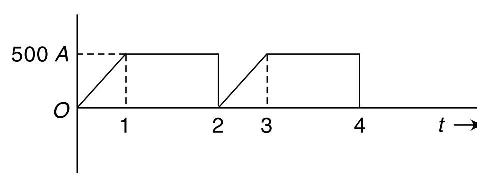

Fig. 5.7 ***Waveform of Ex. 5.8***

$$I_{\text{av}} = \frac{1}{2}\int_0^2 i\, dt = \frac{1}{2}\left[\int_0^1 i_1\, dt + \int_1^2 i_2\, dt\right]$$

$$i_1 = \frac{500}{1}t + 0 = 500t$$

$$i_2 = 500$$

Therefore $$I_{\text{av}} = \frac{1}{2}\left[\int_0^1 500\, t\, dt + \int_1^2 500\, dt\right] = \frac{1}{2}\left[500\left[\frac{t^2}{2}\right]_0^1 + 500\,[t]_1^2\right]$$

$$= \frac{1}{2}\left[500 \times \frac{1}{2} + 500\right] = \frac{1}{2} \times 750 = 375 \text{ A}$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2}\left\{\int_0^1 i_1^2\, dt + \int_1^2 i_2^2\, dt\right\}}$$

$$= \sqrt{\frac{1}{2}\left\{\int_0^1 (500\, t)^2\, dt + \int_0^1 (500)^2\, dt\right\}} = 500\sqrt{\left[\frac{t^3}{3}\right]_0^1 + [t]_1^2}$$

$$= 500\sqrt{\frac{1}{3} + 1} = 500\,\frac{2}{\sqrt{3}} = 577.35 \text{ A.}$$

5.3 PHASOR REPRESENTATION OF AN ALTERNATING QUANTITY

Alternating quantities have varying magnitude and direction. So they are represented by a rotating vector. A phasor is a vector rotating at a constant angular velocity.

Let us consider that an *alternating* or *sinusoidal quantity* be represented by a phasor Oa. It rotates in the counter clockwise direction with a velocity of (ω) radian/s as shown in Fig. 5.8. The projection of this vector on the vertical axis gives the instantaneous value e of the induced emf (i.e sin ωt). When $\omega t = 0$, then instantaneous value = Oa sin $\omega t = 0$. When $\omega t = \pi/2$, the instantaneous value = Oa sin $\pi/2$; $Oa = E_m$ (peak value).

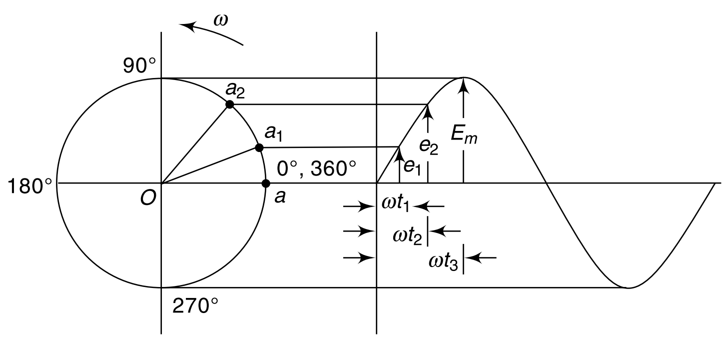

Fig. 5.8 *Phasor representation of alternating quantity*

The instantaneous value of emf at various intervals of time are:

at t_1, $e_1 = E_m \sin \omega t_1$,

at t_2, $e_2 = E_m \sin \omega t_2$

at t_3, $e_3 = E_m \sin \omega t_3$; and so on.

Phasor diagram is one in which different alternating or sinusoidal quantities of the same frequency are represented by phasors with their phase relationship.

Now consider two similar single turn coils A and B displaced from each other by an angle (θ) rotating in a uniform magnetic field with the same angular velocity [Fig. 5.9(a)]. Suppose the emf wave of coil A passes through zero in the positive direction at instant $t = 0$ and at the same instant emf of coil B attains a fixed positive value due to its advancement through an angle (θ) from its zero value [Fig. 5.9(b)]. This can be represented as a still picture with the help of phasors in the phase diagram. [Fig. 5.9(c)] Obviously the angle between the two phasors is the phase difference between the two emfs.

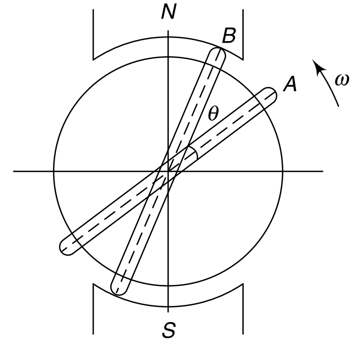

Fig. 5.9(a) *Coil rotating in magnetic field*

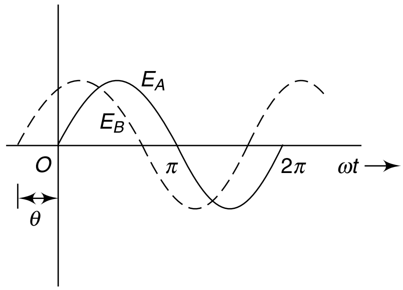

Fig. 5.9(b) *Phasor diagram of ac emf*

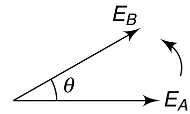

Fig. 5.9(c) ***Phase difference between E_A and E_B***

It should be noted that normally phasors are drawn to represent the *rms* values and the reference phasor is drawn horizontally, e.g, the phasor (E_A). Also the phasors are assumed to rotate in the anticlockwise direction. So the phasor ahead in an anticlockwise direction from a given reference phasor is said to be *leading*, e.g., (E_B) leads phasor (E_A) by angle (θ). The phasor which is behind the reference phasor is said to be *lagging*.

5.3.1 Addition and Subtraction of Sinusoidal Alternating Quantities

Draw the phasor diagram of the alternating quantity and then resolve each phasor into its horizontal and vertical components. Then add or substract the horizontal components and the vertical components separately. Suppose I_x represents the addition or subtraction of the phasors in the horizontal axis and I_y represents the addition and subtraction in the vertical axis. The diagonal of the rectangle formed by I_x and I_y denotes the resultant phasor I as shown in Fig. 5.10. The magnitude of I is given by

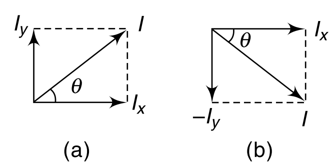

Fig. 5.10 ***(a) Addition of alternating quantities (b) Subtraction of alternating quantities***

$$|I| = \sqrt{I_x^2 + I_y^2}$$

If θ represents the angle between the resultant phasor and the reference phasor (or horizontal line) then

$$\theta = \tan^{-1} \frac{I_y}{I_x}$$

5.3.2 Graphical Method

Let us take an example of adding voltages: $v_1 = 8 \sin (\omega t - 30°)$ and $v_2 = 6 \sin (\omega t + 45°)$.

Magnitude of v_1 is 8 and that of v_2 is 6, i.e, $V_1 = 8$ V and $V_2 = 6$ V. Choose the scale 1 cm = 2 V. Draw one horizontal line OP as the reference line. Draw OA = 8/2 = 4 cm at –30° and OB = 6/2 = 3 cm at an angle of 45° with respect to the reference OP to represent (V_1) and (V_2) respectively. Complete the parallelogram $OACB$. The diagonal of the parallelogram, i.e., OC represents the resultant voltage V_r (Fig. 5.11). By measurement OC = 5.58 cm. So OC = (5.58 × 2) = 11.16 V. The angle θ between OC and OP = 1.2° (by measurement). So, v_r = 11.16 sin (ωt + 1.2°) volts or $v_r = 11.16 \angle 1.2°$ V.

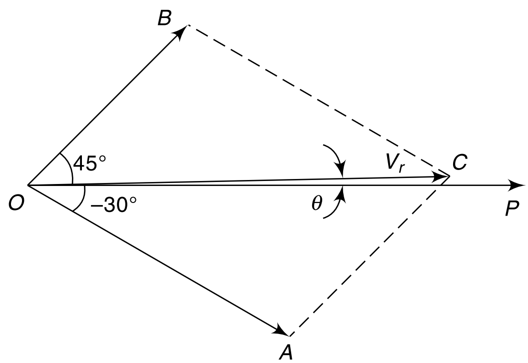

Fig. 5.11 *Addition of two vectors (graphical method)*

5.3.3 Analytical Method

At first draw the phasor diagram. The horizontal component of the resultant voltage

$$V_x = 8 \cos(-30°) + 6 \cos 45° = 8 \times \frac{\sqrt{3}}{2} + 6 \times \frac{1}{\sqrt{2}} = 11.17$$

The vertical component of the resultant voltage

$$V_y = 8 \sin(-30°) + 6 \sin(45°) = -8 \times \frac{1}{2} + 6 \times \frac{1}{\sqrt{2}} = 0.24.$$

So the resultant voltage as shown in Fig. 5.12 is

$$V_r = \sqrt{V_x^2 + V_y^2} = \sqrt{11.17^2 + 0.24^2}$$
$$= 11.1726 \text{ V}.$$

$$\theta = \tan^{-1}\frac{V_y}{V_x} = \tan^{-1}\frac{0.24}{11.17} = 1.23°$$

i.e. $V_r = 11.1726 \sin(\omega t + 1.23°)$ V.

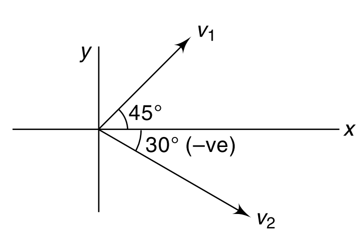

Fig. 5.12 *Addition of two phasors (analytical method)*

5.4 AC VOLTAGE AS APPLIED TO PURE RESISTANCE, PURE INDUCTANCE AND PURE CAPACITANCE

AC through Pure Resistance Alone

When a pure resistance is placed across a sinusoidal emf [Fig. 5.13(a)], the current will be in phase with the emf [Fig. 5.13(b)]. The corresponding phasor diagram is shown in Fig. 5.13(c):

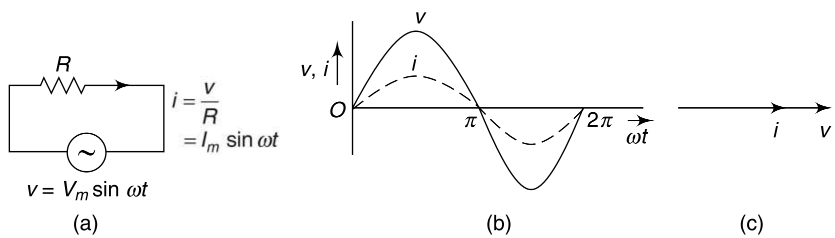

Fig. 5.13 ***(a) AC through pure resistance (b) Phasor diagram of voltage and current through R (c) phasor diagram of voltage and current through R.***

The current is given by, $i = \dfrac{v}{R} = \dfrac{V_m \sin \omega t}{R} = \dfrac{V_m}{R} \sin \omega t = I_m \sin \omega t$

where $\quad I_m = \dfrac{V_m}{R}.$

Also, $\quad I = \dfrac{V}{R}$

where $\quad V$ = rms value of the appplied voltage

$\quad I$ = rms value of current

and $\quad R$ = resistance in ohms.

AC through Pure Inductance Alone

Whenever an alternating sinusoidal voltage is applied to a purely inductive coil [Fig. 5.14(a)] a back emf is produced due to the self-inductance of the coil. The applied voltage has to overcome this self-induced emf and therefore, $v = L\dfrac{di}{dt}$, where L is the self-inductance of the coil, v the back emf and (di/dt) the rate of change of current.

i.e.
$$\frac{di}{dt} = \frac{v}{L} = \frac{V_m}{L} \sin \omega t \qquad [\because \quad v = V_m \sin \omega t]$$

or
$$i = \frac{V_m}{L}\int \sin \omega t\, dt = -\frac{V_m}{\omega L} \cos \omega t = \frac{V_m}{\omega L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

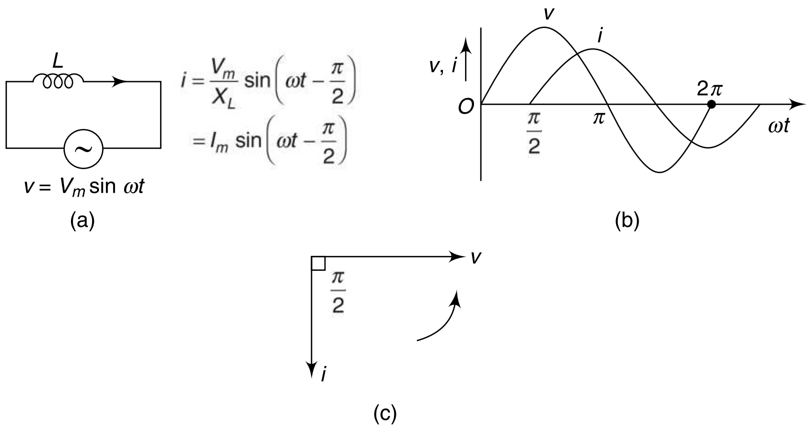

Fig. 5.14 *(a) AC through pure inductance (b) Phasor diagram of voltage and current through L (c) Current lags voltage by 90° in pure inductive circuit*

or $$i = \frac{V_m}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right)$$

where $X_L = \omega L$ (= inductive reactance)

∴ $$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right), \text{ where } I_m = \frac{V_m}{X_L}.$$

So the current *lags* behind the voltage by $\left(\frac{\pi}{2}\right)$ and the phasor diagram is shown in Fig. 5.14(b) and (c).

Also, $I = \frac{V}{X_L}$; where I = rms value of the current

V = rms value of the voltage and

$X_L = \omega L$ = inductive reactance in ohms.

AC throgh Pure Capacitance Alone

If a sinusoidal voltage is applied to the plates of a capacitor [Fig. 5.15(a)] then the instantaneous charge in the capacitors $q = Cv$, where v is the instantaneous value of the applied voltage and C is the capacitance.

If current i is the rate of flow of charge, then

$$i = \frac{dq}{dt} = C\frac{d}{dt}v = C\frac{d}{dt}(V_m \sin \omega t)$$

$$= V_m \omega C \cos \omega t$$

$$= \frac{V_m}{\frac{1}{\omega C}} \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{V_m}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right)$$

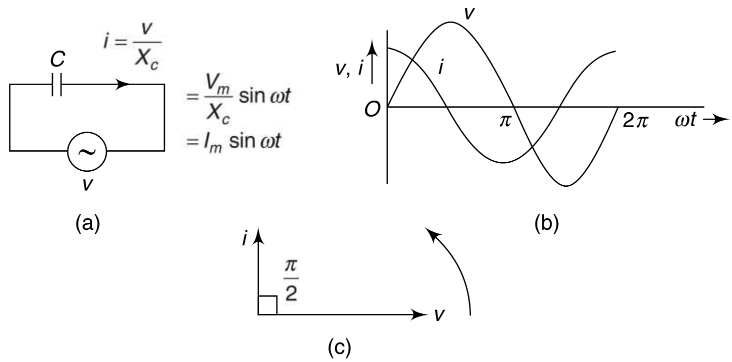

Fig. 5.15 ***(a) AC through pure capacitance (b) Phasor diagram of voltage and current through pure capacitance (c) Current leads voltage by 90° in pure capacitance circuit.***

where $X_C = \dfrac{1}{\omega C}$ (= capacitive reactance).

Also, $i = I_m \sin\left(\omega t + \dfrac{\pi}{2}\right)$, where $I_m = \dfrac{V_m}{X_C}$.

So the current *leads* the applied voltage by $\left(\dfrac{\pi}{2}\right)$ and the phasor diagram is shown in Fig. 5.15(b) and (c).

Also $I = \dfrac{V}{X_C}$, where I = rms value of the current,

V = rms value of the voltage

and X_C = capacitive reactance in ohms.

5.5 SERIES RL CIRCUIT

Consider a coil of resistance R ohms and inductance L henries. The coil is represented by R in series with L [Fig. 5.16(a)]. Let V = rms value of applied voltage, I = rms value of resultant current, V_R = voltage drop across R and V_L = voltage drop across L.

In the phasor diagram of Fig. 5.16(b) the current I flowing in the circuit is drawn in the horizontal axis as reference. V_R is drawn in the same direction as that of I and $V_R = IR$. V_L is drawn leading with respect to I by 90° and $V_L = IX_L$. The resultant of the phasors V_R and V_L gives the supply voltage V. The magnitude of the supply voltage is $|V| = \sqrt{V_R^2 + V_L^2} = \sqrt{(IR)^2 + (IX_L)^2} = |I|\sqrt{R^2 + X_L^2}$ $= |IZ|$, where Z is the *impedance* of the circuit and is expressed in ohms;

also, $|I| = \dfrac{|V|}{|Z|}$ and $|Z| = \sqrt{R^2 + X_L^2}$

or (Impedance)^2 = (Resistance)^2 + (Inductive reactance)^2.

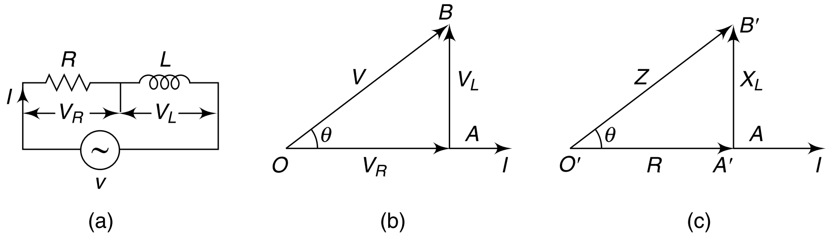

Fig. 5.16 *(a) AC through inductive coil (b) Voltage triangle (c) Impedance triangle.*

Triangle *OAB* [Fig. 5.16(b)] is called the *voltage triangle* and triangle *O′A′B′* [Fig. 5.16(c)] is called the *impedance triangle*. It is noticed that current *I* lags the applied voltage *V* by an angle (θ) where $\theta = \tan^{-1}\dfrac{V_L}{V_R} = \tan^{-1}\dfrac{IX_L}{IR} = \tan^{-1}\dfrac{X_L}{R}$. So, if $v = V_m \sin \omega t$ then, $I = I_m \sin(\omega t - \theta) = \dfrac{V_m}{Z}\sin(\omega t - \theta)$. The phasor diagrams of the applied voltage and current are shown in Fig. 5.17.

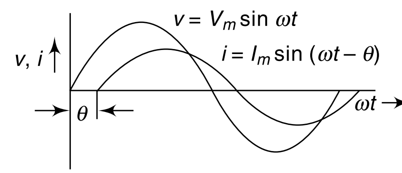

Fig. 5.17 *Phasor diagram of voltage and current through inductive coil.*

5.6 SERIES RC CIRCUIT

Consider a simple ac circuit in which a resistor of *R* ohms and capacitance of *C* farad are connected in series [Fig. 5.18(a)]. Let *V* = rms value of applied voltage, *I* = rms value of resultant current, V_R = voltage drop across *R* and V_C = voltage drop across *C*.

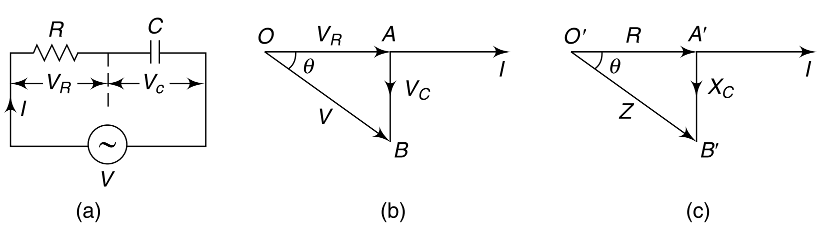

Fig. 5.18 *(a) AC through series R circuit (b) Voltage triangle (c) Impedance triangle.*

In the phasor diagram of Fig. 5.18(b) the current *I* flowing in the circuit is drawn in the horizontal axis as reference V_R is drawn in the same direction as that of *I* and $V_R = IR$. V_C is drawn lagging with respect to *I* by 90° and $V_C = IX_C$. The resultant of the vectors V_R and V_C gives the supply voltage *V*. The magnitude of the supply voltage is

$$|V| = \sqrt{V_R^2 + V_C^2} = \sqrt{(IR)^2 + (IX_C)^2} = |I|\sqrt{R^2 + X_C^2} = |IZ|.$$

where *Z* is the impedance of the circuit and is expressed in ohms.

So, $|I| = \dfrac{|V|}{|Z|}$ and $|Z| = \sqrt{R^2 + X_C^2}$

or (Impedance)^2 = (Resistance)^2 + (Capacitive reactance)^2.

Triangle *OAB* [Fig. 5.18(b)] is called the *voltage triangle* and the triangle *O′A′B′* [Fig. 5.15(c)] is called the *impedance triangle*. It is seen that the current *I* leads the applied voltage *V* by an angle θ where $\theta = \tan^{-1} \dfrac{V_C}{V_R} = \tan^{-1} \dfrac{IX_C}{IR} = \tan^{-1} \dfrac{X_C}{R}$. So, if $v = V_m \sin \omega t$; then $i = I_m \sin(\omega t + \theta) = \dfrac{V_m}{Z} \sin(\omega t + \theta)$. The phasor diagram of the applied voltage and currents are shown in Fig. 5.19.

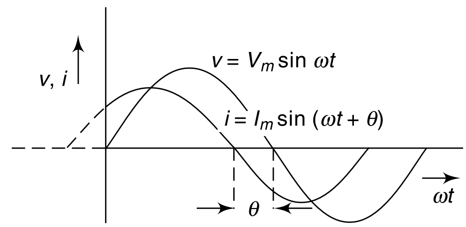

Fig. 5.19 ***Phasor diagram of voltage and current in RC circuit.***

5.7 SERIES RLC CIRCUIT

Consider a simple series ac circuit containing a resistor of resistance *R* ohms, an inductor of inductance *L* henries and a capacitor of capacitance *C* farad across an ac supply of rms voltage *V* volts [Fig. 5.20(a)].

I = rms value of the current flow in the circuit

V_R = rms value of voltage across $R = IR$

V_L = rms value of voltage across $L = IX_L$

and V_C = rms value of the voltage across the capacitor = IX_C.

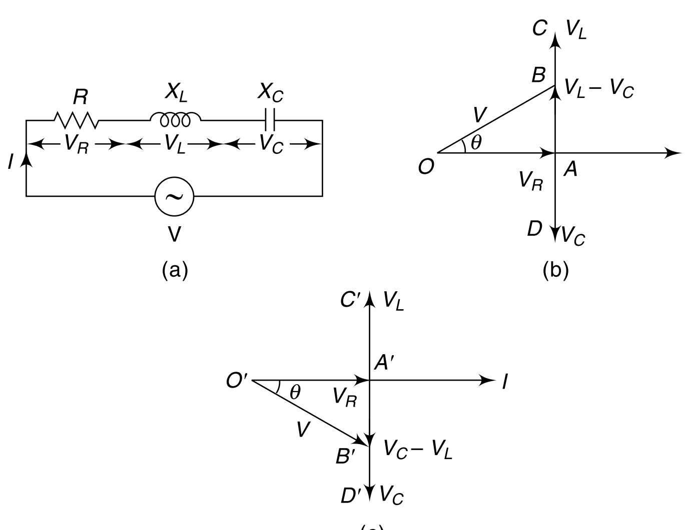

Fig. 5.20 ***(a) AC through RLC series circuit. (b) Voltage triangle for lagging p.f. (c) Voltage triangle for leading p.f.***

In the voltage triangle OAB [Fig. 5.20(b)] OA, AC and AD represents V_R, V_L and V_C repectively. If $|V_L| > |V_C|$ then AB represents the resultant of $(V_L - V_C)$. The vector sum of (V_R) and $(V_L - V_C)$ gives the resultant voltage (V).

Hence, $$|V| = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2} = I\sqrt{R^2 + X^2} = IZ$$

where X = Net reactance in ohm = $(X_L - X_C)$

The phase angle of (V) is given by, $$\theta = \tan^{-1}\frac{(V_L - V_C)}{(V_R)}$$

$$= \tan^{-1}\frac{(IX_L - IX_C)}{IR}$$

$$= \tan^{-1}\frac{(X_L - X_C)}{R}$$

$$= \tan^{-1}\frac{X}{R}\ ;\ X \text{ (say)} = (X_L - X_C),$$

If, $v = V_m \sin \omega t$,

then $$i = \frac{V_m}{Z}\sin(\omega t - \theta) = I_m \sin(\omega t - \theta)$$

Hence, when $|V_L| > |V_C|$, we have
$X_L > X_C$ and current I is lagging with respect to V by an angle less than 90°.

In the voltage triangle $O'A'B'$ in Fig. 5.20(c) $|V_C| > |V_L|$. OA' represents V_R, $A'C'$ represents V_L and $A'D'$ represents V_C. The phasor $(V_C - V_L)$ is represented by $A'B'$ and $O'B'$ denotes resultant voltage V.

Here, $$|V| = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2} = I\sqrt{R^2 + X^2} = IZ$$

where $X = X_L - X_C$ = Net reactance in ohms.

The phase angle of V is given by, $$\theta = \tan^{-1}\frac{(V_L - V_C)}{V_R}$$

$$= \tan^{-1}\frac{IX_L - IX_C}{IR}$$

$$= \tan^{-1}\frac{X_L - X_C}{R} = \tan^{-1}\frac{X}{R}.$$

If $v = V_m \sin \omega t$ then

$$i = \frac{V_m}{Z}\sin(\omega t + \theta) = I_m \sin(\omega t + \theta).$$

When $V_C > V_L$ or $IX_C > IX_L$ or $X_C > X_L$ then current I is leading with respect to the resultant voltage V by an angle less than 90°.

The impedance triangle when $V_L > V_C$ is shown in Fig. 5.21(a) and the impedance triangle when $V_C > V_L$ is shown in Fig. 5.21(b).

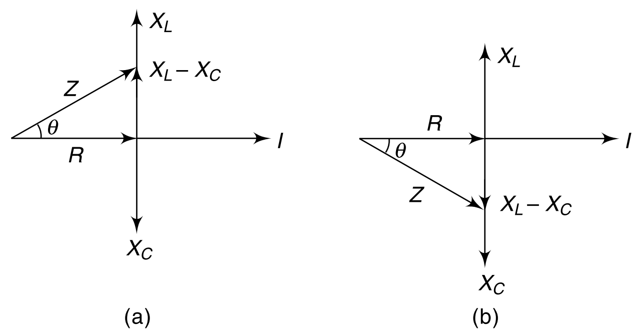

Fig. 5.21 ***(a) Impedance triangle for lagging p.f. (b) Impedance triangle for leading p.f***

5.8 IMPEDANCES IN SERIES

When several impedances are connected in series the net impedance can be found out by using the following steps:

(a) Add all the resistances in the circuit to get total R.
(b) Add all the inductive reactances to get total X_L.
(c) Add all the capacitive reactances to get total X_C.
(d) Total impedance is given by $Z = \sqrt{R^2 + (X_L - X_C)^2}$

[*Note:* all additions in step (b) and (c) are phasor additions.]

5.9 A coil has an inductance of 50 m H and negligible resistance. Find its reactance at 100 Hz.

Solution

$$L = 50 \text{ mH} \quad \text{and} \quad f = 100 \text{ Hz}$$

Inductive reactance $X_L = \omega L = 2\pi f L$, where ω is the angular frequency.

So , $$X_L = 2 \times 3.14 \times 100 \times 50 \times 10^{-3}$$
$$= 31.4159\ \Omega.$$

5.10 If the frequency of applied voltage is 5 kHz, calculate the reactance of a 10 μF capacitor.

Solution

$$f = 5 \text{ kHz} = 5000 \text{ Hz} \quad \text{and} \quad C = 10\ \mu\text{F} = 10 \times 10^{-6}\text{F}$$

$$\therefore \quad \text{Capacitive reactance } (X_C) = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{2 \times 3.14 \times 5000 \times 10 \times 10^{-6}}$$

$$= \frac{10^6}{2 \times 3.14 \times 5000 \times 10} = 3.18\ \Omega$$

5.11 A circuit containing a (a) resistance of 20 Ω alone (b) inductance of 10 mH alone and (c) capacitance of 300 μF alone is connected across an alternating voltage source; write the expressions for the current when $v = 100 \sin 100\ \pi t$.

Solution

$$v = 100 \sin 100\ \pi t, \quad \therefore \quad V_m = 100 \text{ V and } \omega = 100\ \pi \text{ rad/s.}$$

(a) $R = 20\ \Omega \quad \therefore\ i_R = \dfrac{V_m}{R} \sin 100\ \pi t = \dfrac{100 \sin 100\ \pi t}{20} = 5 \sin 100\ \pi t$

(b) $L = 10 \text{ mH} = 0.01 \text{ H}$

Therefore $X_L = \omega L$ (= Inductive reactance) $= 100\ \pi \times 0.01 = 3.14\ \Omega$.

$$i_L = \frac{V_m}{X_L} \sin\left(100\ \pi t - \frac{\pi}{2}\right) = \frac{100}{3.14} \sin\left(100\ \pi t - \frac{\pi}{2}\right)$$

$$= 31.85 \sin\left(100\ \pi t - \frac{\pi}{2}\right)$$

(c) $C = 300\ \mu\text{F} = 300 \times 10^{-6}$ F

$$\therefore \quad \text{capacitive reactance } (X_C) = \frac{1}{\omega C} = \frac{10^6}{100\ \pi \times 300}\ \Omega$$

$$= 10.61\ \Omega$$

$$i_C = \frac{V_m}{X_C} \sin\left(100\ \pi t + \frac{\pi}{2}\right) = \frac{100}{10.61} \sin\left(100\ \pi t + \frac{\pi}{2}\right)$$

$$= 9.425 \sin\left(100\ \pi t + \frac{\pi}{2}\right).$$

5.12 A coil of resistance 100 Ω and inductive reactance 200 Ω is connected across a supply voltage of 230 V. Find the supply current.

Solution

$R = 100\ \Omega$, $X_L = 200\ \Omega$

Impedance $|Z| = \sqrt{R^2 + X_L^2} = \sqrt{(100)^2 + (200)^2} = 223.61\ \Omega$

$\therefore$ supply current $I = \dfrac{V}{Z} = \dfrac{230}{223.61} = 1.028$ A.

5.13 A circuit takes a current $i = 50 \sin\left(314t - \dfrac{\pi}{3}\right)$ when the supply voltage is $v = 400 \sin 314\ t$. Find the impedance, resistance, and the inductance of the circuit.

Solution

$$v = 400 \sin 314\ t$$

$$i = 50 \sin\left(314 - \frac{\pi}{3}\right)$$

$$I_m = 50 \text{ A}$$

and $\theta = \dfrac{\pi}{3}$.

Hence $V_m = 400$ V and $\omega = 314$ rad/s

$\therefore$ Impedance $|Z| = \dfrac{|V|}{|I|} = \dfrac{V_m}{I_m} = \dfrac{400}{50} = 8\ \Omega$

$$\theta = \frac{\pi}{3} = \tan^{-1}\frac{X_L}{R} \quad \text{or,} \quad \frac{X_L}{R} = \tan\frac{\pi}{3} = 1.732 = \sqrt{3}$$

$\therefore$ $X_L = 1.732\ R$ or, $X_L^2 = (1.732)^2 R^2$ or, $Z^2 - R^2 = 3R^2$

or $4R^2 = Z^2 = (8)^2 = 64$. So, $R = \sqrt{\dfrac{64}{4}} = 4\ \Omega$

Thus, $X_L = 1.732 \times 4 = 6.93\ \Omega$

Therefore $L = \dfrac{X_L}{\omega} = \dfrac{6.93}{314} = 0.022$ H = 22 mH.

5.14 When a resistor and coil in series are connected to a 240 V supply, a current of 5 A is flowing lagging 60° behind the supply voltage, and the voltage across the coil is 220 V. Find the resistance of the resistor and the resistance and reactance of coil.

Solution

Let R_L be the resistance of the coil and X_L be the reactance of the coil. If θ be the angle of the current then, cos θ = cos 60° = 0.5 = R/Z where R and Z are the resistance and reactance of the whole circuit respectively.

Therefore, $R = Z \times 0.5$

But, $|Z| = \dfrac{|V|}{|I|} = \dfrac{240}{5} = 48\ \Omega$

$\therefore$ $R = 48 \times 0.5 = 24\ \Omega$

Also, $\dfrac{X_L}{Z} = \sin 60° = 0.866.$

Hence $X_L = 48 \times 0.8666 = 41.57\ \Omega.$

Now, impedance of the coil = $\sqrt{R_L^2 + X_L^2} = \dfrac{220}{5} = 44\ \Omega$

$\therefore$ $R_L = \sqrt{(44)^2 - (41.57)^2} = 14.42\ \Omega.$

Thus resistance of the resistor is (24 – 14.42) = 9.58 Ω, resistance of coil is 14.42 Ω and reactance of coil is 41.57 Ω.

5.15 When a certain inductive coil is connected to a dc supply at 200 V, the current in the coil is 10 A. When the same coil is connected to an ac supply at 200 V, 50 Hz the current is 8 A. Calculate the resistance and reactance of the coil.

Solution

For dc the reactance of the coil is zero ($\because f = 0$).

Hence, resistance of the coil = $\dfrac{200}{10} = 20\ \Omega$

For ac supply, impedance = $\dfrac{200}{8} = 25\ \Omega$

Hence reactance of the coil = $\sqrt{(25)^2 - (20)^2} = \sqrt{625 - 400}$ = 15 Ω

5.16 A 200 V, 120 W lamp is to be operated on 240 V, 50 Hz. supply. Calculate the value of the capacitor that would be placed in series with the lamp in order that it may be used at its rated voltage.

Solution

Let R be the resistance of the lamp as shown in Fig. 5.22. The current flowing through the

$$\text{circuit} = \frac{P}{V} = \frac{120}{200} = 0.6 \text{ A.}$$

It Z be the impedance of the whole circuit,

$$|Z| = \frac{|V|}{|I|} = \frac{240}{0.6} = 400\ \Omega.$$

Now, $$\frac{V^2}{R} = P \text{ or, } R = \frac{(200)^2}{120} = 333.33\ \Omega.$$

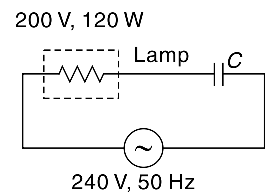

Fig. 5.22 ***Circuit for Ex. 5.16***

Hence the capacitive reactance is

$$X_C = \sqrt{Z^2 - R^2}$$

$$= \sqrt{(400)^2 - (333.33)^2} = 221.11\ \Omega.$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2 \times 3.14 \times 50 \times 221.11} \text{ F}$$

$$= 0.0000144 \text{ F} = 14.4\ \mu\text{F}.$$

Hence the value of the capacitor is 14.4 μF.

5.17 A capacitor and a 50 Ω resistor are connected in series to an alternating current supply. The voltage across the capacitor is 200 V rms and across the resistor is 150 V rms. Determine (a) rms value of supply voltage, (b) peak value of the voltage across the capacitor assuming sinusoidal wave form, (c) power used in the resistor.

Solution

Resistance R = 50 Ω

Voltage acrosss resistor, V_R = 150 V

Voltage across capacitor V_C = 200 V

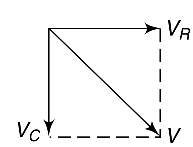

Fig. 5.23 ***Determination of*** $|V|$

$$\therefore \text{ current } |I| = \frac{V_R}{R} = \frac{150}{50} = 3 \text{ A.}$$

$$\text{Supply voltage} = \sqrt{V_R^2 + V_C^2} = \sqrt{(150)^2 + (200)^2} = 250 \text{ V.}$$

Peak value of the voltage across capacitor = $\sqrt{2}\ V_{C\,\text{rms}}$

$= \sqrt{2} \times 200 = 282.8$ V.

Power used in the resistor = $I^2R = (3)^2 \times 50$

= 450 W.

5.18 A resistance of 10 Ω is connected in series with an inductance of 0.05 H and a capacitance of 300 μF to a 100 V ac supply. Calculate the value and phase angle of the current when the frequency is (a) 25 Hz (b) 50 Hz.

Solution

(a) $f = 25$ Hz, $R = 10\ \Omega$, $L = 0.05$ H, $C = 300 \times 10^{-6}$ F;
$V = 100$ V; Hence $X_L = 2\pi fL = 2\pi \times 25 \times 0.05 = 7.85\ \Omega$,

and $$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2 \times 3.14 \times 25 \times 300} = 21.23\ \Omega.$$

$\therefore$ impedance $$|Z| = \sqrt{R^2 + (X_C - X_L)^2}$$
$$= \sqrt{(10)^2 + (21.23 - 7.85)^2} = 16.7\ \Omega;$$

and net reactance $|X| = |X_C - X_L| = 13.38\ \Omega$.

As $X_C > X_L$ so the current is leading.

If θ be the angle of lead then,

$$\theta = \tan^{-1}\frac{X}{R} = \tan^{-1}\frac{13.38}{10} = 53.23°.$$

Current $$|I| = \frac{|V|}{|Z|} = \frac{100}{16.7} = 5.988 \text{ A}.$$

(b) $f = 50$ Hz

So $$X_L = 2\pi \times 50 \times 0.05 = 15.7\ \Omega$$

and $$X_C = \frac{10^6}{2\pi \times 50 \times 300} = 10.61\ \Omega.$$

$\therefore$ $$|X_L - X_C| = 5.09\ \Omega$$

and $$Z = \sqrt{R^2 + (X_L - X_C)^2} = 11.22\ \Omega$$

As $X_L > X_C$ so the current is lagging.

It θ be the angle of lag then,

$$\theta = \tan^{-1}\frac{5.09}{10} = 26.97°$$

and current $$|I| = \frac{100}{11.22} = 8.91 \text{ A}.$$

5.19 A 230 V, 50 Hz voltage is applied to a coil $L = 5$ H and $R = 2\ \Omega$ is in series with a capacitance C. What value must C have in order that the voltage across the coil be 400 V?

Solution

Impedance of the coil $= \sqrt{R^2 + X_L^2} = \sqrt{2^2 + (2\pi \times 50 \times 5)^2} = 1570\ \Omega$

Voltage across the coil = 400 V

$\therefore$ current $I = \dfrac{400}{1570} = 0.2547$ A

Impedance of the circuit $= \dfrac{230}{0.2547} = 903.21\ \Omega$.

If X_C be the capacitive reactance,

$$\sqrt{R^2 + (X_L \sim X_C)^2} = 903.21$$

$\therefore \quad 2^2 + (100\,\pi \times 5 \sim X_C)^2 = (903.21)^2$

$500\,\pi \sim X_C = 903.2$

So $\quad X_C = 666.8\ \Omega$ and $X_L > X_C$

$$\therefore \quad C = \frac{1}{2\pi \times 50 \times 666.8}\ F = 4.77\ \mu\text{F}.$$

5.20 A voltage of 400 V is applied to a series circuit containing a resistor, an inductor and a capacitor. The respective voltages across the components are 250 V, 200 V and 180 V and the current is 5 A. Determine the phase angle of the current.

Solution

Resistance $R = \dfrac{250}{5} = 50\ \Omega.$

Inductive reactance $|X_L| = \dfrac{200}{5} = 40\ \Omega.$

Capacitive reactance $|X_C| = \dfrac{180}{5} = 36\ \Omega$

$$\text{Impedance } |Z| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50)^2 + (40-36)^2}$$

$$= \sqrt{2500 + 16} = 50.61\ \Omega$$

$$\therefore \text{ Phase angle of the current} = \tan^{-1}\frac{X}{R}(X = X_L - X_C) = \tan^{-1}\frac{4}{50} = 4.57° \text{ lagging.}$$

5.9 PARALLEL AC CIRCUIT

Two circuits are said to be connected in parallel if the voltage across them is the same. Consider a parallel ac circuit where an inductive coil is in parallel with a resistor and capacitor in series [Fig. 5.24(a)]. The inductive coil in branch 1 consists of a resistance of $R_L\ \Omega$ and inductance L henry. The resistance in the other branch, i.e branch 2 is R_C and the capacitance is C Farad. So for branch 1 $Z_1 = \sqrt{R_L^2 + X_L^2}$ and $I_1 = V/Z_1$, where V is the supply voltage and $X_L = \omega L$ (the inductive reactance). The phase angle of the current $\theta_1 = \tan^{-1}\dfrac{X_L}{R_L}$.

Similarly, for branch 2

$Z_2 = \sqrt{R_C^2 + X_C^2}$, where $X_C = \dfrac{1}{\omega C}$ is the capacitive reactance and $I_2 = \dfrac{V}{Z_2}$.

The phase angle of the current is $\theta_2 = \tan^{-1}\dfrac{X_C}{R_C}$.

The current I_1 lags behind the applied voltage by θ_1 and current I_2 leads the applied voltage by θ_2 as shown in Fig. 5.24(b).

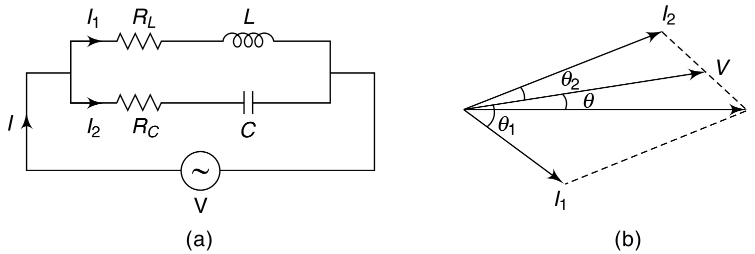

Fig. 5.24 *(a) RL circuit in parallel with RC circuit (b) Phasor diagram of voltage and current of Fig. 5.24(a)*

The resultant current I is the vector sum of I_1 and I_2. Resolving I_1 and I_2 into the X and Y components and then by adding or subtracting [as in Fig. 5.25 (a)] we get

Sum of X axis components of I_1 and $I_2 = I_1 \cos \theta_1 + I_2 \cos \theta_2$.

Sum of Y axis components of I_1 and $I_2 = -I_1 \sin \theta_1 + I_2 \sin \theta_2$.

If θ be the phase angle of the resultant current I then

$$I \cos \theta = I_1 \cos \theta_1 + I_2 \cos \theta_2$$

$$I \sin \theta = -I_1 \sin \theta_1 + I_2 \sin \theta_2.$$

Squaring the above two equations on both sides and then by adding, we get

$$I^2 \cos^2 \theta + I^2 \sin^2 \theta = (I_1 \cos \theta_1 + I_2 \cos \theta_2)^2 + (-I_1 \sin \theta_1 + I_2 \sin \theta_2)^2$$

$$I = \sqrt{(I_1 \cos \theta_1 + I_2 \cos \theta_2)^2 + (-I_1 \sin \theta_1 + I_2 \sin \theta_2)^2}$$

and the phase angle $\theta = \tan^{-1} \dfrac{-I_1 \sin \theta_1 + I_2 \sin \theta_2}{I_1 \cos \theta_1 + I_2 \cos \theta_2}$.

The resultant current is shown in Fig. 5.25(b). If θ is positive the current I leads the applied voltage V and if θ is negtative the current I lags the applied voltage V.

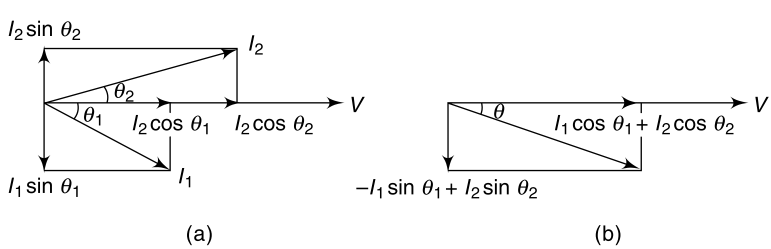

Fig. 5.25 *Branch currents in ac parallel circuit*

5.10 ADMITTANCE, CONDUCTANCE AND SUSCEPTANCE OF AC CIRCUIT

Admittance Y is the reciprocal of impedance Z of an ac circuit.

$$Y = \frac{1}{Z} = \frac{I}{V}$$

Just as impedance has two components viz., resistance R and reactance X, the admittance also has two components, viz. *conductance* G along the horizontal axis and *susceptance* B along the vertical axis [Fig. 5.26].

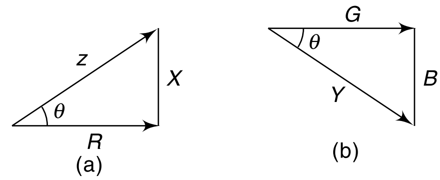

Fig. 5.26 ***(a) Impedance triangle (b) Admittance triangle***

Hence, conductance (G) = $Y \cos \theta = \dfrac{1}{Z} \cdot \dfrac{R}{Z} = \dfrac{R}{Z^2}$

and susceptance (B) = $Y \sin \theta = \dfrac{1}{Z} \cdot \dfrac{X}{Z} = \dfrac{X}{Z^2}$

Admittance (Y) = $\sqrt{G^2 + B^2}$

The units of Y, G and B are mho or ohm^{-1} or Siemens (S). It is to be noted that *inductive susceptance is considered negative and capacitive susceptance is considered positive*.

Use of Admitance in Solving Parallel Circuits

Consider a three branch parallel circuit, as shown in Fig. 5.27.

The total conductance, $G = g_1 + g_2 + g_3$

and total susceptance, $B = -b_1 + b_2$

Current in branch 1, $I_1 = \dfrac{V}{y_1}$

Current in branch 2, $I_2 = \dfrac{V}{y_2}$

Current in branch 3, $I_3 = \dfrac{V}{y_3}$

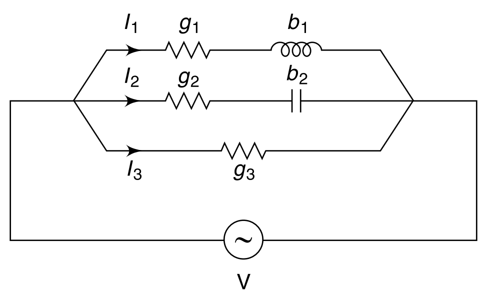

Fig. 5.27 ***Parallel ac circuit***

$$\text{Net admittance } Y = \sqrt{G^2 + B^2} = \sqrt{(g_1 + g_2 + g_3)^2 + (-b_1 + b_2)^2}$$
$$= y_1 + y_2 + y_3.$$

The current, $I = (V \cdot y)$ and the phase angle of the current is, $\theta = \tan^{-1} \dfrac{B}{G}$.

5.11 AVERAGE POWER IN AC CIRCUITS

Power in a dc circuit is given by $P_{dc} = VI = I^2R = \dfrac{V^2}{R}$. In an ac circuit the instantaneous power is the power at any instant of time. It is equal to the product of voltage and current at that instant.

$$p = v.i.$$

Like voltage and current, power is also continuously changing with time. So the average power is given by

$$P = \frac{1}{T}\int_0^T p\, dt.$$

By convention, P always means average power and no subscript is used.

Also, $$P = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} vi\, d\theta$$

Average power is also called *active power* or *real power* or *true power*. Its unit is watts.

5.11.1 Power in a Purely Resistive Circuit

In a purely resistive circuit voltage and current are in phase. Hence, $v = V_m \sin \theta$ and $i = I_m \sin \theta$.

Instantaneous power $p = vi = V_m I_m \sin^2 \theta = \frac{1}{2} V_m I_m (1 - \cos 2\theta)$. The voltage, current and power waveform are shown in Fig. 5.28.

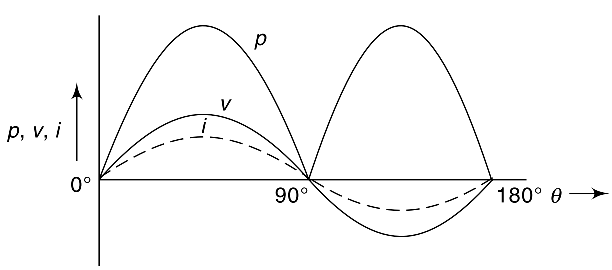

Fig. 5.28 ***Instantaneous power in a pure resistive circuit***

The power waveform in Fig. 5.28 is obtained by multiplying together at every instant the corresponding (instantaneous) values of voltage and current. It is seen that p remains positive throughout the cycle irrespective of the direction of voltage and current in the circuit. This is due to the fact that as voltage and current are in phase so either both voltage and current are positive or both are negative at any instant of time. So their product (p) is always positive. This shows that power flow is only in the direction from the source to the load resistance (R) and this power is called active or real or true power (P_R).

$$\text{Active power } (P_R) = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2} V_m I_m (1 - \cos 2\theta)\, d\theta$$

$$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} d\theta - \frac{V_m I_m}{4\pi}\int_0^{2\pi} \cos 2\theta d\theta$$

$$= \frac{V_m I_m}{4\pi} \times 2\pi - \frac{V_m I_m}{4\pi}\left[\frac{\sin 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{4\pi} \times 0 = \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

In a purely resistive circuit $P_R = VI = (IR)I = I^2R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R}$.

Also, the active or real power in ac circuit is $VI \cos \theta$. A simple reasoning leads to the conclusion that p.f. of a pure resistive circuit is 1(one) ans $\cos \theta$ is 1 ($\because VI \cos \theta = VI$, in pure resistive circuit). $VI = (V^2/R) = I^2R$ is the energy dissipated in resistive circuit.

5.11.2 Power in a Purely Inductive Circuit

In a purely inductive circuit current lags the applied voltage by 90°.

So, $\quad v = V_m \sin \theta$ and $i = I_m \sin\left(\theta - \frac{\pi}{2}\right)$

Instantaneous power $(p) = vi = V_m \sin \theta \, I_m \sin\left(\theta - \frac{\pi}{2}\right)$

$$= \frac{1}{2} \times 2V_m I_m \sin \theta \cos \theta = \frac{V_m I_m}{2} \sin 2\theta.$$

The voltage, current and power waveform are shown in Fig. 5.29.

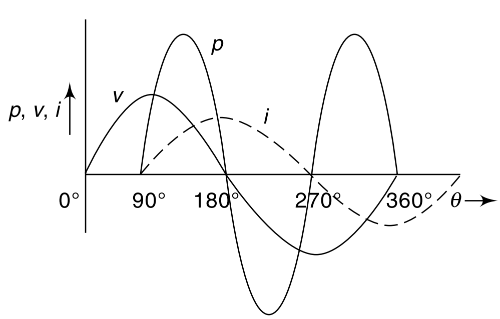

Fig. 5.29 *Instantaneous power in pure inductive circuit*

The power waveform in Fig. 5.29 is obtained by multiplying at every instant together the corresponding (or instantaneous) values of voltage and current.

The power curve is a sine wave of twice the frequency of the current and voltage wave. During the first quarter cycle, the power curve is above the horizontal axis and is positive. The circuit draws energy from the source. This energy is stored in the magnetic field of the inductance. During the second quarter cycle the power curve is below the horizontal axis and is negative. The previously stored energy is now returned to the source. Thus energy stored in the circuit during the first quarter cycle is equal to the energy returned to the source during the second quarter cycle in ideal inductive circuits. So the total energy dissipated, called the *active energy*, during every cycle of the current is zero. The rate of energy dissipated, called the active or average power over the complete cycle of the current in a purely inductive circuit, is also zero.

$$\text{Active power } P_L = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m I_m}{2} \sin 2\theta$$

$$= \frac{V_m I_m}{4\pi} \times \frac{1}{2}[-\cos 2\theta]_0^{2\pi}$$

$$= -\frac{V_m I_m}{8\pi}[\cos 4\pi - \cos 0]$$

$$= -\frac{V_m I_m}{8\pi}[1-1]$$

$$= 0.$$

Thus in a purely inductive circuit the active power over a complete cycle is zero.

The peak value of (p) is $\dfrac{V_m I_m}{2} = \dfrac{V}{\sqrt{2}}\,\dfrac{I}{\sqrt{2}} = VI$

Reactive power Q_L

In a purely inductive circuit $V = V_L = X_L I$, where X_L is the inductive reactance.

Reactive power $Q_L = V_L I = I^2 X_L$

or
$$Q_L = \frac{V_L^2}{X_L} \quad VAR \text{ (volt-ampere reactive).}$$

Q_L is called the *reactive volt amperes* for an inductive circuit. It is measured in VAR. The energy which is continually exchanged between the source and the reactive load is called the *reactive energy*. By convention Q_L is considered positive for inductive circuits.

Also, $Q_L = VI \sin\theta$, (θ) being 90° for pure inductive circuits. Obviously ($\cos\theta$) for inductance is zero.

5.11.3 Power in a Purely Capacitive Circuit

In a purely capacitive circuit the current leads the applied voltage by 90°.

So, $$v = V_m \sin\theta \text{ and } i = I_m \sin\left(\theta + \frac{\pi}{2}\right)$$

Instantaneous power $p = vi = V_m\, I_m \sin\theta \sin\left(\theta + \frac{\pi}{2}\right)$

$$= V_m\, I_m \sin\theta \cos\theta = \frac{1}{2} V_m I_m \cdot 2\sin\theta\cos\theta$$

$$= \frac{V_m\, I_m}{2} \sin 2\theta.$$

The voltage, current and power waveform are shown in Fig. 5.30.

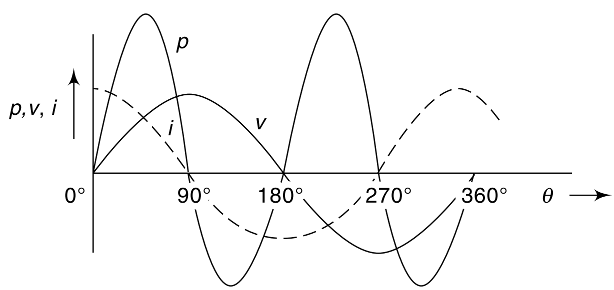

Fig. 5.30 ***Instantaneous power impure capacitive circuit***

The power curve is a sine wave of twice the frequency of the current or voltage curve. During the first quarter cycle the power curve is above the horizontal axis and is positive. The circuit draws energy from the source and the capacitor is charged. The energy is stored in the electric field of the capacitor. During the second quarter cycle the power curve is below the horizontal and is negative. The capacitor is discharged and the energy from the dielectric field is returned to the source. The energy stored in the electric field during the first quarter cycle is equal to the energy returned to the source during the second quarter cycle in a purely capacitive circuit. Therefore the total active energy during each cycle of the current is zero. The active power over a complete cycle of current in a purely capacitive circuit is zero.

Active power $(P_C) = \frac{1}{2\pi}\int_0^{2\pi} p\, d\theta = \frac{1}{2\pi}\int_0^{2\pi} \frac{V_m\, I_m}{2} \sin 2\theta\, d\theta$

$$= \frac{V_m\, I_m}{4\pi}\left[\frac{-\cos 2\theta}{2}\right]_0^{2\pi}$$

$$= \frac{-V_m\, I_m}{8\pi}[\cos 4\pi - \cos 0°]$$

$$= \frac{-V_m\, I_m}{8\pi}[1 - 1] = 0$$

Reactive power (Q_C)

The peak value of (p) is $\dfrac{I_m V_m}{2} = \dfrac{V_m}{\sqrt{2}} \dfrac{I_m}{\sqrt{2}} = VI.$

In a purely capacitive circuit

$$V = V_C = IX_C,$$

$$\therefore \qquad Q_C = V_C I = I^2 X_C = \frac{V_C^2}{X_C} \text{ VAR.}$$

Q_C is called the reactive volt amperes for a capacitive circuit. It is measured in VAR. It is the rate of interchange of reactive energy between a capacitive load and the source. By convention Q_C is considered negative. Obviously, $Q_C = VI \sin \theta$, θ being 90°. Power factor of such a circuit is also zero.

5.11.4 Power in a General Series Circuit

Consider a general case where $v = V_m \sin \theta$ and $i = I_m \sin(\theta - \phi)$, where ϕ is the phase angle of the current with respect to the voltage.

Instantaneous power $p = vi$

$$= V_m I_m \sin \theta \sin(\theta - \phi)$$

$$= \frac{V_m I_m}{2} [\cos \phi - \cos(2\theta - \phi)]$$

Active power

$$P = \frac{1}{2\pi} \int_0^{2\pi} p \, d\theta = \frac{V_m I_m}{2\pi \times 2} \int_0^{2\pi} [\cos \phi - \cos (2\theta - \phi)] \, d\theta$$

$$= \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos \phi \, d\theta - \frac{V_m I_m}{4\pi} \int_0^{2\pi} \cos (2\theta - \phi) \, d\theta$$

$$= \frac{V_m I_m}{4\pi} \cos \phi \, [\theta]_0^{2\pi} - \frac{V_m I_m}{4\pi} \left[\frac{1}{2} \sin (2\theta - \phi) \right]_0^{2\pi}$$

$$= \frac{(\sqrt{2}V)(\sqrt{2}I)}{4\pi} (\cos \phi)(2\pi) - \frac{V_m I_m}{8\pi} [\sin (4\pi - \phi) - \sin (-\phi)]$$

$$= VI \cos \phi - \frac{V_m I_m}{8\pi} [-\sin \phi + \sin \phi]$$

$$= VI \cos \phi.$$

The voltage, current and power waveform are shown in Fig. 5.31.

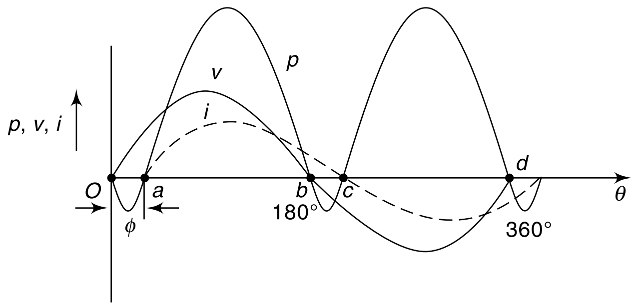

Fig. 5.31 *Instantaneous power in RLC series circuit*

From the power curve it is observed that during the interval *Oa* the power is negative. During the interval *ab* the power is positive. The interval *bc* is the repetition of interval *Oa* and interval *cd* is the repetition of interval *ab*. The negative area under curve *p* between interval *Oa* represents the energy returned from the circuit to the source. The positive area under curve *p* in the interval *ab* represents the energy supplied from the source to the load. So, during each current or voltage cycle a part of the energy called active energy is consumed, while the other part called the reactive energy is interchanged between the source and the load. The rate of energy consumption is the active power. The difference between the total positive and total negative areas during a cycle of current or voltage gives the net active energy of the circuit.

5.11.5 Voltamperes Power (Complex Power)

The product of rms values of voltage and current in a circuit is called the *circuit voltamperes*. It is also called *apparent power* or *complex power*. It is denoted by *S* and is measured in voltamperes VA.

$$S = VI = (IZ)I = I^2Z;\ \text{Also, } S = P + jQ = \sqrt{P^2 + Q^2}\left(\tan^{-1}\frac{Q}{P}\right)$$

5.11.6 Power Triangle

From the previous sections we know that

$P_R = V_R I$ and $Q_R = 0$, for purely resistive circuit

$P_L = 0$ and $Q_L = (V_L I)$ for purely inductive circuit

$P_C = 0$ and $Q_C = (V_C I)$ for purely capacitive circuit.

The net reactive power in the RLC series circuit is $Q = Q_L - Q_C$

[$\because$ Q_L is considered positive and Q_C negative]

$$= I^2X_L - I^2X_C = I^2(X_L - X_C) = I^2X$$

The active power $P = P_R = I^2R$.

The impedance triangle is represented in Fig. 5.32

Multiplying each side of the impedance triangle by I^2 we get the power triangle as shown in Fig. 5.32(b).

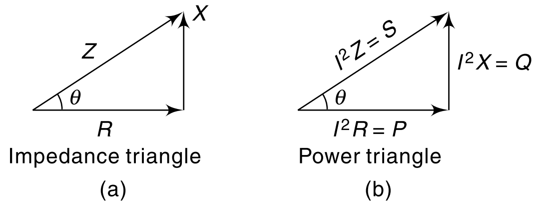

Fig. 5.32 *(a) Impedance triangle(a) (b) Power triangle*

Here, $P = S \cos\theta$ and $Q = S \sin\theta$

As $S = VI$,

Hence, $P = VI \cos\theta$ and $Q = VI \sin\theta$

Also $|S| = \sqrt{P^2 + Q^2}$ and $\theta = \tan^{-1} \dfrac{Q}{P}$.

A power triangle can be obtained from a voltage triangle by multiplying each of its sides by the current as shown in Fig. 5.33.

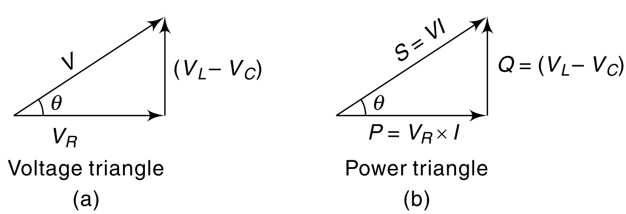

Fig. 5.33 *(a) Voltage triangle and (b) Power triangle*

5.11.7 Power Factor in Resistive, Inductive and Capacitive Circuit

The ratio of the active power to the apparent power in an ac circuit is defined as the *power factor* (p.f.) of the circuit.

$$\text{Power factor} = \frac{P}{S} = \frac{VI\cos\theta}{VI} = \cos\theta$$

So power factor in an ac circuit is also equal to the cosine of the phase angle between the applied voltage and the circuit current.

The power factor is lagging in a circuit in which the current lags the applied voltage. An inductive circuit has lagging power factor.

The power factor is leading in a circuit where the current leads the applied voltage. A capacitive circuit has the leading power factor.

From the impedance triangle, power factor is also given by

$$\cos\theta = \frac{R}{Z}.$$

From voltage triangle power factor can be obtained as

$$\cos\theta = \frac{V_R}{V}.$$

Combining all results,

$$\text{p.f.} = \cos\theta = \frac{P}{S} = \frac{R}{Z} = \frac{V_R}{V}.$$

For purely resistive circuit p.f. = cos 0° = 1.

For purely inductive and capacitive circuits p.f. = cos 90° = 0.

We know that power consumed in a circuit is $VI \cos\theta$

Power consumed in a purely resistive circuit = $VI \cos 0° = VI$

Power consumed in a purely inductive or purely capacitive circuit = $VI \cos 90° = 0$.

Hence, we can conclude that the power is consumed only in the resistor and there is no power consumption in either the pure inductor or the pure capacitor.

5.11.8 Active and Reactive Components of Current

When the current is not in phase with the voltage it lags or leads the applied voltage by an angle θ. The component of the current which is in phase with the voltage namely ($I \cos\theta$) is called the *active component* of current. The other component which is in quadrature with the voltage namely ($I \sin\theta$) is called the *reactive component* of current.

5.21 Two inductive coils A and B are connected in parallel across a 200 V, 50 Hz. supply. Coil A takes 15 A and 0.85 p.f. and the supply current is 30 A and 0.8 p.f. Determine the (a) equivalent resistance and equivalent reactance and (b) resistance and reactance of each coil.

Solution

Considering supply voltage V as the reference phasor,

$$V = 200\angle 0°$$

$$I_1 = 15\angle{-\cos^{-1} 0.85} = 15\angle{-31.79°} \text{ A}$$

The power factor is lagging since the coil is inductive.

Total current $\quad I = 30\angle{-\cos^{-1} 0.8} = 30\angle{-36.86°}$ A.

(a) Equivalent impedance of the circuit

$$Z_{eq} = \frac{V}{I} = \frac{200\angle 0°}{30\angle -36.86°} = 6.67\angle 36.86° \Omega.$$

Hence, equivalent resistance (R_{eq}) = 6.67 cos 36.86° = 5.336 Ω and equivalent reactance (X_{eq}) = 6.67 sin 36.86° = 4 Ω.

(b) If I_2 be the current in coil B lagging by θ angle with respect to the supply voltage, then the horizontal and vertical components of I_2 are

$$I_{2x} = I_2 \cos(-\theta) \text{ and } I_{2y} = I_2 \sin(-\theta)$$

Similarly, $\quad I_{1x} = I_1 \cos(-31.79)° = 15 \cos(-31.79)° = 12.75$ A

and $\quad I_{1y} = I_1 \sin(-31.79°) = -15 \sin 31.79° = -7.9$ A.

The two components of I are

$$I_x = 30 \cos(-36.86°) = 24 \text{ A}$$

and $\quad I_x = 30 \sin(-36.86°) = -18$ A.

Since, $\quad I_x = I_{1x} + I_{2x}$

and $\quad I_y = I_{1y} + I_{2y}$

So $24 = 12.75 + I_{2x}$

or $I_{2x} = 24 - 12.75 = 11.25$ A

and $-18 = -7.9 + I_{2y}$

or $I_{2y} = +7.9 - 18 = -10.1$ A.

Thus the current in coil 2 is $\sqrt{(11.25)^2 + (10.1)^2} \angle \tan^{-1} \dfrac{-10.1}{11.25}$

$= 15.1186\angle{-41.917°}$ A.

The impedance of coil 2 is $(Z_2) = \dfrac{V}{I_2} = \dfrac{200\angle 0°}{15.1186\angle -41.917°}$

$= 13.229\angle 41.917°\ \Omega$.

The resistance of coil 2 is $(R_2) = 13.229 \cos 41.917° = 9.844\ \Omega$ and the reactance of coil 2 is $X_2 = 13.229 \sin 41.917° = 8.837\ \Omega$.

5.22 Find the branch currents, total current, Z and Y, apparent, active and reactive power and power factor in the parallel circuit shown in Fig. 5.34.

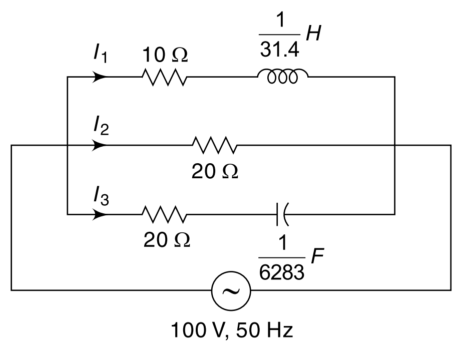

Fig. 5.34 *Circuit diagram for Ex. 5.22*

Solution

Let us consider the supply voltage as reference.

The current in second branch 2,

$$I_2 = \frac{100\angle 0°}{20\,\Omega} = 5\angle 0° \text{ A}.$$

The current in branch 1, $I_1 = \dfrac{V}{z_1}$.

$$Z_1 = \sqrt{(10)^2 + \left(100\pi \times \frac{1}{31.4}\right)^2} \angle \tan^{-1} \frac{100\pi \times \frac{1}{31.4}}{10}$$

$$= 14.14\angle 45°\ \Omega$$

$$\therefore \quad I_1 = \frac{100\angle 0°}{14.14} \angle{-45°} = 7.07\angle 45° \text{ A}.$$

The impedance of branch 3, $Z_3 = \sqrt{(20)^2 + \left(\dfrac{6283}{100\,\pi}\right)^2} \angle{-\tan^{-1}} \dfrac{\frac{6283}{100\,\pi}}{20}$

$= 2829\angle{-45°}\ \Omega$.

The current in branch 3, $I_3 = \dfrac{100\angle 0°}{28.29\angle{-45°}} = 3.53\angle 45°$ A.

Total admittance $y = y_1 + y_2 + y_3$

$$= \frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3}$$

$$= \frac{1}{14.14\angle 45°} + \frac{1}{20} + \frac{1}{28.29\angle{-45°}}$$

$$= 0.0707\angle{-45°} + 0.05 + 0.035\angle 45°.$$

Now, horizontal component of y is

$$y_x = 0.0707 \cos(-45°) + 0.05 \cos \theta° + 0.035 \cos 45°$$
$$= 0.1247 \text{ Siemens}$$

and vertical component of y is

$$y_y = 0.0707 \sin(-45°) + 0.05 \sin 0° + 0.035 \sin 45°$$
$$= -0.025 \text{ Siemens.}$$

Therefore
$$y = \sqrt{y_x^2 + y_y^2}\ \angle\tan^{-1}\frac{y_y}{y_x}$$
$$= \sqrt{(0.1247)^2 + (-0.025)^2}\ \angle\tan^{-1}\frac{-0.025}{0.1247}$$
$$= 0.127\angle{-11.336°} \text{ Siemens}$$

$\therefore$
$$Z = \frac{1}{y} = \frac{1}{0.127\angle{-11.336°}} = 7.87\angle 11.336°\ \Omega.$$

Total current $I = VY = 100\angle 0° \times 0.127\angle{-11.336°} = 12.7\angle{-11.336°}$ A.
Apparent power $(VI) = 100 \times 12.7 = 1270$ VA.
Active power $(VI \cos\theta) = 100 \times 12.7 \cos(11.336°) = 1245$ W.
Reactive power $(VI \sin\theta) = 100 \times 12.7 \sin(11.336°) = 249.63$ VAR (inductive).
Power factor $(\cos\theta) = \cos 11.336° = 0.98$ lagging.

5.23 A lamp rated 400 W takes a current of 4 A when in series with an inductance. (a) Find the value of the inductance connected in series to operate the combination from 240 V, 50 Hz mains (b) Also find the value of the capacitance which should be connected in parallel with the above combination to raise the overall power factor to unity.

Solution

The resistance of the lamp $R = \dfrac{P}{I^2} = \dfrac{400}{(4)^2} = 25\ \Omega.$

Voltage across the lamp $= IR = 4 \times 25 = 100$ V.

(a) The impedance of the circuit $\dfrac{V}{I} = \dfrac{240}{4} = 60\ \Omega.$

If V_L is the voltage across inductance

$$V^2 = V_R^2 + V_L^2$$

or
$$V_L = \sqrt{V^2 - V_R^2} = \sqrt{(240)^2 - (100)^2} = 218.17 \text{ V.}$$

So the inductive reactance $= \dfrac{V_L}{I} = \dfrac{218.17}{4} = 54.54\ \Omega$

and the inductance $= \dfrac{54.54}{\omega} = \dfrac{54.54}{2\pi \times 50} = 0.173$ H.

(b) Let I_C be the current through the capacitance.
The current through the inductive coil is I_L. If the overall power factor is unity the vertical component of total current (I) is 0.

$\therefore$
$$I_C + 4 \sin(-65.37°) = 0$$

or
$$I_C = 4 \sin 65.37° = 3.636 \text{ A.}$$

Hence, capacitive reactance $(X_C) = \dfrac{V}{I_C} = \dfrac{100}{3.636} = 27.5\ \Omega$

and capacitance $= \dfrac{1}{100\pi \times 27.5}$ F = 115.79 μF.

5.24 A fluorescent lamp taking 100W at 0.75 p.f. lagging from a 240 V, 50 Hz. supply is to be corrected to unity p.f. Determine the value of the correcting apparatus required.

Solution

Power = 100 W
p.f. = 0.75 lagging
Voltage = 240 V

$\because \quad VI \cos\theta = P,$

$$\therefore \quad I = \frac{100}{240 \times 0.75} = 0.555\angle -\cos^{-1} 0.75 = 0.555\angle -41.41^\circ \text{ A}$$

and impedance $(Z) = \dfrac{V}{I} = \dfrac{240}{0.555}\ \Omega = 432.43\ \Omega.$

If the power factor becomes unity the net reactive components of current is zero and to improve p.f. from 0.75 lag to 1, a capacitance should be connected in parallel. If I_C be the capacitive current,
net reactive component of current $= I_C + 0.555 \sin(-41.41^\circ) = 0.$
So $I_C = 0.555 \sin 41.41^\circ = 0.367$ A.

$\therefore$ capacitive reactance $(X_C) = \dfrac{V}{I_C} = \dfrac{240}{0.367} = 653.95\ \Omega$

and capacitance $= \dfrac{1}{100\pi \times 653.95}$ F = 4.87 μF.

5.25 A 40 kW load takes a current of 20 A from a 240 V ac supply. Calculate the kVA and KVAR of the load.

Solution

$V = 240$ V
$I = 20$ A
$P = 40$ kW.

If $\cos\theta$ be the power factor then

$$VI \cos\theta = P \quad \text{or, } 240 \times 20 \times \cos\theta = 4000$$

$$\text{or} \quad \cos\theta = \frac{4000}{240 \times 20} = 0.833.$$

Therefore, $\sin\theta = 0.553$

$$\text{kVA } (= VI) = \frac{240 \times 20}{10^3} = 4.8 \text{ kVA}$$

$$\text{and} \quad \text{KVAR } (= VI \sin\theta) = \frac{240 \times 20 \times 0.553}{10^3} \text{ KVAR} = 2.65 \text{ KVAR}.$$

5.26 A 240 V, single phase induction motor delivers 15 kW at full load. The efficiency of the motor at this load is 82% and the p.f. is 0.8 lagging. Calculate (a) the input current of the motor, (b) the kW input and (c) kVA input.

Solution

$$\left.\begin{aligned} V &= 240 \text{ V} \\ \eta &= 82\% \\ \cos\theta &= 0.8 \text{ lag} \end{aligned}\right\} \text{(given)}$$

Output power= 15 kW = 15,000 W

So, input power $(P) = \dfrac{\text{Output power}}{\text{Efficiency}} = \dfrac{15{,}000}{0.82} = 18292.68$ W.

(a) If I be the input current then $VI\cos\theta = P$

$$\text{or} \quad I = \frac{P}{V\cos\theta} = \frac{18292.68}{240 \times 0.8} \text{ A} = 95.27 \text{ A.}$$

(b) kW input $= \dfrac{18292.68}{10^3} = 18.29.$

(c) kVA input $= VI = \dfrac{240 \times 95.27}{10^3} = 22.86.$

5.27 A single phase 50 Hz motor takes 100 A at 0.85 p.f. lagging from a 240 V supply. Calculate the (a) active and reactive components of the current and (b) the power taken from the supply.

Solution

$$\begin{aligned} I &= 100 \text{ A} \\ \cos\theta &= 0.85 \\ V &= 240 \text{ V.} \end{aligned}$$

(a) Active component of current $(I\cos\theta) = 100 \times 0.85 = 85$ A.

Reactive component of current $(I\sin\theta) = 100\sqrt{1-(0.85)^2}$

$= 52.67$ A.

(b) Real power taken from the supply $(VI\cos\theta) = 240 \times 100 \times 0.85$

$= 20400 \text{ W} = 20.4$ kW.

5.28 A series *RL* circuit having $R = 15\ \Omega$ amd $L = 0.03$ H is connected across a 240 V, 50 Hz. supply. Find the (a) rms current in the circuit; (b) average power absorbed by the inductance and (c) the power factor of the circuit.

Solution

$$R = 15\ \Omega$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 0.03 = 9.42\ \Omega$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(15)^2 + (9.42)^2} = 17.71\ \Omega.$$

(a) rms current $|I| = \dfrac{V}{Z} = \dfrac{240}{17.71} = 13.55$ A.

(b) Average power absorbed by the inductance is 0.

(c) Power factor of the circuit $\dfrac{R}{Z} = \dfrac{15}{17.71}$ = 0.847 lag.

5.29 A 200 V, 50 Hz. inductive circuit takes a current of 15 A, lagging the voltage by 45°. Calculate the resistance and inductance of the circuit.

Solution

$$V = 200 \text{ V}$$
$$I = 15 \text{ A}$$
$$\cos\theta = \cos 45° = 0.707$$

Impedance $Z = \dfrac{V}{I} = \dfrac{200}{15} = 13.33\ \Omega.$

Resistance $R = Z\cos\theta = 13.33 \cos 45° = 9.42\ \Omega.$

Inductive reactance $X_L = Z\sin\theta = 13.33 \sin 45° = 9.42\ \Omega.$

Hence the inductance $L = \dfrac{X_L}{\omega} = \dfrac{9.42}{2\pi \times 50}$ H $= 0.03$ H.

5.30 A 2-element series circuit consumes 700 W and has a p.f. of 0.707 leading. If the applied voltage is $v = 141 \sin(314t + 30°)$ find the circuit constants.

Solution

As p.f is leading so the circuit contains a capacitor along with a resistor.

Power $(P) = 700$ W

p.f. $(\cos\theta) = 0.707$, hence $(\sin\theta) = \sin(\cos^{-1} 0.707) = 0.707$

Instantaneous voltage $(v) = 141 \sin(314\,t + 30°)$

rms value of voltage $(V) = \dfrac{141}{\sqrt{2}} = 100$ V.

Angular frequency $(\omega) = 314$ rad/s.

If I be the rms value of current then,

$$P = VI\cos\theta$$

or $$I = \frac{P}{V\cos\theta} = \frac{700}{100 \times 0.707} \text{ A} = 9.9 \text{ A}.$$

Now, Impedance $Z = \dfrac{V}{I} = \dfrac{100}{9.9}\ \Omega = 10.1\ \Omega.$

$\therefore$ Resistance $(R) = Z\cos\theta = 10.1 \times 0.707 = 7.14\ \Omega$ and capacitive reactance $(X_C) = Z\sin\theta = 10.1 \times 0.707 = 7.14\ \Omega.$

Therefore, capacitance $C = \dfrac{1}{\omega X_C} = \dfrac{1}{314 \times 7.14}$ F $= 446\ \mu$F.

Hence the circuit constants are 7.14 Ω and 446 μF.

5.31 A circuit takes a current of 3 A at a p.f of 0.6 lagging when connected to a 115 V, 50 Hz supply. Another circuit takes a current of 5 A at a p.f. of 0.707 leading when connected to the same supply. If the two circuits are connected in series across a 230 V, 50 Hz. supply, calculate (a) the current (b) the power consumed and (c) the p.f. of the circuit.

Solution

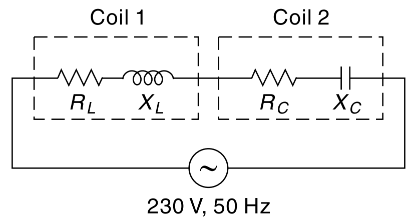

Fig. 5.35 ***Circuit diagram for Ex. 5.31***

As the p.f. is lagging in the first circuit so it contains a resistor along with an inductor. The second circuit contains a resistor along with a capacitor as the p.f. is leading in that circuit. The circuit is shown in Fig. 5.35.

Supply voltage $V = 230$ V,

Frequency $f = 50$ Hz.

For circuit 1,

$$I = 3 \text{ A}, \cos\theta = 0.6 \text{ lag, and } V = 115 \text{ V}$$

$$\therefore \qquad |Z| = \frac{V}{I} = \frac{115}{3} = 38.33\ \Omega$$

$$R_L = Z\cos\theta = 38.33 \times 0.6 = 22.998\ \Omega$$

$$X_L = Z\sin\theta = 38.33 \times \sin(\cos^{-1} 0.6) = 30.664\ \Omega.$$

For circuit 2,

$$I = 5 \text{ A}, \cos\theta = 0.707 \text{ lead and } V = 115 \text{ V}.$$

$$\therefore \qquad |Z| = \frac{V}{I} = \frac{115}{5} = 23\ \Omega$$

$$R_C = Z\cos\theta = 23 \times 0.707 = 16.261\ \Omega$$

$$X_C = Z\sin\theta = 23 \times \sin(\cos^{-1} 0.707)$$

$$= 16.261\ \Omega$$

when the two circuits are connected in sereis,

$$\text{Impedance} = \sqrt{(R_L + R_C)^2 + (X_L - X_C)^2}$$

$$= \sqrt{(39.259)^2 + (14.403)^2} = 41.81\ \Omega$$

(a) The current $(I) = \dfrac{V}{Z} = \dfrac{230}{41.81}$ A $= 5.5$ A.

(b) The power is consumed in the resistors only.

$$\therefore \text{ the power consumed} = I^2(R_L + R_C)$$

$$= (5.5)^2\ (39.259)$$

$$= 1187.58 \text{ W}$$

$$= 1.187 \text{ kW}.$$

(c) Power factor of the circuit $= \dfrac{\text{Net resistance}}{\text{Net impedance}}$

$$= \frac{39.259}{41.81} = 0.94.$$

As $X_L > X_C$ so the p.f. is lagging. Therefore p.f. of the circuit is 0.94 lagging.

5.32 The impedances Z_1 and Z_2 are connected in parallel across a 200 V, 50 Hz single phase ac supply. Z_1 carries 2 A at 0.8 lag p.f. If the total current is 5 A at 0.985 lagging p.f., determine (a) value of Z_1 and Z_2 (b) total power and power consumed by Z_2.

Solution

For the Ist circuit,

$$V = 200 \text{ V}$$

$$I_1 = 2 \text{ A}$$

$$\cos\theta_1 = 0.8 \text{ lag} \quad \text{so} \quad \theta_1 = \cos^{-1} 0.8 = 36.86° \text{ (lag)}.$$

$$|Z_1| = \frac{V}{I} = \frac{200}{2} = 100\ \Omega. \text{ Also, } \sin\theta_1 = 0.6$$

Resistance $R_1 = Z_1 \cos\theta_1 = 100 \times 0.8 = 80\ \Omega$.

Inductive reactance $X_L = Z \sin\theta = 100 \times 0.6 = 60\ \Omega$.

Total current $(I) = 5$ A and net p.f. $(\cos\theta) = 0.985$ lag.

So $\theta = \cos^{-1}(0.985) = 9.93°$

and $\sin\theta = 0.172$

Total power $= VI\cos\theta$
$= 200 \times 5 \times 0.985$
$= 985$ W

$$\text{Total } |Z| = \frac{V}{I} = \frac{200}{5} = 40\ \Omega.$$

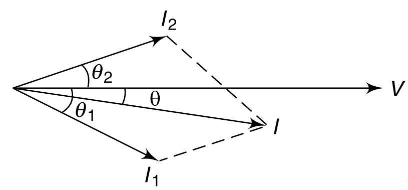

Fig. 5.36 ***Phasor diagram***

The phasor diagram is shown in Fig. 5.36

Horizontal component of I_1 is $I_{1x} = I_1 \cos\theta_1 = 2 \times 0.8 = 1.6$ A

Horizontal component of I is $I_x = I\cos\theta = 5 \times 0.985 = 4.925$ A

Vertical component of I_1 is $I_{1y} = -I_1 \sin\theta_1 = 2 \times 0.6 = -1.2$ A

Vertical component of I is $I_y = -I\sin\theta = -5 \times 0.172 = -0.86$ A.

If I_{2x} and I_{2y} be the horizontal and vertical component of the current in the second circuit then

$$I_x = I_{1x} + I_{2x}$$

and $I_y = I_{1y} + I_{2y}$.

So, $I_{2x} = I_x - I_{1x} = 4.925 - 1.6 = 3.25$

and $I_{2y} = I_y - I_{2y} = -0.86 + 1.2 = +0.34$.

The current in circuit 2 is $(I_2) = \sqrt{I_x^2 + I_y^2} = \sqrt{(3.325)^2 + (0.34)^2}$
$= 3.34$ A.

$$\text{Impedance } Z_2 = \frac{V}{I_2} = \frac{200}{3.34}\ \Omega = 59.88\ \Omega.$$

$$\text{Power factor of circuit 2} = \frac{I_{2x}}{I_2} = \frac{3.325}{3.34} = 0.9955.$$

Hence power consumed by Z_2 is $P_2 = VI_2 \cos\theta_2$
$= 200 \times 3.34\ (0.9955)$
$= 665$ W.

5.33 An iron cored electromagnet has a dc resistance of 7.5 Ω and when connected to a 400 V, 50 Hz supply takes 10 A and consumes 2 kW. Calculate for this value of current (a) power loss in iron core, (b) the inductance of coil, (c) the p.f., and (d) the value of series resistance which is equivalent to the effect of iron loss.

Solution

When the electromagnet is connected to a dc source it is required to consider the resistance of the coil only.

Given, resistance of coil $(R_C) = 7.5\ \Omega$.

When connected to ac source both the resistance of the coil and the equivalent resistance of iron part should be considered.
However, $V = 400$ V; $I = 10$ A and P (= 2 kW) = 2000 W.

Equivalent impedance $Z = \dfrac{V}{I} = \dfrac{400}{10} = 40\ \Omega$.

Power factor $(\cos\theta) = \dfrac{P}{VI} = \dfrac{2000}{400 \times 10} = 0.5$.

Total resistance = $Z \cos\theta = 40 \times 0.5 = 20\ \Omega$
Total reactance = $Z \sin\theta = 40 \sin(\cos^{-1} 0.5) = 34.64\ \Omega$.
$\therefore$ resistance of iron core = $20 - 7.5 = 12.5\ \Omega$.

(a) Power loss in iron core is $I^2 \times 12.5 = (10)^2 \times 12.5 = 1250$ W = (1.25 kW).

(b) Inductance of coil = $\dfrac{34.64}{\omega} = \dfrac{34.64}{100\pi} = 0.1103$ H

(c) Power factor $(\cos\theta) = 0.5$

(d) The value of series resistance is 12.5 ohm which is equivalent to iron loss.

5.34 An iron cored choking coil takes 4 A at p.f. of 0.5 when connected to a 200 V, 50 Hz. supply. When the core is removed and the applied voltage is reduced to 50 V 50 Hz, the current is 8 A and the p.f. 0.8 lag. Calculate the (a) core loss and (b) inductance of the choke with and without the core.

Solution

With core,

$V = 200$ V; $\cos\theta = 0.5$ and $I = 4$ A.

Hence $|Z| = \dfrac{V}{I} = \dfrac{200}{4} = 50\ \Omega$.

Resistance of core along with coil = Z $\cos\theta = 50 \times 0.5 = 25\ \Omega$.
Reactance of the core and coil = Z $\sin\theta = 50 \sin(\cos^{-1} 0.5) = 43.3\ \Omega$.
Without core,

$V = 50$ V; $I = 8$ A and $\cos\theta = 0.8$.

Hence $|Z| = \dfrac{V}{I} = \dfrac{50}{8} = 6.25\ \Omega$.

Resistance of coil $(Z \cos\theta) = 6.25 \times 0.8 = 5\ \Omega$
Reactance of coil $(Z \sin\theta) = 6.25 \times \sin(\cos^{-1} 0.8) = 3.75\ \Omega$.
$\therefore$ Resistance of core = $25 - 5 = 20\ \Omega$.
Core loss = $I^2 \times$ (Resistance of core) = $(4)^2 \times 20 = 320$ W.

Inductance of choke with core = $\dfrac{43.3}{\omega} = \dfrac{43.3}{2\pi \times 50} = 0.13791$

$= 137.9$ mH

Inductance of choke without core = $\dfrac{3.75}{\omega} = \dfrac{3.75}{2\pi \times 50} = 0.01191$ H

$= 11.9$ mH.

5.35 The following loads are connected in parallel:

(a) 100 kVA at 0.8 p.f. lagging,

(b) 250 kVA at 0.8 p.f. leading,
(c) 200 kVA at 0.6 p.f. lagging
(d) 50 kW at unity p.f.

Determine (a) the total kVA, (b) total kW, (c) total KVAR and (d) the overall p.f.

Solution

$$\begin{aligned}\text{Total kW} &= 100 \times 0.8 + 250 \times 0.8 + 200 \times 0.6 + 50 \\ &= 80 + 200 + 120 + 50 = 450 \text{ kW}. \\ \text{Total KVAR} &= 100 \sin(\cos^{-1} 0.8) \\ &\quad - 250 \sin(\cos^{-1} 0.8) + 200 \sin(\cos^{-1} 0.6) + 0 \\ &= 100 \times 0.6 - 250 \times 0.6 + 200 \times 0.8 \\ &= 60 - 150 + 160 = 70.\end{aligned}$$

or Total KVAR is 70 (lagging)

$$\text{Total kVA} = \sqrt{(450)^2 + (70)^2} = 455.4.$$

$$\text{Overall p.f.} = \frac{\text{Total kW}}{\text{Total KVA}} = \frac{450}{455.4} = 0.988.$$

As KVAR is lagging, p.f. is also 0.988 (lagging).

5.12 COMPLEX NOTATION APPLIED TO AC CIRCUITS

For solving complicated ac circuit problems complex algebra is used. In this method a phasor is resolved into two components at right angles to each other. If a phasor V is resolved into two components V_x (horizontal component) and V_y (vertical component) [Fig. 5.37] then $V^2 = (V_x^2 + V_y^2)$ and (V) can be represented in cartesian form as, $V = V_x + j\, V_y = V(\cos\theta + j \sin\theta)$.

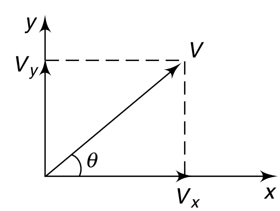

Fig. 5.37 ***Horizontal and vertical components of phasor V***

The symbol j is an operator indicating the anticlockwise rotation of the phasor by 90°. It is assigned by a value $\sqrt{-1}$.

In polar form, the phasor V is represented by

$$V = V\angle\theta, \text{ where } V = \sqrt{V_x^2 + V_y^2} \text{ and } \theta = \tan^{-1}\frac{V_y}{V_x}.$$

Addition and Subtraction of Complex Quantities

Let us consider two phasors v_1 and v_2 which are represented in cartesian form as

$$v_1 = a_1 + jb_1 \text{ and } v_2 = a_2 + j\, b_2.$$

Then, $\quad v_1 + v_2 = (a_1 + a_2) + j(b_1 + b_2)$

and $\quad v_1 - v_2 = (a_1 - a_2) + j(b_1 - b_2)$.

Multiplication and Division of Complex Quantities

$$\begin{aligned} v_1\, v_2 &= (a_1 + jb_1)(a_2 + jb_2) \\ &= (a_1a_2 - b_1b_2) + j(b_1a_2 + a_1b_2).\end{aligned}$$

Also, $$\frac{v_1}{v_2} = \frac{a_1 + jb_1}{a_2 + jb_2} = \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)}$$

$$= \frac{(a_1 a_2 + b_1 b_2) + j(b_1 a_2 - a_1 b_2)}{a_2^2 + b_2^2}$$

$$= \frac{a_1 a_2 + b_1 b_2}{a_2^2 + b_2^2} + j\frac{b_1 a_2 - a_1 b_2}{a_2^2 + b_2^2}.$$

During calculations the horizontal and vertical components of phasors are summed up separaltely in cartesian form. This form is convenient for addition and subtraction while the polar form is convenient for multiplication and division.

Let $v_1 = a_1 + jb_1 = |V_1| \angle\theta_1$

and $v_2 = a_2 + jb_2 = |V_2| \angle\theta_2$

where $|V_1| = \sqrt{a_1^2 + b_1^2}$ and $\theta_1 = \tan^{-1}\dfrac{b_1}{a_1}$

and $|V_2| = \sqrt{a_2^2 + b_2^2}$ and $\theta_2 = \tan^{-1}\dfrac{b_2}{a_2}$

$$v_1\, v_2 = |V_1| \angle\theta_1 \times |V_2| \angle\theta_2 = |V_1||V_2| \angle(\theta_1 + \theta_2)$$

and $$\frac{v_1}{v_2} = \frac{|V_1|\angle\theta_1}{|V_2|\angle\theta_2} = \frac{|V_1|}{|V_2|}\angle(\theta_1 - \theta_2).$$

5.13 SERIES PARALLEL AC CIRCUITS

Consider a series parallel ac circuit as shown in Fig. 5.38.

First, the impedance of the parallel branches 1 and 2 are considered.

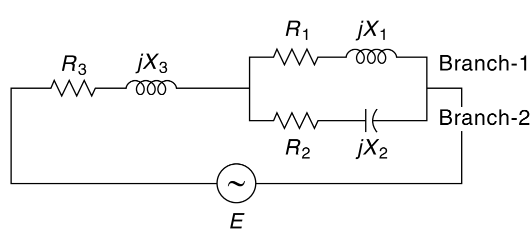

Fig. 5.38 *Series parallel ac circuit*

For branch 1:

$$Y_1 = \frac{1}{Z_1} = \frac{1}{R_1 + j\,X_1}$$

For branch 2: $Y_2 = \dfrac{1}{Z_2} = \dfrac{1}{R_2 - j\,X_2}$

The admittance for parallel circuits 1 and 2 is obtained as

$$Y_{12} = Y_1 + Y_2 = \frac{1}{R_2 + j\,X_1} + \frac{1}{R_2 - j\,X_2}.$$

and impedance $(Z_{12}) = \dfrac{1}{Y_{12}}$.

Total impedance of series parallel ac circuit

$$Z = Z_{12} + Z_3 = Z_{12} + (R_3 + jX_3)$$

Thus, current $I = \dfrac{E}{Z}$.

5.36 The voltage across a circuit is given by (300 + *j*60) V and the current through it by (10 – *j*5) A. Determine the (a) active power, (b) reactive power and (c) apparent power.

Solution

$$V = 300 + j\,60 = \sqrt{(300)^2 + (60)^2}\ \angle\tan^{-1}\frac{60}{300}$$

$$= 305.94\angle 11.31°\ \text{V}$$

$$I = 10 - j5 = \sqrt{(10)^2 + (5)^2}\ \angle -\tan^{-1}\frac{5}{10} = 11.18\angle -26.56°\ \text{A}.$$

Angle between voltage and current is θ = 11.31° – (–26.56°) = 37.87°, and current is lagging with respect to the voltage.

(a) Active power ($VI\cos\theta$) = 305.94 × 11.18 cos 37.87°
= 2700 W = 2.7 kW.

(b) Reactive power ($VI\sin\theta$) = 305.94 × 11.18 sin 37.87°
= 2099.68 VAR
= 2.099 KVAR (lagging).

(c) Apparent power (VI) = 305.94 × 11.18 = 3420.4 VA = 3.42 kVA.

5.37 Three impedances (4 – *j*6) Ω, (6 + *j*8) Ω and (5 – *j*3) Ω are connected in parallel. Calculate the current in each branch when the total supply current is 20 A.

Solution

$$Z_1 = (4 - j6)\ \Omega;\quad y_1 = \frac{1}{Z_1} = \frac{1}{(4-j6)} = \frac{(4-j6)}{(4)^2+(6)^2} = 0.077 + j0.115$$

$$Z_2 = (6 + j8)\ \Omega;\quad y_2 = \frac{1}{Z_2} = \frac{1}{6+j8} = \frac{6-j8}{(6)^2+(8)^2} = 0.06 - j0.08$$

$$Z_3 = (5 - j3)\ \Omega\quad y_3 = \frac{1}{Z_3} = \frac{1}{5-j3} = \frac{5+j3}{(5)^2+(3)^2} = 0.147 + j0.088$$

Total admittance $y = y_1 + y_2 + y_3$ = (0.077 + 0.06 + 0.147) + *j* (0.115 – 0.08 + 0.088)
= 0.284 + *j* 0.123 = 0.31∠23.4°.

$$\text{Supply voltage } V = \frac{I}{y} = \frac{20}{0.31\angle 23.4°} = 64.5\angle -23.4°\ \text{V}.$$

$$I_1 = Vy_1 = 64.5\angle -23.4°(0.077 + j0.115)$$
$$= 64.5\angle -23.4° \times 0.138\angle 56.19°$$
$$I_2 = Vy_2 = 64.5\angle -23.4°\ (0.06 - j0.08)$$
$$= 64.5\angle -23.4° \times 0.1\angle -53.13°$$
$$I_3 = Vy_3 = 64.5\angle -23.4°(0.147 + j0.088)$$
$$= 64.5\angle -23.4° \times 0.171\angle 30.9°$$

i.e.,
$$I_1 = 8.9\angle 32.79°\ \text{A}$$
$$I_2 = 6.45\angle -76.5°\ \text{A}$$
$$I_3 = 11.03\angle 7.5°\ \text{A}.$$

5.38 Find the value of unknown reactance '*X*' so that p.f. of the circuit will be unity in Fig. 5.39. Also calculate the current drawn from the supply.

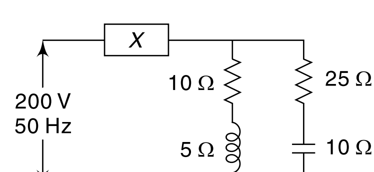

Fig. 5.39 *Circuit diagram for Ex. 5.38*

Solution

The combined impedance of the two parallel branches

$$Z = \frac{(10+j5)(25-j10)}{(10+j5)+(25-j10)} = \frac{(250+50)+j(125-100)}{35-j5}$$

$$= \frac{300+j25}{35-j5} = \frac{(300+j25)(35+j5)}{(35)^2+(5)^2} = \frac{10375+j2375}{1250}$$

$$= (8.3 + j\ 1.9)\ \Omega$$

If the p.f. becomes unity then the net reactance of the circuit should be zero i.e., $X = -j1.9$ or $X = 1.9\ \Omega$ (capacitive).
So, total impedance is 8.3 Ω. Therefore current is 200/8.3 = 24.1 A at u.p.f

5.39 Determine the current drawn by the series parallel circuit shown in Fig. 5.40 and find the overall p.f.

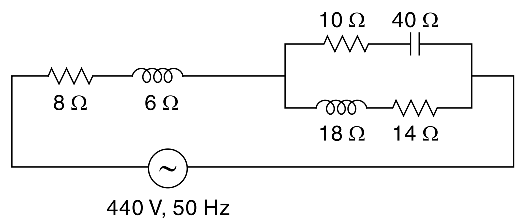

Fig. 5.40 *Circuit diagram for Ex. 5.39*

Solution

The equivalent impedance of the two parallel branches is

$$\frac{(10-j40)(14+j18)}{(10-j40)+(14+j18)} = \frac{860-j380}{24-j22}$$

$$= 27.358 + j9.24\ \Omega.$$

$\therefore$ The impedance of the whole circuit $= 8 + j6 + 27.358 + j9.24$
$= 35.358 + j15.24$
$= 38.5\angle 23.32°$.

Hence the current drawn by the circuit $= \dfrac{440}{38.5\angle 23.3°} = 11.43\angle -23.3°$.

Overall power factor (= cos 23.3° lag) = 0.918 lag.

5.40 In the circuit shown in Fig. 5.41 determine what voltage of 50 Hz frequency is to be applied across *AB* that will cause a current of 10 A to flow in the capacitor?

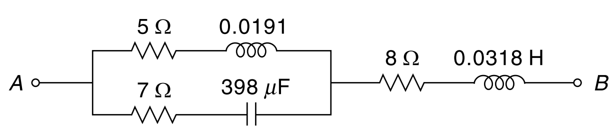

Fig. 5.41 *Circuit diagram for Ex. 5.40*

Solution

The combined impedance of the two parallel branches is

$$\frac{(5+j2\pi\times 50\times 0.0191)\times\left(7+\dfrac{10^6}{j2\pi\times 50\times 398}\right)}{(5+j2\pi\times 50\times 0.0191)+\left(7+\dfrac{10^6}{j2\pi\times 50\times 398}\right)}$$

$$= \frac{(5+j6)(7-j8)}{5+j6+7-j8} = \frac{35+48+j42-j40}{12-j2} = \frac{83+j2}{12-j2}$$

$$= \frac{83.024\angle 1.38°}{12.16\angle -9.46°} = 6.83\angle 10.84° = 6.7 + j1.28\ \Omega.$$

The impedance of the whole circuit

$$(Z) = 6.7 + j\,1.28 + 8 + j\,2\pi \times 50 \times 0.0318$$
$$= 14.7 + j(1.28 + 10)$$
$$= 14.7 + j11.28$$
$$= 102.34\angle 81.74°.$$

The impedance of the capacitor branch = $7 - j8\ \Omega$

The voltage across this branch = $10 \times (7 - j8) = (70 - j80)$ V.

$\therefore$ the current in the other parallel branch

$$= \frac{70 - j80}{5 + j6} = \frac{106.3\angle -48.8°}{7.81\angle 50.19°} = 13.61\angle -99° = -2.129 - j13.44.$$

Thus total current $= 10 - 2.129 - j13.44$

$$= 7.87 - j13.44 = 15.57\angle -59.65°$$

The voltage across the third branch is $= 15.57\angle -59.65° \times (8 + j10)$

$$= 15.57\angle -59.65° \times 12.8\angle 51.34°$$
$$= 200\ \angle -8.31° = (198 - j28)\text{ V}.$$

Hence the supply voltage is $[(70 - j\,80) + (198 - j28)]$ V,

i.e., $(268 - j\,108)$ V or, $288.9\angle -21.95°$ V.

[Note that in this problem we have assumed 10 A current to be the reference phasor having angle $10\angle 0°$. Other currents and voltages are expressed accordingly.]

5.41 Two circuits having the same numerical value of impedance are connected in parallel. The p.f. of one circuit is 0.8 (lead) and the other is 0.6 (lead). What is the p.f. of the combination?

Solution

Let the numerical value of impedance be Z. So Impedance of one circuit is $Z_1 = Z(\cos\theta_1 + j\sin\theta_1)$ and that of the second circuit is $Z_2 = Z(\cos\theta_2 + j\sin\theta_2)$.

However, $\cos\theta_1 = 0.8$ and $\cos\theta_2 = 0.6$

$\therefore$ $\sin\theta_1 = 0.6$ and $\sin\theta_2 = 0.8$.

Hence, $Z_1 = Z(0.8 + j\,0.6)$ and $Z_2 = Z(0.6 + j\,0.8)$.

Now net impedance $= \dfrac{Z_1 Z_2}{Z_1 + Z_2} = \dfrac{(0.8 + j0.6)\,(0.6 + j0.8)}{0.8 + j0.6 + 0.6 + j0.8}$

$$= \frac{1\angle 36.86° \times 1\angle 53.13°}{1.4 + j1.4} = \frac{1\angle 90°}{2.56\ \angle 45°} = 0.31\angle 45°$$

So the p.f. of the circuit is cos 45° = 0.707 (lead).

5.42 A circuit with two branches having admittances $y_1 = 0.16 + j0.12$ and $y_2 = -j0.15$ are in parallel and connected to a 100 V supply. Find the total loss and phase relationship between the branch currents and the supply current.

Solution

$$y_1 = (0.16 + j\,0.12)\text{ S}$$
$$y_2 = (-j\,0.15)\text{ S}$$
$$I_1 = Vy_1 = 16 + j12 = 20\angle 36.87°\text{ A}$$

and $I_2 = Vy_2 = -j15 = 15\angle -90°$ A.

Total current $I = I_1 + I_2 = 16 + j12 - j15 = 16 - j3$

$$= 16.28\angle -10.62°\text{ A}.$$

$\therefore$ total loss $= VI\cos\theta$

$$= 100 \times 16.28 \cos 10.62°$$
$$= 1600\text{ W}.$$

I_1 leads I by $(36.87° + 10.62°) = 47.5°$
amd I_2 lags I by $(90° – 10.62°) = 79.38°$.

5.43 A small single phase 240 V induction motor is tested in parallel with a 160 Ω resistor, the motor takes 2 A and the total current is 3 A. Find the power and p.f. of (a) the whole circuit and (b) the motor.

Solution

(a) The current in the resistor is $(I_1) = \dfrac{240}{160} = 1.5$ A.

The impedance of the motor is $\dfrac{240}{2} = 120\ \Omega$.

Let the motor current is $(a + jb)$ A.

$\therefore \qquad a^2 + b^2 = 2^2 = 4.$

The total current is $(a + jb + 1.5)$

or $\qquad (a + 1.5)^2 + b^2 = 3^2 = 9$

or $\qquad 4 + 2.25 + 3a = 9 \qquad [\therefore\ a^2 + b^2 = 4]$

or $\qquad 3a = 2.75$ or, $a = 0.917$

and $\qquad b = \pm\sqrt{4 - (0.917)^2} = 1.78.$

In induction motor current is lagging. So $b = –1.78$.

The motor current is thus $(0.917 – j1.78)$ A.

The total current $= 1.5 + 0.917 – j1.78$

$= 2.417 – j\ 1.78$

$= 3\angle{–36.37°}$ A.

p.f. of the whole circuit is cos 36.37 = 0.8 lagging.

Power of the whole circuit is $VI \cos\theta = 240 \times 3 \times 0.8 = 576$ W.

(b) p.f. of the motor is $\cos\left(\tan^{-1}\dfrac{1.78}{0.917}\right)$ lagging = 0.458 lagging.

Power of the motor is $240 \times 2 \times 0.458 = 220$ W.

5.44 Find the phase angle of the input impedance of a series circuit consisting of a 500 Ω resistor, a 60 mH inductor and a 0.053 μF capacitor at frequencies of (a) 2000 Hz. and (b) 4000 Hz.

Solution

(a) Phase angle of impedance $\theta = \tan^{-1}\dfrac{\text{Reactance}}{\text{Resistance}} = \tan^{-1}\dfrac{X_L - X_C}{R}$

When $f = 2000$ Hz,

$$(X_L - X_C) = 2\pi \times 2000 \times 60 \times 10^{-3} - \frac{10^6}{2\pi \times 2000 \times 0.53}$$

$$= 753.6 - 1502 = -748.6$$

so $\qquad \theta = \tan^{-1}\dfrac{-748.6}{500} = -56.26°.$

(b) When $f = 4000$ Hz

$$X_L - X_C = 2\pi \times 4000 \times 60 \times 10^{-3} - \frac{10^6}{2\pi \times 4000 \times .053}$$

$$= 1507.2 - 751$$

$$= 756.$$

Therefore phase angle $\theta = \tan^{-1}\dfrac{756}{500} = 56.52°.$

5.45 A resistor R in series with a capacitor C is connected to a 50 Hz. 240 V supply. Find the value of C so that R absorbs 300 W at 100 V. Find also the maximum charge and the maximum stored energy in C.

Solution

Supply voltage = 240 V
Voltage across R is 100 V.
Power across R is 300 W

$\therefore$ Current through (R) is $\dfrac{300}{100} = 3$ A

Voltage across capacitor = $\sqrt{(240)^2 - (100)^2} = 218.17$ V.

Hence, maximum voltage (V_m) = $\sqrt{2} \times 218.17 = 308.54$ V.
Thus maximum charge is ($C\ V_m$)

Now, capacitive reactance $X_C = \dfrac{218.17}{3} = 72.72\ \Omega$.

Hence, $C = \dfrac{1}{72.72 \times 2\pi \times 50}$ F $= 43.79\ \mu$F

Maximum charge ($= CV_m$) $= 43.79 \times 10^{-6} \times 308.54 = 0.0135$ C.

Maximum energy stored = $\dfrac{1}{2} C V_m^2 = \dfrac{1}{2} \times 43.79 \times 10^{-6} \times (308.54)^2 = 2.08$ J.

5.14 SERIES RESONANCE

An ac circuit is said to be in resonance when the circuit current is in phase with the applied voltage. So the power factor of the circuit becomes unity at resonance and the impedance of the circuit consists of only resistance.

In the series RLC circuit in Fig. 5.42 the impedance is given by

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

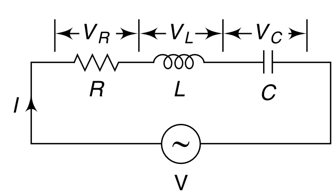

Fig. 5.42 ***RLC series circuit***

The current in the circuit is

$$I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}.$$

The power factor is: $\cos\theta = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$.

At resonance $Z = R$

$$\therefore \qquad \omega L - \frac{1}{\omega C} = 0$$

$$\text{or} \qquad \omega = \frac{1}{\sqrt{LC}}$$

If f_o be the resonant frequency then

$$2\pi f_o = \frac{1}{\sqrt{LC}}$$

or
$$f_o = \frac{1}{2\pi\sqrt{LC}}.$$

The p.f. at resonance: $\cos\theta = \frac{R}{Z} = \frac{R}{R} = 1$ [$\because$ at resonance, $Z = R$]

and the current $I = \frac{V}{Z} = \frac{V}{R}$.

Properties of Series Resonant Circuits

(a) The circuit impedance Z is minimum and equal to the circuit resistance R.
(b) The power factor is unity.
(c) The circuit current $I = \frac{V}{R}$, and the current is maximum.
(d) The power dissipated is maximum, i.e. $P_o = \frac{V^2}{R}$,
(e) The resonant frequency is $f_o = \frac{1}{2\pi\sqrt{LC}}$,
(f) The voltage across inductor is equal and opposite to the voltage across capacitor.

Since the circuit current is maximum at resonance it produces large voltage drops across L and C. But as these voltages are equal and opposite to each other so the net voltage across L and C is zero however large the current is flowing. If R is not present then the circuit would act like a short circuit at resonant frequency. Hence a series circuit is sometimes called an *acceptor circuit* and the series resonance is often referred to as the *voltage resonance*. Figure 5.43 represents variation of R, X_L, X_C, Z and I with frequency. As R is the independent of frequency so it is a straight line. Inductive reactance X_L is directly proportional to the frequency so it is a straight line passing through origin. Capacitive reactance X_C is inversely proportional to frequency and its characteristic is a rectangular hyperbola. At resonant frequency Z is minimum and I is maximum.

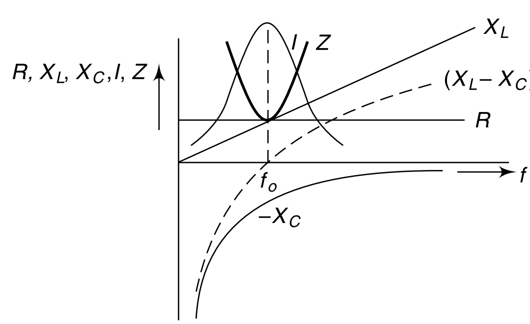

Fig. 5.43 *Variation of R, X_L, X_C, Z, I with frequency*

5.15 *Q* FACTOR IN SERIES RESONANCE

Q factor of a series *RLC* circuit is defined as the voltage magnification produced in the circuit at resonance.

Voltage magnification is the ratio of voltage drop across the inductor or capacitor to the voltage drop across the resistor.

Hence, $$Q \text{ factor} = \frac{IX_L}{IR} = \frac{\omega_o L}{R} = \frac{2\pi f_o L}{R} = \frac{2\pi \dfrac{1}{2\pi\sqrt{LC}} L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}.$$

Q factor is also referred as the *magnification factor* of the circuit.

5.16 DIFFERENT ASPECTS OF RESONANCE

5.16.1 Variation of Current and Voltage Across L and C with Frequency

The voltage across the capacitor is $V_C = I \cdot \dfrac{1}{\omega C}$

In an RLC series circuit $I = \dfrac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$

$$\therefore \qquad V_C = \frac{V}{\omega C\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}$$

or $$V_C^2 = \frac{V^2}{\omega^2 C^2\left\{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2\right\}}$$

$$= \frac{V^2}{\omega^2 C^2\left\{R^2 + \dfrac{(\omega^2 LC - 1)^2}{\omega^2 C^2}\right\}} = \frac{V^2}{R^2\omega^2 C^2 + (\omega^2 LC - 1)^2}.$$

To find the frequency at which V_C is maximum $\left(\dfrac{dV_C}{d\omega}\right)$ should be zero i.e., $\left(\dfrac{dV_C^2}{d\omega}\right)$ should be zero.

As $$\frac{dV_C^2}{d\omega} = 0, \text{ we have}$$

$$\frac{dV_C^2}{d\omega} = V^2\left[\frac{-\{2\omega C^2 R^2 + 2(\omega^2 LC - 1)\, 2\omega LC\}}{\{R^2\omega^2 C^2 + (\omega^2 LC - 1)^2\}^2}\right] = 0$$

or $\quad 2\omega C^2 R^2 + (2\omega^2 LC - 2)\, 2\omega LC = 0$

or $\quad \omega^2 = \dfrac{1}{2L^2 C}(2L - CR^2) = \dfrac{1}{LC} - \dfrac{R^2}{2L^2}.$

Hence the frequency at which V_C is maximum is $f_C = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{2L^2}}.$

To find the frequency at which voltage across L is maximum, we have

$$\frac{dV_L}{d\omega} = 0 \quad \text{or,} \quad \frac{dV_L^2}{d\omega} = 0.$$

Now,
$$V_L = IX_L = \frac{V}{\sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}}\,\omega L$$

$$V_L^2 = \frac{V^2\omega^2 L^2}{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2} = \frac{V^2\omega^4 L^2 C^2}{\omega^2 C^2 R^2 + (\omega^2 LC - 1)^2}$$

By differentiating (V_L^2) with respect to ω and setting $\dfrac{dV_L^2}{d\omega} = 0$, we have on simplification,

$$2\omega^2 LC - \omega^2 C^2 R^2 - 2 = 0$$

or
$$\omega^2[2LC - C^2R^2] = 2$$

or
$$\omega^2 = \frac{2}{2LC - R^2C^2} = \frac{1}{LC - \dfrac{R^2C^2}{2}}$$

or
$$\omega = \frac{1}{\sqrt{LC - \dfrac{R^2C^2}{2}}}.$$

The variation of voltages across the capacitor and the inductor with frequency are shown in Fig. 5.44. It is seen that maximum value of V_C occurs at f_C below f_o (resonant frequency) and maximum value of V_L occurs at f_L which is above f_o.

If R is very small then f_L, f_C and f_o correspond to a single value of freqnency $f_o = \dfrac{1}{2\pi\sqrt{LC}}$.

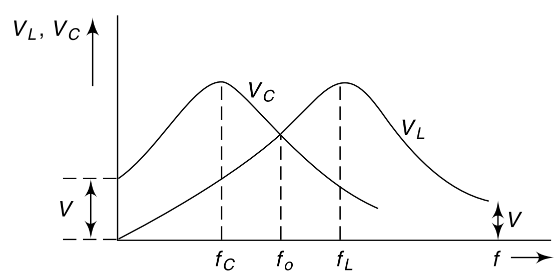

Fig. 5.44 ***Variation of voltage across L and C with frequency***

5.16.2 Effect of Resistance on the Frequency Response Curve

Figure 5.45 shows the nature of resonance curve for different values of R.

As seen from the Fig. 5.45 that when the resistance is small the curve rises steeply while with large resistance value the curve has a low peak.

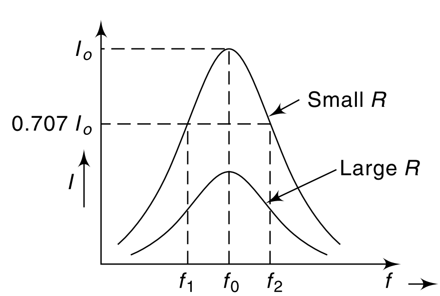

Fig. 5.45 ***Effect of resistance on frequency response curve***

A circuit with a flat frequency response curve will be more responsive and so less selective at frequencies in the neighbourhood of the resonant frequency. On the other hand, a circuit for which the curve has a tall narrow peak will be less responsive and so more selective at frequencies in the neighbourhood of resonant frequency.

5.16.3 Bandwidth and Selectivity in Series Resonance Circuit

The bandwidth of a given circuit is given by the band of frequencies which lies between two points on either side of resonant frequency where the current is $1/\sqrt{2}$ times of the current at resonance and hence the power is half of the power at resonance.

Figure 5.46 shows the variation of circuit current with frequency and this curve is known as the resonance curve. f_1 and f_2 are known as half power frequency where current is $I_o/\sqrt{2}$ (I_o is the current at resonance). f_1 and f_2 are also called *corner* or *edge frequencies*. The power at the two points of frequencies f_1 and f_2 is

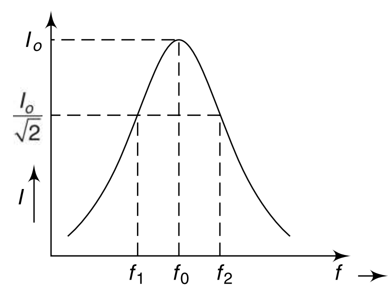

Fig. 5.46 *Variation of current with frequency*

$$P_1 = P_2 = I^2R = \left(\frac{I_o}{\sqrt{2}}\right)^2 R = \frac{I_o^2 R}{2} = \frac{1}{2} \times \text{(power at resonance)}$$

Now $$(I) = \frac{V}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \text{ at any frequency } (\omega)$$

At half power frequencies $I = \dfrac{I_o}{\sqrt{2}} = \dfrac{V}{R\sqrt{2}} \qquad \left(\text{As } I_o = \dfrac{V}{R}\right)$

Squaring both sides and comparing above expressions we have

$$I^2 = \frac{V^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \frac{V^2}{2R^2}$$

or $$\frac{V^2}{2R^2} = \frac{V^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \text{ or, } \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2.$$

or $$\omega L - \frac{1}{\omega C} = \pm R \quad \text{or} \quad \omega = \pm \frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}}\,.$$

As $\left(\dfrac{R^2}{4L^2}\right)$ is much less than $\left(\dfrac{1}{\sqrt{LC}}\right)$ so,

$$(\omega) = \pm\frac{R}{2L} \pm \frac{1}{\sqrt{LC}} = \pm\frac{R}{2L} \pm \omega_o\,.$$

Since only positive values of ω_o is considered so

$$\omega = \pm\frac{R}{2L} + \omega_o$$

or $$\omega_1 = \omega_o - \frac{R}{2L} \quad \text{and} \quad \omega_2 = \omega_o + \frac{R}{2L}$$

or $$f_1 = f_o - \frac{R}{4\pi L} \quad \text{and} \quad f_2 = f_o + \frac{R}{4\pi L}.$$

Bandwidth $$(\Delta\omega) = \omega_2 - \omega_1 = \frac{R}{L} \text{ rad/s},$$

and $$(\Delta f) = f_2 - f_1 = \frac{R}{2\pi L} \text{ Hz}.$$

The ratio of the bandwidth to the resonant frequency is defined as the *selectivity* of the circuit. Thus selectivity $= \dfrac{(f_2 - f_1)}{f_0}$.

Thus, *narrower the bandwidth higher is the selectivity property of the circuit.*

5.46 A circuit consists of a coil of resistance 100 Ω and inductance 1 H in series with a capacitor of capacitance 1 μF. Calculate (a) the resonant frequency, (b) current at resonant frequency and (c) voltage across each element when the supply voltage is 50 V.

Solution

Resistance $R = 100\ \Omega$
Inductance $L = 1$ H
Capacitance $C = 1 \times 10^{-6}$ F.

(a) Resonant frequency $$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{1\times1\times10^{-6}}} \text{ Hz} = 159 \text{ Hz}.$$

(b) Current at resonant frequency $I_o = \dfrac{V}{R} = \dfrac{50}{100} = 0.5$ A.

(c) Voltage across resistance $V_R = I_o \times R = 0.5 \times 100 = 50$ V.

$\therefore$ Voltage across inductance $$V_L = I_o X_L = 0.5 \times 2\pi f_o L = 0.5 \times 2\pi \times 159 \times 1 = 500 \text{ V}$$

and voltage across capacitance $V_C = I_o X_C = 0.5 \times \dfrac{10^6}{2\pi \times 159 \times 1} = 500$ V.

5.47 An inductive coil is connected in series with a 8 μF capacitor. With a constant supply voltage of 400 V the circuit takes minimum current of 80 A when the supply frequency is 50 Hz. Calculate the (a) resistance and inductance of the coil and (b) voltage across the capacitor.

Solution

Supply voltage $V = 400$ V

The current is minimum at 50 Hz, so the resonant frequency is 50 Hz and the current at resonant frequency $(I_o) = 80$ A.

Capacitance $C = 8 \times 10^{-6}$ F.

Hence, resistance $R = \dfrac{V}{I_o} = \dfrac{400}{80} = 5\ \Omega$.

At resonance,

$$X_L = X_C$$

or $$\omega L = \frac{1}{\omega C}$$

or $$\omega = \frac{1}{\sqrt{LC}}$$

or $$2\pi f = \frac{1}{\sqrt{L \times 8 \times 10^{-6}}}$$

or $$2\pi \times 50 = \frac{10^3}{\sqrt{8L}}$$

or $$L = \left(\frac{10^3}{2\pi \times 50}\right)^2 \times \frac{1}{8} = \left(\frac{10}{\pi}\right)^2 \times \frac{1}{8} = 1.266 \text{ H}$$

5.48 The resistor and a capacitor are connected in series with a variable inductor. When a circuit is connected to a 240 V, 50 Hz supply, the maximum current by varying the inductance is 0.5 A. At this current the voltage across the capacitor is 250 V. Calculate R, C and L.

Solution

Supply voltage $V = 240$ V.
Resonant frequency $(f_o) = 50$ Hz.
Current at resonant frequency $I_o = 0.5$ A.
Voltage across capacitor $V_C = 250$ V
Now if X_C and X_L be the capacitive and inductive reactance then

$$I_o X_C = 250$$

and $$I_o X_L = 250.$$

At resonance, $$X_C = X_L = \frac{250}{0.5} = 500\ \Omega$$

Therefore, $$\frac{1}{2\pi f_o C} = 2\pi f_o L = 500$$

or $$C = \frac{1}{2\pi \times 50 \times 500} \text{ F} = 6.37\ \mu\text{F},$$

and $$L = \frac{500}{2\pi \times 50} \text{ H} = 1.59 \text{ H}.$$

Resistance $$R = \frac{V}{I_o} = \frac{240}{0.5} = 480\ \Omega.$$

5.49 A constant voltage at a frequency of 1 MHz is applied to an inductor in series with a variable capacitor. When the capacitor is set at 500 pF, the current has its maximum value, and it is reduced to half of its maximum value when the capacitance is 600 pF. Find (a) the resistance, (b) the inductance and (c) Q factor of the inductor.

Solution

The current is maximum at resonant frequency.

$\therefore \quad f_o = 10^6$ Hz.

It is given that is $C = 500 \times 10^{-12}$ F.

Now, $$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{500 \times 10^{-12}\, L}} = 10^6$$

or $$\frac{10^6}{2\pi\sqrt{500L}} = 10^6$$

or $$2\pi\sqrt{500L} = 1$$

or $$L = \left(\frac{1}{2\pi}\right)^2 \times \frac{1}{500}\,\text{H} = 0.05 \text{ mH}$$

when $C = 600 \times 10^{-12}$ F.

$\therefore$ Capacitive reactance $X_C = \dfrac{1}{2 \times 10^6 \times 600 \times 10^{-12}} = \dfrac{10^6}{2\pi \times 600} = 265\ \Omega$.

Inductive reactance $X_L = 2\pi f_o L = 2\pi \times 10^6 \times 0.05 \times 10^{-3} = 314\ \Omega$.

$\therefore$ Net reactance is $(X_L - X_C) = 314 - 265 = 49\ \Omega$.

Current $I = \dfrac{I_o}{2}\left(= \dfrac{V}{2R}\right)$, where R is the resistance of the circuit and I_o is the current at resonance.

So, $$I = \frac{V}{2R} = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

or $$2R = \sqrt{R^2 + (X_L - X_C)^2}$$

or $$4R^2 = R^2 + (49)^2$$

or $$3R^2 = (49)^2$$

or $$R = 28.29\ \Omega$$

$\therefore$ Q factor of the inductor is $\dfrac{\omega_o L}{R} = \dfrac{2\pi \times 10^6 \times 0.05 \times 10^{-3}}{28.29} = 11.1.$

5.50 A series resonant circuit has an impedance of 500 Ω at resonant frequency and cut off frequencies are 10 kHz and 100 kHz. Determine (a) the resonant frequency, (b) value of R, L and C, (c) quality factor at resonant frequency and (d) p.f. of the circuit at resonant frequency.

Solution

At resonance

Impedance = Resistance or $Z_o = R = 500\ \Omega$.

$$f_1 = 10 \times 10^3 \text{ Hz} \ \& \ f_2 = 100 \times 10^3 \text{ Hz}.$$

$$(f_2 - f_1) = 90 \times 10^3 = \frac{R}{2\pi L} = \frac{500}{2\pi L}.$$

Now $$L = \frac{500}{2\pi \times 90 \times 10^3} = 0.88 \text{ mH}$$

Again, $$f_1 = f_o - \frac{R}{4\pi L}.$$

$\therefore$ Resonant frequency $f_o = f_1 + \dfrac{R}{4\pi L} = 10 \times 10^3 + \dfrac{500}{4\pi \times 0.88 \times 10^{-3}} = 55$ kHz.

As $$f_o = \frac{1}{2\pi\sqrt{LC}},$$

so $$C = \left(\frac{1}{2\pi \times 55 \times 10^3}\right)^2 \times \frac{1}{0.88 \times 10^{-3}} = 0.095 \times 10^{-7} \text{ F}$$

$$Q \text{ factor} = \frac{2\pi f_o L}{R} = \frac{2\pi \times 55 \times 10^3 \times 0.88 \times 10^{-3}}{500} = 0.61$$

and the p.f. of the circuit at resonant frequency is 1.

5.51 A series RLC circuit has $R = 10\ \Omega$, $L = 0.1$ H and $C = 8\ \mu$F. Determine (a) the resonant frequency (b) Q factor of the circuit at resonance (c) half power frequencies.

Solution

(a) Resonant frequency $f_o = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.1 \times 8 \times 10^{-6}}} = 178$ Hz.

(b) $Q_{\text{factor}} = \dfrac{2\pi f_o L}{2} = \dfrac{2\pi \times 178 \times 0.1}{10} = 11.18.$

(c) Lower half power frequency $(f_1) = f_o - \dfrac{R}{4\pi L}$

$$= 178 - \frac{10}{4\pi \times 0.1} = 178 - 7.96 = 170 \text{ Hz}.$$

Upper half power frequency $(f_2) = 178 + 7.96 = 186$ Hz.

5.52 Calculate the half power frequencies of a series resonant circuit when the resonant frequency is 150×10^3 Hz and the bandwidth is 75 kHz.

Solution

Resonant frequency $f_o = 150 \times 10^3$ Hz.

Bandwidth $(\Delta f) = \dfrac{R}{2\pi L} = 75 \times 10^3.$

Lower half power frequency $\left(f_o - \dfrac{R}{4\pi L}\right) = \left(150 - \dfrac{75}{2}\right)10^3 = 112.5$ kHz.

Upper half power frequency $\left(f_o + \dfrac{R}{4\pi L}\right) = \left(150 + \dfrac{75}{2}\right)10^3$ Hz. $= 187.5$ kHz.

5.53 A 25 μF condensor is connected in series with a coil having an inductance of 5 mH. Determine the frequency of resonance, resistance of the coil if a 25V source operating at resonance frequency causes a circuit current of 4 mA. Determine the Q factor of the coil.

Solution

$C = 25 \times 10^{-6}$ F $\quad L = 5 \times 10^{-3}$ H

$I_o = 4 \times 10^{-3}$ A $\quad V = 25$ V.

$$\text{Frequency of resonance } (f_o) = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{5\times10^{-3}\times 25\times10^{-6}}}$$

$$= 450 \text{ Hz.}$$

$$\therefore \text{ Resistance of the circuit} = \frac{V}{I_o} = \frac{25}{4\times10^{-3}} = 6250\ \Omega$$

and $$Q_{\text{factor}} = \frac{2\pi f_o L}{R} = \frac{2\pi\times450\times5\times10^{-3}}{6250} = 2.26\times10^{-3}.$$

5.17 RESONANCE IN PARALLEL CIRCUIT

Let us consider a circuit where a capacitance C is connected in parallel with an inductive coil of resistance R and inductive reactance X_L as shown in Fig. 5.47.

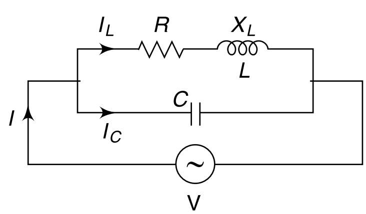

Fig. 5.47 *AC parallel circuit*

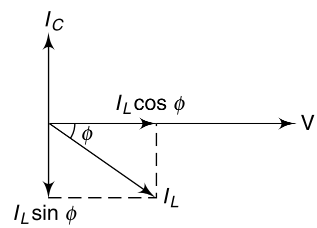

Fig. 5.48 *Branch currents of Fig. 5.47*

If I_L be the current through the coil, I_C be the current through the capacitor and the total current is I, then the vector diagram is shown in Fig. 5.48.

From Fig. 5.48 it is clear that under resonance as the p.f. is unity the reactive component of the total current is zero. The reactive component of the current $(I_C - I_L \sin\phi) = 0$, where ϕ is the power factor angle of the coil. Therefore

$$I_C = I_L \sin\phi$$

or $$\frac{V}{X_C} = \frac{V}{Z_L}\times\frac{X_L}{Z_L}$$

where Z_L is the impedance of the coil and $\left[Z_L = \sqrt{R^2 + X_L^2}\right]$

or $$X_C X_L = Z_L^2$$

or $$\frac{\omega L}{\omega C} = Z_L^2 = R^2 + \omega^2 L^2$$

or $$\omega^2 L^2 = \frac{L}{C} - R^2$$

or $$\text{at resonance } \omega_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

and resonant frequency $$f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}.$$

If resistance is neglected then

$$\omega_o = \frac{1}{\sqrt{LC}}$$

and
$$f_o = \frac{1}{2\pi\sqrt{LC}}.$$

Thus *if resistance is neglected the resonant frequency of the parallel circuit is equal to that of series circuit*. Also at resonance the net susceptance is zero.

$$\text{Net susceptance} = \left(\omega C - \frac{1}{\omega L}\right) = 0$$

$\therefore$
$$\omega_o = \frac{1}{\sqrt{LC}}.$$

As the net reactive component of the current is zero at resonance so the supply current I is equal to the active component of the current.

So
$$I = I_L \cos\phi = \frac{V}{Z_L}\frac{R}{Z_L} = \frac{VR}{Z_L^2} = \frac{VR}{L/C} \qquad (\because Z_L^2 = X_C \cdot X_L = L/C)$$

from previous equation

or
$$I = \frac{V}{L/CR}.$$

Thus at resonance the net impedance is given by L/CR and is known as the *dynamic impedance* of the parallel circuit at resonance. This impedance is resistive only.

The current is minimum at resonance as its reactive part is zero and thus, (L/CR) represent the maximum impedance of the circuit. It is called a *rejector circuit*.

5.18 PROPERTIES OF PARALLEL RESONANT CIRCUITS

(a) At resonance the net reactive component of the line current is zero and the circuit current is equal to the active component of the total current, i.e. $I = I_L \cos\phi$.

(b) The line current is minimum at resonance or $I = \dfrac{V}{L/CR}$.

(c) The power factor is unity at resonance.

(d) Net susceptance is zero at resonance i.e $\left(\omega C - \dfrac{1}{\omega L}\right) = 0$.

(e) The resonant frequency is $f_o = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$.

Since the current at resonance is minimum hence such a circuit is sometimes known as *rejector circuit* because it rejects (or takes minimum current) at resonant frequency. This resonance is often referred to as current resonance because the current, circulating between the two branches, is many times greater than the line current taken from the supply.

Figure 5.49 represents the variation of R, B_L (inductive susceptance), B_C (capacitive susceptance) Z and I with frequency. As R is independent of frequency so it is a straight line. The capacitive susceptance ($B_C = \omega C$) is a straight line passing through the origin and the characteristic of inductive susceptance $\left(B = -\dfrac{1}{\omega L}\right)$ is a rectangular hyperbola. At resonance I is minimum and so Z is maximum.

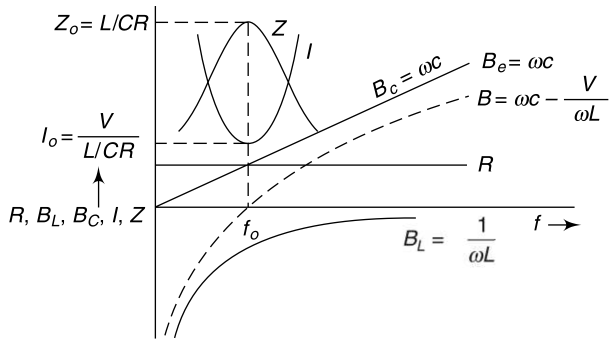

Fig. 5.49 *Variation of R, B_L, B_C, Z, I with frequency*

5.19 *Q* FACTOR IN PARALLEL CIRCUIT

It is defined as the ratio of the current, circulating between the two branches of the parallel circuit to the line current.

$$\therefore \qquad Q_{\text{factor}} = \frac{\frac{V}{XC}}{I_o} = \frac{I_C}{I_o} = \frac{V\omega C}{I_o}$$

$$I_o = \frac{V}{L/CR}.$$

Therefore $$Q_{\text{factor}} = \frac{\omega CL}{CR} = \frac{\omega L}{R}.$$

Now $\quad Q_{\text{factor}}$ at resonance is $\left(\dfrac{\omega_o L}{R}\right) = \dfrac{1}{\sqrt{LC}}\,\dfrac{L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$.

In a series circuit Q_{factor} gives the voltage magnification while in a parallel circuit it gives the current magnification.

5.20 PARALLEL RESONANCE IN RLC CIRCUIT

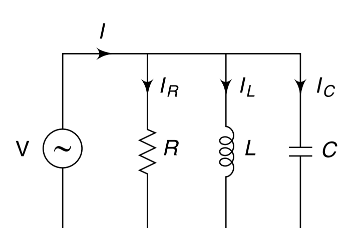

Fig. 5.50 *RLC parallel circuit*

In the circuit of Fig. 5.50 the resonance occurs when the net susceptance is zero.

Admittance $y = G + jB = \dfrac{1}{R} + j\,\omega\,C - j\,\dfrac{1}{\omega L}$

At resonance net suseptance $\left(\omega C - \dfrac{1}{\omega L}\right) = 0$

$$\therefore \qquad \omega_o = \frac{1}{\sqrt{LC}}$$

and
$$(f_o) = \frac{1}{2\pi\sqrt{LC}}.$$

At resonant frequency (f_o) the admittance is minimum so the impedance is maximum and the current is minimum.

5.21 PARALLEL RESONANCE IN *RC-RL* CIRCUIT

A parallel combination of *RL* and *RC* branches connected to a source of emf *E* is shown in Fig. 5.51.

The above circuit will produce parallel resonance when the resultant current is in phase with the applied voltage or the net susceptance of the above circuit is zero.

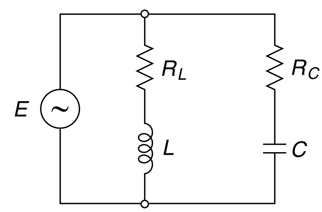

Fig. 5.51 ***RL and RC parallel circuit***

$$\text{Total admittance } Y = y_1 + y_2 = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - j\cdot\dfrac{1}{\omega C}}$$

$$= \frac{R_L - j\omega L}{R_L^2 + \omega^2 L^2} + \frac{R_C - j\cdot\dfrac{1}{\omega C}}{R_C^2 + \dfrac{1}{\omega^2 C^2}}$$

So the net susceptance $\dfrac{-\omega L}{R_L^2 + \omega^2 L^2} + \dfrac{\dfrac{1}{\omega C}}{R_C^2 + \dfrac{1}{\omega^2 C^2}}$

$$\frac{\omega L}{R_L^2 + \omega^2 L^2} = \frac{\omega C}{\omega^2 C^2 R_C^2 + 1}$$

$$\omega^2 R_C^2 C^2 L + L = R_L^2 C + \omega^2 L^2 C$$

or
$$\omega^2 (R_L^2 C^2 L - L^2 C) = R_L^2 C - L$$

or
$$\omega^2 = \frac{R_L^2 C - L}{LC(R_C^2 C - L)} \quad \text{or,} \quad \omega = \sqrt{\frac{1}{LC}\,\frac{R_L^2 C - L}{R_C^2 C - L}}$$

So, Resonant frequency is $\left(\dfrac{1}{\sqrt{LC}}\sqrt{\dfrac{R_L^2 C - L}{R_C^2 C - L}}\right)$ rad/s.

or
$$f_o = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L^2 C - L}{R_C^2 C - L}} \text{ Hz}.$$

5.54 A coil of 10 Ω resistance has an inductance of 0.1 H and is connected in parallel with a 200 μF capacitor. Calculate the frequency at which the circuit will act as a non-inductive resistor of R Ω. Find also the value of R.

Solution

Resistance of coil $R_L = 10\ \Omega$

Inductance of coil $L = 0.1$ H

Capacitance $C = 200 \times 10^{-6}$ F

Resonant frequency $f_o = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}} = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{0.1 \times 200 \times 10^{-6}} - \dfrac{(10)^2}{(0.1)^2}} = 31.8$ Hz.

The value of non-inductive resistor R at resonance is the dynamic impedance of the circuit.

$$\therefore \qquad R = \frac{L}{CR} = \frac{0.1}{200 \times 10^{-6} \times 10} = 50\ \Omega.$$

5.55 In the circuit shown in Fig. 5.52 show that the circulating current at resonance is given by $(I) = V\sqrt{\dfrac{C}{L}}$ for a supply voltage of V volts.

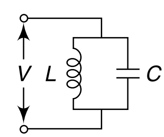

Fig. 5.52 ***Circuit diagram for Ex. 5.55***

Solution

At resonance, Inductive reactance = Capacitive reactance

$$\therefore \qquad X_L = X_C \quad \text{or, } \omega L = \frac{1}{\omega C}$$

Therefore $\qquad \omega_o = \dfrac{1}{\sqrt{LC}}$.

Current through L (or C) $= \dfrac{V}{X_L} = \dfrac{V}{\omega_o L} = \dfrac{V}{\dfrac{1}{\sqrt{LC}} L} = V\sqrt{\dfrac{C}{L}}$.

5.56 Calculate the value of R_C in the circuit shown in Fig. 5.53 which yields resonance.

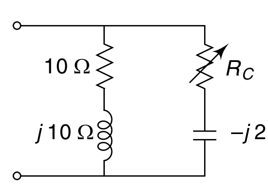

Fig. 5.53 ***Circuit diagram for Ex. 5.56***

Solution

Admittance of the inductive branch

$$y_1 = \frac{1}{10 + j10} = \frac{10 - j10}{100 + 100}$$

Admittance of the capacitive branch

$$y_2 = \frac{1}{R_C - 2j} = \frac{R_C + 2j}{R_C^2 + 4}.$$

Net admittance $y = y_1 + y_2 = \dfrac{10 - j10}{100 + 100} + \dfrac{R_C + 2j}{R_C^2 + 4}$

$$= \left(\frac{10}{200} + \frac{R_C}{R_C^2 + 4}\right) + j\left(\frac{-10}{200} + \frac{2}{R_C^2 + 4}\right).$$

As net susceptance is zero at resonance, so

$$\therefore \qquad \frac{-10}{200} + \frac{2}{R_C^2 + 4} = 0$$

so, $$400 - 40 - 10R_C^2 = 0$$

or $$R_C = \sqrt{\frac{360}{10}} = 6\ \Omega.$$

5.57 Show that no value of R_L in the circuit shown in Fig. 5.54 will make it resonant.

Solution

Net admittance = $\dfrac{1}{R_L + j10} + \dfrac{1}{4 - j5} = \dfrac{(R_L - j10)}{R_L^2 + 100} + \dfrac{(4 + j5)}{16 + 25}$

At resonance net suseptance is 0.

$$\therefore \quad \frac{-10}{R_L^2 + 100} + \frac{5}{16 + 25} = 0$$

or $$\frac{10}{R_L^2 + 100} = \frac{5}{41}$$

or, $$R_L^2 = \frac{41 \times 10}{5} - 100 = (\sqrt{-18})^2.$$

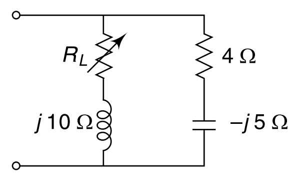

Fig. 5.54 ***Circuit diagram for Ex. 5.57***

This value of R_L is physically impossible and so no value of R_L can make the circuit resonant.

5.58 A 200 V, 50 Hz. source is connected across a 10∠30° Ω branch in parallel with a 10∠–60° Ω branch. Find the impedance of the circuit element which when connected in series with the supply produces resonance.

Solution

$$V = 200 \text{ V}$$
$$Z_1 = 10\angle 30^\circ\ \Omega = 8.66 + j\,5$$
$$Z_2 = 10\angle -60^\circ = 5 - j\,8.66$$

Combined impedance of the parallel branches

$$= \frac{Z_1 Z_2}{Z_1 + Z_2}$$
$$= \frac{10\angle 30^\circ \times 10\angle -60^\circ}{8.66 + j5 + 5 - j8.66}$$
$$= \frac{100\angle -30^\circ}{13.66 - j3.66} = \frac{100\angle -30^\circ}{14.14\angle -15^\circ}$$
$$= 7.07\angle -15^\circ = 6.83 - j\,1.83.$$

At resonance the net reactance of the circuit should be zero. So the element which is connected in series to produce resonance must have reactance of $j1.83$. So inductive reactance $X_L = 1.83\ \Omega$ and inductance

$$L = \frac{X_L}{\omega} = \frac{1.83}{2\pi \times 50} \text{ H} = 5.83 \text{ mH}.$$

5.59 For the circuit shown in Fig. 5.55 find the frequency at which the circuit will be at resonance. If the capacitor and inductor are interchanged what would be the value of new resonant frequency.

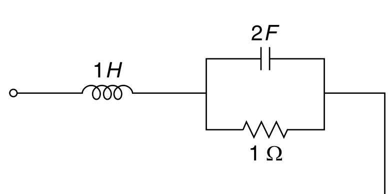

Fig. 5.55 ***Circuit diagram for Ex. 5.59***

Solution

$$\text{Total impedance} = j\omega \times 1 + \frac{1 \times \dfrac{1}{2\,j\omega}}{1 + \dfrac{1}{2\,j\omega}} = j\omega + \frac{1}{1 + 2\,j\omega} = j\omega + \frac{1 - j2\omega}{1 + 4\omega^2}$$

$$= \frac{1}{1+4\omega^2} + j\left(\omega - \frac{2\omega}{1+4\omega^2}\right).$$

$\because$ Net reactance is zero at resonance

Hence, $$\omega - \frac{2\omega}{1+4\omega^2} = 0$$

or $$1 + 4\omega^2 = 2 \quad \text{or,} \quad \omega^2 = \frac{1}{4} \quad \text{or,} \quad \omega = \frac{1}{2} = 0.5 \text{ rad/s}$$

when the capacitor and inductor are interchanged

$$\text{Net impedance is } \frac{1}{2\,j\omega} + \frac{1 \times j\omega}{1 + j\omega} = -\frac{1}{j2\omega} + \frac{j\omega}{1+j\omega}$$

$$= -\frac{j}{2\omega} + \frac{j\omega(1 - j\omega)}{1+\omega^2}$$

$$= -\frac{j}{2\omega} + \frac{j\omega + \omega^2}{1+\omega^2}$$

$$= \frac{\omega^2}{1+\omega^2} + j\left(\frac{\omega}{1+\omega^2} - \frac{1}{2\omega}\right)$$

so $$\frac{\omega}{1+\omega^2} - \frac{1}{2\omega} = 0$$

or $$2\omega^2 = 1 + \omega^2$$

or $$\omega^2 = 1 \quad \text{or,} \quad \omega = 1 \text{ rad/s.}$$

ADDITIONAL PROBLEMS

5.60 A voltage of 400 V is applied across a pure resistor, a pure capacitor and an inductive coil which are in parallel. The resultant current is 6 A and the currents in the above components are 3 A, 4 A and 2 A respectively. Find the power factor of the inductive coil and the power factor of the whole circuit.

Solution

The current in the resistor $I_R = 3$ A.
The current in the capacitor $(I_C) = 4$ A.
The current in the inductive coil $(I_L) = 2$ A.
Let the current in the inductive coil be $(x - jy)$

$\therefore$ $$\sqrt{x^2 + y^2} = 2 \quad \text{or,} \quad x^2 + y^2 = 4. \qquad \text{(i)}$$

Total current $I = 3 + j\,4 + x - jy$

so $$\sqrt{(3+x)^2 + (4-y)^2} = 6 \qquad \text{(ii)}$$

or $$x^2 + y^2 + 6x - 8y + 9 + 16 = 36$$

or $$4 + 6x - 8y = 36 - 25 = 11$$

or $\quad 6x - 8y = 7$

or $\quad x = \dfrac{7+8y}{6}$.

Substituting x in Eq. (1)

$$\left(\frac{7+8y}{6}\right)^2 + y^2 = 4$$

or $\quad 49 + 112y + 64y^2 + 36y^2 = 144$

or $\quad 100y^2 + 112y - 95 = 0$

or $\quad y^2 + 1.12y - 0.95 = 0$

or $\quad y = 0.564$ (taking positive value of y)

so $\quad x = \dfrac{7+8\times 0.564}{6} = 1.92.$

p.f. of the inductive coil = $\dfrac{x}{I_L} = \dfrac{1.92}{2} = 0.96$ lag.

p.f. of the whole circuit = $\dfrac{\text{Real part of total current}}{I} = \dfrac{(3+x)}{6} - = \dfrac{(3+1.92)}{6} = 0.82.$

As the j component of total current is positive, i.e. (4 – 0.564) = 3.436 so the p.f. of the circuit is leading, i.e. p.f. of the whole circuit is 0.82 lead.

5.61 A 400 V single phase ac motor is tested in parallel with a 100 Ω resistor. The motor takes 5 A current at lagging p.f. and the total current is 7 A. Find the p.f. and power of the whole circuit and for the motor alone.

Solution

Current through resistor $\quad (I_R) = \dfrac{400}{100}$ A = 4 A.

Current through motor $\quad (I_m) = 5$ A.

Total current $\quad (I) = 7$ A.

Let the motor current be $(x - jy)$.

$\therefore \quad x^2 + y^2 = 5^2 = 25.$

Total current = $4 + x - jy$.

$\therefore \quad (4 + x)^2 + y^2 = 7^2 = 49.$

or $\quad 16 + x^2 + y^2 + 8x = 49$

or $\quad 25 + 8x = 49 - 16 = 33$

or $\quad 8x = 8$

or $\quad x = 1.$

Thus $\quad y = \sqrt{25-1^2} = \sqrt{24} = 4.9.$

p.f. of the motor = $\dfrac{x}{I_m} = \dfrac{1}{5} = 0.2$ lagging.

The complex part of the total current negative. So the p.f. of the whole circuit is lagging.

$\therefore$ the p.f. of the whole circuit = $\dfrac{4+x}{7} = \dfrac{4+1}{7} = \dfrac{5}{7} = 0.714$ (lag).

Power of the whole circuit = 400 × real part of total current

= 400 × (4 + x) = 400 × 5 = 2000 W.

Power of motor alone = 400 × real part of motor current

= 400 × 1 = 400 W.

5.62 A coil of resistance 5 Ω and inductance 0.1 H is connected in parallel with a circuit containing a coil of resistance 4 Ω and inductance 0.05 *H* in series with a capacitor *C* and a pure resistor *R*. Calculate the values of *C* and *R* so that currents in either branch are equal but differ in phase by 90°.

Solution

The impedance of coil 1 is $Z_1 = 5 + j\omega \times 0.1$

$$= 5 \times j\, 2\pi \times 50 \times 0.1$$

$$= 5 + j\, 31.4.$$

The impedance of coil 2 is $(Z_2) = 4 + R + j\left(2\pi \times 50 \times 0.05 - \dfrac{1}{2\pi \times 50C}\right)$

$$= 4 + R + j\left(15.7 - \frac{1}{314\,C}\right).$$

As the currents in either branch are equal but differ in phase by 90°

$\therefore \quad Z_2 = -\,jZ_1,$

i.e. $\quad Z_2 = 31.4 - j\,5$

Thus, $\quad 4 + R = 31.4 \quad \text{and} \quad 15.7 - \dfrac{1}{314\,C} = -5$

or $\quad R = 31.4 - 4 = 27.4\ \Omega$

and $\quad C = \dfrac{1}{314\,(15.7 + 5)}\ \text{F} = 153.85\ \mu\text{F}.$

5.63 A 230 V 50 Hz supply is applied across a resistor of 10 Ω in parallel with a pure inductor. The total current is 25 A. What should be the value of the frequency if the total current is 36 A?

Solution

Let the inductance of the pure inductor be *L*.

When frequency is 50 Hz the admittance of the circuit is

$$y_1 = \frac{1}{10} + \frac{1}{j \times 2\pi \times 50\,L} = 0.1 - \frac{j}{314L}.$$

$$\therefore \quad \sqrt{(0.1)^2 + \left(\frac{1}{314\,L}\right)^2} = \frac{25}{230}$$

or $\quad 0.01 + \dfrac{1}{98596\,L^2} = 0.0118$

or $\quad L = \dfrac{1}{\sqrt{98596\,(0.0118 - 0.01)}}\ \text{H} = 0.075\ \text{H}$

Let at frequency *f* the total current be 36 A.

Then, $\quad \sqrt{(0.1)^2 + \dfrac{1}{2 \times 3.14 f \times 0.075}} = \dfrac{36}{230} = 0.156$

or $\quad \dfrac{1}{0.471 f} = (0.156)^2 - 0.01 = 0.014$

or $\quad f = \dfrac{1}{0.471 \times 0.014}\ \text{Hz} = 151\ \text{Hz}.$

5.64 In the arrangement shown in Fig. 5.56 calculate the impedance between A and B and the phase angle between voltage and current.

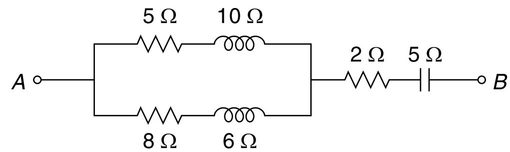

Fig. 5.56 *Circuit diagram for Ex. 5.64*

Solution

$$Z_{AB} = \frac{(5+j10)(8+j6)}{5+j10+8+j6} + (2-j5)$$

$$= \frac{40-60+j(80+30)}{13+j16} + (2-j5)$$

$$= \frac{-20+j110}{13+j16} + (2-j5) = \frac{11.8\angle 100.3°}{20.61\angle 50.9°} + (2-j5)$$

$$= 5.42\angle 49.4° + 2 - j5$$

$$= 5.53 - j\,0.88$$

$$= 5.6\angle -9.04°.$$

The impedance is 5.6 Ω and the angle between voltage and current is 9.04° (current is lagging w.r.t voltage).

5.65 How a current of 50 A is shared among three parallel impedances of $(5 + j\,8)$, $(6 - j\,8)$ and $(8 + j\,9)$ Ω?

Solution

$$Z_1 = 5 + j\,8 = 9.43\angle 58°$$

$$Z_2 = 6 - j\,8 = 9.16\angle -53.13°$$

$$Z_3 = 8 + j\,9 = 12\angle 48.37°.$$

The admittances of the three branches

$$y_1 = \frac{1}{9.43\angle 58°} = 0.106\angle -58°$$

$$y_2 = \frac{1}{9.16\angle -53.13°} = 0.109\angle +53.13°$$

$$y_3 = \frac{1}{12\angle 48.37°} = 0.08\angle -48.37°.$$

Net admittance $y = y_1 + y_2 + y_3 = \dfrac{10}{V}$, where V is the supply voltage.

$$\therefore \qquad (0.056 - j\,0.09) + (0.065 + j\,0.087) + (0.053 - j\,0.06) = \frac{10}{V}$$

or $$0.174 - j\,0.063 = \frac{10}{V}$$

or $$V = \frac{10}{0.185} = 54 \text{ V}.$$

So the currents in the three impedances are

$$I_1 = y_1 V = 0.106 \times 54 = 5.724 \text{ A}$$

$$I_2 = y_2 V = 0.109 \times 54 = 5.886 \text{ A}$$

$$I_3 = y_3 \text{V} = 0.08 \times 54 = 4.32 \text{ A}.$$

5.66 Two coils of resistances 5 Ω and 10 Ω and inductance 0.01 H and 0.03 H respectively are connected in parallel. Calculate (a) the conductance, suseptance and admittance of each coil (b) the total current taken by the circuit, when it is connected to a 230 V, 50 Hz. supply (c) the characteristics of single coil which will take the same current and power as taken by the original circuit.

Solution

(a) Admittance of coil 1, $(y_1) = \dfrac{1}{Z_1} = \dfrac{1}{5 + j100\pi \times 0.01} = \dfrac{1}{5 + j3.14}$

$$= \frac{1}{5.9\angle 32.13^\circ}$$

$$= 0.169\angle -32.13^\circ = 0.143 - j\,0.089.$$

Admittance of coil 2, $(y_2) = \dfrac{1}{Z_2} = \dfrac{1}{10 + j100\pi \times 0.03} = \dfrac{1}{10 + j9.42}$

$$= \frac{1}{13.74\angle 43.3^\circ}$$

$$= 0.073\angle -43.3^\circ = 0.053 - j\,0.05.$$

Conductance of coils 1 and 2 are 0.143 S and 0.053 S respectively, while sesceptance of coils 1 and 2 are 0.089 S (inductive) and 0.05 S (inductive) respectively. Admittance of coils 1 and 2 are 0.169 S and 0.073 S respectively.

(b) Total current taken by the circuit is

$$\begin{aligned} Vy &= 230(y_1 + y_2) \\ &= 230(0.143 - j\,0.089 + 0.053 - j\,0.05) \\ &= 230(0.196 - j\,0.139) \\ &= 55.26 \text{ A.} \end{aligned}$$

(c) $Z = \dfrac{1}{y} = \dfrac{1}{0.196 - j0.139} = \dfrac{1}{0.24\angle -35.34^\circ} = 4.167\angle 35.34^\circ = 3.4 + j2.14.$

The resistance of the single coil is 3.4 Ω and the inductive reactance is 2.14 Ω.

5.67 A coil of resistance 50 Ω and inductance 0.5 H forms part of a series circuit for which the resonant frequency is 60 Hz. If the supply voltage is 230 V, 50 Hz. find (a) the line current, (b) power factor and (c) the voltage across the coil.

Solution

$$60 = \frac{1}{2\pi\sqrt{0.5\,C}}, \text{ where } C \text{ is the capacitance}$$

or $$C = \frac{1}{0.5(2\pi \times 60)^2} = 14\ \mu\text{F}.$$

(a) Line current $= \left(\dfrac{V}{Z}\right) = \dfrac{V}{\sqrt{(50)^2 + \left(314 \times 0.5 - \dfrac{10^6}{314 \times 14}\right)^2}}$

$$= \frac{230}{\sqrt{(50)^2 + (157 - 227.48)^2}}$$

$$= \frac{230}{85.27}\text{ A} = 2.697 \text{ A.}$$

(b) Power factor $= \left(\dfrac{R}{Z}\right) = \dfrac{50}{85.27} = 0.586.$

As capacitive reactance is greater than the inductive reactance so p.f. is leading.

(c) Voltage across coil = $2.697\sqrt{(50)^2 + (2\pi \times 50 \times 0.5)^2}$

$= 2.697 \times 164.77 = 444.38$ V.

5.68 Two impedances $Z_1 = (5 + j\theta)$ and $(Z_2) = (10 - j5)$ are connected in parallel across a 200 V, 50 Hz supply. Find the current through each impedance and total current. What are the angles of phase difference of the branch currents with respect to the applied voltage?

Solution

Current through impedance Z_1 is

$$I_1 = \frac{200}{5 + j8} = \frac{200}{9.43\angle 58°} = 21.21\angle{-58°} \text{ A.}$$

Current through impedance Z_2 is

$$I_2 = \frac{200}{10 - j5} = \frac{200}{11.18\angle -26.56°} = 17.89\angle 26.56° \text{ A.}$$

Total current $(I) = I_1 + I_2 = 2.21\angle{-58°} + 17.89\angle 26.56°$

$= 17.17 + j\,6.12 = 18.23\angle 19.62°$ A.

Current in branch 1 is lagging the applied voltage by 58° and current in branch 2 is leading the applied voltage by 26.56°.

5.69 In a circuit two parallel branches Z_1 and Z_2 are in series with Z_3. The impedances are $Z_1 = 5 + j8$, $Z_2 = 3 - j4$ and $Z_3 = 8 + j11$. The voltage across Z_3 is 50 V. Find currents through the parallel branches and phase angle between them.

Solution

$$\text{Current through } Z_3 = \text{Total current} = \frac{50\angle 10°}{8 + j10} = \frac{50}{12.8\angle 51.34°}$$

$$= 3.9\angle{-51.34°} \text{ A}$$

$$\text{Current through } Z_1 \text{ is } I_1 = 3.9\angle{-51.34°} \times \frac{(3 - j4)}{5 + j8 + 3 - j4}$$

$$= 3.9\angle{-51.34°} \times \frac{5\angle -53.13°}{8.9\angle 26.56°}$$

$$= 2.19\angle{-131.03°} \text{ A}$$

$$\text{Current through } Z_2 \text{ is } I_2 = 3.9\angle{-51.34°} \times \frac{(5 + j8)}{5 + j8 + 3 - j4}$$

$$= 3.9\angle{-51.34°} \times \frac{9.4\angle 58°}{8.9\angle 26.56°}$$

$$= 4.12\angle{-19.9°} \text{ A.}$$

Phase angle between the two currents (131.03° – 19.9°) = 111.13°.

5.70 The total current I in Fig. 5.57 is 15 A at lagging p.f. and the power consumed is 4 kW. The voltmeter reading is 300 V. Find the values of R_1, X_1, and X_2.

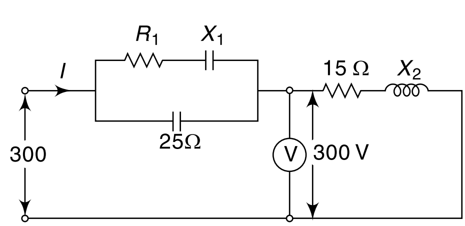

Fig. 5.57 ***Circuit diagram for Ex. 5.70***

Solution

The voltmeter reading is the voltage across 15 Ω and X_2. I is the current through that branch

$$\frac{300}{15} = \sqrt{(15)^2 + X_2^2}$$

or $$X_2 = \sqrt{\left(\frac{300}{15}\right)^2 - (15)^2} = 13.23\ \Omega.$$

Let $(R + jX)$ be the equivalent impedance of the parallel circuit

So net impedancce, $Z = (R + 15) + j(X + 13.23)$

$\therefore$ $$(R + 15)(15)^2 = 4000 \qquad \text{(i)}$$

and $$(R + 15)^2 + (X + 13.23)^2 = \left(\frac{300}{15}\right)^2 = 400 \qquad \text{(ii)}$$

From Eq. (i) $$R = \frac{4000}{225} - 15 = 2.78\ \Omega$$

From Eq. (ii) $$(X + 13.23)^2 = 400 - (2.78 + 15)^2 = 83.87$$

or $$X = -4.072\ \Omega.$$

Net admittance of the parallel branch

$$y = \frac{1}{2.78 - j4.072} = \frac{2.78 + j4.072}{24.31} = 0.114 + j\,0.1675.$$

So, $$\frac{1}{R_1 - jX_1} + \frac{1}{-j25} = 0.114 + j\,0.1675$$

or $$\frac{R_1 + jX_1}{R_1^2 + X_1^2} + \mathrm{j}\ 0.04 = 0.114 + j\,0.1675$$

or $$\frac{R_1}{R_1^2 + X_1^2} = 0.114 \qquad \text{(iii)}$$

and $$\frac{X_1}{R_1^2 + X_1^2} + 0.04 = 0.1675$$

or $$\frac{X_1}{R_1^2 + X_1^2} = 0.1275. \qquad \text{(iv)}$$

Now, $$\frac{R_1}{X_1} = \frac{0.114}{0.1275} = 0.89.$$

From Eq. (iii) $0.89X_1 = 0.114\ \{(0.89)^2 X_1^2 + X_1^2\}$

or $0.204\ X_1 = 0.89$ or $X_1 = 4.35$

and $R_1 = 0.89 \times 4.35 = 3.88\ \Omega.$

5.71 Prove that the impedance of a parallel ac circuit containing R and L in one branch and R and C in the other branch (Fig. 5.58) is equal to R when $R^2 = \frac{L}{C}$.

If $L = 0.01\ H$ and $C = 90\ \mu$F, determine the impedance and current in each branch. Assume supply voltage to be 220 V, 50 Hz.

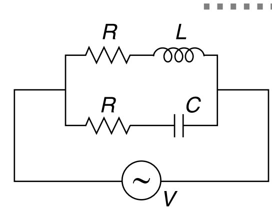

Fig. 5.58 ***Circuit diagram for Ex. 5.71***

Solution

Impedance $$Z = \frac{(R + j\omega L)\left(R - \frac{j}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{R^2 + \frac{L}{C} + j\left(R\omega L - \frac{R}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega C}\right)}$$

Since, $\frac{L}{C} = R^2$ we have

$$Z = \frac{R^2 + R^2 + jR\left(\omega L - \frac{1}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega C}\right)} = R\frac{2R + j\left(\omega L - \frac{1}{\omega C}\right)}{2R + j\left(\omega L - \frac{1}{\omega C}\right)} = R.$$

When $L = 0.01$ H & $C = 90$ μF,

$$Z = R = \frac{L}{C} = \frac{0.01}{90 \times 10^{-6}} = \frac{1}{9} \times 10^3 = 111.11\ \Omega$$

$$I_1 = \frac{220}{111.11 + j\omega 0.01} = \frac{220}{111.11 + j3.14} = 1.98\underline{|-1.62°}\text{ A}$$

$$I_2 = \frac{220}{111.11 - j\frac{10^6}{90\omega}} = \frac{220}{111.11 - j35.385} = 1.98\underline{|17.66°}\text{ A.}$$

5.72 A series connected *RLC* circuit has $R = 15\ \Omega$, $L = 40$ mA and $C = 40$ μF. Find the resonant frequency and under resonant condition calculate the current, power, voltage drops across various elements if the applied voltage is 75 V.

Solution

Resonant frequency $f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2 \times 3.14\sqrt{0.04 \times 0.40 \times 10^{-6}}} = 125.88$ Hz.

Current at resonant condition (I_o) = $\frac{V}{R} = \frac{75}{15} = 5$ A.

Power (= VI_o) = 75 × 5 = 375 W.

Voltage drop across $R = I_oR = 5 \times 15 = 75$ V = Applied voltage.

Voltage drop across $L = I_oX_L = 5 \times 2\pi \times 125.88 \times 0.04 = 158.1$ V.

Voltage drop across $C = I_oX_C = \frac{5 \times 10^6}{2\pi \times 125.88 \times 40} = 158.1$ V.

5.73 A series circuit consists of a capacitor and a coil takes a maximum current of 0.314 A at 200 V, 50 Hz. If the voltage across the capacitor is 300 V at resonance determine the capacitance, inductance, resistance and Q of the coil.

Solution

$$I_o = 0.314\text{ A}$$

$$I_oR = 200 \quad \text{or,} \quad R = \frac{200}{0.314}\ \Omega = 636.943\ \Omega$$

$$I_oX_C = 300\text{ V} \quad \text{or,} \quad X_C = \frac{300}{0.314}\ \Omega = 955.4\ \Omega = \frac{1}{\omega_o C}.$$

$$\therefore \quad C = \frac{1}{2\pi \times 50 \times 955.4}\text{ F} = 3.33\ \mu\text{F}.$$

Again $I_oX_L = 300$ or $X_L = \frac{300}{0.314} = 955.4 = \omega_o L$

$$\therefore \quad L = \frac{955.4}{2\pi \times 50}\text{ H} = 3.04\text{ H}.$$

'Q' of the coil $= \dfrac{\omega_o L}{R} = \dfrac{2\pi \times 50 \times 3.04}{636.943} = 1.5.$

5.74 In a series resonant circuit, the resistance is 6 Ω, the resonant frequency is 4.1×10^6 rad/s and the bandwidth is 10^5 rad/s. Find L and C of the network, half power frequencies and Q_{factor} of the circuit.

Solution

$$R = 6\ \Omega, \quad \omega_o = 4.1 \times 10^6 \text{ rad/s}$$

Bandwidth $(\omega_2 - \omega_1) = \Delta\omega = 10^5$ rad/s

$$\therefore \quad Q = \frac{\omega_o}{\Delta\omega} = \frac{4.1 \times 10^6}{10^5} = 41$$

Also,
$$Q = \frac{\omega_o L}{R} = 41$$

or
$$\frac{4.1 \times 10^6 \times L}{R} = 41$$

or
$$L = \frac{41 \times 6}{4.1 \times 10^6} = 6 \times 10^{-5} \text{ H.}$$

As at resonance, $X_L = X_C$,

$$C = \frac{1}{L\omega_o^2} = \frac{1}{6 \times 10^{-5} \times (4.1 \times 10^6)^2} = \frac{10^{-6}}{1008.6}$$
$$= 9.91 \times 10^{-10} \text{ F.}$$

Lower half power frequency $(\omega_1) = \left(\omega_o - \dfrac{R}{2\pi L}\right) = 4.1 \times 10^6 - \dfrac{6 \times 10^5}{2 \times 6} = 4.05 \times 10^6$ rad/s.

Upper half power frequency $(\omega_2) = \left(\omega_o + \dfrac{R}{2\pi L}\right)$.

$$= 4.1 \times 10^6 + \frac{6 \times 10^5}{2 \times 6}$$
$$= 4.15 \times 10^6 \text{ rad/s.}$$

5.75 A coil having a resistance of 25 Ω and an inductance of 25 mH is connected in parallel with a variable capacitor. For what value of C will the circuit the resonant if a 90 V, 400 Hz. source is applied? What will be the line current under this condition?

Solution

$$(f_o) = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}, \text{ where } (f_o) = \text{Resonant frequency}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.025 X_C} - \left(\frac{25}{0.025}\right)^2} = 400 \text{ Hz}$$

or
$$\frac{1}{0.025C} - 10^6 = (400 \times 2\pi)^2 = 6310144$$

$$\therefore \quad C = 5.47\ \mu\text{F}$$

Line current
$$(I_o) = \frac{V}{(L/CR)} = \frac{90}{\dfrac{0.025}{5.47 \times 10^{-6} \times 25}} \text{ A} = 0.492 \text{ A.}$$

5.76 Two impedances $Z_1 = (10 + j15)\ \Omega$ and $Z_2 = (20 - j25)\ \Omega$ are connected in parallel and this parallel combination is connected in series with impedance $(Z_3) = (25 + jX)\ \Omega$. Find for what value of X resonance occurs.

Solution

The net impedance of the whole circuit

$$Z = \frac{Z_1 Z_2}{Z_1 + Z_2} + Z_3 = \frac{(10 + j15)(20 - j25)}{10 + j15 + 20 - j25} + 25 + jX$$

$$= \frac{18.03\angle 56.31° \times 32\angle -51.34°}{30 - j10} + 25 + jX$$

$$= \frac{576.96\angle 4.97°}{31.62\angle -18.43°} + 25 + jX$$

$$= 18.247\angle 23.4° + 25 + jX$$

$$= 16.74 + j7.24 + 25 + jX$$

$$= 41.74 + j(X + 7.25).$$

For resonance the reactive part of the impedance must be zero, i.e. net reactance is 0.

$\therefore \qquad X + 7.25 = 0$

or $\qquad X = -7.25\ \Omega;$

5.77 A coil having a resistance of 30 Ω and inductive reactance of 33.3 Ω is connected to a 125 V, 50 Hz. source. A series circuit consisting of 200 Ω resistor and a variable capacitor is then connected in parallel with the coil. For what value of capacitance will the circuit be in resonance? Given that resonant frequency is 60 Hz.

Solution

Here a series R-L circuit is in parallel with a series R-C circuit.

Resonant frequency $\qquad f_o = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC}\left[\dfrac{L - CR_L^2}{L - CR_C^2}\right]}.$

Now at 50 Hz., $\qquad (Xs_L) = 33.3\ \Omega$

or $\qquad L = \dfrac{33.3}{2\pi \times 50}\ \text{H} = 0.106\ \text{H}.$

Again, resonant frequency = 60 = $\dfrac{1}{2\pi}\sqrt{\dfrac{1}{0.106\,C} - \dfrac{0.106 - C\,(30)^2}{0.106 - C\,(200)^2}}$

or $\qquad \dfrac{1}{0.106\,C} - \dfrac{0.106 - 900\,C}{0.106 - 40000\,C} = (376.8)^2 = 141978.24$

or $\qquad \dfrac{0.106 - 40000C - 0.106C\,(0.106 - 900\,C)}{0.106\,C\,(0.106 - 40000C)} = 141978.24$

$\qquad 0.106 - 40000C + 95.4C^2 = 1595.27C - 601987737C^2$

or $\qquad C = 66\ \mu\text{F}.$

5.78 An inductor in series with a variable capacitor is connected across a constant voltage source of frequency 10 kHz. When the capacitor is 700 pF the current is maximum, when the capacitance is 900 pF the current is half of its maximum value. Find the resistance, inductance and Q factor of the inductor.

Solution

Resonant freqency $f_o = \dfrac{1}{2\pi\sqrt{LC}}$

or $\qquad 10 \times 10^3 = \dfrac{1}{2\pi\sqrt{L \times 700 \times 10^{-12}}} = \dfrac{10^6}{2\pi\sqrt{700L}}$

or $\qquad 700\ L = \left(\dfrac{10^6}{10 \times 10^3 \times 2\pi}\right)^2 = 253.56$

or $\qquad L = 0.362$ H.

The maximum current = $\dfrac{V}{R}$, where V is the supply voltage and R is the resistance.

When capacitor value = 900 pF.

Current = $\dfrac{1}{2} \times$ max. value of current

$\therefore \quad$ current = $\dfrac{1}{2} \times \dfrac{V}{R} = \dfrac{V}{2R}$.

Again $\quad \dfrac{V}{2R} = \dfrac{V}{\sqrt{R^2 + \left\{2\pi \times 10 \times 10^3 \times 0.362 - \dfrac{10^{12}}{2\pi \times 10 \times 10^3 \times 900}\right\}^2}}$

or $\qquad 4R^2 = R^2 + 25409139.5 \quad$ or $\quad R = 2910.3\ \Omega$

$$Q_{\text{factor}} = \frac{\omega_o L}{R} = \frac{2\pi \times 10 \times 10^3 \times 0.362}{2910.3} = 7.8.$$

5.79 A coil of resistance 10 Ω and inductance 0.5 H is connected in series with a capacitor across a voltage source. When the frequency is 50 Hz the current is maximum. Another capacitor is connected in parallel with the circuit. What capacitance must it have so that the combination acts as a pure resistor at 100 Hz?

Solution

The current is maximum at resonance. So 50 Hz is the resonant frequency.

$\because \qquad f_o = \dfrac{1}{2\pi\sqrt{LC}}$,

or $\qquad 50 = \dfrac{1}{2\pi\sqrt{0.5\,C}}$

or $\qquad C = \left(\dfrac{1}{50 \times 2\pi}\right)^2 \times \dfrac{1}{0.5} = 20.28\ \mu$F.

At 100 Hz the impedance of the series branch is

$$Z_1 = 10 + j\left(2\pi \times 100 \times 0.5 - \frac{10^6}{2\pi \times 100 \times 20.28}\right).$$

$$= 10 + j\ 235.48 = 235.69\angle 87.57°$$

If C' is the capacitor connected in parallel with the circuit, impedance of the parallel branch is

$$= -jX_C$$

$$= -j\frac{1}{2\pi \times 100\,C'} .$$

Admittance of the combined circuit

$$= \frac{1}{235.69\angle 87.57^\circ} + j200\pi C'$$

$$= 0.0042\angle{-87.57^\circ} + j628C'$$

$$= 0.000178 - j0.0042 + j628C'.$$

Net susceptance should be zero if the circuit acts as a pure resistor.

So, $\quad 0.0042 = 628\ C' \quad$ i.e., $\quad C' = 6.68\ \mu\text{F}.$

5.80 A voltage of 100∠30° V is applied to a circuit having two parallel branches. If the currents are 20∠60° A and 10∠–45° A respectively find the kW, KVAR and kVA in each branch and in the whole circuit. What is the p.f. of the combined load?

Solution

For 1st branch

$$\text{kVA} = \frac{100 \times 20}{10^3} = 2$$

$$\text{kW} = \frac{100 \times 20}{10^3} \cos(60^\circ - 30^\circ) = 1.732$$

$$\text{KVAR} = \frac{100 \times 20}{10^3} \sin(60^\circ - 30^\circ) = 1 \text{ (lead).}$$

For the 2nd branch

$$\text{kVA} = \frac{100 \times 10}{10^3} = 1$$

$$\text{kW} = \frac{100 \times 10}{10^3} \cos(30^\circ + 45^\circ) = 0.2588$$

$$\text{KVAR} = \frac{100 \times 10}{10^3} \sin(30^\circ + 45^\circ) = 0.9659 \text{ (inductive)}$$

$$\text{Total current} = 20\angle 60^\circ + 10\angle{-45^\circ}$$

$$= 10 + j\,17.32 + 7.07 - j\,7.07$$

$$= 17.07 + j\,9.62 = 19.59\angle 29.4^\circ.$$

For the whole circuit

$$\text{kVA} = \frac{100 \times 19.59}{10^3} = 1.959$$

$$\text{kW} = 1.959 \cos(30^\circ - 29.4^\circ) = 28.3$$

$$\text{KVAR} = 1.959 \sin(30^\circ - 29.4^\circ) = 1.175 \text{ (inductive)}$$

P.f. of combined load = cos (30° – 29.4°) = 1.

5.81 A choke coil has a resistance of 40 Ω and Q factor of 5 at 1000 Hz. It is connected in parallel with a variable capacitor to a 10 V, 1000 Hz ac supply. Find (a) X_C for resonance (b) equivalent impedance as seen by the source and (c) current drawn from the supply.

Solution

$$Q \text{ factor} = \frac{\omega_o L}{R} = 5$$

or $$\frac{2\pi \times 1000\,L}{40} = 5$$

so $$L = \frac{40 \times 5}{2\pi \times 1000} = 0.0318 \text{ H}$$

(a) If C is the capacitance then

$$\omega_o = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

or $$(2\pi \times 1000)^2 = \frac{1}{0.0318\,C} - \left(\frac{40}{0.0318}\right)^2$$

or $$\frac{1}{0.0318\,C} = 41020615.89$$

or $$C = 0.766\ \mu\text{F}$$

or $$X_C = \frac{1}{\omega_o C} = (2\pi \times 1000 \times 0.766 \times 10^{-6})^{-1}$$

$$= 207.7\ \Omega.$$

(b) Equivalent impedance as seen by the source is

$$\frac{L}{CR} = \frac{0.0318}{40 \times 0.766} \times 10^{-6} = 1037.86\ \Omega.$$

(c) Current taken from supply $= \dfrac{10}{1037.86} = 9.6$ mA.

5.82 A pure capacitor of 60 μF is in parallel with another single circuit element. If the applied voltage and total current are $v = 100 \sin (2000\,t)$ and $i = 15 \sin (2000t + 75°)$ respectively find the other element.

Solution

$$V = 100 \sin 2000\,t$$
$$i = 15 \sin (2000\,t + 75°)$$

Power factor = $\cos \theta = \cos 75° = 0.2588$ leading

As the total current is leading by an angle less than 90° (i.e. 75°) so the other element must be a resistor.

∴ $$\tan \theta = \tan 75° = 3.73 = \frac{X_C}{R}$$

or $$R = \frac{X_C}{3.73} = \frac{1}{\omega C\, 3.73} = \frac{10^6}{2000 \times 50 \times 3.73}\ \Omega = 2.68\ \Omega.$$

5.83 A resistor and a capacitor are in series with a variable inductor across a 100 V, 50 Hz supply. The maximum current obtained by varying inductance is 5 A. The voltage across capacitance is 200 V. Find the circuit elements.

Solution

At resonance $$(I) = \frac{V}{R} = 5 \text{ A}$$

or $$R = \frac{V}{5} = \frac{100}{5} = 20\ \Omega$$

Voltage across capacitor $(V_C) = I \times \dfrac{1}{\omega C} = 200$

or $$C = \frac{I}{200\,\omega} = \frac{5}{200 \times 2\pi \times 50} \text{ F} = 79.6\ \mu\text{F}$$

At resonance, $$\omega L = \frac{I}{\omega C}$$

So, $$L = \frac{I}{\omega^2 C} = \frac{10^6}{(2\pi \times 50)^2 \times 79.6} \text{H}$$
$$= 0.127 \text{ H.}$$

5.84 For the network shown in Fig. 5.59 determine the resonant frequency and input admittance at resonance.

Solution

The net admittance of the circuit

$$y = \frac{1}{5 \times 10^3} + j\frac{1}{X_C} + \frac{1}{50 + jX_L}$$

$$= \frac{1}{5 \times 10^3} + j\omega_o \times 10^{-6} + \frac{1}{50 + j\omega_o}$$

$$= \frac{1}{5 \times 10^3} + j\omega_o \times 10^{-6} + \frac{50 - j\omega_o}{(50)^2 + \omega_o^2}.$$

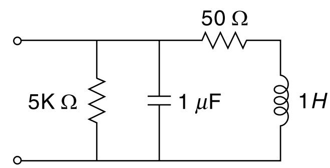

Fig. 5.59 *Circuit diagram for Ex. 5.84*

For resonance the net susceptance should be zero.

so $$\omega_o \times 10^{-6} - \frac{\omega_o}{(50)^2 + \omega_o^2} = 0$$

or $$2500 + w_0^2 = 10^6$$

or $$\omega_o = 998.75 \text{ rad/s}$$

so $$f_o = \frac{998.75}{2\pi}\text{Hz} = 159 \text{ Hz.}$$

Input admittance at resonance is given by

$$y = \frac{1}{5 \times 10^3} + \frac{50}{(50)^2 + (998.75)^2} = 0.00025 \text{ S}$$

5.85 For the network shown in Fig. 5.60 find the maximum value of R_C beyond which the circuit will not resonate.

Solution

Net admittance of the circuit is given as

$$y = \frac{1}{4 + j3} + \frac{1}{R_C - jX_C}$$

$$= \frac{1}{4 + j3} + \frac{1}{R_C - jX_C}.$$

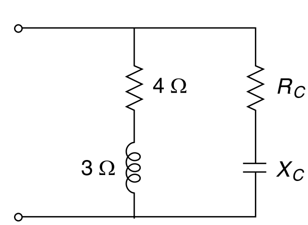

Fig. 5.60 *Circuit diagram for Ex. 5.85*

Under resonance condition,

$$\frac{-3}{16 + 9} + \frac{X_C}{R_C^2 + X_C^2} = 0$$

or $$3X_C^2 + 3R_C^2 = 25\ X_C$$

or $$3X_C^2 - 25X_C + 3R_C^2 = 0$$

or $$X_C = \frac{25 \pm \sqrt{(25)^2 - 4 \times 3 \times 3R_C^2}}{3 \times 2}.$$

If X_C is real then $(4 \times 3 \times 3R_C^2)$ should be less than or equal to $(25)^2$. So maximum value of R_C is

$$R_C = \sqrt{\frac{(25)^2}{4 \times 3 \times 3}} = 4.167\ \Omega.$$

5.86 Find the average and rms value of the waveform shown in Fig. 5.61.

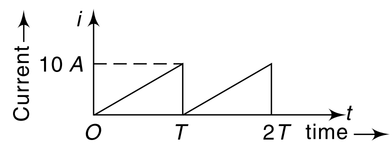

Fig. 5.61 ***Circuit diagram for Ex. 5.86***

Solution

The slope of the curve is obtained as

$$i = \frac{10}{T}t$$

Average value of the waveform is given by

$$I_{\text{av}} = \frac{1}{T}\int_0^T i\,dt = \frac{1}{T}\int_0^T \frac{10}{T}t\,dt = \frac{1}{T}\left[\frac{10}{T}\frac{t^2}{2}\right]_0^T = \frac{10 \times T^2}{T^2 \times 2} = 5\text{ A.}$$

rms value of the waveform is obtained as

$$I_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt} = \sqrt{\frac{1}{T}\int_0^T \frac{100}{T^2}t^2\,dt} = \sqrt{\frac{100}{T^3}\left[\frac{t^3}{3}\right]_0^T}$$

$$= \sqrt{\frac{100 \times T^3}{T^3 \times 3}} = \frac{10}{\sqrt{3}} = 5.77\text{ A.}$$

5.87 Find the average and rms value of the waveform shown in Fig. 5.62.

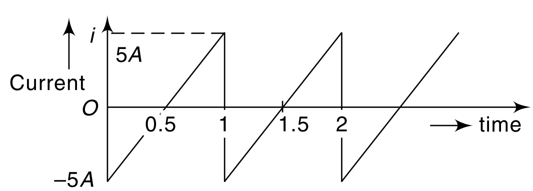

Fig. 5.62 ***Waveform for Ex. 5.87***

Solution

The average value of the waveform over a full cycle ($t = 0$ to $t = 1$) is 0. Considering half cycle ($t = 0.5$ to $t = 1$)

$$I_{\text{av}} = \frac{1}{0.5}\int_{0.5}^{1} i\,dt.$$

Now, $$i = \frac{5}{0.5}t - 5 = 10\,t - 5$$

Hence $$I_{\text{av}} = \frac{1}{0.5}\int_{0.5}^{1}(10t - 5)\,dt = 2\left[\frac{10\,t}{2} - 5\,t\right]_{0.5}^{1}$$

$$= 2[5 \times 1^2 - 5 \times 1 - 5 \times (0.5)^2 + 5 \times 0.5]$$
$$= 2[2.5 - 1.25]$$
$$= 2.5\text{ A.}$$

$$I_{\text{r.m.s.}} = \sqrt{\frac{1}{0.5}\int_{0.5}^{1}(10t-5)^2\,dt}$$

$$= \sqrt{\frac{1}{0.5}\int_{0.5}^{1}(100t^2+25-100t)\,dt}$$

$$= \sqrt{2\left[100\frac{t^3}{3}-50\frac{t^2}{2}+25t\right]_{0.5}^{1}}$$

$$= \sqrt{\begin{aligned}&2\left\{\frac{100}{3}\times 1^3-50\times\frac{1^2}{2}+25\times 1\right.\\ &\left.-\frac{100}{3}(0.5)^3+50\times(0.5)^2\times\frac{1}{2}-25\times(0.5)\right\}\end{aligned}}$$

$= 7.90$ A

5.88 Find the average and rms value of the waveform shown in Fig. 5.63.

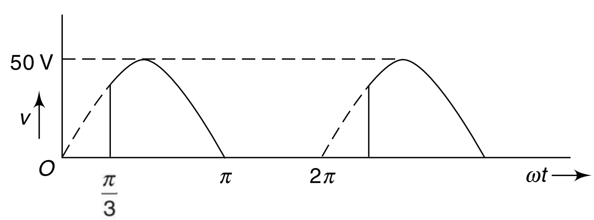

Fig. 5.63 ***Wave form for Ex. 5.88***

Solution

The average value of the waveform

$$I_{\text{av}} = \frac{1}{2\pi}\int_{\frac{\pi}{3}}^{\pi}50\sin\omega t\,d(\omega t) = \frac{50}{2\pi}[-\cos\omega t]_{\pi/3}^{\pi}$$

$$= \frac{50}{2\pi}\left[\cos\frac{\pi}{3}-\cos\pi\right]$$

$$= \frac{25}{\pi}[0.5+1]$$

$= 11.937$ V.

The rms value of the waveform

$$I_{\text{rms}} = \sqrt{\frac{1}{2\pi}\int_{\frac{\pi}{3}}^{\pi}(50)^2\sin^2\omega t\,d(\omega t)}$$

$$= \frac{50}{\sqrt{2\pi}}\sqrt{\int_{\frac{\pi}{3}}^{\pi}\frac{1}{2}(1-\cos 2\omega t)\,d(\omega t)}$$

$$= \frac{1}{2} \times 19.95 \sqrt{\left[\omega t - \frac{\sin 2\omega t}{2}\right]_{\frac{\pi}{3}}^{\pi}}$$

$$= 14.11\sqrt{\pi - \frac{\sin 2\pi}{2} - \frac{\pi}{3} + \frac{\sin \frac{2\pi}{3}}{2}}$$

$$= 14.11\sqrt{\pi - \frac{\pi}{3} + 0.433} = 22.142 \text{ A.}$$

5.89 If the waveform of a current has form factor 1.2 and peak factor 1.7 find the average and rms value of the current if the maximum value of the current is 100 A.

Solution

$$\text{Form factor} = \frac{I_{\text{rms}}}{I_{\text{av}}} = 1.2$$

$$\text{Peak factor} = \frac{I_m}{I_{\text{rms}}} = \frac{100}{I_{\text{rms}}} = 1.7$$

Hence $I_{\text{r.m.s.}} = \dfrac{100}{1.7} = 58.82 \text{ A}$

and $I_{\text{av}} = \dfrac{I_{\text{rms}}}{1.2} = \dfrac{58.82}{1.2} \text{ A} = 49 \text{ A.}$

5.90 A sinusodial signal has a value of (–5) A at $t = 0$ and reached its first negative maximum of (–10) A at 2 ms. Write the equation for the current.

Solution

Let us consider the equation for the current is

$$i(t) = I_m \sin(\omega t + \theta)$$

At $t = 0$, $i(t) = I_m \sin \theta = -5$

At $t = 2 \times 10^{-3}$ s, $i(t) = I_m \sin(\omega \times 2 \times 10^{-3} + \theta) = -10$

Now, $I_m = 10$ A (negative and positive maximum values are same)

Hence $\sin \theta = -\dfrac{5}{10} = -0.5 = \sin\left(-\dfrac{\pi}{6}\right) = \sin 210°$

i.e. $\theta = 210°$

Again $\sin(\omega \times 2 \times 10^{-3} + 210°) = -\dfrac{10}{10} = -1 = \sin 270°$

Hence $\omega \times 2 \times 10^{-3} = 270° - 210° = 60° = \dfrac{\pi}{3}$

i.e. $\omega = \dfrac{\pi \times 10^3}{6} = 523.6$

$$i = 10 \sin(523.6t + 210°) \text{ A.}$$

5.91 In a circuit the voltage and impedance are given by $v = (100 + j80)$ V and $Z = (10 + j8)\ \Omega$. Find the active and reactive power of the circuit.

Solution

It is given that,

$$Z = 10 + j\,8 = 12.8\angle 36.86°\ \Omega$$
$$v = 100 + j\,80 = 128\angle 36.86°\ \text{V}.$$

Hence current $\quad i = \dfrac{v}{Z} = \dfrac{128\angle 36.86°}{12.8\angle 36.86°} = 10\angle 0°$ A.

Power factor angle = angle between v and i = 36.86°

Active power of the circuit

$$v.i \cos\theta = 128 \times 10 \cos(36.86°) = 1024 \text{ W}.$$

Reactive power of the circuit

$$v.i \sin\theta = 128 \times 10 \sin(36.86°) = 768 \text{ VAR}.$$

5.92 Find an expression for the current and calculate the power when a voltage for v = 283 sin (100 πt) is applied to a coil having R = 50 Ω and L = 0.159 H.

Solution

Impedance $\quad (Z) = R + j\omega L$

$$\omega = 100\pi = 314$$

Hence $\quad Z = 50 + j \times 314 \times 0.159$

$$= 50 + j\,49.95$$
$$= 70.67\angle 44.97°.$$

Now $\quad v = 283 \sin 100\,\pi t$

rms value of $v = \dfrac{283}{\sqrt{2}} = 200$ V.

$\therefore$ Rms value of current $(i_{rms}) = \dfrac{283\angle 10°}{\sqrt{2} \times 70.67\angle 44.97°} = 2.83\angle -34.97°$ A

Now $\quad (i_{max}) = \sqrt{2} \times 2.83 = 4$ A.

The expression for current i = 4 sin (100 πt – 44.97°) A

Power = $VI \cos\theta$, where V and I are the rms values of voltage, and current and θ is the power factor angle.

Hence $\quad$ Power = 200 × 2.83 cos (44.97°) = 400 W.

5.93 An emf given by $100 \sin\left(314t - \dfrac{\pi}{4}\right)$ V is applied to a circuit and the current is 20 sin (314t – 1.5708) A. Find (a) frequency and (b) circuit elements.

Solution

$$V = 100 \sin\left(314t - \frac{\pi}{4}\right)$$

$\therefore \quad \omega = 314$ rad/s

(a) Frequency $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2 \times 3.14} = 50$ Hz.

(b) $i = 20 \sin(314\,t - 1.5708)$ A $= 20 \sin\left(314t - \dfrac{\pi}{4}\right)$ A

Current i is lagging w.r.t v by an angle of $\dfrac{\pi}{2} - \dfrac{\pi}{4}$ i.e. $\dfrac{\pi}{4}$.

Hence the circuit contains R and L only. If Z is the impedance then

$$R = Z\cos\theta = \frac{100}{20}\cos\left(\frac{\pi}{4}\right) = 3.356\ \Omega$$

and $$\omega L = Z\sin\theta = \frac{100}{20}\sin\left(\frac{\pi}{4}\right) = 3.536\ \Omega$$

or, $$L = \frac{3.536}{314} = 11.26 \text{ mH.}$$

5.94 A two element series circuit consumes 700 W and has a power factor of 0.707 leading. If the applied voltage is $v = 141 \sin(314\,t + 30°)$, find the circuit elements.

Solution

Active power = $VI\cos\theta = 700$ W.

Power factor $\cos\theta = 0.707$ (lead).

Hence $$VI = \frac{700}{0.707} = 990.$$

Now $$V = \text{rms value of voltage} = \frac{141}{\sqrt{2}} = 99.7 \text{ V.}$$

Hence, $$I = \frac{990}{99.7} = 9.93 \text{ A}$$

and $$Z = \frac{V}{I} = \frac{99.7}{9.93}\,\Omega = 10\ \Omega.$$

As the power factor is leading so the circuit contains resistance R and capacitance C.

Hence $$R = Z\cos\theta = 10 \times 0.707 = 7.07\ \Omega$$

and $$\frac{1}{\omega C} = Z\sin\theta = 10 \times 0.707 = 7.07\ \Omega$$

or $$C = \frac{1}{314 \times 7.07} \text{ farad} = 45.03\ \mu\text{F.}$$

5.95 For the circuit shown in Fig. 5.64 determine the equivalent admittance at terminals AB if $f = 50$ Hz.

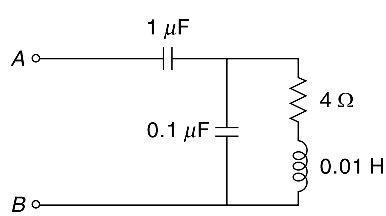

Fig. 5.64

Solution

Equivalent admittance of the parallel branches:

$$(y) = y_1 + y_2$$

$$= \frac{1}{4 + j100\pi \times 0.01} + \frac{1}{\dfrac{-j \times 10^6}{100\pi \times 0.1}}$$

$$= \frac{1}{4 + 3.14j} + j\frac{31.4}{10^6}$$

$$= \frac{1}{5.08\angle 38.13°} + j\frac{31.4}{10^6}$$

$$= 0.1968\angle{-38.13°} + j31.4 \times 10^{-6}$$

$$= 0.155 - j0.1215 + j0.0000314$$

$$= 0.155 - j0.12147 \text{ s.}$$

Now admittance of 1 μF capacitor (y_3) $= j\omega C$

$= j100\ \pi \times 1 \times 10^{-6}$

$= j0.000314.$

In the circuit y and y_3 are in series.

Hence equivalent admittance is

$$\frac{y\,y_3}{y+y_3} = \frac{j0.000314\,(0.155 - j0.12147)}{j0.000314 + 0.155 - j0.12147}$$

$$= \frac{0.000038 + j0.00004867}{0.155 - j0.121}$$

$$= \frac{0.00006\angle 52^\circ}{0.01967\angle -47.98^\circ} = 0.003\angle 99.98^\circ \text{ S.}$$

5.96 A series circuit consists of a 10 Ω resistor, a 30 mH inductor and a 1 μF capacitor is supplied from a 10 V variable frequency supply. Determine the frequency for which voltage developed across the capacitor is maximum and the magnitude of this maximum voltage.

Solution

$$\text{Resonant frequency } f_o = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2 \times 3.14 \times \sqrt{30 \times 10^{-3} \times 1 \times 10^{-6}}} \text{ Hz}$$

$$= 919 \text{ Hz.}$$

Hence voltage across capacitor will be maximum when frequency is 919 Hz.

$$\text{Voltage across capacitor} = (I_o X_C) = \frac{V}{R} X_C = \frac{10}{10} \times \frac{1}{2\pi \times 919 \times 10^{-6}}$$

(where I_o is the current at resonance or maximum current).

$= 173.27$ V.

5.97 A coil of resistance 10 Ω and inductance 10 mH is connected in parallel with a 25 μF capacitor. Calculate the frequency at resonance and the effective impedance of the circuit.

Solution

$R = 10\ \Omega \quad L = 10 \text{ mH} \quad C = 25 \times 10^{-6}$ F.

$$\text{Resonant frequency } f_o = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{10 \times 10^{-3} \times 25 \times 10^{-6}} - \frac{(10)^2}{(10 \times 10^{-3})^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{10^8}{25} - 10^6}$$

$$= 275.8 \text{ Hz.}$$

$$\text{The effective impedance of the circuit } \left(\frac{L}{CR}\right) = \frac{10 \times 10^{-3}}{25 \times 10^{-6} \times 10} = 40\ \Omega.$$

5.98 Two coils are connected in parallel across 200 V, 50 Hz mains. One coil takes 0.8 kW and 1.5 kVA and the other coil takes 1 kW and 0.6 KVAR. Calculate the resistance and reactance of a single coil that would take the same current and power as the original circuit.

Solution

For the 1st coil

$$VI_1 = 1.5 \times 10^3$$

and $\quad VI_1 \cos\theta_1 = 0.8 \times 10^3$

where I_1 and θ_1 are the current and power factor angle of the coil.

Now, $\quad V = 200$

Hence, $\quad I_1 = \dfrac{1.5\times10^3}{200} = 7.5 \text{ A}$

For the 2nd coil

$$VI_2 = 0.6 \times 10^3$$

and $\quad VI_2 \cos\theta_2 = 1 \times 10^3$

(where I_2 and θ_2 are the current and power factor angle of the 2nd coil).

$$I_2 = \frac{0.6\times10^3}{200} = 3 \text{ A.}$$

Total current $\quad I = I_1 + I_2 = (7.5 + 3) \text{ A} = 10.5 \text{ A.}$

Total power = (0.8 + 1) kW = 1.8 kW.

Let the resistance and the reactance of the single coil be R and X.

Now, $\quad V = 200 \text{ V.}$

$$I = 10.5 \text{ A.}$$

Hence $\quad Z = \dfrac{200}{10.5} = 19.07$

Also, $\quad VI \cos\theta = 1.8 \times 10^3$

where $\cos\theta$ is the power factor of the single coil.

Hence $\quad \cos\theta = \dfrac{1800}{200\times10.5} = 0.857$

$\therefore \quad R = Z\cos\theta = 19.07 \times 0.857 = 16.34\ \Omega$

and $\quad X = Z\sin\theta = 19.07 \times 0.515 = 9.82\ \Omega.$

5.99 A capacitor is placed in parallel with two inductive loads, one 20 A at 30° lag and another one of 40 A at 60° lag. What must be the current in the capacitor so that the current from the external circuit shall be at unity power factor?

Solution

$$I_1 = 20\angle{-30°}\text{A}$$

$$I_2 = 40\angle{-60°}\text{A}$$

Horizontral component of

$$I_1 = I_{1x} = 20\cos(-30°) \text{ A} = 17.32 \text{ A.}$$

Vertical component of

$$I_1 = I_{1y} = 20\sin(-30°) \text{ A} = -10 \text{ A.}$$

Horizontal component of

$$I_2 = I_{2x} = 40\cos(-60°) \text{ A} = 20 \text{ A.}$$

Vertical component of

$$I_2 = I_{2y} = 40\sin(-60°) \text{ A} = -34.64 \text{ A.}$$

If I be the total current vertical component of

$$I = I_y = (-10 - 34.64) \text{ A} = -44.64 \text{ A}.$$

If the power factor is unity then vertical component of I should be zero.

Hence if I_c be the current from the capacitor then, $I_c + I_y = 0$, i.e. $I_c = -I_y = 44.64$ A.

5.100 Two circuits having the same numerical ohmic impedance are joined in parallel. The power factor of one circuit is 0.8 lag and that of the other is 0.6 lag. Find the power factor of the whole circuit.

Solution

Let the supply voltage be V and the numerical value of impedance of each circuit be Z.

Current in the Ist circuit $I_1 = \dfrac{V}{Z} \angle -\cos^{-1}(0.8) = \dfrac{V}{Z} \angle -36.87°.$

Current in the 2nd circuit $I_2 = \dfrac{V}{Z} \angle -\cos^{-1}(0.6) = \dfrac{V}{Z} \angle -53.13°.$

The resultant current is $I_1 + I_2$

$$= \frac{V}{Z}[\cos(-36.87°) + j\sin(-36.87°) + \cos(-53.13°) + j\sin(-53.13°)]$$

$$= \frac{V}{Z}(1.4 - j\,1.4) = 1.98\,\frac{V}{Z} \angle -45°$$

Hence power factor of the whole circuit is cos (45°) lag = 0.707 lag.

5.101 A cosine wave is represented by $v_{ab} = V_m \cos(\omega t + \theta)$, where the frequency is 50 Hz. In the expression of v_{ab}, the angle ωt and θ are usually expressed in radians. Express the angle in degrees and write down in the expression for V_{ab}.

Solution

$$v_{ab} = V_m \cos(\omega t + \theta) \text{ [given, } \omega t \text{ and } \theta \text{ in radians]}$$

$\therefore$ In degrees we can represent as

$$v_{ab} = V_m \cos(2 \times 180 \times 50\,t + \theta)$$
$$= V_m \cos(18000\,t + \theta);\ \theta \text{ in degree.}$$

[Note that in radian the same expresion can be simplified as

$$v_{ab} = V_m \cos(2\pi f \cdot t + \theta)$$
$$= V_m \cos(2 \times 3.14 \times 50\,t + \theta)$$
$$= V_m \cos(314\,t + \theta),\ \theta \text{ in radians]}$$

5.102 A cosine wave is expressed as $v_{ab} = V_m \cos(360\,ft + \theta)$, where θ is in degree. Convert it to sine wave expression.

Solution

Given, $v_{ab} = V_m \cos(360\,ft + \theta)$

(Note that '360' stands for 2π, where $\pi = 180°$) since we can convert a cosine function into a sine function by adding 90° to the angle (θ), we now can write

$$v_{ab} = V_m \sin(360\,ft + \theta + 90°),\ \theta \text{ in degree.}$$

[Similarly, we can convert a sine function $V_{ab} = V_m \sin(360\,ft + \theta)$ to a cosine function by subtracting 90° from θ.

i.e. $v_{ab} = V_m \cos(360\,ft + \theta - 90°)$, θ in degree.]

5.103 A voltage wave is applied across a load at input terminals marked "$x - y$". The voltage wave is denoted by a conventionl cosine wave $v_{x-y} = V_m \cos(360\,ft + \theta)$, θ being expressed in degree and has a fixed value of 60° in the present case. If $V_m = 50$ V, find the voltage at $t = 0$, $t = 0.01$, $t = 0.05$ and $t = 1$ sec. Assume a 50 Hz frequency system.

Solution

$$v_{x-y} = V_m \cos(360\, ft + \theta),\ \theta \text{ in degree}$$

Here $v_{x-y} = 50 \cos(360 \times 50 \times 0 + 60°)$

$= 50 \cos 60° = 25$ Volts [at $t = 0$ sec.]

Similarly, for we get $t = 0.01$, 0.05 and 1.0 sec,

$$v_{x-y} = 50 \cos(360 \times 50 \times 0.01 + 60°)$$
$$= -25 \text{ volts [at } t = 0.01 \text{ sec.]}$$
$$v_{x-y} = 50 \cos(360 \times 50 \times 0.05 + 60°)$$
$$= -25 \text{ volts[at } t = 0.05 \text{ sec.]}$$
$$v_{x-y} = 50 \cos(360 \times 50 \times 1 + 60°)$$
$$= +25 \text{ volts [at } t = 1.0 \text{ sec]}$$

Observing the above result we may infer that v_{x-y} is +ve at $t = 0$ and 1.0 sec while v_{x-y} is –ve at $t = 0.01$ and $t = 0.05$ sec. This clearly indicates that at $t = 0$ or at $t = 1.0$ sec, terminal 'x' of that load is +ve with respect to terminal 'y' while for $t = 0.01$ or $t = 0.05$ sec., the terminal 'y' is +ve with respect to terminal 'x'.

[Note that the instantaneous values of v_{x-y} alternate in magnitude and sign depending on the instant of 't' while the rms value remains the same which can be calculated for the given problem as $(V_{\text{rms}}) = \dfrac{V_m}{\sqrt{2}} = \dfrac{50}{\sqrt{2}} = 35.36$ V.]

5.104 A voltage applied across a load terminal 1-2, is expressed as $v_{1-2} = V_m \cos(2\pi ft + \theta)$ in a system having 50 Hz frequency and $\theta = 30°$. Obtain the value of instantaneous voltage at $t = 27.15$ sec. and find how many cycles the voltage wave corresponds when $t = 27.15$ sec. Assume the peak voltage of the wave to be 100 V.

Solution

It is given that

$$v_{1-2} = V_m \cos(2\pi ft + \theta)$$

when π and θ are expressed in degrees.

Here $v_{1-2} = 100 \cos(360 \times 50 \times 27.15 + 30°)$

$= 100 \cos(488700 + 30°)$

$= -86.6$ V.

∴ The instantaneous value of v_{1-2} is (–86.6) volts at $t = 27.15$ sec. and terminal 1 is negative with respect to 2.

For the second part of the given problem we would like to check the number of cycles that corresponds to $t = 27.15$ sec. It may be noted here that an angle of (488700 + 30)° corresponds to (488730 ÷ 360°) cycles (or, 1357.5833 cycles). Then at $t = 27.15$ sec, the system completed 1357 full cycles and 0.5833 cycles. The latter corresponds to 0.5833 × 360° = 209.988°. [Also, 100 cos (209.988°) = (–86.6) volts].

5.105 AC voltage of 230 V (rms) is applied across a load that takes a current of 10 A (rms) from the source. The current lags the voltage by 30°. Assuming the source ac voltage to be a sine wave of 50 Hz frequency, write down the expression for the voltage and current. Draw the respective wave shapes as well as the phasors.

Solution

Given: $V_{\text{rms}} = 230$ V.

$\therefore \qquad V_{max} = V_{rms} \times \sqrt{2} = 325.22$ V.

Also, $\qquad I_{max} = 10 \times \sqrt{2} = 14.14$ A

$[\because I_m = I_{rms} \times \sqrt{2}$ and $I_{rms} = 10$ A]

The voltage expression can be written as

$$v = V_{max} \sin(\omega t)$$
$$= 325.22 \sin (314\, t) \text{ V},$$

when ω is expressed in rad/sec.

Also, $\qquad v = 325.22 \sin (2 \times 180 \times 50t)$

(when the angle is expressed in degree.)

i.e. $\qquad v = 325.22 \sin (18000\, t)$ V.

Since the current lags voltage by 30°, we can write

$$i = I_m \sin (\omega t - \phi)$$
$$= 14.14 \sin \left(314t - \frac{\pi}{6}\right) \text{ A}$$

when the angle is expressed in radian

and $\qquad i = 14.14 \sin (2 \times 180 \times 50t - 30°)$

$$= 14.14 \sin (18000t - 30°),$$

when the angle is expressed in degree.

The phasor representation is given in Fig. 5.65

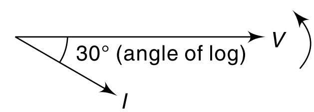

Fig. 5.65 ***I lagging V by 30°***

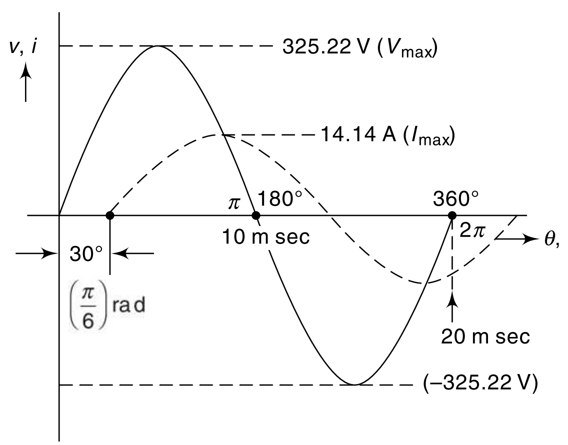

Fig. 5.66 ***V, I waves shapes***

5.106 Obtain the average value of the current waveform $i = I_m \sin \omega t$ for (a) one complete period T and (b) half period $T/2$.

Solution

(a) $$I_{av} = \int_0^T I_m \sin\omega t \, dt = \frac{1}{T}\int_0^T I_m \sin \frac{2\pi}{T} \cdot t.\, dt$$

$$= -\frac{I_m}{2\pi}\left[\cos \frac{2\pi}{T} \cdot t\right]_0^T = 0.$$

Thus average value of a sine wave (or even a cosine wave) for one complete period is zero.

(b)
$$I_{av} = \frac{1}{T/2} \int_0^{T/2} I_m \sin \frac{2\pi}{T} \cdot t \cdot dt$$

$$= -\frac{\text{Im}}{\pi} \left[\cos \frac{2\pi}{T} \cdot t \right]_0^{T/2} = +\frac{2I_m}{\pi}.$$

It may be noted here that we have assumed the current waveform starting from the origin. As a general case we may think of the ac waveform starting from $t = t_o$ in the time scale on x-axis instead of assuming it starting from $t = 0$.

This gives us

$$I_{av} = \frac{1}{T} \int_{t_0}^{t_0+T} I_m \sin \frac{2\pi}{T} \cdot t \cdot dt$$

$$= -\frac{I_m}{2\pi} \left[\cos \frac{2\pi}{T} \cdot t \right]_{t_0}^{t_0+T} = 0, \text{ for the complete cycle.}$$

For the half cycle,

$$I_{av} = \frac{1}{T/2} \int_{t_0}^{t_0+T} I_m \sin \frac{2\pi}{T} \cdot t \cdot dt$$

$$= -\frac{I_m}{\pi} \left[\cos \frac{2\pi}{T} \cdot t \right]_{t_0}^{t_0+T/2}$$

$$= -\frac{I_m}{\pi} \left(-\cos \frac{2\pi t_0}{T} - \cos \frac{2\pi t_0}{T} \right)$$

$$= \frac{2I_m}{\pi} \cos \omega t_o.$$

Check: if we assume $t_o = 0$, I_{av} for the half cycle of the ac wave becomes $\left(\frac{2I_m}{\pi} \right)$.

It may be noted here we could have written I_{av} for full cycle as

$$I_{av} = \frac{1}{2\pi} \int_0^{2\pi} I_m \sin \omega t \, d(\omega t)$$

and for half cycle as

$$I_{av} = \frac{1}{\pi} \int_0^{\pi} I_m \sin \omega t \, d(\omega t)$$

5.107 Draw the power triangles for inductive and capacitive loads.

Solution

We start drawing with voltage and current phasors and in next step we draw power triangles (Fig. 5.67) and (Fig. 5.68). The steps are shown as follows:

For inductive loads

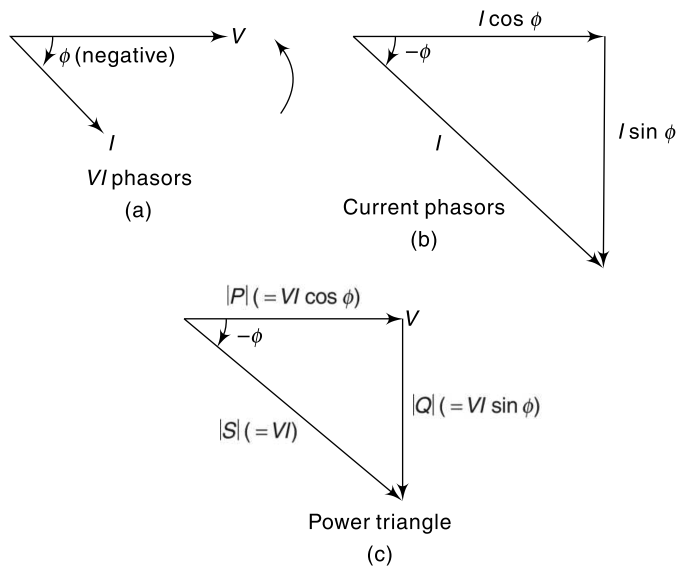

Fig. 5.67

For capacitive loads

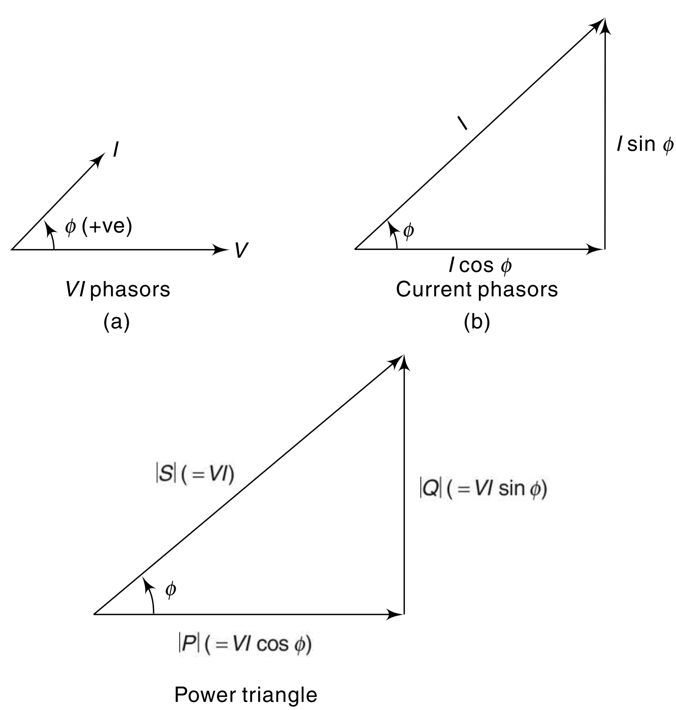

Fig. 5.68

Note: $$P = VI \cos \phi = (IZ)\, I \times \frac{R}{Z} = I^2R$$

$$Q = VI \sin \phi = (IZ) \times I \times \frac{X}{Z} = I^2X$$

$$|S| = \sqrt{P^2 + Q^2}. \qquad \angle\phi = \tan^{-1}\frac{Q}{P}$$

$$\cos \phi = \frac{P}{\sqrt{P^2+Q^2}} = \frac{P}{|S|}.$$

5.108 AC voltage $(v) = V_m \sin \omega t$ is applied across a load R. Obtain the expressions for instantaneous and average power.

Solution

$$\text{Instantaneous power } (p) = \frac{v^2}{R}$$

or $$p = \frac{V_m^2}{R} \cdot \sin^2 \omega t = \frac{V_m^2}{2R}(1 - \cos 2\,\omega t)$$

In the expression of p, the constant part is $\left(\dfrac{V_m^2}{2R}\right)$ and hence it represents the magnitude of average power.

$\therefore$ $$P_{\text{av}} = \frac{V_m^2}{2R}.$$

5.109 A voltage and a current in a circuit are expressed as

$$v = V_m \sin \omega t$$
$$i = I_m \sin(\omega t - \theta).$$

Find the instantaneous power.

Solution

Instantaneous power p is given by

$$p = v.i = V_m \sin \omega t \times I_m \sin(\omega t - \theta)$$
$$= V_m \times I_m \times \frac{1}{2}[\cos(\omega t - \omega t + \theta) - \cos(\omega t + \omega t - \theta)]$$
$$= \frac{1}{2} V_m I_m [\cos \theta - \cos(2\omega t - \theta)].$$

[It can be observed that

$$P_{\text{av}} = \frac{1}{2} V_m I_m \cos \theta,$$

but, $$V_m = \sqrt{2} \times V_{\text{rms}};\ I_m = \sqrt{2}\, I_{\text{rms}}$$

$\therefore$ $$P_{av} = \frac{1}{2} \times 2\, V_{\text{rms}} I_{\text{rms}} \cos\theta$$
$$= V_{\text{rms}} \times I_{\text{rms}} \times \cos\theta$$

'$\cos\theta$' being known as power factor.]

5.110 The following voltages are expressed in time domain. Convert it to phasor domain.

(a) $100 \sin\left(\omega t + \frac{\pi}{3}\right)$ (b) $100 \sin \omega t$ (c) $75 \cos \omega t$

(d) $170 \sin\left(\omega t - \frac{2\pi}{9}\right)$ (e) $\sqrt{2} \times 165 \sin\left(\omega t - \frac{7\pi}{18}\right)$

Solution

(a) $100 \sin\left(\omega t + \frac{\pi}{3}\right) = \frac{100}{\sqrt{2}} \angle 60° = 70.7\angle 60°.$

(b) $100 \sin \omega t = \frac{100}{\sqrt{2}} \angle 0° = 70.7\angle 0°.$

(c) $75 \cos \omega t = 75 \sin\left(\omega t + \frac{\pi}{2}\right) = \frac{75}{\sqrt{2}} \angle 90° = 53 \angle 90°.$

(d) $170 \sin\left(\omega t - \frac{2\pi}{9}\right) = \frac{170}{\sqrt{2}} \angle{-40°} = 120\angle{-40°}.$

(e) $\sqrt{2} \times 165 \sin\left(\omega t - \frac{7\pi}{18}\right) = \frac{\sqrt{2} \times 165}{\sqrt{2}} \angle{-70°} = 165\angle{-70°}$

5.111 (a) Why the p.f. if ac circuit is always positive?
(b) Discuss when the reactive power is positive or negative.

Solution

(a) p.f. is represented by cos θ. Since cos θ is an even function [cos θ = cos $(-\theta)$] hence the power factor is always positive. Moreover, it is obviously always ≤ 1.0.

(b) Q (reactive power) = $VI \sin \theta$
The sine wave is an odd function [sin $\theta = -\sin(-\theta)$]. Hence Q is +ve when θ is negative Q is –ve when θ is +ve.
$\therefore$ Lagging current (inductive circuit) produces +ve and leading current (capacitive circuit) produces $(-Q)$. In resistive circuit $\theta = 0$ giving $Q = 0$.

5.112 A saw-tooth current waveform is shown in Fig. 6.69. Determine the average value, rms value and form factor.

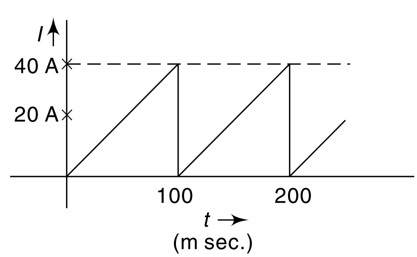

Fig. 5.69 *Waveform for Ex. 5.112*

Solution

The average value is obtained as

$$I_{av} = \frac{1}{T}\int_0^T i\,dt.$$

Here, $T = 100 \times 10^{-3}$ sec.

and $i = \frac{40 \times t}{100 \times 10^{-3}}$, a linear function

$$\therefore \quad I_{av} = \frac{1}{100 \times 10^{-3}} \int_0^{100 \times 10^{-3}} \frac{40}{100 \times 10^{-3}} \cdot t\,dt$$

$$= \frac{40}{(100 \times 10^{-3})^2} \cdot \left.\frac{t^2}{2}\right|_0^{100 \times 10^{-3}} = 20 \text{ A.}$$

rms value can be obtained as

$$I_{\text{rms}} = \left[\frac{1}{T}\int_0^T i^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{100\times10^{-3}}\int_0^{100\times10^{-3}}\left(\frac{40\times t}{100\times10^{-3}}\right)^2 dt\right]^{1/2}$$

$$= \left[\frac{1}{100\times10^{-3}}\times\int_0^{100\times10^{-3}}(400\,t)^2\, dt\right]^{1/2}$$

$$= \left[\frac{(400)^2}{100\times10^{-3}}\times\frac{t^3}{3}\bigg|_0^{100\times10^{-3}}\right]^{1/2}$$

$$= 23.1 \text{ A.}$$

The form factor is given by $\left(\dfrac{I_{\text{rm}}}{I_{\text{av}}}\right)$. Here form factor $\left(=\dfrac{23.1}{20}\right)$ = 1.155.

5.113 For the waveform shown in Fig. 5.70, find the average and rms values for full cycle.

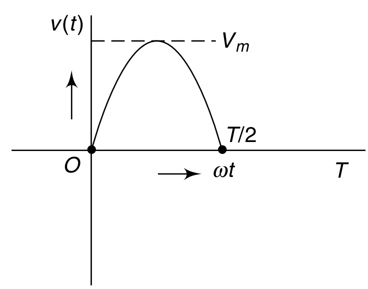

Fig. 5.70 ***Waveform for Ex. 5.113 [T = 2π; T/2 = π]***

Solution

$$V_{\text{av}} = \frac{1}{T}\int_0^{T/2} V_m \sin \omega t\, dt$$

[$\because$ voltage between internal $T/2$ and T is zero]

$$= \frac{V_m}{\omega T}[-\cos \omega t]_0^{T/2} = \frac{V_m}{\pi}.$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T V_m^2 \sin^2 \omega t\, dt}$$

$$= \sqrt{\frac{1}{2T}\int_0^{T/2} V_m^2\,(1-\cos 2\,\omega t)\, dt}$$

$$= \frac{V_m}{2}.$$

[The form factor can be calculated as $V_{\text{rms}}/V_{\text{av}}$ i.e, $(V_m/2)/(V_m/\pi)$ or, 1.57.]

5.114 Find the rms value of the resultant current in a wire that simultancously carries a direct current of 10 A and a sinusoidal alternating current of peek value 10 A.

Solution

As per the given question,

$$I = I_{\text{dc}} + I_{\text{ac}} = 10 + 10 \sin \omega t$$

$$= 10\,(1 + \sin \omega t).$$

$$\therefore \quad I_{\text{rms}} = \left[\frac{1}{2\pi}\int_0^{2\pi}\{10\,(1+\sin\omega t)\}^2\,d\,\omega t\right]^{1/2}$$

$$= \left[\frac{10^2}{2\pi}\int_0^{2\pi}(1+2\sin\omega t+\sin^2\omega t)\,d\,\omega t\right]^{1/2}$$

$$= 10\left\{\frac{1}{2\pi}\left[(\omega t)-2\cos(\omega t)+\frac{\omega t}{2}-\frac{\sin 2\,\omega t}{4}\right]_0^{2\pi}\right\}^{1/2}$$

$$= 10\left\{\frac{1}{2\pi}\times 3\pi\right\}^{1/2} = 12.25 \text{ A.}$$

It may be noted here that we may replace 2π by T, the time period of the function and may write as

$$I_{\text{rms}} = \left[\frac{1}{T}\int_0^{T}\{10\,(1+\sin\omega t)\}^2\,dt\right]^{1/2}$$

where $\omega = \dfrac{2\pi}{T}$.

5.115 Obtain average and effective value of waveform shown in Fig. 5.71.

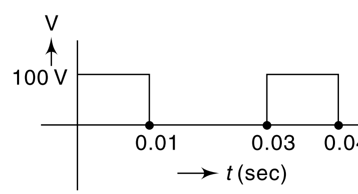

Fig. 5.71 ***Waveform for Ex. 5.115***

Solution

$$V_{\text{av}} = \frac{1}{T}\int_0^{T/3} v\,dt$$

$$= \frac{1}{0.03}\int_0^{0.01} 100.\,dt \qquad [\because\ v(t) = 100 \text{ for } 0 \le t \le 0.01;\ v(t) = 0 \text{ for } 0.01 \le t \le 0.03]$$

$$= \frac{1}{0.03}\times 100\,[t]_0^{0.01}$$

$$= 33.33 \text{ V.}$$

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^{T/3} v^2\,dt}$$

$$= \sqrt{\frac{1}{0.03}\int_0^{0.01} 100^2\,dt}$$

$$= \sqrt{\frac{1}{0.03}\times 100^2\times[t]_0^{0.01}}$$

$$= 57.74 \text{ V.}$$

5.116 Obtain the average and rms values of the waveform shown in Fig. 5.72.

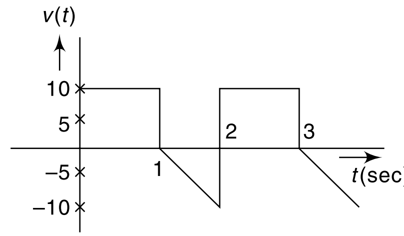

Fig. 5.72 *Waveform for Ex. 5.116*

Solution

From the given figure,

$$v(t) = 10, \text{ for } 0 \le t \le 1 \text{ sec}$$

$$v(t) = -10t + 10, \text{ for } 1 \le t \le 2 \text{ sec}$$

$$\therefore \quad V_{av} = \frac{1}{2}\left[\int_0^1 10\,dt + \int_1^2 (-10\,t + 10)\,dt\right]$$

$$= \frac{1}{2} \times 10 \times 1 + \frac{1}{2} \times (-10) \times \left[\frac{2^2 - 1^2}{2}\right] + \frac{1}{2} \times 10 \times [2 - 1]$$

$$= 5 - 7.5 + 5 = 2.5 \text{ V.}$$

$$V_{rms} = \sqrt{\frac{1}{2}\left[\int_0^1 10^2\,dt + \int_1^2 (-10\,t + 10)^2\,dt\right]}$$

$$= \frac{1}{\sqrt{2}}\left[100 \times t\,|_0^1 + \int_1^2 10^2\,(1 - t)^2\,dt\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}} \times \left[100 + 100\int_1^2 (1 + t^2 - 2t)\,dt\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[100 + 100\left\{(t|_1^2) + \frac{t^3}{3}\bigg|_1^2 - 2 \cdot \frac{t^2}{2}\bigg|_1^2\right\}\right]^{1/2}$$

$$= \frac{1}{\sqrt{2}}\left[100 + 100 + \left\{1 + \frac{7}{3} - 3\right\}\right]^{1/2}$$

$$= 8.17 \text{ V.}$$

5.117 Determine the average and the rms values of the waveform shown in Fig. 5.73.

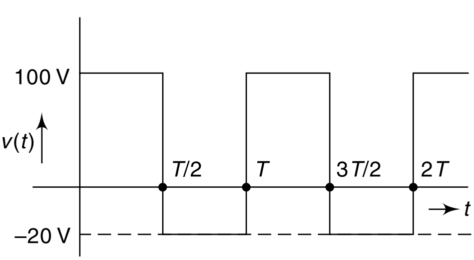

Fig. 5.73 *Waveform for Ex. 5.117*

Solution

It is evident that

$$v(t) = 100, \text{ for } 0 \le t \le \frac{T}{2}$$

$$= -20 \text{ for } \frac{T}{2} \le t \le$$

$$V_{av} = \frac{1}{T}\int_0^T v(t)\,dt = \frac{1}{T}\left[\int_0^{T/2} 100\,dt + \int_{T/2}^{T} -20\,dt\right]$$

$$= \frac{100}{T}[t]_0^{T/2} + \frac{-20}{T}[T]_{T/2}^{T}$$

$$= 50 - 10 = 40 \text{ V.}$$

$$V_{\text{rms}} = \left[\frac{1}{T}\int_0^T v^2(t)\,dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^{T/2}(100)^2\,dt + \int_{T/2}^{T}(-20)^2\,dt\right]^{1/2}$$

$$= \left[\frac{10^4}{T}\cdot t\Big|_0^{T/2} + \frac{400}{T}\cdot t\Big|_{T/2}^{T}\right]^{1/2}$$

$$= [5 \times 10^3 + 200]^{1/2} = (5200)^{1/2} = 72.1 \text{ V}.$$

5.118 Obtain the rms value of the clipped waveform shown in the Fig. 5.74

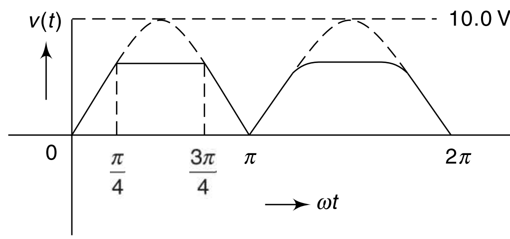

Fig. 5.74 ***Waveform for Ex. 5.118***

Solution

By observation we can say (with $\theta = \omega t$),

$$v(\theta) = 10 \sin \theta, \text{ when } 0 < \theta < \frac{\pi}{4}$$

$$v(\theta) = 7.07, \text{ when } \frac{\pi}{4} < \theta < \frac{3\pi}{4}$$

$$v(\theta) = 10 \sin \theta, \text{ when } \frac{3\pi}{4} < \theta < \pi.$$

[It may be noted that at $\theta = \frac{\pi}{4}$, since $10 \sin \theta = 10 \sin \frac{\pi}{4} = 7.07$, hence the equation of the wave for the period $\frac{\pi}{4}$ to $\frac{3\pi}{4}$ is 7.07 (constant magnitude).]

$$\therefore \quad V_{\text{rms}} = \left[\frac{1}{\pi}\int_0^{\pi/4}(10\sin\theta)^2\,d\theta + \int_{\pi/4}^{3\pi/4}(7.07)^2\,d\theta + \int_{3\pi/4}^{\pi}(10\sin\theta)^2\,d\theta\right]^{1/2}$$

$$= \left[\frac{1}{\pi}\left\{\int_0^{\pi/4}100\,\frac{(1-\cos 2\theta)}{2}\,d\theta + \int_{\pi/4}^{3\pi/4}50\,d\theta + \int_{3\pi/4}^{\pi}100\,\frac{1-\cos 2\theta}{2}\,d\theta\right\}\right]^{1/2}$$

$$= \left[\frac{50}{\pi}(\pi - 1)\right]^{1/2} = (34.1)^{0.5}$$

$$= 5.75 \text{ V}.$$

5.119 The voltage and currents in a circuit element are given as $v = 141 \sin (314\,t + 30°)$ V and $i = 14.1 \sin (314\,t - 60°)$ A. Identify the element and find its value.

Solution

Since the angle associated with voltage is +ve while that for the current is –ve, hence the voltage leads the current. The angle of lead is 30–(–60), i.e. 90°. Hence the element is an inductor.

$$X_L = \frac{V_m}{I_m} = \frac{141}{14.1} = 10\ \Omega$$

or $$\omega L = 10, \text{ giving } L = \frac{10}{2\pi f} = \frac{10}{314} = 31.8 \text{ mH.}$$

5.120 The voltage and current in an element are

$$v = 100 \sin(314\,t - 20°) \text{ V}$$
$$i = 10 \sin(314\,t - 20°) \text{ A.}$$

Identify the element and find its values.

Solution

It may be observed that the voltage and current are in same phase. Thus the element is a resistor.

$$R = \frac{V_m}{I_m} = \frac{100}{10} = 10\ \Omega.$$

5.121 The voltage and current in a circuit element are $v = 100 \cos(314\,t - 80°)$ V and $i = 100 \cos(314\,t + 10°)$ A. Identify the element and find its value.

Solution

It may be observed that the current leads the voltage (angle associated with current is +ve while that associated with voltage is –ve).

The angle of lead for the current is 10 – (–80°) = 90°.

Thus the element is a capacitor.

$$X_C = \frac{V_m}{I_m} = \frac{100}{100} = 1\ \Omega = \frac{1}{\omega C}.$$

$$\therefore \quad C = \frac{1}{314 \times 1} = 3184.7\ \mu\text{F.}$$

5.122 What is the reactance of a 10 μF capacitor at $f = 0$ Hz (dc) and $f = 50$ Hz?

Solution

At dc, $X_C = \dfrac{1}{2\pi \times 0 \times 10 \times 10^{-6}}$ = (infinite) Ω i.e. across a dc voltage, the capacitor would act as an open circuit at steady state.

When $f = 50$ Hz,

$$X_C = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.47\ \Omega.$$

[If the element is an inductor, the reactance X_L at $f = 0$ Hz would have been $X_L = 2\pi \times 0 \times L = 0\ \Omega$, indicating that the inductor acts as a short circuit across a dc voltage at steady state.]

5.123 At what frequency will a 100 μF capacitor offer a reactance of 100 Ω?

Solution

$$X_C = \frac{1}{\omega C}$$

$$\therefore \quad \omega = \frac{1}{X_C \times C} = \frac{1}{100 \times 100 \times 10^{-6}}$$
$$= 100 \text{ rad/sec.}$$

$\therefore \quad f = \dfrac{100}{2\pi} = 15.9$ Hz.

At 15.9 Hz, the given capacitor will offer a reactance of 100 Ω.

5.124 An inductor draws a current $i = I_m \sin \omega t$. Obtain the expression of instantaneous voltage across it.

Solution

$v_L = L\dfrac{di}{dt}$; (v) being the voltage developed across the inductor opposing the supply voltage.

Here,
$$v_L = L \cdot \frac{d}{dt}(I_m \sin \omega t)$$
$$= \omega L\, I_m \cos \omega t = V_m \cos \omega t$$
$$= V_m \sin(\omega t + 90°).$$

$\therefore$ If (v) be the applied voltage
$$v = V_m \sin(\omega t - 90°)$$
$$i = I_m \sin \omega t$$
$$v_L = V_m \sin(\omega t + 90°).$$

[(v_L) is in direct opposition to v as phase angle between (v) and (v_L), is 180°].

5.125 In an ac circuit
$$v = 200 \sin 314\, t \text{ V}$$
$$i = 20 \sin(314\, t - 30°)$$

Determine (a) the power factor
- (b) True or active power
- (c) Apparent or total power
- (d) Reactive power.

Solution

The phase angle ϕ between the voltage and current is 30° while the current lags the voltage.

Here,
$$V_{rms}\,(=V) = \frac{200}{\sqrt{2}} = 141.44 \text{ V}.$$
$$I_{rms}\,(=I) = \frac{20}{\sqrt{2}} = 14.14 \text{ A}.$$

$\therefore$ True power $= V_{rms} \times I_{rms} \times \cos\phi$ [$\cos\phi$ being the power factor]
$$= 141.44 \times 14.14 \times \cos 30°$$
$$= 1732 \text{ W}.$$

Hence we have observed that p.f. of the circuit is 0.866 while the true or active power consumed is 1732 W. The apparent (or total power) being given by ($V \times I$), we find that its value in the given problem is (141.44 × 14.14), i.e. 2000 VA(2 kVA). The reactive power is obtained as $VI \sin\phi$, i.e. 141.44 × 14.14 × sin 30° or 1 kAR (app).

5.126 AC voltage of $79\angle 71.50°$ V is applied across a load which draws $2.83\angle 45°$ A. Find the value of active power.

Solution

$$P = Re\,[VI^*] = Re\,[(25 + j75)(2 - j2)]$$
$$= 200 \text{ W}$$

$[\because\ 79\angle 71.50^\circ = 25 + j75$

and $\quad 2.83\angle 45^\circ = 2 + j2$

while $\quad (2.83\angle 45^\circ)^* = 2 - j2]$

Also to have a check we see

$$P = VI\cos\phi = 79 \times 2.83 \times \cos(71.5^\circ - 45^\circ) = 200 \text{ W.}$$

5.127 An inductive coil consumes active power of 500 W and draws 10 A from a 60 Hz AC supply of 110 V. Obtain the values of resistance and inductance of the coil.

Solution

$\because \quad I^2R = 500 \text{ W};\quad I = 10 \text{ A}$

we find $\quad R = 500/10^2 = 5\ \Omega.$

However, $\quad Z = \dfrac{V}{I} = \dfrac{110}{10} = 11\ \Omega$

$\therefore \quad Z = \sqrt{R^2 + (\omega L)^2}.$

Here $\quad (\omega L)^2 = (11)^2 - (5)^2 = 96$

$\therefore \quad L = \dfrac{\sqrt{96}}{\omega} = \dfrac{\sqrt{96}}{2 \times \pi \times 60} = 26 \text{ mH.}$

Thus, the given coil has 5 Ω resistance and 26 mH inductance.

5.128 In the given circuit currents through r_1 and C are equal in magnitude. If power consumed by the circuit is 12 kW, calculate the values of r_1, r_2 and C. Assume current through the circuit as $5\angle{+45^\circ}$ A.

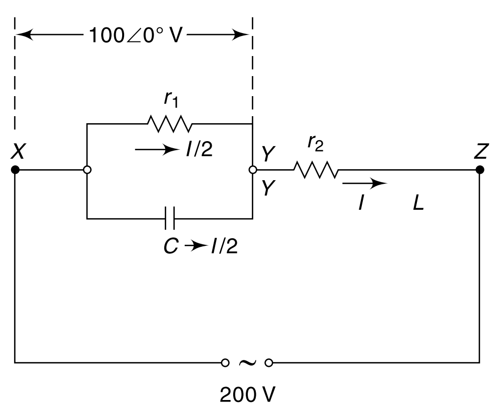

Fig. 5.75 *Circuit diagram for Ex. 5.128*

Solution

$$Z_{x-y} = \frac{r_1 \times (-jX_C)}{r_1 - jX_C} = \frac{-jr_1^2}{r_1 - jr_1}$$

[$\because$ Currents in r_1 and C are equal hence R_1 must be equal to $|X_C|$]

i.e. $$Z_{x-y} = \frac{-jr_1^2(r_1 + jr_1)}{r_1^2 + r_1^2} = \frac{r_1^3 - jr_1^3}{r_1^2}$$

$$= \frac{r_1 - jr_1}{2} = \frac{r_1}{\sqrt{2}}\angle -45^\circ\ \Omega.$$

$\therefore \quad V_{x-y} = IZ_{x-y}$

or $\quad I = \dfrac{V_{x-y}}{Z_{x-y}}$

or $\quad 5\angle{+45^\circ} = \dfrac{100\angle 0^\circ}{\dfrac{r_1}{\sqrt{2}}\angle -45^\circ} = \dfrac{100\sqrt{2}}{r_1\angle -45^\circ}$

$\therefore \quad r_1 = 100\sqrt{2}\ \ \Omega$

given that $P = 12$ kW

i.e . $I^2(r_1 + r_2) = 12000$

$$\therefore \quad r_1 + r_2 = \frac{12000}{25} = 480\ \Omega$$

$$\therefore \quad r_2 = 400 - 100\sqrt{2} = 259\ \Omega$$

Also we have seen, $r_1 = X_C$

$$\therefore \quad X_C = 100\sqrt{2} = 141\ \Omega$$

$$\text{or} \quad \frac{1}{2\pi f C} = 141$$

$$C = \frac{1}{2\pi \times 50 \times 141} = 22.6\ \mu\text{F}$$

$$\therefore \quad r_1 = 141\ \Omega;\ r_2 = 259\ \Omega$$

$$C = 22.6\ \mu\text{F}.$$

5.129 Calculate the real and reactive power to each load and the total complex power provided by the source (Fig. 5.76)

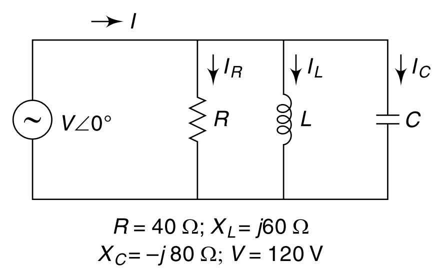

Fig. 5.76 ***Circuit diagram for Ex. 5.129***

Solution

$$I_R = \frac{V}{R} = \frac{120\angle 0°}{40} = 3\angle 0°\ \text{A}$$

$$I_L = \frac{V}{X_L} = \frac{120\angle 0°}{j60} = 2\angle -90°\ \text{A}$$

$$I_C = \frac{V}{X_C} = \frac{120\angle 0°}{-j80} = 1.5\angle 90°\ \text{A}.$$

$\therefore$ total current $(I) = I_R + I_L + I_C$ (vector sum)

$$= 3\angle 0° + 2\angle -90° + 1.5\angle 90°$$

$$= 3 - j0.5 = 3.041\angle -9.46°\ \text{A}.$$

Active power in R being given by P, we find

$$P = V \times I_R = 120 \times 3 = 360\ \text{W}$$

$$P = I_R^2 \times R = 3^2 \times 40 = 360\ \text{W}$$

$$P = \frac{V^2}{R} = \frac{(120)^2}{40} = 360\ \text{W}.$$

Since $\phi = 0$ in the expression of current I_R i.e, as I_R and V are in phase, hence reactive power consumed in R is zero.

Again for L and C elements, ϕ is either –90° or +90°, i.e $\cos\phi = 0$ in both the cases. Hence reactive power consumption by L or C element is zero.

Reactive power for L is obtained as

$$Q_L = V \times I_L = 120 \times 2 = 240\ \text{VAR}$$

$$Q_L = I_L^2 X_L = 2^2 \times 60 = 240\ \text{VAR}$$

$$Q_L = \frac{V^2}{X_L} = \frac{120^2}{60} = 240\ \text{VAR}.$$

Reactive power for C element is obtained as

$$Q_C = V \times I_C = 120 \times 1.5 = 180\ \text{VAR}$$

$$Q_C = I_C^2 \times X_C = (1.5)^2 \times 80 = 180\ \text{VAR}$$

$$Q_C = (V^2/X_C) = 120^2/80 = 180\ \text{VAR}.$$

For total complex power we can write

$$S = P + j\,(Q_L - Q_C).$$

[Q_L is +ve in inductive circuit while Q_C is negative for capacitive circuit]

$$= 360 + j\,(240 - 180) = (360 + j60)\text{ VA}$$

[Also, $S = VI^* = 120\angle 0° \times 3.041\angle 9.46°$

$= 364.9\angle 9.46°\text{ VA} = 360 + j60\text{ VA}$.]

The real power provided by the source is 360 W. The reactive power provided by the source is 60 VAR (inductive circuit requirement is actually 240 VAR but capacitor generates 180 VAR hence net requirement is only 60 VAR).

5.130 A resistor and a capacitor are connected in series across a 150 V AC 40 Hz supply. The current in the circuit is measured as 5 A. If the frequency of the supply be raised to 50 Hz, the current becomes 6 A. Find the values of the resistance and capacitance.

Solution

When $V = 150\text{ V}, f_1 = 40\text{ Hz}, I_1 = 5\text{ A}$

$$\therefore \quad Z_1 = \frac{V}{I_1} = \frac{150}{5} = 30\ \Omega.$$

[Z_1 is the circuit impedance of the *RC* series circuit at 40 Hz supply frequency.]

When $V = 150\text{ V},\quad f_2 = 50\text{ Hz},\quad I_2 = 6\text{ A}.$

$$\therefore \quad Z_2 = \frac{V}{I_2} = \frac{150}{6} = 25\ \Omega.$$

[Z_2 is the circuit impedance of the same R_C series circuit at 50 Hz supply frequcy.]

$\because$ In a series *RC* circuit,

$$Z = \sqrt{R^2 + (1/2\pi \times f \times C)^2}\ ,$$

for the first case we can write

$$\sqrt{R^2 + \left(\frac{1}{2\pi \times 40 \times C}\right)^2} = 30$$

and in the second case we can write

$$\sqrt{R^2 + \left(\frac{1}{2\pi \times 50 \times C}\right)^2} = 25.$$

Solving these two equations for two unknown R and C, we get

$$R = 30\text{ ohm} \quad \text{and} \quad C = 492\ \mu\text{F}.$$

5.131 For the circuit shown in Fig. 5.77, find the value of C such that the input current is 45° out of phase with the input voltage. Assume $\omega = 2 \times 10^3$ rad/sec.

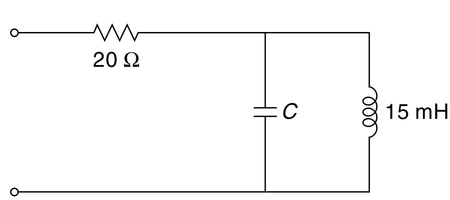

Fig. 5.77 *Circuit diagram for Ex. 5.131*

Solution

For the input current to be 45° out of phase with the input voltage, the net reactance of the parallel portion of *LC* circuit must be ±20 Ω.

i.e. $$\pm\frac{j}{20} = -\frac{j}{\omega L} + j\omega C$$

or $$C = \frac{1}{\omega}\left(\frac{1}{\omega L} \pm \frac{1}{20}\right)$$

$$= \frac{1}{2000}\left(\frac{1}{2000 \times 15 \times 10^{-3}} \pm \frac{1}{20}\right)$$

= (neglecting –ve value)

The feasible value is $C = 41.67 \times 10^{-6}$ μF.

5.132 Given, $I = 11.81\angle{-7.12}°$
$R_1 = 10\ \Omega$
$R_2 = 15\ \Omega$
$X_C = -j15\ \Omega$
$V = 220\angle 0°$.

Obtain the reactance offered by the inductance L at 50 Hz in Fig. 5.78.

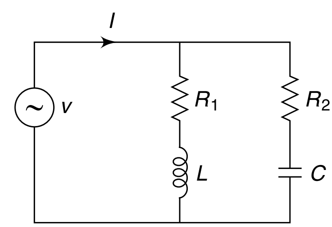

Fig. 5.78 *Circuit diagram for Ex. 5.132*

Solution

Current through ($R_2 - C$) circuit is given by

$$I_{R-C} = \frac{V}{Z_{ac}} = \frac{220\angle 0°}{15 - j15} = \frac{220\angle 0°}{21.2\angle{-45}°}$$

$= 10.35\angle 45° = (7.34 + j7.34)$ A.

∴ Current through $R - L$ circuit is

$I_{R-L} = I - I_{R-C}$

$= 11.81\angle{-7.12} - 10.35\angle 45° = 4.38 - j8.8$

$= 9.83\angle{-63.6}°$ A.

However, $$I_{R-L} = \frac{V}{Z_{R-L}} = \frac{220\angle 0°}{10 + jX_L}$$

or $$(4.38 - j8.8) = \frac{220 + j.0}{10 + jX_L}.$$

∴ $X_L = 20\ \Omega$.

We can now find L as shown below;

$X_L = \omega L = 20$, where $(\omega) = 2\pi f = 314$

∴ $$L = \frac{20}{314} = 63.7 \text{ mH}.$$

5.133 A choke coil having a resistance of 4 Ω and inductance of 2 H is in parallel to a capacitor C. The supply voltage applied to this parallel combination is $(v) = 10 \cos 2t$ V.

(a) Obtain the current passing through the choke.

(b) Obtain the value of C such that the supply voltage V and the supply current I are in same phase.

Solution

Let us first draw the circuit as per given data (Fig. 5.79)

Here, $$I_1 = \frac{(V_m/\sqrt{2})}{(4 + j4)} \text{ A}.$$

(∵ for the choke coil, the impedance is $4 + j \times \omega \times L = 4 + j \times 2 \times 2 = 4 + j4\ \Omega$)

i.e. $$I_1 = \frac{10/\sqrt{2}}{4+j4} = \frac{7.072\angle 0°}{5.66\angle 45°} = 1.25\angle{-45°}$$
$$= (0.884 - j0.884) \text{ A.}$$

$$I_2 = \frac{10/\sqrt{2}}{1/j \times 2 \times C} \quad [\because X_C = (j \times \omega \times C)]^{-1} = (j \times 2 \times C)^{-1}]$$

$$= j\frac{20}{\sqrt{2}} \times C.$$

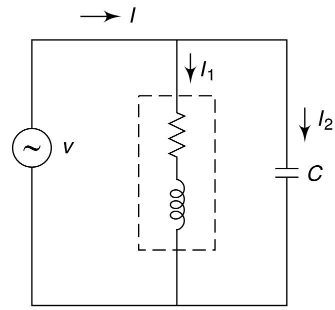

Fig. 5.79 *Circuit diagram for Ex. 5.133*

For I to be in same phase with V, the imaginary parts of I_1 and I_2 must cancel each other;

i.e., $$j\frac{20}{\sqrt{2}} \cdot C = j\,0.884$$

$\therefore \quad C = 0.0625$ F.

Then for $C = 0.0625$ F, the input current will be in phase with input voltage.

5.134 An inductive circuit takes 50 A of current at of power factor 0.8 (lag) from a 250 V, 50 Hz supply. Calculate the value of the capacitance that is required to be connected across the inductive circuit to make the power factor unity.

Solution

When the capacitor is in parallel to the inductive circuit, the net power factor would be unity provided the capacitor's capacitive current cancel the inductive current of the inductive circuit i.e., $I_C = I_L \sin\phi$ [I_C being the capacitor current while I_L the inductive circuit current; ϕ is the p.f. of the inductive circuit]

or $I_C = 50 \times 0.6 = 30$ A.

Again, $I_C = \dfrac{V}{X_C}$, X_C being the capacitive reactance.

$$\therefore X_C = \frac{V}{I_C} = \frac{250}{30} = 8.33\ \Omega.$$

Also, $X_C = \dfrac{1}{2\pi f C}$

$$\therefore \quad C = \frac{1}{2\pi f\ X_C} = \frac{1}{2\pi \times 50 \times 8.33}$$
$$= 382.32\ \mu\text{F}.$$

5.135 In the series circuit shown in Fig. 5.80 the source impedance is $(5 + j3)\ \Omega$ while the source frequency is 2 kHz. At what values of C the power in the 10 Ω resistor will be maximum?

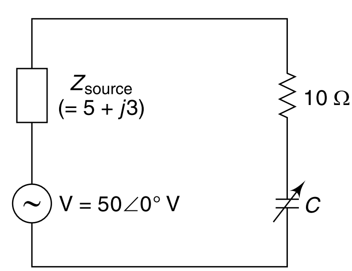

Fig. 5.80 *Circuit diagram for Ex. 5.135*

Solution

Z (total impedance of the circuit) $= Z_{\text{source}} + Z_{\text{load}}$
$$= (5 + j3) + (10 - jX_C)\ \Omega$$
$$= [15 + j(3 - X_C)]\ \Omega$$

At resonance of any series circuit, the current is maximum. Hence to achive flow of maximum power in the 10 Ω resistor, we need to make this series circuit resonant. However, Z being the total impedance of the circuit, it will be purely resistive at resonance. So, the imaginary part of Z is zero, at resonance.

i.e. $(3 - X_C) = 0$

$\therefore$ $X_C = 3$

Since $f = 2$ kHz and $\frac{1}{\omega C}$ $(= X_C) = 3$

$\therefore$ $\frac{1}{2\pi \times 2 \times 10^3 \times C} = 3$

$\therefore$ $C = 26.54$ μF,

Thus at $C = 26.54$ μF, maximum power will flow in the 10 Ω resistor.

5.136 Obtain the values of I_1, I_2 and I for the circuit shown in Fig. 5.81.

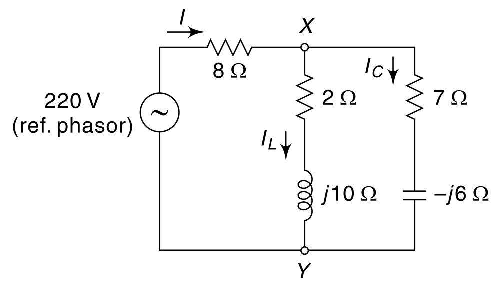

Fig. 5.81 *Circuit diagram for Ex. 5.136*

Solution

$Z_{L-R} = 2 + j10 = 10.2\angle 78.7°$ Ω

$Z_{R-C} = 7 - j5 = 8.6\angle -35.6°$ Ω

Between terminals X-Y in the given circuit, Z_{L-R} and Z_{R-C} are in parallel.

i.e. $$\frac{1}{Z_{X-Y}} = \frac{1}{Z_{L-R}} + \frac{1}{Z_{R-C}}$$

$$= \frac{1}{2+j10} + \frac{1}{7-j5}$$

$= 0.0194 - j0.0960 + 0.0943 + j0.0695$

$= 0.1137 - j0.0285.$

$\therefore$ $$Z_{X-Y} = \frac{1}{0.1137 - j0.0285} = 8.54\angle 14.5° \ \Omega$$

$= (8.27 + j2.08)$ Ω.

Total impedance of the circuit is then given by

$Z = Z_{8\Omega} + Z_{X-Y} = (16.27 + j2.08)$ Ω

$= 16.43\angle 7.3°.$

$\therefore$ $$I = \frac{V}{Z} = \frac{220\angle 0°}{16.43\angle 7.3°} = 18.4\angle -7.3° \text{ A}$$

$V_{X-Y} = Z_{X-Y} \times I = 1142\angle 6.8°$ A

$I_1 = V_{X-Y}/Z_{L-R} = 11.2\angle -71.9$ A

$I_2 = V_{X-Y}/Z_{R-C} = 13.3\angle 42.4$ A.

5.137 A resistance R_1 is in series with a choke coil having resistance R_2 and inductance L as shown in Fig. 5.82. The circuit current is given by $I = 3\angle -37°$ A, while the supply voltage is $V = 240\angle 0°$ volts. If the voltage drop across the choke is 171 volts, Find R_1, R_2 and X_L.

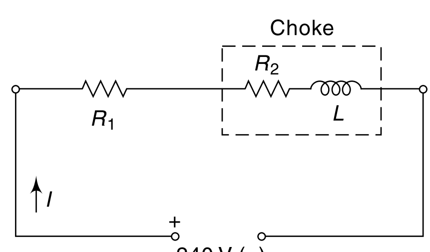

Fig. 5.82 *Circuit diagram for Ex. 5.137*

Solution

$$|Z| = \frac{240}{3} = 80 \ \Omega = [(R_1 + R_2)^2 + (X_L)^2]^{1/2}$$

Since current lags source voltage by 37°, we find p.f. (cos ϕ) is cos 37° = 0.798.

$\therefore$ $(R_1 + R_2) = Z \cos \phi = 80 \times 0.798 = 63.7$ Ω

But we have obtained from above

$$80 = [(R_1 + R_2)^2 + (X_L)^2]^{1/2}$$

$$\therefore \quad X_L = [(80)^2 - (R_1 + R_2)^2]^{1/2} = [(80)^2 - (63.7)^2]^{1/2}$$
$$= 48.2\ \Omega.$$

Again the impedance of the choke coil is given by

$$|Z_C| = \frac{\text{Drop across the choke}}{\text{Current though the choke}}$$

$$= \frac{171}{3} = 57\ \Omega.$$

However, for the choke, $R_2^2 + X_L^2 = Z_C^2$.

$\therefore$ we find $\quad R_2^2 = Z_C^2 - X_L^2 = (57^2 - 48.2^2)$ or, $R_2 = 30.4\ \Omega$.

But $\quad (R_1 + R_2) = 63.7\ \Omega$ (as obtained earlier)

$\therefore \quad R_1 = (63.7 - R_2) = 63.7 - 30.4 = 33.3\ \Omega$

5.138 A 0.5 HP induction motor operates at an efficiency of 89%. If the operating p.f. is 0.8 lag, find the reactive power taken by the motor.

Solution

$$0.5 \text{ HP} = 0.5 \times 746 = 373 \text{ W} = P_{\text{out}}.$$

$$\because \quad P_{\text{input}} = \frac{P_{\text{out}}}{\eta}; \ \eta \text{ being efficiency (89\%)}$$

Here, $\quad P_{\text{input}} = \dfrac{373}{0.89} = 419.10$ W

Since $\quad$ PF = 0.8 (= cos ϕ),

hence, $\quad \sin\phi = 0.6$ (reactive power factor).

Here, $\quad Q_{\text{in}} = VI \sin\phi = \dfrac{P_{\text{in}}}{\cos\phi} \sin\phi$

$$\left[\because P = VI\cos\phi_1 \text{ hence } VI = \frac{P}{\cos\phi}\right]$$

$$= \frac{419.10}{0.8} \times 0.6 = 314.325 \text{ VAR.}$$

$\therefore$ Reactive power taken by the motor is 314.325 VAR.

5.139 Determine R_1 and R_2 which would make the circuit (Fig. 5.83) resonant for all frequencies.

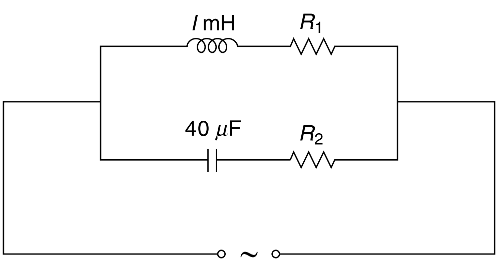

Fig. 5.83 *Circuit diagram for Ex. 5.139*

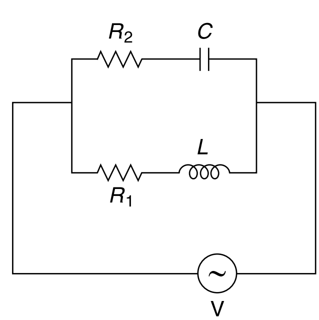

Fig. 5.83(a) ***General parallel circuit diagram for Ex. 5.139***

Solution

Let us assume a general parallel circuit as shown in Fig. 5.83(a).

The total admittance of the parallel branch is $y = y_1 + y_2$, where y_1 is the admittance for $(L - R_1)$ branch and y_2 is the admittance for the $(R_2 - C)$ branch.

$$\therefore \quad y = \frac{1}{R_1 + jX_L} + \frac{1}{R_2 - jX_C}$$

$$= \left(\frac{R_1}{R_1^2 + X_L^2} + \frac{R_2}{R_2^2 + X_C^2}\right) + j\left(\frac{X_C}{R_2^2 + X_C^2} - \frac{X_L}{R_1^2 + X_L^2}\right).$$

The circuit will be at resonance when the imaginery parts of the capacitive and inductive admittances are equal.

This gives
$$\frac{X_C}{R_2^2 + X_C^2} = \frac{X_L}{R_1^2 + X_L^2}$$

or
$$X_2 (R_1^2 + X_L^2) = X_L \left(R_2^2 + \frac{1}{\omega^2 C^2}\right).$$

Now, if $R_1^2 = R_2^2 = \dfrac{L}{C}$, then we find

$$\frac{1}{\omega C}\left(\frac{L}{C} + \omega^2 L^2\right) = \omega L\left(\frac{L}{C} + \frac{1}{\omega^2 C^2}\right)$$

or
$$\frac{L}{\omega C^2} + \frac{\omega L^2}{C} = \frac{\omega L^2}{C} + \frac{L}{\omega C^2}$$

$\therefore$ L.H.S = R.H.S for any value of ω provided.

This indicates that the circuit will be in resonance for all frequencies provided $R_1^2 = R_2^2 = \dfrac{L}{C}$.

Then, for the given problem, $R_1 = R_2 = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{1 \times 10^{-3}}{40 \times 10^{-6}}} = 5\ \Omega$.

Hence for $R_1 = R_2 = 5\ \Omega$, the given circuit is always resonant.

5.140 A current of $8.59\angle 26.56°$ passes through a circuit having series connection of two elements. The voltage drop across element 1 is $v_1 = 80 \sin \omega t$ while that across element 2 is $v_2 = 40 \sin (\omega t - 35°)$ V. Find the complex power in the circuit.

Solution

$$v_1 = 80 \sin \omega t$$

$$\therefore \quad V_1(\text{rms}) = \frac{80}{\sqrt{2}} \angle 0° = 56.57\angle 0°$$

$$= (56.57 + j0) \text{ V}.$$

Also, $\quad v_2 = 40 \sin (\omega t - 35°)$

$$\therefore \quad V_2 = \frac{40}{\sqrt{3}} \angle -35° = (23.17 - j16.22) \text{ V}.$$

$$\therefore \ V_1 + V_2 \text{ (phasor sum)} = (56.57 + j\,0 + 23.17 - j\,16.22) \text{ V}$$

$$= (79.74 - j\,16.22) \text{ V} = 81.37\angle -11.49° \text{ V}.$$

Since the current is $I = 8.95\angle 26.56°$, hence we can write,

Complex power $S = VI^*$

$$= 81.37\angle -11.49° \times (8.95\angle\ 26.56°)^*$$

$$= 81.37\angle -11.49° \times 8.95\angle -26.56°$$

$$= 727.78\angle -38° \text{ VA} = (572.71 - j449.10) \text{ VA}.$$

5.141 A circuit consists of resistance of 35 Ω in series with an unknown coil impedance Z. For a sinsusoidal current of 2 A, the observed voltages are 200 V across R and Z together and 150 V across the impedance Z. Find the value of the impedance. Draw the phasor diagram.

Solution

$$V_R = 35 \times 2 = 70 \text{ V} \quad [\because R = 35\ \Omega, I = 2 \text{ A}]$$

$$V_Z = 150 \text{ V (given)}$$

$$\therefore \quad Z = \frac{150}{2} = 75\ \Omega.$$

In the phasor diagram, Fig. 5.84 V is the reference phasor, I the lagging current while V_R and V_Z are the drops across R and the coil impedance Z respectively. As per the given and obtained data, $|V_R| = 70$ V, $|V_Z| = 150$ V while $|V| = 200$ V.

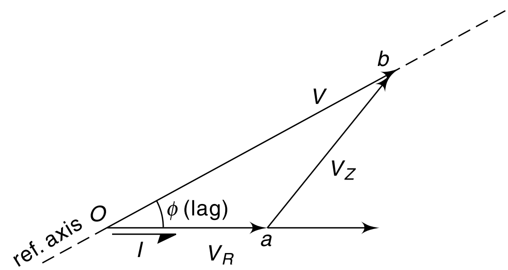

Fig. 5.84 *Phasor diagram for Ex. 5.141*

If ϕ be the angle of lag for the current (and thus for V_R), we can write in ΔOab,

$$V^2 = V_R^2 + V_Z^2 + 2V_R\, V_Z \cos\phi$$

or $$(200)^2 = (70)^2 + (150)^2 + 2 \times 70 \times 150 \times \cos\phi.$$

$$\therefore \quad \cos\phi = 0.6.$$

In the coil let us assume the resistance is r while the inductance is l so that

$$(Z) = r + j\omega l.$$

Since the power factor is 0.6, hence $\sin\phi = 0.8$.

$$\therefore \quad \frac{X_l}{Z} = 0.8,$$

while x_l is the inductive reactance of the coil.

Here, $$x_l = 0.8 \times 75 = 60\ \Omega$$

and $$\left(\frac{R + r}{Z}\right) = 0.6$$

or $$(R + r) = 0.6 \times 75 = 45\ \Omega$$

or $$r = 45 - 35 = 10\ \Omega.$$

Then the coil has a 10 Ω resistance and 60 Ω inductive reactance [the value of the impedance being $(10 + j45)$ Ω].

EXERCISES

1. Explain with the help of a diagram how alternating current is generated.
2. Define the following:
   (a) Amplitude of an alternating quantity
   (b) Instantaneous value of an alternating quantity
   (c) Frequency (d) Phase
   (e) Phase difference (f) Time period
3. Define rms and average value of an alternating quantity. Explain how these value can be obtained.
4. Define form factor and peak factor of an alternating quantity.
5. Explain with the help of diagrams what you understand by in phase, lagging and leading as applied to simusoidal quantities.

6. Define power factor as applied to ac circuits. What do you mean by active power, reactive power and apparent power?
7. Explain the meaning of the following terms in connection with alternating current:
   (a) inductance (b) capacitance (c) reactance
   (d) impedance (e) admittance (f) susceptance
   (g) conductance.
8. Show that power consumed in a purely inductive circuit and purely capacitive circuit is zero when sinusoidal voltage is applied across it.
9. Explain with the help of a diagram the phenomenon of resonance in series *R-L-C* circuit.
10. Derive an expression for the resonant frequency of a parallel circuit, one branch consisting of a coil of inductance L and resistance R and the other branch of capacitance C.
11. Derive the quality factor of a series *R-L-C* circuit at resonance.
12. Define quality factor in a series *R-L-C* circuit. Determine the half power frequencies in terms of quality factor and the resonant frequency for series *R-L-C* circuit.
13. Why is a series resonant circuit called an acceptor circuit and parallel resonant circuit a rejector circuit?
14. Explain dynamic impedance in connection with parallel resonant circuit.
15. Define bandwidth in a series *R-L-C* circuit. Prove that in a series *R-L-C* circuit $(Q_0) = \dfrac{\omega_o L}{R} = \dfrac{f_o}{\text{Bandwidth}}$.
16. Find the average value, rms value, form factor and peak factor of the waveform shown in Fig. 5.85
    [*Ans:* 15 A, 17.8 A, 1.187, 1.685]

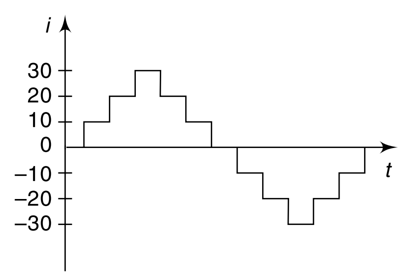

Fig. 5.85 ***Waveform for Ex. 5.16***

17. The maximum value of a sinusoidally alternating voltage is 100 volts. Find the instantaneous value at $\frac{1}{9}$ cycle and $\frac{1}{18}$ cycle. [*Ans:* 64.28 V, 34.2 V]
18. The voltage given by $(v_1) = 50 \sin (377t - 30°)$ and $(v_2) = 20 \sin (377t + 45°)$ act in the series circuit. Determine the frequency and rms value of the resultant voltage.
19. Define phasors as used in the study of ac circuits. Use phasors to find the sum of the sinusoids $40 \sin 314t$ and $30 \cos\left(314t - \frac{\pi}{4}\right)$.

    [*Ans:* $64.78 \sin (314\,t + 19.11°)$]

    [*Hint:*
$$a = 40 \sin 314t$$
$$b = 30 \cos\left(\frac{\pi}{4} - 314t\right)$$
$$= 30 \sin\left(\frac{\pi}{2} - \frac{\pi}{4} + 314t\right)$$
$$= 30 \sin\left(314t + \frac{\pi}{4}\right)$$

Using the method described in Art 5.3.3, we get the required sum is 64.78 sin $(314t + 19.11°)$].

20. The equation of an alternating current is $i = 62.35 \sin 323t$ A.
Determine its
(a) maximum value (b) frequency (c) r.m.s. value
(d) average value and (e) form factor

[*Ans:* $I_{max} = 62.35$ A; $f = 51.41$ Hz; $I_{rms} = 44.1$ A;
$I_{av} = 39.69$ A, FF = 1.11]

[*Hint:*
(i) Maximum value 62.35 A
(ii) Frequency $= \dfrac{323}{2\pi}$ Hz = 51.41 Hz
(iii) r.m.s. value $= \dfrac{62.35}{\sqrt{2}} = 44.1$ A
(iv) Average value $= \dfrac{62.35}{\dfrac{\pi}{2}} = 39.69$ A
(v) Form factor $= \dfrac{\text{r.m.s. value}}{\text{average value}} = \dfrac{44.1}{39.69} = 1.11$]

21. An ac series circuit consisting of a pure resistance of 25 Ω, inductance of 0.15 H and capacitance of 80 μF is supplied from a 230 V, 50 Hz ac source.
(a) Find the impedance of the circuit, the current, the power drawn by the circuit and the power factor.
(b) Draw the phasor diagram. [*Ans:* $Z = 26.04\angle 16.25°\,\Omega$ $|I| = 8.83$ A;
$P = 1950.2$ W; $\cos\phi = 0.96$ (lag)]

[Hint: (a) $Z = 25 + j\,100\pi \times 0.15 - j\dfrac{1}{100\pi \times 80 \times 10^{-6}}$

$$= 25 + j\,7.29 = 26.04\ \underline{|16.25°}\ \Omega$$

$$I = \frac{230}{26.04}\ \text{A} = 8.832\ \text{A}.$$

Power factor $\cos\theta = \cos 16.25° = 0.96$ lagging
Power = 230 × 8.832 × 0.96 = 1950.2 W]

22. Figure 5.86 shows a circuit in which a coil having resistance R and inductance L is connected in series with a resistance of 80 Ω. The combination is fed from a sinusolidal source.

The following measurements (rms value) are taken at 50 Hz. When the circuit is in steady state.

$|V_S| = 145$ V, $|V_R| = 50$ V
and $|V_C| = 110$ V
Find the value of R and L.

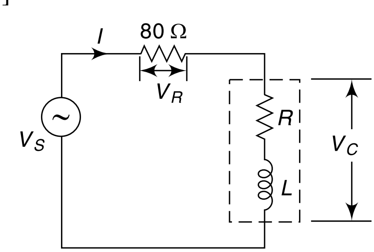

Fig. 5.86

[*Ans:* $R = 102.8\ \Omega$; $L = 45.5$ mH]

[*Hint:* $I = \dfrac{50}{80}\ \text{A} = \dfrac{5}{8}\ \text{A}$

$$Z^2 = R^2 + X_L^2 = \left(\frac{110}{\frac{5}{8}}\right)^2 = (176)^2 \qquad \text{(i)}$$

$$\text{Again } \left(\frac{145}{\frac{5}{8}}\right)^2 = (80 + R)^2 + X_L^2 \qquad \text{(ii)}$$

Solving (i) and (ii)

$R = 102.8\ \Omega$ and $X_L = 142.857$ or $L = \dfrac{142.857}{2\pi \times 50}$ H = 0.455 H].

23. A coil of resistance 10 Ω and inductance 0.02 H is connected in series with another coil of resistance 6 Ω and inductance 15 mH across a 230 V, 50 Hz supply. Calculate (i) impedance of the circuit, (ii) voltage drop across each coil, (iii) the toal power consumed by the circuit.

[*Ans:* $Z = 19.41\angle 34.48°\ \Omega$; $V_{drop(1)} = 19139.92\angle -2.35°$; $V_{drop(2)} = 90.38\angle 3.65°$V; $P = 2.25$ kW]

[Hints: (i) Impedance $= 10 + j2\pi \times 50 \times 0.02 + 6 + j2\pi \times 50 \times 0.015$

$= 10 + j6.28 + 6 + j\,4.71 = 16 + j\,10.99$

$= 19.41\ \underline{|34.48°}\ \Omega.$

(ii) Voltage across the 1st coil

$$(10 + j6.28) \times \frac{230}{19.41\underline{|34.48°}} = 139.92\underline{|-2.35°}\text{ V}$$

Voltage across the 2nd coil is

$$(6 + j\,4.71) \times \frac{230}{19.41\underline{|34.48°}} = 90.38\underline{|3.65°}\text{ V}$$

(iii) Total power $= \left(\dfrac{230}{19.41}\right)^2 \times 16 = 2246.6$ W]

24. A circuit consists of three parallel branches. The branch currents are represented by $i_1 = 10 \sin \omega t$ and $i_2 = 20 \sin (\omega t + 60°)$. If the supply frequency is 50 Hz. Calculate the resultant current at $t = 0$ and at $t = 1$ ms.

[*Ans:* 13.55 A and 22.33 A]

25. A 100 V, 80 W lamp is to be operated on a 240 V, 50 Hz supply. Calculate the value of (a) non-inductive resistor, (b) pure inductor to be connected in series with the lamp so that it can be used at its rated voltage.

[*Ans:* 175 Ω, 0.868 H]

26. A resistance of 12 Ω and inductance of 0.15 H and a capacitance of 130 μF are connected in series across a 100 V, 50 Hz supply. Determine the impedance, current and power factor of the circuit.

[*Ans:* 25.6 Ω, 3.9 A, 0.4687 lag]

27. A coil takes a current of 10 A when connected to a dc supply of 100 V. When connected to an ac supply of 100 V, the current is 5 A. Find the reactance of the coil. [*Ans:* 17.32 Ω]

28. The voltage across a circuit is given by (300 + j 60) V and the current through it is (10 – j5) A. Determine the (i) active power, (ii) reactive power and (iii) apparent power. [*Ans:* (i) 2.7 kW, (ii) 2.1 KVAR (lagging) and (iii) 3.42 kVA]

29. A coil of resistance 15 Ω and reactance 25 Ω is connected in parallel with a capacitor of reactance 10 Ω and series resistance of 12 Ω to a 100 V, 50 Hz supply. Determine the supply current and the circuit phase angle.
[*Ans:* 6.782 A and 9.82°]
30. Three impedances $(4 - j6)$, $(6 + j8)$ and $(5 - j3)$ Ω are connected in parallel. Calculate the current in each branch when the total supply current is 20 A.
[*Ans:* $8.96\angle 32.79°$, $6.46\angle -76.65°$], $11.08\angle 7.44°$]
31. Determine the total current, power factor and power consumed by the circuit shown in Fig. 5.87.
[*Ans:* $30.03\angle 123.5°$ A, 0.552 lead, 1657.4 W]

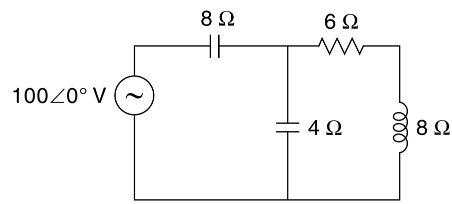

Fig. 5.87 *Circuit diagram for Ex. 5.26*

32. Find the current through each element in the circuit shown in Fig. 5.88.
[*Ans:* $I_1 = 10.23\angle -67.3°$, $I_2 = 3.93\angle 53.14°$, $I_3 = 8.91\angle -44.96°$]

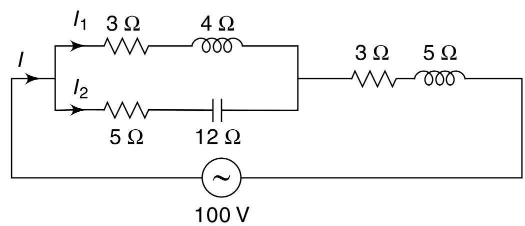

Fig. 5.88 *Circuit diagram for Ex. 5.27*

33. A resistor and a capacitor are connected in series with a variable inductor. When the circuit is connected to a 240 V, 50 Hz supply the maximum current obtained by varying the inductance is 0.5 A. At this the voltage across the capacitor is 250 V. Determine the values of resistance, capacitance and inductance. [*Ans:* 480 Ω, 6.36 μF, 1.59 H]
34. A coil of 20 Ω resistance has an inductance of 0.2 H and is connected in parallel with a 100 μF capacitor. Calculate the frequency at which the circuit will act as a non-inductive resistance of *R* Ω. Find also the value of *R*.
[*Ans:* 31.8 Hz, 100 Ω]
35. A series circuit consists of a resistance of 10 Ω, an inductance of 8 mH and a capacitance of 500 μF. A sinusoidal emf of constant amplitude 5 V with variable frequency is applied. At what frequencies will the current be (i) maximum (ii) half the maximum?
[*Ans:* (i) 79.6 kHz, (ii) 79.872 kHz and 79.528 kHz.]
36. A coil of inductance 9 Hz and resistance 50 Ω in series with a capacitor is supplied at constant voltage from a variable frequency source. If the maximum current of 1 A occurs at 75 Hz, find the frequency when the current is 0.5 A. [*Ans:* 75.75 Hz]

6 AC NETWORK ANALYSIS

6.1 SUPERPOSITION THEOREM (AC APPLICATION)

Just like the dc application, superposition theorem applied to linear ac networks eliminates the need for solving simultaneous linear equations considering the effects of each source independently. The only variation in applying the principle of superposition to the ac networks with independent source is that the circuit will have to be worked out with ac voltage or current sources and impedances involving phasors (i.e. operation with complex numbers) instead of just real numbers (i.e. operation with resistors).

The statement of superposition theorem for ac network is as follows:

If a number of voltage or current sources act simultaneously in a linear network, the resultant current (or voltage) in any branch is the phasor sum of the currents or (voltages) that would be produced in it, when each source acts alone replacing all other independent sources by their internal impedances.

6.1 Find the current through inductive reactance ($j2\ \Omega$) in the circuit of Fig. 6.1 using the superposition theorem.

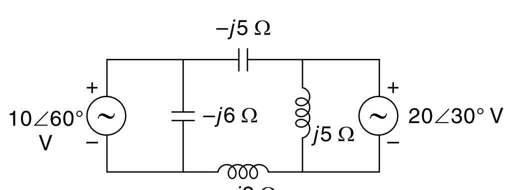

Fig. 6.1 *Circuit of Ex.6.1*

Solution

Considering $10\angle 60°$ V source acting alone in the circuit and removing the other source as shown in Fig. 6.1(a), the current through the ($j2\ \Omega$) reactor is

$$I_1 = \frac{10\angle 60^\circ}{-j5 + j2} = \frac{10\angle 60^\circ}{-j3}$$

$= 3.33\ \angle 150^\circ$ A (from B to A).

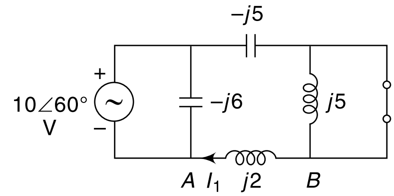

Fig. 6.1(a) ***10∠60°V source acting alone***

Considering the 20∠30° V source acting alone in the circuit and removing the other source as shown in Fig. 6.1(b), the current through ($j2\ \Omega$) reactor is

$$I_2 = \frac{20\angle 30^\circ}{-j5 + j2} = \frac{20\angle 30^\circ}{-j3}$$

$= 6.66\angle 120^\circ$ A (from A to B).

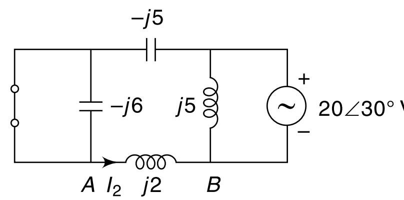

Fig. 6.1(b) ***20∠30°V source acting alone***

According to the superposition theorem, when both the sources are acting simultaneously the current through the ($j2\ \Omega$) inductive reactance is

$$3.33\angle 150^\circ - 6.67\angle 120^\circ$$
$$= 0.455 - j\,4.11$$
$$= 4.126\angle -83.72^\circ$$

A (from B to A).

6.2 Using the superposition theorem find the current flowing in the branch AB of the circuit shown in Fig. 6.2.

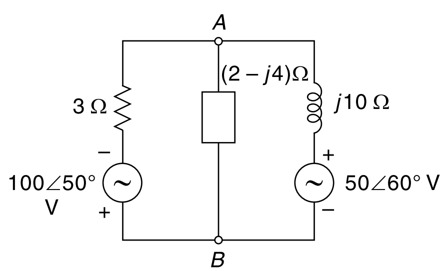

Fig. 6.2 ***Circuit of Ex. 6.2***

Solution

Considering the 100∠50° voltage source acting alone and removing the other, the current through branch AB as shown in Fig. 6.2(a) is given by

$$I_1 = \frac{100\angle 50^\circ}{3 + \dfrac{j10(2 - j4)}{j10 + 2 - j4}} \times \frac{j10}{j10 + 2 - j4}$$

$$= \frac{100\angle 50^\circ}{3 + 10\dfrac{4 + j2}{2 + j6}} \times \frac{j10}{2 + j6}$$

$$= \frac{100\angle 50^\circ}{6 + j18 + 40 + j20} \times j\ 10$$

$$= \frac{1000\angle 140^\circ}{46 + j38}$$

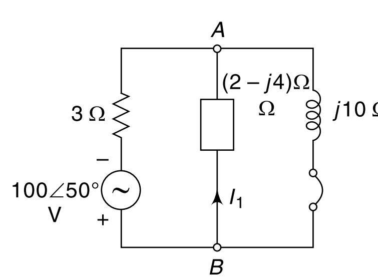

Fig. 6.2(a) ***100∠50°V source acting alone***

$$= \frac{1000\angle 140^\circ}{59.66\angle 39.56^\circ}$$

$= 16.76\angle 100.44°$ A (from B to A).

Considering $50\angle 60°$ V source acting alone and removing the other, the current through branch AB as shown in Fig. 6.2(b) is obtained as

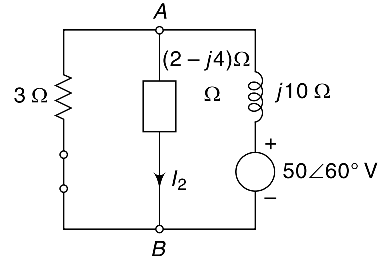

Fig. 6.2(b) ***50∠60°V source acting alone***

$$I_2 = \frac{50\angle 60^\circ}{j10 + \dfrac{3(2-j4)}{3+2-j4}} \times \frac{3}{3+2-j4}$$

$$= \frac{150\angle 60^\circ}{j50+40+6-j12}$$

$$= \frac{150\angle 60^\circ}{46+j38}$$

$$= \frac{150\angle 60^\circ}{59.66\angle 39.56^\circ} = 2.51\angle 20.44^\circ \text{ A (from } A \text{ to } B).$$

According to the superposition theorem when both the sources are acting simultaneously the current through AB is obtained as

$2.51\angle 20.44° - 16.76\angle 100.44° = 5.388 - j\,15.6 = 16.5\angle -70.96°$ A (from A to B).

6.3 Find the current through the capacitor of ($-j5\ \Omega$) reactance in Fig. 6.3 using superposition theorem.

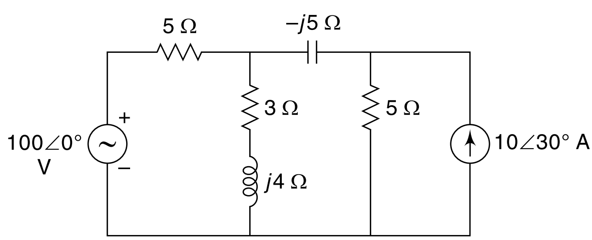

Fig. 6.3 ***Circuit of Ex. 6.3***

Solution

Considering the voltage source acting alone in the circuit and removing the current source, as shown in Fig. 6.3(a), the current through ($-j5\ \Omega$) reactance is given by

$$I_1 = \frac{100\angle 0^\circ}{5 + \dfrac{(5-j5)(3+j4)}{5-j5+3+j4}} \times \frac{3+j4}{3+j4+5-j5}$$

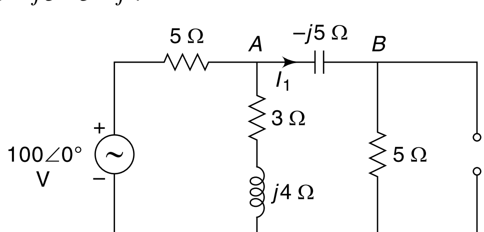

Fig. 6.3(a) ***Voltage source acting alone***

$$= \frac{100\angle 0^\circ \times 5\angle 53.13^\circ}{5(8-j)+(15+20+j20-j15)}$$

$$= \frac{500\angle 53.13^\circ}{40-j5+35+j5} = \frac{500\angle 53.13^\circ}{75} = 6.67\angle 53.13^\circ \text{ A (from } A \text{ to } B).$$

Considering the current source acting alone and removing the voltage source the current through ($-j5\ \Omega$) reactance, as shown in Fig. 6.3(b) and Fig. 6.3(c), is found as

$$I_2 = 10\angle 30^\circ \times \frac{5}{5-j5+(2.5+j1.25)}$$

$$= \frac{50\angle 30^\circ}{7.5-j3.75}$$

$$= \frac{50\angle 30^\circ}{8.385\angle -26.56^\circ} = 5.963\angle 56.56^\circ \text{ A (from } B \text{ to } A).$$

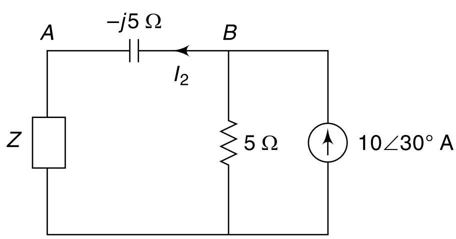

Fig. 6.3(c) *Simplified circuit* $\left[Z = \frac{5(3+j4)}{5+3+j4} = (2.5+j1.25)\ \Omega \right]$

According to the superposition theorem when both the sources are acting simultaneously the current through the ($-j5\ \Omega$) reactance is obtained as

$$I = 6.67\angle 53.13^\circ - 5.963\angle 56.56^\circ$$
$$= 0.801\angle 26^\circ \text{ A (from } A \text{ to } B).$$

6.4 Find the current in the resistor of the 10 Ω resistance in Fig. 6.4 using the principle of superposition theorem.

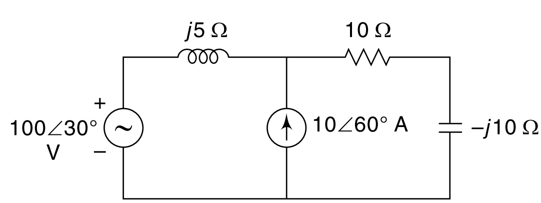

Fig. 6.4 *Circuit of Ex. 6.4*

Solution

When the voltage source 100∠30° V acts alone in the circuit (the corresponding figure being shown in Fig. 6.4(a)), we have

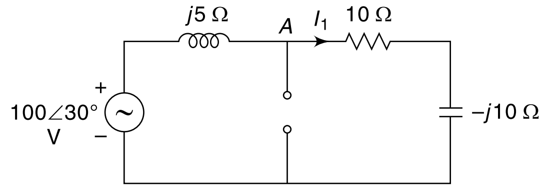

Fig. 6.4(a) ***Voltage source acting alone***

$$I_1 = \frac{100\angle 30^\circ}{j5 + 10 - j10}$$

$$= \frac{100\angle 30^\circ}{10 - j5} = \frac{100\angle 30^\circ}{11.18\angle -26.56^\circ} = 8.94\angle 56.56^\circ \text{ A (from } A \text{ to } B).$$

When the current source $10\angle 60^\circ$ A acts alone in the circuit (the corresponding figure being shown in Fig. 6.4(b)), we can write

$$I_2 = 10\angle 60^\circ \times \frac{j5}{j5 + 10 - j10}$$

$$= 10\angle 60^\circ \frac{j5}{10 - j5}$$

$$= \frac{50\angle 150^\circ}{11.18\angle -26.56^\circ}$$

$$= 4.47\angle 176.56^\circ \; A \text{ (from } A \text{ to } B).$$

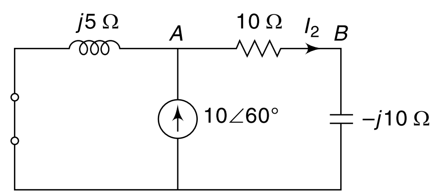

Fig. 6.4(b) ***Current source acting alone***

When both the sources are acting simultaneously following the superposition theorem, the current through 10 Ω resistor is

$$I = 8.94\angle 56.56^\circ + 4.47\angle 176.56^\circ$$
$$= 4.92 + j\,7.46 - 4.46 + j\,0.268$$
$$= 0.46 + j\,7.728$$
$$= 7.74\angle 86.59^\circ \text{ A (from } A \text{ to } B).$$

6.2 THEVENIN'S THEOREM (AC APPLICATION)

Similar to the dc network, Thevenin's theorem is equally applied to ac networks since they also contain linear circuit elements like resistors, capacitors and inductors. In dc circuits, voltage sources are replaced by their internal resistances while evaluating Thevenin's equivalent resistance; in ac circuits voltage sources need to be replaced by their internal impedances while obtaining Thevenin's equivalent impedance. Since reactances of a circuit are frequency dependent, the Thevenin's theorem is applicable to a particular network at a particular frequency.

Thevenin's theorem in ac circuits can be stated as follows:

Any active, two terminal, linear network can be replaced by an equivalent voltage source in series with an impedance, the voltage being equal to the open circuit voltage between the terminals and the impedance being equal to the impedance between the terminals with all independent sources being replaced by their internal impedances.

6.5 Find the Thevenin's equivalent circuit of the network shown in Fig. 6.5.

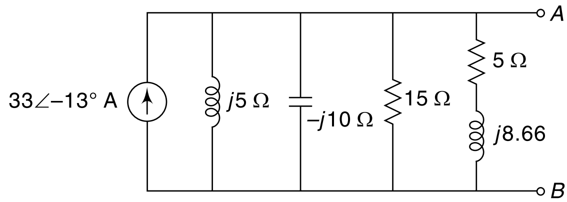

Fig. 6.5 *Circuit of Ex. 6.5*

Solution

The admittances of the branches are

$$y_1 = \frac{1}{j5} = (-0.2\,j) \text{ Siemens (S)}$$

$$y_2 = \frac{1}{-j10} = (0.1\,j) \text{ S}$$

$$y_3 = \frac{1}{15} = (0.067) \text{ S}$$

$$y_4 = \frac{1}{5 + j8.66} = (0.05 - j\,.0866) \text{ S}$$

The equivalent admittance

$$Y = -0.2\,j + 0.1\,j + 0.067 + 0.05 - j\,0.0866$$
$$= (0.117 - j\,0.1866) \text{ S}$$

$\therefore$ the voltage across the open circuited terminals is given by

$$V_{AB} = V_{\text{Th}} = \frac{33\angle -13^\circ}{0.117 - j.1866} = \frac{33\angle -13^\circ}{0.22\angle -57.9^\circ}$$
$$= 150\angle 44.9^\circ \text{ V}$$

Thevenin's equivalent impedance Z_{th} can be found from Fig. 6.5(a).

$$Z_{\text{Th}} = \frac{1}{\dfrac{1}{j5} + \dfrac{1}{-j10} + \dfrac{1}{15} + \dfrac{1}{5 + j866}}$$

$$= \frac{1}{0.117 - j.1866} = \frac{1}{0.22\angle -57.9^\circ} = 4.545\angle 57.9^\circ\ \Omega$$

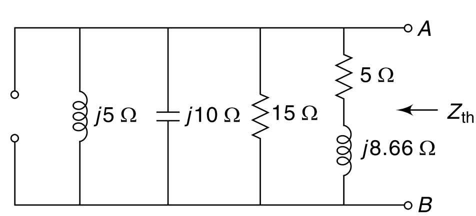

Fig. 6.5(a) *Determination of* $\mathbf{Z}_{th}$

Hence the Thevenin's equivalent circuit can be obtained with V_{th} = 150∠44.9° V and Z_{th} = 4.545∠57.9° Ω.

6.6 Find the current through the 10 Ω resistor using Thevenin's theorem in the network shown in Fig. 6.6.

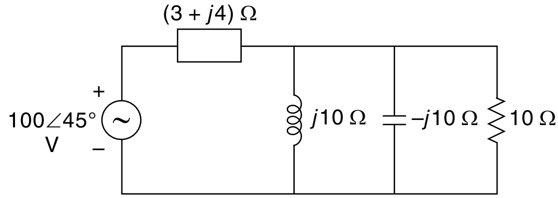

Fig. 6.6 *Circuit of Ex. 6.6*

Solution

Removing the 10 Ω resistor, the open circuit voltage V_{oc} can be found out from Fig. 6.6(a). Let us consider two mesh currents I_1 and I_2.

The mesh equations are as follows:

$$100\angle 45° - I_1(3 + j4) - (I_1 - I_2)j10 = 0 \tag{i}$$

and $$-j10(I_2 - I_1) - I_2(-j10) = 0. \tag{ii}$$

Solving equations (i) and (ii)

$$I_1 = 0$$

and $$I_2 = \frac{100\angle 45°}{-j10} = 10\angle 135° \text{ A.}$$

Hence $$V_{oc} = (-j10) \times 10\angle 135° = 100\angle 45° \text{ V.}$$

Therefore, Thevenin's equivalent voltage $V_{Th} = 100\angle 45°$ V.

Removing the voltage source from Fig. 6.6(a), Thevenin's equivalent impedance (Z_{Th}) can be found out from Fig. 6.6(b).

$$Z_{Th} = \frac{1}{\frac{1}{j10} + \frac{1}{-j10} + \frac{1}{3+j4}} = (3 + j4)\,\Omega$$

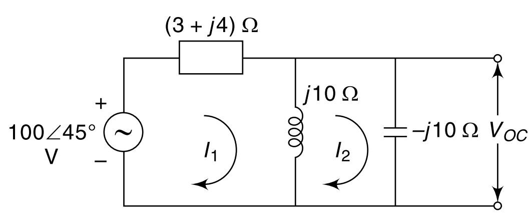

Fig. 6.6(a) *Determination of* V_{OC}

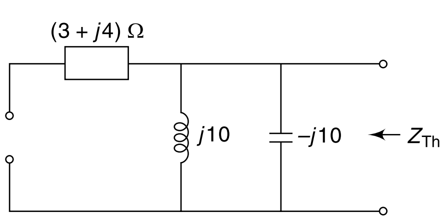

Fig. 6.6(b) *Determination of* $\mathbf{Z}_{th}$

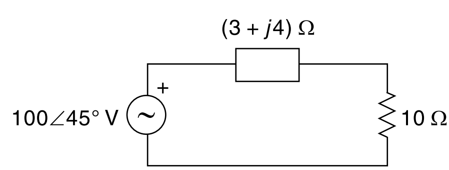

Fig. 6.6(c) *Thevenin's equivalent circuit of Ex. 6.6*

The Thevenin's equivalent circuit is shown in Fig. 6.6(c).

Hence the current through 10 Ω resistor is

$$= \frac{100\angle 45°}{3+j4} = \frac{100\angle 45°}{5\angle 53.13°} = 20\angle -8.13° \text{ A.}$$

6.7 Find the current through the impedance of $(10 + j5)\ \Omega$ in Fig. 6.7.

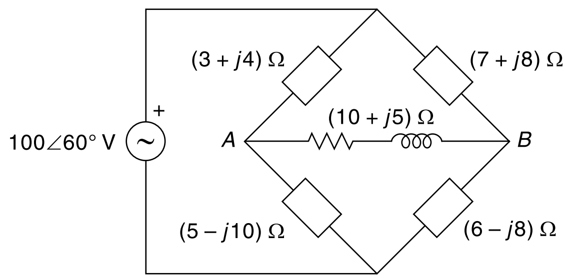

Fig. 6.7 ***Circuit of Ex. 6.7***

Solution

Open circuiting the terminals of $(10 + j5)\ \Omega$ as shown in Fig. 6.7(a), Thevenin's equivalent voltage can be obtained.

Here,
$$I_1 = \frac{100\angle 60^\circ}{3 + j4 + 5 - j10}$$
$$= \frac{100\angle 60^\circ}{8 - j6} = \frac{100\angle 60^\circ}{10\angle -36.87^\circ} = 10\angle 96.87^\circ \text{ A}$$
$$I_2 = \frac{100\angle 60^\circ}{7 + j8 + 6 - j8} = \frac{100\angle 60^\circ}{13} = 7.69\angle 60^\circ \text{ A.}$$
$$V_{\text{Th}} = V_{AB} = V_{PB} - V_{PA}$$
$$= I_2(7 + j8) - I_1(3 + j4)$$
$$= 7.69\angle 60^\circ \times 10.63\angle 48.8^\circ - 10\angle 96.87^\circ \times 5\angle 53.13^\circ$$
$$= 81.745\angle 108.8^\circ - 50\angle 150^\circ$$
$$= 16.96 + j52.38 = 55.06\angle 72.06^\circ \text{ V.}$$

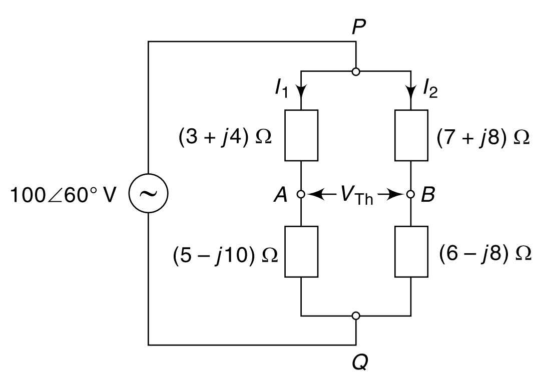

Fig. 6.7(a) ***Determination of V_{Th}***

Thevenin's equivalent impedance can be found out from Fig. 6.7(b) and Fig. 6.7(c) by removing voltage source,

$$Z_{\text{Th}} = \frac{1}{\dfrac{1}{3 + j4} + \dfrac{1}{5 - j10}} + \frac{1}{\dfrac{1}{7 + j8} + \dfrac{1}{6 - j8}}$$

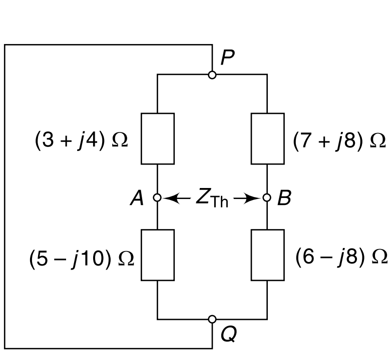

Fig. 6.7(b) *Determination of* Z_{th}

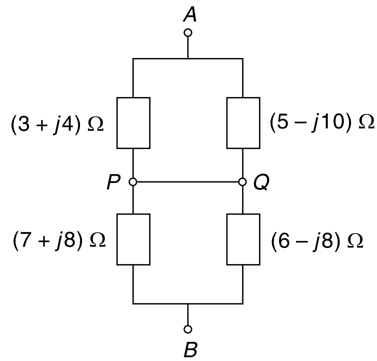

Fig. 6.7(c) ***Simplified circuit of Fig. 6.7(b)***

$$= \frac{15+40+j(20-30)}{8-j6} + \frac{42+64+j(48-56)}{13}$$

$$= \frac{55-j10}{8-j6} + \frac{106-j8}{13}$$

$$= \frac{55.9\angle-10.3°}{10\angle-36.87°} + 8.15 - j0.615$$

$$= 5.59\angle 26.57 + 8.15 - j0.615$$

$$= 5 + j2.5 + 8.15 - j0.615 = (13.15 + j1.885)\ \Omega$$

Thevenin's equivalent circuit is shown in Fig 6.7(d).

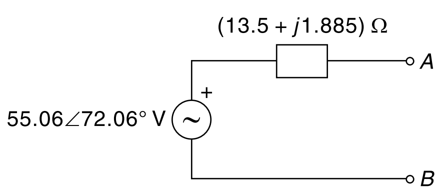

Fig. 6.7(d) ***Thevenin's equivalent circuit of Ex. 6.7***

The current through $(10 + j5)\ \Omega$ impedance is obtained as

$$I = \frac{55.06\angle 72.06°}{13.15 + j1.885 + 10 + j5}$$

$$= \frac{55.06\angle 72.06°}{23.15 + j6.885} = \frac{55.06\angle 72.06°}{24.15\angle 16.56°} = 2.28\angle 55.5°\ \text{A}.$$

6.8 Find the Thevenin's equivalent circuit of the network shown in Fig. 6.8.

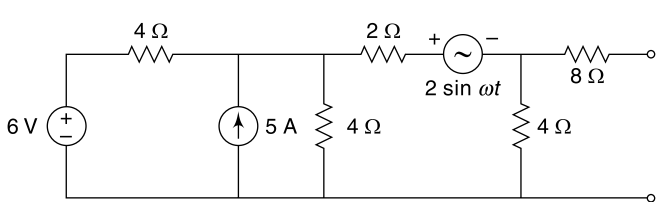

Fig. 6.8 ***Circuit of Ex. 6.8***

Solution

Let us redraw the given network with mesh currents indicated in Fig. 6.8(a). The 5 A source has been converted to equivalent voltage source as shown in the figure.

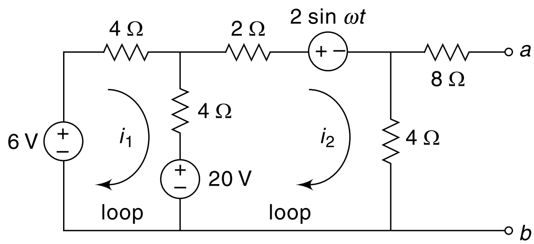

Fig. 6.8(a) ***Determination of V_{Th}***

In loop 1 mesh analysis gives

$$4i_1 - 6 + 20 + 4(i_1 - i_2) = 0$$

or $$8i_1 - 4i_2 = -14 \qquad \text{(i)}$$

In loop 2 the mesh analysis gives

$$2i_2 + 4(i_2 - i_1) - 20 + 4i_2 + 2 \sin \omega t = 0$$

or $$-4i_1 + 10i_2 = 20 - 2 \sin \omega t$$

or $$-2i_1 + 5i_2 = 10 - \sin \omega t \qquad \text{(ii)}$$

Solving equations (i) and (ii)

$$i_2 = 1.625 - 0.25 \sin \omega t.$$

Therefore Thevenin's equivalent voltage is obtained as

$$V_{\text{Th}} = V_{ab} = 4i_2 = 6.5 - \sin \omega t.$$

To find Thevenin's internal impedance the sources are replaced by their internal impedance as shown in Fig. 6.8(b).

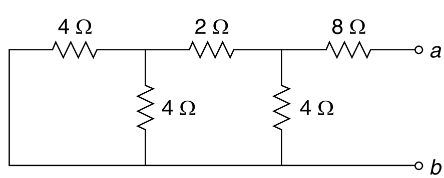

Fig. 6.8(b) ***Determination of Z_{Th}***

Hence

$$Z_{\text{Th}} = \{(4 \parallel 4) + 2\} \parallel 4 + 8 = (4 \parallel 4) + 8 = 2 + 8 = 10\ \Omega.$$

Thevenin's equivalent circuit is shown in Fig. 6.8(c).

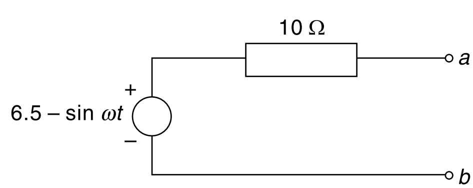

Fig. 6.8(c) ***Thevenin's equivalent circuit of Ex. 6.8***

6.3 NORTON'S THEOREM (AC APPLICATION)

In application of Norton's theorem in ac circuits the resistances of dc circuits are replaced by impedances, the circuit variables being current or voltage phasors.

The statement of Norton's theorem for ac network is as follows:

Any two terminal active network containing voltage sources and impedances when viewed from its output terminals is equivalent to a constant current source and a parallel impedance. The constant current is equal to the current which should flow in a short circuit placed across the terminals and the parallel impedance is the impedance of the network when viewed from open circuited terminals after independent energy sources have been replaced by their internal impedances (if any).

6.9 Use Norton's theorem to find current in the load connected across terminals *a* and *b* of the circuit shown in Fig. 6.9.

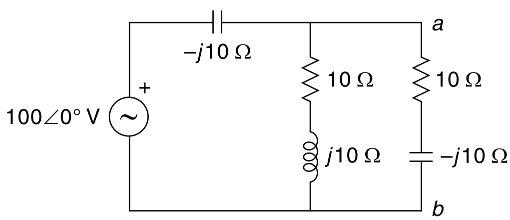

Fig. 6.9 *Circuit of Ex. 6.9*

Solution

Short circuiting the terminals *ab* of the circuit shown in Fig. 6.9 Norton's equivalent current can be found out. The corresponding circuit is shown in Fig. 6.9(a).

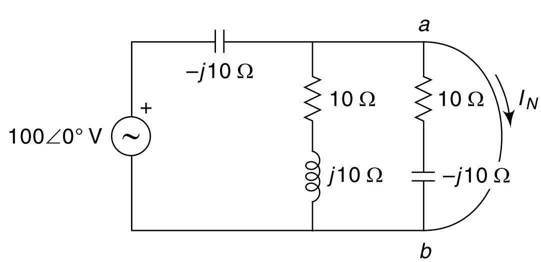

Fig. 6.9(a) *Determination of Norton's current*

The current through the short circuited path

$$I_N = \frac{100\angle 0^\circ}{-j10} = 10\angle 90^\circ \text{ A.}$$

Removing the voltage source and open circuiting the terminals *a* and *b* Norton's equivalent impedance can be found out as shown in Fig. 6.9(b).

Hence $\quad Z_N = \dfrac{-j10(10+j10)}{-j10+10+j10} = (10 - j10)\ \Omega$

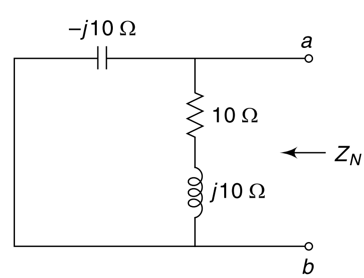

Fig. 6.9(b) *Determination of Z_N*

The Norton's equivalent circuit is shown in Fig. 6.9(c). Hence the current across terminals *a* and *b* is

$$I = 10\angle 90^\circ \times \frac{1}{2} \quad \text{(as both the impedances in parallel are of equal value)}$$

$$= 5\angle 90^\circ \text{ A.}$$

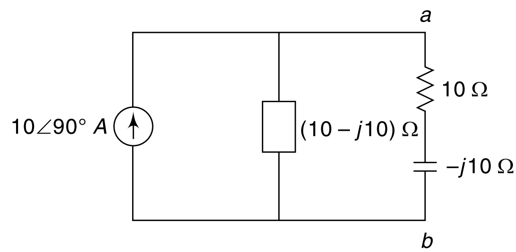

Fig. 6.9(c) *Noton's equivalent circuit of Ex. 6.9*

6.10 Find Norton's equivalent circuit for the network shown in Fig. 6.10 across terminals $a - b$. Assume $I = 5\angle 0°$ A.

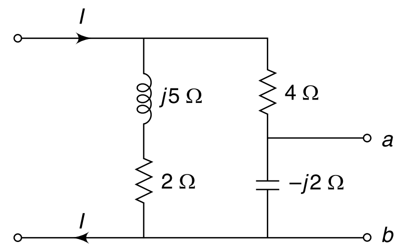

Fig. 6.10 *Circuit of Ex. 6.10*

Solution

Short circuiting terminals a and b as shown in Fig. 6.10(a), Norton's equivalent current (I_N) can be found out.

$$I_N = I \times \frac{j5+2}{j5+2+4}$$

$$= 5\angle 0° \times \frac{2+j5}{6+j5}$$

$$= 5\angle 0° \times \frac{5.385\angle 68.2°}{7.81\angle 39.8°} = 3.45\angle 28.4° \text{ A}$$

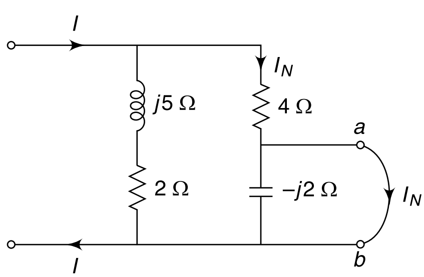

Fig. 6.10(a) *Determination of I_N*

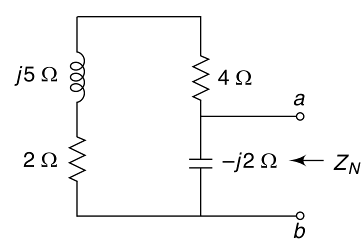

Fig. 6.10(b) *Determination of Z_N*

Open circuiting the terminals $a - b$ as shown in Fig. 6.10(b), Norton's equivalent impedance can be found out.

$$Z_N = \frac{-j2(4+j5+2)}{-j2+4+j5+2}$$

$$= \frac{-j2(6+j5)}{6+j3}$$

$$= \frac{10-j12}{6+j3}$$

$$= \frac{15.62\angle -50.2°}{6.71\angle 26.56°}$$

$$= 2.328\angle -76.76° \ \Omega$$

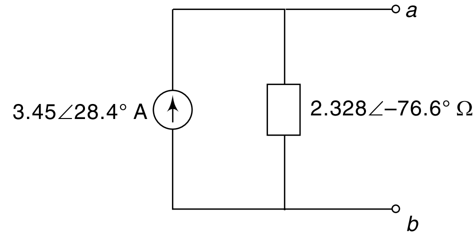

Fig. 6.10(c) *Norton's equivalent circuit of Ex. 6.10*

Norton's equivalent circuit is shown in Fig. 6.10(c).

6.11 Find current through the 1 Ω resistor in the circuit shown in Fig. 6.11 using Norton's theorem.

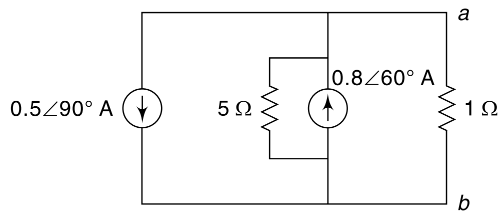

Fig. 6.11 *Circuit of Ex. 6.11*

Solution

Removing 1 Ω resistor in the circuit shown in Fig. 6.11 and short circuiting the terminals, Norton's equivalent current can be found out. The corresponding circuit is shown in Fig. 6.11(a).

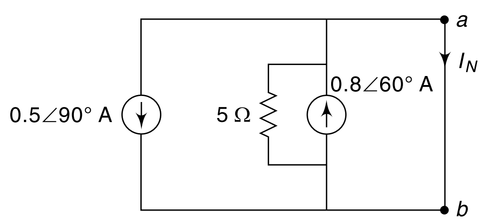

Fig. 6.11(a) *Determination of I_N*

The current through the short circuited path *ab* in Fig. 6.11(a) is the Norton's equivalent current (I_N).

Therefore $\quad I_N = 0.8\angle 60° - 0.5\angle 90°$

$= 0.4 + j0.69 - j0.5 = 0.4 + j0.19 = 0.443\angle 25.4°$ A.

To find out Norton's equivalent resistance R_N terminal *ab* is open circuited and current sources are removed as shown in Fig. 6.11(b).

Therefore, $\quad R_N = 5\ \Omega$.

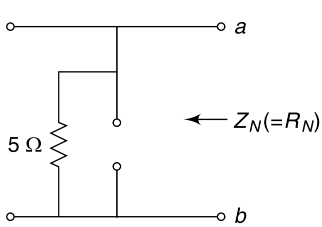

Fig. 6.11(b) *Determination of Z_N*

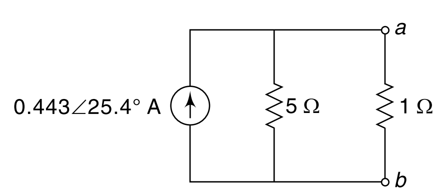

Fig. 6.11(c) *Norton's equivalent circuit of Ex. 6.11*

Norton's equivalent circuit is shown in Fig. 6.11(c).

Current through 1 Ω resistor is given by

$$= 0.443\angle 25.4° \times \frac{5}{5+1}$$

$$= 0.37\angle 25.4° \text{ A.}$$

6.12 In the circuit of Fig. 6.12, find Norton's equivalent circuit across *AB*.

Solution

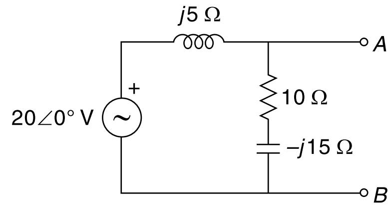

Fig. 6.12 *Circuit of Ex. 6.12*

Terminals *A* and *B* are short circuited and the current through the short circuited path is found out as shown in Fig. 6.12(a).

Hence Norton's equivalent current is obtained as

$$I_N = \frac{20\angle 0^\circ}{j5}\text{ A} = 4\angle{-90^\circ}\text{ A}.$$

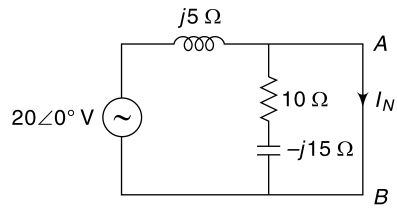

Fig. 6.12(a) *Determination of I_N*

Removing the voltage source and open circuiting terminals *AB*, Norton's equivalent impedance can be found out from Fig. 6.12(b).

Here Norton's equivalent impedance is

$$Z_N = j5 \parallel (10 - j15) = \frac{j5(10-j15)}{j5+10-j15}$$

$$= \frac{75+j50}{10-j10} = \frac{15+j10}{2-j2} = \frac{18\angle 33.7^\circ}{2.828\angle{-45^\circ}}$$

$$= 6.36\angle 78.7^\circ\ \Omega = (1.246 + j6.23)\ \Omega$$

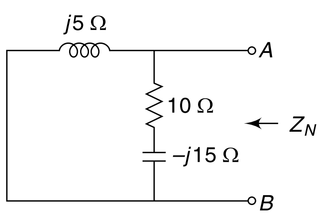

Fig. 6.12(b) *Determiantion of Z_N*

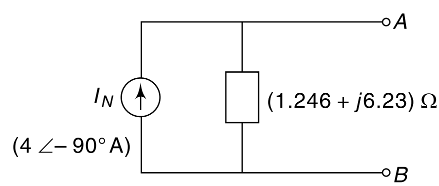

Fig. 6.12(c) *Norton's equivalent circuit of Ex. 6.12*

Norton's equivalent circuit is shown in Fig. 6.12(c).

6.4 MAXIMUM POWER TRANSFER THEOREM (AC APPLICATION)

This theorem finds useful application while evaluating the impedance of load to be connected to a two-terminal active network so that maximum power gets transferred from the network to the load. Three different cases have been considered here.

Case1:

When the load is purely resistive

We assume a circuit as shown in Fig. 6.13 where load resistance is R_L, source impedance $Z_g = R_g + jX_g$ and source voltage is E_g.

The current delivered to the load is

$$I = \frac{E_g}{R_g + R_L + jX_g} \tag{6.1}$$

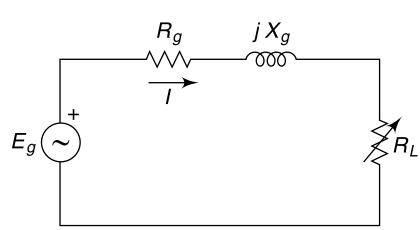

Fig. 6.13 *Power transfer in pure resistive load*

Power delivered to the load

$$P_L = (\text{Magnitude of current})^2 \times R_L = \frac{E_g^2}{(R_g + R_L)^2 + X_g^2} R_L \tag{6.2}$$

If maximum power is to be delivered

$$\frac{dP_L}{dR_L} = 0$$

or $$\frac{d}{dR_2}\left\{\frac{E_g^2}{(R_g + R_L)^2 + X_g^2} R_L\right\} = 0$$

or $$\{(R_g + R_L)^2 + X_g^2\}E_g^2 - E_g^2 R_L\{2(R_g + R_L)\} = 0$$

or $$R_g^2 + R_L^2 + X_g^2 + 2\, R_g R_L - 2\, R_g R_L - 2R_L^2 = 0$$

or $$R_g^2 + X_g^2 = R_L^2$$

or $$R_L = \sqrt{R_g^2 + X_g^2}\ . \tag{6.3}$$

Therefore load resistance $R_L = |Z_g|$, for maximum power transfer to resistive load

If $$X_g = 0 \text{ then } R_L = R_g \tag{6.4}$$

Case 2:

When *the load constitutes of variable reactance and fixed resistance* as shown in Fig. 6.14.

The load impedance $Z_L = R_L + jX_L$ where R_L is fixed and X_L is variable as shown in Fig. 6.14.

The net circuit impedance

$$Z = Z_g + Z_L = (R_g + R_L) + j(X_g + X_L)$$

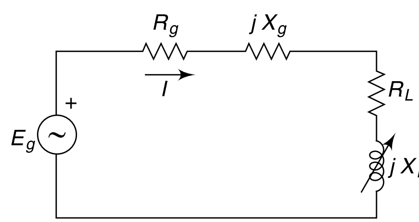

Fig. 6.14 ***Power transfer to R-L load (L is variable)***

The circuit current

$$I_L = \frac{E_g}{\sqrt{(R_g + R_L)^2 + (X_g + X_L)^2}} \tag{6.5}$$

The power delivered to the load

$$P_L = I_L^2 R_L = \frac{E_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2} \tag{6.6}$$

Since R_L is fixed and X_L is variable so for maximum power transfer is obtained when $\frac{dP_L}{dX_L} = 0$.

i.e. $$\frac{d}{dX_L}\left\{\frac{E_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2}\right\} = 0$$

or $$-2(X_g + X_L) = 0$$

or $$X_L = -X_g. \tag{6.7}$$

Therefore for maximum power transfer, the reactance of the load is of the same magnitude as the source reactance of the network, but is of opposite sign.

Case 3:

When the load reactance is of fixed magnitude and load resistance is variable as shown in Fig. 6.15.

The load impedance $Z_L = R_L + jX_L$ where X_L is fixed and R_L is variable.

The power delivered to the load

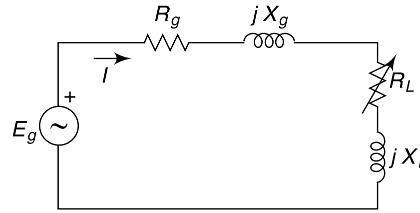

Fig. 6.15 ***Power transfer to R-L load (R is variable)***

$$P_L = I_L^2 R_L$$

$$= \frac{E_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2} \tag{6.8}$$

If X_L is of fixed magnitude and R_L is variable the condition for maximum power transfer is $\frac{dP_L}{dR_L} = 0$.

Therefore $$\frac{d}{dR_L}\left[\frac{E_g^2 R_L}{(R_g + R_L)^2 + (X_g + X_L)^2}\right] = 0$$

or $$(R_g + R_L)^2 + (X_g + X_L)^2 - R_L\{2(R_g + R_L)\} = 0$$

or $$R_g^2 + (X_g + X_L)^2 - R_L^2 = 0$$

or $$R_L^2 = R_g^2 + (X_g + X_L)^2$$

or $$R_L = \sqrt{R_g^2 + (X_g + X_L)^2} \tag{6.9}$$

Equation (6.9) gives the condition of maximum power transfer when R_L is variable.

General Case

If the load is such that both its resistance and reactance are variable, maximum power would be delivered when $Z_L = Z_g^*$ i.e. load impedance is the complex conjugate of the network impedance (i.e., $R_L = R_g$ and $X_L = -X_g$).

Proof:

We have seen that when the load reactance X_L alone is variable the condition for maximum power transfer is $X_L = -X_g$.

Maximum power transferred

$$P_{\max} = I_L^2 R_L = \frac{E_g^2 R_L}{(R_L + R_g)^2 + (X_L + X_g)^2} \tag{6.10}$$

Substituting $X_L = -X_g$

$$P_{\max} = \frac{E_g^2 R_L}{(R_g + R_L)^2} \tag{6.11}$$

If the load resistance R_L is also variable then

$$\frac{dP_{\max}}{dR_L} = 0$$

or $$(R_g + R_L)^2 - R_L^2(R_g + R_L) = 0$$

or $$R_L^2 = R_g^2 \text{ or } R_L = R_g \tag{6.12}$$

Statement of the Theorem:
The power delivered by an active network to a load connected across its terminals is maximum when the impedance of the load is the complex conjugate of the active network impedance.

The maximum power transferred under this condition is given by $P_{max} = \dfrac{E_g^2}{4R_L}$ as $R_g = R_L$ and $X_g = X_L$.

6.13 In the circuit shown in Fig. 6.16 find the value of R_L which results in maximum power transfer. Also calculate the value of the maximum power.

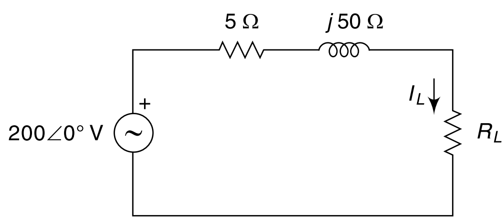

Fig. 6.16 *Circuit of Ex. 6.13*

Solution

As the load consists of resistance only hence for maximum power transfer

$$R_L = |Z_g| = \sqrt{R_g^2 + X_g^2} = \sqrt{(5)^2 + (50)^2} = 50.25\ \Omega$$

The current flowing in the circuit

$$I_L = \frac{200\angle 0^\circ}{5 + 50.25 + j50}$$

$$= \frac{200\angle 0^\circ}{55.25 + j50}$$

$$= \frac{200\angle 0^\circ}{74.52\angle 42.14^\circ} = 2.684\angle{-42.14^\circ}\ \text{A}.$$

Maximum power = $I_L^2 R_L = (2.684)^2 \times 50.25 = 362$ W.

6.14 In the network shown in Fig. 6.17 the load consists of a fixed inductance having reactance 20 Ω and a variable resistance. Find the value of R_L for which power transfer is maximum and also find the value of the maximum power.

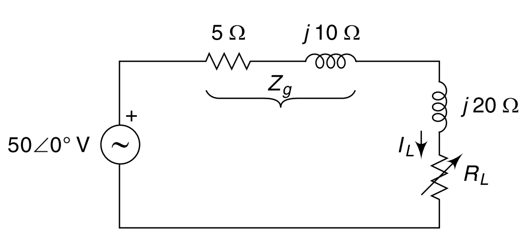

Fig. 6.17 *Circuit of Ex. 6.14*

Solution

When the load reactance is fixed but load resistance is variable then under maximum power transfer condition load resistance R_L is given by

$$R_L = \sqrt{(R_g)^2 + (X_g + X_L)^2}$$

$$= \sqrt{(5)^2 + (20+10)^2} = \sqrt{25+900} = 30.414\ \Omega$$

The circuit current $I_L = \dfrac{50\angle 0°}{(5+30.414) + j(10+20)} = 1.1\angle{-40.26°}$ A.

The maximum power transferred is

$$I_L^2 R_L = (1.1)^2 \times 30.414 = 36.8 \text{ W}.$$

6.15 If in Problem 6.14 the load reactance X_L is variable and the load resistance R_L is fixed having value 10 Ω, find the value of X_L for which power transfer is maximum and find the value of the maximum power.

Solution

When the load reactance is variable but load resistance is fixed then for maximum power transfer,

$$X_L = -X_g.$$

Hence load reactance

$$X_L = -j10\ \Omega.$$

The circuit current I_L is obtained as

$$I_L = \frac{50\angle 0°}{(5+10) + j(10-10)} = \frac{50\angle 0°}{15} = 3.33 \text{ A}.$$

Maximum transferred power $P_{\max} = I_L^2 R_L = (3.33)^2 \times 10 = 111.09$ W

ADDITIONAL PROBLEMS

6.16 Find the current through the load in Fig. 6.18 using Thevenin's theorem.

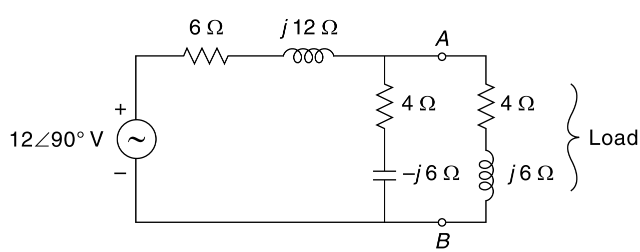

Fig. 6.18 ***Circuit of Ex. 6.16***

Solution

Removing the load impedance the terminals *AB* is open circuited and the open circuit voltage is found out as shown in Fig. 6.18(a).

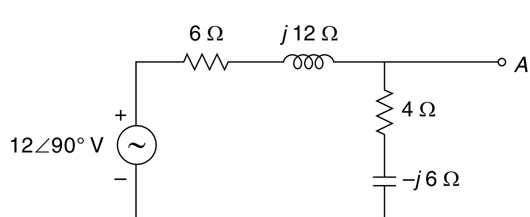

Fig. 6.18(a) ***Determination of V_{Th}***

Thevenin's equivalent voltage:

V_{Th} = Voltage across terminals AB

$$= \frac{12\angle 90^\circ}{6 + j12 + 4 - j6} \times (4 - j6) = \frac{12\angle 90^\circ}{10 + j6}(4 - j6) = 7.42\angle 2.73^\circ \text{ Volts.}$$

Now removing the voltage source, Thevenin's equivalent impedance (Z_{Th}) is found from the circuit shown in Fig. 6.18(b).

$$Z_{Th} = (6 + j12) \parallel (4 - j6)$$

$$= \frac{(6 + j12)(4 - j6)}{10 + j6}$$

$$= 8.3\angle -23.83^\circ$$

$$= (7.6 - j3.35)\ \Omega$$

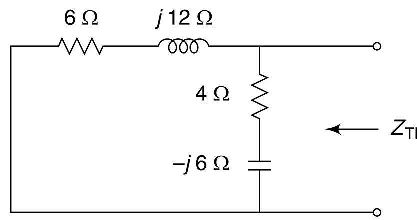

Fig. 6.18(b) ***Determination of Z_{Th}***

Hence current through the load = $\dfrac{7.42\angle 2.73^\circ}{7.6 - j3.35 + 4 + j6}$ = $0.624\angle -10.14^\circ$ A.

6.17 Solve Example 6.16 using Norton's theorem.

Solution

The load impedance is removed and the terminals AB is open circuited as shown in Fig. 6.18(c).

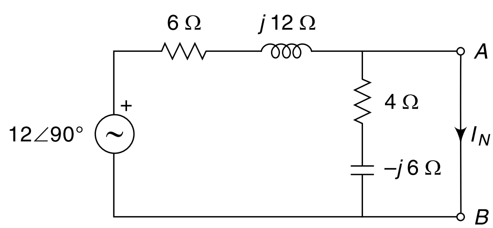

Fig. 6.18(c) ***Determination of I_N***

The current through the short circuited path AB is the Norton's equivalent current I_N.

Hence $$I_N = \frac{12\angle 90^\circ}{6 + j12}\text{ A} = \frac{12\angle 90^\circ}{13.41\angle 63.43^\circ}\text{ A} = 0.89\angle 26.56^\circ \text{ A.}$$

Norton's equivalent impedance (Z_N) can be found in the same way as that of Thevenin's equivalent impedance.

Hence, $$Z_N = \frac{(6 + j12)(4 - j6)}{10 + j6}$$

$$= \frac{96.74\angle 7.125^\circ}{11.66\angle 30.96} = 8.3\angle -23.83^\circ = (7.6 - j3.35)\ \Omega$$

$\therefore$ The current through the load (I) is given by

$$I = 0.89\angle 26.56^\circ \times \frac{7.6 - j3.35}{7.6 - j3.35 + 4 + j6} = 0.62\angle -10.06^\circ \text{ A.}$$

6.18 Find the current through the capacitor in Fig. 6.19 using superposition theorem.

Solution

Considering the current source of 15∠30° A acting alone in the circuit and removing the voltage source, the current through the capacitor [as shown in Fig. 6.19(a)] is given by

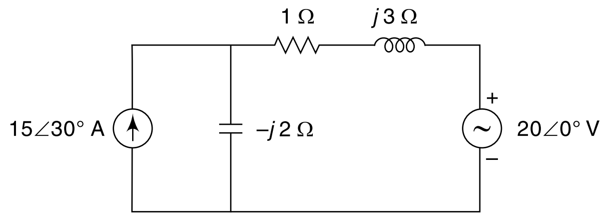

Fig. 6.19 *Circuit of Ex. 6.18*

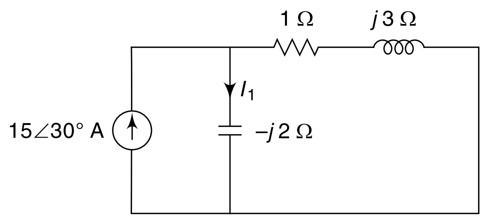

Fig. 6.19(a) *Current source acting alone*

$$I_1 = 15\angle 30^\circ \times \frac{1+j3}{1+j3-j2}$$

$$= 15\angle 30^\circ \times \frac{1+j3}{1+j} = 15\angle 30^\circ \times \frac{3.16\angle 71.56^\circ}{1.414\angle 45^\circ} = 33.54\angle 56.56^\circ \text{ A}$$

Next considering the voltage source of 20∠0° V acting alone in the circuit and removing the current source, the current through the capacitor [as shown in Fig. 6.19(b)] is obtained as

$$I_2 = \frac{20\angle 0^\circ}{1+j3-j2}\text{ A} = \frac{20\angle 0^\circ}{1+j} = 14.14\angle{-45^\circ} \text{ A.}$$

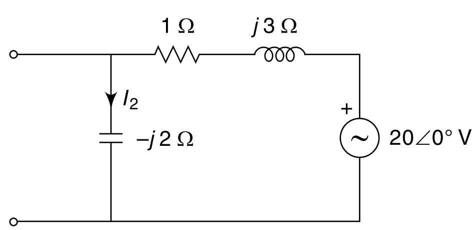

Fig. 6.19(b) *Voltage source acting alone*

According to superposition theorem when both the sources are acting simultaneously the current through the capacitor (I) is

$$I = I_1 + I_2 = 33.54\angle 56.56^\circ + 14.14\angle{-45^\circ}$$
$$= 28.47 + j17.97 = 33.68\angle 32.28^\circ \text{ A}$$

6.19 In the circuit shown in Fig. 6.20 find the load resistance and load reactance if maximum power is transferred to the load considering both the load resistance and load reactance to be variable.

Solution

When both the load resistance and reactance are variable, the load impedance (Z_L) is the complex conjugate of the internal impedance of the network (Z_g) under maximum power transfer condition.

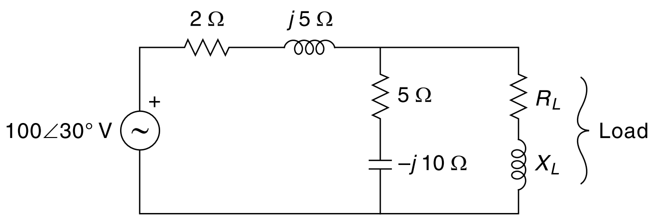

Fig. 6.20 *Circuit of Ex. 6.19*

Now removing the source and open circuiting the terminals of the load impedance as shown in Fig. 6.20(a), (Z_g) can be found out.

$$Z_g = (5 - j10) \parallel (2 + j5)$$

$$= \frac{(5 - j10)(2 + j5)}{5 - j10 + 2 + j5}$$

$$= \frac{60 + j5}{7 - j5}$$

$$= \frac{60.2\angle 4.76^\circ}{8.6\angle -35.53^\circ}$$

$$= 7\angle 40.29^\circ = (5.34 + j4.53)\ \Omega.$$

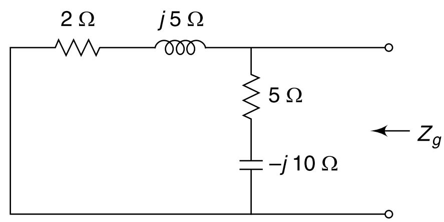

Fig. 6.20(a) *Determination of (Z_g)*

Hence $\quad Z_L = Z_g^* = (5.34 - j4.53)\ \Omega$

or, $\quad R_L = 5.34\ \Omega$ and $X_L = 4.53\ \Omega$ (Capacitive)

6.20 What is the Thevenin's equivalent circuit with respect to the terminals A and B of the circuit shown in Fig. 6.21.

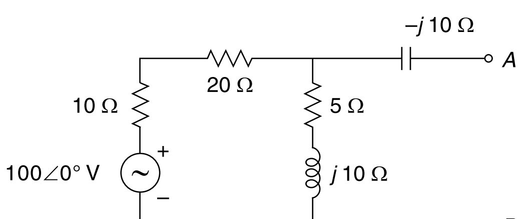

Fig. 6.21 *Circuit of Ex. 6.20*

Solution

Thevenin's equivalent voltage (V_{Th}) is the voltage across the terminals AB of the network;

$$V_{\text{Th}} = V_{AB} = \frac{100\angle 0^\circ}{10 + 20 + 5 + j10}(5 + j10)$$

$$= \frac{100\angle 0^\circ}{35 + j10}(5 + j10) = 30.7\angle 47.48^\circ$$

Removing the voltage source, Thevenin's equivalent impedance Z_{Th} can be found.

$$Z_{\text{Th}} = -j10 + \frac{(5 + j10)(20 + 10)}{5 + j10 + 20 + 10}$$

$$= -j10 + \frac{150 + j300}{35 + j10} = -j10 + 9.21\angle 47.48^\circ = (6.22 - j3.21)\ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.21(a).

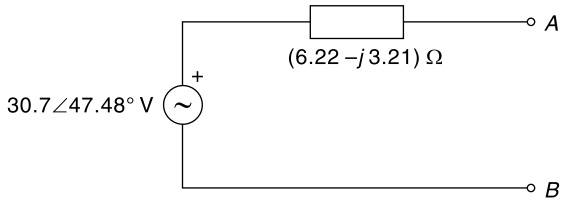

Fig. 6.21(a) ***Thevenin's equivalent circuit of Ex. 6.20***

6.21 In the circuit shown in Fig. 6.22 the circuit consists of fixed inductance and variable load resistance (R_L). Find (R_L) for maximum power transfer and the value of the maximum power.

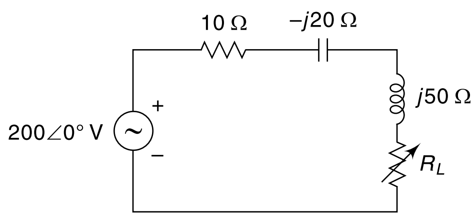

Fig. 6.22 ***Circuit of Ex. 6.21***

Solution

The internal impedance of network is obtained as

$$Z_g = (10 - j20)\ \Omega = (R_g + jX_g)\ \Omega.$$

For maximum power transfer when R_L is variable and X_L is fixed,

$$R_L = \sqrt{R_g^2 + (X_g + X_L)^2} \quad \text{(Equation (6.9))}$$

$$= \sqrt{(10)^2 + (-20+50)^2} = \sqrt{100+900} = 31.62\ \Omega.$$

Under maximum power transfer condition the current through the circuit is obtained as

$$I = \frac{200\angle 0^\circ}{10 - j20 + j50 + 31.62} = \frac{200\angle 0^\circ}{41.62 + j30} = 3.9\angle -35.78^\circ\ \text{A}.$$

Maximum transferred power

$$P_{\max} = I^2 R_L = (3.9)^2 \times 31.62\ \text{W} = 481\ \text{W}.$$

6.22 Find Norton's equivalent circuit with respect to terminals A and B of the network shown in Fig. 6.23.

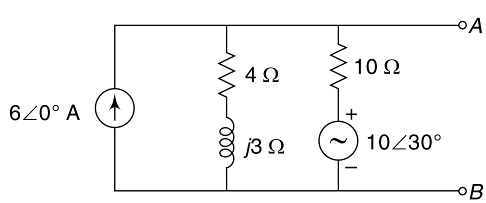

Fig. 6.23 ***Circuit of Ex. 6.22***

Solution

Short circuiting terminals A and B and removing the voltage source, the current through the short circuited path (as shown in Fig. 6.23(a)) due to current source is

$$I_{sc_1} = 6\angle 0^\circ\ \text{A (from } A \text{ to } B)$$

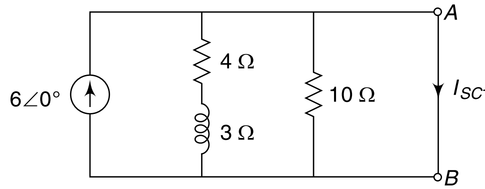

Fig. 6.23(a) *Determination of* I_{SC1}

Now removing the current source from the network in Fig. 6.23 and short circuiting the terminals *AB*, the current through the short circuited path [as shown in Fig. 6.23(b)] due to the voltage source is

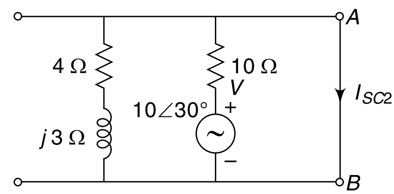

Fig. 6.23(b) *Determination of* I_{SC2}

$$I_{sc_2} = \frac{10\angle 30°}{10} \text{ A}$$
$$= 1\angle 30° \text{ A (from } A \text{ to } B).$$

According to superposition theorem, when both the sources are acting simultaneously the current through the short circuited path is given by

$$I_{sc} = I_{sc_1} + I_{sc_2}$$
$$= 6\angle 0° + 1\angle 30°$$
$$= 6 + 0.866 + j0.5$$
$$= 6.866 + j0.5 = 6.88\angle 4.16° \text{ A}.$$

Hence Norton's equivalent current $I_N = 6.88\angle 4.16°$ A.

Next, removing the sources, Norton's equivalent impedance Z_N can be found as shown in Fig. 6.23(c).

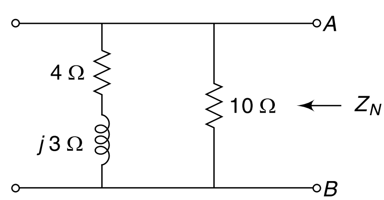

Fig. 6.23(c) *Determination of* (Z_N)

$$Z_N = 10 \parallel (4 + j3)$$
$$= \frac{10(4 + j3)}{14 + j3}$$
$$= 3.49\angle 24.77° \text{ A}$$
$$= (3.17 + j1.46)\ \Omega.$$

Norton's equivalent circuit is shown in Fig. 6.23(d).

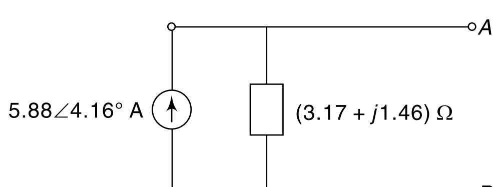

Fig. 6.23(d) *Norton's equivalent circuit of Ex. 6.22*

6.23 Using the superposition theorem find the current in branch *AB* of the circuit given in Fig. 6.24.

Solution

Let us consider the 100∠0° V source acting alone and 50∠0° V source removed. The circuit is shown in Fig. 6.24(a).

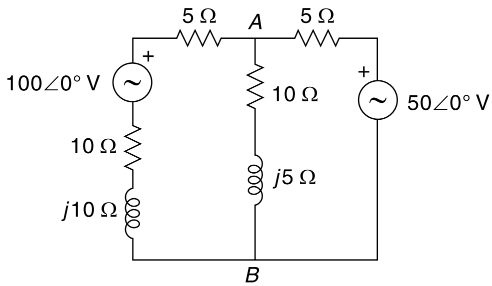

Fig. 6.24 *Circuit of Ex. 6.23*

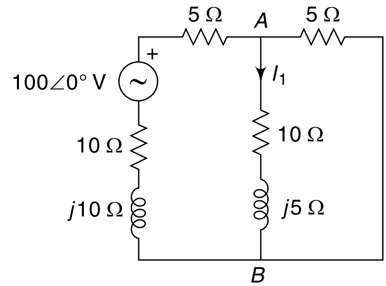

Fig. 6.24(a) ***100∠0° V source acting alone***

$$\text{Total impedance} = 5 + 10 + j10 + \frac{5(10 + j5)}{5 + 10 + j5}$$

$$= 15 + j10 + \frac{10 + j5}{3 + j}$$

$$= 15 + j10 + \frac{11.18\angle 26.56^\circ}{3.16\angle 18.43^\circ}$$

$$= 15 + j10 + 3.54\angle 8.125^\circ$$

$$= 18.5 + j10.5 = 21.27\angle 29.58^\circ\ \Omega.$$

$$\text{Total current} = \frac{100\angle 0^\circ}{21.27\angle 29.58^\circ} = 4.7\angle{-29.58^\circ}\ \text{A}.$$

Hence current through AB is

$$I_1 = 4.7\angle{-29.58^\circ} \times \frac{5}{5 + 10 + j5}$$

$$= 4.7\angle{-29.58^\circ} \times \frac{1}{3 + j}$$

$$= 4.7\angle{-29.58^\circ} \times 0.316\angle{-18.43^\circ} = 1.486\angle{-48.02^\circ}\ \text{A}$$

Now let us consider $50\angle 0^\circ$ V source acting alone while $100\angle 0^\circ$ V source is removed. The corresponding circuit is shown in Fig. 6.24(b).

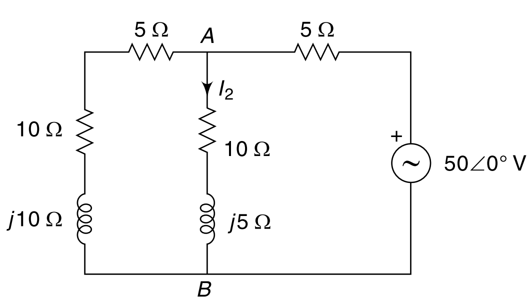

Fig. 6.24(b) ***50∠0°V source acting alone***

$$\text{Total impedance} = \frac{(5 + 10 + j10)(10 + j5)}{5 + 10 + j10 + 10 + j5} + 5$$

$$= \frac{(15 + j10)(10 + j5)}{25 + j15} + 5$$

$$= \frac{(3+j2)(2+j)}{5+j3} + 5$$

$$= \frac{4+j7}{5+j3} + 5$$

$$= \frac{8.062\angle 60.25°}{5.83\angle 30.96°} + 5$$

$$= 1.383\angle 29.29° + 5$$

$$= 6.243\angle 6.23°\ \Omega$$

$$\text{Total current} = \frac{50\angle 0°}{6.243\angle 6.23°} = 8\angle -6.23°\ \text{A}$$

Hence current through AB is obtained as

$$I_2 = 6.243\angle 6.23° \times \frac{5+10+j10}{5+10+j10+10+j5}$$

$$= 6.243\angle -6.23° \times \frac{18.03\angle 33.69°}{29.15\angle 30.96°} = 3.86\angle 8.96°\ \text{A.}$$

Applying superposition theorem the current through branch AB of the circuit when both the sources are acting simultaneously is

$$I = I_1 + I_2 = 1.486\angle -48.02° + 3.86\angle 8.96°$$

$$= 4.807 - j0.504 = 4.833\angle 5.985°\ \text{A}$$

6.24 Apply superposition theorem to find the current through the capacitor of $(-j5)$ in Fig. 6.25.

Solution

Let us consider $20\angle 90°$ A source acting alone in the circuit. The corresponding circuit is

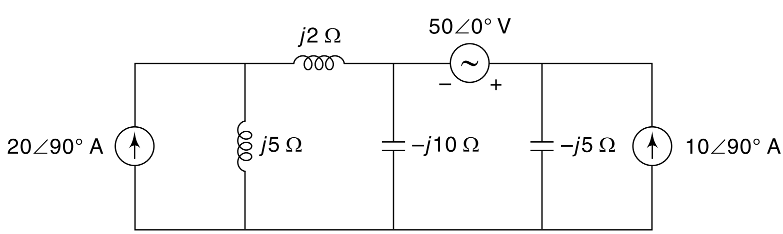

Fig. 6.25 ***Circuit of Ex. 6.24***

shown in Figs 6.25(a) and 6.25(b).

$$X = \frac{(-j10)(-j5)}{-j10-j5} = \frac{-50}{-j15} = -j\frac{50}{15} = -j3.33\ \Omega$$

Current through X is found as

$$I_X = 20\angle 90° \times \frac{j5}{j5+j2-j3.33}$$

$$= 20\angle 90° \times \frac{5\angle 90°}{3.67\angle 90°} = 27.25\angle 90°\ \text{A.}$$

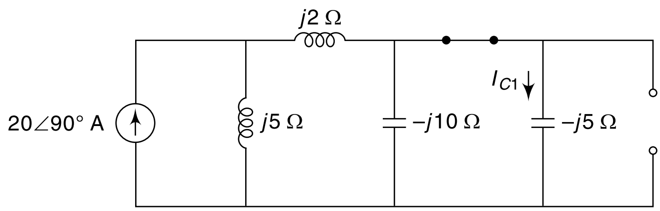

Fig. 6.25(a) ***Current source 20∠90°A acting alone***

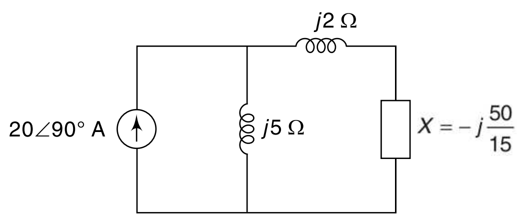

Fig. 6.25(b) ***Simplified circuit of Fig. 6.25(a)***

Hence from Fig. 6.25(a) current through capacitor of (–*j*5) Ω is

$$I_{C1} = 27.25\angle 90° \times \frac{-j10}{-j10 - j5} = 18.167\angle 90° \text{ A.}$$

Now let us consider 50∠0° V source acting alone as shown in Fig. 6.25(c).

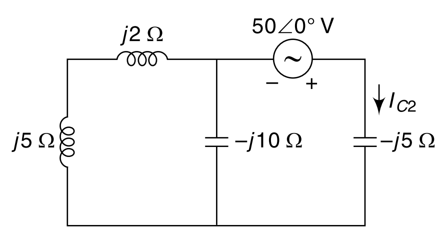

Fig. 6.25(c) ***Voltage source considered alone***

The total impedance is $\left[-j5 + \frac{(-j10)(j5 + j2)}{-j10 + j5 + j2}\right]\ \Omega$

i.e., $Z = -j5 + \frac{70}{-j3} = -j5 + j23.33 = j18.33 = 18.33\angle 90°\ \Omega.$

Hence current through capacitor of (–*j*5) Ω is

$$I_{C2} = \frac{50\angle 0°}{18.33\angle 90°} = 2.73\angle -90° \text{ A.}$$

Let us consider 10∠90° A source acting alone as shown in Fig. 6.25(d).

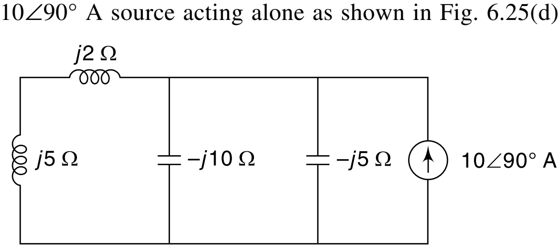

Fig. 6.25(d) ***10∠90°A source acting alone***

Combined impedance of $j5$, $j2$ and $-j10$ is

$$= \frac{j7(-j10)}{j7 - j10} = \frac{70}{-j3} = j23.33\,\Omega.$$

Hence current through capacitance of $(-j5)$ Ω is

$$I_{C3} = 10\angle 90° \times \frac{j23.33}{j23.33 - j5} = 12.73\angle 90° \text{ A}.$$

Using the superposition theorem the current I through the capacitor of $(-j5)$ Ω is $I = I_{C1} + I_{C2} + I_{C3} = 18.167\angle 90° + 2.73\angle -90° + 12.73\angle 90° = 28.167\angle 90°$ A.

6.25 Find the current through Z_L in Fig. 6.26 using the superposition theorem.

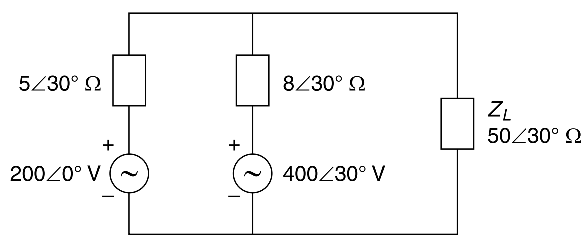

Fig. 6.26 ***Circuit of Ex. 6.25***

Solution

Considering the 200∠0° V source acting alone in the circuit, the total current supplied by the source is

$$\frac{200\angle 0°}{5\angle 30° + \dfrac{8\angle 30° \times 50\angle 30°}{8\angle 30° + 50\angle 30°}} = \frac{200\angle 0°}{5\angle 30° + \dfrac{400\angle 60°}{57.99\angle 30°}}$$

$$= \frac{200\angle 0°}{5\angle 30° + 6.89\angle 30°} = \frac{200\angle 0°}{10.297 + j5.945}$$

$$= \frac{200\angle 0°}{11.89\angle 30°} = 16.8\angle -30° \text{ A}.$$

Hence, current through $Z_L = 16.8\angle -30° \times \dfrac{8\angle 30°}{8\angle 30° + 50\angle 30°}$

$$= \frac{16.8 \times 8}{58\angle 30°} = 2.317\angle -30° \text{ A}.$$

Now considering 400∠30° V source acting alone in the circuit the total current supplied by the source is

$$\frac{400\angle 30°}{8\angle 30° + \dfrac{50\angle 30° \times 5\angle 30°}{50\angle 30° + 5\angle 30°}} = \frac{400\angle 30°}{8\angle 30° + 4.545\angle 30°}$$

$$= \frac{400\angle 30°}{12.545\angle 30°} = 31.88 \text{ A}$$

Hence current through $(Z_L) = 31.88 \times \dfrac{5\angle 30°}{5\angle 30° + 50\angle 30°}$

$$= \frac{31.88 \times 5}{55} = 2.898 \text{ A}$$

Using the superposition theorem the current through Z_L, when both the sources are acting simultaneously, is

$$(2.317\angle{-30^\circ} + 2.898)\text{ A} = 4.898 - j1.1585$$
$$= 5.033\angle{-13.31^\circ}\text{ A.}$$

6.26 Find current through Z_L in Fig. 6.27 applying Thevenin's theorem.

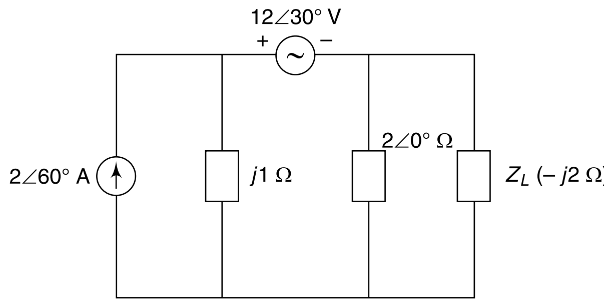

Fig. 6.27 *Circuit of Ex. 6.26*

Solution

Let us open circuit the terminals of Z_L to find the open circuit voltage V_{oc} as shown in Fig. 6.27(a).

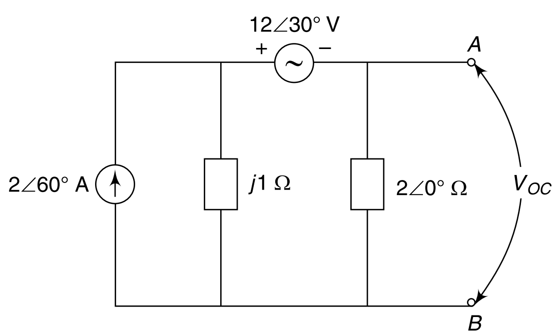

Fig. 6.27(a) *Determination of V_{OC}*

Current through the 2 Ω impedance due to current source only is

$$I_1 = 2\angle 60^\circ \times \frac{j1}{j1 + 2\angle 0^\circ}$$

$$= \frac{2\angle 60^\circ \times 1\angle 90^\circ}{2 + j}$$

$$= \frac{2\angle 150^\circ}{2.236\angle 26.56} = 0.89\angle 123.44^\circ\text{ A} \quad (\text{from } A \text{ to } B).$$

Current through the 2 Ω impedance due to voltage source acting alone is

$$I_2 = \frac{12\angle 30^\circ}{j1 + 2\angle 0^\circ}$$

$$= \frac{12\angle 30^\circ}{2.236\angle 26.56^\circ} = 5.367\angle 3.44^\circ\text{ A (from } B \text{ to } A).$$

Applying the superposition theorem current through the 2 Ω impedance is

$$I = 5.367\angle 3.44^\circ - 0.89\angle 123.44^\circ$$
$$= 5.86\angle{-4.11^\circ} \text{ A.}$$

Hence $V_{oc} = 2\angle 0^\circ \times 5.86\angle{-4.11^\circ} = 11.72\angle{-4.11^\circ}\text{V} \; (= V_{Th})$

Now, Thevenin's equivalent impedance is obtained as

$$Z_{Th} = \frac{j1 \times 2\angle 0^\circ}{j1 + 2\angle 0^\circ} = \frac{2\angle 90^\circ}{2.236\angle 26.56^\circ} = 0.89\angle 63.44^\circ \ \Omega$$

Thevenin's equivalent circuit is shown in Fig. 6.27(b).

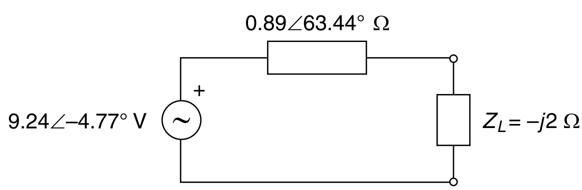

Fig. 6.27(b) ***Thevenin's equivalent circuit of Ex. 6.26***

Hence current through Z_L is obtained as

$$I = \frac{11.72\angle -4.11^\circ}{0.89\angle 63.44^\circ - j2} = 9.243\angle 67.54^\circ \text{ A.}$$

6.27 Find the voltage drop across 2 Ω resistor in the circuit shown in Fig. 6.28 using Thevenin's theorem.

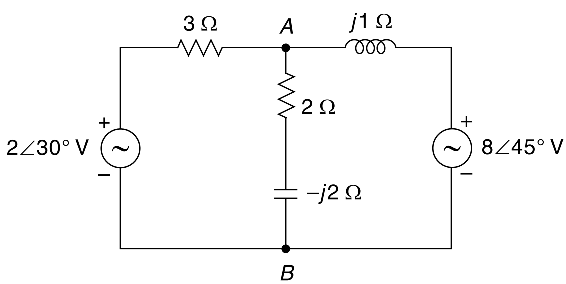

Fig. 6.28 ***Circuit of Ex. 6.27***

Solution

Let us open circuit the terminals *AB* to find the Thevenin's equivalent voltage as shown in Fig. 6.28(a).

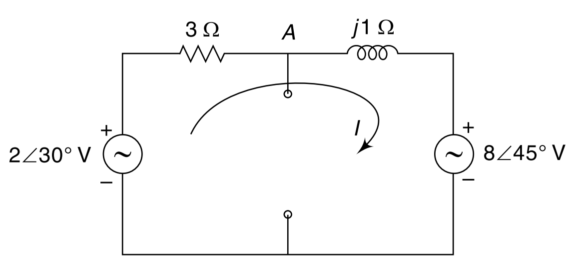

Fig. 6.28(a) ***Determination (V_{Th})***

The current flowing through the circuit is

$$I = \frac{2\angle 30° - 8\angle 45°}{3 + j1}$$

$$= \frac{-3.925 - j4.65}{3.162\angle 18.43°}$$

$$= \frac{6.09\angle -130°}{3.162\angle 18.43°} = 1.92\angle -148.6° \text{ A.}$$

V_{Th} = Voltage across V_{AB}

$= j1 \times 1.92\angle -148.6° + 8\angle 45°$

$= 1.92\angle -58.6° + 8\angle 45°$

$= 6.657 + j4.02 = 7.77\angle 31.127°$ V

Thevenin's equivalent impedance is obtained as

$$Z_{Th} = \frac{3 \times j1}{3 + j1} = \frac{3\angle 90°}{3.162\angle 18.43°} = 0.9487\angle 71.57° \ \Omega.$$

Hence current through 2 Ω resistor is

$$I_{2\Omega} = \frac{7.77\angle 31.127°}{0.9487\angle 71.57° + 2 - j2}$$

$$= \frac{7.77\angle 31.127°}{2.3 - j1.1}$$

$$= \frac{7.77\angle 31.127°}{2.55\angle -25.56°} = 3.05\angle 56.687° \text{ A.}$$

6.28 Find Thevenin's equivalent circuit across the load in the network shown in Fig. 6.29.

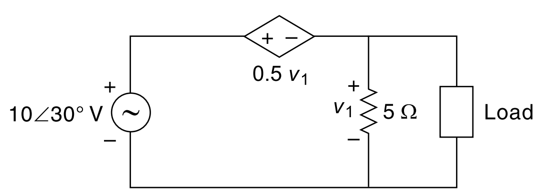

Fig. 6.29 *Circuit of Ex. 6.28*

Solution

Removing the load it is seen that open circuit voltage $V_{oc} = v_1$ as shown in Fig. 6.29(a).

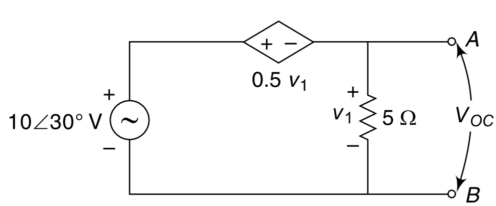

Fig. 6.29(a) *Determination of V_{oc}*

Applying KVL

$10\angle 30° - 0.5v_1 - v_1 = 0$

or $1.5v_1 = 10\angle 30°$

i.e. $v_1 = 6.67\angle 30°\ V\ (= V_{oc})$.

Apply short circuit across terminals AB as shown in Fig. 6.29(b); short circuit current I_{sc} is obtained as

$$I_{sc} = \frac{10\angle 30^\circ}{0} \text{ A [Since there is no impedance in the loop]}$$

$$\therefore \qquad I_{SC} = \infty.$$

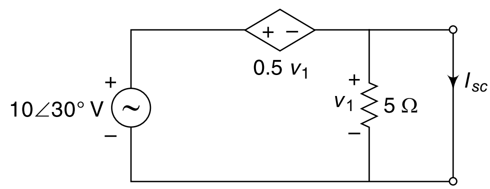

Fig. 6.29(b) ***Determination of (I_{sc}) and (Z_{in})***

Hence internal impedance of the circuit is given by

$$Z_{in} = \frac{V_{oc}}{I_{sc}} = 0\ \Omega.$$

Thevenin's equivalent circuit is shown in Fig. 6.29(c).

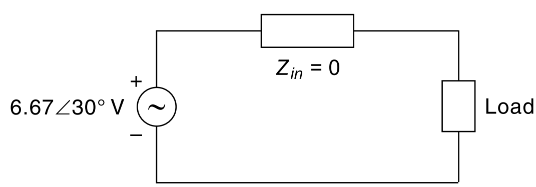

Fig. 6.29(c) ***Thevenin's equivalent circuit of Ex. 6.28***

6.29 Find Thevenin's equivalent network across AB in Fig. 6.30.

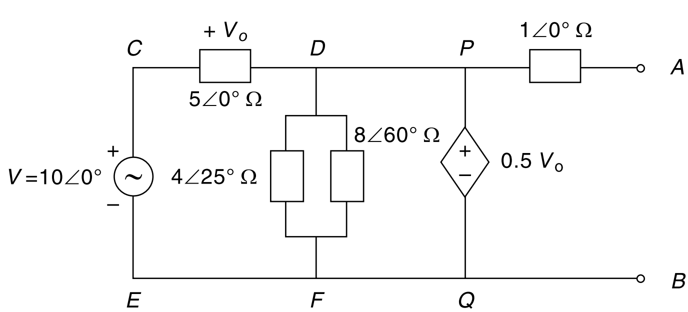

Fig. 6.30 ***Circuit of Ex. 6.29***

Solution

The equivalent impedance of $4\angle 25°$ and $8\angle 60°$ in parallel is

$$\frac{4\angle 25^\circ \times 8\angle 60^\circ}{4\angle 25^\circ + 8\angle 60^\circ} = 2.78\angle 36.5^\circ\ \Omega$$

Open circuit voltage $V_{AB} = V_{oc} = 0.5\ V_o$.

Considering loop *CPQE*

$$10\angle 0° - V_o - 0.5\ V_o = 0$$

or $\quad V_o = 6.67\angle 0°$ V

$\therefore \quad V_{Th} = V_{oc} = 6.67\angle 0°$ V.

Next, terminals *AB* is short circuited as shown in Fig. 6.30(a).

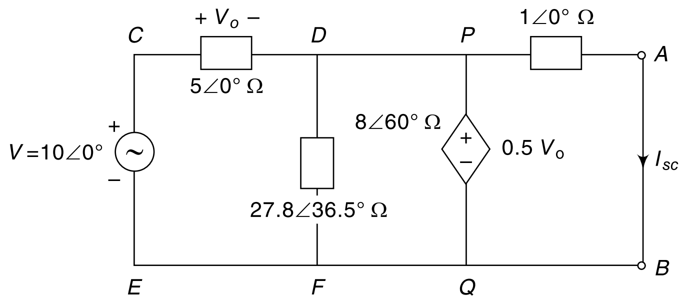

Fig. 6.30(a) ***Determination of (I_{SC}) and (Z_{in})***

Applying KVL in loop *CABE*, we get

$$10\angle 0° - 6.67\angle 0° - 1\angle 0° \times I_{sc} = 0$$

i.e. $\quad I_{sc} = 3.33$ A.

Hence internal impedance of the circuit is obtained as

$$Z_{in} = \frac{V_o}{I_{sc}} = \frac{6.67\angle 0°}{3.33} = 2\ \Omega.$$

Thevenin's equivalent network is shown in Fig. 6.30(b).

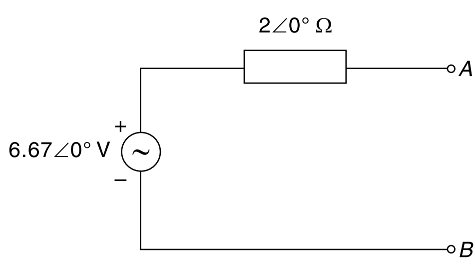

Fig. 6.30(b) ***Thevenin's equivalent circuit of Ex. 6.29***

6.30 Find the current through 10 Ω resistor shown in Fig. 6.31 using Norton's theorem.

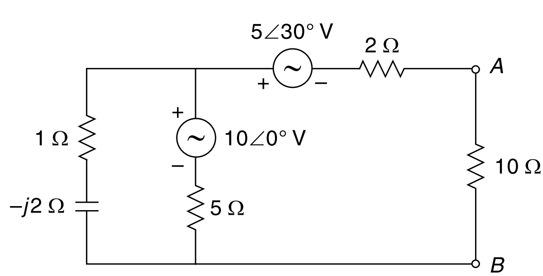

Fig. 6.31 ***Circuit of Ex. 6.30***

Solution

To find out the Norton's equivalent current source let us short circuit terminals *A* and *B* as shown in Fig. 6.31(a).

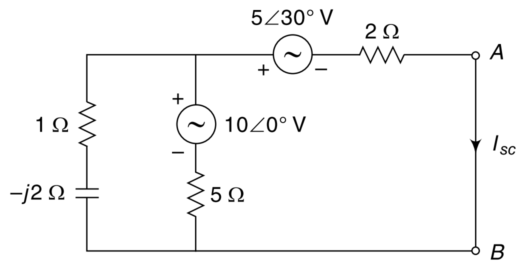

Fig. 6.31(a) ***Determination of (I_{SC})***

Short circuit current due to 10 V source acting alone is

$$I_{sc_1} = \frac{10\angle 0^\circ}{5+\dfrac{(1-j2)2}{1+2-j2}} \times \frac{1-j2}{1-j2+2}$$

$$= \frac{10\angle 0^\circ}{5+\dfrac{2-j4}{3-j2}} \times \frac{2.23\angle -63.43^\circ}{3.6\angle -33.69^\circ}$$

$$= \frac{10\angle 0^\circ}{6.08\angle -5.76^\circ} \times \frac{2.23\angle -63.43^\circ}{3.6\angle -33.69^\circ}$$

$= 1.016\angle{-23.98^\circ}$ A (from *A* to *B*).

Short circuit current due to 5 V source acting alone is

$$I_{sc_2} = \frac{5\angle 30^\circ}{2+\dfrac{5(1-j2)}{5+1-j2}}$$

$$= \frac{5\angle 30^\circ}{2+\dfrac{5-j10}{6-j2}}$$

$$= \frac{5\angle 30^\circ}{2+1.768\angle -45^\circ}$$

$$= \frac{5\angle 30^\circ}{3.48\angle -21^\circ} = 1.436\angle 51^\circ \text{ A (from } B \text{ to } A).$$

Hence $I_{sc} = (1.436\angle 51^\circ - 1.016\angle{-23.98^\circ})$ A (from *B* to *A*).

$= 1.53\angle{-91.05^\circ}$ A ($= I_N$).

Norton's equivalent impedance is found out removing all the sources and looking into the network from terminals *AB* as shown in Fig. 6.31(b).

$$Z_N = 2 + \frac{5(1-j2)}{5+1-j2}$$

$= 2 + 1.768\angle{-45^\circ} = 3.25 - j1.25 = 3.48\angle{-21^\circ}\ \Omega.$

Norton's equivalent circuit is shown in Fig. 6.31(c).

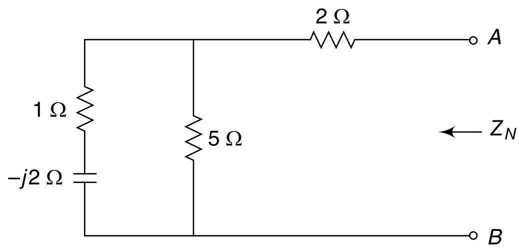

Fig. 6.31(b) *Determination of* (Z_N)

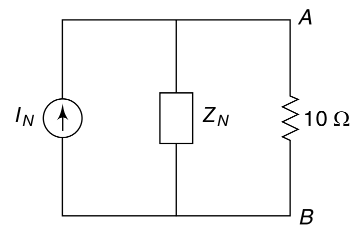

Fig. 6.31(c) ***Norton's equivalent circuit of Ex. 6.30***

Hence current through the 10 Ω resistor

$$= 1.53\angle{-91.05°} \times \frac{3.48\angle{-21°}}{10 + 3.48\angle{-21°}}$$

$$= 1.53\angle{-91.05°} \times \frac{3.48\angle{-21°}}{13.3\angle{-5.39°}}$$

$$= 0.4\angle{-106.66°} \text{ A.}$$

6.31 Find Norton's equivalent circuit across terminals *AB* for the network shown in Fig. 6.32.

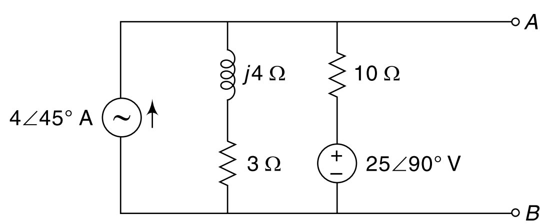

Fig. 6.32 *Circuit of Ex. 6.31*

Solution

Short circuiting the terminals *AB*, Norton's equivalent current is found out from Fig. 6.32(a).

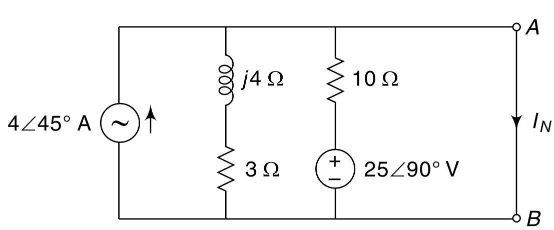

Fig. 6.32(a) *Determination of* (I_N)

$$I_N = 4\angle 45° + \frac{25\angle 90°}{10}$$

$$= 4\angle 45° + 2.5\angle 90° = 2.83 + j5.328 = 6\angle 62° \text{ A.}$$

Removing the sources and looking into the network from terminals *AB*, Norton's equivalent impedance is obtained from Fig. 6.32(b).

$$Z_N = \frac{10(3+j4)}{10+3+j4}$$
$$= \frac{30+j40}{13+j4}$$
$$= \frac{50\angle 53.13°}{13.6\angle 17.1°}\ \Omega$$
$$= 3.676\angle 36°\ \Omega.$$

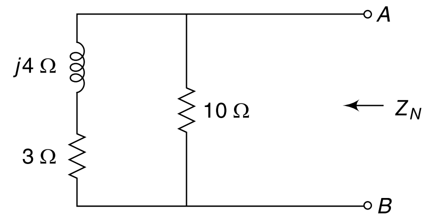

Fig. 6.32(b) *Determination of (Z_N)*

Norton's equivalent circuit is shown in Fig. 6.32(c).

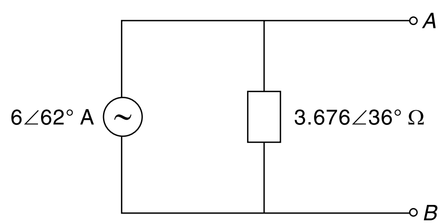

Fig. 6.32(c) *Norton's equivalent circuit of Ex. 6.31*

6.32 Obtain Norton's equivalent network across terminals *AB* of the network shown in Fig. 6.33.

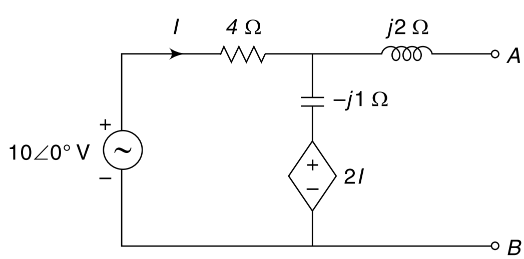

Fig. 6.33 *Circuit of Ex. 6.32*

Solution

Applying KVL in the closed loop

$$10\angle 0° - 4I - I(-j1) - 2I = 0$$
$$6I - jI = 10\angle 0°$$
$$I = \frac{10}{6-j1} = 1.64\angle 9.46°\text{ A.}$$

Now, open circuit voltage across *AB* is

$$V_{oc} = 2I + I(-j1)$$
$$= (2-j)1.64\angle 9.46° = 3.67\angle -17°\text{ V.}$$

To find the internal impedance of the circuit, terminals *AB* are short circuited as shown in Fig. 6.33(a).

Applying KVL in loop *CDQP*

$$10\angle 0° - 4I - (I - I_{sc})(-j1) - 2I = 0$$

i.e. $$10\angle 0° - I(6-j) - jI_{sc} = 0. \qquad \text{(i)}$$

Applying KVL in loop *DABQ*

$$2I + (I - I_{sc})(-j1) - j2I_{sc} = 0$$

or $$(2-j)I - jI_{sc} = 0. \qquad \text{(ii)}$$

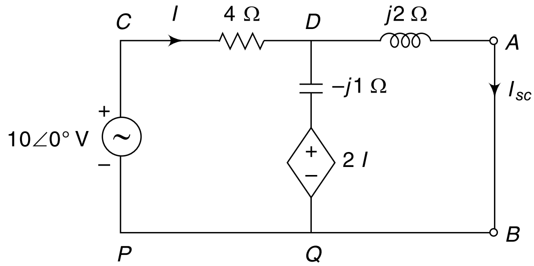

Fig. 6.33(a) ***Determination of (I_{SC})***

Solving the two equations (1) and (2), we get

$$10\angle 0° - I(6 - j) - (2 - j)I = 0$$

$$8I = 10\angle 0°$$

$$I = 1.25\angle 0° \text{ A.}$$

$$\therefore \qquad I_{sc} = \frac{2-j}{j}\, 1.25\angle 0° = \frac{2.236 \times 1.25}{\angle 90°}\angle{-26.56°} = 2.79\angle{-116.56°} \text{ A.}$$

Hence, $$Z_{int} = \frac{3.67\angle -17°}{2.79\angle -116.56°} = 1.315\angle 99.56° \ \Omega.$$

Norton's equivalent circuit is shown in Fig. 6.33(b).

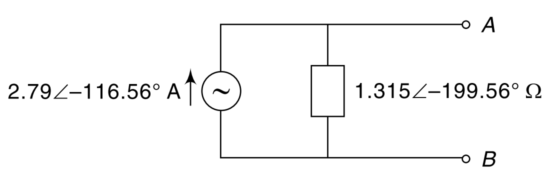

Fig. 6.33(b) ***Norton's equivalent circuit of Ex. 6.32***

6.33 Obtain Norton's equivalent circuit across terminals xy in Fig. 6.34.

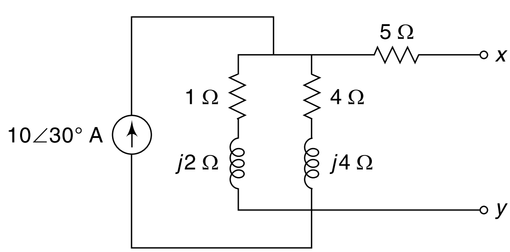

Fig. 6.34 ***Circuit of Ex. 6.33***

Solution

Let us short circuit the terminals xy to find the short circuit current from Fig. 6.34(a). The combined impedance of $(1 + j2)$ Ω and $(4 + j4)$ Ω in parallel gives

$$\frac{(1+j2)(4+j4)}{5+j6} = \frac{12.649\angle 108.43°}{7.81\angle 50.19°} = 1.62\angle 58.24° \ \Omega.$$

Hence $$I_{sc} = 10\angle 30° \times \frac{1.62\angle 58.24°}{5+1.62\angle 58.24°} = 10\angle 30° \times \frac{1.62\angle 58.24°}{6\angle 13.245°} = 2.7\angle 75° \text{ A.}$$

Now removing the source and open circuiting terminals xy, (Z_{int}) is obtained from Fig. 6.34(b).

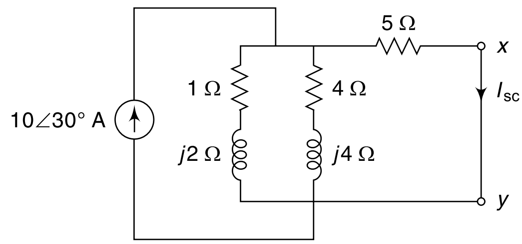

Fig. 6.34(a) ***Determination of (I_{SC})***

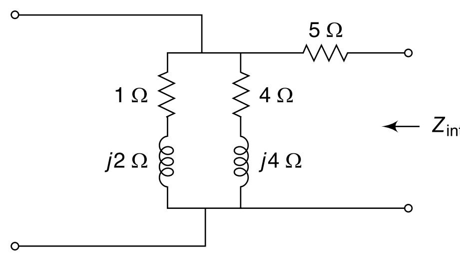

Fig. 6.34(b) ***Determination of (Z_{int})***

Here,
$$Z_{int} = 5 + \frac{(1 + j2)(4 + j4)}{1 + j2 + 4 + j4}$$
$$= 5 + 1.62\angle 58.24°$$
$$= 5.853 + j1.377$$
$$= 6\angle 13.24°\ \Omega.$$

Norton's equivalent circuit is shown in Fig. 6.34(c)

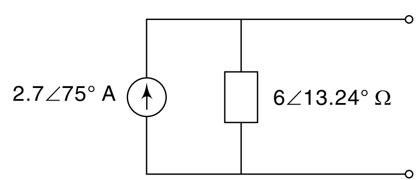

Fig. 6.34(c) ***Norton's equivalent circuit of Ex. 6.33***

6.34 Find Norton's equivalent circuit across terminals *ab* and find current through 10 Ω resistor in Fig. 6.35.

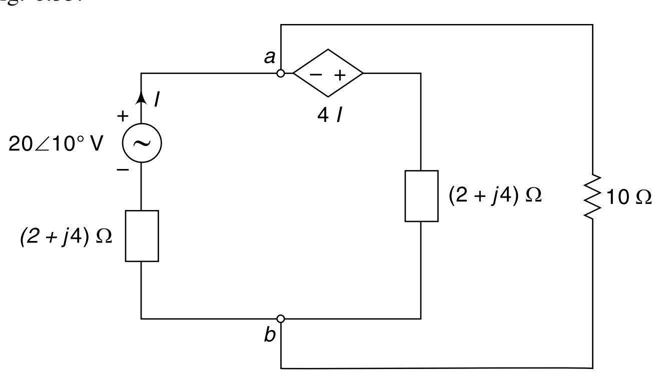

Fig. 6.35 ***Circuit of Ex. 6.34***

Solution

Let us remove 10 Ω resistor and find out the open circuit voltage from Fig. 6.35(a). Applying KVL in the closed loop

$$20\angle 10° + 4I - (2 + j4)I - (2 + j4)I = 0$$
$$j8I = 20\angle 10°$$

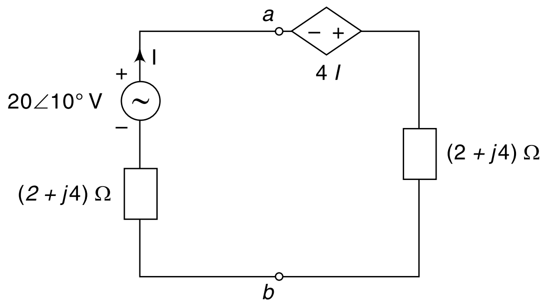

Fig. 6.35(a) *Determination of V_{ab} (V_{oc})*

Hence $\quad I = 2.5\angle{-80°}$ A.

Hence $\quad V_{ab} = 20\angle 10° - (2 + j4)2.5\angle{-80°}$

$= 19.696 + j3.47 - 10.71 + j3.186 = 8.986 + j0.28 = 8.99\angle 1.78°$ V.

Now let us short circuit terminals *ab* as shown in Fig. 6.35(b).

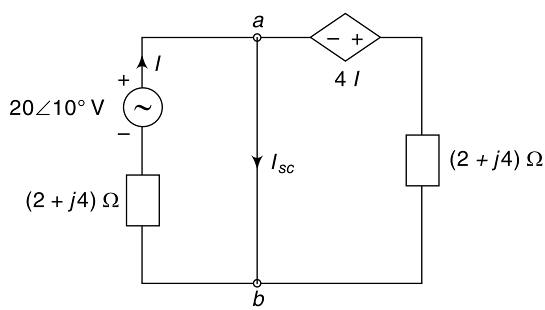

Fig. 6.35(b) *Determination of (I_{SC})*

$$I = \frac{20\angle 10°}{2 + j4} = \frac{20\angle 10°}{4.47\angle 63.43°} = 4.47\angle{-53.43°} \text{ A.}$$

Hence
$$I_{sc} = I - \frac{4I}{2 + j4} = 4.47\angle{-53.43°} - \frac{4 \times 4.47\angle -53.43° \text{ A}}{4.47\angle 63.43°}$$

$$= 4.47\angle{-53.43°} - 4\angle{-116.86°} = 4.47 - j0.02 = 4.47\angle{-0.26°} \text{ A.}$$

Therefore, $Z_{int} = \dfrac{8.99\angle 1.78°}{4.47\angle -0.26°} = 2\angle{-2.04°}\ \Omega.$

Norton's equivalent circuit is shown in Fig. 6.35(c).

Hence current through 10 Ω resistor is given by

$$I_{10\Omega} = 4.47\angle{-0.26°} \times \frac{2\angle{-2.04°}}{10}$$

$$= 0.894\angle 1.78° \text{ A.}$$

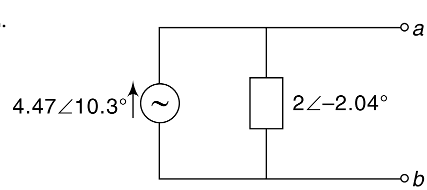

Fig. 6.35(c) *Norton's equivalent circuit of Ex. 6.34*

6.35 In the network of Fig. 6.36 find the value of Z_L so that the power transfer from the source is maximum.

Solution

Let us open circuit the terminals *XY* removing Z_L the voltage source. The corresponding network is shown in Fig. 6.36(a) and Fig. 6.36(b).

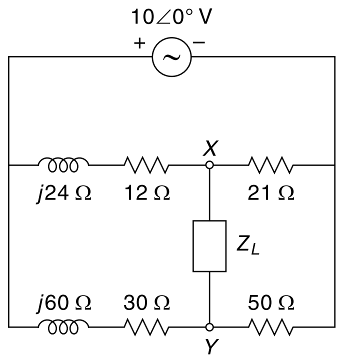

Fig. 6.36 *Circuit of Ex. 6.35*

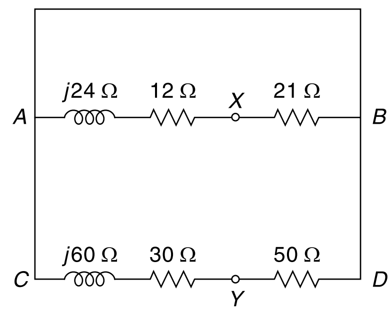

Fig. 6.36(a) *Determination of (Z_{int})*

The internal impedance of the circuit

$$Z_{\text{int}} = \frac{21(12+j24)}{21+j24+12} + \frac{50(30+j60)}{50+30+j60}$$

$$= \frac{252+j504}{33+j24} + \frac{1500+j3000}{80+j60}$$

$$= \frac{563.489\angle 63.43^\circ}{40.8\angle 36^\circ} + \frac{3354\angle 63.43^\circ}{100\angle 36.87^\circ}$$

$$= 13.81\angle 27.43^\circ + 33.54\angle 26.56^\circ$$

$$= (42.258 + j21.36)\ \Omega.$$

According to maximum power transfer theorem (Z_L) should be complex conjugate of Z_{int}.

Hence, $Z_L = (42.258 - j21.36)\ \Omega$.

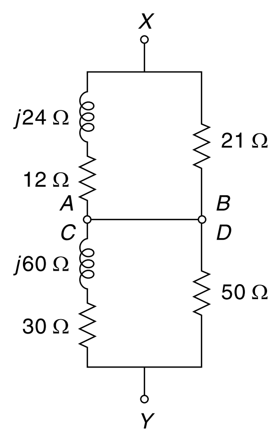

Fig. 6.36(b) *Simplified circuit of Ex. 6.36(a)*

EXERCISES

1. State and prove maximum power transfer theorem when applied to ac circuits.
2. State and explain Thevenin's theorem when applied to ac circuits.
3. State and explain Norton's theorem when applied to ac circuits.
4. State and explain the superposition theorem when applied to ac circuits.
5. Find current I in Fig. 6.37 using superposition theorem.

[*Ans:* $3.33\angle 50^\circ$ A]

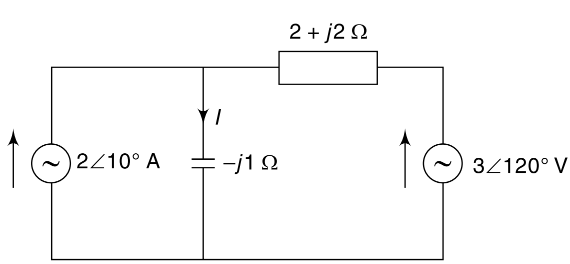

Fig. 6.37

6. Find the power loss in R_L in Fig. 6.38 using superposition theorem. [*Ans:* 35 W]

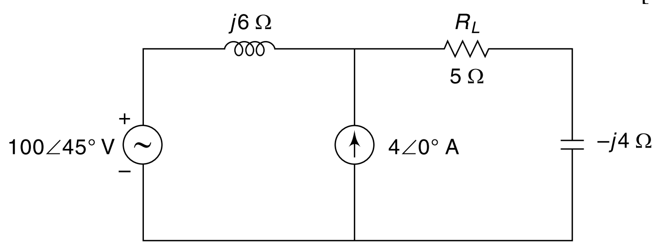

Fig. 6.38

7. Use Thevenin's theorem to find the current in the impedance connected to terminals *x* and *y* of the network shown in Fig. 6.39. [*Ans:* 15.42 A]

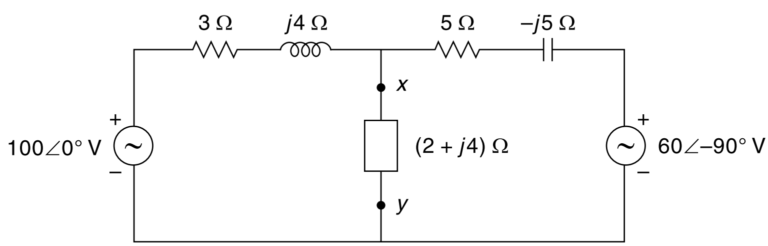

Fig. 6.39

8. Find the amount of maximum power transfer in Fig. E6.40 when Z_L is variable. [*Ans:* 1.81 W]

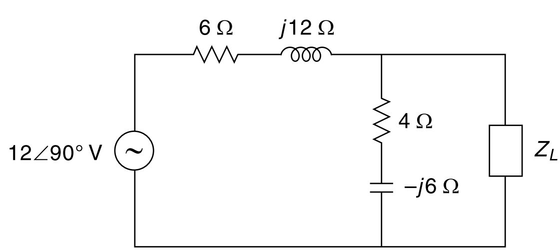

Fig. 6.40

9. In the network of Fig. 6.41 find Norton's equivalent circuit across *AB*. [*Ans:* $I_N = 5.78\angle 20.22°$ A, $Z_N = (2.08 + j1.44)$ Ω]

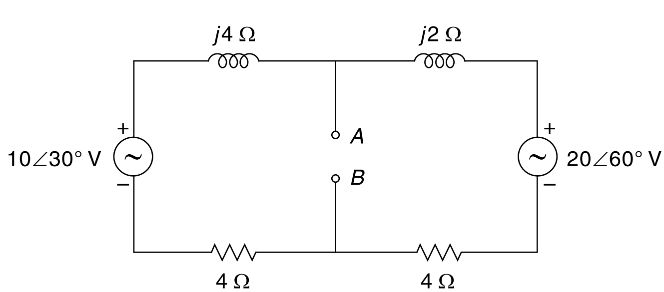

Fig. 6.41

10. Using the superposition theorem find the current through Z_3 in the network shown in Fig. 6.42. [*Ans:* 47.6∠–9.7° mA]

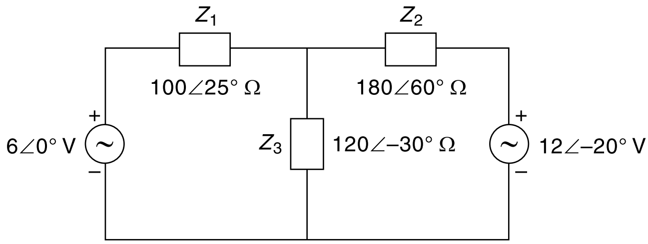

Fig. 6.42

11. Solve example number 6.7 in the text using Thevenin's theorem.
12. A voltage source has an equivalent circuit consisting of R = 500 Ω in series with C = 0.01 μF. Calculate the required component values for a series LR circuit that will draw maximum power from the source when the signal frequency is 1 MHz. [*Ans:* R = 500 Ω, L = 0.00253 mH]
13. Replace the network of Fig. 6.43 at terminals (a–b) with the Norton's equivalent circit. [*Ans:* I_N = 0.439∠105.26° A, Z_N = 8.37∠–69.23° Ω]

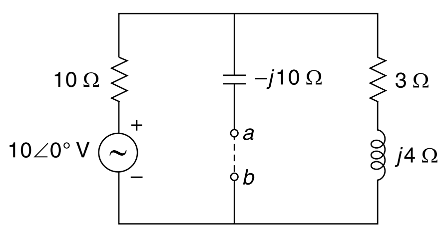

Fig. 6.43

14. Obtain Thevenin's and Norton's equivalent circuits at terminals (a–b) for the network of Fig. 6.44.
[*Ans:* V_{Th} = 11.5∠–95.8° V, I_N = 1.39∠–80.6° A, Z = 8.26∠–15.2° Ω]

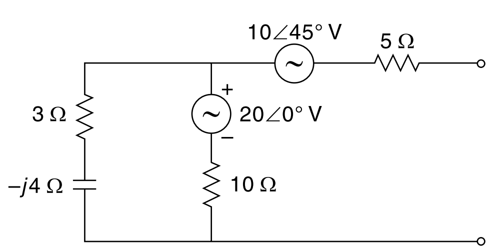

Fig. 6.44

7 THREE-PHASE CIRCUITS

7.1 THREE-PHASE SYSTEM

A three-phase electric system may be considered as three separate single-phase systems displaced from each other by 120° [Fig. 7.1].

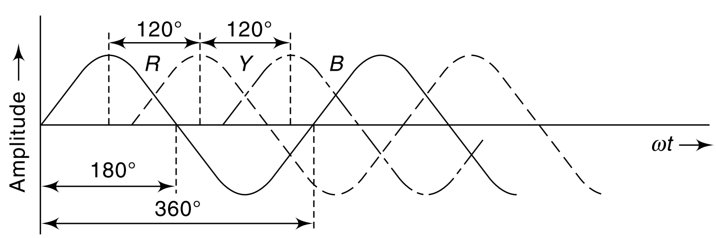

Fig. 7.1 *Graphical representation of a three-phase system*

7.2 ADVANTAGES OF A THREE-PHASE SYSTEM

The advantages of a three-phase system over a single-phase system are:

(a) The amount of conductor material required is less for three-phase system,
(b) domestic power and industrial/commercial power can be provided from the same source,
(c) voltage regulation of a three-phase system is better, and
(d) three-phase motors are self-starting while single-phase motors are not self-starting.

7.3 GENERATION OF A THREE-PHASE SUPPLY

When three identical coils are placed with their axes at 120° displaced from each other and rotated in a uniform magnetic field, a sinusoidal voltage is generated across the coil. Figure 7.2 shows three sets of coils *RR′*, *YY′* and *BB′* displaced from each other by 120° and rotating in an anticlockwise direction with angular velocity ω in a uniform magnetic field. Since the three coils are identical the generated voltages have the same magnitude. The generated voltages in the coils are given by

$$v_R = V_m \sin \omega t$$
$$v_Y = V_m \sin (\omega t - 120°)$$
$$v_B = V_m \sin (\omega t - 240°)$$
$$= V_m \sin (\omega t + 120°).$$

[Here voltage generated in coil R is taken as reference. So v_Y lags v_R by 120° and v_B lags v_R by 240°.]

In polar form

$$v_R = |V| \angle 0°$$
$$v_Y = |V| \angle -120°$$
$$v_B = |V| \angle -240° = |V| \angle 120°.$$

The three phases may be numbered a, b, c or 1, 2, 3 or R, Y and B as customary and they may be given three colours—red, yellow and blue. The phase sequence is usually RYB. Vector rotation is usually anticlockwise.

The voltage waveform is shown in Fig. 7.1 and the phasor diagram is shown in Fig. 7.3. It can be shown that the phasor sum of three-phase emfs is zero.*

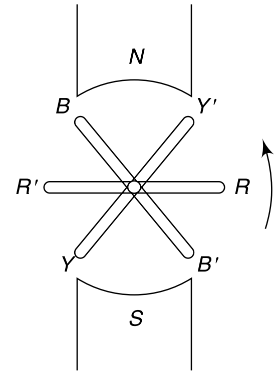

Fig. 7.2 *Three-phase emf generation*

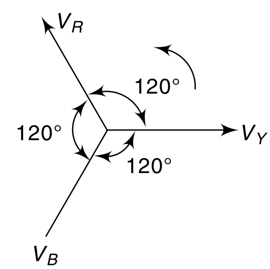

Fig. 7.3 *Vector representation of phase voltages*

7.4 INTERCONNECTION OF PHASES

If the three coils RR', YY' and BB' are not interconnected but kept separate as shown in Fig. 7.4 then each phase would require two conductors and so the total number of conductors would be six. This would make the whole system complicated. Hence the three phases are usually interconnected which results in substantial saving of copper.

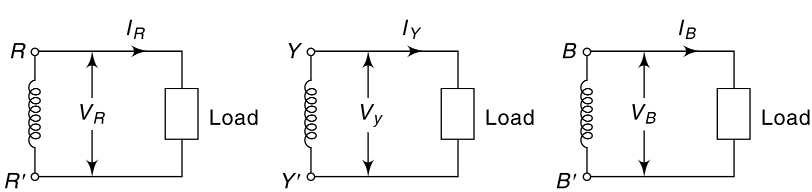

Fig. 7.4 *Three-phase coils not interconnected*

*Resultant instantaneous emf

$$= v_R + v_Y + v_B = V_m \sin \omega t + V_m \sin (\omega t - 120°) + V_m \sin (\omega t - 240°)$$
$$= V_m [\sin \omega t + 2 \sin (\omega t - 180°) \cos 60°]$$
$$= V_m [\sin \omega t - 2 \sin \omega t \times \frac{1}{2}] = 0.$$

The general methods of interconnections are
  (a) *Star (or Y) connection*
  (b) *Mesh or delta (Δ) connection.*

7.4.1 Star (or Y) Connection

Here the similar ends of the coils, i.e. either *a*, *b* and *c* are joined together (or *a′*, *b′* and *c′* are joined together) at point *N* known as "*neutral*" point or *star* point. In Fig. 7.5 a conductor is connected at point *N* which is known as the *neutral conductor*. Such a system is known as a *three-phase four wire* system.

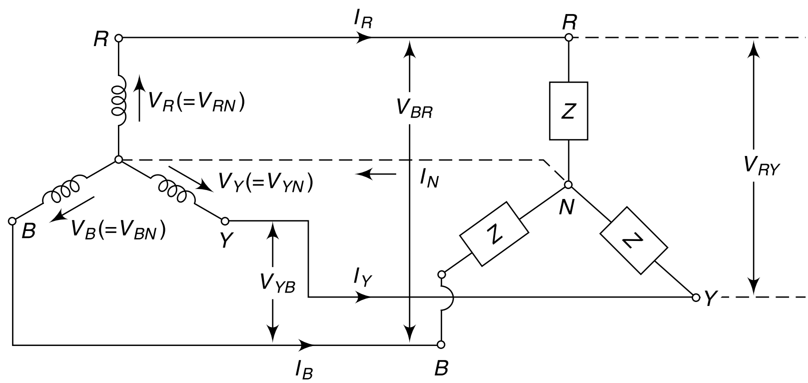

Fig. 7.5 ***Diagrammatic view of star connection***

If a balanced symmetrical load Z is connected across terminals RY, YB and BR then the currents in each phase will be exactly equal in magnitude but displaced 120° from each other (provided the supply voltage is balanced).
The resultant current is then given by

$$i_R + i_Y + i_B = I_m \sin \omega t + I_m \sin (\omega t - 120°) + I_m \sin (\omega t - 240°).$$

The current through the neutral in case of balanced load is zero

i.e. $\quad I_N = I_R + I_Y + I_B = 0.$

The potential difference between any terminal and neutral gives the *phase voltage* and that between any two line terminals, i.e. R, Y, B gives the *line voltage*. In Fig. 7.5, V_R, V_Y and V_B are phase voltages of phases R, Y and B respectively while V_{RY}, V_{YB} and V_{BR} are the line voltages. If these voltages are equal in magnitude and displaced from each other by 120° (elect.) then they are called balanced voltages.

Relation between line and phase voltages in star connection: The potential difference between lines R and Y is

$$V_{RY} = V_R - V_Y \text{ (vector difference).}$$

V_{RY} can be found by compounding V_R and V_Y (reversed). Its value is given by the diagonal of the parallelogram of Fig. 7.6. Obviously, the angle between V_R and V_Y (reversed) is 60°.

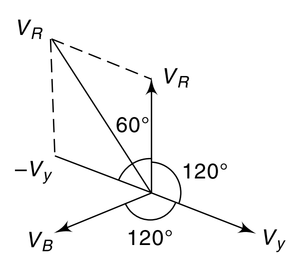

Fig. 7.6 ***Vectorial addition of phase voltages***

Assuming $|V_R| = |V_Y| = |V_B| = |V_{Ph}|$(the phase emf),

$$|V_{RY}| = \sqrt{V_R^2 + V_Y^2 + 2V_R V_Y \cos 60°} = \sqrt{V_{Ph}^2 + V_{Ph}^2 + 2V_{Ph}^2 \times \frac{1}{2}}$$

$$= \sqrt{3}\, V_{Ph} \text{ (i.e., } \sqrt{3} \text{ times magnitude of } V_{Ph}).$$

Similarly, $V_{YB} = V_Y - V_B$ (vector difference)

$= \sqrt{3}\, V_{Ph}$ and

$V_{BR} = V_B - V_R = \sqrt{3}\, V_{Ph}$.

However, $|V_{RY}| = |V_{YB}| = |V_{BR}| =$ Line voltage $|V_L|$.

Hence in star connection

$$|V_L| = \sqrt{3}\, |V_{Ph}|. \qquad (7.1)$$

It may be noted from Fig. 7.7 that

(a) Line voltages are also 120° apart.

(b) Line voltages are 30° ahead of their respective phase voltages.

(c) The angle between the line currents and the corresponding line voltages is (30° + θ) assuming current lagging by an angle θ° (for lagging loads)

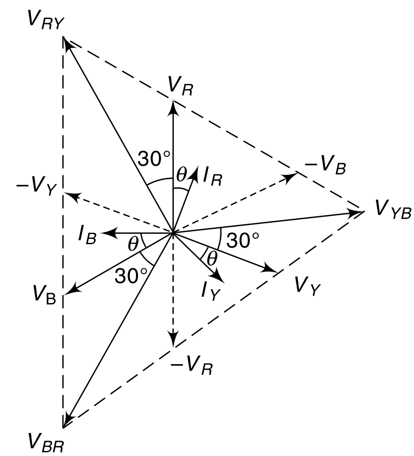

Fig. 7.7 *Complete vector diagram for voltages in star connection*

Relation between line and phase currents in star connection: Observation of Fig. 7.5 reveals that each line is in series with its individual phase winding. Hence the line current in each line is the same as the current in the phase winding to which the line is connected.

Let current in line R be I_R, current in line Y be I_Y and current in line B be I_B.

Since $|I_R| = |I_Y| = |I_B| = |I_{Ph}|$ (say phase current), in star connection, *line current is same as the phase current* i.e. $|I_L| = |I_{Ph}|$.

Power in star connection: The total *active* or *real power P* in the circuit is the sum of the three phase powers. Hence total active power

$$P = 3 \times \text{individual phase power} = 3V_{Ph}I_{Ph} \cos\theta$$

$$= 3 \times \frac{V_L}{\sqrt{3}} I_L \cos\theta$$

$$= \sqrt{3}\, V_L I_L \cos\theta \; [\because V_L = \sqrt{3}\, V_{Ph} \text{ and } I_L = I_{Ph}]. \qquad (7.2)$$

It should be noted that θ is the angle between phase voltage and phase current and V_L, I_L are magnitude vectors.

Similarly, total *reactive power Q* is given by $Q = \sqrt{3}\, V_L I_L \sin\theta$ (7.3)

[By convention reactive power of an inductive coil is taken as positive and that of a capacitor as negative]

S, the *total apparent power* or *complex power* of the three phases is

$$S = \sqrt{P^2 + Q^2} = \sqrt{3}\; V_L I_L. \qquad (7.4)$$

Also, $S = P + jQ$. (7.4(a))

7.4.2 Delta (Δ) or Mesh Connection

In this configuration, the dissimilar ends of three phase windings are joined together, i.e. R is joined with Y', Y with B' and B with R' (or R is joined with B', B with Y' and Y with R'). In other words, the three windings are joined in series to form a closed mesh as shown in Fig. 7.8. The leads are taken out from the three junctions for external connection. If the system is balanced then the sum of the three voltages round the closed mesh is zero, hence no current (of fundamental frequency) can flow around the mesh when the terminals are open. At any instant, the emf of one phase is equal and opposite to the resultant of those in the other two phases. This type of connection is referred to as a three-phase, three-wire delta connection.

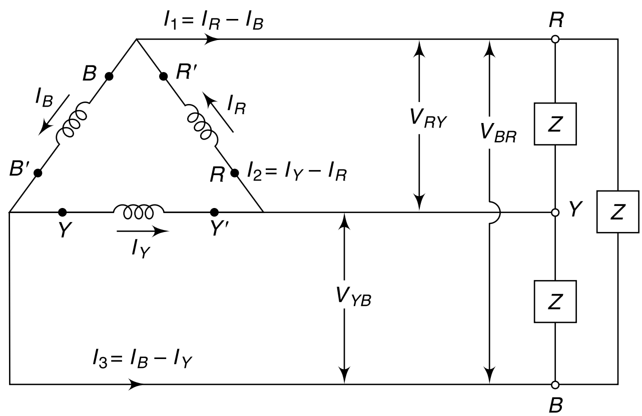

Fig. 7.8 *Delta connection*

Relation between Line Voltages and Phase Voltages in Delta Connection It is seen from Fig. 7.8 that there is only one phase winding completely included between any pair of terminals. Hence in Δ connection, the voltage between any pair of lines is equal to the corresponding phase voltage. Since the common phase sequence is RYB, V_{RY} leads V_{YB} by 120°, V_{YB} leads V_{BR} by 120° (as shown in Fig. 7.9).

If $|V_{RY}| = |V_{YB}| = |V_{BR}|$ = line voltage $|V_L|$, then it is seen that $|V_L| = |V_{\text{Ph}}|$.

Relation between line currents and phase currents in delta connection It is seen from Fig. 7.8 that current in each line is the vector difference of the two-phase currents flowing through that line (i.e. vector difference of corresponding phase currents).

Hence,
$$\left.\begin{aligned}&\text{Current in line } R \text{ is } I_1 = I_R - I_B\\&\text{Current in line } Y \text{ is } I_2 = I_Y - I_R\\&\text{Current in line } B \text{ is } I_3 = I_B - I_Y\end{aligned}\right\}\quad \text{vector difference}$$

Current in line R is found by compounding I_R and I_B (reversed) and its value is given by the diagonal of the parallelogram shown in Fig. 7.9. The angle between I_R and I_B (reversed) is 60°.

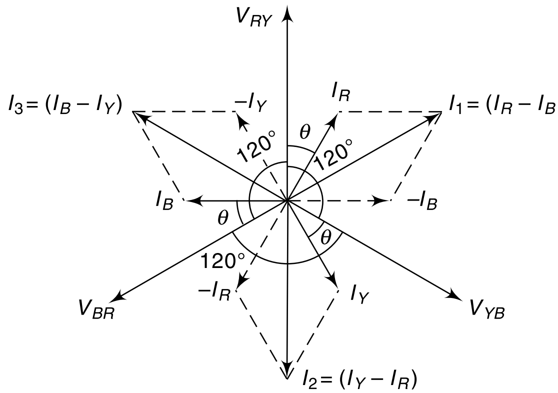

Fig. 7.9 ***Vector resolution of currents***

The current in line R is: $I_1 = \sqrt{I_R^2 + I_B^2 + 2\,I_R\,I_B\,\cos 60°}$

$$= \sqrt{I_{\text{Ph}}^2 + I_{\text{Ph}}^2 + 2\,I_{\text{Ph}}^2 \times \frac{1}{2}}$$

$$= \sqrt{3}\,I_{\text{Ph}} \text{ (i.e., } \sqrt{3} \text{ times magnitude of phase current).}$$

Current in line Y is: $I_2 = I_Y - I_R$ (vector difference) = $\sqrt{3}\,I_{\text{Ph}}$ and current in line B is: $I_3 = I_B - I_Y$ (vector difference) = $\sqrt{3}\,I_{\text{Ph}}$.

Assuming all the line currents are equal in magnitude,

$$[|I_1| = |I_2| = |I_3| = |I_L|]\; I_L = \sqrt{3}\,I_{\text{Ph}}. \tag{7.5}$$

With reference to Fig. 7.9 it should be noted that:

(a) line currents are 120° apart
(b) line currents are 30° behind the respective phase currents
(c) the angle between the line currents and corresponding line voltages is (30° + θ) with the current lagging by an angle θ.

Power in Delta Connection

Three-phase power

$$P = 3 \times \text{individual phase powers}$$
$$= 3\,V_{\text{Ph}}I_{\text{Ph}} \cos\,\theta$$
$$= 3V_L \frac{I_L}{\sqrt{3}} \cos\,\theta = \sqrt{3}\,V_L I_L \cos\,\theta \tag{7.6}$$

$$(\because \quad V_{\text{Ph}} = V_L \text{ and } I_L = \sqrt{3}\,I_{\text{Ph}})$$

[Here, V_{Ph}, I_{Ph}, V_L and I_L are magnitude of the respective phasors.]

Similarly, the total reactive power is given by

$$Q = \sqrt{3}\;V_L I_L \sin\,\theta \tag{7.7}$$

and the total apparent or complex power of the 3-phase delta circuit is given by

$$S = \sqrt{P^2 + Q^2} = \sqrt{3}\;V_L I_L \tag{7.8}$$

Also, $S = P + jQ$ [7.8(a)]

7.5 ONE LINE EQUIVALENT CIRCUIT FOR BALANCED LOADS

We know that a set of three equal impedances in delta connection is equivalent to another set of three equal star connected impedances. Therefore, a more direct computation of the star circuit is possible for balanced three phase loads of either type.

The one line equivalent circuit is one phase of the three phase, four wire, star connected circuit in Fig. 7.10 except that a voltage is used which has the line to neutral magnitude and a zero phase angle. The line current calculated for this circuit has a phase angle with respect to the phase angle of zero on the voltage. Then the actual line currents I_R, I_Y and I_B will lead or lag their respective line to neutral voltages by this same phase angle.

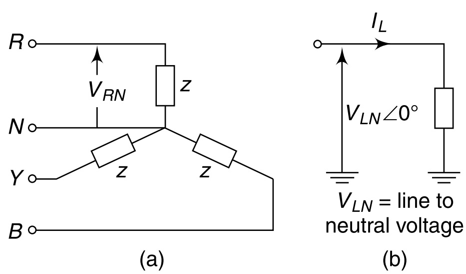

Fig. 7.10 *One line equivalent of one phase*

7.1 Three chokes each of resistance 40 Ω and reactance 30 Ω are connected in star to a 3-phase 440 V balanced supply. What is the line current and the total power dissipated?

Solution

Given, Resistance $R = 40\ \Omega$

Reactance $X = 30\ \Omega$.

$\therefore$ impedance $Z = \sqrt{R^2 + X^2} = \sqrt{(40)^2 + (30)^2} = 50\ \Omega$

Also, line voltage $V_L = 440$ V.

$\therefore$ In a star connected system

Phase voltage $|V_{Ph}| = \dfrac{|V_L|}{\sqrt{3}} = \dfrac{440}{\sqrt{3}} = 254.04$ V:

phase current $|I_{Ph}| = \dfrac{|V_{Ph}|}{|Z|} = \dfrac{254.04}{50} = 5.08$ A.

In a star connected system

Line current = Phase current = 5.08 A.

Total power dissipated is ($\sqrt{3}\ |V_L|\ |I_L| \cos\theta$), where

$$\cos\theta = \text{power factor} = \frac{|R|}{|Z|} = \frac{40}{50} = 0.8.$$

Hence total power dissipated is ($\sqrt{3} \times 440 \times 5.08 \times 0.8$) = 3097.18 W or 3.097 kW.

7.2 The load in each branch of a star connected three-phase circuit consists of 10 Ω resistance and 0.06 H inductance in series. The line voltage is 430 V. Calculate the phase voltage and the phase current.

Solution

Resistance $R = 10\ \Omega$

Reactance $X = \omega L = 2\pi f L = 2\pi \times 50 \times .06 = 18.85\ \Omega$

∴ Impedance $|Z| = \sqrt{R^2 + X^2} = \sqrt{(10)^2 + (18.85)^2} = 21.34\ \Omega$

Line voltage $|V_L| = 430$ V (given).

In a star connected system, line voltage = $\sqrt{3}$ × phase voltage

So phase voltage $|V_{Ph}| = \dfrac{|V_L|}{\sqrt{3}} = \dfrac{430}{\sqrt{3}} = 248.27$ V

Phase current $|I_{Ph}| = \dfrac{|V_{Ph}|}{Z} = \dfrac{248.27}{21.34} = 11.63$ A.

7.3 Three similar coils each having series resistance of 20 Ω and capacitance 100 μF are connected in star to a 3-phase, 400 V, 50 Hz balanced supply. Find the line current, power factor, total KVA and total kW.

Solution

Resistance $R = 20\ \Omega$

Capacitance $C = 100 \times 10^{-6}$ F.

∴ Capacitive reactance $|X_C| = \dfrac{1}{\omega C} = \dfrac{1}{2\pi \times 50 \times 100 \times 10^{-6}} = \dfrac{100}{\pi}\ \Omega$

$= 31.83\ \Omega.$

Impedance $|Z| = \sqrt{R^2 + X_c^2}$

$= \sqrt{(20)^2 + (31.82)^2} = 37.59\ \Omega.$

Line voltage $|V_L| = 400$ V (given)

∴ Phase voltage is $|V_{Ph}| = \dfrac{|V_L|}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.95$ V,

while, phase current $|I_{Ph}| = \dfrac{|V_{Ph}|}{Z} = \dfrac{230.95}{37.59} = 6.144$ A.

∴ Line current $|I_L| = |I_{Ph}| = 6.144$ A

Power factor $(\cos\theta) = \dfrac{R}{|Z|} = \dfrac{20}{37.59} = 0.53$

Total power $(\text{KVA}) = 3|V_{Ph}|\,|I_{Ph}| = (3 \times 230.95 \times 6.144 \times 10^{-3})$ KVA

$= 4.257$ KVA

Total active power $(\text{kW}) = 3|V_{Ph}|\,|I_{Ph}| \cos\theta$

$= (3 \times 230.95 \times 6.144 \times 0.53 \times 10^{-3})$ kW $= 2.256$ kW.

7.4 Each phase of a delta connected load comprises a resistor of 50 Ω and a series capacitor of 50 μF. Calculate the line and phase currents when the load is connected to a 440 V, 3 phase, 50 Hz supply.

Solution

Load resistance $R = 50\ \Omega$

Capacitive reactance $|X_C| = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fc} = \dfrac{1}{2\pi \times 50 \times 50 \times 10^{-6}}$

$= 63.67\ \Omega.$

$\therefore$ Load impedance per phase $|Z| = \sqrt{R^2 + X_C^2} = 80.96\ \Omega.$

Phase current $|I_{Ph}| = \dfrac{|V_{Ph}|}{|Z|}$, where V_{Ph} is the phase voltage.

In delta connected system line voltage $|V_L|$ = phase voltage $|V_{Ph}|$

Here, $|V_{Ph}| = |V_L| = 440$ V

Therefore, $|I_{Ph}| = \dfrac{440}{80.96} = 5.434$ A.

In delta connected system line current $|I_L| = \sqrt{3}\, I_{Ph}$

$\therefore$ $|I_L| = \sqrt{3} \times 5.434 = 9.412$ A.

7.5 The load in each branch of a delta connected balanced three-phase circuit consists of an inductance of 0.0318 H in series with a resistance of 10 Ω. The line voltage is 400 V (balanced) at 50 Hz. Calculate the (i) line current and (ii) the total power in the circuit.

Solution

Inductive reactance $|X_L| = \omega L = 2\pi f L = 2\pi \times 50 \times 0.0318 = 10\ \Omega$

Resistance $R = 10\ \Omega.$

So load impedance, $|Z| = \sqrt{(10)^2 + (10)^2} = 14.14\ \Omega.$

Line voltage, $|V_L| = 400$ V (given)

$\therefore$ Phase voltage $|V_{Ph}| = |V_L| = 400$ V [$\because$ connection is delta]

Phase current $|I_{Ph}| = \dfrac{|V_{Ph}|}{|Z|} = \dfrac{400}{14.14}$ A = 28.288 A.

(i) Line current $|I_L| = \sqrt{3}$ × phase current = $\sqrt{3} \times 28.29 = 49$ A.

(ii) Total power in the circuit $(P) = \sqrt{3}\, |V_L|\, |I_L| \cos\theta$

$\therefore$ $\cos\theta$ = power factor = $\dfrac{R}{|Z|} = \dfrac{10}{14.14} = 0.707.$

Also, total power in the circuit

$(P) = \sqrt{3} \times 400 \times 49 \times 0.707 = 24{,}000.37\ \text{W} \simeq 24\ \text{kW}.$

7.6 A star connected three-phase load draws a current of 20 A at a lagging p.f. of 0.9 from a balanced 440 V, 50 Hz supply. Find the circuit elements in each phase if the elements are connected in series.

Solution

Line current, $|I_L| = 20$ A

Power factor, $\cos\theta = 0.9$ (lagging)

Line voltage = $|V_L| = 440$ V (given).

$\therefore$ Phase voltage, $|V_{Ph}| = \dfrac{|V_L|}{\sqrt{3}} = \dfrac{440}{\sqrt{3}}$ V .

As the power factor is lagging hence the circuit contains resistance R in series with inductance L.

If X_L be the inductive reactance and Z be the impedance then, $\dfrac{R}{|Z|} = 0.9$

and $$|Z| = \frac{|V_{Ph}|}{|I_{Ph}|} = \frac{\frac{440}{\sqrt{3}}}{|I_L|} = \frac{440}{\sqrt{3} \times 20} \ \Omega = 12.7 \ \Omega$$

($|I_{Ph}| = |I_L|$ as the load is star connected).

$\therefore$ $$R = 12.7 \times 0.9 = 11.43 \ \Omega$$

and $$|X_L| = \sqrt{Z^2 - R^2} = \sqrt{(12.7)^2 - (11.43)^2} = 5.536 \ \Omega$$

Therefore, $$L = \frac{X_L}{\omega} = \frac{5.536}{2\pi f} = \frac{5.536}{2\pi \times 50} = 0.0176 \text{ H.}$$

The circuit elements are resistance of 11.43 Ω and inductor of 0.0176 H connected in series in each phase.

7.7 Three coils are connected in star to a balanced three-phase, 3-wire, 440 V, 50 Hz supply and takes a line current of 10 A at 0.9 p.f. lagging. Calculate the resistance and reactance of the coils assuming they are series connected in each phase. If the coils are delta connected to the same supply calculate the line current and the real power.

Solution

Line voltage $|V_L| = 440$ V

Line current $|I_L| = 10$ A

When the coils are star connected,

phase voltage $$|V_{Ph}| = \frac{|V_L|}{\sqrt{3}} = \frac{440}{\sqrt{3}} = 254 \text{ V},$$

phase current $|I_{Ph}| = |I_L| = 10$ A

If R, X and Z be the resistance, reactance and impedance respectively then,

$$|Z| = \frac{|V_{Ph}|}{|I_{Ph}|} = \frac{254}{10} = 25.4 \ \Omega$$

$$\text{p.f.} = \cos\theta = 0.9 = \frac{|R|}{|Z|}.$$

So $$R = 25.4 \times 0.9 = 22.86 \ \Omega.$$

Therefore, $$|X| = \sqrt{Z^2 - R^2} = \sqrt{(25.4)^2 - (22.8)^2}$$
$$= 11.19 \ \Omega.$$

The resistance and reactance of the coil are 22.86 Ω and 11.9 Ω respectively.

Line voltage of supply system $|V_{L_1}| = \sqrt{3}\,|V_{Ph_1}|$

$= \sqrt{3} \times 230$ V

$= 398.36$ V (as it is star connected).

Line voltage of load $|V_{L_2}| = |V_{L_1}| = 398.36$ V

and phase voltage of load $|V_{Ph_2}| = |V_{L_2}| = 398.36$ V (as load is delta connected).

Hence phase current in load is given by $$|I_{Ph_2}| = \frac{|V_{Ph_2}|}{Z}$$

$$= \frac{398.36}{10} = 39.8 \text{ A.}$$

Line current of load is obtained as $|L_{L_2}| = \sqrt{3}\,|I_{Ph_2}|$

$= \sqrt{3} \times 39.836 = 69$ A.

Now, when the coils are delta connected to the same supply

$$|V_L| = 440 \text{ V}, |Z| = 25.4\ \Omega \text{ and } (\cos\theta) = 0.9.$$

$$\text{Phase voltage } |V_{\text{Ph}}| = |V_L| = 440 \text{ V}$$

$$\text{Phase current } |I_{\text{Ph}}| = \frac{|V_{\text{Ph}}|}{|Z|} = \frac{440}{25.4} \text{ A} = 17.32 \text{ A}.$$

$$\text{Line current } |I_L| = \sqrt{3}\,|I_{\text{Ph}}| = \sqrt{3} \times 17.32 = 30 \text{ A}.$$

$$\text{Real power } (P) = \sqrt{3}\,|V_L|\,|I_L| \cos\theta$$

$$= \sqrt{3} \times 440 \times 30 \times 0.9 = 20576.32 \text{ W} = 20.577 \text{ kW}.$$

7.8 For a three-phase delta connected load, each phase of which has series resistance of 8 Ω and reactance of 6 Ω is supplied from a star connected balanced system having a phase voltage of 230 V. Calculate (a) magnitude of the phase current of the load and the supply system (b) active power taken by the load and the power factor of the load.

Solution

Impedance of load $|Z| = \sqrt{8^2 + 6^2} = 10\ \Omega$.

(The system is shown in Fig. 7.11).

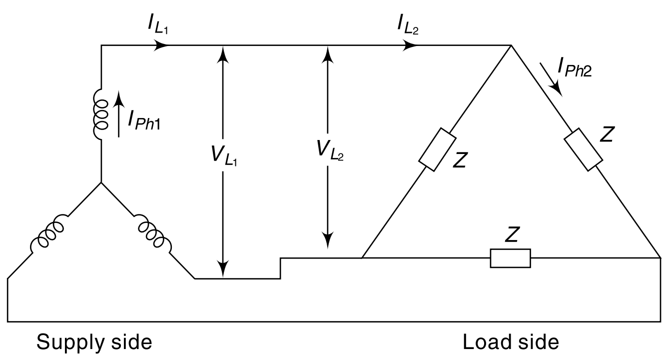

Fig. 7.11 ***Circuit of Ex. 7.8 [Z = (8 + j6) Ω]***

Phase voltage of supply system $|V_{\text{Ph}_1}| = 230$ V.

(a) The line current of the supply system $|I_{L_1}| = |I_{L_2}| = \left(\dfrac{\sqrt{3} \times 230}{\sqrt{8^2 + 6^2}} \times \sqrt{3}\right) = 69\text{A}.$

(b) Power factor of the load is $\cos\theta = \dfrac{R}{|Z|} = \dfrac{8}{10} = 0.8$

$$\text{Active power taken by the load } = \sqrt{3}\,|V_{L_2}|\,|I_{L_2}| \cos\theta$$

$$= \sqrt{3} \times 398.36 \times 69 \times 0.8 \text{ W}$$

$$= 38086.88 \text{ W} = 38.08 \text{ kW}.$$

7.9 A star connected load has an impedance of (3 + *j*4) Ω in each phase and is connected across a balanced 3-phase delta connected alternator having line voltage of 120 V. Obtain the line currents of both the load and generator.

Solution

For the load, $Z_{\text{Ph}} = 3 + j4 = 5\angle 53.13°\ \Omega.$

Magnitude of phase voltage $|V_{Ph}| = \dfrac{120}{\sqrt{3}} = 69.28$ V.

The line currents (as the load is star connected hence line and phase currents are equal) in phase *R*, *Y* and *B* of the load are

$$I_R = \frac{69.28}{5\angle 53.13°}\angle 0° = 13.86\angle -53.13° \text{ A}$$

$$I_Y = \frac{69.28}{5\angle 53.13°}\angle -120° = 13.86\angle -173.13° \text{ A}$$

and $$I_B = \frac{69.28}{5\angle 53.13°}\angle -240° = 13.86\angle 66.87° \text{A}$$

As the star connected load is connected with the delta connected alternator hence the line current of the alternator is same as that of the line current of the load, i.e. $13.86\angle -53.13°$ A, $13.86\angle -173.13°$ A and $13.86\angle 66.87°$ A.

7.10 A balanced star connected load has an impedance of $(2 + j3.46)\ \Omega$ between line and neutral. If the voltage across phase *R* be $20\angle 30°$ volts, find current in phases *Y* and *B*. What is the voltage from line *Y* to neutral. Also obtain V_{RB}.

Solution

$$V_{RN} = 20\angle 30° \text{ V}$$

$$Z_{ph} = 2 + j3.46 = 4\angle 60°\ \Omega.$$

Hence $$I_{RN} = \frac{20\angle 30°}{4\angle 60°} \text{ A} = 5\angle -30° \text{ A}.$$

If current in phase *R* is $5\angle -30°$ A, currents in phases *Y* and *B* will be lagging by 120° and 240° respectively with respect to current in phase *R*.

Hence current in phase $Y = 5\angle(-30° - 120°)$ A $= 5\angle -150°$ A

And current in phase $B = 5\angle(-30° - 240°) = 5\angle -270° = 5\angle 90°$ A.

Also, voltage across phase *B* is lagging with respect to phase *R* by 240°.

Hence $V_{BN} = 20\angle(30° - 240°) = 20\angle -210° = 20\angle 150°$ V.

Therefore, $$\begin{aligned} V_{RB} &= V_{RN} - V_{BN} = 20\angle 30° - 20\angle 150° \\ &= 20\{0.866 + j0.5 + 0.866 - j0.5\} \\ &= 34.64\angle 0° \text{ V.} \end{aligned}$$

7.6 MEASUREMENT OF POWER IN A THREE-PHASE THREE-WIRE SYSTEM

Case (a): Star connected balanced load (with neutral point accessible). If a wattmeter *W* be connected with its current coil in one line and the voltage coil between that line and the neutral point, as shown in Fig. 7.12, the reading on the wattmeter gives the power per phase. Therefore, total active power = 3 × wattmeter reading.

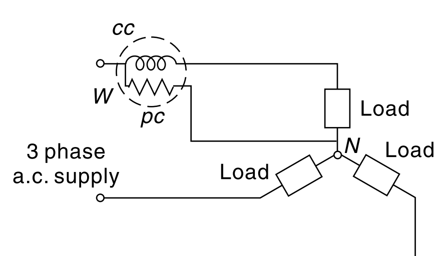

Fig. 7.12 *Measurement of power in a star connected balanced load*

Case (b): Balanced or unbalanced load (star or delta) *The two wattmeter* *method*. Suppose the loads Z_1, Z_2 and Z_3 are connected in star as shown in Fig. 7.13. Current coils of the two wattmeters are connected in any two lines, say the 'red' and 'blue' lines, and the voltage circuits are connected between these lines and the yellow phase. Suppose v_{RN}, v_{YN} and v_{BN} be the instantaneous values of the potential differences across the loads, these p.d.s are assumed positive. Also suppose I_R, I_Y and I_B to be the corresponding instantaneous values of the line (and phase) currents.

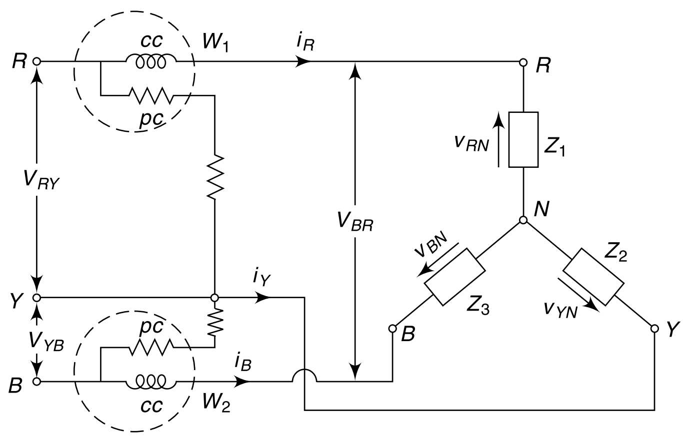

Fig. 7.13 ***Measurement of three-phase power by two wattmeters***

Therefore, instantaneous power in load $Z_1 = i_R \cdot v_{RN}$,

instantaneous power in load $Z_2 = i_Y \cdot v_{YN}$

instantaneous power in load $Z_3 = i_B \cdot v_{BN}$

Total instantaneous power $= i_R \cdot v_{RN} + i_Y \cdot v_{YN} + i_B \cdot v_{BN}$. (7.9)

From Fig. 7.13 it is seen that:

Instantaneous current through current coil of $W_1 = i_R$.

Instantaneous p.d. across voltage coil of $W_1 = v_{RN} - v_{YN}$.

Hence instantaneous power measured by W_1 is $i_R(v_{RN} - v_{YN})$.

Similarly, instantaneous power measured by W_2 is $i_B(v_{RN} - v_{YN})$.

Sum of the instantaneous powers of W_1 and W_2 being W,

$$\begin{aligned} W &= i_R(v_{RN} - v_{YN}) + i_B(v_{RN} - v_{YN}) \\ &= i_R \cdot v_{RN} + i_B \cdot v_{BN} - (i_R + i_B)v_{YN} \end{aligned} \tag{7.10}$$

From Kirchhoff's first law, the algebraic sum of the instantaneous current at N being zero,

we have $i_R + i_Y + i_B = 0$ or $i_R + i_B = -i_Y$

Therefore, sum of the instantaneous powers of W_1 and W_2 becomes

$$\begin{aligned} W &= i_R \cdot v_{RN} + i_B \cdot v_{RN} + i_Y \cdot v_{YN} \\ &= \text{total instantaneous power.} \end{aligned} \tag{7.11}$$

Actually, the power measured by each wattmeter varies from instant to instant, but the inertia of the moving system causes the pointer to read the average value of the power. Hence the sum of the wattmeter readings gives the average value of the total power absorbed by the three phases.

Since the above proof does not assume a balanced load or sinusoidal waveforms, it follows that the sum of the two wattmeter readings gives the total power under all conditions. The above proof was derived for a star-connected load and it is a useful exercise to prove that the same conclusion holds for a delta connected load.

7.7 MEASUREMENT OF POWER FOR A THREE-PHASE SYSTEM USING TWO WATTMETERS (ASSUMING BALANCED LOAD AND SINUSOIDAL VOLTAGES AND CURRENTS)

Suppose Z in Fig. 7.13 represents three similar loads connected in star and let V_{RN}, V_{YN} and V_{BN} be the rms values of the voltages per phase. Since the voltages and currents are assumed sinusoidal, they can be represented by phasors as shown in Fig. 7.14, while the currents being assumed to lag by an angle θ behind the corresponding phase voltages.

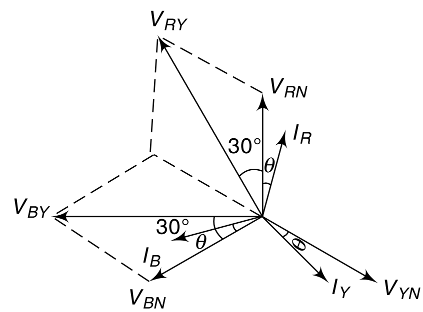

Fig. 7.14 *Phasor diagram for Fig. 7.13*

Current through current coil of W_1 is I_R and p.d. across voltage coil of W_1 is $V_{RN} - V_{YN} = V_{RY}$ (line voltage).
Phase difference between I_R and V_{RY} is $(30° + \theta)$.
Therefore, reading on $W_1 = I_R V_{RY} \cos(30° + \theta)$.
Current through current coil of W_2 is I_B.
P.d. across voltage coil of W_2 is $V_{BN} - V_{YN} = V_{BY}$.
Phase difference between I_B and $V_{BY} = (30° - \theta)$.
Therefore reading on W_2 is $I_B V_{BY} \cos(30° - \theta)$.
Since the load is balanced,

$$I_R = I_Y = I_B = I_L \text{ (numerically)}$$

and $$V_{RY} = V_{BY} = V_L \text{ (numerically)}$$

hence $$P_1 = I_L V_L \cos(30° + \theta) = W_1 \tag{7.12}$$

and $$P_2 = I_L V_L \cos(30° - \theta) = W_2 \tag{7.13}$$

$$\begin{aligned} P_1 + P_2 &= I_L V_L [\cos(30° + \theta) + \cos(30° - \theta)] \\ &= I_L V_L [\cos 30° \cos\theta - \sin 30° \sin\theta \\ &\qquad + \cos 30° \cos\theta + \sin 30° \sin\theta] \\ &= \sqrt{3}\, I_L V_L \cos\theta = W_1 + W_2. \end{aligned} \tag{7.14}$$

This is the expression deduced for the total power in a balanced 3-phase system. This is an alternative method of proving that the sum of the two wattmeter readings gives the total power, but it should be noted that this proof assumes a balanced load and sinusoidal voltages and currents.
From Eqs (7.12), (7.13) and (7.14), $P_2 - P_1 = I_L V_L \sin\theta$ and

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \sqrt{3}\,\frac{P_2 - P_1}{P_2 + P_2} = \sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1}. \tag{7.15}$$

From the above it may be noted that if $\theta = 0°$, $W_1 = W_2$; for $\theta < 60°$, both W_1 and W_2 are positive. On the other hand, if $\theta = 30°$, $W_2 = 0$; for $\theta > 60°$, W_1 becomes negative. In such a case, the connection of either the current coil on the voltage coil are made reverse and then the reading is recorded. Under this condition the sign of W_1 is taken as negative in the calculations.

7.8 UNBALANCED FOUR-WIRE STAR CONNECTED LOAD

With such a system the neutral conductor will carry current; also the voltage across each of the load impedances remains fixed with the same magnitude as the line to neutral voltage. The line currents are unequal and do not have a 120° phase difference.

7.9 UNBALANCED DELTA CONNECTED LOAD

If the three (line) voltages across the terminals of an unbalanced delta connected load are fixed, the voltage drop across each phase impedance is known. The currents in each phase can, therefore, be determined directly. The line currents can then be found from the phasor sum of the two component currents coming towards or flowing away from the line terminal. It will be noted that in this case (unlike with the balanced loads) the line currents will not be equal nor will they have a 120° phase difference.

7.10 UNBALANCED THREE-WIRE STAR CONNECTED LOAD

With such a system the common point O in Fig. 7.15 of the three load impedances will not be at the potential of the neutral and the voltages across the three impedances can vary considerably from line to neutral magnitude. (The displacement of O from N is known as the *displacement neutral voltage*).

There can be two approaches to the problem. The first is to determine with reference to Fig. 7.15 the three line currents I_A, I_B and I_C by writing the mesh currents (based on KVL) and then determining the voltages across the three impedances V_{AO}, V_{BO} and V_{CO} as the product of line current and the corresponding impedances (i.e. $V_{AO} = I_A Z$ etc.). The displacement neutral voltage V_{ON}, O being the common point of the impedances and N the neutral in Fig. 7.16, is then $V_{ON} = V_{OA} + V_{AN} = (V_{OB} + V_{BN} = V_{OC} + V_{CN})$, where V_{OA}, V_{OB} and V_{OC} have all been determined already and V_{AN}, V_{BN}, V_{CN} are known system phase voltages.

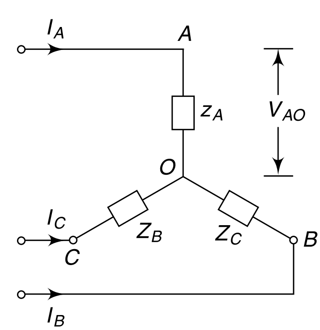

Fig. 7.15 ***Unbalanced three-wire star connected load***

The second is to obtain the displacement neutral voltage in the very beginning. For this we write line currents in terms of the load voltages and load admittances. Referring to Fig. 7.16 again,

$$I_A = V_{AO}Y_A,\ I_B = V_{BO}Y_B,\ I_C = V_{CO}Y_C \quad (7.16)$$

Applying Kirchhoff's current law at point O in Fig. 7.15 we may write

$$I_A + I_B + I_C = 0 \quad (7.17)$$

or $$V_{AO}Y_A + V_{BO}Y_B + V_{CO}Y_C = 0 \quad (7.18)$$

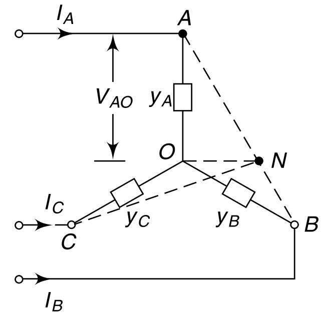

Fig. 7.16 ***Unbalanced three-wire star connected load***

Referring to Fig. 7.16 the voltages V_{AO}, V_{BO} and V_{CO} can be expressed in terms of their two component voltages, i.e.

$$\left.\begin{aligned} V_{AO} &= V_{AN} + V_{NO} \\ V_{BO} &= V_{BN} + V_{NO} \\ V_{CO} &= V_{CN} + V_{NO} \end{aligned}\right\} \text{(all phasor additions)} \quad (7.19)$$

Substituting the expression of (7.19) in (7.18) we obtain

$$(V_{AN} + V_{NO})Y_A + (V_{BN} + V_{NO})Y_B + (V_{CN} + V_{NO})Y_C = 0 \quad (7.20)$$

from which, $$V_{ON} = \frac{V_{AN}\,Y_A + Y_{BN}\,Y_B + V_{CN}\,Y_C}{Y_A + Y_B + Y_C}. \quad (7.21)$$

The voltages V_{AN}, V_{BN} and V_{CN} in Eq. 7.21 are known voltages in a given problem and Y_A, Y_B and Y_C are also known as they are reciprocals of Z_A, Z_B and Z_C. Hence displacement neutral voltage may be computed.

The voltage across the impedances may then be easily obtained (e.g. $V_{AO} = V_{AN} + V_{NO}$, etc.). Finally the line currents may be obtained as product of voltage across admittance and the corresponding admittance (e.g. $I_A = V_{AO}Y_A$ etc.).

7.11 The phase voltage and current of a star connected load is 100 V and 10 A. The power factor of the load is 0.8 (lag). Assuming that the system is 3 wire, 3 phase and power is measured by two wattmeters, find the readings of the wattmeters.

Solution

Phase voltage $|V_{Ph}| = 100$ V.
Phase current $|I_{Ph}| = 10$ A.
As the load is star connected so,

line voltage $|V_L| = \sqrt{3}\,|V_{Ph}| = 173.2$ volts and
line current $|I_L| = |I_{Ph}| = 10$ A.
$\cos\theta$ (Power factor) = 0.8 (lag)
Power factor angle $\theta = \cos^{-1}0.8 = 36.87°$ (lag).
Reading of one wattmeter

$$W_1 = |V_L|\,|I_L| \cos(30° + \theta)$$
$$= 173.2 \times 10 \cos(30° + 36.87°) = 680.36 \text{ W}.$$

Reading of the other wattmeter

$$W_2 = |V_L|\,|I_L| \cos(30° - \theta)$$
$$= 173.2 \times 10 \cos(30° - 36.87°) = 1719.56 \text{ W}.$$

7.12 A three-phase 230 V load has a power factor of 0.7. Two wattmeters are connected to measure the power which shows the input to be 10 kW. Find the reading of each wattmeter.

Solution

Line voltage $|V_L| = 230$ V

Power factor angle $\theta = \cos^{-1}0.7 = 45.57°$

Input power = total power = 10 kW = 10,000 W

If W_1 and W_2 be the readings of the wattmeters then

$$W_1 + W_2 = 10{,}000 \qquad \text{(i)}$$

Now $\sqrt{3}\,|V_L|\,|I_L| \cos\theta = 10{,}000$, where I_L is the line current

$$\therefore \quad |I_L| = \frac{10{,}000}{\sqrt{3}\times 230\times 0.7} = 35.86 \text{ A.}$$

Also, $W_1 - W_2 = |V_L|\,|I_L| \sin\theta = 230 \times 35.86 \sin 45.57° = 5889.8$ W. (ii)

Equating Eq. (i) and Eq. (ii)

$$W_1 = 7944.9 \text{ W} = 7.94 \text{ kW}$$

and $W_2 = 2055.1$ W = 2.06 kW.

7.13 In a balanced three-phase 200 V circuit, the line current is 115.5 A. When the power is measured by two wattmeter method one of the instruments reads 20 kW and the other zero. What is the power factor of the load?

Solution

Line voltage $|V_L| = 200$ V

Line current $|I_L| = 115.5$ A

1st wattmeter reading $W_1 = 20$ kW = 20,000 W

2nd wattmeter reading $W_2 = 0$

Now, $W_1 + W_2 = \sqrt{3}\,|V_L|\,|I_L| \cos\theta$, where $\cos\theta$ is the power factor

i.e.,
$$\cos\theta = \frac{W_1 + W_2}{\sqrt{3}V_L I_L} = \frac{20{,}000 + 0}{\sqrt{3}\times 200\times 115.5} = 0.5$$

7.14 Two wattmeters are connected to measure the input to a balanced three-phase circuit. The readings of the instruments are 2500 W and 500 W respectively. Find the power factor of the circuit when (a) both readings are positive and (b) when the later reading is obtained after reversing the current coil of the wattmeter.

Solution

(a) $W_1 = 2500$ W $W_2 = 500$ W

If θ be the power factor angle

$$\tan\theta = \sqrt{3}\frac{W_1 ` - W_2}{W_1 + W_2} = \sqrt{3}\frac{2500 - 500}{2500 + 500} = 1.155$$

So power factor in first case, $\cos\theta = \cos(\tan^{-1} 1.155) = 0.655$.

(b) $W_1 = 2500$ W and $W_2 = -500$ W

Hence,
$$\tan\theta = \sqrt{3}\frac{2500 - (-500)}{2500 + (-500)} = \sqrt{3}\frac{3000}{2000} = 2.598$$

Power factor in 2nd case, $\cos\theta = \cos(\tan^{-1} 2.598) = 0.359$.

7.15 Two wattmeters are connected to measure the input to a 400 V, three-phase delta connected motor whose output is 24.4 kW at a power factor of 0.4 (lag) and 80% efficiency. Find the resistance and reactance of the motor per phase and readings of each of the wattmeters.

Solution

Line voltage $|V_L| = 400$ V

$\therefore$ Phase voltage $|V_{Ph}| = |V_L| = 400$ V.

$$\left.\begin{aligned}\text{Output power} &= 24.4 \text{ kW}\\ \text{Efficiency} &= 80\% = 0.8\end{aligned}\right\} \text{(given)}$$

$$\therefore \quad \text{Input power} = \frac{\text{Output}}{\text{Effeciency}} = \frac{24.4}{0.8} = 30.5 \text{ kW.}$$

Also, power factor (cos θ) = 0.4 (lag).

If R, X and Z be the resistance, reactance and impedance per phase then we have

$$0.4 = \frac{|R|}{|Z|}$$

However, $\sqrt{3}\,|V_L|\,|I_L| \cos\theta = 30.5$ kW = 30,500 W where $|I_L|$ is the line current.

$$\text{Hence,} \quad |I_L| = \frac{30{,}500}{\sqrt{3} \times 400 \times .4} = 110.06 \text{ A.}$$

$$\text{Phase current } |I_{Ph}| = \frac{|I_L|}{\sqrt{3}} = \frac{110.06}{\sqrt{3}} = 63.543 \text{ A.}$$

$$\text{and} \quad |Z| = \frac{|V_{Ph}|}{|I_{Ph}|} = \frac{400}{63.543}\ \Omega = 6.29\ \Omega$$

$$\text{i.e.} \quad R = 0.4 \times 6.29\ \Omega = 2.52\ \Omega$$

$$\text{and} \quad X = \sqrt{Z^2 - R^2} = 5.76\ \Omega.$$

If W_1 and W_2 are the readings of the two wattmeters then

$$W_1 + W_2 = 30{,}500 \qquad \text{(i)}$$

$$\text{and} \quad W_1 - W_2 = |V_L|\,|I_L| \sin\theta = 400 \times 110.06 \sin(\cos^{-1} 0.4) = 40{,}348.66. \qquad \text{(ii)}$$

From equations (i) and (ii),

$$W_1 = 35424.33 \text{ W} = 35.42 \text{ kW}$$

$$\text{and} \quad W_2 = -4924.33 \text{ W} = -4.924 \text{ kW.}$$

7.16 The input power to a three-phase motor is measured by two wattmeters both of which indicate 100 kW each. Find the power factor of the motor. If the power factor of the motor is changed to 0.75 leading find the readings of the two wattmeters, the input power remaining the same.

Solution

$$W_1 = 100 \text{ kW and } W_2 = 100 \text{ kW (given)}$$

$$\text{Hence,} \quad \tan\theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} \times \sqrt{3} \times 0 = 0 \text{ i.e. } \theta = 0^\circ$$

$\therefore$ Power factor $\cos\theta = 1$

In the second case power factor becomes 0.75 i.e. $\cos\theta = 0.75$

Total power $W_1' + W_2' = 2 \times 100 = 200$ kW

Also, $$W_1' - W_2' = \frac{(W_1' + W_2')\tan\theta}{\sqrt{3}} = \frac{200\times 0.75}{\sqrt{3}} \text{ kW} = 86.6 \text{ kW}$$

From above we get

$$W_1' = 143.3 \text{ kW and } W_2' = 56.7 \text{ kW.}$$

The readings of two wattmeters when the power factor of the motor is 0.75 leading are 143.3 kW and 56.7 kW.

7.17 The input power to a three-phase motor was measured by two wattmeters whose readings were 12 kW and –4 kW and the line voltage was 200 V. Calculate the power factor and the line current.

Solution

$$W_1 = 12 \text{ kW} \qquad W_2 = -4 \text{ kW}$$

Line voltage $|V_L| = 200$ V

If θ be the power factor angle then

$$\tan\theta = \sqrt{3}\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\frac{12-(-4)}{12+(-4)} = \sqrt{3}\times\frac{16}{8} = 3.464.$$

So, power factor = $\cos\theta = \cos(\tan^{-1} 3.464) = 0.277$.

Now, $\sqrt{3}\,|V_L|\,|I_L|\cos\theta = W_1 + W_2 = 12 + (-4) = 8 \text{ kW} = 8000 \text{ W}$

where I_L is the line current.

So, $$|I_L| = \frac{8000}{\sqrt{3}\times 200\times .277} = 83.37 \text{ A.}$$

7.18 Three coils each having resistance of 10 Ω and series reactance of 10 Ω are connected in star across a 400 V, three-phase line. Calculate the line current and readings on the two wattmeters which are connected to measure the total power.

Solution

$$R = 10\ \Omega,\ X = 10\ \Omega, \quad \text{Impedance } |Z| = \sqrt{R^2 + X^2} = 14.14\ \Omega$$

Power factor $\cos\theta = \dfrac{|R|}{|Z|} = \dfrac{10}{14.14} = 0.707.$

Line voltage $|V_L| = 400$ V, phase voltage $|V_{\text{Ph}}| = \dfrac{|V_2|}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94$ V.

Phase current = $\dfrac{|V_{\text{Ph}}|}{|Z|} = \dfrac{230.94}{14.14} = 16.33$ A.

Line current $|I_L|$ = phase current = 16.33 A.

If W_1 and W_2 are the readings of the two wattmeters then

$$W_1 + W_2 = \sqrt{3}\,|V_L|\,|I_L|\cos\theta$$
$$= \sqrt{3}\times 400\times 16.33\times 0.707 = 7999 \text{ W} \simeq 8 \text{ kW}$$

and $$W_1 - W_2 = |V_L||I_L|\sin\theta = 400\times 16.33\sin(\cos^{-1} 0.707)$$
$$= 4619.5 \text{ W} = 4.62 \text{ kW}$$

Hence, $$W_1 = 6.31 \text{ and } W_2 = 1.69 \text{ kW.}$$

ADDITIONAL PROBLEMS

7.19 A balanced three-phase delta connected load of 100 kW takes a lagging current of 50 A with a line voltage of 11 kV, 50 Hz. Find the circuit constants of the load per phase.

Solution

Total power = 100 kW (= 100,000 W).

Line current $|I_L| = 50$ A

Line voltge $|V_L| = 11$ kV (= 11,000 V).

As the system is delta connected,

$$\text{phase current } |I_{Ph}| = \frac{|I_2|}{\sqrt{3}} = \frac{50}{\sqrt{3}} \text{ A,}$$

and phase voltage $|V_{Ph}| = (|V_L|) = 11,000$ V

$$\text{Impedance per phase } |Z| = \frac{|V_{Ph}|}{|I_{Ph}|} = \frac{11,000}{50/\sqrt{3}} = 381\ \Omega 3.$$

$P = \sqrt{3}\,|V_L|\,|I_L| \cos\theta = 100,000$ where ($\cos\theta$ = power factor)

i.e.
$$\cos\theta = \frac{10,000}{\sqrt{3} \times 11,000 \times \frac{50}{\sqrt{3}}} = 0.0182.$$

Also, $|R|/|Z| = \cos\theta$

$\therefore$ $\frac{|R|}{|Z|} = 0.0182$, where R is the resistance per phase

So, $R = 6.93\ \Omega$.

If (X) be the reactance per phase then $|X| = \sqrt{Z^2 - R^2} = 380.94\ \Omega$.

As the current is lagging so the reactance is inductive in nature, i.e. inductance, $L = \frac{|X|}{\omega}$

$$= \frac{380.94}{2\pi \times 50} = 1.213 \text{ H.}$$

We have assumed R and L to be series connected in each phase and the circuit constants of the load per phase are 3.99 Ω (resistor) and 1.213 H (inductor).

7.20 Three impedances Z_A, Z_B and Z_C are connected in star and are supplied from a 400 V, 50 Hz, three-phase ac source. Determine the line currents. Assume $Z_A = 5\angle 0°\ \Omega$, $Z_B = 40\angle 30°\ \Omega$ and $Z_C = 20\angle{-30°}\ \Omega$.

Solution

Referring to Fig. 7.17

$$I_A = \frac{V_{AN}}{Z_A} = \frac{(400/\sqrt{3})\angle 0°}{5\angle 0°} = 46.19\angle 0° \text{ A}$$

$$I_B = \frac{V_{BN}}{Z_B} = \frac{(400/\sqrt{3})\angle -120°}{40\angle 30°} = 5.77\angle -150° \text{ A}$$

$$I_C = \frac{V_{CN}}{Z_C} = \frac{(400/\sqrt{3})\angle -240°}{20\angle -30°}$$

$= 11.55\angle -270°$ A $= 11.55\angle 90°$ A.

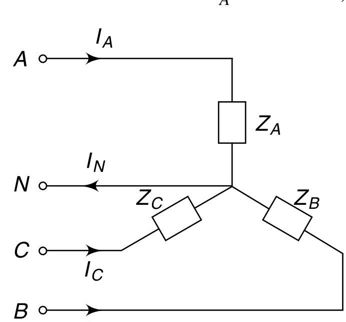

Fig. 7.17 ***Circuit of Ex. 7.20***

7.21 A balanced star connected load with impedances of $2\angle{-30°}\ \Omega$ is supplied from a three-phase, 4-wire system, the voltages to neutral being $V_{AN} = 100\angle{-90°}$, $V_{BN} = 100\angle 30°$, $V_{CN} = 100\angle 150°$ V. Determine the current in the line conductors and the current in the neutral.

Solution

Referring to Fig. 7.18

$$I_A = \frac{V_{AN}}{Z} = \frac{100\angle -90°}{2\angle -30°} = 50\angle{-60°} \text{ A}$$

$$I_B = \frac{V_{BN}}{Z} = \frac{100\angle 30°}{2\angle -30°} = 50\angle 60° \text{ A}$$

$$I_C = \frac{V_{CN}}{Z} = \frac{100\angle 150°}{2\angle -30°} = 50\angle 180° \text{ A.}$$

$$I_N = (I_A + I_B + I_C) = 50(\angle{-60°} + \angle 60° + \angle 180°)$$
$$= 50(0.5 - j0.866 + 0.5 + j0.866 - 1) = 0 \text{ A.}$$

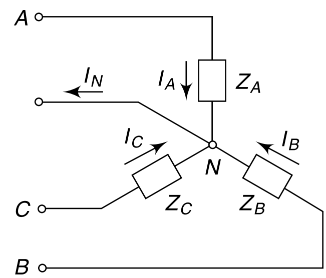

Fig. 7.18 ***Circuit of Ex. 7.21***

7.22 In a three-phase four-wire power distribution system phase *B* is open while currents through phase *R* and *Y* are $100\angle{-30°}$ and $50\angle 60°$. Find the current through the neutral connection.

Solution

The three-phase four-wire system is a star connected system with a neutral wire.

$$I_R = 100\angle{-30°} \text{ A}$$
$$I_Y = 50\angle 60° \text{ A.}$$

As *B* phase is open $\quad I_B = 0$

Neutral current $\quad I_N = (I_R + I_Y + I_B)$

$$= (100\angle{-30°} + 50\angle 60°)$$
$$= (86.6 - j50 + 25 + j43.3)$$
$$= +111.6 - j6.67 = 111.8\angle{-3.414°} \text{ A.}$$

7.23 Figure 7.18 shows a three-phase load connected in star. Here $I_A = 20\angle 100°$ A and $I_B = 10\angle{-50°}$ A. Find the voltage drops across the three impedances of Z_A, Z_B and Z_C where $Z_A = 6\angle 0°$, $Z_B = 6\angle 30°$ and $Z_C = 5\angle 45°$. Assume phasor sum of I_A, I_B and I_C as zero.

Solution

$$I_A = 20\angle 100° \text{ A}$$
$$I_B = 10\angle{-50°} \text{ A.}$$

$\therefore \quad I_A + I_B + I_C = 0,$

$$I_C = -(I_A + I_B) = -(20\angle 100° + 10\angle{-50°}$$
$$= -(-3.47 + j19.69 + 6.43 - j7.66)$$
$$= (-2.96 - j12.03) = 12.38\angle 76.18° \text{ A.}$$

Voltage drop across

$$Z_A = I_A Z_A = 20\angle 100° \times 6\angle 0° = 120\angle 100° \text{ V.}$$

Voltage drop across

$$Z_B = I_B Z_B = 10\angle{-50°} \times 6\angle 30° = 60\angle{-20°} \text{ V.}$$

Voltage drop across

$$Z_C = I_C Z_C = 12.38\angle 76.18° \times 5\angle 45° = 61.9\angle 121.18°.$$

7.24 The input power to a three-phase motor was measured by the two wattmeter method. The readings were 5.2 kW and 1.7 kW, the latter reading has been obtained after reversal of current coil connections. The line voltage was 400 V. Calculate (a) the real power (b) the power factor and (c) the line current.

Solution

$$W_1 = 5.2 \text{ kW} = 5200 \text{ W and } W_2 = 1.7 \text{ kW} = 1700 \text{ W}.$$

As the reading of W_2 has been obtained after reversal of current coil so $W_2 = -1700$ W.

Line voltage (given) $|V_L| = 400$ V

(a) Total power = $W_1 + W_2 = (5200 - 1700)$ W = 3500 W = 3.5 kW.

(b) If θ be the power factor angle then

$$\tan\,\theta = \sqrt{3}\,\frac{W_1 - W_2}{W_1 + W_2} = \sqrt{3}\,\frac{5200 + 1700}{5200 - 1700} = \sqrt{3}\,\frac{6900}{3500} = 3.414$$

∴ Power factor

$$(\cos\,\theta) = \cos(\tan^{-1} 3.414) = 0.28.$$

(c) Total power = $W_1 + W_2 = \sqrt{3}\,|V_L|\,|I_L| \cos\,\theta$, where I_L is the line current.

$$\therefore \qquad |I_L| = \frac{W_1 + W_2}{\sqrt{3}\,V_L\,\cos\theta} = \frac{3500}{\sqrt{3} \times 400 \times .28} = 18 \text{ A}.$$

7.25 Three impedances each of resistance 10 Ω and series inductive reactance of 5 Ω are connected in (i) star (ii) in delta across a 3 phase 400 V supply. Find the line current in each case and the total power.

Solution

$|R| = 10\ \Omega$, $|X| = 5\ \Omega$, so impedance $|Z| = \sqrt{R^2 + X^2} = \sqrt{10^2 + 5^2} = 11.18\ \Omega$.

Line voltage $|V_L| = 400$ V.

$$\text{Power factor } \cos\,\theta = \frac{|R|}{|Z|} = \frac{10}{11.18} = 0.89.$$

(i) In star connection

$$\text{Phase voltage } |V_{\text{Ph}}| = \frac{|V_L|}{\sqrt{3}} = \frac{400}{\sqrt{3}} = 230.95 \text{ V}$$

$$\text{and phase current } |I_{\text{Ph}}| = \frac{|V_{\text{Ph}}|}{|Z|} = \frac{230.95}{11.18} = 20.66 \text{ A}.$$

∴ Line current $|I_L|$ = phase current = 20.66 A.

$$\text{Total power} = \sqrt{3}\,|V_L||I_L| \cos\,\theta = \sqrt{3} \times 400 \times 20.66 \times 89$$
$$= 12739.16\ \Omega = 12.74 \text{ kW}$$

(ii) In delta connection

Phase voltage $|V_{\text{Ph}}| = |V_L| = 400$ V

$$\text{So phase current } |I_{\text{Ph}}| = \frac{|V_{\text{Ph}}|}{Z} = \frac{400}{11.18} = 35.78 \text{ A and}$$

line current $|I_L| = \sqrt{3}\,|I_{\text{Ph}}| = \sqrt{3} \times 35.78 = 61.97$ A

$$\text{Total power} = \sqrt{3}\,|V_L||I_L| \cos\,\theta = \sqrt{3} \times 400 \times 61.97 \times .89$$
$$= 38211.33 \text{ W} = 38.21 \text{ kW}$$

It may be noted here that the arm impedances being same, a delta load consumes more real power than the equivalent star load.

7.26 A three-phase balanced voltage of 230 V is applied to a balanced delta connected load consisting of thirty 40 W lamps connected in parallel in each phase. Calculate the phase and line currents, phase voltages, power consumption of all lamps.

Solution

Line voltage $|V_L|$ = Phase voltage $|V_{\text{Ph}}|$ = 230 V ($\because$ the load is delta connected)

$\therefore$ $|V_{\text{Ph}}| = 230$ V

Power consumption of each lamp = 40 W.

$\therefore$ Total power consumption is $30 \times 40 = 1200$ W

If I_{Ph} be the phase current

$$|V_{\text{Ph}}||I_{\text{Ph}}| = 1200 \text{ W}$$

Hence $$I_{\text{Ph}} = \frac{1200}{230} = 5.2 \text{ A}.$$

Line current = $\sqrt{3}\, I_{\text{Ph}} = \sqrt{3} \times 5.2$ A = 9 A.

Power consumption of all lamps = 3×1200 W = 3600 W = 3.6 kW.

7.27 The load connected to a three-phase supply contains three similar impedances connected in star. The line currents are 50 A and the KVA and kW inputs are 50 and 27 respectively. Find the line and phase voltages, KVAR input and the resistance and reactance of each coil.

Solution

Line current $|I_L| = 50$ A

KVA = 50 and kW = 27

$\therefore$ $$\text{KVAR} = \sqrt{\text{KVA}^2 - \text{KW}^2} = \sqrt{(50)^2 - (27)^2} = 42.$$

As the load is star connected

Phase current $|I_{\text{Ph}}| = |I_L| = 50$ A.

If V_L be the line current and $\cos\theta$ be the power factor then

$$\sqrt{3}\,|V_L||I_L| \cos\theta = 27 \times 10^3$$

or $$\cos\theta = \frac{27000}{\sqrt{3} \times V_L I_L}$$

and $$\sqrt{3}\,|V_L||I_L| \sin\theta = 42 \times 10^3$$

or $$\sin\theta = \frac{42000}{\sqrt{3}\, V_L I_L}$$

$\therefore$ $$\tan\theta = \frac{42}{27} = \frac{X}{R},$$

where X and R are the reactance and resistance of the load.

or $$X = 1.56\, R.$$

Again $$\sqrt{3}\,|V_L||I_L| = 50 \times 10^3$$

or, $$|V_L| = \frac{50 \times 10^3}{\sqrt{3} \times 50} = 577.35 \text{ V}$$

Phase voltage $$|V_{\text{Ph}}| = \frac{|V_L|}{\sqrt{3}} = \frac{577.37}{\sqrt{3}} = 333.33 \text{ V}$$

If Z is the impedance per phase then

$$|Z| = \frac{|V_{\text{Ph}}|}{|I_{\text{Ph}}|} = \frac{333.35}{50}\ \Omega = 6.67\ \Omega$$

or $$R^2 + X^2 = (6.67)^2 = 44.49$$

or $$R^2 + (1.56\, R)^2 = 44.49$$

Hence $$R = 3.6\ \Omega$$

and $$X = 1.56\, R = 1.56 \times 3.6 = 5.62\ \Omega.$$

7.28 Three impedances Z_1, Z_2 and Z_3 are mesh connected to a symmetrical three-phase, 400 V, 50 Hz supply of phase sequence *RYB*. $Z_1 = (10 + j0)\ \Omega$ between *R* and *Y*, $Z_2 = (8 + j6)\ \Omega$ between *Y* and *B* and $Z_3 = (5 - j5)\ \Omega$ between *B* and *R*. Calculate the phase currents and total power consumed. Assume line voltages are balanced and separeted by 120° from each other.

Solution

$$Z_1 = 10 + j0 = 10\angle 0^\circ\ \Omega$$
$$Z_2 = 8 + j6 = 10\angle 36.87^\circ\ \Omega$$
and $$Z_3 = 5 - j5 = 7.07\angle -45^\circ\ \Omega.$$

The corresponding circuit is shown in Fig. 7.19.

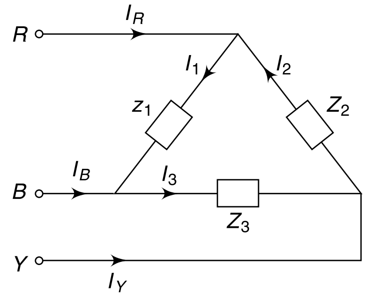

Fig. 7.19 *Circuit of Ex. 7.28*

Line voltage $|V_L| = 400$ V $(=|V_{ph}|)$.

Phase current through Z_1 is $I_1 = \dfrac{400\angle 0^\circ}{10\angle 0^\circ}$ A $= 40\angle 0^\circ$ A.

Phase current through Z_2 is $I_2 = \dfrac{400\angle -120^\circ}{10\angle 36.87^\circ}$ A $= 40\angle -156.87^\circ$ A.

Phase current through Z_3 is $I_3 = \dfrac{400\angle -240^\circ}{7.07\angle -45^\circ}$ A $= 56.58\angle -195^\circ$ A.

The angles of the currents are w.r.t. the corresponding line voltages.

The total power consumed = 400(40 + 40 cos 36.87° + 56.58 cos 45°) = 44.8 kW.

7.29 A balanced delta connected load of $(4 + j3)\ \Omega$ per phase is connected to a three-phase, 230 V supply. Find the line current, p.f., reactive *VA* and total *VA*.

Solution

Impedance $Z = (4 + j3)\ \Omega = 5\angle 36.87^\circ\ \Omega$.

Line voltage $|V_L|$ (= Phase voltage $|V_{Ph}|$) $= 230\angle 0^\circ$ V.

Hence phase current $|I_{Ph}| = \dfrac{|V_{Ph}|}{|Z|} = \dfrac{230}{5}$ A = 46 A.

Line current $|I_L| = \sqrt{3}\,|I_{Ph}| = \sqrt{3} \times 46$ A = 79.67 A.

Power factor angle $\theta = 36.87^\circ$ and power factor $\cos\theta = 0.8$.

Hence reactive $VA = \sqrt{3}\,|V_L||I_L| \sin\theta$

$= \sqrt{3} \times 230 \times 79.67 \sin 36.87^\circ$

= 19042.45 VAR = 19.042 KVAR.

Total $VA = \sqrt{3}\,|V_L||I_L|$ = 31738.27 VA = 31.74 KVA.

7.30 A three-phase 440 V supply feeds a balanced delta connected load. Given: $I_{ab} = 30\angle -30^\circ$ w.r.t. V_{ab}. Find (i) the line current I_a and its phase angle w.r.t. V_{ab}, (ii) total power received by the load and (iii) the resistance per phase. The corresponding figure is given in Fig. 7.20.

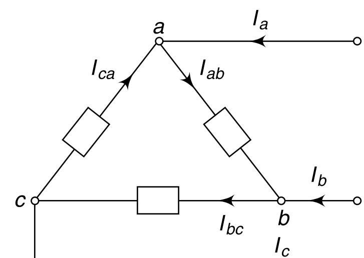

Fig. 7.20 *Circuit of Ex. 7.30*

Solution

(i) Line current $|I_L| = \sqrt{3} \times 30 = 51.96$ A.

Referring to Fig. 7.21(a),

Line current $I_a = I_{ab} - I_{ca}$

$= 51.96\angle-(30° + 30°)$

$= 51.96\angle-60°$

Hence the phase angle of I_a w.r.t. V_{ab} is 60°.

(ii) Total power $= \sqrt{3}\,|V_L||I_L| \cos\theta$

$= \sqrt{3} \times 440 \times 51.96 \cos 30°$

$= 34293.6$ W

$= 34.29$ kW.

(iii) If R is resistance per phase then

$$3I_{\text{Ph}}^2 R = 34293.6$$

or $$R = \frac{34293.6}{3 \times (30)^2}\ \Omega = 12.7\ \Omega.$$

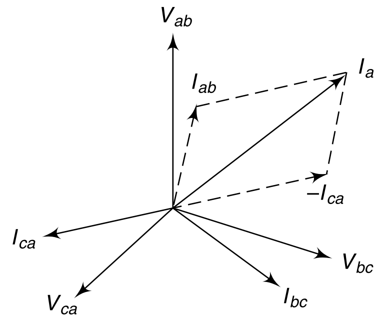

Fig. 7.20(a) ***Phasor diagram for Ex. 7.30***

7.31 A balanced 440 V, three-phase voltage is applied on an unbalanced star connected load. It is given that the supply voltage in phase R is $254\angle-30°$ V and the voltage drop across the load connected in phase R is $200\angle-15°$ V. Calculate the voltage between the star point of the load and the supply neutral. Also find the voltage across the loads in Y and B phases.

Solution

Line voltage of phase R is $V_R = \dfrac{440}{\sqrt{3}} = 254\angle-30°$ V (given)

Voltage drop in phase R is $200\angle-15°$ V.

Let N be the neutral point of the supply and N' be the star point of the load.

Hence $V_{RN} = 254\angle-30°$ volts and $V_{RN'} = 200\angle-15°$ V.

Therefore voltage between star point of load and supply neutral is

$$\begin{aligned} V_{N'N} = V_{RN} - V_{RN'} &= 254\angle-30° - 200\angle-15° \\ &= 220 - j127 - 193.2 + j51.76 \\ &= 26.8 - j75.24 = 79.87\angle-70.39°\text{ V.} \end{aligned}$$

Similarly, for phase Y, voltage across load is given by

$$\begin{aligned} V_{YN'} = V_{YN} - V_{N'N} &= 254\angle-30° - 120° - 79.87\angle-70.39° \\ &= -220 - j127 - 26.8 + j75.24 \\ &= -246.8 - j51.76 = 252.17\angle168.16°\text{ V.} \end{aligned}$$

Also, voltage across load in phase B is

$$\begin{aligned} V_{BN'} = V_{BN} - V_{N'N} &= 254\angle-30° - 240° - 79.87\angle-70.39° \\ &= 330\angle94.65°\text{ V.} \end{aligned}$$

7.32 Three impedances $Z_A = 4\angle30°\ \Omega$, $Z_B = 5\angle-20°\ \Omega$ and $Z_C = 10\angle0°\ \Omega$ are connected in star and are supplied from 50 V, 50 Hz; three-phase balanced source. Obtain line currents and power drawn by each impedance.

Solution

The line currents are

$$I_A = \frac{50\angle0°}{\sqrt{3} \times 4\angle30°} = 7.217\angle-30°\text{ A}$$

$$I_B = \frac{50\angle -120°}{\sqrt{3}\times 5\angle -20°} = 5.772\angle -100° \text{ A}$$

and $$I_C = \frac{50\angle -240°}{\sqrt{3}\times 10\angle 0°} = 2.886\angle 120° \text{ A.}$$

$$\text{Power drawn by } Z_A = \frac{50}{\sqrt{3}} \times 7.217 \cos 30° = 180.43 \text{ W}$$

$$\text{Power drawn by } Z_B = \frac{50}{\sqrt{3}} \times 5.772 \cos 20° = 156.57 \text{ W}$$

$$\text{Power drawn by } Z_C = \frac{50}{\sqrt{3}} \times 2.886 \cos 0° = 83.31 \text{ W.}$$

7.33 A balanced three-phase star connected load of 100 kW takes a leading current of 80 A when connected across a 4 KV, 50 Hz supply. Determine the circuit elements connected in series and power factor of the load.

Solution

Line voltage V_L = 4 KV = 4000 V. Hence phase voltage = $\frac{4000}{\sqrt{3}}$ V .

If I_L be the current (line current and phase current are same in star connected system) we can write,

$\sqrt{3}\, V_L I_L \cos\theta = 100 \times 10^3$, where $\cos\theta$ is the power factor angle of the load.

As $I_L = 80$ A,

Hence $$\cos\theta = \frac{100\times 10^3}{\sqrt{3}\times 4000\times 80} = 0.18 \text{ (as load current is leading)}$$

Now, impedance per phase (Z_{Ph}) = $\frac{\text{Phase voltage}}{\text{Phase current}} = \frac{4000}{\sqrt{3}\times 80}\,\Omega = 28.86\ \Omega$.

If R and X_C be the resistance and capacitive reactance connected in series,

$$\frac{R}{Z_{\text{Ph}}} = 0.18 \text{ or, } R = 0.18 \times 28.86 = 5.2\ \Omega.$$

$\therefore$ From $\sqrt{R^2 + X_C^2} = 28.86$, we have

$$X_C = \sqrt{(28.86)^2 - (5.2)^2} = 28.39\ \Omega.$$

Hence the resistance is 5.2 Ω and capacitive reactance is 28.39 Ω when the elements are connected in series and the power factor of the load is 0.18 lead.

7.33 A three-phase 440 V, 50 Hz line is connected to the identical capacitors connected in delta. If the line current is 10 A, fine the capacitance of each capacitor.

Solution

$$I_C = (I_{\text{Ph}}) = \frac{|I_L|}{\sqrt{3}} = \frac{10}{\sqrt{3}} = 5.774 \text{ A.}$$

V_C, the voltage across each capacitor is 440 V ($\because$ Δ connection, $|V_{\text{Ph}}| = |V_L| = |V_C|$)

$$\therefore \quad X_C \text{ (capacitive reactance of each capacitor)} = \frac{|V_C|}{|I_{\text{Ph}}|} = \frac{440}{5.774} = 76.20\ \Omega.$$

The capacitance of each capacitor is

$$C = \frac{1}{\omega X_C} = \frac{1}{2\times\pi\times 50\times 76.20} = 41.77\ \mu\text{F}.$$

7.34 An industry draws a total of 500 KVA from a balanced 3-phase supply of 3.3 kV ($L - L$) 50 Hz. If the plant power factor is 0.85 lagging, calculate the impedance of the plant (per phase), phase angle between the line to neutral voltage and the line current. Assume Y-connection of the load.

Solution

$$|V_{\text{Ph}}| = \frac{3300}{\sqrt{3}} = 1905.30\ \text{V}.$$

$$I_{\text{Ph}}(= I_L) = \frac{|S|/3}{|V_{\text{Ph}}|} = \frac{500\times 10^3}{3\times 1905.30} = 87.47\ \text{A}$$

$$\therefore \qquad |Z_{\text{Ph}}|\ \text{(impedance/branch)} = \frac{|V_{\text{Ph}}|}{|I_{\text{Ph}}|} = \frac{3300/\sqrt{3}}{87.47} = 21.78\ \text{ohm}.$$

Phase angle between V_{Ph} and the corresponding line current (= 87.47 A) is given by

$$\cos^{-1}(0.85) = 3178°.$$

Thus the line current lags the line voltage by 31.78°.

7.35 A 10 HP 415 V, 3 phase 50 Hz induction motor draws a full load current of 15 A at 0.8 p.f. (lag). Calculate the full load efficiency of the motor.

Solution

Output power being 10 HP, it is equivalent to 7460 W ($\because$ 1 HP = 746 W).

$$\text{Efficiency } \eta = \frac{\text{Output full load power}}{\text{Input power at full load}} \times 100$$

Hence, input power at full load is

$$S = \sqrt{3}\,|V_L||I_L| = \sqrt{3} \times 415 \times 15 = 10782.02\ \text{VA}$$

This is equivalent to input active power of (10782.02 × 0.8) or 8625.61 W (since p.f. is 0.8 and $P = VA$ power × p.f.)

$$\therefore \qquad \eta = \frac{7460}{8625.36} \times 100 = 86.49\%.$$

Then we have obtained the efficiency of the motor at full load as 86.49% while full load input power is 8625.36 W and output power is 7460 W.

7.36 The power factor of each phase of a balanced star connected load is 0.6 lagging. If the impedance per phase is 5 Ω and the load is connected across a balanced 200 V 50 Hz source, find the apparent power drawn by the load.

Solution

$$|V_{\text{Ph}}| = \frac{200}{\sqrt{3}} = 115.47\ \text{V}$$

$$|I_{\text{Ph}}| = \frac{|V_{\text{Ph}}|}{|Z_{\text{Ph}}|} = \frac{115.47}{5} = 23.094\ \text{A}.$$

$$\phi = \cos^{-1}(0.6) = 53.13°\ \text{(lag)}$$

$$\therefore \qquad P = \sqrt{3}\,|V_L||I_L| \cos\phi = \sqrt{3} \times 200 \times 23.094 \times \cos 53.13° = 4800\ \text{W}$$

$$[I_L = I_{\text{Ph}} \text{ and } V_{\text{Ph}} = \sqrt{3}\, V_L \text{ in } Y\text{-connection}]$$

$$Q = \sqrt{3}\,|V_L||I_L| \sin \phi$$
$$= \sqrt{3} \times 200 \times 23.094 \times \sin (\cos^{-1} 0.6) = 6399.99 \text{ VAR}.$$
$$\therefore \quad |S| = \sqrt{P^2 + Q^2} = \sqrt{(4800)^2 + (6399.99)^2} = 7999.99 \text{ VA } (\simeq 8 \text{ KVA}).$$

7.37 A 440 V, three-phase, 50 Hz balanced source supplies electrical energy to the following three phase balanced loads:

(i) A 200 HP, three-phase 50 Hz induction motor operating at 94% efficiency and 0.88 p.f. (lag).
(ii) A 50 kW three-phase electric heating element.
(iii) A mixed load of 40 kW (three-phase) operating at 0.7 lagging p.f.

Obtain

(a) the total load in kW supplied by source
(b) the total KVAR supplied by the source
(c) the total apparent power
(d) the line current.

Solution

(a) *Real power*

(i) For motor: $P_M = \dfrac{\text{HP} \times 746}{\text{Efficiency}} = \dfrac{200 \times 746}{0.94} \times 10^{-3} = 158.72 \text{ kW}$

(ii) For heater: $P_H = 50$ kW.

(iii) For mixed load: $P_X = 40$ kW.

$\therefore$ Total real power supplied = 158.72 + 50 + 40
= 248.72 kW (3-phase).

(b) For motor: $\phi_m = \cos^{-1} 0.88 = 28.36°$

However, $\tan \phi_m = \dfrac{Q_m}{P_m}$ [Q_m = reactive power input to motor
P_m = real power input to motor]

or $Q_M = P_m \tan \phi_m = 158.72 \tan 28.36° = 85.68$ KVAR.

For heating load, $\phi_H = 0$ [as heater is a pure resistive load]

Hence $Q_H = 0$

For mixed load:

$$\phi_X = \cos^{-1}(0.70) = 45.57° \text{ (lag)}$$

$\therefore$ $Q_X = P_X \tan \phi_X = 40 \tan 45.57° = 40.81$ KVAR.

Thus we find total KVAR supplied

$(Q) = 85.68 + 0 + 40.81 = 126.49$ KVAR

(c) Total apparent power $|S|$ is given by

$$|S| = \sqrt{P^2 + Q^2} = \sqrt{248.72^2 + 126.49^2} = 279.04 \text{ KVA}$$

and $\phi = \tan^{-1} \dfrac{Q}{P} = \tan^{-1} \dfrac{126.49}{248.72} = 26.96°$ (lag).

(d) $\because$ $|S| = \sqrt{3}\,|V_L||I_L|$,

$\therefore$ $|I_L| = \dfrac{|S|}{\sqrt{3}|V_L|} = \dfrac{279.04 \times 10^3}{\sqrt{3} \times 440} = 366.15$ A.

7.38 Two wattmeters measure the three-phase power of a load and read 80 and 50 kW (for W_1 and W_2 respectively). Find the total complex power and the power factor. Also find the total power and reactive power.

Solution

$$P = W_1 + W_2 = 80 + 50 = 130 \text{ kW}$$

$$\tan\phi = \sqrt{3}\frac{W_2 - W_1}{W_2 + W_1} = \sqrt{3}\frac{50-80}{80+50} = -0.4$$

$\therefore$ $\phi = -21.78°$ and P.F. ($\cos\phi$) = 0.9286 (lag)

Since real power = (total power) $\times \cos\phi$,
We can write,

$$\text{Complex power } (S) = \frac{P}{\cos\phi} = \frac{130}{0.9286} \simeq 140 \text{ KVA.}$$

Reactive power (Q) is obtained as

$$Q = \sqrt{S^2 - P^2} = \sqrt{(140)^2 - (130)^2} \simeq 52 \text{ KVAR.}$$

7.39 A three phase induction motor, operating from a 400 V, 50 Hz supply, takes 25 A. The power factor of the motor is poor and found to be 0.5 lagging. If the two wattmeter method is used to measure the three-phase power supplied to the motor, what would each wattmeter read?

Solution

The connection diagram is shown in Fig. 7.21 while the phasor diagram in Fig. 7.22. Here, $\phi = \cos^{-1}(0.5) = 60°$ (lagging)

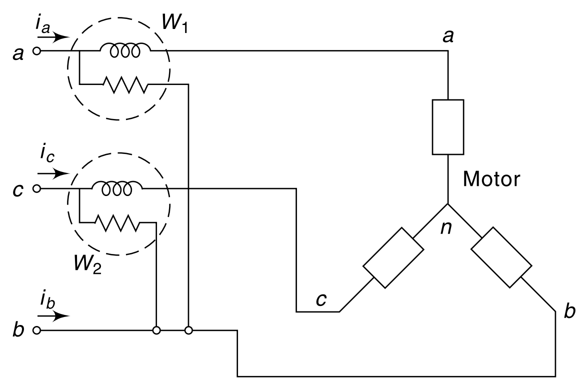

Fig. 7.21 ***Power measurement for the system in Ex. 7.39***

[Voltage across $W_1 = V_{a-c}$ (= $\bar{V}_a - \bar{V}_c$); V_{ac} is lagging I_a by (30° – ϕ). Then the reading of W_1 is $V_{ac}I_a \cos(30° - \phi)$. Similarly, $W_2 = V_L I_L \cos(30° - \phi)$.

$W_1 = V_L I_L \cos(\phi + 30°)$

[Actually $W_1 = V_L I_L \cos(30° - \phi)$
$= V_L I_L \cos(30 - (-\phi))$
$= V_L I_L \cos(30 + \phi)$]

$= 400 \times 25 \times \cos(60° + 30°) = 0.$

$W_2 = V_L I_L \cos(\theta - 30°)$

[$\because$ $W_2 = V_L I_L \cos(30° + \phi)$
$= V_L I_L \cos(30° - \phi)$]

$= 400 \times 25 \times \cos(60° - 30°)$

$= 8.66$ kW.

Thus, W_1 will read zero while W_2 will read 8.66 kW.

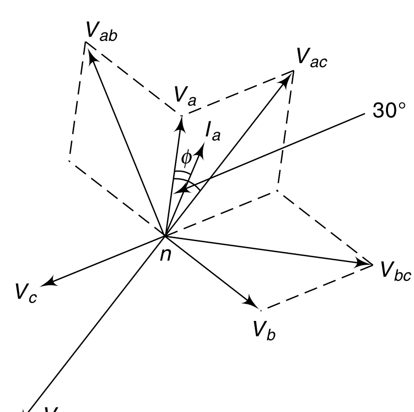

Fig. 7.22 ***Phasor diagram of Ex. 7.39***

7.40 In Fig. 7.23, ammeter A_1 reads 10 A, while A_2 also reads 10 A. Wattmeters W_1 and W_2 read 200 W and 800 W respectively. If $V_1 = V_2 = 400$ V, find the power factor. Also find the KVAR rating of a delta connected capacitor bank to enhance the power factor to 0.85 (lag).

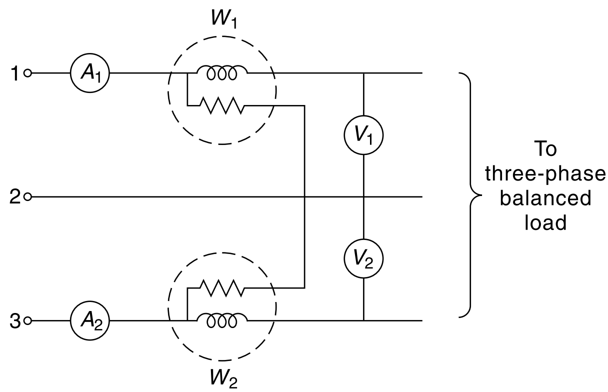

Fig. 7.23 *Circuit of Ex. 7.40*

Solution

$$S = \sqrt{3}\, V_L I_L = \sqrt{3} \times 400 \times 10 = 6928 \text{ VA}$$

$$P = W_1 + W_2 = 200 + 800 = 1000 \text{ W}$$

$$\therefore \quad PF = \frac{P}{S} = \frac{1000}{6928} = 0.144 \text{ (lag)}. \quad [\text{This gives } \phi = \cos^{-1}(0.144) = 81.72°]$$

The reactive load demand is thus

$$Q = P \tan \phi = 1000 \times \tan 81.72° = 6871.54 \text{ VAR.}$$

To improve the p.f. from 0.144 to 0.85 we require that the angle $\phi = 81.72$ becomes $\phi = \cos^{-1}(0.85) = 31.79°$ (lag). Thus the reactive power demand would be $P \tan 31.79°$ i.e., $1000 \times \tan 31.79° = 619.79$ KVAR.

This clearly means that the delta connected capacitor bank is to supply (6871.54 – 619.79) KVAR i.e., 6251.77 KVAR.

Thus the rating of the delta connected KVAR bank needs to be 6251.77 KVAR (3 phase).

[We can find rating of the capacitor, per phase.

$$Q/\text{per phase} = \frac{6251.77}{3} = 2083.92 \text{ KVAR}$$

i.e., $V^2 \omega C = 2083.92 \times 10^3$

or, $(400)^2 \times 314 \times C = 2083.92 \times 10^3$

$\therefore$ $C = 0.04$ F per phase].

7.41 How much active and reactive power are supplied by the source to the given load? Assume line voltages are 120° apart.

Solution

Let us first find out phase currents:

$$I_{xy} = \frac{V_{x-y}}{Z_1} = \frac{400\angle 0°}{(4 - j3)} = 80\angle{-36.87°} \text{ A}$$

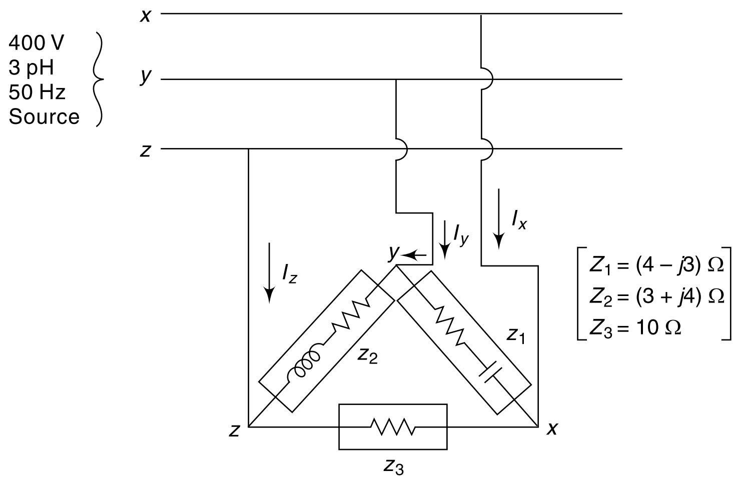

Fig. 7.24 *Circuit of Ex. 7.41*

$$I_{yz} = \frac{V_{y-z}}{Z_2} = \frac{400\angle -120°}{(3 + j\,4)} = 80\angle{-173.13°}\ \text{A}$$

$$I_{zx} = \frac{V_{z-x}}{Z_3} = \frac{400\angle +120°}{10\angle 0°} = 40\angle{-120°}\ \text{A}.$$

We can calculate the active power consumed as follows.

$$P = I_{xy}^2 \times 4 + I_{yz}^2 \times 3 + I_{zx}^2 \times 10$$

or $$P = 80^2 \times 4 + 80^2 \times 3 + 40^2 \times 10 = 60.8\ \text{kW}.$$

Also, the reactive power can be obtained as

$$Q = I_{xy}^2 \times (-3) + I_{yz}^2 \times (4) + I_{zx}^2 \times 0$$
$$= 80^2 \times (-3) + 80^2 \times 4 = 6.4\ \text{KVAR}.$$

[We can also calculate P and Q as follows:

$$P = V_{XY}\, I_{XY} \cos\left(\tan^{-1}\frac{3}{4}\right) + V_{YZ}\, I_{YZ} \cos\left(\tan^{-1}\frac{4}{3}\right) + V_{ZX}\, I_{ZX} \cos 0°$$
$$= 400 \times 80 \times \cos 36.87° + 400 \times 80 \times \cos 53.13° + 400 \times 40$$
$$= 25600 + 19200 + 16000 = 60.8\ \text{kW}$$
$$Q = V_{XY}\, I_{XY} \sin(-36.87°) + V_{YZ}\, I_{YZ} \sin 53.13° + V_{ZX}\, I_{ZX} \sin 0°$$
$$= -19200 + 25600 = 6.4\ \text{KVAR}.$$

7.42 A 1000 HP 6000 V 50 Hz three-phase induction motor operates at an efficiency of 95% at p.f. of 0.85 (lag). Assuming the motor operating at rated load, find the input real and reactive power of the motor. Obtain the value of line current taken by the motor. If a 350 KVAR capacitor bank is installed parallel to the motor, find the new line current and new p.f.

Solution

$$P_{\text{out}} = 1000\ \text{HP} = 746\ \text{kW},$$

$$P_{\text{in}} = \frac{P_{\text{out}}}{\eta} = \frac{746}{0.95} \simeq 785\ \text{kW}.$$

Thus input power at rated load of the motor is 785 kW at 0.85 p.f. (lag).

$$\therefore \qquad |S_{in}| = \frac{P_{in}}{\cos\phi} = \frac{785}{0.85} = 923.5 \text{ KVA}$$

$$|I_L| = \frac{|S_{in}|}{\sqrt{3}\,|V_L|} = \frac{923.5 \times 10^3}{\sqrt{3} \times 6000} = 88.87 \text{ A}$$

$$|Q_{in}| = \sqrt{|S_{in}|^2 - P_{in}^2} = \sqrt{(923.5)^2 - (785)^2} = 486.44 \text{ KVAR.}$$

The capacitor bank being installed now in parallel to the motor, we find Q_C = 350 KVAR and hence it would supply a part of reactive demand of the load (i.e. 350 KVAR).

The motor would now draw less reactive power from the supply and the new reactive demand is [(486.44) – 350)] i.e., 136.44 KVAR.

Obviously, this would reduce the KVA burden on the source. The new KVA demand of the load is

$$|S_{new}| = \sqrt{P_{3\phi}^2 + Q_{new}^2} = \sqrt{785^2 + (136.44)^2} = 796.77 \text{ KVA.}$$

It may be observed that new KVA demand is 796.77 KVA instead of 923.5 KVA.

$$\therefore \qquad |I_L|_{new} = \frac{|S_{new}|}{\sqrt{3}\,|V_L|} = \frac{796.77 \times 10^3}{\sqrt{3} \times 6000} = 76.67 \text{ A.}$$

Thus, with installation of 350 KVAR capacitor bank, the line current also reduces from 88.87 A to 76.67 A. This means lesser loss in cable of the supply feeding the motor and lesser power loss in the motor. Installation of capacitor bank also releases the supply of reactive power burden. The new power factor of the system is now

$$\cos\phi = \frac{|P_{3\phi}|}{|S_{new}|} = 0.985 \text{ (lag).}$$

7.43 A balanced 25 kW load operates at 0.85 p.f. (lag) from a 440 V, 50 Hz, three-phase supply. Calculate the total power, line current and reactive power drawn by the load.

Solution

$$|S_{3\phi}| = \frac{P_{3\phi}}{\cos\phi} = \frac{25 \times 10^3}{0.85} = 29.4 \text{ KVA}$$

$$|I_L| = \frac{|S_{3\phi}|}{\sqrt{3}\,|V_L|} = \frac{29.4 \times 10^3}{\sqrt{3} \times 440} = 38.58 \text{ A}$$

and

$$Q_{3\phi} = \sqrt{3}\,|V_L||I_L| \sin\phi$$
$$= \sqrt{3} \times 440 \times 38.58 \times \sin(\cos^{-1} 0.85) = 15.49 \text{ KVAR.}$$

7.44 In the preceeding problem, let us intend to improve the power factor of the load from 0.85 lag to 0.98 (lag) while the real power demand remains the same. Obtain the value of capacitive KVAR per phase to be inserted in parallel to the load.

Solution

We obtained earlier, for 0.85 p.f. (lag), $|S_{3\phi}|$ = 29.4 KVA while $|Q_{3\phi}|$ is 15.49 KVAR.

However, with improved power factor of 0.98 (lag), the new $|S_{3\phi}|$ would be

$$|S_{3\phi}|_{new} = \frac{P_{3\phi}}{\cos\phi_{new}} = \frac{25 \times 10^3}{0.98} = 25.51 \text{ KVA.}$$

[Observe, with improvement of p.f. from 0.85 to 0.98 (lag), KVA demand reduces.]

$$|Q_{3\phi}|_{\text{new}} = \sqrt{|S_{\text{new}}|^2 - P_{3\phi}^2} = \sqrt{(25.51)^2 - (25)^2} = 5.075 \text{ KVAR}$$

[Note that there is substantial reduction of $|Q_{3\phi}|$ with improvement of p.f.]

If we assume that the capacitor is supplying $|Q_C|$ amount of reactive power to the load such that reactive power flow from the supply is diminished, the capacitor being compensating the reactive burden of the load locally, we can write

$$Q_C = |Q_{3\phi}| - |Q_{3\phi}|_{\text{new}}$$
$$= 15.49 - 5.075 = 10.415 \text{ KVAR}$$

Thus we need to install a 3ϕ capacitor bank of 10.415 KVAR (i.e., 3.472 KVAR/ph) capacity in order to improve the p.f. of the bus from 0.85 to 0.98 (lag) as shown in Fig. 7.25.

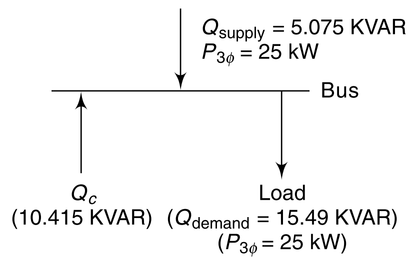

Fig. 7.25 ***System for Ex. 7.44***

[If we need to calculate the capacitance per phase we can write,

$$Q_{\text{cap}} = V \times I_C = V^2 \omega C$$

Assuming Δ connected capacitors,

$$C = \frac{Q_C}{V^2\omega} = \frac{3.472 \times 10}{440^2 \times 2 \times \pi \times 50} = 57.11 \ \mu\text{F}.$$

We need 57.11 μF capacitor per phase].

7.45 At what p.f. will one of the wattmeter reading be zero in a three-phase system?

Solution

Let $W_2 = 0$

Then, $\tan\theta = \sqrt{3}\dfrac{W_1 - 0}{W_1 + 0} = \sqrt{3}$

$\therefore$ $\theta = \tan^{-1}\sqrt{3} = 60°.$

Also, $\cos 60° = 0.5$

Thus, at 0.5 p.f., one of the two wattmeters would show zero reading.

7.46 The total input power in a three-phase circuit, as measured by two wattmeters, is 60 W. At what p.f. will the wattmeter readings be equal?

Solution

If the wattmeter readings are equal, then each wattmeter would read 30 W. Also, $\tan\theta$ being zero, $\theta = 0°$.

Hence $\cos\theta = \cos 0° = 1$

Thus, at u.p.f., the readings of the two wattmeters are equal.

7.47 The input three phase power of a balanced impedance load of $(4 + j3)\ \Omega$ per phase is measured by two wattmeter method. Determine the wattmeter reading if the source is a balanced 220 V three phase 50 Hz source and the loads are first connected in star and then in delta.

Solution

For star connection

$$V_{\text{Ph}} = \frac{220}{\sqrt{3}} = 127 \text{ V}.$$

Also, $I_L \; (= I_{Ph}) = \dfrac{V_{Ph}}{Z_{Ph}} = \dfrac{127\angle 0°}{4 + j3} = 25.4\angle -36.87°\text{A}.$

$\cos\phi = \cos(-36.87°) = 0.8 \text{ (lag)}$

$\therefore \quad P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 220 \times 25.4 \times 0.8$

$= 7742.96 \; (= W_1 + W_2) \text{ W}.$

Also, $\tan\phi \; (= 0.75) = \sqrt{3}\dfrac{W_1 - W_2}{W_1 + W_2}$

This gives $W_1 - W_2 = 3352.80$ W

Thus, $W_1 = 5547.88$ W

$W_2 = 2195.08$ W.

For delta connection

$$|I_{Ph}| = \frac{|V_L|}{|Z_{Ph}|} = \frac{220}{\sqrt{4^2 + 3^2}} = 44 \text{ A}.$$

$\therefore \quad |I_L| = \sqrt{3}\, |I_{Ph}| = 76.21$ A

and $P = \sqrt{3}\, V_L I_L \cos\phi = \sqrt{3} \times 220 \times 76.21 \times 0.8$

$= 23231.93 \text{ W } (= W_1 + W_2)$

But $\tan\phi = \tan(36.87) = 0.75 = \sqrt{3}\dfrac{W_1 - W_2}{W_1 + W_2}$

or $W_1 - W_2 = 10{,}059.72$ W.

Hence, $W_1 + W_2 = 23231.93$ W

$W_1 - W_2 = 10059.72$ W.

This gives $W_1 = 16645.83$ W

$W_2 = 6586.11$ W.

7.48 Three impedances, Z_{ab}, Z_{bc} and Z_{ca}, are connected across a balanced 400 V, 50 Hz, 3-phase supply. Find I_a (Fig. 7.26).

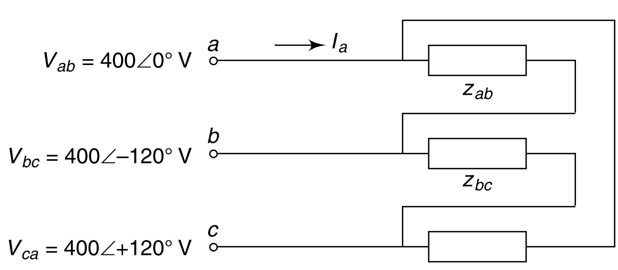

Fig. 7.26 ***Circuit of Ex. 7.48***

Assume $Z_{ab} = 5\angle 30°\ \Omega$; $Z_{bc} = 10\angle 20°\ \Omega$; $Z_{ca} = 10\angle 0°\ \Omega$

Solution

It is evident from the figure that I_a is the phasor sum of currents passing through Z_{ab} and Z_{ca}. Hence we can write

$$I_a = \frac{V_{ab}}{Z_{ab}} + \frac{V_{ac}}{Z_{ca}}$$

$$= \frac{V_{ab}}{Z_{ab}} + \frac{-V_{ac}}{Z_{ca}}$$

$$= \frac{400\angle 0°}{5\angle 30°} + \frac{-400\angle +120°}{10\angle 0°}$$
$$= 80\angle -30° - 40\angle 120°$$
$$= 80(\cos 30° - j \sin 30°) - 40 (\cos 120° + j \sin 120°)$$
$$= 69.28 - j40 + 20 - j34.64$$
$$= (89.28 - j74.64) \text{ A} = 116.37\angle -39.896° \text{ A}.$$

7.49 In the circuit shown in Fig. 7.27 find I_a.

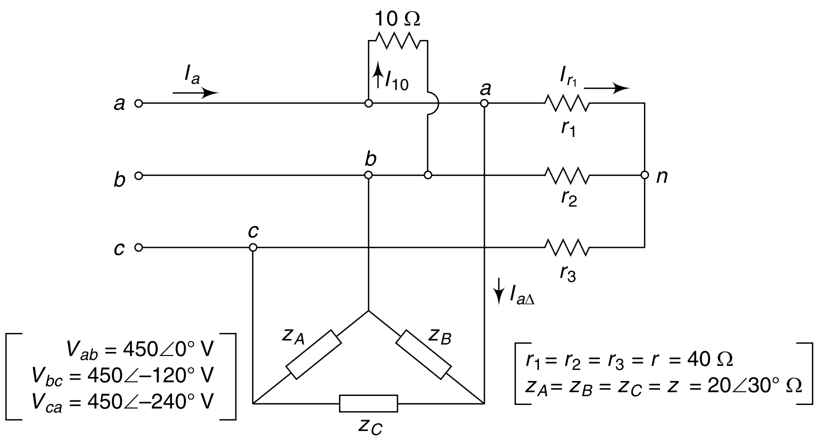

Fig. 7.27 *Circuit of Ex. 7.49*

Solution

It may be observed that

$$I_a = I_{10} + I_{r_1} + I_{a\Delta} \text{ (phasor sum)}$$

Hence $$I_{10} = \frac{V_{ab}}{10} = \frac{450\angle 0°}{10\angle 0°} = 45 \text{ A with phase lag } 0°$$

$$I_{r_1} = \frac{V_{a-n}}{r_1} = \frac{(450/\sqrt{3})\angle -30°}{40\angle 0°} = 6.5\angle -30° \text{ A}.$$

[∵ phase voltage lags by 30° the corresponding line voltage (ref. Fig. 7.28]

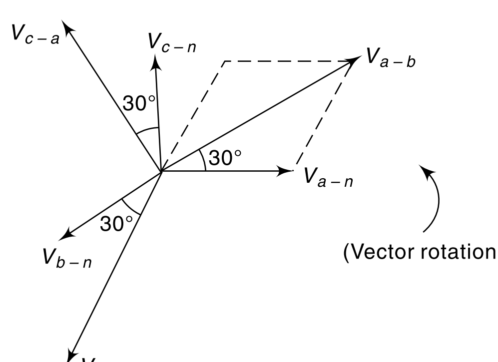

Fig. 7.28 *Voltage phasor diagram for Ex. 7.49*

Again we see that $I_{a\Delta}$ is phasor sum of currents through Z_B and Z_C in the delta circuit.

$$\therefore \quad I_{a\Delta} = \frac{V_{ab}}{Z_B} + \frac{V_{ac}}{Z_C} = \frac{V_{ab}}{Z_B} - \frac{V_{ca}}{Z_C}$$

$$= \frac{450\angle 0^\circ}{20\angle 30^\circ} - \frac{450\angle -240^\circ}{20\angle 30^\circ}$$

$$= \frac{450\angle 0^\circ}{20\angle 30^\circ} - \frac{450\angle +120^\circ}{20\angle 30^\circ}$$

$$= 22.5\angle{-30^\circ} - 22.5\angle{+90^\circ}$$

$$= 22.5\ (\cos 30^\circ - j \sin 30^\circ) - 22.5\ (\cos 90^\circ + j \sin 90^\circ)$$

$$= 22.5(0.866 - j0.5) - 22.5(0 + j)$$

$$= 19.485 - j11.25 - j22.5 = 19.485 - j33.75 = 38.97\angle{-60^\circ}\ \text{A}$$

$$\therefore \quad I_a = I_{ao} + I_{r_1} + I_{a\Delta}$$

$$= 45\angle 0^\circ + 6.5\angle{-30^\circ} + 38.97\angle{-60^\circ}$$

$$= 45 + 5.629 - j3.25 + 19.485 - j33.75$$

$$= (70.114 - j37)\ \text{A} = 79.28\angle{-27.82^\circ}\ \text{A}.$$

7.50 A 2500 HP, star connected three-phase induction motor is connected across a 11 KV (line to line) 50 Hz balanced supply. A delta connected capacitor bank of 400 KVAR is connected across the motor. If the motor produces an output power of 2000 HP at an efficiency of 95% and operates at a power factor of 0.9 (lag), calculate the following:

(a) The input active and total power to the motor.
(b) The reactive power input to the motor.
(c) The reactive power supplied by the source.
(d) The apparent power supplied by the source.
(e) The motor line current.
(f) The impedance of the motor per phase.
(g) Line current drawn by capacitor bank.

Solution

Rated power of the motor = 2500 HP = 2500 × 0.746 = 1865 kW,
while the operating power = 2000 × 0.746 = 1492 kW

$$\therefore \text{ Active operating power input } (P_{\text{in}}) = \frac{1492}{\text{efficiency}} = \frac{1492}{0.95} = 1570.53\ \text{kW} \qquad \text{(i)}$$

Total power absorbed by the motor

$$|S_{\text{in}}| = \frac{P_{\text{in}}}{\cos\phi} = \frac{1492}{0.9} = 1657.78\ \text{KVA} \qquad \text{(ii)}$$

Reactive power input to the motor is obtained as

$$Q_{\text{in}} = \sqrt{|S_{\text{in}}|^2 - |P_{\text{in}}|^2}$$

$$= \sqrt{(1657.78)^2 - (1570.53)^2} = 530.72\ \text{KVAR}. \qquad \text{(iii)}$$

Since the capacitor bank supplies 400 KVAR hence the source would supply (530.72 – 400) 130.72 KVAR. (iv)

Apparent power supplied by the source is

$$|S| = \sqrt{(1570.53)^2 + (130.72)^2} = 1575.96\ \text{KVA} \qquad \text{(v)}$$

Motor line current is

$$|I_m| = \frac{|S_{\text{in}}|}{\sqrt{3}\,|V_L|} = \frac{1657.78 \times 10^3}{\sqrt{3} \times 11000} = 87\ \text{A}. \qquad \text{(vi)}$$

Impedancce of motor/phase is obtained as

$$Z_{\text{in(Ph)}} = \frac{|V_{\text{Ph}}|}{|I_{\text{Ph}}|} = \frac{11000/\sqrt{3}}{87} = 73\ \Omega. \tag{vii}$$

[$\because$ in star connection $|V_{\text{ph}}| = \sqrt{3}\,|V_L|$ and $|I_{\text{ph}}| = |I_L|$]

[Note that this is not static impedance and it changes with motor loading]
The current drawn by the capacitor bank is given by

$$I_C = \frac{Q_c/3}{|V_{\text{Ph}}|} = \frac{(400/3)10^3}{11000/\sqrt{3}} = 21 \text{ A per phase} \tag{viii}$$

$\therefore \qquad I_{C(\text{line})} = \sqrt{3} \times 21 = 36.36$ A.

Figure 7.29 represents the schematic of the system.

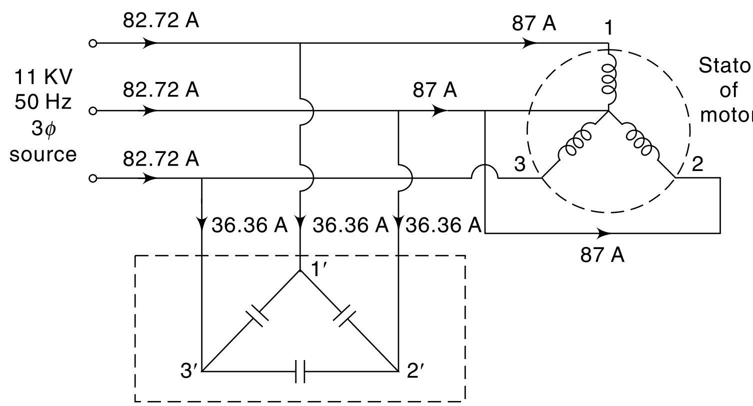

Fig. 7.29 *Delta connected capacitor bank (Ex. 7.50)*

It may be noted here that it is erroneous if one wishes to add the motor current and the capacitor current algebraically to get the supply current. On the other hand, the addition should be done vectorially. If we want to draw the phasor diagram, we should take care of the location of the respective phasors (ref. Fig. 7.30)

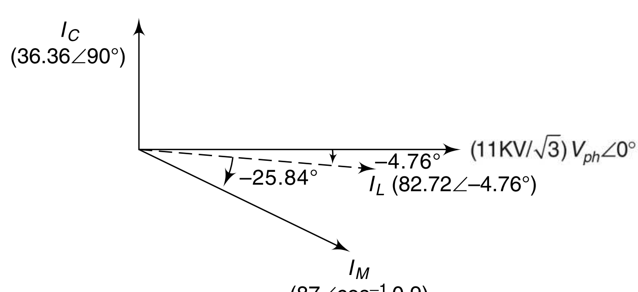

Fig. 7.30 *Phasor diagram of system in Fig. 7.29*

I_L, the source current can be obtained as

$$|I_L| = \frac{|S|}{\sqrt{3}\,V_L} = \frac{1575.96}{\sqrt{3}\times 11} = 82.72 \text{ A}$$

while the phase angle lag can be calculated from,

$$\cos\phi = \frac{P_{\text{in}}}{|S|} = \frac{1570.53}{1575.96} = 0.9966$$

i.e. $$\phi = \cos^{-1}(0.9966) = 4.76°.$$

7.51 A balanced delta connected three-phase load of 30 kW is fed from a 400 V 50 Hz three-phase source. The load power factor is 0.7 (lag). It is desired to improve the power factor to 0.85 (lag) by using three delta connected capacitors. Obtain the value of the resultant current drawn from the supply and the capacitance of each capacitor.

Solution

$\because$ $$P = \sqrt{3}\,|V_L|\,|I_L|\cos\theta,$$

In this case $$|I_L| = \frac{30\times10^3}{\sqrt{3}\times400\times0.7} = 61.86 \text{ A.}$$

Since the load is Δ connected,

$\therefore$ $$|I_L| = \sqrt{3}\,|I_{\text{Ph}}|$$

In this problem, $|I_{\text{Ph}}| = (61.86)/\sqrt{3} = 35.72$ A.

Figure 7.31(a) represents the schematic diagram of the system after the capacitors are connected in Δ across the load in order to improve the p.f. from 0.7 to 0.85 (lag). Figure 7.32(b) rerpresents the circuit diagram per phase basis while Fig. 7.31(c) represents the phasor diagram on per phase basis.

It is evident that with connection of capacitors the power factor is improved from 0.7 (lag) to 0.85 (lag).

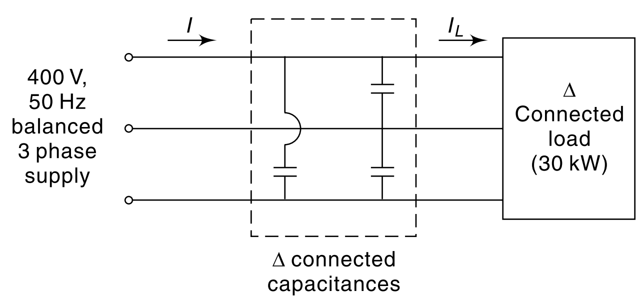

Fig. 7.31(a) *System of Ex. 7.51*

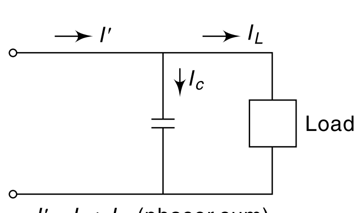

Fig. 7.31(b) *Circuit diagram (per phase) for Fig. 7.31(a)*

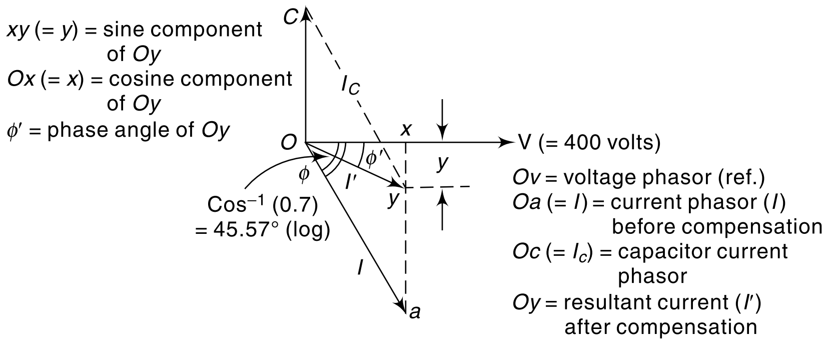

Fig. 7.31(c) ***Phasor diagram for Ex. 7.51***

From 7.31(c), we can write

$$I' \ (= Oy) = \frac{Ox}{\cos\phi'} = \frac{Oa\cos\phi}{\cos\phi'} = \frac{35.72 \times 0.7}{0.85}$$

$= 29.42$ A [∵ p.f. before adding capacitor is 0.7 and after adding capacitor is 0.85]

∴ The current drawn from the supply is 29.42 A after installation of capacitors at load terminals.

Again, $xy = I' \sin\phi' = 29.42 \times \sin(\cos^{-1} 0.85) = 15.5$ A

∴ $I_C = ay = xa - xy = I\sin\phi - xy = 35.72 \times \sin(\cos^{-1} 0.7) - 15.5 = 10$ A.

But $|I_C| = \dfrac{|V|}{|X_C|}$

∴ $X_C = \dfrac{|V|}{|I_C|} = \dfrac{400}{10} = 40\ \Omega$ [∵ capacitors are connected in Δ fachion, hence $|V_L| = |V_{ph}| = 400$ V]

or $\dfrac{1}{\omega C} = 40$

∴ $C = \dfrac{1}{2 \times \pi \times 50 \times 40} = 79.61\ \mu$F (for each capacitor.)

7.52 Two three-phase loads, one star and another in delta connection as shown in Fig. 7.32 are connected to a three-phase 400 V, 50 Hz balanced supply. Assuming $Z_1 = 2\angle 20°\ \Omega$; $Z_2 = 6\angle 75°\ \Omega$; $Z_3 = 30\angle 50°\ \Omega$ and $Z_A = 50\angle 30°\ \Omega$, $Z_B = 33.33\angle{-60°}\ \Omega$, $Z_C = 17.3\angle 90°\ \Omega$, find the current in y-phase. The phase sequence is $(r - y - b)$ (anti-clockwise).

Solution

Let us first see how many loads are connected from phase y. By observation we note that for the Δ load, Z_A is connected across phases $(y - r)$ and Z_B is connected across phases, $(y - b)$. If $I_{y(\Delta)}$ is the line current for Δ-connected load in line y, we can write

$I_{y(\Delta)}$ = Phasor sum of currents due to load across phases $(y - r)$ and $(y - b)$

$$= \frac{V_{y-r}}{Z_A} + \frac{V_{y-b}}{Z_B}$$

$$= \frac{-V_{r-y}}{Z_A} + \frac{V_{y-b}}{Z_B}$$

$$= \frac{-400\angle 0°}{50\angle 30°} + \frac{400\angle -120°}{33.33\angle -60°} = (-8\angle{-30°} + 12\angle{-60°})\text{ A}$$

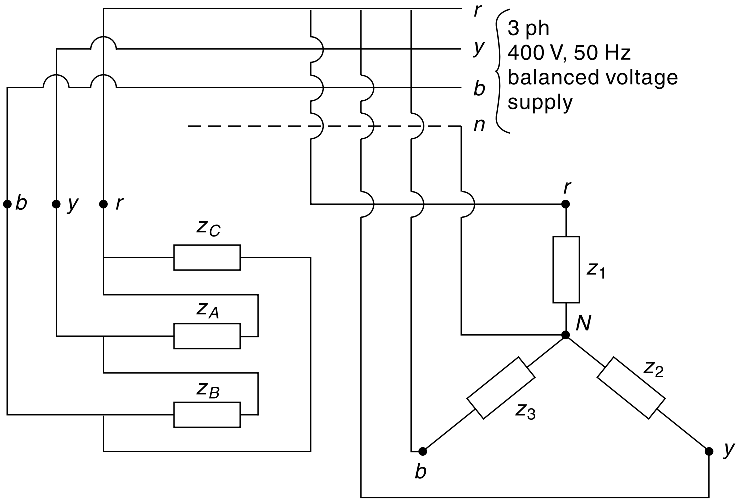

Fig. 7.32 *Circuit of Ex. 7.52*

It may be noted that line-y also supplies star load phase Z_2 for which

$$I_{y(Y)} = \frac{V_{y-n}}{Z_2} = \frac{(400/\sqrt{3})\angle -150°}{6\angle 75°} = 38.49\angle -75° \text{ A.}$$

[$\because$ V_{r-y} (= 400∠0°) is reference phasor and since V_{y-n} will be 120° phase lag than V_{r-n} while V_{r-n} itself lags V_{r-y} by 30°, it is evident that V_{r-n} lags V_{r-y} by 150°.]

$$\therefore \quad I_y \text{ (line current in } y \text{ phase)} = I_{y(\Delta)} + I_{y(Y)}$$
$$= -8\angle -30° + 12\angle -60° + 38.49\angle -75°$$
$$= 31.38\angle 146.4° \text{ A.}$$

7.53 A balanced three-phase load consists of impedances (4 + j3) Ω per phase and is connected to a 400 V source. Assuming V_{1-n} to be the reference phasor calculate the current per phase, power per phase and the total three phase power for a Y-connected load. Repeat the calculation for a Δ-connected load.

Solution

For star (Y) load:

Phase voltages are as follows

$$V_{1-n}\angle 0°,\ V_{2-n}\angle -120°,\ V_{3-n}\angle 120°$$

while line voltages would be

$$V_{1-2}\angle 30°,\ V_{2-3}\angle -90°,\ V_{3-1}\angle 150°.$$

$$\therefore \qquad I_{\text{Ph}_1} = \frac{V_{\text{Ph}_1}}{Z_{\text{Ph}_1}} = \frac{\frac{400}{\sqrt{3}}\angle 0°}{(4+j3)} = 46.2\angle -37° \text{ A}$$

Similarly, for phases 2 and 3, the phase currents would be

$$I_{\text{Ph}_2} = 46.2\angle -37° - 120° = 46.2\angle -157° \text{ A}$$

$$I_{\text{Ph}_3} = 46.2\angle -37° + 120° = 46.2\angle 83° \text{ A.}$$

The phasor diagram is shown is shown in Fig. 7.33. Since the voltages are balanced, the load and the phase currents are balanced. The line currents for the star connected load will be same to those of phase currents.

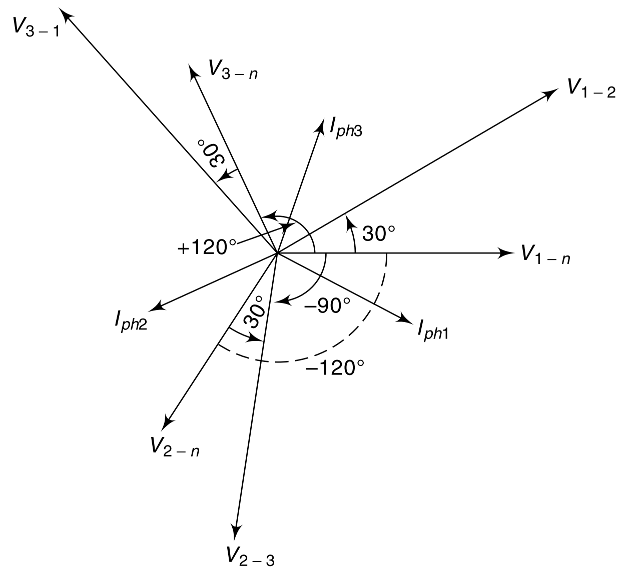

Fig. 7.33 ***Phasor diagram for the star connected system of Ex. 7.53***

The power per phase is obtained as

$$P_{\text{Ph}} = |V_{\text{Ph}}||I_{\text{Ph}}| \cos \phi = 231 \times 46.2 \times \cos(-37°) = 8.523 \text{ kW}.$$

The three-phase power would be

$$P_{3\phi} = 3 \times P_{\text{Ph}} = 3 \times 8.523 = 25.569 \text{ kW}$$

[Check: $P_{3\phi} = \sqrt{3}\,|V_L||I_L| \cos \phi = \sqrt{3} \times 400 \times 46.2 \times \cos(-37°) = 25.563$ kW.]

For Delta (Δ) load

Here the line voltage is equal to the phase voltges. The voltages are as follows:

$$V_{1-2}\angle 30°,\ V_{2-3}\angle -90°,\ V_{3-1}\angle 150°.$$

Phase currents are obtained as

$$I_{\text{Ph}_1} = \frac{V_{1-2}\angle 30°}{(4+j3)} = \frac{400\angle 30°}{5\angle 37°} = 80\angle -7° \text{ A}$$

Similarly, $I_{\text{Ph}_2} = 80\angle -127°$ A

$$I_{\text{Ph}_3} = 80\angle +113° \text{ A}.$$

The phasor diagram is shown in Fig. 7.34.

ϕ, the angle between the voltage and current is found to be [–30° – (7°)] i.e., (–37°).

$\therefore$ $P_{\text{ph}} = |V_{\text{Ph}}||I_{\text{Ph}}| \cos \phi = 400 \times 80 \times \cos(-37°) = 25.556$ kW.

and three-phase power is

$$P_{3\phi} = 3 \times 25.556 = 76.668 \text{ kW}.$$

[Check: $P_{3\phi} = \sqrt{3}\,|V_L||I_L| \cos \phi$

$$= \sqrt{3} \times 400 \times (80 \times \sqrt{3}) \times \cos(-37°) = 76.668 \text{ kW.}]$$

It may be noted here that we can calculate the reactive powers also drawn by the load for both star and delta connection using the formulae

$$Q_{3\phi} = \sqrt{3}\,|V_L||I_L| \sin \phi$$

In star connection $V_L = 400$ V, $I_L = 46.2$ A, $\phi = -37°$.

In delta connection $V_L = 400$ V, $I_L = (\sqrt{3} \times 80)$ A, $\phi = -37°$.

The total power in each case is given by $|S| = \sqrt{3}\,|V_L||I_L|$.]

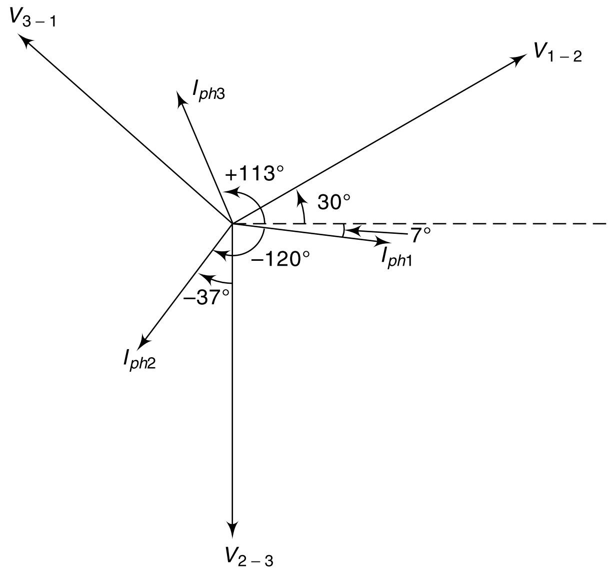

Fig. 7.34 ***Phasor diagram of the delta connected system of Ex. 7.53***

7.54 A 415 V, three-phase, three-wire supply has a phase sequence *RYB*. The following loads are connected in delta across the supply as shown below:

L_1: 6 kW load at upf between lines ($R - Y$)

L_2: 4.5 kW load at 0.8 p.f. (lag) between lines ($Y - B$)

L_3: 2.7 kW load at 0.5 p.f. (lead) between lines ($B - R$).

Assuming V_{RY} as the phasor, find all the three phase currents and line currents.

Solution

Since V_{RY} is reference, we can write for L_1

$$V_{RY} I_{RY}^* = 6000 + j0$$

or $$415\, e^{j0} \times I_{RY}^* = 6000$$

$\therefore$ $$|I_{RY}| = 14.458 \text{ A } (= I_1). \quad \text{(i)}$$

Next we take L_2.

Here, $$V_{YB} I_{YB}^* = \left(\frac{4500}{0.8}\right) e^{j[\cos^{-1}(0.8)]}$$

or $$415 e^{-j120} \times I_{YB}^* = 5625\, e^{j36.86^\circ}$$

$\therefore$ $$I_{YB} = 13.554^\circ\, e^{-j156.87^\circ} = (-12.464 - j5.324) \text{ A}$$

or $$I_2 = (-12.464 - j5.324) \text{ A}.$$

For L_3, we can write,

$$V_{BR} I_{BR}^* = \left(\frac{2700}{0.5}\right) e^{-j\cos^{-1}(0.5)} \quad \text{(ii)}$$

or $$415\, e^{j120} \times I_{BR}^* = 5400\, e^{-j\cos^{-1}(0.5)} = 5400\, e^{-j60^\circ}$$

$\therefore$ $$|I_{BR}| = -13.012 \text{ A } (= I_3).$$

[It may be noted here that in equation (i), the power of the exponential term in right hand side is +ve while that for equation (ii) is –ve because in the former case the p.f. is lagging while for the latter case p.f. is leading.]

We can solve this problem by another method as described below:

Since $\quad I_{RY}\angle\phi_{RY} \times V_{RY}\angle 0° = P_{RY}/\cos\phi_{RY}$

[where ϕ_{RY} is p.f. angle for load connected between $(R - Y)$, which is obviously zero as the load p.f. of load P_{RY} is unity (given). Also, V_{RY} is the reference phasor and P_{RY} is the connected load of 6 kW (L_1). ($P_{RY}/\cos\phi_{RY}$) represents the total power i.e., the VA power]. We can now write,

$$|I_{RY}| = \frac{P_{RY}/\cos\phi}{V_{RY}\angle 0°} = \frac{6000/1}{415\angle 0°} = 14.458 \text{ A } (= I_1)$$

[Please note that, I_{RY} is the current phasor with magnitude of 14.458 A and angle zero.]
Similarly, for L_2 we can write

$$I_{YB}\angle{-\phi_{YB}} \times V_{YB}\angle{-120°} = \frac{P_{YB}}{\cos\phi_{YB}}$$

where ϕ_{YB} is p.f. angle for load L_2 connected between Y and B. V_{YB} is the line voltage (i.e., the phase voltage across the delta load) for line $(Y - B)$ (at 120° lagging angle to V_{RY}).

As per given values, $\phi_{YB} = \cos^{-1}(0.8) = 36.87°$ (lag). It may be noted that I_{YB} lags V_{YB} by 36.87°.

$$\therefore \quad I_{YB}(\cos 36.87° - j\sin 36.87°) = \frac{P_{YB}/\cos\phi_{YB}}{V_{YB}\angle -120°}$$

$$= \frac{4500}{0.8 \times 415\angle -120°}$$

or $\quad I_{YB} = (-12.464 - j5.324) \text{ A } (= I_2)$.

For L_3 we can write

$$I_{BR}\angle{+\phi_{BR}} \times V_{BR}\angle{+120°} = \frac{P_{BR}}{\cos\phi_{BR}} \qquad \text{[p.f. is 0.5 lead hence } \phi_{BR} \text{ is +ve]}$$

or $\quad I_{BR}\angle 60° = \dfrac{2700/0.5}{415\angle +120°}$

$\therefore \quad I_{RB} = 13.01\angle{-180°} = 13.012(\cos 180° - j\sin 180°) = -13.012 \text{ A } (= I_3)$.

Thus we have obtain phase currents of the Δ load as

$$I_1 = 14.458\angle 0° \text{ A} = (14.458 + j0) \text{ A}$$
$$I_2 = (-12.464 - j5.324) \text{ A}$$
$$I_3 = (-13.012 - j0) \text{ A}.$$

The line currents can now be obtained as

$$I_R = I_1 - I_3 = 14.458 + j0 + 13.012 + j0 = 27.47 \text{ A}$$
$$I_Y = I_2 - I_1 = -12.464 - j5.324 - 14.458 - j0 = (-26.922 - j5.324) \text{ A}$$
$$I_B = I_3 - I_2 = (-13.012 - j0 + 12.464 + j5.324) \text{ A} = (-0.548 + j5.324) \text{ A}.$$

EXERCISES

1. A three-phase four wire 208 V, system supplies a star connected load in which $Z_A = 10\angle 0°\ \Omega$, $Z_B = 15\angle 30°\ \Omega$ and $Z_C = 10\angle{-30°}\ \Omega$. Find the line currents, the neutral current and the load power.
   [*Ans:* 12∠90° A, 8∠–60° A, 12∠–120° A, 5.69∠69.4° A, 3519 W]
2. Calculate the active and reactive components for the current in each phase of a star connected 5000 V, three-phase alternator supplying 3000 kW at power factor 0.8. [*Ans:* 346.2 A, 260 A]

3. Three similar coils of resistance 9 Ω and reactance 12 Ω are connected in delta to a three-phase, 440 V, 50 Hz supply. Find the line current, power factor, total KVA and total kW. [*Ans:* 50.8 A, 0.6, 38.7 KVA, 23.23 kW]
4. For the unbalanced delta connected load shown in Fig. 7.35 find the phase currents, line currents and the total power consumed by the load.
[*Ans:* $10\angle{-53.8°}$, $10\angle{-156.5°}$, $20\angle 156.5°$, $29.1\angle{-33.2°}$, $15.73\angle 165.3°$, $14.94\angle 52.3°$ 3000 W]

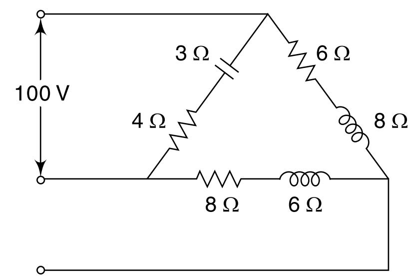

Fig. 7.35

5. A balanced three-phase load consists of three coils each of resistance 4 Ω and inductance 0.02 H. Determine the total power when the coils are
   (i) star connected and
   (ii) delta connected to a 440 V, 3 phase, 50 Hz supply
[*Ans:* 13.99 kW; 41.97 kW]

[*Hint:* $Z = 4 + j(2\pi \times 50 \times 0.02) = (4 + j6.28) = 7.44\underline{|57.5°}\ \Omega$

$\cos\theta = \frac{R}{Z} = \frac{4}{7.44} = 0.5376.$

(i) Star connection

$$V_L = 440 \text{ V} \therefore V_{Ph} = \frac{440}{\sqrt{3}} \text{ V}$$

$$|I_{Ph}| = \frac{440\sqrt{3}}{7.44} = 34.14 \text{ A} = |I_L|$$

$$P = \sqrt{3}\,|V_L|I_L \cos\theta = \sqrt{3} \times 440 \times 34.14 \times 0.5376$$
$$= 13987.855 \text{ W} = 13.988 \text{ kW}.$$

(ii) Delta connection

$$|V_L| = |V_{Ph}| = 440 \text{ V}$$

$$|IPh| = \frac{440}{7.44} = 59.14 \text{ A}$$

$$\therefore \quad P = 3\,V_{Ph}\,I_{Ph}\cos\theta = 3 \times 440 \times 59.14 \times 0.5376$$
$$= 41967.48 \text{ W} = 41.97 \text{ kW}.$$

6. Three equal impedances of $(8 + j12)$ Ω are connected in star across 415 V, 3 phase, 50 Hz supply. Calculate (i) line current (ii) Power factor) (iii) Active and reactive power drawn by the load.
[*Ans:* 6629 W; 9935.28 VAR]

[Hint: $Z = (8 + j12) = 14.42\underline{|56.32°}\ \Omega$

$$|V_L| = 415 \text{ V};\ |V_{Ph}| = \frac{415}{\sqrt{3}} \text{ V}$$

(i) Line current = phase current = $\frac{V_{Ph}}{Z} = \frac{\frac{415}{\sqrt{3}}}{14.42}$ A = 16.616 A.

(ii) Power factor $\cos\theta = \frac{R}{Z} = \frac{8}{14.42} = 0.555.$

(iii) Active power $\sqrt{3} \times 415 \times 16.616 \times 0.555 = 6628.7$ W.

Reative power = $\sqrt{3} \times 415 \times 16.616 \sin (\cos{-1}0.555)$.

= 9935.28 VAR

7. In a balanced three-phase system load 1 draws 60 kW and 80 KVA leading while load 2 draws 160 kW and 120 KVAR lagging. If line voltage of the supply is 1000 V find the line current drawn by each load.

[*Ans:* 57.8 A, 115.5 A]

8. In the network shown in Fig. 7.36 three resistors are connected in star to a three-phase supply of 400 V. The wattmeter *W* is connected as shown. Calculate the currents in the three lines and the readings of the wattmeter.

[*Ans:* 15.9 A, 13.1 A, 9.8 A, 5.82 kW]

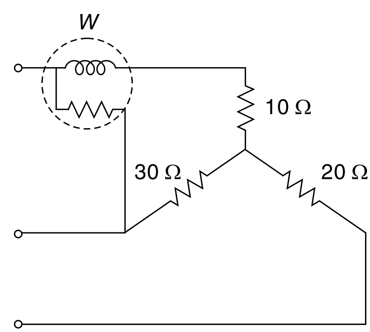

Fig. 7.36

10. A balanced three-phase star connected load draws 10 kW from a three-phase balanced systems of 400 V, 50 Hz while the line current is 75 A (leading). Find the circuit elements of the load.

[*Ans:* 0.6 Ω, 1083 μF]

11. An inductive motor draws a three phase power. Two wattmeter method is applied to find the total power. If W_1 = 10 kW, W_2 = 5 kW, find the total three phase active power, reactive power and power factor.

[*Ans:* 15 kW, 8.66 KVAR, 0.866]

12. A delta connected load has following impedances: $Z_{RY} = j10\ \Omega$, $Z_{YB} = 10\angle 0°\ \Omega$, $Z_{BR} = -j10\ \Omega$. If the load is connected across a three phase 100 V supply find the line currents.

[*Ans:* 8.66 – *j*5 A, –5 + *j*1.34 A, –3.66 + *j*3.6 A]

13. The power in a three-phase circuit is measured by two wattmeters. If the total power is 50 kW, power factor being 0.8 leading, what will be the reading of each wattmeter? For what p.f. will one of the wattmeter reading will be zero? [*Ans:* 35.825 W, –14.175 W, 0.5]

14. Derive the relation between phase and line voltages and currents for (i) star connected load (ii) delta connected load across a three-phase balanced system.

15. Show that sum of three emf's is zero in a three-phase balanced ac circuit.

16. Show that the power in a three-phase circuit can be measured using 2 wattmeters. Draw the circuit diagram and vector diagram.

17. What are the advantages of a polyphase system over the single-phase system?

8 TRANSFORMERS

8.1 DEFINITION

A transformer may be defined as a static electric device that transfers electrical energy from one circuit to another circuit at the same frequency but with changed voltage (or current or both) through a magnetic circuit.

8.2 PRINCIPLE OF OPERATION

When alternating voltage V_1 is applied to the primary winding of a transformer a current (termed as exciting current, I_ϕ) flows through it as shown in Fig. 8.1. The exciting current produces an alternating flux (ϕ) in the core, which links with both the winding (primary and secondary). According to Faraday's laws of electromagnetic induction, the flux will cause self-induced emf E_1 in the primary and mutually induced emf E_2 in the secondary winding. But according to Lenz's law primary induced emf will oppose the applied voltage and in magnitude this primary induced emf is (almost) equal to the applied voltage. Therefore, in brief we can say emf induced in the primary winding is equal and opposite to the applied voltage.*

When a load is connected to the secondary side, current will start flowing in the secondary winding. Voltage induced in the secondary winding is responsible

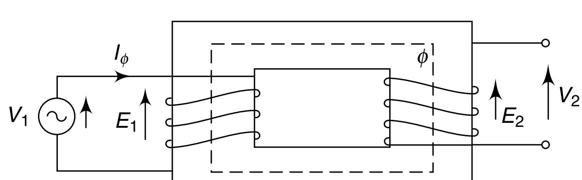

Fig. 8.1 ***Schematic diagram of single phase transformer***

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*If all the losses are neglected, the transformer is said to be ideal and hence for open circuited secondary, we can write $|V_1| = |E_1|$; $|V_2| = |E_2|$.

to deliver power to the load connected to it. In this way power is transferred from one circuit (primary) to another (secondary) winding through a magnetic circuit by electromagnetic induction. This is the working principle of the transformer. The induced emf in the secondary E_2 is also in phase opposition to the applied voltage V_1 at primary. If the secondary is open circuited, terminal voltage V_2 at the secondary is equal in magnitude and in phase with the induced emf at secondary.

8.3 EMF EQUATION

Since the applied voltage is sinusoidal at the primary, the flux produced by the exciting current is also sinusoidal (asuming $\phi \propto I$).

Thus core flux is given by $\phi = \phi_{max} \sin \omega t$. If the coil has N turns then instantaneous value of the induced emf e is given by

$$e = -N\frac{d\phi}{dt}$$

or $$e = -N\frac{d}{dt}(\phi_{max} \sin \omega t)$$

or $$e = -2\pi f\, \phi_{max}\, N \cos \omega t \text{ V} \qquad (\because \quad \omega = 2\pi f)$$

The maximum value of the induced emf will be obtained when ($\cos \omega t$) is 1.

i.e . $$E_{max} = 2\pi f\, \phi_{max}\, N \text{ V} \tag{8.1}$$

Dividing both sides of equation (8.1) by $\sqrt{2}$, we have

$$\frac{E_{max}}{\sqrt{2}} = \frac{2\pi}{\sqrt{2}}\, \phi_{max}\, f\, N$$

or $$E_{rms} = 4.44\, \phi_{max}\, f\, N \text{ V.} \tag{8.2}$$

If N_1 be the primary number of turns, then the rms values of induced voltage at primary is given by

$$E_1 = 4.44\, \phi_{max}\, f\, N_1 \text{ V.} \tag{8.2a}$$

(As the induced voltage in the primary winding is equal and opposite to the applied voltage, so $V_1 = 4.44\, \phi_{max}\, f\, N_1$ V).

Similarly, the rms value of the induced emf at secondary is obtained as

$$E_2 = 4.44\, \phi_{max} f\, N_2 \text{ V} \tag{8.2b}$$

Thus for a single phase *ideal* transformer, the expressions for the induced voltages at the primary as well as at the secondary windings can be obtained from Eqns (8.2a) and (8.2b). In these equations V denotes voltage.

8.4 CONSTRUCTION OF SINGLE-PHASE TRANSFORMERS

A single-phase transformer consists of primary and secondary windings placed on a *magnetic core*. The magnetic core is a stack of thin silicon steel laminations (CRGO steel). The laminations reduce eddy current loss and silicon steel reduces hysteresis loss. There are two general types of transformers, *core type* and *shell type*.

In core type transformers, the windings surround a considerable part of the steel core. The core consists of two vertical legs (or *limbs*) and the horizontal portions (called *yokes*) as shown in Fig. 8.2. For reduction of the leakage flux

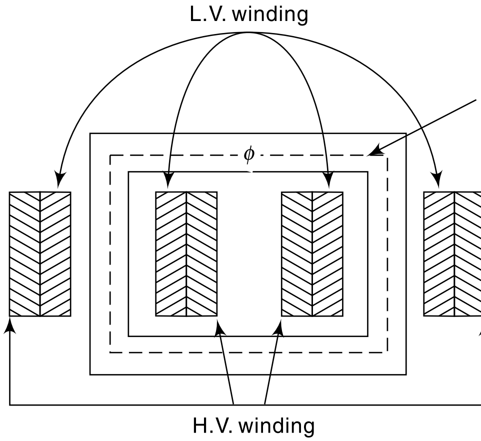


Fig. 8.2 *Core type transformer*

half of each winding is placed on each leg of the core. The low voltage winding is placed usually adjacent to the steel core and high voltage is placed outside in order to minimise the amount of insulation required.

In shell type transformers the steel core surrounds a major part of the winding as shown in Fig. 8.3. The low voltage and high voltage windings are wound over the central limb and are *interleaved* (or) *sandwiched*. The shell type transformer requires more conductor material as compared to core type transformer.

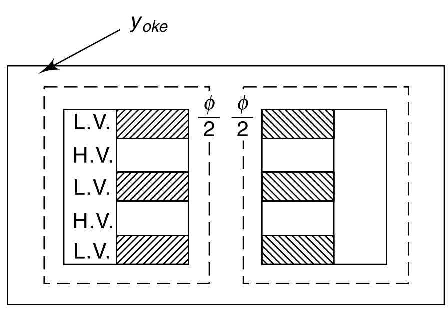


Fig. 8.3 *Shell type transformer*

In core type transformers the flux has a single path around the legs whereas in shell type transformers the flux in the central limb divides equally and returns through the outer two legs. Concentric coils are used for core type transformers and interleaved (or sandwiched) coils are used for shell type transformers.

8.5 TRANSFORMATION RATIO (OR TURNS RATIO)

Let N_1 = Number of turns in the primary winding
N_2 = Number of turns in the secondary winding
E_1 = RMS value of the primary induced emf
E_2 = RMS value of the secondary induced emf.

Using the emf equation, we can write

$$E_1 = 4.44\, f\, N_1\, \phi_m \text{ and } E_2 = 4.44\, f\, N_2\, \phi_m$$

$$\therefore \qquad \frac{E_1}{E_2} = \frac{N_1}{N_2} \tag{8.3}$$

Thus the ratio of primary voltage to secondary voltage is same as the ratio of primary winding turns to the secondary winding turns. The ratio N_1/N_2 is known as the *transformation ratio* (or *turns ratio*). It is usually denoted by K). By selecting this ratio properly, transformation can be done from any input voltage to any convenient output voltage. There can be two cases:

(a) If $N_1 > N_2$, then $E_2 < E_1$; the transformer is known as a *step-down* transformer $(k > 1)$.

(b) If $N_2 > N_1$, then $E_2 > E_1$; the transformer is known as a *step-up* transformer $(k < 1)$.

Let us again consider a two-winding transformer. In the process of transforming electrical power from one voltage to the other, there occurs some losses in the transformer. These losses are actually very small as compared to the total amount of power handled by the transformer. If we neglect these losses for the time being, we must have the same power (volt-ampere) in the primary and in the secondary winding. If I_1 and I_2 are the currents in the primary and secondary windings of an ideal transformer (i.e having no losses), we should have

$$E_1 I_1 = E_2 I_2$$

[E_1I_1 and E_2I_2 are the primary and secondary powers (voltamperes)]

$$\text{or} \qquad \frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{1}{K} \left(= \frac{V_2}{V_1} \right) \tag{8.4}$$

Thus we find that *the current is transformed in the reverse ratio of the voltage*. If a transformer steps up the voltage, it steps down the current. If it steps down the voltage, it steps up the current.

8.6 IMPEDANCE TRANSFORMATION

In Fig. 8.4 an impedance Z_2 is connected across the secondary winding at its output. The primary winding is connected to a voltage source V_1. The number of turns in the two windings are assumed to be N_1 and N_2. Induced emfs E_1 and E_2 are in phase opposition to V_1. Since V_2 is the secondary terminal voltage, it is also in the opposite phase of V_1.

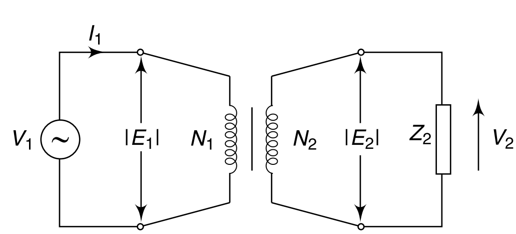


Fig. 8.4 *Schematic diagram of two-winding transformers*

Assuming the transformer to be ideal,

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = \frac{N_1}{N_2} = K \quad \text{(turns-ratio).}$$

The impedance Z_2', as seen from the input side, can be obtained by dividing voltage V_1 by I_1. Thus, we can write

$$Z_2' = \frac{V_1}{I_1} = \frac{V_1 \times (V_2 I_2)}{I_1 \times (V_2 I_2)}$$

$$= \left(\frac{V_1}{V_2}\right) \times \left(\frac{I_2}{I_1}\right) \times \left(\frac{V_2}{I_2}\right) = K \times K \times Z_2 = K^2 Z_2 \text{ i.e,}$$

$$\frac{Z_2'}{Z_2} = K^2. \tag{8.5}$$

Therefore, *impedance transformation ratio is equal to the square of turns ratio*. Referring to the primary or secondary side, this transferred impedance is known as the *equivalent impedance* on that side. In an ideal transformer thus we can note the following.

- Voltages are transformed in the direct ratio of turns. ($V_1/V_2 = K$)
- Currents are transformed in the inverse ratio of turns. ($I_1/I_2 = 1/K$)
- Volt-amperes of two sides are equal. ($V_1 I_1 = V_2 I_2$)
- Impedances are transformed in proportion to the square of turns-ratio. $\left(Z_2' = K^2 Z_2\,;\, Z_1' = \frac{1}{K^2} \times Z_1\right)$

8.1 Find the cross-sectional area of the core of a 10 turns transformer for a voltage of 50 V at 50 Hz. The flux density is 0.9 Wb/m^2.

Solution

Number of turns $N = 10$

Voltage $E = 50$ V

Frequency $f = 50$ Hz

Flux density $B = 0.9$ Wb/m^2.

If A be the cross-sectional area then

Flux $\phi = (0.9A)$ Wb

$\because$ $E = 4.44\, \phi f N$,

Here, $E = 4.44 \times 0.9\, A \times 50 \times 10 = 50$

Hence, $A = \dfrac{50}{4.44 \times 0.9 \times 500}$ m^2 = 0.025 m^2.

8.2 A single-phase transformer has 400 primary and 1000 secondary turns. The net cross sectional area of the core is 60 cm^2. The primary winding is connected to a 500 V supply. Find the (i) peak value of the core flux density and the (ii) emf induced in the secondary winding.

Solution

$N_1 = 400 \quad N_2 = 1000$

$A = 60$ cm^2 = 0.006 m^2

$E_1 = 500$ V (given)

$\because \quad E_1 = 4.44\ \phi_m f N_1,$

So $\quad \phi_m = \dfrac{500}{4.44 \times 50 \times 400} = 0.0056$ Wb.

(i) Peak value of core flux density $= \dfrac{0.0056}{0.006}$ Wb/m^2 = 0.938 Wb/m^2.

(ii) Emf induced in the secondary $E_2 = 4.44\ \phi_m f N_2$

$= 4.44 \times 0.0056 \times 50 \times 1000$ V = 1243.2 V.

8.3 The primary winding of a 50 Hz transformer is supplied from a 440 V, 50 Hz source and has 200 turns. Find the (i) peak value of flux (ii) voltage induced in the secondary winding if it has 50 turns.

Solution

$f = 50$ Hz

$E_1 = 440$ V

$N_1 = 200.$

(i) If ϕ_m is the peak value of flux then

$E_1 = 4.44\ f\ \phi_m\ N_1$

or $\quad \phi_m = \dfrac{440}{4.44 \times 50 \times 200}$ Wb = 0.0099 Wb.

(ii) $N_2 = 50$

Voltage induced in the secondary

$E_2 = 4.44\ f\ \phi_m\ N_2 = 4.44 \times 50 \times 0.0099 \times 50$ V = 110 V.

8.4 A 200 kVA single-phase transformer has 1000 turns in the primary and 600 turns on the secondary. The primary winding is supplied from a 440 V, 50 Hz source. Find the (i) secondary voltage at no load and (ii) primary and secondary currents at the full load.

Solution

Let primary and secondary currents at full load be I_1 and I_2.

Primary kVA = Secondary kVA = 200

$\therefore \quad E_1 I_1 = E_2 I_2 = 200 \times 10^3$ VA

$N_1 = 1000;\ N_2 = 600$

$E_1 = 440$ V.

(i) Now, $\dfrac{E_1}{E_2} = \dfrac{N_1}{N_2}$ or, $E_2 = \dfrac{E_1 N_2}{N_1} = 440 \times \dfrac{600}{1000} = 264$ V.

(ii) $I_1 = \dfrac{200 \times 10^3}{E_1} = \dfrac{200 \times 10^3}{440}$ A = 454.54 A.

$I_2 = \dfrac{200 \times 10^3}{E_2} = \dfrac{200 \times 10^3}{264}$ A = 757.57 A.

8.5 The emf per turn for a single-phase 440/220 V, 50 Hz transformer is approximately 15 V. Find (i) the number of primary and secondary turns and (ii) the net cross sectional area of the core, for a maximum flux density of 1 Wb/m^2.

Solution

$E_1 = 440$ V

$E_2 = 220$ V

$f = 50$ Hz.

Voltage per turn = 15 V.

(i) If N_1 and N_2 be the number of turns in the primary and secondary respectively,

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 15$$

$$\therefore \quad N_1 = \frac{E_1}{15} = \frac{440}{15} = 29.33.$$

As the number of turns cannot be a fraction so N_1 is taken as 30.

$$\therefore \quad \text{voltage per turn} = \frac{440}{30} = 14.67 \text{ V}$$

Also, $N_2 = \dfrac{E_2}{E_1} N_1 = \dfrac{220}{440} \times 30 = 15$

(ii) Flux density $B_m = 1$ Wb/m^2 (given).

If A be the cross-sectional area then, $\dfrac{E_1}{N_1} = 4.44(B_m A)f$, i.e. $4.44\, B_m A\, f = 14.67$

$$\therefore \quad A = \frac{14.67}{4.44 \times 1 \times 50} \text{ m}^2 = 0.066 \text{ m}^2.$$

8.6 A 400/50 V, 60 Hz step down transformer is to be operated at 50 Hz. Find (i) the highest safe input voltage and (ii) transformation ratio in both frequency applications.

Solution

$$E_1 = 400 \text{ V}$$
$$E_2 = 50 \text{ V}$$
$$f = 60 \text{ Hz}$$

$\because \quad E_1 = 4.44\, \phi_m f N_1,$

$\therefore \quad 4.44\, \phi_m N_1 = \dfrac{400}{60} = 6.67.$

(i) If E_1' be the highest safe input at 50 Hz;

$$E_1' = 4.44\, \phi_m N_1 \times 50 = 6.67 \times 50 = 333.5 \text{ V}.$$

(ii) Transformation ratio at 60 Hz

$$\frac{E_1}{E_2} = \frac{400}{50} = 8$$

At 50 Hz, the secondary induced emf is given by

$$E_2' = \frac{N_2}{N_1} E_1' = \frac{N_2}{N_1} \times E_1'$$

Now, $\dfrac{N_2}{N_1} = \dfrac{E_2}{E_1} = \dfrac{50}{400} = \dfrac{1}{8}$

So, $E_2' = \dfrac{1}{8} E_1'$

or $\dfrac{E_1'}{E_2'} = 8.$

Transformation ratio at 50 Hz is also 8.

8.7 A 200/50 V, 50 Hz transformer has a core area of 100 cm^2. The maximum value of the flux density is 1 Wb/m^2. Assuming 9% loss of area due to laminations, find the primary and secondary number of turns and transformation ratio.

Solution

$$A = 100 \times 10^{-4} \quad m^2 = 0.01 \ m^2$$

$$E_1 = 200 \text{ V}; \quad E_2 = 50 \text{ V}; \quad B_m = 1 \text{ Wb/m}^2$$

Assuming 9% loss of area, net area of core = $0.01 \times 0.9 \ m^2 = 0.009 \ m^2$

$$\text{Primary turns } N_1 = \frac{E_1}{4.44 f B_m A} = \frac{200}{4.44 \times 50 \times 1 \times 0.009} = 100$$

$$\text{Secondary turns } N_2 = \frac{E_2}{E_1} N_1 = \frac{50}{200} \times 100 = 25$$

$$\text{Transformation ratio } \frac{E_1}{E_2} = \frac{N_1}{N_2} = \frac{100}{25} = 4.$$

8.8 A 1000 kVA transformer has primary and secondary turns of 400 and 100 respectively and induced voltage in the secondary is 1000 V. Find (i) the primary volt (ii) the primary and secondary full load current and (iii) the secondary current when 100 kW load at 0.8 p.f. is connected at the output.

Solution

Given: kVA = 1000

$N_1 = 400$

$N_2 = 100$

$E_2 = 1000.$

(i) Primary voltage $E_1 = \frac{N_1}{N_2} E_2 = \frac{400}{100} \times 1000 = 4000$ V.

(ii) Primary full load current $I_1 = \frac{VA}{E_1} = \frac{1000 \times 10^3}{4000} = 250$ A

Secondary full load current $I_2 = I_1 \times \frac{N_1}{N_2} = 250 \times \frac{400}{100} = 1000$ A.

(iii) Secondary current at 100 kW and 0.8 p.f. load

$$I_2 = \frac{100 \times 10^3}{0.8 \times 1000} = 125 \text{ A}.$$

8.7 NO LOAD OPERATION OF A TRANSFORMER

A transformer is said to be on no load, if its primary winding is connected to an ac supply source and the secondary is open. The instantaneous flux (ϕ) linking with both the windings is given as $\phi = \phi_m \sin \omega t$.

Therefore, the induced emf in primary winding is given as

$$E_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt} (\phi_m \sin \omega t)$$

$$= -N_1 \ \omega \ \phi_m \cos \omega t = N_1 \ \omega \ \phi_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

Similarly, the induced emf in the secondary winding is given as

$$E_2 = N_2 \ \omega \ \phi_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

We consider the transformer to be ideal (i.e there are no voltage drops in the windings). E_1 and E_2 are in phase opposition to V_1.

It is thus evident that

- The induced emfs in primary and secondary windings (E_1 and E_2) lag behind the main flux ϕ by an angle $\pi/2$, and E_1 and E_2 are in the same phase with each other [as shown in the phasor diagram (Fig. 8.5)].
- Applied voltage to the primary winding V_1, leads the main flux ϕ by an angle $\pi/2$. Also it is in phase opposition to the induced emfs in the primary winding and secondary winding in ideal transformers. In ideal transformers there is no voltage drop in the secondary winding and hence $|V_2| = |E_2|$.

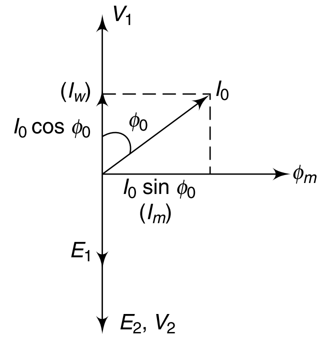


Fig. 8.5 ***Phasor diagram under no load condition ($N_2 > N_1$)***

- *The no load current* or exciting current I_o lags behind the applied voltage by an angle ϕ_o. It has two components I_m and I_w. The magnetising component I_m is in phase with the main flux ϕ, whereas, the other component I_w is in phase with the applied voltage. (This current is required to meet the hysteresis and eddy-current losses occurring in the core.)

  Thus, from the phasor diagram of Fig. 8.5, we have

$$I_o = \sqrt{I_m^2 + I_w^2}\ ;\ I_w = I_o \cos \phi_o,\ I_m = I_o \sin \phi_o$$

and $\phi_o = \tan^{-1} \dfrac{I_m}{I_w}$. (In practice, ϕ_o is close to 90° and is called *no load power factor angle*.)

When a transformer is connected to a supply, there actually occurs eddy-current loss and hysteresis loss in the iron-core and appear as heat. This power is taken from the ac supply at primary.

The no load current component I_m is used in magnetizing the core. (There is no power loss due to this current). The current I_m lags behind the applied voltage V_1 by $\pi/2$. The product of I_m and V_1 does not represent active power. This product is called the *reactive power*. Therefore, the input active power at no-load is

$$P_o = V_1 I_w = V_1 I_o \cos \phi_o \tag{8.6}$$

and, the reactive power is

$$W_o = V_1 I_m = V_1 I_o \sin \phi_o. \tag{8.7}$$

8.8 WORKING OF A TRANSFORMER ON LOAD

When the transformer is loaded, load current I_2 flows in the secondary winding. Secondary number of turns being N_2, the secondary ampere- turns is $I_2 N_2$; it sets up flux ϕ_2 in the core, which opposes the flux ϕ already set up by the no load current. As a result the flux linking with primary is reduced. The difference between applied voltage and induced voltage in the primary will however exist resulting in additional current in the primary (say of magnitude I_1'). The primary turns being N_1 these additional ampere-turns is $I_1' N_1$ and will produce flux ϕ_1 in the opposite direction of ϕ_2 (i.e. in the same direction of the original flux ϕ). Magnetic equilibrium will be achieved when $\phi_1 = \phi_2$, thus leaving behind the initial flux ϕ. Therefore from the above discussion it is evident that when a transformer is loaded the secondary ampere-turns necessitates the production of additional primary ampere-turns, which is equal in magnitude, to the secondary ampere-turns, but opposite in direction (Ref. Fig. 8.6).

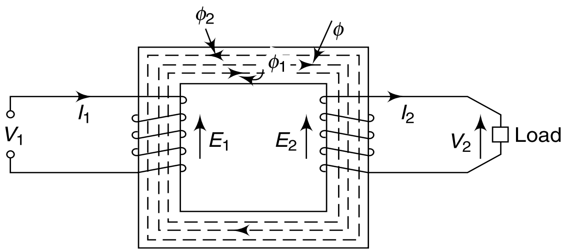


Fig. 8.6 ***Transformer under load***

The phasor diagram of the transformer on load can be drawn for different types of loadings and is explained below.

The no load phasor diagram is drawn and discussed earlier. Let us now consider load current I_2 while the load is of inductive nature. I_2 will be lagging the secondary voltage (i.e. E_2 or V_2). When there is no voltage drop in the transformer (transformer being ideal) then $|V_2| = |E_2|$. To counter balance the secondary ampere-turns additional primary current I_2' will flow and will be 180° out of phase of I_2. The total primary current I_1 is the phasor sum of the no load current I_o and of the additional primary current I_2'. Complete phasor diagram of the transformer on load for inductive load has been shown in Fig. 8.7(b). Figure 8.7(a) and (c) represent phasor diagrams no load when load is having unity power factor and leading power factor respectively.

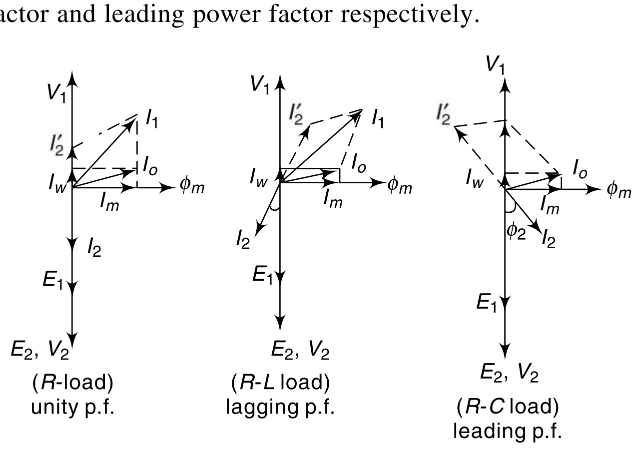


Fig. 8.7 ***Transformer phasor diagram at different p.f. neglecting transformer internal voltage drop ($N_2 > N_1$)***

Fig. 8.8(a) shows the phasor diagram of a transformer under lagging load power factor and considering internal voltage drop (the transformer is now not an ideal one). The voltage ($-E_1$) has been replaced by V_1' for convenience. Alternatively V_1' may be treated as a voltage drop in the primary in the direction of flow of primary current. The primary current I_1 flows through primary resistance R_1 and primary leakage reactance X_1. Hence primary voltage is given as

$$V_1 = V_1' + I_1(R_1 + jX_1), \text{ where } V_1' \text{ is } (-E_1).$$

Similary, $E_2 = V_2 + I_2(R_2 + jX_2)$ where R_2 and X_2 are the resistance and leakage reactance of the secondary side of the transformer.

Here, V_1 = the supply voltage (input voltage at primary)
E_1 = the induced voltage at primary
$V_1' = -E_1$ (phasor E_1 reserved to the primary side in the phasor diagram)
I_o = the no load (magnetising) current at primary ($= I_w + I_m$)
E_2 = the secondary induced voltage
V_2 = the output voltage, i.e. terminal voltage at the secondary
I_2 = the secondary load current
ϕ = the p.f. angle
I_2' = the referred secondary current to primary.

Figures 8.8(b) and 8.8(c) represent the phasor diagrams of the transformer (not an ideal one) operating with unity power factor and leading power factor respectively.

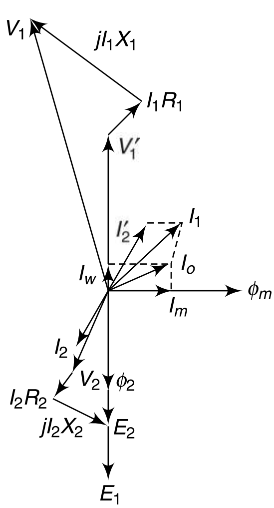


Fig. 8.8(a) ***Exact phasor diagram for lagging power factor I_1 = (primary current); R_1, X_1 and R_2, X_2 = primary and secondary winding resistance and reactances.***

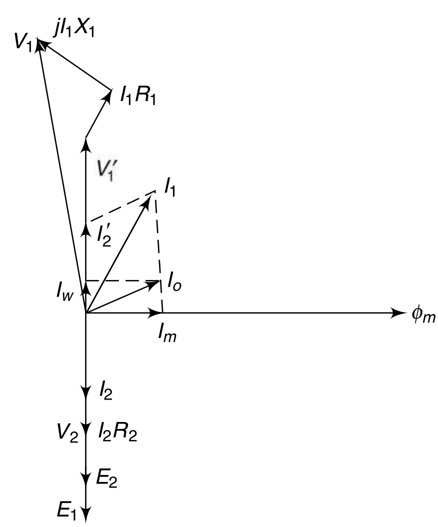


Fig. 8.8(b) ***Exact transformer phasor diagram for unity power factor [I_2 and V_2 in phase]***

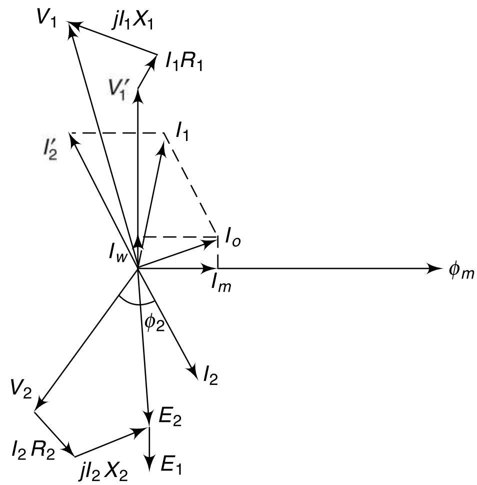


Fig. 8.8(c) ***Exact transformer phasor diagram for leading power factor [I_2 leads V_2]***

8.9 EQUIVALENT CIRCUIT OF TRANSFORMER

Though the ideal transformer winding does not have any resistance, in actual practice, there is always some resistance of the windings. This causes a voltage drop. The resistance of the primary winding is represented by R_1 and that of the secondary by R_2.

The actual transformer has another deviation from the ideal transformer. Not all of the flux produced by the primary winding links with the secondary winding in the actual transformer. Similarly, not all of the flux produced by the secondary winding links with the primary winding. The difference between the total flux linking with the primary winding and the mutual flux ϕ linking with both windings is called the *primary leakage flux*. Similarly, the *secondary leakage flux* can also be expressed.

We can write Kirchhoff's voltage equations for the primary and secondary sides of the transformer (Fig. 8.9) as

$$V_1 = I_1R_1 + jI_1 X_1 - E_1 = I_1(R_1 + jX_1) - E_1 \tag{8.8}$$

and

$$E_2 = I_2R_2 + jI_2 X_2 + V_2 = I_2(R_2 + jX_2) + V_2. \tag{8.9}$$

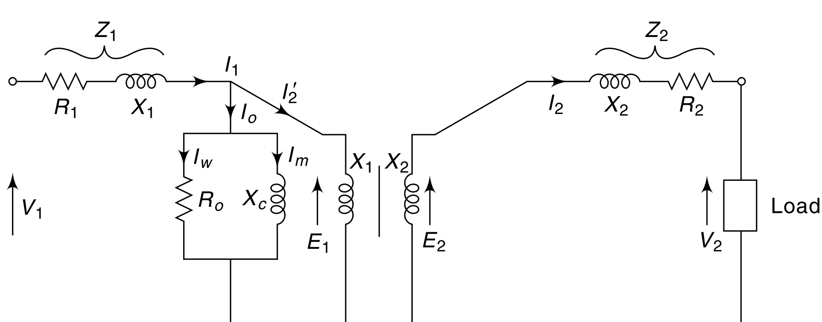


Fig. 8.9 ***Equivalent circuit of transformer***

The equivalent circuit gives the interpretation of the above equations. Further, we know the primary current I_1 is composed of two currents I_2' and I_o. Also, the current I_o consists of two components, I_m and I_w. Therefore, the current I_o can be considered to be split into two parallel branches. The current I_w accounts for the core-loss, and hence is shown to flow through resistance R_o. The current I_m represents the magnetising current and is shown to flow through a pure reactance X_o. This branch consisting of the parallel R_o and X_o is called the *magnetising branch* of the transformer.

Using the impedance transformation, we can draw the simplified equivalent circuit of a transformer, as referred to the primary side only or to the secondary side only.

We have seen earlier that an impedance connected across the secondary appears as K^2 times, when referred to the primary. (Here, $K = N_1/N_2$, where K is the transformation ratio). Therefore to simplify the equivalent circuit of Fig. 8.9, we can transfer the resistance R_2 and the reactance X_2 to the primary side, by simply multiplying each of them by K^2. The total resistance and the total reactance in the primary side then becomes

$$R_{o1} = R_1 + K^2R_2 \quad \text{and} \quad X_{o1} = X_1 + K^2X_2$$

where $R_2' = K^2R_2$ and $X_2' = K^2X_2$.

The equivalent circuit of the transformer now simply reduces to the one as shown in Fig. 8.10(a).

Here,

$$I_2' = \frac{1}{K}I_2$$
$$V_2' = KV_2$$
$$R_{o1} = R_1 + X_2'$$
$$R_2' = K^2R_2$$
$$X_{o1} = X_1 + X_2'$$
$$X_2' = K^2X_2$$
$$I_1 = I_o + I_2'.$$

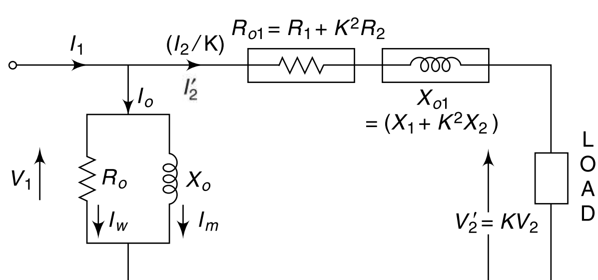


Fig. 8.10(a) ***Equivalent circuit of transformer referred to primary***

Similarly, the equivalent circuit as referred to secondary side can also be drawn. But in this case the equivalent resistance and reactance as referred to secondary side will be

$$R_{o2} = R_2 + (1/K^2)R_1 \text{ and } X_{o2} = X_2 + (1/K^2)X_1.$$

The equivalent circuit referred to the secondary is shown in Fig. 8.10(b). Here R_o' and X_o' represent the core resistance and the magnetizing reactance referred to the secondary.

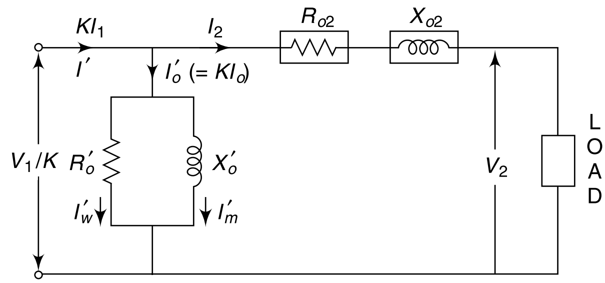


Fig. 8.10(b) *Equivalent circuit of transformer referred to secondary*

8.9.1 Approximate Equivalent Circuit

Since in a transformer the magnitude of I_o is very low (1 – 3% of the full load current), we can neglect the magnetising branch for simplicity. The equivalent circuit neglecting the magnetising branch is called the approximate equivalent circuit (Fig. 8.10(c) and Fig. 8.10(d)).

$$R_{o2} = R_2 + \frac{1}{K^2} R_1 \text{ ; } R_{01} = R_1 + K^2 R_2$$

$$X_{o2} = X_2 + \frac{1}{K^2} X_1 \text{ ; } X_{01} = X_1 + K^2 X_2$$

$$V_1' = \frac{1}{K} V_1 \text{ ; } V_2' = KV_2$$

$$I_1' = KI_1; I_2' = (I_2/K)$$

$$I_o' = KI_o$$

$$R_o' = \frac{1}{K^2} R_o$$

$$X_o' = \frac{1}{K^2} X_o$$

$$KI_1 = I_1' = KI_o + I_2$$

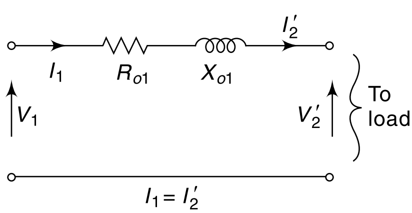


Fig. 8.10(c) *Approximate equivalent circuit of transformer referred to as primary*

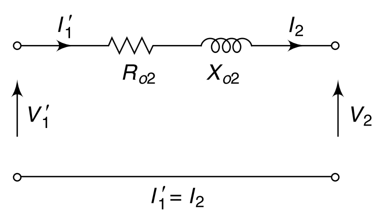


Fig. 8.10(d) *Approximate equivalent circuit of transformer referred to as secondary*

8.10 REGULATION OF A TRANSFORMER

The *regulation* of a transformer (generally expressed as percentage regulation) may be defined as

$$\left[\frac{\text{Secondary no load voltage} - \text{Secondary full load voltage}}{\text{Secondary no load voltage}}\right]\times 100$$

If E_2 = the secondary no load voltage

V_2 = the terminal voltage at secondary

$$\text{then percentage regulation} = \frac{E_2 - V_2}{E_2} \times 100 \tag{8.10}$$

Therefore percentage regulation of a transformer is defined as the *percentage decrease in the terminal voltage of the transformer from no-load to full load condition at a constant applied voltage*.

8.10.1 Expression for Regulation

Let us consider the equivalent circuit of the transformer referred to the secondary (as shown in Fig. 8.11)

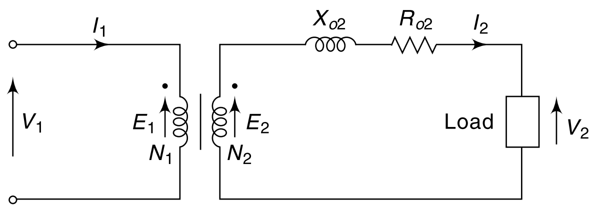


Fig. 8.11 *Approximate equivalent circuit of transformer referred to as secondary*

When the load is connected to the secondary side, current I_2 will start flowing. Depending upon the nature of the load, current I_2 may be lagging the voltage V_2 for inductive load, in phase of the voltage V_2 for resistive load and leading the voltage V_2 for capacitive load.

Hence on the basis of the above current-voltage phasor relation, the load has a lagging power factor, a unity power factor and a leading power factor respectively. Expressions for regulation in each case will be as discussed below.

(i) *Lagging power factor* The phasor diagram of the transformer referred to as the secondary, when supplying a load of lagging power factor load, has been shown in Fig. 8.12, where,

E_2 = the no load voltage

V_2 = the load voltage

I_2R_{o2} = the resistive drop referred to secondary

I_2X_{o2} = the reactive drop referred to secondary

and θ_2 = the angle between V_2 and I_2 i.e. cos θ_2 is p.f. of the load.

[$Oa = V_2$; $ab = I_2R_{o2} \cos \theta_2$; $bc \simeq I_2 X_{o2} \sin \theta_2$; $cd = (I_2 X_{o2}) \cos \theta_2 - I_2R_{o2} \sin \theta_2$]

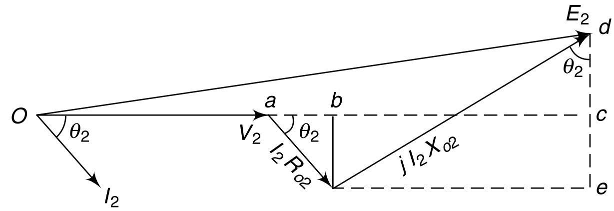


Fig. 8.12 ***Phasor diagram of a transformer on lagging load, referred to as secondary***

From Fig. 8.12

$$E_2^2 = (Oc)^2 + (cd)^2 = (Oa + ab + bc)^2 + (cd)^2$$
$$= (V_2 + I_2 R_{o2} \cos \theta_2 + I_2 X_{o2} \sin \theta_2)^2$$
$$+ (I_2 X_{o2} \cos \theta_2 - I_2 R_{o2} \sin \theta_2)^2$$

where I_2 is the secondary current lagging V_2 by angle θ_2.

As $(I_2 X_{o2} \cos \theta_2 - I_2 R_{o2} \sin \theta_2)$ is very small (being the difference of two quantities) it can easily be neglected.

Hence $\quad E_2 \simeq V_2 + I_2 R_{o2} \cos \theta_2 + I_2 X_{o2} \sin \theta_2$

or $\quad E_2 - V_2 = (I_2 R_{o2} \cos \theta_2 + I_2 X_{o2} \sin \theta_2). \quad (8.11)$

$$\text{Percentage voltage regulation} = \frac{E_2 - V_2}{E_2} \times 100\%$$

$$= \left(\frac{I_2 R_{o2}}{E_2} \cos \theta_2 + \frac{I_2 X_{o2}}{E_2} \sin \theta_2 \right) \times 100\% \quad (8.12)$$

$$= (\mathrm{R}_{\text{p.u}} \cos \theta_2 + X_{\text{p.u}} \sin \theta_2) \times 100\% \quad (8.13)$$

where $(R_{\text{p.u}})$ and $(X_{\text{p.u}})$ are the total p.u resistance and reactance respectively

$$\left[R_{\text{p.u.}} = \frac{I_2 R_{o2}}{E_2}; X_{\text{p.u.}} = \frac{I_2 X_{o2}}{E_2} \right]$$

(ii) *Unity power factor:* Phasor diagram at unity p.f. has been shown in Fig. 8.13. From Fig. 8.13 we have

$$E_2^2 = (V_2 + I_2 R_{o2})^2 + (I_2 X_{o2})^2 \quad (8.14)$$

If the second term is neglected

$$E_2 = V_2 + I_2 R_{o2} \quad (8.15)$$

{The second term $(I_2 X_{02})$ is neglected as it does not contribute much in changing the magnitude of V_2. On the other hand, it is responsible for the phase shift between E_2 and V_2. Hence we can reasonably neglect $(I_2 X_{02})$}.

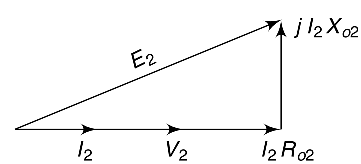


Fig. 8.13 ***Phasor diagram of the transformer for unity p.f. load.***

$$\text{Hence percentage voltage regulation} = \frac{E_2 - V_2}{E_2} \times 100\%$$

$$= \frac{I_2 R_{o2}}{E_2} \times 100\%$$

$$= R_{\text{p.u}} \times 100\% \quad (8.16)$$

Leading power factor: For leading power factor ($\cos\theta_2$),

$$I_2 = I_2\cos\theta_2 + j\,I_2\sin\theta_2$$

$$E_2 = V_2 + I_2 Z_{o2} = V_2 + j.0 + (I_2\cos\theta_2 + jI_2\sin\theta_2)(R_{o2} + j\,X_{o2})$$

or
$$E_2 = V_2 + I_2 R_{o2}\cos\theta_2 - I_2 X_{o2}\sin\theta_2 + j(I_2 R_{o2}\sin\theta_2 + I_2 X_{o2}\cos\theta_2)$$

Hence
$$E_2^2 = (V_2 + I_2 R_{o2}\cos\theta_2 - I_2 X_{o2}\sin\theta_2)^2 + (I_2 R_{o2}\sin\theta_2 + I_2 X_{o2}\cos\theta_2)^2.$$

The phasor diagram is shown in Fig. 8.14.

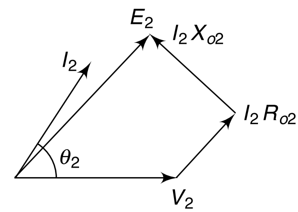


Fig. 8.14 ***Phasor diagram of transformer for leading power factor load.***

Now $(I_2R_{o2}\sin\theta_2 + I_2 X_{o2}\cos\theta_2)$ is very small compared to $(V_2 + I_2 R_{o2}\cos\theta_2 - I_2 X_{o2}\sin\theta_2)$. Hence $(I_2 R_{o2}\sin\theta_2 + I_2 X_{o2}\cos\theta_2)$ is neglected.

$\therefore\ E_2 = V_2 + I_2 R_{o2}\cos\theta_2 - I_2 X_{o2}\sin\theta_2$

or $E_2 - V_2 = I_2 R_{o2}\cos\theta_2 - I_2 X_{o2}\sin\theta_2$ (8.17)

Percentage voltage regulation is

$$\frac{E_2 - V_2}{E_2}\times 100\% = \left(\frac{I_2 R_{o2}}{E_2}\cos\theta_2 - \frac{I_2 X_{o2}}{E_2}\sin\theta_2\right)\times 100\% \tag{8.18}$$

$$= (R_{p.u}\cos\theta_2 - X_{p.u}\sin\theta_2)\times 100\% \tag{8.19}$$

8.11 CONDITION FOR ZERO (MINIMUM) REGULATION

We can use the expression of regulation to find the condition for which the regulation is zero. We can write at zero regulation,

$$I_2R_{o2}\cos\theta_2 + I_2 X_{o2}\sin\theta_2 = 0$$

or
$$\tan\theta_2 = -\frac{R_{o2}}{X_{o2}}. \tag{8.20}$$

The negative sign in the above condition indicates that zero regulation is possible at a leading power factor. Also, if the transformer is not loaded at all, $E_2 = V_2$ and this also gives zero regulation. Thus the regulation is zero if the transformer is open circuited or operated at a leading p.f. so that $\theta_2 = \tan^{-1}\dfrac{R_{o2}}{X_{o2}}$. Also from the expression of regulation, it is evident that for a leading power factor load if the magnitude of θ_2 is high, the magnitude of $(I_2 X_{o2}\sin\theta_2)$ would become more than that of $(I_2 R_{o2}\cos\theta_2)$. The regulation then may become negative. It means, on increasing the load the terminal voltage increases at leading power factor operation of the transformer.

8.11.1 Condition for Maximum Regulation

We can derive the condition for maximum regulation using the expression for regulation. The regulation will be maximum if the differentiation of regulation with respect to phase angle θ_2 is equal to zero. That is

$$\frac{d}{d\theta_2} = (I_2 R_{o2} \cos \theta_2 + I_2 X_{o2} \sin \theta_2) = 0$$

or $$- I_2 R_{o2} \sin \theta_2 + I_2 X_{o2} \cos \theta_2 = 0$$

or $$\tan \theta_2 = \frac{X_{o2}}{R_{o2}}. \tag{8.20}$$

Hence maximum regulation occurs only at lagging power factor and when $\theta_2 = \tan^{-1} \frac{X_{o2}}{R_{o2}}$.

8.9 A single-phase transformer has 200 and 100 turns respectively in its secondary and primary windings. The resistance of the primary winding is 0.05 Ω and that of the secondary is 0.3 Ω. Find the resistance of (i) the primary winding referred to the secondary, (ii) the secondary winding referred to the primary. Also find the equivalent resistance of the transformer referred to the primary.

Solution

Number of turns of primary winding $N_1 = 100$
Number of turns of secondary winding $N_2 = 200$
Resistance of primary winding $R_1 = 0.05\ \Omega$
Resistance of secondary winding $R_1 = 0.3\ \Omega$

(i) Resistance of primary winding referred to secondary

$$= R_1' = R_1 \left(\frac{N_2}{N_1}\right)^2 = 0.05 \times \left(\frac{200}{100}\right)^2 = 0.05 \times 4 = 0.2\ \Omega.$$

(ii) Resistance of secondary winding referred to primary

$$R_2' = R_2 \times \left(\frac{N_1}{N_2}\right)^2 = 0.3 \times \left(\frac{100}{200}\right)^2 = \frac{0.3}{4}\ \Omega = 0.075\ \Omega$$

Equivalent resistance of the transformer referred to the primary

$$R_{01} = R_1 + R_2' = 0.05 + 0.075 = 0.125\ \Omega.$$

[Also, $R_{02} = R_2 + R_1' = 0.3 + 0.2 = 0.5\ \Omega$].

8.10 A 20 kVA, 1000/200 V single-phase transformer has a primary resistance of 1 Ω and a secondary resistance of 0.2 Ω. Find the equivalent resistance of the transformer referred to the secondary and the total resistance drop on full load.

Solution

If N_1 and N_2 be the number of turns of the primary and secondary winding then $\frac{N_1}{N_2} = \frac{1000}{200}$.

Resistance of the primary winding $R_1 = 1\ \Omega$.
Resistance of the secondary winding $R_2 = 0.2\ \Omega$.
Total equivalent resistance in terms of the secondary winding is $(R_1' + R_2)$

i.e. $$R_{o2} = R_1 \left(\frac{N_2}{N_1}\right)^2 + R_2 = 1 \times \left(\frac{200}{1000}\right)^2 + 0.2 = 0.04 + 0.2 = 0.24\ \Omega.$$

Full load secondary current

$$I_2 = \frac{20 \times 10^3}{200} = 100 \text{ A.}$$

Total resistance drop on full load = $I_2 R_{o2}$ = 100 × 0.24 = 24 V.

8.11 A single-phase transformer has turns ratio of 8. The resistances of the high voltage and low voltage windings are 1.5 Ω and 0.05 Ω respectively and the reactances are 10 Ω and 0.5 Ω respectively. Find (i) the voltage to be applied to the high voltage side to obtain a full load current of 100 A on the low voltage winding on short circuit and (ii) the power factor on short circuit.

Solution

If N_H and N_L be the number of turns on the high voltage and low voltage windings then

$$\frac{N_H}{N_L} = 8 \text{ (given).}$$

Resistance of high voltage winding R_H = 1.5 Ω
Resistance of low voltage winding R_L = 0.05 Ω
Reactance of high voltage winding X_H = 10 Ω
Reactance of low voltage winding X_L = 0.5 Ω
Full load current on the low voltage side = 100 A.

$$\therefore \quad \text{full load current on the high voltage side } = 100 \times \frac{N_L}{N_H} = \frac{100}{8} = 12.5 \text{ A.}$$

Equivalent impedance referred to the high voltage side

$$= (R_H + R_L') + j(X_H + X_L')$$

$$= \{1.5 + 0.05(8)^2\} + j\{10 + 0.5(8)^2\} = 4.7 + j42 = 42.26\angle 83.6° \ \Omega.$$

(i) The voltage to be applied to the high voltage side to obtain full load current is 12.5 × 42.26 = 528.25 V

(ii) Power factor on short circuit is cos 83.6° = 0.111.

8.12 A 6600/440 V, 50 Hz single-phase transformer has high voltage and low voltage winding resistances of 0.5 Ω and 0.0007 Ω respectively and reactances of 2 Ω and 0.001 Ω respectively. Find the current and the input power when the high voltage winding is connected to a 220 V 50 Hz supply, the low voltage being short circuited.

Solution

$$\frac{N_H}{N_L} = \frac{6600}{440}, \qquad R_H = 0.5\ \Omega, \qquad R_L = 0.0007\ \Omega,$$

$$X_H = 2\ \Omega, \qquad X_L = 0.001\ \Omega.$$

Equivalent impedance referred to the high voltage side

$$Z_{eH} = R_H + R_L\left(\frac{N_H}{N_L}\right)^2 + j\left\{X_H + X_L\left(\frac{N_H}{N_L}\right)^2\right\}$$

$$= 0.5 + 0.0007\left(\frac{6600}{440}\right)^2 + j\left\{2 + 0.001\left(\frac{6600}{440}\right)^2\right\}$$

$$= 0.6575 + j2.225 = 2.32\angle 73.54° \ \Omega.$$

∴ Current in the high voltage side when low voltage is short circuited is $\frac{220}{2.32}$ A = 94.83 A.

Input power = 220 × 94.83 cos 73.54° = 5911 W = 5.9 kW.

8.13 The equivalent impedance of a 10 kVA, 220/440 V, single-phase, 50 Hz transformer referred to the low voltage side is (0.2 + j0.5) Ω. The core loss resistance and magnetizing reactance are 100 Ω and 150 Ω respectively, both referred to the low voltage side. If the high voltage current is 20 A at a lagging p.f. of 0.8 find the low voltage input current and the high voltage terminal voltage.

Solution

$$\left.\begin{aligned} R_{OL} &= 0.2\ \Omega \\ X_{OL} &= 0.5\ \Omega \end{aligned}\right\} \text{[Low voltage side resistance and reactance]}$$

$$\left.\begin{aligned} R_O &= 100\ \Omega \\ X_O &= 150\ \Omega \end{aligned}\right\} \begin{aligned}&\text{[Magnetizing branch resistance and reactance} \\ &\quad \text{at low voltage side]}\end{aligned}$$

$$\therefore \quad Z_{OL} = 0.5385\angle 68.2°\ \Omega.$$

High voltage current (I_H) = $20\angle -\cos^{-1}0.8 = 20\angle -36.86°$ V

The high voltage current referred to low voltage side = $I_H' = 20 \times \dfrac{440}{220} = 40$ A.

$$\text{The no load component of current } (I_O) = \frac{220}{100} - j\frac{220}{150} = (2.2 - j1.47) \text{ A}$$

$$\begin{aligned}\text{Input current on the low voltage side} &= 40(0.8 - j0.6) + 2.2 - j1.47 \\ &= 34.2 - j25.47 \\ &= 42.64\angle -36.67° \text{ A.}\end{aligned}$$

$$\begin{aligned}\text{High voltage side terminal voltage } (V_2) &= \{220 - 42.64\angle -36.67°(0.2 + j0.5)\}\,\frac{440}{220} \\ &= \{220 - 42.64(0.8 - j0.6)(0.2 + j0.5)\}\,\frac{440}{220} \\ &= \{220 - 42.64(0.46 + j0.28)\}\,\frac{440}{220} \\ &= (200.38 - j11.94\} \times 2 = 401.47\angle -3.41° \text{ V.}\end{aligned}$$

8.14 A 500 kVA, single-phase, 2000/200 V, 50 Hz. transformer has a high voltage resistance 0.2 Ω and a leakage reactance of 0.4 Ω. The low voltage winding resistance is 0.002 Ω and the leakage reactance is 0.008 Ω. Find (i) the equivalent winding resistance and reactance referred to the high voltage side and the low voltage side, (ii) the equivalent resistance and equivalent reactance drops in volts and in percent of the rated winding voltages expressed in terms of high voltage quantities.

Solution

(i) Equivalent winding resistance referred to the high voltage side

$$R_{o1} = R_1 + R_2\left(\frac{N_1}{N_2}\right)^2 \qquad = 0.2 + 0.002 \times \left(\frac{2000}{200}\right)^2 = 0.2 + 0.2 = 0.4\ \Omega.$$

Equivalent reactance referred to the high voltage side

$$X_{o1} = X_1 + X_2\left(\frac{N_1}{N_2}\right)^2 \qquad = 0.4 + 0.008 \times \left(\frac{2000}{200}\right)^2 = 0.4 + 0.8 = 1.2\ \Omega.$$

Equivalent resistance referred to low voltage side

$$R_{o2} = R_{o2} + R_{o1}\left(\frac{N_2}{N_1}\right)^2 = 0.002 + 0.2 \times \left(\frac{200}{2000}\right)^2 = 0.002 + 0.002 = 0.004\ \Omega.$$

Equivalent reactance referred to low voltage side

$$X_{o2} = X_2 + X_1\left(\frac{N_2}{N_1}\right)^2 = 0.008 + 0.4 \times \left(\frac{200}{2000}\right)^2 = 0.008 + 0.004 = 0.012\ \Omega.$$

(ii) Equivalent resistance drop referred to the high voltage side = $I_1 R_{o1} = \frac{500 \times 10^3}{2000} \times 0.4 = 250 \times 0.4 = 100$ V.

Percent equivalent resistance drop = $\frac{I_1 R_{o1}}{V_1} \times 100\% = \frac{100}{2000} \times 100\% = 5\%$.

Equivalent reactance drop referred to the low voltage side

$$I_1 X_{o1} = \frac{500 \times 10^3}{2000} \times 1.2 = 250 \times 1.2 = 300 \text{ V}$$

Percent equivalent reactance drop

$$\frac{I_1 X_{o1}}{V_1} \times 100\% = \frac{300}{2000} \times 100\% = 15\%.$$

8.15 A 5 kVA 440/220 V single-phase transformer has a primary and secondary winding resistance of 2 Ω and 0.8 Ω respectively. The primary and secondary reactances are 10 Ω and 1.5 Ω respectively. Find the secondary terminal voltage at full load, 0.8 p.f. lagging.

Solution

If V_2 be the secondary terminal voltage at full load and E_2 the secondary terminal voltage at no load then

$$E_2 = V_2 + I_2 R_{o_2} \cos\theta_2 + I_2 X_{o_2} \sin\theta_2$$

or $$V_2 = E_2 - I_2 R_{o2}(0.8) - I_2 X_{o2}(0.6)$$

$$R_{o2} = 2 \times \left(\frac{220}{440}\right)^2 + 0.8 = 0.5 + 0.8 = 1.3\ \Omega$$

$$X_{o2} = 10 \times \left(\frac{220}{440}\right)^2 + 1.5 = 2.5 + 1.5 = 4\ \Omega$$

$$I_2 = \frac{5 \times 10^3}{220} = 22.73 \text{ A.}$$

∴ $$V_2 = 220 - 22.73(1.3 \times 0.8 + 4 \times 0.6) = 220 - 22.73 \times 3.44 = 141.8 \text{ V.}$$

8.16 A transformer has 4% resistance and 6% reactance drop. Find the voltage regulation at full load (a) 0.8 p.f. lagging (b) 0.8 p.f. leading and (c) unity p.f.

Solution

(a) Regulation at 0.8 p.f. lagging $= R_{p.u} \cos\theta_2 + X_{p.u} \sin\theta_2$
$= 0.04 \times 0.8 + 0.06 \times 0.6$
$= 0.032 + 0.036 = 0.068$ or 6.8%.

(b) Regulation at 0.8 p.f leading $= R_{p.u} \cos\theta_2 - X_{p.u} \sin\theta_2$
$= 0.04 \times 0.8 - 0.06 \times 0.6$
$= 0.032 - 0.036 = -0.004$ or -0.4%.

(c) Regulation at unity p.f ($= R_{p.u} \cos\theta_2$) $= 0.04 \times 1 = 0.04$ or 4%.

8.17 A 220/1100 V single phase transformer has a resistance of 0.6 Ω and leakage reactance of 1.5 Ω both referred to the high voltage side. Find the p.f. at which regulation is zero. The full load primary current is 30 A.

Solution

Full load secondary current $I_2 = 30 \times \dfrac{220}{1100} = 6$ A

$$R_{\text{p.u}} = \frac{I_2 R_{o2}}{E_2} = \frac{6 \times 0.6}{1100} = 0.0033$$

$$X_{\text{p.u}} = \frac{I_2 X_{o2}}{E_2} = \frac{6 \times 1.5}{1100} = 0.0082$$

Voltage regulation is $(R_{\text{p.u}} \cos \theta_2 + X_{\text{p.u}} \sin \theta_2)$, where $(\cos \theta_2)$ is the lagging p.f.

Hence, $0.0033 \cos \theta_2 + 0.0082 \sin \theta_2 = 0$

or, $\tan \theta_2 = -0.4024$

i.e., $\cos \theta_2 = 0.93$ and the negative sign indicates leading p.f.

The regulation is zero at 0.93 p.f. leading.

8.18 A 10 kVA, 440/200 V, 50 Hz single phase transformer requires 100 V on h.v. side to circulate full load current with l.v. short circuited. The power input is 200 W. Find the maximum possible voltage regulation and p.f. at which it occurs. Also find the secondary terminal voltage under this condition.

Solution

Full load h.v. current = $\dfrac{10 \times 10^3}{440}$ A = 22.73 A.

Power input = 200 W = $(22.73)^2 R_{o1}$

Hence $R_{o_1} = 0.387\ \Omega$.

Also $Z_{o_1} = \dfrac{100}{22.73}\ \Omega = 4.4\ \Omega$, hence $X_{o_1} = \sqrt{Z_{o1}^2 - R_{o1}^2} = 4.38\ \Omega$.

Voltage regulation is $(R_{\text{p.u.}} \cos \theta_2 + R_{\text{p.u.}} \sin \theta_2)$. Maximum voltage regulation occurs when

$$\frac{d}{d\theta_2}(R_{\text{p.u.}} \cos \theta_2 + X_{\text{p.u.}} \sin \theta_2) = 0$$

or, $$-R_{\text{p.u.}} \sin \theta_2 + X_{\text{p.u.}} \cos \theta_2 = 0$$

or, $$\tan \theta_2 = \frac{X_{\text{p.u.}}}{R_{\text{p.u.}}}$$

Now $$R_{\text{p.u.}} = \frac{0.387 \times 22.73}{440} = 0.02$$

and $$X_{\text{p.u.}} = \frac{4.38 \times 22.73}{440} = 0.226$$

Hence, $\tan \theta_2 = \dfrac{0.226}{0.02} = 11.31$ or, $\cos \theta_2$ is 0.088

i.e., Power factor is 0.088 lagging.

Hence maximum voltage regulation $= 0.02 \times 0.088 + 0.226 \times 0.996 = 0.227$ p.u. or, 22.7%.

If (V_2) be the terminal voltage then

$$1 - \frac{V_2}{E_2} = 0.227$$

or, $$V_2 = (1 - 0.227) \times 200 = 154.6 \text{ V}$$

8.19 A 40 kVA, 2500/500 V single phase transformer has the following parameters: $R_1 = 8\ \Omega$, $R_2 = 0.5\ \Omega$, $X_1 = 20\ \Omega$, $X_2 = 0.8\ \Omega$. Find the voltage regulation and the secondary terminal voltage at full load for a p.f. of 0.8 lagging. The primary voltage is held constant at 2500 V.

Solution

Equivalent resistance referred to low voltage side

$$R_{o2} = 0.5 + 8 \times \left(\frac{500}{2500}\right)^2 = 0.5 + 0.32 = 0.82\ \Omega$$

Equivalent reactance referred to low voltage side

$$X_{o2} = 0.8 + 20 \times \left(\frac{500}{2500}\right)^2 = 0.8 + 0.8 = 1.6\ \Omega$$

Full load secondary current $I_2 = \dfrac{40 \times 10^3}{500} = 80$ A

$$\begin{aligned}\text{Voltage regulation} &= (I_2 R_{o2} \cos\theta_2 + I_2 X_{o2} \sin\theta_2)/E_2 \\ &= \frac{80}{500}\{0.82 \times (0.8) + 1.6 \times (0.6)\} \\ &= \frac{129.28}{500} = 0.258 \text{ or } 25.6\%.\end{aligned}$$

If V_2 be the secondary terminal voltage then

$$\frac{E_2 - V_2}{E_2} = 0.256 \quad \text{or,} \quad V_2 = (1 - 0.256)\ 500 = 372 \text{ V.}$$

8.20 A 2000/400 V single phase transformer has an equivalent resistance of 0.03 p.u. and an equivalent reactance of 0.08 p.u. Find the full load voltage regulation at 0.8 p.f. lag if the primary voltage is 1500 V. Find also the secondary terminal voltage at full load.

Solution

$$\text{Voltage regulation} = \frac{E_2 - V_2}{E_2} = R_{\text{p.u.}} \cos\theta_2 + X_{\text{p.u.}} \sin\theta_2$$

or,
$$\frac{E_2 - V_2}{E_2} = 0.03 \times 0.8 + 0.08 \times 0.6 = 0.072$$

So, voltage regulation is 0.072 or, 7.2%.

When primary voltage is 1500 V secondary voltage is $1500 \times \dfrac{400}{2000} = 300$ V at no load or, $E_2 = 300$ V. Hence, secondary terminal voltage $V_2 = E_2(1 - 0.072) = 300 \times 0.928 = 278.4$ V.

8.12 LOSSES AND EFFICIENCY OF TRANSFORMER

Like any other machine, the *efficiency* of a transformer is defined as

$$\begin{aligned}\eta &= \frac{\text{Power output}}{\text{Power input}} \\ &= \frac{\text{Power output}}{\text{Power output + Power losses in the transformer}}\end{aligned}$$

To find the efficiency, we are to know various types of losses. There are two types of losses in a transformer:

(a) *Copper losses* (or *I^2R losses* or *ohmic losses*) in the primary and secondary windings.

(b) *Iron losses* (or *core losses*) in the core. This again has two components: (i) *hysteresis losses* and (ii) *eddy current losses*.

The copper losses (P_C) also have two components: (i) the primary winding copper loss, and (ii) the secondary winding copper loss.

$$\therefore \quad \text{Copper losses, } (P_C) = I_1^2 R_1 + I_2^2 R_2$$
$$= I_1^2 R_1 + I_1^2 R_2' = I_1^2 R_{o_1}$$

Also,
$$P_C = I_2^2 R_2 + I_2^2 R_1' = I_2^2 R_{o_2} \quad (8.22)$$

(For correct determination of copper losses, the winding resistance should be determined at the operating temperature of windings.)

When alternating current flows through the windings, the core material undergoes cyclic processes of magnetisation and demagnetisation.
This process is called *hysteresis*.
The hysteresis losses (in watts) is given as,

$$P_h = K_h B_m^n f v \quad (8.23)$$

where, K_h = hysteresis coefficient whose value depends upon the material (K_h is 0.025 for cast steel, 0.001 for silicon steel and 0.0001 for permalloy)
B_m = maximum flux density (in tesla)
n = a constant, $1.5 \le n \le 2.5$ depending upon the material
f = frequency (in hertz)
v = volume of the core material (in m^3)

The eddy currents are the circulating currents set up in the core. These are produced due to magnetic flux being cut by the core. The loss due to these eddy currents is called *eddy current losses*. This loss (in watts) is given by

$$P_e = K_e B_m^2 f^2 t^2 v \quad (8.24)$$

where K_e = constant dependent upon the material
t = thickness of laminations (in metre)

A comparison of the expressions of hysteresis and eddy current losses reveals that the eddy-current loss varies as the square of the frequency, whereas the hysteresis loss varies directly with the frequency. The hysteresis losses can be minimised by selecting suitable ferromagnetic material for the core. The eddy-current losses can be minimised by using thin laminations in building the core.
The total iron losses (P_i) is given as

$$P_i = P_h + P_e$$

The efficiency of the transformer in thus given as

$$\eta = \frac{\text{Power output}}{\text{Power input}} = \frac{P_o}{P_o + P_c + P_i}$$
$$= \frac{V_2 I_2 \cos \phi_2}{V_2 I_2 \cos \theta_2 + I_2^2 R_{02} + P_i} \quad (8.25)$$

8.13 CONDITION FOR MAXIMUM EFFICIENCY

Dividing the numerator and denominator in the above expression of efficiency by (I_2), we get

$$\eta = \frac{V_2 \cos\theta_2}{V_2 \cos\theta_2 + I_2 R_{02} + P_i / I_2}$$

The transformer being operating at constant terminal voltage and constant power factor, we know the value of (I_2) at which the efficiency is maximum. Obviously, the efficiency will be maximum when ($I_2\ R_{o2}$ + P_i/I_2) is a minimum. (η) is maximum when its first derivative with respect to I_2 is zero.

i.e,
$$\frac{d}{dI_2}\left(I_2 R_{02} + \frac{P_i}{I_2}\right) = 0$$

or
$$R_{o_2} - \frac{P_i}{I_2^2} = 0$$

or
$$I_2^2 R_{o_2} = P_i \tag{8.26}$$

Thus, the efficiency at a given terminal voltage and load power factor is maximum for such a load current (I_2) which makes copper losses equal to the constant iron losses.

8.14 EXPRESSION FOR LOAD AT WHICH EFFICIENCY IS MAXIMUM

Let I_{2fl} = Full load secondary current
I_{2m} = secondary current when efficiency is maximum
R_{o2} = equivalent resistance referred to the secondary
P_i = core loss

Full load copper losses = $I_{2\,fl}^2 R_{o\,2} = P_{cfl}$

Copper losses (when efficiency is maximum) are, $I_{2m}^2 R_{o\,2}$ (= P_c)

So,
$$I_{2m}^2 = \frac{P_C}{R_{o\,2}} = \frac{I_{2fl}^2\ P_C}{I_{2fl}^2 R_{o\,2}} = \frac{I_{2fl}^2 P_C}{P_{cfl}}$$

or,
$$I_{2m} = I_{2fl}\sqrt{\frac{P_C}{P_{cfl}}} = I_{2fl}\sqrt{\frac{P_i}{P_{cfl}}} \tag{8.27}$$

$$\text{Current at maximum efficiency} = \text{current at full load} \times \left(\sqrt{\frac{\text{Core loss}}{\text{Full load copper loss}}}\right)$$

Now,
$$V_2 I_{2m} = V_2 I_{2fl}\sqrt{\frac{P_C}{P_{cfl}}} \tag{8.28}$$

or, (VA) output at maximum efficiency

$$= \text{full load } (VA) \text{ output} \times \sqrt{\frac{\text{Core loss}}{\text{Full load copper loss}}}$$

Hence, if the maximum efficiency occurs at n times the full load, then $n = \sqrt{(P_C / P_{cfl})}$. (8.28a)

$$\text{Maximum efficiency} = \frac{nV_2\, I_{fl} \cos\theta_2}{nV_2\, I_{fl} \cos\theta_2 + 2P_C}\text{, where } (\cos\theta_2) \text{ is the load p.f.}$$

8.21 In a 25 kVA, 2000/200 V transformer the iron and full load copper losses are 350 W and 400 W respectively. Find the efficiency at unity p.f. at (a) full load (b) half load. Determine the load for maximum efficiency.

Solution

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + Losses}}$$

(a) At full load and unity p.f.

$$\text{Output} = 25 \times 10^3 \times 1 = 25 \times 10^3 \text{ W}$$

$$\text{Losses} = 350 + 400 = 750 \text{ W}$$

$$\therefore \quad \text{efficiency} = \frac{25{,}000}{25{,}000 + 750} = 0.97 \text{ or } 97\%$$

(b) At half load and unity p.f.

$$\text{Output} = 25 \times 10^3 \times \frac{1}{2} = 12.5 \times 10^3 \text{ W}$$

$$\text{Iron losses} = 350 \text{ W}$$

$$\text{Copper losses} = \left(\frac{1}{2}\right)^2 \times 400 = 100 \text{ W}$$

$$\text{Total losses} = 450 \text{ W}$$

$$\text{Efficiency} = \frac{12500}{12500 + 450} = 0.965 \text{ or } 96.5\%$$

If maximum efficiency occurs when load is (x) times the full load then copper losses = ($x^2 \times 400$) W

As core losses = copper losses, under maximum efficiency condition then ($x^2 \times 400$) = 350, or $(x) = \sqrt{\frac{35}{40}} = 0.935$

Hence, load for maximum efficiency = 0.935 × 25 kVA = 23.375 kVA.

8.22 Find the efficiency of a 150 kVA transformer at 25% full load at 0.8 p.f. lagging if copper losses are 1600 W at full load and iron losses are 1400 W.

Solution

Output at 25% full load and 0.8 p.f. lagging is

$150 \times 10^3 \times 0.25 \times 0.8 = 30{,}000$ W

$$\text{Copper losses} = 1600 \times (0.25)^2 \text{ W} = 100 \text{ W}$$

$$\text{Iron losses} = 1400 \text{ W}$$

$$\text{Total losses} = (100 + 1400) \text{ W} = 1500 \text{ W}$$

$$\therefore \quad \text{Efficiency} = \frac{\text{Output}}{\text{Output + Losses}} = \frac{30{,}000}{30{,}000 + 1500} = 0.9524 \text{ or } 95.24\%.$$

8.23 Calculate the efficiency at 25% overload for a 100 kVA transformer at 0.7 p.f. The core losses are 800 W and full load copper losses are 1000 W.

Solution

At 25% overload,

$$\text{Output} = 1.25 \times 100 \times 10^3 \times 0.7 \text{ W} = 87500 \text{ W}$$

$$\text{Copper losses} = 1000 \times (1.25)^2 \text{ W} = 1562.5 \text{ W}$$

$$\text{Input} = \text{Output} + \text{total losses} = 87500 + (1562.5 + 800) = 89862.5 \text{ W}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{87500}{89862.5} = 0.974 \text{ or } 97.4\%.$$

8.24 The efficiency of a 10 kVA, 2000/400 V single phase transformer at unity p.f. is 97% at rated load and also at half rated load. Determine the transformer core losses and ohmic losses.

Solution

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{(\text{Input} - \text{Loss})}{\text{Input}} = 1 - \frac{\text{Losses}}{\text{Input}}$$

At full load,

$$0.97 = 1 - \frac{\text{Core losses} + \text{Copper losses}}{\text{Output} + \text{Core losses} + \text{Copper losses}}$$

or
$$0.97 = 1 - \frac{P_C + P_{cu}}{10 \times 10^3 \times 1 + P_c + P_{cu}}$$

or
$$\frac{P_c + P_{cu}}{10000 + P_c + P_{cu}} = 0.03$$

or
$$P_c + P_{cu} = 309.278 \text{ W}. \qquad \text{(i)}$$

At half load,

$$0.97 = 1 - \frac{P_c + \frac{1}{4}P_{cu}}{10 \times 10^3 \times \frac{1}{2} + P_c + \frac{1}{4}P_{cu}}$$

or
$$P_c + \frac{1}{4}P_{cu} = 150 + 0.03\left(P_c + \frac{1}{4}P_{cu}\right)$$

or
$$P_c + \frac{1}{4}P_{cu} = \frac{150}{0.97} = 154.639 \text{ W}. \qquad \text{(ii)}$$

Solving Eqs (i) and (ii)

$$P_{cu} = 206.185 \text{ W}$$

and
$$P_c = 103.1 \text{ W}.$$

8.25 The diagram in Fig. 8.15 shows the equivalent circuit for a single-phase transformer. The ratio of secondary to primary turns is 15. Find the (i) secondary terminal voltage, (ii) the primary current and (iii) efficiency.

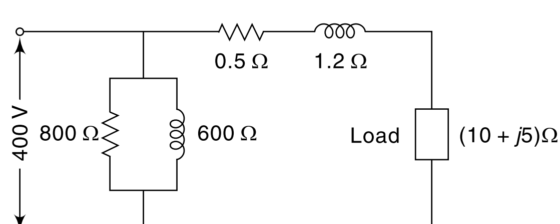


Fig. 8.15 *Circuit diagram for Example 8.25*

Solution

Load impedance = $10 + j5 = 11.18\angle 26.56°\ \Omega$

Total series impedance $= (0.5 + 10) + j(1.2 + 5) = 10.5 + j\,6.2 = 12.194\angle 30.56°\ \Omega$

Load current, $I_2 = \dfrac{400}{12.194\angle 30.56°}$ A $= 32.8\angle -30.56°$ A

(i) Secondary terminal voltage $= I_2 \times 11.18\angle 26.56° = 366.7$ V.

(ii) Magnetising current $= \dfrac{400}{800} - j\dfrac{400}{600} = 0.5 - j0.667 = 0.833\angle -53.14°$ A.

Hence primary current = load current + magnetising current

$= 32.8\angle -30.56° + 0.5 - j0.667$

$= (28.74 - j17.34) = 33.57\angle -31.1°$.

(iii) Input $= (VI \cos\theta) = 400 \times 33.57 \cos(31.1°) = 11497.95$ W.

Total losses = Iron losses + Copper losses

$= 400 \times 0.833 \cos 53.14° + (33.57)^2 \times 0.5 = 763.34$ W

$$\text{Efficiency} = \left(1 - \frac{\text{Loss}}{\text{Input}}\right) = 1 - \frac{763.34}{11497.95} = 0.9336 \text{ or } 93.36\%.$$

8.26 A 20 kVA, 2000/220 V single-phase transformer has a primary resistance of 2.1 Ω and a secondary resistance of 0.026 Ω. If the total iron loss is 200 W find the efficiency on (i) full load and at a p.f. of 0.5 (lagging); (ii) half load and a p.f. of 0.8 (leading).

Solution

Iron losses = 200 W

Full load primary current $= \dfrac{20{,}000}{2000} = 10$ A

Full load secondary current $= \dfrac{20{,}000}{220} = 90.91$ A

Total copper losses at full load $= I_1^2 R_1 + I_2^2 R_2$

$= (10)^2 \times 2.1 + (90.91)^2 \times 0.026$

$= 210 + 214.88 = 424.88$ W.

(i) Output at full load and 0.5 p.f. lag $= 20 \times 10^3 \times 0.5 = 10{,}000$ W

Input = Output + Iron losses + Copper losses

$= 10{,}000 + 200 + 424.88 = 10{,}624.88$ W.

So, efficiency $= \dfrac{\text{Output}}{\text{Input}} = \dfrac{10{,}000}{10{,}624.88} = 0.941 = 94.1\%$.

(ii) Output at half load at 0.8 p.f. leading $= 20 \times 10^3 \times \dfrac{1}{2} \times 0.8 = 8000$ W.

Copper loss at half load $= 424.88 \times \left(\dfrac{1}{2}\right)^2 = 106.22$ W

Input $= 8000 + 106.22 + 200 = 8306.22$ W

Efficiency $= \dfrac{8000}{8306.22} = 0.963 = 96.3\%$.

8.27 The primary resistance of a 440/110 V single-phase transformer is 0.28 Ω and the secondary resistance is 0.018 Ω. If the iron losses is measured to be 160 W when the rated primary voltage is applied, find the kW loading to give maximum efficiency at unity p.f.

Solution

Let at (x) times the full load the efficiency is maximum. If P_{cu} is the copper losses at full load, then $x^2 P_{cu} = 160$, as copper losses = iron losses when efficiency is maximum.

Now $(x^2 \; I_2^2 R_{o_2}) = 160$ where I_2 is the full load secondary current and R_{o2} is the equivalent resistance referred to secondary.

$$\text{Now} \qquad R_{o2} = R_1\left(\frac{N_2}{N_1}\right)^2 + R_2 = 0.28 \times \left(\frac{110}{440}\right)^2 + 0.018$$

$$= 0.0175 + 0.018 = 0.0355 \ \Omega$$

$$\therefore \qquad xI_2 = \sqrt{\frac{160}{0.0355}} \text{ A} = 67.13 \text{ A}$$

The loading at unity p.f. is = $\dfrac{110 \times 67.13 \times 1}{10^3}$ = 7.38 kW.

8.28 A single-phase transformer supplies a load of 20 kVA at a p.f. of 0.81 (lagging). The iron loss of the transformer is 200 W and the copper losses at this load is 180 W. Calculate (i) the efficiency (ii) if the load is now changed to 30 kVA at a p.f. of 0.91 (lagging), calculate the new efficiency.

Solution

Iron loss = 200 W

Copper loss at a load of 20 kVA is = 180 W.

(i) Output of 0.81 p.f. (lag) = $20 \times 10^3 \times 0.81 = 16200$ W

Total losses = 200 + 180 = 380 W

Input = Output + Losses = 16200 + 380 = 16580 W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{16200}{16580} = 0.977 = 97.7\%.$$

(b) New load is 30 kVA at 0.91 p.f. (lag)

Output = $30 \times 10^3 \times 0.91 = 27300$ W

Copper losses at a load of 30 kVA is, $P_{cu} = 180 \times \left(\dfrac{30}{20}\right)^2 = 180 \times \dfrac{9}{4} = 405$ W

Input = Output + Losses = 27300 + 200 + 405 = 27905 W

$$\text{Efficiency} = \frac{27300}{27905} = 0.978 = 97.8\%$$

8.29 The ohmic resistance of the primary and secondary windings of a 27.5 kVA, 450/112 V single-phase transformer are 0.055 Ω and 0.00325 Ω respectively. At the rated supply voltage the iron losses are 170 W. Calculate (i) the full load efficiency at a p.f. of 0.8 lagging, (ii) the kVA output at which efficiency is a maximum at a p.f. of 0.8 (lagging), (iii) the value of maximum efficiency at a p.f. of 0.8 (lagging).

Solution

Full load primary current $I_1 = \dfrac{27.5 \times 10^3}{450} = 61.1$ A.

Full load secondary current $I_2 = \dfrac{27.5 \times 10^3}{112} = 245.53$ A.

Primary copper losses at full load = $(61.1)^2 \times 0.055 = 205.326$ W.

Secondary copper losses at full load = $(245.53)^2 \times 0.00325$ = 195.93 W.
Total copper losses at full load = (205.326 + 195.93) W = 401.25 W.
Iron losses = 170 W.

(i) Full load efficiency at a p.f. of 0.8 lag

$$= \frac{\text{Output}}{\text{Output + Total Losses}} = \frac{27.5 \times 10^3 \times 0.8}{27.5 \times 10^3 \times 0.8 + 401.25 + 170} = 0.975 = 97.5\%.$$

(ii) Let maximum efficiency occurs when load is (x) times the full load. As core losses = copper losses under this condition,
$\therefore$ $x^2 \times 401.25 = 170$ or, $x = 0.65$
kVA output under this condition is = 0.65 × 27.5 = 17.9.

(iii) Maximum efficiency $= \dfrac{17.9 \times 10^3 \times 0.8}{17.9 \times 10^3 \times 0.8 + 170 + (0.65)^2 \times 401.25}$

$$= \frac{14320}{14659.5} = 0.9768 = 97.68\%.$$

8.30 A 50 kVA, 3.3 kV/230 V single-phase transformer has an impedance of 4.2% and a copper loss of 1.8% at full load. Calculate the ohmic value of resistance, reactance and impedance referred to the primary side. Estimate the primary short circuit current, assuming the supply voltage to be maintained.

Solution

Primary full load current = $\dfrac{50,000}{3300}$ A = 15.15 A

Now, $\dfrac{4.2}{100} = \dfrac{I_1 Z_{o1}}{V_1}$

or $0.042 = \dfrac{15.15\, Z_{o1}}{3300}$

or $Z_{o1} = 9.148\ \Omega$

where Z_{o1} = equivalent impedance referred to the primary.

Again, $0.018 = \dfrac{I_1^2 R_{o1}}{V_1 I_1} = \dfrac{I_1 R_{o1}}{V_1} = \dfrac{15.15\, R_{o1}}{3300}$

or $R_{o1} = 3.92\ \Omega$,

where R_{o1} = equivalent resistance referred to the primary.

$\therefore$ equivalent reactance referred to the primary = $\sqrt{(9.148)^2 - (3.92)^2}\ \Omega$ = 8.26 Ω.

Under short circuit condition, the primary current = $\left(\dfrac{V_1}{Z_{o1}}\right) = \dfrac{3300}{9.148}$ A = 360.73 A.

8.31 A 50 kVA, 440/110 V single-phase transformer has an iron loss of 250 W. With the secondary windings short circuited full load currents flow in the windings when 25 V is applied to the primary, and the power input being 500 W. For this transformer determine (i) the percentage voltage regulation at full load, 0.8 p.f. lagging, (ii) the fraction of full load at which the efficiency is maximum.

Solution

Iron loss = 250 W.

Full load primary current = $\frac{50,000}{440}$ A = 113.63 A.

Full load secondary current = $\frac{50,000}{110}$ A = 454.52 A.

Total impedance referred to the primary $Z_{o1} = \frac{25}{113.63}\ \Omega = 0.22\ \Omega$.

Primary input = $(113.63)^2 \times R_{o1} = 500$, where R_{o1} = equivalent resistance referred to the primary.

So $$R_{o1} = \frac{500}{(113.63)^2}\ \Omega = 0.0387\ \Omega$$

Now $$X_{o1} = \sqrt{(0.22)^2 - (0.0387)^2}\ \Omega = 0.2165\ \Omega.$$

If R_{o2} and X_{o2} be the equivalent resistance and reactance referred to the secondary then

$$R_{o2} = (0.0387) \times \left(\frac{110}{440}\right)^2 \Omega = 0.0024\ \Omega$$

$$X_{o2} = (0.2165) \times \left(\frac{110}{440}\right)^2 \Omega = 0.0135\ \Omega$$

(i) Voltage regulation at full load and 0.8 p.f. lagging

$$= \frac{I_2 R_{o_2} \cos\theta_2 + I_2 X_{o_2} \sin\theta_2}{E_2}$$

$$= \frac{454.52}{110}\{0.0024 \times 0.8 + 0.0135 \times 0.6\} = 0.0414 \text{ or } 4.14\%$$

(ii) Let x be the fraction of full load at which efficiency is maximum. For maximum efficiency, core losses = copper losses

$\therefore \quad x^2 I_2^2 R_{o2} = 250$ or, $\quad x^2 = \frac{250}{(454.52)^2 \times 0.0024} = 0.504$

or $\quad x^2 = 0.71.$

$\therefore$ Efficiency is maximum when the load is 0.71 times the full load.

8.15 TESTING OF TRANSFORMERS

The efficiency and regulation of a transformer are calculated by two types of tests, called *open circuit test* and *short circuit test*.

8.15.1 Open Circuit Test

This test is performed to measure the iron losses. The no-load current components I_w and I_m are measured from the open circuit test. From these, R_o (core loss resistance) and X_o (magnetizing reactance) parameters of equivalent circuit can be calculated.

One of the windings of a transformer is open-circuited. The rated voltage at rated frequency is applied to the other winding. Generally the HT side is kept open-circuited and the rated voltage is fed to the LT winding.

The connections are made as shown in Fig. 8.16. The rated voltage is supplied through an auto-transformer (also called *variac*). The readings of the wattmeter, voltmeter and ammeter are noted. Let W_o, V_1 and I_o be their readings. Since the

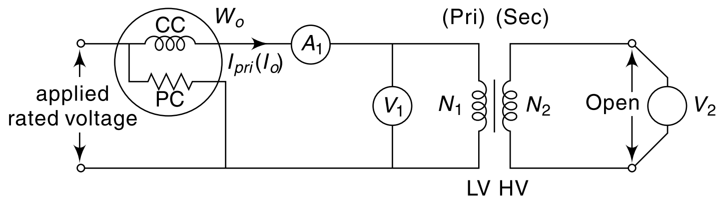


Fig. 8.16 *Circuit diagram for open circuit test*

secondary is open circuited a very small current called the no load current flows in the primary. The ammeter reads no load current I_o. As I_o is very small so the ohmic loss which is proportional to the square of the current can be neglected.

So, Iron losses, $$P_i = W_o - \frac{V_1^2}{r_p} - I_o^2 r_1 \tag{8.29}$$

where r_p = resistance of potential circuit of wattmeter

r_1 = resistance of transformer winding connected to supply.

Since the terms V_1^2/r_p and $I_o^2 r_1$ are very small, the wattmeter reading W_o can be assumed to give the iron losses.

If I_w and I_m be the core loss component and magnetizing component of the no load current I_o and $\cos\theta_o$ is the no load power factor, then

$$P_i = W_o = V_1 I_o \cos\theta_o$$

or $$\cos\theta_o = \frac{W_o}{V_1 I_o} \tag{8.30}$$

$\therefore$ $$I_w = I_o \cos\theta_o \text{ and } I_m = I_o \sin\theta_o$$

$$R_o = \frac{V_1}{I_w} \quad \text{and} \quad X_o = \frac{V_1}{I_m}.$$

8.15.2 Short Circuit Test

This test is carried out to determine the equivalent resistance and the leakage reactance of the transformer. The connections are made as shown in the Fig. 8.17. The LT winding is short-circuited. A low voltage is applied to HT side using an auto-transformer. This voltage is adjusted in such a way that the full-load current flows through the HT and LT windings. Since low voltage is applied the iron loss

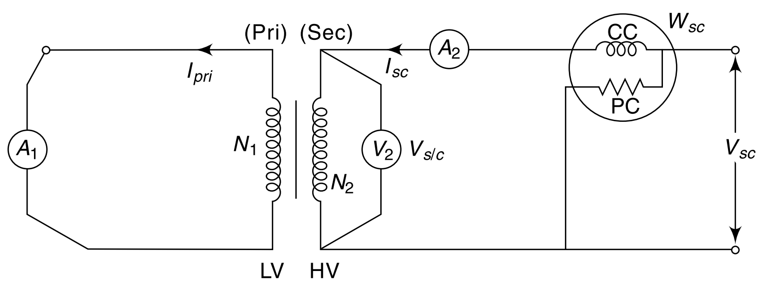


Fig. 8.17 *Circuit diagram for short circuit test.*

which is proportional to the square of the applied voltage is negligibly small as compared to the copper loss. Therefore, the wattmeter reading gives the copper loss. Let the various readings be W_{sc}, V_{sc} and I_{sc}. Then

$$R_{OH} = W_{sc}/ I_{sc}^2 \tag{8.31a}$$

$$Z_{OH} = V_{sc}/I_{sc} \tag{8.31b}$$

$$X_{OH} = \sqrt{(Z_{oH})^2 - (R_{oH})^2} \tag{8.31c}$$

where, R_{OH} is the equivalent resistance, (X_{OH}) is the equivalent leakage reactance and Z_{OH} is the equivalent impedance referred to the h.v. winding. These parameters refer to the winding on which measurements are made, i.e. h.v. side. From these, the various parameters as referred to other winding i.e. l.v. winding can be calculated.

8.15.3 Sumpner's test (Back to back test or Regenerative test)

To determine the maximum temperature rise of a transformer Sumpner's test is performed. This test can also be performed to find out the efficiency of a transformer. Sumpner's test is essentially a load test. It requires two identical transformers whose primaries are connected in parallel. The two secondaries are connected in series with their polarities in phase opposition. The primary windings are supplied at rated voltage and frequency. A voltmeter, ammeter and wattmeter are connected to the input as shown in Fig. 8.18. The range of the voltmeter V_2 connected across the two secondaries should be double the rated voltage of either transformer secondary. As the two secondaries are connected in phase opposition, the two secondary emfs oppose each other and no current can flow in the secondary circuit. A regulating transformer excited by an ac mains supply is used to inject voltage in the secondary winding. The injected voltage is adjusted till the ammeter A_2 reads full load secondary current. The secondary

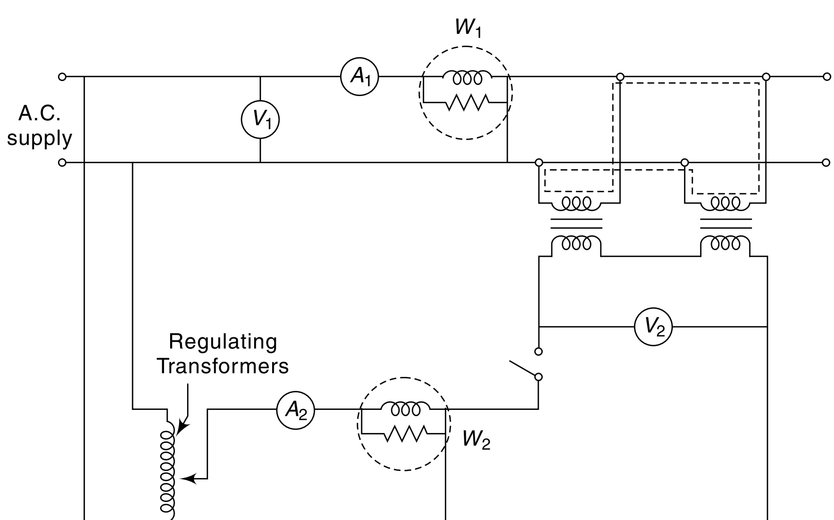


Fig. 8.18 *Sumpner's test*

current causes full load current to flow through the primary windings whereas the primary current remains confined to the dotted path as shown in Fig. 8.18. The wattmeter W_1 indicates total core losses, W_2 indicates total copper losses and ammeter A_1 indicates total no load current of the two transformers. Thus by this method we can load the transformer to full load but the supplying energy is only equal to that required for the losses only. This test can be continued for a long time to determine the maximum temperature rise of a transformer.

8.32 Calculate the values of R_o, X_o, R_1 and X_1 in the diagram shown in Fig. 8.19 of a single-phase 8 kVA, 22/440 V, 50 Hz transformer of which the following are the test results:

Open circuit test

220 V, 0.9 A, 90 W on the low voltage side.

Short circuit test

20 V, 15 A, 100 W on the high voltage side.

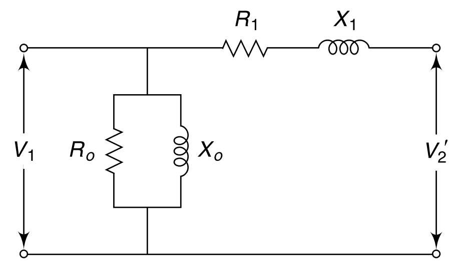


Fig. 8.19 ***Circuit diagram for Example 8.32***

Solution

From the open circuit test data,

No load p.f. $\cos\theta_o = \dfrac{90}{220 \times 0.9} = 0.4545$

$\therefore \quad \sin\theta_o = 0.89$

$$\text{Core loss resistance } R_o = \frac{V_1}{I_o \cos\theta_o} = \frac{220}{0.9 \times 0.4545}\ \Omega = 537.83\ \Omega$$

$$\text{Magnetizing reactance } X_o = \frac{V_1}{I_o \sin\theta_o} = \frac{220}{0.89 \times 0.9} = 274.65\ \Omega$$

From short circuit test data,

$$R_{OH} = R_{o2} = \frac{100}{(15)^2}\ \Omega = 0.444\ \Omega$$

$$Z_{OH} = Z_{o2} = \frac{20}{15}\ \Omega = 1.33\ \Omega$$

where R_{o2} and Z_{o2} are the equivalent resistance and impedance referred to the high voltage side.

Hence, $\quad X_{OH} = X_{o2} = \sqrt{(1.33)^2 - (0.44)^2} = 1.257\ \Omega$

Figure 8.19 shows the equivalent resistance R_1 and reactance X_1 referred to the low voltage side or primary side.

Hence, $\quad R_1 = 0.444 \times \left(\dfrac{220}{440}\right)^2 = 0.111\ \Omega$

and $\quad X_1 = 1.257 \times \left(\dfrac{220}{440}\right)^2 = 0.314\ \Omega$

Also $\quad R_O = 537.83\ \Omega$ and $X_O = 274.65\ \Omega$.

8.33 Short circuit test performed on the h.v. side of a 100 kVA, 6600/440 V, single-phase transformer yields the following results: 100 V, 6 A, 200 W. If the low voltage side is delivering full load current at 0.8 p.f. lag and at 440 V find the voltage applied to the high voltage side.

Solution

From the short circuit test results

$$R_{O1} = \frac{200}{(6)^2}\ \Omega = 5.55\ \Omega$$

$$Z_{O1} = \frac{100}{6}\Omega\ = 16.67\ \Omega$$

$$X_{O1} = \sqrt{Z_{01}^2 - R_{01}^2}\ = 15.71\ \Omega$$

Secondary rated current $I_2 = \dfrac{100 \times 10^3}{440}$ A = 227.27 A.

If E_2 and V_2 be the secondary terminal voltage under no load and full load condition then

$$E_2 - V_2 = I_2\, R_{O2} \cos\, \theta_2 + I_2\, X_{O2} \sin\, \theta_2$$

Now, $$R_{o2} = R_{O1} \times \left(\frac{440}{6600}\right)^2 = 5.55 \times \left(\frac{2}{30}\right)^2 = 0.0247\ \Omega$$

and $$X_{o2} = X_{O1} \times \left(\frac{440}{6600}\right)^2 = 15.71 \times \left(\frac{2}{30}\right)^2 = 0.0698\ \Omega$$

Hence $(E_2 - V_2) = 227.27\ (0.0247 \times 0.8 + 0.0698 \times 0.6) = 14$ V.

For $V_2 = 440$ V, $(E_2) = V_2 + 14 = 454$ V

Hence the voltage applied to the h.v. side is $\left(454 \times \dfrac{6600}{440}\right) = 6810$ V.

8.34 A 8 kVA, 440/2000 V, 50 Hz single-phase transformer gave the following test results:

No load test: 440 V, 0.8 A, 80 W.

Short circuit test: 50 V, 3 A, 20 W.

Calculate (i) the magnetizing current and the component corresponding to iron losses at normal voltage and frequency, (ii) the efficiency on full load at unity p.f., (iii) the secondary terminal voltage on full load at unity p.f.

Solution

(i) From no load test data,

No load p.f. $(\cos\, \theta_o) = \dfrac{80}{440 \times 0.8} = 0.227$

Iron loss component current = 0.8 $(\cos\, \theta_o)$ = 0.8 × 0.227 = 0.182 A

Magnetising current = 0.8 $(\sin\, \theta_o)$ = 0.8 × 0.974 = 0.779 A.

(ii) Iron loss = 80 W

As the secondary side is the h.v. side so a short circuit test is performed on the secondary side.

Rated current of the h.v. side $I_2 = \dfrac{8000}{2000} = 4$ A.

When current is 3 A the wattmeter reading is 20 W.

So, if rated current of 4 A flows through the high voltage winding the wattmeter reading $= 20 \times \left(\dfrac{4}{3}\right)^2 = 35.55$ W.

So rated copper losses = 35.55 W.

Efficiency on full load at unity p.f. $= \dfrac{8 \times 10^3 \times 1}{8 \times 10^3 \times 1 + 35.55 + 80} = 0.9857$ or 98.57%.

(iii) From short circuit test data,

$$R_{O2} = \frac{20}{(3)^2} = 2.22\ \Omega,\ (Z_{O2}) = \frac{50}{3} = 16.67\ \Omega$$

and $$(X_{O2}) = \sqrt{(16.67)^2 - (2.22)^2} = 16.52\ \Omega.$$

If V_2 be the terminal voltage then,

$E_2 - V_2 = I_2 R_{O2} \cos\theta_2 + I_2 X_{O2} \sin\theta_2$, where E_2 is the secondary voltage under no load condition and $\cos\theta_2$ is the load p.f. which is unity in this case.

$\therefore \quad (2000 - V_2) = 4(2.22 \times 1 + 16.52 \times 0)$

or $\quad (V_2) = 2000 - 8.88 = 1991.12$ V.

8.35 The following results were obtained in tests on a 50 kVA, single-phase, 3300/400 V transformer.

Open circuit test: 3300 V, 430 W

Short circuit test: 124 V, 15.3 A, 535 W.

(supply given on h.v. side)

Calculate (i) the efficiency at full load and half full load both at 0.707 p.f. lagging, (ii) the regulation at full load for p.f. of 0.707 (lagging and leading) and (iii) full load terminal voltage under the condition of 0.707 p.f. (lagging).

Solution

For short circuit test data,

$$Z_{e1} = \frac{124}{15.3}\ \Omega = 8.10\ \Omega,\ R_{O1} = \frac{535}{(15.3)^2}\ \Omega = 2.285\ \Omega$$

$$X_{O1} = \sqrt{(8.1)^2 - (2.285)^2} = 7.77\ \Omega$$

(i) Rated current on the h.v. side $= \dfrac{50{,}000}{3300}$ A $= 15.15$ A

So, rated copper loss $= 535 \times \left(\dfrac{15.15}{15.3}\right)^2 = 524.56$ W

Iron loss = 430 W

Efficiency at full load and 0.707 p.f. lagging

$$= \frac{50 \times 10^3 \times 0.707}{50 \times 10^3 \times 0.707 + 524.56 + 430} = 0.9737 \text{ or } 97.37\%.$$

Efficiency at half load and 0.707 p.f. lagging is

$$= \frac{50 \times 10^3 \times 0.707 \times \frac{1}{2}}{50 \times 10^3 \times (0.707) \times \frac{1}{2} + \left(\frac{1}{2}\right)^2 \times 524.56 + 430} = 0.9623 \text{ or } 96.93\%.$$

(ii) Voltage regulation at full load and 0.707 lagging p.f. is

$$= \frac{I_1 R_{01} \cos\theta + I_1 X_{01} \sin\theta}{E_1} = \frac{15.15\,(2.285 \times 0.707 + 7.77 \times 0.707)}{3300}$$

$$= 0.0326 = 3.26\%$$

Voltage regulation at full load and 0.707 p.f. leading

$$= \frac{I_1 R_{01} \cos\theta - I_1 X_{01} \sin\theta}{E_1} = \frac{15.15\,(2.285 \times 0.707 - 7.77 \times 0.707)}{3300}$$

$$= (-0.0178) \text{ or } (-1.78\%).$$

(iii) If V_2 is the terminal voltage at 0.707 p.f. lagging

$$\left[\frac{400 - V_2}{400}\right] = 0.0326 \text{ or, } V_2 = 400\ (1 - 0.0326)\ \text{V} = 386.96\ \text{V}.$$

8.36 A 17.5 kVA, 450/121 V, 50 Hz single-phase transformer gave the following data on test:

Open circuit test (OCT): 450 V, 1.5 A, 115 W

Short circuit test (SCT): 15.75 V, 38.9 A, 312 W.

Estimate the voltage on the secondary terminals and the efficiency of the transformer when supplying full load current, at a p.f. of (0.8) lagging, from the secondary side. Assume the input voltage to be maintained at 450 V.

Solution

Full load primary current = $\dfrac{17500}{450}$ A = 38.89 A

Full load secondary current = $\dfrac{17500}{121}$ A = 144.63 A

As 450 V is the applied voltage in OCT so this test has been performed on the h.v. side and as the current in the SCT is 38.9 A which the rated primary or low voltage current so this test has also been performed on the l.v. side.

From short circuit test data,

$$R_{o1} = \frac{312}{(38.9)^2}\ \Omega = 0.206\ \Omega,$$

$$Z_{O1} = \frac{15.75}{38.9}\ \Omega = 0.405$$

and $$X_{O1} = \sqrt{(0.405)^2 - (0.206)^2} = 0.3487\ \Omega.$$

$\therefore$ $$R_{O2} = 0.206 \times \left(\frac{121}{450}\right)^2 = 0.0149\ \Omega$$

and $$X_{O2} = 0.3487 \times \left(\frac{121}{450}\right)^2 = 0.0252\ \Omega.$$

If V_2 be the secondary terminal voltage then

$$[121 - V_2] = 144.63(0.0149 \times 0.8 + 0.0252 \times 0.6) = 3.91$$

or $$V_2 = 117.09\ \text{V}.$$

From OCT, iron loss = 115 W

From SCT, full load copper loss = 312

So efficiency at full load and 0.8 p.f. lagging

$$= \frac{17.5 \times 10^3 \times 0.8}{17.5 \times 10^3 \times 0.8 + 115 + 312} = 0.97 \text{ or } 97\%.$$

8.16 PARALLEL OPERATION OF TWO-SINGLE PHASE TRANSFORMERS

It is very often necessary to operate transformers in parallel. In substations the total load requirement may be more than the capacity of existing transformers, then another tranformer has to be added in parallel. If there is a breakdown of a single transformer in a group of transformers operated in parallel then the continuity of supply is maintained. For satisfactory parallel operation of transformers the following two conditions are necessary:

(a) The polarities of the transformers must be same.
(b) The turns ratios of the transformers should be equal.

The other two desirable conditions are:

(a) Equal per unit impedance in magnitude and phase angle
(b) Equal voltages at full load across transformer terminals.

Figure 8.20 shows two single-phase transformers in parallel, connected to the same voltage source on the primary side. Terminals with proper polarity markings have been connected both on the h.v. and l.v. sides.

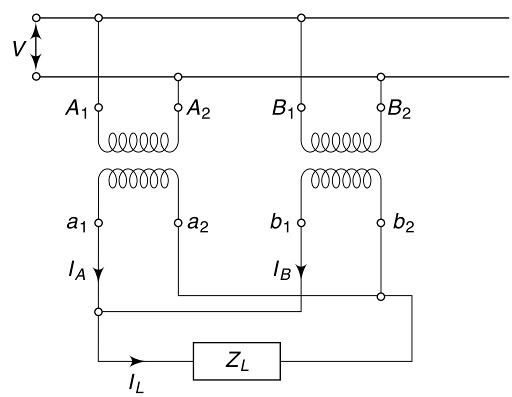


Fig. 8.20 ***Two-single phase transformers in parallel***

Here I_L is the load current and I_A and I_B are the current supplied by the two transformers to the load.

Hence, $I_A + I_B = I_L$.

If K_1 and K_2 be the turns ratio of the two transformers

then $$V_L = \frac{V}{K_1} - I_A Z_A$$

and $$V_L = \frac{V}{K_2} - I_B Z_B = \frac{V}{K_2} - (I_L - I_A) Z_B$$

where, Z_A and Z_B are the equivalent impedance of the two transformers referred to the secondary.

Solving the above two equations

$$I_A = \frac{Z_B I_L}{Z_A + Z_B} + \frac{V(K_2 - K_1)}{K_1 K_2 (Z_A + Z_B)} \tag{8.32}$$

$$I_B = \frac{Z_A I_L}{Z_A + Z_B} - \frac{V(K_2 - K_1)}{K_1 K_2 (Z_A + Z_B)} \tag{8.33}$$

Each of these currents has two components; the first component represents the transformer's share of the load current and the other component is a circulating current in the secondary windings. Circulating currents increase the copper loss and may overload one transformer. In order to eliminate the circulating currents the turns ratios must be identical, i.e. $K_1 = K_2 = K$.

Hence, $I_A = \dfrac{Z_B I_L}{Z_A + Z_B}$ and $I_B = \dfrac{Z_A I_L}{Z_A + Z_B}$

Therefore, $\dfrac{I_A}{I_B} = \dfrac{Z_B}{Z_A}$

Also, $I_A Z_B = I_B Z_B$

or $S_A = S_B$, where (S_A) and (S_B) are the VA of the two transformers.

$$\therefore \quad S_A = \frac{Z_B}{Z_A + Z_B} S_L \tag{8.34}$$

and
$$S_B = \frac{Z_A}{Z_A + Z_B} S_L \tag{8.35}$$

where S_L is the total load VA.

$$\therefore \quad \frac{S_A}{S_B} = \frac{Z_B}{Z_A}. \tag{8.36}$$

8.17 SINGLE-PHASE AUTO TRANSFORMER

An auto transformer is a single winding transformer in which a part of the winding is common to both the high voltage and low voltage side. Figure 8.21 shows a step down auto transformer. The primary winding *AB* has N_1 number of turns and the secondary winding *BC* has N_2 number of turns. The winding *BC* is common to both the primary and secondary. The induced emf in the primary winding *AB* is E_1 and in the secondary winding *BC* is E_2. Hence $\dfrac{E_1}{E_2} = \dfrac{N_1}{N_2} = K$, where K is the turns ratio. The input current is I_1 and the load current is I_2. The mmfs I_1N_1 and I_2N_2 will be equal and opposite. If terminal C is a sliding contact, the output voltage V_2 can be varied. The voltampere delivered to the load $V_2 I_2 = V_2 I_1 + V_2(I_2 - I_1)$. $(V_2 I_1)$ is the voltamperes transferred conductively to the load through winding *AC* and $V_2(I_2 - I_1)$ is the voltamperes transferred inductively to the load through winding *BC* which represents the rating of the equivalent two winding transformer.

Hence,
$$\frac{\text{Output VA of auto transformer}}{\text{Output VA of equivalent a two-winding transformer}}$$
$$= \frac{V_2 I_2}{V_2 (I_2 - I_1)} = \frac{a}{a-1} \tag{8.37}$$

Figure 8.22 represents a step up auto transformer. Here input voltampere $V_1 I_1 = V_1 I_2 + V_1(I_1 - I_2)$.

Hence
$$\frac{\text{Output VA of auto transformer}}{\text{Output VA of equivalent two winding transformer}}$$
$$= \frac{V_1 I_1}{V_1 (I_1 - I_2)} = \frac{a}{a-1} \tag{8.38}$$

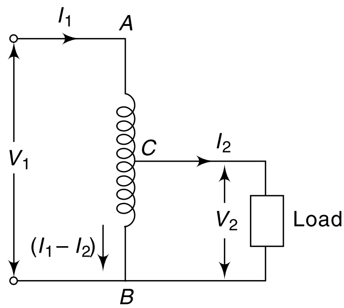


Fig. 8.21 *Step down auto transformer*

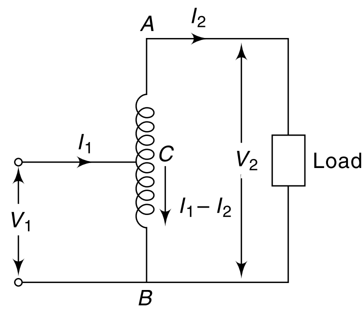


Fig. 8.22 *Step up auto transformer*

8.17.1 Adavantages of an Auto Transformer

(a) For the same capacity and voltage ratio, an auto transformer requires less winding material than a two-winding transformer. Hence there is saving in copper.
(b) An auto transformer is smaller in size and cheaper than a two winding transformer of same output.
(c) An auto transformer has higher efficiency since core loss and ohmic losses are smaller.
(d) Voltage regulation of an auto transformer is better because of reduced voltage drops in the resistance and ractance.
(e) An auto transformer has variable output voltage when a sliding contact is used for the secondary.

8.17.2 Disadvantages of an Auto Transformer

(a) There is direct connection between the high voltage and low voltage side. If there is an open circuit in the winding *BC* (Fig. 8.21) the full primary voltage would be applied to the secondary. This high voltage may cause serious damage to the equipments connected on the secondary side.
(b) The short circuit current is larger for an auto transformer due to reduced internal impedance.

8.17.3 Applications of Auto Transformers

(a) Auto transformers are used for obtaining continuously variable ac voltage.
(b) They are used for interconnections of power systems of different voltage levels.
(c) They are applied for boosting of ac mains voltage by a small amount.
(d) Auto transformers are used for starting the induction motors and synchronous motors.

8.18 TRANSFORMER COOLING

The core and copper losses cause heating of transformers. It is necessary to ensure that the temperature of the transformer does not exceed the maximum value, otherwise it may cause damage to the insulation. The following are the methods for cooling these type of transformers.

(a) **Air Natural Cooling** Small transformers up to 25 kVA are cooled by natural circulation of air surrounding it.

(b) **Air Blast Cooling** In this type of cooling continuous blast of filtered air is forced through the core and windings for better cooling.

(c) **Oil Natural Cooling** A majority of transformers have their core and windings immeresed in oil. Oil is a good insulating material and provides better heat dissipation than air. Oil immersed transformers are enclosed in sheet steel tank. The heat produced in the transformer is passed to the oil. The oil is heated and it becomes lighter and rises to the top and its place is taken by cool air from the bottom of the tank.

The heat of the air is transferred to the tank by natural circulation of air. The heat is then transferred to the surrounding atmosphere.

(d) **Oil Blast Cooling** Here forced air is passed over cooling elements of transformer immersed in oil.

(e) **Forced Oil and Forced Air Cooling** Heated oil is taken from the top of the transformer tank to a cooling plant. Cooled oil is then circulated through the bottom of the tank.

(f) **Forced Oil and Water Cooling** In this type of cooling metallic tubes are situated inside the tank, below the oil level. Water is circulated through these tubes to extract heat from the oil.

8.19 CONSERVATOR AND BREATHER

A *conservator* is an air tight metallic drum supported on a transformer top cover. It takes up the expansion of oil with changes in temperature. When the oil is cold the tank is filled with oil. When the temperature of the oil rises, the oil expands and the expansion is taken up in the conservator. When the transformer cools, the level of oil goes down and the air is drawn in. The incoming air is passed through a device called breather for extracting moisture. A *breather* consists of a small vessel which contains a drying agent like silica gel or calcium chloride.

8.20 DISTRIBUTION TRANSFORMERS AND POWER TRANSFORMERS

Distribution transformers are used to step down the transmission voltage to a lower value suitable for distribution. They are kept in operation all 24 hours in a day whether they carry any load or not. They have better voltage regulation and small leakage reactance.

Power transformers are used in generating stations or substations at each end of transmission line for steping up or steping down the voltage. They are put in service during load periods and are disconnected during light load periods. They have greater leakage reactance and have maximum efficiency at or near full load.

8.21 NAME PLATE AND RATINGS

The specifications of transformers are given by BIS (Bureau of Indian Standard) 2026. As per this standard every transformer must be provided with the following specifications:

Type (power, distribution, auto, etc.), year of mannufacture, number of phases, rated kVA, rated frequency, rated voltage of each winding, connection symbol, percent impedance voltage at rated current, type of cooling, total mass, mass and volume of insulating oil.

8.22 ALL DAY EFFICIENCY

It is usual for the primary of a transformer to be connected permanently to the supply and for the switching of load to be carried out in the secondary circuit. Since the copper loss varies with load but iron loss is constant and the efficiency depending on loading and losses vary througout the day. For transformers which are continuously excited but supply loads only intermittently, a low iron loss is particularly desirable, but a low copper loss is specially important where the load factor is high. Again for a transformer working on full load for greater part of the day, maximum efficiency should be arranged to occur some where around the full load value but for a transformer whose full load value may be supplied for only 1/4 of the day and the unit is only lightly loaded for the rest of the time, it would be desirable to arrange maximum efficiency to occur at about 1/2 full load value.

Considering the above factors the efficiency of a transformer is better estimated on an energy rather than a power ratio and thus we have the term "all day efficiency".

$$\text{All day efficiency} = \frac{\text{Output in KWh for 24 hr.}}{\text{Input in KWh for 24 hr.}}$$

8.23 THREE-PHASE TRANSFORMER

The present day power system is a three-phase system. The change of voltage in a three-phase system is performed either by a single three-phase transformer or by a three single-phase transformers. The advantages of a three-phase transformer over three single-phase transformers are:

(a) The space required is smaller.
(b) It is lighter and cheaper.
(c) It is more efficient.

A single unit three-phase transformer has a three-limbed core, one limb for each phase winding. On each limb the low voltage winding is placed over the core and the high voltage winding is placed over the low voltage winding with suitable insulation between the core and low voltage winding as well as between the two windings. Figure 8.23 and Fig. 8.24 shows the schematic diagram of a three-phase core type and shell type transformer respectively.

The primary and secondary winding of a three-phase transformer can be connected in star or delta. Hence four main connections are possible,

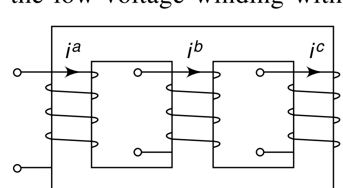


Fig. 8.23 *Three-phase core type transformer*

star-star, star-delta, delta-star and delta-delta. In star-star connection both the primary and secondary windings are connected in star. The neutral point is denoted by N for high voltage winding and n for low voltage winding and the connection is shown in Fig. 8.25. The phase current is equal to the line current but the line voltage is $(\sqrt{3})$ times the phase voltage in both the primary and secondary windings. For delta-delta connection both the primary and secondary windings are connected in delta as shown in Fig. 8.26. Here the line voltage is equal to phase voltage on each side. The phase current is line current divided by $(\sqrt{3})$. As compared to a star-star connection for the same terminal voltage and current, a delta-delta connection has more number of turns in each phase winding but less cross-sectional area of conductors. Hence a delta-delta connection is more economical for large transformers of relatively lower voltage rating. The star-star connection is not used in a three-phase three-wire system due to undesirable effects of a third harmonic current.

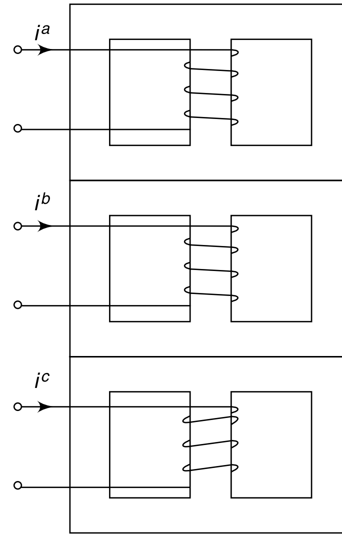


Fig. 8.24 ***Three-phase shell type transformer***

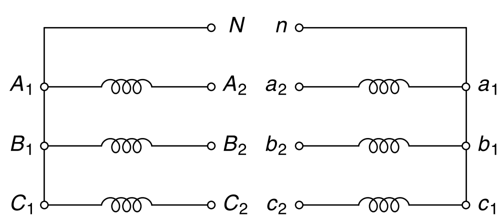


Fig. 8.25 ***Three-phase star-star transformer***

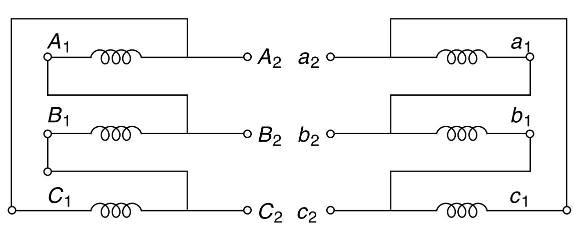


Fig. 8.26 ***Three-phase delta-delta transformer***

Figure 8.27 shows the connection for a three-phase star-delta transformer. On the primary side the line voltage is $(\sqrt{3})$ times the phase voltage while the line and phase voltages are equal on the secondary side. Generally, the high voltage winding is star connected for reducing cost of insulation. This connection is generally used for step down transformers at receiving end substations.

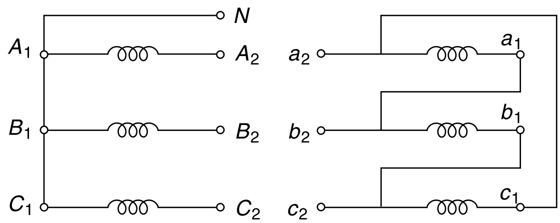


Fig. 8.27 ***Three-phase star-delta transformer***

The connection of a three-phase delta-star transformer is shown in Fig. 8.28. On the primary side the line and phase voltages are equal, but on the seconday side the line voltage is $(\sqrt{3})$ times the phase voltage. These transformers are used at sending and receiving end substations. At power stations the generator feeds the delta winding and the star winding is connected to h.v. transmission lines. In distribution transformers, feeders are connected to delta winding and the star winding supplies three-phase four-wire distributors

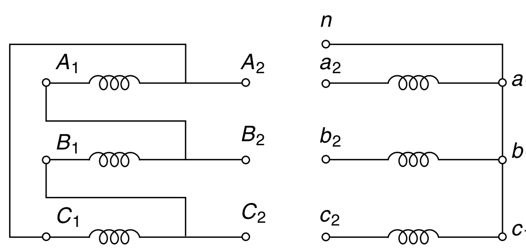


Fig. 8.28 ***Three-phase delta-star transformer***

8.37 A transformer has its maximum efficiency of 0.975 at 20 kVA at unity p.f. During the day it is loaded as follows:

10 hr: 3 kW at 0.6 p.f.
8 hr: 10 kW at 0.8 p.f.
6 hr: 20 kW at 0.9 p.f.

Find the all day efficiency.

Solution

$$\text{kWh output} = (10 \times 3) + (8 \times 10) + (6 \times 20) = 30 + 80 + 120 = 230 \text{ kWh}$$

As maximum efficiency is 0.975 so total losses under this condition is [1 – 0.975] = 0.025 of output power.

At unity p.f. output power = 20 × 1 = 20 kW

Hence losses = 0.025 × 20,000 = 500 W

$$\therefore \text{ core losses} = \text{copper losses} = \frac{500}{2} \text{ W} = 250 \text{ W}$$

As core loss is constant for all p.f. so total core losses in 24 hr.

$$= \frac{250 \times 24}{10^3} \text{ kWh} = 6 \text{ kWh}$$

For the first 10 hr.

$$\text{kVA load} = \frac{3}{0.6} = 5$$

$$\text{Total copper losses} = 10 \times \left(\frac{5}{20}\right)^2 \times \frac{250}{1000} \text{ kWh} = 0.156 \text{ kWh}$$

For the next 8 hr.

$$\text{kVA load} = \frac{10}{0.8} = 12.5$$

$$\text{Total copper losses} = 8 \times \left(\frac{12.5}{20}\right)^2 \times \frac{250}{1000} \text{ kWh} = 0.781 \text{ kWh.}$$

For the last 6 hr.

$$\text{kVA load} = \frac{20}{0.9} = 22.22$$

$$\text{Total copper losses} = 6 \times \left(\frac{22.22}{20}\right)^2 \times \frac{250}{1000} \text{ kWh} = 1.85 \text{ kWh}$$

$$\text{Total copper losses} = 0.156 + 0.781 + 1.85 = 2.79 \text{ kWh}$$

$$\text{Total loss} = 6 \text{ kWh} + 2.79 \text{ kWh} = 8.79 \text{ kWh}$$

$$\text{All day efficiency} = \frac{230}{230 + 8.79} = 0.963 \text{ or } 96.3\%.$$

8.38 A lighting transformer rated at 10 kVA has full load losses of 0.3 kW which is made up equally from the iron losses and the copper losses. The duty cycle consists of full load for 3 hours, half full load for 4 hours and no load for the remainder of a 24 hours period. If the load operates at unity power factor, calculate the all day efficiency.

Solution

The load operates at unity power factor.

For the first three hours,

$$\text{Energy output} = 10 \times 1 \times 3 \text{ kWh} = 30 \text{ kWh}$$

For the next four hours,

$$\text{Energy output} = \frac{1}{2} \times 10 \times 1 \times 4 = 20 \text{ kWh}$$

$$\text{Total energy output} = (30 + 20) \text{ kWh} = 50 \text{ kWh}$$

$$\text{Full load losses} = 0.3 \text{ kW}$$

So, $$\text{iron loss} = \left(\frac{0.3}{2}\right) \text{kW} = 0.15 \text{ kW}$$

$$\text{and full load copper loss} = \frac{0.3}{2} \text{ kW} = 0.15 \text{ kW}$$

$$\text{Iron loss energy} = (0.15 \times 24) = 3.6 \text{ kWh}$$

$$\text{Copper loss energy} = \left(0.15 \times 3 + \frac{0.15}{(2)^2} \times 4\right) \text{ kWh} = (0.45 + 0.15) \text{ kWh} = 0.6 \text{ kWh}$$

$$\text{Energy loss} = (3.6 + 0.6) \text{ kWh} = 4.2 \text{ kWh}$$

$$\therefore \quad \text{All day efficiency} = \frac{50}{50 + 4.2} = 0.922 \text{ or } 92.2\%.$$

8.39 The maximum efficiency of a 100 kVA, single-phase transformer is 95% and occurs at 90% of full load at 0.85 p.f. If the leakage impedance of the transformer is 5%, find the voltage regulation at rated load 0.8 p.f. lagging.

Solution

Output at maximum efficiency = 100 × 0.9 × 0.85 = 76.5 kW

$$\text{Efficiency} = (0.95) = \frac{\text{Output}}{\text{Output} + \text{Losses}} = \frac{76.5}{76.5 + \text{Losses}}$$

$\therefore$ Losses $= \left[\dfrac{76.5}{0.95} - 76.5\right] = 4.026$ kW

At maximum efficiency core losses = copper losses

$\therefore$ Core losses = copper losses = 2.013 kW

2.013 kW is the copper losses at 90% of full load.

So full load ohmic losses = $2.013 \times \left(\dfrac{1}{0.9}\right)^2 = 2.485$ kW

If I_2 be the full load secondary current,

$(I_2^2 R_{O2}) = 2485$, where R_{O2} is the equivalent resistance referred to as the secondary

$\therefore$ $I_2V_2\left(\dfrac{I_2 R_{O2}}{V_2}\right) = 2485$

or $100 \times 10^3 \times R_{P.u} = 2485$, where $R_{p.u.}$ is the p.u resistance

or $R_{P.u} = 0.02485$

Now $Z_{P.u} = 0.05$

$\therefore$ $X_{P.u} = \sqrt{(0.05)^2 - (0.02485)^2} = 0.04338$

Voltage regulation $= (R_{P.u} \cos \theta_2 + X_{P.u} \sin \theta_2) = 0.02485 \times 0.8 + 0.04338 \times 0.6$

$= 0.0459$ or, 4.59%.

8.40 A 10 kVA single-phase transformer has a core loss of 50 W and full load ohmic loss of 120 W. The daily variation of load on the transformer is as follows:

6 a.m to 12 noon:	5 kW at 0.7 p.f.
12 noon to 6 p.m.:	4 kW at 0.8 p.f.
6 a.m to 1 a.m.:	8 kW at 0.9 p.f.
1 a.m to 7a.m.:	No load

Find the all day efficiency of the transformer.

Solution

Core loss = 50 W; Full load ohmic loss = 120 W

From 6 a.m to 12 noon,

Output $= 5 \times 6 = 30$ kWh

kVA load $= \dfrac{5}{0.7} = 7.143.$

Ohmic losses for 6 hours $= \left(\dfrac{7.143}{10}\right)^2 \times 120 = 61.22$ W

Energy lost as ohmic loss $= (61.22 \times 6) = 367.36$ Wh

From 12 noon to 6 p.m.,

Output $= 4 \times 6 = 29$ kWh

kVA load $= \dfrac{4}{0.8} = 5.$

Ohmic losses for 6 hours $= \left(\dfrac{5}{10}\right)^2 \times 120 = 30$ W.

Energy lost as ohmic loss $= (30 \times 6) = 180$ Wh

From 6 p.m. to 1 a.m.,

Output $8 \times 7 = 56$ kWh

kVA load $= \dfrac{8}{0.9} = 8.89.$

Ohmic losses for 7 hours = $\left(\frac{8.89}{10}\right)^2 \times 120 = 94.84$ W

Energy lost as ohmic loss = $94.84 \times 7 = 663.87$ Wh

Daily energy lost as ohmic loss = $(367.36 + 180 + 663.87) \times 10^{-3}$ kWh = 1.211 kWh

Daily energy lost as core loss = $\frac{50 \times 24}{10^3}$ kWh = 1.2 kWh

Total loss = (1.211 + 1.2) = 1.411 kWh

Daily output = (30 + 24 + 56) = 110 kWh

All day efficiency = $\frac{110}{110 + 1.411} = 0.9872$ or 98.73%.

8.41 Two 200 kVA single-phase transformers are to be operated in parallel. The internal impedance of transformer 1 is $(0.006 + j0.08)$ p.u. while transformer 2 has an internal impedance of $(0.008 + j0.05)$ p.u. How will they share a load of 300 kW at 0.8 lagging power factor?

Solution

$$Z_1 = (0.006 + j0.08) = 0.08\angle 85.71°$$

$$Z_2 = (0.008 + j0.05) = 0.0506\angle 80.91°$$

$$\text{Load } S_L = \frac{300}{0.8} \angle -\cos^{-1} 0.8 = 375\angle -36.87° \text{ kVA.}$$

Load shared by transformer 1

$$S_1 = \frac{Z_2}{Z_1 + Z_2} S_L$$

$$\therefore \quad S_1 = \frac{0.0506\angle 80.91°}{(0.006 + 0.008) + j(0.08 + 0.05)} \times 375\angle -36.87°$$

$$= \frac{0.0506\angle 80.91° \times 375\angle -36.87°}{0.13075\angle 83.85°} = 145.12\angle -39.81° \text{ kVA.}$$

Load shared by transformer 2

$$S_2 = \frac{Z_1}{Z_1 + Z_2} S_L$$

$$\therefore \quad S_2 = \frac{0.08\angle 85.71°}{(0.006 + 0.008) + j(0.08 + 0.05)} \times 375\angle -36.87° = 229.44\angle -35° \text{ kVA.}$$

8.42 A 1000 kVA transformer with $(0.02 + j0.1)$ p.u. impedance and a 500 kVA transformer with $(0.015 + j\,0.05)$ p.u. impedance are operating in parallel. The no load secondary voltages of the two transformers are equal. How will they share a load of 1500 kVA at unity p.f. load?

Solution

Let base kVA = 100 kVA

$$Z_1 = (0.02 + j0.1) = 0.102\angle 78.69°.$$

Converting impedance of second transformer to base kVA

$$Z_2 = \frac{1000}{500}(0.015 + j0.05) = (0.03 + j0.1) = 0.104\angle 73.3°$$

$$\text{Load } (S_L) = 1500\angle 0° \text{ kVA.}$$

Hence, load shared by transformer 1

$$S_1 = 1500\angle 0^\circ \times \frac{0.104\angle 73.3^\circ}{(0.02+0.03)+j(0.1+0.1)}$$

$$= \frac{1500 \times 0.104\angle 73.3^\circ}{0.206\angle 75.96^\circ} = 757.28\angle{-2.66^\circ} \text{ kVA.}$$

Load shared by transformer 2

$$S_2 = 1500\angle 0^\circ \times \frac{0.102 \times \angle 78.69^\circ}{(0.02+0.03)+j(0.1+0.1)} = 742.72\angle 2.73^\circ \text{ kVA.}$$

8.43 A 200 VA, 240/120 V two-winding transformer is to be used as an auto transformer. The input voltage is 240 V. Find the secondary voltage. What is the maximum VA rating of an auto transformer?

Solution

The auto transformer is shown in Fig. 8.29

Input V_1 = 240 V across winding *BC*

Hence voltage across winding AC is 120 V.

Therefore, the voltage across winding AB, i.e secondary voltage of the auto transformer is (240 + 120) V i.e 360 V or V_2 = 360 V.

Now current in the secondary winding = $\frac{200}{120}$ A for two-winding transformers.

Hence for auto transformer $I_2 = \frac{200}{120}$ A

The maximum *VA* rating of auto transformer is $V_2 I_2$

i.e $\left(360 \times \frac{200}{120}\right)$ VA or 600 VA.

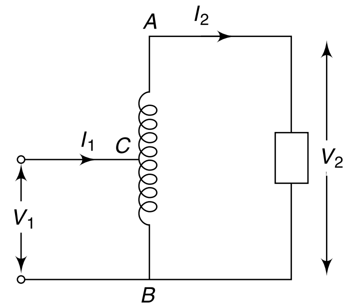


Fig. 8.29 *Circuit diagram for Example 8.43*

8.44 A 10 kVA, 2400/240 V two-winding transformer is reconnected as a step down auto transformer as shown in Fig. 8.30 and excited by a 2640 V source. The transformer is loaded so that the rated currents of the windings are not exceeded. Calculate the currents in different sections of the auto transformer and kVA output.

Solution

Current rating of 2400 V winding is $\frac{10{,}000}{2400}$ A, i.e 4.167 A

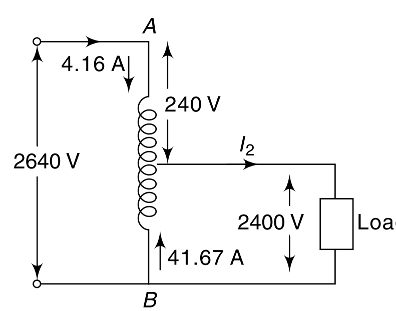


Fig. 8.30 *Circuit diagram for Example 8.44*

Current rating of 240 V winding is $\frac{10{,}000}{240}$ A, i.e 41.67 A

Load current I_L = (4.167 + 41.67) A = 45.837 A

$$\text{kVA output} = \frac{2400 \times 45.837}{1000} = 110$$

The currents in different sections of auto transformer is shown in Fig. 8.30.

8.45 A three-phase step down transformer is connected to 3300 V on the primary side. The ratio of turns per phase is 15 and the line current drawn from the mains is 30 A. Find the secondary line voltage, line current and output when transformer is (i) *YY* and (ii) Δ*Y*.

Solution

Input = $\left(\sqrt{3}V_L I_L\right)$ = $\sqrt{3}$ × 3300 × 30 VA = 171.47 kVA

Neglecting losses, output = 171.47 kVA

Turns ratio $\left(\dfrac{N_1}{N_2}\right)$ = 15.

(i) $\dfrac{V_{1\text{Ph}}}{V_{2\text{Ph}}} = \dfrac{N_1}{N_2}$ = 15, where ($V_{1\text{Ph}}$) and ($V_{2\text{Ph}}$) phase voltage of primary and secondary respectively.

Now, $$V_{1\text{Ph}} = \frac{V_{1L}}{\sqrt{3}} = \frac{3300}{\sqrt{3}}\ \text{V}$$

Hence, $$V_{2\text{Ph}} = \frac{3300}{\sqrt{3}\times 15}\ \text{V} = 127\ \text{V}.$$

Secondary line voltage = $\left(127\times\sqrt{3}\right)$V = 220 V = V_{2L}

For *Y* connection line current = phase current

$$\left(\sqrt{3}V_{2L} I_{2L}\right) = 171.47 \times 10^3$$

i.e, Secondary line current $I_{2L} = \dfrac{171.47\times 10^3}{\sqrt{3}\times 220}$ = 450 A.

(ii) For Δ*Y* connection primary is connected in delta and secondary in star.

Hence, $V_{1\text{Ph}} = V_{1L}$ = 3300 V

∴ $$V_{2\text{Ph}} = \frac{V_{1\text{Ph}}}{15} = \frac{3300}{15}\ \text{V} = 220\ \text{V}$$

Hence secondary line voltage (V_{2L}) = 220$\sqrt{3}$ V

Now, $\left(\sqrt{3}V_{2L} I_{2L}\right)$ = 171.47 × 10^3

∴ secondary line current = $\dfrac{171.47\times 10^3}{\sqrt{3}\times 220\times\sqrt{3}}$ = 260 A.

ADDITIONAL PROBLEMS

8.46 The core of a single-phase 3300/440 V, 50 Hz transformer is of square cross-section, each side being 140 mm. If the maximum flux density in the core is not to exceed 1*T*, find the number of turns required for each winding.

Solution

Flux = Flux density × Area
= 1 × (140 × 10^{-3})^2 = 0.0196 Wb.

If N_1 and N_2 be the number of turns of the primary and secondary windings respectively then,

3300 = 4.44 × 0.0196 × 50 × N_1 (∵ $E_1 = 4.44\ \phi_m f N_1$)

or N_1 = 758.4 or, 758 (say)

As $$\frac{N_1}{N_2} = \frac{E_1}{E_2}$$

So, $$N_2 = N_1 \frac{E_2}{E_1} = 758 \times \frac{440}{3300} = 101.$$

8.47 For the no load test on a transformer, the ammeter was found to read 0.18 A and the wattmeter 12 W. The reading on the primary voltmeter was 400 V and on the secondary voltmeter was 240 V. Calculate the magnetizing component of the no load current, the iron loss component and the transformation ratio.

Solution

Core loss or iron loss component of no load current $I_C = \frac{12}{400} = 0.03$ A.

No load current = 0.18 A

So magnetizing component of no load current $= \sqrt{(0.18)^2 - (0.03)^2} = 0.178$ A.

Transformation ratio $= \frac{400}{240} = \frac{5}{3} = 1.67:1$.

8.48 A single-phase transformer with a ratio of 440/200 V takes a no load current of 8 A at a p.f. of 0.25 (lagging). If the secondary supplies a current 220 A at a p.f. of 0.8 (lagging), estimate the current taken by the primary from the supply.

Solution

Secondary load current $I_2 = 220$ A

$\therefore$ load component of the primary current $(= I_1') = I_2 \times \frac{N_2}{N_1} = 220 \times \frac{200}{440} = 100$ A.

No load component of the primary current $I_o = 8$ A.

Referring to Fig. 8.31, the horizontal and vertical components of I_1' are $(I_1' \sin\theta)$ and $(I_1' \cos\theta)$, where $\cos\theta = 0.8$. Similarly, the horizontal and vertical components of I_o are $(I_o \sin\theta_o)$ and $(I_o \cos\theta_o)$ where $\cos\theta_o = 0.25$. So, the horizontal component of the primary current

$$= (I_1' \sin\theta + I_o \sin\theta_o)$$

i.e. $$I_{1H} = 100 \sin(\cos^{-1} 0.8) + 8 \sin(\cos^{-1} 0.25)$$
$$= 67.75 \text{ A}$$

Vertical component of the primary current

$$I_{1V} = (I_1' \cos\theta + I_o \cos\theta_o)$$
$$= 100 \times 0.8 + 8 \times 0.25 = 82 \text{ A}.$$

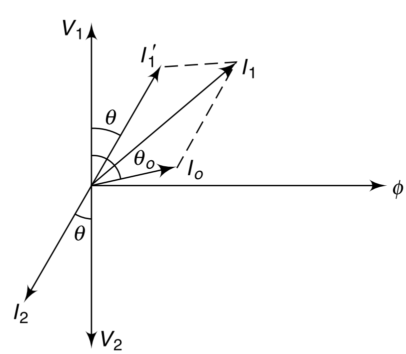


Fig. 8.31 *Figure for Example 8.48*

So the total primary current $I_1 = \sqrt{(82)^2 + (67.75)^2} = 106.37$ A.

8.49 A 6600/440 V single-phase transformer has a primary resistance of 140 Ω and a secondary resistance of 0.25 Ω. Calculate the equivalent resistances referred to the secondary winding and primary winding respectively.

Solution

Primary resistance $R_1 = 140\ \Omega$

Secondary resistance $R_2 = 0.25\ \Omega$

Secondary resistance referred to primary $R_2' = 0.25 \times \left(\frac{N_1}{N_2}\right)^2$

$$= 0.25 \times \left(\frac{6600}{440}\right)^2 = 56.25\ \Omega.$$

So, equivalent resistance referred to primary

$$R_{o1} = r_1 + r_2' = 140 + 56.25 = 196.25\ \Omega$$

Now, primary resistance referred to secondary $R_1' = 140 \times \left(\frac{N_2}{N_1}\right)^2$

$$= 140 \times \left(\frac{440}{6600}\right)^2 = 0.6222\ \Omega.$$

So, equivalent resistance referred to secondary

$$R_{o2} = R_1' + R_2 = 0.622 + 0.25 = 0.872\ \Omega.$$

8.50 A 17.5 kVA, 460/115 V single-phase, 50 Hz transformer has primary and secondary resistances of 0.36 Ω and 0.02 Ω respectively and the leakage reactances of these windings are 0.82 Ω and 0.06 Ω respectively. Determine the voltage to be applied to the primary to obtain full load current with the secondary winding short circuited. Neglect the magnetizing current.

Full load primary current $I_1 = \frac{17500}{460} = 38.04$ A

$$R_1 = 0.36\ \Omega \quad \text{and} \quad R_2 = 0.02\ \Omega$$
$$X_1 = 0.82\ \Omega \quad \text{and} \quad X_2 = 0.06\ \Omega.$$

Equivalent resistance referred to the primary

$$R_{o1} = R_1 + R_2' = 0.36 + 0.02\left(\frac{460}{115}\right)^2 = 0.68\ \Omega.$$

Equivalent reactance referred to the primary

$$X_{O1} = X_1 + X_2' = 0.82 + 0.06 \times \left(\frac{460}{115}\right)^2 = 1.78\ \Omega.$$

Equivalent impedance referred to the primary

$$Z_{O1} = \sqrt{(0.68)^2 + (1.78)^2} = 1.905\ \Omega.$$

As the secondary is short circuited the voltage applied to the primary to obtain full load current = $I_1 Z_{O1}$

$$= (38.04 \times 1.905) = 72.48\ \text{V}.$$

8.51 A 20 kVA, 2000/220 V single-phase transformer has a primary resistance of 2.1 Ω and a secondary resistance of 0.026 Ω. If the total iron loss equals 200 W, find the efficiency on (i) full load and at a p.f of 0.5 lagging (ii) half load and a p.f of 0.8 leading.

Solution

Iron loss = 200 W

Secondary current = $\frac{20 \times 10^3}{220} = 90.91$ A

Equivalent resistance referred to the secondary

$$= 2.1 \times \left(\frac{220}{2000}\right)^2 + 0.026 = 0.0514\ \Omega$$

Total copper losses = $(90.91)^2 \times 0.0514 = 424.8$ W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Output}}{\text{Output + Losses}}$$

(i) Efficiency at full load and 0.5 lagging p.f.

$$= \frac{20 \times 10^3 \times 0.5}{20 \times 10^3 \times 0.5 + 200 + 424.8} \times 100\% = 94.12\%.$$

(ii) Efficiency at half load and p.f. 0.8 leading

$$= \frac{20 \times 10^3 \times 0.8 \times \frac{1}{2}}{20 \times 10^3 \times 0.8 \times \frac{1}{2} + 200 + 424.8 \times \left(\frac{1}{2}\right)^2} \times 100\% = 96.3\%.$$

8.52 The primary resistance of a 440/110 V single-phase transformer is 0.28 Ω and the secondary resistance is 0.018 Ω. If the iron loss is measured to be 160 W when rated voltage is applied, find the kW loading to give maximum effeciency at unity p.f.

Solution

Iron loss = 160 W

For maximum efficiency, iron loss = Copper loss

So, $(I_2^2 R_{O2}) = 160$, where I_2 = Secondary full load current

and R_{O2} is the equivalent resistance referred to the secondary

Now, $R_{O2} = 0.28 \times \left(\frac{110}{440}\right)^2 + 0.018 = 0.0355\ \Omega$

So, $(I_2^2 \times 0.0355) = 160$ or, $I_2 = 67.13$ A

So, kW rating at unity p.f. = $\dfrac{V_2 I_2 \times 1}{10^3} = \dfrac{110 \times 67.13}{10^3} = 7.38.$

8.53 The core of a single-phase transformer has a cross-sectional area of 15000 mm^2 and the windings are chosen to operate the iron at a maximum flux density of 1.1 T from a 50 Hz. supply. If the secondary winding consists of 66 turns estimate the kVA output if the winding is connected to a load of 6 Ω impedance value.

Solution

Area $A = 15{,}000$ mm^2 = 0.015 sqm

Flux density $B_m = 1.1$ Wb/m^2

$f = 50$ Hz.

$N_2 = 66$

$E_2 = 4.44\ \phi_m f N_2 = 4.44\ B_m A f N_2$

$= 4.44 \times 1.1 \times 0.015 \times 50 \times 66 = 241.758$ V.

If load is 6 Ω the current $(I_2) = \dfrac{241.758}{6}$ A = 40.3 A

kVA output = $\left(\dfrac{E_2 I_2}{10^3}\right) = \dfrac{241.758 \times 40.3}{10^3} = 9.743.$

8.54 A 440/220 V single-phase transformer has a primary resistance of 0.29 Ω and a secondary resistance of 0.025 Ω. The corresponding reactance values are 0.44 Ω and

0.04 Ω. Estimate the primary current which would flow if a short circuit was to occur across the secondary terminals.

Solution

$$R_{O1} = 0.29 + 0.025 \times \left(\frac{440}{220}\right)^2 = 0.39\ \Omega$$

$$X_{O1} = 0.44 + 0.04 \times \left(\frac{440}{220}\right)^2 = 0.6\ \Omega$$

$$Z_{O1} = \sqrt{(0.39)^2 + (0.6)^2} = 0.7156\ \Omega$$

So, if a short circuit occurs across the secondary terminals

$$= \frac{440}{0.7156}\ \text{A} = 614.87\ \text{A}.$$

8.55 A 660/220 V single-phase transformer has a primary resistance of 0.3 Ω and a secondary resistance of 0.035 Ω. The corresponding reactance values are 0.5 Ω and 0.06 Ω. Estimate the percentage regulation for a secondary load current of 50 A at a p.f. of 0.8 (lagging).

Solution

$$R_{O2} = 0.3 \times \left(\frac{220}{660}\right)^2 + 0.035 = 0.0683\ \Omega$$

$$X_{O2} = 0.5 \times \left(\frac{220}{660}\right)^2 + 0.06 = 0.1155\ \Omega$$

$$\text{Voltage regulation} = \frac{I_2 R_{O2} \cos\theta_1 + I_1 X_{O2} \sin\theta_2}{E_2}$$

$$= \frac{50}{220}\{0.0683 \times 0.8 + 0.1155 \times 0.6\} = 0.028 \text{ or, } 2.8\%.$$

8.56 A 20 kVA, 2000/220 V, single-phase transformer has a primary resistance of 2.1 Ω and a secondary resistance of 0.026 Ω. The corresponding leakage reactances are 2.5 Ω and 0.03 Ω. Estimate the regulation at full load under p.f. conditions of (i) unity, (ii) 0.5 lagging and (iii) 0.5 leading.

Solution

$$I_2 = \frac{20 \times 10^3}{220} = 90.91\ \text{A}$$

$$R_{O2} = 2.1 \times \left(\frac{220}{2000}\right)^2 + 0.026 = 0.0514\ \Omega$$

$$X_{O2} = 2.5 \times \left(\frac{220}{2000}\right)^2 + 0.03 = 0.06025\ \Omega$$

(i) Voltage regulation at unity p.f. $= \dfrac{I_2}{E_2}\{R_{O2} \cos\theta_2 + X_{O2} \sin\theta_2\}$

$$= \frac{90.91}{220} \times 0.0514 \times 1 = 0.0212 \text{ or } 2.12\%.$$

(ii) Regulation at (0.5 lagging) p.f. $= \dfrac{90.91}{220}\{0.0514 \times 0.5 + 0.06025 \sin(\cos^{-1} 0.5)\}$

$= 0.322$ or 3.22%.

(iii) Regulation at (0.5 leading) p.f. $= \dfrac{90.91}{220}\{0.0514 \times 0.5 - 0.06025 \sin(\cos^{-1} 0.5)\}$

$= (-0.0109)$ or, (-1.09%).

8.57 A single-phase transformer is designed to operate at 2 V per turn and turns ratio of 3:1. If the secondary winding is to supply a load of 8 kVA at 80 V, find (i) the primary supply voltage, (ii) the number of turns on each winding and (c) the current in each winding.

Solution

$$\frac{N_1}{N_2} = \frac{3}{1} = \frac{E_1}{E_2}$$

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 2$$

$$E_2 = 80 \text{ V}$$

where N_1 and N_2 are the number of turns of the primary and secondary windings respectively.

(i) So, $E_1 = 3 \times 80 = 240$ V

Primary voltage = 240 V.

(ii) Now, $\left(\dfrac{E_1}{N_1}\right) = 2$

So, $(N_1) = \dfrac{E_1}{2} = \dfrac{240}{2} = 120$

Again, $(N_2) = \dfrac{E_2}{2} = \dfrac{80}{2} = 40.$

(iii) Secondary current $= \dfrac{8000}{80} = 100$ A ($\because$ load is 8 kVA)

Primary current $= \dfrac{8000}{240} = 33.33$ A.

8.58 A single-phase step down transformer has the following particulars: Turns ratio 4:1, no load current 5 A at 0.3 p.f. lagging. Secondary voltage 110 V. Secondary load 10 kVA at 0.8 p.f. (lagging). Find (i) the primary voltage, neglecting the internal voltage drop, (ii) the secondary current on load, (iii) the primary current and (iv) the primary p.f.

Solution

$$\left(\frac{N_1}{N_2}\right) = \frac{4}{1} = \frac{E_1}{E_2}$$

$$E_2 = 110 \text{ V}$$

(a) $\therefore$ $E_1 = 4 \times 110 = 440$ V.

(b) Secondary load current $(I_2) = \dfrac{10{,}000}{110} = 90.91$ A.

(c) If (I_1') is the load component of the primary current then from $I_1'N_1 = I_2 N_2$ or,

$$I_1' = I_2 \frac{N_2}{N_1} = 90.91 \times \frac{1}{4} = 22.73 \text{ A}$$

if I_1 is the primary current then,

$I_1 \cos \theta_1 = I_o \cos \theta_o + I_1' \cos \theta$, where I_o and I_1' the no load and load component of primary current, $\cos \theta_o$ and $\cos \theta$ are the no load p.f. and secondary load p.f. respectively.

So, $I_1 \cos \theta_1 = 5 \times 0.3 + 22.73 \times 0.8 = 19.684$ A.

Again, $I_2 \sin \theta_1 = 5 \sin(\cos^{-1} 0.3) + 22.73 \sin(\cos^{-1} 0.8)$
$= 18.4$ A

$\therefore$ $I_1 = \sqrt{(19.684)^2 + (18.4)^2} = 26.94$ A.

$\therefore$ Primary current is 26.94 A

(d) Primary p.f. $= \cos \theta = \dfrac{I_1 \cos \theta_1}{I_1} = \dfrac{19.68}{26.94} = 0.73$ lagging.

8.59 A 6.6 kV, 50 Hz single-phase transformer with a transformation ratio (1:0.06) takes a no load current of 0.7 A and a full load current of 7.827 A when the secondary is loaded to 120 A at a p.f. of 0.8 lagging. What is the no load p.f.?

Solution

$I_2 = 120$ A

Load p.f. (cos $\theta = 0.8$ lagging or, $\theta = \cos^{-1} 0.8 = 36.87°$

$$\frac{N_1}{N_2} = \frac{1}{0.06}$$

No load primary current $I_o = 0.7$ A

Load component of primary current (= I_1') = $I_2 \dfrac{N_2}{N_1} = 120 \times 0.06 = 7.2$ A

Full load primary current $I_1 = 7.827$ A

Let no load p.f. angle be θ_o

Referring to Fig. 8.31

$$I_1^2 = I_0^2 + I_1'^2 + 2I_O I_1' \cos(\theta_o - \theta)$$

or $\cos(\theta_o - \theta) = \dfrac{(7.827)^2 - (0.7)^2 - (7.2)^2}{2 \times 0.7 \times 7.2} = 0.886$

or $(\theta_o - \theta) = \cos^{-1} 0.886 = 27.625°$

or $\theta_o = 27.625° + 36.87° = 64.495°$.

8.60 A 1 kVA single-phase transformer has an iron loss of 20 W and a full load copper loss of 40 W. Calculate its efficiency on full load output at a p.f. of (0.8) lagging.

Solution

Efficiency on full load at 0.8 p.f. lagging

$$= \frac{\text{Output}}{\text{Output + Loss}} = \frac{1000 \times 0.8}{1000 \times 0.8 + 20 + 40} \times 100\% = 93\%.$$

8.61 A 25 kVA, 440/110V, 50 Hz single-phase step down transformer is designed to work with 1.5 V per turn with a flux density not exceeding 1.35 T. Calculate (i) the required number of turns on the primary and secondary windings respectively, (ii) the cross-sectional area of the iron core and (iii) the secondary current.

Solution

$$\frac{E_1}{N_1} = \frac{E_2}{N_2} = 1.5$$

$$B_m = 1.35 \text{ Wb/m}^2$$

$$E_1 = 440 \text{ V and } E_2 = 110 \text{ V}$$

(i) $$N_1 = \frac{E_1}{1.5} = \frac{440}{1.5} = 293$$

As $$\frac{E_1}{E_2} = \frac{N_1}{N_2}$$

So, $$N_2 = \frac{E_2}{E_1} \times N_1 = \frac{110}{440} \times 293 = 73$$

Here (N_1) and (N_2) are the number of turns of the primary and secondary windings respectively.

(ii) $E_1 = 4.44\ B_m\ AfN_1$, where A is the cross-sectional area of the iron core.

Here, $$A = \frac{440}{4.44 \times 1.35 \times 50 \times 293} \text{ sqm} = 0.005 \text{ sqm.}$$

8.62 When an open circuit test is made on the primary of a transformer at 440 V and 50 Hz, the iron loss is 2.5 kW. When the test is repeated at 220 V and 25 Hz, the corresponding loss is measured to be 850 W. Determine the hysteresis and eddy current loss at normal voltage and frequency.

Solution

Hysteresis loss, $H \propto f$ and eddy current loss, $E \propto f^2$, where f is the frequency.

At $$H = K_1 f \text{ and } (E) = K_2 f^2$$

At 50 Hz iron loss = 2500 W

As iron loss = $H + E$

$\therefore$ $$2500 = K_1 50 + K_2(50)^2$$

or $$2500\ K_2 + 50\ K_1 = 2500$$

or $$50\ K_2 + K_1 = 50$$

Again at 25 Hz iron loss = 850 W

$\therefore$ $$850 = K_1\ 25 + K_2(25)^2$$

or $$K_1 + 25\ K_2 = 34$$

Solving the above two equations

$$(50 - 25)K_2 = 50 - 34$$

or $$K_2 = \frac{16}{25} = 0.64$$

and $$K_1 = 50 - 50 \times 0.64 = 18.$$

At 50 Hz, hysteresis loss = 18 × 50 = 900 W

and eddy current loss = $0.64 \times (50)^2 = 1600$ W.

8.63 The following results were obtained in tests on a 50 kVA, single-phase, 3300/400 V transformer.

Open Circuit Test:
Primary voltage 3300 V, Secondary voltage 400 V, Input Power 430 W.

Short Circuit Test:
Reduced voltage on primary (124 V) to give full secondary current, primary current is 15.3 A and input power 535 W.
Calculate
(i) The efficiency at half load at 0.707 p.f. lagging.
(ii) The regulation and terminal voltage at full load for p.f. 0.707 leading.

Solution

From open circuit test iron losses = 430 W.
From short circuit test copper losses = 535 W.
Under short circuit test applied voltage = 124 V
and primary current = 15.3 A.
Hence equivalent impedance referred to the primary winding

$$Z_{O1} = \frac{124}{15.3}\ \Omega = 8.1\ \Omega$$

Equivalent impedance referred to the secondary winding

$$Z_{O2} = Z_{O1} \times \left(\frac{N_2}{N_1}\right)^2 = 8.1 \times \left(\frac{400}{3300}\right)^2 = 0.119\ \Omega$$

If I_1 is the primary current and R_{O1} is the equivalent resistance referred to the primary then

$$I_1^2 R_{O1} = 535$$

or

$$R_{O1} = \frac{535}{(15.3)^2} = 2.285\ \Omega.$$

Hence equivalent resistance referred to secondary is

$$R_{O2} = 2.285 \times \left(\frac{400}{3300}\right)^2 = 0.03357\ \Omega$$

$$X_{O2} = \sqrt{(0.119)^2 - (0.03357)^2} = 0.114\ \Omega.$$

Full load secondary current $I_2 = \dfrac{50 \times 10^3}{400}$ A = 125 A

(i) Efficiency at $\left(\dfrac{1}{2}\right)$ load at 0.707 p.f. lagging

$$= \frac{50 \times 10^3 \times \frac{1}{2} \times 0.707}{50 \times 10^3 \times \frac{1}{2} \times 0.707 + 430 + \left(\frac{1}{2}\right)^2 \times 535} \times 100\% = 96.92\%.$$

(ii) Regulation at full load for p.f. 0.707 leading

$$= \frac{I_2}{E_2}\{R_{O2}\cos\theta_2 - X_{O2}\sin\theta_2\}$$

$$= \frac{125}{400}\{0.03357 \times 0.707 - 0.114 \times 0.707\} = -0.0178 \text{ or, } -1.78\%$$

Again, $\left[1 - \dfrac{V_2}{E_2}\right]$ = (–0.0178), where (V_2) is the terminal voltage.

Hence, $V_2 = (1 + 0.0178)\,E_2 = 1.0178 \times 400 = 407.12$ V.

8.64 The daily variation of load on a 100 kVA transformer is as follows:

8 a.m. to 1 p.m.:	65 kW, 45 KVAR
1 p.m .to 7 p.m.:	80 kW, 50 KVAR
7 p.m. to 2 a.m.:	30 kW, 30 KVAR
2 a.m. to 8 a.m.:	No load

The transformer has a no load core loss of 270 W and a full load ohmic loss of 1200 W. Determine the all day efficiency of the transformer.

Solution

From 8 a.m. to 1 p.m.,

$$\text{kVA} = \sqrt{(65)^2 + (45)^2} = 79$$

$$\text{Ohmic loss} = \left(\frac{79}{100}\right)^2 \times 1200 = 749 \text{ W}$$

$$\text{Energy lost as ohmic loss} = \frac{749 \times 5}{10^3} \text{ kWh} = 3.745 \text{ kWh.}$$

From 1 p.m. to 6 p.m.,

$$\text{kVA} = \sqrt{(80)^2 + (50)^2} = 94.34$$

$$\text{Ohmic loss} = \left(\frac{94.34}{100}\right)^2 \times 1200 = 1068 \text{ W}$$

$$\text{Energy lost as ohmic loss} = \frac{1068 \times 6}{10^3} \text{ kWh} = 6.408 \text{ kWh.}$$

From 6 p.m. to 1 a.m.,

$$\text{kVA} = \sqrt{(30)^2 + (30)^2} = 42.426$$

$$\text{Ohmic loss} = \left(\frac{42.426}{100}\right)^2 \times 1200 = 216$$

$$\text{Energy lost as ohmic loss} = \frac{216 \times 7}{10^3} \text{ kWh} = 1.51 \text{ kWh}$$

Daily energy lost as ohmic losses = (3.745 + 6.408 + 1.512) kWh = 11.665 kWh

$$\text{Daily energy lost as core loss} = \frac{24 \times 270}{10^3} \text{ kWh} = 6.48 \text{ kWh}$$

Total energy loss = (11.665 + 6.48) kWh = 18.145 kWh

Daily kWh output = 65 × 5 + 80 × 6 + 30 × 7 + 0 = 1015 kWh

$$\text{All day efficiency} = \frac{\text{Energy output}}{\text{Energy output + Energy loss}} = \frac{1015}{1015 + 18.145} \times 100\% = 98.24\%.$$

8.65 A 200 V, 60 Hz single-phase transformer has hysteresis and eddy current losses of 250 W and 90 W respectively. If the transformer is now energized from 230 V, 50 Hz supply, calculate its core losses. Assume Steinmmetz's constant equal to 1.6.

Solution

If W_h and W_e are the hysteresis and eddy current loss respective then

$$W_h = K_h f B_m^x \quad \text{and} \quad W_e = K_e f^2 B_m^2$$

where K_h and K_e are constants, f is the frequency, B_m is maximum flux density and x is the Steinmetz constant.

We know that $V = \sqrt{2}\,\pi f\, B_m\, A\, N$.

If the cross-sectional area of core, i.e A and no. of turns N remain constant then $V \propto f\, B_m$.

$$\text{So,} \quad \frac{V_1}{V_2} = \frac{f_1 B_{m_1}}{f_2 B_{m_2}}$$

$$\text{or} \quad \frac{200}{230} = \frac{60\, B_{m_1}}{50\, B_{m_2}}$$

$$\text{or} \quad \frac{B_{m_1}}{B_{m_2}} = 0.7246$$

$$\text{Now,} \quad \frac{W_{h_1}}{W_{h_2}} = \frac{f_1\ B_{m1}^x}{f_2\ B_{m2}^x}$$

$$\text{or} \quad \frac{250}{W_{h_2}} = \frac{60}{50} \times (0.7246)^{1.6} = 0.7167$$

$$\text{or} \quad W_{h_2} = 348.82$$

$$\text{Similarly,} \quad \frac{W_{e_1}}{W_{e_2}} = \frac{f_1^2\, B_{m1}^2}{f_2^2\, B_{m2}^2}$$

$$\text{or} \quad \frac{90}{W_{e_2}} = \left(\frac{60}{50}\right)^2 \times (0.7246)^2$$

$$\text{or} \quad W_{e_2} = 119.037.$$

So, the core loss = (348.82 + 119.037) W = 467.857 W.

8.66 A 11,000 V, 500 Hz transformer has a flux density of 1.2 Wb/m^2 at rated voltage and frequency. Now all the linear dimensions of the core are doubled; primary and secondary turns are halved and the new transformer is energised from 22000 V, 50 Hz supply. Both the transformers have the same core material and the same lamination thickness. Calculate the flux density for the new transformer.

Solution

$$V = \sqrt{2}\ \pi f\, B_m\, AN$$

$$\frac{V_1}{V_2} = \frac{11{,}000}{22{,}000} = \frac{50 \times 1.2\, AN}{50 \times B_{m_2} \times 2^2\ A \times \frac{1}{2} N}$$

$$\text{or} \quad \frac{1}{2} = \frac{1.2}{B_{m_2} \times 2} \quad \text{or,}\ B_{m_2} = \frac{1.2 \times 2}{2} = 1.2.$$

So the flux density of the new transformer = 1.2 Wb/m^2.

8.67 A 100 kVA, 2400/240 V, 50 Hz single-phase transformer has an exciting current of 0.64 A and a core loss of 700 W, when its high voltage side is energised at rated voltage and frequency. Calculate the two components of the exciting current.

Solution

Exciting current $I_e = 0.64$

Core loss = ($V_1 I_e \cos\theta_o$) = 700 W, where $\cos\theta_o$ is the no load p.f. and V_1 is the voltage of the primary winding.

So,
$$\cos\theta_o = \frac{700}{2400 \times 0.64} = 0.456$$

The core loss component of exciting current
$$= I_e \cos\theta_o = 0.64 \times 0.456 = 0.292 \text{ A.}$$

The magnetizing component of exciting current
$$= I_e \sin\theta_o = 0.64 \times \sin(\cos^{-1} 0.456) = 0.569 \text{ A.}$$

8.68 A non-sinusoidal voltage $v = 150 \sin 314t - 75 \sin 1570t$ is applied to the 250 turn winding of a transformer. Find the core flux as a function of time.

Solution

If ϕ is the flux then
$$v = -N\frac{d\phi}{dt}, \text{ where } (N) \text{ is the number of turns}$$

or
$$\frac{d\phi}{dt} = -\frac{v}{N}$$

or
$$\phi = -\frac{1}{N}\int v\,dt = -\frac{1}{N}\int (150 \sin 314t - 75 \sin 1570\,t)dt$$
$$= -\frac{1}{250}\left[-\frac{150}{314}\cos 314\,t + \frac{75}{1570}\cos 1570\,t\right]$$

or
$$\phi = (0.0019 \cos 314t - 0.00019 \cos 1570t) \text{ Wb.}$$

8.69 A voltage $v = 200 \sin 314t$ is applied to the transformer winding in a no load test. The resulting current is found to be $i = 3 \sin(314t - 60°)$. Determine the core loss and rms value of the exciting current.

Solution

The instantaneous exciting current $i = 3 \sin(314t - 60°)$

RMS value of exciting current $I_e = \dfrac{3}{\sqrt{2}}$ A = 2.12 A

The instantaneous applied voltage (v) = 200 sin 314 t

rms value of voltage $V_1 = \dfrac{200}{\sqrt{2}}$ V

No load power factor angle = (θ_o) = 60°

$$\therefore \text{ core loss } (= V_1 I_e \cos\theta_o) = \frac{200 \times 3}{\sqrt{2}\sqrt{2}} \cos 60° = \frac{200 \times 3}{2} \times \frac{1}{2} = 150 \text{ W.}$$

8.70 A transformer has the following test data:

Test no.1: 100% voltage, 6% current, p.f. = 0.25

Test no.2: 8% voltage, 100% current, p.f. = 0.4.

Identify the tests. Calculate the efficiency and percentage regulation at full load and unity p.f.

Solution

As in test no. 1, full rated voltage is applied and the current is very less so the test must be no load test or open circuit test.

As in test no. 2, rated current is flowing and the applied voltage is very small so the test must be short circuit test.
From test no. 2

$$Z = \frac{0.8}{100} = 0.08 \text{ p.u.}$$
$$R = Z_e \cos\theta = 0.08 \times 0.4 = 0.032 \text{ p.u.}$$
$$X = 0.08 \times \sin(\cos^{-1} 0.4) = 0.0733 \text{ p.u.}$$

where Z, R and X are the p.u. values of equivalent impedance, resistance and reactance.
The regulation at full load and unity p.f. = (0.032 × 1 + 0.0733 × 0) = 0.032 or 3.2%.
From test 1 core losses = $V_1 I \cos\theta_o = 1 \times 0.06 \times 0.25 = 0.015$ p.u.
From test 2 full load ohmic losses (= I^2R) = $1^2 \times 0.032 = 0.032$ p.u.

Total losses = (0.015 + 0.032) p.u. = 0.047 p.u.

$$\text{Efficiency} = 1 - \frac{\text{Losses}}{\text{Output + Loss}} = 1 - \frac{0.047}{1+0.047} = 0.9551 \text{ or } 95.51\%.$$

8.71 In no load test of a single-phase transformer the following test data were obtained:

Primary voltage = 220 V
Secondary voltage = 110 V
Primary current = 0.5 A
Power input 30 = W.

Find the turns ratio, magnetising component of no load current, loss component of no load current and the iron loss. Resistance of primary winding is 0.6 Ω.

Solution

$$\text{Turns ratio} = \left(\frac{N_1}{N_2}\right) = \frac{V_1}{V_2} = \frac{220}{110} = 2$$

No load current $I_o = 0.5$ A
Power at no load ($V_1 I_o \cos\theta_o$) = 30 W.

Hence, $I_o \cos\theta_o = \frac{30}{220} = 0.136$ A, i.e loss component of no load current is 0.136 A.

Hence, $\cos\theta_o = 0.272$, i.e. $\sin\theta_o = 0.962$
Magnetising component of no load current is

$$I_o \sin\theta_o = 0.5 \times 0.962 = 0.481 \text{ A}$$

Iron loss = Input power – Ohmic loss in primary winding
= $(30 - (0.5)^2 \times 0.6) = 29.85$ W.

8.72 An auto transformer supplies a load of 2.5 kW at 110 V and at unity p.f. If the primary applied voltage is 220 V, calculate power transferred from the mains and power conducted directly from the supply lines to the load.

Solution

$$V_1 = 220 \text{ V}$$
$$V_2 = 110 \text{ V}$$

Power transferred from mains $(V_1 - V_2)I_1$
Now output power = 2.5 kW at unit p.f.
Hence output kVA = 2.5 × 1 kVA = 2.5 kVA
Neglecting loss input kVA = Output kVA = 2.5

Hence primary current $I_1 = \frac{2.5 \times 10^3}{220} \text{A} = \frac{250}{22} \text{A}$

Power transferred from mains $= (220 - 110) \times \dfrac{250}{22} \times 1 \text{ W} = 1.25 \text{ kW}$

Power conducted directly from the supply lines to the load = 2.5 – 1.25 = 1.25 kW.

8.73 Two 2200/110 V transformers are operated in parallel to share a load of 125 kVA at 0.8 p.f. lagging.
Transformers are rated as below:
A : 100 kVA, 0.9% resistance and 1.0% reactance
B : 50 kVA, 0.1% resistance and 0.5% reactance
Find the load carried by each transformer.

Solution

Let base kVA be 100

$$Z_A = (0.009 + j0.01) \text{ p.u.}$$

$$Z_B = (0.001 + 0.005) \times \frac{100}{50} = (0.002 + j0.01) \text{ p.u. converting on 100 kVA.}$$

Load is 125 kVA at 0.8 p.f. lagging

Hence, $S_L = 125\angle{-\cos^{-1} 0.8} \text{ kVA} = 125\angle{-36.87°} \text{ kVA}$

Load carried by transformer *A*

$$= 125\angle{-36.87°} \times \frac{0.002 + j\,0.01}{(0.009 + 0.002) + j(0.01 + 0.01)}$$

$$= \frac{125\angle -36.87° \times 0.0102\angle 78.69°}{0.0228\angle 61.19°}$$

$$= 55.92\angle{-19.37°} \text{ kVA} = 52.75 \text{ kW at lagging p.f.}$$

Load carried by transformer *B*

$$= 125\angle{-36.87°} \times \frac{0.009 + j0.01}{(0.009 + 0.002) + j(0.01 + 0.01)}$$

$$= \frac{125\angle -36.87° \times 0.01345\angle 48°}{0.0228\angle 61.19°}$$

$$= 73.74\angle{-50.06°} \text{ kVA} = 47.34 \text{ kW at lagging p.f.}$$

8.74 A three-phase 50 Hz transformer has a delta connected primary and star connected secondary, the line voltages being 22000 V and 400 V respectively. The secondary has a star connected balanced load at 0.8 p.f. lagging. The line current on the primary is 5 A. Determine the current in each coil of the primary and in each secondary line. What is the output of the transformer in kW?

Solution

Primary line voltage V_{1L} = 22000 V
Secondary line voltage V_{2L}) = 400 V
Primary line current I_{1L} = 5 A
Since the transformer has delta connected primary and star connected secondary,
∴ Primary phase voltage $V_{1\text{Ph}}$ = 22000 V

Secondary phase voltage $V_{2\text{Ph}} = \dfrac{400}{\sqrt{3}}$

$$\text{Turns ratio} = \left(\frac{V_{1\text{Ph}}}{V_{2\text{Ph}}}\right) = \frac{22000\sqrt{3}}{400} = \frac{220\sqrt{3}}{4} = 55\sqrt{3}$$

Primary phase current $I_{1Ph} = \dfrac{5}{\sqrt{3}}$ A

Hence secondary phase current $I_{2Ph} = \dfrac{5}{\sqrt{3}} \times 55\sqrt{3}$ A = 275 A

Secondary line current = 275 A (as secondary is star connected)

$$\begin{aligned}\text{Output of the transformer} &= \sqrt{3}\, V_L\, I_L \cos\theta \\ &= \sqrt{3}\, V_{2L}\, I_{2L} \cos\theta \\ &= \sqrt{3} \times 400 \times 275 \times 0.8 = 152420.47 \text{ W} = 152.42 \text{ kW.}\end{aligned}$$

8.75 A 50 HP, 220 V, three-phase motor has full load efficiency of 0.9 and power factor 0.8. It is fed from a 3300 V system from a 3300/220 delta star transformer. Find the phase current of the primary and secondary winding transformer.

Solution

Output of motor = 50 HP = 50 × 735.5 W = 36775 W

Efficiency = 0.9.

Hence, $\dfrac{\text{Output of motor}}{\text{Input of motor}} = 0.9$

i.e. Input of motor $= \dfrac{36775}{0.9} = 40861$ W

Therefore output of transformer = 40861 W

Power factor ($\cos\theta$) = 0.8

Hence $\left(\sqrt{3} V_L I_L \cos\theta\right) = 40861$, where V_L and I_L are line voltage and current of the secondary winding of transformer

$V_L = 220$

Hence, $I_L = \dfrac{40861}{\sqrt{3} \times 220 \times 0.8}$ A = 134 A.

Since the secondary winding of transformer is star connected the phase current is also 134 A.

Turns ratio of transformer $= \dfrac{3300}{220/\sqrt{3}} = 25.98$

Hence, primary phase current $= \dfrac{134}{25.98}$ A = 5.157 A.

EXERCISES

1. Define a transformer. Discuss the principle of operation of a single phase transformer.
2. Distinguish between core type and shell type transformer. Why is the low voltage winding placed near the core? Why is the core of a transformer laminated?
3. Derive an expression for the emf induced in a transformer winding.
4. Define an ideal transformer. Draw and explain the no load phasor diagram of an ideal single phase transformer.
5. Draw the exact equivalent circuit of a transformer and describe briefly the various parameters involved in it.

6. Draw and explain the phasor diagram of a single-phase transformer under lagging p.f.
7. Define voltage regulation of a transformer. Develop an expression for calculating the voltage regulation of a two winding transformer under (i) lagging p.f., (ii) unity p.f. and (iii) leading p.f.
8. What are the different types of losses in a transformer? Write an expression for efficiency and develop a condition for maximum efficiency.
9. Explain why
   (i) the open circuit test on a transformer is conducted at a rated voltage,
   (ii) usually the low voltage winding is excited and the high voltage winding is open circuited for open circuit test,
   (iii) the open circuit test gives core loss and short circuit test gives copper loss,
   (iv) usually low voltage winding is short circuited and high voltage winding is excited for the short circuit test.
10. Discuss about the Sumpner's test on single-phase transformer.
11. (i) Explain why parallel operation of transformer is necessary.
    (ii) State the essential and desirable conditions which would be satisfied before two single-phase transformers may be operated in parallel.
    (iii) Deduce expressions for the load shared by two transformers connected in parallel.
12. What is an auto transformer? State its merits and demerits over a two winding transformer. What are the applications of an auto transformer?
13. Discuss about the different types of cooling used in transformers. Distinguish between a power transformer and a distribution transformer.
14. Define all day efficiency of a single-phase transformer.
15. What are the advantages of a transformer bank of three single-phase transformers over a unit three-phase transformer of the same kVA rating? What are the distinguishing features of YY, YΔ, ΔY and ΔΔ three-phase connections?
16. The primary winding of a single phase transformer connected to a 500 V, 50 Hz supply takes 1.41 A and absorbs 125 W with the secondary winding open circuited. The secondary open circuit voltage is 250 V. When the secondary winding is short circuited and the primary is connected to a 250 V, 50 Hz supply, the primary current is 15.1 A and the power absorbed is 92 W. Determine the shunt and series components of the equivalent circuit.
    [*Ans:* R_O = 2000 Ω, X_O = 360.23 Ω, r_{e1} = 0.403 Ω, x_{e1} = 1.606 Ω]
17. A 1100/230 V, 150 kVA single-phase transformer has a core loss of 1.4 kW and a full load copper loss of 1.6 kW. Determine (i) the kVA load for maximum efficiency and (ii) the maximum efficiency at unity power factor load. [*Ans:* 140.312 kVA, 98.04%]
18. A 415/220 V transformer takes a no load current of 1 A and operates at a p.f. of 0.19 lagging when the secondary supplies a current of 100 A at 0.8 p.f. lagging; find the primary current. [*Ans.:* 53.27 A]
    [*Hint:*
    No load current, I_o = 1 A

No load power factor angle, $\theta_o = \cos^{-1} 0.19 = 79°$
Secondary current, $I_2 = 100$ A
Load power factor angle, $\theta_2 = \cos^{-1} 0.8 = 36.86°$
Load component of the primary current,

$$I_1' = I_2 \frac{N_2}{N_1} = 100 \times \frac{220}{415} \text{A} = 53 \text{ A.}$$

Vertical component of primary current I_1 is

$$I_1' \cos \theta_2 + I_o \cos \theta_o = 53 \times 0.8 + 1 \times 0.19 = 42.59 \text{ A.}$$

Horizontal component of primary current I_1 is

$$I_1' \sin \theta_2 + I_o \sin \theta_o = 53 \times 0.6 + 1 \times \cos 79° = 32 \text{ A.}$$

Hence $\quad I_1 = \sqrt{(42.59)^2 + (32)^2} = 53.27$ A]

19. The following test data were obtained on a 20 kVA, 50 Hz; 1 ph, 2000/200 V transformer
    No load test: 200 V, 1 A, 120 W
    Short circuit test: 60 V, 10 A, 300 W
    Find (i) efficiency of the transformer at 1/2 of the full load and 0.8 p.f. lagging.
    (ii) maximum efficiency and the load at which it occurs.
    [*Ans:* 97.62%; 63.2% of full load]

    [*Hint:*
    $P_c = 120$ W
    $P_{cu} = 300$ W

    (i) Efficiency $= \dfrac{20 \times 10^3 \times 0.8 \times \frac{1}{2}}{20 \times 10^3 \times 0.8 \times \frac{1}{2} + 120 + \left(\frac{1}{2}\right)^2 \times 300} \times 100\%$

    $= 97.62\%$

    (ii) If maximum efficiency occurs at x times the full load

    $$x^2 \times 300 = 120, \ \therefore \ x = 0.632$$

    Load at maximum efficiency is 20×0.632 kVA = 12.64 kVA
    Maximum efficiency at 0.8 p.f. is

    $$\frac{12.64 \times 10^3 \times 0.8}{12.64 \times 10^3 \times 0.8 + 2 \times 120} \times 100\% = 97.68\%]$$

20. A 100 kVA, single-phase transformer of ratio 10000/200 V requires 300 V at the high voltage winding to circulate full load current with low voltage winding short circuited. The intake power is then 1000 W. Calculate the % regulation and the secondary terminal voltage on full load at 0.8 p.f. lagging.
    [*Ans:* 2.49%]
21. Calculate (i) full load efficiency at 0.8 p.f. and (ii) the approximate voltage at the secondary terminal at full load and power factor of 0.8 lag and 0.8 lead for a 4 kVA, 200/400 V, 50 Hz single-phase transformer with the following test results:
    Open circuit test (L.T. side data) : 200 V, 0.8 A, 70 W
    Short circuit test (H.T. side data) : 17.5 V, 9 A, 50 W
    [*Ans:* 96.05%, 384 V, 406.12 V]

22. The primary and secondary winding resistances of a 30 kVA, 6000/230 V transformer are 10 Ω and 0.016 Ω respectively. The reactance of the transformer as referred to primary is 34 Ω. Calculate the regulation at full load 0.8 p.f. lagging. [*Ans:* 3.33%]
23. A single-phase 20 Hz transformer is required to step down 2200 V to 250 V. The cross-section of the core is 36 sq cm and the maximum value of the flux density is 6 Wb. Determine a suitable number of turns for each winding and the transformation ratio.

$$\left[Ans: 458, 52, \frac{1}{8.8}\right]$$

24. A 20 kVA transformer has 400 turns on the primary and 40 turns on the secondary winding. The primary is connected to a 2 kV, 50 Hz. supply. Find the full load primary and secondary currents, secondary emf and the maximum flux in the core. Neglect leakage drop and no load primary current.
[*Ans:* I_{fl} = 10 A; I_2 = 100 A, E_2 = 200 V; ϕ_m = 22.5 mwb]

[*Hints:*

$$\frac{N_1}{N_2} = \frac{400}{40},$$

$$I_1 = \frac{20}{2}\text{A} = 10 \text{ A}$$

$$I_2 = I_1 \times \frac{N_1}{N_2} = 10 \times \frac{400}{40} = 100 \text{ A}$$

Secondary emf $E_2 = E_1 \times \dfrac{N_2}{N_1} = 2 \times \dfrac{40}{400} = 0.2 \text{ KV} = 200 \text{ V}$

$$E_1 = 4.44 f N_1 \times \phi_m$$

$$\phi_m = \frac{2000}{4.44 \times 50 \times 400} \text{wb} = 0.0225 \text{ wb}]$$

25. A 125 kVA transformer having a primary voltage of 2200 V at 50 Hz has 182 primary turns and 40 secondary turns. Neglecting losses calculate (i) full load primary and secondary currents and (ii) no load secondary induced emf. [*Ans:* 62.5A, 284.4 A, 439.5 V]
26. The secondary windings of a 2 kVA, 2400/120 V, 50 Hz single phase transformer is short circuited and potential difference of 16.2 V produces a primary current of 6 A, the input being 33.5 W. Calculate the total primary impedance, resistance and reactance. [*Ans:* 2.7 W, 0.93 W, 2.54 W]
27. The open circuit and short circuit test data of a 5 kVA, 200/400 V, 50 Hz; 1 phase transformer are:
    (i) *O.C. test:* primary voltage = 200 V, I = 0.75 A, W = 75 W.
    (ii) *S.C. test:* primary voltage = 18 V, S.C. current on the secondary side = 12.5 A, W = 60 W.

    Find the parameters of the equivalent circuit.

[*Ans.:* R_o = 2133.33 Ω; X_o = 1232.66 Ω,
r_e = 0.384 Ω; x_e = 1.39 Ω
(all quantities are referred to h.v. side)]

[*Hint:*
No load current, $I_0 = 0.75$ A

Core loss component of current $I_C = \dfrac{75}{200} = 0.375$ A

Magnetising component of current $I_\phi = \sqrt{(0.75)^2 - (0.375)^2} = 0.649$ A
Parameters referred to h.v. side

Core loss resistance $R_o = \dfrac{200}{0.375} \times \left(\dfrac{400}{200}\right)^2 = 2133.33\ \Omega$

Magnetising reactance $X_\phi = \dfrac{200}{0.649} \times \left(\dfrac{400}{200}\right)^2 = 1232.66\ \Omega$

Equivalent resistance $r_{eH} = \dfrac{60}{(12.5)^2}\Omega = 0.384\ \Omega$

Equivalent impedance $Z_{eH} = \dfrac{18}{12.5}\Omega = 1.44\ \Omega$

Equivalent reactance $x_{eH} = \sqrt{(1.44)^2 - (0.384)^2}\ \Omega = 1.388\ \Omega$]

28. A 10 kVA transformer is given an open circuit test from which the iron losses are found to be 160 W. A short circut test shows the copper losses to be 240 W with full load current flowing. Calculate the all day efficiency if it operates at unity p.f. for 6 hours per day on full load, 2 hours on half load and the remainder of the time on no load. [*Ans:* 92.85%]
29. A 20 kW lighting transformer of ordinary efficiency 95% is on full load for 6 hours per day. Find the all day efficiency if the full load losses are equally divided between copper and iron. [*Ans:* 88.9%]
30. A 100 kVA, 50 Hz, 440/11,000 V, 1-phase transformer has an efficiency of 98.5% when supplying full load current at 0.8 p.f. lagging and an efficiency of 99% when supplying half full load current at unity power factor. Find the core losses and copper losses corresponding to full load current.
[*Ans:* $P_i = 277.33$ W; $P_{cu} = 940.67$ W]

[*Hint:*
At 98.5% efficiency,
Output = 100 × 0.8 kW = 80 kW

Loss $= 80 \times \dfrac{1 - 0.985}{0.985} = 1.218$ kW

If P_i and P_{cu} be the full load iron and copper losses then

$$P_i + P_{cu} = 1218$$

Again, $P_i + \dfrac{1}{4}(P_{cu}) = 100 \times 1 \times \dfrac{1 - 0.99}{0.99} \times 10^3 = 1010.$

Solving, $P_i = 277.33$ W and $P_{cu} = 940.67$ W]

31. Two transformers, one 50 kVA, 2400/240 V having an impedance of 4% and the other 75 kVA, 2400/240 V having an impedance of 6% are connected in parallel to supply a load of 125 kVA. Find the portion of the loads supplied by the individual transformers. [*Ans:* 62.5 kVA each]

32. Two 100 kW, single-phase transformers are connected in parallel. One transformer has an ohmic drop of 0.5% and inductive drop of 8% at full load. The other has an ohmic drop of 0.75% and inductive drop of 4% at full load. Show how will they share a load of 180 kW at 0.9 a power factor.
[*Ans:* 57.9 kW, 121.5 kW]
33. A 25 kVA, 2000/200 V, 2 winding transformer is to be used as an auto transformer with constant voltage of 2000 V. If full load unity p.f. calculate the power output. If the efficiency of the 2 winding transformer at 0.8 p.f. is 95%, find the efficiency of the auto transformer.
[*Ans:* 220 kW, 99.52%]
34. A two-winding 10 kVA, 440/110 V transformer is reconnected as a step down 550/440 V auto transformer. Calculate the VA rating of auto transformer and two winding transformer. [*Ans:* 50 kVA, 10 kVA]
35. A three-phase, 50 Hz transformer has delta connected primary and star connected secondary, the line voltages being 22000 V and 400 V respectively. The p.f. of the load is 0.8 lagging and line current on primary side is 5 A. Determine the current in each coil of the primary and secondary winding. What is the output of the transformer?
[*Ans:* 8.66 A, 275 A, 15.24 kW]

DC MACHINES

9.1 INTRODUCTION

There are two types of *direct current* (dc) machines, the dc generator and the dc motor. The dc generator converts mechanical energy into electrical energy while the dc motor converts electrical energy into mechanical energy. The dc generator is based on the principle that when a conductor is rotated in a constant unidirectional field, a voltage will be induced in the conductor. The dc motor is based on the principle that when a current carrying conductor is placed in a magnetic field a mechanical force is exerted on the conductor. DC generator operation thus follows Fleming's right hand rule while dc motor operation follows Fleming's left hand rule.

9.2 PRINCIPAL PARTS OF A DC MACHINE

1. Magnetic field system
2. Armature
3. Commutator and brushgear.

9.2.1 Magnetic Field System

The *magnetic field system* is usually the stationary part of the machine. It produces the main magnetic field flux. The outer frame (or *yoke*) is a hollow cylinder of cast steel or rolled steel. Even number of poles (say 2, 4, 6 …) are bolted to the yoke. The poles project inwards and they are called *salient poles.* The purpose of the yoke is to support the pole cores and to act as a protective cover to the machine. It also forms a part of the magnetic circuit. Each pole core has a *pole shoe* having a curved surface to support the field coils and to increase the cross-sectional area of the magnetic circuit reducing its reluctance.

The pole cores are made of sheet steel laminations and these laminations are insulated from each other but riveted together. The poles are laminated to reduce eddy-current loss in the fields.

Each pole core has one or more field coils (windings) placed over it and are connected in series with one another such that when the current flows through the coils, alternate north and south poles are produced in the direction of rotation.

9.2.2 Armature

The rotating part of the dc machine is usually called the *armature*. The armature consists of a shaft upon which a laminated cylinder (called armature core) is mounted. The armature core has grooves (or slots) on its outer surface. The laminations are insulated from each other but tightly clamped together. In small machines the laminations may be keyed directly to the shaft. In large machines the laminations are mounted on a special frame. The purpose of using laminations is to reduce eddy-current loss in the armature core.

Insulated conductors (usually copper) are placed in the slots of the armature core and are fastened round the core to prevent them flying under centrifugal forces when the armature rotates. The conductors are suitably connected and this arrangement of conductors is called *armature winding*. Two types of windings are used; *wave* and *lap*. In wave winding the number of parallel paths of armature winding is two and in lap winding the number of parallel paths is equal to the number of poles.

9.2.3 Commutator and Brushgear

Alternating voltage is produced in the coil rotating in a magnetic field. In order to obtain direct current in the external circuit, a *commutator* is used. The commutator that rotates with the armature is made from a number of wedge-shaped hard-drawn copper bars or *segments* insulated from each other as well as from the shaft. The segments form a ring around the shaft of the armature. Each commutator segment is connected to the ends of the armature coils.

Current is collected from the armature winding by means of two or more *carbon brushes* mounted on the commutator. Each brush is supported by a metal holder called *brush holder*. The pressure exerted by the brushes on the commutator can be adjusted through this brush holder and is maintained at a constant value by means of springs. Current produced in the armature winding is passed to the commutator and then to the external circuit through brushes.

9.3 MAGNETIC FLUX PATH IN A DC GENERATOR

The magnetic circuit of a four-pole dc generator is shown in Fig. 9.1. The dotted lines indicate the main flux paths.

9.4 EQUIVALENT CIRCUIT OF A DC MACHINE

The armature of a dc generator can be represented by an equivalent electric circuit. Here E is the generated voltage, R_a is the armature resistance, and V_b is the brush contact voltage drop. The equivalent circuit of the armature of a dc generator is shown in Fig. 9.2(a), while that of a dc motor is shown in Fig. 9.2(b) (In case of dc motor E is the back emf).

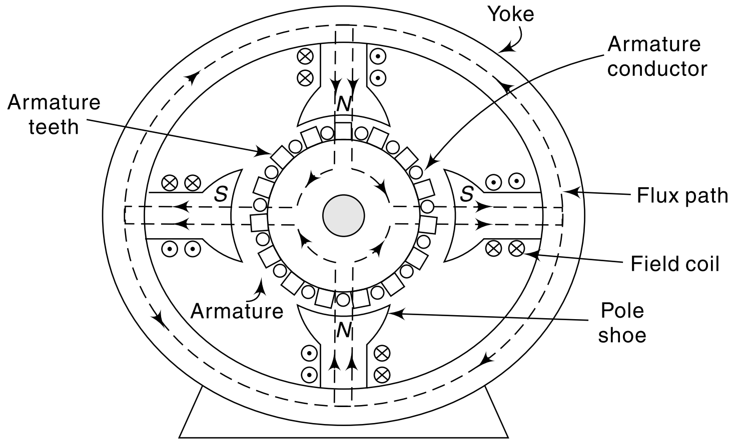


Fig. 9.1 *Magnetic flux path of a four pole dc generator*

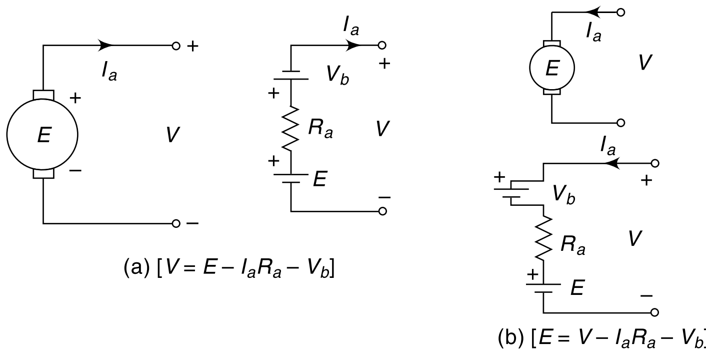


Fig. 9.2 *Equivalent circuits of the armature (a) dc generator (b) dc motor*

9.5 DIFFERENT TYPES OF EXCITATIONS IN DC MACHINE

There are, in general, two methods of exciting the field windings of dc machines.

(a) Separate excitation
(b) Self-excitation.

9.5.1 Separate Excitation

The separately excited field winding consists of several hundred turns of fine wire and is connected to a separate or external dc source as shown in Fig. 9.3(a). The voltage of the external dc source has no relation with the armature voltage, i.e field winding energised from a separate source can be designed for any suitable voltage.

9.5.2 Self-excitation

When the field winding is excited by its own armature the machine is called a *self-excited* dc machine. In these machines the field poles must have residual magnetism. A self-excited dc machine can be classified as follows:

(a) **Shunt excitation** [Fig. 9.3 (b)]. Here field excitation is obtained from the armature voltage. The shunt field excitation ampere turns (AT) is obtained by having a large number of field turns with a small field current. For a generator, shunt excitation is a type of self-excitation when the field winding resistance is high but field exciting current is low.

(b) **Series excitation** [Fig. 9.3 (c)]; The field is wound with a few turns of wire (of low resistance) and is excited in series from the armature current. This excitation varies with the load (current).

(c) **Compound excitation** [Fig. 9.3 (d) and (e)]; Both shunt and series fields are employed in this method.

If the shunt field is connected in parallel with the armature alone the machine is called a *short shunt compound* machine [Fig. 9.3(d)] and if the shunt field is connected in parallel with both the armature and series field the machine is called a *long shunt compound* machine [Fig. 9.3(e)].

If the magnetic flux produced by the shunt field winding aids the flux produced by the series field winding the machine is *cumulatively compounded*. On the other hand, if the series field flux opposes the shunt field flux, the machine is said to be *differentially compounded*.

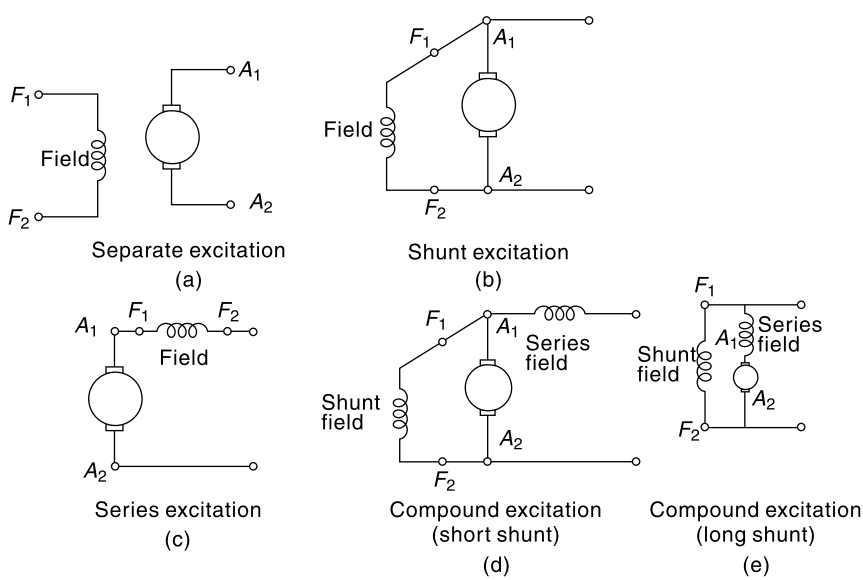


Fig. 9.3 *Different excitations*

Depending upon the number of turns of the series field three types of compound generators can be obtained.

(a) **Over Compounded Generator** the generated voltage increases as the load increases.

(b) **Level or Flat Compounded Generator** the no load voltage is same as that of the full load voltage.

(c) **Under Compounded Generator** the generated voltage decreases as the load increases.

9.6 PROCESS OF VOLTAGE BUILD UP IN SELF-EXCITED GENERATOR

Figure 9.4 shows the process of voltage build-up in a self-excited shunt generator. The line *OA* has a slope equal to the shunt field resistance R_{sh}. When the armature of the machine is rotated, a small voltage *OB* is generated due to residual magnetism in the field poles. This voltage causes field current *OC* to flow. This current *OC* increases the field flux and generates voltage *OD* which in turn results in field current *OE* which will generate a still higher voltage. This process goes on and the generated voltage continues to increase. This process continues till point *P* is reached where the generated voltage is equal to $I_{sh} R_{sh}$, I_{sh} being the shunt field current. If the resistance of shunt field be such that R_{sh} is equal to the slope of the line *OA′* (which is tangent to the curve *BP*) the generated voltage would remain at value *OB* only, so no voltage will build up. The value of R_{sh} corresponding to slope of the line *OA′* is known as *critical field resistance*. The voltage build up is possible only if R_{sh} is less than critical value. If the speed of the generator is decreased the slope of the curve is lower. Hence for each value of R_{sh} there is a value of critical speed. If speed is less than critical speed, no voltage build up will occur.

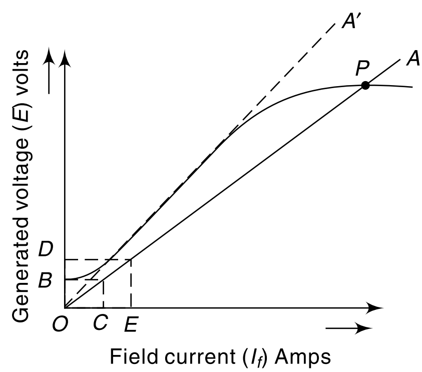


Fig. 9.4 *Voltage build up in a self-excited generator*

The connections of the field circuit should be such that field current strengthens the residual flux. If the connections are such that field current decreases the residual flux voltage will not build up.

For series generator the resistance of the load should be less than critical resistance and load should be connected so that the load current exists. Then only voltage will build up.

Hence the *conditions for voltage build up in self-excited generators* are:

(a) Residual magnetism must be present
(b) Field winding should be properly connected so that field current strengthens the residual magnetism.
(c) The resistance of the field should be less than the critical resistance
(d) The speed of the machine should be higher than the critical speed.
(e) For series generator load should be connected and resistance of load should be less than critical resistance.

9.7 EMF EQUATION OF A DC MACHINE

As the armature of a dc machine rotates, a voltage is generated in its coils. In case of a generator, the emf of rotation E_r is called the generated emf E_g (or armature emf) and $E_r = E_g$. The direction (polarity) of dynamically induced emf can be determmed by Fleming's right hand rule.

In case of a motor, the emf of rotation E_r is known as back emf E_b (or counter emf), and $E_r = E_b$. The expression, however, is the same for both conditions of operation, whether generating or motoring; only the polarity is reversed if the rotation of the machine is in the same direction in both the modes.

Let ϕ = Useful flux per pole in webers (Wb)
P = Total number of poles
Z = Total number of conductors in the armature
n = Speed of rotation of armature in revolutions per second (rps)
A = Number of parallel paths in the armature between brushes of opposite polarity

$\therefore \quad \dfrac{Z}{A}$ = Number of armature conductors in series for each parallel path

Since the flux per pole is (ϕ), each conductor cuts a flux ($P\phi$) in one revolution.
Generated voltage per conductor

$$= \frac{\text{Flux cut per revolution in Wb}}{\text{Time taken for one revolution in seconds}}$$

Since n revolutions are made in one second, one revolution will be made in $1/n$ second. Therefore, the time for one revolution of the armature is $1/n$ second.

The average voltage generated per conductor $= \dfrac{P\phi}{1/n} = nP\phi$ V.

The generated voltage E is determined by the number of armature conductors in series in any one path between the brushes. Therefore, the total voltage generated is obtained as

$$E = (\text{average voltage per conductor}) \times (\text{number of conductors in series per path})$$

i.e. $$E = np\phi \times Z/A$$

$$E = \frac{nP\phi Z}{A} = \frac{P\phi ZN}{60\,A} \quad [N = \text{rpm}]. \tag{9.1}$$

Equation (9.1) is called the *emf equation of a dc machine*.

9.8 TYPES OF WINDINGS

Armature coils can be connected to the commutator to form either lap on wave windings.

Lap Winding

The ends of each armature coil are connected to adjacent segments on the commutators so that the total number of parallel paths (A) is equal to the total number of poles P. Thus for lap winding, $A = P$.

Wave Winding

In this winding, the ends of each of the armature coils is connected to the armature segement some distance apart, and only two parallel paths are provided between the positive and negative brushes. Thus, for wave winding $A = 2$.

In general, lap winding is used in low-voltage, high-current machines and winding is used in high-voltage, low-current machines.

9.1 The armature of a 4-pole 230 V wave wound generator has 400 conductors and runs at 400 rpm. Calculate the useful flux per pole.

Solution

$$\text{Number of poles } P = 4;$$
$$\text{emf } E = 230 \text{ V}$$
$$\text{Number of conductors } Z = 400$$
$$N = 400 \text{ rpm.}$$

As the machine is wave wound the number of parallel paths $A = 2$

$\because \quad E = \dfrac{P\phi ZN}{60 A}$, where ϕ is flux per pole

$$\therefore \quad \phi = \frac{60\, AE}{P\, Z\, N} = \frac{60 \times 2 \times 230}{4 \times 400 \times 400} = 0.043 \text{ Wb.}$$

9.2 A 6-pole lap wound dc generator has 250 armature conductors, a flux of 0.04 Wb per pole and runs at 1200 rpm. Find the generated emf.

Solution

Number of poles $(P) = 6$.

As the machine is lap wound the number of parallel paths, $A\ (= P) = 6$ Also, number of armature conductors $(Z) = 250$

Flux per pole, $\phi = 0.04$ Wb

Speed, $N = 1200$ rpm.

So, generated emf $\quad E = \dfrac{P\,\phi\, ZN}{60\, A} = \dfrac{6 \times 0.04 \times 250 \times 1200}{60 \times 6} = 200$ V

9.3 An 8-pole lap wound dc generator has 1000 armature conductors, flux of 20 m Wb per pole and emf generated is 400 V. What is the speed of the machine?

Solution

Number of poles $(P) = 8$

$\therefore$ Number of parallel paths $A = P = 8$

Number of armature conductors $(Z) = 1000$;

Flux per pole $(\phi) = 20$ m Wb $= 0.02$ Wb

Emf generated $(E) = 400 \text{ V} = \dfrac{P\,\phi\, ZN}{60\, A}$

where N is the speed of the machine in rpm.

$$\therefore \quad N = \frac{60\, A \times 400}{P\,\phi\, Z} = \frac{60 \times 8 \times 400}{8 \times 0.02 \times 1000} = 1200 \text{ rpm.}$$

9.4 A 4-pole generator with 400 armature conductors has a useful flux of 0.04 Wb per pole. What is the emf produced if the machine is wave wound and runs at 1200 rpm? What must be the speed at which the machine should be driven to generate the same emf if the machine is lap wound?

Solution

$$P = 4\ ;\ Z = 400\ ;\ \phi = 0.04 \text{ Wb}\ ;\ N = 1200 \text{ rpm.}$$

So, $\quad \text{emf } (E) = \dfrac{P\,\phi\,ZN}{60\,A} = \dfrac{4 \times 0.04 \times 400 \times 1200}{60 \times 2} = 640$ V (For wave wound $A = 2$).

For lap wound $A = P = 4$. If N_1 be the new speed then

$$N_1 = \frac{60\,AE}{P\phi Z} = \frac{60 \times 4 \times 640}{4 \times 0.04 \times 400} = 2400 \text{ rpm.}$$

9.5 An 8-pole dc generator has 96 slots and 16 conductors per slot. The flux per pole is 40 m Wb and the speed is 960 rpm. Find the emf produced if the machine is (a) wave wound, (b) lap wound.

Solution

Given $\quad P = 8\ ;\ Z = 96 \times 16 = 1536\ ;\ \phi = 0.04$ Wb ; $N = 960$

(a) $A = 2$ (in wave wound)

$$E = \frac{P\phi ZN}{60\,A} = \frac{8 \times 0.04 \times 1536 \times 960}{60 \times 2} = 3932.16 \text{ V.}$$

(b) $A = P = 8$ (in lap wound)

$$E = \frac{3932.16 \times 2}{8} = 983.04 \text{ V.}$$

9.6 A 4-pole wave wound dc generator has 220 coils of 10 turns each. The speed is 400 rpm and resistance of each turn is 0.02 Ω. Find the emf produced and the resistance of armature winding if the flux per pole is 0.05 Wb.

Solution

$$P = 4\ ;\ \text{Total no. of turns} = 220 \times 10 = 2200$$

One turn consists of two conductors. Therefore, total number of conductors (Z) = 2 × 2200 = 4400.

Also. $\quad \phi = 0.05$ Wb ; $N = 400$ rpm ; $A = 2$

Emf produced $\quad E = \dfrac{P\phi ZN}{60\,A} = \dfrac{4 \times 0.05 \times 4400 \times 400}{60 \times 2} = 2933.33$ V

As the number of parallel paths is 2, so conductors per path is $\dfrac{4400}{2} = 2200$ or turns per path is $\dfrac{2200}{2} = 1100$.

∴ Armature resistance per path is 0.02 × 1100 = 22 Ω.

The total resistance of armature winding is thus $\dfrac{22}{2} = 11$ Ω.

9.9 ARMATURE REACTION

When the current flows in the armature conductors, it produces a magnetic field surrounding the conductors. This armature flux reacts with the main flux. The effect of armature flux on the main field flux is called *armature reaction*. The armature flux has two effects on the main flux:

1. It distorts the main flux
2. It weakens the main flux.

Figure 9.5 shows a 2-pole dc generator rotating in a clockwise direction where the brushes are placed in the *geometrical neutral plane* (GNP). The currents in the conductors under the influence of North Pole (i.e alone GNP) carry currents inwards while those under the influence of South Pole (i.e below GNP) carry currents outwards. The direction of the flux due to the armature conductors in the upper and lower half of armature is shown by dotted lines. The resultant flux lies along GNP which is shown by *OA* while *OB* represents the main field flux. The net flux is shown by *OP*. The *magnetic neutral plane* (MNP) coincides with GNP in the absence of armature flux. When armature flux is present, MNP shifts from GNP in the direction of rotation. To facilitate commutator action it is essential to place the brushes along MNP. Figure 9.6 shows brushes placed along MNP. Armature mmf *OA* can be split into two components *OC* and *OD*. The component *OC* is in opposition with the main field flux and called the demagnetising component and *OD* is called the cross-magnetising component. Thus, armature reaction distorts the main field flux by its cross-magnetising flux *OD* and demagnetising flux *OC*.

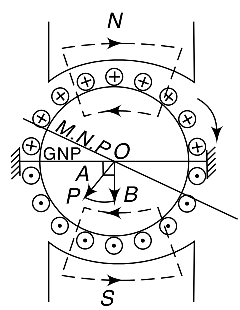


Fig. 9.5 *Two-pole dc generator with brushes in GNP*

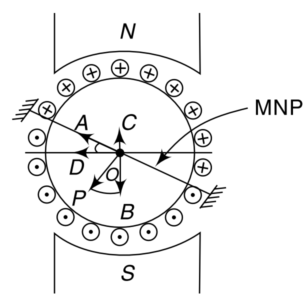


Fig. 9.6 *Two-pole dc generator with brushes at MNP*

9.9.1 Method of Improving Armature Reaction (Compensating Winding)

The demagnetising effect of armature reaction has a detrimental effect on the operation of dc motors whenever there is a sudden change in load. This causes a sudden change in flux/pole resulting in induction of large static emf which can short-circuit the complete commutator (known as *flashover*). Armature reaction AT (Ampere-turns) in dc machines can be compensated by placing a compensating winding in the pole faces with its axis along the brush axis and excited by the armature current in series connection (Fig. 9.7) so that it causes cancellation of armature reaction AT at all values of armature current.

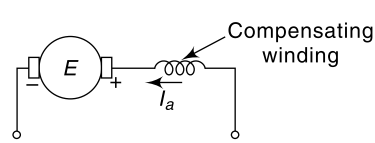


Fig. 9.7 *Compensating winding (dc Motor)*

9.10 COMMUTATION

Commutation is the process of producing a unidirectional or direct current from the alternating current generated in the armature coils.

The currents generated in the armature conductors of a dc generator are alternating. These currents flow in one direction when the armature conductors are under north pole and in the opposite direction when they are under south pole.

As conductors move out of the influence of the north pole and enter south pole, the current in them is reversed. When a brush spans two commutator segments, the winding element connected to those segements is short-circuited. During the period of short circuit of an armature coil by a brush the current in the coil must be reversed and also brought up to its full value in the reversed direction. The time of short-circuit is called the *period of commutation*. The inductive nature of the coil opposes the reversal of current from $(+I)$ to $(-I)$. If t is the time of short-circuit and L is the inductance of the coil, then the average induced voltage in the coil is

$$e_L = -L\frac{di}{dt} = \frac{-L}{t}\,[-I-(+I)] = \frac{2LI}{t}.$$

This induced voltage is called the *reactance voltage*. The sudden reversal of current as the brush leaves the segment may form an arc causing sparking at the commutator and the brush.

9.10.1 Methods of Improving Commutation

The main cause of sparking at the commutator being the reactance voltage, it can be minimised by the following methods:

(a) **Use of High Resistance Carbon Brushes** (use of high contact resistance carbon brushes increases the circuit resistance of coils undergoing commutation. Thus the reactance voltage is reduced.)

(b) **Use of Interpoles** (To reduce sparking at the commutator, small auxiliary poles called *interpoles* are provided in the machine. These are narrow cross-section poles with small cross-sectional area placed in-between the main poles. The interpoles are also called *commutating poles* (or *compoles*). The interpoles are wound with a small number of bigger cross-section conductor turns and are connected in series with the armature. Flux is produced in these poles only when current flows in the armature circuit. The flow of current in the interpole winding is such that the polarity of an interpole in a dc generator is the same as that of the next pole ahead, in the direction of rotation. In a dc motor, the polarity of an interpole is opposite to that of the next main pole in the direction of rotation.

9.11 CHARACTERISTICS OF DC GENERATORS

9.11.1 OCC (Open Circuit Characteristics) of DC Shunt Generator

Figure 9.8(a) shows a dc shunt generator on an open circuit being run at speed n rpm by means of a primemover. The field excitation is varied by regulating the resistance placed in the field circuit. The open circuit characteristic (OCC) so obtained is shown in Fig. 9.8(b). The OCC at any other speed would be a scaled version of the original OCC at rated speed (as $V_{OC} \equiv E_g \propto \omega_n$).

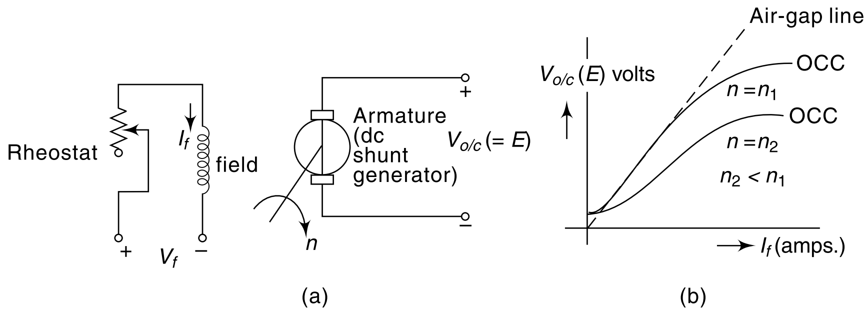


Fig. 9.8 *Open circuit characteristic of a dc generator*

9.11.2 Load Characteristics of DC Shunt Generator

The terminal voltage V versus armature current I_a characteristic is called the *internal characteristic* of a dc shunt generator and is drawn in Fig. 9.9. The load characteristic of a dc generator is called the *external characteristic*. It will only be slightly shifted from the internal characteristic as $I_L = I_a - I_f$. I_f (field current) is usually very small.

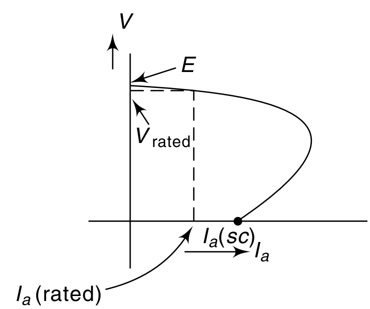


Fig. 9.9 *Internal characterstic of dc shunt generator [terminal voltage V = Induced voltage E – drop within the armature]*

9.11.3 Characteristics of Other Generators

Figure 9.10 (a) shows a series generator with its external characteristic shown in Fig. 9.10(b). The external characteristic of a long shunt compound generator and its connection diagram are drawn in Fig. 9.11(a) and 9.11(b). The characteristic is a combination of the characteristics of shunt and series generators. Series winding turns can be so adjusted that the *OC* (open circuit) voltage equals the full load voltage. The generator is then known as level compound dc generator.

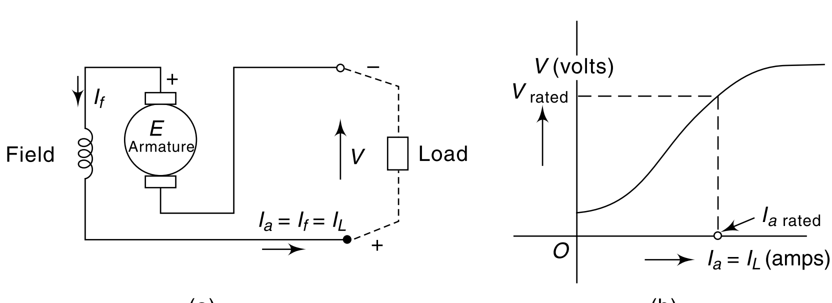


Fig. 9.10 *External characteristic of a dc generator (series)*

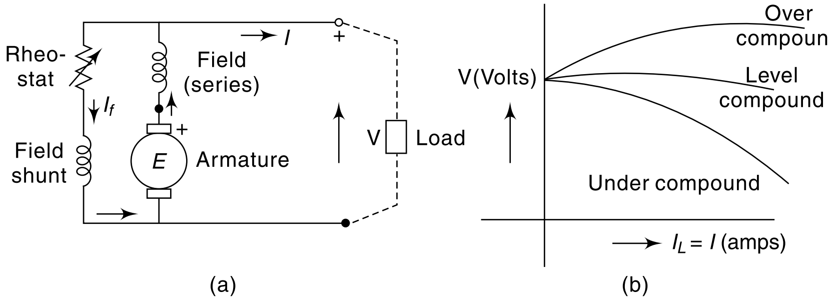


Fig. 9.11 ***External characteristic of a dc generator (compound)***

9.7 A shunt wound dc generator has an induced voltage of 200 V. The terminal voltage is 180 V. Find the load current if the field and armature resistances are 100 Ω and 0.1 Ω respectively.

Solution

The shunt wound machine is shown in Fig. 9.12.

Induced emf E_a = 200 V

Terminals voltage V_t = 180 V

Shunt field resistance r_{sh} = 100 Ω

Armature resistance r_a = 0.1 Ω

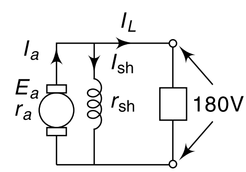


Fig. 9.12 ***A shunt wound a dc generator (Ex. 9.7)***

$$\text{Field current } (I_{sh}) = \frac{V_t}{r_{sh}} = \frac{180}{100} = 1.8 \text{ A}$$

$$E_a = V_t + I_a r_a$$

where I_a is the armature current

or $$I_a = \frac{E_a - V_t}{r_a} = \frac{200-180}{0.1} = 200 \text{ A}$$

∴ Load current $I_L = I_a - I_{sh}$ = 200 – 1.8 A = 198.2 A.

9.8 A 4-pole dc shunt generator having a field and armature resistance of 100 Ω and 0.2 Ω respectively supplies parallel connected 100 number of 200 V, 40 W lamps. Calculate the armature currents and generated emf. Allow 1 V per brush as brush contact drop.

Solution

Given $P = 4$; $r_{sh} = 100\ \Omega$ and $r_a = 0.2\ \Omega$

Current drawn by each lamp $= \frac{40}{200}$ A

Total load current $I_L = 100 \times \frac{40}{200} = 20$ A

Since $V = 200$ V

∴ $$I_{sh} = \frac{200}{100} = 2\ \Omega$$

Armature current $(I_a) = I_L + I_{sh} = 20 + 2 = 22$ A

Generated emf $(E) = V + I_a r_a$ + Brush drop

$= 200 + 22 \times 0.2 + 2 \times 1$

$= 200 + 4.4 + 2 = 206.4$ V.

9.9 A dc shunt generator has an induced voltage of 220 V on open circuit. When the machine is on load the voltage is 200 V. Find the load current if the field resistance is 100 Ω and armature resistance is 0.2 Ω.

Solution

Open circuit voltage $E = 220$ V

Terminal voltage $V = 200$ V

If I_a is be the armature current

$$E = V + I_a\, r_a, \text{ where } r_a \text{ is the armature resistance.}$$

$$\therefore \quad I_a = \frac{E - V}{r_a} = \frac{220 - 200}{0.2}\text{ A} = \frac{20}{0.2}\text{ A} = 100 \text{ A}$$

Field current $I_{sh} = \dfrac{V}{r_{sh}}$, where r_{sh} is the field resistance.

$$\therefore \quad I_{sh} = \frac{200}{100} = 2 \text{ A.}$$

The load current $I_L = I_a - I_{sh} = 100 - 2 = 98$ A.

9.10 A series generator delivers a load current of 50 A at 400 V and has armature and series field resistance of 0.05 Ω and 0.04 Ω respectively. Find the induced emf in the armature if the brush contact drop is 1 V per brush.

Solution

In a series generator (shown in Fig. 9.13), Load current = Armature current

$$I_a = 50 \text{ A}, \quad V = 400 \text{ V},$$
$$r_a = 0.05\ \Omega \text{ and } r_{se} = 0.04\ \Omega,$$

Hence
$$\begin{aligned} \text{emf} &= V + I_a\, r_a + I_a\, r_{se} + \text{Brush drop} \\ &= 400 + 50 \times 0.05 + 50 \times 0.04 + 2 \times 1 \\ &= 406.5 \text{ V} \end{aligned}$$

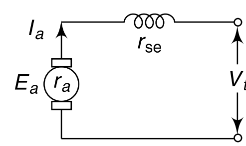


Fig. 9.13 ***A series generator (Ex. 9.10)***

9.11 A long shunt compound wound dc generator delivers a load-current of 100 A at 400 V. The armature, series and shunt field resistances are 0.04 Ω, 0.02 Ω and 200 Ω respectively. Find the armature current and the generated emf.

Solution

Referring to Fig. 9.14

$I_L = 100$ A, $V = 400$ V, $r_a = 0.04\ \Omega$,

$r_{se} = 0.02\ \Omega$ and $r_{sh} = 200\ \Omega$.

$$\therefore \quad I_{sh} = \frac{400}{200} = 2 \text{ A}$$

and $\quad I_a = I_L + I_{sh} = 100 + 2 = 102$ A.

Generated emf
$$\begin{aligned} E &= V + I_a(r_a + r_{se}) \\ &= 400 + 102\ (0.04 + 0.02) \\ &= 400 + 102 \times 0.06 = 406.12 \text{ V.} \end{aligned}$$

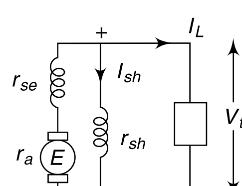


Fig. 9.14 ***A long shunt compound generator (Ex. 9.11)***

9.12 Solve Example 9.11 assuming the machine as short shunt compound.

Solution

Referring to Fig. 9.15,

$$I_L = 100 \text{ A}$$

Voltage across shunt field winding

$= 400 + I_L\, r_{se}$

$= 400 + 100 \times 0.02 = 402$ V

Hence, $I_{sh} = \dfrac{402}{200} = 2.01$ A

and $I_a = I_L + I_{sh} = 100 + 2.01 = 102.01$ A

$E = 402 + I_a\, r_a$

$= 402 + 102.01 \times 0.04 = 406.08$ V.

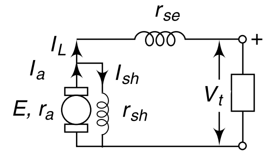


Fig. 9.15 ***A short shunt compound generator***

9.12 PRINCIPLE OF OPERATION OF A DC MOTOR

When a current-carrying conductor is placed in a magnetic field, a force is produced in it. The direction of the force is obtained from Fleming's left hand rule. Let us consider one such conductor is placed in a slot of armature and it is acted upon by the magnetic field developed from a north pole of the motor. By applying Flemming's left-hand rule it can be found that the conductor has a tendency to move to the left-hand side from its axis. Since the conductor is in a slot on the circumference of the rotor, the force acts in a tangential direction to the rotor creating a torque on the rotor. Similar torques will be produced for all the rotor conductors if we assume these conductors are placed in successive slots. The rotor being free to move it then starts rotating in the anticlockwise direction. (left hand side from its axis).

9.13 BACK EMF

When the dc motor armature rotates, its conductors cut the magnetic flux. The emf of rotation E_r is then induced in them. In a motor, this emf of rotation is known as *back emf* (or *counter emf*). The back emf opposes the applied voltage. Since the back emf is induced due to generator action its magnitude is, therefore, obtained from the same expression as that for the generated emf in a dc generator.

$$E_b = \frac{NP\Phi Z}{60\text{ A}} \quad N \text{ being the rpm, } (E_b) \text{ is the back emf } (E_B = E_r).$$

Here other symbols have their usual meanings.

9.14 TORQUE EQUATION OF A DC MOTOR

The expression for torque T is same for the generator and the motor. It can be deduced as follows:

The voltage equation of a dc motor is given by

$$V = E_b + I_a R_a \tag{9.2}$$

Multiplying both the sides of Eq. (9.2) by I_a we obtain

$$VI_a = E_b I_a + I_a^2 R_a. \tag{9.3}$$

However, VI_a = electrical power input to the armature

and $I_a^2 R_a$ = copper loss in the armature.

Since Input = Output + Losses, (9.4)

comparison of Eqns (9.3) and (9.4) shows that

$E_b I_a$ = electrical equivalent of gross mechanical power [developed by the armature (electromagnetic power)]

Let T = average electromagnetic torque developed by the armature in newton metres (Nm)

$\because$ Mechanical power developed by the armature is given by,

$$P_m = \omega T = 2\pi nT \qquad [n \text{ is speed in r.p.s i.e., } (N/60)].$$

$$\therefore \quad P_m = E_b I_a = \omega T = 2\pi nT$$

But $$E_b = \frac{nP\phi Z}{A}$$

$$\therefore \quad \frac{nP\Phi Z}{A} I_a = 2\pi nT$$

or $$T = \frac{1}{2\pi} \cdot \phi Z I_a \cdot \frac{P}{A} \text{ Nm} = 0.159\ \phi\ Z I_a \left(\frac{P}{A}\right) \text{ Nm}$$

and $$T = \frac{PZ}{2\pi A} \Phi\, I_a = \frac{E_b I_a}{2\pi n} = \frac{E_b I_a}{2\pi N} \times 60 \qquad (9.5)$$

Equation (9.5) is called the *torque equation* of a dc motor.

For a given dc machine, P, Z and A are constant, therefore $\left(\frac{PZ}{2\pi A}\right)$ is also a constant.

Let $$\frac{PZ}{2\pi A} = k$$

$$\therefore \quad T = k\ \phi\ I_a$$

or $$T \propto \phi\ I_a \qquad (9.6a)$$

Also $$2\pi nT = E_b I_a$$

$$\therefore \quad T = \frac{1}{2\pi} \cdot \frac{E_b I_a}{n} \text{ Nm} = 0.159 \frac{E_b I_a}{n} \text{ Nm.} \qquad (9.6b)$$

Hence the torque developed by a dc motor is directly proportional to the product of flux per pole and armature current.

9.14.1 Torque Current Characteristic of a Shunt Motor

From the torque expression, we have

$$T \propto \phi I_a.$$

If the effect of armature reaction is neglected, ϕ is nearly constant and we can write

$$T \propto I_a.$$

The graph between T and I_a is thus a straight line passing through the origin (Fig. 9.16). In the high current region, due to saturation of the core, the $T - I_a$ characteristic loses linearity.

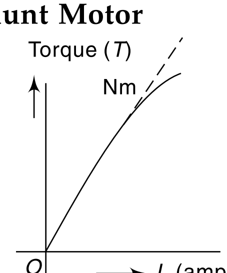


Fig. 9.16 ***Torque current characteristic of dc shunt motor***

9.14.2 Torque Current Characteristic of a Series Motor

$$T \propto \phi\ I_a$$

In series motor, before saturation $\phi \propto I_a$ and hence at rated loads,

$$T \propto I_a^2 \qquad (9.7)$$

The above equation shows that the torque/armature current T/I_a curve of a series motor will be parabolic. When the iron core becomes magnetically saturated, ϕ becomes almost constant, so that at heavy loads

$$T \propto I_a. \tag{9.8}$$

Equations (9.7) and (9.8) shows that the T/I_a characteristic is a parabolic one at light or rated loads and straight line at heavy load. Thus, the torque/current characteristic of a dc series motor is initially parabolic and finally becomes linear when the load current becomes large. This characteristic, up to the rated loading of the motor, is shown in Fig. 9.17.

The characteristic relating the net torque or useful torque T_r to the armature current is parallel to the T/I_a characteristic, but is slightly below it. The difference between the two curves is due to friction and windage losses.

Since the $T - I_a$ characteristic of a dc series motor is parabolic hence the starting torque is high for the motor for a definite starting current. This property is used in traction motors, cranes and hoists where the dc motor is to start with full load. High starting torque (being proportional to square of starting current) helps to overcome the initial inertia of the load and the dc series motor speeds up smoothy with heavy load on it.

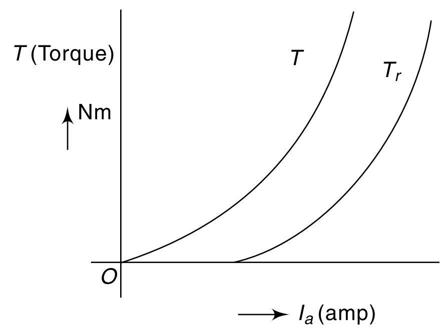


Fig. 9.17 *Torque current characteristic of a dc series motor (up to rated loads)*

9.14.3 Torque Current Characterstic of a Compound Motor

A compound motor has both shunt and series field windings, so its characteristics are intermediate between the shunt and series motors. The cumulative compound motor is generally used in practice. The torque armature current characteristics are shown in Fig. 9.18.

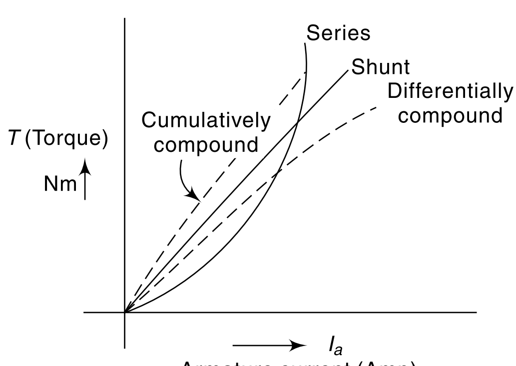


Fig. 9.18 *Torque current characteristic of compound motor*

9.13 A 400 V, 6-pole shunt motor has a two-circuit armature winding with 250 conductors. The armature resistance is 0.3 Ω, field resistance 200 Ω and flux per pole is 0.04 Wb. Find the speed and the electromagnetic torque developed if the motor draws 10 A from the supply.

Solution

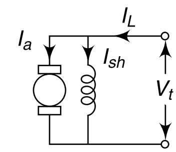


Fig. 9.19 ***A 400 V, 6-pole dc shunt motor (Ex. 9.13)***

Given $P = 6,\ Z = 250,\ r_a = 0.3\ \Omega,\ A = 2$

and $r_{sh} = 200\ \Omega$

Also, $\phi = 0.04$ Wb, $I_L = 10$ A and $V = 400$ V

$\therefore \quad I_{sh} = \dfrac{400}{200} = 2\text{A}$ (from Fig. 9.19)

$I_a = I_L - I_{sh} = 10 - 2 = 8$ A

Back emf $\quad E_b = V_t - I_a r_a = 400 - 8 \times 0.3 = 397.6$ V

i.e $\quad E_b = 397.6 = \dfrac{P\phi ZN}{60\text{ A}}$ where (N) is the speed in rpm

$$\therefore \quad N = \frac{60 \times 2 \times 397.6}{6 \times 0.04 \times 250} = 795 \text{ rpm}$$

Electromagnetic power $P_e = E_b I_a = 397.6 \times 8 = 3180.8$ W.

Electromagnetic torque $T_e = \dfrac{E_b I_a}{\omega}$, where ω is the angular velocity

But $\quad \omega = 2\pi \dfrac{N}{60}$ rad/s $= \dfrac{2\pi \times 795}{60}$ rad/s = 83.21 rad/s

$$\therefore \quad T_e = \frac{397.6 \times 8}{83.21} \text{ Nm} = 38.23 \text{ Nm.}$$

9.14 An 8-pole, 400 V shunt motor has 960 wave connected armature conductors. The full load armature current is 40 A and the flux per pole is 0.02 Wb. The armature resistance is 0.1 Ω and the contact drop is 1 V per brush. Calculate the full load speed of the motor.

Solution

Given $\quad P = 8,\ V = 400\text{ V},\ Z = 960,\ I_a = 40\text{ A}\ \phi = 0.02$ Wb,

$r_a = 0.1\ \Omega$ and $A = 2$. Also total brush drop $= 2 \times 1 = 2$ V

Back emf $\quad E_b = V - I_a r_a -$ brush drop $= 400 - 40 \times 0.1 - 2 = 394$ V

Again, $\quad E_b = \dfrac{P\phi ZN}{60A}$, where ($N$) is the full load speed

$$\therefore \quad N = \frac{60AE_b}{P\phi Z} = \frac{60 \times 2 \times 394}{8 \times 0.02 \times 960} \text{ r.p.m} = 308 \text{ rpm.}$$

9.15 A 42 kW, 400 V dc shunt motor has a rated armature current of 100 A at 1500 rpm. The resistance of armature is 0.2 Ω. Find (i) the internal torque developed and (ii) the internal torque if the field current is reduced to 0.9 times of its original value.

Solution

Given, $\quad V = 400\text{ V},\ I_a = 100\text{ A},\ N = 1500$ r.p.m and $r_a = 0.2\ \Omega$

Back emf $\quad E_b = V - I_a r_a = 400 - 100 \times 0.2 = 400 - 20 = 380$ V

(i) Internal torque developed

$$T_e = \frac{E_b I_a}{\omega}, \quad \text{where} \quad \omega = \frac{2\pi N}{60} \text{ rad/s = angular speed}$$

$$\therefore \qquad T_e = \frac{E_b\, I_a}{2\pi N} \times 60 = \frac{380 \times 100}{2\pi \times 1500} \times 60 \text{ Nm} = 242 \text{ Nm}.$$

(ii) If the field current I_f is reduced to 0.9 times of its original value then ϕ is also reduced by 0.9 times of its previous values (as $\phi \infty I_f$).

$E_b \propto \phi$, if N is constant; hence E_b is also reduced by 0.9 times from its previous value.

Thus, the internal terque (T_e) = 0.9 × 242 = 217.8 Nm.

9.16 A 400 V, 10 kW series motor drives a fan when running at 800 rpm. The motor draws 50 A from the supply. The resistance of the armature and series field are 0.2 Ω and 0.1 Ω respectively. Determine the electromagnetic torque developed by the motor.

Solution

Terminal voltage $V = 400$ V
Armature current $I_a = 50$ A
Armature resistance $r_a = 0.2\ \Omega$
Series field resistance $r_{se} = 0.1\ \Omega$

Back emf $\quad E_b = V - I_a(r_a + r_{se}) = 400 - 50(0.2 + 0.1) = 385$ V

Now, $\quad$ Speed $N = 800$ rpm.

∴ The electromagnetic torque developed is

$$T_e = \frac{E_b\, I_a}{\frac{2\pi N}{60}} = \frac{385 \times 50}{2\pi \times 800} \times 60 = 230 \text{ Nm}.$$

9.17 A four-pole series motor has 944 wave connected armature conductors. At a certain load the flux per pole is 34.6 m Wb and the total mechanical power developed is 4 kW. Calculate the line current taken by the motor and the speed at which it will run with the applied voltage of 500 V. The total motor resistance is 3 Ω.

Solution

Here $\qquad P = 4,\ Z = 944,\ A = 2$ and $\phi = 0.0346$ Wb

Power developed is 4 kW i.e 4000 W

If E_b be the back emf and I_a be the armature current,

$$E_b\, I_a = 4000$$

Now, $V = 500$ V and $r = 3\ \Omega$ where (r) is the motor circuit resistance.

$$\therefore \qquad E_b = V - I_a r = 500 - I_a \times 3$$

$$\text{or} \qquad E_b = 500 - 3 \times \frac{4000}{E_b}$$

$$\text{or} \qquad E_b^2 = 500\, E_b - 12000$$

$$\text{or} \qquad E_b^2 - 500\, E_b + 12000 = 0$$

$$E_b = 25.28 \text{ V or, } E_b = 474.72 \text{ V}$$

$$\text{If} \qquad E_b = 25.28 \text{ V},\ I_a = \frac{4000}{25.28} \text{ A} = 158.22 \text{ A}$$

$$\text{If} \qquad E_b = 474.72 \text{ V},\ I_a = \frac{4000}{474.72} \text{ A} = 8.43 \text{ A}.$$

If I_a is very large the armature of the machine will be damaged. So, I_a is not equal to 158.22 A and the feasible value is $I_a = 8.43$ A and $E_b = 474.72$ V.

If N be the speed, we can write

$$E_b = 474.72 = \frac{P\phi ZN}{60\,A}$$

$$\therefore \qquad N = \frac{60 \times 2 \times 474.72}{4 \times 0.0346 \times 944}\ \text{rpm} = 436\ \text{rpm.}$$

9.18 A dc series motor has an armature resistance of 0.03 Ω and series field resistance of 0.04 Ω. The motor is connected to a 400 V supply. The line current is 20 A when the speed of the machine is 1000 rpm. Find the speed of the machine when the line current is 50 A and the excitation is increased by 20%.

Solution

Given, $r_a = 0.03\ \Omega$, $r_{se} = 0.04\ \Omega$, $V = 400$ V, $I_{L_1} = I_{a_1} = 20$ A

and $N_1 = 1000$ rpm.

When line current is 50 A (i.e $I_{L_2} = I_{a_2} = 50$ A), we assume speed is N_2.

If ϕ be the flux when speed is 1000 rpm, the flux becomes (1.2 ϕ) as this time excitation is increased by 20%.

We know $E_b \propto \phi N$

$$\therefore \qquad \frac{E_{b_1}}{E_{b_2}} = \frac{\phi N_1}{1.2\,\phi N_2}$$

However, $E_{b_1} = V - I_{a_1}(r_a + r_{se}) = 400 - 20(0.03 + 0.04) = 398.6$ V

and $E_{b_2} = 400 - 50(0.03 + 0.04) = 396.5$ V

$$\therefore \qquad N_2 = \frac{N_1 E_{b_2}}{1.2\,E_{b_1}} = \frac{1000 \times 396.5}{1.2 \times 398.6} = 829\ \text{rpm.}$$

9.15 SPEED EQUATION OF A DC MOTOR

The emf equation of a dc machine is given by

We have, $$E = \frac{NP\phi Z}{60\,A}$$

or $$N = \frac{60\,A}{PZ}\frac{E}{\phi}$$

Therefore, $$N = \frac{E}{K\phi}$$

where $$K = \frac{PZ}{60\,A}.$$

This equation shows that *the speed of a dc machine is directly proportional to the emf of rotation E and is inversely proportional to flux per pole* ϕ. Since the expression for emf of rotation applies equally to motors and generators, it gives the speed for both motors and generators.

If the suffixes 1 and 2 denote the initial and final values, we can write

$$N_1 = \frac{E_1}{k\phi_1}$$

$$N_2 = \frac{E_2}{k\phi_2}$$

$$\therefore \qquad \frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\phi_1}{\phi_2}$$

For dc shunt motor, $\phi_1 = \phi_2$ for rated load conditions. Thus for such a motor we can write

$$\frac{N_2}{N_1} = \frac{E_{b2}}{E_{b1}}$$

9.16 SPEED REGULATION OF DC MOTOR

The *speed regulation* is defined as the change in speed from no load N_{nl} to full load N_{fl} expressed as a fraction or a percentage of the full load speed. It can be written as:

$$\text{Per unit speed regulation} = \frac{N_{\text{nl}} - N_{\text{fl}}}{N_{\text{fl}}}$$

$$\therefore \qquad \text{Per cent speed regulation} = \frac{N_{\text{nl}} - N_{\text{fl}}}{N_{\text{fl}}} \times 100.$$

A motor which has a nearly constant speed at all loads is said to have a good speed regulation.

9.17 SPEED VS. ARMATURE CURRENT CHARACTERISTIC OF DC MOTOR (N/I_a CHARACTERISTICS)

9.17.1 Shunt Motor

In a shunt motor, $I_{\text{sh}} = V/R_{\text{sh}}$. If V is constant I_{sh} will also remain constant. Hence the flux is constant at no load. The flux decreases slightly due to armature reaction. If the effect of armature reaction is neglected, the flux ϕ will remain constant. The motor speed being given by

$$N \propto \frac{V - I_a R_a}{\phi} \quad \left(= \frac{E_b}{K\phi} \right)$$

For ϕ remaining constant, the speed can be written as

$$N \propto V - I_a R_a$$

This is an equation of a straight line with a negative slope. That is, *the speed N of the shunt motor decreases linearly with the increase in armature current* as shown in Fig. 9.20.

Since $I_a R_a$ at full load is very small compared to V, the drop in speed from no load to full load is very small in well designed machines. The decrease in speed is partially neutralized by a reduction in ϕ due to armature reaction. Hence for all practical purposes *the shunt motor may be taken as a constant-speed motor*.

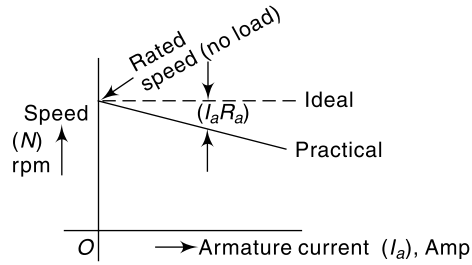


Fig. 9.20 ***Speed vs current characteristic of a dc shunt motor***

9.17.2 Series Motor

The motor speed N for a series motor is given by

$$N \propto \frac{V - I_a\,(R_a + R_{se})}{\phi} \quad \left(= \frac{E_b}{K\phi}\right)$$

At low values of I_a, the voltage drop $[I_a\,(R_a + R_{se})]$ is negligibly small in comparison with V

$$\therefore \qquad N \propto \frac{V}{\phi}$$

Since V is constant,

$$N \propto \frac{1}{\phi}$$

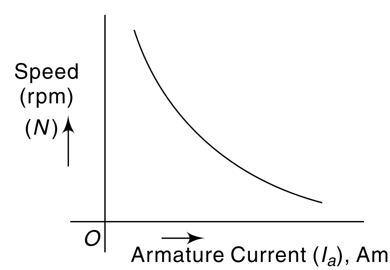


Fig. 9.21 ***Speed vs current characteristic of a dc series motor***

In a series motor, the field flux ϕ is produced by the armature current flowing in the field winding so that $\phi \propto I_a$. Hence the series motor is a variable flux machine.

Also, $$N \propto \frac{1}{I_a}$$

Thus, for *the series motor, the speed is inversely proportional to the armature (load) current*. The speed-load characteristic is a rectangular hyperbola as shown in Fig. 9.21.

The speed equation shows that when the load decreases, the speed will be very large. Therefore at no load (or at light loads) there is a possibility of dangerously high speed, which may damage the series motor due to large centrifugal forces. Hence a series motor should never be run unloaded. It should always be coupled to a mechanical load either directly or through gearing. It should not be coupled by belt, which may slip at any time making the armature unloaded. With the increase in armature current (i.e the field current) the flux also increases and therefore the speed is reduced.

9.17.3 Compound Motor

The speed-armature current characteristics are shown in Fig. 9.22. In differentially compound motor, because of weakening of field, speed increases with increase in armature current while in cummulatively compound, the speed drops because of increase of field flux with armature current.

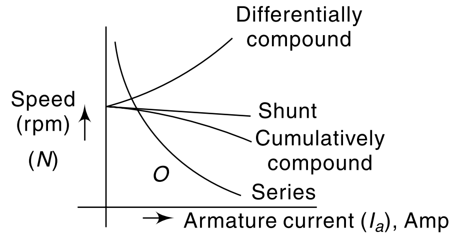


Fig. 9.22 *Speed vs current characteristic of a dc compound motor*

9.18 SPEED TORQUE CHARACTERISTIC OF DC MOTORS

9.18.1 Shunt Motor

Since the torque is proportional to armature current in a dc shunt motor the speed torque characteristic of such a motor will be identical to the speed armature current characteristic. The speed torque characteristic of the shunt motor is shown in Fig. 9.23.

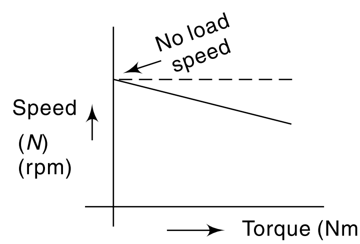


Fig. 9.23 *Speed torque characteristic of a dc shunt motor*

9.18.2 Series Motor

The speed/torque characteristic of a dc series motor can be derived from its speed/armature current N/I_a and torque/armature current T/I_a characteristics. The characteristic (Fig. 9.24) shows that the dc series motor has a high torque at a low speed and a low torque at a high speed. Hence the speed of the dc series motor changes considerably with increasing load. It is a very useful characteristic for traction purposes, hoists and lifts where at low speeds a high starting torque is required to accelerate large masses.

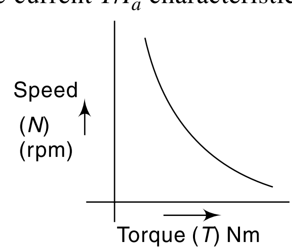


Fig. 9.24 *Speed torque characteristic of a dc series motor*

9.18.3 Compound Motor

Figure 9.25 shows the speed-torque (N/T) characteristic of a dc compound motor. A compound motor has a high starting torque together with a safe no-load speed. These factors make it suitable for use with heavy intermittent loads such as lifts, hoists etc.

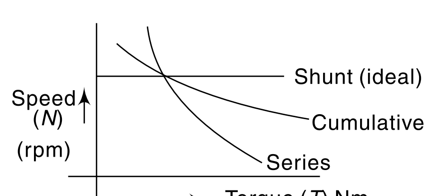


Fig. 9.25 *Speed torque characteristic of a dc compound motor*

9.19 A 240 V shunt motor has an armature resistance of 0.2 Ω and takes armature current of 20 A on full load. The electromagnetic torque being constant, by how much must the flux be reduced to increase the speed by 40%?

Solution

Let N_1 be the speed and ϕ_1 be the flux when armature current I_{a_1} = 20 A; also, r_a = 0.2 Ω; V = 240 V

$$N_2 = 1.4\,N_1 \text{ and flux is } \phi_2$$

$$E_{b1} = V - I_{a_1} r_a = 240 - 20 \times 0.2 = 236 \text{ V}$$

$$\therefore \quad \frac{E_{b_1}}{E_{b_2}} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{\phi_1 N_1}{1.4\,\phi_2 N_1} = \frac{\phi_1}{1.4\,\phi_2} = \frac{236}{E_{b_2}}$$

or, $$\frac{\phi_1}{\phi_2} = \frac{330.4}{E_{b_2}} = \frac{330.4}{V - 0.2\,I_{a_2}}$$

$\because T_e$ is constant,

$$T_{e_1} = T_{e_2}$$

or $$\phi_1 I_{a1} = \phi_2 I_{a_2}$$

or $$20\,\phi_1 = \phi_2 I_{a_2}$$

or $$\frac{\phi_1}{\phi_2} = \frac{I_{a_2}}{20} \quad \text{or,} \quad I_{a_2} = 20\frac{\phi_1}{\phi_2}$$

So, $$\frac{\phi_1}{\phi_2} = \frac{330.4}{240 - 0.2 \times 20\dfrac{\phi_1}{\phi_2}}.$$

Considering $\dfrac{\phi_1}{\phi_2} = x$

$$x = \frac{330.4}{240 - 4x}$$

or $$4x^2 - 240\,x + 330.4 = 0.$$

Taking negative sign,

$$x = \frac{240 - \sqrt{(240)^2 - 4 \times 4 \times 330.4}}{2 \times 4} = 1.41$$

So, $$\phi_1 = 1.41\,\phi_2$$

or $$\phi_2 = 0.71\,\phi_1.$$

So the flux must be reduced by (1 – 0.71)0.29 or 29%.

9.20 A 220 V dc shunt motor takes 4A at no load and 60 A on full load, the shunt field and armature resistance being 220 Ω and 0.2 Ω. Brush contact drop is 2V. If armature reaction weakens the field by 4%, find the percentage change in speed from no load to full load.

Solution

Given, V = 220 V; r_{sh} = 220 Ω; r_a = 0.2 Ω

At no load condition

$$I_{LO} = 4 \text{ A} \quad \text{and} \quad \phi = \phi_1.$$

At full load conditions,

$$I_{Lfl} = 60 \text{ A} \quad \text{and} \quad \phi_2 = (1 - 0.04)\,\phi_1 = 0.96\,\phi_1$$

Let N_1 and N_2 be the speed at no load and full load condition.

Field current $\quad I_{sh} = \frac{220}{220} \text{A} = 1 \text{ A}$

$\therefore \quad I_{ao} = 4 - 1 = 3 \text{ A} \quad \text{and} \quad I_{afl} = (60 - 1) = 59 \text{ A}.$

Now, $\quad E_{bo} = 220 - 3 \times 0.2 = 219.4 \text{ V}$

and $\quad E_{bfl} = 220 - 59 \times 0.2 = 208.2 \text{ V}$

$$\therefore \quad \frac{E_{bo}}{E_{bfl}} = \frac{\phi_1 N_1}{\phi_2 N_2}$$

$$\text{or} \quad \frac{N_1}{N_2} = \frac{E_{bo}}{E_{bfl}} \frac{\phi_2}{\phi_1} = \frac{219.4}{208.2} \times \frac{0.96\phi_1}{\phi_1} = 1.0116.$$

Change of speed from no load to full load is 0.0116 or 1.16%.

9.21 Find the no load and full load speeds of a 220 V, four-pole shunt motor having following data flux 0.04 *Wb*, armature resistance 0.04 Ω, 160 armature conductors, wave connection, full load line current 95 A, no load line current 9 A, field resistance 44 Ω.

Solution

Here $\quad P = 4$, $V = 220$ V, $\phi = 0.04$ *Wb*, $r_a = 0.04\ \Omega$ $Z = 160$ and $A = 2$.

Also, $\quad r_{sh} = 44\ \Omega$ and $I_{sh} = \frac{220}{44} \text{A} = 5 \text{ A}.$

Under full load condition

$$I_{Lfl} = 95 \text{ A}$$

$$\therefore \quad I_{afl} = 95 - 5 = 90 \text{ A}$$

$$E_{bfl} = 220 - 90 \times 0.04 = 216.4 \text{ V}.$$

Under no load condition

$$I_{LO} = 9 \text{ A}$$

$$\therefore \quad I_{ao} = 9 - 5 = 4 \text{ A}$$

and $\quad E_{bo} = 220 - 4 \times 0.04 = 218.4 \text{ V}.$

$$\text{Now,} \quad E_{bo} = \frac{P\phi Z N_o}{60 \text{ A}} \quad \text{or} \quad N_o = \frac{60\ AE_{b_o}}{P\phi Z} = \frac{60 \times 2 \times 218.4}{4 \times 0.04 \times 160} = 1024 \text{ rpm}$$

$$\text{Also} \quad E_{bfl} = \frac{P\phi Z N_{f1}}{60 \text{ A}} \quad \text{or} \quad N_{fl} = \frac{60 \times 2 \times 216.4}{4 \times 0.04 \times 160} = 1014 \text{ rpm}.$$

Hence the no load and full load speeds are 1024 rpm and 1014 rpm.

9.22 A 220 V series motor runs at 400 rpm and takes a line current of 50 A. Find the speed and percentage change in torque if the load is reduced so that the motor takes 20 A. The armature and the field circuit resistance is 0.5 Ω. Assume that the flux is proportional to the field current.

Solution

Given $\quad V = 220\text{V}$, $N_1 = 400$ r.p.m, $I_{L_1} = I_{a_1} = 50$ A

$(r_a + r_{se}) = 0.5\ \Omega$, $I_{L2} = I_{a2} = 20$ A.

Let the torque when line current is 50A be T_{e1} and when the line current is 20 A the torque be T_{e2}

Now $\quad E_{b_1} = V - I_{a1}(r_a + r_{se}) = 220 - 50(0.5) = 195 \text{ V}$

and $\quad E_{b_2} = V - I_{a2}(r_a + r_{se}) = 220 - 20(0.5) = 210 \text{ V}$

Also, $$\frac{E_{b_1}}{E_{b_2}} = \frac{\phi_2 N_1}{\phi_2 N_2} = \frac{I_{a_1} N_1}{I_{a_2} N_2} \quad (\because\ \phi \propto I_a)$$

or $$N_2 = \frac{E_{b_2}}{E_{b_1}} \times \frac{I_{a_1}}{I_{a_2}} N_1 = \frac{210}{195} \times \frac{50}{20} \times 400 = 1077 \text{ rpm}$$

$$\frac{T_{e_2}}{T_{e_1}} = \frac{\phi_2 I_{a_2}}{\phi_1 I_{a_1}} = \frac{I_{a_2}^2}{I_{a_1}^2} \quad (\because\ \phi \propto I_a)$$

or $$\frac{T_{e_2}}{T_{e_1}} = \left(\frac{20}{50}\right)^2 = \frac{4}{25}.$$

Percentage change in torque is $\left(1 - \frac{4}{25}\right) \times 100\%$ or, 84%

9.23 A 400 V dc shunt motor having an armature resistance of 0.3 Ω and shunt field resistance of 200 Ω, draws a line current of 100 A at full load. The full load speed is 1500 rpm and the brush contact drop is 2 V. Find (i) the speed at half load (ii) the speed at 150% of full load.

Solution

Given $V = 400$ V; $r_a = 0.3\ \Omega$; $r_{sh} = 200\ \Omega$

$N_{fl} = 1500$ rpm $\quad I_{Lfl} = 100$ A

$$I_{sh} = \frac{400}{200} = 2 \text{ A}.$$

So $I_{afl} = 100 - 2 = 98$ A

$E_{bfl} = V - I_{afl}\, r_a -$ Brush drop $= 400 - 98 \times 0.3 - 2 = 368.6$ V.

(i) At half load

$$I_{L2} = \frac{100}{2} \text{ A} = 50 \text{ A}$$

So $I_{a_2} = 50 - 2 = 48$ A

$E_{b_2} = V - I_{a_2} r_a - 2 = 400 - 48 \times 0.3 - 2 = 383.6$ V

$$\frac{E_{bfl}}{E_{b_2}} = \frac{N_{fl}}{N_2} \quad (\because\ \phi = \text{constant})$$

or $$N_2 = \frac{383.6}{368.6} \times 1500 = 1561 \text{ rpm}.$$

(ii) At 150% of full load

$I_{L_3} = 100 \times 1.5 = 150$ A

$I_{a_3} = 150 - 2 = 148$ A

So $E_{b_3} = 400 - 148 \times 0.3 - 2 = 353.6$V

Therefore $$N_3 = \frac{E_{b_3}}{E_{bfl}} \times N_{fl} = \frac{353.6}{368.6} \times 1500 = 1439 \text{ rpm}.$$

9.19 SPEED CONTROL OF DC MOTORS

The speed of a dc motor is given by the relationship

$$N = \frac{V - I_a R_a}{K\phi}$$

This speed is thus dependent upon the supply voltage V, the armature circuit resistance R_a, and the field flux ϕ, which is produced by the field current. Thus there are two general methods of speed control of dc motors:

(i) Variation of resistance in the armature circuit. This method is called armature resistance control.

(ii) Variation of field flux (ϕ). This method is called field resistance control.

9.19.1 Armature Resistance Control

In this method a variable series resistor R_e is connected in series with the armature circuit. Figure 9.26 shows the method of connection for a shunt motor. The field is directly connected across the supply and therefore the flux (ϕ) is not affected by variation of R_e.

Figure 9.27 shows the method of connection of external resistance R_e in the armature circuit of a d.c series motor.

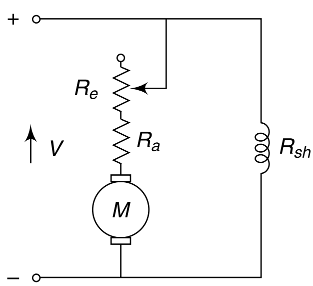


Fig. 9.26 *Speed control of a dc shunt motor by armature resistance control*

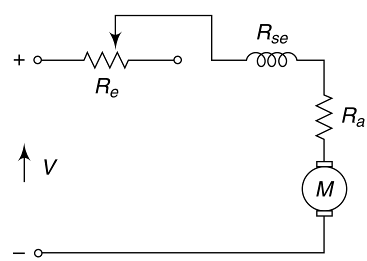


Fig. 9.27 *Speed control of a dc series motor by armature resistance control*

Figures 9.28(a) and (b) show typical speed/current characteristics for shunt and series motors respectively. In both the cases the motor runs at a lower speed as the value of R_e is increased. R_e carries full armature current hence R_e should be designed to carry continuously the full armature current. The main disadvantages of armature control are as under:

(a) A large amount of power is wasted in the external resistance (R_e).

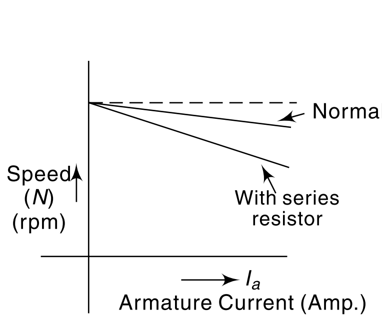


Fig. 9.28(a) *Speed current characteristic for shunt motor*

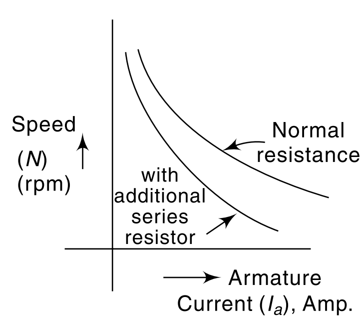


Fig. 9.28(b) *Speed current characteristic for series motor*

(b) Speed control is limited to give speeds below rated and increase of speed is not possible by this method.
(c) For a given value of the external resistance the speed reduction is not constant but varies with the motor load.

This method can only be used for small dc motors.

9.19.2 Variation of Field Flux (ϕ)

The flux in the dc motor being produced by the field current, control of speed is possible by field current variation. In the shunt motor, field current control is acheived by connecting a variable resistor R_C in series with the shunt field winding as shown in Fig. 9.29. The resistor R_C is called the shunt *field regulator*.

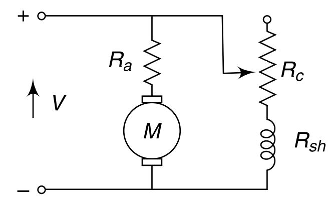


Fig. 9.29 *Speed control of dc shunt motor by field flux control*

The connection of R_C in the field reduces the field current which in turn reduces the flux ϕ. The reduction in flux will result in an increase in the speed. This method of speed control is used to give motor speeds above normal speed. The variation of field current in a series motor is done by any one of the following methods:

(a) A variable resistance R_d is connected in parallel with the series field winding as shown in Fig. 9.30. The parallel resistor is called the *diverter*. A portion of the main current is diverted through R_d, thus the diverter reduces the current flowing through the field winding. This reduces the flux and increases the speed.

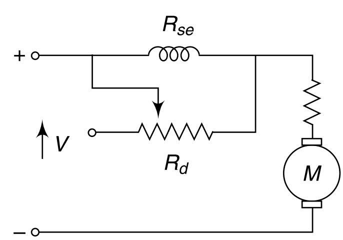


Fig. 9.30 *Speed control of dc series motor by using diverter in the field circuit*

(b) The second method uses a tapped field control as shown in Fig. 9.31.

Here the ampere-turns are varied by varying the number of field turns. This arrangement is used in electric traction.

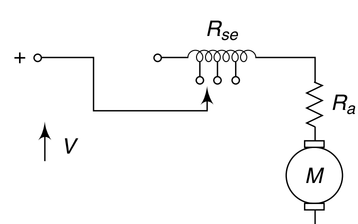


Fig. 9.31 *Speed control of dc series motor by using tapped field control*

Figures 9.32(a) and (b) show the typical speed/torque curves for shunt and series motors respectively, whose speeds are controlled by the variation of the field flux.

The advantages of field control are that this method is easy and convenient and since the shunt field current I_{sh} is very small, the power loss in the shunt field is also small.

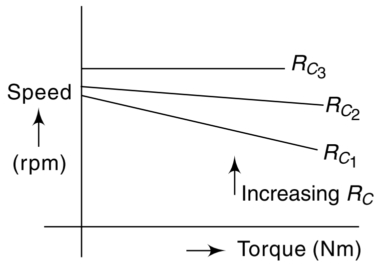


Fig. 9.32(a) ***Speed torque characteristic of a shunt motor***

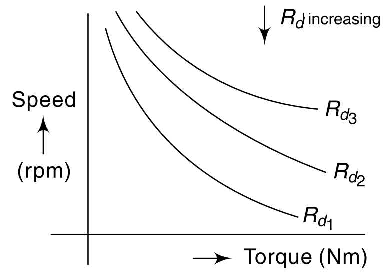


Fig. 9.32(b) ***Speed torque characteristic of a series motor***

9.24 A shunt wound motor with an armature resistance of 0.2 Ω is connected across a 400 V supply. The armature current is 40 A and the speed of the motor is 1000 rpm. Calculate the additional resistance which should be connected in series with the armature to reduce its speed to 700 rpm. Assume that the armature current remains the same.

Solution

Here $r_a = 0.2\ \Omega$, $V = 400$ V, $I_a = 40$ A and $N_1 = 1000$ rpm.

Let the additional resistance connected in series with armature R and $N_2 = 700$ rpm.

$$E_{b1} = V - I_a\, r_a = 400 - 40 \times 0.2 = 392 \text{ V}$$

$$E_{b2} = V - I_a(r_a + R) = 400 - 40\,(0.2 + R)$$

Now for shunt motor $E_b \propto N$ (as ϕ is constant).

Hence $$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} = \frac{1000}{700}$$

or $$E_{b_2} = \frac{7}{10} E_{b_1} = \frac{7 \times 392}{10} = 274.4 \text{ V}$$

$\therefore$ $$274.4 = 400 - 40(0.2 + R)$$

or $$8 + 40\,R = 400 - 274.4 = 125.6 \text{ V}$$

or $$R = 2.94\ \Omega$$

9.25 A series wound dc motor runs at 500 rpm and is connected across 220 V supply. The line current is 10 A and armature circuit resistance is 0.6 Ω. Find the resistance to be inserted in series to reduce the speed of the machine to 400 rpm assuming torque to vary as the square of the speed.

Solution

Given $N_1 = 500$ rpm, $V = 220$ V, $I_{a1} = 10$ A, $(r_a + r_{se}) = 0.6\ \Omega$

$N_2 = 400$ r.p.m and $T_e \propto N^2$

$$E_{b_1} = V - I_{a_1}(r_a + r_{se}) = 220 - 10 \times 0.6 = 214 \text{ V}$$

$$E_{b_2} = V - I_{a_2}(0.6 + R)$$

where R is the resistance to be inserted in series with the armature.

Now, for series motor $T_e \propto I_a^2$.

Hence $$\frac{T_{e_1}}{T_{e_2}} = \frac{I_{a_1}^2}{I_{a_2}^2} = \frac{N_1^2}{N_2^2} = \frac{(500)^2}{(400)^2} = \frac{25}{16}$$

or $$I_{a_2} = \sqrt{\frac{16}{25}}\, I_{a_1} = \frac{4}{5} I_{a_1} = 0.8\, I_{a1} = 0.8 \times 10 = 8 \text{ A}$$

and $$\frac{E_{b_1}}{E_{b_2}} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{I_{a_1} N_1}{I_{a_2} N_2} \quad (\because \text{ in series motor } \phi \propto I_a)$$

or $$\frac{214}{220 - 8\,(0.6 + R)} = \frac{10 \times 500}{8 \times 400}$$

or $(220 - 4.8 - 8R)50 = 32 \times 214 = 6848$

or $215.2 - 8R = 136.96$

or $R = 9.78\ \Omega$.

9.26 A 400 V dc series motor takes 20 A at rated condition and the speed is 1000 rpm. The armature resistance is 0.5 Ω. Calculate the value of the resistance that must be added to obtain the rated torque (i) at starting (ii) at 800 rpm.

Solution

Given, $V = 400$ V, $I_a = 20$ A, $N_1 = 1000$ rpm and $r_a = 0.5\ \Omega$

(i) Since back emf $E_b \propto \phi N$, hence at starting, when $N = 0$ $E_b = 0$.

To obtain rated torque at starting, let R be connected in series with the armature.

$V - I_a(r_a + R) = 0$

or $400 = 20(0.5 + R)$

or $R + 0.5 = 20$

or $R = 19.5\ \Omega$.

(ii) Let R' be the resistance connected in series with the armature when the speed is 800 rpm.

i.e. $N_2 = 800$ rpm

$E_{b1} = V - I_{a1}\, r_a = 400 - 20 \times 0.5 = 390$ V

$E_{b2} = V - I_{a2}(r_a + R') = 400 - 20(0.5 + R') = 390 - 20\, R'$

$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1 \phi_1}{N_2 \phi_2} = \frac{N_1\, I_{a_1}}{N_2\, I_{a_2}} = \frac{N_1}{N_2} \quad (\text{As } I_{a1} = I_{a2} \text{ since the torque is constant})$$

or $$\frac{390}{390 - 20R'} = \frac{1000}{800}$$

or $8 \times 390 = 3900 - 200\, R'$

or $R' = 3.9\ \Omega$.

9.27 A 220 V series motor takes 10 A and runs at 600 rpm. The total resistance is 0.8 Ω. At what speed will it run, when a 5 Ω resistance is connected in series, the motor taking the same current at the same supply voltage.

Solution

Here, $V = 220$ V, $I_a = 10$ A, $N_1 = 600$ r.p.m, $r_a = 0.8\ \Omega$

$R = 5\ \Omega$; $E_1 = V - I_a\, r_a = 220 - 10 \times 0.8 = 212$ V

$E_2 = V - I_a(r_a + R) = 220 - 10(0.8 + 5) = 220 - 58 = 162$ V

Let the speed be N_2 when 5 Ω resistor is added.

$$\therefore \quad \frac{E_1}{E_2} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{N_1}{N_2} \quad (\text{As } I_a \text{ remains same})$$

or $$N_2 = \frac{E_2}{E_1}\, N_1 = \frac{162}{212} \times 600 = 458 \text{ rpm.}$$

9.28 A 500 V dc shunt motor runs at 250 rpm at rated full load condition and takes an armature current of 200 A. The armature resistance is 0.12 Ω. Find the speed of the motor when the field circuit resistance is increased such that the flux is reduced to 80% of the normal value and the motor is loaded for an armature current of 100A.

Solution

Given, $V = 500$ V, $N_1 = 250$ rpm. $I_a = 200$ A and $r_a = 0.12\ \Omega$

$$E_{b_1} = V - I_a r_a = 500 - 200 \times 0.12 = 476 \text{ V}$$

Let initial flux be ϕ.

When armature current I_a' is 100 A, the flux $\phi' = 0.8\ \phi$, and the speed is N, we have

$$E_{b2} = V - I_a' r_a = 500 - 100 \times 0.12 = 488 \text{ V}$$

$$\because \quad \frac{E_{b_1}}{E_{b_2}} = \frac{\phi N_1}{\phi' N_2} \quad \text{or,} \quad N_2 = \frac{E_{b_2}}{E_{b_1}} \frac{\phi}{\phi'} N_1 = \frac{488}{476} \times \frac{\phi}{0.8\phi} \times 250$$

$$\therefore \quad N_2 = 320 \text{ rpm.}$$

9.29 A series motor with an unsaturated magnetic circuit and 0.5 Ω total resistance, when running at a certain speed, takes 60A at 500 V. If the load torque varies as the cube of speed, calculate the resistance required to reduce the speed by 25%.

Solution

Here, $r_a = 0.5\ \Omega$, $V = 500$ V and $I_{a1} = 60$.

Let initial speed be N_1 and when resistance R is connected speed be N_2.

$$N_2 = (1 - 0.25)N_1 = 0.75\ N_1$$

$$T_e \propto N^3$$

$$\frac{T_{e_1}}{T_{e_2}} = \frac{N_1^3}{N_2^3} = \left(\frac{1}{0.75}\right)^3,$$

Also
$$\frac{T_{e_1}}{T_{e_2}} = \frac{I_{a1}^2}{I_{a2}^2} = \frac{(60)^2}{I_{a2}^2} = \left(\frac{1}{0.75}\right)^3$$

$$\therefore \quad I_{a2} = 38.97 \text{ A}$$

$$E_{b1} = 500 - 60 \times 0.5 = 470$$

$$E_{b2} = 500 - 38.97(0.5 + R) = 480.5 - 38.97R$$

Now,
$$\frac{E_{b_1}}{E_{b_2}} = \frac{470}{480.5 - 38.97\,R} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{I_{a_1} N_1}{I_{a_2} N_2} = \frac{60 \times N_1}{38.97 \times 0.75\,N_1} = 2.053$$

$$480.5 - 38.97\,R = 228.9$$

or
$$R = 6.455\ \Omega.$$

9.30 A 400 V dc shunt motor has an armature and shunt field resistance of 0.3 and 200 Ω respectively. It takes full load current of 50 A and runs at 1000 rpm. Calculate the speed of the motor when a 100 Ω resistor is connected in series with the field winding, the load torque remaining same. Assume that field flux is proportional to the field current.

Solution

Here, $V = 400$ V; $r_a = 0.3$; $r_{sh} = 200\ \Omega$; $I_{L_1} = 50$ A

and $N_1 = 1000$ rpm

Let the speed of the motor be N_2 and line current be I_{L2} when 100 Ω resistor is connected in series with the field winding.

Now,
$$I_{a_1} = 50 - \frac{400}{200} = 48 \text{ A} \quad \text{and} \quad I_{sh_1} = \frac{400}{200} = 2 \text{ A}$$

and
$$I_{sh_2} = \frac{400}{200 + 100} = \frac{4}{3} \text{A} = 1.33 \text{ A}$$

$\because \quad T_{e_1} = T_{e_2}$

so, $$\frac{T_{e_1}}{T_{e_2}} = 1 = \frac{\phi_1 I_{a_1}}{\phi_2 I_{a_2}} = \frac{I_{sh_1} I_{a_1}}{I_{sh_2} I_{a_2}} = \frac{2 \times 48}{\frac{4}{3} \times \left(I_{L_2} - \frac{4}{3}\right)}$$

or $$\frac{4}{3}\left(I_{L_2} - \frac{4}{3}\right) = 96$$

or $$I_{L_2} - \frac{4}{3} = 72$$

or $$I_{L_2} = 73.33 \text{ A}$$

$\therefore$ $$I_{a_2} = 73.33 - 1.33 = 72 \text{ A}$$

Now, $$E_{b_1} = 400 - 48 \times 0.3 = 385.6 \text{ V}$$

and $$E_{b_2} = 400 - 72 \times 0.3 = 378.4 \text{ V}$$

$\therefore$ $$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1 \phi_1}{N_2 \phi_2} = \frac{N_1 I_{sh_1}}{N_2 I_{sh_2}}$$

or $$\frac{385.6}{378.4} = \frac{1000 \times 2}{N_2 \times 1.33}$$

$\therefore$ $$N_2 = 1476 \text{ rpm.}$$

9.31 A dc shunt machine connected to a 400 V mains has an armature and field circuit resistance of 0.2 Ω and 250 Ω respectively. Find the ratio of the speed when the machine acts as a generator to the speed when the machine acts as a motor, if the line current in each case is 100 A.

Solution

Given $V = 400$ V, $r_a = 0.2\ \Omega$, $r_{sh} = 250\ \Omega$, $I_L = 100$ A

Also, $$I_{sh} = \frac{400}{250} = 1.6 \text{ A}$$

When the machine acts as a generator

$$I_{a_1} = 100 + 1.6 = 101.6 \text{ A} \quad [\because I_{a(\text{gen})} = I_L + I_f]$$

$$E_1 = V + I_{a_1} r_a = 400 + 101.6 \times 0.2 = 420.32 \text{ V}$$

Let the speed of the generator be N_1 when the machine acts as a motor.

$$I_{a_2} = 100 - 1.6 = 98.4 \text{ A} \quad [I_{a(\text{motor})} = I_L - I_f]$$

So, $$E_2 = V - I_{a_2} r_a = 400 - 98.4 \times 0.2 = 380.32 \text{ V}$$

$\therefore$ $$\frac{E_1}{E_2} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{N_1}{N_2} \quad \text{(As flux is constant)}$$

or $$\frac{N_1}{N_2} = \frac{420.32}{380.32} = 1.105.$$

9.32 A dc shunt motor runs at 1000 rpm and takes an input of 700 W at 220 V under no load conditions. The shunt field current is 1A and armature resistance is 0.2 Ω. Find the speed when the machine is used as a generator if the line current is same in both the cases.

Solution

Speed of motor $N_1 = 1000$ rpm

Terminal voltage $V = 220$ V

Input power $P = 700$ W

Hence input line current $I_L = \frac{700}{220}$ A = 3.18 A

Armature resistance $r_a = 0.2\ \Omega$

Shunt field current $I_{sh} = 1$ A

$\therefore$ Armature current $I_a = 3.18 - 1 = 2.18$ A

Back emf $E_b = V - I_a r_a = 220 - 2.18 \times 0.2 = 219.56$ V

When the machine acts as a generator

Armature current $I_a = 3.18 + 1 = 4.18$ A

$\therefore$ Generated emf $E_g = V + I_a r_a = 220 + 4.18 \times 0.2 = 220.836$ V

If N_2 be the speed of the generator

$$\frac{E_b}{E_g} = \frac{N_1}{N_2}$$

or
$$N_2 = \frac{E_g}{E_b} N_1 = \frac{220.836}{219.56} \times 1000 = 1006 \text{ rpm.}$$

9.33 A 220 V series motor has a total armature resistance of 0.3 Ω. At speed 1500 rpm it draws a line current of 10 A. When a 3 Ω resistor is connected in series with the armature circuit it draws a line current of 6 A. Find the speed of the machine when the 3 Ω resistor is connected and the ratio of two mechanical outputs. Assume the flux at 6 A is 75% of that with 10 A.

Solution

Given $V = 220$ V, $r_a = 0.3\ \Omega$, $N_1 = 1500$ rpm, $I_{L_1} = I_{a_1} = 10$ A, $R = 3\ \Omega$

Back emf without 3 Ω resistor $(E_{b_1}) = 220 - 10 \times 0.3 = 217\ \Omega$

Back emf with 3 Ω resistor $(E_{b_2}) = 220 - 6 \times (3 + 0.3) = 200.2$ V

If ϕ be the (since $I_{a_2} = 6$ A) flux in the first case, then flux with resistance connected is 0.75 ϕ (given).

So
$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1 \phi}{N_2 \times 0.75 \phi} \quad \text{(where } N_2 \text{ is the new speed)}$$

or
$$N_2 = \frac{N_1 E_{b_2}}{0.75\ E_{b_1}} = \frac{1500 \times 200.2}{0.75 \times 217} = 1895 \text{ rpm.}$$

The ratio of the two mechanical outputs $= \frac{E_{b_1} I_{a_1}}{E_{b_2} I_{a_2}} = \frac{217 \times 1500}{200.2 \times 1845} = 0.88.$

9.20 LOSSES IN A DC MACHINE

There are three types of major losses in a dc machine.

(a) **Copper Losses** There are two types of copper losses. One is *armature copper loss* and the other is *field copper loss.* Armature copper loss = $I_a^2 r_a$, where I_a is the armature current and r_a is the armature resistance.

Field copper loss = Shunt field copper loss + Series field copper loss.

$= I_{sh}^2 r_{sh} + I_{se}^2 r_{se}$, where I_{sh} and I_{se} are the shunt and series field current, r_{sh} and r_{se} are the shunt and series field resistance. Brush contact loss is due to resistance of the brush contact. It is included in armature copper losses.

(b) **Iron Losses (Core or Magnetic Losses)** These losses occur in the armature and field core. They are of two types—*hysteresis loss* and *eddy current loss*. Hysteresis loss = $K_h B_m^{1.6} f$ and Eddy current loss = $K_B B_m^2 f$ where B_m = maximum flux density, f = frequency of magnetic reversal and K_h and K_e are constants.

(c) **Mechanical Losses** These losses consist of bearing frictional and windage loss.

In a medium-size motor, armature copper losses are about 30% to 40% of the total full load losses and field copper losses are about 20% to 30% of total full load losses. Iron losses are about 20% and mechanical losses are about 5 to 10% of the total full load losses. For small motors mechanical losses are comparable with full load losses while for larger motors, mechanical losses may be neglected.

Iron losses and mechanical losses are constant for a particular machine and they are together known as *no load rotational losses.*

Besides the above three types of losses there is an additional loss known as stray load loss. All the losses which do not belong to any of the above categories may be included in this group. In most machines stray load loss is taken as 1% of the rated output of the machine and is usually neglected.

Efficiency of dc Machines

Efficiency $\eta = 1 - \dfrac{\text{Losses}}{\text{Input}}$ $\left[\because \eta = \dfrac{\text{Output}}{\text{Input}} = \dfrac{\text{Input} - \text{Losses}}{\text{Input}}\right]$

For a generator, $\eta_g = 1 - \dfrac{\text{losses}}{VI_L + \text{losses}}$, where V is the terminal voltage and I_L is the line current; $V I_L$ is the output power.

For a motor, $\eta_m = 1 - \dfrac{\text{losses}}{V I_L}$; here VI_L is the input power. η may also be expressed in %, if multiplied by 100.

Condition for Maximum Efficiency for a dc Generator

$$\eta_g = \frac{V I_L}{V I_L + I_a^2 r_a + V_f I_f + W_o}$$

Now, $V_f I_f + W_o$ = constant (= C) and as (I_f) is negligible so $I_L = I_a$.

Here, $V I_L$ = output power

$I_a^2 r_a$ = armature copper loss

$V_f I_f$ = shunt field copper loss

W_o = No load rotational loss.

So, $$\eta_g = \frac{VI_L}{VI_L + I_L^2 r_a + C}.$$

Maximum efficiency occurs when

$$\frac{d\eta_g}{dI_L} = \frac{(VI_L + I_L^2 r_a + C)V - VI_L\,(V + 2I_L\, r_a)}{(VI_L + I_L^2 r_a + C)^2} = 0$$

or $$V I_L + I_L^2\, r_a + C = V\, I_L + 2\, I_L^2 r_a$$

or $$C = I_L^2 r_a\,.$$

Hence, constant loss = variable armature circuit loss. Hence a generator has maximum efficiency when variable loss equals constant loss.

The load current at maximum efficiency is given by

$$I_L = \sqrt{\frac{\text{constant loss}}{r_a}}\,.$$

9.34 A 480 V, 20 kW shunt motor takes 2.5 A when running at no load. Taking the armature resistance to be 0.6 Ω, field resistance to be 800 Ω and brush drop 2 V, find the full load efficiency.

Solution

Here, $V = 480$ V, $P = 20 \times 10^3$ W, $I_L = 2.5$ A (no load), $r_a = 0.6$ A, $r_{sh} = 800\ \Omega$

$$\therefore \qquad I_{sh} = \frac{V}{r_{sh}} = \frac{480}{800} = 0.6 \text{ A}$$

Input power at no load = VI_L (no load) = 480 × 2.5 = 1200 W

Field copper loss = $I_{sh}^2\, r_{sh} = (0.6)^2 \times 800 = 288$ W.

Armature current (no load)

$$I_a = I_L - I_{sh} = 2.5 - 0.6 = 1.9 \text{ A.}$$

Armature copper loss (no load) = $I_a^2\, r_a = (1.9)^2 \times 0.6 = 2.166$ W

Brush contact loss = $2 \times I_a = 3.8$ W

So, (Core loss + Frictional losses) = 1200 – 288 – 2.166 – 3.8 = 906.034 W

Under full load condition,

$$I_L = \frac{20 \times 10^3}{480} = 41.67 \text{ A}$$

$$\therefore \qquad I_a = I_L \text{ (full load)} - I_{sh} = 41.67 - 0.6 = 41.07 \text{ A}$$

Armature copper loss at full load = $(41.07)^2 \times 0.6 = 1012.05$ W

Total losses = 1012.05 + 288 + 906.034 + 2 × 41.07 = 2288.22 W

Hence, full load efficiency $= \dfrac{\text{Input} - \text{Loss}}{\text{Input}}$

$$= 1 - \frac{\text{Loss}}{\text{Input}} = 1 - \frac{2288.22}{20{,}000} = 0.8855 \text{ or, } 88.55\%.$$

9.35 A 100 kW, 220V dc shunt generator has the following data:

Armature resistance = 0.1 Ω

Mechanical loss = 5 kW

Iron losses = 5 kW

Shunt field resistance = 220 Ω

Brush contact drop = 1 V per brush
Stray losses are 1% of output.
Find the efficiency at full load. Also find the input torque if the speed is 1000 rpm.

Solution

Given, Output = 100 W
Terminal voltage = 220 V

Line current $I_L = \dfrac{100\times 10^3}{220} = 454.54$ A

Field current $I_{sh} = \dfrac{220}{220} = 1$ A

Armature current $I_a = I_L + I_{sh} = 454.54 + 1 = 455.54$ A

Field copper loss

$$I_{sh}^2\, r_{sh} = 1^2 \times 220 = 220 \text{ W}$$

Armature copper loss

$$I_a^2\, r_a = (455.54)^2 \times 0.1 = 20751.67 \text{ W}$$

Brush contact loss = 1 × 2 × 455.54 = 911.08 W
Mechanical loss = 5000 W.
Iron loss = 5000 W
Stray losses = $0.01 \times 100 \times 10^3 = 1000$ W
Total loss = 220 + 20751.67 + 911.08 + 5000 + 5000 + 1000
= 32882.75 W = 32.88 kW

Input power = Output + Loss = (100 + 32.88) kW = 132.88 kW

$$\therefore \quad \text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100\% = \frac{100}{132.88} \times 100\% = 75.25\%$$

Speed = 1000 rpm

$$\because \quad \text{Angular velocity } \omega = \frac{2\pi \times 1000}{60} \text{ rad/s} = 104.72 \text{ rad/s}$$

$$\text{Input torque} = \frac{\text{Input power}}{\omega} = \frac{132.88 \times 10^3}{104.72} \text{ Nm} = 1269 \text{ Nm.}$$

9.36 A 400 V shunt motor with armature and field resistance of 0.1 Ω and 200 Ω takes no load current of 10 A at 1500 rpm. If full load current is 100 A find the speed and output torque at full load. Assume that the mechanical losses are same at no load and full load.

Solution

Given, $V = 400$ V, $r_a = 0.1\ \Omega$, $r_{sh} = 200\ \Omega$

At no load

$$I_{L_1} = 10 \text{ A}$$

$$N_1 = 1500 \text{ rpm}$$

$$I_{sh} = \frac{V}{r_{sh}} = \frac{400}{200} = 2 \text{ A}$$

$$\therefore \quad I_{a_1} = I_{L_1} - I_{sh} = 10 - 2 = 8 \text{ A}$$

Armature copper loss = $(8)^2 \times 0.1 = 6.4$ W [at no load]

Back emf $E_{b_1} = V - I_{a_1} r_a = 400 - 8 \times 0.1 = 399.2$ V

Input power at no load = 400 × 10 = 4000 W (= total loss)

So, constant losses (i.e mechanical loss + core loss + shunt field copper loss) = 4000 – 6.4 = 3993.6 W

At full load,

$$I_{L_2} = 100 \text{ A}$$

Hence, $\quad I_{a_2} = 100 - 2 = 98$ A

Armature copper loss is $(98)^2 \times 0.1 = 960.4$ W (at full load)

Back emf $\quad (E_{b_2}) = 400 - 98 \times 0.1 = 390.2$ W

If N_2 be the speed at full load

$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2}$$

or

$$N_2 = \frac{E_{b_2}}{E_{b_1}} N_1 = \frac{390.2}{399.2} \times 1500 = 1466 \text{ rpm}$$

Input power at full load = $100 \times 400 = 40000$ W

$\therefore$ Total loss = $3993.6 + 960.4 = 4954$ W (at full load)

Output power = Input power – Loss = 40,000 – 4954 = 35046 W (at full load)

$$\text{Angular velocity } \omega = \frac{2\pi N_2}{60} = \frac{2\pi \times 1466}{60} \text{ rad/s} = 153.52 \text{ rad/s}$$

$$\text{Output torque at full load} = \frac{35046}{153.52} \text{ Nm} = 228.28 \text{ Nm}$$

$$\left[\because \; T_{\text{output}} = \frac{P\,(\text{output})}{\omega}\right].$$

9.37 A 400 V dc shunt motor runs at 1000 rpm and takes an input of 1500 W under no load condition. The armature and shunt field resistance are 0.3 Ω and 200 Ω respectively. Find the efficiency when the machine is used as a generator supplying 100 A at 400 V.

Solution

For motor at no load,

Input power = 1500 W

$$\text{Line current} = \frac{1500}{400} = 3.75 \text{ A [no load]}$$

$$\text{Shunt field current} = \frac{400}{200} = 2 \text{ A}$$

Hence, armature current $I_a = 3.75 - 2 = 1.75$ A,

Armature copper loss = $(1.75)^2 \times 0.3 = 0.9187$ W [no load]

Constant loss = Mechanical loss + core loss + Shunt field copper loss

= 1500 – 0.9187 = 1499.08 W

For generator, output = $100 \times 400 = 40{,}000$ W

Line current = 100 A

Armature current = 100 + 2 = 102 A

Armature copper loss = $(102)^2 \times 0.3 = 3121.2$ W

So, total loss = 3121.2 + 1499.08 = 4620.28 W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Output + Loss}} = \frac{40{,}000}{40{,}000 + 4620.28} = \times 100\% = 89.65\%.$$

9.38 A 200 V dc shunt generator runs at 1000 rpm at no load taking an input of 700 W. The armature resistance is 0.2 Ω and the shunt field current is 2A. Find the line current at which maximum efficiency occurs. Also find the value of maximum efficiency.

Solution

At no load, Input power = 700 W

$$\text{Line current} = \frac{700}{200} = 3.5 \text{ A (no load).}$$

$$\text{Shunt field current} = 2 \text{ A.}$$

Hence, armature current at no load = 3.5 + 2 = 5.5 A

Armature copper loss at no load = $(5.5)^2 \times 0.2 = 6.05$ W

Constant loss = 700 – 6.05 = 693.95 W.

Maximum efficiency occurs when constant loss = variable loss.

In shunt machines variable loss is the armature copper loss.

If I_a be the armature current at which maximum efficiency occurs, then

$$I_a^2 \times 0.2 = 693.95$$

or $$I_a = \sqrt{\frac{693.05}{0.2}} = 58.9 \text{ A}$$

∴ The line current at which maximum efficiency occurs is (58.9 – 2) = 56.9 A.

Output power = 56.90 × 200 = 11380 W;

Total losses = 693.95 × 2 = 1387.9 W (∵ constant loss = variable loss hence total loss = 2 × constant loss)

$$\therefore \text{ Efficiency} = \frac{11380}{11380 + 1387.9} \times 100\% = 89.13\%.$$

9.39 A 400 V dc shunt generator gives a full load output of 50 kW. The armature and field resistance are 0.1 and 250 Ω respectively. The core and frictional losses are together 2000 W. Calculate the generated emf, copper losses and efficiency.

Solution

$$\text{Output} = 50{,}000 \text{ W}$$

$$\text{Line current} = \frac{50{,}000}{400} = 125 \text{ A}$$

$$\text{Shunt field current} = \frac{400}{250} \text{ A} = 1.6 \text{ A}$$

$$\therefore \quad \text{Armature current} = 125 + 1.6 = 126.6 \text{ A}$$

$$\text{Generated emf} = 400 + 126.6 \times 0.1 = 412.66 \text{ V}$$

$$\begin{aligned}\text{Total copper losses} &= \text{Armature copper loss} + \text{Field copper loss}\\ &= (126.6)^2 \times 0.1 + (1.6)^2 \times 250\\ &= 1602.756 + 640 = 2242.756 \text{ W}\end{aligned}$$

$$\begin{aligned}\text{Total losses} &= \text{Copper loss} + \text{Core loss} + \text{Mechanical loss}\\ &= 2242.756 + 2000 = 4242.756 \text{ W}\end{aligned}$$

Hence, $$\text{Efficiency} = \frac{\text{Output}}{\text{Output + Loss}} = \frac{50{,}000}{50{,}000 + 4242.756} \times 100\% = 92.18\%.$$

9.40 A 220 V dc shunt motor takes 3 A at no load. If the armature and shunt field resistance are 0.2 Ω and 110 Ω respectively find the output power and efficiency when motor takes 30 A at full load. Also find the percentage change in speed from no load to full load.

Solution

At no load

$$I_L = 3 \text{ A},$$

Also, $$r_a = 0.2\ \Omega,\ r_{sh} = 110\ \Omega;$$

$\therefore \qquad I_{sh} = \frac{220}{110}\text{ A} = 2\text{A}$

Hence, $\qquad I_a = 3 - 2 = 1\text{A}$

Armature copper loss at no load (= $I_a^2 r_a$) = $(1)^2 \times 0.2 = 0.2$W.

Input at no load = $V \times I_L = 220 \times 3 = 660$ W.

Thus, constant loss = 660 – 0.2 = 659.8 W.

Back emf $E_{b_1} = V - I_a r_a = 220 - 1 \times 0.2 = 219.8$ V

At full load,

$$I_L = 30 \text{ A}$$

So, $\qquad I_a = 30 - 2 = 28$ A

Armature copper loss at full load is $(28)^2 \times 0.2 = 156.8$ W

Total loss at full load = 156.8 + 659.8 = 816.6 W

Input power at full load = 220 × 30 = 6600 W.

$\therefore$ output power at full load = (6600 – 816.6)W = 5783.4 W.

Therefore, efficiency at full load = $\frac{\text{Output}}{\text{Input}} \times 100\% = \frac{5783.4}{6600} \times 100\% = 87.63\%$.

Back emf at full load

$$(E_{b_2}) = 220 - 28 \times 0.2 = 214.4 \text{ V}$$

If N_1 and N_2 be the speed at no load and full load respectively then

$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} = \frac{219.8}{214.4}$$

Change in speed from no load to full load is

$$= \frac{N_1 - N_2}{N_1} \times 100\% = \frac{219.8 - 214.4}{219.8} \times 100\% = 2.46\%.$$

9.41 A 250 V, 200 kW dc generator when at rest takes an armature current of 400 A with 8 V produced across its armature terminals. At no load condition and at rated speed the line and shunt field currents are respectively 36 A and 12 A. Find the generator efficiency at full load and half load.

Solution

When the generator is at rest, back emf $E_b = 0$

$\therefore \qquad 0 = V - I_a r_a,$

or $\qquad r_a = \frac{V}{I_a} = \frac{8}{400} = 0.02\ \Omega.$

At no load condition,

$$I_L = 36 \text{ A and } I_{sh} = 12 \text{ A}$$

$\therefore \qquad I_a = 36 - 12 = 24$ A

Armature copper loss = $I_a^2 r_a = (24)^2 \times 0.02 = 11.52$ W [at no load]

Input = 36 × 250 W.

Constant losses = 36 × 250 – 11.52 ($\because$ Constant losses = input – armature copper loss) = 8988.48 Ω.

At full load,

Output power = 200,000 W

Line current $I_L = \frac{200{,}000}{250}$ A = 800 A.

Armature current (I_a) = 800 + 12 = 812 A.

$\therefore$ Armature copper loss at full load is

$$I_a^2 r_a = (812)^2 \times 0.02 = 13186.88 \text{ W}$$

$$\text{Total losses} = 13186.88 + 8988.48 = 22175.36 \text{ W}$$

$$\therefore \text{ Efficiency at full load} = \frac{200{,}000}{200{,}000 + 22175.36} \times 100\% = 90\%$$

At half load

$$I_L = \frac{800}{2} = 400 \text{ A}$$

$$I_a = 400 + 12 = 412 \text{ A}$$

$$\text{Output} = \frac{200{,}000}{2} \text{ W} = 100{,}000 \text{ W}.$$

Armature copper loss = $(412)^2 \times 0.02 = 3394.88$ W (at half load).

Total loss at half load = 3394.88 + 8988.48 = 12383.36 W.

$$\therefore \text{ Efficiency at half load} = \frac{100{,}000}{100{,}000 + 12383.36} \times 100\% = 88.98\%.$$

9.42 A 1500 kW, 550 V, 16-pole series generator runs at 150 rpm. What must be the useful flux per pole if there are 2500 lap connected armature conductors and full load copper losses are 25 kW?

Solution

In series generator,

Line current = Armature current

$$\text{Hence,} \quad I_a = \frac{1500 \times 10^3}{550} \text{ A} = 2727.27 \text{ A}$$

$$\text{Now, copper loss} = I_a^2 r_a = 25 \times 10^3 \text{ W}$$

$$\therefore \quad r_a = \frac{25 \times 10^3}{(2727.27)^2} = 0.00336\ \Omega$$

Generated emf $E = V + I_a r_a = 550 + 2727.27 \times 0.00336 = 559.167$ V.

If ϕ be the flux per pole, we can write

$$559.167 = \frac{P\phi ZN}{60\text{ A}} = \frac{16\ \phi \times 2500 \times 150}{60 \times 16} = 6250\ \phi$$

$$\therefore \quad \phi = 0.08947 \text{ W}b.$$

9.43 A short shunt compound generator supplies a current of 100 A at a voltage of 220V. The resistance of the shunt field, series field and armature are 50 Ω, 0.025 Ω and 0.05 Ω respectively. The total brush drop is 2 V and the total iron and frictional losses are 1000 W. Find (i) the generated emf, (ii) copper losses and (iii) generator efficiency.

Solution

$I_L = 100$ A, $V = 220$ V, $r_{sh} = 50\ \Omega$, $r_{se} = 0.025\ \Omega$ and $r_a = 0.05\ \Omega$.

Also, brush drop = 2 V.

Total iron and friction losses = 1000 W

Voltage across shunt field (or across armature terminals)

$$= 220 + 100 \times 0.025 = 220 + 2.5 = 222.5 \text{ V}$$

$$\text{Thus,} \quad I_{sh} = \frac{222.5}{50} \text{ A} = 4.45 \text{ A}$$

$$I_a = I_L + I_{sh} = 100 + 4.45 = 104.45 \text{ A}.$$

(i) Generated voltage = 222.5 + 104.45 × 0.05 + 2 = 229.72 V.

(ii) Copper losses = $I_a^2 r_a + I_{sh}^2 r_{sh} + I_L^2 r_{se}$

$= (104.45)^2 \times 0.05 + (4.45)^2 \times 50 + (100)^2 \times 0.025$

= 1785.615 W = 1.78 kW.

(iii) Total loss = Copper loss + Iron and friction loss

= (1785.615 + 1000)W = 2785.615 W

Output power (= VI_L) = 220 × 100 = 22,000 W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100\% = \frac{22{,}000}{22{,}000 + 2785.615} \times 100\% = 88.76\%.$$

9.44 A 400 V dc shunt motor runs at no load at 1500 rpm with input 1000 W. The shunt field current is 1 A and the armature resistance is 0.2 Ω. Find the line current at which maximum efficiency occurs and the value of the maximum efficiency.

Solution

Given, V = 400 V, N = 1500 rpm

Input power = 1000 W, I_{sh} = 1 A and r_a = 0.2 Ω.

$$\text{Line current at no load} = \frac{1000}{400} = 2.5 \text{ A}.$$

Armature current at no load = 2.5 – 1 = 1.5 A.

Armature copper loss at no load (= $I_a^2 r_a$) = $(1.5)^2 \times 0.2$ = 0.45 W

Constant loss = 1000 – 0.45 = 999.55 W.

For maximum efficiency, constant loss = variable loss.

If I_a' be the armature current at maximum efficiency condition then

$$I_a'^2 \times 0.2 = 999.55 \quad \text{or,} \quad I_a' = 70.69 \text{ A}$$

∴ line current = 70.69 + 1 = 71.69 A

Also, at maximum efficiency, total loss = 2 × 999.55 = 1999.1 W.

Input = 400 × 71.69 = 28676 W

$$\therefore \quad \text{Maximum efficiency} = 1 - \frac{1999.1}{28676} = 0.93 \text{ or } 93.03\%.$$

9.45 A 10 kW, 200 V short shunt compound dc generator has a full load efficiency of 90%. If the armature, series and shunt field resistance are 0.2, 0.1 and 50 Ω respectively, find the combined mechanical and core loss of the machine.

Solution

Output power = 10,000 W

$$\text{Line current} = \frac{10 \times 10^3}{200} \text{ A} = 50 \text{ A}$$

η = 0.9

$$\text{Input power} = \frac{\text{Output}}{\eta} = \frac{10{,}000}{0.9} = 11{,}111 \text{ W}$$

Total loss = 11,111 – 10,000 = 1111 W

Voltage across the shunt field or armature terminals is = 200 + 50 × 0.1 = 205 V

$$\text{Hence,} \quad I_{sh} = \frac{205}{100} = 2.05 \text{ A}$$

Shunt field copper loss = $(I_{sh}^2 r_{sh}) = (2.05)^2 \times 50$ = 210.125 W

Series field copper loss = $(I_{se}^2 r_{se}) = (50)^2 \times 0.1$ = 250 W

Also, $I_a = 2.05 + 50 = 52.05$ A

Armature copper loss = $(I_a^2 r_a) = (52.05)^2 \times 0.2 = 541.84$ W

$\therefore$ (Mechanical + Core losses) = 1111 – 541.8 = 569.2 W

= 1111 – 541.84 – 210.125 – 250 = 109.035 W.

9.21 NEED FOR STARTER IN A DC MOTOR

The armature current of a motor is given by

$$I_a = \frac{V - E_b}{R_a}$$

Thus, I_a depends upon E_b and R_a when V is kept constant. When a motor is first switched on, the armature is stationary so the back emf E_b is zero ($\because E_b \propto$ speed).

The initial starting armature current I_a is given by

$$I_{as} = \frac{V - 0}{R_a} = \frac{V}{R_a}.$$

Since the armature resistance of a motor is very small, generally less than one ohm, therefore the starting armature current I_{as} would be very large. For example, if a motor with armature resistance of 0.2 Ω is connected directly to a 230 V supply, then

$$I_{as} = \frac{V}{R_a} = \frac{230}{0.2} = 1150 \text{ A}$$

This large current would damage the brushes, commutator, or windings. Also it may damage supply installation.

As the motor speed increases, the back emf increases and the difference ($V - E_b$) goes on decreasing. This results in the gradual decrease of I_{as} until the motor attains its stable rated speed and the corresponding back emf. Under this condition the armature current reaches its desired rated value. Thus, it is found that the back emf helps the armature resistance in limiting the current through the armature.

Since the starting current is very large, at the time of starting of dc motors (except very small motor) an extra resistance must be connected in series with the armature. This would limit the initial current to a safe value until the motor has built up the stable speed and back emf E_b.

This external starting resistance is basically known as the *starter resistance.*

The series resistance is divided into sections which are cut out one by one as the speed of the motor increases and the back emf builds up. When the speed of the motor attains its normal value, the extra resistance is completely cut out.

9.21.1 Three-point Starter

A three-point starter connected to a dc shunt motor is shown in Fig. 9.33. As only three terminals, i.e. L, A and F are available from the starter it is called a three-point starter. The terminal L of the starter is connected to the supply terminal, A to the armature and F to the field terminal.

When the motor is at rest the starter handle S is kept in the "OFF" position by a spring and motor is disconnected from the supply. When the motor is to be started the handle is moved to stud 1. The shunt field and the holding coil gets the supply and entire starting resistance is connected in series with the armature. The armature starts rotating and the handle is gradually moved through all the studs until it touches the holding magnet. The holding magnet is called the *no volt release* or *low voltage release coil*. In case of power failure the holding coil gets demagnetized and the handle is brought back to "OFF" position by a spring action. Again, if by any chance the shunt field winding gets open circuited the holding magnet gets demagnetized and starter handle returns to the "OFF" position.

There is another coil called *overload release coil* which protects the motor against excessive load current. When armature current exceeds a particular value the overload release coil attracts the soft iron armature and as a result the no volt release coil gets short circuited. The starter is pulled back to the "OFF" position by the spring action as the holding coil gets demagnetized. The motor is thus automatically switched off.

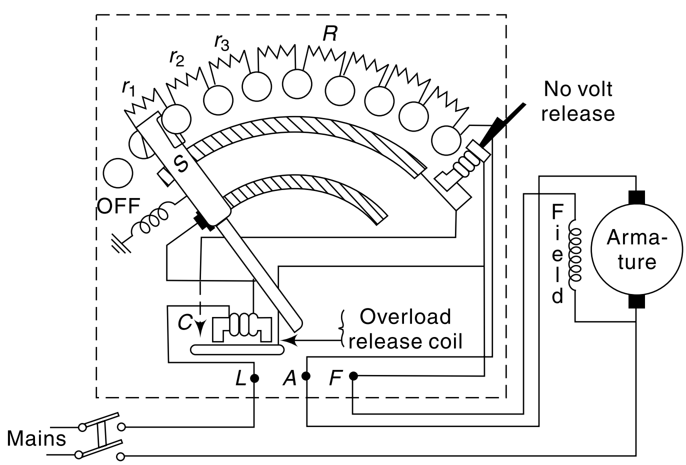


Fig. 9.33 ***Three-point starter for a dc shunt motor***

9.22 REVERSAL OF ROTATION OF DC MOTOR

The direction of rotation of a dc motor can be reversed by reversing the connections of either the field winding or the armature but not both.

It is to be noted that in order to reverse the direction of rotation of a compound motor the reversal of the field connections involves both shunt and series windings.

9.23 DC MACHINE APPLICATIONS

Applications of a dc machine are discussed as follows.

9.23.1 Generator Applications

DC generaters are nowadays used in dynamometers (for measuring torque etc.), for welding purpose, as control type dc generator for closed loop systems, permanent magnet dc generators, etc. Separately excited dc generators are used (a) to serve as an excitation source for large alternators in power generating stations (b) to serve as control generator in Ward-Leonard system of speed control and (c) to serve as auxiliary and emergency power supplies.

9.23.2 Motor Applications

When constant speed service at low speed is required in industries shunt motors are used. When driven load requires a wide range of speed control dc shunt motor is used, e.g. in lathes.

Since series motor have high starting torque it is best suited for driving hoists, trains, cranes, etc. They are widely used in all types of electric vehicles, electric trains, street cars, battery powered portable tools, etc.

Compound motors are used in rolling mill drive, punching press, planning or miling machine. When the supply voltage across the motor terminals is likely to vary considerably such as in traction motors, compound motors are preferred.

As separately excited dc motors are easily adaptable to a wide range of speed and torque control, in high power applications these are widely used in steel and aluminium rolling mills and Ward Leonard method of speed control. In low power applications separately excited dc motor finds use as control motor.

ADDITIONAL EXAMPLES

9.46 A 4-pole dc armature wave winding has 294 conductors;

(i) What flux per pole is necessary to generate 230 V when rotating at 1500 rpm?

(ii) What is the electromagnetic torque at this flux when the rated armature current of 120 A is flowing?

Solution

Here, $P = 4$, $A = 2$ and $Z = 294$

(i) $E = 230$ V, $N = 1500$ rpm

$$E = \frac{P\phi ZN}{60\text{ A}}$$

$$\text{or} \quad \phi = \frac{60\,AE}{PZN} = \frac{60 \times 2 \times 230}{4 \times 294 \times 1500} = 0.0156 \text{ Wb.}$$

(ii) $I_a = 120$ A

$$T_e = K\phi I_a = \frac{PZ}{2\pi A}\,\phi\, I_a = \frac{4 \times 294}{2\pi \times 2} \times 0.0156 \times 120 \text{ Nm} = 175.276 \text{ Nm.}$$

9.47 A dc machine is generating 125 V while delivering 8 A to a load. If its armature circuit resistance is 1.35 Ω, what voltage must be generated internally in the armature?

Solution

Here, $V = 125$ V, $I_L = 8$ A, $r_a = 1.35\ \Omega$

$\therefore$ $E = V + I_a r_a = 125 + 8 \times 1.35 = (125 + 10.8)\text{V} = 135.8$ V.

9.48 A shunt motor has a rated armature current 50A when connected to 200 V. The rated speed is 1000 rpm and armature resistance is 0.1 Ω. Find the speed if total torque is reduced to 70% of that at rated load and a 3Ω resistance is inserted in series with the armature.

Solution

Given, $I_{a_1} = 50$ A, $V = 200$ V, $N_1 = 1000$ rpm,

$r_a = 0.1\ \Omega$, $T_{e2} = 0.7\ T_{e1}$

As $T_e \propto \phi I_a$

So $I_{a_2} = 0.7\ I_{a1}$ (ϕ = constant for shunt motor)

or $I_{a_2} = 0.7 \times 50 = 35$ A

$$E_{b_1} = V - I_{a1}\ r_a = 200 - 50 \times 0.1 = 195 \text{ V}$$

$$E_{b_2} = V - I_{a2}(r_a + 3) = 200 - 35(0.1 + 3) = 200 - 35 \times 3.1 = 91.5 \text{ V}$$

Since $$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} \quad (\text{As } \phi = \text{constant})$$

So $$N_2 = \frac{E_{b_2}}{E_{b_1}} N_1 = \frac{91.5}{195} \times 1000 = 469 \text{ rpm.}$$

9.49 A dc series motor drives a load whose torque varies as cube of the speed. The armature and series field resistance together is 2 Ω. The line current is 10 A when connected to a 400 V supply and the speed is 1500 rpm. Find the resistance to be connected in series with the armature to reduce the speed to 1000 rpm.

Solution

$T_e \propto I_a^2$ (in series motor)

Here, $T_e \propto N^3$. So, $I_a^2 \propto N^3$

Also, $r_a + r_{se} = 2\ \Omega$

$I_{L1} = 10$ A $= (I_{a1})$, $V = 400$ V and $N_1 = 1500$ rpm

When resistance R is connected in series with armature, let speed $N_2 = 1000$ rpm and armature current is I_{a2}

$$\frac{I_{a1}^2}{I_{a2}^2} = \frac{N_1^3}{N_2^3} = \frac{(1500)^3}{(1000)^3}$$

or $$I_{a2} = \left(\frac{1000}{1500}\right)^3 \times (10)^2$$

∴ $$I_{a2}^2 = 5.44 \text{ A}$$

$$E_{b_2} = V - I_{a_2}(r_a + r_{se} + R) = 400 - 5.44\ (2 + R)$$

Now, $$E_{b_1} = V - I_{a_1}(r_a + r_{se}) = 400 - 10 \times 2 = 380 \text{ V}$$

Again $$\frac{E_{b_1}}{E_{b_2}} = \frac{I_{a_1} N_1}{I_{a_2} N_2} \quad (\text{for series motor})$$

Here, $$E_{b_2} = \frac{I_{a_2} N_2}{I_{a_1} N_1} \times 380 = \frac{5.44 \times 1000}{10 \times 1500} \times 380 = 137.897 \text{ V}$$

∴ $400 - 5.44(2 + R) = 137.897$

or $R = 46.18\ \Omega$.

9.50 A 50 kW, 400 V dc shunt generator has field and armature resistance of 200 Ω and 0.1 Ω respectively. The full load speed is 1000 rpm. Find the speed of the machine when running as a motor and taking 50 kW from a 400 V supply. Assume brush contact drop of 1 V per brush.

Solution

Here, $r_{sh} = 200\ \Omega$ and $r_a = 0.1\ \Omega$

For dc shunt generator

$$I_{sh} = (V/R_{sh}) = \frac{400}{200}\text{ A} = 2\text{A}$$

$$I_L = \frac{\text{Power output}}{\text{Terminal voltage}} = \frac{50 \times 10^3}{400} = 125\text{ A.}$$

$\therefore$ $$I_a = (I_L + I_{sh}) = (125 + 2)\text{ A} = 127\text{ A}$$

$$E_g = V + I_a r_a = 400 + 127 \times 0.1 = 412.7\text{ V.}$$

Similarly, for a dc shunt motor,

$$I_L = \frac{50 \times 10^3}{400}\text{ A} = 125\text{ A}$$

$$I_{sh} = \frac{400}{200}\text{ A} = 2\text{ A},\ I_a = (125 - 2)\text{A} = 123\text{ A}$$

$\therefore$ $$E_b = V - I_a r_a = 400 - 123 \times 0.1 = 387.7\text{ V.}$$

If speed of the motor is N_2 then comparing the two cases of operation we have

$$\frac{E_g}{E_b} = \frac{1000}{N_2}$$

or $$N_2 = 1000 \times \frac{387.7}{412.7} = 939\text{ rpm.}$$

9.51 A dc motor takes an armature current of 100 A at 230 V. The armature resistance is 0.03 Ω. The total number of lap connected armature conductors is 500 and the number of poles is 4. The flux per pole is 0.03 W*b*. Find the speed and torque.

Solution

Here, $I_a = 100\text{ A},\ V = 230\text{ V},\ r_a = 0.03\ \Omega$

$$A = P = 4,\ Z = 500 \text{ and } \phi = 0.03\text{ W}b$$

$\therefore$ $$E_b = V - I_a r_a = 230 - 100 \times 0.03 = 227\text{ V}$$

Also, $$E_b = \frac{P\phi ZN}{60\text{ A}}$$

or $$N = \frac{60\,AE_b}{P\phi Z} = \frac{60 \times 227}{0.03 \times 500} = 908\text{ rpm} \quad (\because A = P)$$

$\therefore$ $$T_e = \frac{E_b I_a}{\omega} = \frac{227 \times 100}{\frac{2\pi \times 908}{60}} = 238.85\text{ Nm.}$$

9.52 A 20 kW, 400 V dc shunt generator has no load rotational losses of 800 watts. The armature and shunt field resistances are 0.3 Ω and 200 Ω respectively. Find the input power and efficiency. Also calculate the maximum efficiency and the corresponding power output.

Solution

$$I_{sh} = \frac{V}{R_{sh}} = \frac{400}{200}\text{A} = 2\text{A}$$

$$I_L = \frac{P}{V} = \frac{20 \times 10^3}{400} = 50 \text{ A}$$

Hence, $I_a = (I_L + I_{sh}) = (50 + 2)\text{A} = 52 \text{ A}$

Armature copper loss (= $I_a^2 r_a$) = $(52)^2 \times 0.3$ W = 811.2 W

Shunt field copper loss (= $I_{sh}^2 r_{sh}$) = $(2)^2 \times 200$W = 800 W

No load rotational loss = Core loss + Mechanical loss = 800 W

∴ Total losses = (811.2 + 800 + 800) W = 2411.2 W = 2.41 kW

Output power = 20 kW

Input power = Output power + Losses = (20 + 2.41) kW = 22.41 kW

$$\text{Efficiency} = \left(1 - \frac{\text{Losses}}{\text{Input}}\right) = \left(1 - \frac{2.41}{22.41}\right) = 0.89 \text{ or } 89\%.$$

Maximum efficiency occurs when Variable loss = Constant loss

Now, constant loss = No load rotational loss + Shunt field copper loss = (800 + 800) W = 1600 W.

Variable loss (= $I_a^2 r_a$) = 1600 W

or $$I_a = \sqrt{\frac{1600}{0.3}} = 73 \text{ A}$$

∴ $I_L = I_a - I_{sh} = 73 - 2 = 71$ A

Output power $V \times I_L$ = (400 × 71) W = 28400 W = 28.46 kW

$$\text{Maximum efficiency} = \left(1 - \frac{1600 + 1600}{28400 + 1600 + 1600}\right) = 0.8987 \text{ or } 89.87\%.$$

9.53 A 50 kW belt driven shunt generator is running at 300 rpm and delivers rated load to a 250 V bus bar. The armature and shunt field resistances are 0.025 Ω and 50 Ω respectively. Suddenly the belt breaks and the machine continues to run as a motor taking 5 kW from bus bars. Find the speed of the motor allowing brush drop of 1 V per brush.

Solution

When the machine acts as a generator, we have

P = 50,000 W, N_1 = 300 rpm, V = 250 V, r_a = 0.025 Ω and r_{sh} = 50 Ω.

∴ $$I_L = \frac{P}{V} = \frac{50000}{250}\text{A} = 200 \text{ A}$$

$$I_{sh} = \frac{V}{r_{sh}} = \frac{250}{50}\text{A} = 5 \text{ A}$$

$$I_a = (I_L + I_{sh}) = (200 + 5)\text{A} = 205 \text{ A}$$

$$E_1 = V + I_a r_a + 2 \times 1 = (250 + 205 \times 0.025 + 2)$$

$$E_1 = 257.125.$$

When the machine acts as a motor, we have

$$I_L = \frac{5000}{250}\text{A} = 20 \text{ A}$$

$$I_a = (20 - 5) \text{ A} = 15 \text{ A}$$

$$E_2 = 250 - 15 \times 0.025 - 2 = 247.625 \text{ V}.$$

If N_2 be the speed of the motor

$$N_2 = \frac{E_2}{E_1} N_1 = \frac{247.625}{257.125} \times 300 \text{ rpm} = 289 \text{ rpm}.$$

9.54 The electromagnetic torque developed in a motor is 150 Nm. If the field flux is decreased by 20% and armature current is increased by 15% find the new electromagnetic torque developed.

Solution

Electromagnetic torque $T_e \propto$ flux $\times$ armature current. If ϕ_1 and I_{a_1} be the flux and armature current when the developed torque is 150 Nm then

$$150 = K\,\phi_1\, I_{a1} \quad \text{(where } K \text{ is constant).}$$

If T_{e_2} be the new electromagnetic torque then

$$T_{e_2} = K \times 0.8\ \phi_1 \times 1.15\ I_{a1} = 150 \times 0.8 \times 1.15 = 138 \text{ Nm}.$$

9.55 A 250 kW, 230 V long shunt compound generator supplies 75% of the rated load at rated voltage. The armature and series field resistance are 0.009 Ω and 0.003 Ω. Find the efficiency of the generator if the shunt field current is 13 A. When the machine is run as a motor at no load the armature current is 25A at rated voltage.

Solution

Here, $P = 250 \times 0.75 \text{ kW} = 187.5 \text{ kW}$

$V = 230$ V,

$r_a = 0.009\ \Omega$ and $r_{se} = 0.003\ \Omega$

$$I_{sh} = 13 \text{ A and } I_L = \frac{P}{V} = \frac{187500}{230} \text{ A} = 815.22 \text{ A}.$$

So, $I_a = I_L + I_{sh} = 828.22$ A

Armature copper loss $(= I_a^2 r_a) = (828.22)^2 \times 0.009 \text{ W} = 6173.54$ W.

Field copper loss = shunt field copper loss + series field copper loss

$= \{230 \times 13 + (828.22)^2 \times 0.003\}\text{W} = 5047.8$ W.

When the machine runs at no load as a motor,

$I_a = 25$ A

$I_{sh} = 13$ A

$\therefore$ $I_L = 13 + 25 = 38$ A

Input power $= I_L \times V = (38 \times 230)\text{W} = 8740$ W

Total copper losses $= \{(25)^2 \times 0.009 + 230 \times 13 + (25)^2 \times 0.003\}\text{W} = 2997.5$ W

No load rotational loss = Input power – Copper losses = (8740 – 2997.5) W = 5742.5 W.

For a generator,

Total losses = (6173.54 + 5047.8 + 5742.5) W = 16963.84 W = 16.964 kW

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{\text{Output}}{\text{Output} + \text{Loss}} \times 100$$

$$= \frac{187.5}{187.5 + 16.964} \times 100\% = 91.7\%.$$

9.56 A 600 V dc motor drives a 60 kW load 700 rpm. The shunt field resistance is 100 Ω and armature resistance is 0.16 Ω. If the motor efficiency is 85%, calculate the speed at no load and speed regulation.

Solution

Output = 60000 W, $V = 600$, $N_1 = 700$ rpm, $r_{sh} = 100\ \Omega$,

$r_a = 0.16\ \Omega$, $\eta = 0.85$.

$$\text{Input power} = \frac{\text{Output power}}{\text{Efficiency}} = \frac{60000}{0.85} = 70588 \text{ W}$$

$$\therefore \quad I_L = \frac{\text{Input power}}{\text{Terminal voltage}} = \frac{70588}{600} = 117.65 \text{ A};$$

$$I_{sh} = \frac{V}{r_{sh}} = \frac{600}{100} \text{ A} = 6 \text{ A}$$

$$I_a = I_L - I_{sh} \text{ (for motor)} = 117.65 - 6 = 111.65 \text{ A}$$

Also, $E_{b_1} = V - I_a r_a = 600 - 111.65 \times 0.16 = 582.136$ V

At no load,

$$E_{b_2} = V = 600 \text{ V}$$

It no load speed is N_2 then

$$N_2 = N_1 \frac{E_{b_2}}{E_{b_1}} = 700 \times \frac{600}{582.136} \text{ rpm} = 721 \text{ rpm.}$$

$$\text{Speed regulation} = \frac{721 - 700}{700} \times 100\% = 3\%.$$

9.57 A 200 V shunt motor has $r_a = 0.1\ \Omega$, $r_{sh} = 240\ \Omega$ and rotational loss is 236 W. On full load the line current is 9.8 A with the motor running at 1450 rpm. Find (i) the mechanical power developed (ii) power output (iii) load torque and (iv) full load efficiency.

Solution

Here, $V = 200$ V, $r_a = 0.1\ \Omega$, $r_{sh} = 240\ \Omega$, $I_{fl} = 9.8$ A

$N_{fl} = 1450$ rpm. Rotational loss = 236 W

$$\therefore \quad I_{sh} = \frac{V}{r_{sh}} = \frac{200}{240} = 0.833 \text{ A}$$

and $I_a = (I_{fl} - I_{sh}) = 9.8 - 0.833 = 8.97$ A

Also, $E_b = V - I_a r_a = 200 - 8.97 \times 0.1 = 199$ V

(i) Mechanical power developed is $E_b I_a = 199 \times 8.97 = 1785$ W = 1.785 kW.

(ii) Power output = 1785.9 – 236 ≃ 1549 W = 1.55 kW.

(iii) $$\text{Load torque} = \frac{\text{Power output}}{\omega} = \frac{1549 \times 60}{2\pi \times 1450} = 10.2 \text{ Nm.}$$

(iv) $$\text{Full load efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{1549}{200 \times 9.8} = 0.791 = 79.1\%.$$

9.58 A 200 V shunt motor takes 10 A when running at no load. The brush drop is 2 V at full load and negligible at no load. The stray load loss at line current of 100 A is 50% of the no load loss. Find the efficiency at a line current of 100 A if armature and field resistances are 0.2 Ω and 100 Ω respectively.

Solution

$$V = 200 \text{ V}, I_{LO} = 10 \text{ A}, \text{ Brush drop} = 2\text{V},$$

$$I_{sh} = \frac{V}{r_{sh}} = \frac{200}{100} \text{ A} = 2 \text{ A}$$

At no load, $I_a = I_{LO} - I_{sh} = 10 - 2 = 8$ A

Again, (Input = Loss) = 200 × 10 = 2000 W (at no load)

= No load rotational loss + Shunt field copper loss

+ Armature copper loss

$\therefore$ no load rotational loss = 2000 – 200 × 2 – 8^2 × 0.2 = 1587.2 W

[$\because$ Shunt field copper loss = $V \times I_{sh}$ = (200 × 2) W;

Armature copper loss at no load = I_a^2 (no load) × r_a = (8^2 × 0 0.2) W]

When I_L = 100 A

I_a = 100 – 2 = 98 A

Stray load loss = 0.5 × 2000 = 1000 W

Armature copper loss = $(98)^2$ × 0.2 W = 1920.8 W

Field copper loss = 200 × 2 W = 400 W

Total losses = No load rotational loss + Total copper loss + stray load loss

= 1587.2 + (1920.8 + 400) + 1000 = 4908 W.

$\because$ Input = 100 × 200 = 20,000 W

$$\therefore \quad \text{Efficiency} = \left(1 - \frac{\text{Losses}}{\text{Input}}\right) = \left(1 - \frac{4908}{20{,}000}\right) = 0.7546 = 75.46\%.$$

9.59 A 24 kW, 240 V, 100 A, 1500 rpm dc series motor has the following full load losses expressed in percentage of motor input:

Armature copper loss = 3%

Field copper loss = 2.5%

Rotational loss = 2%

If the motor draws half the rated current at rated voltage determine the speed and shaft power output.

Solution

Input power = 24 × 10^3 = 24,000 W

Rotational loss = 0.02 × 24,000 = 480 W

Armature copper loss = 0.03 × 24,000 = 720 W

Field copper loss = 0.025 × 24,000 = 600 W

$\therefore$ Total copper loss = (720 + 600) W = 1320 W = $I_a^2 (r_a + r_{se})$

$$\text{or} \quad (r_a + r_{se}) = \frac{1320}{(100)^2}\,\Omega = 0.132$$

$$E_{b_1} = V - I_a (r_a + r_{se}) = 240 - 100(0.132) = 226.8 \text{ V}.$$

$$\text{Now,} \quad I_L = \frac{100}{2} = 50 \text{ A}$$

$$\therefore \quad E_{b_2} = 240 - 50 \times 0.132 = 233.4 \text{ V}$$

(i) If N be the required speed

$$\frac{E_{b_2}}{E_{b_1}} = \frac{\phi_2 N}{\phi_1 \times 1500} = \frac{50 \times N}{100 \times 1500} \qquad (\text{As } I_L \propto \phi \text{ in series motor})$$

$$\text{or} \quad N = \frac{100 \times 1500}{50} \times \frac{233.4}{226.8} = 3087 \text{ rpm}.$$

(ii) Power developed = $E_{b_2} I_L$ = 233.4 × 50 = 11670 W

$\therefore$ Shaft power output = 11670 W – Rotational loss

= 11670 – 480 = 11190 W = 11.19 kW.

9.60 A dc series motor runs at 1500 rpm and takes 100 A from 400 V supply. The combined resistance of the armature and field is 0.5 Ω. If an additional resistance of 5 Ω is inserted in series with the armature circuit, find the motor speed if the electromagnetic torque is proportional to the square of the speed.

Solution

$$N = 1500 \text{ rpm};\ I_a = I_L = 100 \text{ A};\ V = 400 \text{ V};$$
$$(r_a + r_{se}) = 0.5\ \Omega$$
$$T_e \propto \phi\, I_a$$

As $\phi \propto I_a$,

$$T_e \propto I_a^2$$

Again $T_e \propto N^2$

Hence, $I_a \propto N$

$\therefore$ I_a = KN (K being a consant)

or, $K = \dfrac{100}{1500} = \dfrac{1}{15}$

Now, $E_{b_1} = V - I_a(r_a + r_{se}) = 400 - 100 \times 0.5 = 350$ V

and $E_{b_2} = V - I_{a2}(r_a + r_{se} + 5) = 400 - I_{a2}(5.5)$

If the speed is N when 5 Ω resistor is connected, then we can write

$$\frac{E_{b_1}}{E_{b_2}} = \frac{N_1}{N_2} \quad \text{or,} \quad \frac{350}{400 - 5.5 I_{a2}} = \frac{1500}{N} \qquad \begin{bmatrix} N_1 = \text{initial speed} \\ \quad = 1500 \text{ rpm} \\ N_2 \equiv N \end{bmatrix}$$

or, $350\,N = 600{,}000 - 5.5 \times K \times N \times 1500.$

or, $N \simeq 667$ rpm.

9.61 A dc shunt motor runs at 750 rpm from 250 V supply and takes a full load line current of 60 A. Its armature and field resistances are 0.4 Ω and 125 Ω respectively. Assuming 2 V brush drop calculate no load speed for a no load line current of 6 A and the resistance to be added series with the armature circuit to reduce the full load speed to 600 rpm.

Solution

Given, $N_{\text{fl}} = 750$ rpm; $V = 250$ V; $I_{\text{fl}} = 60$ A

$$r_a = 0.4\ \Omega,\ r_{\text{sh}} = 125\ \Omega$$
$$I_{nl} = 6 \text{ A}$$
$$I_{a\text{fl}} = 60 - \frac{250}{125} = 58 \text{ A} \quad \left[\because I_{afl} = \left(I_{fl} - \frac{V}{r_{\text{sh}}}\right)\right]$$
$$E_{b\text{fl}} = 250 - 58 \times 0.4 - 2 = 224.8 \text{ V} \quad [\because E_{bfl} = V - I_{afl} \times r_a]$$
$$I_{anl} = 6 - \frac{250}{125} = 4 \text{ A} \quad \left[\because I_{anl} = I_{nl} - I_{\text{sh}} = I_{nl} - \frac{V}{r_{\text{sh}}}\right]$$
$$E_{bnl} = 250 - 4 \times 0.4 - 2 = 246.4 \text{ V} \quad [\because E_{bnl} = V - I_{anl} \times r_a - \text{brush drop}]$$

[Suffix nl stands for no load parameters while suffix fl stands for full load parameters]

If N be the no load speed then we have

$$N = N_{\text{fl}} \times \frac{E_{bnl}}{E_{bfl}} = 750 \times \frac{246.4}{224.8} = 822 \text{ rpm}$$

If R be the resistance connected in series with the armature circuit then we can write,

$$E_b = [250 - I_a(0.4 + R) - 2] \text{ and } N' = 600 \text{ rpm}$$

or $E_b = [248 - 60(0.4 + R)] = 224 - 60\,R.$

$\therefore$ $$\frac{224 - 60R}{246.4} = \frac{600}{822} \quad \left[\because \frac{E_b}{E_{bnl}} = \frac{N'}{N}\right]$$

or $224 - 60\,R = 179.85$

or $R = 0.736\ \Omega.$

9.62 A 400 V series motor runs at 800 rpm and takes a line current of 60 A. Find the speed and percentage change in torque if the load is reduced so that motor takes 50 A. The combined resistance of the armature and field circuit is 0.6 Ω.

Solution

Here, $V = 400$ V, $N_1 = 800$ rpm, $I_{L_1} = 60$ A, $(r_a + r_{se}) = 0.6\ \Omega$

$$E_{b_1} = 400 - 60 \times 0.6 = 364 \text{ V}$$

$$T_e \propto I_a^2 \text{ (for series motor)}$$

$\therefore$ $T_{e_1} = K'(60)^2 = 3600\ K'$ where K' is constant

when $I_L = 50 \text{ A} = I_{L_2}$

$$E_{b_2} = 400 - 50 \times 0.6 = 370 \text{ V.}$$

$\because$ $$\frac{E_{b_1}}{E_{b_2}} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{I_{a_1} N_1}{I_{b_2} N_2} = \frac{I_{L_1} N_1}{I_{L_2} N_2}$$

$\therefore$ $$\frac{E_{b_1}}{E_{b_2}} = \frac{60 \times 800}{50 \times N_2}$$ where N_2 is the speed when $I_L = 50$ A

$\therefore$ $$N_2 = \frac{60 \times 800 \times 370}{50 \times 364} \text{ rpm} = 976 \text{ rpm}$$

Let $T_{e_2} = K'(50)^2$ where T_{e2} is the torque when $I_L = 50$ A

So, percentage change in torque $$= \frac{T_{e_1} - T_{e_2}}{T_{e_1}} \times 100\% = \frac{(60)^2 - (50)^2}{(60)^2}$$

$$= \frac{3600 - 2500}{3600} \times 100\% = 30.56\%.$$

9.63 A 25 H.P., 240 V, 1000 rpm, 4-pole dc shunt motor has 1000 conductors arranged in 2 parallel paths. The armature circuit resistance is 0.3 Ω, field current and line current are 2 A and 100 A respectively. Find (i) flux per pole, (ii) torque developed, (iii) rotational losses, (iv) total losses expressing as a percentage of power.

Solution

Output power $P = 25$ H.P. $= 25 \times 735.5$ W $= 18387.5$ W

$V = 240$ V, $N = 1000$ rpm, $P = 4$, $Z = 1000$,

$A = 2$, $r_a = 0.3\ \Omega$, $I_{sh} = 2$ A, $I_L = 100$ A,

$I_a = I_L - I_{sh} = 100 - 2 = 98$ A

and $E_b = V - I_a r_a = 240 - 98 \times 0.3 = 210.6$ V

(i) Also $$E_b = \frac{P\phi ZN}{60\ A}$$ where ϕ = flux per pole

or $$\phi = \frac{60\ AE_b}{PZN} = \frac{60 \times 2 \times 210.6}{4 \times 1000 \times 1000} = 0.0063 \text{ Wb.}$$

(ii) $$T_e = \frac{P\phi ZI_a}{2\pi A} \text{ Nm} = \frac{4 \times 0.0063 \times 1000 \times 98}{2\pi \times 2} = 196.62 \text{ Nm.}$$

(iii) Power developed by the motor $(= E_b\ I_a) = 210.6 \times 98 = 20638.8$ W

Rotational losses $= E_b I_a$ – output power $= 20638.8 - 18387.5 = 2251.3$ W

(iv) Armature copper loss $(= I_a^2 r_a) = (98)^2 \times 0.3 = 2881.2$ W

Field copper loss $(= V\ I_{sh}) = 240 \times 2 = 480$ W

Total losses = (2881.2 + 480 + 2251.3)W = 5612.5 W

$\therefore \quad \%\text{ loss} = \dfrac{5612.5}{18387.5} \times 100\% = 30.52\%.$

9.64 A 220 V shunt motor takes 10.25 A on full load. The armature resistance is 0.8 Ω and the field resistance is 880 Ω. The losses due to friction, windage and the iron amount to 150 W. Find the output power and the efficiency of the motor on full load.

Solution

Motor input on full load = 220 × 10.25 = 2255 W

Field current = $\dfrac{220}{880}$ = 0.25 A

Armature current = 10.25 A – 0.25 A = 10 A

Armature copper loss = $(10)^2 \times 0.8$ = 80 W

Field copper loss = $(0.25)^2 \times 880$ = 55 W

Total loss = total copper loss + friction, windage and iron loss

= (80 + 55 + 150) = 285 W

Output power = Input – Loss = 2255 – 285 = 1970 W

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100\% = \frac{1970}{2255} \times 100\% = 87.36\%.$$

9.65 A 220 V shunt generator is rated to have a full load current of 200 A. The armature and field resistances are 0.06 Ω and 55 Ω respectively. The rotational losses are 3 kW. Find the input power of the generator and the load current for maximum efficiency.

Solution

Field current $I_{sh} = \dfrac{V}{r_{sh}} = \dfrac{220}{55} = 4$ A

Armature current $I_a = I_{fl} + I_{sh} = 200 + 4 = 204$ A

Armature copper loss ($I_a^2 \times r_a$) = $(204)^2 \times 0.06$ = 2496.96 W

Field copper loss = $I_{sh}^2 \times r_{sh} = (4)^2 \times 55$ = 880 W

Rotational loss = 3000 W

Constant loss = Field copper loss + Rotational losses = 880 + 3000 = 3880 W

Variable losses (= Armature copper loss) = 2496.96 W

Input power of generator = output power + total losses

= 220 × 200 + 3880 + 2496.96

= 50.377 kW

[∵ Total losses = Constant loss + Variable loss]

Condition for maximum efficiency is given by

Variable loss = Constant loss

If I_a be the armature current for maximum efficiency then

$I_a^2 \times 0.06 = 3880$ [∵ Variable loss is ($I_a^2 \times 0.06$) W and

or $I_a = 254.3$ A constant loss is 3880 W]

Hence load current for maximum efficiency is (254.3 – 4) or 250.3 A.

9.66 A dc shunt motor has an armature resistance of 0.9 Ω and takes an armature current of 18A from 230 V dc mains. Calculate the power output and overall efficiency of the motor if the rotational losses are measured to be 112 W and the shunt field resistance is 300 Ω.

Solution

Armature current = 18 A

Field current = $\dfrac{230}{300}$ = 0.767 A

Line current = 18.767 A

Armature copper loss = $(18)^2 \times 0.9$ = 291.6 W

Field copper loss = $(0.767)^2 \times 300$ = 176.48 W

Total losses = 291.6 + 176.48 + 112 = 580.08 W

Output power = Input power – Loss = 18.767 × 230 – 580.08 = 3736.33 W = 3.74 kW.

Overall efficiency = $\dfrac{\text{Output}}{\text{Input}} \times 100\% = \dfrac{3736.33}{18.767 \times 230} \times 100\% = 86.56\%.$

9.67 A 4 kW, 105 V, 1200 rpm shunt motor when running light at normal speed takes an armature current of 3 A at 102 V, nominal voltage being applied to the field winding. The field and armature resistance are 95 Ω and 0.1 Ω respectively. Calculate the output power and efficiency of the motor when operating at 105 V and taking a line current of 40 A. Allow 2 V drop at the brushes.

Solution

Given, line current I_L is 40 A

Field current $I_{sh} = \dfrac{105}{95}$ A = 1.1 A

$\therefore$ Armature current I_a = 40 – 1.1 = 38.9 A [$\because I_a = I_L - I_{sh}$]

Back emf on load $E_b = V - I_a r_a$ – Brush drop = 105 – 38.9 × 0.1 – 2 = 99.11 V

Hence, power developed by the armature = $E_b I_a$ = 99.11 × 38.9 = 3855.4 W

At light load,

Input to armature (= Total loss) = 102 × 3 = 306 W

$\therefore$ Armature copper loss + field copper loss + brush loss + no load rotational losses = 306 W

$$\therefore \text{ No load rotational losses} = 306 - \underbrace{(3)^2 \times 0.1}_{\substack{\text{no load}\\ \text{armature loss}}} - \overbrace{105 \times 1.1}^{\text{field loss}} - \underbrace{3 \times 2}_{\text{brush loss}} = 183.6 \text{ W}$$

At full load, output power = 105 × 40 – 183.6 – 38.9 × 2 – 105 × 1.1 – $(38.9)^2 \times 0.1$
= 3672 W.

[Here, Output power = input power – total losses in the machine.
Total losses = rotational losses + brush loss + field loss + armature loss
= 183.6 + 38.9 × 2 + 105 × 1.1 + $(38.9)^2 \times 0.1$]

$$\therefore \quad \text{Efficiency} = \frac{\text{Output}}{\text{Input}} \times 100 = \frac{3762}{105 \times 40} \times 100\% = 87.42\%.$$

9.68 Calculate the no load current taken by a 100 kW, 460 V shunt motor assuming the armature and field resistances to remain constant and equal to 0.03 Ω and 46 Ω respectively. The efficiency at full load is 88%.

Solution

Field current = $\dfrac{460}{46}$ A = 10 A

At light load,

$$\text{Input (= losses)} = I_a^2 R_a + \text{constant losses}$$

or $$460(I_a + 10) = I_a^2 \times 0.03 + \text{constant losses} \qquad \text{(i)}$$

At full load, Efficiency = $\dfrac{\text{Output}}{\text{Input}}$ = 0.88

$\therefore$ Input = $\dfrac{100 \times 10^3}{0.88}$ = 113.636 kW

Total loss = (input – output) = 113.636 – 100 = 13.636 W

$\therefore$ Input current = $\dfrac{\text{Input power}}{\text{Voltage}} = \dfrac{113.636 \times 10^3}{460}$ = 247 A

$\therefore$ Constant loss = Total loss – full load armature copper loss

= 13636 – $(247)^2 \times 0.03$ = 11805.73 W

This loss remains same at full load as well as at no load.

$\therefore$ from Eq. (i), $460\, I_a + 4600 = I_a^2 \times 0.03 + 11805.73$

or $I_a^2 - 15333.33\, I_a + 240190$

$$I_a = \frac{15333.33 \pm 15302}{2}$$

or I_a = 15.5 A (at no load)

$\therefore$ No load current = 15.5 + 10 = 25.5 A

9.69 A 220 V shunt motor when running light takes 7 A and runs at 1250 rev/min. The armature resistance is 0.1 Ω and the shunt field resistance is 110 Ω. A series winding of resistance 0.05 Ω is added, long shunt and cumulatively connected. This winding increases the flux per pole by 20% when the motor is taking its full load current of 62 A. Assuming the increase in flux proportional to the armature current and neglecting the effect of armature reaction find the speed of the motor (i) when running light (ii) when taking 62A.

Solution

Shunt field current = $\dfrac{220}{110}$ = 2A (= I_{sh})

At light load line curent = 7A (= I_{Lo})

So, armature current is (7 – 2) = 5A (= I_{ao})

$\therefore E_{bo} = 220 - 5 \times 0.1 = 219.5$ V (for shunt motor)

When series winding is added and full load line current is 62 A, the armature current is (62 – 2) = 60 A (= I_{afl}).

This 60 A, when flowing through series field, increases the flux by 20%.

Hence 5 A (at light load) when flowing through series field increases the flux by $\dfrac{5}{60} \times 20$ = 1.66%.

For compound motor $E_{b_0} = 220 - 5 \times 0.1 - 5 \times 0.05 = 219.25$ V

[$\because E_{bo} = V$ – drop in armature – drop in series resistance (for no load current)]

(i) If N_o be the speed of the motor when running light and operated as compound motor, we can write,

$$\frac{N_o}{1250} = \frac{219.25}{219.5}$$

or N_o = 1248 rpm.

(ii) When motor takes 62 A,

Armature current I_a = (62 – 2) = 60 A

$\phi = 1.2\, \phi_0$ where ϕ_0 is the flux of shunt motor when it is running light

Also, Back emf $E_{b_1} = 220 - 60 \times 0.1 - 60 \times .05 = 211$ V (at full load)

If N_1 be the speed at this time then

$$N_1 = 1250 \times \frac{211}{219.5} \times \frac{\phi_0}{1.2\phi_0} = 1000 \text{ rpm.}$$

9.70 A dc shunt machine has armature resistance, including brushes, 0.4 Ω and the field resistance is 160 Ω. The machine takes 5 A while running as a motor at 800 rpm at no load at 220 V. Calculate the speed and efficiency of the machine when taking 45 A at 220 V. Assume armature reaction to weaken the field by 3%.

Solution

Shunt field current $I_{sh} = \frac{220}{160}$ A = 1.375 A

At no load line current 5A (= I_{LO})

So, at no load armature current $I_{ao} = 5 - 1.375 = 3.625$ A.

For motor operation, back emf at no load (E_2) = $V - I_a r_a = 220 - 3.625 \times 0.4 = 218.55$ V.

No load rotational losses $P_{l(nl)} = 5 \times 220 - (3.625)^2 \times 0.4 - (1.375)^2 \times 160$
$= 792.75$ W [$\because P_{l(nl)}$ = No load input power – Armature no load copper loss – Shunt field copper loss]

On load, line current = 45 A (I_L)
Armature current $I_a = 45 - 1.375 = 43.625$ A
Input $VI_L = 220 \times 45 = 9900$ W

Total copper losses $P_{l(fl)} = (43.625)^2 \times 0.4 + (1.375)^2 \times 160$
$= 761.256 + 302.5 = 1063.756$ W.
[$\because P_{l(fl)}$ = Armature copper loss + Field copper loss]

Total loss on load = Total copper loss + No load rotational loss
$= 1063.756 + 792.75 = 1856.506$ W

$$\text{Efficiency} = \left(1 - \frac{\text{Loss}}{\text{Input}}\right) \times \left(1 - \frac{1856.506}{9900}\right) \times 100\% = 81.23\%.$$

Back emf on full load $(E_2') = (220 - 43.625 \times 0.4) = 202.55$ V

If N be the speed at full load with constant flux,

$$\frac{N}{N_o} = \frac{202.55}{218.55} \quad \left[\because \frac{N}{N_o} = \frac{E_2'}{E_2}\right]$$

or
$$N = \frac{202.55}{218.55} \times 800 = 741 \text{ rpm.}$$

If the flux is weaken by 3% the new speed is $\frac{741}{0.97}$ rpm = 764 rpm.

9.71 A dc shunt motor running at 1200 rpm has armature resistance of 0.15 Ω. The current taken by the armature is 60 A when the applied voltage is 220 V. If the load is increased by 30% find the variation in the speed.

Solution

Given $r_a = 0.15$ Ω

When speed $N_1 = 1200$ rpm the back emf is

$$E_{b_1} = 220 - 60 \times 0.15 = 211 \text{ V.}$$

Let the output power be P_1.

If the load is increased by 30% the output power $P_2 = 1.3\ P_1$ and let the back emf and speed be E_{b_2} and N_2 respectively

We have, $E_{b_2} I_{a_2} = 1.3\, E_{b_1} I_{a_1} = 1.3 \times 211 \times 60 = 16458$ W

$$\text{Now} \quad E_{b_2} = 220 - I_{a2} \times 0.15 = 220 - 0.15 \times \frac{16458}{E_{b_2}}$$

$$\text{or} \quad E_{b2}^2 = 220\, E_{b_2} - 2468.7 \text{ or } E_{b2}^2 - 220\, E_{b_2} + 2468.7 = 0$$

$$\text{or} \quad E_{b_2} = \frac{220 \pm \sqrt{48400 - 9874.8}}{2} = \frac{220 \pm 196}{2}$$

$$\text{or} \quad E_{b_2} = 208.14 \text{ V.}$$

$$\text{Speed} = \frac{208.14}{211} \times 1200 = 1184 \text{ rpm.}$$

9.72 A ventilating fan is driven by a 220 V, 10 kW series motor and runs at 800 rpm at full load. The total resistance of the armature circuit is 0.6 Ω. Calculate the speed and percentage change in torque if the current taken by the motor is reduced by 50% of the full load value. The efficiency of the motor is 82%. Assume the flux to be proportional to the field current.

Solution

$$\text{Output} = 10{,}000 \text{ W}$$

$$\therefore \quad \text{Input} = \frac{10{,}000}{0.82} \ (\because \text{ efficiency} = 0.82) = 12195.12 \text{ W.}$$

$$\text{At full load current} \quad I_1 = \frac{\text{Input}}{\text{Voltage}} = \frac{12195.12}{220} = 55.4 \text{ A}$$

and speed $N_1 = 800$ rpm.

Let the flux be ϕ_1 at this speed and load current.

Back emf $(E_{b_1}) = 220 - 55.4 \times 0.6 = 186.76$ V $[\because\ E_{b_1} = V - I_1 r_a]$

When Current $I_2 = 50\%$ of I_1 i.e $I_2 = 27.7$ A,

$$E_{b_2} = 220 - 27.7 \times 0.6 = 203.35 \text{ V.}$$

Let the flux be ϕ_2 at this input current.

If N_2 be the new speed then, $\dfrac{\phi_2 N_2}{\phi_1 N_1} = \dfrac{E_{b_2}}{E_{b_1}}$

$$\text{So,} \quad N_2 = \frac{E_{b_2}}{E_{b_1}} \frac{\phi_1}{\phi_2} N_1 = \frac{E_{b_2}}{E_{b_1}} \frac{I_1}{I_2} N_1 \quad (\because\ \phi \propto I \text{ in the series motor})$$

$$\text{or} \quad N_2 = \frac{203.35}{186.70} \times 2 \times 800 = 1742 \text{ rpm.}$$

If T_1 and T_2 be the torque at full load and 50% of the full load then

$$\frac{T_1}{T_2} = \left(\frac{I_1}{I_2}\right)^2 = \left(\frac{2}{1}\right)^2 = 4.$$

Hence percentage change in torque ($\Delta\, T\%$) is $\dfrac{T_1 - T_2}{T_1} \times 100$

$$\text{or} \quad \Delta T = \frac{T_1 - \frac{T_1}{4}}{T_1} \times 100\% = \frac{3}{4} \times 100\% = 75\%.$$

9.73 An engine room ventilator fan series motor has a total resistance of 0.5 Ω and runs from a 110 V supply at 1000 rpm when current is 28 A. What resistance in series with the motor will reduce the speed to 750 rpm.? The load torque is proportional to the square of the speed and the field strength can be assumed to be proportional to the current.

Solution

When current I_1 = 28 A, speed N_1 = 1000 rpm.

Given, torque $T_1 \propto N_1^2$. If ϕ_1 be the flux then $\phi_1 \propto I_1$

Now for series motor $T \propto I_1^2$

$$\therefore \quad N_1^2 \propto I_1^2 \text{ or } N_1 \propto I_1$$

$$E_{b_1} = 110 - 28 \times 0.5 = 96 \text{ V}$$

Let R be the resistance to be added to reduce speed to 750 rpm, i.e. N_2 = 750 rpm and let flux be ϕ_2.

$$\therefore \quad E_{b_2} = 110 - I_2(R + 0.5),$$

where I_2 is the current at speed 750 rpm

Also, $N_2 \propto I_2$

$$\therefore \quad \frac{I_2}{I_1} = \frac{N_2}{N_1} \quad \text{or,} \quad I_2 = \frac{750}{1000} \times 28 = 21 \text{ A.}$$

Also, $$E_{b_2} = 110 - 21(R + 0.5) \qquad \text{(i)}$$

$$\frac{E_{b_2}}{E_{b_1}} = \frac{N_2}{N_1}\frac{\phi_2}{\phi_1} = \frac{N_2}{N_1}\frac{I_2}{I_1};$$

$$E_{b_2} = \frac{N_2 \times I_2}{N_1 \times I_1} \times E_{b_1} = \frac{750 \times 21}{1000 \times 28} \times 96$$

From Eq. (i) we have

$$110 - 21R - 10.5 = \frac{750}{1000} \times \frac{21}{28} \times 96 = 54$$

or $$R = 2.17 \text{ Ω}.$$

9.74 A 230 V, 10 kW shunt motor with a stated full load efficiency of 85% runs at a speed of 1000 rpm. At what speed should the motor be driven if it is used as a generator to supply an emergency lighting load at 230 V? The armature resistance is 0.2 Ω and the field resistance is 115 Ω. Find the kW rating of the machine under this condition. Assume that the line current is same in both the cases.

Solution

$$\text{Output} = 10 \text{ kW} = 10{,}000 \text{ W}$$

$$\text{Input} = \frac{10000}{0.85} = 11764.7 \text{ W}$$

$$\text{Shunt field current} = \frac{230}{115} \text{ A} = 2\text{A } (= I_{sh}).$$

$$\text{Full load line current} = \frac{11764.7}{230} \text{ A} = 51.15 \text{ A } (= I_{fl}).$$

When the machine runs as a motor

Armature current $[(I_{a(m)}) = I_{fl} - I_{sh})] = 51.15 - 2 = 49.15$ A

Back emf $[(E_{b(m)}) = V - I_{a(m)} \times r_a] = 230 - 49.15 \times 0.2 = 220.17$ V

When used as a generator,

Armature current $[I_{a(g)} = I_{fl} + I_{sh}] = 51.15 + 2 = 53.15$ A

Generated emf $[E_{b(g)} = V + I_{a(g)} \times r_a] = 230 + 53.15 \times 0.2 = 240.63$ V.

If N_g be the speed of the generator then

$$N_g = 1000 \times \frac{240.63}{220.17} = 1093 \text{ rpm} \left[\because \frac{N_g}{N_m} = \frac{E_{b(g)}}{E_{b(m)}}\right]$$

Rating of the machine $(V \times I_{fl} \times 10^{-3}) = \dfrac{230 \times 51.15}{1000}$ kW = 11.76 kW.

9.75 A series dc motor is run on a 220 V circuit with a regulating resistance of R Ω for speed adjustment. The armature and field coils have a total resistance of 0.3 Ω. On a certain load with R being zero, the current is 20 A and the speed is 1200 rpm. With another load and R set at 3 Ω the current is 15 A. Find the new speed and also the ratio of the two values of the power output of the motor. Assume the field strength at 15 A to be 80% of that at 20 A.

Solution

With $R = 0\ \Omega$

Line current $I_1 = 20$ A, $N_1 = 1200$ rpm

Back emf $(E_{b_1}) = 220 - 20 \times 0.3 = 214$ V.

With $R = 3\ \Omega$

$I_2 = 15$ A.

Hence, Back emf $E_{b_2} = 220 - 15 \times 3.3 = 170.5$ V

The new speed $N_2 = 1200 \times \dfrac{170.5}{214 \times 0.8} = 1195$ rpm

$\because$ Power output ∞ torque × speed i.e. Power output ∞ ϕIN

$\therefore$ Ratio of power outputs = $(\phi I_1 N_1 / 0.8 \phi I_2 N_2)$

or, ratio of the two values of output is = $\dfrac{20 \times 1200}{15 \times 1195 \times 0.8} = 1.67$.

EXERCISES

1. Draw a neat sketch of a dc machine showing the different parts. State the function of each part.
2. Derive the emf equation of a dc generator.
3. What are the different types of dc generators according to the ways in which fields are excited. Show the connection diagram of each type.
4. Distinguish between
   (i) self-excited and separately excited dc machines
   (ii) lap connected and wave connected dc machines
   (iii) cumulatively wound and differentially wound dc machines.
   (iv) long shunt and short shunt dc machines.
5. What is armature reaction. Describe the effects of armature reaction on the operation of dc machines. How is the armature reaction minimised?
6. What is commutation in a dc machine? Describe the various methods of improving commutation.

7. Describe the process of voltage build up in a self-excited dc machine. What are the conditions for voltage build up in a dc machine?
8. Draw the external characteristics of various types of dc generators.
9. What is meant by back emf? Explain the principle of torque production in a dc motor.
10. Derive the equation of torque for a dc motor.
11. Describe in details the methods of speed control of dc shunt motor.
12. What is the necessity of a starter in a dc motor? Explain with the help of a neat sketch the principle of operation of a three-point starter. What are the functions of no volt and overload release coils?
13. Draw and explain the speed current, torque current and speed torque characteristics of (i) a dc shunt motor (ii) a dc series motor.
14. What are the losses that occurs in dc machines? Derive the condition for maximum efficiency in a dc generator.
15. A 4-pole dc generator has 51 slots and each slot contains 20 conductors. The machine has a useful flux of 0.007 Wb and runs at 1500 rpm. Find the induced emf if the machine is wave wound. [*Ans.* 357 V]
16. A dc generator has an armature emf of 100 V when the useful flux per pole is 20 m Wb and the speed is 800 rpm. Calculate the generated emf with the same flux and a speed of 1000 rpm. [*Ans.* 125 V]
17. A short shunt compound dc generator delivers 100 A to a load at 250 V. The generator has shunt field, series field and armature resistance of 130 Ω, 0.1 Ω and 0.1 Ω respectively. Calculate the generated voltage. Assume 1 V drop per brush. [*Ans.* 272.2 V]
18. The armature of an 8-pole dc machine has a wave winding containing 664 conductors. Calculate the generated emf when the flux per pole is 0.08 Wb and the speed is 210 rpm. At what speed should the armature be driven to generate 500 V, if the flux per pole is made 0.06 Wb?
[*Ans.* 743.7 V, 188 rpm]
19. A 4-pole, 220 V, dc shunt generator has an armature resistance of 1 Ω, shunt field resistance of 220 Ω. The generator supplies power to a 10 Ω resistor. Calculate the generated emf of the generator if the load voltage is to be maintained at 220 V. Assume brush contact drop = 2 V.
[*Ans.* 245 V]

[*Hint:* Generated emf, $E = 220 + I_a r_a + 2$

$$I_L = \frac{220}{10}\text{ A} = 22\text{ A}$$

$$I_{sh} = \frac{220}{220}\text{ A} = 1\text{ A}$$

$$I_a = 22 + 1 = 23\text{ A}$$

$$\therefore \quad E = 220 + 23 \times 1 + 2 = 245\text{ V}$$]

20. A 4 pole, 220 V, dc shunt motor has armature and shunt field resistances of 0.2 Ω and 220 Ω respectively. It takes 20 A at 220 V from a source while

running at a speed of 1000 rpm. Find the (i) field current, (ii) armature current (iii) back emf (iv) torque developed.

[*Ans.* 1 A; 19 A; 216.2 V; 39.25 Nm]

[*Hint:* (i) Field current $I_{sh} = \frac{220}{220}$ A = 1 A

(ii) Armature current $I_a = 20 - 1 = 19$ A

(iii) Back emf $E_b = 220 - 19 \times 0.2 = 216.2$ V

(iv) Torque $T = \frac{E_b I_a}{\omega} = \frac{216.2 \times 19}{\frac{2\pi \times 1000}{60}}$ Nm = 39.246 Nm]

21. A 400 V shunt generator has a full load current of 200 A, its armature resistance is 0.06 Ω and field resistance 100 Ω, the stray losses are 2000 Watts. Find the input H.P. of the generator. [*Ans.* 117.06 HP]
22. A 100 kW, 460 V shunt generator was run as a motor on no load at its rated voltage and speed. The total current drawn was 9.8 A including shunt current of 2.7 A. The resistance of the armature circuit was 0.11 Ω. Calculate the efficiencies (i) at full load and (ii) at half load.

[*Ans.* 91.1% and 89.5%]
23. A 220 V, dc shunt motor at no load takes a current of 2.5 A. The resistance of the armature and shunt field are 0.8 Ω and 200 Ω respectively. Estimate the efficiency of the motor when the input current is 20 A. [*Ans.* 81.4%]
24. A 250 V, 20 HP shunt motor has a maximum efficiency of 88% and a speed of 700 rpm when delivering 80% of its rated output. The resistance of its shunt field is 100 Ω. Determine the efficiency and speed when the motor draws a current of 78 A from the mains. [*Ans.* 86.9% and 680 rpm]
25. A shunt motor runs at 500 rpm on a 200 V circuit. Its armature resistance is 0.5 Ω and the current taken is 30 A, in addition to field current. What resistance must be placed in series in order that the speed may be reduced to 300 rpm, the current in the armature remaining the same? [*Ans.* 2.466 Ω]
26. A 220 V shunt motor has an armature resistance of 0.5 Ω and takes a current of 40 A on full load. By how much must the main flux be reduced to raise the speed by 50% if the developed torque is constant? [*Ans.* 37.5%]
27. A 120 V dc shunt motor having an armature circuit resistance of 0.2 Ω and field circuit resistance of 60 Ω, draws a line current of 40 A at full load. The brush voltage drop is 3 V and rated full load speed is 1800 rpm. Calculate (i) the speed at half load (ii) the speed at 125% of full load.

[*Ans.* 1864 rpm, 1768 rpm]

[*Hint:* $I_L = 40$ A

$$I_{sh} = \frac{120}{60} \text{ A} = 2 \text{ A} \quad \therefore I_a = 40 - 2 = 38 \text{ A}$$

At full load $E_{b_1} = 120 - 38 \times 0.2 = 112.4$ V

(i) At half load $I_L = \frac{1}{2} \times 40$ A = 20 A and $I_a = 18$ A

$E_{b_2} = 120 - 18 \times 0.2 = 116.4$ V

$$\text{Speed } N_2 = N_1 \frac{E_{b_2}}{E_{b_1}} = 1800 \times \frac{116.4}{112.4} = 1864 \text{ rpm.}$$

(ii) $I_L = 1.25 \times 40 = 50$ A and $I_a = 50 - 2 = 48$ A

$$E_{b_3} = 120 - 48 \times 0.2 = 110.4 \text{ V}$$

$$N_3 = 1800 \times \frac{110.4}{112.4} = 1768 \text{ rpm} \Bigg].$$

28. A 4-pole 240 V dc shunt motor has armature and shunt field resistance of 0.24 Ω and 240 Ω respectively. It takes 20 A from a 240 V dc supply while running at a speed of 1000 rpm. Find the (i) field current, (ii) armature current, (iii) back emf and (iv) torque developed in Nm.

[*Ans*. 1 A; 19 A; 235.44 V; 42.74 Nm]

$\Bigg[$ *Hint:* (i) Field current, $I_{sh} = \frac{240}{240}$ A $= 1$ A

(ii) Armature current, $I_a = 20 - 1 = 19$ A

(iii) Back emf, $E_b = 240 - 19 \times 0.24 = 235.44$ V

(iv) Torque $T = \dfrac{E_b I_a}{\omega} = \dfrac{235.44 \times 19}{2\pi \times \dfrac{1000}{60}}$ Nm $= 42.74$ Nm $\Bigg]$

29. A 220 V separately excited dc machine has an armature resistance of 0.4 Ω. If the load current is 20 A, find the induced emf when the machine operates (i) as a generator (ii) as a motor. [*Ans*. 228 V; 212 V]

[*Hint:* (i) $E = 220 + 20 \times 0.4 = 228$ V

(i) $E = 220 - 20 \times 0.4 = 212$ V]

30. The armature resistance of a 220 V dc shunt motor is 0.4 Ω and it takes a no load armature current of 2 A and runs at 1350 rpm. Find its speed when taking on armature current of 50 A if armature reaction weakens the flux by 2%. [*Ans*. 1257 rpm]

31. The input to a 220 V, dc shunt motor is 11 kW. Calculate (i) the torque developed and (ii) the speed at this load when the particulars of the motor are given as:

No load current = 5 A

No load speed = 1150 rpm

Armature resistance = 0.5 Ω

Shunt field resistance = 110 Ω [*Ans*. 87.1 Nm, 1031 rpm]

32. The full load current in the armature of a shunt motor is 100 A, the line voltage being 400 V, the resistance of the armature circuit is 0.2 Ω, and the speed 600 rpm. What will be the speed if the total torque on the motor is reduced to 60% of the full load value and a resistance of 2 Ω is included in the armature circuit, the field strength remaining unaltered? [*Ans*. 423 rpm]

33. A dc shunt motor runs at 1500 rpm and takes an input of 880 W at 220 V under normal conditions. The shunt field current is 2 A and armature resistance is 0.1 Ω. Find the efficiency when the machine is used as a generator supplying 60 A at 220 V. [*Ans*. 91.26%]

34. A 45 kW, 225 V dc shunt generator runs at 500 r.p.m at full load. The field and armature resistance are 45 Ω and 0.03 Ω respectively. Calculate the speed of the machine when running as a shunt motor and taking 45 kW input at 225 V. Assume brush contact drop of 1 V per brush.

[*Ans.* 465.69 rpm]

35. Find no load and full load speeds of a 220 V, 4 pole shunt motor having following data:

Flux 0.04 m Wb, armature resistance 0.04 Ω, 160 armature conductors, wave connection, full load line current 95 A, no load line current 9 A, field resistance 44 Ω. [*Ans.* 1030.5 rpm, 1014.4 rpm]

10 THREE-PHASE INDUCTION MOTORS

10.1 INTRODUCTION

The whole concept of a polyphase ac, including the *induction motor*, was the idea of the great Yugoslavian engineer, Nikola Tesla.

The induction motor is, by a very considerable margin, the most widely used ac motor in industry. Induction motors normally require no electrical connection to the rotor windings. Instead, the rotor windings are short-circuited. Magnetic flux flowing across the air-gap links these closed rotor circuits. As the rotor moves relative to the air-gap flux, voltages are induced in the short-circuited rotor windings according to Faradays' law of electromegnetic induction causing currents to flow in them. The fact that the rotor current arises from induction, rather than conduction, is the basis for the name of this class of machines. They are also called "asynchronous" (i.e. not synchronous) machines because their operating speed is slightly less than synchronous speed in the motor mode and slightly greater than synchronous speed in the generator mode. Induction machines are usually operated in the motor mode, so they are usually called "induction motors."

Because of its simplicity and ruggedness, relatively less expensive and little maintenance, this motor is often the natural choice, as a drive in industry. The squirrel cage motor is often preferred over when a substantially constant speed of operation is desired, the wound rotor motor is a competitor of the dc motor when adjustable speed is required.

The chief disadvantages of induction motors are:

(a) The starting current may be five to eight times full-load current if direct on line start is allowed.

(b) The speed is not easily controlled.

(c) The power factor is low and also lagging when the machine is lightly loaded.

For most applications, their advantages far outweigh their disadvantages.

10.2 CONSTRUCTION OF INDUCTION MACHINES

Similar to other rotating electrical machines, a three-phase induction motor also consists of two main parts: the *stator* and the *rotor* (the stator is the stationary part and the rotor is the rotating part). Apart from these two main parts, a three-phase induction motor also requires bearings, bearing covers, end plates, etc. for its assembly.

The stator of a three-phase induction motor has three main parts namely, stator frame, stator core and stator windings. The stator frame can either be casted or can be fabricated from rolled steel plates. The stator core is built up of high silicon sheet steel laminations of thickness 0.4 to 0.5 mm. Each lamination is separated from the other by means of either varnish, paper or oxide coating. Each lamination is slotted on the inner periphery so as to house the winding. The laminations for small machines are in the form of complete rings, but for large machines these may be made in sections. The insulated stator conductors are connected to form a three-phase winding, the stator phase windings may be either *star* or *delta-connected*.

The rotor is also built up of their laminations of the same material as the stator. The laminated cylindrical core is mounted directly on the shaft or a spider carried by the shaft. These laminations are slotted on their outer periphery to house the rotor conductors. There are two types of induction motor rotors:

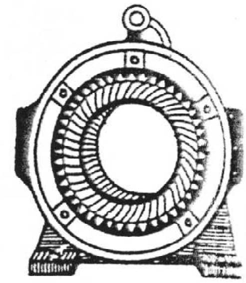
(a) Squirrel cage or simply cage rotor
(b) Phase wound or wound rotor or slip ring rotors.

In either case, the rotor windings are contained in slots in a laminated iron core which is mounted on the shaft. In small machines, the lamination stack is pressed directly on the shaft. In larger machines, the core is mechanically connected to the shaft through a set of *spokes* called a "spider".

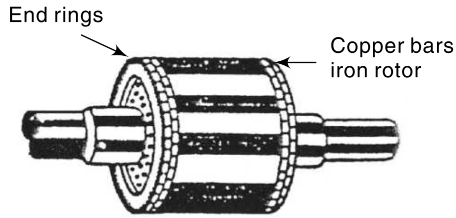
The motor having the first type of rotor is known as a squirrel cage induction motor. This type of rotor is cheap and has a simple and rugged construction. It is cylindrical in shape and is made of sheet steel laminations. Here the slots provided to accommodate the rotor conductors, are not made parallel to the shaft but they are *skewed*. The purpose of skewing is (a) to reduce the magnetic hum and (b) to reduce the magnetic locking. The rotor conductors are short-circuited at the ends by brazing the copper rings, resembling the cage of a squirrel and hence the name squirrel cage rotor.

In present days, 'die-cast rotors' have become very popular. The assembled rotor laminations are placed in a mould. The molten aluminium is forced under pressure to form the bars. Figure 10.1 (a-c) shows a typical stator and rotor (both squirrel cage type and slip ring type) assembly. Figure 10.1(d) shows the schematic of a cage rotor separately.

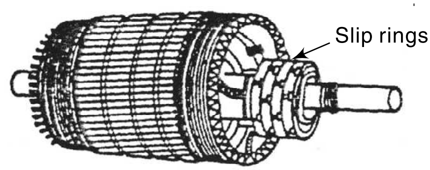
The motor having the second type rotor, i.e. wound type rotor, is named as a slip-ring induction motor. In this motor, the rotor is wound for three-phase, similar to stator winding using open type slots in the rotor lamination. Rotor winding is always star connected and thus only three remaining ends of the windings are brought out and connected to the slip rings as shown in Fig. 10.2. With the help of these slip rings and brushes, additional resistances can also be connected in



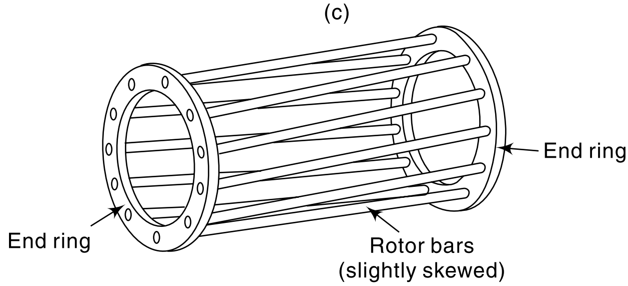
Stator winding
(a)



Squirrel cage rotor
(b)



Slip ring rotor
(c)



Schematic of a cage rotor
(d)

Fig. 10.1 ***Stator and rotor parts***

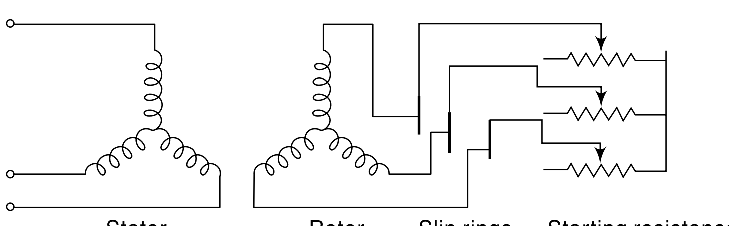


Fig. 10.2 ***Addition of external resistances to the rotor of wound rotor induction motor***

series with each rotor phase (Fig. 10.2). This will increase the starting torque provided by the motor and will also help in reducing, the starting current. When running under normal condition, the external resistances are removed completely from the rotor by short circuiting these additional resistances from the rotor circuit and rotor behaves just like a squirrel cage rotor.

10.3 COMPARISON OF SQUIRREL CAGE AND WOUND ROTORS

The advantages of cage rotor induction motor are as follows:

(a) A rotor is of robust construction and cheaper.
(b) The absence of brushes reduces the risk of sparking.
(c) Squirrel cage rotors require lesser maintenance.
(d) Squirrel cage induction motors have higher efficiency and better power factor.

On the other hand, wound rotors have the following merits:

(a) High starting torque and low starting current.
(b) Additional resistance can be connected in the rotor circuit to control speed.

10.4 ADVANTAGES AND DISADVANTAGES OF A THREE-PHASE INDUCTION MOTOR

Advantages

(a) It is very simple, robust, rugged and capable of withstanding rough use.
(b) It is quite cheap in cost and reliable in operation.
(c) Its maintenance cost is low.
(d) The losses are reasonably small and hence it has sufficiently high efficiency.
(e) It is mostly a trouble-free motor.
(f) Its power factor is reasonably good at full load operation.
(g) It is simple to start (since it has a self starting torque).

An induction motor is equivalent to a static transformer whose secondary is capable of rotating with respect to the primary.

Usually the stator is treated as the primary, while the rotor is treated as the secondary. The induction motor operation is electrically equal even if the rotor is primary and the stator operation is treated as secondary.

Disadvantages

(a) Its speed cannot be varied without sacrificing efficiency.
(b) Its speed decreases with an increase in load.
(c) Its starting torque is inferior to that of a dc shunt motor.
(d) For direct on line starting, the starting current is usually 5 to 8 times of the full-load rated current.
(e) It runs at a low lagging power factor when it is lightly loaded.

10.5 PRINCIPLE OF OPERATION

A three-phase induction motor has a stator winding which is supplied by three-phase alternating balanced voltage and has balanced three-phase currents in the winding. The rotor is not excited from any source and has only magnetic coupling with the stator. Under normal running conditions, the rotor winding (cage or slip-ring) is always short circuited to allow induced currents to flow in the rotor winding. The flow of three-phase currents in the stator winding produces a rotating magnetic field of constant amplitude and rotates at a synchronous speed. Let us assume that the rotor is at standstill initially; the rotating stator field induces an emf in the rotor conductor by transformer action. Since the rotor circuit is a closed set of conductors, a current flows in the rotor circuit. This rotor current then produces a rotor field. The interaction of stator and rotor field produces a torque which causes the rotation of the rotor in the direction of the stator rotating field.

As per Lenz's law, the rotor field will try to oppose the very cause of its production. Thus it speeds up in the direction of the stator field so that relative speed difference between these two fields is zero. In this way, the three-phase induction motor catches up the speed.

When the rotor is at standstill, the relative motion between the stator field and rotor is maximum. Therefore, the emf induced in the rotor and rotor current are reduced. However, the rotor cannot attain the speed of the stator field which is equal to the synchronous speed. This is evidently due to the reason that if the rotor is moving at synchronous speed, there is no relative motion between the stator field and the rotor. Hence the rotor induced emf and current become zero and the torque becomes zero. This would cause the rotor speed to decrease. As the rotor speed falls below the synchronous speed, the rotor emf and current continue to increase. Therefore, the electromagnetic torque continues to increase.

Finally, the rotor speed becomes constant at a value at speed slightly less than that of the stator field, the torque developed equals the sum of load torque and the mechanical losses.

10.6 CONCEPT OF PRODUCTION OF ROTATING FIELD

When a three-phase winding, *displaced in space* by 120°, are supplied by a three-phase currents *displaced in time* by 120°, a magnetic flux is produced which rotates in space. This causes the rotor to rotate. The method of analysis is as follows:

10.6.1 Analytical Method

Let us consider three identical coils placed 120° apart with respect to each other, as shown in Fig. 10.3(a).

The coils are supplied with currents having frequency of supply and varying sinusoidally in time. Each coil will produce an alternating flux along its own axis. Let the instantaneous flux be given by

$$\phi_1 = \phi_m \sin \omega t \qquad 10.1(a)$$

$$\phi_2 = \phi_m \sin (\omega t - 120°) \qquad 10.1(b)$$

$$\phi_3 = \phi_m \sin (\omega t - 240°) \qquad 10.1(c)$$

The resultant flux produced by this system may be determined by resolving the components with respect to the physical axis, as shown in Fig. 10.3(b).

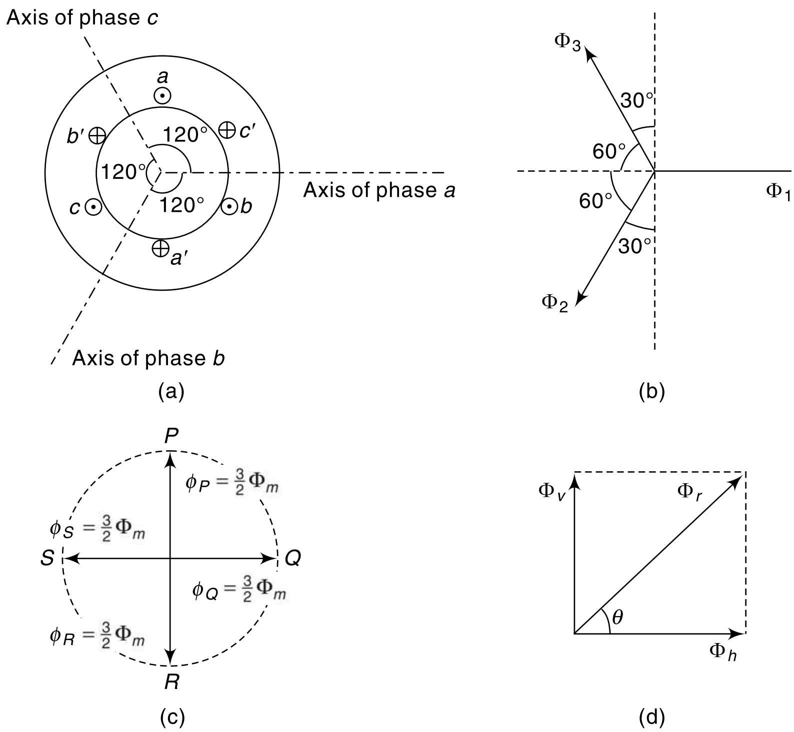


Fig. 10.3 ***Production of Rotating Field in a three-phase induction motor***

Therefore the resultant horizontal component of flux is given by

$$\begin{aligned}\phi_h &= \phi_1 - \phi_2 \cos 60° - \phi_3 \cos 60°\\ &= \phi_1 - (\phi_2 + \phi_3) \cos 60°\\ &= \phi_1 - (\phi_2 + \phi_3) \times \frac{1}{2}\\ &= \phi_m \sin \omega t - \frac{1}{2}[\phi_m \sin (\omega t - 120°) + \phi_m \sin (\omega t - 240°)]\\ &= \phi_m \sin \omega t - \frac{\phi_m}{2} \times (2 \sin \omega t)\left(-\frac{1}{2}\right)\\ &= \frac{3}{2}\phi_m \sin \omega t. \qquad (10.2)\end{aligned}$$

Similarly, the vertical component of flux is given by

$$\begin{aligned}\phi_v &= 0 - \phi_2 \cos 30° + \phi_3 \cos 30° = \frac{\sqrt{3}}{2}[\phi_3 - \phi_2]\\ &= \frac{\sqrt{3}}{2}[\phi_m \sin (\omega t - 240°) - \phi_m \sin (\omega t - 120°)]\\ &= \frac{\sqrt{3}}{2} \times \phi_m \cdot 2 \cos \omega t \times \frac{\sqrt{3}}{2}\\ &= \frac{3}{2}\phi_m \cos \omega t \qquad (10.3)\end{aligned}$$

∴ The resultant flux is (Fig. 10.3(d)),

$$\phi_r = \sqrt{(\phi_h)^2 + (\phi_v)^2} = \frac{3}{2}\phi_m \sqrt{\sin^2 \omega t + \cos^2 \omega t}$$

$$= \frac{3}{2}\phi_m \quad [\because \sin^2 \omega t + \cos^2 \omega t = 1] \tag{10.4}$$

and $\tan \theta = \dfrac{\phi_v}{\phi_h} = \cot \omega t = \tan (90° - \omega t)$.

It implies $\theta = (90° - \omega t)$. (10.5)

The above equation shows that the resultant flux (ϕ_r) is free from time factor. It is a constant flux of magnitude equal to $\left(\dfrac{3}{2}\right)$ times the maximum flux per phase. However, θ is dependent on time and we can calculate θ at different values of (ωt); when (ωt) = 0, $\theta = \pi/2$ corresponding to position P in Fig. 10.3(c).

Similarly, for $\omega t = \pi/2$, $\theta = 0°$, corresponding to position Q,
when $\omega t = \pi$, $\theta = -\pi/2$, corresponding to position R,
when $\omega t = \dfrac{3\pi}{2}$, $\theta = -\pi$, corresponding to position S.

It is thus observed that the resultant flux ϕ_r rotates in space in the clockwise direction with angular velocity of ω radians per second.

Since $\omega = 2\pi f$ and $f = \dfrac{PN_s}{120}$, the resultant flux ϕ_r rotates with synchronous speed (N_s).

10.7 THE CONCEPT OF SLIP

The magnitude and frequency of the rotor voltages depend on the speed of the relative motion between the rotor and the flux crossing the air gap. The difference between the synchronous speed and the rotor speed expressed as a fraction (or percent) of synchronous speed is knows as *slip*, i.e.

Slip speed = $(n_s - n)$ rev/sec

and slip (s) = $\dfrac{n_s - n}{n_s}$ p.u. 10.6(a)

or $n = n_s (1 - s)$ rps 10.6(b)

where n_s = synchronous speed (rev/sec)
n = rotor speed (rev/sec)
s = slip.

When the speed is expressed in rpm, we can write

$$s = \frac{N_s - N}{N_s} \text{ p.u.} = \frac{N_s - N}{N_s} \times 100 \text{ (in \%)}$$

and $N = N_s(1 - s)$ rpm.

This slip s is a very useful quantity in studying induction motors.

The value of slip at full load is about 4 to 5% for small motors and about 2 to 2.5% for large motors. The slip at no load is about 1%. Thus the speed of an induction motor is almost constant from no load to full load. If the machine has P number of poles, the frequency of induced emf in the rotor, i.e. f_2 is given by

$$f_2 = \frac{N_s - N}{N_s} \times f_1 \quad \left[\because\ f_1 = \frac{PN_s}{120};\ f_2 = \frac{P(N_s - N)}{120}\right.$$

$$\left.\text{and hence } (f_2/f_1) = \left(\frac{N_s - N}{N_s}\right)\right]$$

i.e. $\quad f_2 = sf_1$

At standstill of the rotor, $s = 1$, i.e. the frequency of rotor currents is f_1 (the same as the supply frequency).

10.8 FREQUENCY OF ROTOR VOLTAGES AND CURRENTS

Let us consider a typical pair of rotor bars. As the rotor "slips" backward through the flux field, the flux linking these bars will vary cyclically. The voltage induced in the rotor circuit is composed of the voltages in these two bars and the end rings. It is at its peak at the instant when the rate of change of flux linkages is a maximum. Thus one cycle of rotor voltage is generated as a given conductor slips past two poles of the air-gap flux field. In other words, one cycle of rotor voltage corresponds to 360 electrical degrees of "slips". Then the frequency of the rotor voltages and currents is given by

$$f_2 = \text{pole-pairs slipped per second}$$

$$\frac{(n_s - n)}{n_s} \cdot n_s \cdot \frac{P}{2} = s \cdot f_1 \left[\because f_1 = \frac{PN_s}{120} = \frac{PN_s}{2 \times 60} = \frac{Pn_s}{2}\right]. \tag{10.7}$$

i.e., Rotor current frequency = Per unit slip × Supply frequency.

At standstill, rotor speed is zero.

$$\therefore \qquad s = \frac{(n_s - n)}{n_s} = \frac{n_s - 0}{n_s} = 1$$

and $\qquad f_2 = f_1. \qquad\qquad (10.8)$

10.1 A three-phase, 4-pole 50 Hz. induction motor runs at 1450 rpm. Find out the percentage slip of the induction motor.

Solution

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$\therefore \qquad \text{Slip} = \frac{N_s - N}{N_s} = \frac{1500 - 1450}{1500} = 0.033 = 3.33\%.$$

10.2 A three-phase, 50 Hz., 6-pole induction motor runs at 950 rpm. Calculate

(i) the synchronous speed

(ii) the slip and

(iii) frequency of the rotor emf.

Solution

(i) We know, $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{6}$ = 1000 rpm.

(ii) Slip (s) = $\dfrac{N_s - N}{N_s} = \dfrac{1000 - 950}{1000}$ = 0.05.

$\therefore$ percentage of slip = 0.05 × 100 = 5.

(iii) The frequency of rotor emf = $s.f_1$ = 0.05 × 50 = 2.5 Hz.

10.3 The frequency of the emf in the stator of a 4-pole induction motor is 50 Hz., and that in the rotor is 2 Hz. What is the slip and at what speed is the motor running?

Solution

We know $f_2 = s \cdot f_1$

$$\therefore \quad s = \frac{f_2}{f_1} = \frac{2}{50} = 0.04 = 4\%.$$

Again $$f_1 = \frac{P \cdot N_s}{120}$$

$$\therefore \quad N_s = \frac{120 \cdot f_1}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

Speed of the motor

$$N = (1 - s) \cdot N_s = (1 - 0.04) \times 1500 = 1440 \text{ rpm.}$$

10.4 A 10-pole induction motor is supplied by a 6-pole alternator, which is driven at 1400 rpm. If the motor runs with a slip of 2%, what is its speed?

Solution

For induction motor: Synchronous speed is given by

$$N_s = \frac{120\,f}{P} = \frac{120 \times 70}{10} = 840 \text{ rpm} \quad \left[\because f = \frac{PN_A}{120} = \frac{6 \times 1400}{120} = 70\,\text{Hz}.\right]$$

Now slip, $$s = \frac{N_s - N}{N_s} = \frac{840 - N}{840}$$

$$\therefore \quad 0.02 = \frac{840 - N}{840}$$

$$\therefore \quad N = 823.2 \text{ rpm.}$$

10.5 A three-phase 60 Hz induction motor has a no load speed of 890 rpm and a full load speed of 855 rpm. Calculate

(i) the number of poles
(ii) slip s at no load
(iii) slip at full load
(iv) frequency of rotor currents at no load
(v) frequency of rotor currents at full load.

Solution

(i) Since the no load slip of an induction motor is about one percent, the synchronous speed is slightly larger than the no load speed of 890 rpm. For 60 Hz frequency, the number of poles and their corresponding synchronous speeds are

P	2	4	6	8	10
N_s (rpm)	3600	1800	1200	900	720

It is obvious that the synchronous speed can be only 900 rpm and therefore the number of poles is 8.

(ii) No load slip (s) = $\dfrac{900-890}{900} \times 100 = 1.11\%$.

(iii) Full load slip = $\dfrac{900-855}{900} \times 100 = 5\%$.

(iv) At no load, f_2 $(= sf_1) = \dfrac{1.11}{100} \times 60 = 0.66$ Hz.

(v) At full load, $f_2 = \dfrac{5}{100} \times 60 = 3$ Hz.

10.6 A three-phase 6-pole induction motor runs at 760 rpm at full load. It is supplied from an alternator having four poles and running at 1200 rpm. Determine the full-load slip of the induction motor.

Solution

Given the number of poles of alternator $P_A = 4$ and the synchronous speed of the alternator is 1200 rpm, the frequency f is $\dfrac{N \cdot P_A}{120} = \dfrac{1200 \times 4}{120} = 40$ Hz.

∴ Frequency generated by the alternator is 40 Hz.

For the given induction motor, $P = 6$, Speed at full load $N = 760$ rpm, supply frequency from the alternator is $f = 40$ Hz.

∴ Synchronous speed of the motor, $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 40}{6} = 800$ rpm.

∴ The percentage slip, $s = \dfrac{N_s - N}{N_s} \cdot 100 = \dfrac{800-760}{800} \times 100 = 5\%$.

10.7 A three-phase, 400 V, 50 Hz induction motor has a speed of 900 rpm on full-load. The motor has six poles. (i) Find out the slip. (ii) How many complete alternations will the rotor voltage take per minute?

Solution

(i) Given $N = 900$ rpm, $f = 50$ Hz and $P = 60$.

$$\therefore \quad N_s = \frac{120 \times f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$$\therefore \quad \text{slip } (s) = \frac{N_s - N}{N_s} = \frac{1000-900}{1000} = 0.1 \text{ or } 10\%.$$

(ii) Alternation of rotor voltage:

$$f' = s \times f = 0.01 \times 50 = 0.5/\text{sec or } 30/\text{min}.$$

10.8 A three-phase, 6-pole, 50 Hz induction motor has a slip of 0.8% at no load and 2% at full load. Calculate:

(i) the synchronous speed
(ii) the no-load speed
(iii) the full-load speed
(iv) the frequency of rotor current at standstill
(v) the frequency of rotor current at full load.

Solution

(i) $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm.

(ii) Speed at no load = (1 – slip at no load) × N_s = (1 – 0.008) × 1000 = 992 rpm.

(iii) Speed at full load = (1 – slip at full load) × N_s = (1 – 0.02) × 1000 = 980 rpm.

(iv) Frequency of rotor current at standstill $f_2 = sf = 1 \times 50 = 50$ Hz.

(v) Frequency of rotor current at full load, f_2 = (slip at full load) × f
= 0.02 × 50 = 1.0 Hz.

10.9 The voltage applied to the stator of a three-phase, 4-pole induction motor has a frequency of 50 Hz. The frequency of the emf induced in the rotor is 1.5 Hz. Determine slip and speed at which motor is running.

Solution

(i) $N_s = \dfrac{120\,f}{p} = \dfrac{120 \times 50}{4} = 1500$ rpm.

Rotor emf frequency, $f_2 = sf$

or $1.5 = s \times 50$

$\therefore$ slip (s) = $\dfrac{1.5}{50} = 0.03$ or 3.0%.

(ii) Actual speed of motor is $N = (1 - s) \cdot N_s = 1500\,(1 - 0.03) = 1455$ rpm.

10.10 A three-phase, 50 Hz, 6-pole cage motor is running with a slip of 3%. Calculate:

(i) the speed of the rotating field relative to the stator winding
(ii) the motor speed
(iii) the frequency of emf induced in the rotor
(iv) the speed of rotation of rotor mmf relative to rotor winding
(v) the speed of rotation of rotor mmf relative to stator winding.

Solution

(i) $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm.

(ii) $N = N_s(1 - s) = 1000\left(1 - \dfrac{3}{100}\right) = 970$ rpm.

(iii) $f_2 = sf_1 = \dfrac{3}{100} \times 50 = 1.5$ Hz.

(iv) Speed of rotor mmf relative to rotor winding $= \dfrac{120 \times f_2}{P} = \dfrac{120 \times 1.5}{6} = 30$ rpm.

(v) Since the rotor is rotating at 970 rpm and the rotor mmf is revolving at 30 rpm with respect to rotor, therefore speed of the rotor mmf relative to the stationary winding (stator) is (970 + 30) rpm = 1000 rpm.

10.9 TORQUE EXPRESSION OF AN INDUCTION MOTOR

Operating Torque

In the induction motor, the torque T is given by

$$T \propto \phi \cdot I_r \cdot \cos \phi_r \tag{10.9}$$

where ϕ is the stator flux/pole and I_r is the rotor current/phase under running conditions, cos ϕ_r is the rotor power factor.

We have $$I_r = \frac{E_r}{Z_r} = \frac{E_2 \cdot s}{\sqrt{R_2^2 + (sX_2)^2}} \tag{10.10}$$

and $$\cos \phi_r = \frac{R_r}{Z_r} = \frac{R_2}{Z_r} = R_2 \Big/ \sqrt{R_2^2 + (sX_2)^2} \tag{10.11}$$

($\because$ Resistance is independent of relative speed)

where E_r is rotor emf/phase

Z_r is the rotor impedance = $\sqrt{R_2^2 + (sX_2)^2}$

R_r is the rotor resistance/phase.

E_r, Z_r, R_r are the respective parameters of the rotor in running conditions. If s be the slip of the motor, operating at rated speed, we can write

$$X_r = sX_2$$
$$E_r = sE_2$$
and $$R_r = R_2$$

where, X_r is the rotor reactance/phase under running condition.

In the standstill condition,

X_2 is the rotor reactance/phase

E_2 is the rotor emf/phase

and R_2 is the rotor resistance/phase.

From the fundamentals, we have

$$|Z_r| = \sqrt{R_r^2 + X_r^2} = \sqrt{R_2^2 + X_r^2}\ .$$

The equation of the torque can be rewritten as

$$T = K_1 \phi \frac{E_r}{Z_r} \cdot \frac{R_2}{Z_r}$$

[K_1 is the constant of proportionality in Eq. (10.9)]

$$= \frac{K_1 \phi\, sE_2\, R_2}{R_2^2 + X_r^2} = \frac{K_1 \phi\, sE_2\, R_2}{R_2^2 + (sX_2)^2}\ . \tag{10.12}$$

Again, the flux(ϕ) produced by the stator being proportional to the applied phase voltage (E_1), we can write

$$\phi \propto E_1$$

i.e., $$\phi = K_2 E_1$$

Also $$\frac{E_1}{E_2} = \frac{N_1}{N_2} = k.$$

$\therefore$ $$E_2 = \frac{1}{k} \cdot E_1$$

Substituting the expressions for ϕ and E_2 in Eq. (10.12) we get

$$T = \frac{K_1 \cdot K_2 E_1 \cdot s \cdot (E_1/k) \cdot R_2}{R_2^2 + (sX_2)^2} = \frac{KE_1^2 \cdot s \cdot R_2}{R_2^2 + (sX_2)^2} \tag{10.13}$$

where $$K = \frac{K_1 K_2}{k}$$

i.e., $$T \propto \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2} \tag{10.13a}$$

Here T is expressed in watts on per phase basis for $K = 1$. In order to get the value of three-phase torque, the expression obtained in (10.13) or (10.13a) is to be multiplied by a factor 3, provided K is known and all the quantities in the RHS of equation (10.13) are expressed in phase values. We will discuss later how K can be obtained. Actually $K = \frac{3}{\omega_s}$, where $\omega_s = 2\pi n_s$, n_s is expressed in rps i.e., equal to $\left(\frac{N_s}{60}\right)$. For three phase, the electromagnetic torque T is

$$T_{3\phi} = \frac{3}{\omega_s} \cdot \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2} \text{ Nm}$$

i.e. $$T_{3\phi} = 3 \cdot \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2} \text{ W} \left(= \frac{3}{\omega_s} \cdot \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2} \text{ Nm} \right). \tag{10.13b}$$

10.9.1 Starting Torque (T_s)

At starting the rotor is stationary, the slip $s = 1$ and the rotor reactance X_2 is much larger compared to the rotor resistance R_2. So neglecting R_2 in Eq. (10.13 a), we get for $s = 1$,

$$T_s \propto \frac{E_1^2 R_2}{X_2^2} \tag{10.14a}$$

or $T_s \propto R_2$ and $T_s \propto E_1^2$ [assuming (X_2) as constant]

The general expression of starting torque can be obtained from equation (10.13) with $s = 1$.

$$T_s = \frac{KE_1^2 R_2}{R_2^2 + X_2^2} \tag{10.14b}$$

Thus for obtaining large starting torque, the rotor resistance R_2 as well as applied voltage E_1 should be large.

To get the three-phase starting torque, T_s obtained in Eq. 10.14(a) or (b) is to be multiplied by a factor 3.

10.9.2 Effect of Change in Supply Voltage in Torque and Slip

Since $$T \propto \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2},$$

at rated speed, with low values of s we have

$$T \propto \frac{sE_1^2 R_2}{R_2^2} \quad (\because (sX_2) \text{ is very low})$$

i.e. $$T \propto \frac{sE_1^2}{R_2}$$

i.e., torque under normal operating condition is proportional to the square of supply voltage. With drop in supply voltage, running torque T decreases and to maintain same torque, slip must increase (i.e., speed drops.)

10.9.3 Effect of Change in Supply Voltage in Starting Torque

Since $$T_s \propto \frac{E_1^2 R_2}{X_2^2},$$

at constant (X_2) and (R_2), the *starting torque is also proportional to the square of the supply voltage.*

10.9.4 Condition for Maximum Torque

Under normal running conditions,

$$T = \frac{KE_1^2\ R_2 \cdot s}{R_2^2 + X_2^2 \cdot s^2} \tag{10.15}$$

When the motor is operating on constant applied voltage E_1, then

$$T = \frac{K'R_2 s}{R_2^2 + X_2^2 s^2} = \frac{K'R_2}{\dfrac{R_2^2}{s} + X_2^2 s} \quad [K' = KE_1^2]$$

Since the numerator of the right hand side of the expression is constant, thus for getting the condition for maximum torque we differentiate the denominator with respect to slip s and equate the differential co-efficient to zero.

Thus we have, at maximum torque condition,

$$-(R_2^2/s^2) + X_2^2 = 0$$

or $$X_2^2 = R_2^2/s^2$$

or $$s = s_{\max} = \frac{R_2}{X_2} \tag{10.16}$$

(where $s_{\max}$ is *slip for maximum torque*)

From Eq. (10.15) (at constant applied voltage E_1), and (10.16), we have

$$T_{\max} = \frac{K \cdot R_2 \cdot \left(\dfrac{R_2}{X_2}\right) \cdot E_1^2}{R_2^2 + X_2^2 \cdot \dfrac{R_2^2}{X_2^2}} = \frac{K \cdot R_2^2 \cdot E_1^2}{2R_2^2 X_2} = \frac{K''}{X_2} \tag{10.17a}$$

Also, $$T_{\max} = \frac{KE_1^2\,(sX_2)\cdot s}{(sX_2)^2 + (sX_2)^2}$$ [using $R_2 = sX_2$ in Eq. (10.15) to obtain $T_{\max}$]

$$= \frac{KE_1^2}{2X_2} \tag{10.17b}$$

[it may be noted here that $K'' = \frac{K}{2} \cdot E_1^2$]

Thus the *maximum torque of an induction motor, operating on constant applied phase voltage E_1 and constant supply frequency f is inversely proportional to the standstill rotor reactance X_2 but is independent of rotor circuit resistance R_2*. The slip for maximum torque is the ratio of rotor resistance and standstill rotor-reactance, i.e. $S_{max} = \frac{R_2}{X_2}$.

Also,
$$\frac{T}{T_{max}} = \frac{KE_1^2 \cdot R_2 \cdot s}{X_2^2\left(\frac{R_2^2}{X_2^2} + s^2\right)} \Bigg/ \frac{K''}{X_2}$$

$$= \frac{KE_1^2 \cdot R_2 \cdot s}{X_2^2\left(s_{max}^2 + s^2\right)} \frac{X_2}{K''}$$

$$= K''' \cdot \frac{s_{max} \cdot s}{s^2 + s_{max}^2} \left[K''' = \frac{KE_1^2}{K''}\right] \tag{10.18(a)}$$

Also, we can write
$$\frac{T}{T_{max}} = \frac{KE_1^2 R_2 s}{R_2^2 + (sX_2)^2} \Bigg/ \frac{KE_1^2}{2X_2} = \frac{2sR_2 X_2}{R_2^2 + (sX_2)^2} \tag{10.18b}$$

$$= \frac{2s \cdot s_{max}}{s^2 + s_{max}^2} \tag{10.18c}$$

Equations (10.18(a)) and (10.18(c)) are identical if we use the relation $K''' = \frac{KE_1^2}{K''}$ and $K'' = \frac{K}{2} \cdot E_1^2$ in Eq. 10.18(a).

At starting, $s = 1$

$$\therefore \quad \frac{T_s}{T_{max}} = \frac{K''' \cdot s_{max}}{(1 + s_{max}^2)} = \frac{K''' \cdot a}{1 + a^2} \text{ (where } a = s_{max}) \tag{10.19a}$$

[Also for Eq. 10.18(c), at $s = 1$, $\frac{T_s}{T_{max}} = \frac{2s_{max}}{1 + s_{max}^2}$; 10.19(b)

from the equation of torque we can write

$$\frac{T_s}{T} = \frac{KE_1^2 R_2}{R_2^2 + X_2^2} \times \frac{R_2^2 + s^2 X_2^2}{KE_1^2 R_2 \cdot s} = \frac{s^2 + \left(\frac{R_2}{X_2}\right)^2}{s\left[1 + \left(\frac{R_2}{X_2}\right)^2\right]} = \frac{s^2 + s_{max}^2}{s(1 + s_{max}^2)} \tag{10.19c}$$

Case I (Squirrel-cage induction motor): Since the rotor is permanently short-circuited, so no resistance can be inserted in its rotor circuit. Thus, slip for maximum torque is constant, and its value cannot be varied.

Case II (Slip ring induction motor): Since external resistance can be inserted in the rotor circuit, so its slip can be varied for maximum torque. For rotor circuit without external resistance

$$s_{\max}\ (1) = \frac{R_2}{X_2}$$

and if a resistance r per phase is inserted in the rotor circuit, then

$$s_{\max}(2) = (R_2 + r)/X_2. \tag{10.20}$$

10.9.5 Condition for Maximum Starting Torque

At starting, the starting torque (for a given applied voltage) is given by

$$T_s = \frac{KE_1^2 R_2}{R_2^2 + X_2^2} = \frac{K' R_2}{R_2^2 + X_2^2} \tag{10.21}$$

We obtain maximum starting torque by differentiating expression (10.21)

$$\frac{dT_s}{dR_2} = K\left[\frac{1\cdot(R_2^2 + X_2^2) - R_2(2R_2)}{(R_2^2 + X_2^2)^2}\right] = 0$$

$$R_2^2 + X_2^2 - 2R_2^2 = 0$$

$$R_2 = X_2$$

i.e. for a given applied voltage E_1, the starting torque is maximum when the resistance of rotor R_2 equals its reactance X_2.

10.11 A three-phase 4-pole 50 Hz induction motor has a rotor resistance of 0.020 Ω/phase and standstill reactance of 0.5 Ω/phase, calculate the speed at which the maximum torque is developed.

Solution

$$N_s = \frac{120\,f}{P} = \frac{120\times 50}{4} = 1500 \text{ rpm}$$

For maximum torque slip $s_{\max} = \frac{R_2}{X_2} = \frac{0.020}{0.5} = 0.04$

∴ Speed at maximum torque = $N_s(1 - s_{\max})$ = 1500 (1 – 0.04) = 1440 rpm.

10.12 A three-phase 8-pole 50 Hz. induction motor has a full-load slip of 1.5%, the rotor resistance is 0.001 Ω/phase and the standstill reactance is 0.005 Ω/phase. Calculate the ratio of the maximum to full load torque, and the speed at which maximum torque takes place.

Solution

Given f = 50 Hz., P = 8, s = 0.015, R_2 = 0.001 Ω;

X_2 = 0.005 Ω,

$$s_{\max} = \frac{R_2}{X_2} = \frac{0.001}{0.005} = 0.2$$

Now $$\frac{T}{T_{\max}} = \frac{2\cdot s\cdot s_{\max}}{s^2 + s_{\max}^2} = \frac{2\times 0.015\times 0.2}{(0.015)^2 + (0.2)^2} = \frac{6\times 10^{-3}}{4.02\times 10^{-2}}$$

$$\therefore \qquad \frac{T_{max}}{T} = \frac{4.02 \times 10^{-2}}{6 \times 10^{-3}} = 6.7$$

Now, $$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

$\therefore$ Speed at maximum torque = $N_s\,(1 - s_{max})$ = 750(1 – 0.2) = 600 rpm.

10.13 A three-phase, 50 Hz. 6-pole induction motor runs on full load with a slip of 0.04. Determine the available maximum torque in terms of full load torque. Also determine the speed at which the maximum torque takes place. [Given that the rotor standstill impedance per phase is (0.01 + j 0.05) Ω.]

Solution

Given slip (s) = 0.04

$$s_{max} = \frac{R_2}{X_2} = \frac{0.01}{0.05} = 0.2$$

$\therefore$ Speed at maximum torque = $N_s(1 - s_{max})$ = (1 – 0.2) × 1000 = 800 rpm

$$\therefore \qquad \frac{T_{max}}{T} = \frac{s^2 + s_{max}^2}{2 \cdot s \cdot s_{max}} = \frac{(0.04)^2 + (0.2)^2}{2 \times 0.04 \times 0.2} = 2.6$$

or $T_{max} = 2.6 \times T$ [T is full load torque].

10.14 A three-phase, 50 Hz 4-pole induction motor (slip ring) develops a maximum torque of 100 Nm at 1400 rpm. The resistance of the star connected rotor is 0.25 Ω/phase. Determine the value of resistance that must be inserted in series with each rotor phase to produce a starting torque equal to half the maximum torque.

Solution

The synchronous speed $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{4} = 1500$ rpm.

Speed at maximum torque = 1400 rpm (given)

$$\therefore \quad \text{Slip } (s_{max}) \text{ at maximum torque} = \frac{N_s - \text{speed at maximum torque}}{N_s}$$

$$= \frac{1500 - 1400}{1500} = 0.067.$$

Also, $$s_{max} = \frac{R_2}{X_2}$$

$$\therefore \qquad X_2 = \frac{R_2}{s_{max}} = \frac{0.25}{0.067} = 3.73$$

$$T_{max} = \frac{KE_1^2}{2X_2} = \frac{KE_1^2}{2 \times 3.73} = 0.134\,KE_1^2$$

Let r be the external resistance to be inserted per phase in the rotor circuit, then starting torque

$$T_{st} = \frac{K \cdot E_1^2\,(R_2 + r)}{(R_2 + r)^2 + X_2^2} = \frac{K(0.25 + r)\,E_1^2}{(0.25 + r)^2 + (3.73)^2}$$

Again, it is given that $T_{st} = \frac{1}{2} T_{max}$;

$$\therefore \quad \frac{K(0.25+r)\cdot E_1^2}{(0.25+r)^2+(3.73)^2} = \frac{1}{2} \times 0.134\, K \cdot E_1^2.$$

Simplifying,

$$r = 13.67 \text{ ohm or } 0.75 \text{ ohm;}$$

but we ignore the value of r = 13.67 ohm as it corresponds to T_{max} lying in the region where $s > 1$.

$$\therefore \quad r = 0.75 \text{ ohm per phase.}$$

10.15 A three-phase, 24-pole, 50 Hz, 3200 volt star connected induction motor has a slip ring rotor of resistance 0.016 Ω and standstill reactance of 0.270 Ω per phase. Full load torque is obtained at a speed of 247 rpm. Determine:

(a) the ratio of maximum to full-load torque.
(b) the speed at maximum torque, stator impedance being neglected.

Solution

(i) Synchronous speed $N_s = \frac{120 f}{P} = \frac{120 \times 50}{24} = 250$ rpm.

$$\therefore \quad \text{Slip } (s) = \frac{N_s - N}{N_s} = \frac{250-247}{250} = 0.012.$$

Also, $s_{max} = \frac{R_2}{X_2} = \frac{0.016}{0.270} = 0.059.$

We know, $\frac{T}{T_{max}} = \frac{2\cdot s_{max}\cdot s}{s^2 + s_{max}^2}.$

Here, $\frac{T}{T_{max}} = \frac{2\times 0.059 \times 0.012}{(0.012)^2+(0.059)^2}$

or $\frac{T_{max}}{T} = \frac{(0.012)^2+(0.059)^2}{2\times 0.059\times 0.012} = 2.56.$

Let N' be the intended speed at maximum torque

Then, $s_{max} = \frac{N_s - N'}{N_s} = \frac{250-N'}{250}$

$s_{max} = 0.059$ from calculation we have got earlier.

i.e., $0.059 = \frac{250-N'}{250}$

or $N' = 235.25$ rpm.

10.16 A three-phase, 6-pole 50 Hz. induction motor develops a maximum torque of 30 Nm at 960 rpm. Calculate the torque produced by the motor at 6% slip. The rotor resistance per phase is 0.6 Ω.

Solution

Given, f = 50 Hz., P = 6

$$\therefore \quad N_s = \frac{120 f}{P} = \frac{120\times 50}{6} = 1000 \text{ rpm.}$$

Speed at maximum torque = 960 rpm

Slip at maximum torque = $\dfrac{N_s - \text{speed at maximum torque}}{N_s} = \dfrac{1000 - 960}{1000} = 0.04\ (= s_{max})$

Also, $s_{max} = \dfrac{R_2}{X_2}$

$\therefore \quad X_2 = \dfrac{R_2}{s_{max}} = \dfrac{0.6}{0.04} = 15\ \Omega.$

If T is the torque at slip s, $\dfrac{T}{T_{max}} = \dfrac{2s \cdot s_{max}}{s^2 + s_{max}^2}$

here, $s = 0.06$, $T_{max} = 30$ Nm

$\therefore \quad T = \dfrac{2 \times 0.06 \times 0.04}{(0.06)^2 + (0.04)^2} \times 30 = 27.692$ Nm.

10.17 A 746 kW, three-phase, 50 Hz., 16-pole induction motor has a rotor impedance of $(0.02 + j0.15))$ ohm at standstill. Full load torque is obtained at 350 rpm.

Determine (i) the speed at which maximum torque occurs, (ii) the ratio of maximum to full load torque, (iii) the external resistance per phase to be inserted in the rotor circuit to get maximum torque at starting.

Solution

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{16} = 375 \text{ rpm}$$

Speed at full load = 350 rpm

$\therefore$ Slip at full load = $\dfrac{375 - 350}{375} = 0.06$

Slip at maximum torque

$$s_{max} = \frac{R_2}{X_2} = \frac{0.02}{0.15} = \frac{2}{15} = 0.133$$

(i) Speed at which maximum torque occurs = $(1 - s_{max})N_s = \left(1 - \dfrac{2}{15}\right) \times 375 = 325$ rpm.

(ii) $\dfrac{T_{max}}{T} = \dfrac{s_{max}^2 + s^2}{2s \cdot s_{max}} = \dfrac{(0.06)^2 + \left(\dfrac{2}{15}\right)^2}{2 \times 0.06 \times \dfrac{2}{15}} = 1.33.$

(iii) Let the external resistance per phase added to the rotor circuit be 'r' Ω, so that R rotor resistance per phase, $R_2 = (0.02 + r)$.
The starting torque will be maximum when $R_2 = X_2$
$\therefore \quad 0.02 + r = 0.15$
or $\quad r = 0.13\ \Omega$ per phase.

10.10 TORQUE SLIP CHARACTERISTICS OF A THREE-PHASE INDUCTION MOTOR

The torque T of an induction motor (three-phase) is given by (Eq. 10.13)

$$T = \frac{KE_1^2\ R_2 \cdot s}{R_2^2 + X_2^2 s^2}$$

For a constant supply voltage E_1, the value of E_2 is constant. Assuming R_2 as constant, we can write

$$T \propto \frac{s}{R_2^2 + X_2^2 s^2} \tag{10.22}$$

At synchronous speed, slip s is zero, hence torque T is zero; at starting $s = 1$, thus torque T is maximum.

Consequently, the torque slip curve starts from origin (i.e., $s = 0$), and ends at $s = 1$.

Case study I: When s (slip) is very low (at rotor speeds close to synchronous speed), $sX_2 \ll R_2$ and $T \propto \frac{s}{R_2^2}$ (at low-slips).

i.e., *Torque-slip curve at low values of slip is a straight line passing through the origin, and torque is maximum when* $s = \frac{R_2}{X_2}$.

Case study II: When the load on the motor increases, the speed of the motor decreases. When slip s is large, compared to R_2, sX_2 is much large and hence $sX_2 \gg R_2$.

$$\therefore \qquad T \propto \frac{s}{(sX_2)^2} \propto \frac{1}{sX_2^2} \propto \frac{1}{s} \quad \text{(at high slips)}$$

i.e., the torque T slip s curve for larger values of slip is approximately a rectangular hyperbola. Consequently, any further increase in motor load, beyond the point of maximum torque, results in decrease of the torque developed by the motor. Eventually, the motor slows down. The maximum torque developed in an induction motor is called the *pull-out torque* or *break down torque*. This torque is a measure of the short time over loading capability of the motor. Figure 10.4 Shows the torque-slip characteristics of an induction motor operating with constant applied voltage, and constant frequency.

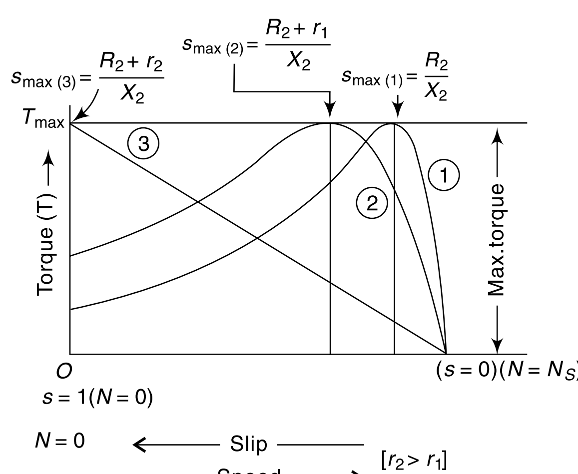


Fig. 10.4 ***Torque-slip characteristics of an induction motor***

Curve 1 represents (T-s) characteristic of an induction motor having low rotor resistance or when no resistance is inserted in the rotor circuit.

Maximum torque is developed at $s_{max}\,(1) = \dfrac{R_2}{X_2}$.

Curve 2 represents the (T-s) characteristic of an induction motor. When an external resistance of r_1 Ω/phase is inserted in the rotor circuit the magnitude of the maximum torque remains unchanged, but the slip for maximum torque in $s_{max}(2) = (R_2 + r_1)/X_2$.

Curve 3 represents the (T-s) characteristic of an induction motor, when an external resistance of r_2 Ω/phase is inserted in the rotor circuit such that $R_2 + r_2 = X_2$, a condition for maximum torque is there at starting.

It may be noted here that $(R_2 + r_2) > (R_2 + r_1) > R_2$

It is also seen that *as the rotor resistance is increased, the pull out speed of the motor decreases, but the maximum torque remains constant*. However, for squirrel cage rotors it is not possible to insert any rotor resistance under normal operating conditions and hence *it is not easily possible to enhance the value of the starting or maximum torque for a squirrel cage induction motor.*

10.11 EQUIVALENT CIRCUIT OF INDUCTION MOTOR

In the case of an ideal induction motor, the equivalent circuit can be represented like that of an ideal transformer. The only difference is that the rotor of induction motor is not static and mechanical power is developed.

Figure 10.5(a) shows the equivalent circuit of an induction motor with all quantities referred to the stator. During shifting of impedance or resistance from the secondary to primary, the secondary quantity is multiplied by (k^2), (where k = transformation ratio = number of stator turns/number of rotor turns). *It is to be remembered that the equivalent circuit is always drawn for the per phase values.*

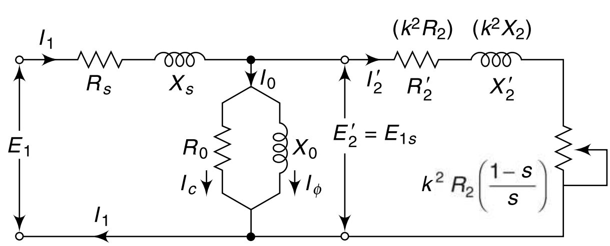


Fig. 10.5(a) *Equivalent circuit of an induction motor*

Looking at the stator side, counter emfs are generated in all the three phases of stator due to rotating air-gap flux wave. The application of a voltage E_1 to the stator winding creates a mutual flux which sets up induced emf E_{1s} in the stator and rotor. Since $E_{1s} < E_1$, so this difference $(E_1 - E_{1s})$ represents the impedance drop $[I_1\,(R_s + jX_s)]$. The effect of no-load current I_0 (= $I_C + I_\phi$, where I_C is the core loss component and I_ϕ is the magnetizing component lagging by an angle $\pi/2$ and is represented by a shunt consisting of R_0 and X_0 connected in parallel).

Thus, R_0 and X_0 account for *working component* and *magnetizing component* of no-load current respectively.

Now, if we block the rotor and make the total rotor resistance equal to R_2/s by inserting an additional resistance in the rotor circuit, then the rotor current, mmf, reactance of rotor on stator, stator current and input to machine would be same as they were when the rotor was running at slip s.

Also, $$\frac{R_2}{s} = R_2 + R_2\left(\frac{1-s}{s}\right)$$

$$= \text{Actual resistance of rotor + Fictitious resistance } \left(R_2 \cdot \frac{1-s}{s}\right).$$

Let us calculate the rotor quantities with respect to the stator. If k is the effective transformation ratio then total rotor resistance R_2 and reactance X_2, when referred to the stator, appear as R_2' and X_2' where ($R_2' = R_2k^2$) and ($X_2' = X_2k^2$). Moreover, the rotor current I_2 when referred to the stator, appears as $I_2'\left(=\frac{I_2}{k}\right)$.

Also, $I_2' + I_0 = I_1$ (stator current). In the above expression $\left[R_2\left(\frac{1-s}{s}\right)\right]$ is the electrical analogue of the variable mechanical load and is the *fictitious resistance* equivalent to load on the motor.

The equivalent circuit can be simplified by transforming no-load current component to the supply side as shown in Fig. 10.5(b).

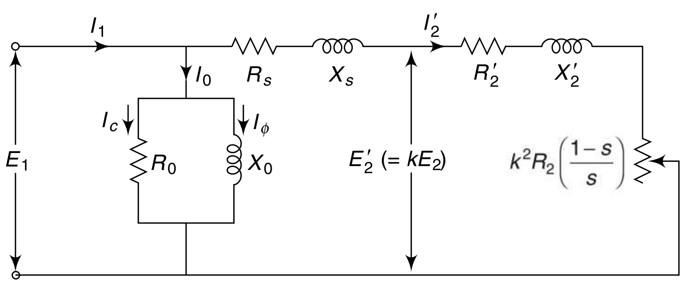


Fig. 10.5(b) ***Simplified equivalent circuit***

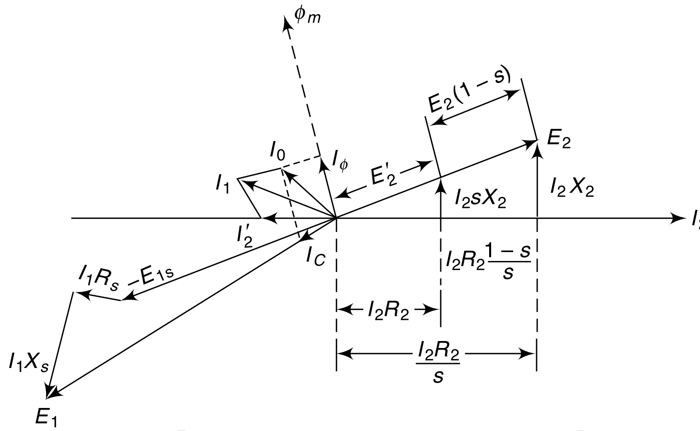
The phasor diagram of the induction motor is shown in Fig. 10.5(c).

10.12 LOSSES AND EFFICIENCY

At starting and during acceleration the rotor core losses are high; with the increase in speed these losses decrease to some extent. The friction and windage losses are zero at start and with increase in speed these losses increase. However, the sum of friction, windage and core losses is roughly constant for a motor even with variable speed. Therefore, these categories of losses are sometimes lumped together and called *constant losses* and are then defined as follows:

$$P_{(\text{constant loss})} = P_{\text{core loss}} + P_{\text{mechanical loss}}$$

$\therefore$ Output power P_0 = Total mechanical power developed – Mechanical losses



$$\left[\begin{array}{l} -E_{1s} = E_2 \cdot R;\, I_2' = I_2/k;\, I_o = \sqrt{I_C^2 + I_\phi^2}\,; \\ I_1 = \sqrt{I_0^2 + I_2'^2}\,;\, \dfrac{I_2 R_2}{s} = I_2 R_2 + I_2 R_2 \dfrac{1-S}{S} \end{array}\right]$$

Fig. 10.5(c) ***Phasor diagram of a 3-phase induction motor on per phase basis***

Losses in a three-phase Induction motor are of two types mainly (a) Fixed losses and (b) Variable losses.

(a) Fixed losses
- Core loss
- Bearing friction loss
- Brush friction loss in wound rotors
- Windage loss

(b) Variable losses
- Stator ohmic loss (I^2R loss in stator)
- Rotor ohmic loss (I^2R loss in rotor)
- Brush contact loss for wound rotor motors only
- Stray load loss.

The rotor output gives rise to the development of *gross torque* or *electromagnetic torque* T_g, which is partly "wasted" (in the form of winding, and frictional losses in the rotor), and partly appears as the *useful shaft torque* T_{sh}. Let n be the actual speed of the rotor (in rps) and T_g be the gross torque (or electromagnetic torque) developed by the rotor, then,

$$T_g \times 2\pi n = \text{Rotor output } (P_0)$$

or
$$\text{Gross torque } T_g = \frac{\text{Rotor output } (P_0)}{2\pi n} \tag{10.23}$$

Since the copper losses in the rotor is negligible, so the input of the rotor equals the output of the rotor.

$$\therefore \quad T_g = \frac{\text{Rotor input } (P_{ag})}{2\pi n_s} \tag{10.24}$$

(n_s being the synchronous speed in rps)

From Eqs (10.23) and (10.24) we can write

$$\text{Rotor output } (P_o) = T_g \times 2\pi n$$

and
$$\text{Rotor input } (P_{ag}) = T_g \times 2\pi n_s$$

$\therefore$ Copper losses (ohmic loss) of rotor = Rotor input – Rotor output

$$= T_g \cdot 2\pi\,(n_s - n)$$

i.e. $P_{\text{rcu}} = T_g \cdot 2\pi \dfrac{(n_s - n)}{n_s}\, n_s = T_g \cdot 2\pi \cdot s \cdot n_s$ = Slip × Rotor input (P_{ag})

$$\therefore \quad P_{\text{rcu}} = s \times P_{ag} \tag{10.25}$$

when (P_{rcu}) is the *rotor copper loss*.

Hence gross mechanical power developed in rotor P_m is equal to (rotor input P_{ag} – rotor copper losses).

i.e., P_m = Rotor input – s × Rotor input = Rotor input $(1 - s)$

$$\text{or,} \quad P_m = P_{ag}(1 - s) \tag{10.26}$$

Hence, rotor efficiency

$$\eta = \frac{\text{Output of rotor}}{\text{Rotor input}}$$

$$= (1 - s) = 1 - \left(\frac{n_s - n}{n_s}\right) = \frac{n}{n_s} = \frac{\text{Actual speed of rotor}}{\text{Synchronous speed of the motor}} \tag{10.27}$$

> [The torque of a polyphase induction motor may be expressed in "Synchronous Watts". It is defined as the torque which develops a power of 1 W at the synchronous speed of the motor.
> $\because$ Rotor input = $T_g \times 2\pi n_s$, hence we can write
> $$T_g \text{ (synchronous W)} = \frac{\text{Rotor input in W}}{2\pi \times n_s},$$
> where n_s is expressed in rps.]

Also, Copper losses of rotor (P_{rcu}) = s × Rotor input (P_{ag})

$$\text{or} \quad P_{\text{rcu}} = s \times \frac{\text{Mechanical power in rotor } (P_m)}{(1 - s)}$$

$$= \left(\frac{s}{1 - s}\right) \times \text{Mechanical power developed in rotor } (P_m) \tag{10.28}$$

$\therefore$ Rotor input : rotor copper loss : mechanical power developed in rotor = 1 : s : $(1 - s)$.

It may be noted here that T or T_g (developed torque/gross torque/electromagnetic torque) can thus be obtained from the following formula:

$T\ (=T_g) = \dfrac{P_{ag}}{\omega_s}$ Nm, where $\omega_s = 2\pi n_s$ and (P_{ag}) is the *air gap power of the motor*, i.e. the power being transferred from the stator to rotor. We have termed it as rotor input earlier where *rotor input* (= air gap power) = (stator input – stator copper loss – stator core loss).

The *shaft output torque* T_{sh} is developed at the output of the motor (i.e., at the shaft) and is due to the output power which is the difference between the air gap power (or rotor input) and the rotor losses. Rotor losses include rotor copper loss and mechanical losses (we neglect the rotor iron loss). Thus the shaft torque is obtained as

$$T_{sh} = \frac{P_0}{\omega} \text{ Nm},$$

where P_o is the motor output and (ω) is $2\pi n$, n_s and n are both expressed in rps, i.e. $n_s = \frac{N_s}{60}$ rps ; $n = \frac{N}{60}$ rps, where N_s and N are expressed in rpm.

The gross mechanical power developed in the rotor being the difference of rotor input P_{ag} and the rotor copper loss P_{rcu}, we can find the gross mechanical torque developed using the relation, $T_m = (P_m/\omega)$ Nm, where T_m is the *gross mechanical torque* developed in the rotor.

Thus finally we have

- P_{rcu} (rotor copper loss in watts) = $s \times P_{\text{ag}}$, where P_{ag} = air gap power (or rotor input) in watts = P_{in} – stator losses.

  [P_{in} (= stator input) = ($\sqrt{3}\, E_L\, I_L \cos\phi$) W and stator losses include stator copper loss and stator core loss]
- P_{rcu} (rotor copper loss in watts) = $\frac{s}{1-s} \times P_m$
- P_m (gross mechanical power developed in the rotor) = $P_{\text{ag}}(1 - s)$; P_m and P_{ag} both being expressed in watts.
- η (motor efficiency)% = $\frac{P_o}{P_{in}} \times 100 = \frac{n}{n_s} \times 100$:

  where P_o = motor output in HP converted to watts, n is the rotor speed in rps and n_s is the synchronous speed of the motor in rps.
- T (in Nm) (= T_g) = $\frac{P_{\text{ag}}}{\omega_s}$; $\omega_s = 2\pi n_s$.

  Also, $T = K\frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}$; $K = \frac{3}{\omega_s}$ when we desire to get three-phase torque; $K = \frac{1}{\omega_s}$ for single-phase expression.
- T_m (mechanical torque developed in rotor) = $\frac{P_m}{\omega}$ Nm; $\omega = 2\pi n$.
- $T_{sh} = \frac{P_o}{\omega}$ Nm

  [if rotational losses are neglected, $P_m = P_o$ and hence $T_m = T_{sh}$]

  [$P_{\text{ag}} = P_{\text{in}} - (P_{\text{scu}} + P_{\text{sc}})$; $P_m = P_{\text{ag}} - P_{\text{rcu}}$; $P_o = P_m - P_{rm}$ where, P_{scu} and P_{sc} are the stator copper loss and stator core loss respectively and P_{rm} in the rotor mechanical loss in watts.]

Let us now extend the discussion to review the aspect of loss in an induction motor.

Fixed loss = (Power input at no load) – (Stator Cu-loss at no load).

This loss can be obtained by performing no load test of the induction motor. Total ohmic losses under variable loss can be obtained using blocked rotor test of induction motor. It should be noted that the brush contact loss for wound rotor induction motor (WRIM) = slip ring current times volt drop in brushes.

Stray load loss occurs in iron and conductors. It is very difficult to measure stray load loss. To account this, the efficiency η is taken as 0.5% less than the calculated value on full load and for other loads.

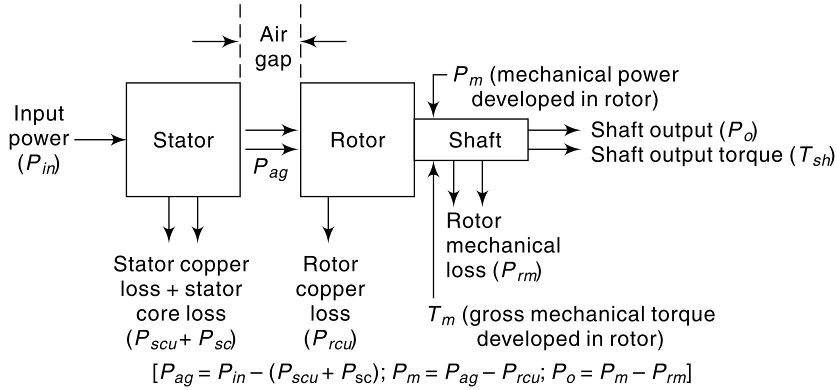


Fig. 10.6 ***Power stages in a three-phase induction motor***

The efficiency η of a three-phase induction motor is given by

$$\eta_{\%} = \frac{P_o}{P_o + P_{fl} + P_{cu}} \times 100 \qquad (10.29)$$

where P_o is the output power, P_{fl} is fixed loss, and P_{cu} is stator and rotor ohomic losses plus brush contact loss.

[Also, Stator input = Stator output + Stator losses.

But the stator output is entirely transferred inductively to the rotor circuit, so Rotor input = Stator output; also Rotor output = Rotor input – Rotor copper losses].

Expression of Torque from Power Input

The rotor input also being termed as air gap power P_{ag} in an induction motor, we can write from Eq. (10.25),

$$\text{Rotor input } P_{ag} = \frac{\text{rotor copper loss}}{s} = \frac{I_2'^2 R_2'}{s} \qquad (10.30a)$$

[when all the quantities are referred to the stator side]

i.e.
$$P_{ag} = \frac{s^2 E_2'^2}{R_2'^2 + (sX_2')^2} \cdot \frac{R_2'}{s} \qquad \left[\because I_2' = \frac{sE_2'}{[R_2'^2 + (sX_2')^2]^{0.5}}\right]$$

$$= \frac{sE_2'^2 R_2 \times k^2}{(k^2 R_2)^2 + (s \cdot k^2 X_2)^2} \qquad [\because R_2' = k^2 R_2;\ X_2' = k^2 \cdot X_2]$$

But $\quad E_2' = kE_2$

$$\therefore \qquad P_{ag} = \frac{sE_2^2 R_2 \cdot k^4}{k^4 [R_2^2 + (sX_2)^2]} = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

Now we refer to the approximate equivalent circuit of Fig. 10.5(b). We observe that if $R_s = 0$; $X_s = 0$ (i.e., we neglect stator resistance and reactance) then E_2' becomes equal to E_1, the applied voltage per phase. With this simplification we can write for the expression of P_{ag} (derived above) as follows:

$$P_{ag} = \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \qquad \text{Torque } T = \frac{P_{\text{ag}}}{\omega_s} = \frac{1}{\omega_s} \cdot \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2} \text{ (where } \omega_s = 2\pi n_s) \qquad (10.30b)$$

If we compare Eq. (10.13) with Eq. (10.30b), we find K in Eq. (10.13) is $\left(\frac{1}{\omega_s}\right)$.

With $K = \frac{1}{\omega_s}$, T gives the expression of electromagnetic torque developed in the rotor on per phase basis and is expressed in Nm.

For three-phase, the expression of torque is $T = \frac{3}{\omega_s} \cdot \frac{sE_1^2 R_2}{R_2^2 + (sX_2)^2}$ Nm. (10.30c)

10.13 DETERMINATION OF MOTOR EFFICIENCY

The efficiency of small induction motors can be determined by directly loading them and by measuring their input and output powers. For larger motors, it may be difficult to arrange loads for them. Moreover, the power loss will be large with direct loading tests. Therefore, indirect methods are used to determine the efficienty of a three-phase induction motors. The following tests are performed on the motor:

(a) No-load test (or open circuit test).
(b) Blocked-rotor test.

The parameters of the equivalent circuit can be found from the no-load and blocked rotor test also.

Open Circuit or No-load Test

This test is similar to the open circuit test on a transformer. A three-phase auto-transformer is used to supply rated voltage at the rated frequency. The motor is run at no load. The power input is measured by two wattmeter method. The power factor under no load condition is generally less than 0.5. Therefore one of the wattmeters will show negative reading. It is, therefore, necessary to reverse the direction of current coil terminals to take the reading.

Let W_o, i.e. the wattmeter reading be equal to the sum of stator core losses and mechanical losses. Let V_o and I_o be the per phase values of voltage and current.

Then no load power factor is

$$\cos\theta_o = \frac{W_o}{3V_o I_o}$$

However $\qquad I_C = I_o \cos\theta_o$

and $\qquad I_\phi = I_o \sin\theta_o$

$$\therefore \qquad R_o = \frac{V_o}{I_c} = \frac{V_o}{I_o \cos\theta_o} \qquad (10.31a)$$

$$X_o = \frac{V_o}{I_\phi} = \frac{V_o}{I_o \sin\theta_o} \qquad (10.31b)$$

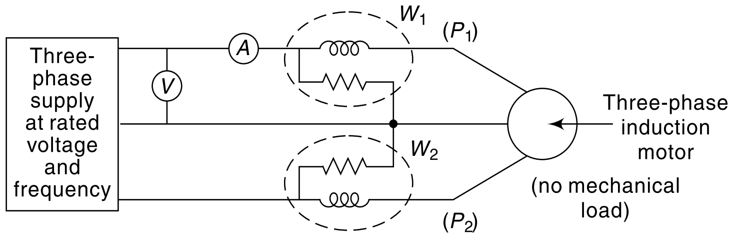


Fig. 10.7(a) *Circuit diagram for no-load test on a three-phase induction motor*

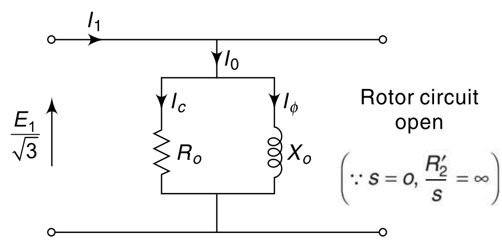


Fig. 10.7(b) *Equivalent circuit at no load*

Blocked Rotor Test The circuit is the same as shown in Fig. 10.8. The motion of the rotor is blocked by a brake (or a belt). This test is analogous to the short-circuit test of a transformer because the rotor winding is short-circuited through slip rings and in cage motors, the rotor bars are permanently short circuited. Only a reduced voltage needs to be applied to the stator at rated frequency. This voltage should be such that the ammeter reads rated current of the motor.

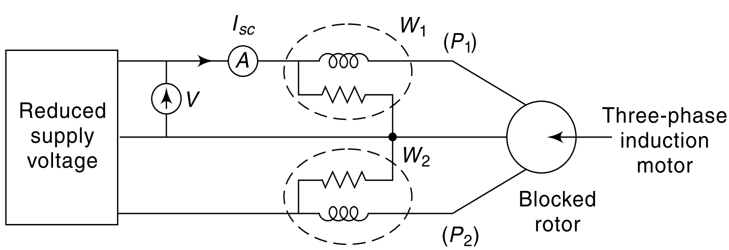


Fig. 10.8(a) *Circuit diagram for blocked rotor test*

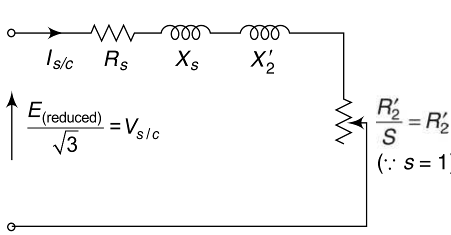


Fig. 10.8(b) *Equivalent circuit during blocked-rotor test*

The total power input on short circuit $W_{s/c}$ is equal to the algebraic sum of the two wattmeter readings, i.e. equals the copper losses of the stator and rotor. Let $V_{s/c}$ and $I_{s/c}$ be the voltage and current per phase; then the power factor under blocked rotor condition is

$$\cos\theta_{s/c} = \frac{W_{s/c}}{3(V_{s/c})(I_{s/c})} \text{ [neglecting the core and mechanical losses].}$$

Since in a R-L circuit, $R = Z\cos\theta$ and $X = Z\sin\theta$, here we can write

$$(R_s + R_2') = \left(\frac{V_{s/c}}{I_{s/c}}\right)\cos\theta_{s/c} \qquad (10.32a)$$

$$(X_s + X_2') = \left(\frac{V_{s/c}}{I_{s/c}}\right)\sin\theta_{s/c} \qquad (10.32b)$$

The stator resistance R_s is measured separately by using a battery, ammeter and a voltmeter. Then R_2' can be found from equation 10.32(a). The reactances (X_s) and (X_2') are generally assumed equal.

10.18 A three-phase, 5 HP, 400 V, 50 Hz induction motor is working at full load with an efficiency of 90% at a power factor of 0.8 lagging.
Calculate: (i) the input power and (ii) the line current.

Solution

Rating of the motor = 5 HP = 5 × 735.5 = 3677.50 watt; V = 400 V (line value); f = 50 Hz; full-load efficiency = 90% (= 0.9) and p.f = 0.8 (lagging)

(i) $\because$ Efficiency $\eta = \dfrac{\text{Output}}{\text{Input}}$,

$\therefore$ Input power $= \dfrac{\text{Output}}{\eta} = \dfrac{5\times 735.5}{0.9}$ = 4.086 kW

(ii) For a three-phase induction motor

Input power = $\sqrt{3}\, V_L I_L \cos\phi$

or $4086 = \sqrt{3} \times 400 \times I_L \times 0.8$

Hence the line current $(I_L) = \dfrac{4086}{\sqrt{3}\times 400\times 0.8}$ = 7.37 A.

10.19 A three-phase, 4-pole induction motor runs at a speed of 1440 rpm on 500 V, 50 Hz mains. The mechanical power developed by the rotor is 20.3 HP. The mechanical losses are 2.23 HP. Determine (i) the slip, (ii) the rotor copper losses (iii) the efficiency.

Solution

(i) $N_s = \dfrac{120\cdot f}{P} = \dfrac{120\times 50}{4}$ = 1500 rpm

$\therefore$ Slip $= \dfrac{N_s - N}{N_s} = \dfrac{1500-1440}{1500}$ = 0.04 or 4%.

(ii) Mechanical power developed in rotor = Power output + rotor losses = 20.3 + 2.23 = 22.53 HP = 22.53 × 735.5 = 16.571 kW

$\therefore$ Power transferred from stator to rotor $(P_{ag}) = \dfrac{16571}{(1-s)} = \dfrac{16571}{(1-0.04)}$ = 17261.46 W

$\therefore$ Rotor copper losses = 17261.46 – 16571.00 = 690.46 W.

(iii) Efficiency (η) = $\dfrac{\text{Output}}{\text{Input}} = \dfrac{20.3 \times 735.5}{17261.46}$ = 0.865 = 86.5%.

10.20 The full-load slip of a 500 HP, 50 Hz three-phase induction motor is 0.03. The rotor winding has a resistance of 0.30 Ω/phase. Determine the slip and the power output, if external resistance of 2 ohms is inserted in each rotor phase. Assume that the torque remains same.

Solution

(i) $R_2 = 0.3\ \Omega$, $R_2' = 2 + 0.3 = 2.3\ \Omega$, $s = 0.03$

$$\therefore \quad \text{Slip} \quad s' = \frac{R_2' \cdot s}{R_2} = \frac{2.3 \times 0.03}{0.3} = 0.23$$

(ii) Let N_s be the synchronous speed, then

$$N = N_s\,(1 - 0.03) = 0.97\,N_s$$

and $N' = N_s\,(1 - 0.23) = 0.77\,N_s$ [N' is the new speed when external resistance of 2 ohm is inserted in each rotor phase]

Since the torque remains same, output is directly proportional to speed.

$$\therefore \text{ New motor output} = 500 \times \frac{0.77\,N_s}{0.97\,N_s} = 397 \text{ HP.}$$

10.21 A three-phase, 50 Hz, 4-pole induction motor has a star connected wound rotor. The rotor emf is 50 V between the slip rings at standstill. The rotor resistance and standstill reactance are 0.4 Ω and 2.0 Ω respectively. Calculate

(i) the rotor current per phase at starting with slip rings short circuited,
(ii) the rotor current per phase at starting if 50 Ω per phase resistance is connected between slip rings,
(iii) the rotor emf when the motor is running at full load at 1440 rpm,
(iv) the rotor current at full load, and
(v) rotor power factor (p.f.) at full load.

Solution

$$N_s = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

(i) $E_2 = \dfrac{50}{\sqrt{3}}$ = 28.867 V.

At standstill with slip rings short circuited

$$I_2 = \frac{E_2}{(R_2^2 + X_2^2)^{0.5}} = \frac{28.867}{\{(0.4)^2 + 2^2\}^{0.5}} = 14.15 \text{ A.}$$

(ii) The total resistance in the rotor circuit is 5.4 ohm per phase.

$$\therefore \qquad I_2 = \frac{28.867}{\{(5.4)^2 + 2^2\}^{0.5}} = 5.01 \text{ A.}$$

(iii) Full load slip = $\dfrac{1500 - 1440}{1500}$ = 0.04

∴ Rotor emf = 28.87 × 0.04 = 1.555 V/Ph.

(iv) $I_2 = \dfrac{sE_2}{[R_2^2 + (sX_2)^2]^{0.5}} = \dfrac{1.155}{[0.4^2 + (0.04 \times 2)^2]^{0.5}}$ = 2.82 A.

(v) Rotor power factor (full load) = $\dfrac{R_2}{Z_2} = \dfrac{0.4}{[0.4^2 + (0.04\,X_2)^2]^{0.5}}$ = 0.98 (lagging).

10.22 A three-phase, 4-pole, 50 Hz induction motor supplies a useful torque of 160 N-m at 4% slip. Determine: (i) rotor input, (ii) motor input, (iii) efficiency. Friction and windage losses are 500 W and stator loss is 1000 W.

Solution

(i) Motor speed, $N = N_s(1 - s) = \dfrac{120\,f\,(1-s)}{P} = \dfrac{120 \times 50\,(1-0.04)}{4} = 1440$ rpm.

Gross power developed in rotor of motor

$$(P_m) = \frac{T_{\text{shaft}} \times 2\pi N}{60} + \text{friction} + \text{windage losses.}$$

or, $$(P_m) = \frac{160 \times 2\pi \times 1440}{60} + 500 = 24615 \text{ W.}$$

$$\therefore \text{ Rotor input } (P_g) = \frac{P_m}{(1-s)} = \frac{24615}{(1-0.04)} = 25640 \text{ W.}$$

(ii) Motor input (P_{in}) = Rotor input (P_{ag}) + stator losses = 25640 + 1000 = 26640 W

(iii) $$\text{Efficiency } (\eta) = \frac{\text{Net motor output } (P_o)}{\text{Motor input } (P_{\text{in}})} = \frac{24615 - 500}{26640} = 0.9052 = 90.52\%$$

10.23 A three-phase, 50 Hz, 4-pole induction motor has a slip of 4%. Determine (i) speed of the motor, (ii) frequency of rotor emf. (iii) if rotor has a resistance of 1 Ω and standstill reactance of 4 Ω, calculate power factor (a) at stand still and (b) at speed of 1400 rpm.

Solution

(i) $$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

Now, slip (s) = 0.04 = $(N_s - N)/N_s = \dfrac{(1500 - N)}{1500}$

$\therefore$ Speed of motor, N = 1440 rpm.

(ii) Frequency of rotor emf, f_2 (= sf_1) = 0.04 × 50 = 2 Hz = 120 rpm.

(iii) (a) at standstill, N = 0, so s = 1

$\therefore$ Rotor reactance = 4 × s = 4 × 1 = 4 Ω

$\therefore$ Rotor impedance = (1 + j4) ohm = 4.123 $\angle 75.96°$ Ω and p.f. (cos ϕ) = cos 75° 96′ = 0.243 (lag).

[Rotor resistance is independent of slip and hence R_2 = 1 Ω]

(b) Slip at 1400 rpm speed is given by

s' = (1500 – 1400)/1500 = 0.067.

$\therefore$ Rotor impedance (Z') = 1 + j (4 × 0.067) = (1 + j0.268) Ω

and $$\text{p.f } (\cos\phi) = \frac{1}{\{1^2 + (0.268)^2\}^{0.5}} = 0.966 \text{ lag.}$$

10.24 The power input to a 6-pole, three-phase, 50 Hz induction motor is 40 kW. Stator loss is 1 kW. Friction and windage loss = 0.2 kW. Speed is 960 RPM. Calculate (i) the slip, (ii) the BHP (iii) the rotor copper loss, and (iv) the efficiency η.

Solution

(i) $$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

$\therefore$ Slip $(s) = \dfrac{N_s - N}{N_s} = \dfrac{1000 - 960}{1000} = 4\%.$

(ii) BHP (Brake Horse Power) = $\dfrac{40}{0.7355} = 54.38$ BHP. [$\because$ 1 HP = 735.5 W]

(iii) Motor input = 40 kW, stator loss = 1 kW
$\therefore$ Rotor input = 39 kW.
Rotor copper loss = slip × rotor input = 0.04 × 39 = 1.56 kW.

(iv) Rotor gross output is $(1 - s)$ × rotor input = 39(1 – 0.04) = 37.44 kW
$\therefore$ Rotor output power = (37.44 – 0.2) kW = 37.24 kW.
$\therefore$ Motor efficiency $(\eta) = \dfrac{37.24}{40} \times 100$
i.e., η = 93% (app.).

10.25 A 18.65 kW, 4-pole, 50 Hz, three-phase induction motor has friction and windage losses of 2.6% of the output and full load slip is 4.2%. Find out (i) the rotor copper loss, (ii) the rotor input, (iii) the output torque and (iv) the gross mechanical torque developed in the rotor.

Solution

Motor output = 18650 W, friction and windage losses = $\dfrac{2.6}{100} \times 18650 = 484.9$ W.

$\therefore$ Rotor gross power developed = 18650 + 484.9 = 19134.9 W $(=P_m)$

(i) Rotor copper loss = Rotor gross power developed × $\left(\dfrac{s}{1-s}\right)$

= 19134.9 × 0.042/(1 – 0.042) = 838.89.

(ii) Rotor input $P_{ag} = \dfrac{\text{Rotor copper loss}}{\text{slip}} = \dfrac{838.89}{0.042} = 19973.5$ W.

[Alternatively: Rotor input = 19134.9 + 838.89 = 19973.79 W]

(iii) $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{4} = 1500$ rpm

$N = N_s(1 - s) = 1500(1 - 0.042) = 1437$ rpm.

Shaft torque, $T_{sh} = \dfrac{\text{output in watts}}{(2\pi N/60)} = \dfrac{(18650 \times 60)}{2\pi \times 1437} = 123.6$ Nm

(iv) Gross mechanical torque T_m

$$= \frac{\text{Gross mechanical power developed in rotor in watts}}{2\pi N/60} = \frac{19134.9 \times 60}{2\pi \times 1437} = 127.2 \text{ Nm.}$$

10.26 A three-phase, 50 Hz, 6-pole induction motor runs at 950 rpm and delivers 8 kW output. What starting torque will the motor develop when switched directly onto the supply if maximum torque is developed at 800 rpm, the friction and windage losses being total of 840 W.

Solution

$$\text{Slip } s = \frac{N_s - N}{N_s} = \frac{1000 - 950}{1000} = 0.05$$

$$s_{max} = \frac{N_s - N_{max}}{N_s} = \frac{1000 - 800}{1000} = 0.2$$

$\therefore$ Motor shaft power (P_{md}) = 8000 + 840 = 8840 W.

We know $P_{md} = 2\pi NT$

$\therefore$ Torque $(T) = \dfrac{8840 \times 60}{2\pi \times 950} = 88.90$ Nm

Also, $T_{st} = \dfrac{s^2 + s_{max}^2}{s(1 + s_{max}^2)} \cdot T$

Here, $T_{st} = \dfrac{(0.05)^2 + (0.2)^2}{0.05\{1 + (0.2)^2\}} \times 88.9 = 72.63$ Nm.

10.27 A three-phase, 440 V, 50 Hz. 6-pole induction motor running at 950 rpm takes 50 kW at a certain load. The friction and windage loss is 1.5 kW and stator losses = 1.2 kW. Determine (i) the slip (ii) the rotor copper loss (iii) the output from the rotor and (iv) efficiency.

Solution

(i) Slip $= \dfrac{N_s - N}{N_s} = \dfrac{1000 - 950}{1000} = 0.05.$

$\left(\text{as } N_s = \dfrac{50 \times 120}{6} = 1000 \text{ rpm}\right).$

(ii) Rotor copper loss = slip × rotor input = 0.05 × 48.8 kW = 2.44 kW.
[rotor input = input – stator loss = 50 – 1.2 = 48.8 kW]

(iii) Rotor output = Rotor input – Rotor copper loss – Friction and windage loss
= 48.8 – 2.44 – 1.5 = 44.86 kW.

(iv) Efficiency $(\eta) = \dfrac{\text{motor output}}{\text{motor input}} \times 100 = \dfrac{44.86}{50} \times 100 = 0.897 = 89.7\%.$

10.28 A three-phase, 415 V, 50 Hz star connected 4-pole induction motor has stator impedance $Z_1 = (0.2 + j0.5)\ \Omega$ and rotor impedance referred to stator side is $Z_2 = (0.1 + j0.5)\ \Omega$ per phase. The magnetizing reactance is 10 Ω and resistance representing core loss is 50 Ω on per phase basis.

Determine (i) the stator current (ii) the stator power factor (iii) the rotor current. Consider slip as 0.04.

Solution

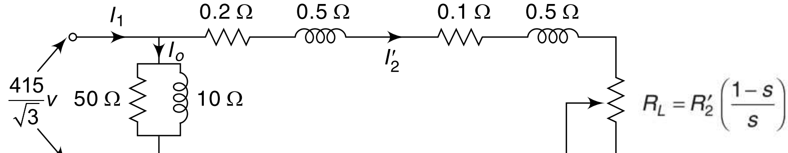


Fig. 10.9 ***Circuit diagram of Ex. 10.28***

Let Z be the total impedance of the circuit (Fig. 10.9).

Load resistance $R_L = R_2'\left(\dfrac{1-s}{s}\right) = 0.1\left(\dfrac{1-0.04}{0.04}\right) = 2.4\ \Omega.$

$\therefore$ Total resistance $R_{1e} = 2.4 + 0.2 + 0.1 = 2.7\ \Omega$

and total reactance $X_1 = 0.5 + 0.5 = 1\ \Omega$

Impedance $Z_1 = \sqrt{(2.7)^2 + 1} = \sqrt{8.29} = 2.88\ \Omega$.

Angle of Z_1 is $\tan^{-1}$ is (1/2.7) i.e., 20.323° (lag.)

Given, $V_L = 415$ V.

$$\therefore \quad V_{\text{phase}} = \frac{415}{\sqrt{3}} = 240 \text{ V}$$

$$\therefore \quad I_2' = \frac{V_{\text{ph}}}{Z_1} = \frac{240\angle 0°}{2.88\angle 20.325°} = 83.36\ \angle{-20.323°} \text{ A}$$

i.e., rotor current (referred to stator) = 83.36 A (Ans. of (iii))

[Also, $I_2' = 83.36\ \angle{-20.323}$

$$\therefore \quad = \frac{V_{\text{ph}}}{Z_1} = \frac{2400°}{2.88/20.325}$$

$\because \quad I_0 = I_C + I_\phi$,

and $\quad I_C = I_0 \cos \phi_0$, we have, $I_C = \dfrac{240}{50} = 4.8$ A

and $\quad I_\phi = I_0 \sin \phi_0 = \dfrac{240}{10} = 24$ A

$\therefore \quad I_0 = (4.8 - j24)$ A

Thus, $\quad I_1 = I_0 + I_2' = (4.8 - j24) + (78.17 - j28.95) = (82.97 - j52.95)$ A

$\therefore \quad |I_1| = = \sqrt{(82.97)^2 + (52.95)^2}\ \ 98.44$ A (Ans. of (i))

Again, $\tan \phi_1 = \dfrac{52.95}{82.97} = 0.63818$

or, $\phi_1 = 32.545°$.

i.e., $\cos \phi_1 = \cos (32.545°) = 0.843$ (lagging) (Ans. of (ii))

10.29 A 20 Hp three-phase, 400 V star connected induction motor gave the following test results:

DC test with the stator windings of two phases in series: 21 V, 30 A.

No load test: Applied voltage 400 V line, line current 8 A, wattmeter reading (2360) W and (–1160) W.

Short circuit test: Applied voltage 140 V, line current 33 A, wattmeter reading 2820 W and –370 W.

Determine the parameters of the equivalent circuit. Assume $X_1 = X_2'$.

Solution

Since two phases of stator windings are in series in the dc test, we have

$$2R_1 = \frac{21}{30} = 0.70\ \Omega$$

or $\quad R_1 = 0.35\ \Omega$.

No load test:

$$V_o = \frac{400}{\sqrt{3}} = 230.95 \text{ V} ;\ I_o = 8 \text{ A}.$$

$$W_o(W_{10} + W_{20}) = 2360 - 1160 = 1200 \text{ W}.$$

$$\therefore \quad \cos \theta_o = \frac{1200}{3 \times 230.95 \times 8} = 0.216$$

$$R_o = \frac{V_o}{I_o \cos \theta_o} = \frac{230.95}{8 \times 0.216} = 133.65\ \Omega.$$

$$X_o = \frac{V_o}{I_o \sin \theta_o} = \frac{230.95}{8 \times 0.976} = 29.57 \text{ W}.$$

Short circuit test:

$$V_{sc} = \frac{140}{\sqrt{3}} = 80.83 \text{ V}$$

$$I_{sc} = 33 \text{ Amps}; W_{sc} = 2450 \text{ W}$$

$$\therefore \quad \cos \theta_{sc} = \frac{2450}{3 \times 80.83 \times 33} = 0.306$$

$$\therefore \quad R_1 + R_2' = \frac{V_{sc}}{I_{sc}} \cdot \cos \theta_{sc} = \frac{80.83}{33} \times 0.306 = 0.745 \ \Omega$$

and $$X_1 + X_2' = \frac{V_{sc}}{I_{sc}} \cdot \sin \theta_{sc} = \frac{80.83}{33} \times \sin \theta_{sc} = \frac{80.83}{33} \times 0.9518 = 2.33 \ \Omega$$

$$\therefore \quad X_1 = X_2' = 0.5 \times 2.33 \simeq 1.666 \ \Omega.$$

and $$R_2' = 0.745 - 0.35 = 0.395 \ \Omega.$$

Hence we can write

$$R_0 = 133.65 \ \Omega$$
$$X_0 = 29.57 \ \Omega$$
$$R_1 = 0.35 \ \Omega$$
$$R_2' = 0.395 \ \Omega$$
$$X_1 = X_2' = 1.166 \ \Omega.$$

10.30 A three-phase, 50 Hz, 500 V induction motor develops 20 BHP at a slip of 4%. The mechanical losses are 1 HP. Calculate the efficiency η, if the stator loss is 1000 W.

Solution

Here $V_L = 500$ V, $f = 50$ Hz, $s = 4\% = 0.04$

Given, BHP = 20, stator losses = 1000 W,

Mechanical loss = 1 HP = 735.5 W

Power output = 20 BHP = 20 × 735.5 W = 14710 W = Rotor net output.

∴ Rotor gross mechanical power developed = Rotor net output + Mechanical loss
= 14710 + 735.5 = 15445.5 W.

$$\text{Hence, Rotor input} = \frac{\text{Rotor gross mechanical power developed}}{(1-s)} = \frac{15445.5}{1-0.04} = 16089.06 \text{ W}$$

∴ Stator input = rotor input + stator losses = 16089.06 + 1000 = 17089.06 W.

10.31 A three-phase, 6-pole induction motor develops 30 HP including 2 HP mechanical losses at a speed of 960 rpm from 550 V, 50 Hz mains. The power factor is 0.9 lagging. Determine (i) the slip (ii) the rotor copper loss (iii) the total input, if stator losses are 2 kW (iv) the efficiency and (v) the line current.

Solution

Here $P = 6$, $N = 960$ rpm, $f = 50$ Hz,

$V_L = 550$ V, p.f = $\cos \phi = 0.90$

(i) $$N_s = \frac{120 f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Slip $s = \dfrac{N_s - N}{N_s} = \dfrac{1000 - 960}{1000} = 0.04 = 4\%$.

(ii) Power transferred from stator to rotor $(P_{ag}) = \dfrac{30 \times 735.5}{1 - s} = \dfrac{30 \times 735.5}{(1 - 0.04)} = 22984.4$ W

$\therefore$ Rotor copper loss = (22984.4 – 28 × 735.5) = 2390.4 W.

(iii) Total input = (30 × 735.5 + 2000) = 24,065 W.

(iv) Efficiency $= \dfrac{30 \times 735.5}{22984.4} = 0.96 = 96\%$.

(v) Line current $(I_L) = \dfrac{\text{Input}}{\sqrt{3}\, V_L \cos\phi} = \dfrac{24065}{\sqrt{3} \times 550 \times 0.9} = \dfrac{24065}{857.365} = 28.06$ A.

10.14 STARTING OF THREE-PHASE INDUCTION MOTORS

A three-phase induction motor has a definite positive starting torque. When switched on to supply it starts itself but draws a high starting current. This is evident from the equivalent circuit. At the time of starting, slip $s = 1$ and hence the resistance $\left[\dfrac{R_2\,(1 - s)}{s}\right]$ becomes zero (The motor behaves as a short circuited transformer). The current in the rotor and the stator windings may be about five times more than full load values.

These high rotor and stator currents cause many problems:

(a) High electromagnetic forces between the conductors on the same part.

(b) High heat generation causing high temperature may damage the insulation.

(c) High current (at low power factor) may cause an appreciable drop in supply voltage causing undesirable effects on other equipments.

Therefore suitable means must be provided with the motor at start, to limit the starting current upto safe value.

The device which is used to start the three-phase induction motor is termed as starter. The function of the starter is to limit the initial rush of current to a predetermined safe value.

The various methods of starting the three-phase induction motor are:

1. By *Direct On Line (DOL) starter*
2. By *Star-delta starter*
3. By *Auto-transformer starter*.

Direct On Line Starting (DOL)

For small size squirrel cage (less than 2 HP) motor or for motors in power system where inrush of high-starting current is permissible, direct start may be used. For these small motors, the starting torque is about twice the full-load torque and the starting period lasts only a few seconds.

Figure 10.10 shows a starter for direct starting with in-built short circuit, overload and under voltage protection. When the motor is to be started, the main switch is put on and start button is pressed. This energises the relay coil S causing the normally open contacts S_1, S_2, S_3 to close. Power is supplied to the motor and

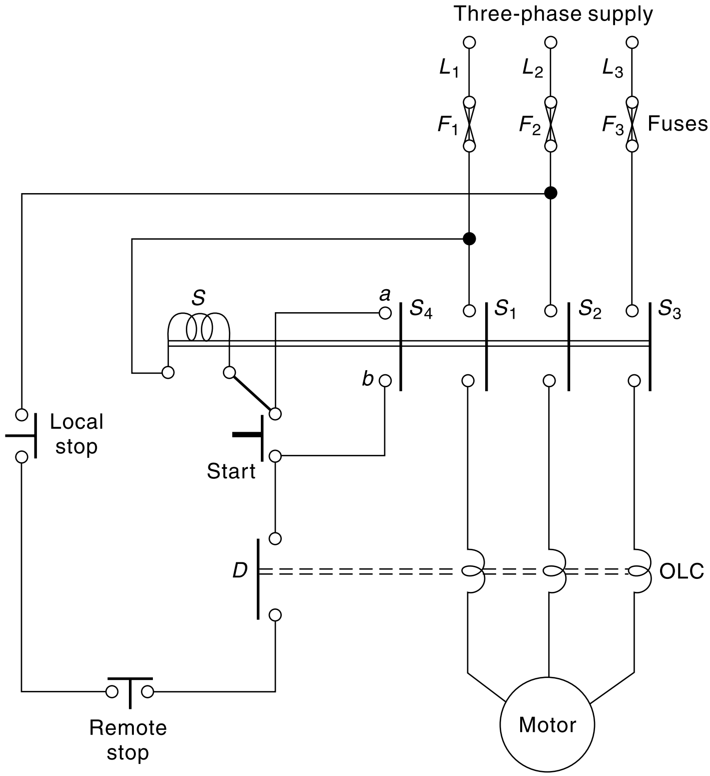


Fig. 10.10 *Diret on-line starter*

it starts. The contact S_4 also shuts, thus shorting out the starting switch allowing the operator to release it without removing power from the S relay. When the stop button is pressed, the S relay is de-energised and the S contacts open, thus stopping the motor. Short circuit protection is provided by fuses F_1, F_2 and F_3. Thermal overload relay (OLC) protects the motor from sustained overloads opening the contact D.

Star-Delta Starter

Figure 10.11 shows the diagram of the star-delta starter. Star-delta starter can be used only for those three-phase induction motors whose stator winding has been designed for delta connection. All the six terminals (of the three phases) are brought out. For starting, the phases are connected in star thereby reducing the voltage of each phase to $\dfrac{1}{\sqrt{3}}$ of its normal value.

From Eq. (10.30a) we have

$$P_{as} = \frac{I_2'^{\,2} R_2'}{s}$$

$$\therefore \qquad T = \frac{P_{as}}{w_s} = \frac{1}{u_s} \cdot I_2'^{2} R_2' \times \frac{1}{s}$$

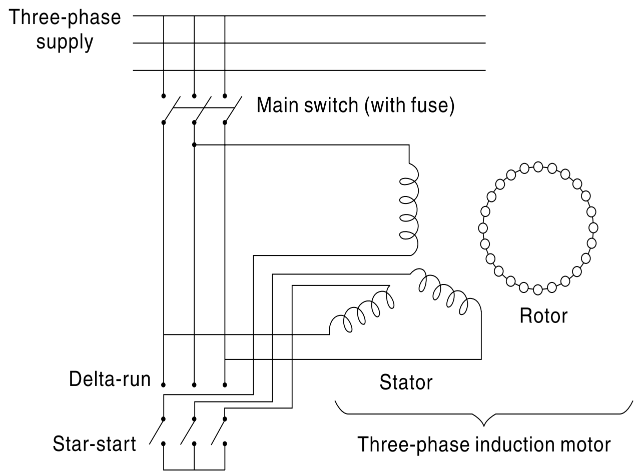


Fig. 10.11 ***Star-delta starter***

At $\quad S = 1$ (i.e., at starting),

$$T_s = \frac{1}{N_s} I_2''^2 \times R_2' \quad [I_2'' \text{ is the rotor current reflected at primary at starting}]$$

$$\therefore \quad \frac{T_s}{T} = \frac{I_2''^2}{I_2'^2}$$

If T represents full load figure, I_2' the full fold rotor current reflected to primary, we have $I_{f.l} = I_2'$, neglecting the magnetizing branch current. Similarly I_2'' represents the starting current (I_3) at stator, the magnetizing branch being neglected.

$$\therefore \quad \text{We can write, } \frac{T_s}{T} = \left(\frac{I_s}{I_{fl}}\right)^2 \times s_{fl} \tag{10.33}$$

The starting line current of the motor with star-delta starter is thus also reduced to $\frac{1}{\sqrt{3}}$ full voltage starting line current. The starting torque which is proportional to $\left(\frac{E_1}{\sqrt{3}}\right)^2$ is reduced to 1/3 of the full load torque. Thus, for star delta start though we are able to reduce the starting current, we sacrifice the torque and the starting torque reduces to 1/3 of the full load torque.

Let us analyse the star delta starting method to find the torque. We assume that the motor first operates with star connection [Fig. 10.12(a)] and when speeds up it operates with delta connection of the stator [Fig. 10.12(b)].

In Fig. 10.12(a),

Starting line (phase) current ($I_{s(\text{star})}$) is given by

$$I_{s(\text{star})} = \frac{E/\sqrt{3}}{Z_s} = \frac{1}{\sqrt{3}} \cdot \frac{E}{Z_s} = \frac{1}{\sqrt{3}} \cdot I_{P(\text{start})} \quad [I_{P(\text{start})} \text{ is the starting phase current}]$$

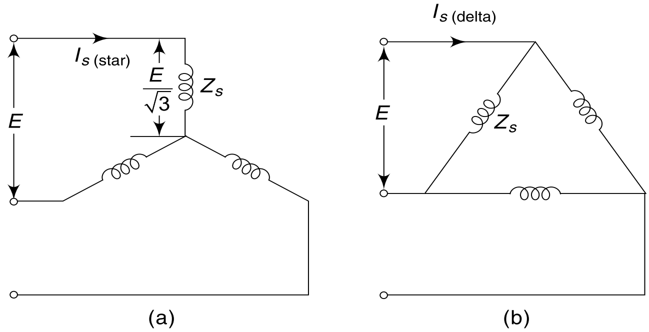


Fig. 10.12 ***Star-delta starting***

In Fig. 10.12(b),

$$\text{Starting phase current } I_{P(\text{start})} = \frac{E}{Z_s}$$

$$\therefore \quad \text{Starting line current } I_{s(\text{delta})} = \sqrt{3}\, I_{P(\text{start})}$$

$$\therefore \quad \frac{I_{s(\text{start})}}{I_{s(\text{delta})}} = \left(\frac{1}{\sqrt{3}} \cdot I_{P(\text{start})}\right) \div (\sqrt{3}\, I_{P(\text{start})}) = \frac{1}{3}.$$

Using relation (10.33) we can write $T_{s(\text{star})}/T_{\text{fl}} = \frac{1}{3}\,(I_{P(\text{start})}/I_{\text{fl}})^2 \times s_{\text{fl}}$. Thus starting torque is $\frac{1}{3}$ of that obtained in DOL starting.

This method is bit economical one but for motors rated beyond 3 KV, this method is not applicable. Like other three-phase motor starters, in this starter also overload coil and no-voltage coils are provided for the protection of the motor (not shown in the star-delta figure). An automatic star-delta starter can also be made by using push button, contactors, time delay relay (TDR), etc.

Auto-Transformer Starter (Fig. 10.13)

In this method reduced voltage is obtained by some fixed tappings on the three-phase auto transformer. Generally 60 to 65% tappings can be used to obtain a safe value of starting current. The full rated voltage is applied to the motor by taking the auto-transformer out of the motor circuit when motor has picked up the speed upto 85% of its normal speed. Figure 10.13 shows the circuit.

Let us assume that the input voltage E is reduced to xE using auto-transformer tappings.

$\therefore$ the motor starting currnet is, $I_s = xI$, where I is the motor starting current when full voltage E is applied. However, the current drawn from the supply $I_{s(\text{line})}$ is obtained from the relation

$$\frac{I_{s(\text{line})}}{I_{s(\text{motor})}} = x$$

$\therefore$ here we have $I_s(\text{line}) = x \cdot I_{s(\text{motor})} = x^2 I.$

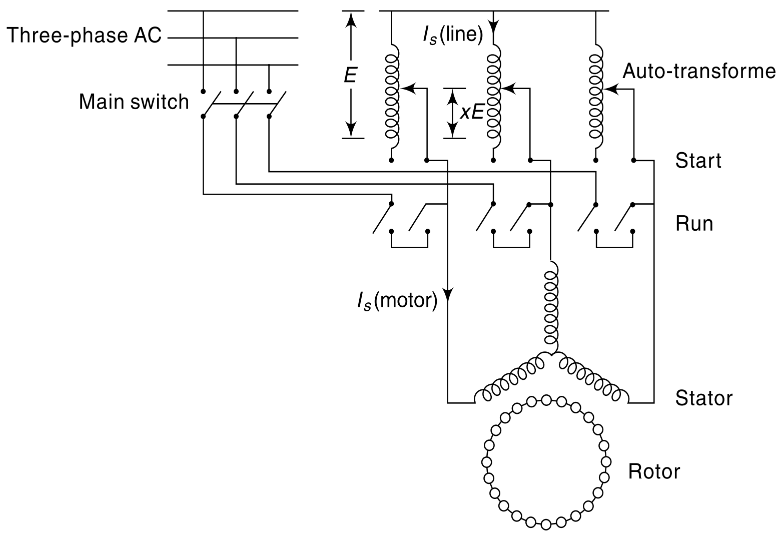


Fig. 10.13 *Auto-transformer starter*

Hence from relation (10.33) we get

$$\frac{T_s}{T_{fl}} = x^2\left(\frac{I}{I_{fl}}\right)^2 \cdot s_{fl}$$

It is found that while the starting torque is reduced by x^2 of that of DOL start, starting line current is also reduced by same fraction.

10.15 COMPARISON AMONG DIRECT ON LINE STARTER, STAR DELTA STARTER AND AUTO- TRANSFORMER STARTER

DOL starter	*Star delta starter*	*Auto-transformer starter*
1. Full voltage is applied to the motor at the time of starting.	1. Each winding gets 58% of the rated line voltage at the time of starting.	1. The starting voltage can be adjusted according to the requirement.
2. The starting current is 5–6 times of the full load current.	2. The starting current is reduced to $\frac{1}{3}$ that of direct on line starting.	2. The starting current can be reduced as desired.
3. The three windings are connected generally in star.	3. The three windings are connected in star at the time of starting, and then in delta at the time of running.	3. The three windings are generally connected in delta.
4. Only three wires are to be brought out from the motor.	4. Six wires to be brought out from the motor.	4. Only three wires are to be brought out from the motor.

(*Contd*)

(*Contd*)

5. Easy to connect motor with direct on line.	5. Identification of three starting leads and three end leads is not so easy.	5. Input and output connections of the auto-transformers are to be made properly.
6. Very easy operation.	6. It is required that connections are first to be made in star, and then in delta either manually or automatically.	6. Skilled operator is needed for connection and starting.
7. Low cost.	7. More cost	7. High cost.
8. Less space required for installation.	8. More space required	8. More space required.
9. Used for motor up to 5 HP.	9. Up to 10 HP	9. Large motors.

10.16 SPEED CONTROL OF A THREE-PHASE INDUCTION MOTOR

The synchronous speed (N_s) of a three-phase induction motor is given by

$$N_s = \frac{N}{1-s} \quad \text{or } N = N_s\,(1-s) = \frac{120\,f}{P}(1-s)$$

The speed N of induction motor can be changed by the three basic methods.

(a) Frequency Control Changing the supply frequency f the speed can be varied directly proportional to the supply frequency of ac supply.

(b) Pole Changing Speed control can also be obtained by changing the number of poles P on the stator (as speed is inversely proportional to the number of poles). This change can be incorporated by changing the stator winding connections with a suitable switch. The change in the number of stator poles P changes the synchronous speed N_s of the rotating flux, thereby the speed of the motor also changes.

(c) By Changing the Slip This can be accomplished by introducing resistance in the rotor circuit, which causes an increase in slip, thereby bringing down the speed of the motor.

Change of Supply Frequency

If the frequency of the supply to the stator of an induction motor is changed its synchronous speed is changed depending on the frequency and hence provides a direct method of speed control. To keep the magnetization current within limits, the applied voltage must be reduced in direct proportion to the frequency. Otherwise the magnetic circuit will become saturated resulting in excessive magnetization current.

The starting torque at reduced frequency is not reduced in the same proportion, because rotor power factor improves with reduction in frequency. The torque that can be produced by the maximum permissible rotor current is equal to that at rated conditions. Since power is the product of torque and speed, operation at reduced speed results in lesser permissible output.

This method of speed control is not a common method and hence this method would be used only as a special case.

In earlier days, the variable frequency was obtained from a motor generator set or mercury arc inverter. In recent days frequency control is used by SCR based inverters or by using IGBT inverters.

Pole Changing

If an induction motor is to run at different speeds, one way is to have different windings for the motor so that it will have different synchronous speeds and the running speeds. Another method is used with suitable connections for a change-over to double the number of poles. The principle of formation of consequent poles is used. The method of changing the number of poles is accomplished by producing two sections of coils for each phase which can be reversed with respect to the other section. It is important to note in this connection that slot angle (i.e. electrical degrees), phase spread, breadth factor and pitch factors will be different for the low and high speed connections. The three phases can be connected in star or delta, thus giving a number of connections. If 50% per pole pitch is used for a high speed connection, a full-pitch winding is obtained for low speed connection. The method of connecting coils of a four pole motor is shown in Fig. 10.14 for one phase and also change over connections to obtain eight poles for the same machine with the same winding.

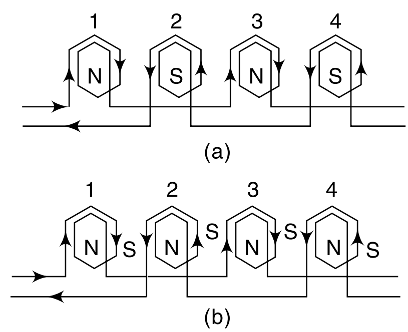


Fig. 10.14 ***Four-pole/eight-pole connections for one phase of induction motor***

The methods of speed control by pole changing are suitable for squirrel cage motors only because, a cage rotor has as many poles induced in it as there are in the stator and can thus adopt when the number of stator poles changes.

By Line Voltage Control

The torque developed by an induction motor is proportional to square of voltage. If the applied voltage to the motor is reduced, the torque is reduced and the slip is increased. Therefore, this method of speed control is applicable over a limited range only. This method is sometimes used on small motors driving fans, whose torque requirement is proportional to square of speed.

10.17 REVERSAL OF ROTATION

The direction of rotation of a three-phase induction motor can be reversed by reversing the direction of the rotation of the magnetic field. This can be done by interchanging the connections of any two of the three wires of the three-phase power supply. This causes the currents in the phases to interchange their relative timings in going positive and negative with the result that the magnetic field produces reversal in direction of rotation.

10.32 A cage motor has a starting current of 40 A when switched on directly. Auto-transformer with 45% tapping is used.

Determine (i) starting current and (ii) ratio of starting torque with auto-transformer to the starting torque with direct switching.

Solution

The ratio of transformation (x) is 0.45

(a) $\therefore$ Starting current with auto-transformer = $(0.45)^2 \times 40 = 8.1$ A.

(b) $$\frac{\text{Starting torque with auto-tranformer}}{\text{Starting torque with direct starting}} = (0.45)^2 = 0.2025.$$

10.33 A three-phase, 10 kW, 6-pole, 50 Hz, 400 V of delta connected induction motor runs at 960 rpm on full load. If it draws 85 A on direct on line starting, calculate the ratio for the starting torque to full load torque with Y-Δ starter. Power factor and full load efficiency are 0.88 and 90% respectively.

Solution

Given: Output = 10 kW
No. of poles = 6
Frequency f = 50 Hz
N = 900 rpm
η = 90%
Full load p.f. = 0.88.

Full-load line current drawn by a three-plane Δ-connected induction motor is given as

$$(I_{\text{fl}}) = \frac{\text{Output in watt}}{\sqrt{3} \cdot V_L \times \text{P.f.} \times \text{efficiency}}$$

$$= \frac{10 \times 1000}{\sqrt{3} \times 400 \times 0.88 \times 0.9} = \frac{10000}{548.71} = 18.22 \text{ A}$$

Now, full-load current per phase (Δ-connection)

$$I_{\text{fl}} = \frac{18.22}{\sqrt{3}} = 10.52 \text{ A.}$$

On direct on line start the current I_{sc} drawn by the motor per phase is given as

$$I_{sc} = \frac{85}{\sqrt{3}} = 49.07 \text{ A}$$

Synchronous speed (N_s) = $\dfrac{120\,f}{P} = \dfrac{120 \times 50}{6}$ = 1000 rpm

Full-load slip (s) = $\dfrac{N_s - N}{N_s} = \dfrac{1000 - 960}{1000}$ = 0.04

$\because$ $$\frac{T_s}{T_{\text{fl}}} = \frac{1}{3}\left(\frac{I_{sc}}{I_{\text{fl}}}\right)^2 \times s_{\text{fl}}$$

$\therefore$ Here $$\frac{T_s}{T_{\text{fl}}} = \frac{1}{3}\left(\frac{49.07}{10.52}\right)^2 \times 0.04 = 0.290.$$

10.34 A three-phase delta connected cage type induction motor when connected directly to a 400 V, 50 Hz supply takes a starting current of 105 A in each stator phase. Find out

(i) the line current for DOL starting

(ii) line and phase starting currents for Y-Δ starting, and

(iii) line and phase starting currents for a 70% tapping on auto-transformer starting.

Solution

(i) Direct on line (DOL) starting current = $\sqrt{3} \times 105 = 181.86$ A

(ii) In Y-Δ starting, phase voltage on starting $= \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.9$ V

Since 400 V produce 150 A in phase winding, $\dfrac{400}{\sqrt{3}}$ will produce $\dfrac{105}{\sqrt{3}} = 60.62$ A

$\therefore$ Starting phase current = 60.62 A.

In Y-connection, line current = phase current

$\therefore$ Starting line current = 60.62 A.

(iii) In auto transformer start, with 70% tapping on auto-transformer, the line voltage across the Δ-connected motor = 0.7×400 V. For Δ-connection,

In Δ-connection, phase voltage = line voltage = $0.7 \times 400 = 280$ V. Since 400 V produces 105 A in phase winding. (0.7×400) V = 280 V produces $0.7 \times 105 = 73.5$ A.

Hence motor phase current is 73.5 A.

$\therefore$ Motor line current = $\sqrt{3} \times 73.5 = 127.30$ A.

But $\dfrac{\text{supply line current}}{\text{motor line current}} = \dfrac{\text{motor applied voltage}}{\text{supply voltage}} = 0.7$

$\therefore$ Supply line current = $0.7 \times 127.30 = 89.11$ A.

10.35 Calculate the suitable tapping on an auto-transformer starter for a three-phase induction motor required to start the motor with 50% of full-load torque. The short circuit current of the motor is 6 times the full load current and full-load slip is 0.035. Also calculate the current drawn from the main supply as a fraction of full-load current.

Solution

Starting torque = $x^2 \cdot \left(\dfrac{I_{sc}}{I_{fl}}\right)^2 \times$ slip at full load $\times$ torque at full load

or $0.5 \times$ Torque at full load = $x^2 \times 6^2 \times 0.035 \times$ torque at full load

$$\therefore \qquad x^2 = \frac{0.5}{6^2 \times 0.035} = 0.396$$

i.e., $x = 0.629$

$\therefore$ Current drawn from the supply mains = $x^2 I_{sc} = 0.396 \times 6 \times I_{\text{full-load}} = 2.736\, I_{\text{fl}}$.

ADDITIONAL EXAMPLES

10.36 A 10-pole, 50 Hz, three-phase star connected slip ring induction motor has a rotor resistance of 0.05 Ω and a standstill rotor reactance of 0.3 Ω per phase. At full load the motor is running at a speed of 585 rpm. Determine the slip at maximum torque and find the ratio of maximum to full load torque.

Solution

$$s_{max} = \frac{R_2}{X_2} = \frac{0.05}{0.3} = 0.167$$

i.e. at 16.7% slip, torque will be maximum.
The full load slip is obtained as

$$s_{fl} = \frac{N_s - N}{N_s} = \frac{600 - 585}{600} = 0.025 \text{ i.e., } 2.5\%$$

$$\left[\because N_s = \frac{120\,f}{P} = \frac{120 \times 50}{10} = 600 \text{ rpm; } N = 585 \text{ rpm.}\right]$$

In Eq. 10.18(c) the ratio of full load torque and maximum torque has been obtained.

$$\frac{T}{T_{max}} = \frac{2 \cdot s \cdot s_{max}}{s^2 + s_{max}^2}.$$

For the given problem, we find

$$\frac{T_{max}}{T} = \frac{(0.025)^2 + (0.167)^2}{2 \times 0.025 \times 0.167} = 3.414.$$

10.37 A 6-pole three-phase induction motor is running at a speed of 950 rpm when the input is 50 kW. At this condition the stator copper loss is 1.5 kW and the rotational loss is 1 kW. Determine the rotor copper loss, electromagnetic power developed by the rotor and the mechanical power output.

Solution

Synchronous speed = $\frac{120 \times 50}{6}$ = 1000 rpm

Rotor speed 950 rpm.

Hence slip $s = 1 - \frac{950}{1000} = 0.05$

Input = 50 kW

Stator copper loss = 1.5 kW

Hence, air gap power (P_{ag}) = (50 – 1.5) = 48.5 kW

∴ Rotor copper loss is (sP_{ag}) (= 0.05 × 48.5) or 2.425 kW

∴ Gross mechanical power developed by the rotor is

[(1 – s) P_{ag}] or, (1 – 0.05) × 48.5 = 46.075 kW.

∴ Shaft power output = gross mech. power – rotational loss

= 46.075 – 1 = 45.075 kW.

10.38 A three-phase, 415 V, 4 kW delta connected induction motor has a short-circuit line current of 20 A at 200 V. The motor is started by a star-delta starter. If the full load efficiency and p.f. are 0.85 and 0.8 respectively determine the starting current drawn by the motor and ratio of starting to full load current.

Solution

Short-circuit line current of the motor at 200 V is 20 A. Hence phase current of the motor is $\frac{20}{\sqrt{3}}$ A, i.e., 11.55 A.

The phase voltage of the motor is $\frac{415}{\sqrt{3}}$ = 239.6 V (when started by star delta starter).

Hence the starting current drawn by the motor is $\left[11.55 \times \frac{239.6}{200} \text{A}\right]$, i.e., 13.83 A.

At full load condition the motor is delta connected.

Hence full load line current is $\dfrac{4000}{\sqrt{3} \times 415 \times 0.85 \times 0.8}$ = 8.184 A.

$\therefore$ The ratio of starting to full load current $\dfrac{13.83}{8.184}$ = 1.689.

10.39 The following are the parameters of the equivalent circuit of a 415 V, three-phase, 6-pole star connected induction motor:

Stator impedance = (0.2 + j0.5) Ω
Magnetizing reactance = 25 Ω
Core loss resistance = 150 Ω.

Equivalent rotor impedance referred to the stator = (0.3 + j0.7) Ω

Determine the stator current, rotor current, mechanical power output and input power at slip of 4% using the exact equivalent circuit.

Solution

The per phase exact equivalent circuit of the motor is shown in Fig. 10.15.

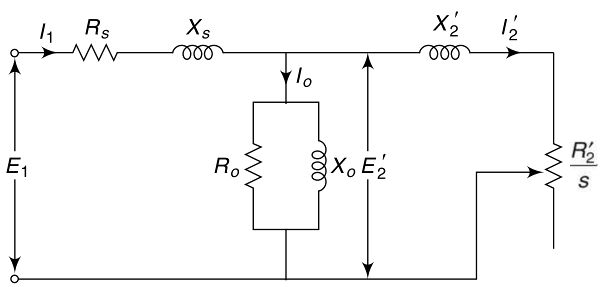


Fig. 10.15 ***Equivalent circuit of Ex. 10.39***

Here, $R_s + jX_s$ = (0.2 + j0.5) Ω
X_o = 25 Ω
R_0 = 150 Ω; s = 0.04

$$\therefore \quad \frac{R_2'}{s} + jX_2' = \frac{0.3}{0.04} + j0.7 = (7.5 + j0.7)\ \Omega$$

The parallel combination of R_o and X_o gives

$$Z_o = \frac{R_o\ (jX_o)}{R_o + jX_o} = \frac{150(j\,25)}{150 + j\,25} = \frac{j150}{6+j}\ \Omega\,.$$

However, Z_o is in parallel with $\left[\frac{R_2'}{s} + jX_2'\right]$

Hence total input impedance is $Z = Z_s + \dfrac{Z_o\left[\frac{R_2'}{s} + jX_2'\right]}{Z_o + \left[\frac{R_2'}{s} + jX_2'\right]}$.

$$\therefore \qquad Z = (0.2 \times j0.5) + \frac{\dfrac{j150}{6+j}(7.5 + j\,0.7)}{\dfrac{j150}{6+j} + (7.5 + j0.7)}$$

$$= (0.2 + j0.5) + \frac{24.67\angle 80.54° \times 7.53\angle 5.33°}{24.67\angle 80.54° + 7.5 + j\,0.7}$$

$$= (0.2 + j0.5) + \frac{185.765\angle 85.87°}{11.55 + j\,25.03}$$

$$= (0.2 + j0.5) + \frac{185.765\angle 85.87°}{27.57\angle 65.23°}$$

$$= 0.2 + j0.5 + 6.74\ \angle 20.64° = (6.51 + j2.87)\ \Omega.$$

$\therefore$ Stator current I_1 is obtained as

$$I_1 = \frac{E_1}{Z} = \frac{\dfrac{415}{\sqrt{3}}\angle 0°}{6.51 + j2.87} = \frac{239.6\angle 0°}{7.11\angle 23.79°} = 33.7\ \angle{-23.79°}\ \text{A}.$$

Voltage across the magnetizing branch obtained as

$$\begin{aligned} E_2' &= E_1 - I_1(R_s + jX_s) \\ &= 239.6\angle 0° - 33.7\angle{-23.79°}\ (0.2 + j0.5) \\ &= 239.6\angle 0° - 18.13\angle 44.4° \\ &= 226.65 - j12.68 = 227\angle{-3.2°}\ \text{V}. \end{aligned}$$

$\therefore$ Current through R_0 is, $I_C = \dfrac{227\angle{-3.2°}}{150} = 1.51\angle{-3.2°}$ A

Current through X_0 is, $I_\phi = \dfrac{227\ \angle{-3.2}}{j25} = 9.08\angle{-93.2°}$ A

Hence no load current $I_0 = 1.51\angle{-3.2°} + 9.08\angle{-93.2°}$

$$\begin{aligned} &= (1.5 - 0.5) + j(-0.08 - 9.06) \\ &= 1 - j9.146 = 9.2\angle{-83.76°}\ \text{A}. \end{aligned}$$

Also, the rotor current referred to stator is

$$\begin{aligned} I_2' = I_1 - I_0 &= 33.7\angle{-23.79°} - 9.2\angle{-83.76°} \\ &= (30.8 - 1) + j(-13.59 + 9.145) \\ &= 29.8 - j4.445 = 30.13\angle{-8.48°}\ \text{A}. \end{aligned}$$

Per phase mechanical power output is

$$I_2'^2 R_2' \left(\frac{1-s}{s}\right) = (30.13)^2 \times 0.3 \left(\frac{1-0.04}{0.04}\right)$$

$$= 907.82 \times 0.3 \times 24 = 6536.3\ \text{W}.$$

Per phase input power is ($3E_1I_1 \cos\theta$)

i.e.,
$$\begin{aligned} P_{\text{in}} &= 3 \times 239.6 \times 33.7 \cos 23.79° \\ &= 22165.286\ \text{W} = 22.165\ \text{kW}. \end{aligned}$$

10.40 A three-phase, 50 Hz, 100 kW induction motor has a full load efficiency of 85%. The stator copper loss and rotor copper loss are each equal to the stator core loss at full load. The mechanical loss is equal to one fourth of the rotor copper loss. Calculate (i) the rotor copper loss, (ii) the air gap power, and (iii) the slip.

Solution

Input power of the motor is $\frac{100}{0.85}$ kW $= 117.65$ kW

Total losses = Input – Output = (117.65 – 100) kW = 17.65 kW

Let stator copper loss = rotor copper loss = stator core loss = P

Mechanical loss = $\frac{1}{4}$ rotor copper loss = $\frac{P}{4}$

Now total loss = stator core loss + stator copper loss + rotor copper loss + mechanical loss

$$= P + P + P + \frac{P}{4} = \frac{13P}{4}$$

Hence $\frac{13P}{4} = 17.65$

or $P = 5.43$

∴ Rotor copper loss is 5.43 kW.

Air gap power = Input power – Stator core loss – Stator copper loss

$= 117.65 - 5.43 - 5.43 = 106.788$ kW.

But Rotor copper loss = Slip × Air gap power

Hence Slip $= \frac{5.43}{106.788} = 0.05.$

10.41 A 440 V, 50 Hz, 8-pole star connected three-phase induction motor has the following test results:

No load test: 440 V, 25 A, 2500 W

Blocked rotor test: 150 V, 115 A, 9000 W

Determine the equivalent circuit parameters of the motor when the per phase stator resistance is 0.2 Ω.

Solution

From no load test we have,

Per phase voltage $E_0 = \frac{440}{\sqrt{3}}$ V $= 254.03$ V

Per phase current $I_C = 25$ A

Per phase power $W_0 = \frac{2500}{3}$ W $= 833.33$ W

Power factor under no load condition $(\cos \theta_0) = \frac{W_o}{V_o I_o}$.

Hence $\cos \theta_0 = \frac{833.33}{254.03 \times 25} = 0.1312$ (lag)

[usually the no load p.f. is very low for induction motors]

Core loss component of the no load current

$$I_C = I_0 \cos \theta_0 = 25 \times 0.1312 = 3.2804 \text{ A},$$

magnetizing component of the no load current

$$I_\phi = I_0 \sin \theta_0 = 25 \sin (\cos^{-1} 0.1312) = 24.783 \text{ A}.$$

Voltage across the magnetizing branch is obtained as $[V_0 - I_0(R_s + jX_s)]$, where $(R_s + jX_s)$ is the per phase stator impedance in ohms.

Again from blocked rotor test data we have

Per phase voltage $E_{s/c} = \frac{150}{\sqrt{3}}$ V = 86.6 V

Per phase current $I_{s/c}$ = 115 A

Per phase power $W_{s/c} = \dfrac{9000}{3}$ W = 3000 W

∴ Per phase impedance $Z_{s/c} = \dfrac{E_{s/c}}{I_{s/c}} = \dfrac{86.6}{115}\ \Omega = 0.753\ \Omega$

Per phase resistance $R_{s/c} = \dfrac{W_{s/c}}{I_{s/c}^2} = \dfrac{3000}{(115)^2}\ \Omega = 0.2268\ \Omega$

Per phase reactance $X_{s/c} = \sqrt{(0.753)^2 - (0.2268)^2} = 0.718\ \Omega$

Per phase rotor resistance referred to the stator,

$$R_2' = R_{s/c} - R_s = 0.2268 - 0.2 = 0.0268\ \Omega.$$

We assume here that per phase stator reactance X_s = Per phase rotor reactance X_2'

$$\therefore \qquad X_1 = X_2' = \frac{X_{s/c}}{2} = \frac{0.718}{2} = 0.359\ \Omega.$$

Voltage across the magnetizing branch is obtained from the formula $[E_0 - I_0(R_s + jX_s)]$. This gives the required voltage as [254.03 – 25 (0.2 + j0.359)] V.

i.e., (249.03 – j8.975) V or (249.192∠–2.064° V)

$$\therefore \qquad \text{Core loss resistance } R_C = \frac{249.192}{I_C} = \frac{249.192}{3.2804}\ \Omega = 75.963\ \Omega.$$

$$\text{Magnetizing reactance } X_o = \frac{249.192}{I_\phi} = \frac{249.192}{24.783}\ \Omega = 10.054\ \Omega.$$

Hence the equivalent circuit parameters are

$R_0 = 75.963\ \Omega$ $\quad X_0 = 10.054\ \Omega$ $\quad R_s = 0.2\ \Omega$

$R_2' = 0.0268\ \Omega$ $\quad X_s = X_2' = 0.359\ \Omega.$

10.42 A 415 V, 50 Hz, 8-pole three-phase delta connected squirrel cage induction motor has a starting current of 30 A when connected directly to the supply. Find (i) the line and phase current drawn by the motor when connected directly on line, (ii) the line current when started by an auto-transformer with 70% tapping and (iii) the line current when started by a star-delta starter.

Solution

(i) Line current when connected directly to the supply is 30 A.
As the motor is delta connected the phase current under direct online supply is $\dfrac{30}{\sqrt{3}}$ A = 17.32 A.

(ii) At 70% auto-transformer tapping the applied line voltage is 415 × 0.7 V = 290.5 V.
As the motor is delta connected phase voltage is 290.5 V.
When phase voltage is 415 V, the phase current is 17.32 A.
When phase voltage is 290.5 V, the current supplied by the auto transformer is $\dfrac{17.32 \times 290.5}{415}$ = 12.124 A.
Hence phase current of the motor = 0.7 ×12·124 = 8.48 A.
Hence the line current when started by auto-transformer starter is 8.48 × $\sqrt{3}$ = 14.7 A.

(iii) When the motor is started by a star delta starter, the motor is connected in star at the instant of starting. Hence, phase voltage of the motor during starting $\frac{415}{\sqrt{3}}$ V = 239.6 V. Phase current at phase voltage of 239.6 V is 17.32 × $\frac{239.6}{415}$ = 10 A.

∴ Line current (= phase current) = 10 A, at start.

10.43 A 5 kW, 4-pole, three-phase star connected inductor motor has slipring rotor resistance of 0.05 Ω and standstill reactance of 0.5 Ω for phase. The full-load speed is 1450 rpm. Determine the ratio of maximum torque to the full-load torque, starting torque to the full-load torque and ratio of starting torque to the full-load torque.

Solution

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

$$N = 1450 \text{ rpm (given)}$$

$$\therefore \; s_{fl}\text{(full load slip)} = 1 - \frac{1450}{1500} = 0.033 \; (= s) \text{ (i.e., 3.3\%)}$$

$$s_{max} \text{ (slip at maximum torque)} = \frac{R_2}{X_2} = \frac{0.05}{0.5} = 0.1 \text{ (i.e., 10\%)}$$

$$\because \quad \frac{T}{T_{max}} = \frac{\text{Full load torque}}{\text{Maximum torque}} = \frac{2 \cdot s \cdot s_{max}}{s^2 + s_{max}^2}$$

Here, $$\frac{T}{T_{max}} = \frac{2 \times 0.033 \times 0.1}{0.033^2 + 0.1^2} = \frac{0.0066}{0.011} = 0.595$$

$$\therefore \quad \frac{T_{max}}{T} = 1.68$$

Also, $$\frac{T_{starting}}{T_{max}} = \frac{2 \cdot s_{max}}{1 + s_{max}^2} = \frac{2 \times 0.1}{1 + 0.1^2} = 0.198.$$

We have seen in the text that

$$T_s = \frac{KE_1^2 R_2}{R_2^2 + X_2^2}$$

and $$T = \frac{KE_1^2 \cdot sR_2}{R_2^2 + (sX_2)^2}$$

$$\therefore \quad \frac{T_s}{T} = \frac{KE_1^2 R_2}{R_2^2 + X_2^2} \times \frac{R_2^2 + (sX_2)^2}{KE_1^2 sR_2}$$

$$= \frac{R_2^2 + (sX_2)^2}{(R_2^2 + X_2^2)s} = \frac{(0.05)^2 + (0.033 \times 0.5)^2}{0.033(0.05^2 + 0.5^2)} = \frac{0.0028}{0.00833} = 0.336$$

i.e. $T_s/T = 0.336$.

10.44 A 4 pole, 50 Hz, three-phase induction motor has a starting torque 17.8% of the full load torque and maximum torque 135% of the full load torque. Determine the full load speed and speed at maximum torque.

Solution

$$\frac{\text{Starting torque}}{\text{Full load torque}} = \frac{T_s}{T} = \frac{17.8}{100} = 0.178.$$

$$\frac{\text{Maimum torque}}{\text{Full load torque}} = \frac{T_{\max}}{T} = \frac{135}{100} = 1.35.$$

Hence, $$\frac{T_s}{T_{\max}} = \frac{0.178}{1.35} = 0.1318.$$

Now, at slip s if the torque be T, then we have $\dfrac{T}{T_m} = \dfrac{2}{\dfrac{s}{s_{\max}} + \dfrac{s_{\max}}{s}}$ where s_m is the slip at maximum torque.

$$\left[\because \text{ From equation 10.18(c), we have } \frac{T}{T_m} = \frac{2 \cdot s \cdot s_{\max}}{s^2 + s_{\max}^2} = \frac{2}{\dfrac{s}{s_{\max}} + \dfrac{s_{\max}}{s}}\right]$$

At starting $s = 1$, hence, from the above relations we can write,

$$\frac{T_{st}}{T_m} = \frac{2}{\dfrac{1}{s_m} + \dfrac{s_m}{1}} = 0.1318$$

or $$0.1318\, s_{\max}^2 - 2s_{\max} + 0.1318 = 0$$

or $$s_{\max} = 0.066.$$

$$\text{Synchronous speed } (N_s) = \frac{120 \times 50}{4} = 1500 \text{ rpm.}$$

If N_1 be the speed at maximum torque

then $$N_1 = (1 - s_{\max})N_s = (1 - 0.066)\ 1500 = 1401 \text{ rpm.}$$

$\because$ $$\frac{T}{T_{\max}} = \frac{2}{\dfrac{s_{fl}}{s_m} + \dfrac{s_m}{s_{fl}}},$$

Here, $$\frac{1}{1.35} = \frac{2}{\dfrac{s_{fl}}{0.066} + \dfrac{0.066}{s_{fl}}}$$

or $$\frac{1}{1.35} = \frac{2 \times s_{fl} \times 0.066}{s_{fl}^2 + 0.004356}$$

or $$s_{fl}^2 - 0.1782\, s_{fl} + 0.004356 = 0$$

$\therefore$ $$s_{fl} = 0.029.$$

Hence full load speed is [(1 – 0.029) 1500] or, 1456.5 rpm.

10.45 An 8-pole, 50 Hz, three-phase induction motor has a full-load torque of 200 Nm when the frequency of the rotor emf is 2.5 Hz. If the mechanical loss is 15 Nm determine the rotor copper loss and the efficiency of the motor. The total stator loss is 1000 W.

Solution

If s be the slip at full load then

$$s.f = 2.5 \text{ (from the given data)}$$

$$\therefore \qquad s = \frac{2.5}{50} = 0.05.$$

The speed of the motor at full load

$$N = (1 - s)\, N_s$$

But synchronous speed $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{8} = 750$ rpm

Hence $\qquad N = (1 - 0.05)\, 750 = 713$ rpm.

or $\qquad \omega = \dfrac{713 \times 2\pi}{60}$ rpm $= 74.665$ rad/s.

Mechanical power developed by the rotor

$$P_m = (200 + 15) \text{ Nm} = 215 \text{ Nm} = 215 \times 74.665$$
$$= 16052.975 \text{ W.} \quad [\because P = \omega_s \times T]$$

If P_{ag} be the air gap power then,

$$(1 - s)\, P_{ag} = P_m$$

or $\qquad P_{ag} = \dfrac{16.053}{1 - 0.05}$ kW $= 16.898$ kW

Rotor copper loss is $(sP_g) = 0.05 \times 16.898 = 0.845$ kW

Motor input = 16.052 + 0.845 + 1 = 17.897 kW

Motor output = 200 × 74.665 W = 14.933 kW

Hence efficiency is $\dfrac{14.933}{17.897} = 0.8344$ or 83.44%.

10.46 The resistance of the rotor winding of a 4-pole, 50 Hz, three-phase induction motor is 0.2 Ω per phase and the maximum torque developed is 15 Nm when the motor is running at a speed of 1350 rpm. Determine (i) the torque at a slip of 4% and (ii) the external resistance to be added to the rotor circuit to obtain 70% of the maximum torque at starting.

Solution

Synchronous speed $N_s = \dfrac{120 \times 50}{4} = 1500$ rpm

Slip at maximum torque $s_{\max} = \left(1 - \dfrac{1350}{1500}\right) = 0.1.$

If P_2 and X_2 be the resistance and reactance of the rotor circuit,

$$\frac{R_2}{X_2} = s_{\max} \text{ or, } X_2 = \frac{R_2}{s_{\max}} = \frac{0.2}{0.1} = 2\ \Omega.$$

Writing the expression for a three-phase maximum torque using equation 10.17(b), we get

$$T_{\max} = \frac{3}{\omega_s} \cdot \frac{E_1^2}{2X_2} = \frac{3E_1^2}{\omega_s \times 2 \times 2} = \frac{3}{4} \cdot \frac{E_1^2}{\omega_s} \qquad \text{(i)}$$

(i) When slip is 4%, we can write, $s = 0.04$. From torque equation (10.13) we have, for three phase torque,

$$T = \frac{3}{\omega_s} \cdot \frac{E_1^2 \cdot s \cdot R_2}{[R_2^2 + (sX_2)^2]} \text{ Nm.} \qquad \text{(ii)}$$

Replacing $\frac{3}{\omega_s} \cdot E_1^2$ from equation (ii) by $4T_{\max}$ (as obtained in equation (i), we get

$$T = \frac{4T_{\max} \cdot s \cdot R_2}{[R_2^2 + (sX_2)^2]} = \frac{4 \times 15 \times 0.04 \times 0.2}{0.2^2 + (0.04 \times 2)^2} = 10.34 \text{ Nm.}$$

(ii) Starting torque $T_{st} = 0.7 \times 15 = 10.5$ Nm (as per question);
at starting, slip is 1.

If R be the net rotor resistance after addition of external resistance then

$$T_s = \frac{3E_1^2 \cdot R}{\omega_s (R^2 + X_2^2)} \text{ Nm} \qquad \text{[using equation (ii) with } R_2 \text{ replaced by } R \text{ and } s = 1]$$

i.e., $10.5 = \frac{3E_1^2}{\omega_s} \cdot \frac{R}{R^2 + X_2^2} = 4T_{\max} \cdot \frac{R}{R^2 + 2^2}$

i.e. $10.5R^2 + 42 = 60R$

i.e., R = either 4.9 Ω or 0.816 Ω

With $R = 4.9\ \Omega$, $s_{\max} = \frac{R}{X_2} = \frac{4.9}{2} = 2.45$, which is an impossible value.

∴ Feasible value of R is 0.816 Ω

Thus external resistance to be added is $(R - R_2)$, i.e., (0.816 – 0.2) or, 0.616 Ω/phase.

10.47 A 6-pole, 50 Hz, three-phase induction motor has a maximum torque of 200 Nm when it is running at a speed of 900 rpm. The resistance of the rotor is 0.25 Ω. Neglecting stator impedance determine the torque at 5% slip.

Solution

Synchronous speed $N_s = \frac{120 \times 50}{6} = 1000$ rpm

At maximum torque slip $s_{\max} = \left[1 - \left(\frac{900}{1000}\right)\right] = 0.1$

$R_2 = 0.25$ (given)

Hence, $X_2 = \frac{R_2}{s_{\max}} = \frac{0.25}{0.1} = 2.5\ \Omega$

Torque at any slip s is given by

$$T = \frac{3}{\omega_s} \cdot \frac{E_1^2\, sR_2}{[R_2^2 + (sX_2)^2]} \text{ Nm}$$

$$= \frac{3}{\omega_s} \cdot E_1^2 \times \frac{0.05 \times 0.25}{(0.25)^2 + (0.05 \times 2.5)^2} = \frac{3E_1^2}{\omega_s}\ 0.16.$$

Maximum torque is given by

$$T_{\max} = \frac{3}{\omega_s} \cdot \frac{E_1^2}{2X_2} = \frac{3E_1^2}{\omega_s} \cdot \frac{1}{2 \times 2.5} = \frac{1}{5} \cdot \frac{3E_1^2}{\omega_s}$$

Using this expression of ($T_{\max}$) in the expression of (T) we get

$$T = 5T_{\max}\ 0.16$$

As $T_{\max} = 200$ Nm, we get

$$T = 5 \times 200 \times 0.16 = 160 \text{ Nm.}$$

10.48 A three-phase squirrel cage induction motor has a full-load torque one third of the maximum torque. The rotor resistance and reactance are 0.25 and 3Ω respectively. Determine the ratio of starting torque to full load torque when it is started by (i) direct on line starter (ii) auto-transformer starter with 60% tapping and (iii) star delta starter.

Solution

Given: $$\frac{T}{T_{\max}} = \frac{1}{3}, \quad \therefore \quad T_{\max} = 3T$$

Slip at maximum torque $$s_{\max} = \frac{R_2}{X_2} = \frac{0.25}{3} = 0.083$$

Also, $$\frac{T}{T_{\max}} = \frac{2s \cdot s_{\max}}{s^2 + s_{\max}^2} = \frac{2}{\dfrac{s}{s_{\max}} + \dfrac{s_{\max}}{s}},$$ where s is the slip.

At any load if T_{st} be the torque at starting then

$$\frac{T_{st}}{T_{\max}} = \frac{2}{\dfrac{1}{s_m} + \dfrac{s_m}{1}} = \frac{2}{\dfrac{1}{0.083} + 0.083}$$

or $$T_{st} = 0.16486\ T_{\max}.$$

(i) During direct on line starting

$T_{st} = 0.16486 \times 3T = 0.49458\ T \quad [\because\ T_{\max} = 3T].$

(ii) During auto-transformer starting with 60% tapping $T_{st} = x^2 .\ T_{st}$ (for direct on line)

i.e. $$T_{st} = (0.49458)\ (0.6)^2 T$$

or $$T_{st} = 0.178\ T.$$

(iii) During starting by a star delta starter

$$T_{st} = \frac{1}{3} \times T_{st} \text{ (for direct for line)}$$

i.e., $$T_{st} = \frac{1}{3} \times 0.49458\ T = 0.165\ T.$$

10.49 A 6-pole, three-phase induction motor develops 35 HP including 3 HP mechanical losses at a speed of 960 rpm when connected to 440 V, 3-phase mains. The power factor is 0.8. Find (i) the slip (ii) the rotor copper loss (iii) the total input if stator loss is 3 kW and (iv) the efficiency.

Solution

Synchronous speed $$N_s = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

Speed of the motor $N = 960$ rpm

(i) Slip $= \left(1 - \dfrac{960}{1000}\right) = 0.04.$

(ii) Gross mechanical power developed is 35 × 735.5 W = 25742.5 W.

or $P_m = 25742.5$ W = 25.742 kW.

$$\therefore \text{ Air gap power } (P_{ag}) = \frac{P_m}{1-s} = \frac{25.742}{1-0.04} = 26.81 \text{ kW}.$$

Hence, rotor copper loss is sP_g, i.e., 0.04 × 26.81 or 1.072 kW.

(iii) Stator loss 3 kW (given).

Hence, total input = 26.81 + 3 = 29.81 kW.

(iv) $\because$ Input $= 29.81$ kW and
Output $= (35 - 3) = 32$ HP $= 32 \times 0.7355$ kW $= 23.536$ kW,

Hence, efficiency is $\dfrac{23.536}{29.81} \times 100\% = 78.95\%$.

10.50 A 15 kW, 4-pole, 50 Hz, three-phase induction motor has a mechanical loss 2% of the output. For a full load slip of 3% determine the rotor copper loss and air gap power.

Solution

Output = 15 kW

Mechanical loss = $\dfrac{2}{100} \times 15 = 0.3$ kW

Slip = 0.03

Power developed by the rotor is $P_m = 15 - 0.3 = 14.7$ kW

If P_{ag} be the air gap power then $(1 - s)P_g = P_m$

or
$$P_g = \frac{14.7}{1-0.03} \text{ kW} = 15.15 \text{ kW}$$

Rotor copper loss $sP_g = 0.03 \times 15.15 = 0.4545$ kW.

10.51 A 10 kW, 440 V, three-phase star connected, 50 Hz, 8-pole squirrel cage induction motor has the following per phase constants referred to the stator.

$$R_1 = 0.2\ \Omega, \quad X_1 = 1\ \Omega, \quad R_2 = 0.18\ \Omega, \quad X_2 = 1.5\ \Omega, \quad X_0 = 30\ \Omega$$

The constant loss is 500 W and the slip is 5%.

Determine the stator current, output torque and efficiency.

Solution

The equivalent circuit of the induction motor is shown in Fig. 10.16,

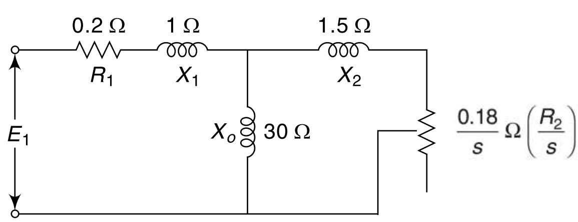


Fig. 10.16 ***Equivalent circuit of Ex. 10.51***

The per phase applied stator voltage $E_1 = \dfrac{440}{\sqrt{3}}$ V = 254 V

Slip $s = 0.05$ (given)

$$\text{The total impedance from input} = (0.2 + j1) + \frac{j30\left(\dfrac{0.18}{0.05} + j1.5\right)}{j30 + \dfrac{0.18}{0.05} + j1.5}\ \Omega$$

$$= 0.2 + j + \frac{-45 + j108}{3.6 + j31.5}\ \Omega$$

$$= 0.2 + j + \frac{117\angle 112.62^\circ}{31.7\angle 83.48^\circ}$$

$$= 0.2 + j + 3.69\angle 29.14^\circ\ \Omega$$

$$= 1.073 + j1.487 + 1.834\angle 54.186^\circ\ \Omega.$$

$\therefore$ Stator current $I_1 = \dfrac{254}{1.834\angle 54.186^\circ} = 138.49\angle{-54.186^\circ}$.

Stator input power $= \sqrt{3}\, E_L\, I_L \cos 54.186^\circ$

$$= \sqrt{3} \times 440 \times 138.49 \cos 54.186^\circ = 61.757 \text{ kW}$$

Air gap power $P_{ag} = 61.757 - \dfrac{3\,(138.49)^2 \times 0.2}{1000} = 50.25$ kW [$\therefore$ P_{ag} = stator input power – stator copper loss, core loss being neglected in stator.]

Mechanical power developed

$$P_m = (1 - s)\, P_g = (1 - 0.05)\, 50.25 = 47.737 \text{ kW}.$$

Power output $P_{out} = \left(47.737 - \dfrac{500}{10^3}\right) = 47.237$ kW.

Output torque $T_{out} = \dfrac{47.237 \times 10^3}{\dfrac{2\pi \times N}{60}}$ Nm, [where N (rpm) is the speed of the motor,

$T_{out} = \dfrac{P_{out}}{\omega_s}$, $\omega_s = 2\pi n_s$, $n_s = \dfrac{N}{60}$ rpm ; $N = (1 - 0.05) \times \dfrac{120 \times 50}{8} = 712.5$ rpm.]

Hence output torque $= \dfrac{47.237 \times 10^3 \times 60}{2\pi \times 712.5} = 633.40$ Nm

$$\text{Efficiency} = \frac{\text{Power output}}{\text{Power input}} = \frac{47.237}{61.757} \times 100\% = 76.48\%.$$

10.52 A three-phase, 15 kW, 440 V, 50 Hz, 6-pole squirrel cage induction motor has a delta connected stator winding. The motor during blocked rotor test yields the following results: 240 V, 25 A, 7 kW.

The dc resistance measured between any two stator terminals is 1 Ω. If the stator core loss at rated voltage is 400 W determine the starting torque when rated voltage is applied.

Solution

If the stator winding resistance per phase is "r" then the resistance between any two terminals (see Fig. 10.17) is

$$\frac{r(r + r)}{r + (r + r)} = 1$$

or $$\frac{2r^2}{3r} = 1$$

or $$r = \frac{3}{2}\,\Omega = 1.5\,\Omega$$

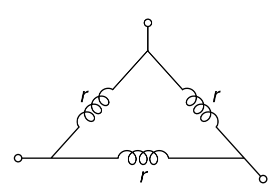


Fig. 10.17 ***Stator winding (Δ)***

At the rated voltage, power input during the blocked rotor test will be $7 \times \left(\dfrac{440}{240}\right)^2$ kW, i.e., 23.52 kW. At rated voltage, stator current during blocked rotor test will be $\left(25 \times \dfrac{440}{240}\right)$ or 45.83 A. Thus at rated voltage the stator copper loss will be $\left[3 \times \left(\dfrac{45.83}{\sqrt{3}}\right)^2 \times 1.5\right]$ or, 3150.768 W.

Air gap power P_{ag} = Power input – (Stator copper loss + Stator Core loss)
= (23520 – 3150.768 – 400) = 19.969 kW

Synchronous speed $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{6}$ = 1000 rpm

Hence starting torque when rated voltage is applied $\dfrac{P_{ag}}{\omega_s} = \dfrac{19969}{\dfrac{2\pi \times 1000}{60}}$ Nm = 190.79 Nm.

$$\left[\because \omega_s = 2\pi n_s = 2\pi \cdot \frac{N_s}{60},\ N_s \text{ being expressed in rpm}\right]$$

10.53 A three-phase squirrel cage induction motor gives a blocked rotor test current of 200% of the rated current when 30% of the rated voltage is applied. The starting torque is 30% of the rated torque. The motor when started by an auto-transformer limits the starting line current to 160% of the rated current. Determine the percentage starting torque with auto-transformer starting.

Solution

At 30% of rated voltage blocked rotor current is 200% (given)
At rated voltage blocked rotor current is

$$I_{sc} = \frac{2}{0.3} \cdot I_{fl} = 6.67\ I_{fl}, \text{ where } I_{fl} \text{ is the full load current.}$$

Now, if x be the fraction of the voltage applied to the stator during auto-transformer starting then per phase starting current would be

$I_{st} = x^2\ I_{sc}$

Here, $I_{st} = x^2\ (6.67\ I_{fl})$ (i)

Again it is given that $(I_{st}) = 1.6\ I_{fl}$ (ii)

From the equations (i) and (ii)

$$x^2 = \frac{1.6}{6.67} = 0.23988.$$

Also, $T \propto \text{voltage}^2$,

here, $0.3\ T_{fl} \propto (0.3V)^2$, where V is the rated voltage, T_{fl} is the full load torque T.

i.e. $V^2 \propto \dfrac{0.3}{0.09} \cdot T_{fl}$.

From the text we know that

$$\frac{\text{Starting torque with auto-transformer starting}}{\text{Starting torque with direct on line starting}} = x^2.$$

Hence starting torque with auto-transformer starting is,

$$x^2 \times \frac{0.3}{0.09}\ T_{fl} = (0.23988)^2 \times \frac{0.3}{0.09} \cdot T_{fl} = 0.1918 T_{fl}$$

i.e., starting torque with auto-transformer starting is 19.18% of the full-load torque.

10.54 The rotor resistance of an 8-pole, 50 Hz. wound rotor induction motor has a resistance of 0.5 Ω per phase. The speed of the rotor is 720 rpm at full load.

Determine the external resistance to be connected with the rotor circuit to reduce the speed to 680 rpm for full-load torque.

Solution

Synchronous speed $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{8}$ = 750 rpm.

$$\text{Slip }(s) = \left[1 - \frac{720}{750}\right] = 0.04$$

If R_2 be the rotor resistance then the rotor copper loss is $I_2^2 R_2$, where I_2 is the rotor current. If P_{ag} be the air gap power or power input to the rotor then

$$sP_{ag} = I_2^2 R_2$$

or
$$0.04 = \frac{I_2^2 R_2}{P_{ag}} = \frac{0.5 I_2^2}{P_{ag}} \tag{i}$$

The new speed $N = 680$ rpm.

$$\therefore \quad s = \left(1 - \frac{680}{750}\right) = 0.093$$

Let R be the total resistance of the rotor circuit.

In order that the full load torque remains same P_{ag} should have the same value as the previous one.

Hence,
$$0.093 = \frac{I_2^2 R}{P_{ag}} \tag{ii}$$

From equations (i) and (ii)

$$0.093 = \frac{0.04}{0.5} \cdot R$$

or
$$R = 1.1625\ \Omega$$

Hence the external resistance to be connected is $(R - R_2)$ or $(1.1625 - 0.5)$, i.e., $0.6225\ \Omega$ per phase.

10.55 A three-phase induction motor has a starting torque 150% of full load torque and maximum torque 200% of full load torque. Determine the slip at maximum torque, full load slip and rotor starting current as a percentage of full load current.

Solution

We know that in any slip s the developed torque can be expressed in terms of maximum torque, i.e.,

$$\frac{T}{T_{\max}} = \frac{2}{\dfrac{s}{s_{\max}} + \dfrac{s_{\max}}{s}},$$

where $T_{\max}$ is the maximum torque at slip $s_{\max}$.

Now,
$$\frac{\text{Starting torque }(T_s)}{\text{Full load torque }(T)} = 1.5 \quad \text{(given)}$$

as well as
$$\frac{T_{\max}}{T} = 2 \quad \text{(given)}$$

Hence,
$$\frac{T_s}{T_{\max}} = \frac{1.5}{2.0} = \frac{3}{4} = \frac{2}{\dfrac{1}{s_{\max}} + s_{\max}}.$$

During starting slip is 1.

$$\therefore \quad \left(\frac{1}{s_{\max}} + s_{\max}\right) = \frac{8}{3} = 2.67.$$

or $$s_{\max}^2 - 2.67\, s_{\max} + 1 = 0$$

or $$s_{\max} = \frac{2.67 \pm \sqrt{(2.67)^2 - 4}}{2}$$

$$= 0.45 \text{ (the other value is rejected as it is greater than 1)}$$

Also, $$\frac{T}{T_{\max}} = \frac{2}{\dfrac{s_{fl}}{s_{\max}} + \dfrac{s_{\max}}{s_{fl}}} = \frac{2}{\dfrac{s_{fl}}{0.45} + \dfrac{0.45}{s_{fl}}} = \frac{1}{2}$$

or $$s_{fl}^2 - 4 \times 0.45 \cdot s_{fl} + (0.45)^2 = 0$$

or $$s_{fl}^2 - 1.8\, s_{fl} + 0.2025 = 0$$

or $$s_{fl} = \frac{1.8 \pm \sqrt{(1.8)^2 - 4(0.2025)}}{2} = 0.12.$$

At full-load rotor current may be obtained as

$$I_{2fl} = \frac{E}{\dfrac{R_2}{s_{fl}} + jX_2}$$

[refer approximate equivalent circuit neglecting the magnetising branch and stator impedance]

or $$I_{2fl}^2 = \frac{E^2}{\left(\dfrac{R_2}{s_{fl}}\right)^2 + X_2^2}$$

Similarly, starting current

$$I_{2st}^2 = \frac{E^2}{R_2^2 + X_2^2}.$$

Hence $$\left(\frac{I_{2fl}}{I_{2st}}\right)^2 = \frac{R_2^2 + X_2^2}{\left(\dfrac{R_2}{0.12}\right)^2 + X_2^2}.$$

Now, $$s_{\max} = \frac{R_2}{X_2} = 0.45$$

Hence $$\left(\frac{I_{2fl}}{I_{2st}}\right)^2 = \frac{\left(\dfrac{R_2}{X_2}\right)^2 + 1}{\left(\dfrac{R_2}{0.12\, X_2}\right)^2 + 1} = \frac{(0.45)^2 + 1}{\left(\dfrac{0.45}{0.12}\right)^2 + 1} = 0.0798$$

Hence $$\frac{I_{2fl}}{I_{2st}} = \sqrt{0.0798} = 0.28$$

or $$\frac{I_{2st}}{I_{2fl}} = 3.54.$$

EXERCISES

1. What are the types of three-phase induction motors as per their rotor construction? Compare between them.
2. What are the advantages of using a three-phase induction motors in industry? What are the disadvantages?
3. Briefly explain the principle of operation of a three-phase induction motor.
4. Analytically justify how a rotating field is created in a three-phase induction motor when a balanced three-phase ac supply is applied at the stator terminals.
5. What is slip? Deduce a relationship between rotor current frequency and supply frequency in terms of slip.
6. Derive the torque equation of a three-phase induction motor in terms of rotor quantities. What is the expression for starting torque?
7. Derive the expression for maximum torque in a three-phase induction motor. Obtain the ratios of the full load torque to maximum torque and starting torque to maximum torque of such a motor.
8. Discuss the role of change of supply voltage on the torque and slip of induction motor.
9. Show that starting torque of a polyphase induction motor is governed by the rotor resistance. Hence discuss how we can improve the starting torque of a slip ring induction motor.
10. Show that for a given applied voltage, the starting torque is maximum when the rotor resistance is equal to the stand-still rotor reactance.
11. Derive the torque-slip characteristic of a three-phase inductor motor. What is the affect of variation of rotor resistance?
12. Derive the exact equivalent circuit of a polyphase induction motor on per phase basis. Draw the approximate equivalent circuit with necessary assumptions.
13. What are the different losses in a three-phase induction motor? How do you find efficiency of such a motor?
14. For a three-phase induction motor, prove the following.
    (i) Rotor copper loss = Slip × Rotor input
    (ii) Mechanical power developed in the rotor = (1 – slip) × Airgap power
    (iii) Rotor copper loss = $\left(\dfrac{\text{slip}}{1-\text{slip}}\right)$ × Mechanical power developed in rotor.
15. Why do we perform no load test and blocked rotor test on induction motors? Describe how we can find the equivalent circuit parameters from these two tests. State the necessary assumptions.
16. Why do we need a starter in starting a three-phase induction motor?
17. Write short notes on:
    (i) Direct on line starting
    (ii) Auto-transformer starting
    (iii) Star delta starter.

18. A 6-pole, 60 Hz induction motor rotates at 3% slip. Find the speed of the stator field, the rotor and the rotor field. What is the frequency of the rotor currents? [*Ans:* 1200 rpm, 1164 rpm, 1200 rpm, 1.8 Hz]

    [*Hint:* $N_s = \dfrac{120 \times 60}{6} = 1200$ rpm.]

    $\therefore$ Stator field rotates at 1200 rpm. Rotor field rotates in the air gap in the same speed.

    N(rotor speed) $= N_s(1 - s) = 1200(1 - 0.03) = 1164$ rpm

    The rotor speed is then 1164 rpm.

    If frequency of rotor current is f_r,

    $$f_r = sf_s = 0.03 \times 60 = 1.8 \text{ Hz.}$$

    Since rotor rotates at 1164 rpm while the speed of the rotor field is 1200 rpm, hence the field speed with respect to the rotor is $(N_s - N)$ i.e., 36 rpm].

19. A three-phase 8-pole squirrel cage induction motor, connected to a 400 V $(L - L)$ 50 Hz supply, rotates at 3% slip at full load. The copper and iron losses at the stator are 2 kW and 0.5 kW respectively. If the motor takes 50 kW at full load, find the full load developed torque at the rotor. [*Ans:* 605 Nm]

    [*Hint:* $P_{ag} = P_{in} - P_{scu} - P_{sc}$

    $= 50 - 2 - 0.5 = 47.5$ kW

    $$\therefore \quad T = \frac{P_{ag}}{\omega_s} = \frac{47.5 \times 10^3}{2\pi \times \dfrac{120 \times 50}{8 \times 60}} = 605 \text{ Nm.]}$$

20. The power input to a three-phase induction motor is 50 kW. Stator loss is 1 kW. Find the gross mechanical power developed in the rotor and the rotor copper loss per phase when the motor has a full load slip of 4%.

    [*Hint:* $P_{\text{in}} = 50$ kW; $s = 0.04$; $P_{scu} = 1$ kW.

    $\therefore \quad P_{ag} = P_{in} - P_{scu} = 49$ kW

    $$P_{rcu} = s \times P_{ag} = 0.04 \times 49 = 1.96 \text{ kW } \left(= \frac{1.96}{3} \text{ kW per phase}\right)$$

    $$\therefore \quad P_m = P_{ag} - P_{rcu} = \left(\frac{49}{3}\right) - \frac{1.96}{3} = 15.68 \text{ kW].}$$

21. The loss at the stator of a three-phase squirrel cage 25 HP, 1500 rpm induction motor is 2 kW. What is rotor mechanical power if the rotor copper loss is 1 kW? What is the running slip? [*Ans:* 15.65 kW; 6%]

    [*Hint:* $P_{in} = 25 \times 746 \times 10^{-3} = 18.65$ kW

    $\therefore \quad P_{ag} = 18.65 - 2 = 16.65$ kW

    $P_m = P_{ag} - P_{rcu} = 16.65 - 1 = 15.65$ kW.

    $$\because \quad P_{rcu} = s \times P_{ag};\ s = \frac{P_{rcu}}{P_{ag}} = \frac{1000}{16650} = 0.06 \text{ i.e., slip is 6\%.]}$$

22. The rotor of a 6-pole, 50 Hz, slip ring induction motor has a resistance of 0.3 Ω/phase and it runs at 960 rpm at full load. How much external resistance/phase must be added to the rotor circuit to reduce the speed to 800 rpm, the torque remaining constant? [*Ans:* 1.2 Ω]

[*Hint:* $N_s = \dfrac{120\,f}{P}$ = 1000 rpm

s_{fl} (full load slip) = $\dfrac{1000 - 960}{1000}$ = 0.04.

If r be the additional resistance per phase in rotor circuit, we can write $\dfrac{s_{\text{new}}}{s_{\text{fl}}} = \dfrac{R_2 + r}{R_2}$.

Since the power input to the rotor and rotor current remain constant for constant torque and hence from the relation, slip = $\dfrac{\text{Rotor Cu loss}}{\text{Rotor input}}$, we have

$$\frac{s_{\text{new}}}{s_{\text{fl}}} = \frac{3I_2^2(R_2 + r)}{3I_2^2 R_2} = \frac{R_2 + r}{R_2}.$$

Substitution of the values of s_{fl} = 0.04,

$s_{\text{new}} = \dfrac{1000 - 800}{1000}$ = 0.2 and R_2 = 0.3, yields r = 1.2 Ω.]

23. A three-phase, 50 Hz induction motor has an output rating of 500 HP, 3.3 kV ($L - L$). Calculate the approximate full-load current at 0.85 p.f., locked rotor current and no-load current. What is the apparent power drawn under locked rotor condition? Assume the starting current to be 6 times full load current and no load current to be 30% of full load current.

[*Ans:* I_{fl} = 76.78 A; $I_{\text{no load}}$ = 23.093 A; $I_{\text{lock rotor}}$ = 460.68 A; $P_{\text{locked rotor}}$ = 2633 KVA]

[*Hint:* $I_{\text{fl}} = \dfrac{500 \times 746}{\sqrt{3} \times 3300 \times 0.85}$ = 76.78 A

∴ $I_{\text{no load}}$ = 0.3 × 76.78 = 23.03 A

∵ $I_{\text{lock rotor}} \equiv I_{\text{start}}$

Hence $I_{\text{start}} = 6 \times I_{\text{fl}}$ = 460.68 A.

Apparent power drawn during locked rotor condition is

$$P_A = \sqrt{3} \times V_L \times I_{\text{st}} = \sqrt{3} \times 3300 \times 460.68 = 2633 \text{ KVA.}]$$

24. A 4-pole, 60 Hz, 460 V, 5HP induction motor has the following equivalent circuit parameters:

R_s = 1.21 Ω; X_s = 3.10 Ω

R_2' = 0.742 Ω; X_2' = 2.41 Ω

X_0 = 65.6 Ω

Find the starting and no-load current of the machine.

[*Ans:* 46.15∠–70.67°A; 3.87∠–89° A]

[*Hint:* With reference to the equivalent circuit of the induction motor, the input impedance looking from the input side is

$$Z_{\text{in}} = (R_s + jX_s) + \frac{jX_o\,(R_2' + jX_2')}{R_2' + jX_2' + jX_o}$$

$$= \left[(1.21 + j3.1) + \frac{j65.6\,(0.742 + j2.41)}{0.742 + j2.41 + j65.6}\right]\Omega = 5.75\angle 70.72°\ \Omega.$$

At start $s = 1.0$. This means the load resistor in equivalent circuit is, shorted, since $1 - s = 0$.

$$\therefore \quad I_{st} = \frac{V_{L-L}}{\sqrt{3}\, Z_{in}} = 46.15\angle{-70.67°} \text{ A.}$$

At no load, $s \simeq 0$, i.e., the load element in the equivalent circuit is open.

$$\therefore \quad Z_{in} \text{ (no load)} = (R_s + jX_s) + jX_0 = (1.21 + j68.7)\ \Omega = 68.71\angle 89°\ \Omega$$

$$\therefore \quad I_{NL} = \frac{V_{L-L}}{\sqrt{3}\, Z_{in(NL)}} = \frac{460\angle 0°}{\sqrt{3}\,(68.71\angle 89°)} = 3.87\angle{-89°}\text{A.}]$$

25. A 30 HP, 3-phase 6 pole, 50 Hz slip ring induction motor runs at full load at a speed of 960 rpm. The rotor current is 30 A. If the mechanical loss in the rotor is 1 kW while 200 W loss is being incurred by the rotor short circuiting system, find the rotor resistance per phase. [*Ans:* $R_2 = 0.287\ \Omega$]

[*Hint:* $N_s = 100$ rpm; $s = \dfrac{N_s - N}{N_s} = 0.04$

$\because$ Rotor Cu loss $= \dfrac{s}{1-s} \times$ gross mech power developed in rotor Hence we can write for this problem,

$$3I_2^2 R_2 + 200 = \frac{0.04}{1-0.04}(30 \times 746 + 1000)$$

or, $\quad 3 \times 30^2 \times R_2 = 774.17$

or, $\quad R_2 = 0.287\ \Omega$]

26. A 10 HP, 400 V($L - L$), 50 Hz, 3-phase induction motor has a full load p.f. of 0.8 and efficiency of 0.9. The motor draws 7 A when a voltage of 160 V is applied directly across the live terminals, the motor being standstill. Determine the ratio of starting to full load current when a star-delta starter is used to start the motor. [*Ans:* $(I_{st}/I_{fl}) = 0.39$]

[*Hint:* $I_{fl} = \dfrac{10 \times 746}{\sqrt{3} \times 400 \times 0.8 \times 0.9} = 15$ A.

$I_{s/c}$ at 160 V input = 7.0 A

$\therefore \quad I_{s/c_f}$ at 400 V($L - L$) is $\dfrac{400}{160} \times 7.0 = 17.5$ A.

With star delta starter,

$$I_{\text{starting}} = \frac{1}{3} \times I_{s/c_f} = \frac{1}{3} \times 17.5 = 5.833 \text{ A.}$$

$$\therefore \quad \frac{I_{\text{starting}}}{I_{\text{fl}}} = \frac{5.833}{15} = 0.39.]$$

27. A 75 kW, three-phase induction motor has 1500 rpm synchronous speed. It is connected across a 440 V($L - L$) supply and rotates at 1440 rpm at full load. The two wattmeter method is applied to measure the power input which shows that the motor absorbs 70 kW while the line current is 80 A. If the stator iron loss is 2 kW and rotor mechanical loss is 1.5 kW, find

(i) the power supplied to the rotor
(ii) the rotor copper loss
(iii) the mechanical power developed at shaft
(iv) the torque developed at rotor
(v) the efficiency of the motor.

Assume stator resistance/phase = 0.2 Ω.

[*Ans:* P_{ag} = 64.16 kW; P_{rcu} = 2.57 kW P_m = 61.59 kW; T = 408.66 Nm; η = 85.84%]

[*Hint:*

(i) P_{in} = 70 kW; P_{scu} (stator Cu loss)

$$= 3I_{fl}^2 \times R_s = 3 \times 80^2 \times 0.2 = 3.84 \text{ kW}.$$

P_{sc}(core loss in stator) = 2 kW

$\therefore P_{ag} = P_{in} - P_{scu} - P_{sc} = 64.16$ kW

(ii) $P_{rcu} = s \times P_{ag} = 0.04 \times 64.16 = 2.57$ kW.

(iii) $P_m = P_{ag} - P_{rcu} = 64.16 - 2.57 = 61.59$ kW.

(iv) $$T = \frac{P_{ag}}{\omega_s} = \frac{64.16 \times 10^3}{2\pi \times \dfrac{1500}{60}} = 408.66 \text{ Nm}.$$

(v) $$\eta = \frac{P_0}{P_{in}} = \frac{P_m - \text{mech loss in rotor}}{P_{in}}.$$

$$= \frac{61.59 - 1.5}{70} = 0.8584 \text{ i.e., } 85.84\%.]$$

28. A 4-pole, three-phase 50 Hz induction motor develops a maximum torque of 20 Nm at 1440 rpm. Obtain the torque exerted by the motor at 5% slip. Assume rotor resistance to be 0.5 Ω. [*Ans:* 19.51 Nm]

[*Hint:* $$N_s = \frac{120\,f}{P} = 1500 \text{ rpm}$$

$$N = 1440 \text{ rpm}$$

$$\therefore \quad s = \frac{N_s - N}{N_s} = 0.04 = s_{max}$$

But $$s_{max} = \frac{R_2}{X_2}$$

$$\therefore \quad X_2 = \frac{R_2}{s_{max}} = \frac{0.5}{0.04} = 12.5\ \Omega$$

If s be the required slip (5%) then from the relation

$$\frac{T}{T_{max}} = \frac{2 \cdot s \cdot s_{max}}{s^2 + s_{max}^2} = \frac{2 \times 0.05 \times 0.04}{(0.05)^2 + (0.04)^2} = 0.9756$$

$$\therefore \quad T = 0.9756 \times 20 = 19.51 \text{ Nm.}]$$

29. You have a 50 HP, three-phase, 60 Hz, 4-pole, 1765 rpm induction motor operating at 400 V ($L - L$). Find the shaft torque. If the mechanical loss in the shaft (rotor) is 500 W, find the gross mechanical power developed in

the shaft and the developed torque. If the rotor copper loss is 800 W, what is the value of air gap power? Also find the electromagnetic torque developed. If the total stator losses are 1 kW, find the motor input power.

[*Ans:* T_{sh} = 201.91 Nm, P_m = 37.8 kW; T_m = 204.62 Nm P_{ag} = 38.6 kW; T = 204.89 Nm; P_{in} = 39.6 kW]

[*Hint:* $$T_{sh} = \frac{P_0}{\omega} = \frac{50 \times 746}{2\pi \times \dfrac{1765}{60}} = 201.91 \text{ Nm}$$

$$P_m = P_0 + \text{mech loss in rotor}$$
$$= 50 \times 746 \times 10^{-3} + 500 \times 10^{-3} = 37.8 \text{ kW}$$

$$T_m = \frac{P_m}{\omega} = \frac{37.8 \times 10^3}{2\pi \times \dfrac{1765}{60}} = 204.62 \text{ Nm}$$

$$P_{ag} = P_m + \text{rotor copper loss}$$
$$= 37.8 \times 10^3 + 800 = 38600 \text{ W} = 38.6 \text{ kW}$$

$$T = \frac{P_{ag}}{\omega_s} = \frac{38.6 \times 10^3}{2\pi \times \dfrac{1800}{60}} = 204.89 \text{ Nm}$$

$$\left(\because N_s = \frac{120 \times 60}{4} = 1800 \text{ rpm} \right)$$

$\because$ Stator copper loss is 1 kW,

$\therefore$ $P_{in} = 1 + P_{ag} = 1 + 38.6 = 39.6$ kW.]

30. A 400 V, 4-pole, 50 Hz, 3 phase squirrel cage induction motor develops 25 HP at 4% slip on full load. If the ratio of motor resistance to standstill reactance is 1:4, find the ratio starting torque to full load torque.

[*Ans:* (T_s/T) = 1.51]

[*Hint:* N_s = 1500 rpm; $N = N_s(1 - s)$ = 1440 rpm ($\because$ s = 0.04)

$$T(\text{full load}) = \frac{25 \times 735.5}{\omega} = \frac{25 \times 735.5}{2\pi \times \dfrac{1440}{60} \times 10^3} = 0.122 \text{ Nm.}$$

At standstill the torque is T_s while at 4% slip it is T.

$$\therefore \quad \frac{Ts}{T} = \frac{\dfrac{s_0 R_2 E_1^2}{R_2^2 + (s_0 X_2)^2}}{\dfrac{sR_2 E_1^2}{R_2^2 + (sX_2)^2}}$$, (s_o) being the slip at starting.

With $s_0 = 1$,

$$\frac{T_s}{T} = \frac{R_2^2 + (sX_2)^2}{s\left[R_2^2 + X_2^2\right]} = \frac{\left(\dfrac{R_2}{X_2}\right)^2 + s^2}{s\left[\left(\dfrac{R_2}{X_2}\right)^2 + 1\right]} = \frac{\left(\dfrac{1}{4}\right)^2 + (0.04)^2}{0.04\left[\left(\dfrac{1}{4}\right)^2 + 1\right]} \simeq 1.51]$$

31. A 415 V, three-phase, 6 pole, 50 Hz induction motor runs at a slip of 4% on full load. The rotor resistance and reactance are 0.01 Ω and 0.05 Ω when the motor is at standstill. Find the ratio of full load torque to maximum torque. Also obtain the speed at which the maximum torque occurs.

[*Ans:* N_{max} = 800 rpm; (T/T_{max}) = 0.385]

[*Hint:* $s_{max} = \dfrac{R_2}{X_2} = \dfrac{0.01}{0.05} = 0.2.$

The speed at a slip of 0.2 would then be 800 rpm as N_s = 1000 rpm.

$$\because \quad \frac{T}{T_{max}} = \frac{2 \cdot s \cdot s_{max}}{s^2 + s_{max}^2}$$

for this problem $\dfrac{T}{T_{max}} = \dfrac{2 \times 0.04 \times 0.2}{(0.04)^2 + (0.2)^2} = 0.385.$]

32. For a 30 HP, 440 V, 50 Hz, three-phase, 6-pole star connected motor, no load test and blocked rotor tests are conducted. The test results are as follows:

No-load test

No load voltage V_0 line to line: 440 V

No load current I_0: 15 A

No load power P_{one} : 1550 W.

Resistance measured for each stator phase winding = 0.5 Ω

Locked Rotor test

Input voltage $(L - L)$	: 160 V
Input current	: 60 A
Input power	: 7000 W.

Determine the equivalent circuit of the motor. Assume stator leakage reactance is equal to rotor stand still reactance referred to stator side.

[*Ans:* R_0 = 130.27 Ω, X_0 = 17.08 Ω, R_s = 0.5 Ω;
R_2' = 0.148 Ω, X_s = 0.7 Ω, X_2' = 0.7 Ω]

[*Hint:* R_s = 0.5 Ω.

$$\cos\theta_o = \frac{W_o}{3V_o I_o} = \frac{1500}{3 \times \frac{440}{\sqrt{3}} \times 15} = 0.13$$

$$I_C = I_0 \cos\theta_0 = 15 \times 0.13 = 1.95 \text{ A}$$

$$I_\phi = I_0 \sin\theta_0 = 15 \times 0.99 = 14.873 \text{ A}$$

$$\therefore \quad R_o = \frac{V_o}{I_c} = \frac{440/\sqrt{3}}{1.95} = 130.27\ \Omega$$

$$X_o = \frac{V_o}{I_\phi} = \frac{440/\sqrt{3}}{14.873} = 17.08\ \Omega.$$

Next, from locked rotor test,

$$\cos\theta_{s/c} = \frac{W_{s/c}}{3(V_{s/c})(I_{s/c})} = \frac{7000}{3 \times \frac{160}{\sqrt{3}} \times 60} = 0.421$$

$$\therefore R_s + R_2' = \frac{V_{s/c}}{I_{s/c}} \cos\theta_{s/c} = \frac{160/\sqrt{3}}{60} 0.421$$
$$= 0.648$$

or $\quad R_2' = 0.648 - R_s = 0.148\ \Omega$

$$X_s + X_2' = \frac{V_{s/c}}{I_{s/c}} \sin\theta_{s/c} = \frac{160/\sqrt{3}}{60} 0.91$$
$$= 1.4\ \Omega.$$

Since $X_s = X_2'$, hence $X_s = X_2' = \frac{1.4}{2} = 0.7\ \Omega$.

The approximate equivalent circuit is shown below.

$R_0 = 130.27\ \Omega$; $X_0 = 17.08\ \Omega$; $R_s = 0.5\ \Omega$; $R_2' = 0.148\ \Omega$, $X_s = 0.7\ \Omega$; $X_2' = 0.7\ \Omega$.]

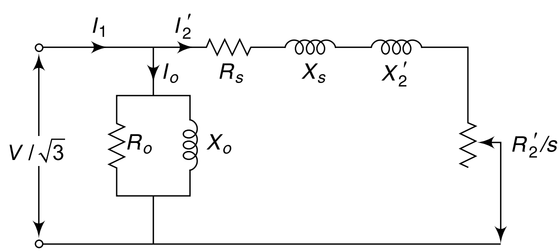


Fig. 10.18 ***Equivalent circuit of Problem 32***

33. An induction motor has a short circuit current of 6 times the full load current at normal supply voltage. It has a full-load slip of 5%.
Calculate the starting torque in terms of the full load torque if started by
   (i) star-delta starter
   (ii) auto-transformer starter with 60% tapping.
   Neglect magnetizing branch currents. [*Ans:* 0.2; 0.648]

[*Hint:* We have $\frac{T_s}{T_{fl}} = \left(\frac{I_s}{I_{fl}}\right)^2 \cdot s_{fl}$

(i) For Y-Δ starting: At start, winding is placed in star and only $\left(\frac{1}{\sqrt{3}}\right)$ of normal voltage is applied. Again, short circuit current being 6 times the full load current (winding in delta connection), we can write short circuit phase current as

$$I_{s/c(ph)} = \frac{6 \times \text{f.l. current}}{\sqrt{3}}$$

$$\therefore \quad I_{\text{starting/phase}} = \frac{1}{\sqrt{3}} \times I_{s/c(ph)}$$

as $\frac{1}{\sqrt{3}}$ of normal voltage is applied per phase.

i.e. $$I_{st/ph} = \frac{1}{\sqrt{3}} \times \frac{6}{\sqrt{3}} \times I_{fl}.$$

$\therefore$ $$\frac{I_{st}}{I_{fl}} = \frac{1}{\sqrt{3}} \times \frac{6}{\sqrt{3}} = 2$$

Hence, $$\frac{T_s}{T_{fl}} = (2)^2 \times 0.05 = 0.2.$$

(ii) Auto-transformer starting with 60% tapping:
At start, stator winding remain in delta connection. However, only 60% voltage is made available at stator.

$\therefore$ $$I_s = \frac{60}{100} \times 6\ I_{fl} = 3.6\ I_{fl}$$

Hence $$\frac{T_s}{T_{fl}} = (3.6)^2 \times 0.05 = 0.648.]$$

11 SYNCHRONOUS MACHINES

11.1 INTRODUCTION

A synchronous generator (or alternator) is the most commonly used machine for generation of electrical power. It generates alternating voltage which is stepped up and transmitted. A synchronous generator, like other electrical rotating machines, has two main components viz. the *stator* and the *rotor*. The component of the machine (the winding or the core) in which the alternating voltage is induced is called the *armature*. The armature winding is placed on the stator slots in the stator core. The rotor consists of the field poles which produce the magnetic lines of force. The poles are excited with dc supply (or may be permanent magnets in small alternator).

A synchronous machine works as an alternator when the rotor is rotated by a primemover. The same machine works as a synchronous motor, when a three-phase ac supply is applied to the armature winding, the field being excited by dc supply in both the cases.

The construction of a synchronous generator depends upon the type of primemover used to rotate the rotor. The types of primemovers are:

(a) In thermal or nuclear power stations, steam turbines are used to rotate at a high speed (3000 rpm for a two pole 50 Hz. machine or 3600 rpm for a two pole 60 Hz. machine) as at high speeds the efficiency of steam turbine is comparitively higher.

(b) For hydroelectric power stations hydraulic turbines of different types are used depending upon the water head available in that station.

(c) In diesel stations, lower rating alternators are used with diesel engines as primemover at low speed.

(d) In captive power plants, high speed gas turbines are employed as primemovers to rotate medium capacity alternators.

11.2 OPERATING PRINCIPLE

A synchronous generator essentially consists of two parts:

1. Field magnet system (rotor)
2. Armature (stator)

A 3-phase distributed winding is placed on the stator in slots of a synchronous generator to act as an armature. This machine works on the principle of electro-magnetic induction (Faraday's law). When there is a cutting of field flux by a conductor or when there is a change of flux linkage in a coil, emf is induced in the conductor.

The field magnet system is usually excited from a separate dc source of 110/125 or 250 V supply. In conventional generators it is provided by a dc shunt or compound generator called an *exciter*, mounted on the shaft of the alternator itself. The field system (also known as rotor) is rotated within the armature (stator). The exciting current is supplied to the alternator rotor through a slip ring and brushes in conventional alternators. In modern alternators *brushless excitation* and *static exciters* are also employed.

11.3 TYPES OF ROTORS

From the construction point of view there are two types of rotors:

(a) Salient pole type
(b) Cylindrical (or non-salient) rotor type.

11.3.1 Salient Pole Type

This type of rotor has a large number of projecting (salient) poles having their cores bolted on to a heavy magnetic core of cast iron or CRGO steel of good magnetic quality.

Alternators with salient pole rotors have the following features:

(a) Salient pole machines are larger in diameter but smaller in axial length.
(b) The pole shoes are wide and usually cover (2/3) pole-pitch.
(c) Poles of these rotors are laminated in order to reduce eddy current loss.
(d) These machines are suitable for medium speed (i.e. 120 to 500 rpm)

11.3.2 Cylindrical (or Non-salient) Type Rotor

A cylindrical rotor consists of a smooth solid forged steel cylinder having a number of slots for accommodating field coils. Such type of rotors are generally designed for 2-pole turbo-generators running at 3000 rpm (or 3600 rpm).

The salient features of non-salient type of alternators are:

(a) They are smaller in diameter and larger in axial length.
(b) Usually the number of poles on the rotor is two.
(c) Windage loss is less.
(d) Mechanically they are more balanced.
(e) These machines are suitable for speeds from 1000 to 3000 rpm.

11.4 STATOR

Constructionally the stator for both types of alternators is identical. The stator is made of laminations of special magnetic iron or steel alloy and can accommodate armature conductors. The whole structure is fitted in a frame. The frame may be of cast iron or welded steel plate. The laminations in the stator are insulated from each other with paper or varnish depending upon the size of the machine. The stampings also have openings which make axial and radial ventilating ducts to provide efficient cooling.

Slots provided on the stator core are mainly of two types (a) Open type and (b) Semi-closed type. Open type slots are commonly used because the coils can be formed, wound and insulated prior to being placed in the slots. These slots also provide facility in removal and replacement maintenance of defective coil. However, these type of slots have the disadvantage of distributing the air gap flux into branches which tends to produce ripples in the emf wave. The semi-closed type slots are better in this respect but do not permit the use of pre-wound coils and lacks in ventilation as well as poses difficulty in maintenance.

11.5 FIELD AND ARMATURE CONFIGURATIONS

There are two arrangements of fields and armatures:

(a) Revolving armature and stationary field.
(b) Revolving field and stationary armature.

11.6 ADVANTAGES OF ROTATING FIELD

In large alternators, rotating field arrangement is usually forward due to the following advantages.

(a) **Ease of Construction** Armature winding of large alternators being complex, the connection and bracing of the armature windings can be easily made for the stationary stator.

(b) **Number of Slip Rings** If the armature be made rotating, the number of slip rings required for power transfer from armature to external circuit is atleast three. Also, heavy current flows through brush and slip rings cause problems and require more maintenance in large alternators. Insulation required for slip rings from rotating shaft is difficult with the rotating armature system.

(c) **Better Insulation to Armature** Insulation arrangement of armature windings can easily be made from core on stator.

(d) **Reduced Rotor Weight and Rotor Inertia** The field system being placed on rotor, insulation requirement is less (for low dc voltage). Also rotational inertia is less. It takes lesser time to gain full speed.

(e) **Improved Ventilation Arrangement** The cooling can be provided by enlarging the stator core with radial ducts. Water cooling is much easier if the armature is housed in the stator.

Hence in almost all of the alternators the armature is housed in the stator while the dc field system is placed in the rotor.

11.7 EXPRESSION OF FREQUENCY

The rotor being rotated at a steady speed by means of a primemover, the magnetic field of the rotors will also rotate to cut the stationary armature conductors. This will change the flux linkage in the stator conductors and induce an alternating emf in stator conductors. The direction of induced emf is given by "Fleming's right hand rule". One cycle of emf will induce in a conductor when one pair of poles passes over it (or when the angular distance travelled by the armature is twice the pole pitch).

Let P = Number of magnetic poles

N = Rotating speed of rotor in rpm.

f = Frequency of generated emf in Hz.

$\therefore$ Number of cycles/revolution = $P/2$ and

number of revolution per second = N/60

$\therefore$ Frequency $f = \dfrac{N}{60} \times \dfrac{P}{2} = \dfrac{PN}{120}$ Hz.

[For 3000 rpm two-pole alternators, thus f becomes 50 Hz. while for 3600 rpm, two-pole alternators, f is 60 Hz.]

11.8 DISTRIBUTED ARMATURE WINDING

The armature consists of the distributed winding.

In the distributed winding, the conductors of the coil are placed in several armature slots under one pole. The induced emf per phase in the distributed winding is to some extent less than the case when winding in such a way that the number of slots is equal to the number of poles.

There are some specific advantages for the distributed winding.

(a) Voltage waveform is improved and wave form is more sinusodial in nature.
(b) Distorting harmonics are eliminated.
(c) Distributed winding is helpful in reducing armature reaction and armature reactance too.
(d) Heat dissipation is better.
(e) The core is better utilised.
(f) It is suitable for higher current density due to even distribution of copper.

11.9 SHORT-PITCH ARMATURE WINDING

In short-pitch windings, two coil sides (forming a coil) are not exactly 180° electrically apart; the emf induced in the two sides are thus not in phase and the resultant of induced emf in the coil is less than the arithmetic sum of the emf induced in the coil sides of the full-pitched winding.

This winding is advantageous as the voltage waveform is improved and distorting harmonic emf is further reduced thus making the output wave more sinusoidal. Also, copper is saved in the coil ends. The inductance of the armature winding is reduced due to lesser length of coil.

Fractional number of slots/pole can be used in this winding which reduces the tooth ripples.

11.9.1 Coil Span (or pitch factor) for short-pitch windings

The total emf induced/phase is lesser in short-pitch winding coils, compared to that of full-pitch winding coils. We define a term "pitch factor" to quantify this reduction of emf.

$$\text{Pitch factor } K_p = \frac{\text{emf induced in a short-pitch coil}}{\text{emf induced in a full-pitch coil}}$$

K_p is always less than unity and it can be shown that

$$K_p = \cos\frac{\alpha}{2}$$

where α = Short-pitch angle

$$= \frac{180 \times \text{No. of slots for which the winding is short pitched}}{\text{No. of slots/No. of poles}}.$$

11.9.2 Distribution (or Breadth or spread) Factor

Distribution factor for short-pitch winding is defined as:

$$K_d = \frac{\text{emf with distributed winding}}{\text{emf with concentrated winding}}$$

$$= \frac{\text{Vector sum of emf induced per coil}}{\text{Arithmetic sum of emf induced per coil}} = \frac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}}.$$

[The value of K_d is less than one for distributed winding].

Here, m = No. of slots per pole per phase.

$$\beta = \text{Angular displacements between slots} = \frac{180°}{\text{No. of slots/No. of poles}}$$

11.10 SINGLE AND DOUBLE LAYER WINDINGS

In a *single layer* winding one coil side occupies one slot completely. Hence the number of slots required is double the number of coils. In *double layer* windings one coil side lies in the upper half of one slot, while the other coil side lies in the lower half of another slot spaced about one pole pitch from the first one. Hence the number of slots is equal to the number of poles. A double layer winding results in a cheaper machine.

11.11 EXPRESSION FOR INDUCED EMF IN A SYNCHRONOUS MACHINE

Let us consider an armature of a synchronous machine having

Number of poles = P

Number of conductors (or coil sides) in series/phase (= Z)
= $2T$ (T is the number of turns)
Frequency of induced emf in Hz = f
Flux per pole in weber = ϕ

Coil span factor $K_p = \cos \dfrac{\alpha}{2}$

$$\text{Distribution factor} = \frac{\sin \dfrac{m\beta}{2}}{m \sin \dfrac{\beta}{2}}$$

Speed of rotor in rpm = N
Form factor (if emf is sinusoidal) $K_f = 1.11$.

In one revolution of rotor (60/N second) each stator conductor is cut by a flux of ($P\phi$) Wb.

$\therefore \qquad d\phi = P\phi$

Time taken by rotor poles to make one revolution $dt = \dfrac{60}{N}$ s.

$\because$ Flux cut/sec. by stator conductor = $\dfrac{d\phi}{dt}$

(according to principle of electromagnetic inductions),

we can writre, $\dfrac{d\phi}{dt} = \dfrac{P\phi}{60/N} = \dfrac{P\phi N}{60}$ Wb/sec.

$\therefore$ Average emf induced in conductor of the armature = $\dfrac{d\phi}{dt}$ = ($P\phi N$/60) V

i.e., Average emf/phase in the armature = $\dfrac{P\phi N}{60} \times (2T)$ V

or E_{av}/phase = $\left(\dfrac{P\phi N}{60} \times Z\right)$ V

[Z being total number of conductors (=$2T$)].

$$\therefore \qquad E_{rms} = \left(1.11 \times \frac{P\phi N}{60} \times 2T\right) \text{ V} = \left(2.22 \frac{P\phi NT}{60}\right) \text{ V}$$

$$\text{or} \qquad E_{rms} = \left(2.22 \times \frac{P\phi}{60} \times \frac{120\,f}{P} \times T\right) \text{ V} \qquad \left[\therefore f = \frac{PN}{120} \text{ Hz or } N = \frac{120\,f}{P}\right]$$

$$\text{Hence} \qquad E_{rms} = (4.44\, f\, \phi\, T) \text{ V}. \qquad (11.1)$$

This would have been the actual value if the winding per phase is full pitched and concentrated. The winding being short pitched and distributed, the actual voltage available/phase

$$V_{ph} = (4.44\, K_p K_d\, \phi f T) \text{ V} = (2.22\, K_p K_d\, \phi f Z) \text{ V}$$

Since most of the industrial synchronous generators are three phase, we can write,

Line voltage $V_L = \sqrt{3}\, V_{ph}$ and $I_L = I_{ph}$ (for star connection) while $V_L = V_{ph}$; $I_L = \sqrt{3}\, I_{ph}$ (for delta connection of armature).

11.1 A three-phase, 8-pole alternator has 96 slots and runs at 1500 rpm. Its coils are short pitch by one-tenth of the pole pitch. Determine the coil span factor, breadth factor, frequency of emf generated by the alternator.

Solution

Let the coils of the alternator be short pitched by an angle (α).

Here $$(\alpha) = \frac{180°}{10} = 18° \qquad [\because 1 \text{ pole pitch} = 180°]$$

Coil span factor (Pitch factor) $K_p = \cos\frac{\alpha}{2} = \cos\frac{18°}{2} = \cos 9° = 0.9877.$

Breadth factor $K_d = \dfrac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}}$, where m is the number of slots per pole per phase and β is the angular displacement between the slots.

Here $m = \dfrac{96}{8\times 3} = 4$ and $\beta = \dfrac{180°}{\text{no. of slots per pole}} = \dfrac{180°}{96/8} = 15°$

$$\text{Hence breadth factor} = \frac{\sin\dfrac{4\times 15°}{2}}{4\sin\dfrac{15°}{2}} = \frac{0.5}{0.522} = 0.9576.$$

If f be the frequency of emf generated by the alternator then

$$1500 = \frac{120 f}{8}$$

or $$f = \frac{8\times 1500}{120}\text{ Hz} = 100\text{ Hz}.$$

11.2 A three-phase, 50 Hz. Alternator has 90 turns per phase. The flux per pole is 0.1 weber. Calculate (i) the emf induced per phase and (ii) emf between the line terminals with star connection. Take distribution factor equal to 0.96 and assume full pitch winding.

Solution

Number of turns per phase $T = 90$
Flux per pole $\phi = 0.1$ Wb
Frequency $f = 50$ Hz
Distribution factor $K_d = 0.96$
Pitch factor $K_p = 1$ ($\because$ full pitch winding);

(i) Hence emf induced per phase

$$E = 4.44\ K_p\ K_d\ \phi f\ T\text{ V} = 4.44 \times 1 \times 0.96 \times 0.1 \times 50 \times 90 = 1918.08\text{ V}.$$

(ii) With star connection the emf between the line terminals is $\sqrt{3} \times 1918.08 = 3322.21$ V.

11.3 The stator of a three-phase, 20-pole alternator has four slots per pole per phase and each slot has 12 conductors. The air gap flux is 0.03 weber and the coil span of the stator winding is 150° electrical. The speed of the alternator is 300 rpm. Find the emf generated per phase.

Solution

No. of slots per pole per phase $m = 4$
Total number of conductors per phase $Z = 12 \times 4 \times 20 = 960$

Hence number of turns per pole per phase $T = \dfrac{960}{2} = 480$

Air gap flux $\phi = 0.03$ weber

The winding is short pitched by an angle $\alpha = 180° - 150° = 30°$

Hence pitch factor $K_p = \cos\dfrac{\alpha}{2} = 0.9659.$

Speed of the alternator $N = 300$ rpm.

Hence frequency $f = \dfrac{20 \times 300}{120} = 50$ Hz.

Distribution factor $K_d = \dfrac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}}.$

Here, m = no. of slots per pole per phase = 4

$$\beta = \frac{180°}{\text{no. of slots per pole}} = \frac{180°}{4 \times 3} = 15°$$

$$\text{Hence} \quad K_d = \frac{\sin\dfrac{4 \times 15°}{2}}{4\sin\dfrac{15°}{2}} = 0.9576.$$

The emf generate per phase

$$E = 4.44\, K_p\, K_d\, \phi f T = 4.44 \times 0.9659 \times 0.9576 \times 0.03 \times 50 \times 480 = \underline{2957 \text{ V.}}$$

11.4 A three-phase 6600 V, 24-pole, 300 rpm star connected alternator has 288 slots on its armature. Coil sides per slot is 2. Number of turns in each coil = 16, coil pitch is 2 slots less to a pole pitch. Find the flux per pole. Assume sinusoidal flux distribution.

Solution

Given: $P = 24$, $N = 300$ rpm

No. of slots $S = 288$

If frequency be f then,

$$f = \frac{PN}{120} = \frac{24 \times 300}{120} = 60 \text{ Hz.}$$

As coil sides per slot is 2, hence number of coils is equal to the number of slots.

No. of coils = 288

Total no. of turns = 16 × 288 = 4608

No. of turns per phase $T = \dfrac{4608}{3} = 1536$

Line voltage = 6600 V.

Hence phase voltage $V_P = \dfrac{6600}{\sqrt{3}}$ V $\quad\because$ star connected = 3810.62 V.

No. of slots per pole per phase $m = \dfrac{288}{24 \times 3} = 4$

$$\text{Slot angle } \beta = \frac{180°}{\text{no. of slots per pole}} = \frac{180°}{\dfrac{288}{24}} = 15°$$

$$\text{Distribution factor } K_d = \frac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}} = \frac{\sin\dfrac{4\times 15^\circ}{2}}{4\sin\dfrac{15^\circ}{2}} = 0.9576.$$

As the coil pitch is 2 slots less to a pole pitch hence the winding is short pitched by an angle $\alpha = 15^\circ \times 2 = 30^\circ$.

$$\text{Pitch factor } K_P = \cos\frac{\alpha}{2} = \cos 15^\circ = 0.9659$$

$$V_P = 4.44\ K_p\ K_d\ \phi f T, \text{ where } \phi \text{ is the flux per pole}$$

$$\therefore \qquad \phi = \frac{3810.62}{4.44\times 0.9659\times 0.95766\times 60\times 1536} = 0.01 \text{ Wb.}$$

11.5 A 4-pole, three-phase 50 Hz. star connected alternator has a single layer winding in 36 slots with 30 conductors per slot. The flux per pole is 0.05 Wb and winding is full pitched. Find the synchronous speed and line voltage.

Solution

Given: $P = 4$, $f = 50$ Hz

Number of slots $S = 36$.

$\therefore$ Number of slots per pole per phase $m = \dfrac{36}{4\times 3} = 3$

Number of conductors z = 30 × 36 = 1080.

Number of turns per phase $T = \dfrac{1080}{3\times 2} = 180$

$\phi = 0.05$ Wb.

As the winding is full pitched, hence pitch factor $K_p = 1$

$$\text{Slot angle } \beta = \frac{180^\circ}{\text{No. of slots per pole}} = \frac{180^\circ}{\dfrac{36}{4}} = 20^\circ$$

$$\text{Hence distribution factor } K_d = \frac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}} = \frac{\sin\dfrac{3\times 20^\circ}{2}}{3\sin\dfrac{20^\circ}{2}} = \frac{0.5}{0.521} = 0.96.$$

Now, if N_s being the synchronous speed,

$$N_s = \frac{120 f}{P} = \frac{120\times 50}{4} = 1500 \text{ rpm.}$$

$\therefore$ Phase voltage $V_P = 4.44\ K_p\ K_d\ \phi\ f\ T$

$= 4.44 \times 1 \times 0.96 \times 0.05 \times 50 \times 180 = 1918.08$ V.

Line voltage $V_L = \sqrt{3} \times 1918.08 = 3322$ V.

11.6 Find the rms value of the phase voltage of a three-phase synchronous machine having 30 poles, 180 slots, single layer winding and full pitch coils. Each coil has 8 turns. Flux per pole is 0.03 Wb.

Solution

Number of slots per pole per phase $m = \dfrac{180}{30\times 3} = 2$

$$\text{Slot angle } \beta = \frac{180°}{\text{No. of slots per pole}} = \frac{180°}{\frac{180}{30}} = 30°$$

$$\text{Distribution factor } K_d = \frac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}} = \frac{\sin\dfrac{2\times 30°}{2}}{2\sin\dfrac{30°}{2}} = \frac{0.5}{2\times 0.2588} = 0.9659.$$

As the coils are full pitched hence pitch factor $K_p = 1$

Since the machine has single layer winding, total number of coils $= \frac{1}{2} \times$ No. of slots $= \frac{1}{2} \times 180 = 90.$

$\therefore$ Total numbers of turns per phase $T = \frac{8\times 90}{3} = 240.$

Hence rms value of the phase voltage V_p is given by

$$V_p = 4.44\ K_p\ K_d\ \phi\ f\ T = 4.44 \times 1 \times 0.9659 \times 0.03 \times 50 \times 240 = 1543.89 \text{ V}.$$

11.12 ARMATURE REACTION AND ARMATURE WINDING REACTANCE

When a three-phase ac supply is given to the stator of the alternator, current I_a flows in each phase of the stator winding. This current I_a produces a rotating magnetic field in the air gap of the alternator. Hence the flux in the air gap is the resultant of the flux produced by field current I_f and armature current I_a. The net air gap flux ϕ is given by

$\phi = \phi_f + \phi_a$, where ϕ_f is the flux produced by I_f and ϕ_a is the flux produced by I_a. As ϕ_f is produced by I_f, ϕ_f and I_f are in phase. The emf induced in the stator winding E_f lags behind the field flux by 90°. If the stator current I_a lags behind E_f by an angle θ then ϕ_a also lags behind E_f by angle θ, as ϕ_a and I_a are in phase. The resultant flux ϕ_r is shown in Fig. 11.1.

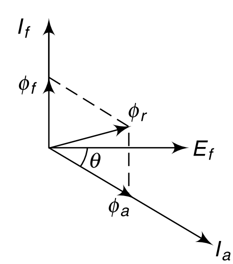


Fig. 11.1 ***Flux-current phasor diagram***

It is clear that the main field flux ϕ_f is greatly affected by the stator flux ϕ_a produced by the stator current. This effect of the stator flux on the main field flux is termed as *armature reaction*. This effect is also dependent on the power factor of the load. The stator flux ϕ_a has a cross-magnetizing and demagnetizing effect upon the main flux ϕ_f. If ϕ_a is resolved into two components, one component will act in direct opposition to ϕ_f and reduces the net flux in the magnetic circuit. This is called the *demagnetizing effect*. The other component of ϕ_a is perpendicular to ϕ_f and it results in shifting the axis of the resultant flux. This is called the *cross-magnetizing effect*. Under lagging power factor operation ϕ_a has both demagnetizing and cross-magnetizing effect. But at unity p.f. load I_a and hence ϕ_a lies along E_f and the effect of ϕ_a on ϕ_f is purely cross-magnetizing. For leading p.f. operation cross magnetizing effect persists but the other effect becomes magnetizing.

A major part of the flux ϕ_a is thus armature reaction flux ϕ_{ar} and a small part of it is the leakage flux ϕ_{al} which links only the stator winding and not the rotor winding. This flux ϕ_{ar} induces a voltage E_{ar} in the stator winding which lags ϕ_{ar} by 90°. As I_a is in phase with ϕ_{ar}, hence E_{ar} lags I_a by 90° or it can be said that I_a lags $-E_{ar}$ by 90°. This voltage $-E_{ar}$ can be represented as a voltage drop across a fictitious equivalent reactance X_{ar} due to I_a.

Hence, $E_{ar} = j\, I_a X_{ar}$. X_{ar} is termed as *armature reaction reactance* or the *magnetizing reactance.*

11.13 ARMATURE LEAKAGE REACTANCE AND SYNCHRONOUS IMPEDANCE

A portion of the air gap flux ϕ_a produced by stator current I_a links only the stator winding and not the rotor winding. This flux ϕ_{al} is called the *leakage flux* which induces an emf E_{al} in the stator winding. The induced emf E_{al} can be considered equivalent to a voltage drop across a fictitious leakage reactance X_{al}.

Hence, $E_{al} = j\, I_a X_{al}$.

The effect of armature reaction and the effect of leakage flux can be represented by considering two fictitious reactances X_{ar} and X_{al}. *Synchronous reactance* X_s is thus the sum of armature reaction reactance and the leakage reactance, i.e.

$$X_s = (X_{ar} + X_{al})$$

As both X_{ar} and X_{al} are fictitious quantities, hence X_s is also a fictitious quantity. If R_a is the resistance of the armature winding then *synchronous impedance* Z_s is obtained as

$$Z_s = R_a + jX_s = \sqrt{R_a^2 + X_s^2} \angle \tan^{-1} \frac{X_s}{R_a}\, \Omega.$$

The magnitude of R_a is very small. Hence for practical purpose, Z_s is assumed to be equal to X_s.

11.14 EQUIVALENT CIRCUIT AND PHASOR DIAGRAM

The equivalent circuit of an alternator is shown in Fig. 11.2.

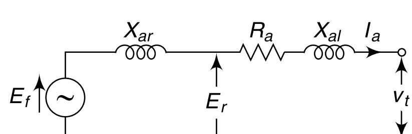


Fig. 11.2 *Equivalent circuit of alternator*

From the circuit diagram it is clear that the open circuit voltage E_f is the summation of the terminal voltage V_t and the drop across the synchronous impedance X_s and the armature resistance.

Hence, $E_f = V_t \times I_a R_a + j\, I_a\, (X_{ar} + X_{al})$ [where, $X_s = (X_{ar} + X_{al})$]

$= V_t \times I_a Z_s$ [Z_s being equal to $\{R_a + j\,(X_{ar} + X_{al})\}$]

Also, $E_f = E_r + j\, X_{ar}\, I_a$

where E_r is the emf induced by the resultant flux ϕ_r.

Figure 11.3(a) and Fig. 11.3(b) show the phasor diagram of an alternator under lagging p.f. and leading p.f. operation respectively.

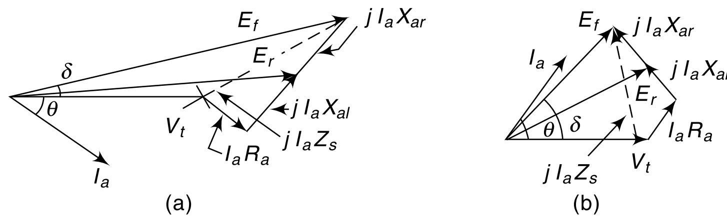


Fig. 11.3 ***Phasor diagram of cylindrical rotor alternator at (a) lagging p.f. (b) leading p.f.***

[θ is the power factor angle and the angle between E_f and V_t, i.e. δ is called the *power angle*].

11.15 VOLTAGE REGULATION OF A SYNCHRONOUS GENERATOR

Voltage regulation of a synchronous generator may be defined as the percentage change in voltage from no load to full load when the field excitation and frequency remain constant.

$$\therefore \qquad \%\text{ regulation } (\Delta V) = \frac{V_o - V_{\text{fl}}}{V_{\text{fl}}} \times 100 \tag{11.2}$$

where, V_o is the no load voltage and V_{fl} is the full load voltage.

Expression for Voltage Regulation

With reference to the phasor diagram of an alternator operating at lagging p.f. load,

$$OH^2 = OC^2 + CH^2$$

or $$V_o = \sqrt{(V_{\text{fl}} + AB + BC)^2 + (DH - DC)^2}$$

$$= \sqrt{(V_{\text{fl}} + IR\cos\phi + IX_s\sin\phi)^2 + (IX_s\cos\phi - IR\sin\phi)^2} \tag{11.3}$$

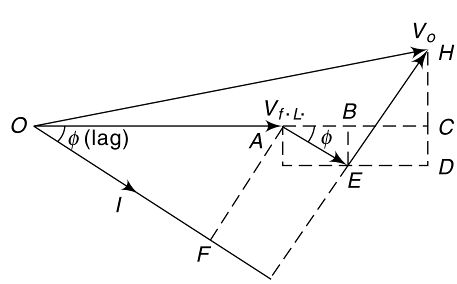


Fig. 11.4 ***Phasor diagram of an alternator operation at lagging load***

Since ($IX_s \cos\phi - IR \sin\phi$) is the difference of two quantities, its square value is small and can be neglected. Hence, the expression for V_o can approximately be written as

$$V_o = V_{\text{fl}} + IR\cos\phi + IX_s \sin\phi \tag{11.4a}$$

or

$$\frac{V_o - V_{\text{fl}}}{V_{\text{fl}}} = \frac{IR}{V_{\text{fl}}}\cos\phi + \frac{IX_s}{V_{\text{fl}}}\sin\phi = \text{p.u. } R\cos\phi + \text{p.u. } X_s\sin\phi \tag{11.4b}$$

where, p.u. $R = \dfrac{R}{\dfrac{V_{\text{fl}}}{I}}$; p.u. $X_s = \dfrac{X_s}{\dfrac{V_{\text{fl}}}{I}}$ [p.u. stands for "per unit"].

When the p.f. is unity, $\cos\phi = 1$, $\sin\phi = 0$

$$\therefore \qquad \Delta V = \frac{V_o - V_{\text{f.l.}}}{V_{\text{fl}}} = \frac{IR}{V_{\text{fl}}}. \tag{11.5}$$

Regulation is zero if $V_o = V_{\text{fl}}$ and this is possible if the alternator runs at no load. At leading p.f., $V_{\text{f.l.}}$ may be more in magnitude than V_o (this is due to magnetizing armature reaction when the field flux is assisted by the magnetizing component of armature reaction flux and terminal voltage is increased than its no load value as because leading p.f. load is increased.) Thus, at leading p.f. operation, regulation $\Delta V \left(= \dfrac{V_o - V_{\text{f.l.}}}{V_{\text{f.l.}}}\right)$ may be negative.

The magnitude of voltage regulation is larger for cylindrical pole alternators. The cost of the alternator increases with decrease in the magnitude of voltage regulation.

Figure 11.5(a) and 11.5(b) may be referred to illustrate about lagging and leading p.f. operation of the synchronous generator (non-salient pole). During laging p.f. operation we find $V_{\text{fl}} < V_o$ while during leading p.f. operation, $V_{\text{fl}} > V_o$.

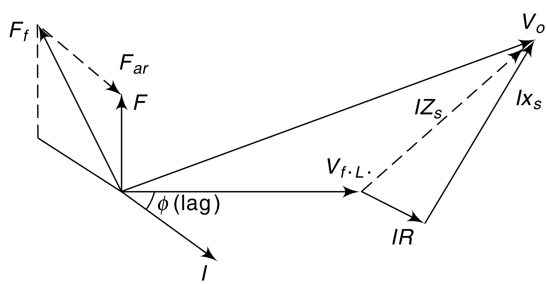


Fig. 11.5(a) *Synchronous generator operating with lagging p.f. load*

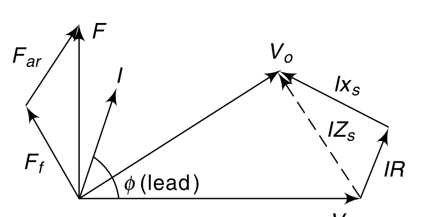


Fig. 11.5(b) *Synchronous generator operating with leading p.f. load*

Field flux mmf (F_f) is opposed by F_{ar}, the armature reaction mmf and hence resultant field mmf F decreases, when the p.f. in lagging. With leading p.f. operation, the resultant field mmf increases due to the magnetising effect of armature reaction mmf F_{ar} being added to main field mmf F_f.

Again, for leading p.f. operation, the expression for V_o can be written as

$$V_o = \sqrt{(V_{\text{fl}} \times IR\cos\phi - IX_s \sin\phi)^2 + (IX_s \cos\phi + IR\sin\phi)^2} \tag{11.6}$$

Also, from Fig. 11.4 an alternate expression for regulation can be derived for operation both in lagging and leading loads. In Fig. 11.4.

$$V_o = \sqrt{(OG)^2 + (GH)^2} = \sqrt{(OF + FG)^2 + (GE + EH)^2}$$

$$= \sqrt{(V_{\text{fl}} \cos\phi + IR)^2 + (V_{\text{fl}} \sin\phi + IX_s)^2}$$

(for lagging loads) and

$$V_o = \sqrt{(V_{\text{fl}} \cos\phi + IR)^2 + (V_{\text{fl}} \sin\phi - IX_s)^2} \tag{11.7}$$

(for leading loads)

Regulation can be determined in practical machines by first measuring the dc resistance of the stator at (which is say R ohms) and then finding Z_s (synchronous impedance) by obtaining the ratio of V_o and $I_{\text{s/c}}$ where V_o is the no load voltage at the machine output where it is being operated with full speed and normal excitation and $I_{s/c}$ is the short circuit current (being equal to the full load current) at the machine terminal for operation at reduced suitable excitation at normal speed when the terminals are shorted. Obviously, $Z_s = \dfrac{V_o}{I_{\text{s/c}}}$, all the quantities being expressed in per phase basis. Now we can find X_s(synchronous reactance) using the formula,

$$X_{\text{s}} = \sqrt{Z_s^2 - R^2}\ .$$

11.7 A 440 V, three-phase alternator, running at rated speed, has a 2 A excitation current when short circuit is applied at its terminals. The short circuit magnitude is 50 A (full load current). At this excitation the open circuit voltage is 150 V/phase. Assuming the armature circuit resistance to be 0.5 Ω per phase, obtain the value of regulation of the alternator at (i) 0.8 p.f. lagging load and (ii) 0.8 p.f. leading load.

Solution

$$Z_s = \frac{150}{50} = 3\ \Omega/\text{ph}$$

$$\therefore \quad X_s = \sqrt{3^2 - 0.5^2} = 2.96\ \Omega/\text{ph.} \qquad (\because Z_s = \sqrt{X_s^2 + R^2}\ )$$

For lagging p.f. operation,

$$V_o = \sqrt{(V_{\text{f.l.}} + IR\cos\phi + IX_s \sin\phi)^2 + (IX_s \cos\phi - IR\sin\phi)^2}$$

$$= \sqrt{\left\{\left(\frac{440}{\sqrt{3}}\right) + 50 \times 0.5 \times 0.8 + 50 \times 2.96 \times 0.6\right\}^2 + (50 \times 2.96 \times 0.8 - 50 \times 0.50 \times 0.6)^2}$$

$$= \sqrt{131654 + 10691.56} = 377.29\ \text{V/ph.}$$

$$\therefore \qquad \Delta V \text{ (regulation)\%} = \frac{V_o - V_{\text{f.l.}}}{V_{\text{f.l.}}} \times 100 = \frac{377.29 - \dfrac{440}{\sqrt{3}}}{\dfrac{440}{\sqrt{3}}} \times 100 = 48.52\%.$$

If we use another formula of regulation, we can write,

$$V_o = \sqrt{(V_{\text{f.l.}} \cos\phi + IR)^2 + (V_{\text{f.l.}} \sin\phi + IX_s)^2}$$

$$= \sqrt{\left(\frac{440}{\sqrt{3}} \times 0.8 + 50 \times 0.5\right)^2 + \left(\frac{440}{\sqrt{3}} \times 0.6 + 50 \times 2.96\right)^2}$$

$$= \sqrt{52087.7 + 90252.46} = 377.286 \simeq 377.29 \text{ V/ph.}$$

Hence V_o obtained is same for both the formula and any one can be used to find regulation of the given alternator at lagging p.f. We will find similarly that any of these formulae can be used for leading p.f. operation too to find V_o.

For leading p.f. operation,

$$V_o = \sqrt{(V_{\text{f.l.}} + IR\cos\phi - IX_s \sin\phi)^2 + (IX_s \cos\phi + IR \sin\phi)^2}$$

$$= \sqrt{\begin{array}{l}(254.04 + 50 \times 0.5 \times 0.8 - 50 \times 0.6 \times 2.96)^2 \\ + (50 \times 2.96 \times 0.8 + 50 \times 0.5 \times 0.6)^2\end{array}}$$

$$= \sqrt{34313.86 + 17795.56} = 228.27 \text{ V/ph}$$

Also, we can find V_o using another formula

$$V_o = \sqrt{(V_{\text{f.l.}} \cos\phi + IR)^2 + (V \sin\phi - IX_s)^2}$$

$$= \sqrt{(254.04 \times 0.8 + 50 \times 0.5)^2 + (254.04 \times 0.6 - 50 \times 2.96)^2}$$

$$= \sqrt{52089.85 + 19.57} = 228.27 \text{ V/ph}$$

$$\because \qquad \Delta V(\%) = \frac{V_o - V_{\text{f.l.}}}{V_{\text{f.l.}}} \times 100 = \frac{228.27 - 254.04}{254.04} \times 100 = -10.14\%.$$

[*Note:* In subsequent examples, to maintain symmetry for the notations, we will replace V_o by E_f and $V_{\text{f.l.}}$ by V_t so that regulation is $\left(\dfrac{E_f - V_t}{V_t} \times 100\right)$ per phase.]

11.8 A 1000 kVA, three-phase star connected alternator has a rated line to line terminal voltage of 3000 V. The resistance per phase is 0.5 Ω and synchronous reactance 5 Ω. Calculate the voltage regulation at full load and 0.8 p.f. lagging.

Solution

Power output $P = 1000 \times 10^3$ VA

Line voltage $V_L = 3000$ V

If I_L be the line current then

$$\sqrt{3}\, V_L I_L = 1000 \times 10^3$$

or
$$I_L = \frac{1000 \times 10^3}{\sqrt{3} \times 3000} \text{ A} = 192.45 \text{ A.}$$

From the phasor diagram (Fig. 11.2), the no load voltage $E_f = V_t + I_a(R_a + jX_s)$ where, $X_s = X_{ar} + X_{al} = 5$ Ω and $R_a = 0.5$ Ω. (V_t) is the terminal voltage per phase i.e. $V_t = \dfrac{3000}{\sqrt{3}}$ V.

Also, $I_a = I_L = 192.45$ A.

Hence, $E_f = \dfrac{3000}{\sqrt{3}} \angle 0° + 192.45\angle -\cos^{-1} 0.8\ (0.5 + j5)$

$= 1732.1 + 192.45\angle -36.87° \times 5.025\angle 84.29°$

$= 1732.1 + 967.06\angle 47.72° = 2386.43 + j715.5$

$\therefore$ $|E_f| = 2491$ V (per phase).

$$\text{Voltage regulation of full load} = \frac{2491 - \dfrac{3000}{\sqrt{3}}}{\dfrac{3000}{\sqrt{3}}} \times 100\% = 43.81\%$$

11.16 PHASOR DIAGRAM FOR A SALIENT POLE ALTERNATOR: TWO-REACTION THEORY

As discussed earlier the rotor of cylindrical pole machines has uniform air gap whereas in salient pole alternators the air gap is non-uniform. Figure 11.6 shows a two-pole salient pole alternator rotating in an anti-clockwise direction. From the figure it is clear that there are two axes of symmetry here. The axis of the rotor pole is called the *direct* or d axis and the axis perpendicular to the rotor pole is called the *quadrature* or q axis. The direct axis flux path includes two small air gaps under pole faces only whereas quadrature axis flux path has two larger air gaps in the inter-polar regions. Hence with direct axis flux path has minimum reluctance and quadrature axis flux path has maximum reluctance. According to the two-reaction theory, the armature mmf F_a produced by stator I_a is resolved into two componenets, one along the direct or d-axis and another along quadrature or q-axis. The direct axis component of F_a is F_{ad} and the quadrature axis component is F_{aq}. The effect of F_{ad} is either magnetizing or demagnetizing (depending whether the p.f. is leading or lagging), whereas the effect of F_{aq} is entirely cross-magnetizing. Similarly, flux ϕ_a produced by stator current I_a is resolved into two components ϕ_{ad} and ϕ_{aq}; ϕ_{ad} induces direct axis armature reaction voltage E_{ad} and ϕ_{aq} induces quadrature axis armature reaction voltage E_{aq}. If I_d and I_q be the two components of current I_a along direct and quadrature axis respectively then

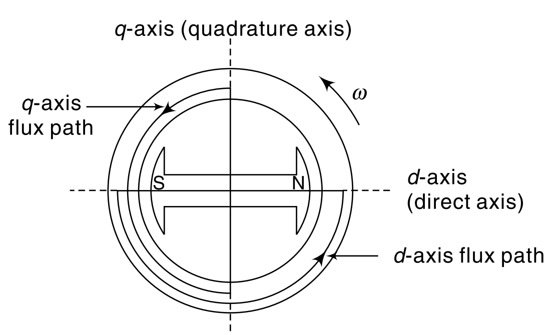


Fig. 11.6 ***2-pole salient pole alternator***

$$E_{ad} = -j\, X_{ad}\, I_d$$

and $$E_{aq} = -j\, X_{aq}\, I_q$$

where X_{ad} and X_{aq} are the direct axis and quadrature axis armature reactance. Figure 11.7 shows the phasor diagram of a salient pole alternator based on the two-reaction theory.

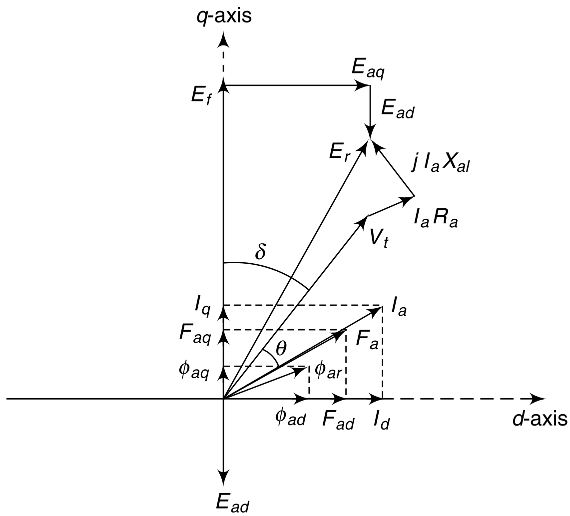


Fig. 11.7 ***Phasor diagram of a salient pole alternator***

The terminal voltage V_t is taken as reference. The stator current I_a is assumed to be lagging with respect to V_t by an angle θ. The two perpendicular components of I_a, i.e I_d and I_q are shown along the *d*-axis and *q*-axis. Similarly, stator mmf F_a and flux ϕ_{ar} has been resolved into *d*-axis and *q*-axis components. The *d*-axis component of armature reaction voltage E_{ad} lags behind ϕ_{ad} by 90° and *q*-axis component of armature reaction voltage E_{aq} lags behind ϕ_{aq} by 90° as shown in Fig. 11.5. The resultant air gap voltage is given by

$$E_r = E_f + E_{ad} + E_{aq}$$

Terminal voltage is given by $V_t = E_r - I_a\, R_a - j\, I_a\, X_{al}$

For obtaining a simplified phasor diagram the voltage drop ($I_a\, X_{al}$) is resolved into two reactangular components ($I_d\, X_{al}$) and ($I_q\, X_{al}$) i.e

$$I_a\, X_{al} = (I_d + I_q)\, X_{al}$$

Also armature reaction reactance drop and leakage reactance drop are combined together, i.e.

$$I_d\, X_{ad} + I_d\, X_{al} = I_d\, X_d$$

and $$I_q\, X_{aq} + I_q\, X_{al} = I_q\, X_q$$

where X_d and X_q are *d*-axis and *q*-axis synchronous reactances.

The simplified phasor diagram is shown in Fig. 11.8(a). Further simplification can be obtained by neglecting the stator resistance; the phasor diagram is shown in Fig. 11.8(b).

Figure 11.8(b) represents the phasor diagram neglecting the stator resistance and in terms of the known parameters. Here

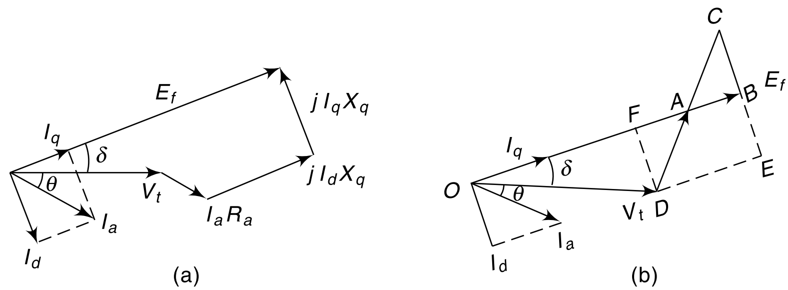


Fig. 11.8 ***Simplified phasor diagram of salient pole alternator***

$OD = V_t,\ DE = FB = I_d\, X_d$

$DA = I_a\, X_q,\ DC = I_a\, X_d$

Hence, $AC = I_a(X_d - X_q)$.

Now, $AB = FB - FA = I_d\, X_d - I_d\, X_q = I_d(X_d - X_q)$

Hence, $E_f = OB = OA + AB = OD + DA + AB = V_t + I_a\, X_q + I_d\,(X_d - X_q)$.

11.17 STEADY STATE PARAMETERS

As discussed in the previous article the armature reaction voltage E_{ad} and E_{aq} induced in the stator winding by fluxes ϕ_{ad} and ϕ_{aq} are

$$E_{ad} = -j\,X_{ad}\,I_d \quad \text{and} \quad E_{aq} = -j\,X_{aq}\,I_q$$

where X_{ad} and X_{aq} are the armature reaction reactance in the d-axis and q-axis respectively and I_d and I_q are the d-axis and q-axis component of the stator current I_a.

Hence the armature reaction reactance drop is $(j\,X_{ad}\,I_d)$ along d-axis and $(j\,X_{aq}\,I_q)$ along the q-axis.

Also the leakage reactance drop along the d-axis is $(j\,X_{al}\,I_d)$ and $(j\,X_{al}\,I_q)$ along the q-axis where X_{al} is the leakage reactance.

Combining X_{ad} and X_{al}, the net reactance along the d-axis is

$$X_d = X_{ad} + X_{al}.$$

Similarly, the net reactance along the q-axis is

$$X_q = X_{aq} + X_{al}$$

X_d and X_q are the direct axis and quadrature axis synchronous reactances. These are the steady state parameters of synchronous machine. For salient pole machines X_d is much greater than X_q as the reluctance along the q-axis is much greater than along d-axis (because of larger air gap along the q-axis). Generally, X_d is 1.25 to 1.7 times X_q for salient pole alternators. But for cylindrical rotor alternator, as the air gap is uniform, $X_d = X_q$.

11.18 POWER ANGLE CHARACTERISTIC OF SALIENT POLE AND CYLINDRICAL POLE MACHINES

The power output per phase of an alternator in terms of d-axis and q-axis voltages and currents are

$$P = V_d I_d + V_q I_q \tag{11.8}$$

From the phasor diagram in Fig. 11.6, we have

$$V_q = V_t \cos\delta \quad \text{and} \quad V_d = V_t \sin\delta \tag{11.9}$$

Also, $V_d = I_q X_q$

and $V_q = E_f - I_d X_d$

Hence, $I_d = \dfrac{E_f - V_q}{X_d}$ and $I_q = \dfrac{V_d}{X_q}$.

Substituting the values of I_d and I_q in Eq. (11.8) we get

$$P = V_d\left(\frac{E_f - V_q}{X_d}\right) + V_q\left(\frac{V_d}{X_q}\right)$$

$$= \frac{V_d E_f}{X_d} - \frac{V_d V_q}{X_d} + \frac{V_d V_q}{X_q} \tag{11.10}$$

Substituting the values of V_d and V_q from Eq. (11.9) in Eq. (11.10), we have

$$P = \frac{V_t E_f \sin\delta}{X_d} - \frac{V_t^2 \sin\delta\cos\delta}{X_d} + \frac{V_t^2 \sin\delta\cos\delta}{X_q}$$

$$= \frac{V_t E_f \sin\delta}{X_d} - \frac{1}{2}V_t^2\left[\frac{1}{X_d} - \frac{1}{X_q}\right]\sin 2\delta$$

$$= \frac{V_t E_f \sin\delta}{X_d} + \frac{1}{2}V_t^2\left[\frac{X_d - X_q}{X_d X_q}\right]\sin 2\delta \tag{11.11}$$

For cylindrical rotor alternator $X_d = X_q$, hence

$$P = \frac{V_t E_f \sin\delta}{X_d} = \frac{V_t E_f \sin\delta}{X_s},$$

where X_s is the synchronous reactance. The power angle curve for cylindrical rotor alternator is shown in Fig. 11.9.

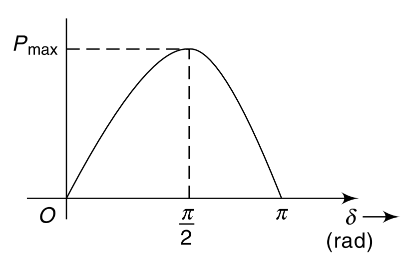


Fig. 11.9 ***Power angle characteristics for a cylindrical rotor alternator***

Equation (11.11) represents the expression of output power for salient pole alternators. The first component, i.e. $\left(\dfrac{V_t E_f}{X_d}\sin\delta\right)$ represents the power output due to field excitation. The second component of the power expression is due to the saliency of the alternator and is independent of the excitation. This component of power is called the *reluctance power* and it is directly proportional to (sin 2δ). The power angle characteristic of the salient pole alternator is the resultant of two curves as shown in Fig. 11.10.

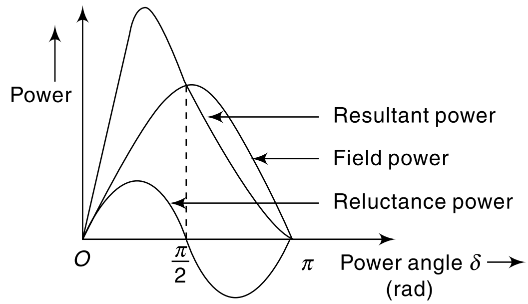


Fig. 11.10 ***Power angle characteristics for salient pole alternator***

11.9 A salient pole synchronous generator is rated at 200 MVA and 11 kV. Its armature winding is star connected having negligible resistance; it is given that $X_d = 0.8$ p.u., $X_q = 0.5$ p.u. The generator is supplying rated MVA at rated voltage and at 0.8 p.f. lagging. Find power angle and open circuit emf in p.u.

Solution

$$X_d = 0.8 \text{ p.u.}; X_q = 0.5 \text{ p.u.}, R_a = 0 \text{ (given)}$$

Taking terminal voltage as the reference phasor, $V_t = 1\angle 0°$

As power factor is 0.8 lagging hence armature current I_a is given by

$$I_a = 1\angle -36.87° \text{ p.u.}$$

From phasor diagram (Fig. 11.8(b))

$$OA = V_t + j\, I_a\, X_q$$
$$= 1\angle 0° + 1\angle -\cos^{-1} 0.8 \times 0.5\angle 90°$$
$$= 1\angle 0° + 0.5\angle 90° - 36.87° = 1 + 0.5\angle 53.13° = 1.3 + j\, 0.4 = 1.36\angle 17.1°$$

Hence power angle $\delta = 17.1°$;

again referring to Fig. 11.8(b),

$$I_d = I_a \sin(\delta + \theta) = 1 \sin(17.1° + 36.87°) = 0.809.$$

Open circuit emf E_f is obtained as

$$E_f = |OA| + |AB| = 1.36 + I_d(X_d - X_q) = 1.36 + 0.809\,(0.8 - 0.5) = 1.6027 \text{ p.u.}$$

11.10 A 70 MVA, 13.8 kV, three-phase star connected salient pole alternator has a direct axis reactance of 1.83 Ω and quadrature axis reactance of 1.2 Ω. It delivers a load of 0.8 p.f. lagging. Calculate the excitation voltage and voltage regulation.

Solution

$$X_d = 1.83\ \Omega$$
$$X_q = 1.2\ \Omega$$

$$\text{Phase voltage } V_t = \frac{13.8 \times 10^3}{\sqrt{3}} \text{ V} = 7967.67 \text{ V}$$

$$\text{Line current } I_L = \frac{70 \times 10^6}{\sqrt{3} \times 13.8 \times 10^3} \text{ A} = 2928.67 \text{ A}$$

From the phasor diagram in Fig. 11.8(b),

$$OA = V_t + j\, I_a\, X_q$$
$$= 7967.67 + 2928.67\angle -36.87° \times 1.2\angle 90°$$
$$= 7967.67 + 3514.4\angle 53.13° = 10076.3 + j\, 2811.5 = 10461.19\angle 15.6°$$

Power angle $\delta = 15.6°$;

$$\therefore \quad I_d = I_a \sin(\delta + \theta) = 2928.67 \sin(15.6° + 36.87°) = 2322.644 \text{ A.}$$

Excitation voltage is given by

$$|E_f| = |V_t + I_a X_q + I_d (X_d - X_q)|$$
$$= 10461.19 + 2322.536(1.83–1.2) = 11924.39 \text{ V} = 11.92 \text{ kV}.$$

Voltage regulation is found to be

$$\Delta V = \frac{11.92 - \dfrac{13.8}{\sqrt{3}}}{\dfrac{13.8}{\sqrt{3}}} \times 100\% = 49.6\%.$$

11.11 A 50 MVA, 11 KV, 50 Hz, 6-pole alternator has an armature resistance of 0.001 p.u. and synchronous reactance of 0.75 p.u. At full load, the rated emf is 1.6 p.u. Find the torque angle and power factor.

Solution

Considering $I_a = 1$ p.u., $V = 1$ p.u.

$R_a = 0.001$ p.u. and $X_s = 0.75$ p.u.,

we have

$$E^2 = (V \cos\theta + I_a R_a)^2 + (V \sin\theta + I_a X_s)^2$$
$$(1.6)^2 = (1 \cos\theta + 1 \times 0.001)^2 + (1 \sin\theta + 1 \times 0.75)^2$$

or $$2.56 = \cos^2\theta + (0.001)^2 + 0.002 \cos\theta + \sin^2\theta + 1.5 \sin\theta + 0.5625$$

or $$1.5 \sin\theta + 0.002 \cos\theta = 2.56 - 1 - 0.5625 - (0.001)^2 = 0.9975.$$

By trial and error method solving the above equation $\cos\theta = 0.75$ lagging or power factor 0.75 lagging.

Hence $E \cos\delta = V + I_a R_a \cos\theta + I_a X_s \sin\theta$

or $1.6 \cos\delta = 1\angle 0° + 1 \times 0.001 \times 0.75 + 1 \times 0.75 \times 0.66 = 1.4968$

or Torque angle (δ) = 20.69°.

[Torque angle is same as to power angle]

Parallel Operation of Synchronous Generators

Generators are run in parallel to cater increase of load demand. Whenever more power is needed, an additional generator is connected to the existing generator bus and synchronised. When a small generator is brought up to an existing network having a large generator, it is referred to as connecting this single generator to the infinite bus. The following conditions must be satisfied for parallel operation of a generator with an existing generator bus:

(a) the terminal voltage of the incoming machine should be identical to the existing bus bar voltage

(b) the frequency of both the generators must be identical during synchronisation

(c) the phase sequence of the incoming generator must be same as that of the existing bus voltage sequence

(d) the phase relations of the incoming generator and the existing one should coincide.

A synchroscope is usually used for synchronising two synchronous generators or a synchronous generator to the infinite bus. The pointer of the synchroscope rotates clockwise when an alternator is running faster and anticlockwise when it runs slower then the existing bus with which it is to be synchronised. When the pointer is stationary and pointing upward, it is the right moment for synchronising the incoming alternator with the existing bus. The synchronising needs to be closed at this instant without any further delay.

11.19 SYNCHRONOUS MOTOR

Any alternator can be operated as a synchronous motor and a synchronous motor can also be operated as an alternator. For a given number of poles and fixed frequency, it operates at only one speed known as *synchronous speed* $\left(N_s = \frac{120f}{P}\right)$, so it works only at a constant speed. Damper winding is provided for making it self-starting and also to prevent hunting.

11.20 PRINCIPLE OF OPERATION

When a three-phase stator winding of a synchronous machine is fed by a balanced three-phase ac supply, then a magnetic flux of constant magnitude but rotating at synchronous speed is produced.

Let us assume that the rotor be rotating round the stator at synchronous speed having the same number of poles as stator. As the rotor is excited by an external dc source the poles of the rotor retain same polarity throughout but the polarity of the stator poles changes as it is connected to an ac supply, i.e. the polarity of the stator as poles are alternating. Two similar poles of stator and rotor repel each other with the result that the rotor tends to rotate in a particular direction. After (T/2) sec. $\left(\text{i.e. } T = \frac{1}{f}\right)$, the polarity of stator poles is reversed but the polarity of the rotor poles remains the same. Under this condition stator N-poles attract rotor S-poles and stator S-poles attract the rotor N-poles and hence the torque produced will be in the reverse direction and thus the rotor starts to rotate in the reverse direction.

For frequency 50 Hz., these changes will occur 100 times in one sec, thus the torque acting on the rotor of the synchronous motor is pulsating and the rotor does not move in any direction and remains stationary. Therefore, the synchronous motor is not self-starting.

Let us now consider that the rotor is rotated in a clockwise direction by some external means so that torque is clockwise. After half a period later, the stator N-pole and S-pole will become S-pole and N-pole. If the rotor speed is such that the N-pole of the rotor also turns by a pole pitch so that it is again under the N-pole then the torque acting on the rotor will again be clockwise. Hence in order to obtain a continuous and unidirectional torque, the rotor must be rotated with such a speed that it advances 1-pole pitch by the time the stator poles interchange their polarity. This means the rotor must rotate at synchronous speed with the stator. At this instant the stator and rotor poles get magnetically interlocked (i.e. N-pole of stator attract S-pole of rotor and vice versa). It is because of this magnetic locking acting between the two, the motor rotates. The motor can rotate at synchronous-speed only.

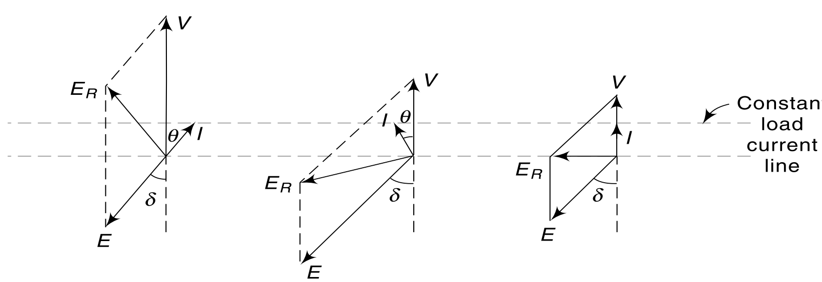
When the mechanical load is applied to a synchronous motor its speed cannot decrease since the rotor must operate at constant speed. Hence speed is independent of load and can be varied only by varying the supply frequency.

11.21 EFFECT OF CHANGE OF EXCITATION OF A SYNCHRONOUS MOTOR (V-CURVES) FOR THE MOTOR DRIVING A CONSTANT LOAD

If a synchronous motor is loaded with a constant load, the input power ($VI \cos \theta$) drawn from the supply will also remain constant. As the supply voltage is constant, hence ($I \cos \theta$) will also remain constant. Under this condition, the effect of change of field excitation on armature current, I can be studied.

As the dc excitation is changed, the magnitude of induced emf E changes. The torque angle δ, i.e the angle of lag of E from the axis of V remains constant as long as the load on the motor remains constant.

Figure 11.11 shows the effect of varying excitation on the power factor of the motor. When the magnitude of E is less than V, the motor works at lagging p.f. (Fig. 11.11(a)), when E is greater than V, the motor works at leading p.f. (Fig. 11.11(b)) and when $E = V$, the motor works at unity p.f. (Fig. 11.11(c)).



$|E| > |V|$, motor works at lagging p.f, I lags V by θ
(a)

$|E| > |V|$, motor works at leading p.f, I leads V by θ
(b)

$|E| = |V|$, motor works at upf, I and V in phase
(c)

Fig. 11.11

The excitation corresponding to the unity p.f. current drawn by the motor is called *normal excitation*. Excitation higher than normal is called *over excitation* and the excitation lower than normal is called *under excitation*. It can be observed that the magnitude of current under normal excitation is the minimum.

Effect of change of excitation on armature current and p.f. at a constant load are shown in Fig. 11.12.

In Fig. 11.12 at normal excitation the p.f. of the motor is unity. The magnitude of armature current at this excitation is minimum. For excitation higher than the normal excitation, the magnitude of the armature current increases and p.f. is leading. For excitation lower than the normal excitation, the magnitude of the armature current still increases but the p.f. is lagging.

As the shape of graph I versus I_f is similar to the English alphabet "V", it is also known as the V-curve of the motor. A series of V-curves can be obtained by changing the load on the motor.

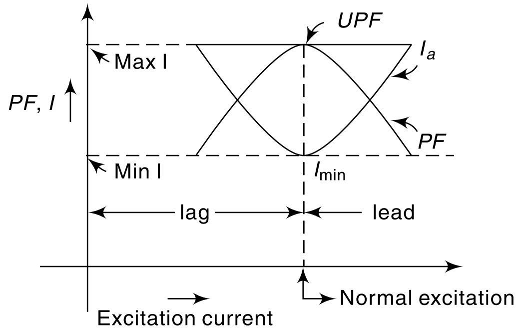


Fig. 11.12 ***Profile of power factor and current against excitation current in synchronous motor***

11.22 STARTING OF SYNCHRONOUS MOTORS

It is now evident that a synchronous motor needs an auxiliary starting arrangement. The methods of starting of a synchronous motor are as follows:

(a) Starting with the help of a damper winding.
(b) Starting with the help of a separate small induction motor.
(c) Starting by using a dc motor coupled to the synchronous motor.
(d) Starting as induction motor and run as synchronous motor.

(a) Starting with the Help of a Damper Winding

In this method a synchronous motor is started independently using a damper winding. The damper winding is provided on the pole face slots in the field. Bars of aluminium, copper, bronze or similar alloys are inserted in slots of pole shoes. These bars are short-circuited by end-rings on each side of the poles. By short-circuiting of these bars, a squirrel cage winding is virtually formed. When a three-phase supply is given to the stator, a synchronous motor with damper winding will start as a three phase induction motor with the speed of rotation near to synchronous speed. Now the d.c. excitation to the field winding of rotor is applied and the rotor will be pulled into synchronism. A reduced supply voltage may be necessary, to limit the starting current drawn by the motor.

In this method since starting is done as an induction motor, the starting torque developed is rather low. Hence a large capacity synchronous motor may not be able to start on full load if damper winding starting is employed.

(b) Starting with the Help of a Separate Small Induction Motor

In this method, a separate induction motor is used to bring the speed of the synchronous motor to synchronous speed. The number of poles of the synchronous motor needs to be more than that of poles of the induction motor to enable the induction motor to rotate at the synchronous speed of the synchronous motor. As the set attains synchronous speed, dc excitation is applied and as the rotor and stator of the synchronous motor are pulled in synchronism, the induction motor is switched off.

(c) Starting by Using a dc Motor Coupled to a Synchronous Motor

In this method, the dc motor drives the synchronous motor and brings it to synchronous speed. Then the synchronous motor is synchronised with the supply.

(d) Starting as Induction Motor

In this method the rotor winding is shorted at start and no dc excitation is given. The stator receives the applied voltage in steps and when near full speed is attained by the rotor, the rotor short circuit is removed and dc voltage is applied. The motor continues to operate as a synchronous motor. Instead of keeping the rotor winding shorted at start, sometime there is one more additional winding which helps the machine to start as an induction motor. This winding remains open-circuited during the run of the machine as a synchronous motor.

Out of these three methods, the method of using a damper winding for starting the synchronous motor is mostly used, because it requires no external motor.

11.23 APPLICATION OF SYNCHRONOUS MOTOR

An over-excited synchronous motor operates at leading p.f. and takes a leading current from the bus bar; so it can be used to raise the overall power factor of the bus bar supplying load.

When the motor is run without load with over excitation for improving the voltage regulation of a transmission line it is called a *synchronous capacitor* or *synchronous condenser*. Synchronous motors can be used in electric clocks as it runs at constant speed.

11.24 COMPARISON BETWEEN SYNCHRONOUS AND INDUCTION MOTOR

Synchronous motor	*Induction motor*
1. It is internally not a self starting motor.	1. It is a self-starting motor.
2. It runs at constant speed called synchronous speed and this speed is independent of the load.	2. The speed of the motor is always less than the synchronous speed and its speed decreases as the shaft load increases.
3. It requires dc source for the field excitation.	3. No dc exciter is needed.
4. It can be operated under a wide range of power factors including lagging a leading p.f.	4. It runs with lagging p.f. only which may be very low at light loads.
5. It runs at synchronous speed only. The only way to change its speed is by varying the supply frequency.	5. Many power electronic methods are available with which speed can be varied.
6. It is used to improve the p.f. and in that case it is called as synchronous capacitor.	6. It is used only to drive a mechanical load.

(*Contd*)

(*Contd*)

7. It is costlier and construction wise more complicated. maintenance is also more.	7. It is cheaper and is commonly used in industry as a robust drive.

11.25 HUNTING OF SYNCHRONOUS MACHINE

When a synchronous motor is loaded mechanically at its shaft, the rotor falls back by certain angle known as *load angle* δ behind the poles of forward rotating magnetic field. If the load is suddenly thrown off, the rotor poles are pulled into almost exact position to the poles of forward field, but due to inertia of rotor, the rotor poles travel beyond. They are then pulled back again and this process may continue. Thus an oscillation of rotor is set up about the equilibrium position corresponding to new load. This oscillation of the rotor about its equilibrium position is known as *hunting*.

Hunting may be caused due to following reasons:

(a) Change in load.

(b) Change in excitation.

(c) Change in supply frequency.

Hunting is an undesirable characteristic of all synchronous motors. It produces severe mechanical stresses as well as great variations in current and power drawn by motor.

11.25.1 Methods of Reducing Hunting

The hunting can be reduced by providing *damper winding* (or grids). These windings consist of short-circuited copper bars embeded in the faces of the field poles of the synchronous motors.

When the rotation at constant load is uniform, there is no relative motion between the rotor and stator forward rotating fields and hence no current is induced in these windings. But when hunting takes place, the relative motion of the rotor sets up eddy currents in these windings which flow such as to suppress the oscillations (as per Lenz's Law). The dampers should have low resistance to be more effective. However this method cannot eliminate hunting completely.

11.26 EQUIVALENT CIRCUIT AND PHASOR DIAGRAM OF SYNCHRONOUS MOTOR

The equivalent circuit and phasor diagram of a cylindrical rotor synchronous motor is similar to that of a synchronous generator. The effect of armature reaction is replaced by the fictitious reactance X_{ar}, while the leakage reactance is X_{al}. The resultant of X_{ar} and X_{al} is called the synchrounous reactance X_s. The equivalent circuit of a cylindrical rotor synchronous motor is shown in Fig. 11.13.

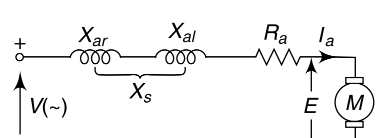


Fig. 11.13 ***Equivalent circuit of a cylindrical rotor synchronous motor***

Here V is the terminal voltage and E is the counter emf

$$V = E + I_aR_a + j\, I_aX_s \tag{11.12}$$

The corresponding phasor diagram is shown in Fig. 11.14.

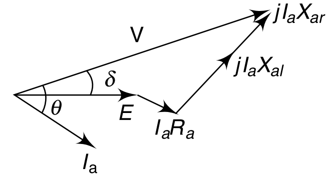


Fig. 11.14 *Phasor diagram of a cylindrical rotor synchronous motor*

The phasor diagram of a salient pole synchronous motor is shown in Fig. 11.15(a). From this phasor diagram the terminal voltage is obtained as

$$V = E + I_aR_a + j\, I_dX_d + j\, I_qX_q \tag{11.13}$$

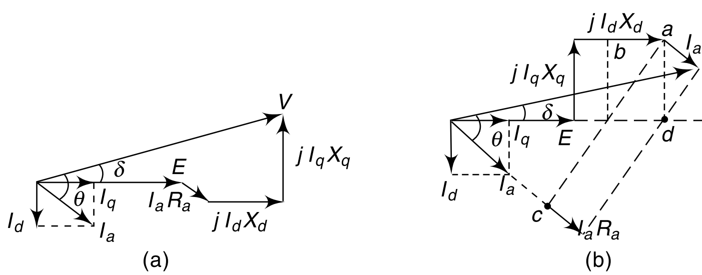


Fig. 11.15 *Phasor diagram of a salient pole synchronous motor*

The armature current I_a can be decomposed into two components: I_d, which is lagging phasor E by 90° and I_q, which is in phase with E.

Figure 11.15(b) shows the phasor diagram in terms of the known parameters. From this phasor diagram it is also evident that $V = E + j\, I_qX_q + j\, I_dX_d + I_aR_a$. Next, ac is drawn from point a perpendicular to I_a and $ac = jI_aX_q$. From point e, i.e the terminal point of V, ed is drawn parallel to ac which meets the extended line of vector E at d.

Hence $\quad cd = jI_dX_q$

Now, $\quad ac = cd + da$

or $\quad jI_aX_q = jI_dX_q + jI_qX_q$

or $\quad j\, I_qX_q = j\, I_aX_q - jI_dX_q$

Hence
$$\begin{aligned} V &= E + jI_qX_q + jI_dX_d + I_aR_a \\ &= E + I_aR_a + jI_dX_d + jI_aX_q - jI_dX_q \\ &= E + I_aR_a + jI_aX_q + jI_d\,(X_d - X_q). \end{aligned} \tag{11.14}$$

The equivalent circuit of the salient pole synchronous motor is shown in Fig. 11.16.

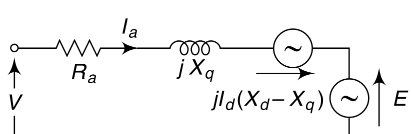


Fig. 11.16 *Equivalent circuit of a salient pole synchronous motor*

11.27 POWER AND TORQUE DEVELOPED IN A CYLINDRICAL ROTOR MOTOR

Let S represent the per phase complex power drawn by the cylindrical rotor synchronous motor.

$$S = V \cdot I_a^* = V \cdot \left|\frac{V - E}{Z_s}\right|^*,$$

where Z_s is the synchronous impedance.

$$= V \cdot \left|\frac{V}{Z_s}\right|^* - V \cdot \left|\frac{E}{Z_s}\right|^*$$

Now $\qquad E = |E| \angle{-\delta}$

and $\qquad Z_s = |Z_s| \angle\phi = \sqrt{R_s^2 + X_s^2}\ \angle\tan^{-1}\frac{X_s}{R_s}.$

Hence $\qquad S = |V|\angle 0° \cdot \dfrac{|V| \angle 0°}{|Z_s| \angle -\phi} - |V|\angle 0° \cdot \dfrac{|E| \angle\delta}{|Z_s| \angle -\phi}$

$$= \frac{|V|^2}{|Z_s|}\angle\phi - \frac{|V||E|}{|Z_s|}\angle\delta + \phi = P + jQ$$

$\therefore$ Active power P per phase is

$$P = \frac{|V|^2}{|Z_s|}\cos\phi - \frac{|V||E|}{|Z_s|}\cos(\delta + \phi) \tag{11.15}$$

and reactive power Q per phase is

$$Q = \frac{|V|^2}{|Z_s|}\sin\phi - \frac{|V||E|}{|Z_s|}\sin(\delta + \phi) \tag{11.16}$$

Neglecting stator resistance (i.e $\phi = 90°$)

$$P = \frac{|V||E|}{|X_s|}\sin\delta$$

where X_s is the synchronous reactance.

Total active power (or total mechanical power developed) by the motor is

$$P = \frac{3|V||E|}{|X_s|}\sin\delta \tag{11.17}$$

$$= P_{\max}\sin\delta, \text{ where } P_{\max} = \frac{3|V||E|}{|X_s|}.$$

Mechanical torque developed by the motor

$$T = \frac{P_{\max}\sin\delta}{\omega_s},$$

where (ω_s) is the synchronous speed

$$\therefore \qquad T = \frac{3|V||E|}{\omega_s |X_s|}\sin\delta \tag{11.18}$$

$$= T_{\max}\sin\delta$$

where $(T_{\max})$ is the *maximum torque* which is also known as *pull-out torque*.

The power angle characteristics is shown in Fig. 11.17.

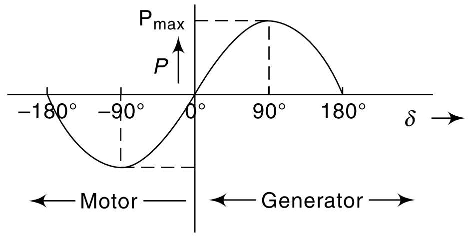


Fig. 11.17 ***Power angle characteristics of a cylindrical rotor synchronous motor***

The synchronous motor can be loaded up to a maximum value of P_{max} which is called the *static stability limit* and after which it will lose synchronism. In order to increase the stability limit at fixed applied voltage V the field current should be increased which in turn will increase the excitation voltage E.

11.28 POWER AND TORQUE DEVELOPED IN SALIENT POLE MOTOR

Neglecting the armature resistance, the simplified phasor diagram of salient pole motor is shown in Fig. 11.18.

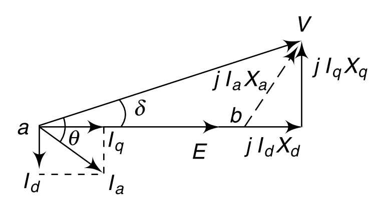


Fig. 11.18 ***Phasor diagram of a salient pole motor (neglecting R_a)***

From the phasor diagram

$$|V|\cos\delta = |E| + I_d X_d$$

and

$$|V|\sin\delta = I_q X_q$$

Hence $I_d = \dfrac{|V|\cos\delta - |E|}{X_d}$ and

$$I_q = \frac{|V|\sin\delta}{X_q} \qquad (11.19)$$

If V_d and V_q are the two components of V, then

$$V_d = -|V|\sin\delta$$

[$-ve$ sign appears as for motor δ is –ve and $\sin(-\delta) = -\sin\delta$]

$$V_d = |V|\cos\delta$$

Per phase active power

$$P = V_d I_d + V_q I_q$$

$$= -|V|\sin\delta \cdot \frac{|V|\cos\delta - |E|}{X_d} + |V|\cos\delta \cdot \frac{|V|\sin\delta}{X_q}$$

$$= \frac{|V||E|\sin\delta}{X_q} + |V|^2 \sin\delta\cos\delta\left(\frac{1}{X_q} - \frac{1}{X_d}\right)$$

$$= \frac{|V||E|}{X_d}\sin\delta + |V|^2\,\frac{X_d - X_q}{2X_d X_q}\sin 2\delta \qquad (11.20)$$

Hence total mechanical power developed

$$P_m = \frac{3|V||E|}{X_d}\sin\delta + 3|V|^2\,\frac{X_d - X_q}{2X_d X_q}\sin 2\delta \qquad (11.21)$$

Total torque developed

$$T = \frac{P_m}{\omega_s} = \frac{3|V||E|}{\omega_s X_d} \sin \delta + 3|V|^2 \frac{X_d - X_q}{2\omega_s X_d X_q} \sin 2\delta \qquad (11.22)$$

$\left(\frac{3|V||E|}{X_d} \sin \delta\right)$ is the power of synchronous motor due to excitation voltage E and $\left(\frac{3|V|^2}{2X_d X_q}(X_d - X_q) \sin 2\delta\right)$ is the power due to saliency. $\left(\frac{3|V|^2}{2\omega_s X_d X_q}(X_d - X_q) \sin 2\delta\right)$ is called the *reluctance torque* and is independent of field excitation. The power angle curve of salient pole synchronous motor is shown in Fig. 11.19.

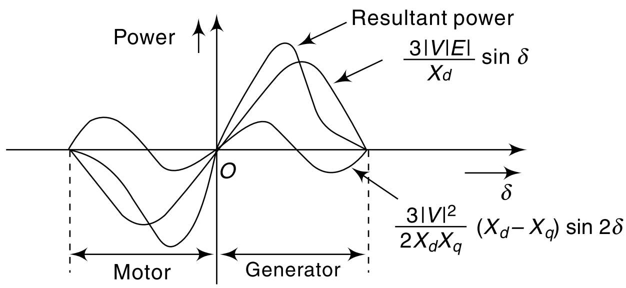


Fig. 11.19 ***Power angle characteristics of a salient pole synchronous motor***

11.12 A 1200 kW load has a power factor of 0.6 lagging. Find the rating of a synchronous condenser to raise the power factor to 0.9 lagging. Also find the total kVA supplied at the new power factor.

Solution

The phasor diagram is shown in Fig. 11.20

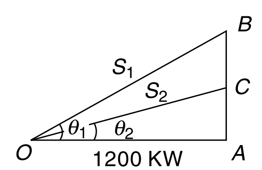


Fig. 11.20 ***Phasor diagram***

$$P = 1200 \text{ kW}$$

At 0.3 p.f. lagging,

$$\text{Apparent power is } S_1 = OB = \frac{1200}{\cos \theta_1} = \frac{1200}{0.6}$$

$$= 2000 \text{ kVA}$$

Reactive power $Q_1 = AB = S_1 \sin \theta_1 = 2000 \sin(\cos^{-1} 0.6) = 1600$ KVAR

When power factor is raised to 0.9 by a synchronous condenser, real power remaining same,

$$P = 1200 \text{ kW},$$

$$\text{Apparent power } S_2 = \frac{1200}{\cos \theta_2} = \frac{1200}{0.9} = 1333.33 \text{ kVA}$$

Reactive power $Q_2 = AC = S_2 \sin \theta_2 = 1333.33 \sin(\cos^{-1} 0.9) = 581.18$ KVAR

Hence reactive power suplied by the condenser is

$$BC = AB - AC = 1600 - 581.18 = 1018.8 \text{ KVAR}.$$

Therefore rating of the synchronous condenser is 1018.8 KVAR and total kVA supplied is 1333.33 kVA.

11.13 The input to an 11000 V, three-phase star connected synchronous motor is 60 A. The effective resistance and synchronous reactance per phase are 1 Ω and 30 Ω respectively. Find the power supplied to the motor and the induced emf for a p.f. of 0.8 leading.

Solution

Per phase input voltage is $\dfrac{11000}{\sqrt{3}}$ V = 6351 V

$$I_a = 60\angle\cos^{-1} 0.8 \text{ A} = 60\angle 36.87° \text{ A}$$

(Note that θ is +ve as I_a is leading)

$$X_s = 30\ \Omega \text{ and } R_a = 1\ \Omega$$

Power supplied to the motor is $\sqrt{3}\, V_L\, I_a \cos\theta$

$$= \sqrt{3} \times 11000 \times 60 \times 0.8 \text{ W} = 914522.82 \text{ W} = 914.52 \text{ kW}$$

Induced emf per phase is obtained as

$$E = V - I_a\,(R_a + j\,X_s)$$
$$= \frac{11000}{\sqrt{3}} - 60\angle 36.87°\ (1 + j30)$$
$$= 7383 - j1476.03 = 7529.1\ \angle{-11.3°} \text{ V}$$

Hence line voltage of induced emf is $\sqrt{3} \times 7529.1\ V$ i.e 13040.38 V.

11.14 A 30 MVA, 11 kV, three-phase star connected synchronous motor has a direct axis reactance of 6 Ω and quadrature axis reactance of 4 Ω. Determine the excitation voltage and power at full load and unity power factor. Neglect the armature resistance.

Solution

Terminal voltage per phase

$$V = \frac{11000}{\sqrt{3}} = 6351 \text{ V}$$

Armature current per phase

$$I = \frac{30000}{\sqrt{3} \times 11} \text{ A} = 1574.638 \text{ A}$$

$$X_d = 6\ \Omega \text{ and } X_q = 4\ \Omega \text{ (given)}$$

From Eq. (11.14), the excitation voltage

$$E = V - jIX_q - jI_d(X_d - X_q),$$

neglecting armature resistance.

From Fig. 11.18

$$ab \simeq V - jIX_q = 6351 - j1574.638\ \angle 0° \times 4$$
$$= 6350 - j6298.55 = 8944\ \angle{-44.767°}$$

∴ Load angle $\delta = 44.767°$

Now, $I_d = I \sin\delta$

$$= 1574.638 \sin 44.767° = 1108.9 \text{ A}$$

Hence excitation voltage

$$E = V - j\,IX_q - j\,I_d(X_d - X_q)$$
$$= 6351 - j\ 1574.638 \times 4 - j\ 1108.9(6 - 4)$$
$$= 6351 - j\ 6298.55 - j\ 2217.8$$
$$= 6351 - j\ 8516.35 = 10623\ \angle{-53.3°} \text{ V}$$

∴ Excitation voltage per phase is 10623 V

From Eqn (11.21) total power at full load

$$P = \frac{3VE}{X_d}\sin\delta + V^2\ 3\frac{X_d - X_q}{2X_d X_q}\sin 2\delta$$

$$= \frac{3\times 6351\times 10623}{6}\ \sin(44.767°) + (6351)^2\times 3\times\frac{6-4}{2\times 6\times 4}\ \sin(2\times 44.77°)$$

$$= 28797.61 \text{ kW}$$

11.15 A 2500 V, three-phase star connected synchronous motor has a resistance of 0.35 Ω per phase and synchronous reactance of 2.2 Ω per phase. The motor is operating at 0.75 p.f. leading with a line current of 250 A. Determine the excitation voltage per phase.

Solution

Per phase voltage $V = \frac{2500}{\sqrt{3}}$ V = 1443.42 V

Resistance per phase $R_a = 0.35\ \Omega$

Synchronous reactance per phase $X_s = 2.2\ \Omega$

Power factor is 0.75 leading

Hence $\cos\theta = 0.75$ and $\sin\theta = 0.66$

Line current $I = 250$ A

The phasor diagram of synchronous motor at leading p.f. is shown in Fig. 11.21.

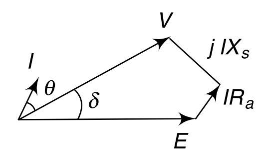


Fig. 11.21 ***Phasor diagram of the synchronous motor at leading p.f.***

Hence excitation voltage per phase can be obtained.

$$E = \sqrt{(V\cos\theta + IR_a)^2 + (V\sin\theta - IX_s)^2}$$

$$= \sqrt{(1443.42\times 0.75 + 250\times 0.35)^2 + (1443.42\times 0.66 - 250\times 2.2)^2}$$

$$= \sqrt{1369052 + 162132.82} = 1237.4 \text{ V}$$

11.16 A 440 V, 50 Hz, three-phase synchronous motor takes 50 A at a power factor of 0.85 lagging. The synchronous reactance per phase is 5 Ω and the resistance is negligible. The number of armature conductors per phase is 40. The distribution factor is 0.966 and the coils are full pitched coils. Determine the flux per pole.

Solution

Terminal voltage per phase is $\frac{440}{\sqrt{3}}$ = 254 V

Armature current $I = 50$

Power factor $\cos\theta = 0.85$

Hence, $\sin\theta = 0.5268$

Given, $X_s = 5\Omega$

$Z = 40$

Distribution factor $K_d = 0.966$

Hence number of turns per phase is

$$T_p = \frac{40}{2} = 20$$

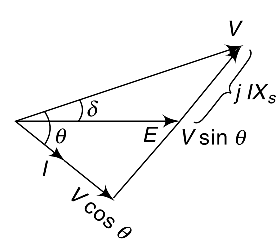


Fig. 11.22 ***Phasor diagram of problem in Ex. 11.16***

From the phasor diagram (Fig. 11.22),

$$E = \sqrt{(V\cos\theta)^2 + (V\sin\theta - IX_s)^2}$$

$$= \sqrt{(254\times 0.85)^2 + (250\times 0.5268 - 50\times 5)^2} = \sqrt{46612.8 + 13995} = 246.19 \text{ V}$$

We know, $E = 4.44\, K_d\, K_p f\, \phi\, T_{\text{ph}}$,

where f is the frequency, ϕ is the flux per pole and K_p is the pitch factor which is 1 in this case. Hence flux per pole is obtained

$$\phi = \frac{246.19}{4.44 \times 0.966 \times 1 \times 50 \times 20} = 57.4 \text{ mwb.}$$

ADDITIONAL EXAMPLES

11.17 A three-phase star connected alternator has 108 slots and 15 conductors per slot. The flux per pole is 0.03 Wb and the total number of poles is 12. Determine the frequency of the emf generated and the emf generated per phase if the speed of the alternator is 500 rpm.

Solution

Synchronous speed $N_s = \dfrac{120 f}{P} = 500$ rpm

Here, $P = 12$

$\therefore$ Frequency $f = \dfrac{500 \times 12}{120} = 50$ Hz.

In the problem it is given that $\phi = 0.03$ Wb,

total number of slots $S = 108$.

$\therefore$ Total number of conductors $= 108 \times 15 = 1620$

Hence number of turns per phase $T_{\text{ph}} = \dfrac{1620}{2 \times 3} = 270$

Number of slots per pole per phase $m = \dfrac{108}{12 \times 3} = 3$

Slot angle $\beta = \dfrac{180°}{\text{No. of slots per pole}} = \dfrac{180°}{\frac{108}{12}} = 20°$,

hence distribution factor

$$K_d = \frac{\sin \dfrac{m\beta}{2}}{m \sin \dfrac{\beta}{2}} = \frac{\sin \dfrac{3 \times 20°}{2}}{3 \sin \dfrac{20°}{2}} = \frac{0.5}{0.521} = 0.9598.$$

Considering pitch factor $K_p = 1$,

Emf generated per phase is

$$E = 4.44\, K_d K_p f\, \phi\, T_{\text{ph}} = 4.44 \times 0.9598 \times 1 \times 0.03 \times 50 \times 270 = 1725.9 \text{ V.}$$

11.18 A 10-pole, three-phase alternator with a single layer winding with full pitch coils has 16 slots per pole. Each coil has 20 turns. The flux per pole is 0.025 Wb and the speed of the machine is 1000 rpm. The current per conductor is 75 A. Neglecting the internal voltage drop find the kVA output of the stator winding.

Solution

Given:

$$P = 10$$
$$\phi = 0.025 \text{ Wb}$$
$$N_s = 1000 \text{ rpm}$$

In a single layer winding machine number of coils $= \frac{1}{2}$(number of slots)

$$= \frac{1}{2} \times 16 \times 10 = 80$$

Total number of turns = 20 × 80 = 1600

Number of turns per phase $T_{ph} = \frac{1600}{3} \simeq 533$

Hence, emf induced per phase is

$$E = 4.44\, f\, \phi\, T_{ph} = 4.44 \times 50 \times 0.025 \times 533 = 2958.15 \text{ V}$$

$$\text{Total kVA} = 3 \times 2958.15 \times 75 \times 10^{-3} = 665.58$$

Hence output of the stator winding is 665.58 kVA.

11.19 A 4 pole three-phase, 50 Hz, 2500 V synchronous machine has 48 slots. Each slot has two conductors. The coil pitch is 46 slots. Determine the flux per pole.

Solution

Number of slots per pole per phase $m = \frac{48}{4 \times 3} = 4$

$$\text{Slot angle } \beta = \frac{180°}{\text{Number of slots per pole}} = \frac{180° \times 4}{48} = 15°$$

$$\text{Distribution factor } K_d = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}} = \frac{\sin \frac{4 \times 15°}{2}}{4 \sin \frac{15°}{2}} = \frac{0.5}{0.522} = 0.9576$$

Total number of slots = 48
but coil pitch = 46 slots.
Hence the coils are short pitched by (48 – 46) i.e., 2 slots
Short pitching angle $\alpha = 2 \times 15° = 30°$

Hence pitch factor $K_p = \cos \frac{\alpha}{2} = \cos \frac{30°}{2} = 0.9659$

Total number of conductors is (48 × 2) i.e 96

Hence number of turns per phase $T_{ph} = \frac{96}{2 \times 3} = 16$

Emf per phase $E = \frac{2500}{\sqrt{3}}$ V = 1443.42 V

If ϕ be the flux per pole,

$$E = 4.44\, K_p K_d f\, (T_{ph})\, \phi.$$

Hence $$\phi = \frac{1443.42}{4.44 \times 0.9659 \times 0.9576 \times 50 \times 16} = 0.439 \text{ Wb.}$$

11.20 Find the number of series turns required for each phase of a three-phase, star connected 50 Hz, 10 pole alternator with 90 slots. The line voltage is 11 kV and the flux per pole is 0.2 Wb.

Solution

Given, $P = 10$ and $S = 90$

$$\text{Voltage per phase} = \frac{11{,}000}{\sqrt{3}} = 6351 \text{ V}$$

Also $\phi = 0.2$ Wb.

If T is the series turns in each phase, we can write

$$6351 = 4.44 \times 0.2 \times 50 \times T$$

or $$T = \frac{6351}{4.44 \times 0.2 \times 50} = 143$$

11.21 A 2000 kVA, 11 kV, 50 Hz star connected synchronous generator has a no load voltage of 13 kV. At full load the power factor is 0.8 lagging. Determine the synchronous reactance, voltage regulation, torque angle and power developed. Neglect the armature resistance.

Solution

$$\text{Rated current } I = \frac{2000 \times 10^3}{\sqrt{3} \times 11 \times 10^3} \text{ A} = 105 \text{ A}$$

Power factor ($\cos\theta$) = 0.8 i.e., $\theta = \cos^{-1} 0.8 = 36.87°$

$$\text{No load voltage per phase } E = \frac{13{,}000}{\sqrt{3}} \text{ V} = 7505.8 \text{ V}$$

$$\text{Full load voltage per phase } V = \frac{11{,}000}{\sqrt{3}} \text{ V} = 6351 \text{ V}$$

If X_s be the synchronous reactance then

$$7505.8\angle\delta = 6351\angle 0° + j105\angle{-36.87°}\, X_s$$

Equating the real and imaginary terms,

$$7505.8 \cos\delta = 6351\angle 0° + 105 \sin 36.87°\, X_s = 6351 + 63\, X_s \quad \text{(i)}$$

and $$7505.8 \sin\delta = 105 \cos 36.87°\, X_s = 84\, X_s \quad \text{(ii)}$$

Squaring equations (i) and (ii) and then adding them

$$(7505.8)^2 = (6351)^2 + 2 \times 6351 \times 63\, X_s + (63)^2 X_s^2 + (84)^2 X_s^2$$

or $11025\, X_s^2 + 800226 X_s - 16001832 = 0$

or $X_s = 1.857\ \Omega$

$$\text{Voltage regulation} = \frac{13-11}{11} \times 100 = 18.18\%$$

Now, from equation (ii) we can write

$$\sin\delta = \frac{84 \times 1.806}{7505.8} = 0.0208$$

$\therefore$ Torque angle (δ) = 1.19°.

Power developed $P = \sqrt{3}\, VI \cos\theta = \sqrt{3} \times 11 \times 105 \times 0.8$ kW = 1600 kW.

11.22 A three-phase alternator has reactance of 8 Ω and armature current of 200 A at unity p.f. when running on 11 kV at constant frequency. If the emf is raised by 20%, input remaining unchanged, find the new value of machine current and power factor.

Solution

When $I = 200$ A, $\cos\theta = 1$,

$$E = \frac{11{,}000}{\sqrt{3}} + j\, 200 \times 8 = 6549.30 \angle 14.14° \text{ V}$$

New value of emf is given by

$$|E'| = 1.2 \times 6549.30 = 7859.16 \text{ V}$$

Input power being constant, if I_1 and $\cos\theta_1$ be the new current and power factor then we have

$$VI\cos\theta = VI_1\cos\theta_1$$

or $$I\cos\theta = I_1\cos\theta_1 = 200\times 1 = 200\text{ A} \qquad (\because \cos\theta = 1)$$

Now, $$7859.16 = \left|\frac{11{,}000}{\sqrt{3}} + jI_1\angle{-\theta_1}\times 8\right|$$

$$= |6351 + j8\,(I_1\cos\theta_1 - jI_1\sin\theta_1)|$$

$$= |6351 + j8\,(200 - jI_1\sin\theta_1)| = \sqrt{(6351+8I_1\sin\theta_1)^2 + (8\times 200)^2}$$

or $$I_1\sin\theta_1 = 197.95$$

Also, $$\tan\theta_1 = \frac{I_1\sin\theta_1}{I_1\cos\theta_1} = \frac{197.95}{200} = 0.989$$

or $$\theta_1 = 44.705°$$

or $$\cos\theta_1 = 0.711$$

and $$I_1 = \frac{200}{0.711} = 281.4\text{ A}$$

Hence new value of machine current is 281.4 A and power factor is 0.711 lagging.

11.23 A three-phase synchronous motor of 1000 kW and 6.6 kV has synchronous reactance of 10 Ω per phase. The efficiency of the motor is 90%. Neglecting armature resistance determine the minimum current and the corresponding induced emf at full load.

Solution

At constant supply voltage and input power the current is minimum when power factor is 1.

$$\text{Output power} = 1000\text{ kW}$$

$$\text{Efficiency} = 90\%$$

$$\text{Hence input power} = \frac{1000}{0.9} = 1111.11\text{ kW}$$

At unity power factor,

$$\text{line current} = \frac{1111.11\times 10^3}{\sqrt{3}\times 6.6\times 10^3\times 1} = 97.2\text{ A}$$

Also at unity power factor,

$$\text{induced emf } E = \frac{6600}{\sqrt{3}}\angle 0° - j97.2\angle 0°\times 10$$

$$= 3810.6\angle 0° - j\,972 = 3932.64\angle{-14.31°}\text{ V.}$$

Hence at full load emf is 3932.64 V per phase.

11.24 A 1000 kVA, 33 kV star connected salient pole synchronous motor has X_d = 20 Ω and X_q = 10 Ω per phase respectively. Determine the excitation emf when the motor is supplying rated load at 0.8 leading p.f. What maximum load the motor can supply if the excitation is cut off without loss of synchronism? Neglect armature resistance and assume load angle to be 25.35°.

Solution

$$\text{Armature current} = \frac{1000}{\sqrt{3}\times 3.3}\text{ A} = 175\text{ A.}$$

$$\text{Terminal voltage per phase } V = \frac{3.3}{\sqrt{3}}\text{ kV} = 1.9053\text{ kV} = 1905.3\text{ V.}$$

Given, $X_d = 20\ \Omega$ and $X_q = 10\ \Omega$.

Here load angle $\delta = 25.35°$.

Now, $I_d = I \sin \delta = 74.92$ A

$\therefore$ Excitation voltage per phase is given as

$$E = V - jIX_q - jI_d(X_d - X_q)$$
$$= 1905.3 - j175 \times 10 - j74.92(20 - 10)$$
$$= 1905.3 - j1750 - j\ 749.2$$
$$= 1905.3 - j2499.2 = 3142.6 \angle -52.68° \text{ V}.$$

When the excitation is cut off, output power is only the reluctance power.

$$\text{Power} = 3V^2 \frac{X_d - X_q}{2X_d X_q} \sin 2\delta$$
$$= 3(1905.3)^2 \frac{20-10}{2 \times 20 \times 10} \sin (2 \times 25.35°)$$
$$= 210687 \text{ W} = 210.687 \text{ kW}.$$

11.25 A 6.6 kV, star connected synchronous motor is operating at constant excitation. The synchronous impedance is $(1 + j8)\ \Omega$. It operates at a power factor of 0.8 lagging and draws 1000 kW from the supply. Determine its power factor when the input is increased to 1200 kW.

Solution

$$Z_s = 1 + j8 = 8.06\angle 82.87°\ \Omega$$
$$V = \frac{6600}{\sqrt{3}} = 3810.62 \text{ V}$$
$$P_1 = \frac{1000}{3} \text{ KW} = 333.33 \text{ kW}$$
$$\cos \theta = 0.8 \text{ lagging}$$

Hence $Q_1 = 333.33 \times \tan \theta = 249.997$ KVAR

From Eqs (11.15) and (11.16)

$$P_1 = 333.33 \times 10^3 = \frac{|V|^2}{|Z_s|} \cos \theta - \frac{|V||E|}{|Z_s|} \cos (\delta + \theta) \qquad \text{(i)}$$

$$Q_1 = 249.997 \times 10^3 = \frac{|V|^2}{|Z_s|} \sin\theta - \frac{|V||E|}{|Z_s|} \sin (\delta + \theta) \qquad \text{(ii)}$$

From Eq. (i)

$$333.33 \times 10^3 = \frac{(3810.62)^2}{8.06} \times 0.8 - \frac{3810.62|E|}{8.06} \cos (\delta + \theta)$$

or $E \cos (\delta + \theta) = 2343.52$ (iii)

From Eq. (ii)

$$249.997 \times 10^3 = \frac{(3810.62)^2}{8.06} \times 0.6 - \frac{3810.62|E|}{8.06} \sin (\delta + \theta)$$

or $E \sin (\delta + \theta) = 1757.59$ (iv)

Dividing Eq. (iv) by Eq. (iii) we get

$$\tan (\delta + \theta) = 0.75$$

or $\delta = 36.87° - 36.87° = 0$

Hence $E = \dfrac{2343.456}{\cos \theta} = 2929.32$ V

Since $P_2 = \frac{1200}{3}$ kW = 400 kW, we can write

$$400 \times 10^3 = \frac{(3810.62)^2}{8.06} \cos\theta - \frac{3810.62 \times 2929.32}{8.06} \cos\theta$$

or $\cos\theta = 0.96$

$\therefore$ Now power factor is 0.96 lagging.

11.26 The full load current of a 6.6 kV, star connected motor is 200 A at 0.8 p.f. lagging. The per phase resistance and reactance of the motor is 1 Ω and 7 Ω respectively. If the mechanical loss is 40 kW find the excitation emf, torque angle, efficiency and shaft output of the motor.

Solution

Given, $R = 1\ \Omega$

$X_s = 7\ \Omega$

$$V = \frac{6600}{\sqrt{3}} = 3810.6 \text{ V}$$

$I = 200$ A, $\cos\theta = 0.85$.

Hence excitation emf is

$$\begin{aligned} E &= 3810.6 - 200 \angle -\cos^{-1}0.8\ (1 + j\,7) \\ &= 3810.6 - 200 \angle -36.87^\circ \times 7.07 \angle 81.87^\circ \\ &= 3810.6 - 1414 \angle 45^\circ \\ &= 2810.75 - j\,999.8 = 2983.29 \angle -19.58^\circ \text{ V.} \end{aligned}$$

$\therefore$ Excitation emf is 2983.29 V and torque angle is 19.58°.

Mechanical power developed

$$\begin{aligned} &= 3\,EI\cos\theta, \text{ where } \theta \text{ is the angle between } E \text{ and } I \\ &= 3 \times 2983.29 \times 200 \cos(-36.87^\circ + 19.58^\circ) \\ &= 3 \times 2983.29 \times 200 \cos(-17.29^\circ) \\ &= 1709089.88 \text{ W} = 1709 \text{ kW.} \end{aligned}$$

The shaft output= 1709 – 40 = 1669 kW

$$\begin{aligned} \text{Power input} &= \sqrt{3}\, V_L I \cos\theta \\ &= \sqrt{3} \times 6600 \times 200 \times 0.8 = 1829045.65 \text{ W} = 1829 \text{ kW} \end{aligned}$$

Hence efficiency $= \frac{1669}{1829} \times 100\% = 91.25\%$.

EXERCISES

1. Derive an expression for the emf induced in an alternator.
2. Explain the principle of operation of an alternator.
3. Explain the essential difference between the cylindrical rotor and salient pole rotor.
4. Define pitch factor and distribution factor. What are the advantages of distributed and short pitch windings?
5. Draw and explain the equivalent circuit of cylindrical rotor alternator. Also draw the phasor diagram.
6. Draw and explain the phasor diagram for a salient pole alternator when the power factor of the load is lagging.
7. Explain the two-reaction theory as applied to a salient pole machine.

8. Derive an expression for power developed as a function of power angle δ for salient pole alternator.
9. Derive an expression for the torque developed in a three-phase cylindrical rotor synchronous motor.
10. Explain the principle of operation of synchronous motor.
11. Explain various methods of starting of a synchronous motor.
12. Compare between an induction motor and synchronous motor.
13. What do you mean by hunting of a synchronous motor? How do you prevent hunting?
14. A three-phase, delta connected, 16-pole, 50 Hz synchronous generator has 144 slots and 10 conductors/slot. Coils are full pitch and the flux/pole is 0.0248 Wb. What is the value of alternator speed and what is the value of no load voltage? [*Ans:* 1270 V]

[*Hints:* $N = \dfrac{120\,f}{P} = 375$ rpm

No. of slots/pole = $\dfrac{144}{16} = 9$

$\therefore \quad m =$ No. of slots/pole/ph = 3

$\beta = 180°/9 = 20°$

$$K_d = \frac{\sin{(3 \times 20/2)°}}{3 \times \sin{(20/2)°}} = 0.96;\ K_p = 1$$

$$Z \text{ (no. of conductors)} = \frac{144 \times 10}{3} = 480/\text{ph.}$$

$$\therefore \quad E = 4.44 \times 0.96 \times 1.0 \times 0.0248 \times 50 \times \frac{480}{2} \left(\because T = \frac{Z}{2}\right)$$

$$= 1268.5 \text{ V/ph.}$$

Hence $E_{L\text{–}L} = E_{\text{ph}} = 1270$ V.]

15. A 500 V, 50 kVA, 1-ph, 50 Hz, synchronous generator has armature resistance of 0.5 Ω/ph. An excitation current of 10 A produces 100 A armature current in any phase on short circuiting its terminals. At this same exciting currents the open circuit voltage is 400 V. Calculate synchronous reactance. [*Ans:* 3.97 Ω/ph]

[*Hints:* $Z_s/\text{ph} = \dfrac{V_{\text{o/c}}}{I_{\text{s/c}}} = \dfrac{400}{100} = 4\ \Omega$

$$\therefore \quad X_s/\text{ph} = \sqrt{Z_s^2 - R^2} = \sqrt{4^2 - 0.5^2} = 3.97\ \Omega]$$

16. A single-phase alternator has six number of slots per pole. Obtain the value of distribution factor if (i) all the slots are wound and (ii) only two-third of the slots are wound. [*Ans:* 0.64; 0.84]

[*Hints:* (i) $\beta = \dfrac{180°}{6} = 30°$, $K_d = \dfrac{\sin\left(6 \times \dfrac{30°}{2}\right)}{6 \sin\left(\dfrac{30°}{2}\right)} = 0.64$

(ii) $\beta = 30°$; $m = \dfrac{2}{3}$ of 6 = 4

$$\therefore \quad K_d = \frac{\sin\left(4 \times \dfrac{30°}{2}\right)}{4 \sin\left(\dfrac{30°}{2}\right)} = 0.84.]$$

17. A Δ-connected, 3 ph, 50 Hz, 8000 $V(L - L)$, 750 rpm alternator has 3 slots/pole/ph. Coil span is 7 slots. There are 10 turns/coil. Find the coil span, distribution factor and flux/pole. [*Ans:* 140°; 0.94; 333 mWb]

[*Hints:* Pole pitch has 3 × 3 = 9 slots.

$$\therefore \text{ Coil span} = \frac{7}{9} \times 180° = 140°$$

$$K_C = \cos\frac{180° - 140°}{2} = 0.94$$

$$m = 3 \text{ (given)}; \beta = \frac{180}{9} = 20°$$

$$\therefore \quad K_d = \frac{\sin\left(3 \times \dfrac{20}{2}\right)}{3 \sin \dfrac{20}{2}} = 0.96$$

$$\because \quad E = 4.44\, \phi\, K_c K_d\, f\, T$$

Hence $\phi = \dfrac{E}{4.44\, \phi\, K_c\, K_d\, fT} = \dfrac{8000}{4.44 \times 0.94 \times 0.96 \times 50 \times T}$

where T = No. of poles × No. of slots/pole/ph × (No. of turns/coil) ÷ 2
= 8 × 3 × 10/2 = 120

Thus, $\phi = 333$ mWb.]

18. A three-phase,16 pole synchronous generator has a resultant air gap flux of 0.06 Wb per pole. The flux is distributed sinusoidally over the pole. The stator has two slots per pole per phase and four conductors per slot are accommodated in two layers. The coil span is 150° electrical. Calculate the phase and line induced voltages when the machine runs at 375 rpm. [*Ans:* 795.3 V, 1377.5 V]

19. A three-phase star connected alternator is rated at 1600 kVA, 13.5 kV. The armature effective resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Determine the percentage regulation for a load of 1280 kW at p.f. of 0.8 leading. [*Ans:* –12%]

20. A 550 V, 55 kVA, 1 – ϕ alternator has dc resistance of 0.2 Ω per phase. A field current of 10 A produces an armature current of 200 A on short circuit and 450 V on open circuit. Determine the synchronous reactance of the alternator armature winding and voltage regulation at full load with 0.8 p.f. (lag) load. [*Ans:* 2.24 Ω; 30.91%]

[*Hints:* $I = \dfrac{55 \times 10^3}{550} = 100$ A

$$Z_s = \frac{V_{o/c}}{I_{s/c}} = \frac{450}{200} = 2.25\ \Omega/\text{ph}$$

$$\therefore \qquad X_s = \sqrt{Z_s^2 - R_s^2} = \sqrt{2.25^2 - 0.2^2} = 2.24\ \Omega$$

$$E_f = \sqrt{(V_t \cos\phi + IR_s)^2 + (V_t \sin\phi + IX_s)^2}$$

$$= \sqrt{(550 \times 0.8 + 100 \times 0.2)^2 + (550 \times 0.6 + 100 \times 2.24)^2}$$

$$= 720 \text{ V/ph}$$

$$\therefore \qquad \Delta V\% = \frac{E_f - V_t}{V_t} \times 100 = 30.91\%.]$$

21. A 40 MVA, 3ph, star connected, 3000 rpm synchronous generator has open circuit voltage of 12 kV while the terminal voltage is 11 kV ($L - L$) at rated load of 0.8 p.f. (lag). If the synchronous reactance is 2.0 Ω/phase, find the power angle when the machine is delivering full load. What is the line current at full load and what is the maximum power output?

[*Ans:* 29°; 2099 A; 66 MW]

[*Hints:* $P_{fl} = 40 \times 10^6 \times \cos\phi = 40 \times 0.8 \times 10^6 = 32$ MW

i.e, $P_{fl}/\text{phase} = \frac{32}{3} = 10.67$ MW

However, $P = \frac{E_f \times V_t}{X_s} \sin\delta$

Hence, $E_f = \frac{12}{\sqrt{3}} \times 10^3$; $V_t = \frac{11}{\sqrt{3}} \times 10^3$

$X_s = 2\Omega/\text{ph}$

$$\therefore \qquad \sin\delta = \frac{PX_s}{E_f \times V_t} = \frac{10.67 \times 10^6 \times 2.0}{6.93 \times 10^3 \times 6.35 \times 10^3} = 0.485$$

$$\therefore \qquad \delta = 29^\circ$$

$$I_L = \frac{S}{V} = \frac{40 \times \frac{10^6}{3}}{11 \times 10^3 \sqrt{3}} \simeq 699.82 \text{ A}$$

$$P_{\max} = \frac{E_f \times V_t}{X_s} = \frac{6.93 \times 10^3 \times 6.35 \times 10^3}{2} = 22 \text{ MW/ph}$$

or $P_{\max}(3\phi) = 66$ MW.]

22. A three-phase alternator has a direct axis reactance of 0.8 p.u. and quadrature axis reactance of 0.4 p.u. Calculate the per unit excitation voltage per phase when the alternator is supplying rated load at 0.8 p.f. lagging at rated voltage. Neglect armature resistance. [*Ans:* 1.59 p.u.]
23. A 25 MVA, 6.6 kV, 50 Hz, 4-pole alternator has a p.u. armature resistance 0.004 p.u. and synchronous reactance 0.67 p.u. When the machine is supplying rated current at rated voltage, the induced emf is 1.5 p.u. Find torque angle and power factor. [*Ans:* 21°, 0.805 lagging]
24. A 3000 kW, star connected, three-phase, 6.6 kV, 50 Hz, 6-pole synchronous motor operates at full load at a leading p.f. of 0.8. If the synchronous reactance is 10 Ω/ph, calculate (i) the apparent power of the motor (per phase), (ii) the line current, (iii) the induced emf, and (iv) the torque angle.

[*Ans:* 1250 kVA; $328\angle 36.9^\circ$; 6347.67 V/ph, 24.41°]

[*Hints:* P = 1000 kW/ph (given),

∴ S (apparent power) = $P/\cos\phi$ = 1000/0.8 = 1250 kVA

$$I = \frac{S}{V_t} = \frac{1250 \times 10^3}{6600/\sqrt{3}} = 328 \text{ A}$$

It may be noted here that I leads V by $\cos^{-1}$ (0.8) i.e., at angle 36.9°.

For the synchronous motor,

$$E_f = V_t - j\, IX_s$$

$$= \frac{6600}{\sqrt{3}} - j \times 328\angle 36.9° \times 10$$

$$= 3811\angle 0° - 328\angle 36.9° \times 10\angle 90°, \quad \text{with } V_t \text{ as reference.}$$

$$\therefore \quad E_f = 3811 - 3280\angle 126.9°$$

$$= 3811 - 3280\,(\cos 126.9° + j \sin 126.9°)$$

$$= 5780.38 - j\,2623 = 6347.67\angle{-24.41°} \text{ V}$$

∴ Torque angle = 24.41°; E_f = 6347.67 V/ph.

25. A 150 kW, 1200 rpm, 460 V, three-phase, delta connected synchronous motor has X_s = 0.8 Ω/ph. If the excitation voltage E_f is fixed at 300 V per phase, determine (i) the power output at torque angle of 30°, (ii) the pull out torque. Assume a lossless machine. [*Ans:* 50 kW; 100 kW]

[*Hints:*

$$V_{\text{ph}} = \frac{V_{L-L}}{\sqrt{3}} = \frac{460}{\sqrt{3}} = 266 \text{ V}$$

$$P = \frac{E_f V_t}{X_s} \sin \delta = \frac{300 \times 266}{0.8} \times \sin 30° \simeq 50 \text{ kW}$$

Pull out torque = Maximum power output.

Maximum power output occurs at δ = 90°

$$\therefore \quad P_{\max} = \frac{E_f V_t}{X_s} = \frac{300 \times 266}{0.8} = 100 \text{ kW.]}$$

26. A three-phase, 400 V, 50 Hz, 4-pole synchronous motor has a synchronous reactance of 8 Ω and negligible resistance. The field excitation is adjusted so that the power factor is unity, when the motor draws 3 kW from the supply. Determine excitation voltage, power angle and mechanical torque developed. [*Ans:* 233.6 V, 8.5°, 19.1 Nm]

27. A 50 MVA, 11 kV, 50 Hz, three-phase salient pole synchronous motor has d-axis reactance of 0.8 p.u. and q-axis reactance 0.4 p.u. per phase. Neglecting the armature resistance, calculate the excitation voltage in p.u, if the motor draws rated current at a p.f of 0.8 lagging. Also calculate the power due to field excitation and that due to saliency.

[*Ans:* 0.723 p.u., 0.35 p.u., 0.446 p.u.]

28. A 15000 kVA, star connected, 6.6 kV salient pole synchronous motor has X_d = 23.2 Ω and X_q = 14.5 Ω/phase. Determine the excitation emf when the motor is supplying rated load at 0.8 p.f. leading. What maximum load the motor can supply without loss of synchronism, if the excitation is cut off? Neglect armature resistance. [*Ans*: 6.1 kV/phase, 563 kW]

12 SINGLE-PHASE INDUCTION MOTORS

12.1 INTRODUCTION

Single-phase induction motors have numerous and diversified applications both in home and industry. It is probably safe to say that single-phase induction motor applications far outweigh the three-phase motor applications in the domestic sector. At home normally only single-phase power is provided, since power was originally generated and distributed to provide lighting. For this reason early motor-driven appliances in the home depended on the development of the single-phase motor. Single-phase induction motors are usually small sized motors of fractional kilowatt rating. They find wide applications in fans, washing machines, refrigerators, pumps, toys, hair dryers, etc. Single-phase induction motors operate at low power factors and are less efficient than three-phase induction motors.

12.2 PRODUCTION OF TORQUE

From the study of three-phase induction motors it is seen that the three-phase distributed stator winding sets up a rotating magnetic field which is fairly constant in magnitude and rotates at synchronous speed. In single-phase induction motor there is only single field winding excited with alternating current and therefore it is not inherently self-starting since it does not have a true revolving field. Various methods have been devised to initiate rotation of the squirrel cage rotor and the particular method employed to start the rotor of single phase motor will designate the specific type of motor.

Consider the behaviour of the magnetic field set up by an ac current in the single-phase winding. With reference to Fig. 12.1 when the current is flowing in the field winding, if the current is sinusoidal, neglecting the saturation effects of the magnetic iron circuit, the flux through the armature will vary sinusoidally with time. The magnetic field created at a particular instant of time, will reverse during the next half cycle of the ac supply voltage. Since the flux is pulsating it will induce currents in the rotor bars which in turn will create a rotor flux which

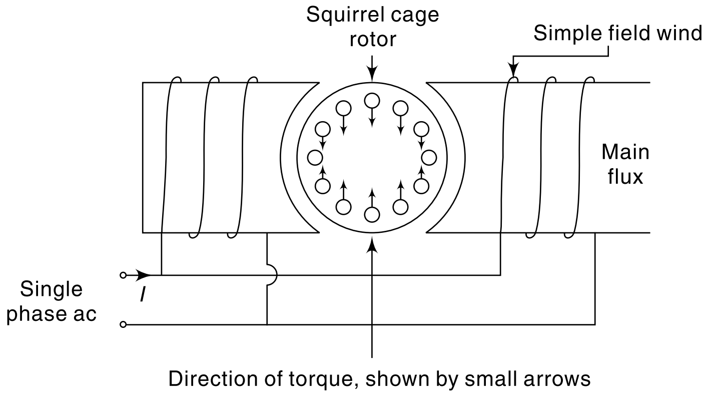


Fig. 12.1 ***Torque produced in the squirrel cage of a single-phase induction motor***

by Lenz's law opposes that of the main field. The direction of the rotor current as well as the torque created can also be determined. It is apparent that the clockwise torque produced is counteracted by the counter-clockwise torque and so no motion results, i.e. the motor is at standstill.

However, any pulsating field can be resolved into two components, equal in magnitude but oppositely rotating phasors as shown in Fig. 12.2(a). The maximum value of the component fields equals half of the main field. A physical interpretation of the two oppositely rotating field components is predicted in Fig. 12.2(b). Each component field guides around the air gap in opposite directions with equal velocities, their instantaneous sum represents the instantaneous resultant magnetic field which changes from (ϕ_{max}) to (ϕ_{min}). This method of field analysis is commonly known as the *double revolving field theory*. Each field component acts independently on the rotor and in a similar fashion as of the rotating field in a three-phase induction motor. The only difference is that here there are two fields, one tending to rotate the rotor clockwise and the other tending to rotate the rotor anticlockwise.

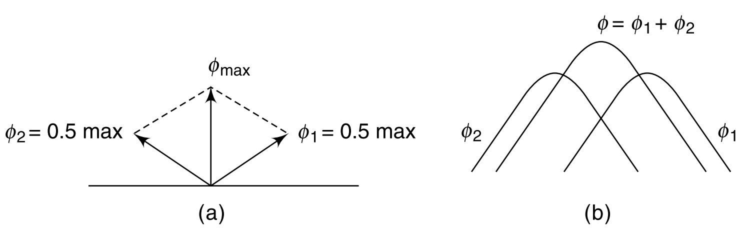


Fig. 12.2 ***Pulsating field resolved into two oppositely rotating fields***

The torque slip curve of the actual motor can be obtained by applying the principle of superposition to the hypothetical constituent motor. The clockwise flux component produces torque called the *forward torque* which is operating at slip "s". The counterclockwise flux component produces a backward torque which operates at slip (2-*s*). Their individual torque slip curves will have the form shown in Fig. 12.3 and the algebraic sum of their ordinates will give the resultant

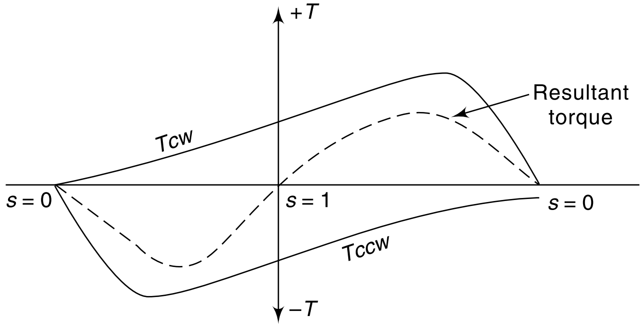


Fig. 12.3 ***Torque-speed characteristic of a single-phase squirrel cage induction motor***

torque. It is observed that at standstill (s = 1) the two torque components produced are equal but acting in opposite direction. Although the net torque produced at standstill is zero, it is seen that if the rotor is advanced in either direction, a net torque will result and the rotor will continue to rotate in the direction in which it has been started. The component torque in the direction of rotation may be termed as forward torque while the other one may be treated as the backward torque.

Thus, once started the single-phase motor having a simple winding will continue to run in the direction in which it is started. The manual self-starting is not a desirable feature in practice, and modifications are introduced to obtain the torque required to start. To accomplish this, a quadrature flux component in time and space with the stator flux must be provided at standstill. *Auxiliary windings* are normally placed on the stator to provide starting torque. The auxiliary winding is also called *starting winding*.

12.3 EQUIVALENT CIRCUIT OF A SINGLE-PHASE INDUCTION MOTOR

At standstill the equivalent circuit of a single-phase induction motor is exactly similar to that of a transformer on short circuit. The equivalent circuit at standstill condition is shown in Fig. 12.4. R_c and X_ϕ represent the core loss and magnetizing reactance. r_1 and x_1 are the resistance and leakage reactance of the stator, r_2' and x_2' are the resistance and leakage reactance of the rotor referred to the stator.

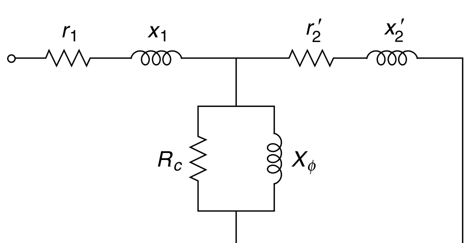


Fig. 12.4 ***Equivalent circuit of a single-phase induction motor at standstill***

The air-gap flux can be resolved into two oppositely rotating components. These components at standstill are equal in magnitude, each one contributing an equal share to the resistive and reactive voltage drops in the rotor circuit. Hence r_2 and x_2 can be split into two parts, each one corresponding to the effects of one of the magnetic fields. E_f and E_b are the voltages set up by the two oppositely rotating fields. viz. *forward* and *backward* rotating fields respectively. The equivalent circuit considering the effect of forward and backward flux component is shown in Fig. 12.5.

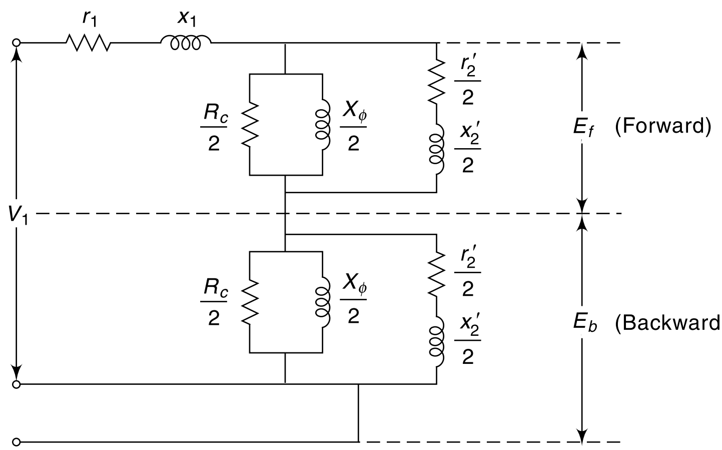


Fig. 12.5 ***Equivalent circuit at standstill showing the effect of forward and backward flux components***

When the motor is running at a slip s, the slip for the forward field is s and for backward field is $(2 - s)$. Hence the resistance in the forward field becomes $\left(\frac{r_2'}{2s}\right)$ and in the backward field becomes $\left(\frac{r_2'}{2(2-s)}\right)$. As s is normally very small, $\left(\frac{r_2'}{2s}\right)$ is much higher than $\left(\frac{r_2'}{2(2-s)}\right)$. Hence E_f is much greater than E_b. The equivalent circuit at any slip (s) is shown in Fig. 12.6.

From Fig. 12.6

$$P_{gf} = \text{air-gap power of forward field} = (I_{2f}')^2 \frac{r_2'}{2s} W$$

$$P_{gb} = \text{air-gap power of backward field} = (I_{2b}')^2 \frac{r_2'}{2(2-s)} W$$

$$T_f = \text{torque due to forward field} = \frac{P_{gf}}{2\pi n_s} \text{ Nm}$$

$$T_b = \text{torque due to backward field} = \frac{P_{gb}}{2\pi n_s} \text{ Nm}$$

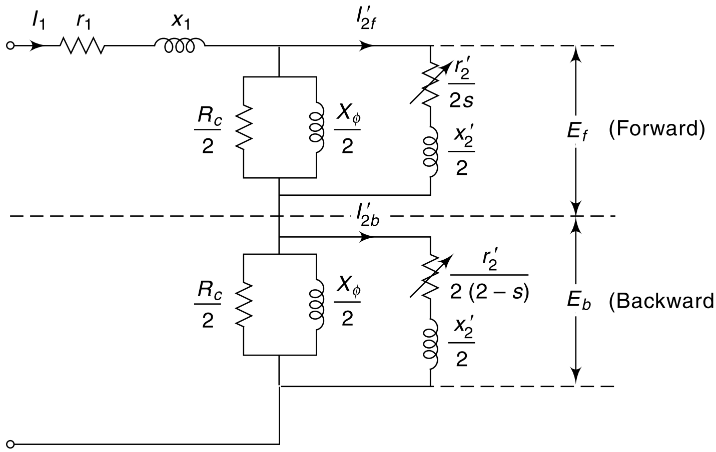


Fig. 12.6 *Equivalent circuit of a single-phase induction motor at any slip* s

Net torque, $T = T_f - T_b$
Rotor copper loss due to forward field ($P_{cu(rot.)f}$) = sP_{gf}
Rotor copper loss due to backward field ($P_{cu(rot.)b}$) = $(2 - s)P_{gb}$
Total rotor copper loss ($P_{cu(rot.)}$)= $sP_{gf} + (2 - s)P_{gb}$
Mechanical power developed (= P_m) = $(1 - s)(P_{gf} - P_{gb})$.

12.4 DETERMINATION OF PARAMETERS OF EQUIVALENT CIRCUIT

The parameters of the equivalent circuit of a single-phase induction motor can be determined from the *no load* and *blocked rotor* test.

12.4.1 Blocked Rotor Test

In this test a very small voltage is applied to the stator and the rotor is blocked (Care is to be taken such that the stator current does not exceed the f.l. current). The voltage, current and power input to the stator are measured. When the rotor is blocked, $s = 1$ and hence parallel combination $\left(\frac{R_c}{2}\right)$ and $\left(\frac{X_\phi}{2}\right)$ is much greater than $\left[\frac{r_2'}{2} + j\frac{x_2'}{2}\right]$ (in Fig. 12.6). Therefore under blocked rotor test the equivalent circuit reduces to that shown in Fig. 12.7. Since $(R_c/2)$ and $(X_\phi/2)$ are of very high values hence they can be neglected in the equivalent circuit.

Let V_{sc}, I_{sc} and W_{sc} be the input voltage, current and power during blocked rotor test.

The total resistance $(r_1 + r_2') = \frac{W_{sc}}{I_{sc}^2} = R_{sc}$

Total impedance, $Z_{sc} = \frac{V_{sc}}{I_{sc}}$

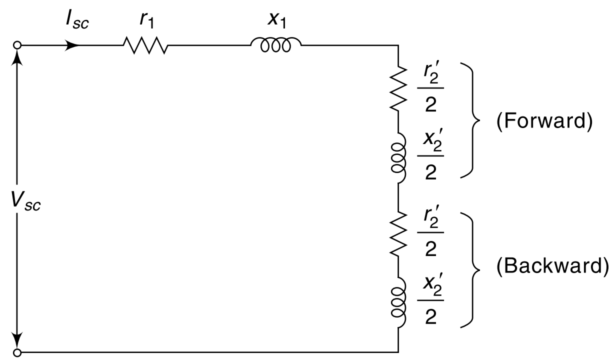


Fig. 12.7 ***Equivalent circuit under blocked rotor condition***

Hence total reactance $(x_1 + x_2') = \sqrt{Z_{sc}^2 - R_{sc}^2}$

Generally, $r_1 = r_2'$ and $x_1 = x_2'$. Hence r_1, r_2', x_1 and x_2' can be determined from this text.

12.4.2 No Load Test

In this test the motor is run on no load condition and voltage V_o, current I_o and power W_o to the stator are measured. At no load s is very small and core loss resistance R_c is neglected. Hence from Fig. 12.6, $\left(\frac{r_2'}{s}\right)$ is much greater than $\left(\frac{X_\phi}{2}\right)$. Also, $\frac{r_2'}{2\,(2-s)}\left(\simeq \frac{r_2'}{4}\right)$ is much smaller than $X_\phi/2$. Therefore under no load condition the equivalent circuit can be reduced to that shown in Fig. 12.8. Here, $\left(\frac{r_2'}{s}\right)$ and $\left(\frac{X_\phi}{2}\right)$ are thus neglected in equivalent circuit.

No load p.f. $(\cos\theta_0) = \dfrac{W_o}{V_o I_o}$

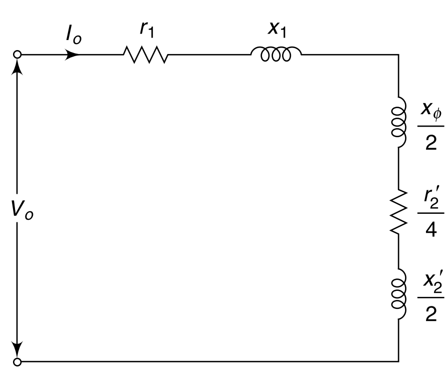


Fig. 12.8 ***Equivalent circuit under no load condition***

Now voltage across $\left(\frac{X_\phi}{2}\right)$ is $\left[V_o - I_o\angle-\theta_o\left\{\left(r_1+\frac{r_2'}{4}\right)+j\left(x_1+\frac{x_2'}{2}\right)\right\}\right]$

Hence, $$\frac{X_\phi}{2} = \frac{V_o - I_o\angle-\theta_o\left[\left(r_1+\frac{r_2'}{4}\right)+j\left(x_1+\frac{x_2'}{2}\right)\right]}{I_o}$$

and X_ϕ can thus be determined.

12.1 A 200 W, 240 V, 50 Hz single phase induction motor runs on rated load with a slip of 0.05 p.u. The parameters are

$$r_1 = 11.4\ \Omega,\ x_1 = 14.5\ \Omega,$$
$$r_2' = 13.8\ \Omega,\ x_2' = 14.4\ \Omega,\ X_\phi = 270\ \Omega$$

Calculate (i) power factor, (ii) input power and (iii) efficiency.

Solution

From Fig. 12.6 neglecting the core loss resistance R_C the total series impedance

$$Z = r_1 + jx_1 + \frac{j\frac{X_\phi}{2}\left(\frac{r_2'}{2s}+j\frac{x_2'}{2}\right)}{\frac{r_2'}{2s}+j\left(\frac{X_\phi}{2}+\frac{x_2'}{2}\right)} + \frac{j\frac{X_\phi}{2}\left(\frac{r_2'}{2(2-s)}+j\frac{x_2'}{2}\right)}{\frac{r_2'}{2(2-s)}+j\left(\frac{X_\phi}{2}+\frac{x_2'}{2}\right)}$$

$$= 11.4 + j14.5 + \frac{-\frac{270\times 14.4}{4}+j\frac{270\times 13.8}{4\times 0.05}}{\frac{13.8}{2\times 0.05}+j\left(\frac{270}{2}+\frac{14.4}{2}\right)} + \frac{-\frac{270\times 14.4}{4}+j\frac{270\times 13.8}{4(2-0.05)}}{\frac{13.8}{2(2-0.05)}+j\frac{270+14.4}{2}}$$

$$= 11.4 + j14.5 + \frac{-972+j18630}{138+j142.2} + \frac{-972+j477.69}{3.538+j142.2}$$

$$= 11.4 + j14.5 + \frac{18655\angle 92.98^\circ}{198\angle 45.86^\circ} + \frac{1083\angle 153.83^\circ}{142.24\angle 88.57^\circ}$$

$$= 11.4 + j14.5 + 94.22\angle 47.12^\circ + 7.6\angle 65.25^\circ$$

$$= (11.4 + 64.11 + 3.18) + j(14.5 + 69 + 6.9) = (78.69 + j90.4)\ \Omega.$$

$\therefore$ Input current $= \frac{240\angle 0^\circ}{78.69+j90.4} = \frac{240\angle 0^\circ}{119.85\angle 48.96^\circ} = 2\angle-48.96^\circ$ A

Hence power factor is (cos 48.96°) lagging, i.e. 0.656 lagging.
Input power = 240 × 2 × 0.656 W, i.e. 314.88 W.
Output power is 200 W.

Hence efficiency $= \frac{\text{Output}}{\text{Input}} = \frac{200}{314.88} = 0.635$, i.e. 63.5%.

12.2 A 230 V, 50 Hz. 4-pole single-phase induction motor has the following parameters:

$$r_1 = 2.51\ \Omega,\ x_1 = 4.62\ \Omega,\ r_2' = 7.81\ \Omega,\ x_2' = 4.62\ \Omega$$

and $X_\phi = 150.88\ \Omega$

Determine the stator main winding current and power factor when the motor is running at a slip of 0.05.

Solution

The total series impedance is obtained as

$$Z = 2.51 + j4.62 + \frac{j\dfrac{150.88}{2}\left(\dfrac{7.81}{2\times 0.05}+j\dfrac{4.62}{2}\right)}{\dfrac{7.81}{2\times 0.05}+j\left(\dfrac{4.62}{2}+\dfrac{150.88}{2}\right)} + \frac{j\dfrac{150.88}{2}\left\{\dfrac{7.81}{2(2-0.05)}+j\dfrac{4.62}{2}\right\}}{\dfrac{7.81}{2(2-0.05)}+j\left(\dfrac{150.88}{2}+\dfrac{4.62}{2}\right)}$$

$$= 2.51 + j4.62 + \frac{-174.26+j5891.86}{78.1+j77.75} + \frac{-174.26+j151.07}{2+j77.75}$$

$$= 2.51 + j4.62 + \frac{5894.40\angle 91.69^\circ}{110.20\angle 44.87^\circ} + \frac{230.62\angle 139.077^\circ}{77.77\angle 88.50^\circ}$$

$$= 2.51 + j4.62 + 53.48\angle 46.82^\circ + 2.965\angle 50.58^\circ$$

$$= (2.51 + 36.596 + 1.88) + j(4.62 + 39 + 2.3)$$

$$= 40.986 + j45.92 = 61.54\angle 48.25^\circ\ \Omega$$

Stator main winding current is $\dfrac{230\angle 0^\circ}{61.54\angle 48.25^\circ}$ i.e. $3.73\angle -48.25^\circ$ A.

Hence power factor is (cos 48.25°) i.e. 0.666 lagging.

12.3 In a 6-pole single-phase induction motor the gross power absorbed by the forward and backward fields are 160 W and 20 W respectively. If the motor speed is 950 rpm and the no load frictional loss is 75 W, find the shaft torque.

Solution

Air-gap power of forward field P_{gf} = 160 W

Air-gap power of backward field P_{gb} = 20 W.

Net power = $P_{gf} - P_{gb}$ = 160 W – 20 W = 140 W.

Synchronous speed $N_s = \dfrac{120\times 50}{6}$ = 1000 rpm.

Speed of motor N_r = 950 rpm.

Hence slip $s = \dfrac{1000-950}{1000}$ = 0.05.

Power output is $(1 - s) \times 140 - 75$ = 58 W (= shaft power).

Shaft torque = $\dfrac{\text{shaft power}}{2\pi\times\dfrac{950}{60}} = \dfrac{58}{2\pi\times\dfrac{95}{6}}$ = 0.58 Nm.

12.5 STARTING OF SINGLE-PHASE INDUCTION MOTORS

Since a single-phase induction motor does not have a starting torque, it needs special methods of starting. The stator is provided with two windings, called *main* and *auxiliary windings*, whose axes are space displaced by 90 electrical degrees. The auxiliary winding is excited by a current which is out of phase with the current in the main winding, both currents derived from the same supply. If the phase difference between the two currents is 90° and the mmfs created by them are equal, maximum starting torque is produced. If the phase difference is not 90° and the mmfs are equal, the starting torque will be small, but in many applications it is still sufficient to start the motor. The auxiliary winding may be disconnected by a centrifugal switch after the motor has achieved about 75% speed.

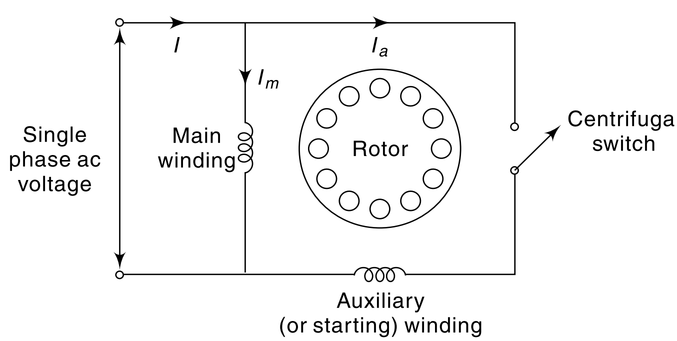
Single-phase induction motors are usually classified according to the auxiliary means used to start the motors. They are classified as follows:

1. Split-phase motor
2. Capacitor start motor
3. Capacitor start capacitor run motor
4. Shaded pole motor.

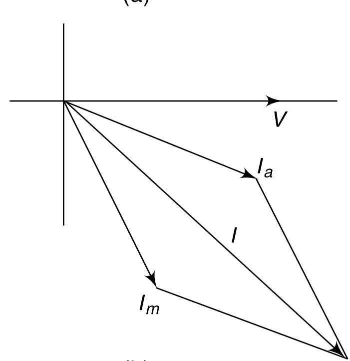
12.6 SPLIT PHASE INDUCTION MOTORS

One of the most widely used types of single-phase motors is the *split phase* induction motor. Its service includes a wide variety of applications such as refrigerators, washing machines, portable hoists, small machine tools, blowers, fan, centrifugal pumps, etc.

The essential parts of the split phase motor is shown in Fig. 12.9(a). It shows the auxiliary winding, also called the starting winding, in space quadrature, i.e. 90 electrical degrees displacement with the main stator winding. The rotor is normally of squirred cage type. The two stator windings are connected in parallel



(a)



(b)

Fig. 12.9 *Split phase motor (a) Schematic representation (b) Phasor diagram* to the ac supply. A phase displacement between the winding currents is obtained by adjusting the winding impedances, either by inserting a resistor in series with the starting winding or as is generally the practice, by using a smaller gauge wire for the starting winding. A phase displacement of 30° between the currents of main winding I_m and auxiliary winding I_a can be achieved at the instant of starting. A typical phasor diagram is shown in Fig. 12.9(b).

When the motor has come to about 70 to 75% of synchronous speed, the starting winding may be opened by a centrifugal switch and the motor will continue to operate as a single-phase motor. At the point where the starting winding is disconnected, the motor develops nearly as much torque with the main winding alone as it was with both windings connected. It can be observed from the typical torque speed characteristic for this type of motor in Fig. 12.10. The starting winding is designed to take the minimum starting current from the required torque. The locked rotor starting current may be typically in the range 5 to 7 times the rated current while the starting torque is also about 1.5 to 2 times the rated torque. The high starting current is not objectionable since once started it drops off almost instantly. The major disadvantages of this type of induction motor are relatively low starting torque and high slip. Moreover the reversal of rotation can be made only when the motor is standstill (by reversing the line connections of either the main winding or the starting winding) but not while running. Also the efficiency is lower.

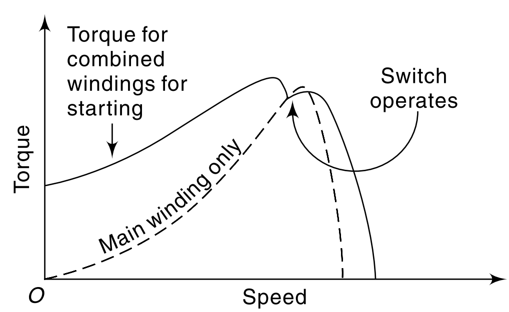
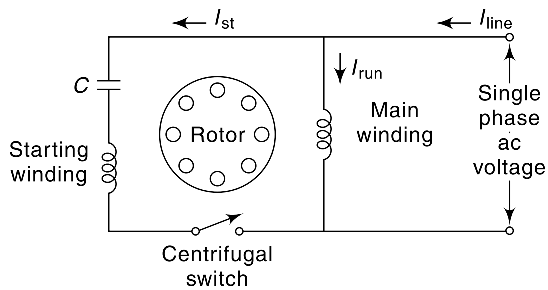


Fig. 12.10 ***Typical torque speed characteristic of a general purpose split phase motor***

12.7 CAPACITOR START MOTOR

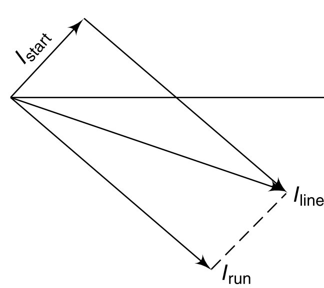
In the split phase motor the phase shift between the stator currents was accomplished by adjusting the impedances of the windings, i.e. by making the starting winding of a relatively higher resistance. This resulted in a phase shift of nearly 30°. Since the developed torque of any split phase motor is proportional to the pole flux produced and the rotor current, it is also dependent on the angle between the winding currents. This implies that if a capacitor is connected in series with the starting winding, the starting torque will increase. By proper selection of the capacitor, the current in the starting winding will lead the voltage across it and a greater displacement between winding currents is obtained.

Figure 12.11 shows the capacitor start motor and its corresponding phasor diagram indicating a typical displacement between winding currents of about 80° – 90°. The value of the capacitor needed to accomplish this is typically 135 pF or a 1/4 h.p. motor and 175 pF for a 1/3 h.p. motor. Contrary to the split phase motor discussed earlier the speed of capacitor start motor under running conditions is reversible. If temporarily disconnected from the supply line, its speed will drop allowing the centrifugal switch to close. The connections to the starting winding are reversed during this interval and the motor is reconnected to the



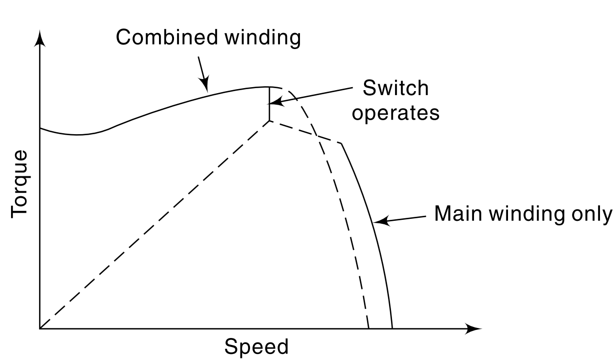
Circuit diagram

(a)



Phases diagram

(b)



Torque speed charge

(c)

Fig. 12.11 ***Capacitor start induction motor***

supply with closed centrifugal switch. The resulting rotating field will now rotate opposite to the direction in which the motor rotates. Since the current displacement between the windings is much larger in this motor compared to the split phase motor, the torque being proportional to this will also be much larger and exceed the torque produced by the rotor. Therefore the motor will slow down, stop and reverse its direction of rotation. When the speed reaches to about 75 to 80% of synchronous speed the centrifugal switch opens and the motor will reach speed as dictated by the load.

Because of higher starting torques, capacitor start motors are used in applications where not only higher starting torques are required but also where reversible motors are needed. Applications of capacitor motors are in washing machines, belted fans and blowers, dryers, pumps and compressors.

12.8 CAPACITOR START CAPACITOR-RUN MOTOR

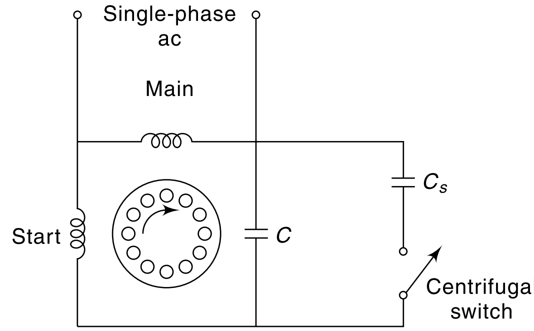
The capacitor start motor discussed above has still relatively low starting torque, although it is considerably better than the split phase motor.

In case where higher starting torques are required best results will be obtained if a large value of capacitance is used at start which is then gradually decreased as the speed increases. In practice, two capacitors are used for starting and one is cut out of the circuit by a centrifugal switch once a certain speed is reached, usually about 75% of full load speed. This starting or intermittent capacitor is of fairly high capacity (usually of the order of 10 to 15 times the value of the running capacitor, which remains in the circuit. Figure 12.12 illustrates the connection diagrams for the capacitor motor showing two methods generally encountered.

The first method shown in Fig. 12.12(a) uses an electrolytic capacitor in the starting circuit whose leakage is too high. The second capacitor is oil filled which remains in the circuit always and has little leakage; it is therefore suitable for continuous operation.

The second circuit (Fig. 12.12(b)) uses an auto transformer and one oil filled high voltage capacitor. This method utilizes the transformer principle of reflected impedance from the secondary to the primary. For instance an auto transformer with 180 turns tapped at the 30 turn point would reflect an 8 μF running capacitor to the primary as $\left(\frac{180}{30}\right)^2 \times 8\ \mu\text{F} = 288\ \mu\text{F}$, representing an increase of about 36 times. Thus running oil filled capacitor may be used for starting purposes as well, thereby eliminating one capacitor in lieu of the auto transformer, which is of comparable cost. Care must be taken to ensure that the capacitor can withstand the stepped up voltage which is 180/30 = 6 times the rated voltage at start.

Like the capacitor start motor, the capacitor run motor may be damaged if the centrifugal switch fails to operate properly. The primary advantage of a capacitor run motor or a two-value capacitor motor is its high starting torque, good running torque and quiet operation. Reversing the line leads to one of the windings in the usual manner causes motor operation in the opposite direction. It is therefore



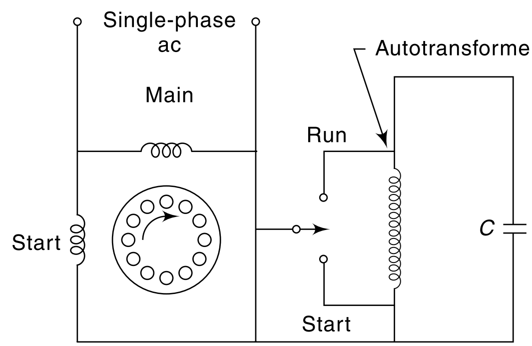


Fig. 12.12 ***Capacitor start capacitor run motor***

classified as a reversible type motor. These motors are manufactured in a number of sizes from 1/8 to 3/4 hp and are used in compressors, conveyors, pumps and other high torque loads.

12.4 A 220 V single-phase induction motor gave the following test results:

Blocked rotor test:	100 V, 9 A, 380 W
No load test:	220 V, 5 A, 120 W

Find the parameters of the equivalent circuit neglecting the core loss resistance. Also find the iron, friction and windage loss.

Solution

From Art. 12.4.1

$$r_1 + r_2' = \frac{380}{9^2}\ \Omega = 4.69\ \Omega;$$

$$Z_{sc}\left(=\frac{V_{sc}}{I_{sc}}\right) = \frac{100}{9} = 11.11\ \Omega$$

Hence $$x_1 + x_2' = \sqrt{(11.11)^2 - (4.69)^2} = 10\ \Omega,$$

$$r_1 = r_2' = \frac{4.69}{2} = 2.345\ \Omega \text{ and}$$

$$x_1 = x_2' = \frac{10}{2} = 5\ \Omega.$$

From Art. 12.4.2

$$\cos\,\theta_o\left(=\frac{W_{o/c}}{V_{o/c}\times I_{o/c}}\right) = \frac{120}{220\times 5} = 0.11 \text{ or, } \theta_o = 83.74°$$

$$\therefore \qquad \frac{X_\phi}{2} = \frac{220 - 5\angle -83.74°\left[\left(2.345+\frac{2.345}{4}\right)+j\left(5+\frac{5}{2}\right)\right]}{5}$$

$$= 44 - 1.0\angle -83.74°(2.93 + j7.5)$$

$$= 44 - 1.0\angle -83.74° \times 8\angle 68.66°$$

$$= 44 - 8\angle -15° = 36.275 + j2 = 36.33\angle 3.156°\ \Omega.$$

Hence $\qquad X_\phi = 72.66\angle 3.156°\ \Omega$

Iron, friction and windage loss (from Fig. 12.8) is given as

$$= 120 - 5^2\left(r_1 + \frac{r_2'}{4}\right) \qquad \left[\text{i.e., } W_{o/c} - I_o^2\left(r_1+\frac{r_2'}{4}\right)\right]$$

$$= 120 - 25\left(2.345 + \frac{2.345}{4}\right) = 46.72 \text{ W}.$$

12.5 A 200 V, 50 Hz. capacitor start motor has the following impedances at standstill.

Main winding $Z_m = (8 + j3)\ \Omega$

Auxiliary winding $Z_a = (10 + j8)\ \Omega$

Find the value of capacitance to be connected in series with auxiliary winding to give phase displacement of 90° between currents in the two windings.

Solution

The phasor diagram is shown in Fig. 12.11(b)

Phase angle of current in main winding is $\left(\tan^{-1}\frac{3}{8}\right) = 20.55°$

With capacitor C in the auxiliary winding the phase angle of current in auxiliary winding is $\left(\tan^{-1}\frac{8-\frac{1}{\omega C}}{10}\right)$.

To give a phase displacement of 90° between the two winding currents,

we can write, $\qquad -\tan^{-1}\frac{8-\frac{1}{\omega C}}{10} - (-20.55°) = 90°$

i.e. $\qquad -\tan^{-1}\frac{8-\frac{1}{\omega C}}{10} = 69.45°$

or $\qquad 8 - \frac{1}{\omega C} = 10\tan(-69.45°) = -26.67$

Hence $\qquad \frac{1}{\omega C} = 34.67\ \Omega$

or $\qquad C = \frac{1}{2\pi\times 50\times 34.67}\text{ F} = 91.84\ \mu\text{F}$

12.9 SHADED POLE MOTORS

Like any other induction motor, the *shaded pole* motor is caused to run by the action of the magnetic field set up by the stator windings. There is, however, one extremely important difference between the poly phase induction motor and the single-phase induction motor discussed so far. As discussed, these motors have a truly rotating magnetic field, either circular, as in three-phase machine, or of elliptical shape as encountered in most of the single-phase motors. In the shaded pole motor the field merely shifts from one side of the pole to the other. In other words, it does not have a rotating field but one that sweeps across the pole faces.

An elementary understanding of how the magnetic field is created may be gained from the simple circuit in Fig. 12.13, illustrating the shaded pole motor. As can be seen, the poles are divided into two parts, one of which is "shaded", i.e., around the smaller of the two areas formed by a slot cut across the laminations, a heavy copper short circuited ring, called the *shading coil*, is placed. That part of the iron around which the *shading coil* is placed is called the shaded part of the pole. When the excitation winding is connected to an ac source, the magnetic field will sweep across the pole face from the unshaded to the shaded portion. This, in effect is equivalent to an actual physical motion of the pole, the result is that the squirrel cage rotor will rotate in the same direction.

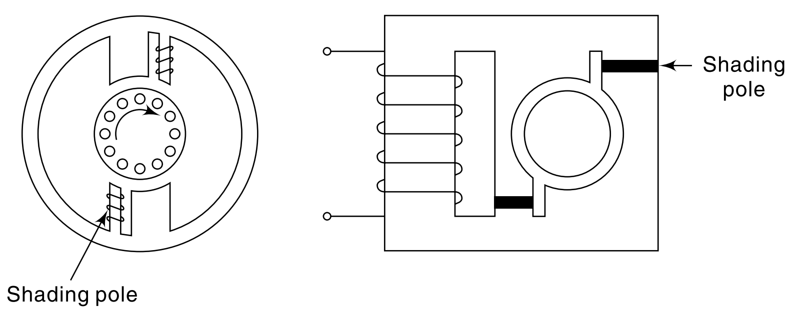


Fig. 12.13 *Shaded pole motor*

To understand how this sweeping action of the field across the pole face occurs, let us consider the instant of time when the current flowing in the excitation winding is starting to increase positively from zero, as illustrated in Fig. 12.14(a). In the unshaded part of the pole the flux will start to build up in phase with the current. Similarly, the flux ϕ, in the shaded portion of the pole will build up, but this flux change induces a voltage in the shading coil which causes current to flow. By Lenz's law, this current flows in such a direction as to oppose the flux change that induces it. Thus the building up of flux ϕ, in the shaded portion is delayed. It has the overall effect of shifting the axis of the resultant magnetic flux into the unshaded portion of the pole. When the current in the excitation coil is at or near the maximum value as indicated in Fig. 12.14(b), the flux does not change appreciably. With an almost constant flux, no voltage is induced in the shading coil and therefore it, in turn, does not influence the total flux. The result is that the resulting magnetic flux shifts to the centre of the pole.

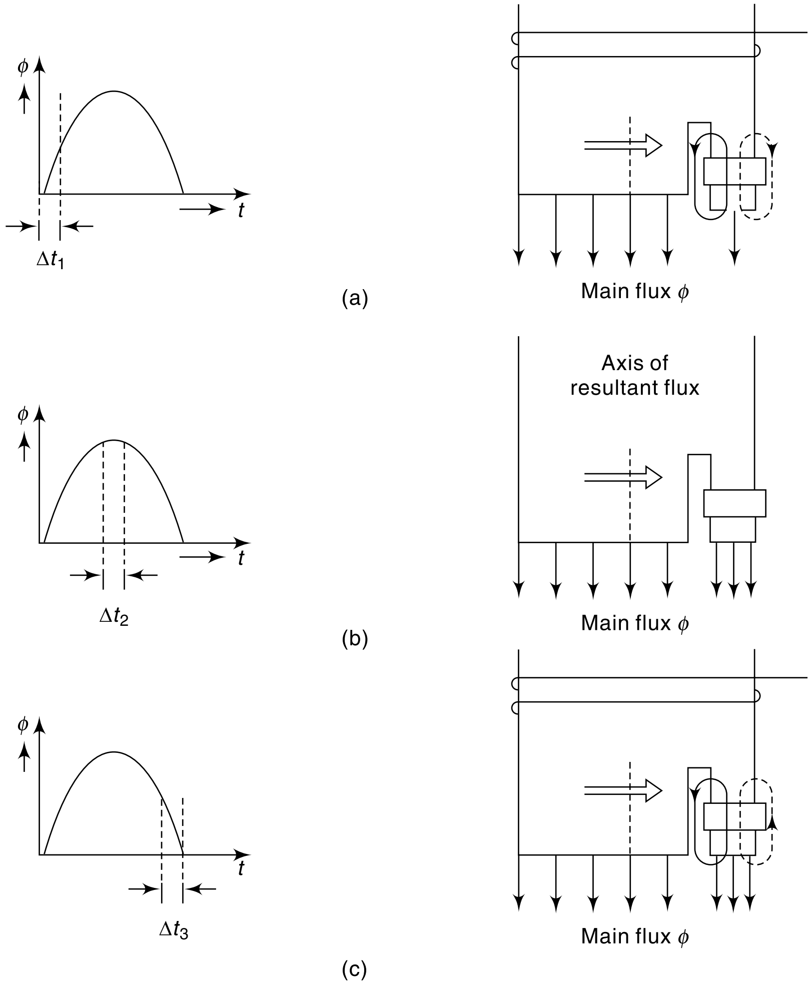


Fig. 12.14 ***Sweeping action of field across the pole***

Figure 12.14(c) shows the current in the excitation coil decreasing. The flux in the unshaded portion of the pole decreases immediately. However, currents induced in the shading coil tend to oppose this decrease in flux; consequently they try to maintain the flux. The result of this action translates into a movement of the magnetic flux axis towards the centre of the shaded portion of the pole. Hence flux ϕ continues to lag behind the flux axis during this part of the cycle.

It can similarly be reasoned that at any instant of the current cycle, the flux ϕ, lags behind in time. The net effect of this time and space displacement is to produce a gliding flux across the pole face and consequently in the air gap, which is always directed towards the shaded part of the pole. Therefore, *the direction of rotation of the shaded pole motor is always from the unshaded towards the shaded part of the pole.*

Simple motors of this type cannot be reversed but must be assembled so that the rotor shaft extends from the correct end in order to drive the load in the proper direction. There are specially designed shaded pole motors which are reversible. One form of design is to use two main windings and a shading coil. For one direction of rotation one main winding is used and for the opposite rotation the other; such an arrangement is adaptable only to distributed windings, hence this necessitates a slotted stator.

Another method employed is to use two sets of open circuited shading coils, one set placed on each side of the pole. A switch is provided to short circuit either the shading coil, depending on the rotational direction desired offsetting the simple construction and a low cost of this motor. This motor has a low starting torque, little overload capacity and low efficiencies (5 to 35%).

These motors are built in sizes ranging from 1/250 hp upto about 1/20 hp. Typical applications of shaded pole motors are where efficiencies are of minor concern such as in toys and fans.

EXERCISES

1. A three-phase induction motor develops a starting torque, but a single phase induction motor does not. Why?
2. Explain the operation of a single-phase induction motor on the basis of double revolving field.
3. Draw and explain a typical torque speed curve of a single phase induction motor on the basis of the double revolving field theory.
4. Discuss the procedure to determine the parameters of equivalent cicuit of single-phase induction motor.
5. Draw and explain the equivalent circuit of a single-phase induction motor.
6. Briefly discuss the different methods for starting single phase induction motors.
7. Discuss the differences between capacitor start, capacitor start capacitor run and permanent split capacitor motors.
8. Describe the construction and working principle of a shaded pole motor.
9. Discuss the procedure to determine the parameters of an equivalent circuit of a single-phase induction motor.
10. A 220 V 50 Hz single-phase induction motor gave the following test results:
    Blocked rotor test: 110 V, 10 A, 400 W.
    No load test: 220 V, 4 A, 100 W.
    Find (a) the parameters of equivalent circuit (b) the iron friction and winding losses. [*Ans:* $r_1 = r_2' = 2\ \Omega$, $x_1 = x_2' = 5.125\ \Omega$, $X_\phi = 88.9\ \Omega$, 60 W]
11. A 250 W, 230 V, 50 Hz capacitor motor has the following impedances at standstill:
    Main winding: $(7 + j5)\ \Omega$
    Auxiliary winding: $(11.5 + j5)\ \Omega$
    Find the value of the capacitor to be connected in series with the auxiliary winding to give a phase displacement of 90° between the currents in the two windings. [*Ans:* 156 μF]

12. A 230 V, 50 Hz, 4 pole single-phase induction motor has the following equivalent circuit parameters:

$$r_1 = r_2' = 8\ \Omega,\ x_1 = x_2' = 12\ \Omega \text{ and } X_\phi = 200\ \Omega$$

If the slip is 4% calculate the input current, input power and developed power. [*Ans:* 2.6 A, 384.8 W, 293.3 W]

13. A 220 V, 50 Hz single-phase induction motor gave the following test results:
Blocked rotor test: 120 V, 9.6 A, 460 W
No load test: 220 V, 4.6 A, 125 W
The stator winding resistance is 1.5 Ω. Determine the equivalent circuit parameters. Also find the core, friction and windage losses.

[*Ans:* $r_1 = 1.5\ \Omega$, $r_2' = 3.49\ \Omega$, $x_1 = x_2' = 5.73\ \Omega$, $X_\phi = 47.46\ \Omega$, 74.8 W]

13 ELECTRICAL MEASURING INSTRUMENTS

13.1 INTRODUCTION

Instruments which measure electrical quantities like voltage, current, power, energy etc. are called electrical instruments. These instruments are generally named after the electrical quantity to be measured. The instruments which measure current, voltage, power and energy are called ammeter, voltmeter, wattmeter and energy meter respectively.

13.2 TYPES OF INSTRUMENTS

The following types of electrical instruments which are commonly used:

(a) Indicating instruments
(b) Integrating instruments
(c) Recording instruments

(a) Indicating Instruments

The instruments which directly indicate the instantaneous value of an electrical quantity at the time when it is being measured are called indicating instruments.

Any indicating instrument has a pointer which sweeps over a calibrated scale and it directly gives the magnitude of the electrical quantity. Ammeters, voltmeters and wattmeters are examples of indicating instruments which are commonly used.

(b) Integrating Instruments

The instruments that measure the total quantity of electricity (like ampere-hours or electrical energy in watt-hours) for a given time are called integrating instruments. In such instruments there are sets of dials and pointers which register the total quantity of electricity or electrical energy supplied to the circuit in a given time.

However, the integrating instruments do not indicate the rate at which the quantity of electricity is flowing but only provide the summation of electrical quantity (or energy) being supplied for any given time.

(c) Recording Instruments

The instrument which gives a continuous record of the changes of the electrical quantity to be measured is called a recording instrument.

In recording type of instruments, the moving system usually bears an ink pen which rests on a paper wrapped over a drum. The drum rotates with a slow uniform speed and the motion of the drum is in a direction perpendicular to the direction of the pointer. The path traced out by the pen indicates the changes in the magnitude of electrical quantity under observation over the given time. An Electrocardiogram (ECG) machine is a typical example of these type of instruments.

13.3 WORKING PRINCIPLE OF ELECTRICAL INSTRUMENTS

Since an electrical quantity cannot be observed physically, it is necessary to convert the given electrical quantity into a mechanical force and then measure that force. This mechanical force moves the pointer on a calibrated scale and indicates the value of electrical quantity to be measured. This conversion of electrical quantity under measurement is achieved by utilising the following effects of electrical current:

(a) Magnetic effect	For voltmeters, ammeters and wattmeters.
(b) Thermal effect	For ammeters and voltmeters.
(c) Electrodynamic effect	For voltmeters, ammeters and wattmeters.
(d) Electromagnetic induction effect	For voltmeters, ammeters, wattmeters and energy meters.
(e) Chemical effect	For dc ampere hour meter.

13.4 DIFFERENT TORQUES IN INDICATING INSTRUMENTS

An indicating instrument indicates the value of electrical quantity at the time when it is being measured. It consists of a pointer attached to the moving system pivoted in jewelled bearings which moves over a graduated scale. In order to have the proper operation of indicating instruments, the following torques are essential:

(a) Deflecting torque (or Operating torque) (T_o)
(b) Controlling (or Restraining torque) (T_c)
(c) Damping torque (T_d)

13.4.1 Deflecting Torque (T_o)

The deflecting torque (or operating torque) is an essential requirement of an indicating instrument in order to initiate the movement of the pointer. The deflecting torque causes the moving system to move from zero position to the required value when the instrument is connected in the circuit to measure the electrical quantity. The deflecting torque is developed by utilising any of the known effects of current (or voltage).

13.4.2 Controlling Torque (T_c)

Once the deflecting torque is developed, the pointer will continue to move and will be independent of the value of electrical quantity to be measured. It is then essential to control the movement of the pointer and this requirement makes that the controlling torque must be provided. The controlling torque opposes the deflecting torque of the moving system so that the pointer comes to the rest position when the two opposing torques are equal. The purpose of providing the controlling torque is three-fold:

(a) to oppose the deflecting torque and get increased with the deflection of the moving system.
(b) to make the pointer to come to rest when $T_c = T_o$.
(c) to bring the pointer back to zero position when the deflecting torque is removed.

13.4.3 Generator of Controlling Torque

The following two methods are commonly used to provide controlling torque in indicating instruments:

(a) Spring control
(b) Gravity control

(a) Spring control

This is the most common method of providing controlling torque in electrical instruments. In this method, generally two spiral springs (or hair springs) P and Q of phosphor bronze are attached to the moving system, as shown in Fig. 13.1 (springs are wound in the opposite direction of each other to compensate for changes in temperature). One end of each spring is attached to the spindle, while the other end is attached to a fixed point in the instrument. The two springs provide the necessary controlling torque as well as they provide electrical connection to the operating coil.

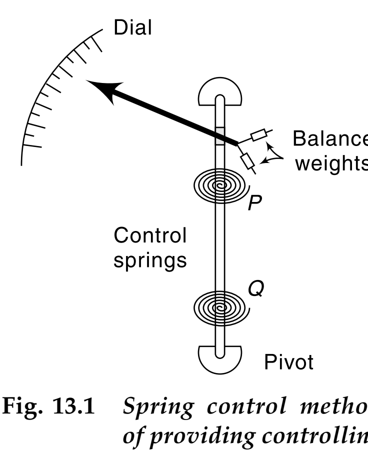


Fig. 13.1 *Spring control method of providing controlling torque*

The moving system is statically balanced in all positions by balance weights and an arrangement, called *zero adjuster*, is provided on the pointer to adjust the zero of the pointer.

When the instrument is not in use, the two springs are in their natural position without any tension or compression and the controlling torque is zero. When the instrument is in the process of measuring of an electrical quantity with production of the deflection torque, the pointer moves and one of the springs is unwound while the other gets twisted. The resultant twist in the springs provides the controlling torque. More the deflection, more is the twist and hence greater will be the controlling torque. Hence the controlling torque becomes directly proportional to the deflection θ of the moving system, i.e.

$$T_c \propto \theta$$

The pointer comes to rest at a position when the controlling torque is equal to deflecting torque (i.e. $T_c = T_o$)

Advantages of Spring Control

(i) It is suitable for portable instrument as it works in any position of the instrument.
(ii) There is practically no increase in the weight of the moving system.

Disadvantages of Spring Control

(i) Change in temperature may affect the spring length and hence the controlling torque.
(ii) The controlling torque cannot be adjusted normally.

(b) Gravity Control

In this method, a small adjustable weight W (control weight) is attached to the spindle (moving system), as shown in Fig. 13.2. It provides the necessary controlling torque. The controlling torque can be varied by changing the position of weight W on the arm.

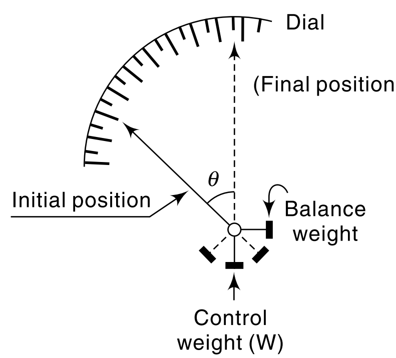


Fig. 13.2 *Gravity control method of providing controlling torque*

In the initial rest (or zero) position of the pointer, the control weight is suspended vertically downwards and, therefore, it does not produce any torque. However, under the action of a deflecting torque, the pointer moves from zero position (from left to right) and the control weight moves in the opposite direction, as shown in Fig. 13.2 (dotted). Due to gravity, the control weight would always try to come to its original position (i.e. vertical) and hence it produces a opposing torque on the moving system. This torque (controlling torque) opposes the deflecting torque and the pointer would come to rest at a position when the magnitude of controlling torque becomes equal to the deflecting torque.

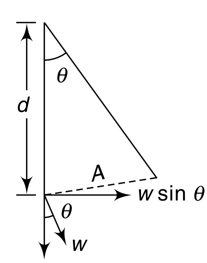


Fig. 13.3 *Direction of movement of pointer in gravity control method*

As the pointer gets deflected through an angle θ from its zero position when measuring an electrical quantity, the control weight will also move through an angle θ but in the opposite direction as shown in Fig. 13.3.

Let, m = control weight

d = distance of control weight from axis of rotation.

In the deflected position, weight m can be resolved into two components; ($m \cos\theta$) and ($m \sin\theta$). Only the component ($m \sin\theta$) provides the controlling torque (T_c).

$$T_c = m \times d \sin\theta$$

If m and d have fixed values, then

$$T_c \propto \sin\theta$$

However, $T_o \propto I$, the current being measured is I.

$\therefore$ for the position at which pointer stops,

$$T_o = T_c$$

i.e. $$I \propto \sin\theta$$

Therefore, *gravity controlled instruments have a non-uniform scale (or cramped scale) being crowded in the beginning of the scale.*

Advantages of Gravity Control

(i) It is a very simple method.
(ii) It is relatively cheaper.
(iii) It is unaffected by ordinary temperature changes.
(iv) It is not normally subjected to fatigue.
(v) The controlling torque can be easily varied.

Disadvantages of Gravity Control

(i) The instrument needs to be kept always in a vertical position.
(ii) The control weight adds to the weight of the moving system.
(iii) Gravity controlled instruments have non-uniform scale.

13.5 DAMPING TORQUE (T_D)

When the moving system in indicating instruments is acted upon simultaneously by deflecting and controlling torques then the pointer due to its inertia will oscillate about the final position before coming to rest. These oscillations are undesirable and need to be prevented in order to avoid these oscillations of the pointer. To bring it quickly to its deflected position, damping torque is provided in indicating instrument which opposes the movement of the pointer and operates only when the system is in motion.

The damping torque acts only when the pointer is in motion, when the pointer is in the final position, both deflecting and controlling torques are still acting on the moving system but the damping torque becomes zero. The pointer is then steady and there is no movement of the moving system. The damping torque acts like a brake on the moving system.

If the instrument is underdamped, the pointer will oscillate about the final position and take some time to come to rest. On the other hand, if the instrument is overdamped, the pointer will become slow to move. However, if the degree of damping is adjusted to such a value that the pointer moves and quickly halts to its final position, the instrument is said to be *dead-beat* or *critically damped*. Figure 13.4 shows the profile of underdamping, overdamping and critical damping.

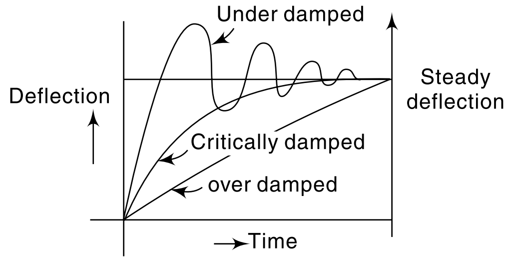


Fig. 13.4 *Deflection profile showing underdamping, overdamping and critical damping conditions*

13.5.1 Methods of Generating Damping Torque

The following are the common methods employed for providing damping torque:
(a) Air friction damping
(b) Eddy current damping

(a) Air Friction Damping

In this method, a light aluminium vane (or a piston) is attached to the spindle and it moves with a small clearance in a rectangular or circular air chamber closed at one end (as shown in Figs 13.5 and 13.6). As the pointer moves forward, the vane comes out of the chamber and a partial vacuum is created in the closed space. The atmospheric pressure then opposes the moving system in its rapid clockwise movement. When the pointer moves backward (i.e. anti-clockwise), the piston is pushed into the air chamber, compressing the air in the closed space. This compressed air tries to push out the piston and thus opposes the movement of the pointer in anti-clockwise direction. In this way, motion (backward or forward) of the pointer is opposed and hence, necessary damping is produced.

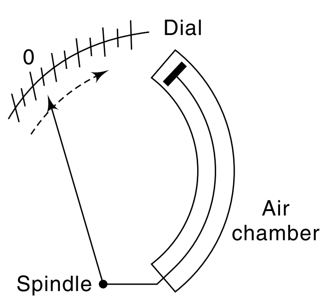


Fig. 13.5 *Air friction damping*

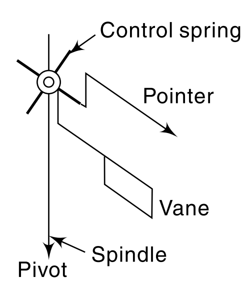


Fig. 13.6 *Movement of pointer in air friction damping*

(b) Eddy Current Damping

It is one of the most effective and efficient method of damping. We know that when a conducting (but non-magnetic material such as aluminium or copper) disc is rotated in a magnetic field, eddy currents are induced in the disc according to Faraday's laws of electromagnetic induction. These eddy currents counteract with the magnetic field to produce a force which opposes the motion providing the necessary damping torque. The eddy currents and hence the damping torque exists as long as the moving system is in motion.

Figure 13.7(a) shows a form of eddy current damping. A thin aluminium disc is attached to the spindle and is allowed to rotate horizontally in the air gap of a permanent magnet. When the pointer (or the spindle) moves, the aluminium disc also moves and cuts the magnetic lines of force produced by the permanent magnet. This induces eddy currents that produces a force opposing the motion of the disc.

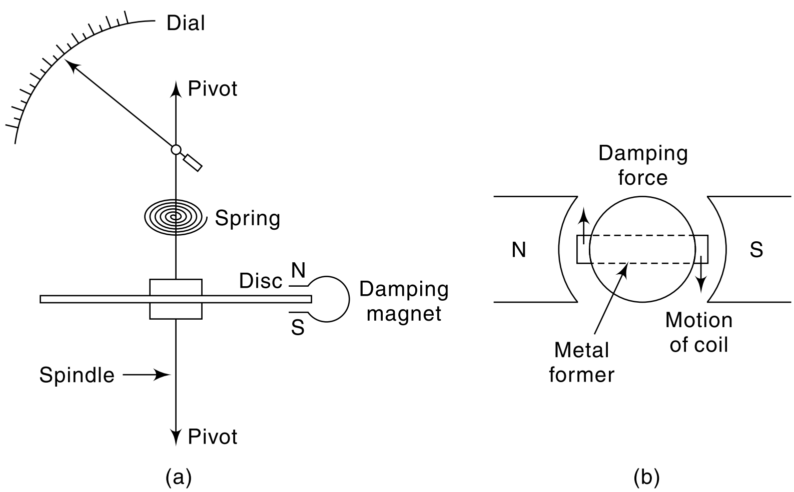


Fig. 13.7 *Eddy current damping*

In Fig. 13.7(b), the operating coil (coil which produces deflecting torque) is wound on an aluminium former. When the coil moves in the field of the permanent magnet, eddy currents are induced in the aluminium former, providing the necessary damping torque.

13.6 TYPES OF INDICATING INSTRUMENTS

The following types of instruments are commonly used:

(a) Moving iron (MI) instruments.
(b) Moving coil (MC) instruments.
(c) Dynamometer instruments.
(d) Electrostatic instruments.
(e) Induction instruments.

13.6.1 Moving Iron Instruments

These instruments are reasonably accurate, cheaper and simple in construction. These instruments are widely used in laboratories and on electric panel boards. Moving iron instruments are usually used either as ammeters or voltmeters.

Moving iron instruments are of two types:

1. Attraction type
2. Repulsion type.

Attraction type Moving Iron Instruments

Principle The operation of these instruments are based on the following principle when an unmagnetised soft iron piece is placed in the magnetic field of a coil, the piece is attracted towards the coil. The moving system of the instrument is attached to a soft iron piece and the operating current is passed through a coil placed adjacent to it. The operating current sets up a magnetic field which attracts the iron piece and thus creates deflecting torque in the pointer to move over the scale.

Construction It consists of a hollow cylindrical coil (or solenoid) that is kept fixed (as shown in Fig. 13.8). An oval shaped soft iron piece is attached to the spindle in such a way that it can move in or out of the coil. The pointer is attached to the spindle so that it is deflected with the motion of the soft iron piece. The controlling torque on the moving system is usually provided by spring control method while damping is provided by air friction.

Working Principle When the instrument is connected in the circuit, the operating current flows through the coil. This current sets up a magnetic field in the coil. The coil then behaves like a magnet and it attracts the soft iron piece towards it. The pointer attached to the moving system moves from zero position across the dial.

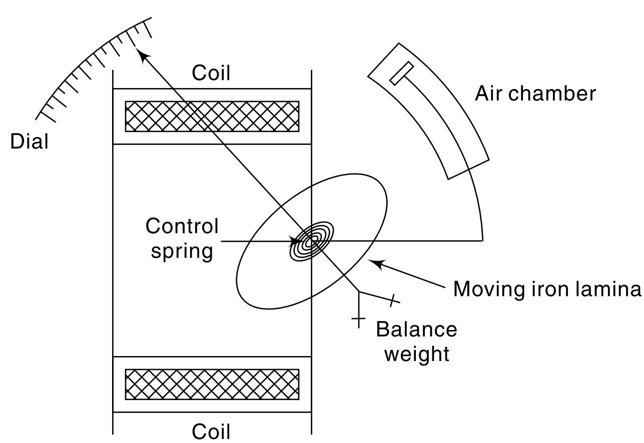


Fig. 13.8 *Attraction type moving iron instruments*

If current in the coil is reversed, the direction of magnetic field also reverses and so does the magnetism produced in soft iron piece. Hence the direction of deflecting torque remains unchanged. Therefore, such instruments can be used both for dc as well as ac measurement of current and voltage.

The force F pulling the soft iron piece towards the coil depends upon:

(i) The field strength H produced by the coil.

(ii) The pole strength M developed by the iron piece

i.e. $F \propto MH$

$\therefore$ $F \propto H^2 \quad (\because M \propto H)$

Thus, deflecting torque $(T_o) \propto F \propto H^2$

If the permeability of iron is assumed to be constant, $H \propto I$

$\therefore \qquad T_o \propto I^2$

Since the controlling torque is provided by the springs, $T_c \propto \theta$ (deflection), and in the steady position of deflection,

$$T_o = T_c$$

$\therefore \qquad \theta \propto I^2$ (for dc)

and $\qquad \theta \propto I_{\text{rms}}^2$ (for ac).

Since the deflection $\theta \propto I^2$, hence scale of such instruments is non-uniform (being crowded in the beginning).

Repulsion type Moving Iron Instruments

Principle These instruments are based on the principle of repulsion between the two iron pieces magnetised with same polarity.

Construction Any repulsion instrument consists of a fixed cylindrical hollow coil that carries the operating current (Fig. 13.9). Inside the coil, there are two soft iron pieces of vanes, one of which is fixed and other is movable. The fixed iron vane is attached to the coil whereas the movable vane is attached to the spindle. Under the action of deflection torque, the pointer attached to the spindle moves over the scale.

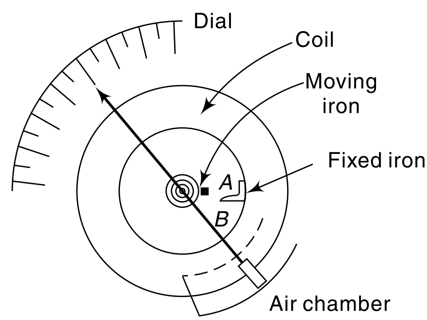


Fig. 13.9 ***Repulsion type moving iron instruments***

The controlling torque is produced by spring control method and damping torque is provided by air friction damping in repulsion type of instruments.

Working Principle When the instrument is connected in a circuit and current is flowing through the circuit, current sets up a magnetic field in the coil within the instrument. The magnetic field magnetises both the iron vanes in the same direction (i.e. both pieces become magnets with the same polarity) they repel each other. Due to this force of repulsion, only movable iron vane can move as the other piece is fixed and cannot move. The result is that the pointer attached to the spindle moves from zero position.

If current in the coil is reversed, the direction of deflection torque remains unchanged. This is because both iron vanes are in the same magnetic field and so they will be magnetized similarly and consequently repel each other irrespective of the direction of magnetic field. Hence, such instruments can be used both for ac and dc measurements.

The deflection torque is generated due to the repulsion between the similarly charged iron pieces. If the two pieces develop pole strengths M_1 and M_2 respectively, we can write

Instantaneous deflecting torque ($\propto$ repulsive force) $\propto M_1 M_2$

or Instantaneous deflecting torque, $T_o \propto H^2$ [Since pole strength developed are proportional to H.]

Assuming constant permeability of iron, $H \propto$ current through the coil.

$\therefore$ Instantaneous deflecting torque, $T_o \propto i^2$

However, controlling torque provided by springs $T_c \propto \theta$.

$\therefore$ In the steady position of deflection, when $T_o = T_c$,

$$\theta \propto i^2$$

i.e. $$\theta \propto I^2 \text{ (for dc)}$$

$$\propto I_{\text{rms}}^2 \text{ (for ac)}$$

Since deflection θ is proportional to I^2, therefore scale of such instruments is non-uniform (being crowded in the beginning). Scale of such instruments may be made uniform by using tonge shaped iron vanes.

13.6.2 Advantages and Disadvantages of Moving Iron Instruments

The moving iron instruments have the following advantages:

(a) These are cheap, robust and simple in construction.
(b) The instruments can be used for both ac as well as dc circuits.
(c) These instruments have a high operating torque.
(d) These instruments are reasonably accurate.

The following are the disadvantages of moving iron instruments:

(a) These instruments have non-uniform scale.
(b) These instruments are less sensitive to changes of operating variables.
(c) Errors are introduced due to change in frequency in case of ac measurement.
(d) Power consumption of these instruments are relatively higher.

13.6.3 Errors in Moving Iron Instruments

The errors which may occur in moving iron instruments can be divided into two categories:

(a) Errors with both dc and ac measurement.
(b) Errors with ac measurement only.

Errors with both dc and ac Measurement

(i) Errors due to Hysteresis Since the iron parts move in the magnetic field, hysteresis loss occurs in them. The effect of this error will result in higher readings when current increases than when it decreases. The hysteresis error can be eliminated by using "mumetal" or "permalloy" which have negligible hysteresis loss.

(ii) Error due to Stray Fields Since the operating magnetic field is comparatively weak, therefore such instruments are susceptible to stray fields. This may give rise to wrong readings. This error is eliminated by shielding the instrument with iron enclosure.

(iii) Error due to Temperature Changes in temperature affect the circuit resistance of the coil and stiffness of the control springs.

(iv) Error due to Friction Due to friction of moving parts, slight error may be introduced. This can be avoided by making torque-weight ratio of the spindle high.

Error with ac Measurement Only

(i) Change in Frequency With the change in frequency, the impedance of the instrument coil changes. This in turn changes the deflecting torque. This is of particular importance in voltmeters.

This error can be eliminated by connecting a capacitor of suitable value in parallel with *swamp resistance* R of the voltmeter. The value of capacitor C is given by: $C = L/R^2$.

13.6.4 Difference between an Ammeter and a Voltmeter

Both ammeter and voltmeter are current operated devices (i.e. deflecting torque is produced when current flows through their operating coils). However, in case of ammeters, the deflecting torque is produced by the current to be measured whereas in the case of a voltmeter deflecting torque is produced by the current proportional to the voltage to be measured.

Also, an ammeter is connected in series with the circuit in which the current is to be measured and, hence it has low resistance. On the other hand, a voltmeter is connected across the circuit whose voltage is to be measured and, hence it must have high resistance.

It is possible that an ammeter is converted into a voltmeter by connecting a suitable high resistance in series with it. We will discuss this issue later in Art 13.6.7.

13.6.5 Moving Coil Instruments: Permanent Magnet Type (for dc measurement only)

Permanent Magnet type Moving Coil Instruments

These instruments are also used either as ammeters or voltmeters and can be used for dc measurement only.

Principle The operation of this type of instrument is based on the principle that when a current carrying conductor is placed in a magnetic field, mechanical force acts on the conductor (as per Fleming's left hand rule). If the current carrying coil placed in magnetic field is attached to the moving system, with the movement of the coil, the pointer moves over the scale.

Construction It consists of a powerful static permanent magnet with a movable light rectangular coil having many turns of fine electrical wire wound on aluminium former inside which there is a soft iron core (as shown in Fig. 13.10). The coil is mounted on the spindle and acts as the moving element. The current enters into the coil and comes out of it by means of the two control hair springs, one above and the other below the coil (these springs also provide the controlling torque). Eddy current damping is provided by the aluminium former.

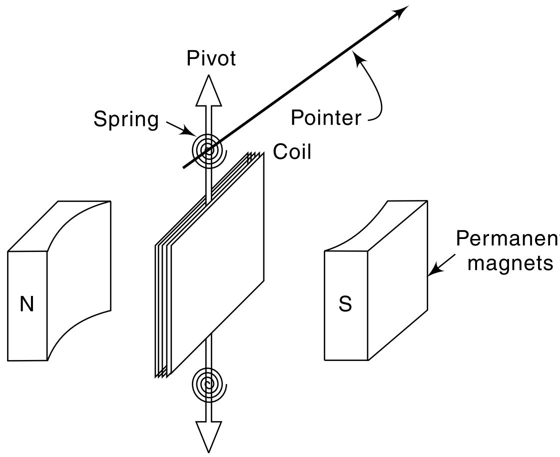


Fig. 13.10 ***Permanent magnet moving coil instrument***

In this instrument if current direction is reversed, the torque will also be reversed while the direction of the field of permanent magnet remains same.

Hence, with the current reversed the pointer will move in the opposite direction (i.e. it will go on the other side of zero or below zero). These instruments work only when current in the circuit is passed in a definite direction (i.e. for d.c. only). Such instruments are called *permanent magnet moving coil (PMMC)* instruments because a coil moves in the field of a permanent magnet.

Damping of the moving system is provided by the aluminium former. As the former rotates, eddy currents are induced in it. These eddy currents react with the field (due to permanent magnet), developing torque that opposes the movement of pointer.

Deflecting torque depends upon the operation current I and the uniform flux ϕ produced by the permanent magnet i.e.

$$T_o \propto \phi I$$

Since field is uniform, ϕ is constant

$$\therefore \qquad T_o \propto I$$

Also, the controlling torque is proportional to angle of deflection i.e.

$$T_c \propto \theta$$

The pointer will come to rest at a position when,

$$T_o = T_c$$

$$\therefore \qquad \theta \propto I$$

The deflection θ is thus directly proportional to the operating current. Therefore, such instruments have a uniform scale.

Advantages of MC Instruments

(a) Uniform scale.
(b) Very effective eddy current damping.
(c) Low power consumption.
(d) No hysteresis loss.
(e) Usually such instruments are not affected by stray fields.

(f) Require small operating current.
(g) Very accurate and reliable.

Disadvantages

(a) These instruments cannot be used for ac measurements.
(b) These instruments are costlier as compared to moving iron instruments.
(c) Errors may be caused due to the ageing of control springs and the permanent magnet.

13.6.6 Difference between Moving Coil and Moving Iron Instruments

(a) Moving iron instruments can be used both ac and dc while moving coils are used only in dc.
(b) Moving iron instruments are relatively cheaper.
(c) Moving iron instruments are more robust and simpler in construction.
(d) Scale of moving iron instruments are not uniform while that of moving coil instruments are linear.
(e) Hysteresis and eddy current errors affect moving iron instruments.

13.6.7 Extension of Instrument Range

The coils in measuring instruments are very delicate and therefore they cannot carry heavy currents. Full scale deflection will occur when rated current flows through the instruments coil. However, in practice, we have to measure heavy currents and voltages. Thus, we need to develop a method by which instrument ranges can be extended.

1. Extension of Ammeter Range

The range of an ammeter can be extended by connecting a low resistance (called *shunt*) across its coil as shown in Fig 13.11. Shunt is a thick copper conductor connected across the instrument terminals. The shunt by-passes the extra current and allows only recommended current to flow through the ammeter.

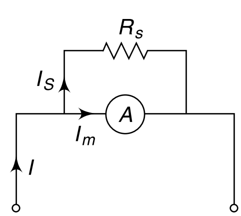


Fig. 13.11 ***Extension of range of ammeter***

Theory

Let us consider the circuit shown in Fig. 13.11. Let I be the circuit current to be measured.

Here, R_m = Ammeter resistance
R_s = Shunt resistance
I_m = Full scale deflection current of ammeter.
I_s = Shunt current.

$\because \quad I = I_s + I_m$

or $\quad I_s = I - I_m$

The voltage across shunt and ammeter being same

We can write

$$I_m R_m = (I - I_m)R_s \quad (\because I_s = I - I_m)$$

or $$I_m (R_m + R_s) = IR_s$$

$$\therefore \quad \frac{I}{I_m} = \frac{R_m + R_s}{R_s}$$

or $$\therefore I = I_m \times \frac{R_m + R_s}{R_s}$$

i.e. Circuit current= Full scale deflection (f.s.d.) current $\times \dfrac{R_m + R_s}{R_s}$.

Instrument Constant The ratio of current to be measured to the full scale deflection current is called instrument constant i.e.,

$$\text{Instrument constant} = \frac{I}{I_m} = \frac{R_m + R_s}{R_s}$$

With different shunts, the same instrument will have different instrument constants.

2. Extension of Voltmeter Range

The range of a voltmeter can be extended by connecting a high resistance (R) in series with its coil as shown in Fig. 13.12. The voltmeter is connected with the two points (A and B) across a resistance (r) whose voltage drop is to be measured.

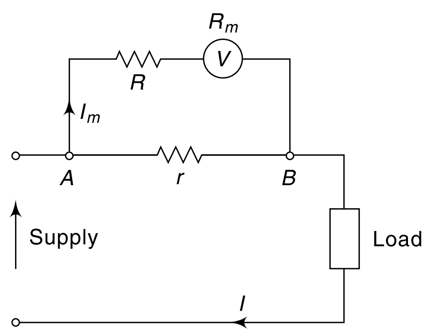


Fig. 13.12 ***Extension of range of voltmeter***

Theory

We consider the circuit shown in Fig. 13.12. Let (V) volts be the maximum voltage to be measured (i.e. the drop across r)

Let R_m = Voltmeter resistance

R = High resistance in series with voltmeter coil.

I_m = Full scale deflection current for voltmeter.

Since voltmeter is connected in parallel with resistance r, hence voltage across AB (i.e. across r) = Voltage across voltmeter

or $$V = I_m (R + R_m)$$

or $$R + R_m = \frac{V}{I_m}$$

$$\therefore \quad R = \frac{V}{I_m} - R_m$$

Hence, required high resistance R

$$= \frac{\text{Max. voltage to be measured}}{\text{f.s.d. voltmeter current}} - \text{Voltmeter resistance.}$$

13.6.8 Dynamometer Type Instruments

Dynamometer instruments are a modified form of moving coil type instruments. Here, the operating field is produced not by a permanent magnet but by another fixed coil. However, the moving system and the control system are similar to those of permanent magnet type moving coil instruments. These instruments can be used for both ac and dc measurements, they can also be used as ammeters and voltmeters but mostly they are used as voltmeters.

Principle These instruments are based on the principle that mechanical force is created between the current carrying conductors.

Construction A dynamometer type instrument essentially consists of a fixed coil and moving coil. The fixed coil is split into two equal parts and are placed close together and parallel to each other as shown in Fig. 13.13. The moving coil is pivoted in between the two fixed coils. The fixed and moving coils may be excited separately or they may be connected in series depending upon the use. The moving coil is attached to the spindle so that under the action of deflecting torque, the pointer moves over the scale.

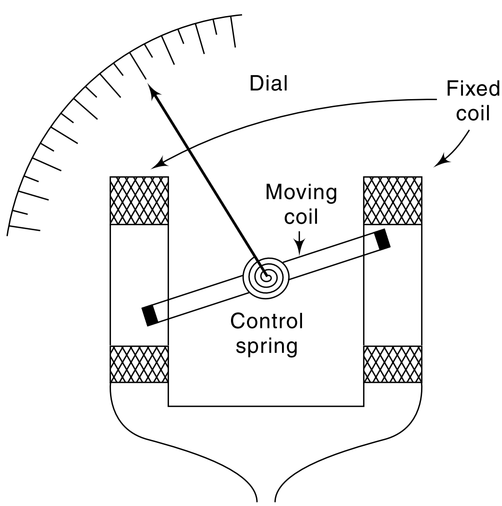


Fig. 13.13 ***Dynamometer type instrument***

The controlling torque is provided by two springs. They also serve the additional purpose of leading the current into and out of the moving coil. Air friction damping is provided in dynamometer instruments.

Working When the instrument is connected in the circuit, operating currents flow through the coils and due to this, mechanical force is developed between the coils. The moving coil moves the pointer over the scale. The pointer comes to rest at a position where deflecting torque is equal to the controlling torque.

By reversing the current, the field due to the fixed coils is reversed as well as the current in the moving coil, so that the direction of deflecting torque remains unchanged. Therefore, such instruments can be used for both dc and ac measurements.

Let

I_F = Current through fixed coil.

I_M = Current through moving coil.

$\therefore \quad T_o \propto I_F I_M$

Since $I_F = I_M = I$; the fixed and moving coils being in series,

$$\therefore \quad T_o \propto I^2 \qquad \text{(i)}$$

The control is by springs and hence the controlling torque is proportional to the angle of deflection i.e.

$$T_c \propto \theta \qquad \text{(ii)}$$

The pointer comes to rest at a position when $T_o = T_c$

Thus comparing equation (i) and (ii), we get,

$$\theta \propto I^2$$

It is clear that deflection of the pointer is directly proportional to the square of the operating current. Hence, the scale of these instruments is non-uniform (being crowded in their lower parts and spread out at the higher side).

Advantages

1. These instruments can be used for both ac and dc measurements.
2. These instruments are free from hysteresis and eddy current errors.

Disadvantages

(a) Since torque/weight ratio is small, therefore, such instruments have frictional errors which reduce the sensitivity.
(b) Scale is not uniform (in case of ammeters and voltmeters)
(c) A good amount of screening of the instrument is required to avoid the effect of study fields.
(d) These instruments are costlier then other types and therefore, they are rarely used an ammeters or voltmeters.

13.6.9 Dynamometer type Wattmeter

A dynamometer type wattmeter is most commonly employed for measurement of the power in a circuit. It can be used to measure power in ac as well as dc circuits.

Construction In a dynamometer type wattmeter (Fig. 13.14) the fixed coils are connected in series with the load and carry the circuit current. These coils are called *current coils*. The moving coil is connected across the load and carries current proportional to the voltage. It is called *potential coil*. Usually a high resistance is connected in series with the potential coil to limit the current through the potential coil.

The controlling torque is provided by springs. Springs also serve the additional purpose of leading current into and out of the moving coil. Air friction damping is employed in such instruments.

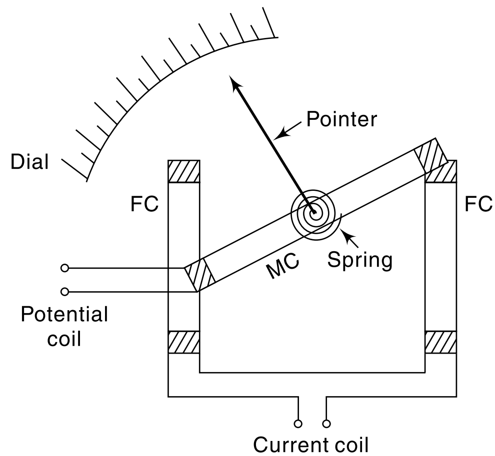


Fig. 13.14 *Dynamometer type wattmeter*

Working When power is to be measured in a circuit, the instrument is suitably connected in the circuit (Fig. 13.15). The current coil is connected in series with the load so that it carries the circuit current. The potential coil is connected across the load so that it carries current proportional to the voltage.

Due to the currents in both the coils mechanical force exists between them. As a result, the moving coil moves the pointer over the scale. The pointer comes to rest at a position where deflecting torque is equal to the controlling torque.

Reversal of the current reverses the field due to fixed coils as well as the current in the moving coil so that the direction of the deflecting torque remains unchanged. Therefore, such instruments can be used for the measurement of ac as well as dc power.

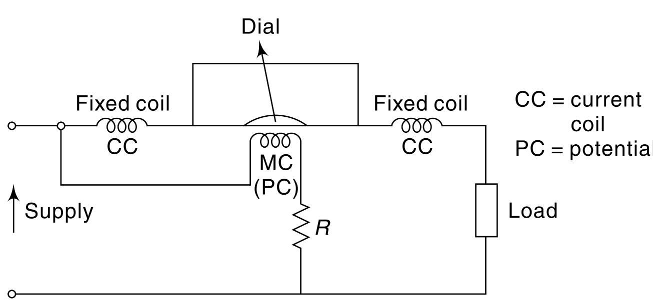


Fig. 13.15 *Connection of a wattmeter in the circuit*

DC Working of Dynamometer Wattmeter

Let us suppose that in a dc circuit

V = voltage across load

I = current through load.

Current through fixed coil, $I_F \propto I$

while current through moving coil $I_M \propto V$

$\therefore$ Deflecting torque T_o is produced due to the currents I_F and I_M,

$$T_o \propto I_M I_F$$

or $T_o \propto VI$

i.e. $T_o \propto$ power

AC Working of Dynamometer Wattmeter

Let us now suppose that in an ac circuit,

e = instantaneous voltage across load

i = instantaneous current through load

If the load has lagging power factor cos ϕ, we can write

$$e = E_m \sin \omega t$$

$$i = I_m \sin (\omega t - \phi)$$

Current through fixed coil, $I_F \propto i$

Current through moving foil, $I_M \propto e$

Due to large inertia of the moving system, the deflection will be proportional to the average torque.

$\therefore$ Mean deflecting torque $\propto$ Average of $I_M I_F$

i.e., $T_o \propto$ Average of $e \times i$

or, $T_o \propto$ Average of $(E_m \sin \omega t) \times (I_m \sin (\omega t - \phi))$

$\propto EI \cos \phi$

$\propto$ Power (ac)

Hence, the dynamometer type wattmeter can be used for the measurement of both ac and dc power.

Since $T_o \propto$ power

and $T_o \propto \theta$

$\therefore$ $\theta \propto$ power

Hence, such instruments have a uniform scale.

Advantages

A dynamometer wattmeter has the following advantages:

(a) It has uniform scale.
(b) By careful design, high accuracy can be obtained.
(c) It can be used for ac as well as dc measurements.

Disadvantages

A dynamometer wattmeter has the following disadvantages:

(a) At low power factors, the inductance of the potential coil causes error in measurement.
(b) The readings of the instruments may be affected by the stray fields acting on the moving coil. In order to prevent it, the instrument is shielded from external fields by enclosing it in a soft iron case.

13.6.10 Induction Type Energy Meter (Single-Phase)

Single-phase induction type energy meter (Fig. 13.16) is extensively used to measure the electric energy supplied to a single-phase ac circuit in a given time. This is suitable only for use with ac application.

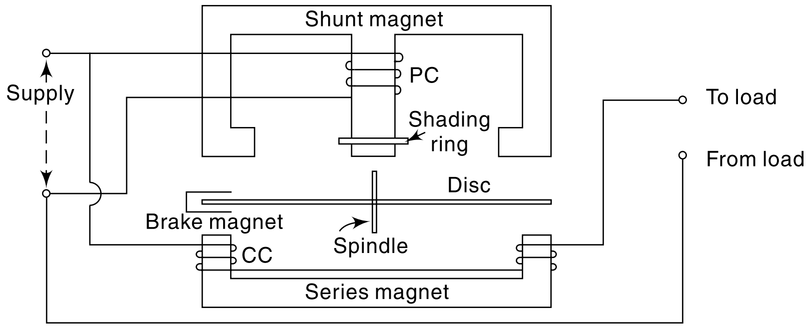


Fig. 13.16 ***Single-phase induction type energy meter***

Operating Principle The operation of induction type energy meter depends on alternating currents through two suitably located coils (current coil and pressure coil) producing rotating magnetic field. This field interacts with a metallic disc pivoted in between the coils and causes the disc to rotate.

The current coil carries the line current and produces a magnetic field in phase with the line current. The pressure coil is made highly inductive so that the current through it lags behing the supply voltage by 90°. Thus, a phase difference of 90° exists between the fluxes produced by the two coils. This sets up a rotating field which interacts with the disc to cause it to rotate.

Construction A single-phase induction type energy meter generally has the following parts:

(a) Moving system.
(b) Operating mechanism.
(c) Recording mechanism.

(i) Moving System

The moving system consists of a light aluminium disc mounted on a vertical spindle. The spindle is supported by a cup-shaped jewelled bearing at the bottom end and has a spring journal bearing at the top end.

In the instrument there is no pointer and control spring so that the disc makes continuous rotation under the action of deflecting torque.

(ii) Operating Mechanism

The operating mechanism consists of

(a) Series magnet.
(b) Shunt magnet.
(c) Braking magnet.

Series Magnet The series magnet consists of a number of U-shaped laminations assembled together to form a core. A thick wire of few turns is wound on both legs of the U-shaped laminated core. The wound coil is known as the *current coil* and is connected in series with the load so that it carries the load current. The series magnet is usually placed beneath the aluminium disc (shown in Fig. 13.16) and produces magnetic field proportional to and in phase with line current.

Shunt magnet The shunt magnet consists of a number of W-shaped laminations assembled together to form a core. A thin wire of large turns is wound on the central limb of this magnet. The wound coil is known as pressure coil and is connected across the load. It carries a current proportional to supply voltage. The shunt magnet is placed above the aluminium disc as shown.

In order to obtain deflecting torque, current in the pressure coil must lag behind the supply voltage by 90°. This necessary phase shift is obtained by placing a copper ring (also known as *compensating loop*) over central limb of shunt magnet. This copper ring acts as a short circuited transformer secondary. As its inductance is high as compared with its resistance, the current circulating in the ring will lag by nearly 90° behind the voltage producing it.

Braking Magnet The speed of the aluminium disc is controlled to the required value by the C-shaped permanent braking magnet. The magnet is mounted so that the disc revolves in the air gap between its polar extremities. As the disc rotates, voltages are induced in the disc (because it cuts the flux produced by the braking magnet). The direction of the current in the disc is such that it opposes the rotation of the disc (Lenz's law). Since the induced voltage (hence induced current) in the disc is proportional to the speed of the disc, therefore braking torque is proportional to the disc speed. As speed increases, braking torque increases.

(iii) Recording Mechanisms

The number of revolutions of the disc is a measure of the electrical energy passing through the meter and is recorded on dials with the help of a suitable gearing arrangement.

Working: When the energy meter is connected in circuit to measure electrical energy, the current coil carries the load current whereas the pressure coil carries current proportional to the supply voltage. The magnetic field due to the current coil is in phase with the line current whereas the magnetic field produced due to pressure coil (being highly inductive) lags approximately 90° behind the supply voltage.

The current coil field produces eddy currents in the disc which react with the field due to the pressure coil. Thus, a driving force is created which causes the disc to rotate.

The braking magnet provides the braking torque on the disc. By altering the position of this magnet, the desired speed can be obtained

The spindle is geared to the recording mechanism so that electrical energy consumed in the circuit is directly registered in kWH.

Theory

The current coil carries current proportional to the load current whereas the pressure coil carries current proportional to the supply voltage.

∴ Average deflecting torque

$$T_o \propto \text{Average power}$$

or $$T_o \propto VI \cos\phi$$

∴ $$T_o = K_1\, VI \cos\phi \qquad \text{(i)}$$

The braking torque T_B is proportional to the disc speed N i.e.

$$T_B \propto N$$

or $$T_B = K_2 N \qquad \text{(ii)}$$

The disc achieves steady speed N when the braking torque is equal to the deflecting torque.

From equations (i) and (ii),

$$K_2 N = K_1 \; VI \cos \phi$$

Multiplying, both sides by time 't'

$$K_2 N t = K_1 (VI \cos \phi) t$$

or $$Nt = K_3 P t \qquad \left[P = VI \cos \phi \text{ and } K_3 = \frac{K_1}{K_2} \right]$$

Since the product Nt represents the number of revolutions of the disc in time 't' and the product Pt represents the energy passing through the meter in time t, therefore, number of revolutions of the disc is proportional to the energy passing through the meter i.e.

Number of revolutions of disc $\propto$ Electrical energy passing through the meter.

13.6.11 Errors of induction type energy-meters

The following are the common errors which may creep in an energy meter:

1. Phase and speed errors
2. Frictional error
3. Creeping error
4. Temperature error
5. Frequency error.

1. Phase and Speed Errors

(a) Phase Error This error is introduced because the shunt magnet flux does not lag behind the supply voltage by exactly 90° (due to some resistance of the coil and iron losses in the core).

In order to remove this error, flux due to the shunt magnet should be made to lag behind the supply voltage by exactly 90°. This is accomplished by adjusting the position of the shading ring filled on the central limb of the shunt magnet. Since the inductance of the shading loop is high as compared with its resistance, the current circulating in the loop will lag behind the supply voltage by nearly 90°. By altering the position of this ring on the central limb of the shunt magnet, 90° displacement can be adjusted,

(b) Speed Error Sometimes, the speed of the disc of the energy meter is either faster or slower, introducing an error.

The speed of the energy meter can be adjusted to the desired value by changing the position of the braking magnet. If the braking magnet is moved towards the center of spindle, the braking torque is reduced, increasing the speed of the disc and vice versa.

2. Frictional Errors

This error is introduced due to the friction in the bearing of the rotating system. It depends upon the amount of load in the circuit.

The error is compensated by placing two short-circuit loops placed on the outer limbs of the shut magnet. The field in loops tends to produce a torque from shunt winding alone (i.e. with no current in the series coil).

The loops are adjusted so that when no current is passing through the current coil, the torque produced by the loops is just sufficient to overcome the friction in bearings, without actually rotating the disc.

3. Creeping Error

Often the disc of the energy meter makes slow but continuous rotation when only the pressure coil of the meter is excited but with no current flowing in the load. This is called *creeping*.

This error is eliminated by drilling two holes in the disc on the opposite sides of the spindle. This causes sufficient distortion of the field. The result is that the disc tends to remain stationary when one of the holes comes under one of the poles of the shunt magnet.

4. Temperature Errors

Due to the change in temperature, parameters of the coils (of the meter) change slightly so that a very small error is introduced. Swamping resistor is connected to the potential coil to eliminate this error.

5. Frequency Error

The meter is designed to give minimum error at a particular frequency (generally 50 Hz). If frequency is changed, the reactance of the coils will change, resulting in small errors.

13.7 MEASUREMENT OF RESISTANCE

An *ohm meter* is an instrument that directly measures a resistance. It consists of a battery, an MC (moving coil) instrument and a variable resistance R (Fig. 13.17). The unknown resistance R being connected across the output terminals of the meter, a circuit current flows and the MC instrument gives the deflection.

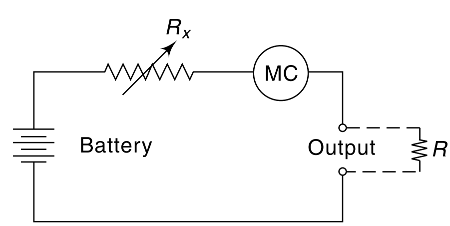


Fig. 13.17 *Ohm meter*

The infinity ohm position is fixed at the position of the pointer when the output is open circuited. The zero position of the pointer gives zero ohm and is obtained by shorting the outputs of the meter.

Ohm meters are available in a number of ranges from milli ohm to mega ohms. Multi range ohm meters have a number of shunts and selector switches in order to connect a particular shunt in parallel with the meter. Every time a resistance is measured, the zero setting should be done by short circuiting the output terminals and by varying R_x.

13.8 RECTIFIER TYPE INSTRUMENTS

AC measurements can be achieved by using a diode rectifier to convert ac into dc and its movements indicate the value of rectified AC. This method is preferable due to its higher sensitivity than electrodynamometer or moving iron instruments.

The rectifier instruments use PMMC movement and usually consists of a semiconductor bridge rectifier, as shown in (Fig. 13.18). A multiplier resistance may also be attached.

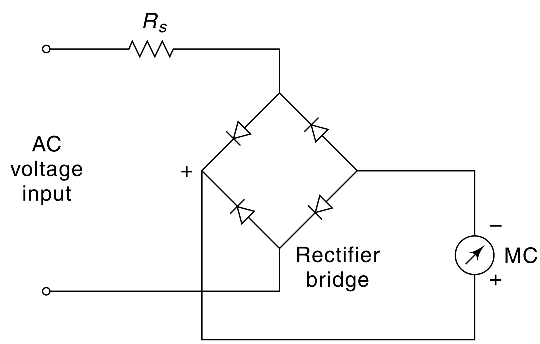


Fig. 13.18 ***Rectifier instrument***

During voltage measurement, the moving coil meter (in the dc side of the bridge) exhibits steady deflection proportional to the average value of the current. But the meter scale is calibrated in terms of rms value of the input waveform so it indicates the corresponding ac input voltage.

The major drawback of this instrument is that the resistance of the diodes may change with temperature. There may also be some variation of reading with change of operating frequency.

13.1 A moving iron voltmeter reads correctly on 200 V dc. If 200 V, 50 Hz ac is applied on it, determine the reading of the voltmeter. The instrument coil has a resistance of 300 Ω and inductance of 2 H while the series noninductive resistance is 1000 Ω.

Solution

When the instrument is used on dc only, resistance will affect the reading whereas if it is used in ac the reading will be affected by both resistance and inductance.

Total resistance of the instrument = 300 + 1000 = 1300 Ω

The impedance of the instrument at 50 Hz is obtained as

$$Z = \sqrt{(1300)^2 + (2\pi \times 50 \times 2)^2} = 1443.88\ \Omega$$

$$\therefore \qquad \text{Current} = \frac{V}{Z} = \frac{200}{1443.88}\ \text{A} = 0.1385\ \text{A, when connected to ac supply.}$$

When connected to dc supply, current $= \dfrac{200}{1300}$ A = 0.1538 A

Since the voltmeter reads correctly on dc supply, reading of the volt meter when connected to 200 V, 50 Hz ac is $\dfrac{200}{0.1538} \times 0.1385$, i.e. 180 V.

13.2 The full-scale deflecting torque of a 20 A moving iron ammeter is 6×10^{-5} Nm. Determine the rate of change of self-inductance of the instrument at full scale if the full scale deflection torque is obtained for the relation

$$T_o = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

Solution

Full scale deflecting torque = 6×10^{-5} Nm.
Full scale current I = 20 A

Deflecting torque = $\frac{1}{2} I^2 \frac{dL}{d\theta}$

Hence $\frac{dL}{d\theta} = \frac{2 \times 6 \times 10^{-5}}{(20)^2}$ H/rad = 0.3 μH/rad.

13.3 In a gravity controlled instrument the controlling weight is 0.008 kg. and it acts at a distance 2 cm from the axis of the moving system. Determine the deflection in degrees corresponding to deflecting torque of 1.5×10^{-4} kg.m.

Solution

Deflecting torque $T_o = 1.5 \times 10^{-4}$ kg.m
If the deflection be θ, then
controlling torque: $T_c = Wl \sin\theta = 0.008 \times 2 \times 10^{-2} \sin\theta$ $[\because W = 0.008 \text{ kg}]$
Since $T_d = T_c$ $[l = 2 \times 10^{-2} \text{ m}]$
Hence $1.5 \times 10^{-4} = 2 \times 0.008 \times 10^{-2} \sin\theta$

or $\sin\theta = \frac{1.5 \times 10^{-4}}{2 \times 0.008 \times 10^{-2}} = 0.9375$

i.e. $\theta = 69.64°$.

13.4 A moving coil meter has a resistance of 3 Ω and gives full scale deflection with 30 mA. Show how it can be used to measure voltage up to 300 V?

Solution

Resistance of the meter R_m = 3 Ω.
Current $I = 30 \times 10^{-3}$ A = 0.03A.
Voltage to the measured V = 300 V.

Total resistance within the meter circuit should be $\frac{300}{0.03}$ Ω or, 10,000 Ω

Hence external resistance to be connected in series with the instrument to measure 300 V is (10,000 – 3) Ω i.e. 9997 Ω.

13.5 A moving coil instrument has a resistance of 15 Ω and gives a full-scale deflection when carrying 60 mA. Show how it can be used to measure current up to 50 A.

Solution

Resistance of the instrument is 15 Ω.
Current through the instrument is 60 mA i.e., 0.06 A.
Current to be measured is 50 A.
An external resistance should be connected in shunt with the instrument.
The shunt resistance should carry (50 – 0.06) A = 49.94 A.
Now, voltage across the shunt resistance = voltage across the instrument only = 15 × 0.06 = 0.9 V.

Hence resistance of the external resistor (shunt) = $\frac{0.9}{49.94}$ Ω = 0.018 Ω.

EXERCISES

1. What is meant by (i) an indicating type instrument, (ii) an integrating type instrument and (iii) recording type instrument? Give an example of each.
2. What are the different methods of obtaining the controlling torque in an indicating instrument? Discuss briefly about each of them bringing out the advantages and disadvantages.
3. What are the various techniques by which damping torque is produced in an electrical measuring instrument? Explain them.
4. Differentiate between
   (i) Moving iron and moving coil instruments.
   (ii) Deflecting torque and controlling torque.
5. Explain the following:
   (i) On what principle is eddy current damping based?
   (ii) What are advantages of gravity control over spring control?
6. In a gravity controlled instrument if the controlling weight is 0.006 kg. and the deflecting torque corresponding to a deflection of 60° is 1.25×10^{-4} kg.m, determine the distance of controlling weight from the spindle. [*Ans:* 24 mm]
7. An instrument spring of length 0.6 m breadth 0.75 mm and thickness 0.08 mm has Young's modulus of elasticity 1.2×10^4 kg/mm^2. Determine the approximate torque created by the spring when it is turned through an angle of 90°. [*Ans:* 100.5×10^{-6} kg.m]
8. Describe the constructional details and working of a moving iron attraction type meter. Derive its torque equation
9. With the help of neat diagrams explain the working principle of a repulsion type moving iron instrument.
10. Describe the construction and working principle of PMMC type of instrument.
11. Explain with the help of a neat sketch the construction of an electro-dynamic type instrument.
12. Describe the construction and principle of operation of shaded pole type induction energy meter.
13. What are different types of errors of induction type energy meters? How they are minimized?
14. Write short notes on
    (i) Rectifier instruments
    (ii) Creeping error in induction type energy meter
    (iii) Extension of ammeter range
    (iv) Extension of volt meter range.
15. The full-scale deflecting torque of a 0.5 A moving iron ammeter is 1.5×10^{-5} Nm. Estimate in micro Henry per radian the rate of change of self inductance of the instrument at full scale. [*Ans.* 1.2 μH/radian]

14 REVIEW PROBLEMS

In this chapter different types of worked-out examples are set covering the full syllabus of the basic electrical engineering course. The reader is referred to the following section for the worked-out examples.

14.1 A coil has a resistance of 10 Ω at 0°C and 15 Ω at 100°C. What is the temperature co-efficient of the resistance of the coil? At what temperature will its resistance be 30 Ω?

Solution

Since $R = R_o(1 + \alpha_0 T)$ (i)

$\therefore$ $15 = 10[1 + \alpha_0 \times 100]$

$\therefore$ $\alpha_o = 0.005$ per °C at 0°C

Also, using Eq. (i) we have,

$30 = 10(1 + 0.0005 \times T)$

i.e. $T = 400°C$

14.2 A copper coil is found to have its resistance as 90 Ω at 20°C and is connected to a 230 V supply. By how much must the voltage be increased to keep the current constant, if the temperature of the coil rises to 60°C. Take the temperature co-efficient of copper as 0.00428 per °C at 0°C.

Solution

Since $R_{20} = R_o(1 + \alpha_0 \times 20)$ and

$R_{60} = R_o(1 + \alpha_0 \times 60)$,

we have $$\frac{R_{20}}{R_{60}} = \frac{R_o(1+\alpha_o \times 20)}{R_o(1+\alpha_o \times 60)}$$

$$\therefore \quad R_{60} = \frac{R_{20}[1+(60\times 0.00428)]}{[1+(20\times 0.00428)]} = \frac{90\times[1+(60\times 0.00428)]}{[1+(20\times 0.00428)]} = 104.4\ \Omega$$

Current taken by coil at 20°C is thus, $I_{20°C} = \dfrac{230}{90} = 2.56$ A.

At 60°C to keep the current constant, the voltage must be (2.56 × 104.4) or, 267.26 V therefore the voltage must be raised by (267.26 – 230) i.e., 37.26 V.

14.3 The resistance of the field coil of a motor is 200 Ω at 15°C. After the motor worked for a few hours on full load, the resistance increases to 240 Ω. Calculate the temperature rise of the field coil assuming the temperature co-efficient of resistance is 0.0042 per °C at 0°C.

Solution

We can write,
$$\frac{R_2}{R_1} = \frac{R_o(1+\alpha_o \times T_2)}{R_o(1+\alpha_o \times T_1)}$$

$$\therefore \quad R_L = \frac{R_1(1+\alpha_o \times T_2)}{(1+\alpha_o \times T_1)}$$

or
$$(1 + \alpha_0\, T_2) = \frac{R_2}{R_1}(1 + \alpha_o\, T_1) = \frac{240}{200}[1 + 0.0042 \times 15]$$

$$\therefore \quad \alpha_0\, T_2 = 1.2 + 0.0756 - 1 = 0.2756$$

or
$$T_2 = \frac{0.2756}{0.0042} = 65.6°\text{C}.$$

∴ Temperature rise = 65.6 – 15 = 50.6°C.

14.4 Two resistors are made of different materials and have temperature coefficients α_1 and α_2/°C at 0°C. They are connected in parallel across voltage source and consume equal power at 25°C. If $\alpha_1 = 2\alpha_2$, while $\alpha_2 = 0.005$ per °C, find the ratio of the power consumed for both the resistors at 60°C.

Solution

At 25°C, the resistors consume same power. Since both the resistors are connected across a single source hence we can write at 25°C, $V^2/R_1 = V^2/R_2$.

i.e. $R_1 = R_2$

Hence we have

$$R_{o1}(1 + 25\alpha_1) = R_{o2}(1 + 25\alpha_2)$$

i.e.
$$\frac{R_{o1}}{R_{o2}} = \frac{1+25\alpha_2}{1+25\alpha_1}$$

However, at 60°C we can write

$$\frac{V^2/R_2}{V^2/R_1} = \frac{R_1}{R_2} = \frac{R_{o1}(1+60\alpha_1)}{R_{o2}(1+60\alpha_2)}$$

or
$$\frac{P_2}{P_1} = \frac{(1+25\alpha_2)(1+60\alpha_1)}{(1+25\alpha_1)(1+60\alpha_2)},$$

where P_2 and P_1 are the power consumed by the second and first resistor at 60°C respectively.

Since $\alpha_1 = 2\alpha_2$ we have

$$\frac{P_2}{P_1} = \frac{(1+25\alpha_2)(1+60\times 2\alpha_2)}{(1+25\times 2\alpha_2)(1+60\alpha_2)},$$

$$= \frac{1+25\alpha_2+120\alpha_2+3000\alpha_2^2}{1+50\alpha_2+60\alpha_2+3000\alpha_2^2} = \frac{3000\alpha_2^2+145\alpha_2+1}{3000\alpha_2^2+110\alpha_2+1}$$

with $\alpha_2 = 0.005$/°C, we have at 60°C

$$\frac{P_2}{P_1} = \frac{3000\times 0.005^2+145\times 0.005+1}{3000\times 0.005^2+110\times 0.005+1}$$

or
$$\frac{P_2}{P_1} = 1.1076.$$

14.5 A cylindrical copper conductor has a resistivity ρ while the resistance between the opposite ends is (R). Assuming its volume to be v, length L and diameter D, find an expression for the length and diameter in terms of the resistivity.

Solution

$$R = \rho\frac{L}{A} = \frac{\rho L^2}{\frac{\pi D^2}{4}\cdot L} = \frac{\rho L^2}{v} \qquad \left[\because \text{volume } (v) = \frac{\pi D^2 L}{4}\right]$$

Hence,
$$L = \sqrt{\left(\frac{Rv}{\rho}\right)}$$

Also,
$$R = \rho\frac{L}{\frac{\pi D^2}{4}} = \frac{\rho L\frac{\pi D^2}{4}}{\left(\frac{\pi D^2}{4}\right)^2} = \frac{\rho v}{\frac{\pi^2 D^4}{16}} = \frac{16\,\rho v}{\pi^2 D^4}$$

$\therefore$
$$D = \left[\frac{16\,v\rho}{\pi^2 R}\right]^{1/4}.$$

14.6 A solution of resistivity 22 Ω cm is poured to fill a glass container whose top and bottom ends are made up of two electrodes (Fig. 14.1). A dc voltage of 220 V is applied across the electrodes when the power absorbed by the liquid solution is 22 kW. If the area of each electrode is circular and of 100 cm^2 value, find the distance between the electrodes. Neglect the thickness and resistance of the electrodes.

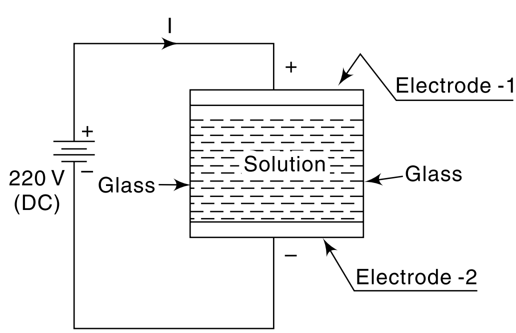


Fig. 14.1

Solution

Current I through the solution is obtained as

$$I = \frac{P}{V} = \frac{22\times 10^3}{220} = 100 \text{ A}$$

From $\quad R = \rho\dfrac{L}{A}$, we get

$$L = \frac{R\times A}{\rho} = \frac{(V/I)\times A}{\rho} = \frac{\frac{220}{100}\times 100}{22} = 10 \text{ cm}.$$

Thus the length of the liquid path is 10 cm.

14.7 A resistor is wound using a wire of resistivity ρ_1 whose length is l_1, and diameter d_1. Due to some effect, it is subsequently needed to replace the original wire by another wire of resistivity ρ_2. Assuming the diameter of the new wire being 1.5 times of the diameter of the original one, find the ratio of the resistivities of these two wires assuming the resistance and surface area of both as same at the operating temperature.

Solution

As per the question,

$$\frac{\rho_1 l_1}{A_1} = \frac{\rho_2 l_2}{A_2}$$

i.e. $$\frac{\rho_2}{\rho_1} = \frac{l_1}{A_1} \times \frac{A_2}{l_2} = \frac{l_1}{\pi d_1^2/4} \times \frac{\pi d_2^2/4}{l_2} = \frac{l_1}{l_2} \times \frac{d_2^2}{d_1^2}$$

Also, $\pi d_2 l_2 = \pi d_1 l_1$ (surface area being same).

i.e. $$\frac{l_1}{l_2} = \frac{d_2}{d_1}$$

$\therefore$ $$\frac{\rho_2}{\rho_1} = \frac{d_2}{d_1} \times \frac{d_2^2}{d_1^2} = \frac{d_2^3}{d_1^3} = \frac{(1.5\,d_1)^3}{d_1^3} = 3.375.$$

Thus the resistivity of the second wire is 3.375 times more than the resistivity of the first one.

14.8 Determine the equivalent capacitance of the network shown in Fig. 14.2. All values shown in the figure are in μF.

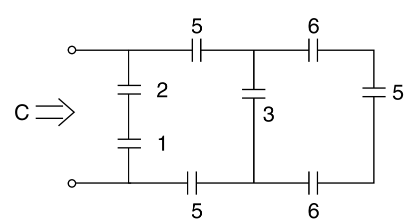


Fig. 14.2

Solution

The network shown in Fig. 14.2 is reduced to a simpler network as shown in Fig. 14.3.
Here,

$$C_1 = \frac{1}{\frac{1}{6}+\frac{1}{5}+\frac{1}{6}} = \frac{15}{8}\ \mu\text{F}$$

$$C_2 = \frac{2 \times 1}{2+1} = \frac{2}{3}\ \mu\text{F}$$

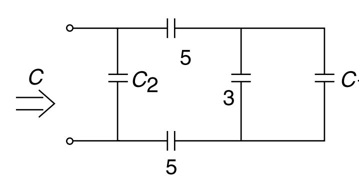


Fig. 14.3

Further network reduction is shown in Fig. 14.4 where, $C_3 = \frac{15}{8} + 3 = 4.875$ μF.

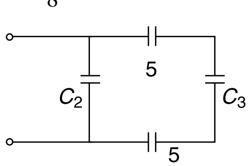


Fig. 14.4

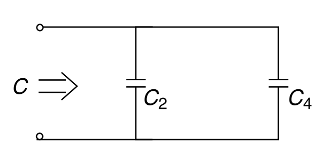


Fig. 14.5

Since two capacitances of 5 μF each and 4.875 μF are in series, we have the equivalent capacitance of these three capacitance as

$$C_4 = \frac{1}{\frac{1}{5}+\frac{1}{5}+\frac{1}{4.875}} = 1.65\ \mu\text{F. (Fig. 14.5)}$$

Hence C, the equivalent capacitance is

$$C = C_2 + C_4 = \frac{2}{3} + 1.65 = 2.3167\ \mu\text{F}.$$

14.9 A parallel plate capacitor has three identical parallel plates. The outer two plates are connected by metals. When the inner plate is at the middle of the two outer plates the capacitance is C_1. When the inner plate is four times near to one plate as the outer, the capacitance is C_2. Determine the ratio C_1/C_2.

Solution

Let us assume the distance between the two outer plates be $5d$, area of cross-section of each plate A and the permitivity of the medium is ε.

$$\therefore \qquad C_1 = \frac{A\varepsilon}{2.5\,d} + \frac{A\varepsilon}{2.5\,d} = \frac{2A\varepsilon}{2.5\,d}$$

Again, $$C_2 = \frac{A\varepsilon}{d} + \frac{A\varepsilon}{4\,d} = \frac{5A\varepsilon}{4\,d}$$

Hence, $$\frac{C_1}{C_2} = \frac{2\times 4}{2.5\times 5} = \frac{8}{12.5} = 1 : 1.56.$$

14.10 A capacitor consists of two parallel circular metal discs of 15 cm radius and 20 mm apart. Between the discs there are three different layers of dielectrics having different thicknesses given as follows:

Thickness d	ε
7 mm	2
5 mm	5
8 mm	7

Determine the voltage gradient in each dielectric when 1000 V (dc) is applied across it.

Solution

Area of each plate $A = \pi r^2 = \pi(0.15)^2 = 0.0706$ sq m.

$$\text{Capacitance } C = \frac{A\varepsilon_0}{\dfrac{d_1}{\varepsilon_1} + \dfrac{d_2}{\varepsilon_2} + \dfrac{d_3}{\varepsilon_3}} = \frac{A\times\varepsilon_0}{\dfrac{7}{2} + \dfrac{5}{5} + \dfrac{8}{7}}\times 10^3$$

$$= \frac{0.0706\times 8.854\times 10^{-12}\times 10^3}{\dfrac{7}{2} + 1 + \dfrac{8}{7}}$$

$$= \frac{0.625}{5.643}\times 10^{-9}\text{ F}$$

$$= 0.111\times 10^{-9}\text{ F} = 111\ \mu\text{F}.$$

Charge $Q = VC = (1000 \times 0.111 \times 10^{-9})$ Coulumb

$$\text{Charge density } D\left(=\frac{Q}{A}\right) = \frac{1000\times 0.111\times 10^{-9}}{0.0706}\text{C/m}^2 = 1.572\ \mu\text{C/m}^2.$$

$\therefore$ Voltage gradient for the

$$\text{1st dielectric, } E_1 = \frac{D}{\varepsilon_o\,\varepsilon_1} = \frac{1.572\times 10^{-6}}{8.854\times 10^{-12}\times 2}\text{ V/m} = 0.0888\times 10^6\text{ V/m}$$

$$\text{2nd dielectric, } E_2 = \frac{D}{\varepsilon_o\,\varepsilon_2} = \frac{1.572\times 10^{-6}}{8.854\times 10^{-12}\times 5}\text{ V/m} = 0.0355\times 10^6\text{ V/m}$$

3rd dielectric, $E_3 = \dfrac{D}{\varepsilon_o \varepsilon_3} = \dfrac{1.572 \times 10^{-6}}{8.854 \times 10^{-12} \times 7}$ V/m $= 0.025 \times 10^6$ V/m.

14.11 An air capacitor consists of two parallel square plates of 60 cm side. When the two plates are 2 mm apart the capacitor is charged to a voltage of 240 V. Determine the work done in separating the plates from 2 mm to 5 mm.

Solution

When the separation between the plates is 2 mm;

$$C_1 = \frac{A\varepsilon}{d_1} = \frac{(0.6 \times 0.6) \times \varepsilon_o}{2 \times 10^{-3}} = 180\ \varepsilon_o$$

Energy stored $W_1 = \frac{1}{2} C_1 V^2 = \frac{1}{2} \times 180 \times \varepsilon_o \times (240)^2 = \varepsilon_o \times 5184$ kJ.

When the separation between the plates is 5 mm;

$$C_2 = \frac{A\varepsilon}{d_2} = \frac{(0.6 \times 0.6) \times \varepsilon_o}{5 \times 10^{-3}} = 72\ \varepsilon_o$$

Energy stored $W_2 = \frac{1}{2} C_2 V^2 = \frac{1}{2} \times 72\ \varepsilon_o \times (240)^2 = \varepsilon_o \times 2073.6$ kJ

Work done in separating the plates from 2 mm to 5 mm is

$$(W_1 - W_2) = (5184 - 2073.6) \times 10^3 \times 8.854 \times 10^{-12} \text{ J} = 27.54\ \mu\text{J}.$$

14.12 Two capacitors A and B have capacitances of 50 μF and 30 μF respectively. When 230 V, 50 Hz voltage is applied, find the current and maximum energy stored. Assume A and B are connected in (i) series and (ii) parallel.

Solution

(i) When the capacitances are connected in series the equivalent capacitance is

$$C = \frac{50 \times 30}{50 + 30}\ \mu\text{F} = \frac{150}{8}\ \mu\text{F}$$

The current $I = V\omega C = 230 \times 2\pi \times 50 \times \frac{150}{8} \times 10^{-6} = 1.355$ A.

Maximum energy stored $= \frac{1}{2} C V_{\max}^2 = \frac{1}{2} \times \frac{150}{8} \times 10^{-6} \times (230 \times \sqrt{2})^2 = 0.992$ J.

(ii) When the capacitances are connected in parallel the equivalent capacitance is

$$C = (50 + 30)\mu\text{F} = 80\ \mu\text{F}$$

Current $I = V\omega C = 230 \times 2\pi \times 50 \times 80 \times 10^{-6}$ A $= 5.78$ A

Maximum energy stored $= \frac{1}{2} C V_{\max}^2 = \frac{1}{2} \times 80 \times 10^{-6} \times (\sqrt{2} \times 230)^2 = 4.232$ J.

∴ When the capacitances are connected in series, the current is 1.355 A while the energy stored is 0.992 J. In parallel connection, the current is 5.78 A and the energy stored is 4.232 J.

14.13 A 10 μF capacitor is connected through a 2 MΩ resistance to a direct current source. After remaining on charge for 30s the capacitor is disconnected and discharged through a resistor. Find what percentage of the energy input from the supply is dissipated in the resistor.

Solution

Given, $C = 10 \times 10^{-6}$ F

$R = 2 \times 10^{6}\ \Omega$

λ = C.R = 20 s (time constant)

$\because$ $v = V(1 - e^{-t/\lambda})$, we have at t = 30 s,

$v = V(1 - e^{-30/20}) = 0.7768$ V.

Energy stored in the capacitor (at t = 30 s) is given by

$$\frac{1}{2}Cv^2 = \frac{1}{2} \times 10 \times 10^{-6} \times (0.7768)^2\ V^2$$

$$= 3.0326V^2 \times 10^{-6} \text{ (= energy dissipated in the resistor)}$$

But $$i = \frac{dq}{dt} = \frac{d}{dt}(Cv)$$

$$= \frac{d}{dt}\{CV(1 - e^{-t/\lambda})\} = CV \times \frac{1}{\lambda}e^{-t/\lambda} = \frac{V}{R}e^{-t/\lambda} \quad [\because \lambda = RC]$$

$$\therefore \text{ Total energy input} = \int_0^{30} Vidt = \frac{V^2}{R}\int_0^{30} e^{-t/20}\,dt = -\frac{V^2}{R} \times 20\,[e^{-t/20}]_0^{30}$$

$$= \frac{10V^2}{10^6}(0.7768) = 7.768 \times 10^{-6}\ V^2.$$

Percentage of the energy input from the supply dissipated in the resistor =

$$\frac{3.0326 \times 10^{-6}\ V^2}{7.768 \times 10^{-6}\ V^2} = 0.3904 \text{ or, } 39.04\%$$

14.14 Calculate the maximum energy stored in the capacitor and energy dissipated in the resistor in the time interval 0 < t < 1 s in the circuit shown in Fig. 14.6.

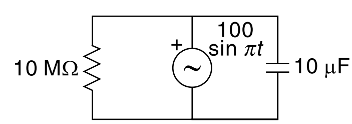


Fig. 14.6

Solution

The expression for the energy stored in the capacitor is,

$$E = \frac{1}{2}Cv_c^2 = \frac{1}{2} \times 10 \times 10^{-6} \times (100 \sin \pi t)^2$$

$$= \frac{1}{2} \times 10^{-1} \sin^2 \pi t = 0.05 \sin^2 \pi t. \text{ J}$$

The energy will be maximum when ($\sin^2 \pi t$) = 1. Hence maximum energy stored in the capacitor is 0.05 J.

The current through the resistor $i_R = \dfrac{100 \sin \pi t}{10 \times 10^6} = (10^{-5} \sin \pi t)$ A

Energy dissipated in the resistor in the time interval 0 < t < 1s

$$= \int_0^1 i_R^2 R\,dt$$

$$= \int_0^1 (10^{-5} \sin \pi t)^2 \times 10 \times 10^6\,dt$$

$$= \int_0^1 10^{-3} \sin^2 \pi t\,dt$$

$$= \frac{10^{-3}}{2}\int_0^1 (1-\cos 2\pi t)dt$$

$$= \frac{10^{-3}}{2}\left[t-\frac{\sin 2\pi t}{2\pi}\right]_0^1 = \frac{10^{-3}}{2}\left[1-\frac{\sin 2\pi}{2\pi}\right] = 0.5 \text{ mJ.}$$

14.15 Determine the voltage across the capacitor when a current waveform shown in Fig. 14.7 is applied to a 10 μF capacitor.

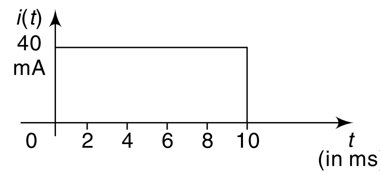


Fig. 14.7

Solution

We know, voltage across capacitor $v_C = \frac{1}{C}\int i\,dt$

Here, i = 40 mA = 0.4 A (from 0 to 10 ms).

Hence, $v_C = \frac{1}{C}\int 0.04\,dt$ (from 0 to 10 ms).

or $v_C = \frac{1}{10\times 10^{-6}}\int 0.04\,dt$

$= 10^5 \times 0.04t + K = 4000\,t + K$, where K is constant

At $t = 0$, $v_C = 0$; this gives $K = 0$.

With this the general expression become $v_C = 4000t$

∴ At t = 10 ms (= 0.01 s), v_C = 40 V.

When $t \geq 10$ ms then

$i = 0$

Hence, $v_C = \frac{1}{C}\int 0\times dt + 40$ [∵ At t = 10 ms, v_C = 40 V]

or v_C = 40 volts at $t \geq 10$ ms.

The voltage waveform across the capacitor is represented in Fig. 14.8.

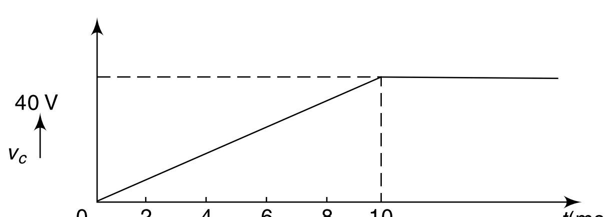


Fig. 14.8

14.16 A triangular voltage wave (shown in Fig. 14.9) with a peak amplitude of 150 V and frequency 75 KHz. is applied across a 0.03 μF capacitor. Determine the rms value of the current flowing in the capacitor.

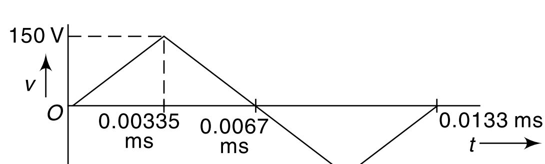


Fig. 14.9

Solution

From the given data,
Frequency $f = 75 \times 10^3$ Hz.

Time period $T = \dfrac{1}{f} = 0.0133$ ms

The voltage rises from 0 to 150 V in 0.00335 ms. Also, $C = 0.03 \times 10^{-6}$ F.

The peak value of the current flowing through the capacitor $\left(= C\dfrac{dv}{dt}\right) = 0.03 \times 10^{-6} \times \dfrac{150}{0.00335} = 0.001343$ A.

The current wave through the capacitor is rectangular when a triangular voltage waveform is applied.

Hence rms value of the current flowing through the capacitor is 0.001343 A.

14.17 Determine the current through a 20 μF capacitor when a voltage waveform shown in Fig. 14.10 is applied.

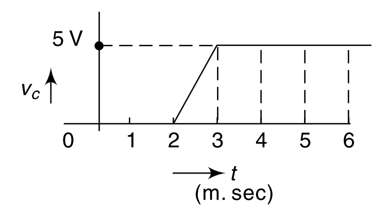


Fig. 14.10

Solution

$$i = C\frac{dv_c}{dt}$$

From the given figure, for $t < 2$ ms,

$$v_C = 0.$$

Hence, $\quad i = 0$ A.

For $2 \le t \le 3$ ms, we find $v_C = \left(\dfrac{5}{1 \times 10^{-3}} \times t\right)$ V

$$\therefore \quad i\left(= C\frac{dv_c}{dt}\right) = 20 \times 10^{-6}\ \frac{d}{dt}\left(\frac{5}{1 \times 10^{-3}}\, t\right) = 20 \times 10^{-6} \times 5000 = 0.1 \text{ A}.$$

For $t > 3$ ms

$$i = 20 \times 10^{-6}\ \frac{d}{dt}(5) = 0 \text{ A}. \qquad [\because v_C = 5 \text{ V (constant)}]$$

The current waveform is shown in Fig. 14.11.

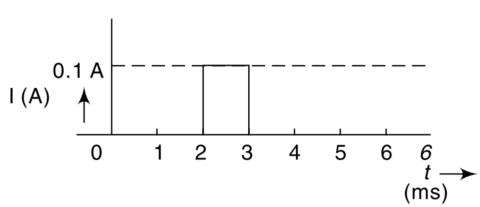


Fig. 14.11

14.18 The current waveform through a 0.5 H inductor is shown in Fig. 14.12. Calculate the voltage across the inductor at 0 ms, 8 ms and 14 ms.

Solution

We know that the voltage across the inductor is

$$v_L = L\frac{di}{dt}$$

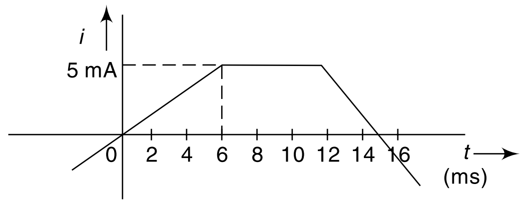


Fig. 14.12

For $t < 6$ ms, $\quad i = \dfrac{5 \times 10^{-3}}{6 \times 10^{-3}} t$

$$\therefore \quad v_L = 0.5 \, \frac{d}{dt} \left(\frac{5 \times 10^{-3}}{6 \times 10^{-3}} t \right) = \frac{0.5 \times 5 \times 10^{-3}}{6 \times 10^{-3}} = 0.4167 \text{ V.}$$

For $6 \leq t \leq 12$ ms

$$v_L = 0.5 \, \frac{d}{dt} (5 \times 10^{-3}) = 0$$

For $12 \leq t \leq 15$ ms, $i = -\dfrac{5 \times 10^{-3}}{3 \times 10^{-3}} t$

$$v_L = 0.5 \frac{d}{dt} \left(-\frac{5 \times 10^{-3}}{3 \times 10^{-3}} t \right) = -\frac{0.5 \times 5}{3} \text{ V} = -0.833 \text{ V.}$$

$\therefore$ At $\quad t = 0$, $v_L = 0.4167$ V

At $\quad t = 8$ ms, $v_L = 0$

At $\quad t = 14$ ms, $v_L = -0.833$ V.

14.19 Simplify the diagram shown in Fig. 14.13.

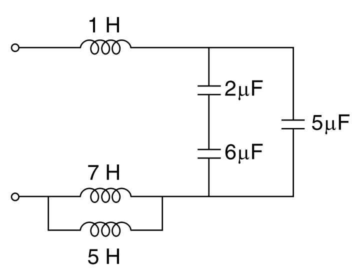


Fig. 14.13

Solution

Combining 2 μF and 6 μF capacitors in series, equivalent capacitance is $\left(\dfrac{1}{\frac{1}{2} + \frac{1}{6}} \right)$ or, $\dfrac{3}{2}$ μF; therefore we see that the capacitances 5 μF and $\dfrac{3}{2}$ μF are in parallel. The equivalent capacitance is $\left(5 + \dfrac{3}{2} \right)$ = 6.5 μF.

Again, it is observed that 7 H and 5 H inductors are in parallel. Their combined inductance becomes $\left(\dfrac{7 \times 5}{7 + 5} \right)$ or, $\dfrac{35}{12}$ H. However, $\left(\dfrac{35}{12} \text{ H} \right)$ and (1 H) inductors are in series.

The equivalent inductance is then $\left(1 + \dfrac{35}{12} \right)$ or, $\dfrac{47}{12}$ H (=3.91 H).

The simplified diagram is shown in Fig. 14.13(a).

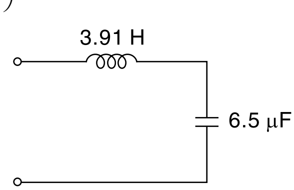


Fig. 14.13(a)

14.20 The resistance and inductance of a coil are 8 Ω and 35 H respectively. The current increase is at the rate of 6 A/s at the instant of closing the supply switch. Determine the applied voltage and the rate of growth of current when 10 A flows in the circuit. Also find the stored energy.

Solution

Given, $R = 8\ \Omega$

$L = 35$ H

Also, at $t = 0,\quad \dfrac{di}{dt} = 6$ A/s and $i = 0$ (at $t = 0$).

If V be the applied voltage, we can write,

$$V = R \cdot i + L \cdot \frac{di}{dt}$$

or $V = 8 \times 0 + 35 \times 6$

Hence, $V = 210$ V.

When $i = 10$ A, we can write using the same relation,

$$210 = 8 \times 10 + 35 \cdot \frac{di}{dt}$$

or, $$\frac{di}{dt} = \frac{210 - 80}{35} = 3.71 \text{ A/s.}$$

$\therefore$ Energy stored (when $i = 10$ A) is $\dfrac{1}{2}Li^2$ i.e., $\dfrac{1}{2} \times 35 \times (10)^2$, or, 1750 J.

14.21 A coil of resistance 25 Ω and inductance 0.75 H is switched on to a dc 230 V supply. Calculate the rate of change of current (i) at the instant of closing the switch and (ii) when $t = L/R$. Also find the final steady state value of the current.

Solution

Given, $R = 25\ \Omega$, $L = 0.75$ H, $V = 230$.

Hence, $\lambda = \dfrac{L}{R} = \dfrac{0.75}{25}$ s $= 0.03$ s (time constant).

From $i = \dfrac{V}{R}(1 - e^{-t/\lambda})$, we get $i = \dfrac{230}{25}(1 - e^{-t/0.03}) = 9.2(1 - e^{-33.33t})$ A

$\therefore$ $$\frac{di}{dt} = 33.33 \times 9.2e^{-33.33t} = 306.67\, e^{-33.33t} \text{ A/s.}$$

(i) At the instant of closing the switch (i.e. at $t = 0$),

$$\frac{di}{dt} = 306.67e^{-33.33 \times 0} = 306.67 \text{ A/S.}$$

(ii) When $t = \dfrac{L}{R}$ i.e. $t\ (= \lambda) = 0.03 = \dfrac{1}{33.33}$ s,

$$\frac{di}{dt} = 306.67e^{-33.33 \times \frac{1}{33.33}} = 112.82 \text{ A/s}$$

Final steady state value of the current is $\dfrac{V}{R}\left(= \dfrac{230}{25}\right) = 9.2$ A.

14.22 The resistance and inductance of a coil are 50 Ω and 20 H respectively. If the applied voltage is 200 V find the time taken by the current to reach 0.98 of its final steady value. If the current is reduced to zero in 50 ms calculate the average e.m.f. induced.

Solution

Given $R = 50\ \Omega$, $L = 20$ H

$V = 200$ V

$\therefore \qquad \lambda = \left(= \frac{L}{R}\right) = \frac{2}{5} = 0.4$ s.

Since $\qquad i = \frac{V}{R}(1 - e^{-t/\lambda})$, we have

$$0.98\left(\frac{V}{R}\right) = \frac{V}{R}(1 - e^{-t/0.4}) \qquad \left[\because \frac{V}{R} \text{ is steady state current}\right]$$

$\therefore \qquad e^{-t/0.4} = 1 - 0.98 = 0.02$

Hence $\qquad t = 1.568$

The average induced emf

$$e = L\frac{di}{dt} = 20 \times \frac{\frac{V}{R} - 0}{5 \times 10^{-3}} = 20 \times \frac{\frac{200}{50}}{50 \times 10^{-3}}$$

$= 1600$ V (in the direction to oppose the applied voltage).

14.23 A dc voltage applied to a coil of inductance 2 H and resistance 20 Ω is suddenly changed from V_1 to V_2. Determine an expression for the current during the transient. Calculate the value for the current when $t = 0.75$ s if $V_1 = 120$ V and $V_2 = 180$ V.

Solution

Given, $\qquad L = 2$ H,

$\qquad R = 20\ \Omega$

$\therefore \qquad \lambda = \frac{L}{R} = \frac{2}{20} = \frac{1}{10} = 0.1$ s

When supply voltage is V_2, i the circulating current

then we have, $R \cdot i + L\frac{di}{dt} = V_2$

or $\qquad \frac{di}{dt} = \frac{V_2 - R \cdot i}{L} = \frac{R}{L}\left(\frac{V_2}{R} - i\right)$

or $\qquad \frac{di}{\frac{V_2}{R} - i} = \frac{R}{L} \cdot dt$

or $\qquad \ln\left(\frac{V_2}{R} - i\right) = -\frac{tR}{L} + K_1 \qquad$ (where K_1 is constant)

or $\qquad \frac{(V_2/R) - i}{K_2} = e^{\frac{-t}{L/R}} \qquad$ (where $K_1 = \ln K_2$)

$\therefore \qquad i = \left(\frac{V_2}{R}\right) - \left(K_2 e^{\frac{-t}{L/R}}\right)$

At $t = 0$, $\qquad i = \frac{V_1}{R}$

hence $\qquad \frac{V_1}{R} = \frac{V_2}{R} - K_2 \quad$ or, $\quad K_2 = \frac{V_2}{R} - \frac{V_1}{R}$.

or $\qquad i = \frac{V_2}{R} - \frac{1}{R}(V_2 - V_1)e^{\frac{-t}{L/R}} \qquad$ (expression of current during the transient)

At $t = 0.75$s, $\qquad i = \frac{180}{20} - \frac{1}{20}(180 - 120)e^{-\frac{0.75}{0.1}} = 8.998$ A

14.24 Two identical coils A and B having 500 turns each lie in parallel planes. 80% of the magnetic flux produced by one coil links the other. When 6 A current flows through coil A, a flux of 0.04 mwb is produced in it. Determine the emf induced in B if the current in coil A changes from 5A to –5A in 0.03 s. Also calculate the self-inductance of each coil and the mutual inductance.

Solution

Self-inductance (L) of each coil is $\dfrac{N\phi}{I}$

i.e., $$L = \frac{500 \times 0.04 \times 10^{-3}}{6} = 0.0033 \text{ H.}$$

Mutual inductance $M = \left(\dfrac{80}{100} \times 0.0033\right)$ H.

$\therefore$ emf induced in coil B is, $e = M\dfrac{di}{dt} = \dfrac{80}{100} \times 0.0033 \times \dfrac{5-(-5)}{0.03}$ V $= 0.88$ V.

14.25 Assuming $M = 15$ H and $i_1 = 5e^{-3t}$, determine the voltage v_2 for (i) Fig. 14.4 (a) (ii) Fig. 14.14(b).

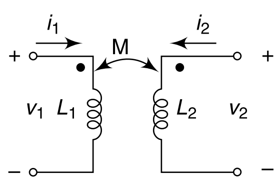


Fig. 14.14(a)

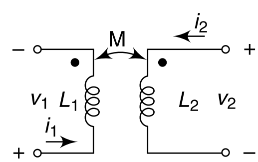


Fig. 14.14(b)

Solution

(i) In Fig. 14.14(a), M is +ve as current in both the windings enter from the dotted sides. [dot indicates higher polarity at any instant]

$$v_2 = M\frac{di_1}{dt} = 15\frac{d}{dt}(5e^{-3t}) = -225\ e^{-3t} \text{ V}$$

(ii) In Fig. 14.14(b), M is –ve as i_1 enters from the non-dotted side while i_2 enters from the dotted side.

$$v_2 = -M\frac{di_1}{dt} = -15\frac{d}{dt}(5e^{-3t}) = 225\ e^{-3t} \text{ V.}$$

14.26 For the circuit shown in Fig. 14.15 if $v_s = 30\ e^{-500t}$ V, obtain the mesh equations for the two meshes.

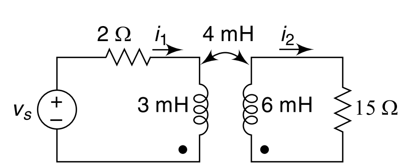


Fig. 14.15

Solution

Applying KVL to the left mesh we can write

$$30e^{-500t} = 2i_1 + 0.003\frac{di_1}{dt} - 0.004\frac{di_2}{dt} \qquad [\because L_1 = 3 \text{ mH}; M = 4 \text{ mH and } M \text{ is –ve}]$$

Applying KVL to the right mesh we have

$$0 = 15i_2 + 0.006\frac{di_2}{dt} - 0.004\frac{di_1}{dt}. \qquad [\because L_2 = 6 \text{ mH}; M = 4 \text{ mH and } M \text{ is –ve}]$$

14.27 Determine the total energy stored in the passive network shown in Fig. 14.16 at $t = 0$. Assume $K = 0.5$ and terminals x and y (i) open circuited (ii) short circuited.

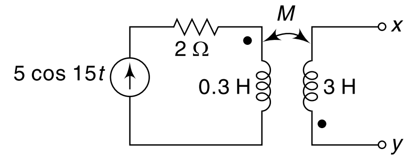


Fig. 14.16

Solution

$$M = K\sqrt{L_1 L_2} = 0.5\sqrt{0.3 \times 3}\text{ H} = 0.474\text{ H.}$$

Let us consider the two mesh currents i_1 and i_2 are flowing the clockwise direction in the two meshes.

From Fig. 14.16, we have

$$i_1 = 5\angle 0^\circ\text{ A.}$$

(i) When x and y are open circuited $i_2 = 0$

Hence total energy stored is $\frac{1}{2}L_1 i_1^2 = \frac{1}{2} \times 0.3 \times 5^2 = 3.75$ J.

(ii) When x and y are short circuited,
$i_1(t) = 5 \cos 15t$ and voltage v_{xy} across xy is 0.

Hence, $$v_{xy} = 3\frac{di_2}{dt} + 0.474\frac{di_1}{dt} = 0$$

or, $$\frac{di_2}{dt} = -\frac{0.474}{3}\frac{d}{dt}(5\cos 15t) = \frac{0.474}{3} \times 5 \times 15 \sin 15t = 11.85 \sin 15t.$$

Hence $$i_2(t) = \int_{-\infty}^{t} 11.85 \sin 15t\, dt = -0.75 \cos 15t \text{ (assuming zero initial point)}$$

Energy stored is

$$\left[\frac{1}{2} \times 0.3 \times 5^2 + \frac{1}{2} \times 3 \times (0.75)^2 + 0.474 \times 5(-0.75)\right] = 2.817\text{ J.}$$

14.28 In the circuit shown in Fig. 14.17 $L_1 = 2$ H, $L_2 = 5$ H and $M = 1.8$ H. Find the expression for the energy stored after the circuit is connected to a dc voltage of 30 V. Assume M to be positive.

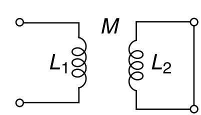


Fig. 14.17

Solution

If i_1 and i_2 be the currents in the two coils, we can write

$$30 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt} \qquad \text{(i)}$$

$$0 = L_2\frac{di_2}{dt} + M\frac{di_1}{dt} \qquad \text{(ii)}$$

From Eq. (ii), we get

$$\frac{di_2}{dt} = -\frac{M}{L_2}\frac{di_1}{dt}$$

$\therefore$ From Eq. (i), we get

$$30 = \frac{di_1}{dt}L_1 + M\left(-\frac{M}{L_2}\frac{di_1}{dt}\right) = \frac{di_1}{dt}\left(L_1 - \frac{M^2}{L_2}\right) = \frac{di_1}{dt}\frac{L_1 L_2 - M^2}{L_2}$$

The equivalent inductance $= \dfrac{L_1 L_2 - M^2}{L_2} = \dfrac{2 \times 5 - (1.8)^2}{5} = 1.352$ H.

Hence we have, $30 = (1.352)\dfrac{di_1}{dt}$

or $$\frac{di_1}{dt} = \frac{30}{1.352} = 22.19 \text{ A/s}.$$

$\therefore$ $$i_1 = \int_0^t 22.19dt = 22.19t.$$

Energy stored is, $\dfrac{1}{2}Li_1^2 = \dfrac{1}{2} \times 1.352 \times (22.19t)^2 = 332.86t^2.$

14.29 The value of the inductance shown in Fig. 14.18 is 10 mH. Determine (i) v_L at t = 10 ms if $i = 20te^{-50t}$, (ii) i at t = 0.2s if $v_L = 10e^{-15t}$ and $i(0)$ = 15 A, and (iii) power delivered to the inductor at t = 20 ms if $i = 10(1 - e^{-20t})$.

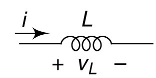


Fig. 14.18

Solution

(i) $$v_L = L\frac{di}{dt} = 10 \times 10^{-3}\frac{d}{dt}(20te^{-50t})$$
$$= 10 \times 10^{-3}\{20t \times (-50)e^{-50t} + 20 \times e^{-50t}\}$$
$$= 10^{-2} \times 20\{-50te^{-50t} + e^{-50t}\}.$$

At t = 10 ms(= 0.01s), we have
$$v_L = 10^{-2} \times 20\{-50 \times 0.01 \times e^{-50 \times 0.01} + e^{-50 \times 0.01}\}$$
$$= 0.2\{-0.5 + 1\}e^{-0.5} = 0.06 \text{ V}.$$

(ii) $$i = \frac{1}{L}\int_0^{0.2} v_L\, dt + 15 = \frac{1}{10 \times 10^{-3}}\int_0^{0.2} 10e^{-15t}\, dt + 15$$

or $$i = 10^3\left[\frac{e^{-15t}}{-15}\right]_0^{0.2} + 15$$

or $$i = -\frac{10^3}{15}[e^{-15 \times 0.2} - e^0] + 15$$

or $$i = \frac{10^3}{15} \times 0.95 + 15 = 78.35 \text{ A}.$$

(iii) Power delivered to the inductor
$$v_L \times i = L\frac{di}{dt} \times i$$
$$= 0.01 \times 10(1 - e^{-20t}) \times \frac{d}{dt}[10(1 - e^{-20t})]$$
$$= 0.1(1 - e^{-20t}) \times 10(20e^{-20t}) = 20e^{-20t}(1 - e^{-20t})$$

At t = 20 ms(= 0.02s),

we get the power delivered to the inductor as
$$v_L \times i = 20e^{-20 \times 0.02}(1 - e^{-20 \times 0.02}) = 0.67 \times 0.3297 \times 20 = 4.4 \text{ W}.$$

14.30 In the circuit shown in Fig. 14.19, find the instantaneous value of voltage drop across the resistor and the inductance. What is the amount of power dissipated in the resistor. What is the energy dissipated?

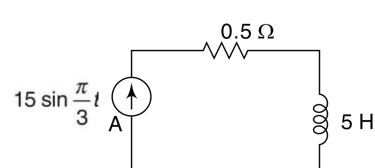


Fig. 14.19

Solution

Current $\quad i = \left(15 \sin \frac{\pi}{3} t\right) \text{A}$

$\therefore$ Voltage (instantaneous) across the resistor is $\left(0.5 \times 15 \sin \frac{\pi}{3} t\right) \text{V}$.

i.e. $\quad v_R = \left(7.5 \sin \frac{\pi}{3} t\right) \text{V}$.

Also, voltage across the inductor is given by

$$v_L = L \cdot \frac{di}{dt} = 5 \frac{d}{dt}\left(15 \sin \frac{\pi}{3} t\right) = 75 \cdot \frac{\pi}{3} \cos \frac{\pi}{3} t = \left(25\, \pi \cos \frac{\pi}{3} t\right) \text{V}.$$

Power across the resistor is

$$i^2 R = \left(225 \sin^2 \frac{\pi}{3} t\right) 0.5 = 112.5 \sin^2\left(\frac{\pi}{3} t\right) \text{W}.$$

The energy stored by the inductor is maximum when the current through it is maximum.

Current is maximum when $\left(\sin^2 \frac{\pi}{3} t\right) = 1$

i.e. $1 - \cos\left(\frac{2\pi}{3} t\right) = 2$ or, $\cos \frac{2\pi}{3} t = -1 = \cos \pi$

$\therefore \quad \frac{2\pi}{3} t = \pi$ or, $t = \frac{3}{2}$ s.

Hence energy stored in the inductor is maximum at t = 3/2s. In another 3/2s energy will be recovered from the inductor.

Hence in $\left(\frac{3}{2} + \frac{3}{2} = 3\,\text{s}\right)$ energy dissipated in the resistor is $\int_0^3 \left(112.5 \sin^2 \frac{\pi}{3} t\right) dt$

$$= \frac{112.5}{2} \int_0^3 \left(1 - \cos \frac{2\pi}{3} t\right) dt$$

$$= \frac{112.5}{2}\left[t - \frac{\sin \frac{2\pi}{3} t}{\frac{2\pi}{3}}\right]_0^3 = \frac{112.5}{2}\left[3 - \frac{\sin 2\pi}{\frac{2\pi}{3}}\right] = 168.75 \text{ J.}$$

14.31 Two coils having self-inductances of 0.3 H and 0.5 H are connected in series across a 230 V, 50 Hz supply. What current will flow if the coupling co-efficient of the coils is 0.45?

Solution

Mutual inductance $M = \sqrt{L_1 L_2} = 0.45 \sqrt{0.3 \times 0.5} = 0.1743$

When connected in series the equivalent impedance is given by

$$L = L_1 + L_2 \pm 2M = 0.3 + 0.5 \pm 2 \times 0.1743 = 1.1486 \text{ H or } 0.4514.$$

Hence $\quad X_L = 100\pi \times 1.1486 = 360.84\ \Omega$

or $\quad X_L = 100\pi \times 0.4514 = 141.8\ \Omega$

Current is, $\frac{230}{360.84}$ A $= 0.6374$ A

or $\frac{230}{141.8}$ A $= 1.622$ A.

14.32 Two coils are connected in series with same polarities and the combined inductance is found to be 0.567 H. When the coils are connected in series with reverse polarities then the combined inductance is 0.267 H. The self-inductance of one coil is 0.3 H. Determine the mutual inductance and the coupling coefficient.

Solution

Let L_1 and L_2 be the self-inductances of the two coils and M the mutual inductance. Then

$$L_1 + L_2 + M = 0.567$$

and $$L_1 + L_2 - M = 0.267$$

Hence $$L_1 + L_2 = 0.417$$

But $$L_1 = 0.3 \text{ H},$$

$\therefore$ $$L_2 = 0.417 - 0.3 = 0.117 \text{ H}$$

and $$M = 0.417 - 0.267 = 0.15 \text{ H}$$

We know, $M = K\sqrt{L_1 L_2}$, where K is the coupling co-efficient

Hence $$K = \frac{0.15}{\sqrt{0.3 \times 0.117}} = 0.8.$$

14.33 Write three mesh equations for the circuit shown in Fig. 14.20.

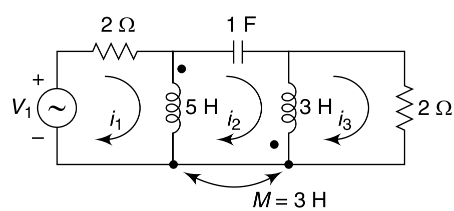


Fig. 14.20

Solution

The mutual inductance and the self inductances are replaced by their impedances and the corresponding circuit is shown in Fig. 14.21.

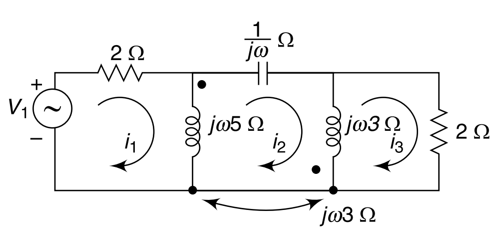


Fig. 14.21

Applying KVL in the first mesh (leftmost mesh),

$$2i_1 + j\omega 5(i_1 - i_2) + 3j\omega(i_3 - i_2) = V_1$$

or $$(2 + 5j\omega)i_1 - 8j\omega i_2 + 3j\omega i_3 = V_1 \quad \text{(i)}$$

Applying KVL in the second mesh (middle mesh),

$$5j\omega(i_2 - i_1) + 3j\omega(i_2 - i_3) + \frac{1}{j\omega}i_2 + 3j\omega(i_2 - i_3) + 3j\omega(i_2 - i_1) = 0$$

or $$-8j\omega i_1 + \left(14\,j\omega + \frac{1}{j\omega}\right) i_2 - 6j\omega i_3 = 0 \qquad \text{(ii)}$$

Applying KVL in the third mesh (rightmost mesh),

$$3j\omega(i_3 - i_2) + 3j\omega(i_1 - i_2) + 2i_3 = 0$$

or $$3j\omega i_1 - 6j\omega i_2 + (2 + 3j\omega)i_3 = 0. \qquad \text{(iii)}$$

14.34 When a coil of 1200 turns is linked with a flux of 4m wb, a certain value of current flows through the circuit. When the circuit gets opened, the flux falls to its residual value of 1.5 m wb in 40 m secs. Find the average value of the induced emf.

Solution

Average emf in volts = Rate of change of flux linkages.

Change of flux = $(4 - 1.5) \times 10^{-3} = 2.5 \times 10^{-3}$ wb. (= $d\phi$)

Time for the change (dt) = 0.04 secs (given)

$\therefore$ Rate of change of flux linkage $\left(N\frac{d\phi}{dt}\right) = 1200 \times \dfrac{2.5 \times 10^{-3}}{4 \times 10^{-2}} = 75$ V.

14.35 A one-turn coil of axial length 0.4 m and a diameter of 0.2 m rotates at a speed of 500 rpm in a uniform flux density of 1.2 T. Calculate the induced emf.

Solution

Diameter of the armature = 0.2 m.

Circumference (= $2\pi r$) = $\pi d = \pi \times 10^{-1} \times 2 = 0.628$ m

In one second the armature turns (500/60) revolutions.

$\therefore$ In one second a coil side travels $\dfrac{500}{60} \times 0.628$ m.

i.e. $v = 5.233$ m/s.

$\therefore$ induced emf (E) = $Blv = 1.2 \times (2 \times 0.4) \times 5.233$ V

= 5.024 V, in the entire coil having 2 turns.

14.36 A straight horizontal conductor carries a current of 150 A at right angles to a uniform magnetic field of 0.6 Tesla. Find the force developed per meter length and the direction in which it acts.

Solution

$$F = (BIl)\text{ N} = 0.6 \times 150 \times 1 = 90 \text{ N/m}.$$

[Assuming the current flowing away from the observer, the force acts from right to left to move the conductor horizontally].

14.37 An armature conductor has an effective length of 400 m and carries a current of 25 A. The flux/pole is 0.5 Tesla. Determine the force in Newtons exerted on the armature conductor.

Solution

$$F = BIl\text{ N} = 0.5 \times 25 \times 400 \times 10^{-3} = 5 \text{ N}.$$

14.38 An iron ring has an air-gap of 1.5 mm, 250 turns and a length of 100 cm. Determine the flux density when a current of 2A flows through the coil. Assume permeability of iron to be 300. Assume = $\mu_o = 4\pi \times 10^{-7}$ H/m.

Solution

The reluctance of the iron path and the air gap

$$S = \left(\frac{1}{A\mu_o\ 300} + \frac{1.5\times10^{-3}}{A\mu_o}\right) AT/\text{Wb} = \frac{0.0048}{A\mu_o}\ \text{AT/Wb.}$$

$$\text{Flux } (\phi) = \frac{AT}{S} = \frac{NI}{S} = \left(\frac{250\times 2}{0.0048}\times A\,\mu_o\right)\text{Wb}$$

$$\therefore \text{ Flux density } (B) = \frac{\phi}{A} = \frac{250\times 2}{0.0048} \times \mu_o = 0.131\ \text{Wb/m}^2.$$

14.39 Determine the flux density at the surface and 15 cm away from the center of the long straight conductor of 0.4 cm diameter carrying dc current of 150 A.

Solution

At the surface,

$$\text{Radius } r = \frac{0.4}{2}\text{cm} = 0.2 \times 10^{-2}\ \text{m}$$

$$\text{Flux density } B = \frac{\mu_o I}{2\pi r} = \frac{4\pi\times10^{-7}\times150}{2\pi\times0.2\times10^{-2}} = \frac{300}{0.2}\times 10^{-5} = 15\ \text{m Wb/m}^2$$

At 15 cm away from the center,

$$r = 0.15\ \text{m.}$$

$$\therefore \text{ flux density } (B) = 15 \times \frac{0.2\times10^{-2}}{0.15}\ \text{m wb/m}^2 = 0.2\ \text{m Wb/m}^2.$$

14.40 A magnetic ring has a mean circumference of 20 cm and a cross-section of 20 cm^2 and has 800 number of turns of wire. When the exciting current is 5 A, the flux is 2 m Wb. Determine the relative permeability of iron.

Solution

Flux density $\quad B = \mu\text{H} = \mu_o\mu_r\text{H}$ and $\text{H} = \dfrac{NI}{l}$

Now, $\quad B = \dfrac{\phi}{A} = \dfrac{2\times10^{-3}}{20\times10^{-4}} = 4\pi\times10^{-7}\,\mu_r\times\dfrac{800\times5}{0.2}$

or, $\quad \mu_r = \dfrac{0.2\times10^{7}}{4\pi\times800\times5} = 39.81.$

Hence, relative permeability of the ring is 39.81.

14.41 An iron core having a mean circumference of 1.25 m and a cross-sectional area of 1500 mm^2, is wound with 400 turns of wire. An exciting current of 2.5 A produces a flux of 0.75 m Wb in the iron ring. Determine (i) the permeability (relative) of the iron (ii) the reluctance of the iron (iii) the mmf of the exciting winding.

Solution

(i) H = Ampere-turns per meter $\left(=\dfrac{F}{l}\right) = \dfrac{400\times2.5}{1.25}$

Thus $\quad H = \dfrac{1000}{1.25} = 800$ AT/m.

Also, $\quad B = \dfrac{\phi}{A} = \dfrac{0.00075}{1500 \times 10^{-6}} = 0.5$ T

Again, $\quad B = \mu$H $\quad$ or, $\mu = \dfrac{B}{H} = \dfrac{0.5 \times 1.25}{1000} = \dfrac{0.625}{1000}$

Also $\quad \mu = \mu_r \cdot \mu_o \quad \therefore \mu_r = \dfrac{\mu}{\mu_o} = \dfrac{0.625}{1000 \times 4\pi \times 10^{-7}} = \dfrac{6250}{4\pi}$.

Thus relative permeability of the iron sample = 497.5.

(ii) Reluctance of iron $S\left(= \dfrac{l}{\mu A}\right) = \dfrac{1.25 \times 10^3}{0.625 \times 1500 \times 10^{-6}} = 1.33 \times 10^6$ AT/Wb

(iii) Since (Hl) is the m.m.f is we have the mmf in the given problem as (800 × 1.25) = 1000 AT.

14.42 An iron core has a cross-section of 500 mm^2 and having a length of 100 cm. A magnetizing force of 500 AT in it produces a magnetic flux of 400 μ wb. Determine (i) the relative permeability of the material and (ii) the reluctance of the magnetic circuit. $\mu_o = 4\pi \times 10^{-7}$ H/m.

Solution

(i) $\quad B = \dfrac{\phi}{A}$; Here $B = \dfrac{400 \times 10^{-6}}{500 \times 10^{-6}} = 0.8$T.

Also, as H is given by the ratio of total mmf and length hence,

$$H = \frac{F}{l} = \frac{500}{1} = 500 \text{ AT/m.}$$

Also, since $B(= \mu\text{H}) = \mu_o \cdot \mu_r \cdot H$, we have

$$\mu_r = \frac{B}{\mu_o H} = \frac{0.8}{4 \times \pi \times 10^{-7} \times 500} = 1275.$$

(ii) Reluctance $= \dfrac{\text{Length}}{\mu \times \text{Area}} = \dfrac{l}{\mu_o \mu_r \times A}$

$$= \frac{1}{4 \times \pi \times 10^{-7} \times 1275 \times 500 \times 10^{-6}} = 1.25 \times 10^6 \text{ AT/Wb.}$$

14.43 An air-gap of 3 mm thickness and area of 650 mm^2 is made by a cut in the iron of a magnetic circuit. If a flux of 0.05 Wb is required in the air-gap, find the ampere-turns required for the air-gap to produce the necessary flux. Assume $\mu_o = 4\pi \times 10^{-7}$ H/m.

Solution

Flux density in the air-gap is $B = \dfrac{0.05}{650 \times 10^{-6}} = \dfrac{5 \times 10^2}{6.5}$ T

Also we know $B = \mu_o \times H$

$$\therefore \quad H\left(= \frac{B}{\mu_o}\right) = \frac{5 \times 10^2}{6.5 \times 4 \times \pi \times 10^{-7}} = 6.12 \times 10^7 \text{ AT/m}$$

Air-gap (= 3 mm) = 3×10^{-3} m.

$\therefore$ Required-ampere turns = $6.12 \times 10^7 \times 3 \times 10^{-3}$ = 183600 AT.

14.44 Two long parallel copper bars are placed vertically 0.8 m apart (being supported on insulators); each bar is carrying a current of 2000 A. What is the force/meter acting on the bars due to this?

Solution

Magnetising force H of a long straight conductor

$$= \frac{I}{2\pi r} = \frac{2000}{2 \times \pi \times 0.8} = \frac{1000}{\pi \times 0.8} \text{ AT/m}$$

$$\therefore \quad B(= \mu_o \text{ H}) = \frac{4 \times \pi \times 1000 \times 10^{-7}}{\pi \times 0.8} = \frac{10^{-3}}{2} \text{ T} \qquad [\because \mu_o = 4\pi \times 10^{-7} \text{ H/m}]$$

$$\therefore \quad F(= BIl) = \frac{10^{-3}}{2} \times 2000 \times 1 = 1 \text{ Nw/meter length.}$$

14.45 A magnetic circuit consists of two cores and two yoke without an air gap. Each core is cylindrical (5 cm diameter and 16 cm long) and each yoke is of square cross-section (47 × 47 mm), and is 18 cm long. Determine the AT necessary to get a flux density of 1.2 Wb/m^2 in the cores. The magnetic characteristics of the material are:

B(T)	0.9	1.0	1.05	1.1	1.15	1.2
H(AT/m)	200	260	310	380	470	650

Neglect magnetic leakage.

Solution

Since the flux density B value in the cores is to be 1.2T, then the AT/m required will be 650. The total magnetomotive force for the cores will be, $2 \times 160 \times 10^{-3} \times 650 = 208$ AT.

In the yokes, the flux is the same as that for the cores but the flux density will be different (as the areas are different).

Thus flux, $\phi = 1.2 \times \frac{\pi}{4} \times 50^2 \times 10^{-6}$ Wb.

$$(B) \text{ in yokes} = \left(\frac{\phi}{A}\right)_{\text{yoke}} = 1.2 \times \frac{\pi}{4} \times \frac{25 \times 10^{-4}}{47 \times 47 \times 10^{-6}} = 1.066 \text{ T.}$$

14.46 A dc machine has an useful flux of 0.05 Wb/pole. The effective area of air gap is 60,000 mm^2, and mean length of air gap is 5 mm. Effective area of pole is 40000 mm^2, mean length of pole 250 mm, effective area of teeth is 25000 mm, mean length of teeth is 45 mm.

Determine the ATs per field coil required for the air-gap, the armature teeth and the pole of the d.c. machine.

Given: Magnetic leakage co-efficient = 1.2; Magnetic characteristics of the materials are:

B(T)	1.3	1.4	1.5	1.6	1.8	2.0
H(AT/m):	2000	1500	2000	3000	8500	24000

Solution

Air-gap: Useful flux = 0.05 wb/pole

$$\text{Flux density in air gap } \left(B = \frac{\phi}{A}\right) = \frac{0.05}{60000 \times 10^{-6}} = 0.833 \text{ T}$$

We know $B = \mu_o H$.

$$\therefore H \text{ value (for air)} = \frac{B}{\mu_o} = \frac{0.833}{4\pi \times 10^{-7}} = 66.2 \times 10^4 \text{ AT/m.}$$

MMF for air gap = $66.2 \times 10^4 \times 5 \times 10^{-3}$ = 3310 AT.
Pole: Total flux = 0.05×1.2 = 0.06 Wb.

$$\text{Flux density in pole} = \frac{0.06}{40000 \times 10^{-6}} \qquad \left[\because B = \frac{\phi}{A}\right]$$

or $\qquad B = 1.5$ T.

From the given magnetic characteristics, a flux density (B) of 1.5 T gives an H value of 2000 AT/m.

Thus mmf required for pole = $2000 \times 250 \times 10^{-3}$ = 500 AT.
Teeth: Total flux = 0.05 Wb (same as the air-gap).

$$\text{Flux density in teeth} = \frac{0.05}{25000 \times 10^{-6}} = 2\text{T}.$$

From the given characteristic, a flux density of $2T$ gives an H value of 24000 AT/m.
Thus mmf required for teeth = $24000 \times 45 \times 10^{-3}$ = 1080 AT.
$\therefore$ Total field-coil mmf = 3310 + 500 + 1080 = 4890 AT.

14.47 Two co-axial magnetic poles, each of 10 cm. in diameter, are separated by an air gap of 2.5 mm and the flux crossing the air gap is 0.004 Wb. Determine.
(i) the energy stored in the air gap in joules
(ii) the pull force in Newtons between the poles (Ignore fringing).

Solution

(i) Area of air-gap = $\dfrac{\pi \times 100^2}{4} \times 10^{-6} = \dfrac{3.14 \times 10^{-2}}{4}$ sq.m.

Volume of air-gap = $\dfrac{3.14 \times 10^{-2}}{4} \times 2.5 \times 10^{-3} = \dfrac{3.14 \times 10^{-4}}{16}$ cubic meter.

Flux density in gap = $\dfrac{0.004 \times 4}{\pi \times 100^2 \times 10^{-6}} = \dfrac{1.6}{\pi}$ Tesla = 0.508 T.

(i) Energy stored in joules = $\dfrac{B^2}{2\mu_o} \times$ volume

$$= \frac{0.508^2}{2 \times 4\pi \times 10^{-7}} \times \frac{3.14}{16} \times 10^{-4} = 2 \text{ Joules}$$

(ii) The pull force (Newtons) = $\dfrac{B^2 A}{2\mu_o} = \dfrac{0.508^2 \times 3.14 \times 10^{-2}}{2 \times 4\pi \times 10^{-7} \times 4}$ = 806 N.

14.48 Two non-identical coils P and Q of 1000 and 500 turns are magnetically coupled. A current of 2 A is flowing in coil P and produces a flux of 18 m wb, of which 80% is linked with coil Q. If the current of 2 A is reversed uniformly in 0.1 sec, calculate the average emf in each coil.

Solution

Coil P is linked with flux (ϕ) = 18×10^{-3} Wb

Associated flux linkages during reversal (= turns × flux) decrease to zero and then it builds up to full value ϕ in reversed direction. Thus the rate of charge of flux is $N\left(\dfrac{d\phi}{dt}\right)$ = 1000[0.018 – (–0.018)] = 36 Wb turns.

Time of reversal = 0.1 sec (given)

and induced emf = rate of charge of flux linkages = $\dfrac{36}{0.1}$ = 360 V (average value)

Only 80% flux is associated with coil Q and turns are 500.

$\therefore$ Associated flux-linkages during reversal = 500 × 0.8 × 2 × 0.018 = 14.4 Wb-turns

Induced emf = $\dfrac{14.4}{0.1}$ = 144 V (average value).

14.49 A solenoid 1.5 m long is wound uniformly with 400 turns and a small 50 turns coils of 10 mm diameter is placed inside and at the center of the solenoid. The axes of the solenoid and the coil are same.

Determine (i) the flux linked with the small coil when the solenoid carries a current of 6 amps. (ii) the average emf induced in the small coil when the current in the solenoid is reduced from 6 A to zero in 50 ms.

Solution

(i) Ampere-turns of solenoid = 400 × 6 = 2400 AT

The magnetizing force H at the center = ampere-turns/meter = $\dfrac{2400}{1.5}$ = 1600 AT/m

The flux density B at the centers of the solenoid and small coil
$(= \mu_o H) = 4 \times \pi \times 10^{-7} \times 1600 = 64\pi \times 10^{-5}$ T

Area of small coil $\left(= \dfrac{\pi d^2}{4}\right) = \dfrac{\pi \times (10 \times 10^{-3})^2}{4} = \dfrac{\pi \times 10^{-4}}{4}$ sq. m

So flux linked = $64\pi \times 10^{-5} \times \dfrac{\pi \times 10^{-4}}{4}$ Wb = 0.158 μWb.

(ii) Average induced emf (= rate of change of flux-linkages)

$$e = N\frac{d\phi}{dt} = 50\frac{0.158 - 0}{50 \times 10^{-3}} = 0.158 \text{ mV}$$

14.50 Determine the difference of potential between the points A and B in the network shown in Fig. 14.22

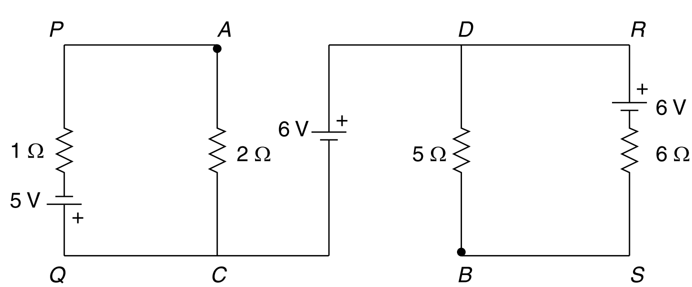


Fig. 14.22

Solution

The current flowing in loop $ACQPA$ is $I_1 \left(= \dfrac{5}{2+1}\right) = \dfrac{5}{3}$ A , in the anticlockwise direction.

The current flowing in loop $DRSBD$ $I_2 \left(= \dfrac{6}{6+5}\right) = \dfrac{6}{11}$ A in the anticlockwise direction.

Considering the path *ACDB*

Potential difference between points *A* and *B* is

$$v_{ab} = -2 \times \frac{5}{3} - 6 + 5 \times \frac{6}{11} = -\frac{10}{3} - 6 + \frac{30}{11} = -6.6 \text{ V}.$$

Hence *B* is at higher potential and potential between points *A* and *B* is 6.6 V.

14.51 A battery consists of 15 cells each having an electromotive force of 3V and internal resistance of 0.015 Ω. Determine the power developed in an external circuit of resistance 0.3 Ω if the cells are connected in (i) series (ii) parallel.

Solution

(i) When the cells are connected in series

Total emf = 3 × 15 = 45 V.

Total internal resistance of all the cells = 0.015 × 15 = 0.225 Ω.

The circuit current = $\dfrac{45}{0.225 + 0.3}$ A = 85.71 A.

Power developed across 0.3 Ω resistance is $I^2R = (85.71)^2 \times 0.3 = 2203.86$ W.

(ii) When the cells are connected in parallel

Total emf = 3 V

Total internal resistance of all the cells = $\dfrac{0.015}{15}$ = 0.001 Ω.

The circuit current = $\dfrac{3}{0.001 + 0.3}$ = 9.967 A

Power developed across 0.3 Ω resistance is $I^2R = (9.967)^2 \times 0.3 = 29.8$ W.

14.52 Determine the currents I_{cd}, I_{ef} and I_{ij} in Fig. 14.23, if $I_{fj} = 5$ A, $I_{de} = 3$ A, $I_{be} = 0$ A and $I_{hd} = -8$ A.

Solution

Applying KCL at node *C*, we get

$I_{cd} + 10 - 4 = 0$ or, $I_{cd} = -6$ A

Applying KCL at node *f* we have

$6 - I_{ef} + I_{fj} = 0$

or $I_{ef} = 6 + 5 = 11$ A $[\because I_{fj} = 5 \text{ A}]$

Applying KCL at node *e*, we can write

$-I_{de} + I_{ei} + I_{ef} = 0$ $[\because I_{be} = 0 \text{ A}]$

or $I_{ei} = 3 - 11 = -8$ A.

Applying KCL at node *j* we get

$I_{ij} + 5 - 15 = 0$

$\therefore$ $I_{ij} = 10$ A

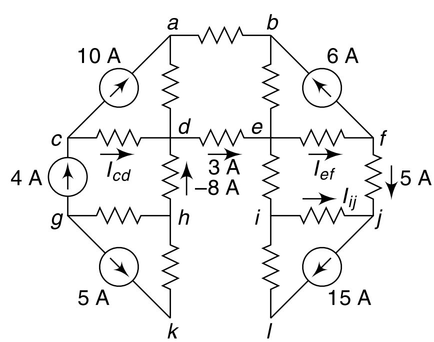


Fig. 14.23

14.53 A voltage of 400 V is applied to a variable rheostat of 800 Ω. A load is connected as shown in Fig. 14.24. The load voltage is 50 V while the load current is 0.5 A. Determine the resistance between *A* and *B* and the total power consumed.

Solution

Let us assume that the resistance between *A* and *B* is *R*. Hence resistance between *A* and *C* is (800 – *R*).

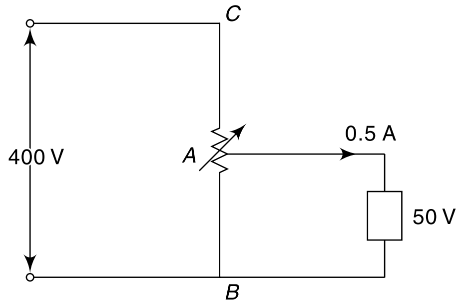


Fig. 14.24

Current through AC is

$$0.5 + \frac{50}{R} = \frac{400-50}{800-R}$$

or $(0.5R + 50)(800 - R) = 350\ R$

or $400\ R - 50\ R - 0.5R^2 + 40{,}000 = 350\ R$

or $0.5R^2 = 40{,}000$

or $R^2 = 80{,}000$ or, $R = 282.84\ \Omega$.

Hence, resistance between A and B is 282.84 Ω.

$$\text{Total power consumed} = 400 \times \frac{400-50}{800-282.84} = 270.7\ \text{W}.$$

14.54 Using mesh analysis technique determine the voltage across 4 Ω resistor in the circuit shown in Fig. 14.25.

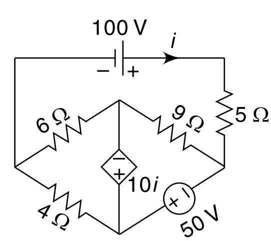


Fig. 14.25

Solution

Let us consider two mesh currents i_1 and i_2 as shown in Fig. 14.26.

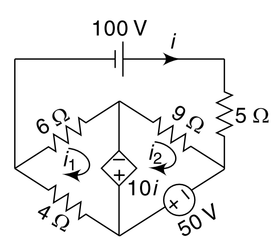


Fig. 14.26

The three mesh equations are as follows

$$-100 + 5i + 9(i - i_2) + 6(i - i_1) = 0$$

$$9(i_2 - i_1) - 50 + 10i = 0$$

and $6(i_1 - i) - 10i + 4i_1 = 0$

i.e., the equations are, $20i - 6i_1 - 9i_2 = 100$ (i)

$10i - 9i_1 + 9i_2 = 50$ (ii)

and $16i - 10i_1 = 0$ (iii)

Adding equations (i) and (ii)

$$30i - 15i_1 = 150$$

or $i_1 = 2i - 10$ (iv)

Substituting the value of i_1 from equation (iv) in equation (iii) we get

$$16i - 10(2i - 10) = 0$$

or $i = 25$ A.

∴ From equation (iii) we get

$$i_1 = \frac{16}{10}i = \frac{16}{10} \times 25 = 40\ \text{A}.$$

Hence voltage across 4 Ω resistor is $(4 \times 40) = 160$ V.

14.55 Determine the mesh currents in the circuit shown in Fig. 14.27

Solution

From Fig. 14.27 it is clear that $i_1 = 25$ A. Considering mesh 1 and 2 we get,

$$i_2 - i_1 = 0.5\ v$$

or $$i_2 = 25 + 0.5\ v \quad (1)$$

Considering mesh 3 we have

$$2(i_3 - i_1) + 2(i_3 - i_2) + 4i_3 = 0 \quad [\because i_1 = 25 \text{ A}]$$

or $$8i_3 - 2i_2 = 50 \quad \text{or,} \quad i_2 = 4i_3 - 25 \quad \text{(ii)}$$

Now $$v = 2(i_2 - i_3)$$

or $$i_2 = i_3 + \frac{v}{2} \quad \text{(iii)}$$

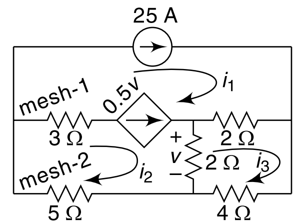


Fig. 14.27

Subtracting Eq. (iii) from Eq. (i)

$$25 - i_3 = 0$$

or $$i_3 = 25.$$

From Eq. (ii) we get

$$i_2 = 4 \times 25 - 25 = 75.$$

Hence, $i_1 = 25$ A, $i_2 = 75$ A and $i_3 = 25$ A.

14.56 Determine the values of i_1, i_2, i_3 and i_4 for the circuit shown in Fig. 14.28.

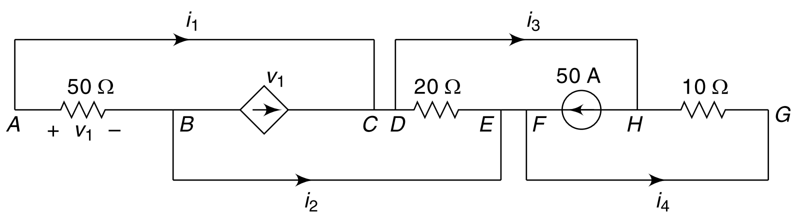


Fig. 14.28

Solution

The circuit shown in Fig. 14.28 has been redrawn as shown in Fig. 14.29(a).

Applying KCL at node *B*,

$$-\frac{v_1}{50} + v_1 + i_2 = 0$$

or $$i_2 = -\frac{49}{50}v_1 \quad \text{(i)}$$

Applying KCL at node *F*,

$$-50 + i_4 - i_2 - \frac{v_1}{20} = 0$$

or $$-50 + i_4 + \frac{49}{50}v_1 - \frac{v_1}{20} = 0$$

or $$-50 + i_4 + \frac{93}{100}v_1 = 0 \quad \text{(ii)}$$

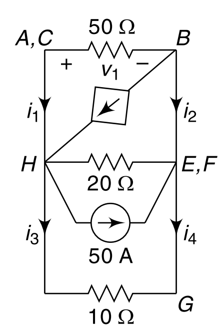


Fig. 14.29(a)

Now $$i_1 = -\frac{v_1}{50},$$

and $$i_4 = -\frac{v_1}{10}.$$

Hence from the Eq. (ii) we have

$$-50 - \frac{v_1}{10} + \frac{93}{100}v_1 = 0$$

or $\quad 0.93v_1 - 0.1v_1 = 50$

or $\quad v_1 = 60.241$ V.

$\therefore \quad i_1 = -\dfrac{60.241}{50} = -1.2$ A

$$i_2 = -\frac{49}{50}(60.241) = -59.036 \text{ A}$$

$$i_4 = -6.024 \text{ A}$$

Next, applying KCL node H we have

$$-i_1 - v_1 + \frac{v_1}{20} + 50 + i_3 = 0$$

or $\quad 1.2 - 60.241 + \dfrac{60.241}{20} + 50 + i_3 = 0$

or $\quad 1.2 - 60.241 + 3.012 + 50 + i_3 = 0$

$\therefore \quad i_3 = 6.029$ A.

14.57 Calculate the value of v and the power supplied by the independent source of Fig. 14.29(b).

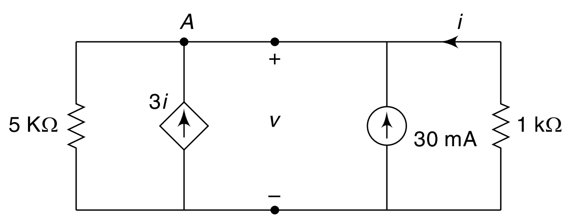


Fig. 14.29(b)

Solution

Applying KCL at node A we get

$$\frac{v}{5000} - 3i - 0.03 - i = 0$$

Now, $\quad i = -\dfrac{v}{1000}$.

Hence, $\quad \dfrac{v}{5000} - 4\left(-\dfrac{v}{1000}\right) - 0.03 = 0$

or $\quad \dfrac{v}{5000} + \dfrac{4v}{1000} - 0.03 = 0$

or $\quad v + 20v = 0.03 \times 5000 = 150$

or $\quad v = \dfrac{150}{21} = 7.143$ V

The independent source is a 30 mA current source. The power supplied by this source is $v \times 0.03$ (= 7.143×0.03) = 0.2143 W.

14.58 In the circuit shown in Fig. 14.30 determine the power absorbed by each element of the circuit.

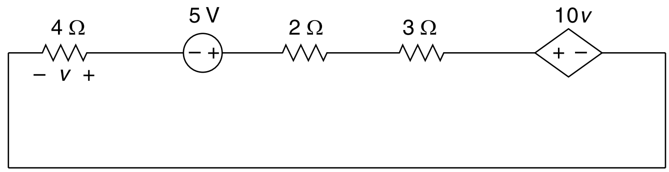


Fig. 14.30

Solution

Let us assume the current i is flowing in the loop in a clockwise direction.

Hence, $4i = -v$

or $v = -4i$.

Applying KVL we have

$$-v - 5 + 2i + 3i + 10v = 0$$

or $-5 - 31i = 0$ $[\because v = -4i]$

$\therefore$ $i = -\dfrac{5}{31}$ A .

$\therefore$ Power absorbed by 5 V source is $(-5i)$ i.e., $-5\left(-\dfrac{5}{31}\right) = 0.806$ W.

Power absorbed by 10 V source is $(10vi) = 10(-4i)i = -40i^2 = -40\left(-\dfrac{5}{31}\right)^2 = -1.04$ W.

i.e., Power delivered by this source is 1.04 W.

Power absorbed by 4 Ω source is $= 4 \times \left(-\dfrac{5}{31}\right)^2 = 0.104$ W

Power absorbed by 2Ω source is $= 2 \times \left(-\dfrac{5}{31}\right)^2 = 0.052$ W

Power absorbed by 3Ω source is $= 3\left(-\dfrac{5}{31}\right)^2 = 0.078$ W.

[It may be observed that power is supplied by the dependent source and it is 1.04 W while the power is absorbed by the voltage source and the resistors. Check that power delivered is same as power absorbed.]

14.59 Calculate the power delivered by the sources and power dissipated by the resistors in the network shown in Fig. 14.31.

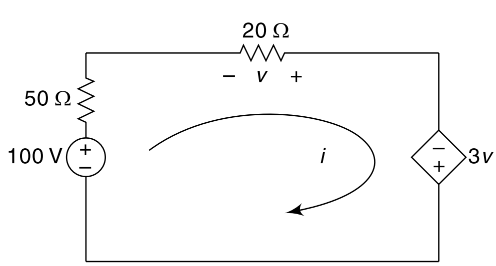


Fig. 14.31

Solution

Let as assume that current i is flowing through the loop. Applying KVL in the loop,

$$-100 + 50i + 20i - 3v = 0$$

i.e. $100 - 70i + 3v = 0$ (i)

Also, $(-20i) = v$ (ii)

Using Eq. (ii) in (i) we get

$$100 - 70i + 3(-20i) = 0$$

or $$i = \frac{100}{130}\text{ A}$$

$\therefore$ $$v = -20\left(\frac{100}{130}\right) = -\frac{200}{13}\text{ V.}$$

Now, power delivered by (100 V) source is $\left(100 \times \dfrac{100}{130}\right)$ W $= 76.92$ W

Power delivered by the dependent source is $(3vi)$ i.e., $3\left(-\dfrac{200}{13}\right)\dfrac{100}{130} = -35.5$ W.

Power dissipated by 50 Ω resistor is $\left[50 \times \left(\dfrac{100}{130}\right)^2\right]$ i.e., 29.58 W.

Power dissipated by 20 Ω resistor is $\left[20 \times \left(\dfrac{100}{130}\right)^2\right]$ i.e., 11.83 W.

14.60 In the circuit shown in Fig. 14.32 determine v_1 and v_2.

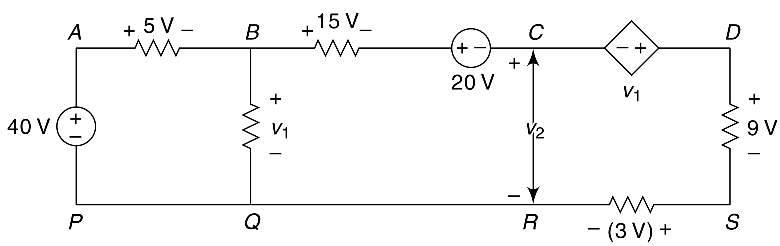


Fig. 14.32

Solution

Applying KVL in loop *ABQPA*

$$+5 + v_1 - 40 = 0$$

or $$v_1 = 35\text{ V.}$$

Applying KVL in loop *ABCRQPA*

$$+5 + 15 + 20 + v_2 - 40 = 0$$

or $$v_2 = 40 - 40 = 0\text{ V.}$$

14.61 Determine the voltage across a 500 Ω resistor using source transformation in the circuit shown in Fig. 14.33.

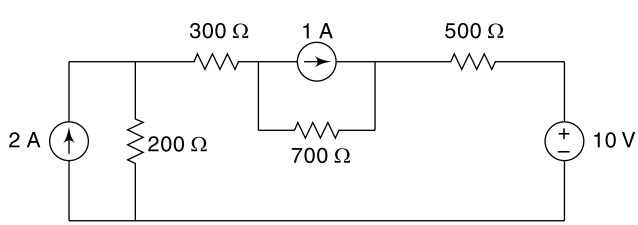


Fig. 14.33

Solution

Transforming 2A current source and 1A current source into voltage sources a new circuit is obtained (Fig. 14.34).

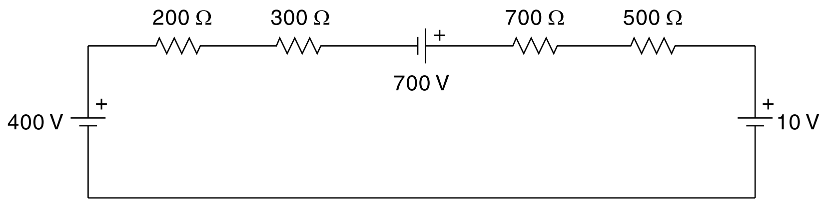


Fig. 14.34

Now, current through 500 Ω resistor is

$$\frac{400 + 700 - 10}{200 + 300 + 700 + 500} = \frac{1090}{1700}\text{ A } = 0.641176\text{ A.}$$

The voltage across the 500 Ω resistor is (500 × 0.641176) = 320.59 V

14.62 By using source transformation technique find the current through the 8 Ω resistor in Fig. 14.35.

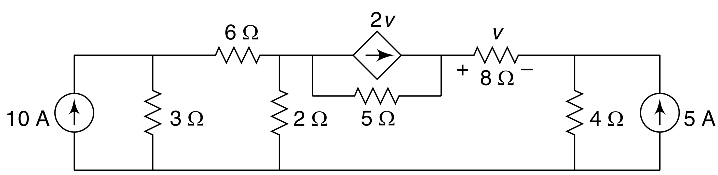


Fig. 14.35

Solution

As 3 Ω is in parallel with 10 A source and 4 Ω is parallel with 5 A source, the current sources can be transformed into voltage sources as shown in Fig. 14.35(a).

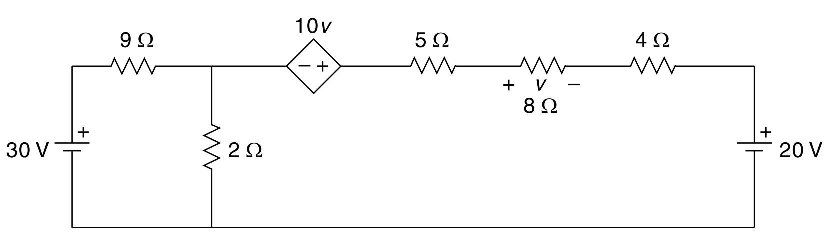


Fig. 14.35(a)

Transforming 30 V source into current source the circuit shown in Fig. 14.36 is obtained.

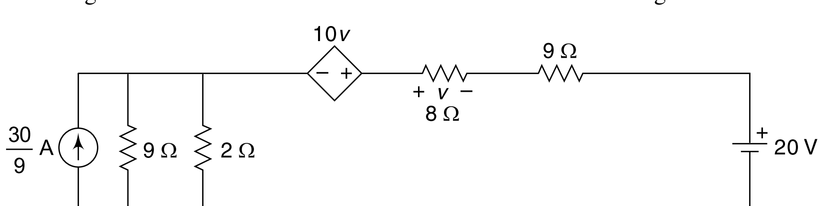


Fig. 14.36

Now, 9 Ω and 2 Ω are in parallel. Their combined resistance is $\left(\dfrac{9 \times 2}{9+2}\right)$ or, $\dfrac{18}{11}$ Ω.

Transforming the current source into voltage source the circuit in Fig. 14.36(a) is obtained.

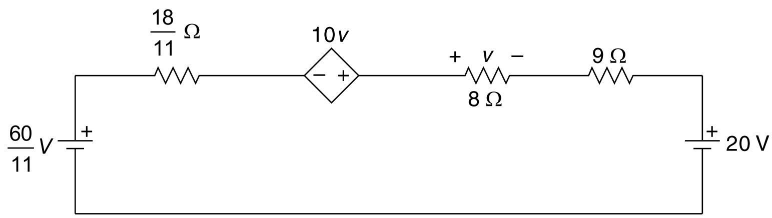


Fig. 14.36(a)

If current (i) flows through 8Ω resistor then applying KVL

$$-\frac{60}{11} + \frac{18}{11}i - 10v + v + 9i + 20 = 0$$

or $$9v - \frac{117}{11}i = 20 - \frac{60}{11} \qquad \text{(i)}$$

However from the circuit it is clear that $v = 8i$

Hence from Eq. (i) we have

$$9 \times 8i - \frac{117}{11}i = 20 - \frac{60}{11}$$

or $$\frac{792 - 117}{11}i = \frac{220 - 60}{11}$$

or, $$i = 0.237 \text{ A.}$$

14.63 In the circuit shown in Fig. 14.37(a) determine

(i) the power delivered in R_L, when (R_L) = 500 Ω

(ii) the maximum power that can be delivered to (R_L) and value of (R_L) for maximum power transfer

(iii) the possible values of R_L so that power across R_L is 0.5 W.

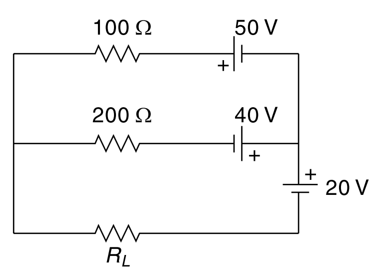


Fig. 14.37(a)

Solution

Let us first find the Thevenin's equivalent circuit. Removing the load resistance R_L the corresponding circuit is shown in Fig. 14.37(b).

The mesh current $i = \dfrac{50+40}{100+200} = \dfrac{9}{20}$ A.

Hence Thevenin's voltage is

$$V_{\text{Th}} = 20 - 40 + 200 \times \frac{9}{30} = 40 \text{ V.}$$

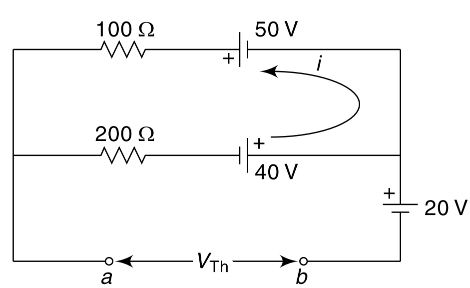


Fig. 14.37(b)

Removing the sources, Thevenin's equivalent resistance (R_{Th}) is

$$(R_{Th}) = \frac{200 \times 100}{200 + 100} = \frac{200}{3}\ \Omega.$$

Hence the original circuit can be redrawn as shown in Fig. 14.37(c).

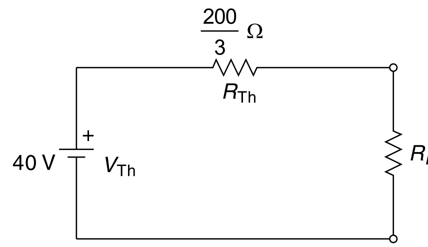


Fig. 14.37(c)

(i) When $R_L = 500\ \Omega$

Current through (R_L) is,

$$I_L = \frac{40}{\frac{200}{3} + 500} = 0.07\ \text{A}.$$

Hence power delivered to R_L is $(0.07)^2 \times 500 = 2.49$ W.

(ii) For maximum power across R_L the value of R_L should be equal to the internal resistance of the network.

Hence $R_L = \frac{200}{3}\ \Omega$

Current through R_L is, $I_L = \dfrac{40}{\frac{200}{3} + \frac{200}{3}} = 0.3$ A.

Hence maximum power that can be delivered to (R_L) is $\left[(0.3)^2 \times \frac{200}{3}\right] = 6$ W.

(iii) Power through (R_L) is

$$P_L (= I_L^2 R_L) = 0.5\ \text{W}$$

or $$\left(\frac{40}{\frac{200}{3} + R_L}\right)^2 R_L = 0.5$$

or $$\frac{1600\ R_L}{R_L^2 + \frac{400}{3} R_L + \frac{40000}{9}} = 0.5$$

or $$0.5 R_L^2 + \frac{200}{3} R_L + \frac{20000}{9} = 1600\ R_L$$

or $$0.5 R_L^2 - 1533.33 R_L + 2222.22 = 0$$

or $$R_L^2 - 3066.66\ R_L + 4444.44 = 0$$

or $$R_L = \frac{3066.66 \pm \sqrt{(3066.66)^2 - 4 \times 4444.44}}{2}$$

$$= \frac{3066.66 \pm 3063.76}{2} = 3065.2\ \Omega \text{ or } 1.45\ \Omega.$$

Hence the two possible values of R_L are 3065.2 Ω and 1.45 Ω.

14.64 In Fig. 14.38, find the current through the ammeter having 10 Ω internal resistance. Use the principle of superposition.

Solution

Let us first remove the 10 V battery, shown in Fig. 14.39.

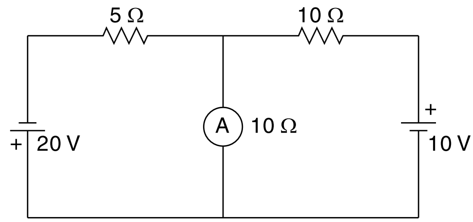


Fig. 14.38

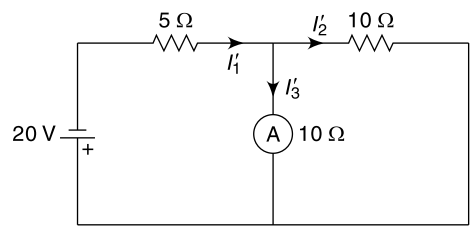


Fig. 14.39

Here $$I_1' = -\frac{20\text{ V}}{5+\dfrac{10\times 10}{10+10}} = -2\text{A}$$

Also, $I_3' = I_2' = 1$ A (–ve).

Next we remove 20 V battery, shown in Fig. 14.40

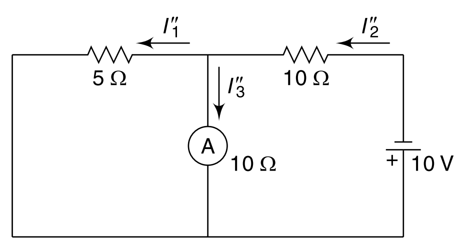


Fig. 14.40

Here, $$I_2'' = \frac{10}{10+\dfrac{5\times 10}{5+10}} = \frac{10}{13.33}\text{ A}$$

$$I_3'' = I_2'' \times \frac{5}{10+5} = \frac{10}{13.33}\times\frac{5}{15} = 0.25\text{ A}$$

$\therefore$ Using superposition principle, the current through the ammeter is

$$I_3 = I_3' + I_3'' = -1 + 0.25 = -0.75\text{ A}$$

(negative sign indicates that the direction of the current is upwards through the ammeter in the corresponding figure.)

14.65 In the circuit of Fig. 14.41, find (I) using superposition theorem.

Solution

Let us first remove 2A source. Sec Fig. 14.42.

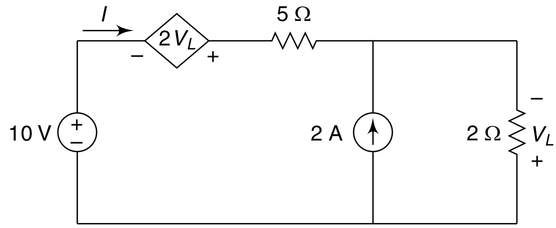


Fig. 14.41

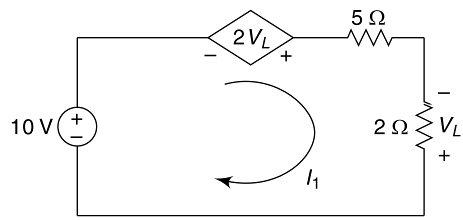


Fig. 14.42

Here, $\quad -10 - 2V_L - 5I_1 - V_L = 0 \quad$ (i)

and $\quad V_L = -2I_1 \quad$ (ii)

Using (ii) in (i) we get $I_1 = 0.91$ A.

Next we remove the 10 V source. See Fig. 14.43.

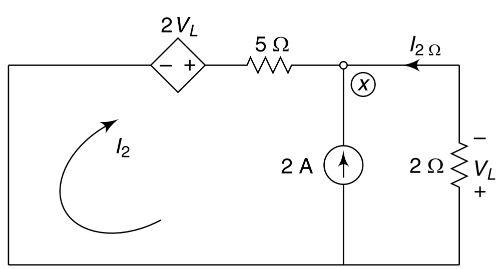


Fig. 14.43

Here, at node x,

$$-I_2 - 2 - I_{2\Omega} = 0$$

or
$$I_2 + 2 + I_{2\Omega} = 0$$

$$\frac{2V_L - (-V_L)}{5} + 2 + \frac{V_L}{2} = 0$$

or
$$V_L = -\frac{20}{11}\text{ V}$$

∴
$$I_2 = \frac{3V_L}{5} = -1.09 \text{ A.}$$

Using superposition theorem, $I = I_1 + I_2 = 0.91 - 1.09 = 0.18$ A (–ve)

Hence magnitude of I is 0.18 A and directed opposite to the direction shown in the given Figure (Fig. 14.41).

14.66 Determine the Thevenin's equivalent and Norton's equivalent of the circuit shown in Fig. 14.44.

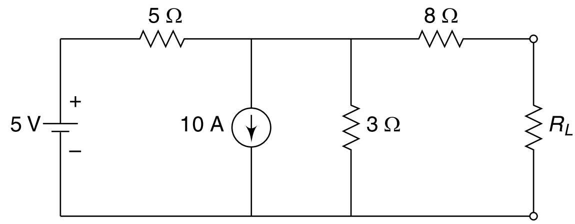


Fig. 14.44

Solution

Removing the load resistance R_L and transforming the 10 A current source into voltage source circuit shown in Fig. 14.45 is obtained.

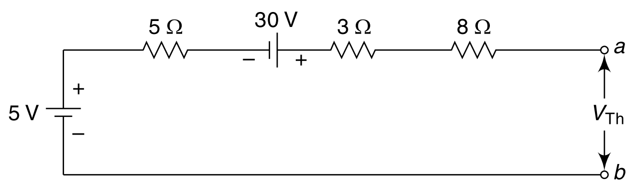


Fig. 14.45

The open circuit voltage is the Thevenin's equivalent voltage V_{Th}.

Here, $V_{Th} = V_{ab} = 5 + 30 = 35$ V.

Removing the sources, Thevenin's equivalent resistance can be obtained and the corresponding circuit is obtained as shown in Fig. 14.46.

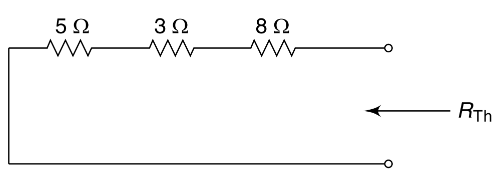


Fig. 14.46

Hence, $R_{Th} = 8 + 3 + 5 = 16\ \Omega$.

Therefore Thevenin's equivalent circuit can be obtained as shown in Fig. 14.47.

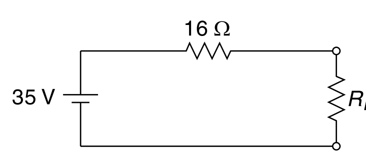


Fig. 14.47

Now, to find Norton's equivalent current I_N the load resistance R_L is short circuited as shown in Fig. 14.48.

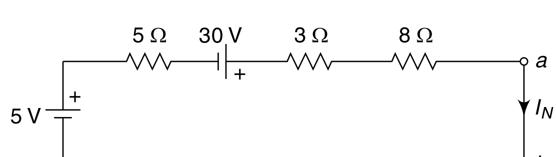


Fig. 14.48

Here, $I_N = \dfrac{30+5}{5+3+8} = \dfrac{35}{16}$ A = 2.1875 A

Norton's equivalent resistance R_N (=R_{Th}) = 16 Ω.
Norton's equivalent circuit is shown in Fig. 14.49.

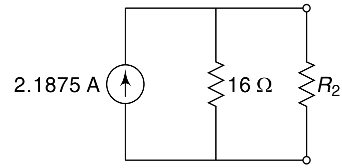


Fig. 14.49

14.67 In Fig. 14.50, find the power loss in 1 Ω resistor by Thevenin's theorem.

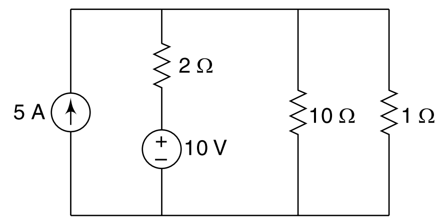


Fig. 14.50

Solution

Let us first remove the 1 Ω resistor (Fig. 14.50(a)). At node *x*, we have

$$\frac{V_{o/c}}{10} + \frac{V_{o/c} - 10}{2} = 5$$

∴ $V_{o/c}$ = 16.67 V.

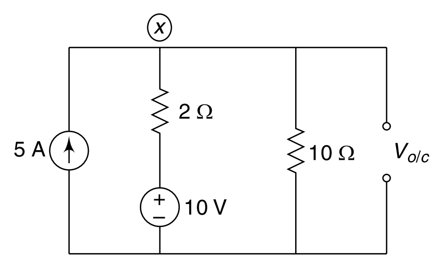


Fig. 14.50(a)

Next we deactivate the constant current source (Fig. 14.51).

$$R_{Th} = \frac{10 \times 2}{10+2} = 1.67\ \Omega$$

The Thevenin's equivalent circuit is shown in Fig. 14.52.

$$I = \frac{V_{o/c}}{R_{Th} + R} = \frac{16.67}{1.67+1} = 6.24\ \text{A}.$$

∴ Power loss in 1Ω resistor is [$(6.24)^2 \times 1$] W or, 39.94 W.

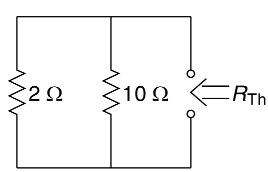


Fig. 14.51

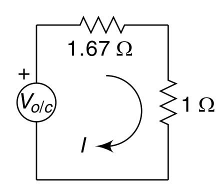


Fig. 14.52

14.68 In Fig. 14.53, find the maximum power transfer in 12 Ω resistor.

Solution

Let us first remove the 12 Ω resistor (Fig. 14.54). In loop *abcd*,

$$2(I + 45) - 30 + (24 + 6)I - 180 + 4(I + 30) = 0$$

∴ $I = 0$

i.e., $V_{o/c}$ = 180 V, with terminal *C* as +ve.

Next we deactivate all independent sources and find internal resistance (R_{Th}) with reference to Fig. 14.55.

$$R_{Th} = \frac{12 \times 24}{12+24} = 8\ \Omega$$

$$\therefore \quad P_{max} = \frac{V_{o/c}^2}{4R_{Th}} = \frac{(180)^2}{4 \times 8} = 1012.5\ \text{W}.$$

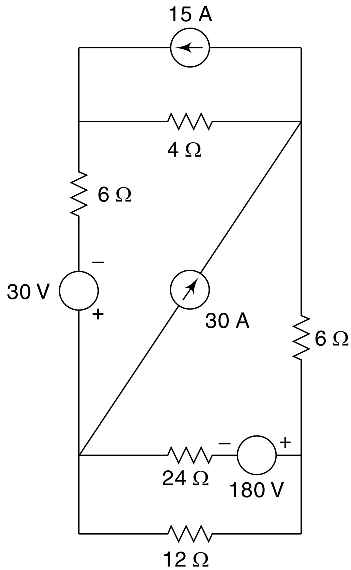


Fig. 14.53

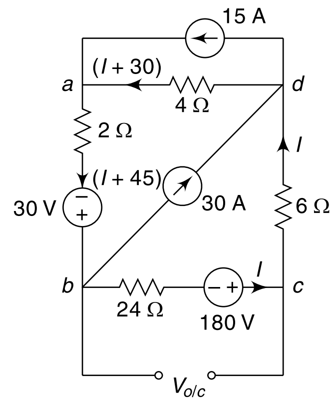


Fig. 14.54

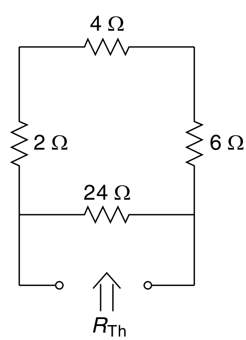


Fig. 14.55

14.69 In Fig. 14.56, find the current through the 2 Ω resistor and obtain power loss. Use Norton's theorem.

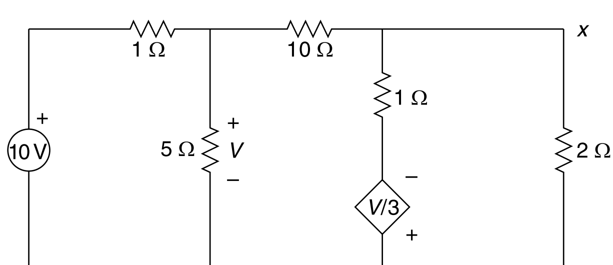


Fig. 14.56

Solution

Let us remove 2Ω resistor and short the terminals *x-y* (Fig. 14.57).

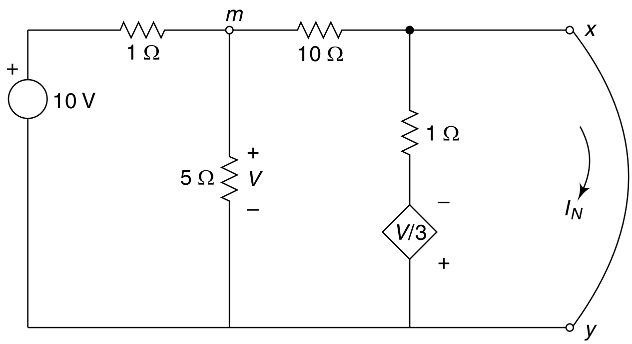


Fig. 14.57

At node x,

$$\frac{0+\frac{V}{3}}{1}+\frac{0-V}{10}+I_N=0$$

[Voltage at shorted terminal x is zero]

or $$-\frac{V}{3}+\frac{V}{10}=I_N$$

or $$I_N=-\frac{7}{30}.$$

However at node m, we have

$$\frac{V}{5}+\frac{V-10}{1}+\frac{V-0}{10}=0$$

or $$\frac{V}{5}+V+\frac{V}{10}=10 \quad \text{or,} \quad V=\frac{100}{13}$$

Hence $$I_N=-\frac{7}{30}\times\frac{100}{13}=-\frac{70}{39}\text{ A}$$

Next, to find R_{Th} we remove the independent source (Fig. 14.58)

Here we apply a voltage V_o so that input current is I. The value of (V_o/I) would give R_{Th}. Obviously.

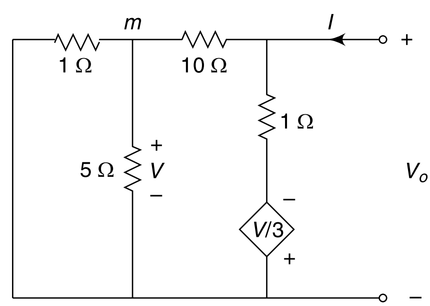


Fig. 14.58

$$I=\frac{V_o-V}{10}+\frac{V_o+V/3}{1}=V_o+\frac{V_o}{10}-\frac{V}{10}+\frac{V}{3}=\frac{11}{10}V_o-\frac{7}{30}V \qquad \text{(i)}$$

Also, $$\frac{V}{5}+\frac{V}{1}+\frac{V-V_o}{10}=0 \qquad \text{(at node } m\text{)}$$

or, $$\frac{6V}{5}+\frac{V}{10}=\frac{V_o}{10} \quad \text{or,} \quad \frac{13V}{10}=\frac{V_o}{10}$$

∴ $$V=\frac{1}{13}V_o \qquad \text{(ii)}$$

Using (ii) in (i) we get

$$I=\frac{11}{10}V_o-\frac{7}{30}\times\frac{1}{13}V_o=\frac{39\times 11-7}{390}V_o$$

∴ $$V_o/I=R_{Th}=\frac{390}{422}\,\Omega.$$

Thus for the given circuit, the equivalent Norton circuit can be obtained.

$$I_N = -\frac{70}{39}\text{ A} \quad \text{and} \quad R_{Th} = \frac{390}{422}\ \Omega.$$

Hence the current through 2 Ω resistor is

$$I_{2\Omega} = -\frac{70}{39} \times \frac{\frac{390}{422}}{\frac{390}{422}+2} = 0.567\text{ A}.$$

Power loss in 2Ω resistor is

$$P_{2\Omega} = (I_{2\Omega})^2 \times 2 = 0.644\text{ W}.$$

14.70 A power of 50 W is to be dissipated in a resistor of R Ω. The resistor is connected across the terminals of a battery having an electromotive force of 35 V and internal resistance 0.15 Ω. Determine the value of R and the terminal voltage of the battery.

Solution

Total current, $$I = \frac{35}{0.15+R}$$

Power $$P = I^2R = \left(\frac{35}{0.15+R}\right)^2 R = 50$$

or $$\frac{1225R}{0.0225+0.3R+R^2} = 50$$

or $$R^2 + 0.3R + 0.0225 = 24.5R$$

or $$R^2 + 0.3R - 24.2 = 0$$

or $$R = \frac{-0.3 \pm \sqrt{(0.3)^2 + 4 \times 24.2}}{2} = -0.15 \pm \frac{9.84}{2}$$

Hence, $R = 4.77\ \Omega$

Terminal voltage of the battery is then

$$4.77 \times \frac{35}{0.15+4.77} = 33.93\text{ V}.$$

14.71 An inductive coil A having an inductance of 250 mH and a resistance of 120 Ω is connected in series with other inductive coil B having an inductance of 400 mH and a resistance of 100 Ω across a 230 V, 50 Hz supply. Calculate the (i) the line current (ii) the phase difference between the supply voltage and current (iii) the voltage across A and B (iv) the phase difference between these voltages.

Solution

Given:
$$R_A = 120\ \Omega,$$
$$R_B = 100\ \Omega,$$
$$X_A = 2\pi fL = 2 \times 3.14 \times 50 \times 250 \times 10^{-3} = 78.5\ \Omega,$$
$$X_B = 2\pi fL = 2 \times 3.14 \times 50 \times 400 \times 10^{-3} = 125.6\ \Omega.$$

∴ $$Z_A = 100\sqrt{(1.20)^2 + (0.785)^2} = 143.4\ \Omega.$$

$$Z_B = 100\sqrt{1^2 + (1.25)^2} = 160.5\ \Omega.$$

Total resistance (R) of the circuit is given by $R = 120 + 100 = 220\ \Omega$.

Total circuit reactance (X) is then $(78.5 + 125.6) = 204.1\ \Omega$

∴ Z (total impedance of the circuit) = $\sqrt{(220)^2 + (204.1)^2} = 300\ \Omega$.

(i) $I = \dfrac{230}{300} = 0.766$ A.

(ii) P.f.(= $\cos \phi$) = $\dfrac{220}{300} = 0.733$ (lagging) and $\phi = 42°36'$.

(iii) Voltage across A, $v_A = 0.766 \times 143.4 = 109.8$ V.
Voltage across B, $v_B = 0.766 \times 160.5 = 122.9$ V.

(iv) $\cos \phi_A = \dfrac{120}{143.4} = 0.837$ (lagging) or, $\phi_A = 33.175°$

and $\cos \phi_B = \dfrac{100}{160.5} = 0.623$ (lagging) or, $\phi_B = 51.46°$.

$\therefore$ The phase difference $\phi = 51.46° - 33.175° = 18.285°$.

14.72 Two coils are connected in series. When 2A d.c. is passed through the circuit, the voltage drop across the coils are 20 V, and 30 V respectively and when 2 A ac is passed at 40 Hz, the drop becomes 140 V and 100 V respectively. If two coils are connected to 230 V, 50 Hz mains, calculate the current flowing through the coils.

Solution

DC condition

$$R_A = \frac{20}{2} = 10\ \Omega, \quad R_B = \frac{30}{2} = 15\ \Omega.$$

AC condition

$$Z_A = \frac{140}{2} = 70 \text{ ohm}, \quad Z_B = \frac{100}{2} = 50\ \Omega.$$

$$\therefore \quad X_A = \sqrt{70^2 - 10^2} = \sqrt{4900 - 100} = 69.3\ \Omega.$$

$$\therefore \quad X_B = \sqrt{50^2 - 15^2} = \sqrt{2500 - 225} = 47.7\ \Omega.$$

Since X is proportional to frequency, therefore at 50 Hz,

$$X_A = 69.3 \times \frac{5}{4} = 86.6\ \Omega.$$

$$X_B = 47.7 \times \frac{5}{4} = 59.7\ \Omega.$$

For the total series circuit (R) = 10 + 15 = 25 Ω
(since resistance is independent of frequency) and

$$X = 86.6 + 59.7 = 146.3\ \Omega \text{ (at 50 Hz)}.$$

$$\therefore \text{ Now} \quad Z = \sqrt{25^2 + 146.3^2} = 148.1\ \Omega.$$

$$\therefore \text{ Current} \quad I = \frac{230}{148.1} = 1.55 \text{ A (at 50 Hz)}.$$

14.73 A resistor of 8 Ω is connected in series with an inductive load and the combination is placed across a 100 V supply mains. A voltmeter when connected across the load and then across the resistor and indicates 48 V and 64 V respectively.

Find

(i) the power consumed by the load.
(ii) the power consumed by the resistor
(iii) the total power taken by the supply
(iv) the power factors of the load and the whole circuit.

Solution

(i) Current in 8 Ω resistor (i.e, the current in the circuit) = 64/8 = 8 A.

(ii) Power consumed in resistor = $(I^2R) = 8^2 \times 8 = 512$ W.
The connection is displayed in Fig. 14.59(a).

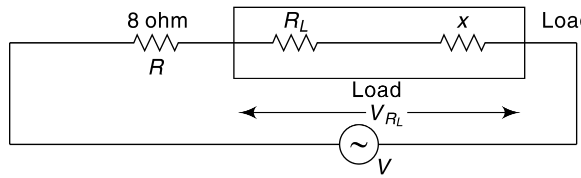


Fig. 14.59(a)

[V_{R_L} = drop across $R_L(CB)$
V_R = drop across 8 Ω resistor = (OC)].
Now from Fig. 14.59(b),

$$OA^2 = OC^2 + CA^2 + 2 \times OC \times CA \times \cos\theta$$

or $\quad 100^2 = 64^2 + 48^2 + 2 \times 64 \times 48 \times \cos\theta$

$\therefore \quad \cos\theta = 0.586.$

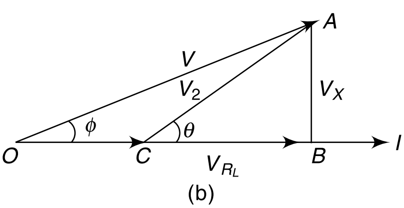


Fig. 14.59(b)

(iv) $\therefore$ Power factor of the load = 0.586 (lagging)
Voltage drop in resistance of inductive load $(=V_{R_L}) = V_2 \cos\theta = 48 \times 0.586$ = 28.128 V (= $I \cdot R_L$)

$$\therefore \qquad R_L = \frac{28.128}{8} = 3.52\ \Omega.$$

(i) Power consumed by load (= I^2R_L) = $8^2 \times 3.52 = 225.3$ W.

(iii) Total power = (512 + 225.3) = 737.3 W.

(iv) Power factor of the circuit, $(\cos\phi) = \dfrac{P}{VI} = \dfrac{737.3}{100 \times 8} = 0.92$ (lagging).

14.74 A coil having a capacitance of 10 μF is connected in series with an inductance of 0.5 H and a resistance of 60 Ω. This coil is connected to 100 V supply. Find the source frequency and current flowing in the circuit under resonance condition. Draw the phasor diagram and find voltage across the resistance.

Solution

At resonance, $\quad X_L = X_C$;

i.e. $\qquad 2\pi fL = \dfrac{1}{2\pi fC}$

$$\therefore \qquad f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi \times \sqrt{0.5 \times 10 \times 10^{-6}}} = 71.21 \text{ Hz.}$$

Here I can be obtained form $I = \dfrac{V}{R}$

Thus $\qquad I = \dfrac{100}{60} = 1.67$ A

Fig. 14.60 represents the phasor diagram.

Here, $\qquad V_R = IR = 1.67 \times 60 = 100.2$ V.

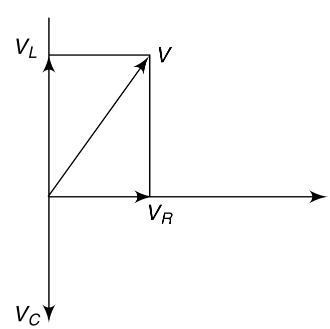


Fig. 14.60

14.75 An inductance coil of 8 Ω resistance and 0.02 H inductance is connected in parallel with another inductive coil of 10 Ω resistance 0.05 H inductance. The combination is connected across a 10 V, 50 Hz supply. A capacitor of 80 μF is connected in series with a 20 Ω resistor and the combination is connected in parallel with the same supply. Calculate the total current taken from the mains and its phase angle with respect to the applied voltage.

Solution

Let the branches be *A*, *B*, *C* respectively.

then: $X_A = 2\pi fL = 2 \times 3.14 \times 50 \times 0.02 = 6.28\ \Omega$

$$Z_A = \sqrt{8^2 + 6.28^2} = 10.2\ \Omega.$$

and $\cos\phi_A = \dfrac{8}{10.2} = 0.785$ (lagging)

$$\sin\phi_A = \frac{6.28}{10.2} = 0.616.$$

$$I_A = \frac{100}{10.2} = 9.8\ \text{A}.$$

Next, $X_B = 2 \times 3.14 \times 50 \times 0.05 = 15.7\ \Omega.$

$$Z_B = \sqrt{10^2 + 15.7^2} = \sqrt{100 + 246.5} = 18.6\ \Omega.$$

$$\cos\phi_B = \frac{10}{18.6} = 0.537 \text{ (lagging)}$$

$$\sin\phi_B = \frac{15.7}{18.6} = 0.845$$

$$I_B = \frac{100}{18.6} = 5.37\ \text{A}.$$

In branch *C*, $X_C = \dfrac{1}{2\pi fC} = \dfrac{10^6}{2 \times 3.14 \times 50 \times 80} = 39.8\ \Omega.$

then, $Z_C = \sqrt{20^2 + 39.8^2} = 44.54\ \Omega.$

$$\cos\phi_C = \frac{20}{44.54} = 0.449 \text{ (leading)}.$$

$$\sin\phi_C = \frac{39.8}{44.54} = 0.894$$

$$I_C = \frac{100}{44.54} = 2.24\ \text{A}.$$

Adding the active and reactive current components (I_a and I_r respectively)

$$\begin{aligned} I_a &= I_A \cos\phi_A + I_B \cos\phi_B + I_C \cos\phi_C \\ &= (9.8 \times 0.785) + (5.37 \times 0.537) + (2.24 \times 0.449) = 11.59\ \text{A}. \\ I_r &= -I_A \sin\phi_A - I_B \sin\phi_B + I_C \sin\phi_C \\ &= -(9.8 \times 0.616) - (5.37 \times 0.845) + (2.24 \times 0.894) \\ &= -6.04 - 4.54 + 2.04 = -8.54. \end{aligned}$$

Then $(I) = \sqrt{I_a^2 + I_r^2} = \sqrt{11.59^2 \times 8.54^2} = 14.38$ A;

$$\cos\phi = \frac{11.59}{14.38} = 0.805 \text{ (lagging)}, \ \phi = 36° \text{ (approx)}.$$

14.76 Three loads, 2 Ω, 4 Ω and 5 Ω are connected across the ac lines as shown in Fig. 14.61. The resistances of the lines as well as the neutral has value of 1 Ω each. Obtain the amount of power delivered to each of the three loads as well as the power loss in the neutral wire and the lines.

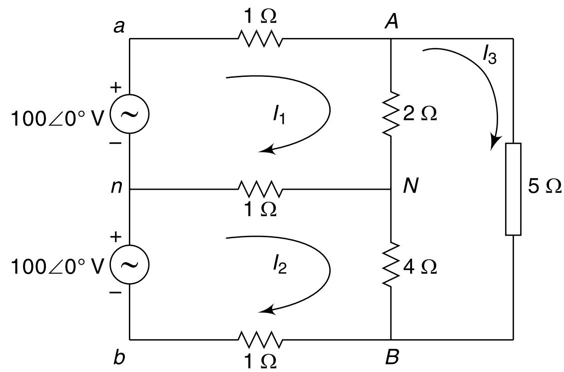


Fig. 14.61

Solution

The three loads in the circuit are 2 Ω, 4 Ω and 5 Ω. The two lines are *aA* and *bB* having resistance of 1 Ω each and neutral *nN* has resistance of 1 Ω.

The mesh currents are respectively I_1, I_2 and I_3.

The three mesh equations are

$$-100\angle 0° + I_1 + 2(I_1 - I_3) + (I_1 - I_2) = 0$$
$$-100\angle 0° + (I_2 - I_1) + 4(I_2 - I_3) + I_2 = 0$$
$$5I_3 + 4(I_3 - I_2) + 2(I_3 - I_1) = 0$$

The above equations can be rearranged as

$$4I_1 - I_2 + 2I_3 = 100\angle 0°$$
$$-I_1 + 6I_2 - 4I_3 = 100\angle 0°$$
$$2I_1 + 4I_2 - 11I_3 = 0$$

Solving the equations we get

$$I_1 = 30.23 \angle 0° \text{ A}$$
$$I_2 = 22.1 \angle 0° \text{ A}$$

and $\quad I_3 = 0.59 \angle 0°$ A.

∴ Current through 1 Ω resistor in line *aA* is 30.23 ∠0° A, current through 1 Ω resistor in line *bB* is –22.1∠0° A.

Also, I_n (current though neutral) = $I_2 - I_1 = -8.13\angle 0°$ A.

Hence power loss in the two lines and the neutral wire are:

$$P_{aA} = (30.23)^2 \times 1 = 913.853 \text{ W.}$$
$$P_{bB} = (22.1)^2 \times 1 = 488.4 \text{ W.}$$
$$P_{nN} = (8.13)^2 \times 1 = 66.097 \text{ W.}$$

Power delivered to the loads:

$$P_{2\Omega} = (I_1 - I_3)^2 \times 2 = (30.23 - 0.59)^2 \times 2 = 1757 \text{ W.}$$
$$P_{4\Omega} = (I_2 - I_3)^2 \times 4 = (22.1 - 0.59)^2 \times 4 = 1850.7 \text{ W.}$$
$$P_{5\Omega} = I_3^2 \times 5 = (0.59)^2 \times 5 = 1.7405 \text{ W.}$$

14.77 Determine the value of v in the circuit shown in Fig. 14.62.

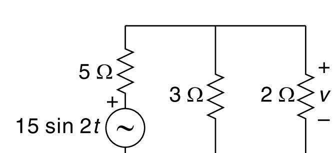


Fig. 14.62

Solution

As 3Ω and 2Ω are in parallel, voltage across each of the resistances is v and combining them the circuit is redrawn as shown in Fig. 14.63.

Current $i = \dfrac{15 \sin 2t}{5+1.2} = \dfrac{15 \sin 2t}{6.2}$ A

Appling KVL in the loop we get

$$15 \sin 2t - 5 \times \frac{15 \sin 2t}{6.2} - v = 0$$

or $\quad 2.9 \sin 2t = v$

Hence $\quad v = (2.9 \sin 2t)$ V.

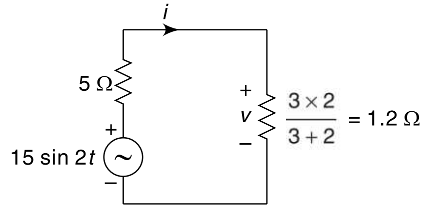


Fig. 14.63

14.78 In the network of Fig. 14.64, find the value of Z_L for maximum power transfer. Also obtain the amount of power transfer.

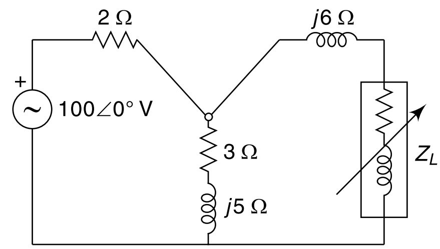


Fig. 14.64

Solution

The active source 100∠0° V is first deactivated and Z_L is removed (Fig. 14.65). Here

$$Z_{Th} = j6 + \frac{2(3+j5)}{2+3+j5} = (1.6 + j6.4)\ \Omega.$$

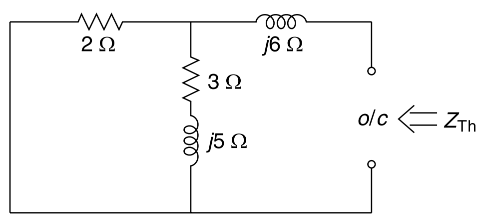


Fig. 14.65

Also, following Fig. 14.66, we can write

$$I = \frac{100\angle 0^\circ}{2+3+j5}$$

$$= 14.14\angle -45^\circ \text{ A}$$

$\therefore \quad V_{o/c} = I(3+j5) = 82.44\angle 14^\circ$ V.

As per maximum power transfer theorem,

$$Z_{Th}^{*} = Z_L$$

$\therefore$ Here, $\quad Z_L = (1.6 - j6.4)\ \Omega.$

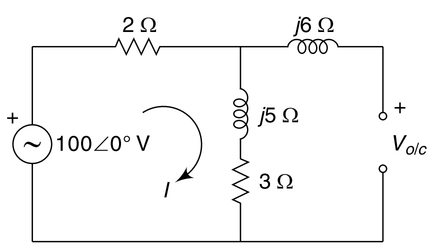


Fig. 14.66

Load current $(I_L) = \dfrac{V_{o/c}}{Z_{Th} + Z_L} = \dfrac{82.44\angle 14^\circ}{3.2\angle 0^\circ} = 25.76\angle 14^\circ$ A.

$\therefore$ Maximum, power transfer $P_{max} = (25.76)^2 \times 1.6 = 1.06$ kW.

14.79 Obtain Thevenin's equivalent across *P-Q* in Fig. 14.67.

Solution

Here, $\quad I = \dfrac{10\angle 0^\circ \times j15}{5 + j15 - j5} = \dfrac{150\angle 90^\circ}{5 + j10} = \dfrac{150\angle 90^\circ}{11.18\angle 63.43^\circ} = 13.42\angle 26.57^\circ$ A.

$\therefore V_{o/c}$ (open circuit voltage across *P-Q*)

$$= I(-j5) = 5\angle -90^\circ \times 13.42\angle 26.57^\circ = 67.08\angle -63.43^\circ \text{ V}.$$

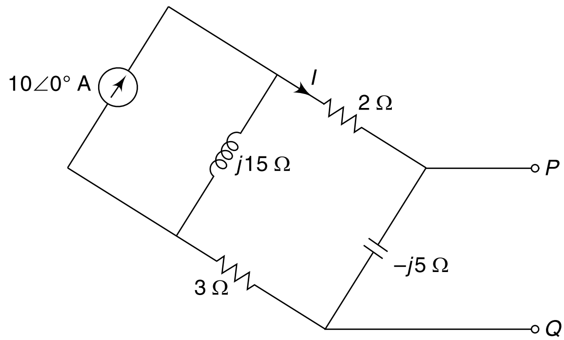


Fig. 14.67

Next we deactivate the constant source and look into the circuit from *P-Q* terminals to find Z_{Th}.

$$Z_{Th} = \frac{(-j5)(5+j15)}{-j5+5+j15} = \frac{75-j25}{5+j10} = (1-j7)\ \Omega.$$

∴ For Thevenins equivalent circuit,

$$V_{o/c} = 67.08\angle{-63.43°}\ \text{V}$$
$$Z_{Th} = 7.07\angle 81.86°\ \Omega.$$

14.80 Obtain Norton's equivalent network (Fig. 14.68) between $(x - y)$ terminals.

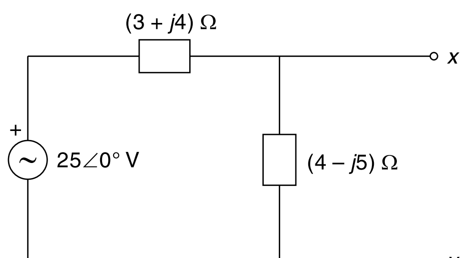


Fig. 14.68

Solution

Let us first short terminals $x - y$ (Fig. 14.69).

$$I_N = \frac{25\angle 0°}{3+j4} = \frac{25\angle 0°}{5\angle 53.13°}$$
$$= 5\angle{-53.13°}\ \text{A}.$$

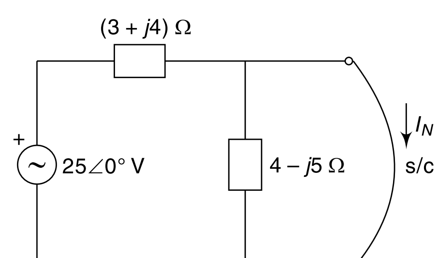


Fig. 14.69

Next we deactivate the constant source and find Z_{Th} looking from open circuited $x - y$ terminal (Fig. 14.70).

Here,

$$Z_{Th} = \frac{(3+j4)(4-j5)}{(3+j4)+(4-j5)} = 4.53\angle 9.52°\ \Omega$$

Norton's equivalent network is shown in Fig. 14.71.

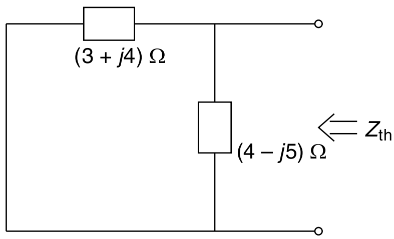


Fig. 14.70

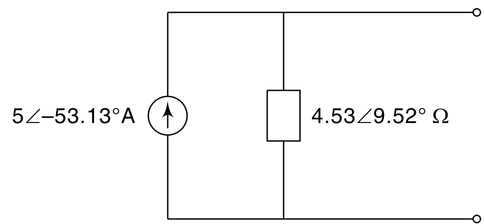


Fig. 14.71

14.81 Determine the value of R for which power consumed in it is maximum when it is connected in series with a coil of resistance 3 Ω and reactance 20 Ω across 230 V, 50 Hz ac supply.

Solution

Current $I = \dfrac{230}{\sqrt{(3+R)^2+(20)^2}}$ A

Power $P = I^2R = \dfrac{(230)^2\ R}{(3+R)^2+(20)^2}$ W

For power consumed to be maximum $\dfrac{dP}{dR} = 0$

$\therefore \quad (230)^2R\{2(3 + R)\} = (230)^2\{(3 + R)^2 + (20)^2\}$

or $\quad 6R + 2R^2 = 9 + R^2 + 6R + 400$

or $\quad R = 20.22\ \Omega.$

14.82 Find the expression for $i(t)$ in the circuit shown in Fig. 14.72.

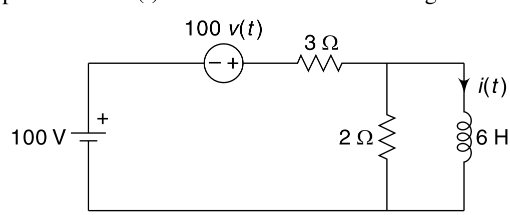


Fig. 14.72

Solution

Let us first find out the Thevenin's equivalent of the circuit shown in Fig. 14.72. Thevenin's equivalent resistance $R_{Th} = \dfrac{3\times 2}{3+2} = \dfrac{6}{5}\ \Omega$.

Thevenin's voltage

$$(V_{Th}) = \left[\frac{100+100\,v(t)}{3+2}\right]\times 2$$

$$= 40 + 40\ v(t)$$

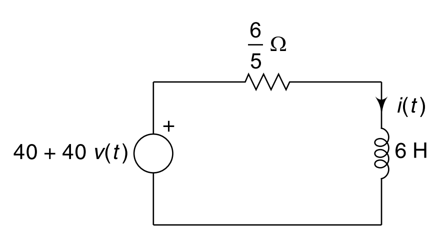


Fig. 14.73

Thevenin's equivalent circuit is shown in Fig. 14.73

Time constant of the circuit $(\lambda) = \dfrac{6}{6/5} = 5$

Hence $\quad i(t) = \dfrac{40+40\,v(t)}{6/5}(1 - e^{-t/5}) = 33.33\{1 + v(t)\}\ (1 - e^{-t/5}).$

14.83 Determine v and i_1 at t = (0+) for the circuit shown in Fig. 14.74 if $v(0^-) = V_o$.

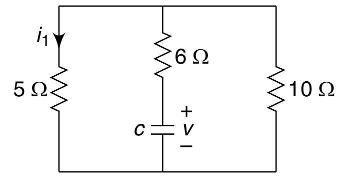


Fig. 14.74

Solution

5 Ω and 10 Ω resistors are in parallel and the combination is in series with 6 Ω resistor. The equivalent resistance is thus $\left[6+\frac{5\times 10}{5+10}\right]$ i.e., 9.33 Ω. The simplified circuit is shown in Fig. 14.75.

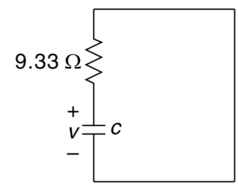


Fig. 14.75

Now, $v = V_o e^{-\frac{t}{9.33\,C}}$

At $t = (0\,+),\ v = V_o$

$$i = C\frac{dv}{dt} = CV_o\left(-\frac{1}{9.33C}\right)e^{-\frac{t}{9.33\,C}}$$

or $$i = -\frac{V_o}{9.33}\,e^{-\frac{t}{9.33\,C}}$$

At $t = 0\,+,\ i = -\dfrac{V_o}{9.33}$.

14.84 Find the current i through the 50 Ω resistor in Fig. 14.76 at (i) $t = (0^-)$, (ii) $t = (0^+)$, (iii) $t = \infty$ and (iv) t = 2 ms.

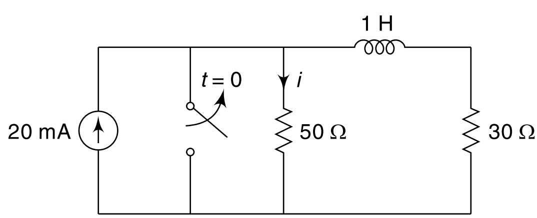


Fig. 14.76

Solution

(i) At $t = 0^-$

$i = 0$ (∵ 20 mA will flow through the short circuited path i.e., through the switch)

(ii) At $t = 0^+$

There are two parallel paths: One is a 50 Ω resistor, another is a 30 Ω resistor in series with 1 H inductor. According to the property of the inductor no current will flow through it at t = 0+. Hence all current will flow through the 50 Ω resistor.

∴ At $t = 0+,\ i = 20$ mA.

(iii) At $t = \infty$, inductor will act as a short circuit.

Hence current i at $t = \infty$ is $20 \times \dfrac{30}{30+50} = \dfrac{600}{80}$ = 7.5 mA (flowing through 50 Ω resistor).

(iv) Current through the 30 Ω resistor at $t = \infty$ is $I_o = 20 \times \dfrac{50}{50+30}$ = 12.5 mA.

At any time t, current through the inductor is, $I_o e^{-t/\lambda} = 12.5\,e^{-\frac{t}{(1/30)}} = 12.5e^{-30t}$ mA. At t = 2 ms current through the 30 Ω resistor is thus $12.5\,e^{-30\times 2\times 10^{-3}}$ mA i.e., 11.77 mA. Thus current through 50 Ω resistor at t = 2 ms, is (20 mA – 11.77 mA) i.e., 8.23 mA.

14.85 What is reading shown by the wattmeter in the circuit of Fig. 14.77? Identify the source *s* generating this power. The positive terminal of the potential coil of the wattmeter is connected separately to (i) *x*, (ii) *y* and (iii) *z*. Check your result in each case.

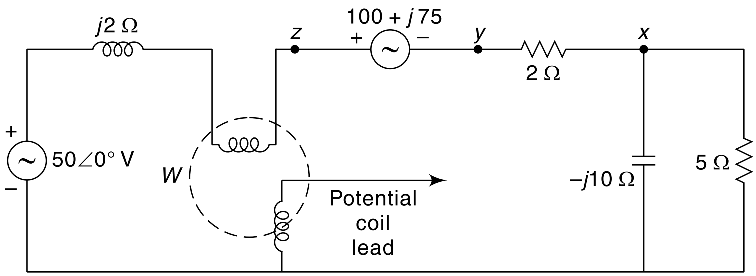


Fig. 14.77

Solution

The parallel combination of 5 Ω and $-j10$ Ω is

$$\frac{5(-j10)}{5-j10} = \frac{50\angle -90^\circ}{11.18\angle -63.43^\circ} = 4.472\angle -26.56^\circ\ \Omega = (4 - j2)\ \Omega.$$

Let us consider that a current *I* flows through the mesh. Hence applying KVL we get
$-50\angle 0^\circ + j2I + (100 + j75) + 2I + (4 - j2)I = 0$

or $\quad -50 - j75 - 6I = 0$

or $\quad I = -\dfrac{50 + j75}{6} = -8.33 - j12.5 = 15\angle -123.68^\circ$ A.

(i) When the potential coil lead is connected at point *x* the wattmeter measures the potential given by

$$\frac{5(-j10)}{5-j10} \times 15\angle -123.68^\circ = 4.472\angle -26.56^\circ \times 15\angle -123.68^\circ$$
$$= 67.08\angle -150.24^\circ \text{ V}.$$

The wattmeter reading is: $67.08 \times 15 \cos(-150.24^\circ + 123.68^\circ) = 900$ W.

As the potential coil is in parallel with the 5 Ω resistor this power is absorbed by 5 Ω resistor.

(ii) When the potential coil lead is connected at point *y* wattmeter measures the potential:

$$50\angle 0^\circ - j2I - (10 + j75) = 50 - j2 \times 15\angle -123.68^\circ - 100 - j75$$
$$= -50 - j75 + \text{j}16.636 - 24.96$$
$$= (-74.96 - j58.364) = 95\angle -142.1^\circ \text{ V}.$$

The wattmeter reading is thus, $[95 \times 15 \cos(-142.1^\circ + 123.68^\circ)]$ or 1352 W.
This power is absorbed by the 2 Ω and 5 Ω resistor.

(iii) When the potential coil lead is connected at point *Z*, the wattmeter measures the potential:

$(50 - j2I)$ V, i.e., $50 - j2 \times 15\angle -123.68^\circ$ V.

The measured potential is thus $(50 - 30\angle -33.68^\circ)$ or, $(25 + j16.63)$ V or, $30\angle 33.65^\circ$ V.

The wattmeter reading is therefore $[30 \times 15 \cos(33.53^\circ + 123.68^\circ)]$ W or, (-415.172) W.

As the reading is negative this power is absorbed by a 50 V source.

14.86 Determine the complex power absorbed by all the elements in the circuit shown in Fig. 14.78.

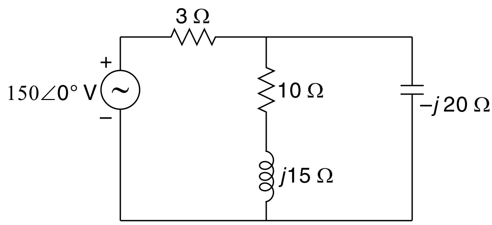


Fig. 14.78

Solution

The equivalent impedance of the circuit is

$$Z = 3 + \frac{(10 + j15)(-j\,20)}{10 + j15 - j20} = \frac{-j200 + 300}{10 - j5} + 3 = \frac{60 - j40}{2 - j} + 3$$

$$= 3 + \frac{72.11\angle -33.69^\circ}{2.236\angle -26.565^\circ}$$

$$= 3 + 32.25\angle -7.215^\circ = (35 - j4)\ \Omega = 35.23\angle -6.52\ \text{A}.$$

Current through 3 Ω resistor is $I = \dfrac{150\angle 0^\circ}{35.23\angle -6.52^\circ} = 4.258\ \angle 6.52^\circ$ A.

It absorbs a complex power of $[(4.258)^2 \times (3 + j0)]$, or, $(54.39 + j0)$ VA.

Current through the capacitor is, $I_C = 4.258\angle 6.52^\circ \times \dfrac{10 + j15}{10 + j15 - j20}$

$$= 4.258\angle 6.52^\circ \times \frac{2 + j3}{2 - j}$$

$$= 4.258\angle 6.52^\circ \times \frac{3.6\angle 56.31^\circ}{2.236\angle -26.56^\circ} = 6.855\angle 89.39^\circ\ \text{A}.$$

∴ Voltage drop across capacitor is

$$v_C = 150\angle 0^\circ - 3 \times 4.258\angle 6.52^\circ$$
$$= 150 - 12.774\angle 6.52^\circ = 137.3 - j1.45 = 137.3\angle -0.61^\circ\ \text{A}.$$

The complex power across capacitor is

$$(VI^*) = 137.3\angle -0.61^\circ \times 6.855\angle -89.39^\circ = 941.19\angle -90^\circ\ \text{VA}.$$

Also, current through the inductor

$$I_L = 4.258\angle 6.52^\circ \times \frac{-j20}{-j20 + 10 + j15}$$

$$= 4.258\angle 6.52^\circ \times \frac{-j4}{2 - j} = 4.258\angle 6.52^\circ \times 1.789\angle -63.43^\circ = 7.617\angle -56.91^\circ\ \text{A}.$$

The complex power across the inductor branch

∴ $(= VI^*) = 137.3\angle -0.61^\circ \times 7.617\angle 56.91^\circ = 1045.81\angle 56.3^\circ$ VA.

14.87 A voltage source v_s is connected across a 5 Ω resistor. Determine the average power absorbed by the resistor if v_s is equal to (i) 10 sin 100t V, (ii) [10 sin 100t – 8 cos (100t – 30°)], (iii) [10 sin 100t – 5 sin 50t] (iv) [10 sin 100t – 8 cos (100t – 30°) – 5 sin 50t + 10 V].

Solution

(i) Average power = $\dfrac{V^2}{R} = \dfrac{\left(\dfrac{10}{\sqrt{2}}\right)^2}{5}$ = 10 W.

(ii) Instantaneous voltage

$$v_s = 10 \sin 100t - 8 \cos (100t - 30°)$$
$$= 10 \cos (100t - 90°) - 8 \cos (100t - 30°) \text{ V.}$$

In phasor representation

$$V_s = \frac{10}{\sqrt{2}} \angle{-90°} - \frac{8}{\sqrt{2}} \angle{-30°}$$
$$= -7.07j - 4.89 + j2.83 = -4.89 - j4.24 = 6.472\angle{-139.14°}$$

Average power = $\dfrac{(6.472)^2}{5}$ = 8.38 W.

(iii) Average power is obtained as

$$\frac{\left(\dfrac{10}{\sqrt{2}}\right)^2}{5} + \frac{\left(\dfrac{5}{\sqrt{2}}\right)^2}{5} = 10 + 2.5 = 12.5 \text{ W.}$$

(iv) Combing the two sinusoidal terms of voltage having equal frequency of (ω = 100 rad/s)

$$V_{S_1} = \frac{10}{\sqrt{2}} \angle{-90°} - \frac{8}{\sqrt{2}} \angle{-30°} = \frac{1}{\sqrt{2}} (-j10 - 6.92 + j4) = 6.472\angle{-139.14°} \text{ V.}$$

Hence average power is $\dfrac{(6.477)^2}{5} + \dfrac{\left(\dfrac{5}{\sqrt{2}}\right)^2}{5} + \dfrac{(10)^2}{5}$ = 8.39 + 2.5 + 20 = 30.89 W.

14.88 Determine the effective value of each of the periodic voltages (i) 8 cos (50t), (ii) 8 cos (50t) + 5 sin (50t + 45°), (iii) 6 cos (25t) + 5 cos^2 (25t).

Solution

(i) Effective value of voltage is $\dfrac{8}{\sqrt{2}}$ V = 5.657 V.

(ii) The voltage waveform is 8 cos [(50t) + 5 cos (50t + 45° – 90°)]

Hence voltage in phasor form is

$$\frac{8}{\sqrt{2}} \angle 0° + \frac{5}{\sqrt{2}} \angle{-45°} \qquad = 5.657 + 2.5 - j2.5 = (8.157 - j2.5) \text{ V}$$

$\therefore$ Effective value = 8.53 V.

(iii) The effective value of voltage [6 cos (25 t) + 5 cos^2 (25 t)] is

$$\left[\frac{1}{T}\int_0^T \{6\cos(25\,t) + 5\cos^2(25\,t)\}^2\, dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T \{36\cos^2(25\,t) + 60\cos^3(25\,t) + 25\cos^4(25\,t)\}\, dt\right]^{1/2}$$

$$= \left[\frac{1}{T}\int_0^T 36\cos^2(25\,t)\,dt + \frac{1}{T}\int_0^T 60\cos^3(25\,t)\,dt + \frac{1}{T}\int_0^T 25\cos^4(25\,t)\,dt \right]^{1/2}$$

$$= \left[\frac{36}{T}\int_0^T \left(\frac{1}{2} + \frac{1}{2}\cos(50\,t)\,dt\right) + \frac{60}{T}\int_0^T \left(\frac{1}{2}\cos(25\,t) + \frac{1}{4}\cos(75\,t) + \frac{1}{4}\cos(25\,t)\right)dt \right.$$

$$\left. + \frac{25}{T}\int_0^T \left(\frac{1}{4} + \frac{1}{2}\cos(50\,t) + \frac{1}{8} + \frac{1}{8}\cos(100\,t)\right)dt \right]^{1/2}$$

$$= \left[\frac{36\times 25}{2}\left(\frac{1}{25}\right) + 60\times 25\times 0 + 25\times 25\left(\frac{1}{4}+\frac{1}{8}\right)\times\frac{1}{25} \right]^{1/2} \qquad \left[\because\ T = \frac{1}{25}\,\text{s}\right]$$

$$= \sqrt{18 + \frac{75}{8}} = 5.232\ \text{V}.$$

14.89 For the circuit shown in Fig. 14.79 determine the power factor of the combined load if load impedance $Z_L = 15\ \Omega$.

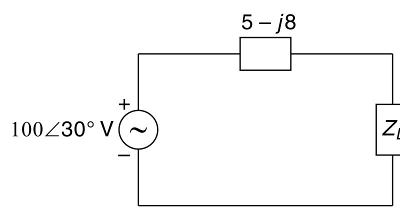


Fig. 14.79

Solution

Current through the circuit, $I = \dfrac{100\angle 30^\circ}{5 - j8 + 15}$

$$= \frac{100\angle 30^\circ}{20 - j8}$$

$$= \frac{100\angle 30^\circ}{21.54\angle -21.8^\circ} = 4.64\angle 51.8^\circ\ \text{A}.$$

Hence power factor is cos (51.8° – 30°) = 0.928 leading.
(since current is leading w.r.t. voltage)

14.90 Calculate the average power delivered to each passive element for the circuit shown in Fig. 14.80.

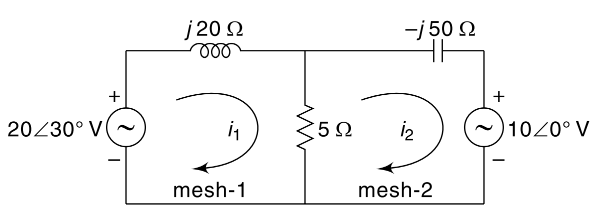


Fig. 14.80

Solution

Applying KVL in mesh 1,

$$-20\angle 30^\circ + j20i_1 + 5(i_1 - i_2) = 0 \qquad \text{(i)}$$

Applying KVL in mesh 2,

$$-j50i_2 + 10\angle 0^\circ + 5(i_2 - i_1) = 0 \qquad \text{(ii)}$$

Solving Eqs (1) and (2) we get

$$i_1 = \frac{39.06\angle 123.32}{40.45\angle 171.47^\circ} = 0.965\angle{-48.146^\circ} \text{ A}$$

and $\quad i_2 = 0.153\angle{-68.96}$ A.

Average power across 5 Ω resistor is $[5(i_1 - i_2)^2]$ W.

Here $\quad (i_1 - i_2) = 0.965\angle{-48.146^\circ} - 0.153\angle{-68.96^\circ}$

$$= 0.644 - j0.72 - 0.549 + j0.143$$
$$= 0.095 - j0.577 = 0.585\angle{-80.65^\circ} \text{ A}$$

Hence, average power across 5 Ω resistor is $[5 \times (0.585)^2$ W] or 1.711 W.
Average power across each of the inductor and capacitor is zero.

14.91 A circuit is composed of a sinusoidal voltage source [5 cos (50t + 10°)] V in series with a 100 Ω resistor, 50 mH inductor and an unknown impedance. Determine the value of the unknown impedance if the source is delivering maximum power to it.

Solution

According to the maximum power transfer theorem, power across an impedance is maximum when it is equal to the complex conjugate of the internal impedance of the circuit.

Now, the internal impedance of the circuit is $(100 + j\omega 50 \times 10^{-3})$ or $(100 + j\omega 0.05)$ Ω

Hence the value of the unknown impedance is

$$Z = (100 - j\omega 0.05) \ \Omega, \text{ to have maximum power transfer.}$$

$\because \quad \omega = 50$ rad/s (given),

hence, $\quad (Z) = [100 - j50 \times 0.05] = (100 - j2.5)$ Ω.

Hence the unknown impedance contains a resistor of 100 Ω and a series capacitor of capacitive reactance 2.5 Ω [i.e. value of capacitor is $\left(\frac{1}{50 \times 2.5} \text{F}\right)$ or 8 mF].

14.92 Determine the average power delivered to a 10 Ω resistor by a current $(i) = (2 \cos 15t - 3 \cos 25t)$ A.

Solution

The current consists of two cosine terms of different frequencies. The average power should be calculated separetely and then added.

The power delivered to 10 Ω resistor is thus

$$\left(\frac{2}{\sqrt{2}}\right)^2 \times 10 + \left(\frac{3}{\sqrt{2}}\right)^2 \times 10 = 65 \text{ W}.$$

14.93 Determine the current (i) through the 5 Ω resistor using superposition theorem in Fig. 14.81.

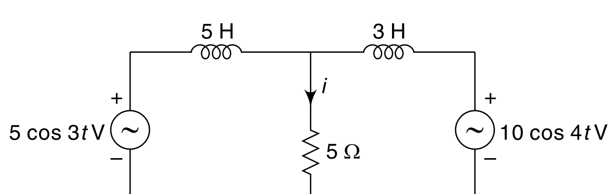


Fig. 14.81

Solution

Considering (5 cos 3*t*) V source acting alone the corresponding circuit along with impedances (Fig. 14.82)

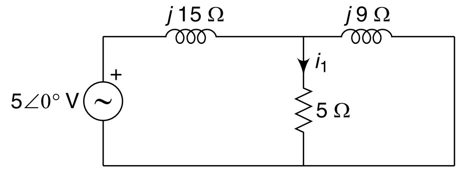


Fig. 14.82

$$\text{Now,}\quad i_1 = \frac{5\angle 0^\circ}{j15 + \dfrac{5\times j9}{5+j9}} \times \frac{j9}{j9+5}$$

$$= \frac{j45}{j75 - 135 + 45j}$$

$$= \frac{45\angle 90^\circ}{-135 + j120} = \frac{45\angle 90^\circ}{180.6\angle 138.36} = 0.25\angle{-48.37^\circ}\ \text{A}.$$

Considering the (10 cos 4*t*) source acting alone the corresponding circuit is shown in Fig. 14.83.

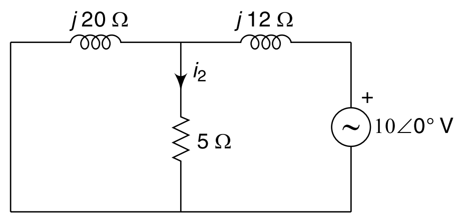


Fig. 14.83

$$i_2 = \frac{10\angle 0^\circ}{j12 + \dfrac{5\times j20}{5+j20}} \times \frac{j20}{5+j20}$$

$$= \frac{200\angle 90^\circ}{j60 - 240 + j100}$$

$$= \frac{200\angle 90^\circ}{-240 + j160}$$

$$= \frac{200\angle 90^\circ}{288.44\angle 146.3^\circ} = 0.6933\angle{-56.3^\circ}\ \text{A}.$$

Hence the current (i) through the 5 Ω resistor when both the source are acting simultaneously is $[0.25\angle{-48.37^\circ} + 0.6933\angle{-56.3^\circ}] = 0.942\ \angle{-54.21^\circ}$ A.

14.94 A voltage, $v = \left(6\cos\dfrac{\pi}{3}t\right)$ V is applied to a circuit having $Z = 3\angle 45^\circ\ \Omega$. Determine the average power and the expression for the instantaneous power if voltage phasor is taken as the reference.

Solution

$$\text{Current } I = \frac{(6\sqrt{2})\angle 0^\circ\ \text{A}}{3\angle 45^\circ\ \text{A}} = 1.414\angle{-45^\circ}\ \text{A}.$$

$$\text{The average power} = \frac{6}{\sqrt{2}} \times \frac{2}{\sqrt{2}}\cos 45^\circ = 4.24\ \text{W}.$$

$$\text{Instantaneous voltage, } v(t) = 6\cos\frac{\pi}{3}t$$

$$\text{Instantaneous current, } i(t) = 2\cos\left(\frac{\pi}{3}t - 45^\circ\right)$$

Hence expression for instantaneous power is

$$p(t) = 6\cos\frac{\pi}{3}t \times 2\cos\left(\frac{\pi}{3}t - 45^\circ\right)$$

$$= 12\cos\frac{\pi}{3}t\ \cos\left(\frac{\pi}{3}t - 45^\circ\right)$$

$$= 6\left\{\cos\left(\frac{\pi}{3}t + \frac{\pi}{3}t - 45°\right) + \cos\left(\frac{\pi}{3}t - \frac{\pi}{3}t + 45°\right)\right\}$$

$$= 6\left\{\cos\left(\frac{2\pi}{3}t - 45°\right) + \frac{1}{\sqrt{2}}\right\} = 4.24 + 6\left(\cos\frac{2\pi}{3}t - 45°\right)\text{W.}$$

14.95 The phasor voltage of 150 $\sqrt{2}\angle 30°$ V is applied across an impedance of Z (= $20\angle 60°$ Ω). Find the expression for the instantaneous power and determine the average power if ω = 50 rad/sec.

Solution

$$V_m = 150\sqrt{2}\angle 30° \text{ V}$$

$$I_m = \frac{\sqrt{2}\,150\angle 30°}{20\angle 60°} = 10.6\angle{-30°} \text{ A.}$$

Instantaneous power

$$p(t) = 150\sqrt{2}\sin(\omega t + 30°) \times 10.6 \sin(\omega t - 30°)$$
$$= 75\sqrt{2} \times 10.6\{\sin 2\omega t + \sin 60°\} = 1124.3 \sin(2\,\omega t) + 973.50$$

The average power is obtained as $\left[\frac{150\sqrt{2}}{\sqrt{2}} \times \frac{10.6}{\sqrt{2}}\cos(30° + 30°)\right]$ W, i.e. 562.15 W.

14.96 A current source of (5 + j12) A delivers current to a circuit having impedance of $10\angle 30°$ Ω. Calculate the average power delivered.

Solution

$$I = (5 + j12) = 13\angle 67.38° \text{ A}$$
$$Z = 10\angle 30° \ \Omega$$
$$\therefore \quad V = 13\angle 67.38° \times 10\angle 30° = 130\angle 97.38° \text{ V}$$

Average power = 130 × 13 cos∠(97.38° – 67.38°) = 1463.58 W

[Check: $Z = 10\angle 30°$ Ω = 8.66 + j5 Ω.

Hence average power is = $(13)^2 \times 8.66$ = 1463.5 W]

14.97 Using superposition theorem determine the value of V_1 (in Fig. 14.84).

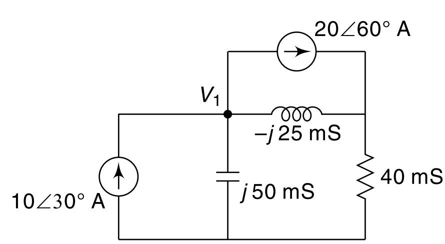


Fig. 14.84

Solution

Considering 10∠30° A source acting alone and removing the 20∠60° A source the inductor and the resistor are in series and the capacitor is in parallel with the combination.

Hence the equivalent admittance y_1

$$= j50 + \frac{1}{\frac{1}{40} + \frac{1}{-j25}} = 33.94\angle 70.67° \text{ mS.}$$

$$\therefore \text{ The equivalent impedance } (Z_{eq}) = \frac{10^3}{33.94\angle 70.67°} = 29.47\angle{-70.67°}\ \Omega$$

Hence $V_1 = 10\angle 30° \times 29.47\angle{-70.67°} = 294.7\angle{-40.67°}$ V.

Next considering $20\angle 60°$ A source acting alone and removing $10\angle 30°$ A source the resistor and the capacitor are in series as shown in Fig. 14.85.

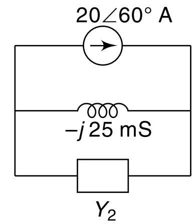


Fig. 14.85

Here, $$y_2 = \frac{1}{\frac{1}{40}+\frac{1}{j50}}\text{ mS} = \frac{1}{0.025 - j0.02}\text{ mS}$$

Hence $$Z_2 = \frac{1}{y_2} = (0.025 - j0.02)10^3\ \Omega$$
$$= (25 - j20)\ \Omega = 32\angle{-38.66°}\ \Omega$$

Now Z_2 and $\left(\frac{10^3}{-j25}\right)$, i.e. $j40\ \Omega$ are in parallel with $20\angle 60°$A current source.

Hence voltage across each branch (v_d) is $\left[20\angle 60° \times \frac{32\angle -38.66°}{j40+25-j20}\right] \times (j40)$ V.

i.e. $$v_d = \frac{25600\angle 110.34°}{32\angle 38.66°} = 800\angle 71.68°\text{ V}.$$

When both the sources are acting simultaneously
$$V_1 = 294.7\angle{-40.67°} - 800\angle 71.68°$$
$$= 223.52 - j192 - 251.46 - j759 = -27.94 - j951 = 951.4\angle{-91.68°}\text{ V}.$$

14.98 Determine i_1 and i_2 in the circuit shown in Fig. 14.86 using mesh analysis technique.

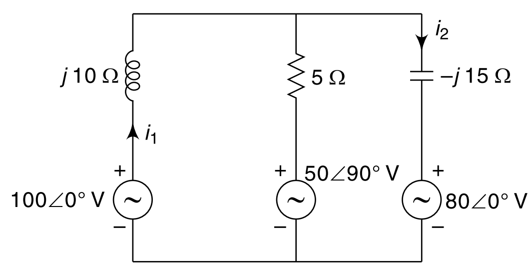


Fig. 14.86

Solution

The mesh currents i_1 and i_2 are shown in Fig. 14.87.

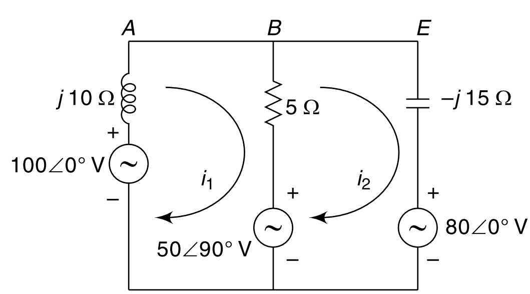


Fig. 14.87

Applying KVL in mesh *ABCDA*

$$5(i_1 - i_2) + 5\angle 90° - 100\angle 0° + j10i_1 = 0$$

or, $$i_2 = \frac{(5 + j10)\, i_1 - 100 + j5}{5} \qquad \text{(i)}$$

Appling KVL in mesh *BEFCB*,

$$-j15i_2 + 80\angle 0° - 50\angle 90° + 5(i_2 - i_1) = 0$$

or $$-5i_1 + (5 - j15)i_2 = -80 + j50 \qquad \text{(ii)}$$

Substituting the value of i_2 from Eq. (i) in Eq. (ii) we get

$$-5i_1 + (1 + j3)\ \{(5 + j10)i_1 - 100 + j5\} = -80 + j50$$

or $$5i_1 - 5(1 + j3)\ (1 + j2)i_1 + 5(1 + j3)\ (20 - j) = 80 - j50$$

or $$5i_1 - 35.35\angle 135°,\ i_1 = 80 - j50 - 316.62\angle 68.7$$

or $$i_1 = \frac{80 - j50 - 115 - j295}{5 + 25 - j25} = \frac{-35 - j345}{30 - j25} = \frac{346.77\angle -95.79°}{39.05\angle -39.8°}$$

$$= 8.88\angle -56° \text{ A.}$$

Hence $$(i_2) = \frac{11.18\angle 63.43° \times 8.88\angle -56° - 100 + j5}{5}$$

$$= 19.85\angle 7.435° - 20 + j1$$

$$= 19.68 + j2.568 - 20 + j1 = -0.32 + j3.568 = 3.58\angle 95.12° \text{ A.}$$

14.99 In the network shown in Fig. 14.88, find I_1 and I_2.

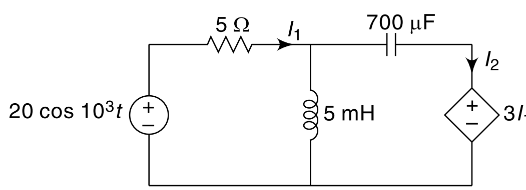


Fig. 14.88

Solution

Let us consider the currents in the two meshes to be i_1 and i_2.

Using KVL for the two loops we can write,

$$-20 \cos 10^3 t + 5i_1 + j10^3 \times 5 \times 10^{-3}\ (i_1 - i_2) = 0$$

[$\because$ drop in 5 mH inductor is $\{i_1 - i_2) \times j\omega \times L\}$]

and $$3i_1 + \frac{1}{j10^3 \times 700 \times 10^{-6}}\, i_2 + j \times 10^3 \times 5 \times 10^{-3}\ (i_2 - i_1) = 0$$

In polar co-ordinate voltage source (20 cos $10^3\ t$) can be represented as $20\angle 0°$ V with $\omega = 10^3$.

Simplifying the above two equations we get

$$7.07\angle 45°\ i_1 - j5i_2 = 20\angle 0° \qquad \text{(i)}$$

and $$5.83\angle -59° i_1 + 3.57\angle 90° i_2 = 0$$

or $$i_1 = \frac{-3.57\angle 90°}{5.83\angle -59°} i_2 \qquad \text{(ii)}$$

Substituting the value of (i_1) from Eq. (2) in Eq. (1) we have

or $$(-4.329\angle 194° - j5)\ i_2 = 20\angle 0°$$

or $$(4.2 - j3.95)i_2 = 20\angle 0°$$

or $$i_2 = \frac{20\angle 0°}{5.76\angle -43.24°} = 3.47\angle 43.24° \text{ A} = I_2.$$

Also $\qquad I_1 = i_1 = -0.612\angle 149° \times 3.47\angle 43.24°$

$\qquad = -2.123\ \angle 192.24° = 2.07 + j0.45 = 2.12\angle 12.26°$ A.

$\therefore \qquad I_1 = 2.12\angle 12.26°$ A.

$\qquad I_2 = 3.47\angle 43.24°$ A.

14.100 Determine the node voltages V_1 and V_2 in the circuit shown in Fig. 14.89.

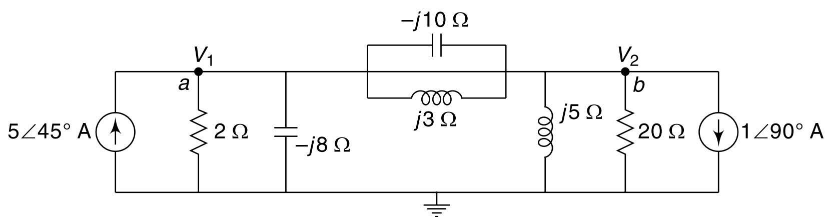


Fig. 14.89

Solution

Applying KCL at node (a),

$$-5\angle 45° + \frac{V_1}{2} + \frac{V_1}{-j8} + \frac{V_1 - V_2}{-j10} + \frac{V_1 - V_2}{j3} = 0$$

or $$\frac{V_1}{2} - j\frac{13V_1}{120} + j\frac{7}{30}V_2 = 5\angle 45°$$

or $$(0.5 - j0.108)V_1 + j0.233V_2 = 5\angle 45° \qquad \text{(i)}$$

Applying KCL at node (b)

$$1\angle 90° + \frac{V_2}{20} + \frac{V_2}{j5} + \frac{V_2 - V_1}{-j10} + \frac{V_2 - V_1}{j3} = 0$$

or $$j0.233V_1 + (0.05 - j0.433)V_2 = -1\angle 90° = -j$$

or $$V_1 = \frac{-j - (0.05 - j0.433)V_2}{j0.233}$$

or $$V_1 = -4.29 + (j0.2146 + 1.858)V_2 \qquad \text{(ii)}$$

Substituting the value of V_1 from Eq. (ii) in Eq. (i) we get

$$(0.95 + j0.14)V_2 = 5\angle 45° + 2.192\angle -12.188°$$

or $$V_2 = \frac{6.44\angle 28.216°}{0.96\angle 8.38°} = 6.71\angle 19.83° \text{ V.}$$

Hence $\qquad V_1 = -4.29 + (1.858 + j0.2146) \times 6.71\angle 19.83°$

$\qquad = -4.29 + 1.87\angle 6.588 \times 6.71\angle 19.83°$

$\qquad = 6.95 + j5.58 = 8.915\angle 38.76°$ V

$\therefore \qquad V_1 = 8.915\angle 38.76°$ V

$\qquad V_2 = 6.71\angle 19.83°$ V.

14.101 Find the equivalent impedance of the network shown in Fig. 14.90 when the frequency is 10 rad/s.

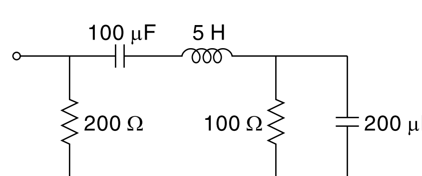


Fig. 14.90

Solution

Let us replace each element by their impedances and the corresponding network is shown in Fig. 14.91.

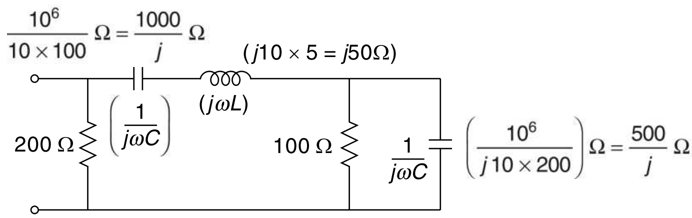


Fig. 14.91

Here, 100 Ω and 500/j Ω (or $-j500$ Ω) are in parallel. Their combined impedance is $\left(\frac{100\,(-j500)}{100 - j500}\right)$ or, $\left(\frac{-j500}{1 - j5}\right)$.

Also, $j50$ Ω and $\left(\frac{1000}{j}\right)\Omega$ or, ($-j1000$ Ω) are in series with $\left(\frac{-j500}{1-j5}\right)\Omega$.

Their combined impedance is obtained as

$$Z_1 = j50 - j1000 - \frac{j500}{(1 - j5)}$$

$$= -j950 - \frac{500\angle 90°}{5.099\angle -78.69°}$$

$$= -j950 - 98.058\angle 168.69° = (96.15 - j969.23)\ \Omega = 973.99\ \angle -84.33°\ \Omega.$$

However, Z_1 is in parallel with 200 Ω. hence the equivalent impedance of the network is

$$\frac{200 \times 973.99\angle -84.33°}{200 + 973.99\angle -84.33°} = \frac{200 \times 973.99\angle -84.33°}{296.15 - j969.23} = 192.2\angle -11.32°\ \Omega.$$

14.102 Determine the voltage across the series combination of a resistor of 100 Ω and inductor of 100 mH when a current of $10e^{j4000t}$ mA flows through them.

Solution

The phasor form of current of ($10e^{j4000t}$) mA can be written as

$$I = 10\angle 0°\ \text{mA with angular velocity of 4000 rad/s.}$$

Impedance $Z = R + j\omega L = 100 + j \times 4000 \times 0.1 = 100 + j400 = 412.3\angle 75.96°\ \Omega$.

Voltage across the series combination is $= 10\angle 0° \times 10^{-3} \times 412.3\angle 75.96° = 4.123\angle 75.96°$ A.

or, we can say the voltage across the series combination is $4.123e^{\,j(4000t\,+\,75.96°)}$ V.

14.103 Determine the values of i_1, i_2 and i_3 in the circuit shown in Fig. 14.92 at $t = 0.2$s.

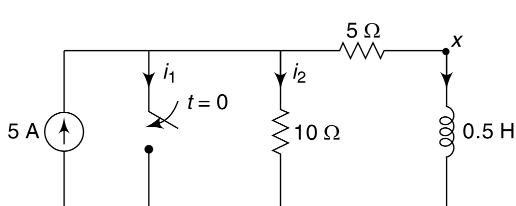


Fig. 14.92

Solution

For $t < 0$,

$$i_1 = 0$$

$$i_2 = 5 \times \frac{5}{5+10} = \frac{25}{15} = \frac{5}{3} \text{ A}$$

$$i_3 = 5 \times \frac{10}{10+5} = \frac{50}{15} = \frac{10}{3} \text{ A}$$

For $t > 0$, total current of 5 A from current source flows through the short circuited path and 10 Ω resistor is shorted out.

Hence for $t > 0$ (at $t = 0.2$ s) $i_2 = 0A$. Now, at $t < 0$ current through the inductor is

$$I_0 = i_3 = \frac{10}{3} \text{ A}.$$

Hence, at $t > 0$,

$$i_3 = I_0 e^{-t/\lambda}, \text{ where } \lambda = \frac{0.5}{5} = 0.1 \text{ s}.$$

∴ At $t = 0.2$ s

$$i_3 = \frac{10}{3} e^{-\frac{0.2}{0.1}} = 0.45 \text{ A}.$$

The current 0.45 completes its path through the short circuited path.

Hence, $i_1 = 5 - 0.45 = 4.55$ A at $t = 0.2$ s.

[At $t = 0+$, i_3 will continue to flow in the same direction before switching as the current through the inductance will not change instantly following switching, thus i_3 will have direction flowing from node x to node y during $t = 0(-)$ as well as during $t = 0(+)$.]

14.104 A voltage $v = 3000 \sin \omega t + 500 \sin 3\omega t + 200 \sin 5\, \omega t$ is applied to a series *R-L-C* circuit having a resistance of 15 Ω, capacitance of 50 μF and a variable inductance. Determine the value of the inductance that will give resonance with the third harmonic. Find the rms values of voltage and current with this value of inductance in the circuit. Assume $\omega = 300$ rad/s. Also find the rms value of the total current.

Solution

For resonance with the third harmonic,

$$(3\omega)^2 = \frac{1}{LC} \qquad \left[\because\ \omega_n = \frac{1}{\sqrt{LC}};\ \text{here } \omega_n = (3\omega)\right]$$

or $$(3 \times 300)^2 = \frac{1}{LC}$$

or $$L = \frac{1}{(900)^2 \times 50 \times 10^{-6}} = 0.0247 \text{ H}.$$

Rms value of voltage = $\frac{1}{\sqrt{2}}\sqrt{(3000)^2 + (500)^2 + (200)^2} = 2155.23$ V.

For the third harmonic

$$X_L\ (=X_C) = 3 \times 300 \times 0.0247 = 22.23\ \Omega$$

Peak value of third harmonic current is $\frac{500}{15}$ A, or, 33.33 A.

For fundamental frequency

$$Z_1 = 15 + j\left(\omega L - \frac{1}{\omega C}\right) = 15 + j\left(\frac{22.23}{3} - 3 \times 22.23\right)$$

$$= 15 - j59.28 = 61.15\angle{-75.8°} \text{ W}$$

Peak value of fundamental current is thus $\frac{3000}{61.15}$ A , 49.06 A.

For the fifth harmonic, we have

$$Z_5 = 15 + j\left(\frac{22.23}{3} \times 5 - \frac{3 \times 22.23}{5}\right)$$

$$= 15 + j23.712 = 28.06\angle 57.68°\Omega.$$

Peak value of the fifth harmonic current is $\left(\frac{200}{28.06}\text{ A}\right)$ or, 7.127 A.

$\therefore$ Rms value of the total current

$$\frac{1}{\sqrt{2}}\sqrt{(33.33)^2 + (49.06)^2 + (7.127)^2} = 42.25 \text{ A}.$$

14.105 A voltage $v = 100 \sin \omega t + 75 \sin\left(3\omega t + \frac{\pi}{3}\right) + 40 \sin\left(5\omega t + \frac{5\pi}{6}\right)$ is applied to a circuit of resistance 30 Ω and inductance 0.075 H. Derive (i) expression for current (ii) the rms value of current and voltage (iii) total power supplied and power factor. Assume $\omega = 314$ rad/s.

Solution

Fundamental reactance, $X_1(= \omega L) = 314 \times 0.075\ \Omega = 23.55\ \Omega$.

Fundamental impedance, $Z_1 = \sqrt{(30)^2 + (23.55)^2}\angle\tan^{-1}\left(\frac{23.55}{30}\right) = 38.14\angle 38.14°\ \Omega$.

Fundamental current, $i_1 = \frac{100}{38.14}\sin(\omega t - 38.14°) = 2.622 \sin(\omega t - 38.14°)$ A

rms value of fundamental current is $\frac{2.622}{\sqrt{2}} = 1.854$ A.

rms value of fundamental voltage is $\frac{100}{\sqrt{2}}$ V $= 70.71$ V.

Fundamental power $(= i_1^2 R) = (1.854)^2 \times 30 = 103.12$ W.

Similarly for third harmonic X_3 $(= 3\ \omega L) = 3 \times 23.55\ \Omega = 70.65\ \Omega$.

$$Z_3 = \sqrt{(30)^2 + (70.65)^2}\angle\tan^{-1}\left(\frac{70.65}{30}\right) = 76.755\angle 67°\ \Omega$$

$$i_3 = \frac{75}{76.755}\sin\left(3\omega t + \frac{\pi}{3} - 67°\right) = 0.977 \sin(3\omega t - 7°) \text{ A}.$$

rms value of current is $\frac{0.977}{\sqrt{2}}$ A $= 0.691$ A.

rms value of voltage is $\frac{75}{\sqrt{2}}$ A $= 53$ V.

Power $i_3^2 R = (0.691)^2 \times 30$ W $= 14.32$ W

For the 5th harmonic,

$$X_5(= 5\ \omega L) = 5 \times 23.55 = 117.75\ \Omega$$

$$Z_5 = \sqrt{(30)^2 + (117.75)^2}\angle\tan^{-1}\frac{117.75}{30} = 121.51\angle 75.7°\ \Omega$$

$$i_5 = \left(\frac{40}{121.51}\right)\sin\left(5\omega t + \frac{5\pi}{6} - 75.7°\right) = 0.329 \sin(5\omega t + 74.3°)$$

rms value of current is $\dfrac{0.329}{\sqrt{2}}$ A $= 0.2326$ A.

rms value of voltage is $\dfrac{40}{\sqrt{2}}$ A $= 28.28$ V.

Power $(i_5^2 R) = (0.2326)^2 \times 30$ W $= 1.623$ W.

(i) Expression for total current in amp. is $2.622 \sin(\omega t - 38.14°) + 0.977 \sin(3\omega t - 7°) + 0.329 \sin(5\omega t + 74.3°)$

(ii) rms value of total current is $= \dfrac{1}{\sqrt{2}}\sqrt{(2.622)^2 + (0.977)^2 + (0.329)^2} = 1.99$ A.

rms value of voltage is $= \dfrac{1}{\sqrt{2}}\sqrt{(100)^2 + (75)^2 + (40)^2}$ or, 92.8 V.

(iii) Total power supplied,

$$P = (103.12 + 14.32 + 1.623) = 119.063 \text{ W}$$

$$\text{Power factor} = \left(\frac{P}{V \times I}\right) = \frac{119.063}{92.8 \times 1.99} = 0.645.$$

14.106 A series *R-L-C* circuit consists of a resistor of 15 Ω, inductor 0.05 H and capacitor 100 μF. The current through the circuit is (5 sin 300 t). Determine the current, induced voltage across the inductor, and terminal voltage in the circuit at $t = 0.02$ s.

Solution

Current $i = 5 \sin(300t)$ A, $\omega = 300$ rad/s. $[\because i = i_{\max} \sin(\omega t)]$

Impedance of the circuit $Z = R + j\left(\omega L - \dfrac{1}{\omega C}\right)$

$$= 15 + j\left(300 \times 0.05 - \frac{10^6}{300 \times 100}\right)$$

$$= 15 + j(-18.33) = 23.68\angle{-50.70°}\ \Omega.$$

Terminal voltage $v\ (= Zi)$ $= 23.68 \times 5 \sin\left(300t - \dfrac{3.14}{180°} \times 50.7°\right)$

$$= 118.4 \sin(300t - 0.885) \text{ V}.$$

Voltage induced in the inductor is $\left(-L\dfrac{di}{dt}\right) = -0.05 \times 5 \times 300 \cos(300t)$

$$= -75 \sin\left(\omega t - \frac{\pi}{2}\right) \text{V}.$$

At $t = 0.02$ s, current is given by

$$i = 5 \sin 300 \times 0.02 = 5 \sin 6° = 0.523 \text{ A}.$$

Induced voltage across the inductor is $-75 \sin\left(300 \times 0.02 - \dfrac{\pi}{2}\right)$ V or, (72.02 V).

Terminal voltage, $v = 118.4 \sin(300 \times 0.02 - 0.885)$

$$= 118.4 \sin \frac{5.115 \times 180°}{3.14} = 118.4 \sin 293.2° = -108.812 \text{ V}.$$

14.107 A circuit consists of a coil of 20 μH inductance and 2 Ω resistance in parallel with a 1 μF capacitor. Determine the current input to the circuit for resonant frequency and 90% of resonant frequency when the supply is 230 V.

Solution

In a parallel ac circuit resonance occurs when the net susceptance of the circuit is zero.

$$\text{However, admittance } y = \frac{1}{R + j\omega L} + j\omega C$$

$$= \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C = \frac{R - j\omega L + j\omega C\,(R^2 + \omega^2 L^2)}{R^2 + \omega^2 L^2}$$

∴ At resonance,

$$C(R^2 + \omega^2 L^2) = L$$

or $$\omega^2 = \frac{L - CR^2}{CL^2} = \frac{1}{LC} - \frac{R^2}{L^2}$$

or $$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$\therefore \text{ At resonance, } y = \frac{R}{R^2 + \omega^2 L^2} = \frac{R}{R^2 + L^2\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)} = \frac{R}{R^2 + \left(\frac{L}{C} - R^2\right)}.$$

Here, $R = 2\ \Omega$, $L = 20 \times 10^{-6}$ H and $C = 1 \times 10^{-6}$ F.

Hence $$(y) = \frac{2}{2^2 + \left(\frac{20 \times 10^{-6}}{1 \times 10^{-6}} - 2^2\right)} = \frac{2}{4 + (20 - 4)} = 0.1 \text{ S}.$$

Current input for resonance frequency is $I = Vy = 230 \times 0.1 = 23$ A.
At 90% of resonant frequency, the frequency is

$$\omega_1 = 0.9\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} = 0.9\sqrt{\frac{10^{12}}{20 \times 1} - \frac{2^2}{400 \times 10^{-12}}}$$

$$= 0.9 \times 10^6 \sqrt{\frac{1}{20} - \frac{4}{400}} = 0.18 \times 10^6 \text{ rad/s}.$$

$$\therefore \quad y_1 = \frac{R + j\omega_1 (R^2 C + \omega_1^2 L^2 C - L)}{R^2 + \omega_1^2 L^2}$$

$$= \frac{2 + j\omega_1\{(4 \times 10^{-6} + (0.18 \times 20)^2\ 10^{-6} - 20 \times 10^{-6}\}}{2^2 + (0.18 \times 20)^2}$$

$$= (0.1179 - j0.032) \text{ S} = 0.122\angle{-15.185°} \text{ S}$$

Hence current input is Vy_1 (= 230 × 0.122) or, 28.06 A.

14.108 Determine the impedance of the circuit shown in Fig. 14.93 and the power consumed in each branch.

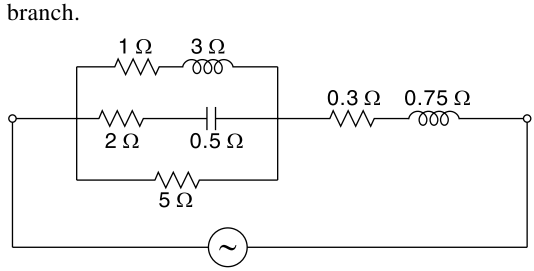


Fig. 14.93

Solution

The equivalent admittance of the three parallel branches is

$$(y) = \frac{1}{1+j3} + \frac{1}{2-j0.5} + \frac{1}{5}$$
$$= 0.316\angle -71.56° + 0.485\angle 14.036° + 0.2$$
$$= 0.77 - j0.182 = (0.79\angle -13.298°) \text{ S.}$$

The impedance of the whole circuit,

$$Z = \frac{1}{0.79\angle -13.298°} + 0.3 + j0.75$$
$$= 1.2658\angle 13.298° + 0.3 + j0.75 = (1.5318 + j1.041)\ \Omega$$

Total current , $(I) = \dfrac{230}{1.5318 + j1.041} = \dfrac{230}{1.852\angle 34.2°} = 124.2\angle -34.2°$ A.

Power consumed in (0.3 + j0.75) Ω branch is $(1124.2)^2 \times 0.3 = 4627.69$ W

Voltage across each parallel branch is, $\dfrac{I}{y} = \dfrac{124.2}{0.79} = 157.215$ V.

Current in three parallel branches

$$|I_1| = \frac{157.215}{\sqrt{1^2 + 3^2}} \text{ A} = 49.716 \text{ A}$$

$$|I_2| = \frac{157.215}{\sqrt{2^2 + (0.5)^2}} = 76.26 \text{ A}$$

$$|I_3| = \frac{157.215}{5} = 31.443 \text{ A.}$$

Power in three parallel branches

$$P_1 = (49.716)^2 \times 1 \text{ W} = 2471.68 \text{ W } (= 2.47 \text{ kW})$$
$$P_2 = (76.26)^2 \times 2 \text{ W} = 11631.17 \text{ W } (= 11.63 \text{ kW})$$
$$P_3 = (31.443)^2 \times 5 \text{ W} = 4943.3 \text{ W } (= 4.943 \text{ kW})$$

14.109 Two impedances $(10 + j8)\ \Omega$ and $(20 + j15)\ \Omega$ are connected in parallel across a 400 V, 50 Hz supply. Determine (i) the admittance of each branch and of the whole circuit factor (ii) the total current, power and power (iii) the capacitance which when connected in parallel with the whole circuit makes the power factor of the whole circuit unity.

Solution

(i) Admittance of the first branch $y_1 = \left(\dfrac{1}{10 + j8}\right)$ S

$$= 0.078\angle -38.66° = (0.061 - j0.0487) \text{ S}$$

Admittance of the second branch

$$y_2 = \left(\frac{1}{20 + j15}\right) \text{Siemens} = 0.04\angle -36.87° = (0.032 - j0.024) \text{ S}$$

Admittance of the whole circuit

$$y = (y_1 + y_2) = 0.061 + 0.032 - j(0.0487 + 0.024)$$
$$= 0.093 - j0.0727 = (0.118\angle -38°) \text{ S}$$

(ii) Total current $I (= Vy) = 400\angle 0° \times 0.118\ \angle -38° = 47.2\angle -38°$ A.

Power $P (= VI \cos\theta) = 400 \times 47.2 \cos 38° = 14877.64$ W.

Power factor $(\cos\theta) = \cos 38° = 0.788$ lagging.

(iii) If the current through capacitance branch is I_C then in this case to have unity p.f.,

$$I_C = I \sin \theta = 47.2 \sin 38° = 29.06 \text{ A.}$$

If X_C be the capacitive reactance then

$$X_C = \frac{V}{I_C} = \frac{400}{29.06} = 13.7\ \Omega$$

Hence capacitance, $C = \dfrac{1}{\omega X_C} = \dfrac{1}{2\pi \times 50 \times 13.76} = 231.37\ \mu\text{F}.$

14.110 A circuit has two parallel branches and the current through them is $15\angle 60°$ A and $25\angle -45°$ A. If the supply voltage is $230\angle 0°$ V, determine the kVA, KVAR and kW in each branch and in the whole circuit. Also find out the power factor of the combined load.

Solution

In the first branch

$$\text{kVA}_1 = \frac{230\angle 0° \times 15\angle 60°}{1000} = 3.45\angle 60°$$

$$\text{KVAR}_1 = 3.45 \sin 60° = 2.99 \text{ (leading)}$$

$$\text{kW}_1 = 3.45 \cos 60° = 1.725.$$

In the second branch

$$\text{kVA}_2 = \frac{230\angle 0° \times 25\angle -45°}{1000} = 5.750\angle -45°$$

$$\text{KVAR}_2 = 5.75 \sin 45° = 4.066 \text{ (lagging)}$$

$$\text{kW}_2 = 5.75 \cos 45° = 4.066.$$

For the whole circuit

$$\text{kVA} = 3.45\angle 60° + 5.75\angle -45° = 5.79 - j1.078 = 5.8895\angle -10.55°$$

$$\text{KVAR} = 5.8895 \sin (-10.55°) = 1.077 \text{ (lagging)}$$

[Also $\quad \text{KVAR} = 2.99 - 4.066 = -1.077$ (or 1.077 lagging)]

$$\text{kW} = 5.8895 \cos 10.55° = 5.79$$

[Also, $\quad \text{kW} = 1.725 + 4.066 = 5.79$].

14.111 A coil has a resistance of 8 Ω and an inductance of 0.05 H. Determine the parameters of a shunt circuit such that the total current is 25 A at 150 V for all frequencies.

Solution

The impedance of the coil $Z_1 = (8 + j\omega 0.05)\ \Omega$

If the total current is same for all frequencies the circuit will operate under resonant condition. Hence the parallel circuit must be capacitive. Let the impedance of the parallel circuit be $Z_2 = \left(R - \dfrac{j}{\omega C}\right)\Omega$

The parallel combination is having impedance:

$$\frac{(8 + j\omega \cdot 0.05)\left(R - \dfrac{j}{\omega C}\right)}{8 + R + j\left(\omega \cdot 0.05 - \dfrac{1}{\omega C}\right)} = \frac{8R + \dfrac{0.05}{C} + j\left(0.05\, r_\omega - \dfrac{8}{\omega C}\right)}{8 + R + j\left(\omega \cdot 0.05 - \dfrac{1}{\omega C}\right)}$$

This will be independent of frequency if

$$\left(0.05\, R\omega - \frac{8}{\omega C}\right) = 0 \quad \text{and} \quad \left(\omega (0.05) - \frac{1}{\omega C}\right) = 0.$$

i.e. $$R = \frac{8}{0.05\,\omega^2 C} \text{ and } \omega^2 C = \frac{1}{0.05}.$$

Hence, $$R = \frac{8}{0.05 \times \dfrac{1}{0.05}} = 8\ \Omega$$

Total impedance is $\left[\dfrac{8R + \dfrac{0.05}{C}}{8+R}\right]$ i.e., $\left[\dfrac{64 + \dfrac{0.05}{C}}{8+8}\right]\Omega.$

$\therefore$ $$Z = \frac{64 + (0.05/C)}{16}\ \Omega.$$

Also, $$Z = \frac{V}{I} = \frac{150}{25} = 6\ \Omega$$

$\therefore$ $$6 = \frac{64 + (0.05/C)}{16}$$

or $C = 1.56$ mF.

Hence the resistance and capacitance of the shunt circuit is 8 Ω and 1.56 mF respectively.

14.112 Two impedances having the same magnitude are joined in parallel. The p.f of one impedance is 0.7 and that of other is 0.6. Determine the p.f. of the combination.

Solution

Let (Z) be the magnitude of each impedance. For the first impedance we have

$$\cos\theta_1 = 0.7 \text{ and hence } \sin\theta_1 = 0.714$$

Hence, $Z_1 = Z(0.7 + j0.714)$

Similarly, $Z_2 = Z\{0.6 + j\sin(\cos^{-1} 0.6)\} = Z(0.6 + j0.8)$

The impedance of the parallel combination

$$\frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{Z^2(0.7 + j0.714)(0.6 + j0.8)}{Z(0.7 + 0.6 + j0.714 + j0.8)}$$

$$= Z \cdot \frac{0.999\angle 98.7^\circ}{1.995\angle 49.35^\circ} = Z(0.5\angle 49.35^\circ)\ \Omega$$

Hence the power factor of the combination is cos 49.35° = 0.65.

14.113 An inductive coil is connected across a variable frequency alternating current source of 230 V. When the frequency is 100 Hz, the current is 20 A and when the frequency is 60 Hz, the current is 25 A. Determine the coil parameters and the time constant of the coil.

Solution

Let R and L be the resistance and inductance of the coil.

When the frequency is 100 Hz

$$\frac{230}{20} = \sqrt{R^2 + (2\pi \times 100L)^2}$$

or $$R^2 + 394784.2\,L^2 = 132.25 \qquad \text{(i)}$$

When the frequency is 60 Hz

$$\frac{230}{25} = \sqrt{R^2 + (2\pi \times 60L)^2}$$

or $$R^2 + 142122.3L^2 = 84.64 \qquad \text{(ii)}$$

Solving equation (i) and equation (ii) we get

$$L = 0.01373 \text{ H}$$

and $$R = 7.61\ \Omega.$$

Time constant of the coil is $\left(\dfrac{L}{R}\right) = \dfrac{0.01373}{7.61}\text{ s} = 0.0018 \text{ s} = 1.8 \text{ ms}.$

14.114 An inductive coil takes 20 A and dissipates 1500 W when connected to a 230 V, 50 Hz. supply. Determine the impedance, resistance, reactance and power factor of the circuit.

Solution

Given
$$I = 20 \text{ A}$$
$$P = 1500 \text{ W}$$
$$V = 230 \text{ V}$$

Impedance, $$Z| = \frac{|V|}{|I|} = \frac{230}{20} = 11.5\ \Omega.$$

$$P = VI \cos \theta.$$

Hence p.f. $$(\cos \theta) = \frac{P}{VI} = \frac{1500}{230 \times 20} = 0.326.$$

Resistance, $R = Z \cos \theta = 11.5 \times 0.326 = 3.749\ \Omega$

Reactance, $X = Z \sin \theta = 11.5 \times \sin (\cos^{-1} 0.326) = 10.87\ \Omega$

14.115 An iron cored coil of resistance 8 Ω takes 12 A when connected to a 230 V, 50 Hz mains. The power dissipated is 1500 W. Determine the iron loss, inductance and power factor. Assume the coil to be equivalent to a series impedance.

Solution

Resistance of the coil, $R = 8\ \Omega$.
$$I = 12 \text{ A}$$
$$V = 230 \text{ V}$$

Impedance $$Z = \frac{|V|}{|I|} = \frac{230}{12}\ \Omega = 19.17\ \Omega$$

Ohmic loss is, $I^2R = (12)^2 \times 8 = 1152$ W

Total power dissipated is 1500 W.

Hence iron loss = (1500 – 1152) = 348 W.

Total resistance (resistance of coil + resistance of core) = $\dfrac{1500}{(12)^2}\ \Omega = 10.42\ \Omega.$

Inductive reactance $X = \sqrt{(19.17)^2 - (10.42)^2} = 16.09\ \Omega.$

Inductance = $\dfrac{16.09}{2\pi \times 50} = 0.05$ H.

p.f. of the circuit = $\dfrac{10.42}{19.17} = 0.543.$

14.116 An alternating voltage of (100 + j80) is applied to a circuit and current through the circuit is (10 + j15) A. Determine the impedance of the circuit, power consumed and the phase angle.

Solution

Impedance $$Z = \frac{V}{I} = \frac{100 + j80}{10 + j15} = \frac{128.06\angle 38.66^\circ}{18.03\angle 56.31^\circ} = 7.103\angle{-17.65^\circ}\ \Omega.$$

Power consumed $P = VI \cos\theta = 128.06 \times 18.03 \cos 17.65° = 2200$ W.
Phase angle is 17.65°.

14.117 An attracted armature type of relay operating at 250 V, 50 Hz single phase ac supply draws a current of 4 A at p.f. 0.15 lag to attract the plunger from open to closed position. The same relay draws 1.5 A at p.f. of 0.07 lag when the armature operates (i.e. in closed position). How much energy is spent in operating the relay?

Solution

When the plunger is at open position, the impedance

$$Z = \frac{V}{I} = \frac{250}{4}\,\Omega = 62.5\ \Omega.$$

Power factor $(\cos\theta) = 0.15$
Inductive reactance $X = Z \sin\theta = 62.5 \sin(\cos^{-1} 0.15) = 61.8\ \Omega$.

Inductance $L = \dfrac{61.8}{2\pi \times 50} = 0.196$ H

Hence energy stored is $\left(\dfrac{1}{2}LI^2\right) = \dfrac{1}{2} \times 0.196 \times (4)^2 = 1.57$ J

When the plunger is at closed position

Impedance $Z = \dfrac{V}{I} = \dfrac{250}{1.5}\,\Omega = 167\ \Omega$

Power factor $(\cos\theta) = 0.07$.
Inductive reactance $X = Z \sin\theta = 167 \sin(\cos^{-1} 0.07) = 166.5\ \Omega$.

Inductance $L = \dfrac{166.5}{2\pi \times 50}$ H $= 0.53$ H.

Hence energy stored is $\left(\dfrac{1}{2}LI^2\right) = \dfrac{1}{2} \times 0.53 \times (1.5)^2 = 0.59$ J.

Energy spent in operating the relay is (1.57 – 0.59) or, 0.98 J.

14.118 The load taken from a single-phase supply consists of a filament lamp load of 10 kW at unity power factor, motor load of 80 kVA at 0.8 p.f. (lagging) and motor load of 40 kVA at 0.7 p.f. (leading). Find the total load taken from the supply in kW and in kVA and the p.f. of the combined load. Also calculate the main current if the supply voltage is 250 V.

Solution

Load (a): Apparent power, $S_a = \dfrac{\text{Active power}}{\text{Power factor}} = \dfrac{10}{1} = 10$ kVA.
Active power $P_a = 10 \times 10 =$ kW
Reactive power, $Q_a = S_a \times \sin\theta = 10 \times 0 = 0$ KVAR.

Load (b): Apparent power, $S_b = 80$ kVA at a power factor of 0.8 (lagging).
Active power, $P_b = 80 \times 0.8 = 64$ kW
Reactive power, $Q_b = 80 \times 0.6 = 48$ KVAR (lagging).

Load (c): Apparent power, $S_c = 40$ kVA. at a power factor of 0.7 (leading).
Active power, $P_c = 28$ kW
Reactive power, $Q_c = 40 \times 0.7143 = 28.57$ KVAR

$\therefore$ Total power taken from the supply (the net apparent power)
$= 10 + 80 + 40 = 130$ kVA.
Total active power $= 10 + 64 + 28 = 102$ kW
Total reactive power $= 0 + 48 - 28.57 = 19.40$ KVAR

Power factor of the combined load is

$$\frac{\text{Total active power}}{\text{Total apparent power}} = \frac{102}{130} = 0.785 \text{ (lag)}$$

$$\text{Main current} = \frac{\text{Total apparent power}}{\text{voltage}} = \frac{130 \times 10^3}{250} = 520 \text{ A.}$$

14.119 A balanced three-phase three-wire distribution system is terminated with two delta connected loads in parallel. Load 1 draws 25 kVA at a lagging p.f. of 0.85 and load 2 absorbs 20 kW at lagging p.f. of 0.8. Assuming no line resistance and line voltage V_{ab} to be $400\angle 30°$ V, determine the total power drawn by the loads, load current I_{ab_1} for the first load and I_{ab_2} for the second load.

Solution

Total power drawn by the loads = 25 × 0.85 + 20 = 41.25 kW.

The first load draws an apparent power of 25/3 = 8.33 kVA per phase at 0.85 p.f. lagging.

$$\text{Hence,} \qquad I_{ab_1} = \left(\frac{8.33 \times 10^3 \angle \cos^{-1} 0.85}{400\angle 30°}\right) = 20.825\angle{-1.788°} \text{ A.}$$

The second load draws an apparent power of $\dfrac{20}{3 \times 0.8}$ = 8.33 kVA per phase at 0.8 p.f. lagging.

$$\text{Hence,} \qquad I_{ab_2} = \left(\frac{8.33 \times 10^3 \angle \cos^{-1} 0.8}{400\angle 30°}\right) = 20.825\angle{-6.87°} \text{ A.}$$

14.120 In a power system each phase of a balanced three-phase delta connected load is having a 0.1 H inductor in series with the parallel combination of 10 μF capacitor and a 150 Ω resistance. Assuming a phase voltage magnitude of 230 V at 50 Hz, determine the phase and line current and total power absorbed by the load.

Solution

$$\text{Capacitive reactance } X_C = \frac{1}{j2\pi \times 50 \times 10 \times 10^{-6}} \,\Omega = -j318.31 \,\Omega$$

Inductive reactance $X_L = j2\pi \times 50 \times 0.1 = j31.416\ \Omega$

$$\text{Net impedance } Z = j31.416 + \frac{150(-j318.31)}{150 - j318.31}$$

$$= j31.416 + \frac{47746.5\angle{-90°}}{351.88\angle{-64.768°}}$$

$$= j31.416 + 135.69\angle{-25.232°} = 122.74 - j26.426 = 125.55\angle{-12.15°}\ \Omega.$$

$$\text{Phase current } I_p = \frac{230}{125.55} \text{ A} = 1.832 \text{ A.}$$

Line current $I_L = \sqrt{3} \times 1.832 = 3.173$ A.

Total power absorbed by the load = 3 × 230 × 1.832 cos (12.15°) = 1235.76 W.

14.121 Find the amplitude of the line current in a three-phase power system with a line voltage of 420 V, 50 Hz. when it supplied 1500 W to a delta connected load at a lagging power factor of 0.85 lagging.

Solution

If I_L is the line current then we have $\sqrt{3}\, V_L I_L \cos\theta = 1500$

or $$I_L = \frac{1500}{\sqrt{3}\times 420\times 0.85} = 2.4258 \text{ A.}$$

The amplitude or the maximum value of the line current is $(\sqrt{2} \times 2.4258) = 3.43$ A.

14.122 In a balanced three-phase four wire-power distribution system three balanced star connected loads are connected. The first load draws power of 10 kW at unity p.f., second load draws 15 kVA at 0.8 p.f. lagging and the third load draws 12 kW at 0.9 p.f. lagging. The phase voltage applied at the load terminals is 230 V, 50 Hz. Each line has a resistance of 0.2 Ω while the neutral has a resistance of 1.5 Ω. Determine the total power drawn by the loads, the combined p.f. of the loads, total power lost in the four lines, phase voltage and power factor of the source.

Solution

Total power drawn by the loads = 10 kW + (15 × 0.8) kW + 12 kW = 34 kW.

The complex power drawn by the loads

$$= (10 + j0) + [(15 \times 0.8) + j15 \sin(\cos^{-1} 0.8)] + [12 + j12 \sin(\cos^{-1} 0.9)]$$
$$= (10 + j0) + (12 + j9) + (12 + j5.23) = (34 + j14.23) = 36.858\angle 22.71° \text{ kVA.}$$

Hence the combined p.f. of the loads is (cos 22.71°) or, 0.922 lagging.

The per phase complex power is given by

$$S = \frac{36.858}{3}\angle 22.71° = 12.286\angle 22.71° \text{ kVA.}$$

Again phase voltage at the loads is 230 V.

Hence, phase current (= line current) = $\frac{12.286}{230} \times 10^3 = 53.417$ A.

Power loss in each of the three lines (P_{loss}) = $(53.417)^2 \times 0.2 = 570.675$ W.

Total power loss in the three lines ($3P_{\text{loss}}$) = 3 × 570.675 = 1712.025 W.

As the loads are balanced, current in the neutral wire is zero and hence power in the neutral wire is zero.

The phase voltage at the source is (230 + 53.417 × 0.2) or, 240.683 V.

The equivalent load impedance is given by

$$Z_L = \frac{V_{\text{ph}}}{I_p} = \frac{230}{\dfrac{12.286\times 10^3}{230}\angle -22.71°} = 4.3057\angle 22.71° \ \Omega.$$

$$\left[\begin{aligned} &\because\ S = VI^* \\ &\text{or, } I^* = \frac{S}{V} = \frac{12.286\times 10^3\ \angle 22.71°}{230} \\ &\because\ I = \frac{12.286\times 10^3}{230}\angle -22.71° \end{aligned}\right]$$

But the line is having a resistance of 0.2 Ω.

∴ Net impedance across the source is

$$Z = (0.2 + j0) + [4.3057 (\cos 22.71° + j\sin 22.71°)]$$
$$= (4.172 + j1.66)\ \Omega = 4.491\angle 21.697\ \Omega$$

∴ The source is operating at a p.f. of cos 21.697° or, 0.929 (lag.)

14.123 A balanced three-phase three-wire star connected system has a line voltage of 440 V. One inductive load of (3 + j6) and another capacitive load of (5 – j3) Ω are connected in parallel in each phase. Determine the phase voltage, line current, total power drawn by the load and the power factor at which the load is operating.

Solution

The phase voltage is $\left(\dfrac{440}{\sqrt{3}}\right)$V = 254 V.

Impedance per phase is $\dfrac{(3+j6)(5-j3)}{3+j6+5-j3} = \dfrac{33+j21}{8+j3} = \dfrac{39.115\angle 32.47^\circ}{8.54\angle 20.55^\circ} = 4.58\angle 11.92^\circ$ Ω.

Line current (= Phase current) = $\dfrac{254}{4.58}$ A = 55.458 A.

Total power drawn by the load = 3 × phase voltage × phase current × power factor
= 3 × 254 × 55.458 cos (11.92°) = 41347.76 W
= 41.348 kW.

Power factor (cos 11.92°) = 0.978, lagging.

14.124 A balanced three-phase three-wire system has a star connected load. Each phase contains 50 Ω and (j100 Ω) in parallel. Assume positive phase sequence with V_{ab} = 400∠0°. Determine V_{an}, I_a and the total power drawn by the load.

Solution

The phasor diagram of V_{an} V_{bn}, V_{cn}, V_{ab} are shown in Fig. 14.94.

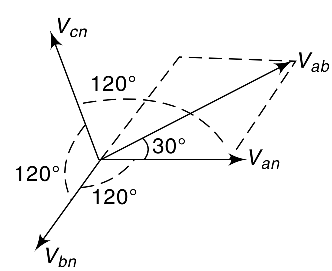


Fig. 14.94

$$V_{ab} = 400\angle 0^\circ \text{ V}$$

Hence $$V_{an} = \frac{400}{\sqrt{3}}\angle -30^\circ = 230.9\angle -30^\circ \text{ V}$$

$$Z = \frac{(50\times j100)}{(50+j100)} = \frac{100\angle 90^\circ}{2.236\angle 63.43^\circ}$$
$$= 44.7\angle -26.565^\circ\ \Omega.$$

As the load is star connected

$$I_a = \frac{230.9\angle -30^\circ}{44.7\angle -26.565^\circ} = 5.1655\angle -3.435^\circ \text{ A.}$$

Power absorbed by the load of each phase = 230.9 × 5.1655 cos (–30° + 3.435°)
= 1066.796 W

Hence total power drawn by load in all phases = (3 × 1066.796 × 10^{-3}) kW = 3.2 kW.

14.125 A three-phase, three-wire supply is connected to a load consisting of three identical resistors. By how much is the power reduced if one of the resistors be removed (i) when the load is star connected (ii) when the load is delta connected.

Solution

(i) *When the load is star connected*

Line current = Phase current

$$I = \frac{V}{\sqrt{3}R}, \text{ where } V \text{ is the line voltage.}$$

$$\text{Total power } \left[= 3\left(\frac{V}{\sqrt{3}R}\right)^2 R\right] = \frac{V^2}{R}.$$

When one of the resistors is removed as shown in Fig. 14.95(a) and Fig. 14.95(b).

Line current $I = \dfrac{V}{2R}$

Total power $= 2\left(\dfrac{V}{2R}\right)^2 R = \dfrac{V^2}{2R}.$

Hence, reduction in power is

$$\left(\frac{V^2}{R}\right)-\left(\frac{V^2}{2R}\right)=\frac{V^2}{R} \text{ or } 50\%.$$

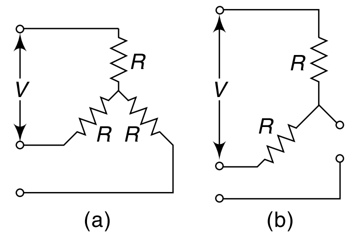


Fig. 14.95

(ii) *When the load is delta connected,*

Phase current $I = \dfrac{V}{R}$

Total power is $\left[3\left(\dfrac{V}{R}\right)^2 R\right]$ or $\dfrac{3V^2}{R}.$

When one resistor is removed as shown in Fig. 14.96(a) and Fig. 14.96(b), then current in each resistor is $\dfrac{V}{R}$.

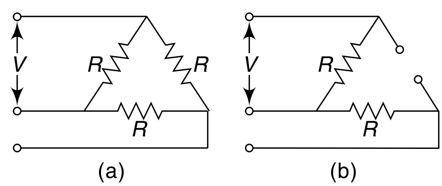


Fig. 14.96

Total power is $2\left(\dfrac{V}{R}\right)^2 R$ or $\dfrac{2V^2}{R}$

Hence reduction in power is $\left(\dfrac{3V^2}{R}\right)-\left(\dfrac{2V^2}{R}\right)$, or, $\dfrac{V^2}{R}$, i.e. 33.3%.

14.126 A three-phase star connected alternator supplied a 1500 H.P. delta connected induction motor having a power factor 0.8 and efficiency 90%. Determine the active and reactive components of the phase current of alternator and motor. Assume line voltage to be 1000 V.

Solution

Output of the induction motor is = (1500 × 735.5) W = 1103250 W

Input of the induction motor is $\dfrac{1103250}{0.9}$ W, or 1225.83 kW.

If the line current of induction motor be I then

$$\sqrt{3}\ VI \cos\theta = 1225.83 \times 10^3$$

or
$$I = \frac{1225.83\times10^3}{\sqrt{3}\times1000\times0.8}\text{ A } = 884.69 \text{ A}$$

Line current of the alternator is also (884.69) A

As alternator is star connected, hence phase current of alternator (= line current) = 884.69 A

Active component of current is (884.69 × 0.8) = 707.752 A

Reactive component of current is [884.69 sin ($\cos^{-1}$ 0.8)] = 530.8 A

As motor is delta connected, we have,

Phase current of motor $= \dfrac{884.69}{\sqrt{3}}$ A = 510.776 A

Active component of current = (510.776 × 0.8) 408.62 A.

Reactive component of current = (510.776 × 0.6) 306.46 A.

14.127 Determine the readings in each wattmeter in the circuit shown in Fig. 14.97 when the network is connected to 440 V, 50 Hz supply.

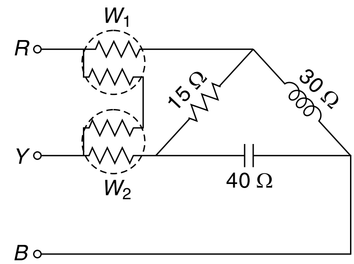


Fig. 14.97

Solution

The current measured by wattmeter W_1 is

$$I_R = \frac{E_{RY}}{15} + \frac{E_{RB}}{j30}$$

$$= \frac{440\angle 0^\circ}{15} + \frac{(-440\angle 120^\circ)}{30\angle 90^\circ}$$

$$= 29.33 - 14.67\angle 30^\circ = 16.625 - j7.335 = 18.17\angle -23.8^\circ \text{ A.}$$

The current measured by wattmeter W_2 is

$$I_Y = \frac{E_{YR}}{15} + \frac{E_{YB}}{-j40} = \frac{-440\angle 0^\circ}{15} + \frac{440\angle -120^\circ}{40\angle -90^\circ}$$

$$= -29.33 + 11\angle -30^\circ = -19.8 - j5.5 = 20.55\angle -164.47^\circ \text{ A.}$$

The voltage measured by wattmeter W_1 is

$$V_1 = \frac{1}{2}E_{RY} = \frac{1}{2} \times 440\angle 0^\circ = 220\angle 0^\circ \text{ V.}$$

The voltage measured by wattmeter W_2 is

$$V_2 = \frac{1}{2}E_{YR} = -220\angle 0^\circ \text{ V} = 220\angle 180^\circ \text{ V.}$$

Hence reading $W_1 = I_R V_1 \cos\theta_1 = 18.17 \times 220 \cos(23.8^\circ) = 3657.46$ W = 3.657 kW.

Reading $W_2 = I_Y V_2 \cos\theta_2 = 20.55 \times 220 \cos(180^\circ - 164.47^\circ) = 4355.94$ W = 4.356 kW.

14.128 A resistor of 500 Ω and capacitance of 10 μF are connected in series across lines *R* and *Y* of a 440 V, 50 Hz mains. Find out the voltage between the junction of the resistor and capacitor and line *B*.

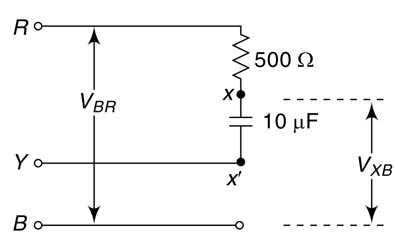


Fig. 14.98

Solution

The circuit diagram is shown in Fig. 14.98.

Taking E_{RY} as the reference, the current through the resistance and capacitance is

$$I = \frac{440\angle 0^\circ}{500 + \dfrac{10^6}{j2\pi \times 100 \times 10}}$$

$$= \frac{440}{500 - j159.23} = 0.8385\angle 17.66^\circ \text{ A.}$$

Considering the loop *R XX′ BR*,

$$500 \times 0.8385\angle 17.66^\circ + V_{XB} + V_{BR} = 0$$

or $\quad V_{XB} = -V_{BR} - 419.25\angle 17.66^\circ$

$$= \frac{-440}{\sqrt{3}}\angle 120^\circ - 419.25\angle 17.66^\circ = (-272.5 - j347.187) = 441.35\angle -128.13^\circ \text{ V.}$$

14.129 A symmetrical 440 V, three-phase system supplies a star connected load with phase impedances $Z_R = j10$ Ω, $Z_Y = -j15$ Ω and $Z_B = 20$ Ω. Determine the voltage drop across each branch and the potential of the neutral point to earth.

Solution

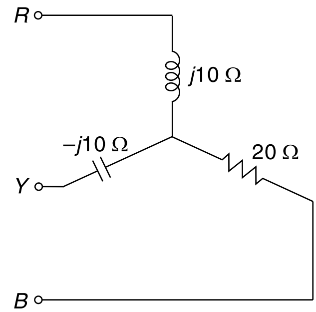


Fig. 14.99

The three-phase system is shown in Fig. 14.99. Considering E_R as reference, we can write

$$E_R = \frac{440}{\sqrt{3}}\angle 0° \text{ V}; \quad E_Y = \frac{440}{\sqrt{3}}\angle -120°$$

and $$E_B = \frac{440}{\sqrt{3}}\angle -240°.$$

Current through the R branch $I_R = \frac{440}{\sqrt{3}}\angle 0°/j10$

$= 25.4\angle -90°$ A.

Current through the Y branch $I_Y = \left(\frac{440}{\sqrt{3}}\angle -120°\right)\Big/(-j15) = 16.935\angle -30°$ A.

Current through the B branch $I_B = \frac{440}{\sqrt{3}}\angle -240°/20 = 12.7\angle -240°$ A.

The resultant current $I = I_R + I_Y + I_B = 25.4\angle -90° + 16.935\angle -30° + 12.7\angle -240°$

$= -j25.4 + 14.67 - j8.467 - 6.35 + j11$

$= 8.32 - j22.87 = 24.336\angle -70°$

Potential of the neutral point to earth

$$E_N = \frac{24.336\angle -70°}{\frac{1}{j10} + \frac{1}{-j15} + \frac{1}{20}} = \frac{24.336\angle -70°}{0.05 - j0.1 + j0.067} = \frac{24.336\angle -70°}{0.05 - j0.033}$$

$= 404.975\angle -36.575°$ V.

Voltage across load in phase R is $(E_{RN}) = E_R - E_N = \frac{440}{\sqrt{3}}\angle 0° - 404.975\angle -36.575°$

$= -73.22 + j241.31$

$= 252.17\angle 106.88°$ V

Voltage across load in phase Y is $(E_{YN}) = E_Y - E_N = \frac{440}{\sqrt{3}}\angle -120° - 404.975\angle -36.575°$

$= 450.724\angle 177.29°$ V.

Voltage across load in phase B is

$$(E_{BN}) = E_B - E_N = \frac{440}{\sqrt{3}}\angle 240° - 404.975\angle -36.575° = 646\angle 134.43° \text{ V}.$$

14.130 Three equal inductors connected in star take 10 kW at power factor of 0.85 when connected to a 440 V, 3-wire supply. If one inductor is short circuited, determine the line currents.

Solution

Line current before short circuit is $I_L = \frac{10{,}000}{\sqrt{3}\times 440\times 0.85} = 15.437$ A

Impedance in each phase $Z = \frac{440/\sqrt{3}}{15.437}\,\Omega = 16.456\ \Omega$

Power factor is $\cos\theta = 0.85$.

Hence, $\theta = 31.788°$.

Suppose phase B is short-circuited. Hence star point N and phase B are now at same potential (ground potential).

The three line voltages before short are

$$V_{12} = 440\angle 0° \text{ V}$$
$$V_{23} = 440\angle -120° \text{ V}$$
$$V_{31} = 440\angle -240° \text{ V}$$

Since one inductor is shorted,

Hence, $$I_1 = \left(= \frac{V_{13}}{Z}\right) = \frac{-440\angle -240°}{16.456\angle 31.788°} \quad [\because V_{13} = -V_{31}]$$
$$= -26.738\angle -271.788°$$
$$= -0.834 - j26.72 = 26.738\angle -91.787° \text{ A}$$
$$I_2 = \frac{V_{23}}{Z} = \frac{440\angle -120°}{16.456\angle 31.788°} = 26.738\angle -151.788° \text{ A}$$

and $$I_3 = -(I_1 + I_2) = 0.834 + j26.72 + 23.56 + j12.64$$
$$= 24.394 + j39.36 = 46.3\angle 58.21° \text{ A.}$$

14.131 Three equal impedances of 10 Ω each and with a phase angle of 30° (lagging) makes a load on a three-phase alternator generating 100 V per phase. Calculate the current per line and the total power when connected as (i) alternator in star and load in star, (ii) load in delta but alternator in star, (iii) alternator as well as load in delta and (iv) alternator in delta but load in star.

Solution

(i) Given: Phase voltage = 100 V, impedance per phase of load = 10 Ω.

$\therefore$ Load current per phase = $\frac{100}{10}$ = 10 A

Line current (= phase current) = 10 A

Total power $P = \sqrt{3}\, VI \cos\phi$.

But, $V(= \sqrt{3}\, V_{Ph})$ = 1.732 × 100 = 173.2 V and I = 10 A.

$$\therefore \quad P = \frac{\sqrt{3} \times 173.2 \times 10 \times \cos 30°}{1000} = 2.598 \text{ kW.}$$

(ii) Line voltage = ($\sqrt{3}$ × 100) V

$\therefore$ Voltage per phase of load = $\sqrt{3}$ × 100 V.

Current per phase of load = $\frac{V_{ph}}{Z_{ph}} = \frac{\sqrt{3} \times 100}{10}$ A

Line current (= phase current) = $\sqrt{3} \times \sqrt{3} \times 10$ = 30 A

Total Power, $P = \sqrt{3}\, VI \cos\phi = \frac{\sqrt{3} \times \sqrt{3} \times 100 \times 30 \times 0.866}{1000}$ = 7.794 kW.

(iii) Line voltage = 100 V

Votage per phase of load = 100 V

Current per phase of load = $\frac{100}{10}$ = 10 A

Line current ($\sqrt{3}\, I_{ph}$) = 1.732 × 10 = 17.32

Total power = $\sqrt{3}\, VI \cos\phi = \frac{\sqrt{3} \times 100 \times \sqrt{3} \times 10 \times 0.866}{1000}$ = 2.598 kW

(iv) Line voltage = 100 V

Votage per phase at load terminals = $\dfrac{100}{\sqrt{3}}$ V

Current per phase of load = $\dfrac{100}{\sqrt{3}\times 10} = \dfrac{10}{\sqrt{3}}$ A

Line current (= Phase current) = $\dfrac{10}{\sqrt{3}}$ = 5.77 A

Total power $P = \sqrt{3}\, VI \cos\phi = \dfrac{\sqrt{3}\times 100\times 10\times 0.866}{1000\times\sqrt{3}}$ = 0.866 kW.

14.132 A three-phase 500 V, 50 Hz, star connected alternator supplies a star connected induction motor which develops 45 kW. The efficiency of the motor is 88% and the p.f. is 0.9 (lagging). The efficiency of the alternator at this load is 80%. Calculate (i) the line current (ii) the output power of the alternator, and (iii) the power output of the prime mover.

Solution

Output from motor (= 45 kW) = 45000 W.
η (efficiency) of motor = 88%

Input to motor = 45000 × $\dfrac{100}{88}$ = 51140 W.

Since $P = \sqrt{3}\, VI \cos\phi$,

(i) Line current $I = \dfrac{51140}{\sqrt{3}\times 500\times 0.9}$ = 65.6 A

(ii) Output from the alternator (= input to motor) = 51.14 kW

(iii) Input to alternator = $\dfrac{51140\times 100}{80}$ Watts = 64 kW

This is the output power of the primemover.

14.133 The balanced load in Fig. 14.100 is fed by a balanced three-phase system having $V_{ab} = 200\angle 0°$ V (rms). Determine the reading of each wattmeter and the total power drawn by the load.

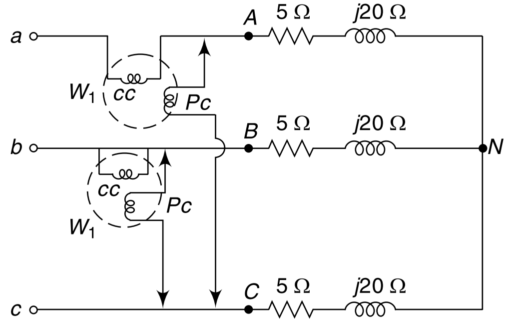


Fig. 14.100

Solution

The current coil of W_1 is measuring current I_{aA} and the potential coil is measuring voltage V_{AC} or V_{aC}.

Also, $V_{ab} = 200\angle 0°$ V

$V_{bc} = 200\angle -120°$ V

$V_{ca} = 200\angle 120°$ V

(or $V_{ac} = 200\angle -60°$ V).

The phasor diagram of the line and phase voltages is shown in Fig. 14.101.

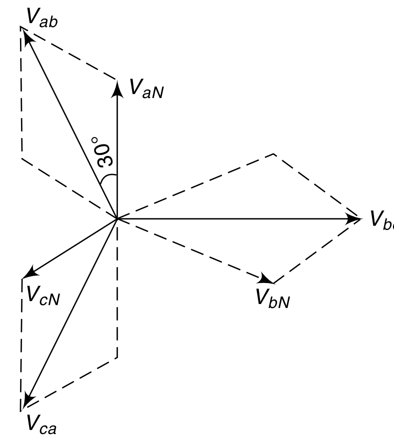


Fig. 14.101

The phase current

$$I_{aA} = \frac{V_{aN}}{(5 + j20)} = \frac{200/\sqrt{3}\angle -30°}{20.615\angle 75.96°}$$

$$= 5.6\angle -105.96° \text{ A}$$

Similarly, $$I_{bB} = \frac{V_{bN}}{5 + j20} = \frac{200/\sqrt{3}\angle -150°}{20.615\angle 75.96°}$$

$$= 5.6\angle -225.96° \text{ A}$$

Power measured by W_1 is

$$W_1 = |V_{aC}|\ |I_{aA}| \cos(-60° + 105.96°)$$

$$= 200 \times 5.6 \cos 45.96° = 778.58 \text{ W}$$

Power measured by W_2 is

$$W_2 = |V_{bC}|\ |I_{bB}| \cos(-120° + 225.96°) = 200 \times 5.6 \cos 105.96° = -307.96 \text{ W}$$

Total power drawn by the load is 778.58 W – 307.96 W = 470.62 W.

14.134 A dc genrator generates an emf of 200 V when driven at 1000 rpm where the flux per pole is 20 mwb. If the speed is now increased to 1100 rpm and at the same time the flux per pole is reduced to 19 mwb, what will be the induced emf?

Solution

Let E_1, ϕ_1 and N_1 be the initial values and E_2, ϕ_2 and N_2 be the new values of emf, flux and speed respectively. We know for a dc generator,

$$E \propto (\phi N)$$

or $$E = K\phi N, \text{ } K \text{ being the machine constant.}$$

In our case,

$$\frac{E_2}{E_1} = \frac{K\,\phi_2\,N_2}{K\,\phi_1\,N_1} \quad \therefore E_2 = \frac{E_1\,\phi_2\,N_2}{\phi_1\,N_1}$$

$$\therefore \quad E_2 = \frac{200 \times 19 \times 10^{-3} \times 1100}{20 \times 10^{-3} \times 1000} = 209 \text{ V.}$$

14.135 The open circuit voltage of a dc shunt generator is 130 V. Under load condition, the terminal voltage drops to 125 V. Calculate the load current if the field and armature resistances are 20 ohm and 0.03 ohm respectively. Ignore armature reaction and brush drop.

Solution

Let the load current be I_L.

$\therefore$ The current flowing through the field winding when the generator is loaded can be obtained as

$$I_{sh} = \frac{V}{R_f} = \frac{125}{20} = 6.25 \text{ A.}$$

Armature current $I_a = I_L + I_{sh} = (I_L + 6.25)$ A

$\therefore$ Armature drop $= I_a \cdot R_a = (I_L + 6.25) \cdot R_a = (130 - 125)$ V

or $$I_L + 6.25 = \frac{5}{R_a} = \frac{5}{0.03} = 166.66 \text{ A}$$

$\therefore$ $$I_L = 166.66 - 6.25 = 160.41 \text{ A.}$$

14.136 A four-pole, wave wound, 220 V dc shunt generator has an armature resistance of 0.1 Ω and field resistance of 50 Ω. The machine has 700 armature conductors. Determine the flux per pole if the machine runs at 800 rpm and is supplying a load of 38 kW.

Solution

Shunt current $I_{sh} = \dfrac{220}{50} = 4.4$ A

and load current $I_L = \dfrac{38 \times 10^3}{220} = 172.72$ A

$\therefore$ Armature current $I_a = 172.72 + 4.4 = 177.12$ A [$\because$ $I_a = I_L + I_{sh}$ for dc shunt generator]

For dc generator,

$$E = V + I_a R_a$$

$\therefore$ $$E = 220 + (177.12 \times 0.1) = 237.72 \text{ V}$$

Also, $$E = \frac{\phi Z N}{60} \cdot \frac{P}{A}$$

$\therefore$ $$237.72 = \frac{\phi \times 700 \times 800 \times 4}{60 \times 2} = \frac{56 \times 10^3 \times \phi}{3}$$

$\therefore$ $$\phi = \frac{237.72 \times 3}{56 \times 10^3} = 12.8 \times 10^{-3} = 12.8 \text{ mWb.}$$

14.137 A 250 kW, 440/480 V over compounded generator requires 0.055 wb of flux per pole to generate 440 V at 620 rpm. The resistance of the series field, inter poles, and armature are 0.005 Ω, 0.005 Ω and 0.01 Ω respectively. Calculate the flux per pole required at full load if speed is 600 rev per minute at full load. Ignore the current taken by the shunt field.

Solution

Open circuit emf = 440 V = $K\phi N$ (given)

$\therefore$ $$K = \frac{440}{0.055 \times 620} = 12.9$$

Full load current $= \dfrac{250 \times 10^3}{480} = 520$ A [Terminal voltage rises from 440 to 480 V due to over compounding as for over compounded generator, the full load voltage at terminals of the machine is more than the no load voltage.]

$\therefore$ Total voltage drop on full-load = 520 (0.01 + 0.005 + 0.005) = 520 × 0.02 = 10.4 V

$\therefore$ emf generated on full load = 480 + 10.4 = 490.4 V

i.e, $$490.4 = K \cdot \phi_2 N_2 = 12.9 \times \phi_2 \times 600$$

$\therefore$ $$\phi_2 = \frac{490.4}{600 \times 12.9} = 63.34 \text{ mwb.}$$

14.138 A four-pole, lap-wound dc generator with 90 slots runs at 1500 rpm. Each slot contains 6 conductors and the flux per pole is 0.03 Wb.

(i) Determine the generated emf from the first principle.

(ii) If the machine is operated as a shunt generator with the same field flux, the armature and field resistance being 1.0 Ω and 200 Ω, determine the output current when the armature current is 25 A.

(iii) If due to a drop in speed the emf becomes 380 V, determine the load current if a 40 Ω load is connected at its terminals.

Solution

(i) Here $Z = 90 \times 6 = 540$

$$\therefore \quad E = \frac{540 \times 0.03 \times 1500}{60} \times \frac{4}{4} = 405 \text{ V}$$

$\left[\because E = \frac{\phi ZN}{60} \cdot \frac{P}{A} \text{ and for lap wound } A = P\right]$

(ii) If $I_a = 25$ A, the armature voltage drop is $(25 \times 1.0) = 25$ V. Since the same field flux and speed are to be assumed, then the same emf is being generated.

From $V = E - I_aR_a$, we have

$$V = 405 - 25 \times 1 = 380 \text{ V}$$

$$\therefore \quad I_{sh} = \frac{380}{200} = 1.9 \text{ A.}$$

Hence machine output current $= (I_a - I_{sh}) = (25 - 1.9) = 23.1$ A.

(iii) Let I_L = the load current

then $I_L \times 40 = V$ (the terminal voltage)

Also, $V = E - I_aR_a$

or $V = 380 - I_a \times 1.0$

or $V = 380 - 1.0\,(I_{sh} + I_L)$ $\quad [\because I_a = I_L + I_{sh}]$

$\therefore$ $I_L \times 40 = 380 - I_{sh} - I_L$

or $41I_L = 380 - I_{sh}$ $\quad$ (i)

Also, $$I_{sh} = \frac{V}{R_f} = \frac{V}{200} = \frac{40I_L}{200} = \frac{I_L}{5} \quad \text{(ii)}$$

$\therefore$ we have, $41I_L = 380 - \frac{I_L}{5}$ $\quad$ [using (ii) in (i)]

i.e. $$I_L = \frac{1900}{206} = 9.22 \text{ A}$$

14.139 A short shunt dc compound generator develops 250 at its terminals and delivers a load current of 50 A. the resistances of the armature, series field and shunt field are 0.05 Ω, 0.04 Ω and 100 Ω respectively. Find the emf generated and armature current if there is a drop of 1 volt at each of its brushes.

Solution

Refer Fig. 14.102

Here, $V = 250 \text{ V} = V_{XY}$ (in Fig. 14.102); $I = 50$ A

$R_{se} = 0.04\ \Omega$; $R_{sh} = 100\ \Omega$; $R_a = 0.05\ \Omega$.

$$\therefore \quad V_{ZY} = V_{XY} + IR_{se} = 250 + 50 \times 0.04 = 252 \text{ V}$$

$$I_{sh} = \frac{V_{ZY}}{R_{sh}} = 2.52 \text{ A}$$

$$\therefore \quad I_a = I_{load} + I_{sh} = 50 + 2.52 = 52.52 \text{ A.}$$

Hence, generated emf $E = V_{ZY} + I_aR_a$ + brush drop

$= 252 + 52.52 \times 0.05 + 2 \times 1 = 256.626$ V

Hence emf generated = 256.626 V

and armature current = 52.52 A.

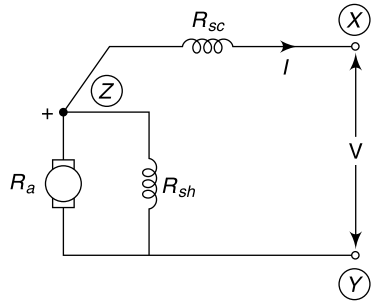


Fig. 14.102

14.140 Determine the input power required to drive a dc shunt generator when giving an output of 50 kW at 230 V; under these conditions the total bearing friction, windage and core loss is 1.6 kW and the total voltage drop at the brushes is 0.034 Ω and that of the field circuit is 55 Ω.

Solution

Output current $I_L = \dfrac{KW \times 10^3}{V} = \dfrac{50 \times 1000}{230} = 217.39$ A

Shunt field current $I_{sh} = \dfrac{V}{R} = \dfrac{230}{55} = \dfrac{46}{11} = 4.18$ A.

$\therefore$ Armature current $I_a = 217.39 + 4.18 = 221.57$ A.

Armature voltage drop = $(I_aR_a) = 221.57 \times 0.034 = 7.53$ V.

Induced emf $E = 230 + 7.53 + 2 = 239.53$ V [$\because E = V + I_aR_a$ + brush drop].

$\therefore$ Electrical power required to be generated

$P_g = 239.53 \times 221.57$ W = 53.07 kW. [$\because P_g = E \times I_a$]

$\therefore$ Total input power = Electrical power input + Mechanical loss = 53.07 + 1.6 = 54.67 kW.s

$\therefore$ The input power is 54.67 kW.

14.141 A dc shunt generator has an open circuit voltage of 500 V, armature and shunt field resistances are 0.2 Ω and 300 Ω respectively. If the machine supplies a full load current of 250 A, the flux being 93% of no load flux while the prime mover speed drops by 4%, find the terminal voltage.

Solution

Shunt current = $\dfrac{V}{R_f} = \dfrac{500}{300} = 1.66$ ($\equiv I_a$ on open circuit)

$\therefore \quad E_1 = V + I_aR_a = 500 + 1.66 \times 0.2 = 500.33$ V.

Since for a dc generator, $E \propto \phi N$, we have $\dfrac{E_2}{E_1} = \dfrac{\phi_2 N_2}{\phi_1 N_1}$

$\therefore \quad E_2 = (500.33) \times \dfrac{93}{100} \times \dfrac{96}{100} = 446.69$ V

[ϕ_2 = 93% and N_2 = 96% of no load condition].

This gives, $V = E_2 - I_aR_a = 446.69 - 250 \times 0.2 = 396.69$ V.

14.142 A 24 kW, 240 V dc self-excited shunt generator driven at 1000 rpm has an armature resistance of 0.1 Ω while the field current is 2.5 A. The rotational losses are 750 W. Assuming constant speed and neglecting armature reaction, determine

(i) the armature induced emf
(ii) the developed full load torque
(iii) the full load efficiency.

Solution

(i) At full load, $I_L = \dfrac{\text{Power (KW)} \times 10^3}{V} = \dfrac{24 \times 10^3}{240} = 100$ A.

$$I_a = I_L + I_{sh} = 100 + 2.5 = 102.5 \text{ A}$$
$$E = V + I_aR_a = 240 + 102.5 \times 0.1 = 250.25 \text{ V}.$$

(ii) Electrical power developed

$$P_g = E \times I_a = 250.25 \times 102.5 = 25650.625 \text{ W}.$$

$$\therefore \text{ Torque developed } T = \frac{E \cdot I_a}{2\pi N} = \frac{25650.625}{2\pi \times \dfrac{1000}{60}} = 244.84 \text{ Nm}.$$

(iii) Total losses $= P_{(\text{rot})} + I_a^2R_a + V \cdot I_{sh}$

$$= 750 + (102.5)^2 \times 0.1 + 240 \times 2.5 = 2400.63 \text{ W}.$$

$$\therefore \text{ Efficiency } (\eta) = \frac{\text{output}}{\text{output + losses}} = \frac{24 \times 10^3 \times 100}{24 \times 10^3 + 2400.63} = 90.90\%.$$

14.143 A 400 V long shunt compound generator has a constant loss (rotational + shunt excitation losses) of 4 kW. The armature, series field and shunt field resistances are 0.08, 0.02 and 100 Ω respectively. Calculate the maximum efficiency and the load at which it occurs.

Solution

For maximum efficiency, we know that Constant loss = Variable losses. Let the armature current for the maximum efficiency load be I_a.

Here, $$\underbrace{I_a^2\,(0.08 + 0.02)}_{\text{variable loss}} = \underbrace{4000}_{\text{constant loss}}$$

$$\therefore \qquad I_a = \sqrt{\frac{4000}{0.1}} = 200 \text{ A}$$

$\therefore$ Corresponding load current is $I_L = I_a - I_{sh} = 200 - \dfrac{400}{100} = 196$ A

$\therefore$ Required load = 196 × 400 = 78.4 kW. Hence the output is 78.4 kW.

$$\therefore \quad \text{Maximum efficiency} = \frac{\text{Output}}{\text{Output + Losses}} = \frac{78400 \times 100}{78400 + 2 \times 4000} = 90.74\%.$$

14.144 An over-compounded dc generator is supplying a load at 220 V. If the machine terminal voltage is 220.2 V when the load current is 10 A, what will be the machine terminal voltage when the load is 600 A?

Solution

When the load current is 10 A, the voltage drop in the cables, between machine terminals and load = (220.2 – 220) = 0.2 V.

By proportion, the voltage drop for 600 A would be $\left(0.2 \times \dfrac{600}{10}\right) = 12$ V.

If the load voltage is still to be 220 V, the terminal voltage would need to be raised to (220 + 12) = 232 V.
∴ The machine terminal voltage is 232 V.

14.145 A dc shunt generator delivers 50 kW at 250 V when the speed is 400 rpm. The field resistance and armature resistance are 50 Ω and 0.02 Ω respectively. Determine the machine's speed when running as a shunt motor taking 50 kW input at 250 V. Consider 2 V for brush-contact drop.

Solution

As a shunt generator:

50 kW at 250 V gives a load current of $(I_L) = \dfrac{50 \times 10^3}{250} = 200$ A

Field current $I_{sh} = \dfrac{250}{50} = 5$ A $\quad \left[\because \; I_{sh} = \dfrac{V}{R_f}\right]$

∴ Armature current $(I_a) = I_L + I_{sh} = 200 + 5 = 205$ A.
$\because \; E = V + I_aR_a$ + brush voltage drop,
we get, $E = 250 + (205 \times 0.02) + 2.0 = 250 + 4.1 + 2.0 = 256.1$ V.

As a shunt motor:

Input current $= \dfrac{250}{50} = 5$ A.

∴ Armature current $I_a = I_L - I_{sh} = 200 - 5 = 195$ A

Back emf, $E_b = V - I_aR_a$ – brush voltage drop
$= 250 - 3.9 - 2 = 244.1$ V.

Again, E and E_b being proportional to flux and speed, we can write,

$$E = K\,\phi\,N \quad \text{and} \quad \text{also } E_b = K\,\phi\,N$$

Thus $\quad \dfrac{E}{E_b} = \dfrac{5 \times 400}{5 \times N} \quad$ or $\quad N = 382$ rpm.

[ϕ can be substituted by field current as flux is assumed to be proportional to be field ampere turns and hence the exciting current.]

14.146 The emf induced in the armature of a 450 kW, 250 V dc shunt generator is 260 V when the field current is 20 A. The armature circuit resistance is 0.004 Ω. Calculate:

(i) The load current I_L
(ii) The power generated P_g
(iii) The power output
(iv) The electrical efficiency η.

Solution

(i) Let the load current be I_L

∴ Armature current $I_a = I_L + 20 \quad [\because \; I_a = I_s + I_{sh}]$

∴ Induced emf E = Terminal voltage $V + I_aR_a$ drop, we have

$$260 = 250 + (I_L + 20) \times 0.004$$

$$\therefore \qquad I_L = \frac{260 - 250}{0.004} - 20 = 2480 \text{ A.}$$

(ii) Power generated $P_g = EI_a = 260\,(2480 + 20) = 650$ kW.

(iii) Power output $VI_L = (250 \times 2480)$ W = 620 kW.

(iv) Electrical efficiency $(\eta) = \dfrac{\text{Output}}{\text{Input}} \times 100 = \dfrac{620.0}{650.0} \times 100 = 95.38\%$

14.147 A 4-pole dc motor whose armature is 0.36 m in diameter and has 720 conductors has effective length of 0.3 m. The flux density of the field under the poles is 0.7*T*. Each conductor carries 30 A. If the armature rotates at 680 rpm, calculate the torque in Nm and the power developed if only two-thirds of the conductors are effective.

Solution

Force developed by one conductor is given by $F = (BIL) = 0.7 \times 30 \times 0.3 = 6.3$ N.

Number of conductors in the field at any instant = $2/3 \times 720 = 480$.

Total force = $480 \times 6.3 = 3024$ N.

Torque = force × radius = $3024 \times 0.18 = 544.32$ Nm

i.e., Torque exerted = 544 N-m.

$$\text{Power developed} = \frac{2\pi NT}{60} = \frac{2 \times 3.14 \times 680 \times 544}{60}$$

$$= 3.14 \times 68 \times 181.333 \text{ W} = 38.7 \text{ kW}.$$

14.148 A 220 V dc shunt motor draws a current of 3 A on light load at 1250 rpm and draws of current of 40 A on full load at the same speed. Calculate the speed on full-load, if the armature resistance is 0.29 Ω and the field resistance is 165 Ω. Due to the armature reaction, the flux per pole is 4% less than the no-load value.

Solution

At no load

Voltage across the shunt field = 220 V.

Current through the shunt field, $I_{sh} = \dfrac{220}{165} = 1.33$ A.

Armature current $I_a = I - I_{sh} = 3.0 - 1.33 = 1.67$ A.

Voltage drop across the armature $(= I_aR_a) = 1.67 \times 0.29 = 0.484$ V

Back emf $E_{b0} = 220 - 0.484 = 219.515$ V. $[\because E_b = V - I_aR_a]$

At full load:

Current through shunt field remains same and hence $I_{sh} = 1.33$ A.

Armature current $I_a = 40 - 1.33 = 38.67$ A.

Voltage drop across armature $= I_aR_a = 38.67 \times 0.29 = 11.23$ V.

Back emf is obtained as $E_{b_1} = 220 - 11.23 = 208.77$ V.

Since, $E_b \propto \phi N$, we can write $E_b = K \phi N$

Hence for no-load and full load condition, $\dfrac{E_{b_0}}{E_{b_1}} = \dfrac{\phi_0 N_0}{\phi_1 N_1}$.

But $\phi_1 = 0.96\, \phi_0$

$$\therefore \quad \frac{E_{b_0}}{E_{b_1}} = \frac{\phi_0 N_0}{0.96 \times \phi_0 \cdot N_1} \quad \text{or,} \quad N_1 = \frac{N_0 \times E_{b_1}}{0.96 \times E_{b_0}}$$

$$= \frac{1250 \times 208.77}{0.96 \times 219.515} = 1238.3 = 1238 \text{ rpm}$$

∴ speed on full load is 1238 rpm.

14.149 A dc shunt motor is rated for 220 V and has an armature winding resistance of 0.4 Ω. It draws armature current of 2 A at no-load and 50 A at full-load, the full-load speed being 500 rpm. Assuming the flux to be constant for no-load and full-load operations, find the no-load speed of the motor.

Solution

At no-load, $E_{b_0} = V - I_a R_a = 200 - (2 \times 0.4) = 199.2$ V

At full-load, $E_{b_1} = 200 - (50 \times 0.4) = 180$ V.

Since this is a shunt motor, the field current is constant and therefore the same constant flux can be assumed for the no-load and full-load conditions.

Since $E_{b_0} = K\phi_0 N_0$ and $E_{b_1} = K\phi_1 N_1$, while $\phi_0 = \phi_1$,

we can write,

$$\frac{E_{b_0}}{E_{b_1}} = \frac{K\phi_0 N_0}{K\phi_1 N_1} \quad \therefore \quad N_0 = \frac{N_1 \times E_{b_0}}{E_{b_1}}$$

$$N_0 = 500 \times \frac{199.2}{180} = 553 \text{ rpm.}$$

14.150 A 200 V, 4-pole lap wound dc motor has 600 conductors in the armature winding and has a resistance of 0.3 Ω. The resistance of the shunt field circuit is 100 Ω while the flux per pole is 0.02 Wb. At no load, the input current is 3 A, while the normal full-load current in the armature is 50 A. Determine the speed regulation of the motor from no-load to full-load. Ignore the armature reaction effect.

Solution

Back emf on no-load $E_{b_0} = 200 - I_{a_0} R_a$

Shunt field current $I_{sh} = 2$ A

$$I_{a_0} = I_L - I_{sh} = 3 - 2 = 1 \text{ A}$$

$$\therefore \quad E_{b_0} = 200 - (1 \times 0.3) = 199.7 \text{ V.}$$

No load speed of the motor is given by (N_0) where $E_{b_0} = \dfrac{Z \cdot \phi_0 N_0}{60} \cdot \dfrac{P}{A}$

$$\text{or} \quad 199.7 = \frac{600 \times 0.02 \times N_0}{60} \qquad [\because \text{ for lap-winding } A = P]$$

$$\text{or} \quad 199.7 = 0.2 \times N_0$$

$$\therefore \quad N_0 = \frac{1997}{2} = 998.5 \text{ rpm.}$$

Again, Back emf E_{b_1} on full-load is given by:

$$E_{b_1} = 200 - I_{a_1} R_a = 200 - (50 - 2) \times 0.3 = 185.6 \text{ V.}$$

Since, $E_{b_1} = K\phi_1 N_1$ assuming a constant flux and with $\phi_0 = \phi_1$,

we have

$$\frac{E_{b_1}}{E_{b_0}} = \frac{K\phi_1 N_1}{K\phi_0 N_0}, \quad \text{where} \quad N_1 = \frac{N_0 \cdot E_{b_1}}{E_{b_0}}$$

$$\therefore \quad N_1 = \frac{998.5 \times 185.6}{199.7} = 928 \text{ rpm}$$

$\therefore$ Full load speed = 928 rpm.

$$\text{Thus speed regulation} = \frac{\text{No-load speed} - \text{Full load speed}}{\text{Full-load speed}} \times 100$$

$$= \frac{998.5 - 928}{928} \times 100 = 7.6\%$$

14.151 A 105 V, 3 kW dc shunt motor has a full-load efficiency of 82%. The field and armature resistances are 90 Ω and 0.25 Ω respectively. The full-load speed of the motor is 1000 rpm. Determine the speed at which the motor will run at no-load if the line current at no load is 3.5 A. Also determine the resistance to be added to the armature circuit, in order to reduce the speed to 800 rpm, the torque remaining constant at full-load value. Ignore the armature reaction and brush drop.

Solution

On no-load,

$$I_{sh}\left(=\frac{V}{R_f}\right) = \frac{105}{90} = 1.17 \text{ A}$$

No-load current I_{L_0} = 3.5 A (given)

$\therefore \quad I_{a0} = 3.5 - 1.17 = 2.33$ A $\quad [\because I_a = I_L - I_{sh}]$

and $\quad E_{b0} = V - I_{a0}R_a = 105 - (2.33 \times 0.25) = 105 - 0.58 = 104.42$ V.

On full-load, output = 3 × 1000 W.

$$\therefore \quad \text{Input} = \frac{\text{Output}}{\text{Efficiency}} = \frac{3 \times 1000 \times 100}{82} = 3660 \text{ W}$$

Input line current $I_{L_1} = \dfrac{3660}{105} = 34.86$ A.

$\therefore \quad I_{a1} = 34.86 - 1.17 = 33.7$ A and

$\quad E_{b1} = 105 - (33.7 \times 0.25) = 96.57$ V.

$\because \quad E \propto \phi N$ or $E = K\phi N$, we can write $\dfrac{E_{b_0}}{E_{b_1}} = \dfrac{K\phi_0 N_0}{K\phi_1 N_1}$

or, $$N_0 = \frac{E_{b_0} \times \phi_1 \times N_1}{E_{b_1} \times \phi_0} = 1080 \text{ rpm.}$$

Again, since T is constant and $T \propto \phi I_a$, we can write

$$T_2 = K\phi_2 I_{a_2} \text{ and } T_1 = K\phi_1 I_{a_1}$$

$$\therefore \quad \frac{T_2}{T_1} = \frac{K \cdot \phi_2 \cdot I_{a_2}}{K \cdot \phi_1 \cdot I_{a_1}} \quad \text{but}$$

But $\quad T_2 = T_1$ and $\phi_2 = \phi_1$.

$\therefore \quad I_{a_2} = I_{a_1} = 33.7$ A.

If R is the added resistance to reduce speed then we can write

$$E_{b_2} = V - I_{a_2} \times (R + R_a) = 105 - 33.7\,(R + 0.25)$$

Also $E_b \propto \phi N$; since flux is constant, we can write

$$\frac{E_{b_2}}{E_{b_1}} = \frac{800}{1000} \quad \text{or,} \quad E_{b_2} = 96.57 \times \frac{800}{1000}$$

$\therefore$ Back emf (at reduced speed) = 77.26 V,

thus we have $\quad 77.26 = 105 - 33.7\,(R + 0.25)$

$$\therefore \quad R = \frac{19.31}{33.7} = 0.57\ \Omega.$$

14.152 A 4-pole dc shunt motor has a wave wound armature with 294 conductors. The flux per pole is 0.03 Wb and the resistance of the armature is 0.35 Ω. Determine (i) the speed of the armature and (ii) the torque developed, when the armature current is 200 A and the supply is 230 V dc.

Solution

(i) $V = E_b + I_a R_a$;

or $\quad E_b = V - I_a R_a = 230 - (200 \times 0.35) = 160$ V

Since $\quad E_b = \dfrac{Z\phi N}{60} \times \dfrac{P}{A}$

Here, $\quad N = \dfrac{E_b}{Z\phi} \times \dfrac{60 \cdot A}{P} = \dfrac{160 \times 60 \times 2}{294 \times 0.03 \times 4} = 544$ rpm.

(ii) Again torque is given by:

$$T = 0.159 \times Z\,\phi\,I_a \left(\frac{P}{A}\right) \text{ Nm}$$
$$= 0.159 \times 294 \times 0.03 \times 200 \times 4/2 = 560.95 \text{ Nm.}$$

14.153 A 230 V dc shunt motor runs at 600 rpm when taking a line current of 50 A. The field and armature resistances of the motor are 104.5 Ω and 0.4 Ω respectively. Determine (i) the no-load speed if the no-load line current is 5 A. (ii) The resistance to be placed in the armature circuit in order to reduce the speed to 500 rpm when taking a line current of 50 A. (iii) The percentage reduction in the flux per pole in order that the speed may be 750 rpm, when taking an armature current of 30 A with no added resistance in the armature circuit. Ignore the effect of armature reaction but allowing a brush drop of 2 V.

Solution

$$I_{sh_0} = \frac{230}{104.5} = 2.2 \text{ A}; \; I_{a_0} = 5 - 2.2 = 2.8 \text{ A}$$

Also, $\quad I_{sh_1} = 2.2$ A; $I_{a_1} = 50 - 2.2 = 47.8$ A

Again, $\quad E_{b_1} = 230 - (47.8 \times 0.4) - 2 = 208.88$ V

and $\quad E_{b_0} = 230 - (2.8 \times 0.4) - 2 = 226.88$ V.

(i) Since $\dfrac{E_{b_1}}{E_{b_0}} = \dfrac{K\,\phi_1\,N_1}{K\,\phi_0\,N_0}$, here $N_0 = \dfrac{E_{b0} \cdot N_1}{E_{b1}} = \dfrac{226.88 \times 600}{208.88} = 651$ rpm.

(ii) At 600 rpm, $E_{b_1} = 208.88$ V.

Assuming a constant flux, then for 500 rpm.

$$E_{b_2} = 208.88 \times \frac{5}{6} = 174.07 \text{ V.}$$

∴ The voltage across the armature has to be reduced by (230 – 174.07) = 55.93 V

Since $\quad V = E_b + I_a(R_a + R) + 2$

$$V - E_b = I_a(R_a + R) + 2$$

or $\quad 55.93 = I_a R_a + I_a R + 2$

∴ $\quad I_a \cdot R = 55.93 - 2 - (47.8 \times 0.4) = 34.81$ V.

∴ $\quad R = \dfrac{34.81}{47.8} = 0.73\ \Omega$.

(iii) Let the reduced flux be ϕ_3.

∴ $\quad \dfrac{E_{b_3}}{E_{b_1}} = \dfrac{K\,\phi_3\,N_3}{K\,\phi_1\,N_1}$

or $\quad E_{b_3} = 230 - (30 \times 0.4) - 2 = 216$ V.

∴ $\quad \dfrac{216.0}{208.88} = \dfrac{I_{sh_3} \times 750}{2.2 \times 600}$ or $I_{sh_3} = 1.82$ A

Thus current (and therefore flux) is to be reduced to $\dfrac{1.82}{2.2} = 82.7\%$.

14.154 A 250 V dc shunt motor drives a load whose torque varies as the cube of the speed. This motor takes 20 A when running at 1200 rpm. If the field current is kept constant and the speed is varied by external resistance in the armature circuit, determine the resistance needed to bring the speed down to 900 rpm. Ignore the armature resistance.

Solution

From the given conditions, we can write $T \propto \phi I_a \propto N^3$. Here ϕ is constant.

$$\therefore \qquad \frac{I_{a_1}}{I_{a_2}} = \left(\frac{N_1}{N_2}\right)^3$$

$$\therefore \qquad I_{a_2} = \left(\frac{N_2}{N_1}\right)^3 \times I_{a_1} = \left(\frac{900}{1200}\right)^3 \times 20 = 8.4375.$$

$$\because \qquad \frac{E_{b_2}}{E_{b_1}} = \frac{\phi_2 N_2}{\phi_1 N_1} = \frac{N_2}{N_1}$$

Here, $$\left(\frac{E_{b_2}}{E_{b_1}}\right) = \left(\frac{900}{1200}\right) = 0.75.$$

or, $$E_{b_2} = 0.75 \times E_{b_1} = 0.75 \times 250 = 187.5 \text{ V}.$$

But $$E_b = V - I_a R_a$$

Here, $$187.5 = V - I_{a_2}(R_a + R_e) = 250 - 8.4375\,(R_a + R_e)$$

Since $$R_a = 0 \text{ (given)}$$

$$\therefore \qquad R_e = 7.40\ \Omega.$$

14.155 A shunt connected dc motor draws a total current of 56 A from a 650 V supply when delivering its rated output at its rated speed of 1000 rpm. The resistance of the field circuit and armature are 300 Ω and 0.50 Ω respectively and the mechanical losses are 1.90 kW. Determine the useful power output, torque and efficiency.

Solution

Field current $(I_{sh}) = \dfrac{V}{R_{sh}} = \dfrac{650}{300} = 2.16$ A.

Hence armature current $(I_a) = I_L - I_{sh} = 56 - 2.16 = 53.83$.

Let the suffix 1 denote the initial condition of rated output and rated speed,

then $$E_1 = V - I_{a_1} R_a = 650 - 53.83 \times 0.50 = 623.08 \text{ V}$$

Gross power output $E_1 I_{a1} = 623.08 \times 53.83 = 33.54$ kW

Thus net power output = Gross power – Mechanical loss = 33.54 – 1.90 = 31.64 kW

(i.e. the useful power output is 31.64 kW)

Since speed = 1000 rpm = $2\pi \times \dfrac{1000}{60} = 104.7$ rad/sec.

Hence torque $T = \dfrac{31.64 \times 10^3}{104.7} = 302.3$ N-m.

i.e. useful torque is 302 N-m.

Again, input power = $(V \times I_{L_1}) = 650 \times 56 = 36.4$ kW, while output power = 31.64 kW.

$$\therefore \qquad \text{Efficiency } \eta = \frac{31.64}{36.4} \times 100$$

$\therefore$ the percentage efficiency is 86.92%.

14.156 A dc shunt motor of 500 V has a full load armature current of 20 A. 3% of the input power to the motor is dissipated as heat energy. What would be the armature current on starting if 500 V is supplied across the armature?

Calculate also the value of starting resistance required to limit the starting current to twice the full load current.

Solution

Since the field current is small, we can ignore the field current.

∴ Input power (= VI) = 500 × 20 = 10,000 W.

Again, 3% of the input power (i.e. (3/100) × 10,000.00) is 300 W which is dissipated as heat in the armature. This is copper (or I^2R loss).

$$\therefore \quad I^2 R_a = 300 \quad \text{or,} \quad 20^2 \cdot R_a = 300$$

$$\therefore \quad R_a = \frac{300}{400} = \frac{3}{4} = 0.75\ \Omega.$$

The starting current with only armature resistance to limit the armature current is given by, $(I_{as}) = \dfrac{500}{0.75} = 666.6$ A.

∴ The twice full load current will be 20 × 2 = 40 A.

Total resistance required in the armature circuit to limit starting current to 40 A is $\left(\dfrac{500}{40}\right)$ or 12.5 Ω. Since R_a = 0.75 Ω, the series resistance would be 12.5 – 0.75 = 11.75 Ω.

14.157 A belt driven 250 kW dc shunt generator running at 500 rpm which supplies full load power to a 500 V bus bar. When the belt breaks down, it continues to run as motor taking 20 kW from the bus bar. What would be new speed and torque developed? Given:

$$R_a = 0.02\ \Omega$$
$$R_{sh} = 100\ \Omega.$$

Assume a constant voltage drop in each brush as 1.5 V. Ignore the armature reaction effect.

Solution

I_{L_1} (as shunt generator) $= \dfrac{250 \times 10^3}{500} = 500$ A and $I_{sh} = \dfrac{500}{100} = 5$ A.

$$\therefore \quad I_{a_1} = 500 + 5 = 505 \text{ A} \qquad [\because I_{a_1} = I_{L_1} + I_{sh}]$$

$$E_1 = 500 + (505 \times 0.02) + (2 \times 1.5) = 513.10 \text{ V}.$$

$$[\because E = V + I_{a_1} R_a + \text{brush drop for generator}]$$

$$\therefore \quad I_{L_2} \text{ (as motor)} = \frac{20{,}000}{500} = 40 \text{ A}.$$

Also $\quad I_{a_2}\ (= I_{L_2} - I_{sh}) = 40 - 5 = 35$ A.

$$\left[\begin{aligned} \because E_2 &= V - I_a R - \text{brush drop} \\ &= 500 - 350 \times 0.02 - 3 \\ &= 496.3 \text{ V} \end{aligned}\right]$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \text{or} \quad \frac{513.10}{496.3} = \frac{500}{N_2}$$

$$\therefore \quad N_2 = 483.63 \text{ rpm}$$

$$\therefore \quad T = \frac{E_b \cdot I_A}{2\pi N}$$

$$\therefore \quad T = \frac{E_2 \cdot I_{a_2}}{\dfrac{2\pi \cdot N_2}{60}} = \frac{496.3 \times 35 \times 60}{2\pi \times 483.63} = 342.84 \text{ Nm}.$$

14.158 A 440 V dc shunt motor runs at a speed of 1000 rpm when on no-load. After a few hours of loading its temperature rises by 28°C and the supply voltage falls to 436 V.

Calculate the new speed of the motor when the armature current is 85 A. The cold armature resistance is 0.04 Ω. Temperature co-efficient of the armature and field conductor is 0.4% per degree centigrade.

Solution

Let $R_{sh(c)}$ be the cold resistance of the field winding and $R_{sh(H)}$ and $R_{A(H)}$ be the hot field winding and armature winding resistances respectively.

$\therefore \quad R_{sh}(H) = R_{sh(c)}\ (1 + 28 \times 0.0044) = 1.1232\ R_{sh(c)}$

and $\quad R_A(H) = 0.04\ (1 + 28 \times 0.0044) = 0.0449\ \Omega.$

Assuming armature current on no load to be zero, we can write

$$E_1 = V_1 = 440 \text{ V (at no load-condition)}$$

$$E_2 = V_2 - I_A R_{A(H)} = 436 - 85 \times 0.0449 = 432.1835 \text{ V.}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{\phi_1 N_1}{\phi_2 N_2} = \frac{I_{sh1}}{I_{sh2}} \times \frac{N_1}{N_2} = \frac{V_1/R_{sh\,(c)}}{V_2/R_{sh\,(H)}} \times \frac{N_1}{N_2}$$

$$\therefore \quad \frac{440}{432.1835} = \frac{440/R_{sh(c)}}{436/1.1232R_{sh(c)}} \times \frac{1000}{N_2}$$

$$\therefore \quad N_2 = 1113 \text{ rpm.}$$

14.159 A 240 V dc series motor develops a shaft torque of 200 N-m at 92% efficiency while running at 600 rpm. Calculate the motor current.

Solution

$$\text{Power output} = 2\pi NT = 2\pi \times \frac{600}{60} \times 200 = 12566 \text{ W.}$$

$$\text{Power input } W = \frac{12566}{0.92} = 13659 \text{ W}$$

$$\therefore \text{ The motor current } (I) = \frac{13659}{240} = 56.91 \text{ A.}$$

14.160 A 220 V dc series motor takes 60 A. The armature resistance is 0.1 Ω and the series field resistance is 0.08 Ω. If the iron and friction losses are equal to the copper losses at this load, calculate the BHP (brake horsepower) and electrical efficiency.

Solution

$$\text{Motor resistance} = 0.1 + 0.08 = 0.18\ \Omega.$$

$$E_b = V - I_a R_m = 220 - (60 \times 0.18) = 209.2 \text{ V.}$$

$$\text{Input power} = 220 \times 60 = 13200 \text{ W.}$$

$$\text{Copper losses} = 648 \text{ W } [(60)^2 \times 0.18 = 648 \text{ W}]$$

$$\text{Output} = 11904 \text{ W } [13200 - 2 \times 648 = 11904 \text{ W}]$$

$$\therefore \quad \text{BHP} = \frac{11904}{735.5} = 16.18.$$

$$\text{Efficiency } (\eta) = \frac{11904}{13200} \times 100 = 90.18\%.$$

14.161 A dc series motor develops 40000 W and takes a current of 80 A when running at 1200 rpm. Calculate starting torque if the starting current is 150% of the rated current. Magnetic circuit remains unsaturated.

Solution

Output of dc series motor = 40000 W (given).

$$\therefore \quad 2\pi NT = 2\pi \times \frac{1200}{60} \times T = 40000$$

$$T = \frac{40000 \times 60}{2\pi \times 1200} = \frac{1000}{\pi} \text{ Nm.}$$

For a series motor (with unsaturated field), $T \propto I_a^2$

$$\therefore \quad \frac{1000}{\pi} = K \cdot (80)^2$$

At starting I_A = 120 A (starting current = 150% rated current)

$$\therefore \quad \frac{T_s}{1000/\pi} = \left(\frac{120}{80}\right)^2 \quad [T_s \text{ being the starting torque.}]$$

$$\therefore \quad T_s = \left(\frac{120}{80}\right)^2 \times \frac{1000}{\pi} = 715.90 \text{ Nm.}$$

14.162 A 220 V dc series motor is working with an unsaturated field taking a current of 100 A at 800 rpm. Calculate the speed of the motor when it develops half the torque. [Given: the total resistance of the motor is 0.1 ohm]

Solution

For dc series motor,

$$\text{Torque } T = KI_a^2$$

$$\therefore \quad \frac{T_1}{T_2} = \frac{T_{a_1}^2}{T_{a_2}^2}$$

But $\quad T_2 = 0.5T_1$ (as per the given condition)

$$\therefore \quad \frac{T_1}{0.5T_1} = \frac{100^2}{I_{a2}^2} \quad \text{or} \quad I_{a2}^2 = 100^2 \times 0.5 = 5000.$$

$$\therefore \quad I_{a_2} = 10\sqrt{50} = 10 \times 7.07 = 70.7 \text{ A.}$$

Under the first condition,

$$E_{b_1} = V - I_{a_1}(R_a + R_{se}) = 220 - (100 \times 0.1) = 210 \text{ V.}$$

Under the second condition

$$E_{b_2} = V - I_{a_2}(R_a + R_{se}) = 220 - 70.7 \times 0.1 = 212.93 \text{ V.}$$

However, $\quad E_b = K\,\phi\,N.$

$$\therefore \quad \frac{E_{b_1}}{E_{b_2}} = \frac{\phi_1 N_1}{\phi_2 N_2} \quad \text{and also } \phi \propto I_a$$

i.e
$$\frac{E_{b_1}}{E_{b_2}} = \frac{I_{a_1} \cdot N_1}{I_{a_2} \cdot N_2}$$

$$\therefore \quad N_2 = \frac{I_{a_1}\,N_1\,E_{b_2}}{I_{a_2}\,E_{b_1}} = \frac{100 \times 800 \times 212.93}{70.7 \times 210} = 1147 \text{ rpm}$$

$\therefore$ Speed at half torque = 1147 rpm.

14.163 A 250/6600 V, 50 Hz single-phase transformer has a core of cross section 600 cm^2 and the flux density in it is 0.707 Tesla. Determine the number of turns in each winding of the transformer.

Solution

Here the maximum flux density (B_m) is given by

$$(B_m) = \sqrt{2} \cdot B = 0.707 \times \sqrt{2} = 1 \text{ Tesla.}$$

$$\therefore \quad 250 = 4.44 \times 1 \times 600 \times 10^{-4} \times 50 \times N_1$$

$$\therefore \quad N_1 = 18.77 \simeq 19 \text{ turns.}$$

$$\therefore \quad \frac{E_1}{E_2} = \frac{N_1}{N_2} \quad \therefore \quad N_2 = \frac{6600}{250} \times 19 = 501.6 \simeq 502 \text{ turns.}$$

14.164 A 50 Hz single-phase core transformer having 550 turns in primary with primary voltage of 3.3 kV has a laminated core of mean length of 1.5 m. The joints cause an effective air gap of 0.1 mm. The maximum flux density is 1.2 Tesla, corresponding magnetic flux strength H for sheet steel is 840 AT/m and the specific loss from the manufacturer's curve is 3 W/kg. The density of the steel is 7800 kg/m^3.

Calculate (i) the no-load current in the primary, and (ii) the corresponding power factor.

Solution

(i) Ampere turns for the steel core = 1.5 × 840 = 1260.

Ampere turns for the air-gap = $H \cdot L_A = \dfrac{B_m}{\mu_0} \cdot L_A = \dfrac{1.2 \times 0.1 \times 10^{-3}}{4\pi \times 10^{-7}} = 95.5.$

$\therefore$ The total ampere-turns required = 1260 + 95.5 = 1355.5

The peak value of the magnetizing current = $\dfrac{1355.5}{550} = 2.46$ A.

$\therefore$ rms value of the magnetizing current is $\dfrac{2.46}{\sqrt{2}} = 1.74$ A.

Let A_i be the net area of cross-section of the core, then

$$E_1 = 4.44\, B_m\, A_i \cdot f \cdot N_1$$

$$3300 = 4.44 \times 1.2 \times A_i \times 50 \times 550$$

$$\therefore \quad A_i = 225 \text{ cm}^2 = 0.0225 \text{ m}^2.$$

The volume of the steel core = 1.5 × 0.0225 = 0.03375 m^3.

Mass of the core = 7800 × 0.03375 = 263.25 kg.

Total iron loss = 263.25 × 3 = 789.75 W

$\therefore$ Eddy current (I_e) = $\dfrac{789.75}{3300} = 0.239$ A

Hence no-load current (I_0) = $\sqrt{I_m^2 + I_c^2} = \sqrt{(1.74)^2 + (0.239)^2} = 1.756$ A

(ii) No-load power factor $\left(= \dfrac{I_C}{I_0}\right) = \dfrac{0.239}{1.756} = 0.136.$

14.165 A single-phase transformer has a transformation ratio 2. If the winding constants are given below, determine the equivalent impedance with respect to primary. What voltage should be applied across the primary when the secondary is shorted so that 12 A flows in the primary. Ignore no-load current. $R_1 = 1\ \Omega$, $X_1 = 2\ \Omega$, $R_2 = 0.20\ \Omega$ and $X_2 = 0.70\ \Omega$.

Solution

When the secondary resistance is transferred to the primary side,

$$R_{01} = R_1 + K^2 \cdot R_2 \text{ where } \left(K = \frac{V_1}{V_2} = \frac{I_2}{I_1} \right) = 2$$

$\therefore$ $$R_{01} = 1 + (0.20) \times 2^2 = 1.8\ \Omega$$

and $$X_{01} = 2 + (0.7) \times 2^2 = 4.8\ \Omega$$

Hence, $$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{(1.8)^2 + (4.8)^2} = 5.126\ \Omega.$$

Voltage (on short circuit) = 5.126 × 12 = 61.5 V.

14.166 The open circuit voltage ratio of a 6.3 kVA single-phase transformer is given as 230/460 V. It has a primary resistance of 0.2 Ω and a reactance of 0.5 Ω and corresponding values for secondary are 0.75 Ω and 1.8 Ω respectively. Determine (i) the secondary voltage on full load and (ii) the voltage regulation for

(a) 0.8 p.f. (lagging)
(b) 0.8 p.f. (leading).

Solution

Here, $$\frac{N_2}{N_1} = \frac{460}{230} = 2 = \frac{1}{K} \quad \text{(where } K \text{ is the turns ratio).}$$

$\therefore$ $$R_{02} = R_2 + \left(\frac{1}{K}\right)^2 \cdot R_1 = 0.75 + 0.2 \times 2^2 = 1.55\ \Omega.$$

$$X_{02} = X_2 + \left(\frac{1}{K}\right)^2 \cdot X_1 = 1.8 + 0.5 \times 2^2 = 3.80\ \Omega.$$

(a) *When 0.8 p.f. lagging:*

(i) Secondary voltage $(V_2) = E_2 - (I_2R_{02} \cos \phi + I_2X_{02} \sin \phi)$

$$= 460 - 13.7\ (1.55 \times 0.8 + 3.8 \times 0.6) = 411.78 \text{ V.}$$

(ii) $$\%\text{ Regulation} = \frac{E_2 - V_2}{V_2} \times 100 = \frac{460 - 411.78}{411.78} \times 100 = 11.71\%$$

[∵ the load is inductive type]

(b) *When 0.8 p.f. (leading):*

(i) Secondary voltage $(V_2) = E_2 - (I_2R_{02} \cos \phi - I_2X_{02} \sin \phi)$

$$= 460 - 13.7\ (1.55 \times 0.8 - 3.8 \times 0.6) = 474.25 \text{ V.}$$

(ii) $$\%\text{ Regulation} = \frac{E_2 - V_2}{V_2} \times 100 = \frac{460 - 474.25}{474.25} \times 100 = -3\%$$

[∵ the load is capacitive type].

14.167 A 11 kV/240 V, 10 kVA, single-phase transformer is connected to an 11 kV supply. It has the following parameters:

Primary winding: Resistance = 70 Ω
Leakage reactance = 350 Ω.
Secondary winding: Resistance = 0.016 Ω
Leakage reactance = 0.097 Ω.

Under no load condition, the given transformer consumes 50 W of power. Calculate the load current, voltage regulation and efficiency of the transformer when it is working at maximum efficiency and a power factor of 0.9 (lagging). Find the power factor at which regulation would be zero. The voltage drop across the resistance and reactance of the secondary winding is 5.07 V.

Solution

After referring the primary winding parameters to the secondary side and then combining with the secondary parameters, we have

$$R_{02} = R_2 + \left(\frac{1}{K}\right)^2 \cdot R_1 = 0.016 + \left(\frac{240}{11000}\right)^2 \times 70 = 0.0493\ \Omega.$$

$$X_{02} = X_2 + \left(\frac{1}{K}\right)^2 \cdot X_1 = 0.097 + \left(\frac{240}{11000}\right)^2 \times 350 = 0.2636\ \Omega.$$

At maximum efficiency, we know Copper loss = Iron loss.
From the problem, iron loss = 50 W

i.e. $I_2^2 \cdot R_{02} = 50.$

$$I_2 = 31.8 \text{ A.}$$

$$\text{Regulation} = \frac{E_2 - V_2}{E_2} \times 100 = \frac{5.07}{240} \times 100 = 2.11\%$$

Here $E_2 = 240$ V

Again $E_2 - V_2 = 5.07$ V

$\therefore$ $V_2 = 240 - 5.07 = 234.93$ V

This is the voltage across the load. Thus for power factor 0.9 lagging.

Output power = 234.93 × 31.8 × 0.9 = 6723.7 W.

$\therefore$ The efficiency η is found to be $\dfrac{\text{Output}}{\text{Output + Losses}} = \dfrac{6723}{6723 + 50 + 50} = 0.985 = 98.5\%$

For zero regulation, $(E_2 - V_2) = 0$, thus

$$I_2R_{02} \cos\phi + I_2X_{02} \sin\phi = 0$$

or
$$\tan\phi = \left(-\frac{R_{02}}{X_{02}}\right) = \left(-\frac{0.0493}{0.2636}\right)$$

$\therefore$ $\phi = 10.6°$ (leading).

14.168 A single-phase 5 MVA 11/220 kV transformer gives full-load efficiency η of 90% at 0.8 p.f. Calculate the full load copper loss and iron loss, if the two are equal at 80% of the load.

Solution

We know, efficiency

$$\eta = \frac{\text{Output power}}{\text{Output power + Copper loss + Iron loss}}$$

$$\therefore \quad \frac{90}{100} = \frac{5 \times 10^6 \times 0.8}{5 \times 10^6 \times 0.8 + W_c + W_i}$$

or $W_C + W_i = 444.44$ kW (i)

Since copper loss at 80% of the rated load = iron loss,

$\therefore$ $W_C \times (0.8)^2 = W_i$ (ii)

Solving Eq. (i) and (ii), we have $W_i = 166.67$ kW and $W_C = 277.78$ kW.

14.169 In a single-phase transformer, the full-load voltage drop across the resistance is 1.5% and the full-load voltage drop is 5.5% across the leakage reactance. Calculate its efficiency at half-load unity power factor. Assume maximum efficiency of the transformer occurs at full load.

Solution

Here full-load resistance drop = 1.5% = $\frac{IR}{V} \times 100$.

and $\quad W_C = I^2R = I^2 \times 0.015 \times \frac{V}{I} = 0.015\ VI = 1.5\%$ of VI.

Also, full load copper loss = 1.5%

$\therefore$ Copper loss at half load = $\left(\frac{1}{2}\right)^2 \times 1.5 = 0.375\%$

Since maximum efficiency also occurs at full load, therefore, iron loss are also = 1.5%.

Hence efficiency η at half load = $\frac{50}{50+1.5+0.375} \times 100 = 96.39\%$.

14.170 A single-phase transformer of 500 kVA capacity has an iron loss of 4.5 kW and a full-load copper loss of 7.8 kW.

Calculate (i) The efficiency of the transformer at half-load for a power factor of unity. (ii) Also calculate the load for maximum efficiency and the value of maximum efficiency when the power factor is (a) unity (b) 0.7 (lagging).

Solution

Since copper loss α (current)^2

$\therefore$ Copper loss α (kVA)^2

(i) Half-load copper loss at unity power factor = $7.8 \times 0.5^2 = 1.95$ kW

Iron loss = 4.5 kW (given).

$\therefore$ The total losses = 1.95 + 4.5 = 6.45 kW

Transformer output (at unity p.f.) = $0.5 \times 500 = 250$ kW.

$$\therefore \quad \text{Efficiency } \eta = \frac{\text{Output power}}{\text{Input power}} = \frac{250}{250+6.45} \times 100 = 97.5\%$$

We know the condition of maximum efficiency occurs when copper loss equals the iron loss. Let the maximum efficiency occurs at $1/x$ of the full-load kVA, thus

$$\text{Iron loss} = \frac{\text{Full-load copper loss}}{x^2}$$

$$\text{Thus} \qquad x = \sqrt{\frac{\text{Full-load copper loss}}{\text{Iron loss}}} = \sqrt{\frac{7.8}{4.5}} = 1.317.$$

$\therefore$ kVA for maximum efficiency = $500 \times \frac{1}{1.317} = 380$ kVA.

and total losses at maximum efficiency = $2 \times$ Iron loss = 9.0 kW.

(ii) (a) When p.f. is unity

Output power = kVA $\times$ 1 = 380 kW

Total losses = 9 kW

$$\therefore \quad \text{Efficiency} = \frac{380}{380+9} \times 100 = 97.7\%.$$

(b) When p.f. is 0.7 lagging

Output power = $380 \times 0.7 = 266$ kW

Total losses = 9 kW.

$$\therefore \quad \text{efficiency} = \frac{266}{(266+9)} \times 100 = 96.7\%.$$

14.171 A (240/24) V single-phase transformer has the following parameters:

$$R_1 = 6.0\ \Omega, \quad X_1 = 20\ \Omega$$
$$R_2 = 0.05\ \Omega, \quad X_2 = 0.18\ \Omega$$

When the transformer is supplied at 240 V on no-load, the primary current is 0.020 A at p.f. of 0.3 (lagging). Calculate the current in the primary side, the voltage regulation and the efficiency when the secondary of the transformer is connected to a load of $(3.9 + j1.9)\ \Omega$.

Solution

Referring secondary winding impedances to the primary gives

$$R'_{02} = 0.05 \times \left(\frac{240}{24}\right)^2 = 5\ \Omega$$

and $$X'_{02} = 0.18 \times \left(\frac{240}{24}\right)^2 = 18\ \Omega.$$

Referring load impedances to the primary gives

$$R'_L + jX_L' = (3.9 + j1.9)\left(\frac{240}{24}\right)^2 = 390 + j190 = 434\angle 26.0°\ \Omega$$

$\therefore$ I_{01} = secondary current referred to the primary

$$= \frac{V_1}{R_1 + R'_{02} + R'_L + j\,(X_1 + X'_{02} + X'_L)}$$

$$= \frac{240\angle 0}{(401 + 228)} = 0.520\angle{-29.6°}\ \text{A} = 0.452 - j0.257\ \text{A}$$

Given no-load current $(I_0) = 0.02\angle{-\cos^{-1}}\,(0.3) = 0.006 - j0.0191$ A.

$\therefore$ The total primar current I_1 is

$$I_1 = I_{01} + I_0 = 0.458 - j0.276 = 0.535\angle{-31.1°}\ \text{A}.$$

i.e. the total primary current is 0.54 A.

Again, $$V_2' = I_{01}\,(R'_L + jXL') = 0.520\angle{-29.6} \times 434\angle{-26.0°} = 226\ \text{V}.$$

$\therefore$ The regulation $= \dfrac{240 - 226}{240} = 0.060 = 6\%$

Again copper losses $= I_{01}^2 \cdot R_{01} = 0.520\,(R_1 + K^2 \cdot R_2) = 0.520\,(6 + 5) = 5.72$ W.

$$\left[\because\ K = \frac{E_1}{E_2}\right]$$

The iron is obtained from the no-load current:

$$\text{Iron loss} = V_1 \cdot I_0 \cos\phi_0 = 240 \times 0.020 \times 0.3 = 1.44\ \text{W}.$$

$$\text{Ouptut power} = I_{01}^2 \cdot R'_L = (0.520)^2 \times 390 = 105.5\ \text{W}.$$

$$\therefore \quad \text{Efficiency}\ (\eta) = \frac{\text{output power}}{\text{output power} + \text{iron loss} + \text{copper loss}}$$

$$= \frac{105.5}{105.5 + 1.44 + 5.72} = 0.9364 = 93.64\%.$$

14.172 How much current is drawn by the primary of a transformer which steps down 220 V to 22 V to operate a device having an impedance of 220 Ω.

Solution

Primary voltage $V_1 = 220$ V
Secondary voltage $V_2 = 22$ V

Secondary current $I_2 = \dfrac{22}{220} = 0.1$ A.

Primary current I_1 is to be found out.

Now for an 'ideal' transformer, Output power = Input power.

$$\therefore \qquad V_2 \cdot I_2 = V_1 \times I_1$$

$$\therefore \qquad I_1 = \frac{V_2 \cdot I_2}{V_1} = \frac{22 \times 0.1}{220} = 0.01 \text{ A.}$$

14.173 A transformer with 100% efficiency has 200 turns in the primary and 40000 turns in the secondary. It is connected to a 220 V, 50 Hz main supply and the secondary feeds to a 100 kΩ resistance. Calculate the secondary potential difference per turn and the power delivered to the load.

Solution

Primary turns $N_1 = 220$

Secondary turns $N_2 = 40000$

Primary voltage $V_1 = 220$ V

Load resistance = 100 kΩ = 10^5 Ω.

$$\because \qquad \frac{V_2}{V_1} = \frac{N_2}{N_1},$$

$$\therefore \qquad V_2 = V_1 \cdot \frac{N_2}{N_1} = 220 \times \frac{40000}{200} = 44000 \text{ V.}$$

$\therefore$ The secondary potential difference per turn is: $\dfrac{V_2}{N_2} = \dfrac{44000}{40000} = 1.1$ V.

The power delivered by the 'ideal' (100% efficiency) transformer to the load is

$$V_2 \times I_2 = V_2 \times \frac{V_2}{\text{load resistance}} = \frac{V_2^2}{\text{load resistance}} = \frac{(44000)^2}{10^5} = 19.36 \text{ kW}$$

14.174 A residential area requires 800 kW of electric power at 220 V and is 3 km away from the generating source. The source is generating power at 440 V, the resistance of the line carrying power is 0.5 ohm/km. The area gets power from the line through a (4000/220) V step-down transformer at a sub-station in the load area.

Determine:

(i) the line ohmic power loss

(ii) how much power must the plant supply

(iii) find the ratio of the step-up transformer at the plant.

Solution

The resistance of the two-wire line is:

$$R = 0.5 \times 3 = 1.5 \ \Omega.$$

The 800 kW power is transmitted at 4000 V through the line. Thus, the rms current in the line is:

$$I_{\text{rms}} = \frac{P}{V_1} = \frac{800 \times 10^3}{4000} = 200 \text{ A.}$$

(i) The line power loss is

$$(I_{\text{rms}})^2 \cdot R = (200)^2 \times 1.5 = 60 \text{ kW}$$

(ii) The power to be supplied by the plant is power required + power loss = (800) kW + (60) kW = 860 kW.

(iii) The voltage drop across the line is

$$I_{rms} \times R = 200 \times 1.5 = 300 \text{ V}$$

The plant generates power at 440 V. It has to be stepped up so that after suffering a line drop of 300 V it reaches the substation in the town at 4000 V.

Hence the step-up transformer at the plant should be (440/4300) V voltage ratio.

14.175 A single-phase step down transformer of (1200/400) V, 50 Hz, have the following parameters:

$$R_1 = 0.2\ \Omega,\ X_1 = 0.5\ \Omega.$$
$$R_2 = 0.02\ \Omega,\ X_2 = 0.06\ \Omega.$$
$$R_0 = 12000\ \Omega, \text{ and } X_0 = 2000\ \Omega.$$

Calculate:

(i) The primary current and power factor of a transformer when a load impedance of $(8 + j6)\ \Omega$ is connected across its secondary terminals and its primary is connected to a 1200 V supply mains.

(ii) The short-circuit current and short circuit power factor of the transformer.

Solution

$$I_0 = I_w + I_m = \frac{V_1}{R_0} + \frac{V_1}{jX_0} = \frac{1200}{12000} + \frac{1}{j} \times \frac{1200}{2000} = (0.1 - j0.6) \text{ A}.$$

$$\therefore \quad R_{01} = R_1 + K^2 \cdot R_2 + K^2 R_L$$

$$= 0.2 + 0.02 \times \left(\frac{1200}{400}\right)^2 + \left(\frac{1200}{400}\right)^2 \times 8 = 72.38\ \Omega.$$

Similarly,

$$X_{01} = X_1 + K^2 X_2 + K^2 X_L$$

$$= 0.5 + 3^2 \times 0.06 + 3^2 \times 6 = 55.04\ \Omega. \qquad \left[\because K = \frac{1200}{400} = 3\right]$$

$$\therefore \quad I_{01} = \frac{V_1}{R_{01} + jX_{01}} = \frac{1200}{72.38 + j55.04} = (10.5 - j7.99) \text{ A}$$

$\therefore$ Primary current $I_1 = I_0 + I_{01} = (0.1 - j0.6) + (10.5 - j7.99) = (10.6 - j8.59)$ A

The magnitude of the primary current is $I_1 = 13.6$ A while the power factor is $\left(\frac{10.6}{13.6}\right)$ or 0.78 (lag)

Again, $\quad R_L' = X_L' = 0 \qquad$ (given)

and Z_{eq}' (Z referred to primary) $= (R_1 + K^2 \cdot R_2) + j(X_1 + K^2 X_2)$

$$= R_{eq'} + jX_{eq'} = (0.38 + j1.04)\ \Omega.$$

$$\therefore\ I_{01} \text{ (on short circuit)} = \frac{1200}{0.38 + j1.04} = \frac{1200}{1.107} = 1085.8 \text{ A}.$$

This is the primary short circuit current as I_0 is extremely negligible in comparison to this. Power factor (on short circuit) is $\left(\frac{R_{eq}}{Z_{eq}'}\right)$ i.e., $\frac{0.38}{1.107}$, or, 0.343.

14.176 A 2300/575/230 V three-winding (P/S/T) transformer has a total of 300 turns on its high voltage primary side. Each of the two secondary windings (say A and B) is rated for 200 kVA. Calculate the primary current, (i) when the 230 V secondary winding carries its rated current at unity power factor. (ii) when the 575 V winding carries its rated current at 0.5 lagging. Ignore the magnetizing current, internal drops and losses.

Solution

Let 2300/575/230 V be the voltages of windings P, S, T respectively.

$\therefore$ The rated current for S is $I_{2 \cdot A} = \dfrac{200000}{575} = 347.8$ A

$\therefore$ The rated current for T is $I_{2 \cdot B} = \dfrac{200000}{230} = 869.6$ A

Since $I_{2.A}$ is at p.f. 0.5 (lagging),

$$\therefore \quad I_{2A} = (173.9 - j301.2) \text{ A.}$$

I_{2B} is at unity p.f.

$$I_{2B} = (869.6 - j0) \text{ A.}$$

$$\therefore \quad I'_{2 \cdot A} = (173.9 - j301.2) \times \frac{2300}{575} = (695.6 - j1204.8) \text{ A [referred to } P \text{ side]}$$

$$\text{and} \quad I'_{2 \cdot B} = (869.6 - j0.0) \times \frac{2300}{230} = 8696 \text{ A} \qquad \text{[(referred to } P \text{ side)]}$$

$$\therefore \quad I'_2 = I'_{2A} + I'_{2B} = 695.6 - j1204.8 + 8696 = 9391.6 - j1204.8$$

$$\therefore \quad |I'_2| = 9468.46 \text{ A} \qquad \text{[total primary load current]}$$

14.177 A transformer has maximum efficiency η at 95% at a load of 100 kW. Determine the constant loss of the transformer.

Solution

We know at maximum efficiency.

$$\text{Constant loss } W_i = \text{copper loss } W_c$$

$$\therefore \quad (\text{efficiency})_{\max} = \frac{\text{outout power}}{\text{output power} + 2W_i}$$

$$\text{or} \quad 0.95 = \frac{100 \times 10^3}{100 \times 10^3 + 2 \cdot W_i}$$

$$\text{or} \quad 2 \cdot W_i \times 0.95 = 100 \times 10^3 - 0.95 \times 100 \times 10^3$$

$$W_i = \frac{100 \times 10^3 (1 - 0.95)}{2 \times 0.95} = 2631.58 \text{ W} = 2.63 \text{ kW.}$$

14.178 A single-phase transfoprmer on open circuit gave the following test results:

216 V	45 Hz	58.2 W
264 V	55 Hz	73.2 W

Calculate the eddy current and hysteresis losses separately at 240 V, 50 Hz.

Solution

For the first test, $\dfrac{V}{f} = \dfrac{216}{45} = 4.8$

For the second test, $\dfrac{V}{f} = \dfrac{264}{55} = 4.8$

At 240 V, $\dfrac{V}{f} = \dfrac{240}{50} = 4.8$

Since $V = E = 4.44 \, f N \phi_m$

then $\dfrac{V}{f} = K\phi_m$ where K is a constant ($K = 4.44$ N).

As V is constant, the flux and flux density are constant.

We know, hysteresis loss $(P_h) \propto f$ or, $P_h = af$

Eddy current loss (P_c) $\propto f^2$ or, $P_e = bf^2$

Total iron loss = $P_h + P_e = af + bf^2$

$$\therefore \quad 58.2 = 45a + 2025b \quad \text{(i)}$$

$$73.2 = 55a + 3025b \quad \text{(ii)}$$

Solving Eq. (i) and (ii), we have

$$a = 1.124$$

$$b = 0.0038$$

$\therefore$ at 50 Hz, $P_h = 50 \times a = 50 \times 1.124 = 56.2$ W

$$P_e = 2500 \times 0.0038 = 95 \text{ W}.$$

14.179 A 30 kW induction motor is operated at 0.85 lagging p.f. The supply voltage is 230 V. Determine the external circuit element that should be connected with the motor so that it operates at a power factor of 0.9 lagging.

Solution

The external circuit element must be connected in parallel with the motor so that the supply voltage to the motor does not change. To improve the power factor from 0.85 lagging to 0.9 lagging a capacitor must be connected in parallel as shown in Fig. 14.103(a).

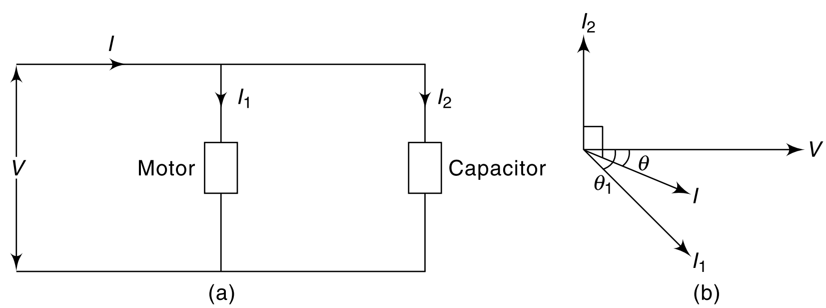


Fig. 14.103

The corresponding phasor diagram is shown in Fig. 14.103(b).

Now $\theta_1 = \cos^{-1} 0.85 = 31.78°$

$$\theta = \cos^{-1} 0.9 = 25.84°$$

From the phasor diagram

$$I_1 \cos \theta_1 = I \cos \theta$$

or $VI_1 \cos \theta_1 = VI \cos \theta$

or $30{,}000 = 230 \times I \times 0.9$

or $I = 144.92$ A

Again $I_1 = \dfrac{30{,}000}{230 \times 0.85} = 153.45$ A

From the phasor diagram

$$I_1 \sin \theta_1 - I_2 = I \sin \theta$$

or $I_2 = 153.45 \sin 31.78° - 144.92 \sin 25.84° = 80.816 - 62.34 = 18.472$ A.

Hence the capacitive reactance (X_C) = $\dfrac{230}{18.472} = 12.45\ \Omega$.

If C be the capacitance then

$$X_C = \frac{1}{\omega C} = 12.45$$

or $C = \dfrac{1}{2\pi \times 50 \times 12.45}$ F = 255.67 μF.

The required capacitance is 255.67 μF.

14.180 A three-phase induction motor having an input power of 30 kW is running at a slip of 4%. The stator loss is 1 kW and friction and windage loss is 1.5 kW. Determine the power input to the rotor, rotor copper loss, mechanical power developed and motor efficiency.

Solution

Air gap power P_{ag} = Power input – Stator loss = 30 – 1 = 29 kW.

$\therefore$ Power input to the rotor = 29 kW.

Rotor copper loss $P_{cur} = s \cdot P_{ag} = 0.04 \times 29 = 1.16$ kW

Mechanical power developed

$$P_m = P_{ag} - P_{cur} = 29 - 1.16 = 27.84 \text{ kW}$$

$$\text{Power output} = 27.84 - 1.5 = 26.34 \text{ kW}$$

$$\text{Efficiency} = \frac{26.34}{30} \times 100\% = 87.8\%.$$

14.181 A 440 V, 8-pole, 50 Hz star connected three-phase induction has a full-load slip of 4%. The ratio of stator to rotor turns is 6. The rotor resistance and standstill reactance per phase is 0.3 and 1 Ω respectively. Determine the full-load torque developed.

Solution

$$\text{Per phase rotor voltage } E_2 = \frac{440}{\sqrt{3}} \text{ V}.$$

$$\text{Synchronous speed } N_S = \frac{120 \times 50}{8} = 750 \text{ rpm}.$$

$$\text{Torque developed } T = \frac{3s\, E_2^2 R_2}{(R_2^2 + s^2 X_2^2)\, w_s} \quad \left(\omega_s = \frac{2\pi Ns}{60} \text{ rad/s}\right)$$

$$= \frac{3 \times 0.04 \times \left(\dfrac{440}{\sqrt{3}}\right)^2 \times 0.3}{\{(0.3)^2 + (0.04 \times 1)^2\}\, \dfrac{2\pi \times 750}{60}} = 323.09 \text{ Nm}.$$

14.182 A three-phase, 40 V, 5-pole, 60 Hz induction motor has a star connected stator and rotor. The ratio of stator to rotor turns in 1.35. The full-load slip is 4% and the rotor resistance and standstill reactance is 0.3 Ω and 1.5 Ω respectively. Determine the torque and power developed at full load. Also find the maximum torque and the speed at which it occurs.

Solution

$$\text{Per phase rotor voltage } E_2 = \frac{400}{\sqrt{3} \times 1.35} = 171.06 \text{ V}.$$

$$\omega_s = \frac{2\pi}{60} \times \frac{120 \times 50}{6} = 104.67 \text{ rad/s}.$$

$$\text{Torque developed } T = \frac{3s\, E_2^2 R_2}{\omega_s \{R_2^2 + s^2 X_2^2\}} = \frac{3 \times 0.04 \times (171.06)^2 \times 0.3}{104.67\{(0.3)^2 + (0.04 \times 1.5)^2\}} = 107.52 \text{ N-m}.$$

Power developed at full load = 107.52 × 104.67 W = 11254.12 W = 11.254 kW

$$\text{Maximum torque } T_{max} = \frac{3\, E_2^2}{2\, \omega_s X_2} = \frac{3 \times (171.06)^2}{2 \times 104.67 \times 1.5} = 279.56 \text{ Nm}.$$

Slip at maximum torque $s_m = \frac{R_2}{X_2} = \frac{0.3}{1.5} = 0.2.$

$\therefore$ Speed N_r at maximum torque $= N_s(1 - s_m) = \frac{120 \times 50}{6}$ (1 – 0.2) = 800 rpm.

14.183 An 8-pole, 50 Hz squirrel cage induction motor has a rotor resistance and standstill reactance of 0.05 Ω and 0.5 Ω per phase respectively. Determine the speed at maximum torque condition and the external rotor resistance per phase to give two thirds of maximum torque at starting.

Solution

Slip at maximum torque $s_m = \frac{R_2}{X_2} = \frac{0.05}{0.5}. = 0.1$

$\therefore$ Speed (N_r) at maximum torque $= (1 - 0.1) \times \frac{120 \times 50}{8}$ = 675 rpm.

Maximum torque $T_m = \frac{3}{2} \frac{E_2^2}{2\omega_s X_2}$ [ω_s is synchronous speed.]

The required torque $T = \frac{2}{3} T_m = \frac{E_2^2}{2\omega_s X_2}$; at starting i.e. at $s = 1$,

$$\therefore \quad T = \frac{3s\, E_2^2\, R_2'}{\omega_s \left(R_2'^2 + s^2 X_2^2\right)},$$

where R_2 is the new rotor resistance

$$= \frac{3\, E_2^2\, R_2}{\omega_s\, (R_2'^2 + X_2^2)}$$

Now,
$$\frac{3\, E_2^2\, R_2'}{\omega_s\, (R_2'^2 + X_2^2)} = \frac{E_2^2}{2\, \omega_s\, X_2}$$

$$\therefore \quad 6R_2' X_2 = R_2'^2 + X_2^2$$

or $$R_2'^2 - 6 \times 0.5\, R_2' + (0.5)^2 = 0$$

or $$R_2'^2 - 3R_2' + 0.25 = 0$$

or $$R_2' = \frac{3 \pm \sqrt{9-1}}{2} = 1.5 \pm 1.414 = 0.086\ \Omega \text{ or } 2.914\ \Omega$$

$\therefore$ External rotor resistance per phase is (0.086 – 0.05) Ω = 0.036 Ω or (2.914 – 0.05) Ω = 2.864 Ω.

14.184 A 11 kV star connected squirrel cage induction motor has full load slip of 4% and standstill impedance of 8 Ω. The full load current is 30 A and the maximum starting current which can be drawn from the line is 75 A. Find the tapping on the auto transformer used for starting the motor. Also give the value of the starting torque in terms of the full load torque.

Solution

Without auto transformer the current at standstill condition is $\frac{11000/\sqrt{3}}{8}$ = 793.85 A.

Let the tapping on the auto transformer be x.

Hence, $793.85\ x^2 = 75$

$\therefore \quad x = 0.307$

Therefore the tapping on the autotransformer is 30.7%

If T_{st} is the starting torque and T_{fl} is the full load torque then,

$$\frac{T_{\text{st}}}{T_{\text{fl}}} = \left(\frac{75}{30}\right)^2 \times 0.04 = 0.25.$$

14.185 The input power to the rotor of a three-phase, 50 Hz, 4-pole induction motor is 50 kW. The rotor emf makes 120 alternations per minute. Determine (i) slip (ii) speed (iii) mechanical power developed (iv) rotor copper loss per phase (v) rotor resistance per phase if rotor current is 50 A and (vi) torque developed.

Solution

(i) If (s) be the slip then

$$sf = \frac{120}{60}$$

or $$s = \frac{120}{60 \times 50} = 0.04.$$

(ii) Speed $= (1 - s)\ N_s = (1 - 0.04) \times \dfrac{120 \times 50}{4} = 1440$ rpm.

(iii) Now, $P_{\text{ag}} = 50$ kW.

$\therefore$ Mechanical power developed (P_m) $= (1 - s)\ P_{\text{ag}} = (1 - 0.04) \times 50 = 48$ kW.

(iv) Rotor copper loss (P_{cur}) $= sP_{\text{ag}} = 0.04 \times 50 = 2$ kW

Hence rotor copper loss per phase $= \dfrac{2}{3}$ kW $= 0.667$ kW $= 667$ W

(v) Rotor copper loss per phase $(= I_2^2 R_2) = (50)^2 \times R_2 = 667$

$\therefore$ Rotor resistance $R_2 = \dfrac{667}{(50)^2} = 0.2668\ \Omega.$

(vi) Torque developed $T = \dfrac{P_{\text{ag}}}{\omega_s} = \dfrac{50{,}000}{2\pi \dfrac{120 \times 50}{4 \times 60}} = 318.47$ N-m. $\quad \left[\because \omega s = \dfrac{2\pi N_s}{60}\right]$

14.186 The power input to a three-phase, 6-pole, 440 V, 50 Hz induction motor is 50 kW at 970 rpm. The mechanical loss and the total stator losses are 1.5 kW and 2 kW respectively. Determine the rotor copper loss, gross torque and the rotor resistance per phase if the rotor phase current is 110 A.

Solution

Air-gap power $P_{\text{ag}} = 50 - 2 = 48$ kW

$$\text{Slip } (s) \left(1 - \frac{N_r}{N_s}\right) = 1 - \frac{970}{\dfrac{120 \times 50}{6}} = 1 - 0.97 = 0.03.$$

$\therefore$ Rotor copper loss $P_{\text{cur}} = sP_{\text{ag}} = 0.03 \times 48 = 1.44$ kW

Mechanical power developed $P_m = (1 - s)P_{\text{ag}} = 0.97 \times 48 = 46.56$ kW.

Power output $= 46.56 - 1.56 = 45.06$ kW.

$\therefore$ Gross torque $= \dfrac{45.06 \times 10^3}{2\pi \times \dfrac{2 \times 50}{6}} = 430.5$ Nm. $\quad \left[\because \omega_s = \dfrac{2\pi N_s}{60} \text{ and } T = \dfrac{P}{\omega_s}\right]$

If rotor resistance is R_2,

$$I_2^2 R_2 = \frac{1440}{3}$$

$$\therefore \quad R_2 = \frac{1440}{3 \times (110)^2}\ \Omega = 0.039\ \Omega.$$

14.187 A 50 Hz slip ring induction motor having star connected rotor gives 750 V at standstill between the slip rings. Rotor resistance and reactance per phase are 0.3 Ω and 5 Ω respectively. Calculate the rotor current and p.f. at standstill when an external impedance of (3 + j10) Ω is connected with the rotor. Also calculate the current and p.f. when the slip rings are short-circuited and the motor is running at a slip of 3%.

Solution

$$E_2 = \frac{750}{\sqrt{3}}\ \text{V} = 433\ \text{V}.$$

When external impedance is connected the total rotor impedance at standstill is (Z_2) = $\sqrt{(3+0.3)^2 + (5+10)^2} = 15.3\ \Omega$.

At standstill, rotor current = $\frac{433}{15.3}$ A = 28.3 A.

$$\text{p.f.} \left(= \frac{R}{Z}\right) = \frac{3+0.3}{15.3} = 0.216.$$

when the slip rings are short circuited and the motor is running at a slip of 3%,

$$Z_2 = \sqrt{(0.3)^2 + (0.03 \times 5)^2} = 0.3354\ \Omega.$$

$$\text{Rotor current} \quad I_2 = \frac{s E_2}{Z_2} = \frac{0.03 \times 433}{0.3354}\ \text{A} = 38.73\ \text{A}$$

$$\text{Power factor} = \frac{0.3}{0.3354} = 0.89\ \text{(lag)}.$$

14.188 The rotor of a three-phase induction motor has 0.03 Ω resistance and 0.5 Ω standstill reactance per phase. Determine the external resistance to be connected in the rotor circuit to get one third of the maximum torque at starting. Neglect stator impedance. By what percentage will the external resistance change the current and p.f. at starting?

Solution

$$\frac{T_{st}}{T_m} = \frac{2}{\frac{1}{s_m} + s_m} = \frac{1}{3}$$

$$\therefore \quad \frac{1}{s_m} + s_m = 6$$

$$\text{or} \quad s_m^2 - 6s_m + 1 = 0$$

$$\text{or} \quad s_m = \frac{6 \pm \sqrt{36-4}}{2} = 5.82 \text{ or } 0.18$$

As induction motor cannot have slip more than 1, hence value of s_m is 0.18.

For negligible stator impedance $\frac{R_2}{s_m} = X_2$

$$\text{or} \quad R_2 = 0.18 \times 0.5 = 0.09$$

∴ External resistance to be connected in the rotor circuit is (0.09 – 0.03) = 0.06 Ω.

Without external resistance,

Starting current $(I_{st})\left(=\dfrac{V}{Z_2}\right) = \dfrac{V}{\sqrt{(0.03)^2+(0.5)^2}} = (1.996\ \text{V})$

(where V is the supply voltage per phase).

Power factor $\left(=\dfrac{R_2}{Z_2}\right) = \dfrac{0.03}{\sqrt{(0.03)^2+(0.5)^2}} = 0.0599.$

With external resistance

$$I_{st} = \frac{V}{\sqrt{(0.09)^2+(0.5)^2}} = (1.968\ \text{V}).$$

and $$\text{p.f.} = \frac{0.09}{\sqrt{(0.09)^2+(0.5)^2}} = 0.177.$$

$\therefore$ Percentage reduction in starting current $\dfrac{1.996\,V - 1.968\,V}{1.996\,V} \times 100\% = 1.4\%$

and percentage improvement in p.f. $= \dfrac{0.177 - 0.0599}{0.0599} \times 100\% = 195.5\%$

14.189 A three-phase, 50 Hz, 8-pole induction motor with shaft output of 15 kW runs at 730 rpm. The total stator losses and mechanical losses are 800 W and 250 W respectively. Calculate the input power to the motor and the air gap power. If maximum torque is developed at 740 rpm calculate the starting torque with the rated voltage starting.

Solution

$$N_S = \frac{120 \times 50}{8} = 750 \text{ rpm}$$

Full load slip $s_{fl} = \dfrac{750-730}{750} = 0.0267.$

Mechanical power developed $P_m = 15{,}000 + 250 = 15250$ W

$\therefore$ Air-gap power $P_{ag} = \dfrac{P_m}{1-s_{fl}} = \dfrac{15250}{1-0.0267} = 15668.34$ W

Hence power input to the motor $= P_{ag}$ + Stator losses $= 15668.34 + 800 = 16468.34$ W

Now, slip at maximum torque $s_m = \dfrac{750-740}{750} = 0.0133.$

Torque at full load $T_{fl} = \dfrac{P_{ag}}{\omega_s} = \dfrac{15668.34}{2\pi \times \dfrac{750}{60}} = 199.59$ Nm $\qquad \left[\because \omega_s = 2\pi \cdot \dfrac{N_s}{60}\right]$

$\because \qquad \dfrac{T_{fl}}{T_m} = \dfrac{2}{\dfrac{s_{fl}}{s_m} + \dfrac{s_m}{s_{fl}}}$, here, $T_{fl}/T_m = \dfrac{2}{\dfrac{0.0267}{0.0133} + \dfrac{0.0133}{0.0267}} = 0.798.$

$\therefore$ Maximum torque $T_m = \dfrac{199.59}{0.798} = 250$ Nm.

If T_{st} be the starting torque, $\dfrac{T_{st}}{T_m} = \dfrac{2}{\dfrac{1}{s_m} + \dfrac{s_m}{1}} = \dfrac{2}{\dfrac{1}{0.0133} + 0.0133} = 0.0266.$

$\therefore \qquad T_{st} = 0.0266 \times 250 = 6.649$ Nm.

14.190 In a three-phase induction motor the maximum torque is thrice the full load torque and the starting torque is twice the full load torque. If the full load slip is 4%, calculate the percentage reduction in rotor circuit resistance. Neglect stator impedance.

Solution

$$\frac{T_m}{T_{fl}} = 3 \quad \text{and} \quad \frac{T_{st}}{T_{fl}} = 2,$$

$$\therefore \quad \frac{T_{st}}{T_m}\left(=\frac{2}{3}\right) = \frac{2}{\dfrac{1}{s_m} + s_m}, \quad \text{where } s_m \text{ is the slip at maximum torque.}$$

$$\therefore \quad s_m^2 + 1 = 3s_m$$

or $\quad s_m^2 - 3s_m + 1 = 0$

or $$s_m = \frac{3 \pm \sqrt{9-4}}{2} = 1.5 \pm 1.118 = 2.618 \text{ or } 0.382.$$

Neglecting $s_m = 2.618$, the feasible value of s_m is 0.382
Now neglecting stator impedance, we have

$$\frac{R_2}{X_2} = s_m = 0.382$$

or $\quad R_2 = 0.382X_2.$

For a full-load slip of 0.04, if slip at maximum torque is s_m',

we have, $$\frac{T_{fl}}{T_m}\left(=\frac{1}{3}\right) = \frac{2}{\dfrac{s_m'}{s_{fl}} + \dfrac{s_{fl}}{s_m'}}$$

or $$\frac{s_m'}{0.04} + \frac{0.04}{s_m'} = 6$$

or $\quad s_m'^2 - 0.24\,s_m' + 0.016 = 0$

$$\therefore \quad s_m' = \frac{0.24 \pm \sqrt{0.0576 - 0.0064}}{2} = 0.12 \pm 0.113 = 0.233 \text{ or } 0.00686.$$

Considering $\quad s_m' = 0.233$

$$\frac{R_2'}{X_2} = 0.233,$$ where R_2' is the new rotor circuit resistance for obtaining full load slip of 4%.

$$\therefore \quad R_2' = 0.233X_2$$

Hence reduction in the rotor circuit resistance is $\dfrac{0.382X_2 - 0.233X_2}{0.382X_2} \times 100\% = 39\%.$

14.191 A 8 kW, 440 V, 6-pole delta connected squirrel cage induction motor gave the following results:

No load test: 440 V, 10 A, 300 W

Blocked rotor test: 100 V, 30 A, 1500 W

The dc resistance of the stator winding is 0.5 Ω. Determine the rotational losses and the equivalent circuit parameters.

Solution

The stator resistance $R_1 = 0.5 \times 1.2 = 0.6\ \Omega$

No load rotational loss $= 300 - \left(\dfrac{10}{\sqrt{3}}\right)^2 \times 0.6 \times 3 = 240$ W.

From no-load test,

$$Z_{nl} = \frac{440}{10/\sqrt{3}} = 76.2\ \Omega$$

$$R_{nl} = \frac{440}{3(10/\sqrt{3})^2} = 3\ \Omega$$

$$\therefore \qquad X_{nl} = \sqrt{Z_{nl}^2 - R_{nl}^2} = 76.14\ \Omega$$

From blocked rotor test

$$Z_{br} = \frac{100}{30/\sqrt{3}} = 5.77\ \Omega \text{ and } R_{br} = \frac{1500}{3 \times (30/\sqrt{3})^2} = 1.67\ \Omega$$

We have, $\quad X_{br} = \sqrt{(5.77)^2 - (1.67)^2} = 5.52\ \Omega.$

$$\therefore \qquad X_1 = X_2 = \frac{1}{2} X_{br} = \frac{1}{2} \times 5.52 = 2.76\ \Omega$$

$$X_0 = 76.14 - 2.76 = 73.38\ \Omega.$$

Per phase rotor resistance

$$R_2 = (R_{br} - R_1)\left(\frac{X_2 + X_0}{X_0}\right)^2 = (1.67 - 0.6)\left(\frac{2.76 + 73.38}{73.38}\right)^2 = 1.152\ \Omega.$$

The parameters of the induction motor equivalent circuit are:
$R_1 = 0.6\ \Omega$, $R_2 = 1.152\ \Omega$, $X_1 = X_2 = 2.76\ \Omega$, $X_0 = 73.38\ \Omega$.

14.192 A three-phase squirrel cage induction motor has maximum torque equal to thrice the full-load torque. Calculate the ratio of the starting torque to the full load torque when it is started by (i) direct online starter (ii) star delta starter and (iii) auto transformer with 60% tapping. Neglect stator impedance. The rotor resistance and standstill reactance referred to the stator are 0.3 Ω and 5 Ω per phase respectively.

Solution

(i) Slip at maximum torque $(s_m) = \dfrac{R_2}{X_2} = \dfrac{0.3}{5} = 0.06$ (as star impedance is neglected)

Now $\quad \dfrac{T_{st}}{T_m} = \dfrac{2}{\dfrac{1}{s_m} + s_m} = \dfrac{2}{\dfrac{1}{0.06} + 0.06} = 0.119$

$\therefore \qquad T_{st} = 0.119\ T_m$; But $T_m = 3\ T_{fl}$.

Hence, $\quad T_{st} = 0.119 \times 3T_{fl} = 0.357\ T_{fl}$.

(ii) With star delta starter starting torque is one third of the starting torque obtained by direct on line starter.

$\therefore \qquad T_{st} = \dfrac{1}{3} \times 0.357\ T_{fl} = 0.119\ T_{fl}$.

(iii) With auto transformer starter of 60% tapping

$$T_{st} = (0.6)^2 \times 0.357\ T_{fl} = 0.1285\ T_{fl}.$$

14.193 A three-phase, 8-pole induction motor has a full-load speed of 730 rpm with its slip rings short circuited. The motor drives a constant load torque. If the rotor speed is reduced to 700 rpm and then to 400 rpm, compare the rotor copper loss at these two speeds with that at full load.

Solution

As the motor drives constant load torque the electromagnetic torque developed by the motor is constant. Hence the air gap power P_{ag} is constant. As rotor copper loss is sP_{ag}, the rotor copper loss is proportional to the slip.

$$\text{Slip at full load}\left(=1-\frac{N_r}{N_s}\right) = 1 - \frac{730}{\frac{120\times 50}{8}} = 0.0267.$$

$$\text{Slip at 700 rpm}\left(=1-\frac{700}{750}\right) = 0.067.$$

$$\text{Slip at 400 rpm}\left(=1-\frac{400}{750}\right) = 0.467.$$

Here, $$\frac{\text{Rotor copper loss at 700 rpm}}{\text{Rotor copper loss at full load}} = \frac{\text{Slip at 700 rpm}}{\text{Slip at full load}} = \frac{0.067}{0.0267} = 2.51.$$

and $$\frac{\text{Rotor copper loss at 400 rpm}}{\text{Rotor copper loss at full load}} = \frac{0.467}{0.0267} = 17.49.$$

14.194 A salient pole, 750 rpm, 50 Hz, three-phase alternator has a flux density of 1 Wb/m^2 in the poles. The stator has two layer three-phase winding and total number of slots is 160. Each coil has 3 turns. The coils are connected in 63° phase groups and the coil span is 15 slots. The length and diameter of the stator winding is 50 cm and 150 cm respectively. Determine the line voltage if the winding is delta connected.

Solution

$$N = 750 \text{ rpm and } f = 50 \text{ Hz}.$$

$$\therefore \text{ No. of poles} \quad P = \frac{120\times 50}{750} = 8.$$

$$\text{Pole pitch area} = \frac{\pi\times 150}{8}\times 50\times 10^{-4}\text{ m}^2 = 0.2945\text{ m}^2.$$

$$\text{Flux/pole } \phi = 1\times 0.2945 = 0.2945\text{ Wb}.$$

Now, number of slots per pole $= \dfrac{160}{8} = 20.$

$$\text{Slot angle } \beta = \frac{180^\circ}{20} = 9^\circ.$$

Number of slots in a phase group $= \dfrac{63^\circ}{9^\circ} = 7.$

$\therefore$ Slots/pole/phase (m) = 7.

$$\text{Pitch factor } K_P = \cos\frac{\alpha}{2} = \cos\left[\frac{9^\circ\times(20-15)}{2}\right] = \cos\frac{45^\circ}{2} = 0.92388.$$

$$\text{Distribution factor } K_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin\left(\frac{7\times 9^\circ}{2}\right)}{7\sin\left(\frac{9^\circ}{2}\right)} = 0.9513.$$

Number of turns/phase = $\frac{160}{3} \times 3 = 160$ (Since each coil has 3 turns)

Phase voltage $V_P = 4.44\ K_P K_d\ \phi f T$

$$= 4.44 \times 0.92388 \times 0.9513 \times 0.2945 \times 50 \times 160 = 8746.52 \text{ V}.$$

Since the winding is delta connected line, the voltage is 8746.52 V.

14.195 A delta connected three-phase, 750 rpm, 6 kV, 50 Hz alternator has 3 slots per pole per phase. The coil span is 7 slots and flux per pole is 0.33 wb. Determine the distribution factor and the number of turns per phase.

Solution

Number of slots/pole/phase (m) = 3

$\therefore$ Number of slots/pole = (3 × 3) = 9

Hence coil span $\left(=\frac{7}{9}\times 180°\right) = 140°$

Pitch factor $K_P = \cos\left[\frac{180° - 140°}{2}\right] = 0.938$

Slot angle $\beta = \frac{180°}{9} = 20°$

$\therefore$ Distribution factor $K_d = \dfrac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \dfrac{\sin\left(\frac{3\times 20°}{2}\right)}{3\sin\left(\frac{20°}{2}\right)} = 0.9597.$

Flux per pole $\phi = 0.33$ Wb.

$\therefore \quad V_P = 4.44\ K_d K_P\ \phi f T$, where ($T$) is the number of turns per phase.

Hence, $\qquad T = \dfrac{6000}{4.44\times 0.9597\times 0.938\times 0.33\times 50} = 91.$

14.196 A delta connected, three-phase, 50 Hz alternator has 16 poles, 144 slots and 10 conductors per slot. The open circuit line voltage is 1500 V. Determine the flux per pole.

Solution

Number of slots/pole/phase $m = \dfrac{144}{16\times 3} = 3.$

Slot angle $\beta = \dfrac{180°}{\text{slots/pole}} = \dfrac{180°}{\frac{144}{16}} = 20°$

Distribution factor $(K_d) = \dfrac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \dfrac{\sin\left(\frac{3\times 20°}{2}\right)}{3\times\sin\left(\frac{20°}{2}\right)} = 0.9598.$

Number of conductors per phase is $\dfrac{144\times 10}{3} = 480.$

Number of turns per phase $T = \dfrac{480}{2} = 240.$

If ϕ be the flux per pole, we have

$$V_P = 4.44\ K_P K_d\ \phi f T$$

i.e., $1500 = 4.44 \times 1 \times 0.9598 \times \phi \times 50 \times 240$

or, $\phi = \dfrac{1500}{4.44 \times 0.9598 \times 50 \times 240} = 0.0293$ Wb.

14.197 A 12-pole 50-Hz single-phase alternator has 144 slots, one third of which are wound. Each slot has 6 conductors. The flux per pole is 0.036 Wb. The breadth factor is 0.79 and the coils are of full pitch. Determine the generated emf.

Solution

$P = 12$, $S = 144$ (given).
Number of conductors $Z = 6 \times 144 = 864$

As one-third of the slots are wound, the number of turns is $T = \dfrac{864}{2} \times \dfrac{1}{3} = 144.$

Also, $\phi = 0.036$ Wb, $K_d = 0.79$ and $K_P = 1$

$\therefore$ Generated emf $E = 4.44\ K_P K_d\ \phi f T = 4.44 \times 1 \times 0.79 \times 0.036 \times 50 \times 144 = 909.17$ V.

14.198 A 1500 V, single-phase alternator while running at normal speed requires an excitation current of 5 A to give a short circuit-current equal to full-load current of 120 A. An emf of 400 V is produced by the same excitation current. The resistance of the armature circuit is 0.3 Ω. Determine the regulation of the alternator at 0.8 p.f. lagging and 0.75 p.f. leading.

Solution

Synchronous impedance $Z_s = \dfrac{400}{120}\,\Omega = 3.33\ \Omega.$

Synchronous reactance $X_s = \sqrt{(3.33)^2 - (0.3)^2} = 3.32\ \Omega.$

Open circuit emf at 0.8 p.f. lagging is

$$V_0 = \sqrt{(V_{\text{fl}} \cos \phi + IR)^2 + (V_{\text{fl}} \sin \phi + IX_s)^2}$$

$$= \sqrt{(1500 \times 0.8 + 120 \times 0.3)^2 + (1500 \times 0.6 + 100 \times 3.32)^2} = 1745.14 \text{ V}.$$

Hence regulation at 0.8 lagging p.f. is $\left[\dfrac{1745.14 - 1500}{1500} \times 100\%\right]$

or, 16.34%.
Open circuit voltage at 0.75 p.f. leading is

$$(V_0) = \sqrt{(1500 \times 0.75 + 120 \times 0.3)^2 + (1500 \times 0.66 - 100 \times 3.32)^2} = 1334.5 \text{ V}.$$

Hence regulation at 0.75 p.f. leading is $\left[\dfrac{1334.5 - 1500}{1500} \times 100\%\right]$ or, (–11%).

14.199 A star connected, 50 Hz, 20 pole, three-phase alternator has a single layer winding with full pitch coils. The number of stator slots is 180 and the coils are connected in 60° phase groups. The flux per pole is 0.03 wb and each coil has 2 turns. Determine the open circuit line voltage.

Solution

Given, $P = 20$, $S = 180$, $\phi = 0.03$ Wb.

Since the alternator has a single layer winding, number of coils is $\dfrac{180}{2} = 90.$

Number of turns (T) = (2×90) = 180.

$$\text{Slots/pole} = \frac{180}{20} = 9$$

$$\text{Slot angle } \beta = \frac{180°}{9} = 20°.$$

$$\text{Number of slots in a phase group} = \frac{60°}{20°} = 3.\ (= m)$$

$$\therefore \quad \text{hence } (K_d) = \frac{\sin \frac{m\beta}{2}}{m \sin \frac{\beta}{2}} = 0.9598.$$

$$\therefore \quad V_{Ph} = 4.44\ K_P K_d\ \phi f T = 4.44 \times 1 \times 0.9598 \times 0.03 \times 50 \times 180 = 1150.6 \text{ V}.$$

$\therefore$ Open circuit line voltage $V_L = \sqrt{3} \times 1150.6 = 1992.85$ V.

14.200 An 11 kV star connected synchronous motor has an input current of 100 A. The synchronous reactance and resistance are 25 Ω and 0.5 Ω respectively. Determine the power input to the motor and the induced emf for a p.f. of 0.75 lagging.

Solution

Line voltage V_L = 11 kV.

Armature current $I_a = 100\angle{-\cos^{-1} 0.75°} = 100\angle{-41.41°}$ A.

X_s = 25 Ω and R_a = 1 Ω.

$$\therefore \text{ Power input to the motor } = \sqrt{3}\, V_L\, I_a \cos\theta = (\sqrt{3} \times 11000 \times 100 \times 0.75)$$
$$= 1428941.916 = 1428.94 \text{ kW}.$$

Induced emf per phase

$$E = V_{Ph} - I_a(R_a + jXs)$$
$$= \frac{11000}{\sqrt{3}} - 100\angle{-41.41°}\ (0.5 + j25)$$
$$= 6351 - 100\angle{-41.41°} \times 25.005\angle{88.25°}$$
$$= 6351 - 2500.5\angle{47.44°} = 5010.7\angle{-21.567°} \text{ V}.$$

14.201 The full-load current of a 3.3 kV star connected synchronous motor is 100 A at 0.8 p.f. lagging. The mechanical loss is 25 kW and the per phase resistance and reactance of the motor is 0.3 Ω and 10 Ω respectively. Determine the excitation emf, torque angle, efficiency and shaft output of the motor.

Solution

Given :

$$R_a = 0.3\ \Omega$$
$$X_s = 10\ \Omega$$
$$V_{Ph} = \frac{3300}{\sqrt{3}} \text{ V} = 1905.3 \text{ V}$$
$$I_a = 100 \text{ A}; \cos\theta = 0.8 \text{ (lagging)}.$$

The excitation emf is $E = 1905.3 - 100\angle{-\cos^{-1} 0.8}\ (0.3 + j10)$

$$= 1281.41 - j782.09 = 1501.22\angle{-31.397°} \text{ V/ph}$$

$\therefore$ Excitation emf is 1501.22 V per phase and the torque angle is 31.397°.

Let ϕ be the angle between E and I_a. Mechanical power developed is ($3\ E\ I_a \cos\phi$),

i.e.,
$$P = 3 \times 1501.22 \times 100 \cos(-36.87° + 31.397°)$$
$$= 448312.89 \text{ W} = 448.31 \text{ kW}$$

The shaft output = 448.3 – 25 = 423.3 kW.

$$\text{Power input} = \sqrt{3}\, V_L I_a \cos\theta = \sqrt{3} \times 3300 \times 100 \times 0.8 = 457261 \text{ W} = 457.26 \text{ kW}.$$

Hence, $\text{Efficiency} = \dfrac{423.3}{457.26} \times 100\% = 92.57\%.$

14.202 A 1500 kW load has a power factor of 0.7 lagging. Find the rating of a synchronous condenser to raise the power factor to 0.85 lagging. Also find the total kVA supplied at the new power factor.

Solution

Refer to the phasor diagram shown in Fig. 11.20 (in text)

$$P = OA = 1500 \text{ kW}.$$

At 0.7 p.f. lagging the apparent power $S_1 = OB = \dfrac{1500}{0.7}$ kVA = 2142.857 kVA

Reactive power (AB) = $Q_1 = S_1 \sin\theta_1$ = 2142.857 sin ($\cos^{-1}$ 0.7) = 1530.306 KVAR.

When p.f. is raised to 0.85 lagging by a synchronous condenser, the real power remaining the same (i.e. P = 1500 kW) we have apparent power

$$S_2 = \frac{1500}{0.85} = 1764.7 \text{ kVA}$$

and reactive power (Q_2) = $AC = S_2 \sin\theta_2$ = 1764.7 sin ($\cos^{-1}$ 0.85) = 929.613 KVAR.

Hence, reactive power supplied by the condenser is

$$BC = AB - AC = 1530.306 - 929.613 = 600.693 \text{ KVAR}.$$

∴ Rating of the synchronous condenser is 600.93 KVAR and the total KVAR supplied at new p.f. is 1764.7 KVAR.

MULTIPLE CHOICE QUESTIONS

15.1 CIRCUIT ELEMENTS, KIRCHHOFF'S LAWS, AND NETWORK THEOREMS

1. Identify the passive elements among the following.
   (a) voltage source (b) current source
   (c) inductor (d) transistor
2. Determine the total inductance of a parallel combination of 100 mH, 50 mH and 10 mH.
   (a) 7.69 mH (b) 160 mH (c) 60 mH (d) 110 mH
3. If the voltage across a given capacitor is increased, the amount of stored charge
   (a) increases (b) decreases
   (c) remains same (d) is exactly doubled
4. How much energy is stored by a 100 mH inductance with a current of 1 A?
   (a) 100 J (b) 1 J (c) 0.05 J (d) 0.01 J
5. The following voltage drops are measured across each of three resistors in series 5.2 V, 8.5 V and 12.3 V. What is the value of the source voltage to which these resistors are connected?
   (a) 8.2 V (b) 12.3 V (c) 5.2 V (d) 26 V
6. A certain series circuit has 100 Ω, 270 Ω and 330 Ω resistors in series. If the 270 Ω resistor is removed, the current will
   (a) increase (b) become zero
   (c) decrease (d) remain constant
7. A series circuit consists of a 4.7 kΩ, 5.6 kΩ, 9 kΩ and 10 kΩ resistors. Which resistor has the highest voltage across it?
   (a) 4.7 kΩ (b) 5.6 kΩ (c) 9 kΩ (d) 10 kΩ
8. The total power in a series circuit is 10 W. There are five equal value resistors in the circuit. How much power does each resistor dissipate?
   (a) 10 W (b) 5 W (c) 2 W (d) 1 W

9. When a 1.2 kΩ resistor, 100 Ω resistor, 1 kΩ resistor and 50 Ω resistor are in parallel, the total resistance is less than
   (a) 100 Ω (b) 50 Ω (c) 1.2 kΩ (d) 1 kΩ
10. If one of the resistors in a parallel circuit is removed, what happens to the total resistance?
   (a) decreases (b) increases
   (c) exactly doubles (d) remains constant
11. Six light bulbs are connected in parallel across 110 V. Each bulb is rated at 75 W. How much current flows through each bulb?
   (a) 0.682 A (b) 0.7 A (c) 75 A (d) 110 A
12. Superposition theorem is valid only for
   (a) linear circuits (b) non-linear circuits
   (c) both (a) and (b) (d) neither (a) nor (b)
13. When superposition theorem is applied to any circuit, the dependent voltage source is always
   (a) opened (b) shorted
   (c) active (d) none of the above.
14. Maximum power is transferred when the load resistance is
   (a) equal to source in resistance
   (b) equal to half of the source resistance
   (c) equal to zero
   (d) none of the above
15. The superposition theorem is not valid for
   (a) voltage responses (b) current responses
   (c) power responses (d) all the above.
16. Determine the current I in the circuit (Fig. 15.1)
   (a) 2.5 A (b) 1 A
   (c) 12 A (d) 4.5 A

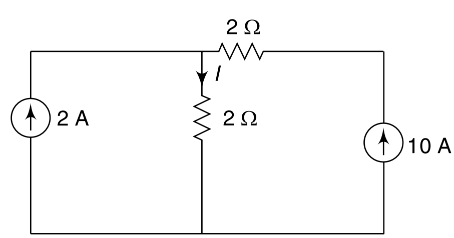


Fig. 15.1

17. The reciprocity theorem is applicable to
   (a) linear networks only
   (b) bilateral networks only
   (c) both (a) and (b)
   (d) neither (a) nor (b)
18. Thevenin voltage in the circuit shown in Fig. 15.2 is
   (a) 3 V (b) 2.5 V
   (c) 2 V (d) 0.1 V

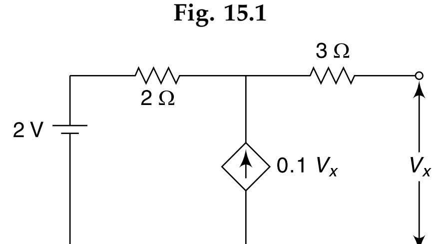


Fig. 15.2

19. Three equal resistances of 3 Ω are connected in star what is the resistance in one of the arms in an equivalent delta circuit?
   (a) 10 Ω (b) 3 Ω
   (c) 9 Ω (d) 27 Ω
20. Three equal resistances of 5 Ω are connected in delta. What is the resistance in one of the arms of the equivalent star circuit?
   (a) 5 Ω (b) 1.67 Ω (c) 10 Ω (d) 15 Ω

21. Norton's current in the circuit (Fig. 15.3) is given by
    (a) $(2i/5)$ (b) zero
    (c) infinite (d) none

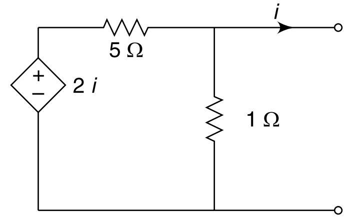


Fig. 15.3

22. The nodal method of circuit analysis is based on
    (a) KVL and Ohm's law
    (b) KCL and Ohm's law
    (c) KVL and KCL
    (d) both (a) and (b)
23. A practical voltage source consists of an ideal voltage source in
    (a) series with an internal resistance
    (b) parallel with an internal resistance
    (c) both (a) and (b)
    (d) neither (a) nor (b)
24. Find the voltage between A and B in a voltage divider network (Fig. 15.4)
    (a) 90 V (b) 9 V
    (c) 100 V (d) 0 V

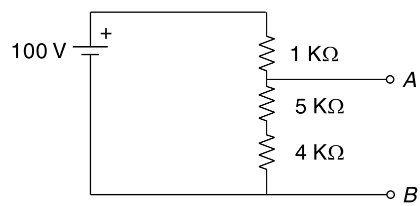


Fig. 15.4

25. The algebraic sum of all the currents meeting a junction is equal to
    (a) 1 (b) −1 (c) zero (d) can't say

Answer (15.1)

1. (c)	2. (a)	3. (a)	4. (c)	5. (d)	6. (a)	7. (d)	8. (c)
9. (b)	10. (b)	11. (a)	12. (a)	13. (c)	14. (a)	15. (c)	16. (c)
17. (c)	18. (b)	19. (c)	20. (b)	21. (a)	22. (b)	23. (a)	24. (a)
25. (c)							

15.2 ELECTROMAGNETIC INDUCTION AND INDUCTANCE

1. In electrical machine laminated cores are used with a view to reduce
   (a) copper loss (b) hysteresis loss
   (c) eddy current loss (d) all of the above
2. The unit of retentivity is
   (a) dimension less (b) ampere turn
   (c) ampere turn/meter (d) ampere turn/weber
3. The permeability of all non-magnetic materials including air is
   (a) $2\pi \times 10^{-7}$ H/m (b) $4\pi \times 10^{-7}$ H/m
   (c) $\pi \times 10^{-7}$ H/m (d) $6\pi \times 10^{-7}$ H/m
4. A coil of 400 turns has a flux of 0.5 mWb linking with it when carrying a current of 2A. What is the value of inductance if the coil?
   (a) 100 H (b) 10 H (c) 0.001 H (d) 0.1 H

5. The magnetism left in the iron after exciting field has been removed is known as
   (a) reluctance (b) performace
   (c) susceptance (d) residual magnetism
6. The initial permeability of an iron rod is
   (a) the permeability almost in non-magnetised state
   (b) the lowest permeability of the iron rod
   (c) the highest permability of iron rod
   (d) the permeability at the end of the rod
7. A crack in the magnetic path of the inductor will
   (a) not affect the inductance of the coil
   (b) increase the inductance value
   (c) decrease the inductance value
8. Magnetic moment is the
   (a) pole strength (b) vector quantity
   (c) scalar quantity (d) universal constant
9. A conductor of length 1 m moves at right angles to a magnetic field of flux density 1 wb/m^2 with a velocity of 20 m/s. The induced emf in conductor will be
   (a) 100 V (b) 20 V (c) 2 V (d) 40 V
10. The tubes of force within the magnetic material are known as
    (a) tubes of induction (b) electric flux
    (c) lines of force (d) none of the above
11. A magnetic field exists around
    (a) copper (b) iron
    (c) moving charges (d) aluminium
12. Reciprocal of permeability is
    (a) conductivity (b) reluctivity
    (c) susceptibility (d) permitivity
13. When the current in a circuit is constant, what will be the value of induced voltage?
    (a) same value as current (b) can't say
    (c) half of the current value (d) zero
14. Presence of magnetic flux in a magnetic circuit is due to
    (a) mmf (b) emf
    (c) low reluctance path (d) none of the above
15. In the left hand rule, forefinger always represents
    (a) voltage
    (b) current
    (c) magnetic field
    (d) direction of force on the conductor
16. At radio frequencies, the iron core material of inductors
    (a) has a low permeability
    (b) is laminated
    (c) is called ferrite
    (d) reduces inductance as well as losses

17. Suceptibility is positive for
   (a) ferromagnetic substances (b) Paramagnetic substances
   (c) non-magnetic substances (d) none of these
18. The area of the face of a pole is 1.5 m^2 and the total flux is 0.18 webers. The flux density in the air gap
   (a) 0.12 tesla (b) 120 tesla
   (c) 1.2 tesla (d) 1.2×10^{-2} tesla
19. A conductor of 0.2 m long carries a current of 3 A. at right angle to a magnetic field of 0.5 tesla. The force acting on the conductor will be
   (a) 30 N (b) 3.0 N (c) 1.0 N (d) 0.3 N
20. Reluctivity is analogous to
   (a) resistivity (b) permeability
   (c) conductivity (d) none of these
21. Which of the following magnetic path will require the largest mmf?
   (a) iron core (b) air-gap
   (c) filament (d) inductance coil
22. Two parallel long conductors will carry 100 A. If the conductors are separated by 200 m, the force per meter of the length of the each conductor will be
   (a) 100 N (b) 10 N (c) 1 N (d) 0.1 N
23. The reciprocal of reluctance is
   (a) permeance (b) conductance
   (c) suceptance (d) admittance
24. The area of hysteresis loop is the measure of
   (a) permitivity (b) permeance
   (c) energy loss per cycle (d) magnetic flux
25. Sparking occurs when a load is switched off because the circuit has
   (a) high capacitance (b) high impedance
   (c) high inductance (d) high resistance

Answers (15.2)

1. (c)	2. (d)	3. (b)	4. (d)	5. (d)	6. (a)	7. (c)	8. (b)
9. (b)	10. (a)	11. (c)	12. (b)	13. (d)	14. (a)	15. (c)	16. (c)
17. (a)	18. (a)	19. (d)	20. (a)	21. (b)	22. (d)	23. (a)	24. (c)
25. (a)							

15.3 FUNDAMENTALS OF AC CIRCUITS

1. Peak value being the same, which of the following will have the highest rms value?
   (a) sine wave
   (b) half wave rectified sine wave
   (c) triangular wave
   (d) square wave
2. The peak value of a sine wave is 400 V. The average value is
   (a) 254.8 V (b) 5656 V (c) 282.8 V (d) 400 V

3. An inductor supplied with 100 V ac with a frequency of 10 KHz passes a current of 10 mA. The value of inductor is
   (a) 1.7 H (b) 16 mH (c) 1 mH (d) 160 mH
4. When two sinusoidal quantities are said to be in phase quadrature, their phase difference is
   (a) 90° (b) 0° (c) 45° (d) 30°
5. Two sinusoidal currents are given by $i_1 = 10 \sin (\omega t + \pi/3)$ and $i_2 = 15 \sin (\omega t - \pi/4)$. Phase difference between them is
   (a) 150° (b) 50° (c) 105° (d) 60°
6. The period of a sinusoidal wave is
   (a) the same as frequency
   (b) time period to complete one cycle
   (c) expressed in ampere
   (d) none of these
7. In a generator maximum value of emf is generated within the coil axis is at
   (a) zero degree with field axis
   (b) 45° with field flux
   (c) 180° with field axis
   (d) 90° with field flux
8. For which of the following sinusoids, the rms value and mean value is the same?
   (a) sine wave
   (b) triangular wave
   (c) square wave
   (d) half wave rectified sine wave
9. The time period of a sinusoid with 200 Hz frequency will be
   (a) 0.05 s (b) 0.005 s (c) 0.53 (d) 0.00053
10. Admittance is the reciprocal of
    (a) inductive reactance (b) reactive power
    (c) capacitive reactance (d) impedance
11. In any ac circuit always
    (a) apparent power is more than the actual power
    (b) actual power is more than the reactive power
    (c) reactive power is more than the apparent power
    (d) reactive power is more than the actual power.
12. Zero degree phase difference exists between voltage and current in an ac current
    (a) when current is maximum and voltage is zero
    (b) when voltage and current both reach zero and maximum at the same time.
    (c) when voltage is maximum and current is zero or vice versa
    (d) when voltage is maximum, current is neither zero or maximum and vice versa
13. The period of a sine wave is 1/5 m seconds, its frequency is
    (a) 5000 Hz (b) 40 Hz (c) 20 Hz (d) 30 Hz

14. When the current and voltage in a circuit are out of phase by 90°, the power is
    (a) zero (b) undefined (c) maximum (d) minimum
15. The positive maximum of a sine wave occurs at
    (a) 180° (b) 90° (c) 45° (d) 0°
16. In purely inductive circuit
    (a) reactive power is zero (b) apparent power is zero
    (c) actual power is zero (d) none of above
17. Form factor of a sine wave is
    (a) 0.637 (b) 0.707 (c) 1.11 (d) 1.414
18. What is the periodic time of a system with a frequency of 50 Hz?
    (a) 0.2 s (b) 2 s (c) 0.02 s (d) 20 s
19. For the same peak value, which wave will have the least rms value?
    (a) square wave (b) triangular wave
    (c) sine wave (d) full wave rectified sine wave
20. Inductive reactance of a circuit is more when
    (a) inductance is more and frequency of supply is more
    (b) inductance is more and frequency is less.
    (c) inductance is less and frequency is less
    (d) inductance is less and frequency is more
21. Inductance affects the direct current flow at the time of
    (a) turning on and off (b) operation
    (c) turning on (d) turning off
22. In a series RC circuit as frequency increases
    (a) current decreases (b)current remains unaltered
    (c) current increases
23. Which of the following waves has unity form factor?
    (a) triangular (b) square (c) sine wave (d) square wave
24. The current in a circuit is given by $i = 50 \sin \omega t$. If the frequency be 25 Hz, how long will it take for the current to rise to 25 amp?
    (a) 0.02 sec (b) 0.05 sec
    (c) 3.33×10^{-3} sec (d) 0.033 sec
25. In a parallel RC circuit, the supply current always _______ the applied voltage
    (a) lags (b) leads
    (c) remains on phase with (d) none of the above
26. For a given power factor of the load, if the p.f. of the load decreases, it will draw from the supply
    (a) less current (b) more current
    (c) same current
27. In a series circuit on resonance, the following will occur:
    (a) $X_L = X_C$ (b) $V_L = V_C$
    (c) $Z = R$ and $V = V_R$ (d) all above
28. P.F. of following circuit will be zero when the circuit contains
    (a) capacitance only (b) resistance only
    (c) inductance only (d) capacitance and inductance

29. In ac circuit the power curve is a sine wave having
    (a) half the frequency of voltage
    (b) double the frequency of voltage
    (c) same frequency of voltage
    (d) three times the frequency of voltage
30. In an ac circuit, a low value of KVAR compared with kW indicates
    (a) maximum load current (b) low efficiency
    (c) high p.f. (d) unity p.f.

Answers (15.3)

1. (d)	2. (a)	3. (d)	4. (a)	5. (c)	6. (b)	7. (d)	8. (c)
9. (b)	10. (d)	11. (a)	12. (b)	13. (a)	14. (a)	15. (b)	16. (c)
17. (c)	18. (c)	19. (b)	20. (a)	21. (a)	22. (c)	23. (b)	24. (c)
25. (b)	26. (b)	27. (d)	28. (d)	29. (b)	30. (c)		

15.4 DC MACHINE

1. As the load of a dc shunt motor is increased, its speed
   (a) increases proportionately (b) remains constant.
   (c) increases slightly (d) reduces slightly
2. The direction of rotation of a dc motor can be reversed by reversing the connection to
   (a) armature (b) series field (c) shunt field (d) any of the above
3. A dc series motor is running at rated speed. If a resistance is placed in series, the speed of the motor
   (a) increases (b) decreases
   (c) remains unchanged (d) increases very much
4. Plugging of a dc motor is normally executed by
   (a) reversing the field polarity
   (b) reversing the armature polarity
   (c) reversing both armature and field polarity
   (d) connecting a resistance across the armature
5. When the direction of power flow reverses, a cumulatively compounded motor becomes.
   (a) a differentially compounded generator
   (b) a shunt generator
   (c) a commulatively compounded generator
   (d) a series generator
6. The resistance of armature winding depends on
   (a) no. of conductor (b) conductor length
   (c) cross sectional area (d) all the above
7. In shunt generator interpole winding carries ______________ current
   (a) armature (b) shunt field (c) full load (d) short circuit
8. The output voltage of a simply dc generator is
   (a) ac square wave (b) ac sinusodial wave
   (c) pulsating dc (d) pure dc

9. Which of the following factors doesn't govern the iron losses in a dc machine?
   (a) speed (b) load (c) voltage (d) speed & voltage
10. Brushes of dc machine are usually made of
   (a) carbon (b) hard copper (c) soft copper (d) iron
11. In a unsaturated dc machine armature reaction is
   (a) cross magnetising (b) demagnetising
   (c) magnetising (d) none of these
12. A dc generator has
   (a) wave winding (b) duplex winding
   (c) lap winding (d) any of these
13. With given power rating for lower current and higher voltage rating of a dc machine, one should prefer
   (a) wave winding (b) lap winding
   (c) (a) or (b) (d) duplex winding
14. What is the nature of the current flowing in the armature of dc machine
   (a) pulsating dc (b) pure dc
   (c) rectified ac (d) alternating current
15. Why is the armature of a dc machine made of silicon steel stampings?
   (a) to reduce hysteresis loss
   (b) to reduce eddy current loss
   (c) to achieve high premeability
   (d) for the ease with which the slots can be created
16. Commutation in a dc generator causes
   (a) dc changes to ac
   (b) ac changes to dc
   (c) dc changes to dc
   (d) ac changes to high voltage dc
17. In Ward Leonard control, the dc motor is
   (a) series motor (b) shunt motor
   (c) compound motor (d) separately excited motor
18. If the voltage at a dc shunt motor terminal is halved, load torque being proportional to the armature current will be
   (a) halved (b) doubled (c) zero (d) unaltered
19. If the number of poles in a lap wound generator be doubled,than the generated emf will
   (a) double (b) half
   (c) remain constant (d) increase to four times
20. Which type of dc generator is used to charge the batteries?
   (a) shunt generator
   (b) long shunt compound generator
   (c) series generator
   (d) any of these
21. The characteristic drawn between no-load generated emf and field current is known as
   (a) magnetising current (b) internal characteristic
   (c) external characteristic (d) total characteristic

22. In the armature, dc generator generates
    (a) ac voltage (b) oscillating emf
    (c) dc voltage (d) ac superimposed over dc
23. The commutator segments of a dc machine are made of
    (a) iron (b) carbon
    (c) stainless steal (d) hard copper
24. In dc machines, lap winding is used for
    (a) high voltage low current (b) low voltage low current
    (c) high voltage high current (d) low voltage high current
25. Fractional pitch winding is used in dc machine
    (a) to reduce sparking
    (b) to increase the generated voltage
    (c) to save the copper because of shorter end connection
    (d) due to (a) and (c) above
26. Magnetic field in a dc generator is produced by
    (a) electromagnets (b) both (a) and (c)
    (c) permanent magnet (d) none of these
27. The sparking at the brushes of a dc generator is due to
    (a) reactance voltage (b) armature reaction
    (c) light load (d) high resistance of the brushes
28. In dc generators, current to the external circuit from armature comes out from
    (a) commutator (b) slip rings
    (c) brush connection (d) none of above
29. What type of compounding would be desirable in a dc generator feeding a long transmission line?
    (a) over compounding (b) flat compounding
    (c) under compounding (d) any one of the above
30. Which would have the highest percentage of voltage regulation?
    (a) a shunt generator (b) a series generator
    (c) a compound generator (d) a separately excited generator
31. The ripples in a dc generator are reduced by
    (a) using equalizer rings
    (b) using conductor of unnealed copper
    (c) using carbon brushes of superior quality
    (d) using commutator with large number of segments
32. Which of the following speed control methods of dc motor require auxiliary motor?
    (a) flux control (b) voltage control
    (c) armature control (d) Ward Leonard control
33. Compensation winding in a dc machine is connected
    (a) in series of field winding (b) directly across the supply
    (c) in series of interpole winding (d) in series of armature winding
34. While pole flux remains constant, if the speed of the generator is doubled, the emf generated will be
    (a) twice (b) half
    (c) nominal value (d) slightly less than nominal

35. The critical resistance of a dc generator refers to the resistance of
    (a) load (b) brushes (c) field (d) armature
36. If the supply voltage in a shunt motor is increased, which of the following will decrease?
    (a) full load current (b) starting torque
    (c) full load speed (d) none of the above
37. Which device changes the alternating emf generated by the dc generator in its armature coil, to dc?
    (a) rectifier (b) rotary converter
    (c) commutator (d) slip ring
38. In a dc generator probable cause of failure to build up voltage is
    (a) imperfect brush contact
    (b) field resistance higher than critical resistance
    (c) no residual magnetism in the generator due to faulty shunt connection
    (d) all of the above
39. Stray losses in dc machine are
    (a) magnetic losses (b) mechanical losses
    (c) windage loss (d) all of these
40. Which of the following parts of a dc motor can sustain the maximum temperature rise?
    (a) slip rings (b) armature winding
    (c) commutator (d) field winding
41. If the resistance of the field winding of a dc generator is increased the output voltage will
    (a) decrease (b) increase
    (c) remain same (d) fluctuate heavily
42. The voltage between commutator segments should exceed than
    (a) 2 V (b) 15 V (c) 0.002 V (d) 10 V
43. What is the flux in the armature core section of dc machine if the air gap flux be f
    (a) ϕ (b) 1.5 ϕ (c) $\phi/2$ (d) 0.1 ϕ
44. Full load speed of a dc motor being 1000 rpm, and speed regulation being 9%, no load speed will be
    (a) 900 rpm (b) 1000 rpm (c) 1110 rpm (d) 1200 rmp
45. The conventional exciter of a turbo generator is basically a
    (a) shunt generator (b) separately excited generator
    (c) series generator (d) compound generator
46. The flux set up by armature current has a
    (a) magnetizing affect (b) demagnetizing effect
    (c) cross-magnetizing effect (d) both (b) and (c)
47. If residual magnetism is not present in a dc generator, the induced emf at zero speed
    (a) 10% of rated voltage (b) impracticable
    (c) zero (d) the same as rated voltage
48. Which motor will have the least percentage increase of input current for the same percentage increase in torque?
    (a) shunt motor (b) separately excited motor
    (c) series motor (d) cumulatively compound motors

49. A series generator can self-excite provided
    (a) the speed is low (b) the load current is not zero
    (c) the interpole is present (d) the load current is zero
50. Change of dc excitation of shunt motor changes
    (a) motor speed (b) applied voltage
    (c) direction of rotation of motor (d) none of these
51. In dc generators, the brushes remain in
    (a) the interpolar axis (b) under north pole
    (c) under south pole (d) none of the above position
52. The function of the brushes and the commutator in a dc motor is to
    (a) reduce sparking
    (b) produce unidirectional torque
    (c) help in changing the direction of rotation of armature
    (d) produce unidirectional armature current
53. The series field of short shunt dc generator is excited by the
    (a) armature current (b) shunt current
    (c) external current (d) load current
54. The direction of rotation of dc shunt motor can be reversed by interchanging
    (a) the field terminals (b) armature terminals
    (d) the supply terminals (d) either (a) or (b)
55. If the no load voltage of a generator is 220 V and the rated voltage is 200 V, then the voltage regulation is nearly
    (a) 0% (b) 20% (c) 15% (d) 10%
56. Once residual magnetism of a shunt generator is lost, it may be restored by connecting its shunt field
    (a) to earth (b) to a external battery
    (c) in reverse (d) to an alternator
57. Which of the following tests can be used to measure stray losses of a dc motor?
    (a) Field test (b) Brake test
    (c) Running down test (d) Swimburne's test
58. Which one of the following types of generators doesn't need equalties for satisfactory parallel operation?
    (a) series (b) over compound
    (c) flat compound (d) under compound
59. In a dc machine, there are as many commutator bars as the number of
    (a) slots (b) poles
    (c) winding elements (d) armature conductors
60. No volt release coil of dc shunt motor starter is connected in series with the
    (a) line (b) series winding
    (c) shunt field winding (d) armature winding
61. A small air-gap between stator and armature
    (a) reduces noise (b) facilitate high speed operation
    (c) provides high ventilation (d) provides stronger magnetic field
62. Which of the dc generator has poorest voltage regulation?
    (a) shunt (b) compound
    (c) over compound (d) series

63. In dc machine, the brushes are placed on
    (a) M.N.P (b) G.N.P
    (c) any of (a) and (b) (d) none of these
64. Power developed by dc motor is maximum when the ratio of back emf and applied voltage is
    (a) double (b) zero (c) unity (d) half
65. In a shunt generator the voltage builds up till constrained by
    (a) speed limitation (b) armature heating
    (c) saturation of iron (d) insulation restrictions

Answers (15.4)

1. (d)	2. (a)	3. (b)	4. (b)	5. (a)	6. (d)	7. (a)	8. (c)
9. (b)	10. (a)	11. (a)	12. (d)	13. (a)	14. (d)	15. (b)	16. (b)
17. (b)	18. (b)	19. (c)	20. (a)	21 (a)	22. (a)	23. (d)	24. (d)
25. (d)	26. (a)	27. (a)	28. (c)	29. (a)	30. (a)	31. (d)	32. (d)
33. (d)	34. (a)	35. (c)	36. (a)	37. (c)	38. (d)	39. (d)	40. (d)
41. (a)	42. (b)	43. (c)	44. (c)	45. (a)	46. (d)	47. (c)	48. (c)
49. (b)	50. (a)	51. (a)	52. (b)	53. (d)	54. (d)	55. (d)	56. (b)
57. (a)	58. (d)	59. (c)	60. (c)	61. (d)	62. (d)	63. (a)	64. (d)
65. (c)							

15.5 TRANSFORMER

1. Maximum flux established in an ac excited iron core of a transformer is determined by
   (a) impressed frequency only
   (b) impressed voltage only
   (c) both impressed voltage and frequency
   (d) reluctance of the core
2. A coil wound on a magnetic core is excited from an ac voltage source. The source voltage and its frequency are both doubled. The eddy current loss in the core will become
   (a) half (b) remains same
   (c) double (d) four times
3. A 25 KVA transformer has a voltage ratio of 3300/400 V. Find the primary current.
   (a) 6.25 A (b) 75.8 A (c) 62.5 A (d) 7.58 A
4. When a two winding transformer is operated with lower copper loss, its efficiency is
   (a) same (b) decreased (c) increased (d) rises to 100%
5. In a transformer, the flux phasor
   (a) leads the induced emf by 90°
   (b) lags the induced emf by 90°
   (c) leads the induced emf by slightly less than 90°
   (d) lags the induced emf by slightly less than 90°

6. Distribution transformers have core losses
   (a) more than full load copper loss
   (b) equal to full load copper loss
   (c) less than full load copper loss
   (d) negligible compared to full load copper loss
7. The high frequency hum in a transformer is mainly due to
   (a) lamination is not proper (b) magnetostriction
   (c) tank walls (d) oil of the transformer
8. A single-phase transformer when supplied from 220 V, 50 Hz has eddy current loss of 50 W. If the transformer is connected to a voltage of 330 V, 50 Hz, the eddy current loss will be
   (a) 168.75 W (b) 112.5 W (c) 75 W (d) 50 W
9. Low voltage windings are placed nearer the core in the case of concentric windings because
   (a) it reduces leakage flux (b) it reduces hysteresis loss
   (c) it reduces eddy current loss (d) it reduces insulation requirement
10. Continuous disc winding is suitable for
    (a) high voltage winding of large transformer
    (b) low voltage winding of large transformer
    (c) high voltage winding of small transformer
    (d) low voltage winding of small transformer.
11. Short heat run test on transformers is performed by means of
    (a) short circuit test
    (b) half time on short circuit and half time an open circuit
    (c) Sumpner's test
    (d) open circuit test
12. The amount of leakage flux in the transformer windings depends upon
    (a) mutual flux (b) load current (c) applied voltage (d) turn ratio
13. The efficiency of a transformer at full load 0.85 p.f. (lag) is 95%. Its efficiency at full load 0.85 p.f. (lead) will be
    (a) 95% (b) more than 95%
    (c) 100% (d) less than 95%
14. For small power transformers, it is preferable to use
    (a) corrugated tank (b) tubed tanks
    (c) radiator tank (d) tank with separate coolers.
15. The core in a large power transformer is built of
    (a) mild steel (b) ferrite (c) cast iron (d) silicon steel
16. For all sizes of distribution transformers, it is preferable to use
    (a) radiator tank (b) plain sheet steel tank
    (c) tubed tanks (d) corrugated tank
17. When a two-winding transformer is connected as an auto transformer its efficiency (full load)
    (a) increases (b) remains same
    (c) decreases (d) rises to 100%

18. In an ideal transformer, the impedance transforms from one side to the other:
    (a) in direct ratio of turns
    (b) in direct square ratio of turns
    (c) in inverse ratio of turns
    (d) in direct ratio of square root of turns.
19. Distribution transformers have core losses
    (a) negligible compared to full load copper loss
    (b) more than full load copper loss
    (c) less than full load copper loss
    (d) equal to full load copper loss
20. Cross-over winding is suitable for
    (a) high voltage winding of large transformer
    (b) high voltage winding of small transformer
    (c) low voltage winding of small transformer
    (d) low voltage winding of large transformer
21. Transformer oil is used as
    (a) an inert medium (b) an insulant only
    (c) a coolant only (d) both as an insulant and a coolant
22. A transformer may have two or more ratings depending upon the type of
    (a) insulation used (b) winding used
    (c) core used (d) cooling used
23. Breather mounted on transformer tank contains
    (a) calcium (b) oil (c) water (d) liquid
24. Buchholz relay is a
    (a) gas actuated device (b) voltage sensitive device
    (c) frequency sensitive device (d) current sensitive device
25. A transformer opearates most efficiently at 3/4th full load. Its iron loss (Pi) and full load copper loss (Pc) are related as
    (a) Pi/Pc = 4/3 (b) Pi/Pc = 16/9
    (c) Pi/Pc = 3/4 (d) Pi/Pc = 9/16
26. A 2/1 ratio, two winding transformer is connected as an auto transformer. Its KVA rating as an auto transformer compared to a two-winding transformer is
    (a) 3 times (b) 2 times (c) same (d) 1.5 times
27. The load of a certain transformer is reduced to 50%; the copper loss would become
    (a) 0.25 times (b) 0.5 times
    (c) same as before (d) 2 times
28. A transformer has negligible resistance and a p.u. reactance of 0.1. Its voltage regulation on full load will a leading p.f. angle of 30° leading is
    (a) –10% (b) 10% (c) 5% (d) –5%
29. During SC test at full load current the power input to a transformer comprises predominantly
    (a) eddy current loss (b) copper loss
    (c) core loss (d) both (b) & (c)
30. The power transformer is a constant ______ device.
    (a) current (b) voltage (c) main flux (d) power

31. The colour of fresh dielectric oil for a transformer is
    (a) grey (b) dark brown
    (c) pale yellow (d) colourless
32. A conservator is used
    (a) for better cooling
    (b) to act as an oil storage
    (c) to take up the expansion of oil due to temperature rise
    (d) none of the above.
33. On the two sides of a star/delta transformer
    (a) the voltage and currents both differ in phase by 30°
    (b) the voltage & currents are both in phase
    (c) the currents differ in phase by 30° but voltages are in phase
34. With peaked emf in the transformer windings, the hysteresis loss is,
    (a) reduced (b) increased (c) constant (d) zero
35. R = equivalent resistance, X = equivalent reactance, P_i = core loss. The load current for maximum efficiency operation of a transformer is given by
    (a) $\sqrt{P_i/R}$ (b) P_i/X (c) $\sqrt{P_i/X}$ (d) P_i/R
36. When the supply frequency to the transformer is increased, the iron loss will
    (a) increase (b) fluctuate (c) decrease (d) not change
37. The best method to obtain the efficiency of two identical transformers under load conditions is
    (a) a open circuit test (b) a back to back test
    (c) a short circuit test (d) none of the above
38. Spacers are provided between the adjacent coils
    (a) to provide free passage to the cooling oil
    (b) to insulate the coils from each other
    (c) due to both reasons
    (d) no use at all
39. If full load Cu loss of a transformer is 1600 W, its Cu loss at half load will be
    (a) 400 W (b) 800 W (c) 1600 W (d) 200 W
40. Which of the following gives the highest secondary voltage for a given primary voltage?
    (a) Y-Δ connection (b) Y-Y connection
    (c) Δ-Δ connection (d) Δ-Y connection
41. The main purpose of using magnetic core in a transformer is to
    (a) prevent eddy current loss
    (b) eliminate magnetic hysteresis
    (c) decrease iron losses
    (d) decrease reluctance of the common magnetic flux path
42. Greater the secondary leakage flux
    (a) less will be the primary induced emf
    (b) less will be the secondary induced emf
    (c) less will be the secondary terminal voltage

43. If the magnetizing current in a transformer is sinusoidal, induced voltage
    (a) will also be sinusoidal (b) will be cosine wave
    (c) will contain 3^{rd} harmonics (d) will be zero
44. Which of the transformer part is most subjected to damage from overheating?
    (a) copper winding (b) frame or case
    (c) winding insulation (d) iron core
45. Buchholz relay gives warning
    (a) 1 ms after fault
    (b) when fault is going to take place
    (c) before the fault
    (d) when actually fault has taken place
46. Transformer cores are laminated in order to
    (a) reduce hysteresis loss (b) reduce cost
    (c) minimise eddy current loss (d) simplify its construction
47. In a 2-winding transformer, the resistance between its primary and secondary should be
    (a) zero (b) 500 Ω (c) infinity (d) nearly 2 kΩ
48. Friction loss in a transformer is
    (a) 20% of total loss (b) equal to iron loss
    (c) nil (d) 10% of total loss
49. A Δ/Y transformer works satisfactorily on
    (a) unbalanced load only (b) balanced load only
    (c) both (a) and (b) (d) there is no such condition
50. If the secondary terminals of a 5 : 1 stepdown transformer is connected to the primary of a 2 : 1 stepdown transformer then the total stepdown ratio of both transformers is
    (a) 2.5 : 1 (b) 3 : 1 (c) 10 : 1 (d) 7 : 1
51. In tap changing transformer, taps are usually provided on.
    (a) H-V side (b) L-V side (c) any side
52. Transformer core is laminated to reduced
    (a) core loss (b) eddy current loss
    (c) copper loss (d) hysteresis loss
53. Star-star power transformers protected by current transformer having ______ connection.
    (a) Δ/Δ (b) Y/Δ (c) Δ/Y (d) Y/Y
54. The phase relationship between primary and secondary voltage of a transformer is
    (a) 180° out of phase
    (b) in same phase
    (c) primary voltage leading the secondary voltage by 90°
    (d) primary voltage leading the secondary voltage by 90°
55. If a transformer is switched on to a voltage more than rated voltage
    (a) p.f. will decrease (b) p.f. will increase
    (c) p.f. becomes zero (d) there will be no effect on p.f.
56. Most suitable connection of three phase distribution transformer is
    (a) delta/delta (b) delta/star (c) star/star (d) star/delta

57. An ideal transformer has
    (a) no losses and magnetic leakage
    (b) a core of stainless steel
    (c) a common core for primary and secondary windings
    (d) interleaved primary and secondary winding
58. Laminations of the iron core are insulated from each other by
    (a) paper (b) thin coat of varnish
    (c) iron oxide (d) (b) and (c) both
59. As compared to ordinary efficiency, the all day efficiency of a 2 winding transformer is always.
    (a) lower (b) higher (c) infinite (d) moderate
60. Efficiency of a power transformer is of the order of
    (a) 100% (b) 98% (c) 75% (d) 50%
61. What is the purpose of using oil in the transformer?
    (a) insulation (b) cooling
    (c) cooling and insulation (d) lubrication
62. The efficiency of a given transformer is maximum when
    (a) it runs overlaod (b) it runs at full load
    (c) it runs at half full load (d) its cu loss equals iron loss
63. In a transformer voltage regulation is negative when its load p.f. is
    (a) unity (b) leading (c) lagging (d) zero
64. When a 50 Hz transformer is operated at 400 Hz, its KVA rating is
    (a) increased by 8 times (b) reduced by 8 times
    (c) unaffected (d) determined by load on secondary
65. Stranded conductors in transformer are used primarily
    (a) to reduce eddy current
    (b) to take heavy current
    (c) to give flexibility to the conductor
66. Silicon steel is preferred for transformer core because
    (a) it decreases the tensile strength
    (b) it decreases permeability of core
    (c) it reduces resistivity of core
    (d) it reduces both eddy current and hysteresis loss
67. The core in a large power transformer is unit of
    (a) mild steal (b) ferrite
    (c) cast iron (d) silicon steal
68. For large power transformers, best utilisation of avaliable core space can be made by using ______ cross section.
    (a) square (b) triangle (c) stepped (d) rectangular
69. Cruciform shape is used in transformer core to reduce
    (a) core reluctance (b) winding copper
    (c) core loss (d) mechanical strength
70. The applied voltage is increased by 50% and frequency is reduced by 50%. The maximum core flux density will become
    (a) 1.5 times (b) 3 times (c) same (d) 5 times

Answer (15.5)

1. (c)	2. (d)	3. (d)	4. (c)	5. (a)	6. (c)	7. (b)	8. (d)
9. (d)	10. (a)	11. (c)	12. (b)	13. (a)	14. (c)	15. (d)	16. (c)
17. (a)	18. (b)	19. (c)	20. (b)	21. (d)	22. (d)	23. (a)	24. (a)
25. (d)	26. (a)	27. (c)	28. (d)	29. (b)	30. (c)	31. (c)	32. (c)
33. (a)	34. (a)	35. (a)	36. (a)	37. (b)	38. (a)	39. (a)	40. (d)
41. (d)	42. (b)	43. (c)	44. (c)	45. (b)	46. (c)	47. (c)	48. (c)
49. (c)	50. (c)	51. (a)	52. (b)	53. (a)	54. (a)	55. (a)	56. (b)
57. (a)	58. (d)	59. (b)	60. (b)	61. (c)	62. (d)	63. (b)	64. (d)
65. (a)	66. (d)	67. (b)	68. (d)	69. (a)	70. (b)		

15.6 INDUCTION MOTOR

1. For a 3-phase induction motor, what frequency of rotor currents would you observe?
   (a) slip frequency (sf) (b) same as stator frequency (f)
   (c) $sf + f$ (d) $f - sf$
2. For 4% drop in supply voltage, the torque of an induction motor decreased by
   (a) 4% (b) 8% (c) 16% (d) 2%
3. The power factor of star connected induction motor is 0.5. On being connected in delta, the power factor will
   (a) become zero (b) remain the same
   (c) reduces (d) increases
4. In a double cage induction motor, the inner cage has
   (a) Low R and low X (b) Low R and high X
   (c) High R and high X (d) High R and low X
5. The direction of rotation of a three phase induction motor is reversed by
   (a) rewinding the motor
   (b) adding a capacitor in any phase
   (c) interchanging the connetion of any two phases
   (d) interchanigng the connection of all the three phases
6. The resistance R_0 of the exciting branch of the equivalent ckt. of a 3ϕ induction motor represents
   (a) stator core loss (b) stator copper loss
   (c) friction and windage losses (d) rotor copper loss.
7. For controlling the speed of an Induction motor the frequency of supply is increased by 10%. For magnetizing current to remain the same, the supply voltage must
   (a) remain constant (b) be increased by 10%
   (c) by reduced by 10% (d) reduced by 20%
8. For maximum starting torque in an induction motor
   (a) $r_2 = x_2$ (b) $r_2 = 5x_2$ (c) $r_2 = 0.5x_2$ (d) $2r_2 = 3x_2$
9. The double cage rotors are used to
   (a) increase pull out torque (b) increase starting torque
   (c) improve efficiency (d) reduce rotor core losses

10. Cogging of induction motor occurs due to
    (a) harmonic synchronous torque only
    (b) vibration torques
    (c) harmonic induction torque only
    (d) both synchronous and induction torque
11. Induction generator runs at
    (a) very low sub-synchronous speed
    (b) super synchronous speed
    (c) synchronous speed
    (d) sub-synchronous speed very near to synchronous speed
12. Rotor core losses of a 3-phase. Induction motor are small because
    (a) rotor flux density is small (b) rotor frequency is small
    (c) rotor is laminated (d) none of the above
13. At low slip, torque-slip characteristic is
    (a) $T \infty S$ (b) $T \infty S^2$ (c) $T \infty \frac{1}{S}$ (d) $T \infty \frac{1}{S^2}$
14. In a 3ϕ induction motor, slip for maximum torque in terms of rotor resistance r_2 is
    (a) proportional to $(r_2)^2$
    (b) directly proportionmal to r_2
    (c) inversely proportional to r_2
    (d) independent of r_2
15. An induction motor is analogus to
    (a) two winding transformer with secondary open circuit
    (b) two winding transformer with short circuit secondary
    (c) 3 winding transformer
    (d) auto transformer
16. For a slipring induction motor if rotor resistance is increased then
    (a) starting torque increases but efficiency decreases
    (b) both starting torque and efficiency increases
    (c) both decreases
    (d) starting torque decreases and efficiency increases
17. If the mechanical load is increased from no load
    (a) p.f. becomes zero (b) p.f. increases
    (c) p.f. decreases (d) p.f. remains constant
18. The relative speed between the stator and rotor fluxes is equal to
    (a) $(n_s + n_r)$ rpm (b) $(n_s - n_r)$ rpm
    (c) zero rpm (d) n_s rpm
19. At a slip of 4% the maximum possible speed of 3ϕ squirrel cage induction motor is
    (a) 1440 rpm (b) 1500 rpm (c) 2880 rpm (d) 3000 rpm
20. In a 3ϕ induction motor, with increase in load from light load
    (a) both stator and rotor p.f. increase
    (b) both stator and rotor p.f. decrease
    (c) stator p.f. increases and rotor p.f. decreases
    (d) stator p.f. decreases and rotor p.f. increases

21. If stator impedance is neglected, the maximum torque of an induction motor occurs at a slip of
    (a) $s = 1$
    (b) $s = r_2/x_2$
    (c) $s = \sqrt{\dfrac{r^2}{x^2}}$
    (d) torque-slip curve doesn't exhibit a maximum.
22. In dynamic braking
    (a) any two stator terminals are earthed
    (b) a dc voltage is injected in the rotor circuit
    (c) stator terminals are switched over to a dc source from the ac supply
    (d) the supply terminals of any two stator phases are interchanged
23. Speed control by variation of no. of stator poles is applicable to
    (a) squirrel cage motors only
    (b) both squirrel cage and wound rotor motors
    (c) wound-rotor motor of large rating only
    (d) wound-rotor motor of small rating only
24. Star-delta starting is equivalnt to auto-transformer starting with
    (a) 58% tapping (b) 85% tapping
    (c) 33% tapping (d) 52% tapping
25. Variation of supply frequency for speed control is generally carried out with
    (a) $V_1 f$ constant (b) V_1/f constant
    (c) V_1 constant (d) none of above
26. Regenerative braking occurs when
    (a) the load is lifted by a hoisting machine
    (b) the load is lowered by a hoisting machine
    (c) the number of poles is decreased in a pole changing motor
    (d) limits starting current only
27. When it is required to control the speed of the motor during deceleration, it is preferable to employ
    (a) plugging (h) mechanical branch
    (c) dc dynamic braking (d) regenerative braking
28. Speed control by supply voltage variation is not done because
    (a) the range of speed control is limited
    (b) it reduces pull out torque and also the range of speed control is limited
    (c) it reduces pull out torque only
    (d) none of the above
29. In applications where reversal of direction of rotation is required, the type of braking used is
    (a) ac dynamic braking (b) regenerative braking
    (c) dc dynamic braking (d) plugging
30. If stator impedance is neglected the maximum torque in an induction motor occurs at a rotor resistance of
    (a) $(1 - s)x_2$ (b) x_2 (c) sx_2 (d) $(1 + s)x_2$

31. Plugging is executed when
    (a) The supply terminals of any two stator phases are interchanged
    (b) two stator terminals are shorted together
    (c) any two stator terminals are connected to dc source
    (d) any two stator terminals are earthed
32. During the blocked-rotor test on an induction motor, the power is drawn mainly for
    (a) core loss
    (b) windage and friction loss
    (c) copper loss
    (d) both (a) and (c)
33. Rotor impedance seen from stator is
    (a) $r_2 + jsx_2$ (b) $r_2/s + jx_2$ (c) $x_2'/s + jx_2'$ (d) $r_2 + j/sx_2$
34. During shortcircuit test on a a slip ring induction motor
    (a) the rotor is open circuited but free to rotate
    (b) the rotor is short circuited but is blocked from rotation
    (c) the rotor is open circuited but ius blocked from rotation
    (d) the rotor is short circuited but is free to rotate
35. In a 3ϕ induction regulator, the output line voltages are in phase with the supply line voltage in
    (a) maximum boost position only
    (b) maximum boost position
    (c) both (a) and (b)
    (d) none of the above
36. A synchronous induction motor
    (a) has combined characteristic of synchronous motor and induction motor at starting
    (b) starts as synchronous motor but runs as an induction motor
    (c) starts as an induction motor but runs as a synchronous motor
    (d) none of the above
37. For controlling the speed of an induction motor the frequency of the supply is increased by 10%. For maximum torque to remain constant, the supply voltage must.
    (a) be decreased by 10% (b) be increased by 10%
    (c) be increased by 20% (d) remain constant
38. In an induction motor if the ratio of the motor output to rotor input is 0.96, then the percent slip is
    (a) 0.04 (b) 4.17% (c) 4% (d) 9.6%
39. The ratio between rotor input, rotor output and rotor copper losses is
    (a) $1 : (1 - s) : s$ (b) $1 : (s - 1) : s$
    (c) $1 : (s) : 1$ (d) $s : (1 - s) : 1$
40. A 3ϕ induction motor, while supplying a constant load, has the fuse of one line suddenly blown off. The motor will run as a single phase induction motor with line current nearly increased to
    (a) 3 times (b) $\sqrt{3}$ times
    (c) 6 times (d) 9 times

41. In a three phase induction motor iron losses occur in
    (a) rotor and stator winding (b) stator core and teeth
    (c) stator and rotor (d) rotor core and teeth
42. Just at start the slip of a 3ϕ induction motor is
    (a) zero (b) infinite
    (c) one (d) any value both 0 and 1
43. If two of the power supply terminals to a three phase inducitn motor got interchanegd during reconnection after maintenance, the motor will
    (a) fail to start
    (b) stall immediately after start
    (c) rotate in reverse direction to that prior to maintenance
    (d) no rotate in some direction as previous.
44. In running condition the rotor reactance of 3ϕ induction motor is proportional to
    (a) slip (b) supply voltage
    (c) induced emf (d) rotor resistance
45. Speed of 3ϕ induction motor field in space is
    (a) $N_r + sN_S$ (b) N_S
    (c) anyone of (a) & (b) (d) none of the above
46. Slip of an induction motor is obtained by following method(s)
    (a) Stroboscopic method
    (b) zero centre galvanometre across any two slipring
    (c) magnetic needle method
    (d) all above method
47. In high voltage test of an inducitn motor, full test voltage is maintained for
    (a) 1 minute (b) 2 minutes (c) 2 seconds (d) 1 seconds
48. The total number of slots on the stator of a 3-ϕ, 6 pole induction motor having 3 slots per pole per phase is
    (a) 18 (b) 54 (c) 9 (d) 27
49. Relation between stator and rotor slot is
    (a) stator slots are exact multiple of rotor slots
    (b) stator slots are not exact multiple of rotor slots
    (c) stator slots = rotor slots
    (d) no such relation
50. Rotor coppper loss of a 3 phase inductin motor is directly proportional to
    (a) slip (b) load current
    (c) input of motor (d) output of motor
51. Rotating magnetic field is produced in
    (a) 3 phase induction motor (b) dc series motor
    (c) ac series motor (d) transformer
52. Maximum torque is developed by an induciton motor when phase difference between stator flux and rotor current is
    (a) 30° electrical (b) 45° electrical
    (c) 0° electrical (d) 90° electrical
53. A 10 H.P, 3 phase, 400 V induction motor at full load will take ______ current.
    (a) 20 amp (b) 10 amp (c) 15 amp (d) 7.5 amp

54. When line voltage of an induction motor is reduced to 75% of its rated value, its starting current is reduced by
    (a) 75% (b) 86.6% (c) 100% (d) 25%
55. When voltage and frequency of a 3 phase induction motor are halved
    (a) air gap flux is halved (b) slip increases greatly
    (c) maximum torque is halved (d) maximum torque remains same
56. Operating the three phase induction motor at higher than rated voltage will cause its p.f. to
    (a) increase
    (b) remain same
    (c) decrease
57. Burning of fuses will give protection to induction motor from single phasing in case of induction motor having stator
    (a) star connected (b) both (a) and (c)
    (c) delta connected (d) none of above
58. Slip rings are made of
    (a) steel (b) copper (c) bakelite (d) mica
59. The no load current of 3ϕ induction motor, as compared to transformer is
    (a) twice (b) same (c) less (d) more
60. In a 3ϕ induction motor, the variable mechanical load is electrically represented by
    (a) variable capacitance only
    (b) variable resistance only
    (c) combination of variable resistance and inductance
    (d) variable inductance only
61. Starter is connected on rotor side in
    (a) squirrel cage induction motor
    (b) synchronous motor
    (c) slipring induction motor
    (d) none of the above
62. If supply voltage is changed by 5% for a 3ϕ induction motor, its torque will have a change of
    (a) 5% (b) 25% (c) 10% (d) 20%
63. The crawling of an induction motor is due to
    (a) low supply voltage
    (b) high starting load
    (c) presence of 7^{th} harmonic in the flux wave
    (d) fluctuation of supply voltage
64. Rotor reactance of induction motor is more in
    (a) squirrel cage
    (b) slip ring
    (c) both have equal reactance
65. In a 3ϕ, 6 pole winding of a 3ϕ induction motor, the space angle between two consecutive phase is
    (a) 40° mechanical (b) 120° mechanical
    (c) 180° mechanical (d) none of these

Answers (15.6)

1. (a)	2. (c)	3. (c)	4. (b)	5. (c)	6. (a)	7. (b)	8. (a)
9. (a)	10. (a)	11. (b)	12. (b)	13. (a)	14. (b)	15. (b)	16. (a)
17. (b)	18. (c)	19. (c)	20. (c)	21. (b)	22. (c)	23. (a)	24. (a)
25. (b)	26. (b)	27. (c)	28. (b)	29. (d)	30. (c)	31. (a)	32. (c)
33. (c)	34. (b)	35. (c)	36. (c)	37. (b)	38. (c)	39. (a)	40. (b)
41. (b)	42. (c)	43. (c)	44. (a)	45. (c)	46. (d)	47. (a)	48. (b)
49. (b)	50. (a)	51. (a)	52. (c)	53. (c)	54. (d)	55. (d)	56. (c)
57. (a)	58. (b)	59. (d)	60. (b)	61. (c)	62. (c)	63. (c)	64. (b)
65. (a)							

15.7 SYNCHRONOUS MACHINE

1. To have two alternators in parallel which of the following factors should be identical for both?
   (a) voltage (b) phase sequence
   (c) frequency (d) all of the above
2. In an alternator, the armature reaction influences the magnitude of
   (a) no load loss (b) speed of the machine
   (c) terminal voltage/phase (d) waveform of voltage generated
3. The back e.m.f. set up in the armature of a synchronous motor depends on
   (a) rotor speed only (b) rotor excitation only
   (c) rotor excitation and rotor speed
   (d) coupling angle, rotor speed and excitation
4. The armature curent of a synchronous motor may have large values for
   (a) unity power factor loads (b) high excitation only
   (c) low excitation only (d) both low and high excitation
5. Hunting in a synchronous motor is not due to
   (a) variable frequency (b) variable supply voltage
   (c) windage friction (d) variable load
6. In a three phase synchronous motor, the magnitude of field flux
   (a) varies with load (b) is independent of load
   (c) varies with speed (d) varies with power factor
7. Synchronous motors for power factor correction operate at
   (a) normal load with no excitation
   (b) normal load with low excitation
   (c) no load and over excited
   (d) no load and under excited
8. Harmonic component of generated e.m.f. will be more in
   (a) long pitch coil (b) short pitch coil
   (c) full pitch coil (d) none of above coil
9. Why the synchronous motor is not starting?
   (a) as there is no slip (b) because starter is not used
   (c) as the direction of instantaneous torque on the rotor reverses after half cycle
   (d) because starting winding is not provided

10. At leading power factor operation, the armature flux in an alternator
    (a) aids the rotor flux (b) opposes the rotor flux
    (c) distorts the rotor flux (d) doesn't affect the rotor flux
11. Which of the following is the probable cause if a synchronous motor fails to pull into synchronism even if dc field is applied?
    (a) low field current (b) high core losses
    (c) high field current (d) low short circuit ratio
12. If the field flux is leading the armature field axis the machine will work as
    (a) synchronous motor (b) asynchronous motor
    (c) asynchronous generator (d) synchronous generator
13. Conventional rotating exciter is basically
    (a) dc sereis generator (b) dc shunt motor
    (c) dc shunt generator (d) dc sereis motor
14. With reduction of load on alternator
    (a) the frequency increases (b) the frequency decreases
    (c) the frequency remains same (d) the frequency oscillates
15. The armature current of a synchronous motor may have
    (a) unity power factor loads (b) high excitation only
    (c) low excitation only (d) both low and high excitation
16. A synchronous motor is said to be 'floating' when it operates
    (a) on pulsating load (b) on no load and without losses
    (c) on constantly varying load
    (d) on high load and variable supply voltage
17. Driving torque of two alternators running in parallel being changed; this will result in changes in
    (a) frequency (b) back emf
    (c) generated voltage (d) all of the above
18. When two alternators run in exact synchronous the synchronising power will be
    (a) unity (b) zero
    (c) sum of the output of two (d) none of the above
19. In a 3 phase synchronous motor, the magnitude of field flux
    (a) varies with load (b) varies with speed
    (c) is independent of load (d) varies with p.f.
20. Negative phase sequence current in a 3ϕ synchronous motor may exist in the stator when the stator is
    (a) over loaded (b) hot
    (c) under loaded (d) supplied with unbalanced voltage
21. Damper winding are provided on
    (a) stator frame (b) pole faces
    (c) motor shaft (d) separate armature
22. V curves for synchronous motor represent relation between
    (a) p.f and speed (b) field current and speed
    (c) field current and speed (d) armature current and field current
23. For under excited operation of a synchronous motor the p.f. will be
    (a) lagging (b) zero (c) unity (d) leading

24. The advantage of salient pole alternator is that it
    (a) reduce noise (b) reduce bearing loads & noise
    (c) reduce windage loss
    (d) be suitable for low and medium speed operation
25. If the excitation current of synchronous motor be increasd, the p.f. of the motor will
    (a) improve (b) decrease
    (c) remainconstant (d) depend on other factor

Answers (15.7)

1. (b)	2. (c)	3. (b)	4. (d)	5. (c)	6. (b)	7. (c)	8. (c)
9. (c)	10. (a)	11. (a)	12. (d)	13. (c)	14. (a)	15. (d)	16. (b)
17. (a)	18. (b)	19. (c)	20. (d)	21. (b)	22. (d)	23. (a)	24. (d)
25. (a)							

15.8 BASIC CIRCUIT; CAPACITANCE; AC CIRCUIT

1. The resistance of a conductor is __________ proportional to its length.
2. The __________ is the property of conductor to oppose the flow of __________ through it.
3. The unit of resistivity in SI unit is __________.
4. The temperature co-efficient of resistance of carbon is __________.
5. The inverse of conductivity is __________.
6. Unit of conductance is __________.
7. Capacitance of two capacitors increases when they are connected in __________.
8. The capacitance of an isolated spherical conductor is given by C = __________ $\times$ r farads.
9. Energy stored in the electric field produced by the application of 10 V across a capacitor of 2 F is __________ joules.
10. Polarized electrolytic capacitors are for use on __________ circuits.
11. Thinner the dielectric __________ will be its capacitance __________ will be its breakdown voltage.
12. Capacitor __________ dc but __________ ac
13. What will be the resultant capacity and voltage rating of capacitors in following combination.
    (a) When two capacitors each of 100 μF and 220 V rating are in sereis __________ μF __________ V.
    (b) When two capacitors each of 100 μF and 220 V rating are in parallel __________ μF __________ V.
14. Dielectric strength of material __________ with temperature.
15. When two capacitors are in sereis, capacitance of the group __________.
16. The permitivity of evacuated space is __________ f/m.
17. The potential gradient at which the insulation just gets damaged is called __________.

18. Current in polar form in a circuit containing $R = 3\Omega$, $X_C = 4\ \Omega$, $X_e = 8\ \Omega$ having applied voltage $220\angle 0$ will be __________.
19. Time period of a square wave is 10 μsec. The frequency of the wave will be __________.
20. In pure capacitive circuit, voltage __________ the current.
21. In inductor, energy is stored in __________ field and in capacitor, it is stored in __________ field.
22. For a given power in ac circuit, magnitude of current __________ as the pf decreases.
23. In resistive circuit the phase difference in current and voltage is __________.
24. In an inductive circuit, current __________ its maximum value immediately on the application of voltage.
25. In an inductive circuit __________ leads the __________.
26. Unit of inductance is __________.
27. Pure inductive circuit takes us real power from ac supply even __________ rises.
28. In RLC series circuit, voltage across capacitance happens to be maximum just __________ resonance of frequency.
29. Maximum value of pf of an electrical circuit is __________.
30. Apparent power of a circuit ______, true power is __________ and reactive power is __________.

Answer (15.8)

1. directly	2. resistance, current	3. ohm-mt
4. negative	5. resistivity	6. mho
7. parallel	8. $tr/9 \times 10^9$	9. 100
10. dc	11. more, less	12. blocks, passes
13. 50, 440, 220, 110	14. reduces	15. decrease
16. 8.854×10^{-12}	17. breakdown potential	18. $44\angle -53.13°$
19. 100 KHz	20. leads by 90°	21. magnetic, electrostatic
22. increases	23. zero	24. does not reach
25. voltage, current	26. Henry	27. current
28. before	29. 1	30. VI, VI cos ϕ, VI sin ϕ

15.9 MAGNETIC CIRCUITS

1. Relative permeability of diamagnetic material is less than __________.
2. __________ permeability is pure number.
3. If the current in conductor reverses, the magnetic field __________
4. __________ is inversely proportional to the permeabilty of the magnetic material.
5. Unit of MMF is __________.
6. The profile of flux density with magnetising force is known as __________.
7. Energy stored in a magnetic field is measured in __________.
8. The algebraic sum of fluxes at a point is __________.
9. Hysteresis loss is __________ proportional to frequency

10. Reluctance of the magnetic circuit is measured in ________.
11. The ratio of total flux to useful flux is called ________.
12. Longer the air gap ________ the fringing effect.
13. Permeability of evacuated space is ________ H/m.
14. Transformer works on the principle of ________ induction.
15. Hysteresis loop represents magnetic ________.

Answers (15.9)

1. one	2. relative	3. reverses
4. Reluctance	5. AT	6. BH curve
7. Joules	8. zero	9. directly
10. AT/wb	11. leakage factor	12. larger
13. $4\pi \times 10^{-7}$	14. mutual	15. energy

15.10 ELECTRICAL MACHINES

1. In a double cage induction motor the cages are in ________.
2. At the time of starting a three phase induction motor, the slip is ________.
3. The speed of an induction motor operating in ________ region decreases with increase in load.
4. 5 H.P. 400 V, 3 phase induction motor on full load will take ________ amp.
5. Greater the slip ________ will be rotor copper losses.
6. Discoloured rotor bar is an indication of ________.
7. The ________ current of a 3ϕ induction motor is about 20% of its full load current.
8. A 3ϕ induction motor should have small air gap length to have ________.
9. Better speed control can be obatined with ________ induction motor.
10. Greater the number of poles ________ will be the speed of the motor.
11. The stator of a 3ϕ induction motor is laminated to reduced ________ loss.
12. The power factor of a 3ϕ induction motor at no load is ________.
13. Iron losses in the motor of a 3ϕ induction motor are ________.
14. In a 3ϕ induction motor iron losses occur in ________ core and ________.
15. The rotor current frequency of 3ϕ induction motor is ________ proportional to the slip.
16. Power factor of an induction motor is ________ as compared to that of transformer.
17. Induction motor has ________ air gap to prevent it from drawing power at very low lagging P.F.
18. In a slip ring induction motor, with the addition of external resistance in the rotor circuit speed ________ normal speed can be obtained.
19. To achieve high ________ torque in a polyphase squirrel cage induction motor, doiuble cage rotor is used.
20. Starting torque of ________ motor is more than developed by ________ motor.

21. Rotor core laminations may be __________ than stator core laminations.
22. The rotor winding of a phase wound 3ϕ induction motor is mostly connected in __________.
23. When applied voltage is less than rated voltage, induction motor will take __________ current than the rated current at full load.
24. Rotor bars of outer cage in a double cage induction motor are made up of __________.
25. __________ connected armature offers the advantage of having neutral of fourth wire.
26. Distribution of the armature winding has the effect of __________the harmonic component of emf generated.
27. Synchronous motor is provided with __________ airgap as comapred to that of induction motor
28. In place of long pitch coil, short pitch coil is used to improve __________.
29. While conducting short circuit test, we have apply __________value of field excitation.
30. The characteristics between field current and armature current for constant load is known as __________.
31. For the same rating efficiency of synchronous motor is __________ than the efficiency of induction motor.
32. Amortisseur windings of synchronous motors are placed __________.
33. The poles when they are flush with the surface of either stator or rotor are called __________ poles.
34. Damper widing is maily used to reduce __________ of synchronous machine.
35. More the no. of slots per pole per phase __________ will be the value of distribution factor.
36. An over excited synchronous motor is used for __________.
37. Voltage rating of exciter of an synchronous motor is usually __________ V.
38. If the terminal voltage of an alternator is to be maintained constant, with the increase in the load current, it will require __________ in the excitation current.
39. Armature core of synchronous motor is laminated to reduce __________.
40. Damper winding is usually located in __________.
41. An alternator requires dc for its __________.
42. Greater the condition of saturation of the magnetic circuit __________ will be the demagnatisation effect of armature reaction.
43. In a transformer of given voltage rating, greater the cross section area of the core __________ will be the magnitude of magnetizing current.
44. Transformer efficiency is __________ than generator.
45. Transformer can't work on __________ supply.
46. Transformer works on the principle of __________ induction.
47. For maximum efficiency __________ must be equal to __________losses.
48. The core of a transformer is laminated to reduce __________ loss.
49. If the iron core of a transformer is repalced by wooden core it will draw __________ magnetising current.
50. The cooling of __________ type transformer is poor as compared to __________ type transformer.

51. Maximum voltage regulation occurs when P.F. of the load is equal to __________.
52. For sinusoidal applied voltage, magnetizing current is non-sinusoidal due to __________.
53. A transformer is used to increase or decrease __________.
54. Iron loss in a transformer __________ with the variation in a load.
55. In Scott connections, transformer which has __________ tapping in primary is called teaser transformer.
56. Induced voltage per turn of the primary and the secondary of the transformer are __________.
57. Usually __________ connections are used for stepping down the voltages.
58. To avoid mixing of air with oil, trasnsformer is always tilled from __________.
59. Short circuit test on a transformer gives the __________.
60. Applied voltage and primary induced emf in a transformer are in __________.
61. Magnetising inrush current of a transformer is __________ full load current.
62. __________ losses will take place in the core of a transformer.
63. Transformer oil should have __________ dielectric strength.
64. In a power trasnsformer, oil is used for __________ of the transformer.
65. Viscosity of transformer oil is __________.
66. Usually __________ connections are used for stepping up the voltages.
67. In a transformer __________ winding is used nearer to the core.
68. All day efficiency of a transformer is also called as __________.
69. Viscosity of the cooling oil in a transformer should be __________.
70. Need of elaborate cooling becomes more as the rating of transformers becomes __________.
71. Sludging of transformer oil __________ its property of cooling.
72. Transformer oil should be wax free to ensure that it does not solidify at __________ temperature.
73. Percentage resistance and reactance is __________ times the per unit values of resistance and reactance.
74. By short circuit test we find out equivalent __________ and __________.
75. Auto transformer __________ safe to supply very low voltage from high voltage source.
76. Greater the leakage fluxes, __________ will be the voltage regulation.
77. Zero voltage regulation occurs when the p.f. of the load is equal to __________.
78. Where flux wave is steep, emf induced is __________.
79. Transformer secondary connected in parallel with wrong polarities will result __________.
80. The fluctuations in the output voltage of a simple dc generator are called __________.
81. DC shunt motor is a __________ speed motor.
82. The commutator of a dc generator serves the purpose of changing the __________ into __________.
83. DC series motor should not be run without __________.

84. DC __________ motors are not suitable for belt drives.
85. In generating action, current flows in __________ direction as that of induced __________.
86. An arc welding dc generator is basically __________ compounded generator.
87. In dc generator, iron losses occur in __________ and __________.
88. In motoring action current flow in __________ direction that of induced emf.
89. Frictional torque always acts in opposite direction to the direction of __________.
90. The saturation curve of a dc generator is known as __________ curve.
91. A series wound motor is a __________ excited motor.
92. DC series generator is used for __________ up supply voltage.
93. The most suitable motor for hoist drive is a __________ motor.
94. The nature of the current flowing the armature of dc machine is __________.
95. __________ motors are employed with fly wheels.
96. A series motor has __________ speed and __________ torque.
97. On loading the speed of dc series motor __________.
98. The electrical characteristic of a dc motor is curve between __________ and armature __________.
99. The terminal voltage of a dc shunt generator on loading __________slightly.
100. For high current and low voltage the dc generator armature should be __________ wound.
101. Brushes are provided in dc generator for providing a path for the __________.

Answers (15.10)

1. Parallel	2. one	3. stable
4. 7.5	5. more	6. excessive rotor heating
7. no load	8. better power factor	9. slipring
10. less	11. eddy circuit	12. low
13. negligible	14. stator, teeth	15. directly
16. less	17. small	18. below
19. starting	20. slipring, squirrel cage	
21. thicker	22. star	23. more
24. brass/Aluminium	25. star	26. reducing
27. larger	28. power factor	29. reduced
30. V-curve	31. more	32. on rotor pole
33. salient	34. hunting	35. less
36. synchronous phase modifier		37. 110 V
38. increase	39. eddy current loss	40. pole face
41. rotor field excitation	42. more	43. less
44. greater	45. dc	46. electromagnetic mutual
47. copper losses, iron	48. eddy current	49. more
50. shell, core	51. zero (lag)	52. voltage
53. voltage	54. remains constant	55. 86%

56. same
57. star/delta
58. bottom drain value
59. copper losses
60. phase opposition
61. more than
62. eddy current
63. high
64. cooling
65. low
66. delta/star
67. L.T.
68. energy efficiency
69. less
70. high
71. reduces
72. low
73. 100
74. resistance, reactance
75. is not
76. more
77. leading
78. maximum
79. short circuit
80. ripples
81. constant
82. ac, dc
83. load
84. shunt
85. same, emf
86. cumulative
87. armature, pole shoe
88. opposite
89. rotation
90. magnetisation
91. self
92. boosting
93. dc series
94. alternating
95. shunt
96. high, high
97. decreases
98. torque, current
99. decreases
100. lap
101. flow of current

INDEX

Absolute permittivity 37
Acceptor circuit 383
Active component 367
Active element 181
Active energy 362
Active power 360
Air blast cooling 570
Air capacitors 56
Air friction damping 793
Air gap power 683
Air natural cooling 570
All day efficiency 571
Alternator 728
Ampere (A) 2
Ampere's circuital law 167
Amplitude (Peak Value) 337
Approximate equivalent circuit 543
Armature 599, 728
Armature copper loss 629
Armature reaction 605
Armature reaction reactance 738
Armature resistance control 623
Armature winding 599
Asynchronous 660
Attraction type 794
Auto transformer 568
Auto-transformer starter. 695
Auxiliary windings 772, 777, 778
Average (or Mean) Value 339
Average Power 359

B-H 145
Backward torque 771
Bandwidth 386
Bilateral element 181
Bilateral network 232
Biot Savart's 117
Blocked rotor 774
Blocked Rotor Test 687
Board of Trade Unit (BOT) 14
Braking magnet 807
Branch 180
Break down torque 679
Breather 570
Brush holder 599
Brushless excitation 729
By changing the slip 700

Cage rotor 661
Candela 3
Capacitance 55
Capacitance of an isolated sphere 60
Capacitance of concentric spheres 60
Capacitive reactance 348
Capacitive susceptance 359
Capacitor 54
Capacitor start motor 780
Carbon brushes 599
Ceramic capacitors 57
CGS system 1
Change of supply frequency 700
Charging 75
Coercivity 157
Coil span 732
Commutating poles 607
Commutation 606

Commutator 599
Compoles 607
Compound excitation 601
Compound motor 618
Concentric coils 532
Conductance 13, 188, 359
Conductivity 14
Conservator 570
Constant-speed motor 617
Controlling 789
Cooling 569
Copper losses 553
Core 630
Core losses 160, 553
Core type 531
Corner 386
Coulombs 3, 36
Coulomb's First Law 36
Coulomb's Law 110
Coulomb's Second law 36
Counter emf 611
Creeping 809
Creeping error 809
Crest or amplitude or peak factor Ka 339
Critical field resistance 602
Cross-magnetising component 606
Cumulatively compounded 601
Curie Law 111
Curie temperature 111
Curie-Weiss Law 111
Current divider circuits 188
Current limiting resistor 184
Current magnification 393
Current source 231, 225
Cycle 337
Cylindrical 729

D axis 743
Damper winding 751
Damping torque 789
Datum node 201
dc generator 598
dc motor 598
Decimal system 1
Deflecting torque 789
Delta-delta 572
Delta-star 572
Demagnetising component 606
Demagnetization 111
Dependent voltage 231
Derived units 1
Diagonal elements 215
Dielectric strength 56
Differentially compounded 601
Dipole 109
Direct current (dc) machines 598
Direct On Line (DOL) starter 695
Discharging 77
Distributed armature winding 731
Distribution transformers 570
Diverter 624
Domains 146
Dot covention 131
Double revolving field theory 771
Dynamic impedance 392
Dynamometer instruments 794
Dynamometer type wattmeter 803
Dynamometers 640

Eddy current damping 793
Eddy current loss 386, 630
Eddy current losses 553
Eddy currents 159
Efficiency 552
Efficiency of dc machines 630
Electric dipole 45
Electric field strength 40
Electric flux density 37
Electric potential 38
Electrical instruments 788
Electrolytic capacitors 57
Electromagnetic induction 123
Electromagnetic torque 672
Electrostatic instruments 794
Electrostatics 36
Emf 11, 611
Emf equation of a dc machine 603
Energy stored 82
Equipotential surface 39
Equivalent circuit 542, 680
Equivalent impedance 534
Equivalent resistance 181
Exciting current 530
Extension of ammeter range 800
External characteristic 608

Farad 3, 55
Faraday's laws 336
Ferromagnetism 146
Fictitious resistance 681

Field coils (windings) 599
Field control 624
Field copper loss 629
Field intensity 40
Field regulator 624
Field strength 120
Fixed losses 682
Fleming's left hand rule 112, 598
Fleming's right hand rule 598
Forced oil and forced air cooling 570
Forced oil and water cooling 570
Form factor 339
Forward and backward rotating fields 773
Forward torque 771
Foucault's 159
Foucault's currents 159
FPS system 1
Frequency 337
Frequency control 700
Frequency error 809
Frictional errors 809
Fringing 156
Fundamental units 1

Gauss' Law 44
Gauss's Law of Magnetism 110
Geometrical neutral plane 606
Gravity control 790, 791
Gross torque 682

Hardness constant 145
Henry 3
Hertz 3
High slips 679
Hysteresis cycle 157
Hysteresis loss 158, 538, 553, 630

I2R losses 553
Impedance 348
Impedance transformation ratio 534
Impedance triangle 349, 350
Independent sources 231
Indicating instruments 788
Induction instruments 794
Induction motor 660
Induction type energy meter 805
Inductive reactance 347
Inductive susceptance 359
Infinity 184
Instantaneous induced emf 336
Instantaneous value 337
Instantaneous value of the current 336
Instrument constant 801
Integrating instruments 788
Intensity of magnetism 111
Interleaved 532
Internal characteristic 608
International system of units 1
Interpoles 607
Iron losses 160, 553, 630

Joule 14

Kelvin (K) 2
Kilogram 2
Kirchhoff's current and voltage laws 13
Kirchhoff's current law 196
Kirchhoff's voltage laws 196
KWhr 14

Lagging 344
Lap 599
Laplace's 117
Leading 344
Leakage factor 155
Leakage flux 155
Lenz's Law 124
Lenz's law 336
Level or flat compounded generator 602
Limbs 531
Linear 232
Linear element 180
Load 11
Load angle 753
Loop 180
Loop analysis 207
Lorentz force 111
Low voltage release coil 639
Low-slips 679

Magnet 109
Magnetic coupling 126
Magnetic energy density 130
Magnetic field intensity 140
Magnetic field system 598
Magnetic induction 109
Magnetic leakage 155
Magnetic losses 630
Magnetic neutral plane 606
Magnetising branch 542
Magnetising force 140
Magnetization 111, 145

Magnetization curve 145, 156
Magnetizing reactance 738
Magneto motive force 139
Magnification factor 384
Main 777
Main stator winding 778
Maximum efficiency 554
Maximum power transfer theorem 268
Maximum power transfer theorem (AC application) 457
Maximum torque 673, 674
Measurement of power in a three-phase three-wires 496
Mechanical losses 630
Mesh 180, 207
Mesh or delta (D) connection. 487
Metre 2
Mho 13
Mica capacitors 57
Microfarad 55
Mole 3
Monopole does 109
Motor efficiency 684
Moving coil instruments 794
Moving iron instruments 794
Multi-plate capacitor 58
Mutual inductance 126

Negative plate 54
Network elements 180
No load 774
No load current 538
No load rotational losses 630
No volt release 639
No-load Test 686
Nodal analysis 201
Node 180
Non-linear element 180
Nortonizing procedure 256
Norton's equivalent circuit 256
Norton's equivalent current source 256
Norton's equivalent resistance 256
Norton's theorem 256
Norton's theorem for ac network 454

OCC 607
Off-diagonal elements 215
Ohm 3, 13
Ohm meter 809
Ohmic losses 553
Ohm's law 13
Oil blast cooling 570
Oil natural cooling 570
Open circuit 184, 686
Open circuit test 560
Open circuits 189
Operating torque 789
Orbital magnetic 146
Over compounded generator 601
Overload release coil 639

Paper capacitors 56
Parallel 187
Parallel AC circuit 357
Parallel operation 567
Parallel-platecapacitor 57
Passive element 181
Per unit (p.u.) 4
Percentage regulation 544
Period of commutation 607
Permanent magnet type moving coil instruments 798
Permeability of 140
Permeance 140
Permittivity 37
Phase 337
Phase angle 337
Phase difference 337
Phase error 808
Phase wound 661
Pitch factor 732
Pole Changing 700
Pole cores 598
Pole shoe 598
Positive plate 54
Potential difference 11, 38
Potential divider 182
Potential gradient 42
Potentiometer 183
Power 14, 183
Power angle 739
Power curve 363, 365
Power factor 364, 366
Power transformers 570
Power waveform 361
Pull-out torque 679

Q axis 743

Radian 3
Reactance voltage 607
Reactive component 367

Reactive energy 362
Reactive power 362, 488, 538
Reactive volt amperes 362
Real power 360
Recording instruments 788
Rectifier instruments 810
Reference 201
Regulating transformer 562
Regulation 544
Rejector circuit 392
Relative permeability 141
Relative permittivity 37
Reluctance 140
Reluctance power 746
Reluctance torque 757
Repulsion type 794
Resistance 13
Resistance matrix 215
Resistivity 14
Resonance in parallel circuit 391
Resonant frequency 383
Restraining torque 789
Retentivity 157
Roots Mean Square (RMS value) 338
Rotating field 664
Rotor 661, 728
Rotor copper loss 683
Rotor input 683

Salient pole type 729
Salient poles 598
Sandwiched 532
Second 2
Segments 599
Selectivity 386, 387
Self-excitation 600
Self-inductance 125
Self-resistance 215
Separate excitation 600
Series 181
Series aiding 182
Series excitation 601
Series field copper loss 630
Series magnet 806
Series motor 618
Series opposing 182
Series parallel ac circuits 377
Series RC circuit 349
Series resonance 382
Series RL circuit 348
Series RLC circuit 350
Shaded pole motor 784
Shading coil 784
Shell type 531
Short circuit test 560
Short circuits 189
Short-pitch 732
Short-pitch windings 731
Shunt compound machine 601
Shunt excitation 601
Shunt field copper loss 630
Shunt magnet 807
Shunt motor 617
SI 1
Siemens 3, 13
Single-phase induction motors 770
Sinusoidal quantity 343
Skewed 661
Slip 666
Slip for maximum torque 673
Slip ring rotors 661
Solenoid 120
Source 11
Source conversion 227
Speed control 700
Speed control of DC motors 622
Speed equation 616
Speed error 808
Speed regulation 617
Speed torque characteristic 619
Spin magnetic field 146
Split phase 778
Spring control 790
Square loop 157
Squirrel cage 661
Stacking 156
Stacking factor 156
Star (or Y) connection 487
Star delta conversion 217
Star-delta 572
Star-delta starter 695
Star-star 572
Starter 638
Starter resistance 638
Starting torque 672
Starting winding 772, 778
Static exciters 729
Static stability limit 756
Stator 661, 728
Stator input 684
Steady state current 161

Step-down 533
Step-up 533
Stray load loss 684
Sumpner's 562
Superposition theorem 232
Superposition theorem for ac network 444
Susceptance 359
Swamp resistance 798
Synchronous capacitor 752
Synchronous condenser 752
Synchronous impedance 738
Synchronous machine works 728
Synchronous motor 749
Synchronous reactance 738
Synchronous reactance 741
Synchronous speed (Ns) 700
Synchronous Watts 683

Temperature coefficient of resistance 17
Temperature errors 809
Tesla 3
Thevenin's equivalent circuit 243
Thevenin's equivalent resistance 243
Thevenin's theorem 242
Thevenin's theorem in ac circuits 448
Thevenin's voltage 243
Thevenizing procedure 244
Three-phase system 485
Three-phase transformer 571
Three-point starter 638
Time constant 161
Time period 337
Torque 611, 670
Torque equation 612
Torque slip characteristics 678
Total apparent power or complex power 488
Transformation ratio 533
Transformer 530
True power 360
Two-reaction theory 743

Under compounded generator 602
Unilateral element 181
Unit charge 37
Useful shaft torque 682

V-curve 750
VAR 364
Variable losses 682
Variation of field flux 624
Volt 3
Voltage divider 182
Voltage divider theorem 183
Voltage magnification 384
Voltage regulation 739
Voltage resonance 383
Voltage sources 225
Voltage triangle 349, 350

Ward-Leonard 640
Watt 3
Wave 599
Wound rotor 661
Wound rotor induction motor (WRIM) 684

Yokes 531

Zero adjuster 790
Zero potential node 201