THERMODYNAMICS

Basic and Applied

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V. GANESAN, born in Nagercoil in 1945, had a successful career of 48 years in the Department of Mechanical Engineering, Indian Institute of Technology Madras. He retired from the institute in 2017 after serving as professor emeritus for five years after his superannuation in 2012. He has published more than 450 papers and has guided 44 PhD and 20 MS scholars. So far, he has received 52 national and international awards. To mention a few, he is the recipient of Anna University National Award for Outstanding Academic for the Year 1997, Khosla National Award in 2004, ISTE Periyar Award for Best Engineering College Teacher in 2004, Excellence in Engineering Education (Triple 'E') Award by SAE International, USA, in 2007, Shri P. K. Das Memorial Best Faculty Award by Nehru Educational and Charitable Trust, Coimbatore, in 2009, Life Time Achievement award from SAE India, Southern Section in 2017.

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He is the Fellow of Indian National Academy of Engineering, National Environmental Science Academy, Fellow of SAE International, USA, and Institution of Engineers (India). He has also been felicitated by International Combustion Institute for lifetime contribution in the field of IC engines and combustion. Dr. Ganesan has authored several books such as *Internal Combustion Engine*, *Gas Turbines*, *Computer Simulation of Four-Stroke Spark-Ignition Engines*, and *Computer Simulation of Four-Stroke Compression-Ignition*, *Engines and Heat Transfer*. Further, he has edited several proceedings of the national and international conferences.

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McGraw Hill Education (India) Private Limited

Published by McGraw Hill Education (India) Private Limited
444/1, Sri Ekambara Naicker Industrial Estate, Alapakkam, Porur, Chennai 600 116

Thermodynamics: Basic and Applied

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This edition can be exported from India only by the publishers,
McGraw Hill Education (India) Private Limited.

1 2 3 4 5 6 7 8 9 D102739 22 21 20 19 18

Printed and bound in India.

Print-Book Edition

ISBN (13): 978-93-5316-127-9
ISBN (10): 93-5316-127-4

E-Book Edition

ISBN (13): 978-93-5316-128-6

ISBN (10): 93-5316-128-2

Director—Science & Engineering Portfolio: *Vibha Mahajan*

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Typeset at Text-o-Graphics, B-1/56, Aravali Apartment, Sector-34, Noida 201 301, and printed at

Cover Design: V Ganesan

Cover Printer:

Visit us at: www.mheducation.co.in
Write to us at: info.india@mheducation.com
CIN: U22200TN1970PTC111531
Toll Free Number: 1800 103 5875

Dedicated to my Punctilious, Hardworking and Devoted Scholar

Dr. Vijayashree

PREFACE

There are many books in the market on the subject *Thermodynamics*. The logical question will be why another one? The main reason for bringing this book is to disseminate the knowledge that I have gained over past 50 years while teaching undergraduate and postgraduate students at IIT Madras. Further, I felt that the available books that I have consulted do not cover the syllabi effectively; and hence, this book.

I have collated the syllabi of various major universities across India and tried to cover most of the topics in a lucid and comprehensive manner. Thermodynamics, per se, is a simple subject but a little difficult to comprehend because most of the theories were evolved over a period by means of experiments and measurements. This book is expected to serve as a textbook to help students understand and appreciate the basics of thermodynamics starting from the fundamentals. The subject matter has been organised in 14 chapters in a logical sequence which covers both basic and applied thermodynamics. I have taken care to explain the theory in a lucid manner with practical examples, wherever necessary. In every chapter, 25 examples have been worked out along with review questions, exercise problems and MCQs, thereby, each chapter has a pedagogy of 100 questions. Twenty five well designed exercise problems are included in every chapter with answers so that the inquisitive students may solve these problems and compare with answers given. It is hoped that this book will satisfy the needs of both basic and applied thermodynamics.

As there are large number of equations in several chapters, maximum care has been taken to minimise the typing errors. I would be obliged to the readers for finding any such errors and would be grateful for the constructive criticism for the improvement of the book. The readers may send their suggestions to my email id so as to enable me to improve the next edition.

I wish to express my sincere gratitude to Dr. Vijayashree, who has put extraordinary efforts in bringing out this book. The entire book has been made camera ready by her. Without her unstinted support, this book would not have been made possible. I am thankful to my family members who have borne with me for the last four years.

Last but not the least, I am indebted to the Continuing Education Centre of IIT Madras for the support provided for this project.

V GANESAN

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CONTENTS

NOMENCLATURE

Greek Symbols

β	coefficient of volume expansion
ΔS	entropy change
ℓ	distance
ϵ	emissivity [1] [1]; strain [2]; effectiveness [11]; heat exchanger effectiveness [13]
η	efficiency
η_{CV}	clearance volumetric efficiency
η_{hg}	mercury cycle efficiency
η_{II}	second law efficiency
η_P	Pump efficiency
η_s	steam cycle efficiency
η_T	turbine efficiency
η_{th}	thermal efficiency
η_{tv}	total volumetric efficiency
γ	ratio of specific heats
γ_s	specific weight
$\dot{I}$	irreversibility rate
μ	degree of saturation [9]; Joule–Thomson coefficient [14]
$\dot{m}$	mass flow rate
$\dot{m}_a$	mass flow rate of air
$\dot{m}_f$	mass flow rate of fuel
$\dot{m}_p$	mass flow rate of products of combustion
$\dot{m}_s$	mass flow rate of steam
μ_i	chemical potentials
ω	angular velocity [radian/sec]
$\overline{\rho}$	molal density
ϕ	availability function [7]; relative humidity [9];

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[1] Numbers in square bracket indicates the chapter number in which it is first referred

	number of phases present in equilibrium [8]
Ψ	availability
$\overline{R}$	universal gas constant = 8.314 Nm/kg mol K.
ρ	density
ρ_{cr}	density of the substance at critical point
σ	Stefan-Boltzmann constant [1]; stress [2]
τ	torque [2]; intensive coordinate [2]; Time [8]
θ	angle of rotation of the shaft
$\dot{V}$	volume flow rate
$\mathbb{V}$	velocity
$\overline{\mathbb{V}}$	average velocity
$\overline{v}$	molal specific volume [1]; molar volume [10]
$\dot{W}$	rate of work done

Alphabet

A	area
a	Helmholz function
A_{min}	minimum exergy
a_d	area of the indicator diagram
ADP	apparatus dew point
AE	available energy
b	correction factor
BF	by-pass factor
$bmep$	Brake mean effective pressure
BP	brake power
C	constant [2]; components [8, 14]; clearance ratio [12];
c	specific heat
c_{pf}	specific heats at constant pressure for liquids
c_{pg}	specific heats at constant pressure for gas
c_{pm}	specific heat at constant pressure for humid air
c_{ps}	constant pressure specific heats for solids
c_{pw}	specific heat of water vapor
$c_1 \ldots c_6$	regression coefficients
c_1	constant heat capacity
c_p	specific heat at constant pressure
c_V	specific heat at constant volume
COP	coefficient of performance
CV	calorific value of the fuel
d, D	diameter
DBT	dry-bulb temperature
DPT	Dew Point Temperature
ds	displacement
dS_e	entropy increase due to the external heat interaction
dS_i	entropy increase due to internal irreversibility
dt	small interval of time

e	Specific energy
E	total energy; modulus of elasticity [2]
e_{tot}	total energy per unit mass flow
F	force [1]; Helmholz free energy [1]; degrees of freedom of the system [14]
g	acceleration due to gravity
G	Gibbs free energy
g	Gibbs function
H	enthalpy; field strength [2]
h	specific enthalpy; height [1]
h_{fg}	Latent heats of vaporisation
h_{sf}	latent heat of fusion
h_{vapour}	Specific enthalpy of vapour
h_a	enthalpy of dry air
h_f	enthalpy of saturated liquid
h_g	enthalpy of saturated vapour/steam
H_m	Enthalpy of the mixture
HE	heat engine
Hg	mercury
HP	heat pump
I	electric current
$imep$	Indicated mean effective pressure
IP	indicated power
j^*	thermal radiation or irradiance
K	isothermal compressibility [14]; indicator spring constant [2]; number of cylinders in the engine [2]; spring constant [2]; constant [3];
KE	Kinetic energy
L	displacement [2]; extensive coordinate [2]; length [1]; stroke of piston [2];
l_d	length of the indicator diagram
m	mass
M	molecular weight; molar mass [10];
m_{cv}	mass of control volume
M_a	molecular weight of air
M_v	molecular weight of water vapour
n	exponent of compression or expansion [2]; constant [3]; number of independent variable [8];
N	number of revolutions per minute of the crankshaft [2]; speed [2]
n, N	number of moles [1]
N_i	Molecular weight of individual gas
N_m	Molar Fraction [10]; Molecular weight of mixture [10]
P	Power; thermodynamic property; phases [14]
p	pressure
p_{bm}	brake mean effective pressure
p_{cr}	critical pressure
p_{gauge}	gauge pressure

p_{im}	indicated mean effective pressure
p_{sat}	Saturated Vapour Pressure
p_{vac}	vacuum pressure
p_{vs}	partial pressure of water vapour when air is fully saturated
p_a	partial pressure of dry air
p_c	compressor cylinder pressure
p_d	discharge pressure
p_i	partial pressure
p_m	mean effective pressure [10]; mixture pressure [10]
p_r	reduced pressure
p_s	shaft power output [2]; suction pressure [12]
p_t	total pressure of moist air
p_v	partial pressure of water vapour
PE	Potential energy
Q	heat transfer
Q_{added}	heat addition
Q_{in}	heat input
Q_{out}	heat rejected
$Q_{rejected}$	heat rejected
Q_L	latent heat removed
Q_R	Heat rejected
Q_S	Heat added
r	compression ratio
R	gas constant [8.314 kJ/k mol K]
R	resistance
R_0, R_u	universal gas constant
R_a	gas constants of air
r_c	cut-off ratio
R_m	gas constant of the mixture
R_n	Heat extracted
r_o	Intermolecular distance
r_p	pressure ratio
R_v	gas constants of water vapour
RH	relative humidity
s	entropy; distance [2]; Stress [2];
S	Entropy
s_f	entropy of saturated liquid
s_g	entropy of saturated vapour
S_m	entropy of gas mixture
S_o	entropy of the system at standard state
$SFEE$	steady state steady flow energy equation
SG	specific gravity
SHF	sensible heat factor
SHR	sensible heat ratio
SP	shaft power
SSC	Specific steam consumption
SSF	Specific steam flow rate
T	Temperature

t	Time; time interval; Temperature
t_{db}	dry bulb temperature [9]; dew point temperature [9]
T_{sat}	saturation temperature
t_{wb}	wet bulb temperature
T_0	atmospheric temperature
T_a	solar collector temperature
TR	Ton of Refrigeration
U	internal energy
u	internal energy
UE	unavailable energy
v	specific volume
V	volume
v_a	specific volume of dry air
v_f	molar volume of the liquid phase
v_f	specific volume of saturated liquid
V_f	volume of the saturated liquid
v_g	specific volume of saturated vapour [8]; molar volume of vapor phase [14]
V_g	volume of the saturated vapour
V_m	volume of mixture
v_v	specific volume of water vapour
w	specific weight of the liquid
W	weight [1]; work [1]; specific humidity or humidity ratio [9]
W_{act}	actual work
W_{comp}	work of compression
W_{exp}	work of expansion
$W_{net,out}$	net work output
W_a	acceleration work
W_s	Shaft work [Nm] [2]; specific humidity of air [9]
W_T	turbine work
W_u	useful work
WBT	wet -bulb temperature
x	dryness fraction
x_i	mass fraction
y_i	mole fraction
Z	compressibility factor
z	elevation from the datum

Subscript

0	atmospheric
1 and 2	inlet and exit
a	acceleration
a	air
abs	absolute
act	actual
atm	atmosphere
bm	brake mean

c	cold [7]; cylinder [12]
$comp$	compression
cr	critical
CV	clearance volume
cv	control volume
d	discharge
db	dry-bulb ; dew point
e	exit; out
exp	expansion
f	final [6]; saturated liquid [8]; fuel [11]; liquid [14]
g	saturated vapour [8]; steam / vapour [8]; vapour [14];
gen	entropy generation
H	high
hg	mercury
HP	heat pump
i	inlet [4]; in [6]; input [7]; partial [10]
im	indicated mean effective
in	supplied
$irrev$	irreversible
L	Low
m	mean effective
m	mixture
$mech$	mechanical
o	standard state
out	rejected
P	power
r	reduced; reservoir [7]
rev	reversible
s	shaft [2]; saturated[8]; saturated mixture [9]; steam [11] suction [12]
s	steam
sat	Saturated Vapour
SL	second law
sup	superheated
t	total
T	turbine
tv	total volumetric
u	useful [7]; universal [9]
v	vapour [1]; partial [9]
v	vapour
vac	vacuum
wb	wet bulb

1

INTRODUCTION TO BASICS

1.1 PREAMBLE

Let us begin this chapter with a question: *What is Thermodynamics*? You will get the answer step by step in 14 chapters of this book. Thermodynamics can be considered as a branch of both science and engineering. It is a science, because it deals with fundamental concepts. It is engineering because this science is applied to design and analyse various energy conversion systems. Therefore, thermodynamics applies to a wide variety of topics in science and engineering.

In technical terms, thermodynamics can be defined as the science of energy and entropy. As with all sciences, the basis of thermodynamics is experimental observations. Looking at the definition, the word, energy is quite familiar to all of us whereas the word entropy is not so familiar.

The ability of a system to perform work is the common description of energy. Energy is the property of an object, which can be transferred to other objects or converted into different forms. However, energy cannot be created or destroyed. It is a conserved property.

Entropy describes the tendency of a system to go from a higher organised state to lowest organised state. It can be viewed as the available energy for doing useful work in a thermodynamic process. It is not a conserved property.

To go into the history, the word, thermodynamics, has originated from the two Greek words *therme* meaning heat and *dunamis* meaning power. In short, thermodynamics relates heat and work transfer. It provides an understanding of the nature and degree of energy transformations. Thermodynamics is mainly concerned with energy conversion devices, engines or machines. Such devices can only be driven by convertible energy. Note that energy cannot be completely converted into work. Devices have a theoretical maximum efficiency while converting energy to work. During this process of conversion, a property called entropy accumulates in the system. It then dissipates in the form of waste heat.

The principles of thermodynamics have been in existence since the creation of the universe. Thermodynamics did not emerge as a science until the construction of the first successful atmospheric steam engines in England by Thomas Savery in 1697 and Thomas Newcomen in 1712. These engines were very slow and inefficient but they opened the way for the development of a new science called 'Thermodynamics'. The first thermodynamic textbook was written in 1859 by William Rankine, a professor at the University of Glasgow, U.K. *Thermodynamics is a simple and easy subject if you understand the fundamental concepts.* This book is intended to help you to understand the basics concepts and also explains its applications to engineering equipments. Thus, both basic and applied thermodynamics are covered in this book. The details are explained right from the fundamentals.

1.1.1 Who Should Study Thermodynamics?

Thermodynamics has a few fundamental laws and principles. These laws can be applied to a wide range of problems. Thermodynamics is core to all branches of engineering and allows understanding of the mechanism of energy conversion. It is next to impossible to identify any area where there is no interaction in terms of energy and matter. It is a science having its relevance in every occupation. Scientists are interested in gaining a fundamental understanding of the physical and chemical behaviour of fixed quantities of matter at rest. They use the principles of thermodynamics to relate the properties of matter. On the other hand, engineers are generally interested in studying the systems and their interaction with their surroundings.

Thermodynamics can be divided into *classical thermodynamics* and *statistical thermodynamics.* In engineering systems analysis, classical thermodynamics is employed. Therefore, the subject thermodynamics is of interest to students of *all branches of engineering.* In fact, it is useful to entire humankind as it deals with the most basic processes occurring in nature.

1.1.2 Basic Concepts to Understand

Every science has a unique vocabulary associated with it and thermodynamics is no exception. Precise definition of basic concepts forms a sound foundation for the development of a science and prevents possible misunderstandings. In order to understand the subject, to start with, the reader should be able to appreciate the basic concepts of the following:

(i) the dimensions and units that we will come across in thermodynamics,

(ii) the definition of system, surrounding and boundary,

(iii) thermodynamic property, state, path, process and cycles,

(iv) thermodynamic equilibrium,

(v) the basics of force, pressure, volume and temperature,

(vi) various pressure measuring and temperature measuring devices and

(vii) zeroth law of thermodynamics.

1.2 DIMENSIONS AND UNITS

Before going into the details of thermodynamics, it is worthwhile to introduce the dimension and units that we would come across quite often with the subject. Therefore, let us understand the dimension and units associated with thermodynamics.

Dimension refers to certain fundamental physical concepts that are involved in the process of nature. They are more or less directly evident to our physical senses. Any physical quantity can be characterised by *dimensions.* The magnitudes assigned to the dimensions are called *units.* Some basic dimensions such as mass (m), length (L), time (t) and temperature (T) are selected as *primary* or *fundamental dimensions.* Others, such as velocity $\mathbb{V}$, energy E and volume V are expressed in terms of the primary dimensions and are called *secondary dimensions* or *derived dimensions.*

Units assigned to ‘primary dimensions’ are called *base units* whereas units assigned to ‘secondary dimensions’ are called *derived units.* At present, SI system (System-International) of units has been accepted worldwide and therefore, the same is followed in this book.

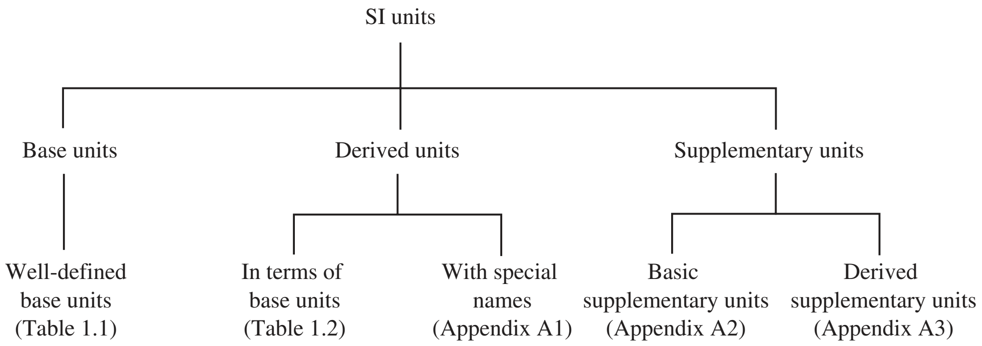


Fig. 1.1 Classes of SI units

1.3 INTRODUCTION TO SI UNITS

The international system (SI) of units are divided into three classes: (i) Base units (ii) Derived units and (iii) Supplementary units. The details are shown in Fig.1.1.

From the scientific point of view, division of SI units into these classes is to a certain extent arbitrary, because it is not essential to the physics of the subject. Nevertheless, considering the advantages of a single, practical, worldwide system for teaching and for scientific work, the international system has seven well-defined base units, as given in Table 1.1.

The second class of SI units contains derived units. These units are formed by combining base units according to the algebraic relations linking the corresponding quantities. Several of these algebraic expressions in terms of base units can be replaced by special names and symbols. Derived units may, therefore, be classified under two headings which we will quite often come across are given in Tables 1.2. Appendix C1 gives the details of derived units with special names.

The SI units assigned to third class are called *supplementary units*. It may be represented in terms of either in base units or in derived units. Refer Appendix C2 and Appendix C3 for details. The SI units is based on a decimal relationship between units. In order to express the multiples of various units, we use prefixes. The details are listed in Appendix C4. They are standard for all units. In this digital era, the students are encouraged to memorise them because of their widespread use.

1.3.1 Principal Features of SI Units

The principal features of SI units can be summarised as follows:

(i) Minimum number of base units (only seven).

(ii) Derived units are obtained in terms of base units based on definition or physical laws.

(iii) Derived units do not have any arbitrary constants.

(iv) Internationally accepted names for derived units.

(v) Multiples and sub-multiples of units (Appendix A4) can be employed to cover wide range of values.

Table 1.1 SI base units

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	kelvin	K
Luminous intensity	candela	cd
Amount of substance	mole	mol

Table 1.2 Typical examples of SI derived units expressed in terms of base units

Quantity	Unit	Symbol
Acceleration	metre per second squared	m/s^2
Activity (radioactive)	1 per second	s^{-1}
Area	square metre	m^2
Concentration (amount of substance)	mole per cubic metre	mol/m^3
Current density	ampere per square metre	a/m^2
Density, mass density	kilogram per cubic metre	kg/m^3
Luminance	candela per square metre	cd/m^2
Magnetic field strength	ampere per metre	A/m
Specific volume	cubic metre per kilogram	m^3/kg
Speed, velocity	metre per second	m/s
Volume	cubic metre	m^3
Wave number	reciprocal metre	m^{-1}

1.4 MACROSCOPIC AND MICROSCOPIC VIEWS OF THERMODYNAMICS

Study of thermodynamics can be undertaken either from a *macroscopic* or a *microscopic* point of view. The macroscopic approach to thermodynamics deals with the gross or overall behaviour. This is also called *classical thermodynamics.* Structure of matter at the molecular, atomic and subatomic levels are *not* considered in classical thermodynamics. It allows important aspects of system behaviour to be evaluated from the observations of the *overall system.*

The *microscopic approach* to thermodynamics deals directly with the structure of matter. This is also called *statistical* thermodynamics. The objective of statistical thermodynamics is to characterise the average behaviour of the particles by statistical methods. It then relates this information to the observed macroscopic behaviour of the system.

For the great majority of engineering applications, classical thermodynamics, namely macroscopic approach is more appropriate. It is a more direct approach for design and analysis. Further, it involves far fewer mathematical complications. For these reasons, the macroscopic viewpoint is the one adopted in this book. What it means is, relativity effects are not considered significant for the systems under consideration. However, to promote understanding, wherever necessary, concepts are interpreted from the microscopic point of view also.

1.5 SYSTEM, SURROUNDINGS AND BOUNDARY

In general, a system is nothing but a collection of matter. There is a lot of subtlety in the way it is defined. However, in thermodynamics, it is a much more straightforward concept. In this section, we will discuss the details of *system, surroundings and boundary.*

1.5.1 System

A system is defined as a fixed quantity of matter or a region in space chosen for study. The region in space occupied by the matter is called the control volume. To define it more precisely, *a system is a region containing energy and/or matter that is separated from its surroundings by arbitrarily imposed walls or boundaries.* A typical thermodynamic system along with boundary and surroundings is illustrated in Fig.1.2.

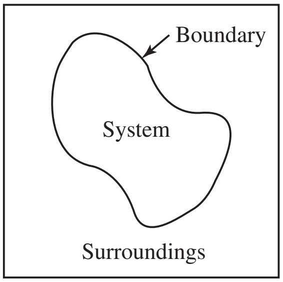


Fig. 1.2 A typical thermodynamic system

1.5.2 Surroundings

All matter and space external to a system is called surroundings (refer Fig.1.2). However, it is usually restricted to those particles of matter external to the system which may be affected by changes within the system. Note that surroundings themselves may form another system. Thermodynamics is concerned with the interactions of a system and its surroundings or one system interacting with another. A system is said to interact with its surroundings when it starts transferring energy or mass or both across its boundary.

1.5.3 Boundary

Real or imaginary closed surface that separates the system from its surroundings is called the boundary. A typical example is the wall of a beaker, which acts as the boundary since it separates the system from surroundings. Boundaries can be classified into three groups:

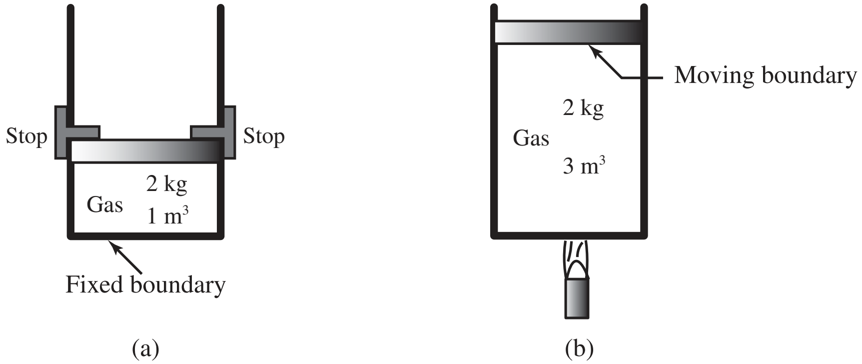
(i) Closed system with fixed and moving boundaries [Fig.1.3(a) and (b)],

(ii) Open system with fixed and imaginary boundaries [Fig.1.3(c)],

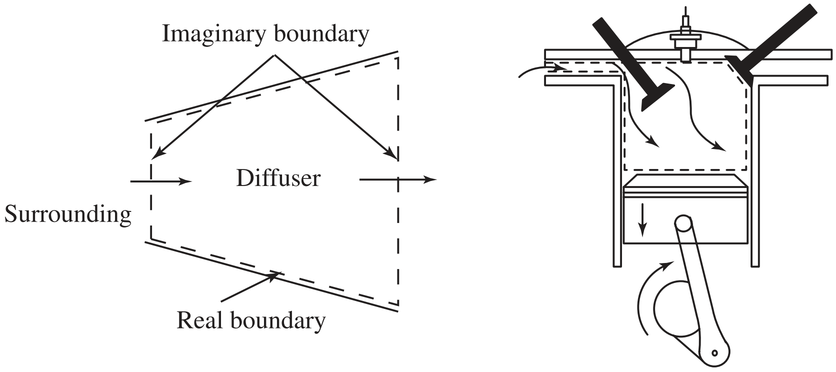
(iii) Open system with moving boundaries [Fig.1.3(d)].

For example, consider Fig.1.3(a) which consists of a cylinder piston arrangement for a closed system where boundaries are fixed. Only energy can enter or leave the system. When we supply heat and stops are removed, the gas expands and the piston moves up. This is a typical moving boundary problem in a closed system. In this also only energy can enter or leave the system [Fig.1.3 (b)].

Consider another example, viz a diffuser [Fig.1.3(c)], which is an open system. The inner surface of the diffuser forms the real part of the boundary. The entrance and exit areas form the imaginary part, since there are no physical surfaces there. Figure 1.3(d) shows a cylinder piston arrangement used in an internal combustion (IC) engine. This is a typical example of an open system with moving and imaginary boundaries. As can be seen in the figure, the imaginary boundary would be the one drawn around a system by dotted lines. Fresh mixture enters the cylinder during suction stroke together with the last part of the cylinder charge after the exhaust process [Fig.1.3(d)]. It is not difficult to envisage a real boundary. *But to imagine an imaginary boundary requires a little bit of imagination.* Fortunately, control volumes in engineering applications have fixed boundaries and thus do not involve any varying boundaries with time.



A closed system with fixed and moving boundary in a
cylinder-piston arrangement



(c) An open system with real and imaginary boundaries in a diffuser

(d) An open system with moving boundary in a cylinder-piston arrangement

Fig. 1.3 Illustration of various boundary types

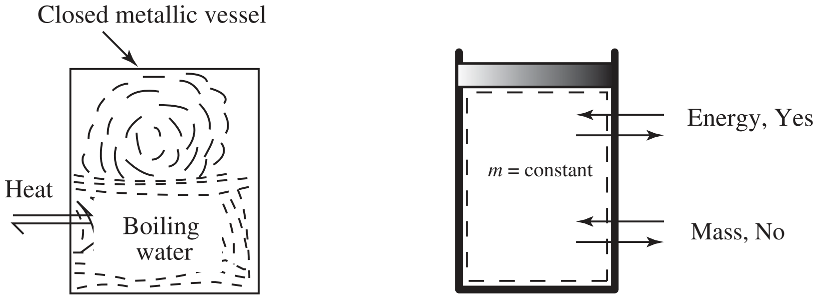
Note that the boundary is the contact surface shared by both the system and the surroundings (Fig.1.2). Theoretically, the boundary has zero thickness and thus it can neither contain any mass nor occupy any volume in space. Further, to three types of boundaries, they can be further divided into three more types, namely:

(i) permeable or porous (ii) impermeable or diathermal (iii) adiabatic

Permeable or *porous* boundaries allow both mass and energy transfer to occur whereas *impermeable* or *diathermal* boundaries do not allow mass transfer but allow energy transfer. *Adiabatic* boundaries do not allow heat transfer but allow mass transfer. A typical example is dewar flask in which the boundary is impermeable to heat flux.

1.6 CLASSIFICATION OF A SYSTEM

Having seen the details regarding the system, surrounding and boundary, let us see how systems are classified. Thermodynamic systems are classified into three types as:
(i) closed system; (ii) open system; and (iii) isolated system.



(a) Aclosed system with fixed boundary

(b) A closed system with moving boundary

Fig. 1.4 Types of closed system

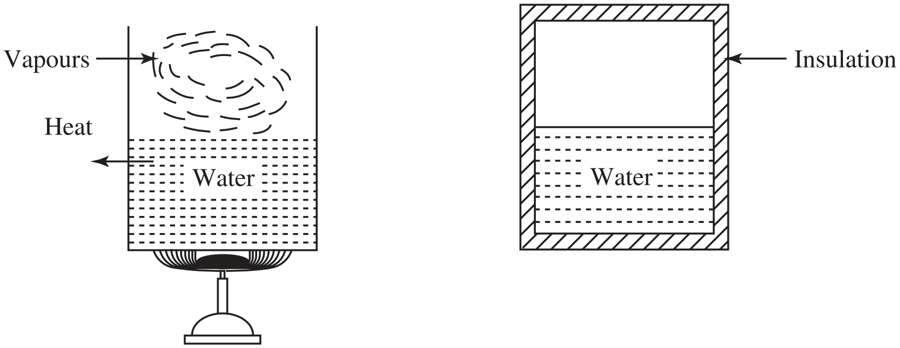
1.6.1 A Closed System

Another name for a closed system is *control mass* since a closed system will contain a fixed amount of mass (Fig.1.4). In a closed system, no mass transfer takes place. In other words, no mass can enter or leave a closed system. But energy, in the form of heat or work, can cross the boundary. The volume of a closed system need not have fixed volume as shown in Fig.1.4(a). It can have varying volume as shown in Fig.1.4(b), because of moving piston.

1.6.2 An Open System

It is also known as a *control volume* and it is a properly selected region in space. It usually encloses a device that involves both mass and energy transfer. A typical example is the diffuser shown in Fig.1.3(c). A large number of engineering problems involve mass flow in and out of a system. Therefore, they are modelled as control volumes. For example, a car radiator, a compressor, a turbine and a water heater in a bathroom all involve mass flow and should be analysed as control volumes or open systems.

In general, any arbitrary region in space can be selected as a control volume. There are no hard and fast rules for the selection of control volumes. However, the proper choice certainly makes the analysis much easier. For example, if we were to analyse the flow of air through a diffuser, a good choice for the control volume would be the region within the diffuser. The inner surface of the diffuser forms the real part of the boundary and the entrance and exit areas form the imaginary part, since there are no physical surfaces existing out the entrance or exit [refer Fig.1.3(c)]. A control volume can be fixed in size and shape, as in the case of a diffuser or it may involve a moving boundary, as in the case of an internal combustion engine as shown in Fig.1.3(d). A simple open system is heating of water in a container as shown in Fig.1.5(a). Exchange of both matter and energy takes place in this system.



(a) Open system (b) Isolated system

Fig. 1.5 An open and isolated system

1.6.3 An Isolated System

It is a special case of a closed system where neither energy nor mass is allowed to cross the boundary [Fig.1.5(b)]. Therefore, they do not exchange energy or matter with the surrounding. So to say, there is no interaction between the system and surrounding. Therefore, it is called an *isolated system*. It may be noted that such systems are extremely slow. *Except for the universe as a whole, truly isolated systems do not exist in nature.*

1.7 UNIVERSE

We have so far seen the details regarding system, surrounding and boundary. Let us see what is meant by universe. As we know everything outside the real or hypothetical boundary is termed as the *surroundings*. Thus, surroundings are, everything surrounding the system. System and surroundings when put together is called the universe.

$$\text{Universe} \quad = \quad \text{System} + \text{Surroundings}$$

1.8 THERMODYNAMIC PROPERTY, STATE, PATH, PROCESS AND CYCLES

In order to appreciate thermodynamics, first we must understand certain basic terms. In the following sections, we will discuss the details regarding property, state, path, process and cycles.

1.8.1 Property

A property is any macroscopic characteristic associated with a system. Thermodynamic properties can be observed, measured or calculated. Some of the familiar *properties* that are commonly encountered in thermodynamics are: *energy, enthalpy, mass, pressure, temperature and volume* to which a numerical value can be assigned at a given time. Note that knowledge of the previous behaviour (history) of the system is not required to define a property. So to say, a property depends upon its state but not upon how the state is reached.

To put it in a nut shell, a quantity is said to be property, if and only if, its change in value between two states is independent of the process. Property has following characteristics:
(i) does not depend on the process; (ii) depends only on end states; and (iii) its differential is exact.

1.8.2 Fundamental and Derived Properties

Pure substances[1] have only five *fundamental* or *primitive* thermodynamic properties. They cannot be derived from other thermodynamic properties. These properties with their common symbols are given below:

(i) pressure, p
(ii) volume, V
(iii) temperature, T
(iv) internal energy, U
(v) entropy, S

In addition, the following three derived thermodynamic properties are combinations of the fundamental properties:

(vi) Helmholz free energy, $F = U - TS$
(vii) Gibbs free energy, $G = H - TS$
(viii) Enthalpy, $H = U + pV$

The above eight basic thermodynamic properties, can be classified as intensive, extensive or specific.

1.8.3 Intensive Property

A property which **is independent of the mass of the system** is called *intensive property.* Typical examples are pressure and temperature.

1.8.4 Extensive Property

A property which **is dependent on the mass of the system** is called *extensive property.* Typical examples are mass, volume, enthalpy and entropy.

To illustrate the difference between extensive and intensive properties, consider an amount of matter that is uniform in temperature. Imagine that it is composed of several parts, as illustrated in Fig.1.6. The total mass of the system is the sum of the masses of the individual parts and the overall volume is the sum of the volumes of the parts. However, the temperature of the whole is not the sum of the temperatures of the parts; it is the same for each part. Thus, mass and volume are extensive properties but temperature is intensive property.

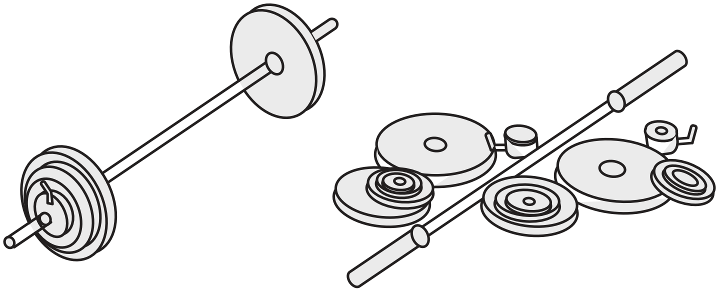


Fig. 1.6 Illustration of extensive and intensive properties

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[1] A pure substance is one that has a homogeneous and invariable chemical composition even though there is a change of phase.

1.8.5 Specific Property

Specific properties are extensive properties per unit mass. If an extensive property is divided by mass associated with them then it is called *specific property.* For example, if the volume of a system, V, has a mass, m, then the specific volume of matter within the system is v, given by V/m which is a specific property. This feature permits extensive properties, F, G, H, S, U, V etc. to be made intensive/specific. This is achieved by simply dividing the extensive property by the quantity of the substance in the system. Quantity can be measured in terms of mass, moles or number of molecules. The lower-case designations are reserved for intensive/specific properties. *The only exception is pressure which is designated by lower case* p. In this book, we use lower case p for pressure.

1.8.6 State

State is the unique condition of the system at an instant of time. It can be described or measured by its properties. In other words, it may be viewed as the complete description of a system in terms of its properties. Note that all the properties of the system have fixed values and there are connecting relations among themselves. Therefore, the state of a system can be specified by providing the values for the minimum required number of properties. Then, all other properties can be determined in terms of these few properties specified. However, if a system exhibits the same values of its properties at different times, then it is said to be at *steady* state and none of its properties changes with time.

1.8.7 State Function

Thermodynamic properties are sometimes called *state functions* because they depend only on the *state or condition* of the system. They do not depend on the *process* or the *path* by which the particular state was reached. For example, properties of water vapour at a specified pressure and temperature is the same whether created by evaporating liquid water by heating or by reacting H_2 and O_2.

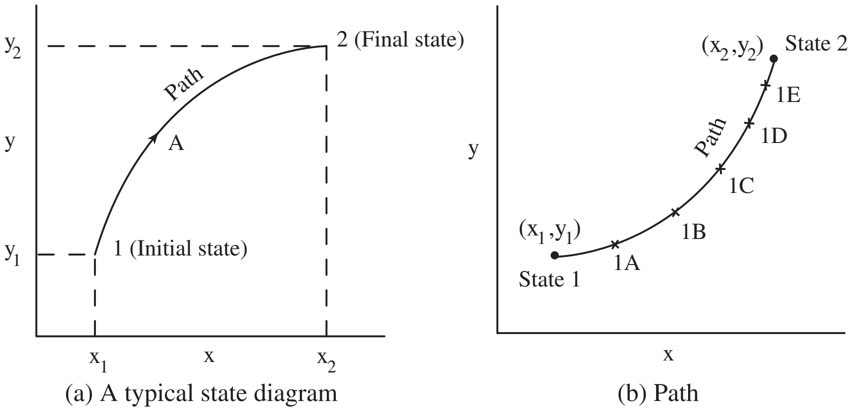
1.8.8 State Diagram

If the state of a system is represented in a diagram then that diagram is called the *state diagram* [Fig.1.7(a)]. The state diagram will be normally drawn in Cartesian (x-y) coordinates. Any two independent properties can be represented in the x and y axes. This diagram can reveal the state of the system at different instants. A point on the state diagram showing the condition of a system at a given instant is called *state point.* The point will be represented by two independent properties selected for x and y axes. At point 1 in the diagram, the state is represented by (x_1, y_1) and at 2 it is (x_2, y_2).

1.8.9 State Postulate

As explained in the previous section, the state of the system can be described by its properties. Note that all the properties need not be specified in order to describe the state. Then, the question is, what is the minimum number of properties required to describe the state. The answer is given by the state postulate. It is stated as *'the state of a simple compressible substance can be specified by* **two** *independent intensive properties'.*

Now, the question turns to what is meant by *a simple compressible substance*? The substance of the system in which electrical, gravitational, magnetic, motion and surface tension effects are absent is called simple compressible system. Note that as per state postulate, two



(a) A typical state diagram (b) Path

Fig. 1.7 A simple state diagram and path

properties must be specified and they must be independent. The two properties are said to be independent if one can be varied while the other is held constant. For example, the temperature T and the specific volume v are always independent properties. Together they can fix the state of a simple compressible system. For a single phase system, the pressure p and temperature T are the two independent properties whereas *for a multiphase system, they are not.*

To illustrate the above point, let us consider the boiling of water. At atmospheric pressure and at sea level, water boils at 100 °C but at higher altitudes, water boils at a lower temperature since pressure is lower. That is, $T = f(p)$ during the phase-change process. Thus, p and T are **not** sufficient to describe the state of a two-phase system.

1.8.10 Path

The path refers to the series of state changes through which the system passes during a process. Referring to Fig.1.7(b), initially the system is at state 1 with properties (x_1, y_1). It reaches to state 2 with properties (x_2, y_2). It passes through various intermediate states (1A to 1E) as shown in the figure. Therefore, the path is the line joining successive state points on a state diagram during a change of state. In other words, *it is the loci of various intermediate states passed through, by a system during a process.*

1.8.11 Process

If any of the properties of a system change, the state of the system is said to change. *Therefore, a process is that, which causes the system to change its state.* In other words, a process is nothing but a transformation from one state to another.

A process may be a *non-flow process* in which a fixed mass within the defined boundary is undergoing a change of state. A typical example is the heating of water in a closed cylinder or container [Fig.1.4(a)].

A process may be a *flow process* in which mass enters and leaves through the boundary of an open system. In a steady flow process [Fig.1.3(c)], mass crosses the boundary from surroundings at entry and an equal mass will cross the boundary at the exit so that the total mass of the system remains constant. In an open system, the following points are to be noted:

(i) the work is delivered from the surroundings to the system at entry to cause the mass to enter,

(ii) the work delivered by the system to the surroundings at the exit to cause the mass to leave and

(iii) heat and/or work can cross the boundary of the system.

1.8.12 Quasi-static Process

What does quasi mean? Quasi means almost. Therefore, quasi-static process denotes a process that is almost static. It is also called a *reversible process* . When a process proceeds in such a way that the system always remains infinitesimally close to an equilibrium state, it is called *quasi-static* or *quasi-equilibrium* process. It can be viewed as a sufficiently slow process that allows the system to adjust itself internally. Further, properties in one part of the system do not change any faster than those of other parts. This process is a locus of all equilibrium points passed through by the system. *It is to be noted that infinite slowness is the characteristic feature of the quasi-static process.*

Let us consider a system shown in Fig.1.8(a) where a gas is contained in a cylinder by a piston on which a weight, W, is placed. Assume that the system is initially in equilibrium state at point 1 [Fig.1.8(c)]. Let the properties be represented by p_1, V_1, T_1. The weight on the piston just balances the upward force exerted by the gas pressure. If the weight is *suddenly removed*, then there will be a force imbalance between the system and surroundings. This will cause the piston to move up until it hits the stops shown in Fig.1.8(b). The system again comes to an equilibrium state with properties, say, p_2, V_2, T_2. The point to note is that the intermediate states the system passes through from 1 to 2 are non-equilibrium states. This cannot be described by thermodynamic coordinates. Figure 1.8(c) illustrates the process 1 to 2. Points 1 and 2 are the initial and final equilibrium states. We have just joined them as shown in the figure and it need not be unique. It has got no meaning otherwise.

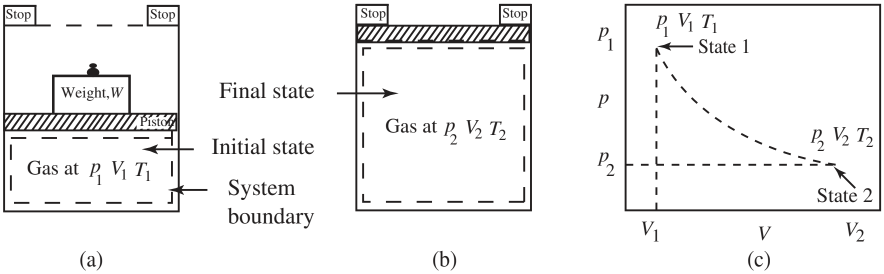
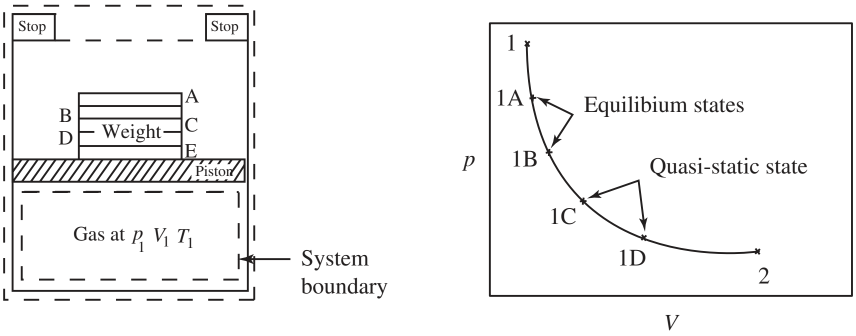


Fig. 1.8 Transition between two equilibrium states due to force imbalance

Now, assume the single weight on the piston is replaced by a number of extremely small pieces of weight as shown in Fig.1.9(a) and assume that the system is isolated. If these small pieces of weights are removed one by one extremely slowly then, the departure of the state of the system from thermodynamic equilibrium will be extremely small. When the first piece A is removed, the system at state 1 moves very slowly to state 1A [1.9(b)]. Since the process is extremely slow, it can be assumed that practically there is no change in equilibrium state. Similarly, when every piece is removed every state passed through by the system will be an equilibrium state. Such a process is nothing but the loci of all the equilibrium points passed through by the system. This type of process is known as quasi-static process which is illustrated in Fig.1.9(b). The process is reversible because, now if we add one extremely small piece, say E, again the gas pressure will rise to p_{1D} from p_2 and the volume will reduce from V_2 to V_{1D}.



(a) Infinitely slow transition between the equilibrium states of a system caused by infinitesimally small force

(b) Illustration of a quasi-state process on a p-V diagram

Fig. 1.9 Quasi-static process

It should be noted that the quasi-static process is only an idealisation. It is not a true representation of any actual process. However, many actual processes can be approximated to quasi-static. Especially engineers, are interested in quasi-static processes. This is due to two reasons:

(i) they are easy to analyse and

(ii) work-producing devices delivers the maximum work during the quasi-static process.

Therefore, quasi-static processes can serve as standards for comparison.

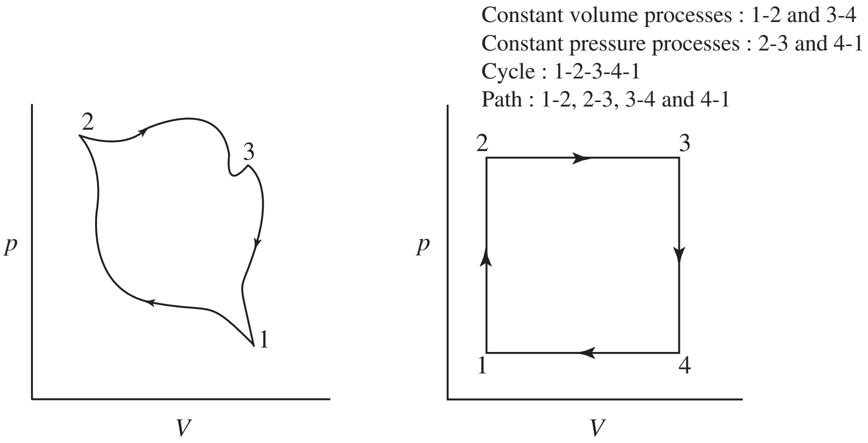
1.8.13 Thermostatics or Thermodynamics

Infinitely slow quasi-static process executed by systems are meaningful only in thermodynamic plots. Then, you may think that the name thermodynamics may be a misnomer, since, it does not deal with the dynamics of heat/work, which is not quasi-static. Then, the name thermostatic may be more appropriate. However, most of the real processes are dynamic but not quasi-static even though the initial and final states of the system might be in equilibrium. Definitely, we can deal with such processes successfully using the concepts of thermodynamics. Therefore, the term, thermodynamics, is *not inappropriate.*

1.8.14 Cycle

Any process or series of processes whose end states are identical is termed as a *cycle*. In other words, a cycle is the one in which the processes occur one after the other so as to finally bring the system to the same initial state. The processes through which the system has passed can be shown on a state diagram. Figure 1.10(a) shows a generalised cycle in which a system with condition 1 changes in pressure and volume through a path 123 and returns to its initial condition 1. Thermodynamic path in a cycle is in closed loop form. After the occurrence of a cyclic process, system shall show no sign of the processes having occurred.

Mathematically, it can be said that the cyclic integral of any property in a cycle is zero, i.e. $\oint dP = 0$, where P is any thermodynamic property. Typical thermodynamic processes, path and cycle are shown on a p-V diagram in Fig.1.10(b).



Constant volume processes : 1-2 and 3-4
Constant pressure processes : 2-3 and 4-1
Cycle : 1-2-3-4-1
Path : 1-2, 2-3, 3-4 and 4-1

(a) Generalised representation of a cycle of operations

(b) Illustration of thermodynamic process, path and cycle

Fig. 1.10 Cycle

1.8.15 Phase

A phase is defined as the quantity of matter which is homogeneous throughout, both in chemical composition and physical structure. We are aware that matter can exist in any of the following three phases, viz solid, liquid and gas.

A pure substance can exist physically in all of the above three different phases. For example, water can be found as solid (ice), liquid (water) and gas (vapour or steam). Another example is allotropic forms of sulphur. It can be found in rhombic form (lemon yellow in colour) and monoclinic form (orange in colour). Similarly, solid carbon can exist in two different phases, namely graphite and diamond.

1.8.16 Homogeneous System

If a system consists of only single phase then it is called a homogeneous system. Typical examples are mixture of air and water vapour, water plus nitric acid and octane plus heptane.

1.8.17 Heterogeneous System

If a system consists of more than one phase then it is called a heterogeneous system. Typical examples are water plus steam, ice plus water and water plus oil.

1.9 THERMODYNAMIC EQUILIBRIUM

Equilibrium means a state of balance. *Thermodynamics deals with equilibrium states.* In a state of equilibrium, the properties of a system are uniform and only one value can be assigned to it. The concept of equilibrium implies that there is no tendency for any spontaneous change in any macroscopic property when the system is isolated. In short, a system is said to be in equilibrium when it involves no changes with time.

To illustrate with an example, assume that one end of an aluminum rod is dipped in a boiling water bath while the other end is placed in a melting ice box. After keeping the system for a sufficiently long time, if you measure the temperature of the rod, it will vary from point to point along the length. Once the aluminum rod has reached the steady state, remove both the melting ice box and boiling water bath and isolate the system. Again, initially you will notice the variation of temperature along the length of the rod. However, after a sufficiently long time, you will notice no change in the temperature at any location of the rod. At this situation, the system is said to have reached a state of equilibrium.

Broadly, there are three types of equilibriums that we will consider in thermodynamics, viz (i) Chemical equilibrium, (ii) Mechanical equilibrium and (iii) Thermal equilibrium

1.9.1 Chemical Equilibrium

If there is no transfer of matter from one part of the system to another or if there is no chemical reaction taking place, then the system is said to be in a state of chemical equilibrium. During chemical equilibrium, chemical composition of the system does not change.

1.9.2 Mechanical Equilibrium

If there is no force imbalance within the system and also between the system and surrounding, then the system is said to be in mechanical equilibrium. For example, if there is no change in pressure at any point in a system with time then the system is said to be in mechanical equilibrium. However, if there is any imbalance within the system or between the system and surroundings, then the system will undergo a change of state until mechanical equilibrium is reached.

1.9.3 Thermal Equilibrium

Consider a system which is in chemical and mechanical equilibrium. Assume that it is separated from its surrounding by a wall which is diathermic. If there is no spontaneous change in any of its properties, then the system is said to be in thermal equilibrium. In short, a system is in thermal equilibrium if the temperature is the same throughout the entire system.

A system is said to be in thermodynamic equilibrium when all the above three equilibrium conditions are simultaneously satisfied.

1.10 THE CONCEPT OF CONTINUUM

In engineering thermodynamics, focus lies upon the gross behaviour of the system. We postulate that the matter in our system or control volume is a continuum. The question is, what is continuum? Let us elaborate. The concept of continuum is *just the idea* in macroscopic approach of thermodynamics. It says that do not look too closely at a substance. If we agree to this, then we can assume that the substance does not behave as if it were made up of individual molecules and atoms in random motion. The entire volume of matter can be assigned with averaged properties of a very large number of individual molecules. This implies that we cannot look at a volume of a substance smaller than, say 100 micrometres.

In macroscopic approach of thermodynamics, the substance is considered to be continuous. It means that the material is continuously distributed throughout the region of interest. Such a postulate allows us to describe a system or control volume using only a few measurable properties. When we want to understand the concept of continuum, we should disregard the atomic nature of a substance. We should view it as a continuous and homogeneous matter without any discontinuity. The concept of continuum is thus a *convenient fiction* which remains valid for most of engineering problems where only macroscopic or phenomenological information is desired.

To understand the concept, let us refer Fig.1.11(a). All of us know that matter is made up of atoms. In gas phase, such atoms are widely spaced. To appreciate the distance involved at the molecular level, consider a container filled with oxygen at atmospheric conditions. The diameter of the oxygen molecule is about 3×10^{-10} m and its mass is 5.3×10^{-26} kg. Also, the mean free path of oxygen at 1 atm pressure and 20 °C is 6.3×10^{-8} m. That is, an oxygen molecule travels, on an average, a distance of 6.3×10^{-8} m (about 200 times of its diameter) before it collides with another molecule. Also, there are about 3×10^{16} molecules of oxygen in the tiny volume of 1 mm^3 at 1 atm pressure and 20 °C [Fig.1.11(a)]. The continuum model is applicable as long as the characteristic length of the system (such as its diameter) is much larger than the mean free path of the molecules. At very high vacuums or very high elevations, the mean free path may become large (for example, it is about 0.1 m for atmospheric air at an elevation of 100 km). For such cases, the rarefied gas flow theory should be used and the impact of individual molecules should be considered. In this book, we will limit our consideration to substances that can be modelled as a continuum.

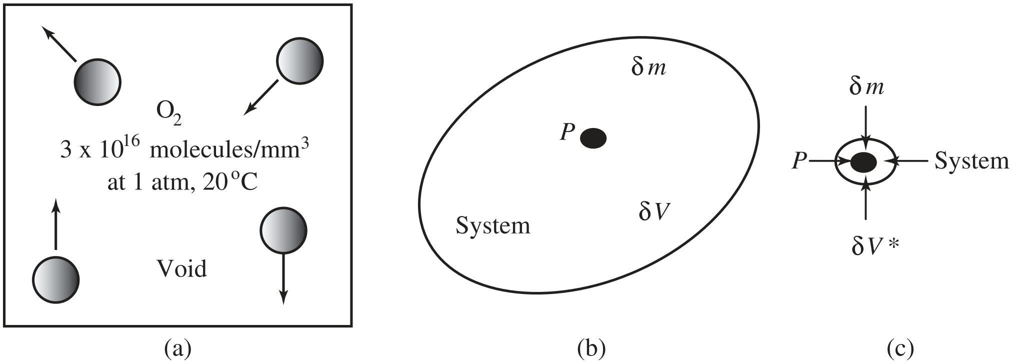


Fig. 1.11 The concept of continuum

Now, let us consider a system shown in Fig.1.11(b). Let the mass be δm and volume be δV, surrounding a point P. The ratio $\dfrac{\delta m}{\delta V}$ is called the average mass density of the system. Initially, assume that δV is quite large. Now, start shrinking the volume δV around point P. When δV becomes so small as δV^* [Fig.1.11(c)], there will be relatively very few molecules. Then, the average density start fluctuating substantially with time. The molecules will move into and out of the volume in random motion. At this point, it is impossible to assign a definite value for $\delta m/\delta V$. Let the smallest volume until which there is no oscillation be designated as δV^* as shown in Fig.1.12. If you further start shrinking the volume, $\delta m/\delta V$ starts oscillating. Mathematically, the density, ρ, of the system at a point can be defined as

$$\rho = \lim_{\delta V \to \delta V^*} \frac{\delta m}{\delta V}$$

It may be noted that, in most engineering applications, the assumption of continuum is valid. Therefore, it is convenient to study thermodynamics from the macroscopic point of view which is the subject matter of this book. The continuum idealisation allows us to treat properties as *point functions*. It means that the properties vary continually in space with no jump discontinuities.

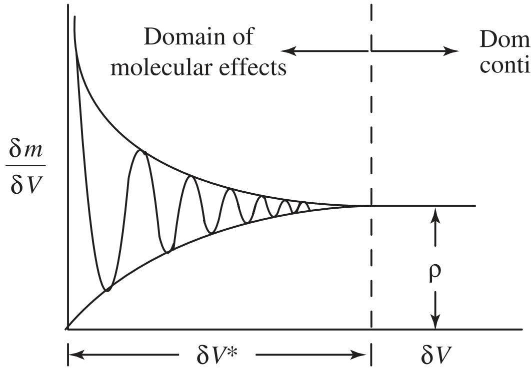


Fig. 1.12 Definition of density as the macroscopic property

The above idealisation holds good as long as the size of the system we deal with is large relative to the space between the molecules. In other words, continuum refers to the size of a system. The size should be more than the mean free path of the molecules. What is important to note is that a system can be treated as continuum if it were to contain a large number of particles to make the statistical averages meaningful. It is the general practice to consider a system as a continuum if its size to be greater than either 10 times the mean free path or 10 times the mean molecular path. Practically this is the case in all engineering problems, except some specialised ones. The continuum idealisation is implicit in many statements we make. For example, the assumption that the density of water in a glass is the same at all points.

As already stated, this viewpoint can be inaccurate for rarefied gases because molecules are so few and far between. Therefore, the random behaviour of individual molecules significantly affects the properties of the substance. If you consider a relatively small number of molecules, the average properties you observe may not be the true average properties for the substance.

1.11 FORCE, PRESSURE, VOLUME AND TEMPERATURE

Many properties exist to describe substances in thermodynamics. A basic understanding of theory is required for their definition and application. A detailed discussion on some important properties such as force, pressure, volume and temperature will be introduced now.

Force

The concept of force was originally defined by Sir Isaac Newton in his three laws of motion. He explained gravity as an attractive force between bodies that possessed mass. Force is a vector having both magnitude and direction. A force is anything that changes an object's state of rest or motion, a push or pull of an object. In other words, a force (F) is any influence that causes an object to undergo a change in speed or a change in direction or a change in shape. The unit of force is newton (N). *A force of one newton produces an acceleration of one* m/s^2 *when applied to a mass of one kg.*

The weight of a body (W) is the force with which the body is attracted towards the centre of the earth. It is the product of the mass (m) and the local acceleration due to gravity (g). The value of g, at sea level is 9.80665 m/s^2. For all practical purposes, it can be approximated to 9.81 m/s^2. The mass of the substance remains constant with altitude where as the weight varies with altitude because of change in gravitational force.

1.12 PRESSURE

Pressure is defined as a normal force exerted by a fluid per unit area. In this section, we will discuss absolute/gauge pressures, variation of pressure with depth, barometer, manometers and pressure gauges.

Pressures are exerted by gases, vapours and liquids. You must understand the concept of pressure correctly, since it is one of the very important properties quite often used in thermodynamics. The counterpart of pressure in solids is called stress. Mathematically,

$$p = \frac{F}{A}$$

where p is the pressure, F is the normal force and A is the area on which the force is acting. The unit of pressure is N/m^2 which is equal to the force of one newton acting on one square meter area. It is named as Pascal (Pa).

$$1 \text{ N/m}^2 = 1 \text{ Pa}$$

For engineering applications, the unit of pascal is very small. Hence, we use quite often *kilopascal* (kPa) and *megapascal* (MPa).

$$1 \text{ kPa} = 10^3 \text{ Pa} \quad \text{and} \quad 1 \text{ MPa} = 10^6 \text{ Pa}$$

Pressure can also be expressed in terms of *atmosphere* (atm) and *bar*.

$$\begin{aligned}
1 \text{ bar} &= 10^5 \text{ Pa} = 100 \text{ kPa} = 0.1 \text{ MPa} \\
1 \text{ atm} &= 101327 \text{ Pa} = 101.327 \text{ kPa} = 1.01327 \text{ bar} \\
1 \text{ kgf/cm}^2 &= 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ Pa} \\
&= 9.807 \times 10^4 \text{ N/m}^2 = 0.9807 \text{ bar} = 0.9679 \text{ atm}
\end{aligned}$$

Pressure as a Function of Depth

Consider a homogeneous fluid having a uniform density, ρ, in static equilibrium in a cylindrical container having an area A as shown in Fig.1.13. The fluid will exert the hydrostatic pressure due to the weight of the fluid. The pressure exerted by the column height h of the fluid can be obtained by equating the force to the weight of the fluid column above. Referring to Fig.1.13:

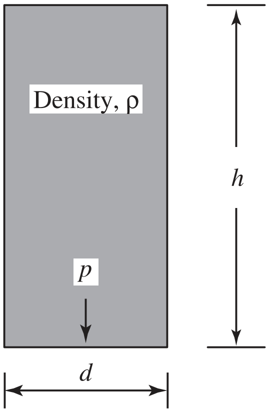
$$\begin{aligned}
\text{Volume, } V &= Ah \\
\text{Mass, } m &= \rho V = \rho A h
\end{aligned}$$

Note that the weight of the fluid is exerting a force. This force, F, is given by,

$$\text{Force, } F = mg = \rho A h g$$

$$\text{Pressure} = \frac{F}{A} = \rho A h \frac{g}{A} = \rho g h$$

Note that the hydrostatic pressure will be same at all point in a horizontal plane. *It varies only with height*. Therefore, the *height of the fluid column can be taken as another unit* to represent pressure. The pressure exerted by one mm column of mercury (Hg) is called torr.



Area, $A = (\pi/4)\, d^2$

Fig. 1.13 Homogeneous fluid having uniform density

$$1 \text{ mm Hg} = 1 \text{ torr} = 133.322 \text{ Pa}$$

The pressure at a point in a fluid *(liquid)* increases with depth due to the increase in weight of the fluid. However, the pressure in a gas tank can be considered to be more or less uniform since the weight of the gas is too small to cause any significant difference. The pressure of the fluid can be measured by a pressure gauge. The most common instruments used are either a manometer or a Bourdon pressure gauge. They, however, record pressure as the *difference between two pressures.*

1.13 TYPES OF PRESSURE

We will come across four types of pressure in thermodynamics. They are
(i) Atmospheric pressure (ii) Absolute pressure; (iii) Gauge pressure; and (iv) Vacuum pressure. The pictorial representation of above pressures is shown in Fig.1.14.

1.13.1 Atmospheric Pressure

It is defined as the pressure exerted by the atmosphere. It is said to be equal to 760 mm (0.76 m) of mercury column at 0 °C for mercury density of 13.5951 kg/m^3, with the gravitational acceleration of 9.80665 m/s^2 and has magnitude of 1.0132 bar (= 1.0132 $\times 10^5$ N/m^2).

The instrument used for measuring atmospheric pressure is called barometer. Italian scientist Torricelli was the first to construct the barometer to measure the pressure. In his honour, the pressure exerted by one millimeter column of mercury under atmospheric conditions is known as Torr (1 atm = 760 Torr).

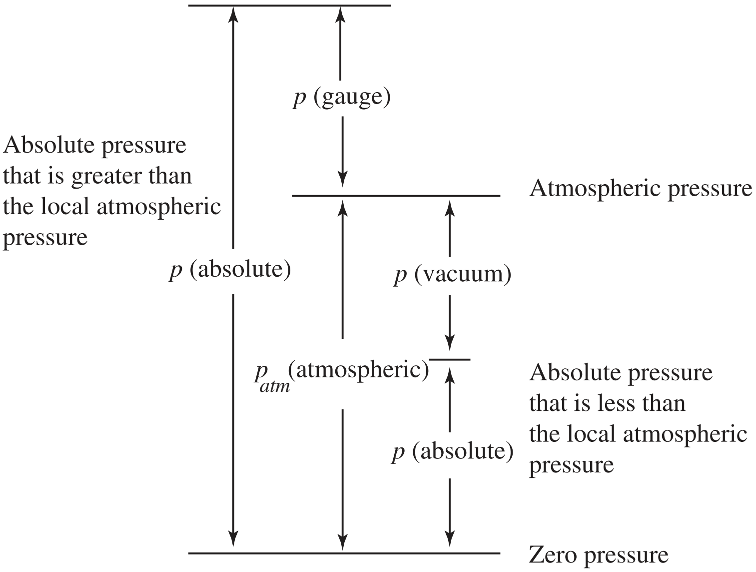


Fig. 1.14 Pictorial representation of absolute, gauge and vacuum pressure

1.13.2 Absolute Pressure

The actual pressure at a given position which is greater than the local atmospheric pressure is called the *absolute pressure* (p_{abs}). It is measured *relative to absolute vacuum, i.e. absolute zero pressure* (refer Fig.1.14).

1.13.3 Gauge Pressure

When the absolute pressure of a system is greater than the atmospheric pressure (p_{atm}), the difference between the absolute pressure and the atmospheric pressure is called gauge pressure (p_{gauge}). This is because, a gauge connected to the system will read this pressure. Hence,

$$p_{gauge} = p_{abs} - p_{atm}$$

1.13.4 Vacuum Pressure

For pressures below atmospheric pressure, the gauge pressure will be negative and is called sub-atmospheric pressure. If the absolute pressure of the system is less than the atmospheric pressure then the difference between the atmospheric pressure and the absolute pressure is called the vacuum pressure (p_{vac}). Mathematically,

$$p_{vac} = p_{atm} - p_{abs}$$

1.13.5 Practical Units of Pressure

By now you are aware that the unit for pressure is Pascal. As already mentioned a Pascal which is 1 N/m^2 is too small an unit for engineering applications. Therefore, a several other bigger units are used. They are listed in Table 1.3.

Table 1.3 Various units of pressure

Unit of pressure	Abbreviation	Equivalent N/m^2
pascal	Pa	1
kilopascal	kPa	10^3
megapascal	MPa	10^6
bar	Bar	10^5
atmosphere	Atm	1.0132×10^5

1.14 PRESSURE MEASURING DEVICES

Pressure is usually measured either by indicating gauges or recorders. These instruments may be mechanical, electro-mechanical, electrical or electronic in operation.

1.14.1 Mechanical-type Instruments for Measuring Pressure

The mechanical-type instruments are classified as
(i) barometer; (ii) manometer and (iii) pressure gauge.

1.14.2 Barometer

Barometer is a device which is used to measure *atmospheric pressure.* Therefore, atmospheric pressure is also called barometric pressure. Barometer was invented by Torricelli in seventeenth century and hence, it is usually called Torricelli Barometer. The construction details are as follows.

A typical barometer is shown in Fig. 1.15 (a). Initially, a long glass tube is filled with mercury to the brim. It is then turned upside down keeping the open end temporarily closed, so that mercury does not spill out. The temporarily closed end of the tube is then dipped in pool of mercury and then opened holding the tube in vertical position. Some mercury in the tube will slide into the pool and the height h of the mercury in the vertical tube stabilises [Refer Fig.1.15(a)]. A small length of the tube in the top portion, above the mercury meniscus, which appears empty, is under absolute vacuum. Therefore, the pressure at point 2 can be taken as zero. The pressure at point 1 is atmospheric since the mercury pool in the tub is open to atmosphere. Now, the height between 1 and 2 is a measure of atmospheric pressure. Force balance in the vertical direction gives

$$p_{atm} = \rho g h$$

where ρ is the density of the fluid (here Hg) and g is local gravitational acceleration. What is to be noted is that the length or the cross-sectional area of the tube [Fig.1.15(b)] has no effect on the height of the fluid column of a barometer. However, the tube diameter must be large enough to avoid surface tension (capillary) effects. The measuring column will reach the same height in all the tubes as shown in Fig.1.15(b).

1.14.3 Manometers

Small and medium pressures are generally measured by *manometers* which employ liquid columns. Their use for such purposes are *quite accurate.* The liquids most commonly used in manometers are mercury and water. *Mercury is used for the measurement of medium pressures and water for small pressures.* A low viscosity liquid will be highly suitable because it can

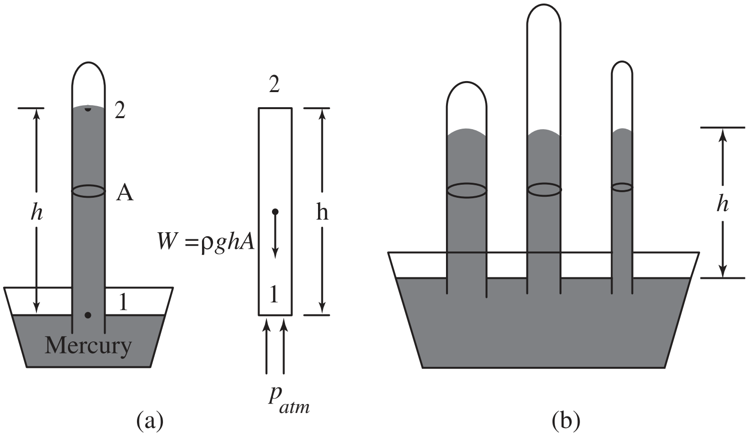


Fig. 1.15 Barometer

adjust itself quickly. Further, it must have a low coefficient of thermal expansion, so that density changes with temperature are minimum. Manometers are further divided into

(i) U-tube manometer, (ii) Cistern manometer and (iii) Micro-manometer.

1.14.4 U-tube Manometer

An U-tube manometer is in the form of 'U' and is made of glass [Fig.1.16 (a)]. When no pressure is applied, the height of the liquid in the two legs will be the same. It is normally adjusted to zero in the scale. When a pressure which is above atmospheric is applied to one leg (say left leg), keeping the other open to the atmosphere, the liquid in the left leg will go down and will rise in the right leg [Fig.1.16(a)]. As the right leg is open to the atmosphere, the pressure on this side will be atmospheric. As can be seen, the pressure of the gas in the tank which is connected to right leg has pushed down the manometric liquid to a height, h. Now, the pressure applied to the left leg can be calculated using the force balance as follows:

$$p_{abs}A = W + p_{atm}A \tag{1.1}$$

where A is the cross-sectional area of the tube and W is the weight of the fluid column h in the manometer. The weight, W is given by:

$$W = mg = \rho V g = \rho A h g \tag{1.2}$$

where V is the volume of the fluid column h and ρ is the manometric fluid density. Thus,

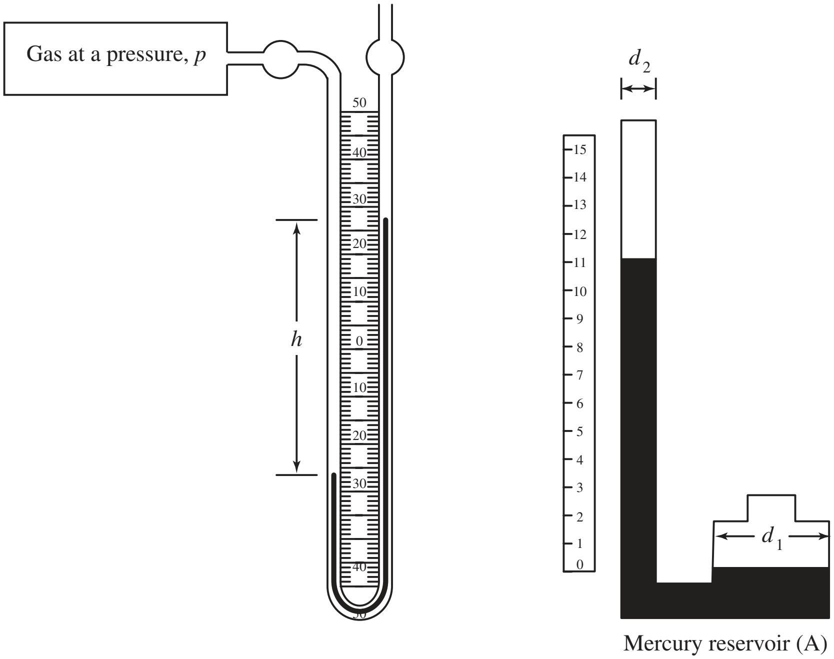
$$p_{abs}A = \rho A h g + p_{\text{atm}}A$$

$$p_{abs} = \rho g h + p_{\text{atm}}$$

The pressure difference, Δp, between the tank pressure and the atmospheric pressure can be expressed as

$$\Delta p = p_{abs} - p_{atm} = \rho g h \tag{1.3}$$

If the manometer is connected to a tank whose pressure is less than the atmospheric pressure then Δp, will be given by,



(a) U-tube manometer (b) Cistern manometer

Fig. 1.16 Principle of U-tube manometer

$$\Delta p \quad = \quad p_{atm} - p_{abs} = \rho g h$$

Note that the unit of Δp will be in N/m^2 (pascal), ρ will be kg/m^3, g will be m/s^2 and h will be in m. The U-tube manometer shown in Fig. 1.16 (a) is of the simplest form. However, readings have to be taken at two different places to arrive at h. Further, the *deflection of the two columns may not be the same.* To avoid this difficulty, cistern or well type manometer was developed.

1.14.5 Cistern Manometer

Figure 1.16(b) shows a typical cistern manometer. It is quite similar to U-tube manometer. The mercury reservoir A having a diameter, d_1, is made large enough so that change of level in the reservoir is negligible. This form of manometer is generally used for measuring pressures above atmospheric level. In this case, *only one reading of the level in the column is required.* However, accurate zero setting is absolutely essential.

1.14.6 Micro-manometer or Inclined Tube Manometer

The U-tube manometer discussed in previous sections is not suitable for measuring very low pressures. Therefore, for such purposes, precision type manometers are required. They are called *micro-manometers.* Sometimes it is called *multiplying manometer* because they multiply the movement of the level of the liquid. By far, the most widely used type of multiplying manometer is the *inclined tube manometer.* A typical inclined tube manometer is shown in Fig.1.17. The tube is inclined at an angle of θ, as shown in Fig.1.17. This helps to increase the sensitivity of the manometer compared to keeping it vertical. Inclined tube *causes a* *larger displacement of the liquid along the tube for a given pressure difference.* The measuring principle of the manometer can be explained with the help of Fig.1.17.

Initially, the pressure p_1 in the limb 1 and the pressure p_2 in the limb 2 are same as they are open to atmosphere. Then, the level of the liquid would be as shown by the dotted line XY (Fig.1.17). However, when p_1 is slightly greater than p_2, the level in the reservoir goes down by a height, h_1 as shown in the figure. At the same time, level in the inclined tube will rise by a greater height, h_2 as shown in Fig.1.17. If h is the vertical distance between the two surfaces due to difference of pressure, then

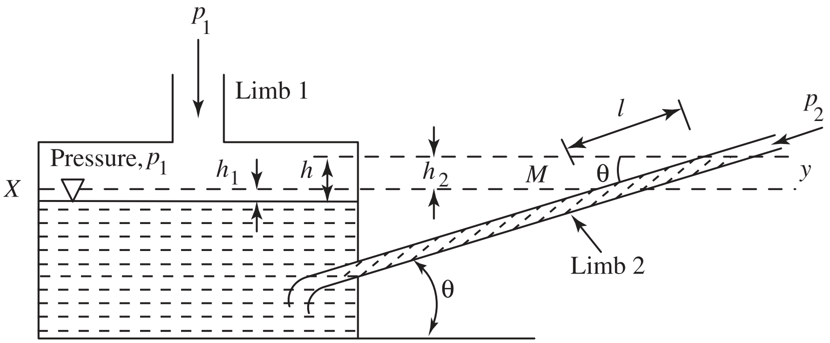


Fig. 1.17 Principle of inclined manometer

$$h = h_1 + h_2$$

$$h_2 = \ell \sin\theta$$

where ℓ is the distance moved by the manometric fluid in the inclined tube (Fig.1.17) and

$$h_1 \times A = \ell \times a$$

$$h_1 = \ell \times \frac{a}{A}$$

where A is the area of cross-section of the reservoir and a is the area of cross-section of the inclined tube. Let the pressure difference, $(p_1 - p_2)$, is denoted by Δp:

$$\Delta p = hw = (h_1 + h_2)w = \left(\ell\frac{a}{A} + \ell\sin\theta\right)w$$

where w is the specific weight of the liquid in the manometer.

$$h = d\left(\sin\theta + \frac{a}{A}\right)$$

$$\frac{h}{\ell} = \left(\sin\theta + \frac{a}{A}\right) \qquad (1.4)$$

The sensitivity of the instrument can be varied by *changing the slope of the inclined tube.* Usually the position of the inclined tube is arranged in such a way that $\left(\sin\theta + \frac{a}{A}\right)$ is an integer. The multiplication factor of the gauge is given by

$$\frac{\ell}{h} = \frac{1}{\sin\theta + \frac{a}{A}}$$

Thus, the multiplication factor depends on θ and $\frac{a}{A}$. The smaller the value of θ and $\frac{a}{A}$ the *greater is the multiplication factor.*

1.14.7 Advantages and Disadvantages of Manometers

There are certain advantages as well as disadvantages in using manometers.
Advantages:

(i) Easy to fabricate,

(ii) Good accuracy and sensitivity,

(iii) Particularly suitable for measuring low pressures and low differential pressures,

(iv) Relatively inexpensive and requires very little maintenance and

(v) Sensitivity can be changed by changing manometric fluids.

Disadvantages:

(i) Calibration is affected by changes in gravitational force, density of fluids, altitude and temperature,

(ii) Determination of exact meniscus height is a problem,

(iii) Large, bulky and fragile,

(iv) Not suitable for recording, and

(v) Surface tension of manometric fluid may create a capillary effect and possible hysteresis.

In order to obtain accurate measurement there are certain desirable properties of manometric liquid used. They are:

(i) Chemically inert,

(ii) Clear visible interface,

(iii) Low freezing and high boiling point,

(iv) Low surface tension and non-wetting characteristics and

(v) Stability of density in the temperature range of interest.

1.15 PRESSURE GAUGES

For medium and high pressures, manometers become quite bulky and heavy as they are filled with heavy liquids like mercury. Therefore, for measuring such pressures, elastic pressure gauges are normally used. These gauges employ different forms of elastic systems such as tubes, diaphragms or bellows. The elastic deformation property of these elements is used to measure the pressure. Since, these elements are deformed only within the elastic limit, they are called elastic gauges. They are also called secondary instruments. They must be calibrated with primary instruments such as a manometer or other standard pressure measuring devices. Some of the most commonly used pressure gauges are discussed in the following sections. They can be classified as:

(i) Bourdon pressure gauge (ii) Diaphragm gauge (iii) Vacuum gauge.

1.15.1 Bourdon Pressure Gauge

Bourdon pressure gauge can be used for measuring both *higher* and *lower pressures*. Bourdon tube was invented by E. Bourdon in 1852. The principle of Bourdon gauge can be explained as follows. Consider a tube having internal cross-section that is *not a perfect circle*. If it is bent or distorted by means of internal pressure variation then there will be a *free end deflection* of the tube. This deflection can be used to measure the change in pressure inside it.

Construction and working of Bourdon gauge: Bourdon pressure gauges use different types of Bourdon springs. A typical C-shaped Bourdon tube, formed by winding the tube to form a segment of a circle having arc length of about 270 degrees is shown in Fig.1.18. As can be seen, a curved tube having an elliptical cross-section with one end [A] is attached to the system whose pressure is to be measured. The other end [B] is connected to a pointer by a mechanism shown in the figure. When fluid under pressure fills the tube, the internal pressure makes the tube to expand. This causes circumferential stress, i.e. hoop tension to set up. Because of this, the elliptical section tends to become circular and the tube tries to straighten itself. This motion is transmitted by a suitable mechanism (Fig.1.18) to the pointer. The mechanism converts the linear motion of the tip into proportional rotary motion of the pointer. A hair spring is usually employed to eliminate any backlash. The deflection of the pointer can be calibrated for known pressures. A graduated scale in suitable units is usually fixed from which any applied pressure can be read. Note that the Bourdon tube measures the pressure relative to the pressure of the surroundings. Accordingly, the dial will read zero when the pressures are the same both at inside and outside of the tube.

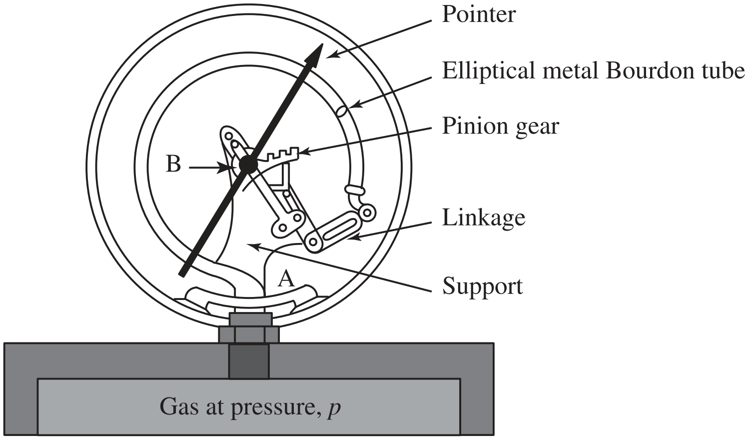


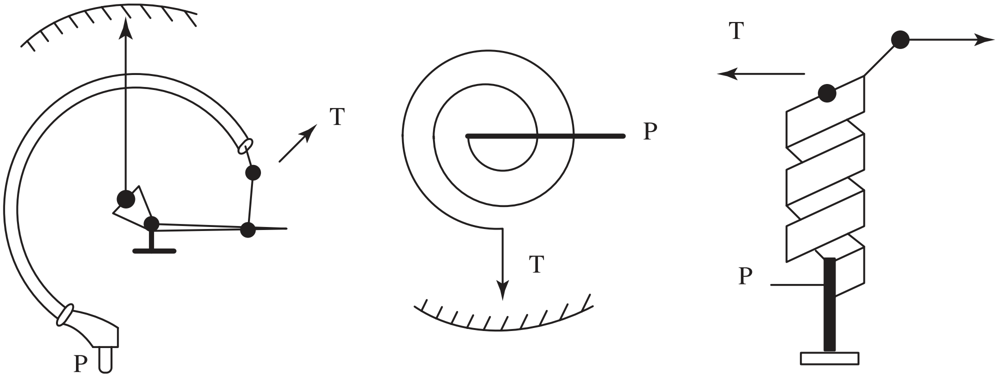
Fig. 1.18 Schematic diagram of a Bourdon gauge

A simplified sketch of C-type Bourdon gauge is shown in Fig.1.19(a). Typical pressure ranges that can be measured using this gauge are given in Table 1.4. There are other types of gauges such as spiral and helix. In spiral type, number of turns is wound in the shape of a spiral about a common axis. In helix type, a number of turns are wound to form a helix. Typical spiral and helix gauges are shown in Figs.1.19(b) and (c), respectively, in which 'P' indicates direction of application of pressure, while 'T' indicates tip travel for rise in pressure. As helical and spiral types bourdon tubes have many number of turns and hence, the movements for given change in pressure is much larger than that of a single turn C shaped tube.

Bourdon tube material: A bourdon tube is usually made up of such metals or alloys that

Table 1.4 Normal pressure range

C-shaped tube	0 to 700	MPa
Gauge pressure	0 to 100	MPa
Absolute pressure	0 to 1	MPa
Vacuum pressure	0 to 1000	mm Hg



(a) C-shaped (b) Spiral type (c) Helix type

Fig. 1.19 Types of gauges

exhibit satisfactory elastic properties. Typical materials used are: beryllium, brass, bronze, copper, phosphor monel, stainless steel and so on.

Gauge pressure: When the bourdon tube is connected to unknown pressure and its outside is exposed to atmosphere, the reading would be in gauge units and is called gauge pressure.

Absolute pressure: When the bourdon tube is connected to unknown pressure and its outside (instrument case) is evacuated, then reading would be in absolute units.

Vacuum pressure: It is similar to gauge pressure measurement. However, the accuracy is very poor. Compound Bourdon tube are employed for measuring pressures both above and below atmospheric. Double Bourdon tube is used where severe vibrations are encountered.

Calibration: Bourdon gauge are usually calibrated using dead weight tester or by comparison calibration.

1.15.2 Advantages and Limitations

Advantages:

(i) good accuracy
(ii) low cost
(iii) simple in construction
(iv) wide range of pressure measurement

Limitations:

(i) low spring gradient
(ii) susceptible to shock and vibration
(iii) Bourdon tube material exhibits some hysteresis in a pressure cycle

1.15.3 Diaphragm Gauge

This type of gauge employs a metallic disc or diaphragm instead of a bent tube used in Bourdon gauge. Disc or diaphragm is used for actuating the indicating device, namely the pointer (Fig.1.20). When the pressure of a fluid is applied on the lower side of the diaphragm, elastic deflection of the diaphragm takes place. The amount of deflection is proportional to the pressure applied on the diaphragm. The movement of the diaphragm is transmitted to a rack and pinion. This in turn causes the linear displacement of a linkage rod attached to the internal side of the diaphragm. The linkage rod further translates the linear movement of the gauge's pointer into angular movement by a series of gears. The latter is attached to the spindle needle, moving on a graduated dial. The dial can be graduated conveniently on a suitable scale. The diaphragm also serves to isolate the fluid from the internals of the gauge. Therefore, diaphragm-type pressure gauges are better suited for use on most fluids.

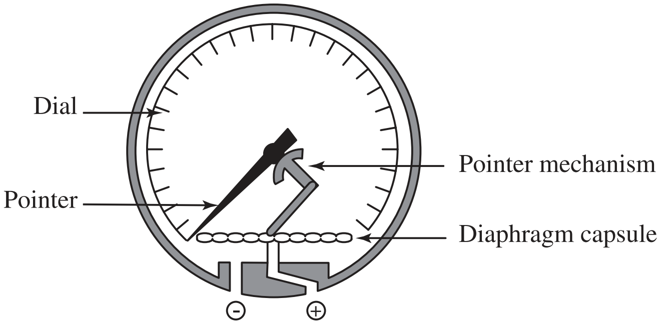


Fig. 1.20 Schematic diagram of a diaphragm pressure gauge

1.15.4 Vacuum Gauge

Bourdon gauges discussed in the earlier section can be used even to measure vacuum. Only slight changes in the design are required for this purpose. In this case, the tube is bent inwards instead of outward as in pressure gauge. Vacuum gauges are graduated in millimetres of mercury below atmospheric pressure. Accordingly, absolute pressure in millimetres of mercury is the difference between barometer (atmospheric pressure) reading and vacuum gauge reading. Vacuum gauges are normally used to measure the vacuum in condensers. One must be very careful in avoiding leak, otherwise, the vacuum will drop. While installing the vacuum gauge, the following points are to be taken care of:

(i) Flexible copper tubing and compression fittings are to be used during installations.

(ii) The installation of a gauge cock and tee ('T') in the line close to the gauge should be provided. This helps in testing or replacement without shutting down the system.

(iii) The gauge and its connecting line should be filled with an inert liquid. Further, liquid seals should be provided. It is to be noted that trapped air at any point in the gauge lines can cause serious errors in pressure reading.

1.15.5 Modern Method of Pressure Measurement

There are modern methods by which pressure can be measured nowadays. They use sensors. An important class of sensors utilise the *piezoelectric effect*.

Piezoelectric effect: A charge is generated within certain solid materials when they are deformed. This mechanical input (say pressure) is converted to electrical output in piezoelectric materials. This forms the basis for pressure measurement as well as displacement and force measurements. Another important type of sensor employs a diaphragm that deflects when a force is applied, altering an inductance, resistance or capacitance. Figure 1.21 shows a piezoelectric pressure sensor together with an automatic data acquisition system.

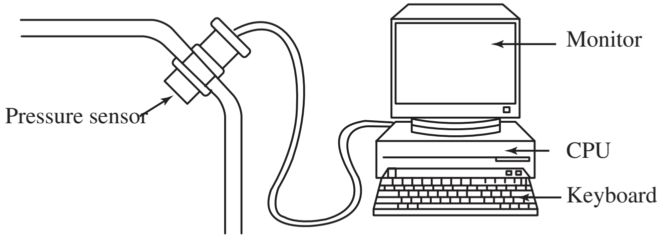


Fig. 1.21 Modern pressure measuring system

1.16 VOLUME, DENSITY, SPECIFIC VOLUME, SPECIFIC GRAVITY AND SPECIFIC WEIGHT

Volume is one of the important extensive properties we quite often come across in thermodynamics. In this section, we will discuss the details of volume, density, specific volume, specific gravity and specific weight as they are most often used in the analysis.

1.16.1 Volume

In thermodynamic systems, volume normally means the volume of the working fluid. It describes its thermodynamic state. A typical example is the working fluid within a cylinder and piston arrangement of an internal combustion engine as shown in Fig.1.22. Volume is a function of state and is *interdependent* with other thermodynamic properties, namely pressure and temperature. For example, volume of an ideal gas is related to the pressure and temperature by the ideal gas law. The physical volume of a system may or may not coincide with a control volume used to analyse the system. A typical volume measuring cup for liquids is shown in Fig.1.23.

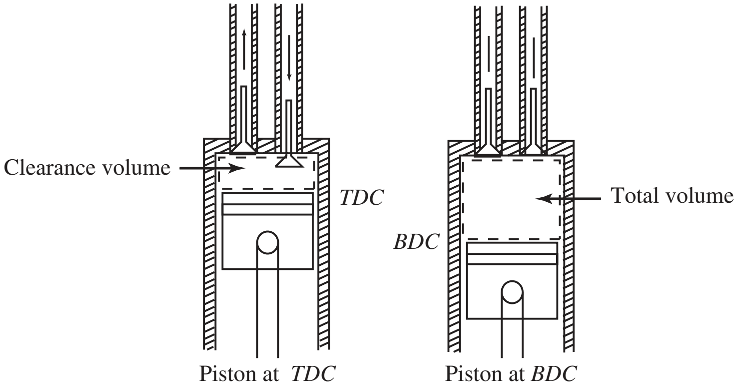


Fig. 1.22 Cylinder-piston arrangement

1.16.2 Density

It is the amount of matter contained in a certain volume of space. It is defined as *mass per unit volume*. In other words, it is the mass of the object divided by the volume of the object. Mathematically, density, ρ, is written as $\rho = \dfrac{m}{V}$. It is normally expressed in grams (or kilograms) per cubic centimeter (or cubic meter or liter). Consider the cylinder-shaped regions of space shown in Fig.1.24.

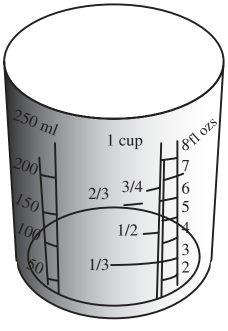


Fig. 1.23 Volume measuring cup

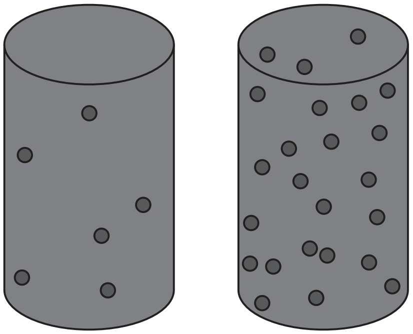


Fig. 1.24 Illustration of density

As can be seen that the right-side volume contains four times as many particles as the one on the left side. Therefore, it has four times the mass and therefore, four times the density since the volume is the same. An object that is very dense is usually very heavy for its size. For example, a cricket ball is more dense than a tennis ball. Essentially, the more mass you pack in to a given volume, the higher will be the density.

While density is calculated as mass per volume, it can depend on temperature and pressure. This is mostly the case for *gases*, such as our atmosphere. Higher temperatures cause the gas particles to move around more quickly. This causes larger spaces between molecules. Therefore, the same mass takes up a larger volume of space, causing a lower density. Higher pressures can push molecules together, making the space between particles smaller. Therefore, the same mass takes up a smaller volume of space, causing a higher density. Liquids and solids, on the other hand, are essentially incompressible substances and the variation of their density with pressure is usually negligible. Therefore, the density of liquids and solids are assumed to be constant for all practical purposes.

1.16.3 Specific Volume

By now you are aware that mass and volume are directly related to each other under static conditions. Now, we will define a new property called the specific volume. The specific volume of a substance is defined as the volume per unit mass and is designated by the symbol v. Note that the specific volume is an intensive property. It is the reciprocal of density. Mathematically, the specific volume v, is given by:

$$v = \frac{V}{m} = \frac{1}{\rho}$$

The specific volume of a system in a gravitational field may vary from point to point. For example, if the atmosphere is considered a system, the specific volume increases with altitude. Therefore, the definition of specific volume involves the specific volume of a substance at a point in a system.

Consider a small volume δV of a system and let the mass be designated δm. The specific volume is defined by the relation:

$$
\begin{aligned}
v &= \lim_{\delta V \to 0} \frac{\delta V}{\delta m} \\
\delta V &\to \delta V^*
\end{aligned}
$$

where δV^* is the smallest volume for which the principle of continuum can be applied. Volumes smaller than this will lead to the condition where mass is not evenly distributed in space but is concentrated in particles as molecules, atoms, electrons, etc. This is tentatively indicated in Fig.1.25, where in the limit of a zero volume, the specific volume may be infinite. This is because as the volume does not contain any mass or infinitesimally small as in the case of the volume of a nucleus. Thus, in a given system, we should speak of the specific volume or density at a point in the system and recognise that this may vary with elevation. However, most of the systems that we consider in our analysis are relatively small and the change in specific volume with elevation is not very significant. Therefore, assumption of value of specific volume or density for the entire system under consideration will not cause any big error in the calculation.

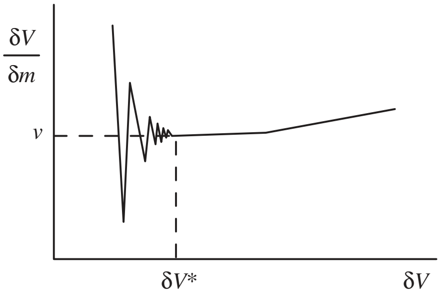


Fig. 1.25 The continuum limit for the specific volume

In this book, the specific volume and density will be given either on a mass or a mole basis. A bar over the symbol (lowercase) will be used to designate the property on a mole basis. Thus, $\overline{v}$ will designate molal specific volume and $\overline{\rho}$ will designate molal density. In SI units, specific volume can be given either as m^3/kg or m^3/mol or m^3/kmol. For density, the corresponding units are kg/m^3 and mol/m^3 or kmol/m^3. Although the SI unit for volume is cubic metre, a commonly used unit for volume is liter (ℓ), which is a special name given to a volume of 0.001 cubic metres, that is, $1\ \ell = 10^{-3}\ \text{m}^3$ or $1\ \text{m}^3 = 1000\ \ell$. Either the specific volume or the density can be used in defining the state of the gas using only intensive variables which means that it does not depend on the mass of the system. Let us illustrate this by means of a simple example as illustrated in Fig.1.26. Assume usual units.

The state of a gas is defined by various properties which we can observe with our senses, including the gas pressure (p), temperature (T), mass (m), number of moles n and volume (V) of the vessel which contains gas. It is observed that, if we have a certain amount (mass or volume) of gas present, the value of the temperature and pressure does not depend on the amount of gas which we examine. For example, suppose we have gas contained in gas tank A, if we insert a plate into the tank (gas tank B) which cuts the volume in half, the temperature in each half remains the same, as does the pressure. The value of pressure and temperature does not depend on the amount of gas used in the measurement. The mass of the gas, on the

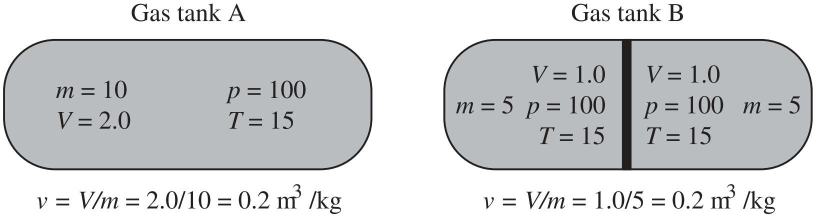


Fig. 1.26 Specific volume

other hand, does depend on the volume. Cutting the volume into two, cuts the mass also into two. The mass in each section of the tank is one-half the mass of the entire tank. The mass depends on the volume and in turn, the volume depends on the mass.

Consider maintaining the pressure and temperature of this gas and fill an object which can vary its volume, like a balloon or a cylinder with a sliding end. The final volume depends directly on the amount of the gas that we fill. Recall that properties which depend on the amount of gas are called extensive properties, while properties that do not depend on the amount of gas are called intensive properties. When performing a thermodynamic analysis, it is much easier to deal with intensive properties since we are able to eliminate the mass from the analysis.

With the numbers given in Fig.1.26, the specific volume of the gas in tank A is given by $v = V/m = 2.0/10 = 0.2\ \text{m}^3/\text{kg}$. If you partition the volume by a thin membrane, then the volume corresponding to the figure of the gas tank will be as shown in figure for gas tank B. Now, the specific volume of gas in tank B is $v = V/m = 1.0/5 = 0.2\ \text{m}^2/\text{kg}$. This shows that specific volume does not depend on the amount of gas. Note that for many fluid dynamic (moving) applications, the mass varies from one location to another and aerodynamicists normally use the density as the intensive property.

1.16.4 Specific Gravity

Sometimes the density of a substance is expressed relative to the density of a well-known substance. Then, it is called specific gravity or relative density. *It is defined as the ratio of the density of a substance to the density of some reference (standard) substance at a specified temperature.* Usually water at 4 °C, for which $\rho_{\text{H}_2\text{O}} = 1000\ \text{kg/m}^3$ is taken as reference subject. That is, specific gravity, SG, can be written as: $SG = \rho/\rho_{\text{H}_2\text{O}}$. Note that the specific gravity of a substance is a dimensionless quantity. Specific gravities of various substances at 0 °C are given in Table 1.5. Note that substances with specific gravities less than 1 are lighter than water and thus they would float on water. Typical example is ice floating on water.

Table 1.5 Specific gravities of various substances at 0 °C

Substance	SG	Substance	SG
Water	1.0	Blood	1.05
Seawater	1.025	Gasoline	0.7
Ethyl alcohol	0.79	Mercury	13.6
Wood	0.3-0.9	Gold	19.2
Bones	1.7-2.0	Ice	0.92
Air (at 1 atm)	0.0013		

1.16.5 Specific Weight

The weight of a unit volume of a substance is called specific weight or weight density. It is expressed as $\gamma_s = \rho g$, where g is the gravitational acceleration. The densities of liquids do not change much and therefore, specific weight are considered essentially constant. They are often approximated as being incompressible substances during most processes without sacrificing much of accuracy.

1.17 TEMPERATURE

A concept for temperature originates from the perceptions of our sense from the notion of *hotness* or *coldness* of a body. We use our sense of touch to distinguish hot bodies from cold bodies. However, sensitive, the human body may be, physiological sensation is not sufficient for the definition or evaluation of a scientific property, namely temperature. Temperature is actually a measure of molecular activity. The big question is how to define temperature? A satisfactory definition of temperature cannot be given at this stage. We will discuss the thermodynamic scale of temperature in Chapter 5 when we deal with second law of thermodynamics. However, a theoretical definition for temperature can be given as follows:

Temperature is a measure of the tendency of a body to spontaneously give up energy to its surroundings. When two bodies are in thermal contact, the one that tends to spontaneously lose energy is at higher temperature. Therefore, at this stage take, *temperature* as a property which is used for the quantification of energy interaction.

In classical thermodynamics, the quantities of interest are defined in terms of macroscopic observations only. Therefore, a definition of temperature using molecular (microscopic) measurements is not useful from the macroscopic point of view. Again, at the moment, take that *temperature is a thermal state of a body which distinguishes a hot body from a cold body.* On the basis of the relative degree of coldness/hotness concept, it is concluded that the absolute value of temperature is difficult to describe. Hence, it is the practice to make temperature estimations in reference to certain widely acceptable known thermal states of the substances.

Temperature is thus the intensive parameter and requires reference states. The acceptable known thermal states are such as the boiling point of water commonly called *steam point*, freezing point of water commonly called *ice point,* etc. These are taken as reference states since they are easily reproducible and universally acceptable. The temperature values assigned to them are called *reference temperatures.* Since, these reference points and reference temperatures maintain their constant value, they are also called *fixed points and fixed temperatures* , respectively. A list of these fixed points is given in Table 1.6.

Instruments for measuring lower temperatures are known as *thermometers* and for measuring high temperatures are known as *pyrometers.* It has been found that a gas will occupy zero volume at a certain temperature. This temperature is known as *absolute zero temperature.* The temperatures measured with respect to absolute zero temperature are called *absolute temperatures.* Absolute temperature is stated in degrees centigrade. The point of absolute temperature is found to occur at 273.15 °C below the freezing point of water. Then, absolute temperature = thermometer reading in °C + 273.15. Absolute temperature is denoted by kelvin, K in SI unit.

Equality of Temperature

By now we are aware that, although temperature is a familiar property, defining it exactly is difficult. For understanding the concept of temperature, we have to evolve the *concept of*

Table 1.6 Some fixed points used for international practical temperature scale

Reference State		Temperature °C
Gold point	(normal freezing point)	1064.43
Ice point		0
Neon point	(normal boiling point)	−246.05
Oxygen point	(normal boiling point)	−182.96
Silver point	(normal freezing point)	961.93
Steam point		100
Sulphur point	(normal boiling point)	444.60
Triple point of hydrogen		−259.34
Triple point of oxygen		−218.79
Triple point of water		0.01
Zinc point	(normal freezing point)	419.58

equality of temperature. We know that when a hot body and a cold body are brought into contact, the hot body becomes cooler and the cold body becomes warmer. If these bodies remain in contact for sufficiently long time, they usually appear to have the same hotness or coldness. However, we also realise that our sense of hotness or coldness is not so reliable. Sometimes, for somebody, cold bodies may seem hot. Further, bodies of different materials that are at the same temperature may appear to be at different temperatures due to sensory perception.

Now, let us consider two blocks of copper, one hot and the other cold. Let each one of them be in contact with mercury-in-glass thermometer. Let the two blocks be brought in thermal contact with each other and assume that the blocks are isolated from the surroundings. You will notice the mercury column of the thermometer in the hot block drops and in the cold block rises. After sufficiently long time, no further change in the mercury column height in both thermometers are observed and both show the same height. Therefore, we can conclude that the two bodies have reached *equality of temperature*. When this condition is reached, there will be no change in any observable property while they are kept in thermal contact over a period of time.

1.18 ZEROTH LAW OF THERMODYNAMICS

From the concept of equality of temperature, we can now enunciate a law which is called the zeroth law of thermodynamics. *Zeroth law of thermodynamics states that if two bodies are in thermal equilibrium with a third body separately then all the three bodies are in thermal equilibrium with each other.* It may look silly that such an obvious fact is called one of the basic laws of thermodynamics. It is because the above simple fact could not be concluded from the other laws of thermodynamics. Zeroth law serves as a basis for the validity of temperature measurement. If the third body we consider is a thermometer, the zeroth law can be restated as follows:

Two bodies are said to be in thermal equilibrium if both have the same temperature reading even if they are not in contact.

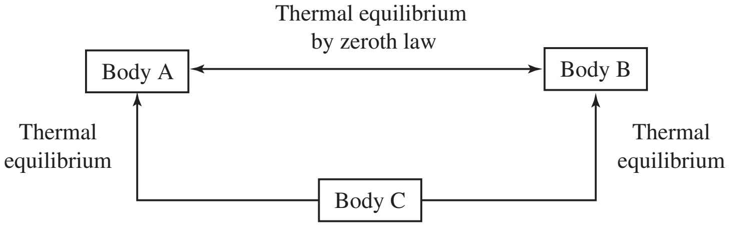
The zeroth law was first formulated and labelled by R. H. Fowler in 1931. It is named as the zeroth law since its value as a fundamental physical principle was recognised more than half a century after the formulation of the first and the second laws of thermodynamics. Therefore, it was designated as *zeroth law* so that it *precedes* the first and second laws to *form a logical sequence.*

Consider again the two blocks of copper and assume that the third block is a thermometer. Let one block of copper be brought into contact with the thermometer until equality of temperature is established and then remove it. Now, let the second block of copper be brought into contact with the same thermometer. If there is no change in the mercury level in the thermometer during this operation with the second block, then both blocks are said to be in thermal equilibrium with the given thermometer. Now, let us see what is meant by thermal equilibrium? Thermal equilibrium is a theoretical concept. It can be visualised in two ways:

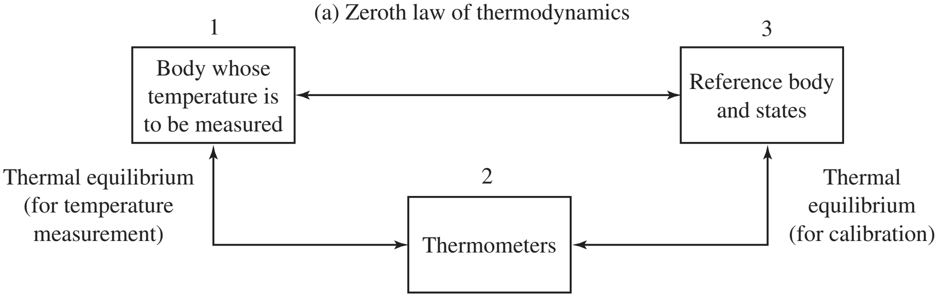
(i) Systems are said to be in thermal equilibrium if they are in a position to transfer heat between each other (for example by conduction or radiation) *but do not do so.*

(ii) Systems can also be said to be in thermal equilibrium if they are not in a position to transfer heat to each other but *would not do so even if they are able to.*

What it means is that all temperatures of interest are unchanging in time and space for all systems under consideration. This is the principle used in temperature measurement. Block diagrams shown in Fig.1.27(a) and (b) illustrate the zeroth law of thermodynamics and its application for temperature measurement, respectively. This may seem quite obvious to us because we are so familiar with this experiment. Thus, zeroth law can be thought of the basis for temperature measurement. It enables us to compare temperatures of two bodies '1' and '2' with the help of a third body '3'. In practice, body '2' in the zeroth law is called the *thermometer* (Fig.1.27). The word, thermometer, originates from two Greek words, *thermos* meaning *hot* and *metron* meaning *measure.*



(a) Zeroth law of thermodynamics



(b) Application of zeroth law of thermodynamics

Fig. 1.27 Zeroth law of thermodynamics

The thermometer is usually calibrated in terms of the height of mercury column. The height of mercury column in a thermometer, therefore, becomes a *thermometric property.* First, the thermometer is brought into thermal equilibrium with a set of reference temperatures [say, ice point and steam point, (refer Table 1.6)]. Let us call this reference body as of a body '3' and thus the thermometer gets calibrated. Later, when any other body, say, '1' is brought in thermal communication with the thermometer, we say that the body '1' has attained equality of temperature with the thermometer and hence, with body '2'. This way, the body '1' has the temperature of body '2' given for example by, say the height of mercury column in the thermometer '2'. Typical thermometric properties of various thermometers are given in Table 1.7.

Table 1.7 Thermometric property

Thermometer	Thermometric property
Alcohol or mercury-in-glass	Length (L)
Constant pressure gas	Volume (V)
Constant volumes gas	Pressure (p)
Electric resistance	Resistance (R)
Radiation (pyrometer)	Intensity of radiation (I or J)
Thermocouple	Electromotive force (E)

Every time a body has equality of temperature with the thermometer, we can say that the body has the temperature we read on the thermometer. The main problem to be addressed is that how to relate temperatures that we might read on different mercury thermometers or obtain from different temperature-measuring devices, such as thermocouple and resistance thermometers. Therefore, there is a need for a standard scale for temperature measurements.

1.19 TEMPERATURE SCALES

From time to time, a number of temperature measuring scales came up. The following section provides a comprehensive information regarding the different temperature scales used in thermometry. Different temperature scales have different names based on the names of persons who invented them and have different numerical values assigned to the reference states.

1.19.1 Celsius Scale or Centigrade Scale

Anders Celsius gave this Celsius or Centigrade scale using ice point of 0 °C as the lower fixed point and steam point of 100 °C as upper fixed point for developing the scale. It is denoted by letter C. Ice point refers to the temperature at which freezing of water takes place at standard atmospheric pressure. Steam point refers to the temperature of water at which its vaporisation takes place at standard atmospheric pressure. The interval between the two fixed points was equally divided into 100 equal parts and each part represents 1 °C or 1 degree Celsius.

1.19.2 Fahrenheit Scale

Fahrenheit gave a temperature scale known as Fahrenheit scale and has the lower fixed point as 32 °F and the upper fixed point as 212 °F. The interval between these two is equally divided into 180 parts. It is denoted by letter F. Each part represents 1 °F or 1 degree Fahrenheit.

1.19.3 Rankine Scale

Rankine scale was developed by William John MacQuorn Rankine, a Scottish engineer. It is denoted by letter R. It is related to Fahrenheit scale as shown below

$$T_R = T_F + 459.67$$

Normally it is taken as $T_R = T_F + 460$

1.19.4 Kelvin Scale

Kelvin scale proposed by Lord Kelvin is very commonly used in thermodynamic analysis. It also defines the absolute zero temperature. Zero degree Kelvin or absolute zero temperature is taken as −273.15 °C. It is denoted by letter K. Comparison of different temperature scales is illustrated in Fig.1.28.

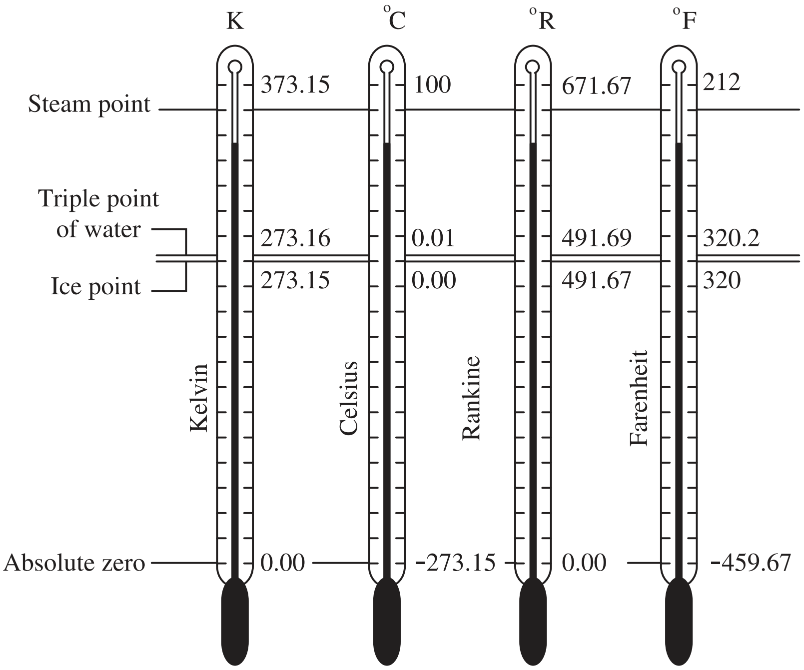


Fig. 1.28 Comparison of temperature scales

1.20 TEMPERATURE MEASUREMENT

A number of thermometers/instruments are available using different thermometric properties for measurement of temperature. Typical examples are: length, volume, pressure, resistance, emf, etc. (Table1.7). Thermometers and instruments developed using these thermometric properties are discussed in the following sections. Thermometers are usually classified as follows:

- (i) Expansion thermometers
  - (a) Liquid-in-glass thermometers
  - (b) Bimetallic thermometers
- (iii) Gas thermometers
  - (a) Constant volume thermometer
  - (b) Constant pressure thermometer

(ii) Pressure thermometers
  (a) Vapour pressure thermometers
  (b) Liquid-filled thermometers
  (c) Gas-filled thermometers

(iv) Thermocouple thermometers
(v) Resistance thermometers
(vi) Radiation pyrometers
(vii) Optical pyrometers

Further, temperature measuring instruments may be divided into two broad categories:

(i) Non-electrical type
  (a) using the principle of change in volume of a liquid with temperature,
  (b) using the principle of change in pressure of a gas with temperature, and
  (c) using the principle of change in the vapour pressure with temperature.

(ii) Electrical type
  (a) using the principle of thermo-electricity,
  (b) using the principle of change in resistance of material with change in temperature,
  (c) comparing the colours of filament and the object and
  (d) using the principle of energy reception by radiation.

1.20.1 Expansion Thermometers

Expansion thermometers use the principle of the differential expansion of two different substances. A typical example is the liquid-in-glass thermometer. The principle employed in this is the difference in expansion of liquid and the containing glass. There is another type called bimetallic thermometer. Principle employed is the difference in expansion of the two solids.

1.20.2 Liquid in Glass Thermometer

Even though this type of thermometer is used much less frequently nowadays than a few decades ago, it is still a very commonly used device. It employs liquids as the thermometric substance. The principle used is the change in volume of liquid with heat interaction. Two commonly used liquids are mercury and alcohol. Figure 1.29 shows a typical mercury-in-glass thermometer. Mercury is found to be the most suitable liquid and can be used from −39°C to about 600 °C. An inert gas like nitrogen is introduced above the mercury to prevent boiling. The thermometers used in the laboratory have the scale engraved directly on the glass stem. An expansion bulb is usually provided at the top of the stem to allow room for expansion of mercury, in case the thermometer is subjected to temperature above its range. In this thermometer, change in volume of the mercury causes the rise or fall in the level of mercury column in the glass tube. Instead of mercury as the thermometric fluid, alcohols can also be used. However, mercury is preferred over alcohol due to the following reasons:

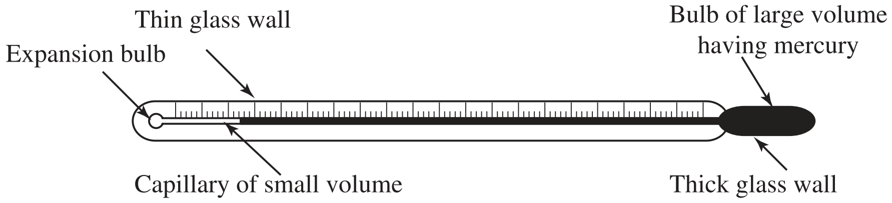


Fig. 1.29 Mercury in glass thermometer

(i) can be clearly and conveniently seen in a fine capillary tube,
(ii) comparatively a good conductor of heat,
(iii) does not wet the wall of the tube,
(iv) low specific heat and hence, absorbs little heat from body,
(v) remains as liquid over a large range as its freezing and boiling points, and
(vi) uniform coefficient of expansion over a wide range of temperature.

Pentane, ethyl alcohol and toluene are the other liquids which can be used for liquid-in-glass thermometers. Since these liquids are normally colourless, a dye is added for easy reading.

Main advantages of this thermometer are:

(i) compatibility with most environments,
(ii) easy portability,
(iii) independence of auxiliary equipment,
(iv) low cost,
(v) Reasonable ruggedness, and
(vi) wide range (temperatures as low as -200 °C and as high as 1000 °C).

Disadvantages include:

(i) a large sensing element,
(ii) awkward dimensions,
(iii) difficult to use for continuous or automatic readout, and
(iv) long time constant.

For calibration, usually the ice point (can also be any other reference) is always taken first. Before calibration, the thermometer which is to be calibrated must be kept at room temperature for at least 72 hours. Then, the thermometer must be allowed to attain thermal equilibrium with the ice and should be gently tapped with the fingernail or a pencil to ensure a non-sticking meniscus before reading. Following the ice-point checks, the thermometer is calibrated from the lowest to the highest temperature against a laboratory standard in a suitable bath.

1.20.3 Bimetallic Thermometers

Figure 1.30 shows the basic principle of the bimetallic thermometer. All of us know that metals when exposed to temperature experience a change in dimension. They either expand or contract when there is a change in temperature. The rate of change of this expansion or contraction depends on the coefficient of expansion of the metal (Fig.1.30). It may be noted that the temperature coefficient of expansion is different for different metals. Hence, the difference in thermal expansion rates is used to produce deflection which is proportional to temperature changes. This principle is used in bimetallic thermometers which is illustrated in Fig.1.30.

1.20.4 Construction

The essential element in a bimetallic expansion thermometer is a bimetallic strip. A simplified picture of thermometer which consists of two layers of different metals fused together is shown

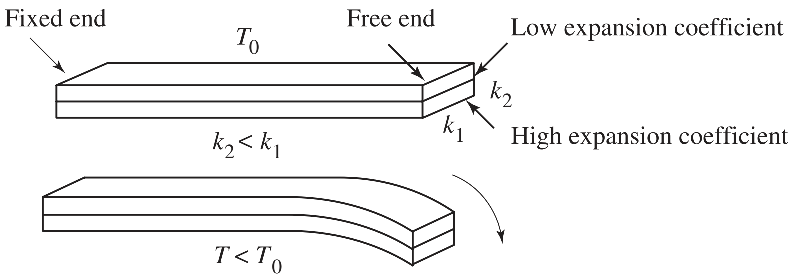


Fig. 1.30 Bimetallic strip

in Fig.1.31. When such a strip is subjected to temperature changes, one layer expands or contracts more than the other, thus tending to change the curvature of the strip. As shown in Fig.1.31, one end of a straight bimetallic strip is fixed in place. As the strip is heated, the other end tends to curve away from the side that has the greater coefficient of linear expansion. When used in thermometers, the bimetallic strip is normally wound into a flat spiral (Fig.1.32), a single helix or a multiple helix. The end of the strip that is not fixed in position is fastened to the end of a pointer that moves over a circular scale. Bimetallic thermometers are easily adapted for use as recording thermometers; a pen is attached to the pointer and positioned so that it marks on a revolving chart.

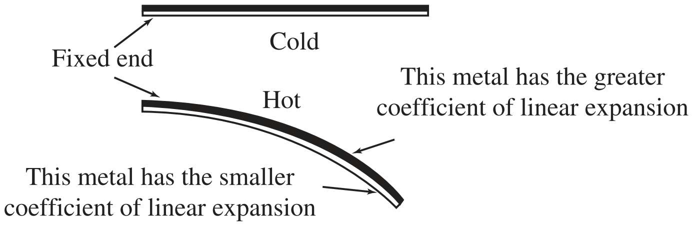


Fig. 1.31 Bimetallic thermometer (flat strip)

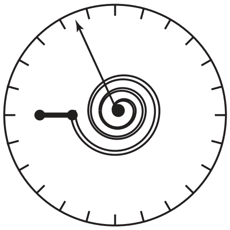


Fig. 1.32 Bimetallic thermometer (spiral strip)

1.20.5 Thermoelectric Thermometer or Thermocouple

For higher range of temperature, i.e. above 1000 °C, liquid or gas filled thermometers are not suitable. For such high temperatures, thermoelectric thermometer (thermocouple) and pyrometers are used. Thermoelectric thermometer works on the principle based on the thermoelectric effect, namely Seebeck effect.

1.20.6 Seebeck Effect

Seebeck effect can be stated as follows. A current will flow or emf will be generated in a circuit of two dissimilar metals when one junction is kept in hot condition while other is kept cold. Current produced in this way is called thermoelectric current while the emf produced is called thermo emf Such developed e.m.f should be calibrated with respect to known temperature.

For calibration, one junction is kept at ice point and other in oil bath having any temperature as shown in Fig.1.33. Upon heating the oil bath, a thermo emf will be generated because of Seebeck effect. Temperature of the oil bath is measured by some calibrated thermometer of any other type. Further, the temperature of oil bath is changed to different known temperatures. Corresponding emf is noted and a calibrated graph is plotted between the temperature of bath and emf

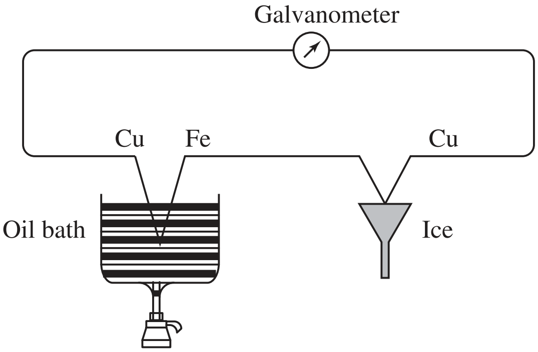


Fig. 1.33 Thermoelectric thermometer using a galvanometer

Now, for using this thermocouple, the cold junction shall still be maintained at the ice point while the hot junction may be kept in contact with an object whose temperature is to be measured. To get the temperature depending upon the emf produced the calibrated graph is used and corresponding temperature is noted from the graph. In these thermometers, the potentiometers may also be connected as shown in Fig.1.34.

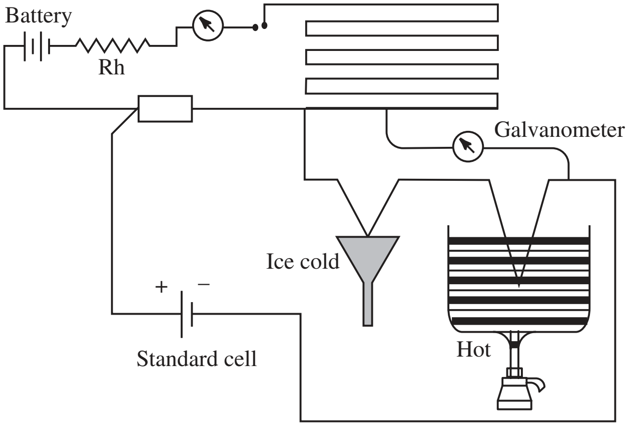


Fig. 1.34 Thermoelectric thermometer using a potentiometer

Here, also the one junction is maintained at ice point while the other junction is put at the temperature to be measured. The potentiometer wire is directly calibrated to measure temperature. Here, the length of the potentiometer wire at which the balance point is obtained is used for getting temperature.

In thermoelectric thermometer, instead of potentiometer a sensitive galvanometer can be connected with thermocouple as shown in Fig.1.35. Reference junction usually called cold junction is kept at some constant temperature, such as 0 °C. Figure 1.35 shows a simple circuit of a thermocouple and the temperature measuring device. In many industrial installations, the instruments are equipped with automatic compensating devices for temperature changes of the reference junction, thus eliminating the necessity of maintaining this junction at constant temperature. Table 1.8 gives the composition, useful temperatures range and temperature versus emf relationship for some commercial thermocouples.

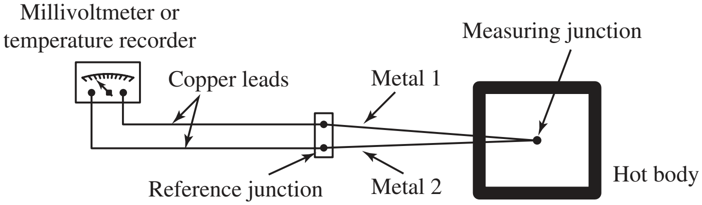


Fig. 1.35 Thermocouple

1.21 PYROMETER

A pyrometer is used to determine the surface temperature of an object. The word 'pyrometer' comes from the two Greek words, *pyro* meaning *fire* and *meter*, meaning to *measure*. In many manufacturing processes, the ability to measure surface temperature without contacting the product with any measuring instruments becomes critical to ensure quality and yield. For this purpose, nowadays pyrometers are employed. A pyrometer is a non-contacting device that intercepts and measures thermal radiation. This process is known as pyrometry. Pyrometer was originally coined to denote a device capable of measuring temperatures of objects above incandescence (i.e. objects too bright to the human eye). Basically, two types of non-contact pyrometers are available. The first are the radiation pyrometers and the second, optical pyrometers (also known as brightness pyrometers). The two groups use different technologies to measure temperature.

1.21.1 Radiation Pyrometer

A radiation pyrometer has an optical system and a detector. The optical system focuses the thermal radiation onto the detector. The output signal of the detector (temperature T) is related to the thermal radiation or irradiance, j^*, of the target object given by the Stefan-Boltzmann law:

$$j^* = \epsilon\sigma T^4$$

where the constant of proportionality, σ, is called the Stefan-Boltzmann constant and ϵ is the emissivity of the object. This output, j^*, is used to infer the object's temperature. Thus, there is no need for any direct contact between the pyrometer and the object, which is common with thermocouples and resistance temperature detectors (RTDs). A simplified sketch of a total radiation pyrometer is shown in Fig.1.36. Pyrometer collects the radiation from the object

Table 1.8 Composition, useful temperatures range and temperature versus emf relationship for some commercial thermocouples

Thermocouple	Composition	Temperature (°C)		Thermoelectric power		Remarks
		Useful range	Max.	°C	Millivolt	
Platinum vs Platinum-rhodium	Pure platinum vs Pt +10 or 13% Rh	400 to 1450	1700	0	0.0	Used for high temperature measurements
				500	4.219	
				1000	9.569	
				1500	15.498	
Chromel vs alumel	90%Ni + l0% Cr vs 95% Ni + 5% (AI + Sn) Mn	−200 to 1200	1450	−200	−5.75	High resistance to oxidation
				0	0.0	
				300	12.21	
				600	24.90	
				900	37.36	
				1200	48.85	
Iron vs constantan	Pure iron vs 45-60% Cu+ 55-40% Ni	−200 to 750	1000	−200	−8.27	
				0	0.0	
			300	16.59	–	
				600	33.27	
				900	52.29	
Copper vs constantan	Pure copper vs Cu-Ni constantan	−200 to 350	600	−200	−5.539	Not suitable in air due to excessive oxidation
				0	0.0	
				200	9.285	
				400	20.865	

(hot body) whose temperature is to be measured. A mirror is used to focus this radiation on a thermocouple. The energy which is focused and concentrated on the thermocouple raises its temperature. This in turn generates an emf. This emf is then measured either by the galvanometer or potentiometer method. Thus, *rise of temperature is a function of the amount of radiation emitted from the object.*

1.21.2 Optical Pyrometer

An optical pyrometer works on the principle that matters glow above 480 °C and the colour of visible radiation is proportional to the temperature of the glowing matter. The amount of light radiated from the glowing matter (solid or liquid) is measured and employed to determine the temperature. Figure 1.37 shows a disappearing filament pyrometer.

Operation : The optical pyrometer is sighted and focused at the hot body. In the beginning, filament will appear dark as compared to the background which is bright (being hot). By varying the resistance (R) in the filament circuit, more and more current is fed into it. Because of this, filament becomes equally bright as the background and hence, disappears. The current flowing in the filament at this stage is measured with the help of an ammeter which is calibrated directly in terms of temperature. If the filament current is further increased, the filament appears brighter than the background which then looks dark. An optical pyrometer can

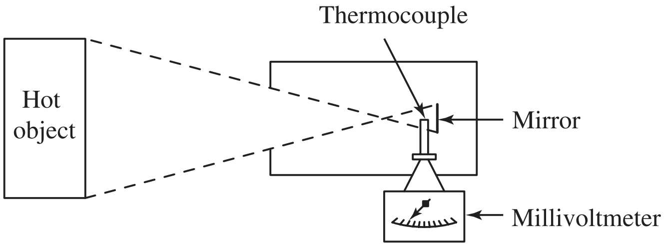


Fig. 1.36 A schematic diagram of radiation pyrometer

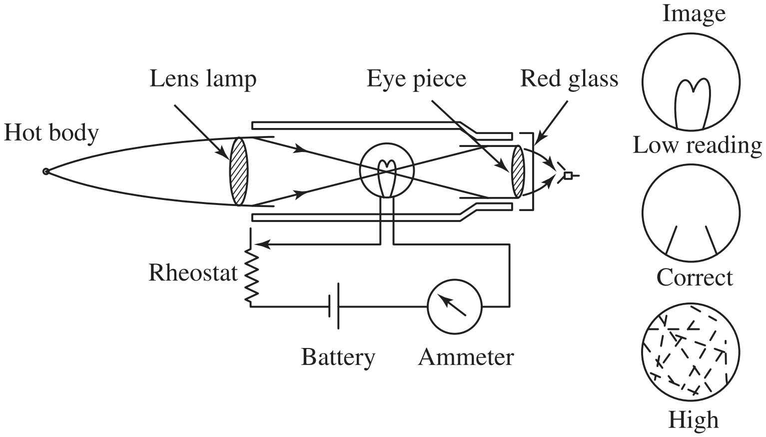


Fig. 1.37 Optical pyrometer

measure temperatures ranging from 1000 to 4000 °C.
Advantages of the pyrometers:

(i) The temperatures of moving objects can be easily measured.

(ii) Much higher temperatures can be measured compared to thermocouples.

(iii) The temperature of the objects which are not easily accessible can be measured.

(iv) The average temperatures of the extended surface can be measured.

Disadvantages:

(i) It is based on an observer's judgement of two colours to be the same.

(ii) It requires direct line of sight to the object whose temperature is to be measured.

1.22 A NOTE ON ENGINEERING CALCULATIONS

Engineering calculations are reported with an accuracy of certain number of significant digits. A three-digit accuracy is considered reasonable. Reporting results in more significant digits will imply greater accuracy than exists. *It should be avoided*. Let us illustrate by an example. Consider a container of volume 3.75 ℓ filled with gasoline whose density is 0.855 kg/ℓ. Now, try to determine its mass which is the product of volume and density. Probably, we will report the result as

$$m = \rho V = 0.855 \times 3.75 = 3.20625$$

as displayed in the calculator screen. What does the above result *falsely imply* is that mass calculated is accurate to six significant digits. What you should understand is that mass calculated cannot be more accurate than three significant digits since both volume and density are accurate to three significant digits only. Therefore, the result should be rounded off to three significant digits. Hence, mass should be reported as 3.21 kg. Further, what you should note is that by giving volume as 3.75 ℓ we imply that the volume is accurate within $\pm 0.01\ell$ and therefore, volume cannot be either 3.74 or 3.76. However, it can be 3.746, 3.75, 3.754, etc. which can be rounded off to 3.75 ℓ with three significant digit accuracy. It is appropriate to retain all the digits during intermediate calculations and do the rounding off in the final step. This is how the computer calculation does.

While solving engineering problems, it is reasonable to assume that the given information is accurate to at least three significant digits. For example, if the length is given as 50 m we should assume as 50.0 m in order to justify our assumption of three significant digits in the final results. You need not have any reservation in rounding the final results to a reasonable number of significant digits. Further, having a few per cent uncertainty in the engineering analysis are usually the norm and need not be the exception.

Worked-Out Examples

1.1 In what units force, pressure, energy and power are expressed? Give their units and their dimensions. Also establish the relation between their base units and derived units.

Solution

All of the above are expressed in terms of derived units. $\overset{\textbf{Ans}}{\Longleftarrow}$

• Force

According to the Newton 's second law of motion, $F = ma$, where m is mass and a is acceleration:

$$F = ma \quad = \quad m\left(\frac{d\mathbb{V}}{dt}\right) = m\left(\frac{d^2L}{dt^2}\right)$$

where L is the distance and t is the time. Now, the units of force is given by,

$$(F) = \text{kg}\left(\frac{\text{m}}{\text{s}^2}\right) \quad = \quad \text{kg m s}^{-2} = N$$

The force has the dimension MLT^{-2}. $\overset{\textbf{Ans}}{\Longleftarrow}$

The unit of force in SI units is newton (N). This is in honour of the great scientist Sir Isaac Newton. One newton is the force required to produce the acceleration of 1 m/s^2 in a body of mass of 1 kg.

• Pressure

Pressure p is defined as the force per unit area. Mathematically,

$$p \quad = \quad \left(\frac{F}{A}\right)$$

where F is force and A is area.

Units of force is given by:

$$p \quad = \quad \frac{\text{kg m s}^{-2}}{\text{m}^2} = \text{kg m}^{-1}\text{s}^{-2} = \text{ Pa} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Pressure has the dimension $\text{ML}^{-1}\ \text{T}^{-2}$ $\overset{\textbf{Ans}}{\Longleftarrow}$

In SI units, the unit for pressure is pascal (Pa). It can also be expressed as N/m^2.

- **Energy**

If a force, F acting on a body moves through a distance L in the direction of applied force, then the work transfer to the body is given by

$$W \quad = \quad FL$$

Units of energy can now be written as;

$$W \quad = \quad \text{kg m s}^{-2}\ \text{m} = \text{kg m}^2\ \text{s}^{-2} = \text{ J} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

As work is in a form of energy (E), it has the dimension $\text{ML}^2\ \text{T}^{-2}$ $\overset{\textbf{Ans}}{\Longleftarrow}$.

In SI Units the unit for energy is joule (J). It can also be expressed as Nm.

- **Power**

Power is the rate at which energy is delivered or consumed per unit time. It can be written as:

$$\text{Power, } P = \text{energy/time} = \frac{E}{t}$$

where E is energy and t is time.

Unit of power is given by $\text{kg m}^2\ \text{s}^{-2}/\text{s} = \text{kg m}^2\ \text{s}^{-3}$ = Watts = W.

Dimensions of power is $\text{ML}^2\ \text{T}^{-3}$.

In SI units, the unit for power is watt (W). It can also be expressed as J/s.

1.2 Express the following quantities in terms of base SI units: (i) heat flux, (ii) power and (iii) specific weight.

Solution

The base SI units are kg, m and s.

$$\begin{aligned}
\text{Heat flux} = \text{Heat transfer/Time} &= \text{J/s} = \text{N m/s} = (\text{kg m/s})(\text{m/s}) = \text{kg m}^2/\text{s}^2 \ \overset{\textbf{Ans}}{\Longleftarrow}\\
\text{Power} = \text{Force} \times \text{Velocity} &= \text{N} \times (\text{m/s}) = (\text{kg m/s}^2)\ (\text{m/s}) = \text{kg m}^2/\text{s}^3 \ \overset{\textbf{Ans}}{\Longleftarrow}\\
\text{Specific weight} = \text{Weight/Volume} &= \text{N/m}^3 = (\text{kg m/s}^2)\ /\text{m}^3 = \text{kg}/(\text{m}^2\text{s}^2) \ \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

1.3 Identify each of the following as either control mass (closed system) or control volume (open system).

(i) The product of combustion in an I C engine cylinder during expansion.
(ii) The product of combustion in an I C engine cylinder during exhaust stroke.
(iii) An automobile tyre being heated during driving.
(iv) A balloon exhausting air.

(v) A pressure cooker during pressure release operation.

Solution

(i) Control mass since both valves will be closed and there is no mass transfer.
(ii) Control volume since exhaust valve will be open and there is mass transfer.
(iii) Control mass since there is no mass transfer.
(iv) Control volume since there is mass transfer.
(v) Control volume since there is mass transfer.

1.4 Consider the following properties. (i) a volume of 20 m^3, (ii) a pressure of 100 kPa and (iii) kinetic energy of 120 J. Classify the above as intensive or extensive properties with reason. Convert the extensive properties to specific properties assuming $m = 60$ kg.

Solution

Note: Extensive properties are *mass dependent* whereas intensive properties are *mass independent.*

(i) Volume is dependent on mass. For example, if the mass doubles the volume increases. Therefore, it is an extensive property. $\overset{\textbf{Ans}}{\Longleftarrow}$
(ii) Pressure is independent of mass. Therefore, it is an intensive property. $\overset{\textbf{Ans}}{\Longleftarrow}$
(iii) Kinetic energy is dependent on mass. If mass doubles kinetic energy also doubles. Therefore it is an extensive property. $\overset{\textbf{Ans}}{\Longleftarrow}$

Volume and kinetic energy are extensive properties. To convert them into specific properties, we should divide them by mass. Now,

$$\begin{array}{llllll} V = & 20\ \text{m}^3 & v = \text{specific volume} = & V/m = 20/60 & = & 0.33\ \text{m}^3/\text{kg} \quad \overset{\textbf{Ans}}{\Longleftarrow} \\ E = & 120\ \text{J} & e = \text{specific energy} = & E/m = 120/60 & = & 2.00\ \text{J/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow} \end{array}$$

1.5 Consider a thermodynamic quantity X defined as $dX = pdV$, where $pV = RT$. Establish whether X is a property or not.

Solution

First of all, let us recollect the characteristics of property. *Property is a point function and its differential is exact.* For example, if P is a property, then the change in the property in going from the initial state i to the final state f is given by

$$\int_i^f dP = P_f - P_i$$

If P is a function of x and y, then, Mathematically, it is expressed as $P = P(x, y)$:

$$\begin{aligned} dP &= \left(\frac{\partial p}{\partial x}\right)_y dx + \left(\frac{\partial p}{\partial y}\right)_x dy = Mdx + Ndy \\ M &= \left(\frac{\partial p}{\partial x}\right)_y \quad \text{and} \quad N = \left(\frac{\partial p}{\partial y}\right)_x \end{aligned}$$

if $\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$, then dP is said to be an exact differential and P is called the point function. Also, if P is a point function then it is *property.* Now, consider the example,

$$X = X(p, V)$$

$$dX = \left(\frac{\partial X}{\partial p}\right)_V dp + \left(\frac{\partial X}{\partial V}\right)_p dV = Mdp + NdV \tag{1}$$

$$dX = pdV \text{ (Given)} \tag{2}$$

Therefore, from (1) and (2), $M = 0$ and $N = p$. For dX to be an exact differential, it should satisfy the condition

$$\left(\frac{\partial M}{\partial V}\right)_p = \left(\frac{\partial N}{\partial p}\right)_V$$

As this condition is not satisfied by $dX = pdV$, it is not a property.

1.6 A piston/cylinder arrangement in Fig.1.38 has a cross-sectional area of 0.01 m^2. Piston mass is 100 kg and is resting on the stops, as shown in the figure. With an outside atmospheric pressure of 101 kPa, what should be the minimum oil pressure required to lift the piston? Take $g = 9.8$ m/s^2. Note that the downward force acting on the piston comes from gravitational force and the outside atmospheric pressure acting over the top surface. Oil pressure is acting on the bottom surface of the piston.

Solution

$$\text{Downward force, } F\downarrow \quad = \quad m_p g + p_0 A$$

where m_p is mass of the piston, A is the area of the piston and p_0 is the atmospheric pressure.

$$\text{Upward force, } F\uparrow \quad = \quad pA$$

where p is the oil pressure and A is the area of the piston. Force balance gives:

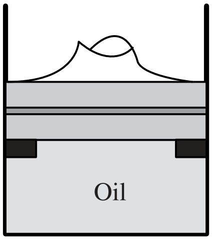


Fig. 1.38

$$F\uparrow = F\downarrow$$

$$pA = m_p g + p_0 A$$

$$p = \frac{m_p g}{A} + p_0 = \frac{100 \times 9.8}{0.01} + 101 \times 10^3 = 199,000 \text{ Pa} = 199 \text{ kPa} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

1.7 In the condenser of a steam power plant the vacuum recorded is 75 cm of mercury. Estimate the absolute pressure in the condenser in P_a. Take the barometric reading as 76 cm Hg. Note that 1 mm of Hg = 133.4 N/m^2.

Solution

$$\text{Absolute pressure in condenser,} \quad = \quad \text{Barometeric reading} - \text{Vacuum in condenser}$$

$$p_{ab} = 760 - 750 = 10 \text{ mm of Hg} = 10 \times 133.4 \text{ N/m}^2$$

$$= 1334 \text{ N/m}^2 = 1334 \text{ Pa} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

1.8 An apple is kept in a refrigerator at 8 °C. It weighs 120 g and has a volume of 150 cm^3. Find the values of two intensive and two extensive properties of the apple.

Solution

$$\text{Density of the apple, } \rho = \frac{m}{V} = \frac{120}{150} = 0.8 \text{ g/cc} = 800 \text{ kg /m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Intensive property is independent of mass. The values of two intensive properties are:

$$\text{Specific volume, } v = \frac{1}{\rho} = \frac{1}{800} = 0.00125 \text{ m}^3\text{/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Temperature, } T = 8 \text{ °C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

The values of two extensive properties, m and V are:

$$\text{Mass, } m = 120 \text{ g } = 0.12 \text{ kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Volume, } V = 150 \text{ cm}^3 = 0.15\ell = 0.00015 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

1.9 A thick piston is initially held in a cylinder by a pin as shown in Fig.1.39. Assume that the cylinder contains carbon dioxide at 220 kPa and take ambient temperature as 300 K. The density of the piston is 8000 kg/m^3. Assume atmospheric pressure as 100 kPa. If the pin is removed then the piston will move up. Assume that after a while the gas returns to ambient temperature under these circumstances, ascertain whether the piston will touch the stop at the top while moving up.

Solution

$$\begin{aligned}
\text{Pressure of gas, } p_1 &= 220 \text{ kPa} \\
\text{Volume of CO}_2\text{, } V_1 &= \frac{\pi}{4} d^2 h = \frac{\pi}{4} \times 0.1^2 \times 0.1 \\
&= 0.785 \times 10^{-3} \text{ m}^3 \\
m_{piston} &= V\rho \\
&= \frac{\pi}{4} \times 0.1^2 \times 0.1 \times 8000 \\
&= 6.28 \text{ kg}
\end{aligned}$$

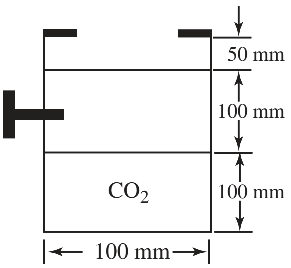


Fig. 1.39

$$\text{Force exerted by the piston, } mg = 6.28 \times 9.81 = 61.61 \text{ Nm}$$

Initial pressure exerted by the piston on the gas,

$$p_1 = \frac{F}{A} = \frac{61.61}{\frac{\pi}{4} \times 0.1^2} = 7844.4 \text{ Pa} = 7.84442 \text{ kPa}$$

After removal of pin, if the piston touches the stop, it would travel a distance of 0.05 m.

$$\text{Volume of CO}_2,\ V_2 = \frac{\pi}{4} \times 0.1^2 \times [0.1 + (0.1 - 0.05)]$$

$$= 1.18 \times 10^{-3}\ \text{m}^3$$

Now, $p_1 V_1 = p_2 V_2$

$$p_2 = \frac{p_1 V_1}{V_2} = \frac{220 \times 0.785}{1.18} = 146.4\ \text{kPa}$$

Pressure exerted on the gas by the piston = Atmospheric pressure + Pressure exerted by piston

$$= 100 + 7.844 = 107.844\ \text{kPa} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Pressure exerted by the gas on the piston} = 146.4\ \text{kPa}$$

As pressure exerted by the gas on the piston is greater than the pressure exerted by the piston on the gas, when the pin is removed, the piston will touch the stop.

1.10 A hydraulic lift has a maximum fluid pressure of 500 kPa. What is the diameter of the piston-cylinder required to lift a mass of 800 kg. Take $g = 9.81$ m/s^2.

Solution

$$\text{Upward force } F \uparrow = p \times A$$

$$\text{Downward force } P \downarrow = m \times g$$

$$F \uparrow = F \downarrow$$

$$p \times \frac{\pi}{4} d^2 = mg$$

$$d = \sqrt{\frac{4mg}{\pi p}} = 2 \times \sqrt{\frac{800 \times 9.81}{\pi \times 500 \times 1000}} \times \frac{1}{1000} = 0.141\ \text{m} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

1.11 An one cubic meter container is filled with 0.12 m^3 of granite, 0.15 m^3 of sand and 0.2 m^3 of liquid at 25 °C water; the rest of the volume, 0.53 m^3, is air with a density of 1.15 kg/m^3. Find the overall (average) specific volume and density. Take the density of granite, sand, water and air as 2750 kg/m^3, 1500 kg/m^3, 997 kg/m^3 and 1.15 kg/m^3, respectively.

Solution

From the definition of specific volume and density, we have

$$v = \frac{V}{m} \quad \rho = \frac{m}{V} = \frac{1}{v}$$

We need to find the total mass, taking into consideration the given density:

$$\begin{aligned}
m_{\text{granite}} &= \rho_{\text{granite}} V_{\text{granite}} = 2750 \times 0.12 = 330 \text{ kg} \\
m_{\text{sand}} &= \rho_{\text{sand}} V_{\text{sand}} = 1500 \times 0.15 = 225 \text{ kg} \\
m_{\text{water}} &= \rho_{\text{ water}} V_{\text{ water}} = 997 \times 0.2 = 199.4 \text{ kg} \\
m_{\text{air}} &= \rho_{\text{ air}} V_{\text{ air}} = 1.15 \times 0.53 = 0.61 \text{ kg} \\
\text{Overall mass} &= m_{\text{granite}} + m_{\text{sand}} + m_{\text{water}} + m_{\text{air}} \\
&= 330 + 225 + 199.4 + 0.61 = 775.01 \text{ kg} \\
\text{Average specific volume} &= \frac{V}{m} = \frac{1}{775.01} = 0.00129 \text{ m}^3/\text{kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

1.12 An U-tube mercury manometer connected to a system shows a deflection of 20 cm when the other limb is open to atmosphere. Assume g as 9.8 m/s^2 and density of mercury as 13.5×10^3 kg/m^3. Find the pressure.

Solution

Pressure at a point with respect to atmosphere is the pressure difference, Δp.

$$\begin{aligned}
\Delta p &= \rho g h \quad \text{where } h \text{ is in meters.} \\
\Delta p &= 13.5 \times 10^3 \times 9.8 \times 20 \times 10^{-2} = 26,460 \text{ Pa} = 26.46 \text{ kPa} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

1.13 Compressed air is filled in a tank. The manometer shows a reading of 40 cm of mercury column. The atmospheric pressure is 100 kPa. Find the actual pressure of air in kPa. Take the density of mercury as 13, 560 kg/m^3, $g = 9.81$ m/s^2.

Solution

$$\begin{aligned}
\text{Gauge pressure} &= \rho g h = 13,560 \times 9.8 \times 40 \times 10^{-2} = 53,209.44 \text{ Pa} = 53.21 \text{ kPa} \\
\text{Actual pressure} &= \text{Gauge pressure} + \text{Atmospheric pressure} \\
&= 53.21 + 100 = 153.21 \text{ kPa} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

1.14 A tank is filled with oil of specific gravity 0.8. Determine the pressure at a depth of 2 m. Assume density of water as 1000 kg/m^3 and $g = 9.81$ m/s^2.

Solution

$$\begin{aligned}
\text{Specific gravity of oil} &= \text{Density of oil/density of water} \\
\text{Density of oil} &= \text{Specific gravity of oil} \times \text{Density of water} \\
&= 0.8 \times 1000 = 800 \text{ kg/m}^3 \\
\text{Gauge pressure} &= \rho_{\text{oil}} g h = 800 \times 9.8 \times 2 \times 10^{-3} \text{ N/m}^2 = 15.7 \text{ kPa} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

1.15 5 kg of gas is contained in a cylindrical vessel of dimension 60 × 80 cm ($d \times h$). The pressure measured with a manometer reads 600 mm of Hg above atmospheric pressure when barometer reads 760 mm of Hg. Calculate

(i) the absolute pressure of the gas,
(ii) specific volume and density of the gas.

Solution

$$\text{Volume of the vessel} = \frac{\pi}{4}d^2h = \frac{\pi}{4} \times 60^2 \times 10^{-4} \times 80 \times 10^{-2} = 0.226 \text{ m}^3$$

$$\text{Total pressure on vessel} = 760 + 600 = 1320 \text{ mm Hg} = 1320 \times 133.3$$

$$= 1760 \approx 1.760 \times 10^5 \text{ N/m}^2$$

$$= 1.76 \text{ bar } (1 \text{ bar} = 10^5 \text{ N/m}^2) \quad \overset{\text{Ans}}{\Longleftarrow}$$

$$\text{Specific volume, } v = \frac{0.226}{5} = 0.0452 \text{ m}^3\text{/kg} \quad \overset{\text{Ans}}{\Longleftarrow}$$

$$\text{Density, } \rho = \frac{1}{v} = \frac{1}{0.0452} = 22.17 \text{ kg/m}^3 \quad \overset{\text{Ans}}{\Longleftarrow}$$

1.16 A cylindrical jar of 40-cm diameter is resting on a flat table. It is evacuated with a vacuum pump until the pressure reduces to 20 Pa. If the atmospheric pressure is 100 kPa, calculate the minimum force required to lift the jar from the table.

Solution

Minimum force required depends on the force exerted by the atmosphere on the jar:

$$A_{\text{jar}} = \frac{\pi}{4} \times \frac{d^2}{10^4} = \frac{\pi}{4} \times \frac{40^2}{10^4} = 0.1257 \text{ m}^2$$

Net pressure acting on the jar = Atmospheric pressure − the pressure inside the jar:

$$p_{\text{net}} = p_{\text{atm}} - p_{\text{jar}} = 10,000 - 20 = 99,980 \text{ Pa}$$

$$\text{Force, } F = p_{\text{net}} \times A_{\text{jar}} = 99,980 \times 0.1257 = 12,567 \text{ N} \quad \overset{\text{Ans}}{\Longleftarrow}$$

Minimum force required to lift the jar from the table is 12.567 kN.

1.17 Assume that water is flowing through the pipe as shown in Fig.1.40. Determine the pressure drop, taking the density of water as 1000 kg/m^3 and $g = 9.81$ m/s^2.

Solution

At the surface B-B the pressure is same. The pressure exerted by the water in the pipe in the left limb AB is the same as the pressure exerted by water and mercury in the right limb CB. Note that in the left limb, pressure by a water column $h + l$ is acting at B where as in the right limb, pressure at B is by the two fluids, viz a mercury column of height h and a water column of height ℓ:

$$p_A + \rho_w g(h + l) = p_B + \rho_{\text{Hg}} gh + \rho_w gl$$

$$p_A - p_B = \Delta p = \rho_{\text{Hg}} gh - \rho_w gh = \left(\rho_{\text{Hg}} - \rho_w\right) gh$$

$$\Delta p = \left(13.6 \times 10^3 - 1 \times 10^3\right) \times 9.80 \times 30 \times 10^{-2} = 37.044 \text{ kPa} \quad \overset{\text{Ans}}{\Longleftarrow}$$

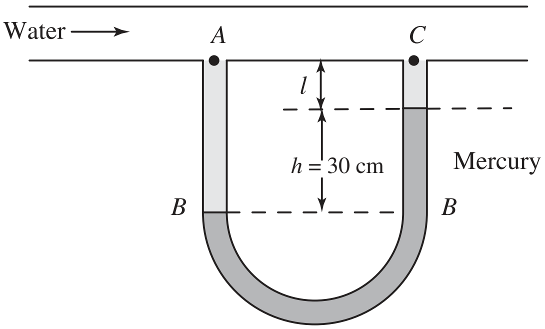


Fig. 1.40

1.18 Consider the cylinder piston arrangement shown in Fig.1.41. Assume that the cylinder contains 0.18 m^3 of gas at a pressure of 101 kPa. Assume that the spring is just touching the piston but does not exert any force on it. The gas is heated until it volume is doubled. Assume that the force exerted by the spring is proportional to the displacement of the piston during this process. If the spring constant is 60 kN/m and cross-sectional area of the piston is 0.06 m^2, calculate the final pressure of the gas in the cylinder.

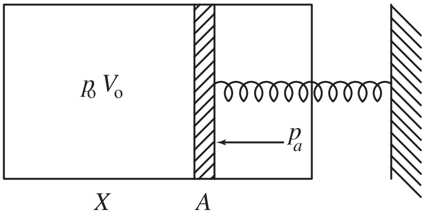


Fig. 1.41

Solution

A force balance on the piston will give:

$$pA = p_aA + kx = p_aA + k(V - V_0)/A$$

$$p = p_a + k(V - V_0)/(A \times A)$$

where p_a = atmospheric pressure, A = cross-sectional area, k = spring constant, V = volume of the gas at any instant during expansion, V_0 = initial volume of the gas.

$$p = p_a + k\frac{(V - V_0)}{A^2} = 101 + 60 \times \frac{(0.36 - 0.18)}{0.06^2} = 3018 \text{ kPa} = 3.018 \text{ MPa} \overset{\textbf{Ans}}{\Longleftarrow}$$

1.19 Normally pressure gauges are calibrated in kg$_f$/cm^2. Such a gauge is connected to a system which reads 10 kg/cm^2. Estimate the absolute pressure (in bar). Take $g = 9.8$ m/s^2 and atmospheric pressure as 100 kPa.

Solution

The pressure kg$_f$/cm^2 is the force exerted by 1 kg mass which is placed on an area of 1 cm^2.

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{1 \times 9.8}{1 \times 10^{-4}} = 98 \times 10^3 \text{ N/m}^2 = 98 \text{ kPa}$$

$$10 \text{ kg}_f/\text{cm}^2 = 10 \times 9.8 = 980 \text{ kPa}$$

The absolute pressure = atmospheric pressure + gauge pressure

$$p_{abs} = p_{\text{atm}} + p_{gauge} = 100 + 980 = 1080 \text{ kPa} = 10.8 \text{ bar}$$

1.20 It is proposed to measure the atmospheric pressure at a place using a barometer shown in Fig.1.42. On the day of measurement, if the barometer reads 736 mm of Hg, calculate the atmospheric pressure in kPa. Also estimate the pressure in bar. Take $g = 9.8$ m/s^2. Assume density of mercury ρ_{Hg} as 13, 595 kg/m^3.

Solution

$$\begin{aligned} p &= \rho g h \\ &= 13,595 \times 9.8 \times 736 \times 10^{-3} \\ &= 98,058 \text{ Pa} \\ &= 98.058 \text{ kPa} \\ &= 0.98058 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

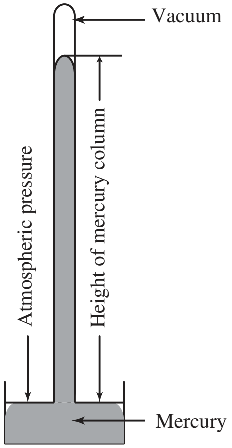


Fig. 1.42

1.21 An aeroplane at the airport which is at sea level weighs 5000 N. Show that its weight change is less than 0.5% when flying at an altitude of 10 km from mean sea level. Assume that the g varies with h as $g = \left(980.1 - 3.1 \times 10^{-6} h\right)$ cm/s^2.

Solution

$$g_{\text{sea}} = 980.1 - 3.1 \times 10^{-6} \times 0 = 980.1 \text{ cm/s}^2$$

Weight of the plane at sea level = 5000 N

$$\begin{aligned} m \times g &= 5000 \\ m &= \frac{5000}{g} = \frac{5000}{9.801} = 510.15 \text{ kg} \\ g_{\text{at 10 km}} &= 980.1 - 3.1 \times 10^{-6} \times (10 \times 1000 \times 100) \\ &= 977 \text{ cm/s}^2 = 9.77 \text{ m/s}^2 \end{aligned}$$

$$\begin{aligned}
\text{Weight of the plane at 10 km} &= 510.15 \times 9.77 = 4984.16 \text{ N} \\
\text{\% difference in weight} &= \frac{(5000 - 4984.16)}{5000} \times 100 = 0.3168\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

which is less than 0.5%.

1.22 Nowadays multistorey buildings are quite common. The height of such buildings can be measured by using a barometer. If the barometer readings at the top and bottom are 675 and 695 mm of Hg, respectively, determine the height of the building. Take $\rho_{air} = 1.18$ kg/m^3 and $\rho_{Hg} = 13,600$ kg/m^3.

Solution

$$\begin{aligned}
p_{top} &= (\rho g h)_{top} \\
&= 13,600 \times 9.81 \times 0.675 \\
&= 90,055.8 \text{ Pa} = 90.06 \text{ kPa} \\
p_{bottom} &= (\rho g h)_{bottom} \\
&= 13,600 \times 9.81 \times 0.675 = 92,724.12 \text{ Pa} \\
&= 92.72 \text{ kPa}
\end{aligned}$$

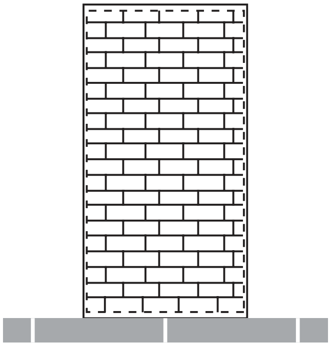


Fig. 1.43

Taking the air column between the top and bottom of the building and writing the force balance per unit base area, we have:

$$\frac{W_{air}}{A} = p_{bottom} - p_{top}$$

$$\begin{aligned}
\frac{W_{air}}{A} &= \frac{mg}{A} = \frac{\rho A h g}{A} = \rho g h \\
(\rho g h)_{air} &= p_{bottom} - p_{top} \\
1.18 \times 9.81 \times h_{air} &= (92.72 - 90.06) \\
h &= \frac{92.72 - 90.06}{1.18 \times 9.81} \times 10^3 \approxeq= 230 \text{ m} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

This will be the height of the building.

1.23 Refer the manometer shown in Fig.1.44, having bulbs A and B containing water ($\rho =$ 1000 kg/m^3) at different pressures. These bulbs are connected to two mercury manometers. The open end of the manometers are connected by an inverted U-tube manometer containing oil ($\rho_o = 875$ kg/m^3). Calculate the pressure difference between the bulbs A and B in kPa. Take the density of mercury as 13, 600 kg/m^3 and $g = 9.81$ m/s^2. Note that all numbers in Fig.1.44 is in cm.

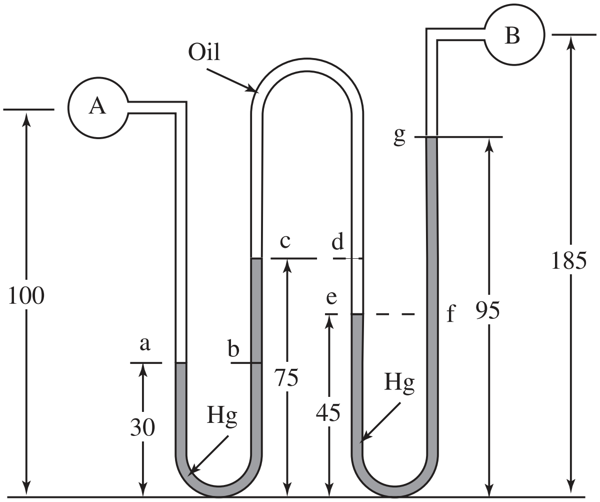


Fig. 1.44

Solution

$$p_a = p_A + \text{Pressure due to } (100 - 30) = 70 \text{ cm of H}_2\text{O column}$$

$$p_a = p_A + \rho_w g h_w = p_a + 1000 \times 9.81 \times \frac{70}{100} = p_A + 6,867 \text{ N/m}^2$$

As per hydrostatic principle, the pressure at all points at the same height of a stationary column of a single fluid is the same. Therefore, the pressure at b is:

$$p_b = p_a = p_a + 6,867 \text{ N/m}^2$$

$$p_b - p_c = \text{Pressure due to } (75 - 30) = 45 \text{ cm of Hg column}$$

$$= \rho_g g h_g = 13,600 \times 9.81 \times \frac{45}{100} = 60,037.2 \text{ N/m}^2$$

$$p_c = (p_A + 6,867) - 60,037.2 = p_A - 53,170.2$$

$$p_d = p_c = p_A - 53,170.2$$

$$p_e = p_d + \text{Pressure due to } (75 - 45) = 30 \text{ cm of oil column}$$

$$= (p_A - 53,170.2) + 875 \times 9.81 \times \frac{30}{100} = p_A - 50,595 \text{ N/m}^2$$

$$p_f = p_e = p_A - 50,595 \text{ N/m}^2$$

$$p_f - p_g = \text{Pressure due to } (95 - 45) = 50 \text{ cm of Hg column}$$

$$
\begin{aligned}
&= \rho_{\mathrm{Hg}} g h_{\mathrm{Hg}} = 13,600 \times 9.81 \times \frac{50}{100} = 66,708 \ \mathrm{N/m^2} \\
p_g &= p_f - 66,708 = p_A - 50,595 - 66,708 = p_A - 117,303 \ \mathrm{N/m^2} \\
p_g - p_B &= \text{Pressure due to } (185 - 95) = 90 \text{ cm of water column} \\
&= \rho_{\mathrm{w}}\ g\ h_{\mathrm{w}} = 1000 \times 9.81 \times \frac{90}{100} = 8,829 \ \mathrm{N/m^2} \\
p_A - 117,303 &= p_B + 8,829 \\
p_A - p_B &= 117,303 + 8,829 = 20,559 \ \mathrm{N/m^2} \qquad \xLeftarrow{\mathbf{Ans}}
\end{aligned}
$$

1.24 Assume pressure p and specific volume, v of the atmosphere are related by $pv^{\gamma} = 80,350$, where p is in $\mathrm{N/m^2}$ and v in $\mathrm{m^3/kg}$ and $\gamma = 1.4$. Assuming g to 9.81 and $p_{atm} = 10^3$ kPa, what is the atmospheric height required to impart a pressure of one standard atmosphere at the earth surface.

Solution

Considering a small element of atmospheric column of thickness dh at a depth of h from the top of the atmosphere, where $p = 0$, the pressure difference, dp, across the element is:

$$dp = \rho g dh \tag{i}$$

$$
\begin{aligned}
pv^{1.4} &= 80,350 \\
v &= \left(\frac{80,350}{p}\right)^{1/1.4} \\
\rho &= \frac{1}{v} = \left(\frac{p}{80,350}\right)^{0.7143}
\end{aligned}
$$

Substituting ρ in dp,

$$
\begin{aligned}
dp &= \left(\frac{p}{80,350}\right)^{0.7143} \times g \times dh \\
\frac{dp}{p^{0.7143}} &= (80,350)^{-0.7143} \times 9.81 \times dh = 3.076 \times 10^{-3} dh \\
p^{-0.7143} dp &= 3.076 \times 10^{-3} dh \qquad \text{(ii)}
\end{aligned}
$$

Considering the boundary conditions, $p = 0$ at $h = 0$ and $p = p_{atm}$ at $h = h_{atm}$. Integrating (ii) between the above boundary conditions:

$$
\begin{aligned}
3.076 \times 10^{-3} \int_{h=0}^{h=h_{atm}} dh &= \int_{p=0}^{p=p_{atm}} p^{-0.7143} dp \\
3.076 \times 10^{-3} h_{atm} &= \frac{p^{1-0.7143}}{1 - 0.7143} = \frac{p_{atm}^{0.2857}}{0.2857}
\end{aligned}
$$

$$= \frac{(1.01 \times 10^5)^{0.2857}}{0.2857} = 94.15$$

$$h_{atm} = \frac{94.15}{3.076} \times 10^3 = 30,608 \text{ m} = 30.608 \text{ km} \quad \overset{\text{Ans}}{\Longleftarrow}$$

1.25 Consider a U-tube manometer. One limb of the manometer is connected to the test cell while the other end is evacuated and sealed. If the silicon oil filled in the manometer has a specific gravity of 0.8 and the difference in height of the oil in the two limbs is 40 cm, determine the pressure of the gas in the test cell. Draw the schematic diagram of the arrangement.

Solution

The schematic diagram is shown in Fig.1.45. Pressure at a = Pressure at b and Pressure above c is zero since it is evacuated:

$$p_b = \rho_{\text{oil}} g h_{\text{oil}}$$

$$= 800 \times 9.81 \times 0.4$$

$$= 3139.2 \text{ Pa} = 3.14 \text{ kPa}$$

The pressure of the gas in the test cell is 3.14 kPa

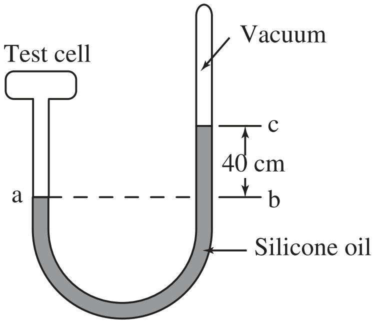


Fig. 1.45

Review Questions

1.1 Define thermodynamics.

1.2 What do you understand by the term 'dimension' and what is meant by units?

1.3 What is a thermodynamic system? Explain the various types of thermodynamic systems.

1.4 Define boundary. Explain different types of boundaries.

1.5 Explain the term 'property'. What are fundamental and derived properties?

1.6 Distinguish between the three terms 'change of state', 'path' and 'process'.

1.7 Explain the following: (i) State function (ii) State diagram (iii) State postulate

1.8 Explain with figures what do you understand by quasi-static process.

1.9 What is meant by a thermodynamic cycle?

1.10 What is the difference between thermal equilibrium and thermodynamic equilibrium?

1.11 With figures briefly explain the concept of continuum.

1.12 Define pressure. What are the four type of pressures that we come across in thermodynamics? Indicate them in a sketch.

1.13 Explain with the help a sketch the working principle of a barometer to measure atmospheric pressure.

1.14 Explain with a neat sketch the following bringing out their advantages and disadvantages.
(i) U-tube manometer (ii) Cistern manometer (iii) Micro-manometer

1.15 Explain the working of the Bourdon pressure gauge with a neat sketch. Mention the advantages, limitations and application of Bourdon pressure gauge.

1.16 Explain the concept of temperature.

1.17 State zeroth law of thermodynamics. Explain why is called zeroth law.

1.18 Explain how zeroth law can be used to measure temperature.

1.19 Explain what is meant by equality of temperature.

1.20 Explain by means of a sketch a liquid in glass thermometer. What are its advantages and disadvantages?

1.21 Explain the principle of operation of a bimetallic thermometer. Mention its advantages and limitations.

1.22 Explain the principle of operation of the following thermometers: (i) constant volume thermometer, (ii) constant pressure thermometer, and(iii) vapour pressure thermometer

1.23 Explain with a sketch the electric resistance thermometer bringing out its advantages and disadvantages.

1.24 Define Seebeck effect. Explain how it can be used for the measurement of temperature.

1.25 What is pyrometer? Explain with a sketch the principle of working of the following pyrometers: (i) radiation pyrometers (ii) optical pyrometers.

Exercise

1.1 In what units area, volume, density and velocity are expressed. Give their units and indicate their dimensions. Establish the relation between their base units and derived units. *Ans:* (i) $A = L^2 = \text{m}^2$; (ii) $V = L^3 = \text{m}^3$; (iii) $\rho = M/L^3 = \text{kg/m}^3$; (iv) $\mathbb{V} = M/L^3 = \text{kg/m}^3$;

1.2 Express the following quantities in terms of base SI units. Refer appropriate tables.

(i) energy density (ii) heat capacity (iii) moment of force (iv) specific energy (v) electric field strength *Ans:* (i) energy density : kg/m s^2; (ii) heat capacity : m^2 kg/s^2 K; (iii) moment of force : m^2 kg/s^2; (iv) specific energy : m^2 /s^2; (v) electric field strength : m kg/s^3 A

1.3 An orange weighs 100 g and geometry can be approximated to a perfect sphere. If the diameter is 6 cm and it is kept in a refrigerator at 15 °C, identify appropriate intensive and extensive properties of the orange and the values.

Ans: extensive property : volume = 113.04 cm^3;
intensive property : density = 0.8846 kg/mm^3

1.4 Classify the following, $P, F, V, \mathbb{V}, \rho, T, a, m, L, t$ and V into intensive, extensive and non-properties. *Ans:* intensive properties : $P, \mathbb{V}, \rho, T$; extensive properties : V, m; non-properties: F, a, L, t, V

1.5 Convert the following readings of pressure to kPa assuming that the barometer reads 760 mm of Hg. Take density of mercury as 13.6×10^3 kg/m^3: (i) 50 cm Hg vacuum (ii) 96 cm Hg gauge (iii) 125 cm H_2O gauge. Also estimate the corresponding absolute pressure.

Ans: (i) 66.7 kPa, 34.65 kPa; (ii) 128 kPa, 226.325 kPa; (iii) 12.62 kPa, 223.95 kPa

1.6 A cylindrical vessel with a diameter of 40 cm and height of 80 cm is filled with 5 g of gas. Estimate the specific volume and density of the gas in the cylinder. *Ans:* 0.02 cc/g; 50 g/cc

1.7 An one tonne car moving at 20 km/h is to be accelerated at a constant rate of 5 m/s^2 up to a speed of 80 km/h. Estimate the force and time required. *Ans:* 3.333 s; 5000 N

1.8 Express the following quantities as indicated

(i) 20 Åin micrometer and nanometer (ii) 50 g/cc in kg/m^3 (iii) 0.38 J in mJ and MJ *Ans:* (i) 20 nm; (ii) 5×10^3 kg/m^3; (iii) 0.38×10^{-6} MJ

1.9 Consider a quantity z. which is the function of x and y. It is given by the expression:

$$dz = \big(4xy + 4y^2\big)dx + \big(2x^2 + 8xy\big)dy$$

Check whether z is a property. If so find z as a function of x and y.

Ans: Yes z is a property; $z = 2x^2y + 4xy^2 + f(y)$; $z = 2x^2y + 4xy^2 + f(x)$

1.10 In a pipe-line, the pressure of gas is read using a mercury manometer. One limb of the manometer is open to atmosphere. The difference is height is measure as 500 mm of Hg. Find the gas pressure, $\rho_{Hg} = 13,640$ kg/m^3; $g = 9.8$ m/s^2. Take barometric pressure as 760 mm of Hg. *Ans:* 168.43 bar

1.11 Classify the following properties:

(i) Pressure
(ii) Temperature
(iii) Volume
(iv) Internal energy
(v) Volume per mole
(vi) Mass
(vii) Enthalpy per unit mass

Ans: Extensive property : (iii), (iv), (vi); Intensive property : (i), (ii); Specific property : (vii); Molar property : (v)

1.12 Express the following quantities as indicated.

(i) 200, 000, 000 W in kW and in MW;
(ii) 20 kPa in Torr and in bar;
(iii) 10 mm of Hg in kPa;
(iv) 20 bar in MPa;
(v) 1 electronvolt in J;
(vi) 30 dm^3 in litre and in m^3.

Ans: (i) 200, 000 kW, 200 MW; (ii) 150 torr; (iii) 1.333 kPa; (iv) 2 MPa; (v) 1.602 $\times 10^{-19}$; (vi) 30 litre; 0.03 m^3

1.13 Assume that you have started filling a helium balloon having a full capacity of 1.2 ℓ. While filling the balloon, you stop to make sure that it is not going to explode by checking, the pressure. At the time of checking the pressure is found to be 6.7 kPa while the temperature at the place of filling is 17 °C when the balloon is 1 ℓ full. When the balloon is fully inflated the pressure is 7.2 kPa. However, when you take the balloon outside, it explodes. What would be the temperature to cause this explosion? *Ans:* 374 K

1.14 A U-tube manometer using mercury as the manometeric fluid has one arm open to atmosphere is used for measuring pressure in a steam pipe. Some of the steam condenses in the manometer arm connected to the pipe. The height of the column is 34 mm. The level of mercury in the open arm is 100 mm. Atmospheric pressure is 760 mm of Hg. Calculate the absolute pressure of the steam. Draw a schematic diagram. *Ans:* 1.144 bar

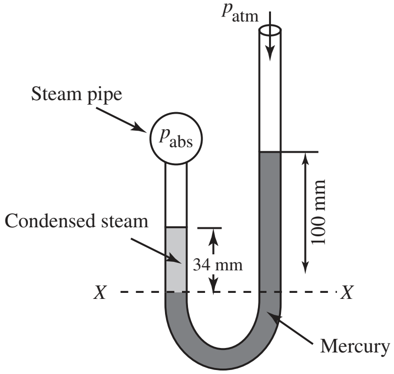


Fig. 1.46

1.15 The following two equations give the expression for differential pressure in terms of equation of state. Using the concept of properties, identify the correct expression and determine the equation of state.

$$dp_1 = \frac{2(v-b)}{RT}dv + \frac{(v-b)^2}{RT^2}dT \qquad (1)$$

$$dp_2 = -\frac{RT}{(v-b)^2}dv + \frac{R}{(v-b)}dT \qquad (2)$$

Ans: $p = \dfrac{RT}{v-b} + C$

1.16 The differential pressure, p, of a thermodynamic system is expressed by the equation:

$$dp = \frac{R}{V-b}dT - \left[\frac{RT}{(V-b)^2} - \frac{2a}{V^3}\right]dV$$

where V is the volume, T is the temperature and a, b, R are constants. Check whether the given differential is exact. If so, obtain the relation between p, V and T.

Ans: $p = \dfrac{RT}{V-b} - \dfrac{a}{V^2} + C$

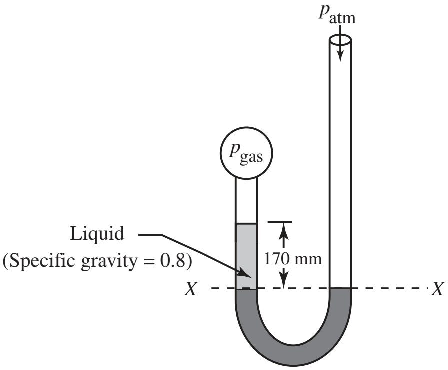


Fig. 1.47

1.17 Read Fig.1.47 with the details given. Estimate the gas pressure in bar. *Ans:* 1 bar

1.18 Estimate the mass of the piston that can be supplied by the gas entrapped under the piston in a 200 mm diameter vertical cylinder when a manometer indicates a difference of 117 mm of Hg column for the gas pressure. *Ans:* 490.4 kg

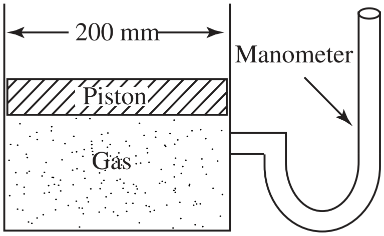


Fig. 1.48

1.19 A 10-m high cylindrical jar open to atmosphere as shown in Fig.1.49 is filled with water having a density of 1000 kg/m^3. The upper half is filled with oil having a specific gravity of 0.85. Find the pressure difference between top and bottom of the cylinder in torr. Assume that the density of mercury is 13, 550 kg/m^3 and $g = 9.8$ m/s^2. *Ans:* 683 torr

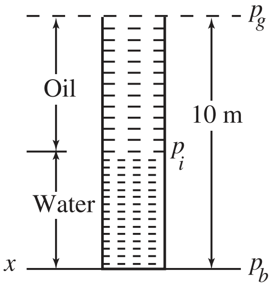


Fig. 1.49

1.20 Consider an evacuated cubical vessel having a side of 25 cm. It is closed by a square lid. Find the effort (force) required to lift the lid. Take atmospheric pressure as 76 cm of mercury and $g = 9.8$ m/s^2 and $\rho_{HG} = 13,350$. *Ans:* 90, 828.4 N

1.21 Water is flowing through a pipe. A mercury manometer is used to find the pressure drop. If the difference in the mercury level between two limbs is 40 cm, estimate the pressure drop. Take $\rho_{\text{Hg}} = 13.6 \times 10^3$ kg/m^3 and $g = 9.8$ m/s^2. Draw schematic diagram showing the manometer connection. *Ans:* 49.4424 kPa

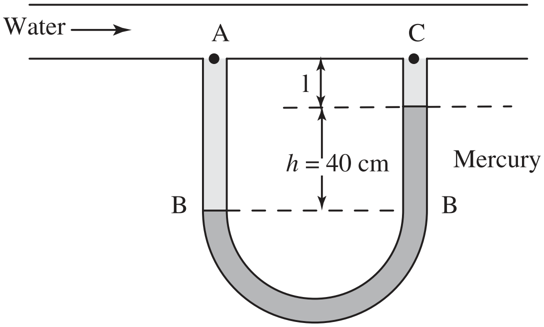


Fig. 1.50 Manometric connections

1.22 Gas is flowing through a pipeline. Pressure is measured by a water manometer. The difference in the water level between both limbs are 950 mm. Calculate the pressure with which gas is flowing. The atmospheric pressure is 760 mm of mercury, $\rho_{\text{Hg}} = 13.6 \times 10^3$ kg/m^3, $g = 9.8$ m/s$^2 and \rho_w = 1000$ kg/m^3. *Ans:* 1.1 bar

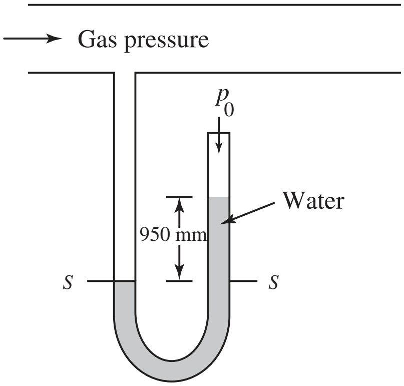


Fig. 1.51

1.23 The water tank shown in Fig.1.52 is kept in a mountain of an altitude 1.5 km. The water is pressurised by air. If the atmospheric pressure at that altitude is 85.5 kPa. Determine the air pressure in the tank, if $h_1 = 0.1$ m, $h_2 = 0.2$ m and $h_3 = 0.3$ m. Take the densities of water, oil and mercury to be 1000 kg/m^3, 850 kg/m^3 and 13, 600 kg/m^3, respectively. *Ans:* 122.88 kPa

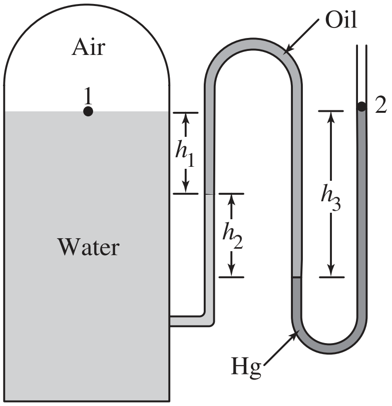


Fig. 1.52

1.24 A steam turbine is supplied with steam at a gauge pressure if 1.5 MPa. After expansion in the turbine, the steam flows into a condenser which is maintained at 700 mm of Hg. The barometric pressure is 770 mm of Hg. Find the steam pressure at inlet and exhaust in pascals (absolute). Take $\rho_{\text{Hg}} = 13.6 \times 10^3$ kg/m^3 and $g = 9.8$ m/s^2. *Ans:* 9.339kPa

1.25 Consider Fig.1.53. An inclined tube manometer is connected to the duct whose pressure is to be measured. The measuring arm of the manometer is inclined at an angle of 45° from horizontal. Assume that the manometeric liquid has a density of 0.81 kg/ℓ. The vertical distance between the fluid level is 12 cm as shown in the figure. Estimate the gauge pressure of air in the duct and the length (L) of the fluid column in the inclined arm above the fluid level in the vertical arm. Explain the purpose of using an inclined manometer.
Ans: 954 Pa; 17 cm; compared to the vertical height of 12 cm, the inclined tube registers 17 cm provides better accuracy in reading.

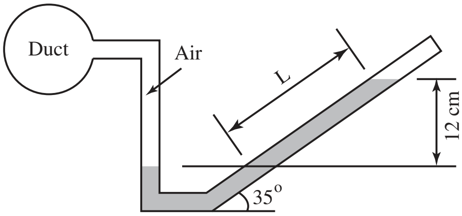


Fig. 1.53

Multiple Choice Questions (choose the most appropriate answer)

1. Path of the system refers to
   (a) loci of various intermediate state
   (b) no changes in the system during a process
   (c) state diagram during a process which is quasi-static
   (d) interaction with boundaries

2. In a cycle
    (a) end states are different
    (b) end states are identical
    (c) there is no connection between starting and ending state
    (d) cyclic integral of any property is nonzero
3. In macroscopic approach of thermodynamics
    (a) the substance is considered to be continuous
    (b) a system can be described using a few measurable properties
    (c) a system is viewed as homogeneous matter without any discontinuity
    (d) all of the above
4. A fixed quantity of matter or a region in space chosen for study is known as
    (a) thermodynamic cycle
    (b) thermodynamic system
    (c) thermodynamic process
    (d) thermodynamic law
5. A system is considered as open system when
    (a) heat and work cross the boundary but not the mass of the working substance
    (b) mass of working substance crosses the boundary but not heat and work
    (c) neither heat and work nor mass of the working substances cross the boundary
    (d) both the heat and work as well as mass of the working substances cross the boundary
6. An isolated system
    (a) is one which cannot transfer either energy or mass to or from the surroundings
    (b) is a region where transfer of energy and/or mass can take place
    (c) is a region of constant mass and only energy is allowed to cross the boundaries
    (d) is one where mass within the system need not be constant
7. The heating and expanding of a gas is called
    (a) thermodynamic system
    (b) thermodynamic cycle
    (c) thermodynamic process
    (d) thermodynamic law
8. A series of operations that take place in a certain order and restores initial condition is
    (a) reversible cycle
    (b) irreversible cycle
    (c) thermodynamic cycle
    (d) all of the above

9. The series of state changes through which the system passes during a process is called
   (a) cycle
   (b) path
   (c) state
   (d) phase

10. In thermodynamics, state means
    (a) general condition of the system
    (b) not measurable by its properties
    (c) complete description of a system in terms of its properties
    (d) none of the above

11. As per the zeroth law of thermodynamics, if the two bodies A and B are in thermal equilibrium with a third body C separately, then the two bodies A and B shall also be in thermal equilibrium with each other. Using this law as basis of temperature measurement of the body A, the thermometer shall be referring to
    (a) Body A
    (b) Body B
    (c) Body C
    (d) none of the above

12. The instrument used for measuring atmospheric pressure is called
    (a) U-tube manometer
    (b) cistern manometer
    (c) micro-manometer
    (d) barometer

13. Gauge pressure is written as
    (a) $p_{gauge} = p_{abs} - p_{atm}$
    (b) $p_{gauge} = p_{abs} + p_{atm}$
    (c) $p_{gauge} = p_{abs} \times p_{atm}$
    (d) $p_{gauge} = p_{abs}/p_{atm}$

14. Small and medium pressures are measured by
    (a) barometer
    (b) mercury manometer
    (c) pressure gauge
    (d) water or alcohol manometer

15. Choose the correct answer.
    (a) manometers are suitable for high pressure measurements
    (b) manometers are used for recording pressures

(c) manometers requires very little maintenance

(d) manometers are compact and sturdy

16. In Fahrenheit scale
    (a) ice point is 32 °F and steam point is 212 °F
    (b) ice point is 0 °F and steam point is 100 °F
    (c) ice point is 0 °C and steam point is 100 °C
    (d) ice point is 32 °C and steam point is 212 °C

17. When two bodies are in thermal equilibrium with a third body, they are also in thermal equilibrium with each other. This statement is called
    (a) zeroth law of thermodynamics
    (b) first law of thermodynamics
    (c) second law of thermodynamics
    (d) Kelvin Planck's law

18. Which one in the following cannot be used for temperature measurement
    (a) liquid-in-glass manometer
    (b) pressure thermometer
    (c) resistance thermometers
    (d) radiation pyrometers

19. In a constant volume gas thermometer the thermometric property used is
    (a) length
    (b) volume
    (c) mass
    (d) resistance

20. The temperature at which the volume of a gas becomes zero is called
    (a) absolute scale of temperature
    (b) absolute zero temperature
    (c) absolute temperature
    (d) none of the above

21. Mercury is the preferred manometric fluid, because
    (a) it can be seen clearly
    (b) it is a good conductor of heat
    (c) It does not wet the wall
    (d) all of the above

22. Celsius scale is between
    (a) 0 °C to 100 °C
    (b) 32 °C to 212 °C

   (c) 273 °C to 373 °C
   (d) 492 °C to 582 °C

23. A thermocouple
   (a) works on Seebeck effect
   (b) contains only one junction of dissimilar metals
   (c) does not require any calibration
   (d) can be used for pressure measurements also

24. An optical pyrometer is preferred because
   (a) measurements depend upon observers judgements
   (b) no direct line of sight is required
   (c) temperature of a moving object can be measured
   (d) it can be used to measure very low temperature

25. Zeroth law provides the concept for
   (a) enthalpy
   (b) entropy
   (c) energy
   (d) temperature

Ans: 1. – (a) 2. – (b) 3. – (d) 4. – (b) 5. – (d)
6. – (a) 7. – (c) 8. – (d) 9. – (a) 10. – (c)
11. – (c) 12. – (d) 13. – (a) 14. – (d) 15. – (c)
16. – (a) 17. – (a) 18. – (a) 19. – (a) 20. – (b)
21. – (d) 22. – (a) 23. – (a) 24. – (c) 25. – (d)

2

ENERGY, WORK AND HEAT

2.1 INTRODUCTION

In the last chapter, we introduced basic concepts of thermodynamics. By now you must be aware that thermodynamics is a basic science that principally deals with *energy.* It is mainly concerned with the following two important aspects:

(i) the study of energy and its various forms and

(ii) the process of its transformation.

Even though it is easy to feel energy, like temperature, it is difficult to define. The term, 'energy' was coined in 1807 by Thomas Young. Its use in thermodynamics was proposed in 1852 by Lord Kelvin. Before going further, it is essential to understand clearly the meaning and the applications of the three terms, viz energy, work and heat. This chapter is devoted for the same.

In this chapter, we will try to understand and appreciate the meaning of energy, work and heat and also go through various forms of energy. We will see the difference between mechanical work and thermodynamic work. The difference between the flow and non-flow work and the sign convention adopted to designate work and heat will be explained.

2.1.1 Energy, Work and Heat

Energy is normally viewed as the capacity to do work and has the ability to cause changes. The concept of energy evolved very slowly over a very long period of time. It has finally led to the establishment of the general principle of the *law of conservation of energy.* It states that during an interaction, energy can change from one form to another but the total amount remains the same. It simply means that *energy is neither created nor destroyed. It only changes form.*

Work is nothing but an energy interaction between a system and surroundings. First, you should understand that energy can cross the boundary of a closed system in the form of *work* and/or *heat.* Note that *work* and *heat* are the only two mechanisms which can change the energy of a closed system. Therefore, if there is no heat transfer then the energy change in a closed system must be only due to work transfer. This point we will emphasise again and again in this chapter. As work is a form of energy, it is expressed in terms of joules (J). The work done during a process between states 1 and 2 is denoted by W_{12} or simply W. The work done per unit time is called power and denoted by $\dot{W}$. The unit for power is J/s or W.

Heat is also a form of energy. It can be transferred between two systems or a system and its surrounding by virtue of a temperature difference between them. The meaning of the term 'heat' in thermodynamics is different from that in day-to-day usage. In normal usage, heat is often used to mean internal energy. The typical example is heat content of a fuel. *Understand that heat and internal energy are two different entities.* **Energy is a property but heat is not**. A body contains energy but not heat. Energy is associated with a state and heat is associated with a process. In short, *heat is nothing but the energy in transition.* It is recognised only when it crosses the boundary of a system.

2.2 ENERGY AND ITS FORMS

Energy is a general term which includes *energy in transition* and *stored energy.* As already stated, it is indeed quite difficult to give a precise definition for the term 'energy'. Let us recall the two points which we have already made.

(i) Energy may be considered as the ability to do mechanical work.

(ii) Energy may be referred to as the capacity for producing effect.

Normally, energy is in the stored form. The stored energy of a substance may be broadly classified into:

(i) mechanical energy

(ii) internal energy

Other forms of stored energy are *chemical energy* and *electrical energy.*

Part of the stored energy may take the form of either as potential energy due to height above a chosen datum line, or kinetic energy due to velocity. The balance part of the energy is known as *internal energy.*

In a **non-flow process***, usually there is no change of potential or kinetic energy and hence, change of mechanical energy will not enter in to the calculations. In a* **flow process***, however, there may be changes in both potential and kinetic energy and these must be taken into account while considering the changes in stored energy.*

To reiterate again, note the following:

(i) Heat and work are the forms of energy in transition.

(ii) These are the only forms by which energy can cross the boundaries of a system.

(iii) Neither heat nor work can exist as stored energy.

Total energy at any moment may be the algebraic sum of the different forms of energy. Conversion of energy from one form to another is possible. In thermodynamics, we are primarily interested in studying the change in total energy of a system. Thus, for analysis, relative value of energy is considered instead of absolute value. From the above description, we can classify energy in the following general categories:

(i) *Energy in transition*: It refers to the energy that is in process of transition between substances or regions because of some driving potential. Typical driving potentials are *difference* of force or of temperature or of electrical potential. The difference is usually called *gradient.*

(ii) *Energy stored in particular mass*: It refers to the potential and kinetic energy associated with mass which are at a height with respect to a datum or moving with respect to the earth.

Apart from above broad classification, energy can also be categorised into the following two forms:

(i) *Macroscopic energy*: It refers to the energy possessed by a system considered at macroscopic level such as kinetic energy and potential energy.

(ii) *Microscopic energy*: It refers to the energy defined at molecular level.

Summation of energy at molecular level (microscopic energy) results in internal energy.

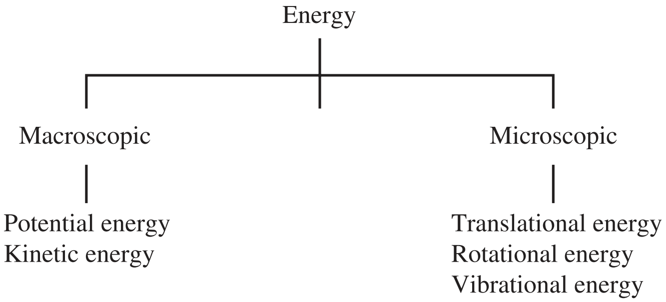


Fig. 2.1 Category of energy

For example, if we ask the question, does a glass of water sitting on a table (Fig.2.2) has any energy? The answer will be, at room temperature, a glass of water placed on a table has no apparent macroscopic energy, either potential or kinetic. But on the microscopic scale, there is a large mass of high-speed molecules travelling at hundreds of meters per second having kinetic energy. Further, there is a good amount of molecular attractive forces associated with potential energy. All depends upon how do we look at it. Now, let us see the various forms of energy in detail.

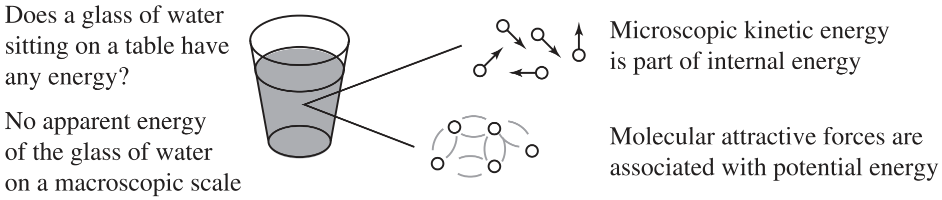


Fig. 2.2 Macroscopic and microscopic energy

Potential energy : This type of energy is based on relative position of bodies in a system, i.e. elevation in a gravitational field. Potential energy for a mass m at elevation h is given as:

$$PE = mgh \tag{2.1}$$

where m is the mass, g is the gravitational acceleration and h is the elevation measured from some reference point.

Kinetic energy: It is based on the relative movement of bodies. For a mass m moving with certain velocity, it could be mathematically expressed as:

$$KE = \frac{1}{2}m\mathbb{V}^2 \tag{2.2}$$

where m is the mass and $\mathbb{V}$ is the linear velocity. This means that the kinetic energy of a system is by virtue of its velocity.

Total energy: The total energy (E) is the sum of all forms of energy (E_i). It is the sum of energy associated with micro- and macro-level. Mathematically,

$$E = \sum_i E_i \tag{2.3}$$

E_i may include sensible energy, latent energy, chemical energy, nuclear energy and so on, where
Sensible energy refers to the kinetic energy associated with molecules.
Latent energy refers to the energy associated with phase of a substance.
Chemical energy refers to the energy associated with molecular bonds.
Nuclear energy refers to the energy associated with the bonds within nucleus of atom itself.

Internal energy: Internal energy can be viewed as the energy needed to create the system. It involves energy on the microscopic scale. At microscopic level, a system can be viewed as atoms and molecules. Internal energy refers to the invisible microscopic energy on the atomic and molecular scale. The energy associated with the disordered random motion of molecules is called internal energy. Therefore, in most of the engineering analysis, it will be broadly divided into potential energy (U_{PE}) and kinetic energy (U_{KE}) components of the moving atoms and molecules. Mathematically, it can be written as:

$$U = U_{PE} + U_{KE} \tag{2.4}$$

The potential energy (U_{PE}) includes all energies given by the *mass* of atomic particles. whereas the kinetic energy (U_{KE}) is the sum of the *motions* of all the system's atomic particles.

Note that internal energy does not include the energy due to motion of a system as a whole. It further excludes any kinetic or potential energy the body may have because of its location. The internal energy is an extensive property. It depends on the size of the system or on the amount of substance it contains. To put it in a nut shell, internal energy of a system is the energy associated with the molecular structure at molecular level.

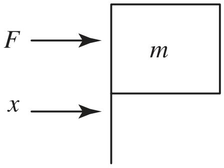
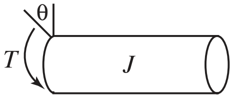
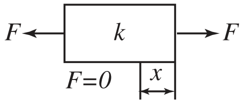
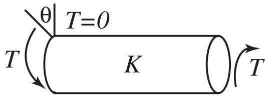
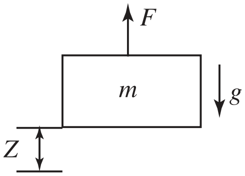
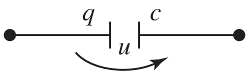
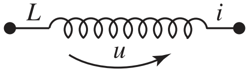
Total energy (TE) of a system may be given as summation of different forms of energy at a moment. Mathematically,

$$TE = KE + PE + IE = mgh + \frac{1}{2}m\mathbb{V}^2 + U_{PE} + U_{KE} \tag{2.5}$$

Some different forms of energy interaction and associated work interactions with block diagram are given in Table 2.1. To give a practical example, let us consider the fall of a 'weight' from certain height on the floor taking datum as the floor. Upon hitting the floor, assume that the 'weight' comes to dead stop. Its potential energy and kinetic energy of the system become zero.

Now, one interesting question arises. Where does the vanishing potential energy and kinetic energy go when the 'weight' touched the floor? If we touch the point of contact of the 'weight' and floor, we will observe that both these points of contact on the floor and on the weight will be *slightly* hotter than that before impact. Thus, it is obvious that the energy has changed its form from *potential and kinetic to internal energy* and causes rise in temperature of 'weight' and 'floor' at the points of contact.

Table 2.1 *Some forms of energy and the associated work interactions*

Macroscopic form of energy	Governing equation	Energy interaction	Work interaction	Block diagram
Kinetic energy (translational)	$F = m\dfrac{d\mathbb{V}}{dt}$	$\Delta E = \frac{1}{2}m\left(\mathbb{V}_2^2 - \mathbb{V}_1^2\right)$	$-F\,dx$	
Kinetic energy (rotational)	$T = J\dfrac{d\omega}{dt}$	$\Delta E = \frac{1}{2}J\left(\omega_2^2 - \omega_1^2\right)$	$-T\,d\theta$	
Spring stored energy (translational)	$F = kx$	$\Delta E = \frac{1}{2}k\left(x_2^2 - x_1^2\right)$	$-F\,dx$	
Spring stored energy (rotational)	$T = K\theta$	$\Delta E = \frac{1}{2}K\left(\theta_2^2 - \theta_1^2\right)$	$-T\,d\theta$	
Gravitational energy	$F = mg$	$\Delta E = mg\left(Z_2^2 - Z_1^2\right)$	$-F\,dz$	
Electrical energy (capacitance)	$u = m\dfrac{q}{c}$	$\Delta E = \frac{1}{2}\,\frac{q^2}{c} = \frac{1}{2}cu^2$	$-u\,dg$	
Electrical energy (inductance)	$\phi = Li$	$\Delta E = \frac{1}{2}Li^2 = \frac{1}{2}\,\frac{\phi^2}{L}$	$-i\,d\phi$	

2.3 WORK

We are aware that thermodynamics is the science that deals with the interactions between work and heat. Therefore, before going further, let us concentrate a little more on the two terms, *work* and *heat*.

Recall that, work is nothing but an energy interaction between a system and its surroundings. We have already seen that energy can cross the boundary of even a closed system in the form of work or heat. To emphasise again, note that *energy of a closed system can be changed* **only** *by work or heat transfer*. Therefore, if it is **not work** then it is **only heat**. Again, it is reiterated here that neither heat nor work can exist as stored energy. Now, let us see the various forms of work and how work is treated in thermodynamics.

2.3.1 Definition of Work

Work can be defined in two ways:

(i) by using the principle of mechanics and

(ii) by using the principle of thermodynamics.

2.3.2 Definition Based on Mechanics

The term 'work' was first coined in 1826 by the French mathematician Gaspard-Gustave Coriolis. *Work is said to be done when the point of application of a force moves through a certain distance in the direction of force.* The magnitude of work done is equal to the product of the force and the distance through which the point of application of force moves. *It is important to note that the point should move in the direction of force* (refer example 2.14 in this chapter).

To put it mathematically, if a constant force of magnitude F acts on a point that moves through a distance s in the direction of the force, then the work W done by this force, F, is:

$$W = Fs \tag{2.6}$$

For example, if a constant force of 12 newtons ($F = 12$ N) acts along a path of 1.5 metres, ($s = 1.5$ m), then the work done W is equal to:

$$W = (12\text{ N})(1.5\text{ m}) = 18\text{ N m} = 18\text{ J}$$

2.4 MECHANICAL WORK

If the force F is not constant, then the work done is obtained by adding (i.e. integrating) the differential amounts of work (force times the differential displacement ds). That is:

$$W = \int_1^2 F ds \tag{2.7}$$

In most of the thermodynamic analyses, mechanical work is the main form of work involved. It is associated with the movement of the boundary of a system or with the movement of the entire system as a whole. Let us see the various forms of mechanical work before going into the details of thermodynamic work. Mechanical work can be arranged alphabetically into the following:

(i) Acceleration work
(ii) Displacement or moving boundary work
(iii) Gravitational work
(iv) Shaft work
(v) Spring work

2.4.1 Acceleration Work

Acceleration work is defined as the work associated with a change in velocity of a system. According to Newton's second law of motion:

$$F \quad = \quad ma \tag{2.8}$$

where F is the force and m is the mass of the system and a is the acceleration. Acceleration, a in terms of velocity, $\mathbb{V}$ can be written as:

$$a = \frac{d\mathbb{V}}{dt} \tag{2.9}$$

Therefore,

$$F = m\frac{d\mathbb{V}}{dt} \tag{2.10}$$

Velocity in terms of displacement is given by:

$$\mathbb{V} = \frac{ds}{dt} \tag{2.11}$$

or

$$ds = \mathbb{V}dt \tag{2.12}$$

The acceleration work W_a can be written as:

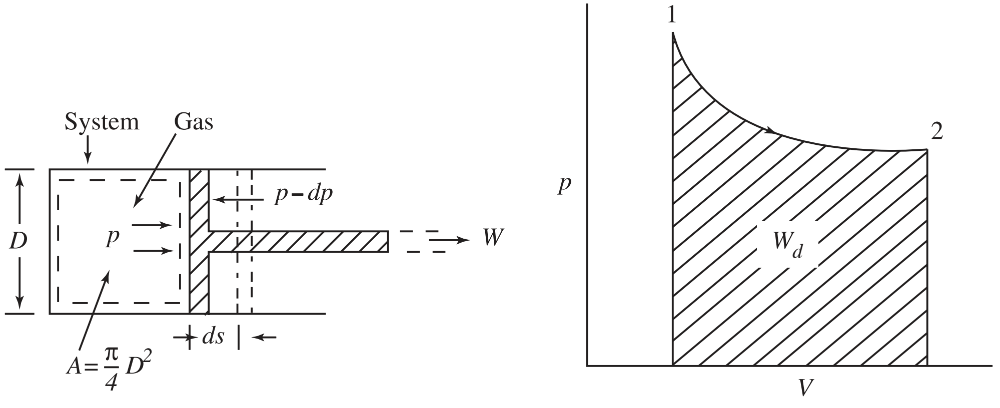
$$W_a = \int_1^2 F ds = \int_1^2 \left(m\frac{d\mathbb{V}}{dt}\right)(\mathbb{V}dt) \quad m\int_1^2 \mathbb{V}d\mathbb{V} = \frac{1}{2}m(\mathbb{V}_2^2 - \mathbb{V}_1^2) \tag{2.13}$$

From the above equation, it is clear that work done to accelerate a body is independent of path and is equivalent to the change in kinetic energy of the body.

2.4.2 Displacement or Moving Boundary Work

Mechanical work frequently encountered in practice is the displacement or moving boundary work. When a system boundary sweeps a volume while interacting with its surroundings, it is called the *moving boundary* or *displacement work*. Typical examples are the compression and expansion of a gas in a cylinder-piston device. It may be noted that the volume swept can be against the surroundings or against the system itself. It depends upon whether the work is done by the system or on the system.

Before going into the details for evaluating displacement work, the reader should note that displacement work can be evaluated only for a *quasi-static process*. We have already dealt with quasi-static process in Chapter 1. It is a process which is *extremely* slow. For example,



(a) Example of workdone during a reversible expansion process

(b) Typical path 1-2 and displacement work in a quasi-static process

Fig. 2.3 Displacement or moving boundary work

consider a certain amount of gas trapped in a piston-cylinder assembly as shown in Fig.2.3(a).

Let A be the piston cross-sectional area and ds be its displacement during a small interval of time dt. The infinitesimal work done by the system is given by:

$$\delta W \quad = \quad \text{Force} \times \text{Distance} \quad = \quad pA(ds) = pdV \tag{2.14}$$

If the gas is allowed to expand extremely slowly or In other words, *quasistatically (reversibly)* from the initial pressure, p_1, to final pressure, p_2 [refer Fig.2.3(b)], the total work done by the gas during the expansion process is:

$$W_d \quad = \quad \int_1^2 pdV \tag{2.15}$$

It is due to this relation, displacement work is also called *pdV work*. The process is reversible because during the expansion, the external pressure is always infinitesimally smaller $(p - dp)$ than the gas pressure (p) [see Fig.2.3(a)]. The expansion process can be reversed at any time by $(p - dp)$ increasing the external pressure infinitesimally more than the gas pressure. In other words, during this expansion process, the forces on the moving boundary are balanced. Therefore, the system is said to have undergone a reversible process. This process can be represented on a pressure-volume diagram as shown in Fig.2.3(b) in Cartesian coordinates. The displacement work is given by Eq.2.15.

At any instant during the process, the uniform system pressure exerted on the moving part of the system boundary is equal to the external resisting pressure. It means that the system is in mechanical equilibrium with its surroundings all through the process. Thus, the path of a quasi-static process on any state diagram is the line joining the successive equilibrium states from the initial to the final state. A quasi-static process is also called a *resisted process* or a *reversible process.*

As can be seen in Fig.2.3(b), p is represented in y-axis and V in x-axis. Area under the curve (hatched portion) is given by:

$$\int pdV = \int ydx \tag{2.16}$$

Now, it can be generalised that the area under the curve in a quasi-static process on a p-V plane [Fig.2.3(b)] is always equal to the displacement work. Note that the path shown in the figure is arbitrary. It can follow a different path also. In this context, we have to understand two important terms, viz *path function and point function.* Let us see the details in the following section.

2.4.3 Path Function and Point Function

Consider the change of state of a system from initial state 1 defined by properties, p_1 and V_1, to final state defined by properties p_2 and V_2 as shown in Fig.2.4. As can be seen, state 2 can be reached by means of any one of the following paths, viz 1-A-2 or 1-B-2 or 1-C-2 or 1-D-2 or 1-E-2. Now, you should note one important point. State 2 will have the same properties p_2 and V_2 irrespective of the path it follows to reach state 2 from state 1. This means that the values of properties of a system in a given state are independent of the path it follows to reach that state. Because of this, characteristic, the *state is a point function.* Now, it should be clear that the *change* in the value of a property between any two given state is the same. This is true irrespective of the path or process followed between any two given states. Now, referring to Fig.2.4, we have

$$\int_1^2 dp \quad = \quad p_2 - p_1 \quad \text{and} \int_1^2 dV \quad = \quad V_2 - V_1 \tag{2.17}$$

are always the same between the states 1 and 2 irrespective of the path followed. Therefore, between two state points, the change in any property is independent of the path. It is due to this fact that properties of a system are called *point functions.*

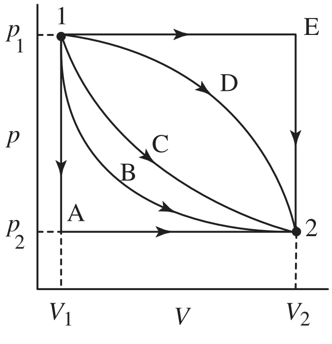


Fig. 2.4 Path and point functions

You should appreciate that the converse of the above is also true. For example, *any quantity whose change is independent of the path is a point function and hence, it is a property.* It is to be emphasised here that we have arrived at *a very important criterion* in thermodynamics. This has been arrived at, by mere logic. We shall use this criterion to show later the existence of two very important thermodynamic properties, viz *internal energy* and *entropy.*

Again consider the five quasi-static processes 1-A-2, 1-B-2, 1-C-2, 1-D-2 or 1-E-2. Note that all the five processes occur between the same two state points, 1 and 2, of a given system. The area below each path must be equal to the corresponding displacement work. Notice that in Fig.2.4, all five areas have different values. Accordingly, the displacement work for each path is different. Hence, between two given state points of a system, the displacement work is not *unique.* It should be noted that the displacement work depends upon the path. It is for this reason that work is said to be a *path function.*

Path functions have *inexact differentials* designated by the symbol δ, the magnitude of a path function depends on the path followed during a process as well as on the end states. Therefore, a differential amount of work is represented by δW instead of dW.

By now, it should be clear that properties are *point functions.* That is, they depend on the state only and not on how it reaches that state. Property changes are designated by the symbol d. For example, a differential amount of pressure or temperature is represented by dp or dT.

A small change in volume is represented by dV and the total volume change during a process between states 1 and 2 is given by:

$$\int_1^2 dV \quad = \quad V_2 - V_1 = \Delta V \tag{2.18}$$

The total work done during the process 1-2 is given by:

$$\int_1^2 \delta W \quad = \quad W_{1-2} \textbf{ and not } \Delta W \tag{2.19}$$

This is because, *work is not a property and systems do not possess work at a state.* Similarly, heat is also a path function for the same reason. Hence, differential heat transfer, which is also an inexact differential, is symbolised by δQ. In summary, between two given state points (refer Fig.2.5):

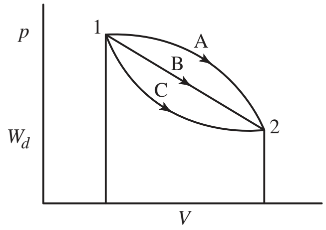


Fig. 2.5 Displacement work in a quasi-static process

(i) $\int \delta W$ along $A \neq \int \delta W$ along $B \neq \int \delta W$ along C where A, B, C refer to different paths.

(ii) For a given path, $\int \delta W \neq (W_2 - W_1)$.

(iii) $\int dp$ along $A = \int dp$ along $B = \int dp$ along $C = (p_2 - p_1)$.

(iv) $\int dV$ along $A = \int dV$ along $B = \int dV$ along $C = (V_2 - V_1)$.

Being a path function, work at a point has no meaning, That is, W_2 and W_1 have no meaning. In general, $\int \delta W$ along a given path is represented as ${}_1W_2$ or W_{1-2}.

2.5 DISPLACEMENT WORK OF RESISTED PROCESSES WITH UNIFORM PRESSURE

We will discuss in the following section the displacement work of various resisted processes with *uniform pressure* undergone by a system which are commonly encountered in engineering practice. They can be classified into the following categories:

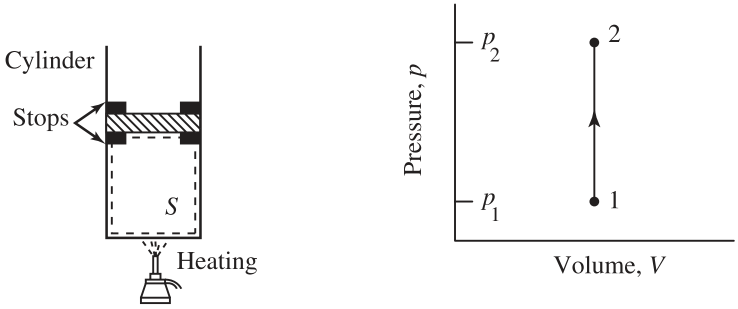
(i)	Constant volume	or	isochoric process
(ii)	Constant pressure	or	isobaric process
(iii)	$p \times V = C$	or	hyperbolic process
(iv)	$p \times V^n = C$	or	Polytropic process

2.5.1 Constant Volume or Isochoric Process

Consider a fluid undergoing a thermodynamic process in a fixed enclosed space shown in Fig.2.6(a). As the system boundary does not sweep any volume, the process is called *constant volume* or *isochoric process.* Let us consider heating of this gas at constant volume. Figure 2.6(b) illustrates the process on a p-V diagram. As can be seen, this process is represented by a vertical line. Mathematically, work done, W, is given by:

$$W = \int_1^2 p dV = 0 \tag{2.20}$$

as change in volume $dV = 0$.



(a) Constant volume process

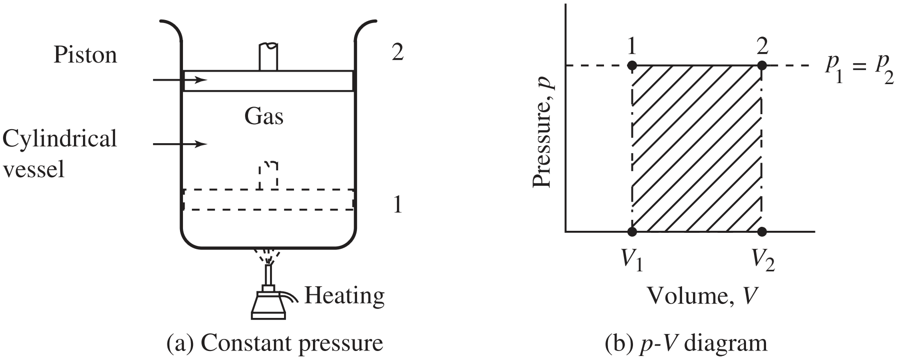
(b) p-V diagram

Fig. 2.6 Constant volume or isochoric process

Area under the process line is zero which indicates that there is only rise in pressure. As there is no change in volume, work involved will be zero. A typical example is the heat addition in an *ideal* Otto cycle (gasoline engine).

2.5.2 Constant Pressure or Isobaric Process

It refers to the thermodynamic process in which there is no change in pressure. Such a process is called *constant pressure or isobaric process.* Consider a cylindrical vessel having gas in it. A piston is placed above the gas as shown in Fig.2.7(a). Assume that piston is free to move up or down in the cylindrical wall. Under normal situation, piston shall be subjected to atmospheric pressure. Let the pressure and volume be p_1 and V_1, respectively. Now, let the gas be heated from bottom of cylinder. Assume that this energy addition is a reversible process. Further, assume that, system always remains in equilibrium. Because of heat addition, the gas will expand. Expansion of gas results in moving the piston up to position 2. Let the pressure and volume be p_2 and V_2, respectively. The process is shown on p-V diagram in Fig.2.7(b). Note that since the pressure above the piston is atmospheric the pressure within the cylinder will also be atmospheric. Thus, the whole process has taken place at constant pressure.



(a) Constant pressure (b) p-V diagram

Fig. 2.7 Constant pressure or isobaric process

The work involved in lifting the piston is given by,

$$W_d = \int_1^2 pdV = p\int_1^2 dV = p(V_2 - V_1) \tag{2.21}$$

If $V_2 > V_1$, it is an expansion process (or a heating process) and W for the fluid is taken as positive and if $V_2 < V_1$ it is a compression process (or a cooling process) and W for the fluid is taken as negative. The path 1-2 shown on the p - V diagram in Fig. 2.7(b) is for an expansion process. The area below this path, shown hatched, is equal to the displacement work.

2.5.3 Hyperbolic Process (pV = Constant)

In this process, the pressure of the system will vary inversely with respect to its volume. It implies that as pressure increases volume will decrease and vice versa so that $p \times V$ is always constant and is called a *hyperbolic process*. Figure 2.8 is the path of one such expansion process shown in the p-V diagram. It is worthwhile to note that hyperbolic process becomes an isothermal process for an ideal gas for which, at constant temperature, $pV = RT = C$. During this process, pV = constant. This means,

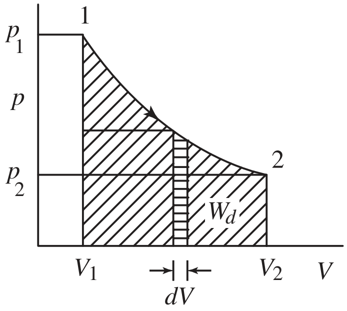


Fig. 2.8 p-V diagram for a hyperbolic process

$$pV = p_1V_1 = p_2V_2 = C \tag{2.22}$$

where C is a constant and hence, $p = \dfrac{C}{V}$.

$$W_d = \int_1^2 pdV = \int_1^2 \left(\frac{C}{V}\right) dV = C\int_1^2 \frac{dV}{V} = C\ln\left(\frac{V_2}{V_1}\right) \tag{2.23}$$

$$W_d = p_1V_1\ln\left(\frac{V_2}{V_1}\right) = p_2V_2\ln\left(\frac{V_2}{V_1}\right) \qquad (\because p_1V_1 = p_2V_2) \tag{2.24}$$

2.5.4 Polytropic Process (pV^n = Constant)

In this process, the pressure of a system varies inversely with V^n and is called a *polytropic process* where n is the exponent of compression or expansion. The path of one such expansion process is shown on p-V coordinates in Fig.2.9.

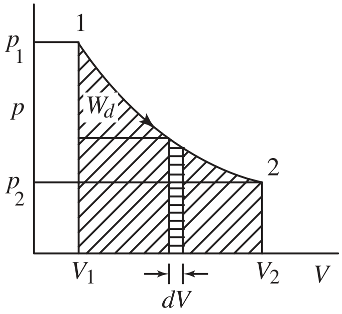


Fig. 2.9 p-V diagram for a polytropic process

During this process, $pV^n = C$. This means,

$$pV^n = p_1V_1^n = p_2V_2^n = C \tag{2.25}$$

where C is a constant and hence, $p = \dfrac{C}{V^n}$.

$$W_d = \int_1^2 pdV = \int_1^2 \left(\frac{C}{V^n}\right) dV = C\int_1^2 V^{-n}dV \tag{2.26}$$

$$= \frac{C}{-n+1}\left[V^{-n+1}\right]_1^2 = \frac{1}{1-n}\left[CV^{1-n}\right]_1^2 = \frac{1}{1-n}\left[pV^nV^{1-n}\right]_1^2$$

$$= \frac{1}{1-n}[pV]_1^2 = \frac{(p_2V_2 - p_1V_1)}{1-n} = \frac{(p_1V_1 - p_2V_2)}{n-1} \tag{2.27}$$

For engineers, especially for mechanical engineers, the work interaction in a reciprocating piston-cylinder device is most important. It is because, of late it has become impossible to live without automobiles which use reciprocating internal combustion engines. These engines do displacement work. For more details, readers may refer the book on, '*Internal Combustion Engine,* 4th Edition' by V Ganesan, McGraw Hill Education, New Delhi, 2012.

The net work output from the engine is called *indicated work*. It is the algebraic sum of the work interactions the piston will have with the working substance during various strokes corresponding to one engine cycle. This work is equal to $\int pdV$ over one engine cycle. Pressure acting on the piston during each stroke varies with the piston position in the cylinder. Hence, to evaluate $\int pdV$ it is necessary to know the pressure acting on the piston at each of its positions during a cycle.

2.5.5 Engine Indicator Diagram

An engine indicator is a device which measures the pressure against each of piston positions (volume) during an engine cycle. For this, a device called indicator will be attached to the engine being tested. The indicator draws a closed curve on the indicator card available in the device (Fig.2.10). The height of the diagram is proportional to the pressure and the length is proportional to the stroke (volume) of the engine. The area of the diagram, thus, is proportional to the net work on the piston in an engine cycle.

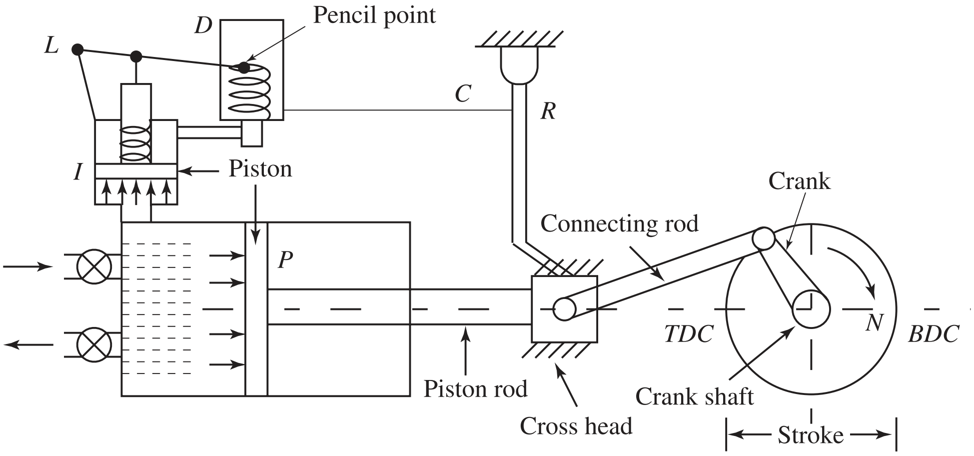


Fig. 2.10 Engine indicator

An indicator diagram is a trace made by a pressure gauge, which has recording facility, is attached to the cylinder of a reciprocating engine. Figure 2.10 shows a typical engine connected with an indicator. The gas pressure is the same on both the engine piston P and the indicator piston I. The indicator piston is loaded by a spring. It moves in direct proportion to the change in pressure. The motion of the indicator piston causes a pencil held at the end of the linkage L to move upon a strip of paper wrapped around drum D. The drum is rotated about its axis by cord C. It is connected through a reduction mechanism R to the piston P of the engine. The surface of drum D moves horizontally under the pencil while the pencil moves vertically over the surface and a plot of pressure upon the piston versus piston travel is obtained. Before tracing the engine indicator diagram, a pressure reference line is recorded. This is achieved by subjecting the indicator to the atmospheric pressure and tracing a line at a constant pressure of one atmosphere.

The area of the indicator diagram represents the magnitude of the net work done by the system(expansion work)in one engine cycle. The area under the path 1–2 represents work done by the system and the area under the path 2–1 represents work done upon the system(compression work)(Fig.2.11). The area of the diagram, a_d, is measured by means of a planimeter and the length of the diagram, l_d, is also measured. The mean effective pressure, p_m, is defined in the following way:

$$p_m = \frac{a_d}{l_d} \times K \tag{2.28}$$

where K is the indicator spring constant (N/cm^2)× travel.

$$\text{Work done in one engine cycle} = (p_m A)\, L \tag{2.29}$$

where A is cross-sectional area of the cylinder $= \frac{\pi}{4}D^2$, where D is the cylinder diameter and L is stroke of piston.

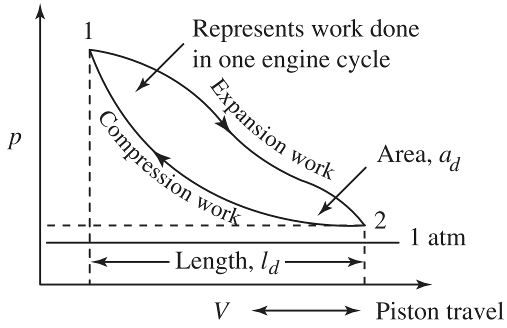


Fig. 2.11 Indicator diagram

Let N be the revolutions per minute (rpm) of the crankshaft. In a two-stroke cycle, the engine cycle is completed in two-strokes of the piston or in one revolution of the crankshaft. In a four-stroke cycle, the engine cycle is completed in four-strokes of the piston or two revolutions of the crankshaft.

For a two-stroke engine, work done in one minute$= p_m LAN$ and for a four-stroke engine, work done in one minute $= p_m LAN/2$.

The power developed inside the cylinder of the engine is called indicated power (IP),

$$IP = \frac{p_m LAnK}{60} \text{ kW} \tag{2.30}$$

where p_m is in kPa, K is the number of cylinders in the engine and $n = N$ for a two-stroke engine and $N/2$ for a four-stroke engine. N is the speed of the engine in rpm.

The power available at the crankshaft is always less than this value (IP) due to friction and other losses and is called as *brake power* (BP). It is also called as *shaft power* (SP). If ω is the angular velocity of the crankshaft in radian/sec, then:

$$BP = \tau\omega \tag{2.31}$$

where τ is the torque transmitted to the crankshaft in Nm. Therefore,

$$BP = \frac{2\pi\tau N}{60} \tag{2.32}$$

where N is the number of revolutions per minute (rpm).

The mechanical efficiency of the engine, η_{mech} is defined as:

$$\eta_{mech} = \frac{BP}{IP} \tag{2.33}$$

An engine is said to be *double-acting*, if the working fluid is made to work on both sides of the piston. Such an engine theoretically develops twice the amount of work developed in a single-acting engine. Most reciprocating steam engines are double-acting and so are many marine diesel engines. Internal combustion engines for road transport are always single-acting.

2.5.6 Mean Effective Pressure

Mean effective pressure (abbreviated usually either as *mep* or p_m) is the average pressure inside the cylinder of the engine based on the measured or the calculated power output. *Mean effective pressure is that pressure which, when multiplied by the displacement volume gives the same net work as actually produced with the varying pressure.* Mean effective pressure is classified into two categories:

(i) Indicated mean effective pressure (*imep*)

(ii) Brake mean effective pressure (*bmep*)

Indicated power and brake power are obtained when they are multiplied by displacement volume.

2.6 DISPLACEMENT WORK IN UNRESISTED PROCESSES

Let the gas, enclosed initially in the boundary S_1 be considered as the system [refer Fig.2.12(a)]. Assume that the right side of the boundary is evacuated. Suppose the diaphragm is suddenly ruptured by some means, the gas in the left part of the vessel will immediately start moving towards right to fill the cylinder completely and the final system boundary will be S_2, as shown in Fig.2.12(b). Although in this process, the system boundary sweeps a volume, it does so against zero pressure. Hence, the process is unresisted. The displacement work in such a process is given by $W = \int pdV = 0$, as the resisting pressure $p = 0$.

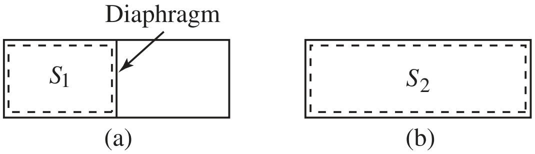


Fig. 2.12 Unresisted process

The above process can be imagined in a different way. Consider the initial as the total internal volume of the cylinder, comprising the left and right portions. This system boundary still contains the same quantity of gas in the left portion, as the right portion has no material. When the diaphragm is ruptured and the gas expands rapidly to occupy the entire cylinder, the imagined system boundary does not change. Thus, the system does not sweep any volume. As $dV = 0$, there is no displacement work.

2.7 GRAVITATIONAL WORK

Gravitational work can be defined as the work done by or against the gravitational force. In a gravity field, the force acting on a body of mass m is given by:

$$F = mg \tag{2.34}$$

where g is the acceleration due to gravity. The work required to raise the body of mass m from elevation z_1 to z_2 is given by:

$$W_g = \int_1^2 Fdz = mg\int_1^2 dz = mg(z_2 - z_1) \tag{2.35}$$

The potential energy of a system increases when gravitational work is done on it.

2.8 SHAFT WORK

The importance of shaft work in engineering lies in the fact that most power plants deliver their output through rotating shafts. Shaft work, W_s, is evaluated as follows:

$$W_s = \int \delta W_s = \int \tau d\theta = \tau\theta \tag{2.36}$$

where τ is the steady torque on the shaft and θ is the angle of rotation of the shaft under the influence of τ. The unit of W_s is Nm. If a shaft is considered as a system and is rotated by a motor then there is a work transfer into the system. If τ is the torque applied to a shaft running at N rpm (angular velocity ω) then the rate of work done would be:

$$\dot{W} = \omega\tau = 2\pi N\tau \tag{2.37}$$

Shaft power output can be calculated from:

$$P_s = \frac{2\pi N\tau}{60,000}\ \text{kW} \tag{2.38}$$

2.9 SPRING WORK

When the length of a spring changes by a differential amount dL under the influence of a force F, the work done is given by:

$$\delta W_{\text{spring}} = FdL \tag{2.39}$$

To determine the total spring work, the functional relation between F and L should be known. For linear elastic springs, the displacement L is proportional to the force applied. That is:

$$F = KL \tag{2.40}$$

where K is spring constant. Therefore, the spring work for a linear elastic spring is given by:

$$W_{\text{spring}} = \int_1^2 FdL = \int_1^2 KLdL = \frac{1}{2}K\left(L_2^2 - L_1^2\right) \tag{2.41}$$

where L_1 and L_2 are the initial and final displacements of the spring, respectively.

2.10 OTHER MODES OF WORK

So far we have seen the details regarding mechanical work. In the following sections, we will deal with other modes of work.

2.10.1 Electrical Work

Consider a battery having an emf, E, connected to an external circuit as shown in Fig.2.13. Assume that an electric current I is flowing. Let dQ_e represent the change in the charge of the cell because of the flow of the electric current. The work done by the battery is given by:

$$\delta W_{bat} = -E\ dQ_e \tag{2.42}$$

Negative sign indicates that the battery discharges dQ_e. This is negative and therefore, work is done by the battery. The work done by the battery can be written in the following convenient form in terms of current flowing through the battery as:

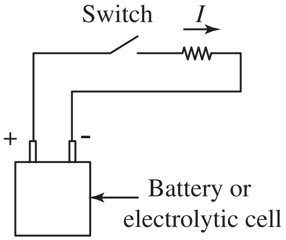


Fig. 2.13 Electrolytic cell

$$I = \frac{dQ_e}{dt} \tag{2.43}$$

where t is the time. Therefore, the work done by the battery can be written as:

$$\delta W_{bat} = -EIdt \tag{2.44}$$

2.10.2 Extension of an Elastic Rod with Both Ends Free

Solids elongate or contract under the action of a force. As long as the force applied is within the elastic limits, the rod will come back to its original length. This is similar to a spring when the force is removed. Therefore, they can be modelled as a linear spring. Equations pertaining to linear springs can be applied to elastic solid bars also. Now, consider a solid rod of cross-section A as shown in Fig.2.14. Let a normal stress $\sigma = (F/A)$ is applied on the rod in the horizontal direction. Then, the force exerted on the rod is $F = \sigma A$. Let dL represent the *total elongation* of the rod. The work done on the rod is:

$$\delta W = -\sigma A dL \tag{2.45}$$

Negative sign implies that when dL is positive (elongation of the rod), work must be done on the system. The strain (ϵ) which is defined as the deformation per unit length is given as:

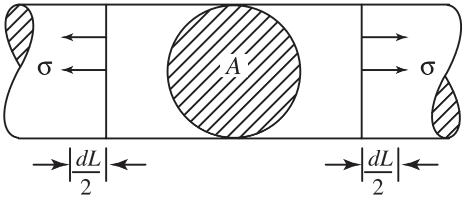


Fig. 2.14 Extension of an elastic rod

$$d\epsilon = \frac{dL}{L} = A\frac{dL}{LA} = A\frac{dL}{V} \tag{2.46}$$

$$dL = d\epsilon \frac{V}{A} \tag{2.47}$$

Substituting dL in Eq.2.45, $\delta W = -V\sigma d\epsilon$ (2.48)

Work done per unit volume of the rod is given by:

$$\frac{\delta W}{V} = -\sigma d\epsilon \tag{2.49}$$

For an elastic bar, the intensive coordinate is stress (σ) and the extensive coordinate is strain (ϵ).

2.10.3 Extension of an Elastic Rod with One End Fixed

Consider Fig.2.15, in which one end is fixed and is subjected to a force F. In this case, also, the work associated with expansion or contraction of an elastic bar can be calculated by replacing pressure p, with normal stress $\sigma = F/A$, where A is cross-sectional area of the rod. Now,

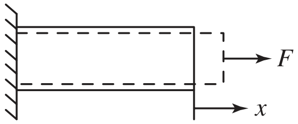


Fig. 2.15 A solid bar subjected to tension

$$W_{\text{elastic}} = -\int_1^2 \sigma dV = -\int_1^2 \sigma A dL \tag{2.50}$$

2.10.4 Paramagnetic Material

The work done per unit volume on a magnetic material through which the magnetic and magnetisation fields are uniform is given by:

$$\delta W = -HdI \tag{2.51}$$

$$W_{1-2} = -\int_{I1}^{I2} HdI \tag{2.52}$$

where H is field strength and I is component of magnetisation field in the direction of the field. The negative sign implies that with an increase in magnetisation (positive dI), work must be done on the system.

2.10.5 Stretching of a Liquid Film

Consider a liquid film held between two wire frames. Assume that one side of the frame can be moved using the sliding wire as shown in Fig.2.16. The liquid film on the surface will experience a surface tension (σ), which is the property of the liquid and the surroundings. *Surface tension is the force per unit length normal to a line on the surface.* The total surface tension force exerted on the wire is equal to $2\sigma b$ where b is the width of the liquid film. The factor 2 appears because of the two surfaces of the film. If the sliding wire is moved through a distance dL then the work done is given by:

$$\delta W = -2\sigma b dL \tag{2.53}$$

The total change in the surface area of the film is $dA = 2bdL$. Therefore,

$$\delta W = -\sigma dA \tag{2.54}$$

$$W = \int_1^2 \sigma dA \tag{2.55}$$

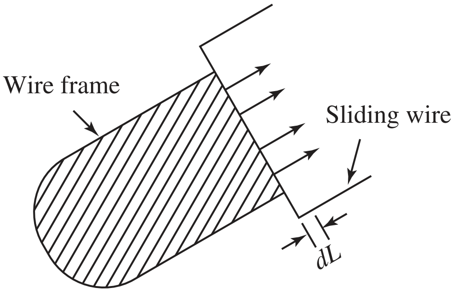


Fig. 2.16 Stretching of a liquid film

2.10.6 Stretching of a Wire

Let us consider the wire as a system. Let the tension applied to the wire be τ. Let the length of the wire changes from L to $L + dL$ which means there is a change of length dL. The infinitesimal amount of work that is done is equal to:

$$\delta W = -\tau dL \tag{2.56}$$

Note that τ is the intensive coordinate and L is the extensive coordinate. Negative sign indicates the fact that for positive dL (extension of wire, work has to be done on the system.

If we consider the situation to within the elastic limit, where E is the modulus of elasticity, s is the stress, ϵ is the strain and A is the cross-sectional area, then ϵ:

$$\tau = sA = E\epsilon A \qquad (\because E = \frac{s}{\epsilon}) \tag{2.57}$$

$$d\epsilon = dL/L \tag{2.58}$$

$$\delta W = -\tau dL = -E\epsilon ALd\epsilon \tag{2.59}$$

$$W = -AEL\int_1^2 \epsilon d\epsilon = -\frac{AEL}{2}(\epsilon_2^2 - \epsilon_1^2) \tag{2.60}$$

2.10.7 Stirring of Fluids

Consider the system shown in Fig.2.17 in which a paddle wheel is connected to a pulley. A weight is suspended as shown in the figure. As the weight is lowered, the paddle wheel starts rotating. This causes the fluid in the system to get stirred. The volume of the system is not changed and therefore, $\int pdV = 0$. Let the mass of the weight lowered be m. Let the distance moved be dL. If τ is the torque transmitted by the shaft in rotating through an angle $d\theta$, the differential work transfer is given by:

$$\delta W = mgdL = \tau d\theta \tag{2.61}$$

The total work done is given by:

$$W = \int_1^2 mgdL = \int_1^2 WdL = \int_1^2 \tau d\theta \tag{2.62}$$

For stirring work, the intensive coordinate is the torque τ and the extensive coordinate is the angle θ.

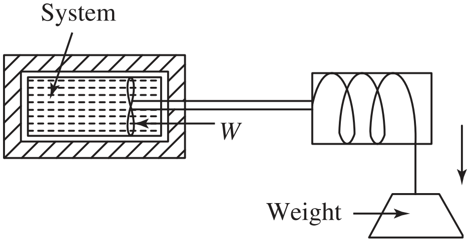


Fig. 2.17 Paddle wheel work

2.10.8 Generalised Equation for Work Transfer

It is worth noting that in all the above cases, the work done by a system is equal to the product of an intensive coordinate and the difference of an extensive coordinate. Now, we can write the work done in a generalised form as:

$$\delta W \quad = \quad F_k dX_k \tag{2.63}$$

where F_k is generalised force and dX_k is the generalised displacement. The generalised forces and displacements for various work modes are given in Table 2.2.

Table 2.2 Generalised forces and displacements for various work modes

System	Generalised force		Generalised displacement		Work done
Compression/Expansion	Pressure	(p)	Volume	(dV)	pdV
Solid rod (elastic)	Stress	(σ)	Strain	$(d\epsilon)$	$-\sigma d\epsilon$
Stretching of wire	Tension	(τ)	Extension	(dL)	$-\tau dL$
Liquid film	Surface tension	(τ)	Surface area	(dA)	$-\tau dA$
Reversible cell	emf	(E)	Charge	(dQ_e)	$-EdQ_e$
Polarisation	Voltage	(V)	Charge	(dQ_e)	$-VdQ_e$
Magnetisation	Magnetic field	(H)	Magnetisation	(dM)	$-HdM$

2.11 FLOW WORK AND NON-FLOW WORK

Work interaction taking place in a system can be classified as flow work or non-flow work based on the nature of process. Two basic types of processes are:

(i) Flow process (ii) Non-flow process

2.11.1 Flow Process

Flow process is the one in which fluid enters the system and leaves it after work interaction. Such processes occur in the systems having open boundary. Therefore, there will be a mass interaction across the system boundary. Such systems are called open systems. Open systems are common in many engineering devices such as blower, compressor and pumps. In such systems, fluid flows across the control volume. Flow work becomes significant in such cases. Flow work is analogous to displacement work. Flow processes can be further classified into steady flow and non-steady flow processes. Examples of engineering systems having steady flow processes are flow through nozzles, turbines, compressors and so on. Examples of non-steady flow processes are the filling or emptying of any vessel. To understand the concept, consider Fig.2.18, which shows the flow process occurring in a system.

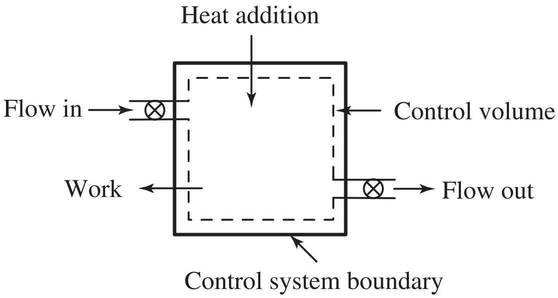


Fig. 2.18 Flow process

It indicates various energy and mass interactions taking place across the system boundary. Note that mass interaction takes place here because it is an open system. Therefore, for every fraction of mass crossing the boundary, work is done *on or by* the system. Thus, a portion of work is always required to push the fluid mass into or out of the system. This amount of work is called flow work. It can be defined as the work required for causing flow of fluid into or out of a system.

Consider the fluid element of volume V, as shown in Fig.2.19. The force F on the face of the imaginary piston due to the fluid element with pressure p is given by:

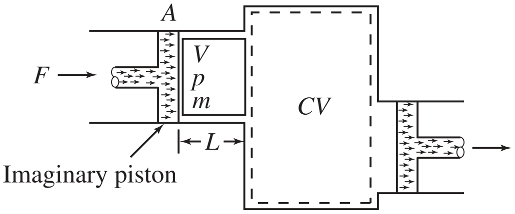


Fig. 2.19 A fluid element entering a control volume

$$F = pA \tag{2.64}$$

where A is the cross-sectional area of the piston. In order to push the entire mass of the fluid element into the control volume, the force F must act through a distance L. The work done in pushing the fluid element across the boundary is called the flow work and is given by:

$$W_{flow} = FL = pAL = pV \tag{2.65}$$

where AL is the volume of the fluid element. The flow work per unit mass can be obtained dividing both side of the above equation by mass of the fluid element:

$$w_{\text{flow}} = pv \tag{2.66}$$

Note that unlike other mode of work, flow work is expressed in terms of properties. As can be seen from the above equation, it is the product of two properties of the fluid. For this reason, it is viewed as a combination property. *It is often referred to as convected energy or flow energy or transport energy instead of flow work.*

2.11.2 Non-flow Process

During a process, when there is no mass interaction across the system boundaries it is called non-flow process. Figure 2.20 shows the block diagram of a piston-cylinder arrangement in which there is a fixed mass of fluid inside the cylinder. Let the fluid undergo a compression process. Thus, during compression the process will be non-flow process. The work interaction is considered as non-flow work. If the force exerted by piston is F and cross-section area of piston being A, the elemental work done in compressing along the length dL is given by:

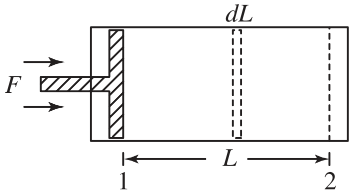


Fig. 2.20 Non-flow process

$$\delta W = FdL \tag{2.67}$$

If pressure of fluid is p then

$$F = pA \tag{2.68}$$

so that

$$\delta W = pAdL \tag{2.69}$$

The total work done in piston displacement, from 1 to 2 shall be:

$$\int_1^2 \delta W = \int_1^2 pAdL = \int_1^2 pdV \tag{2.70}$$

$$W_{1-2} = \int_1^2 pdV \tag{2.71}$$

Thus, this is called the *non-flow work* or *displacement work.*

2.12 DEFINITION OF WORK BASED ON THERMODYNAMICS

In previous sections, we have gone through various systems carrying out work transfer based on the mechanics definition of work. The question now is what is the necessity for another definition for work, viz thermodynamic definition? Note that the mechanics definition of work has certain inherent limitations. They are:

(i) We have to physically observe the point of application of the force and

(ii) We must be able to measure the distance it moves.

In many situations, it is not possible to measure either of them or both of them. A simple example is the flow of current through a wire which involves the electrical work interaction. It is not possible, in macroscopic level, to either find the point of application of force or the distance it moves. Therefore, a necessity arises to define work in thermodynamics in a more universal manner. This will also include the mechanics definition of work stated earlier and will also take into account of all other possible modes of work interaction.

2.12.1 Thermodynamic Definition of Work

Work can be looked upon as the energy transfer across the boundary of a system, without the transfer of mass. During a given operation, work is said to be done by a system on its surroundings, if, *the sole effect, external to the system, could be reduced to the raising of a weight* through a distance. This definition was suggested by Max Planck and further developed by J.H.Keenan. The words in italics in the definition have very significant and important meaning. Let us elaborate:

(i) The *sole effect* means the one and only effect. It bars any other effect besides the raising of a weight.

(ii) *External to the system* conveys the meaning that work is an interaction between a system and its surroundings across the system boundary.

(iii) *Could be reduced to the raising of a weight* implies that work interaction need not necessarily result in the raising of a weight in the surroundings.

Consider a system interacting with its surroundings. If the consequent change in the surroundings does not happen to be the raising of a weight as the only effect then, the interaction may or may not be work. To assess whether it is work or not, a specific exercise might be needed. This exercise is carried out to find out whether the *same interaction* could have resulted in the raising of a weight as the sole effect in a suitably imagined fictitious surroundings. If the answer to this proposition is 'YES', the interaction is work and if not, the interaction may not be work.

The magnitude of work interaction is calculated as the product of the weight ($m \times g$) raised and the distance it moves up vertically (against the gravitational force) in the surroundings. To illustrate, let us consider the expansion of a gas inside a cylinder. As we see from Fig.2.21 or the better Fig.2.22, through the proper selection of linkages, we can make use of the expansion process to lift a weight. Therefore, work can be said to be done by the gas.

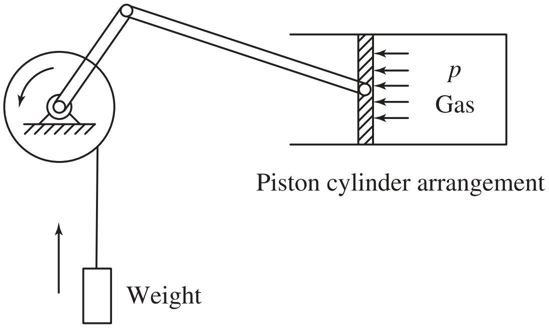


Fig. 2.21 Lifting of a weight due to work done by a gas in an expansion process

Let us see another example of storage battery as our system. Referring to Fig.2.23, the terminals of the battery are connected to a resistance through a switch. As the switch is closed, current will flow through the resistor. Note that circuit external to the battery is the surrounding. What will happen? We will notice that the resistor (surrounding) will become warmer and charge in the battery (system) starts decreasing. The question is can this interaction be called work? From the point of view of mechanics definition, the answer is

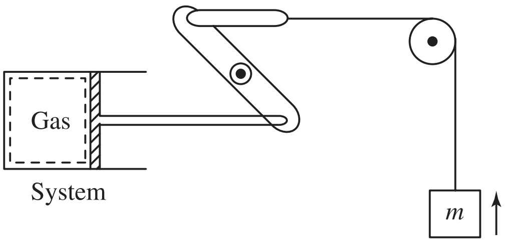
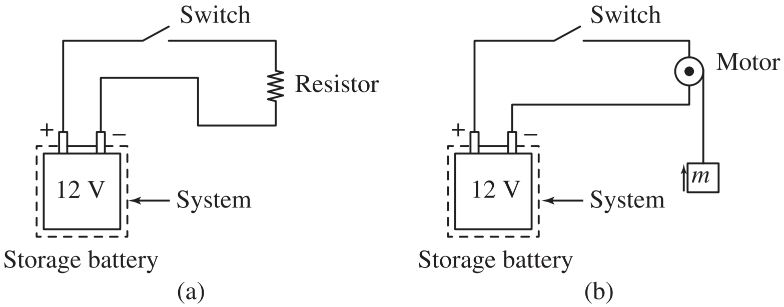


Fig. 2.22 Visualisation of work done by a system as equivalent to raising of weight

NO. From the point of view of thermodynamic definition, the answer is YES. Let us see how? Now, modify the external circuit with a motor instead of the resistor as shown in Fig.2.23. When the switch is closed, the motor rotates and the mass is moved up against gravity. In this example, external to the system (battery), the sole effect is the lifting the mass. Hence, the interaction is called work.



(a) (b)

Fig. 2.23 Visualisation of battery interaction as work done

As far as the battery is concerned, the situation is the same as in the previous case, in which it was discharging through a resistor. Under these circumstances, the interaction of battery with resistor can be called work, even though no mass has been raised through any distance. This is how the thermodynamic work is interpreted. Once again, recall that, work is nothing but energy in transit. When this form of energy crosses the boundary of the system, it becomes part of energy content of the system or surroundings as the case may be. It is to be understood that there is nothing like a system having a certain amount of work at a given state. It means that quantity of work is not a thermodynamic property. There is a possibility of work interaction when a change of state takes place.

2.12.2 Sign Convention

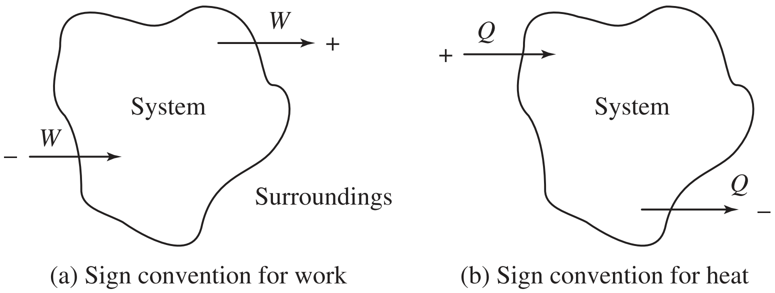
We are aware that a system cannot do work on itself, because, work is an interaction across the system boundary. The commonly used sign convention for work is:

(i) Work done by a system (work out) is positive (+ve).

(ii) Work done on a system (work in) is negative (−ve).

The sign convention for work is illustrated in Fig.2.24(a). Figure 2.25 schematically shows the sign convention adapted for heat and work. For a given process, the magnitudes of work interaction for both the system and the surroundings are the same except for their direction. Therefore, we can write:

$$W_{\text{system}} + W_{\text{surroundings}} = 0$$



(a) Sign convention for work (b) Sign convention for heat

Fig. 2.24 Sign convention for work

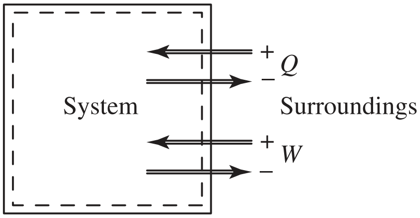


Fig. 2.25 Sign convention for work and heat

2.13 HEAT

Heat is the interaction between a system and surroundings. This interaction is solely due to a temperature difference existing between the two. Therefore, heat is considered a form of energy in transit which is similar to work. It is a boundary phenomenon and it occurs only at the boundary of a system. Consider two systems at different temperatures brought in contact with each other. Spontaneously the process of heat transfer will start. The direction of heat transfer will be from higher temperature system to lower temperature system. The driving potential for heat transfer is the temperature difference. Thus, heat is an occurrence and not something stored in a system. The temperature difference is called the 'potential' or 'force' and heat transfer is a flux.

For heat transfer to occur, there must be a driving force. For example, consider a hot piece of iron ball being dropped into oil in a container which is at atmospheric temperature. Immediately the process of heat transfer will set in. Heat will be transferred from the iron ball to oil. You will notice that the temperature of the iron ball starts decreasing and that of oil keeps increasing due to the process of heat transfer. After sufficiently long time, the two temperatures reach a common value. From this instant, heat transfer ceases to occur since there is no temperature difference which is the driving force. There are three modes of heat transfer. They are (i) conduction, (ii) convection and (iii) radiation.

The transfer of heat between two bodies in direct contact is called *conduction.*

The transfer of heat between a wall and fluid system in motion is called *convection*.

The transfer of heat between two bodies separated by empty space or gases through electromagnetic waves is called *radiation*.

2.13.1 Sign Convention and Unit

Similar to work, heat is also a path function. While we use W to denote work, it is customary to use Q to denote heat transfer. We have already shown the sign convention for heat in Fig.2.24(b). To emphasis again, note that the heat flow into the system is considered positive while heat flow out of the system is considered as negative. Refer Fig.2.24(b) for details.

The unit of heat transfer in SI is joule (J). Since joule is a very small quantity, various practical units such as kJ (10^3 J) and MJ (10^6 J) are used as units. The rate of heat transfer measured J/s is called watt (W). The practical units of heat transfer are kW (10^3 J/s) and MJ (10^6 J/s).

Worked-Out Examples

2.1 A two wheeler of mass 100 kg is travelling at a speed of 50 km/h. What is its kinetic energy?

Solution

The kinetic energy, KE, is given by

$$KE = \frac{1}{2}m\mathbb{V}^2 = \frac{1}{2} \times 100 \times \left(\frac{50 \times 1000}{3600}\right)^2 = 9645 \text{ J} = 9.645 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.2 An aircraft having a mass of 20 tonnes flies at a speed of 1000 km/h at an altitude of 10 km. Calculate the kinetic energy of the aircraft and estimate its potential energy.

Solution

The kinetic energy, KE is given by:

$$KE = \frac{1}{2}m\mathbb{V}^2 = \frac{1}{2} \times 20 \times 10^3 \times \left(\frac{1000 \times 1000}{3600}\right)^2 = 771,604,938 \text{ J} = 7.716 \text{ GJ} \overset{\textbf{Ans}}{\Longleftarrow}$$

The potential energy, PE is given by:

$$PE = mgh = 20 \times 10^3 \times 9.81 \times 10 \times 10^3 = 1.962 \times 10^9 \text{ J} = 1.962 \text{ GJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.3 An iron ball of mass 10 kg is dropped from the roof of a building which has ten floors of each 4 m height. Estimate the velocity with which the ball hits the ground ($g = 9.8$ m/s^2).

Solution

Let the height of the building be h

$$h = 10 \times 4 = 40 \text{ m}$$

Initially, the iron ball has a potential energy, PE, given by:

$$PE = mgh = 10 \times 9.8 \times 40 = 3920 \text{ J} = 3.92 \text{ kJ}$$

When the ball hits the ground, its $PE = 0$ and it is completely converted into KE.

$$\begin{aligned} KE &= \frac{1}{2} m\mathbb{V}^2 = 3920 \\ \mathbb{V}^2 &= \frac{3920 \times 2}{10} = 784 \\ \mathbb{V} &= \sqrt{784} = 28 \text{ m/s} \qquad \overset{\text{Ans}}{\Longleftarrow} \end{aligned}$$

The ball will hit the ground with a velocity of 28 m/s.

2.4 Water is ejected vertically out of a fountain through a nozzle. Determine the height to which the water can reach if the velocity of water at the tip is 35 m/s.

Solution

Assume that 1 kg of water is released at the tip of the nozzle. Then,

$$KE = \frac{1}{2} m\mathbb{V}^2 = \frac{1}{2} \times 1 \times 35^2 = 612.5 \text{ J} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

As the water is ejected vertically up, its kinetic energy, KE, decreases while the potential energy, PE, increases. However, the total energy $(KE + PE)$ should be constant. Therefore, at the maximum height the kinetic energy will be zero, whereas

$$\begin{aligned} PE &= mgh = 612.5 \text{ J} \\ h &= \frac{612.5}{1 \times 9.81} = 62.44 \text{ m} \qquad \overset{\text{Ans}}{\Longleftarrow} \end{aligned}$$

2.5 A two wheeler of mass 100 kg moves with a speed of 60 km/h. Find the work done by an opposing force on the vehicle to reduce the speed to half of the original speed.

Solution

$$\begin{aligned} \text{Initial velocity of the two wheeler, } \mathbb{V}_1 &= \frac{60 \times 1000}{3600} = 16.66 \text{ m/s} \\ \text{Final velocity of the two wheeler, } \mathbb{V}_2 &= \frac{30 \times 10^3}{3600} = 8.33 \text{ m/s} \end{aligned}$$

Work to be done to reduce the speed to half

$$\begin{aligned} &= \frac{1}{2} m \left(\mathbb{V}_1^2 - \mathbb{V}_2^2\right) \\ &= \frac{1}{2} \times 100 \times \left(16.66^2 - 8.33^2\right) \\ &= 10408 \text{ J} = 10.41 \text{ kJ} \qquad \overset{\text{Ans}}{\Longleftarrow} \end{aligned}$$

2.6 Consider a hollow sphere of mass m. Its volume is V and is immersed in a liquid of density ρ. If the sphere is raised through a distance of h in the liquid by an external agent. Determine the work done by the external agent. Is there any energy transfer between the sphere and the liquid as work? What happens to the energy of the fluid?

Solution

The schematic sketch of the problem under consideration is given in Fig.2.26.

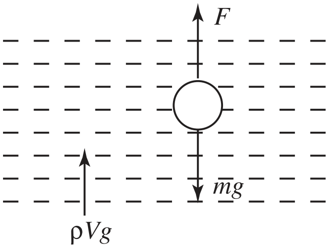


Fig. 2.26

The force, F, required by the external agent is given by:

$$F \quad = \quad mg - \rho V g \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

where $\rho V g$ is the buoyancy force acting on the sphere.

$$\text{Work done by external agent, } W_e \quad = \quad \int F dh = (mg - \rho V g)\, h \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Work done by the fluid, } W_f \quad = \quad \rho V g h \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

The potential energy of the fluid decreases by the amount of work done by the fluid.

2.7 In the above example, if the mass of the sphere is 5 kg and if the external agent raises it by 50 cm, find the work done by the external agent. Take the density of the fluid to be 800 kg/m^3. Diameter of the sphere is 10 cm.

Solution

From the above example, the work done by the fluid is given by

$$W_f \quad = \quad \rho V g h = 800 \times \frac{4}{3} \times \pi \times 0.05^3 \times 9.8 \times 0.5 = 2.05$$

Work done by the external agent

$$W_e \quad = \quad mgh - W_f = 5 \times 9.81 \times 0.5 - 2.05 = 22.475 \text{ J} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.8 During an adiabatic compression, if the volume reduces by half and initial pressure is 100 kPa and volume is 2 m^3, calculate the work done by the gas. Take $\gamma = 1.4$.

Solution

$$\begin{aligned} pV^{\gamma} &= C \\ p_1V_1^{\gamma} &= p_2V_2^{\gamma} \\ 100\times 0.2^{1.4} &= p_2\times 0.1^{1.4} \\ p_2 &= 264 \text{ kPa} \\ W &= \frac{p_2V_2-p_1V_1}{1-\gamma} \\ \text{Work done} &= \frac{264\times 0.2-100\times 0.1}{1-1.4} = -107 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

The negative sign indicates that work is done on the gas.

2.9 Consider a balloon which is initially flat and collapsed. It is very slowly filled with nitrogen from a cylinder forming a balloon into a sphere of 6-m diameter. Assume ambient pressure is 100 kPa. Assuming that filling process takes place at a constant temperature of 300 K. Calculate the work done by the cylinder-balloon system.

Solution

Initially, nitrogen is contained in the cylinder. The filling process is carried out very slowly. Nitrogen enters the balloon at a pressure of 100 kPa which is ambient pressure. The process can be considered as the equilibrium process. The work done by nitrogen is given by:

$$W = \int_1^2 pdV = p\,(V_2 - V_1)$$

Since balloon is initially flat and collapsed, $V_1 = 0$.

$$\text{Radius of ballon} = \frac{6}{2} = 3 \text{ m}$$

$$W = pV_2 = 100\times 10^3\times\frac{4}{3}\times\pi\times 3^3 = 11,309,733.6 \text{ J} = 11.31 \text{ MJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.10 Helium is contained in a cylinder of 10 litres at a pressure of 10 MPa and 300 K. Helium starts leaking into the atmosphere until the gas pressure in the cylinder becomes half. Assume that the temperature of the cylinder and the gas remains at 300 K all the time. If the atmospheric pressure and temperature are 100 kPa and 300 K, respectively, is there any energy transfer as work? If yes, determine the work done by helium. Assume that helium obeys the relation $pV = nRT$, where $R = 8.314$ kJ/k mol K and n is the number of moles.

Solution Initial quantity of helium in the cylinder, N_i is given by

$$N_i = \frac{p_iV}{RT} = \frac{10\times 10^6\times 0.01}{8.314\times 10^3\times 300} = 0.04 \text{ kmol}$$

Final quantity of helium in the cylinder, N_f is given by,

$$N_f = \frac{p_f V}{RT} = \frac{5 \times 10^6 \times 0.01}{8.314 \times 10^3 \times 300} = 0.02 \text{ kmol}$$

Leaked out quantity of helium

$$N_l = N_i - N_f = 0.04 - 0.02 = 0.02 \text{ kmol}$$

The volume occupied by leaked helium in the atmosphere is given by,

$$V = N_l \frac{RT}{p} = \frac{0.02 \times 8.314 \times 10^3 \times 300}{100 \times 10^3} = 0.4988 \text{ m}^3$$

Work done by helium is given by,

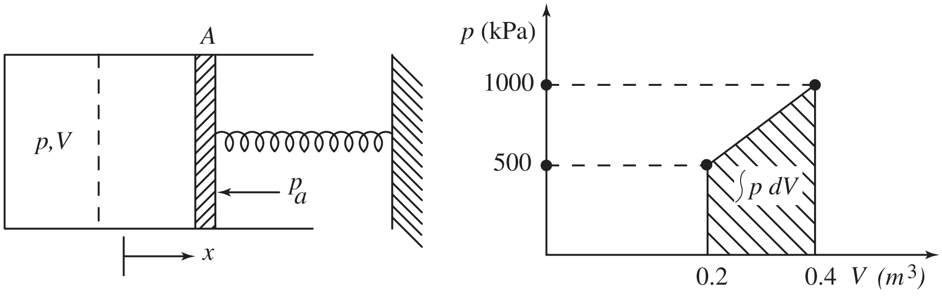
$$\begin{aligned} W &= \int_1^2 p dV = p\left(V_2 - V_1\right) = 100 \times 10^3 \times 0.4988 - 0.01 \\ &= 49,879.99 \text{ J} = 49.88 \text{ kJ} \end{aligned} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

As W is positive, work is done by helium in the atmosphere.

2.11 Consider a system of cylinder and piston arrangement containing gas. Initially, the gas is at 500 kPa and occupies a volume of 0.2 m^3. The force exerted by the spring is proportional to the displacement from its equilibrium position. Take ambient pressure as 100 kPa. The gas is heated until the volume becomes 0.4 m^3 and the pressure attained is 1 MPa. Determine the work done by the gas. Draw the schematic and p-V diagram.

Solution

The schematic diagram and the corresponding process p-V diagram is given in Figs.2.27(a) and (b). The force exerted by the spring is proportional to L



(a) (b)

Fig. 2.27

$$F_s = KL$$

The force balance on the piston in the initial state gives

$$p_1 A = p_a A + KL = p_a A + \frac{K}{A}\left(V_1 - V_2\right) \qquad (1)$$

The force balance on the piston in the final state gives

$$
\begin{aligned}
p_2 A &= p_a A + KL = p_a A + \frac{K}{A}\left(V_2 - V_n\right) \qquad (2)\\
p_2 &= p_a + \frac{K}{A^2}\left(V_2 - V_n\right)
\end{aligned}
$$

where A is cross-sectional area of piston; p_a is atmospheric pressure; V_n is volume of the gas when the spring is in its natural length; p_1 and p_2 are initial and final pressure of the gas; V_1 and V_2 are initial and final volume of the gas.

Consider the gas contained in the cylinder as a system. Then, work done, W, by the gas is given by

$$
\begin{aligned}
W &= \int_1^2 pdV = \int_1^2 \left[p_a + \frac{K}{A^2}\left(V_2 - V_n\right)\right] dV\\
&= p_a\left(V_2 - V_n\right) + \frac{K}{2A^2}\left[\left(V_2 - V_n\right)^2 - \left(V_1 - V_n\right)^2\right] \qquad (3)
\end{aligned}
$$

Equation (3) can be arranged as

$$
\begin{aligned}
W &= p_a\left(V_2 - V_1\right) + \frac{k}{2A^2}\left(V_2 + V_1 - 2V_n\right)\left(V_2 - V_1\right)\\
&= \left[p_a + \frac{K}{2A^2}\left(V_2 + V_1 - 2V_n\right)\right]\left(V_2 - V_1\right) \qquad (4)
\end{aligned}
$$

From Eqs (1) and (2), we have

$$
p_a + \frac{K}{2A^2}\left(V_2 + V_1 - 2V_n\right) = \frac{p_1 + p_2}{2} \qquad (5)
$$

Substituting Eq.(5) in Eq. (4), we get

$$
W = \left(\frac{p_1 + p_2}{2}\right)\left(V_2 - V_1\right) = \frac{(500 + 1000) \times 10^3}{2} \times 0.2 = 150 \text{ kJ} \qquad \overset{\text{Ans}}{\Longleftarrow}
$$

2.12 A spherical balloon of 2 m diameter is filled with a gas at 200 kPa and 300 K. The gas inside the balloon is heated. Finally the pressure reaches 1 MPa. During the process of heating, assume that the pressure is proportional to the diameter of the balloon. Find the work done by the gas inside the balloon.

Solution

Given $p \propto D$ or $p = kD$

$$
\begin{aligned}
p_1 &= kD_1\\
200 \times 10^3 &= k \times 2\\
k &= 10^5 \text{ Pa/m}\\
p_2 &= kD_2\\
1000 \times 10^3 &= 10^5 D_2
\end{aligned}
$$

$$D_2 \quad = \quad 10 \text{ m}$$

Volume of the spherical balloon is given by:

$$\begin{aligned} V &= \frac{4}{3} \times \pi \times R^3 = \frac{4}{3} \times \pi \times \left(\frac{D}{2}\right)^3 = \frac{\pi}{6} D^3 \\ dV &= \frac{\pi}{6} \times 3 \times D^2 \times dD = \frac{\pi}{2} \times D^2 \times dD \end{aligned}$$

Work done by the gas is given by,

$$\begin{aligned} W &= \int_1^2 pdV = \int_1^2 kD \times \frac{\pi}{2} \times D^2 \times dD \\ &= \int_1^2 k \times \frac{\pi}{2} \times D^3 \times dD = \frac{\pi}{8} \times k \times \left(D_2^4 - D_1^4\right) \\ &= \frac{\pi}{8} \times 10^5 \times \left(10^4 - 2^4\right) \times 10^{-6} = 392.07 \text{ MJ} \end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

2.13 Air in a cylinder at an initial volume of 0.1 m^3 and initial pressure of 10 MPa expands following a polytropic process given by $pV^{1.3}$ = constant. If the final volume of the gas is 0.25 m^3, calculate the work done by the gas.

Solution

The work done by the gas is given by

$$W_{12} = \int pdV$$

Given that $pV^{1.3} = C$, then,

$$\begin{aligned} p_1 V_1^{1.3} &= p_2 V_2^{1.3} = pV^{1.3} = C \\ p_2 &= p_1 \left[\frac{V_1}{V_2}\right]^{1.3} = 10 \times \left(\frac{0.1}{0.25}\right)^{1.3} = 3.0386 \text{ MPa} \end{aligned}$$

The work of expansion becomes

$$\begin{aligned} W_{12} &= C \int_1^2 pdV = \frac{C}{1-1.3}\left(V_2^{-0.3} - V_1^{-0.3}\right) = \frac{p_2 V_2 - p_1 V_1}{1-n} \\ W_{1-2} &= \frac{(3.0386 \times 0.25 - 10 \times 0.1)}{1-1.3} \times 10^3 = 801.17 \text{ kJ} \end{aligned}$$

2.14 A block of wood is resting on an inclined plane as shown in Fig.2.28. The block is acted upon by a horizontal force, F, of 200 N. Because of this, the block moves a distance, s, of 15 m from position 1 to position 2. Find the work done by the force in kJ.

Solution

$$F \quad = \quad \text{Force } \times \text{Distance } = F \times L$$

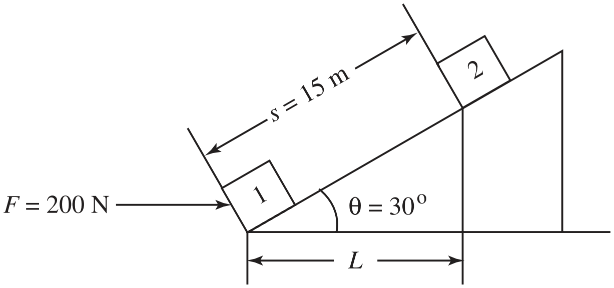


Fig. 2.28

where L is the distance moved by the point of application of force in the direction of force.

$$L = s\cos\theta = 12 \times \cos 30 \quad \xleftarrow{\textbf{Ans}}$$

$$W = F \times L = 200 \times 12 \times \cos 30 = 2078.46 \text{ J} = 2.08 \text{ kJ} \quad \xleftarrow{\textbf{Ans}}$$

2.15 A hydraulic turbine is operated by water from a reservoir at a height of 300 m. If the power developed by the turbine is 30 MW, find the mass flow through the turbine. Take $g = 9.81$.

Solution

The hydraulic turbine works under gravitational force. The gravitational work is given by

$$\dot{W}_g = \dot{m}g(z_2 - z_1)$$

$$\dot{W}_g = 30 \text{ MW}$$

$$\dot{m} = \frac{\dot{W}}{g(z_2 - z_1)} = \frac{30 \times 10^6}{9.81 \times (300 - 0)} = 10193 \text{ kg/s} \quad \xleftarrow{\textbf{Ans}}$$

2.16 Estimate the power required to accelerate a car which weight 1 tonne from rest to 60 km/h in 30 s on a level road. All resistance to acceleration may be discarded.

Solution

The acceleration work is given by:

$$W_a = \frac{1}{2}m(\mathbb{V}_2^2 - \mathbb{V}_1^2)$$

where $\mathbb{V}_1$ and $\mathbb{V}_2$ are initial and final velocities. Here, $\mathbb{V}_2 = 60$ km/h

$$\mathbb{V}_1 = 0 \text{ km/h}$$

$$W_a = \frac{1}{2} \times 1000 \times \left[\left(\frac{60,000}{3,600}\right)^2 - 0^2\right] = 138.9 \text{ kJ} \quad \xleftarrow{\textbf{Ans}}$$

The average power required for acceleration is given by:

$$\dot{W} = \frac{W}{\Delta t} = \frac{138.9}{30} = 4.6 \text{ kW} \quad \xleftarrow{\textbf{Ans}}$$

2.17 A school van weighs 1200 kg. It has the capability to carry 20 students. The average weight of a student may be approximated to 40 kg. Estimate the minimum power required to accelerate the car from rest to 60 km/h. Make suitable assumptions with justification.

Solution

Assumption : Air drag, friction, rolling and other resistance may be neglected. Since we have to calculate only minimum energy, the acceleration work is given by:

$$\begin{aligned} W_a &= \frac{1}{2}m\left(\mathbb{V}_2^2 - \mathbb{V}_1^2\right) \\ &= \frac{1}{2} \times (1200 + 20 \times 40) \times \left[\left(\frac{60 \times 10^3}{3600}\right)^2 - 0^2\right] = 277.8 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

2.18 The power rating of the 1000 kg automotive engine is 80 kW. Calculate the time required to accelerate the vehicle from 30 km/h to 60 km/h at full power on a level road. Neglect the air drag, friction and other resistances.

Solution

Acceleration work is given by:

$$\begin{aligned} W_a &= \frac{1}{2}m\left(\mathbb{V}_2^2 - \mathbb{V}_1^2\right) \\ &= \frac{1}{2} \times 1000 \times \left[\left(\frac{60 \times 10^3}{3600}\right)^2 - \left(\frac{30 \times 10^3}{3600}\right)^2\right] = 104.17 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

Power rating of the vehicle = 80 kW

$$\begin{aligned} W_a &= \dot{W}_a \times \Delta t = 80 \\ \Delta t &= \frac{W_a}{\dot{W}_a} = \frac{104.87}{80} = 1.31 \text{ s} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

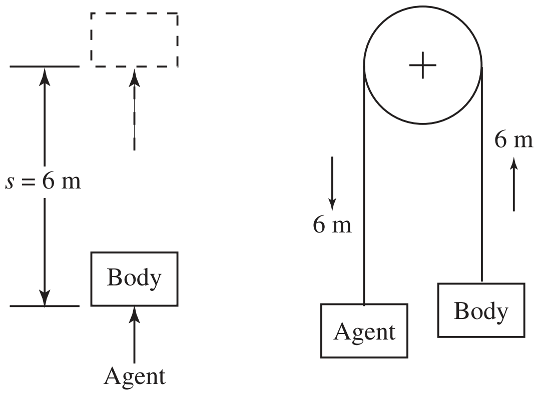
2.19 An agent raises a cube of mass of 3 kg very slowly through a vertical distance of 6 m at a place where $g = 9.81$ m/s^2. Estimate the magnitude and direction of work for the system agent and the body from mechanics definition of work and also thermodynamics definition of work.

Solution

Let us consider the agent as a system. As can be seen from Fig.2.29(a), the agent raises the body against the gravitational force. To overcome the downward pull on the body by gravity, the agent has to do work on the body. Therefore, according to mechanics, definition of work is as follows:

Mechanics definition of work:

$$\begin{aligned} W_{agent} &= \text{Resisting force } \times \text{Distance moved} = \text{Weight of the body } \times \text{Distance} \\ &= (mg) \times s = (3 \times 9.81) \times 6 = 176.58 \text{ Nm} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$



(a) Actual (b) Imaginary

Fig. 2.29

Thermodynamics definition of work:

Figure 2.29 (b) shows the equivalence from the thermodynamics point of view. In this, the agent and the body are imagined to be suspended over a frictionless pulley by a light rigid cable. With the slow rotation of the pulley, the body is lifted up. In order to lift the body slowly, mass of the body must be equal to the mass of the agent. As can be seen from Figs.2.29(a) and (b), the agent is subjected to the same interaction, viz the raising of the same weight through same distance. Considering the agent as the system, the only effect external to it is raising of the body through a distance of 6 m. As the only effect external to the agent is the raising of the weights, by the thermodynamic definition of work.

$$W_{agent} \quad = \quad \text{Weight raised} \times \text{Distance} = (mg) \times h = 176 \text{ Nm} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Note: It is recommended to present the answer in one of the following formats:

$W_{agent} = +176$ Nm or 176 Nm of work is done by the agent (Do not use both + and −). As $W_{agent} = W_{body} = 0$.

$$W_{body} \quad = \quad -W_{agent} = -176 \text{ Nm} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_{body} \quad = \quad -176 \text{ Nm} \quad \text{or} \quad 176 \text{ Nm of work is done on the body.} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.20 A current of 1 amp flows through a resister when it is connected to a 12 V battery. Explain the interaction between the battery and resistor.

Solution

As can be seen, the battery supplies the same current as in Fig.2.30(a) to the imaginary systems in Fig.2.30(b), which has an equivalent resistance offered by the winding of a motor. Thus, the battery has the interaction with its surroundings in both cases.

The motor shaft is fixed with a pulley. An inextensible, light cable wound around the pulley carries the weight at its end. As motor is connected to the battery, the pulley on the motor shaft rotates. This makes the weight to be lifted. Neglecting the friction in the motor pulley and heating of the motor windings, the only effect external to the battery is the raising of

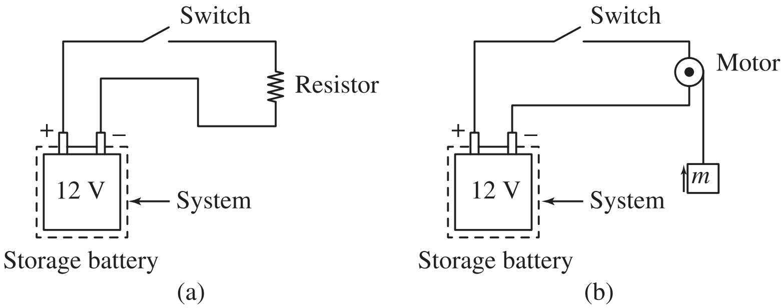


Fig. 2.30

a weight. Hence, the interaction between the battery and the motor (and thus the original resistor) is work. The work is positive for the battery in conformity with the definition.

2.21 A spherical balloon has a diameter, D, 30 cm and contains air at a pressure of 3 bar. The diameter of the balloon increases to 40 cm in a certain process. It is observed that the pressure in the balloon is proportional to the diameter.

(i) Calculate the magnitude and direction of work for air inside the balloon.

(ii) Assuming that the atmospheric pressure is 1 bar, find the magnitude and direction of work for the atmosphere and the balloon.

Solution

$$p \propto D \quad \text{or} \quad p = kD$$

where k is the proportionality constant and d is the diameter of the balloon.

$$p_1 = kD_1$$

or
$$k = \frac{p_1}{D_1} = \frac{3 \times 10^5}{0.3} = 10^6 \text{ N/m}^3$$

$$p_2 = kD_2 = 10^6 \times 0.4 = 4 \times 10^5$$

As it is a special balloon, $V = \frac{\pi}{6}D^3$

$$dV = \frac{3\pi D^2 dD}{6} = \left(\frac{\pi D^2}{2}\right)dD$$

$$W_{air} = \int_1^2 pdV = \int_1^2 kD\left(\frac{\pi D^2}{2}\right)dD = \int_1^2 \left(\frac{k\pi}{2}\right)D^3 dD$$

$$= \left(k \times \frac{\pi}{2}\right)\frac{\left(D_2^4 - D_1^4\right)}{4} = 10^6 \times \left(\frac{\pi}{8}\right) \times \left(0.4^4 - 0.3^4\right) = 6,872 \text{ Nm}$$

W_{air} is positive as air does work on the balloon fabric to increase the diameter of the balloon.

(b) As the balloon expands, it pushes atmospheric air with it at a constant pressure of $p_{atm} = 10^5$ N/m^2. Considering atmosphere as the system, its volume decreases during the process due to air going into the balloon and expanding it. This decrease is equal to the increase in the volume of the balloon. Therefore,

$$\begin{aligned} dV_{atm} &= -dV_{balloon} \\ W_{atm} &= \int pdV = p_{atm}\int dV = p_{atm}[-(V_2 - V_1)] = p_{atm}[(V_1 - V_2)] \\ &= 10^5 \times \left(\frac{\pi}{6}\right) \times \left(0.3^3 - 0.4^3\right) = -1937.3 \text{ Nm} \end{aligned}$$

The work done by the air in the balloon is 6872 Nm and the work done on the atmosphere is 1937.3 Nm. As $W_{air} + W_{balloon} + W_{atm} = 0$,

$$W_{balloon} = -(W_{air} + W_{atm}) = (6872 - 1937.3) = 4934.7 \text{ Nm} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.22 A pure substance expands from 0.15 m^3 at 10 bar to 0.48 m^3 at 2 bar while going through a quasistatic process. Calculate the work interaction for the pure substance. Make required assumptions.

Solution

Let us assume that the expansion process follow to law pV^n = constant. Now,

$$\begin{aligned} p_1V_1^n &= p_2V_2^n \\ \frac{p_1}{p_2} &= \left(\frac{V_2}{V_1}\right)^n \\ \ln\left(\frac{p_1}{p_2}\right) &= n\ln\left(\frac{V_2}{V_n}\right) \\ n &= \frac{\ln\left(\frac{p_1}{p_2}\right)}{\ln\left(\frac{V_2}{V_1}\right)} = \frac{\ln\left(\dfrac{10}{2}\right)}{\ln\left(\dfrac{0.48}{0.15}\right)} = 1.3836 \\ W_d &= \frac{(p_1v_1 - p_2v_2)}{(n-1)} = \frac{\left(10 \times 10^5 \times 0.15 - 2 \times 10^5 \times 0.48\right)}{(1.3836 - 1)} \\ &= 140,771.6371 \text{ J} = 140.77 \text{ kJ} \end{aligned} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.23 A certain mass of air is compressed adiabatically from 1 bar at 0.1 m^3 to 5 bar in a cylinder piston arrangement. Find the

(i) work of compression, assuming adiabatic compression and

(ii) work done on the air, assuming hyperbolic compression from the same initial state to the final pressure of 5 bar.

Solution

If the compression is adiabatic, then $p_1V_1^\gamma = p_2V_2^\gamma$, where $\gamma = 1.4$.

$$V_2 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} V_1 = \left(\frac{p_1}{p_2}\right)^{\frac{1}{1.4}} V_1 = \left(\frac{1}{5}\right)^{0.7143} \times 0.1 = 0.0317\ \text{m}^3$$

$$W_d = \frac{(p_1v_1 - p_2v_2)}{n-1} = \frac{10^5 \times (1 \times 0.1 - 5 \times 0.0317)}{0.4} = -14,625\ \text{J} = 14.625\ \text{kJ}$$

For hyperbolic compression, $p_1V_1 = p_2V_2$

$$V_2 = \left(\frac{p_1}{p_2}\right) V_1 = \left(\frac{1}{5}\right) \times 0.1 = 0.02\ \text{m}^3$$

$$W_d = p_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 1 \times 10^5 \times 0.1 \times \ln\left(\frac{0.2}{0.1}\right) = 6931.47\ \text{J} = 6.931\ \text{kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

2.24 A four-cylinder four-stroke internal combustion engine running at 1000 rpm has equal bore and stroke of 10 cm. Indicator diagram obtained from the engine has an area of 8 cm^2 and a length of the indicator diagram is 6 cm. The indicator spring constant is 2 bar/mm. Find the mean effective pressure in kPa and the indicated power in kW.

Solution

Assume adiabatic compression. *imep* from the diagram is given by:

$$imep = \frac{a}{l} \times s$$

where a is the area of the indicator diagram, l is the length of the indicator diagram and s is the spring constant.

$$\begin{aligned} imep &= \left(\frac{8 \times 10^{-4}}{6 \times 10^{-2}}\right) \times 2 \times 10^8 = 0.0267 \times 10^5 \times 10^3 \\ &= 2.67 \times 10^6\ \text{N/m}^2 = 2670\ \text{kPa} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\ \text{Area of cylinder} &= \frac{\pi}{4} \times D^2 = \frac{\pi}{4} \times 0.1^2 = 7.854 \times 10^{-3}\ \text{m}^2 \\ ip &= \frac{PLAnK}{60,000} \end{aligned}$$

where $n = N/2$ for a four-stroke engine and K is the number of cylinders

$$ip = \frac{2.67 \times 10^6 \times 0.1 \times 7.854 \times 10^{-3} \times \left(\frac{1000}{2}\right) \times 4}{60,000} = 69.9\ \text{kW} \overset{\textbf{Ans}}{\Longleftarrow}$$

2.25 A four-stroke, six-cylinder engine has a bore of 10 cm and stroke of 12 cm. It develops brake power of 25 kW. The torque in the crankshaft is 500 Nm. Indicator diagram obtained from the engine has an area of 9 cm^2 and length 5 cm. The spring constant is 0.5 bar/mm. Find the ratio of brake power to indicated power. What is their ratio called?

Solution

$$imep = \frac{aS}{l} = \frac{9 \times 10^{-4} \times 0.5 \times 10^{8}}{0.05} = 9 \times 10^{5}\ \mathrm{N/m^2}$$

$$ip = \frac{PLAnK}{60,000}$$

where $n = N/2$ and K is the number of cylinders.

$$ip = \frac{9 \times 10^{5} \times 0.12 \times \left(\frac{\pi}{4}\right) \times 0.1^{2} \times N \times 6}{2 \times 60,000} = 0.0424\ \mathrm{N} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$bp = \frac{2 \times \pi \times N \times T}{60,000} = \frac{2 \times \pi \times N \times 500}{60,000} = 0.0524\ \mathrm{N} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\frac{bp}{ip} = \frac{0.0424\ N}{0.0524\ N} = 0.81 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

This ratio is called mechanical efficiency.

Review Questions

2.1 What do you understand by (i) energy, (ii) work and (iii) heat.

2.2 What are various forms of energy? Explain.

2.3 Define work based on the principle of (i) mechanics and (ii) thermodynamics.

2.4 What are the different forms of work? Arrange them alphabetically.

2.5 Derive the expression for acceleration work.

2.6 What is displacement work? Show that displacement work is expressed by $\int pdv$.

2.7 Explain path function and point function.

2.8 Mention the various resisted processes for thermodynamic applications.

2.9 With a sketch, explain isochoric process.

2.10 What do you understand by isobaric process. Explain with a sketch.

2.11 With p-V diagrams explain polytropic and hyperbolic processes.

2.12 Derive the expression for work when a system undergoes polytropic process.

2.13 What is an indicator? With an indicator diagram how will you calculate the mean effective pressure of an engine?

2.14 Develop an expression for gravitational work.

2.15 What is shaft work? Derive an expression for power for rotating mechanics.

2.16 What is spring work? Show that spring work, $W_{spring} = \frac{1}{2}K\left(L_2^2 - L_1^2\right)$, *where* K *is the spring constant and* L_2 *and* L_1 *are the final and initial displacement length.*

2.17 With a sketch, explain electrical work show that $\delta W_{bat} = -EIdt$

2.18 For an elastic rod subjected to a force when both ends are free, show that $\frac{\delta W}{V} = -\sigma d\epsilon$, *where* σ *is the normal stress and* ϵ *is strain.*

2.19 Derive an expression for elastic work for a rod which is fixed at one end and subjected to a force, F.

2.20 Derive the expression for work for stretching of a liquid film.

2.21 What is meant by paddle wheel work? Derive an expression for work done.

2.22 What do you understand by flow and non-flow? Explain and derive the appropriate expression for work.

2.23 Explain clearly what do you understand by thermodynamic definition of work. Why such a definition is required?

2.24 Explain the sign convention adopted in work definition.

2.25 What is heat and explain the sign convention adopted.

Exercise

2.1 An artificial satellite revolves round the earth with a relative velocity of 1 km/s. If the acceleration due to gravity is 9.2 m/s^2 and gravitational force is 3680 N. Calculate the kinetic energy of the satellite. *Ans:* 200 MJ

2.2 An object of 50 kg mass falls freely under the influence of gravity from a height of 150 m on the earth's surface. The initial velocity is directed towards ground is 150 m/s. Ignoring the effect of air resistance, what is the magnitude of velocity in m/s of the object just before it strikes the ground. Take $g = 9.81$ m/s^2. *Ans:* 159.5 m/s

2.3 Consider a locomotive which consumes 1.8 tonne of coal. Assume that 15% of heat generated by the coal is converted into coal gas and then into mechanical work. The tractive effort required is 25 N/tonne of the dead mass of the locomotive. Take the dead mass of the locomotive as 2500 tonnes. Take calorific value of the coal as 35 MJ/kg. Under these conditions, calculate the distance to which the locomotive can move. *Ans:* 151.2 km

2.4 A fluid is continuously stirred in a cylinder piston arrangement as shown in Fig.2.31. The cylinder diameter is 40 cm. During the 15 minutes of the stirring process, the piston moves slowly through a distance of 500 mm against the atmospheric pressure of 1 bar. The work done on the fluid during the process is 2000 Nm. Speed of electric motor is 1000 rpm.

(i) Determine the torque in the driving shaft and shaft power output of the motor.

(ii) The electric motor in (i) is supplied with a current of 1 amp from a 24 V battery. Compute the net work for the battery and the motor.

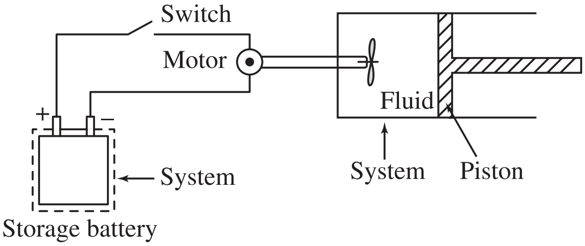


Fig. 2.31

Ans: (i) 87.85 Nm (ii) $W_{bat} = = 21,600$ Nm; $W_{motor} = -13,316.8$ Nm

2.5 The specific heat capacity of a system during certain process is given by the equation $c_s = 0.42 + 0.0042T$ kJ/kg °C. If the mass of the gas is 5 kg and its temperature changes from 27 °C to 127 °C, find the (i) heat transferred and (ii) mean specific heat.
Ans: (i) 371.7 kJ; (ii)0.743

2.6 Gas from a bottle of compressed helium is used to fill an inelastic balloon, originally folded completely flat to a volume of 0.6 m^3. If the barometer reads 750 mm Hg, what is the amount of work done upon the atmosphere by the balloon? Sketch the system before and after the process and express the details of the process. Figure 2.32 shows the system before and after the process. The full line P1 shows the boundary of the system before the process and the dotted line P2 shows the boundary after the process.
Ans: 60.80 kJ

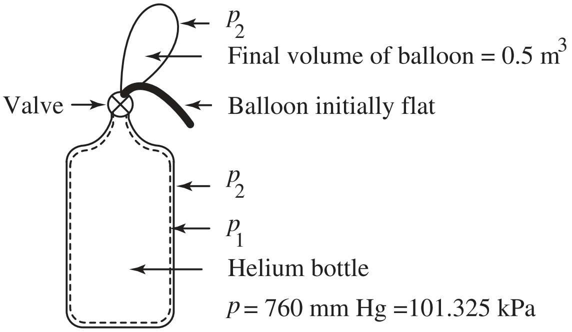


Fig. 2.32

This is positive since work is done by the system. The work done by the atmosphere is −60.80 kJ. Since the wall of the bottle is rigid, there is no *pdV* work involved. It is assumed that the pressure in the balloon is atmospheric at all times and the balloon fabric is light, inelastic and unstressed, had the balloon is elastic and stressed during the filling process, the work done by the gas would be greater than 60.80 kJ by an amount equal to the work done in stretching the balloon, although the displacement work done by the atmosphere is still -60.80 kJ. However, if the system includes both the gas and balloon, the displacement work would be 60.80 kJ as estimated alone.

2.7 Consider an evacuated bottle shown in Fig.2.33. When it is opened, atmospheric air pushes into the bottle. If the atmospheric pressure is 1 bar and 0.5 m^3 of air (measured atmospheric conditions) enters into the bottle, calculate the work done by the air and determine the sign. *Ans:* 50.6625 kJ; surroundings do positive work on the system.

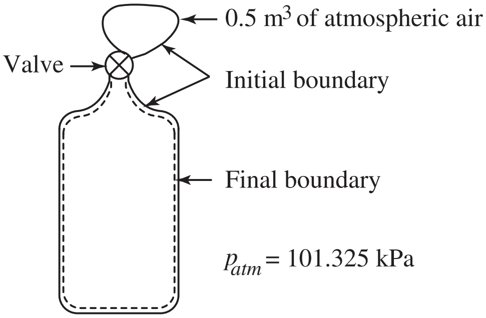


Fig. 2.33

2.8 Consider a piston-cylinder with stirrer arrangement as shown in Fig.2.34. The piston is frictionless and is held down against the fluid due to the atmospheric pressure of 1 bar. The stirring device is turned at speed of 9000 rev/minute. The average torque against the fluid of 1.5 Nm. Assume that in this process, the piston whose diameter is 0.75 m travels a distance of 1 m. Find the net work transfer for the system. *Ans:* -40 kJ

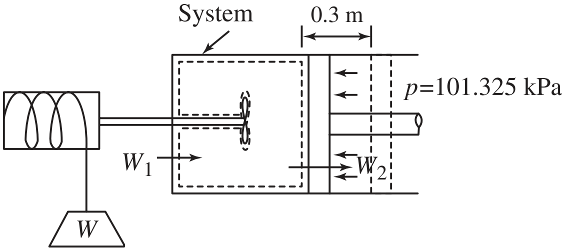


Fig. 2.34

2.9 Consider a 12-cylinder, single-acting two-stroke marine diesel engine with following specifications:

Speed	:	100 rpm
Cylinder diameter	:	1 m
Stroke	:	1.2 m
Area of the indicator diagram	:	6×10^{-4} m^2
Length of the diagram	:	0.06 m
Spring constant	:	150 MPa/m

Find the net rate of work transfer from the gas to the piston. *Ans:* 28200 kW

2.10 It is required to melt 3 tonnes/h of iron from a charge at 20 °C to molten metal at 1620 °C. The melting point of iron can be taken as 1540 °C and latent heat is 270 kJ/kg. The specific heat in kJ/kg of solid iron is 0.5 and at liquid state is 30/atomic weight. An elective furnace is used to melt the iron has an efficiency of 75%. Estimate the kW rating needed. If the density of molten state is 7000 kg/m^3 and the furnace bath volume is 2.5 times the hourly melting rate. Find the dimensions of the cylindrical furnace if the height to diameter ratio is 3. Take the atomic weight of iron as 56. *Ans:* $d = 0.7686$ m; $h = 2.306$ m

2.11 It is proposed to melt aluminum in a furnace. Solid aluminum has a specific heat of 0.9 kJ/kg and at liquid state, it is given by 30/atomic weight. Latent heat is 400 kJ/kg. Atomic weight 27, density at molten state 2424 kg/m^3. The final temperature is 800 °C. Estimate how much metal can be melted per hour with the given below kW rating. Also find the mass of aluminum alone that the furnace can hold. The melting point of aluminum is 650 °C. Take the efficiency of furnace as 80%. Assume a kW rating of 2.2×10^3 kW and volume as 1 m^3. *Ans:* 5, 575 tonnes/g; 2.424 tonnes

2.12 A cooling tower nozzle disperses water into a stream of droplets. Average diameter of the droplets is 50 micron. Calculate the work required to atomise 2 kg of water isothermally in the ambient conditions. Take surface tension of water in contact with air = 0.08 Nm. Density of water as 1000 kg/m^3. Assume that the water enters the nozzle through a pipe of 25 mm. *Ans:* 19.17 J

2.13 An electric motor drives a stirrer fitted in a horizontal cylinder. The cylinder of 50 cm diameter contains fluid restrained by a frictionless piston. During the stirring operation for 15 min, the piston moves outward slowly by distance of 25 cm against the atmospheric pressure of 1 bar. The current supplied to the motor is 1 amp from a 12 V battery. If the conversion efficiency of the motor is 90%. Estimate the work done on the motor stirrers and atmosphere. *Ans:* 9.72 kJ; 4.908 kJ

2.14 Compressed nitrogen gas from a cylinder is employed to inflate an inelastic flexible balloon. Originally the balloon was folded completely flat. When inflated, the volume was $0.4m^3$. If the barometric reading is 760 mm of Hg, calculate the amount of work done upon the atmosphere by the balloon? Sketch the system before and after the process. *Ans:* 40.53 kJ

2.15 A piston engine cylinder has a diameter of 5 cm and moves by 6 cm during the part of the friction stroke. During this operation 110 cc of free air is drawn in. The cylinder pressure is 0.8×10^5 N/m^2 and the inlet manifold pressure is 1.013×10^5 N/m^2. The difference between the friction and atmospheric pressure is accounted for the resistance in the intake pipe and inlet volume, etc. Find the net work done during the process. *Ans:* -1.718 Nm

2.16 Consider the piston-cylinder arrangement shown in Fig.2.35. Assume the piston is frictionless and is held down against the fluid of 100 cc due to the atmospheric pressure of 101.3 kPa. The stirring device rotates at a speed of 12, 000 rpm with an average torque of 2 Nm. The diameter of the piston is 0.4 and during the process, the piston moves by twice the diameter. Find net work transfer for the system. Find the net work done. *Ans:* -140.616 kJ

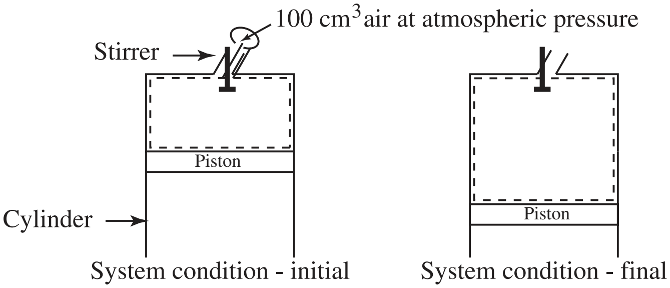


Fig. 2.35

2.17 Assume that the properties of a closed system can be characterised by $pV = 2.8$ where p is in bar and V is in m^3. Calculate the work done where the pressure increases from 1.4 bar to 7 bar. *Ans:* −450.6 kJ

2.18 In a closed system, 200 kJ of work is supplied. If the initial volume is 0.5 m^3 and pressure changes as per the law $p = 10 - 5V$ where p is in bar and V is m^3. What will be the final volume and pressure of the system. *Ans:* 0.2536 m^3; 8.732 bar

2.19 Consider a room of size $10 \times 12 \times 5$ m^3 is cooled electrically from an initial temperature of 37 °C – 17 °C. Assume the air pressure inside the room is same as the surroundings. The surrounding air pressure is 75 cm of Hg. Assume that pressure remains constant during the cooling process. The heat absorbing capacity of walls and furniture is 40 kJ/s. Take the specific heat of air is 1 kJ/kg. Calculate the amount of air leak from the room and estimate the amount of electrical energy needed for cooling the room. *Ans:* 47.67 kg; 4.195 kW

2.20 A 1000-litre rigid tank with air at 1 MPa and 400 K is connected to an air line as shown in Fig.2.36 The valve is opened to fill the tank. When the tank pressure reaches 5 MPa, the valve is closed. At this time, the temperature of the gas in the tank is 450 K. Calculate:

(i) the mass of air in the tank before and after the valve is closed and

(ii) the pressure inside the tank when the gas cools to the room temperature of 300 K.

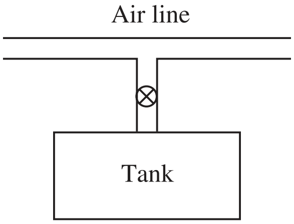


Fig. 2.36

Ans: (i) 8.72 kg; 38.8 kg; (ii) 3.34 MPa

2.21 30 people attended a farewell party in a club room whose size is $10 \times 10 \times 5$ m. Each person emits about 300 kJ of heat per hour. Assuming no leak of air from the room and the room to be perfectly insulated, calculate the rise in temperature of air in 15 minutes.

Take c_V of air as 0.717 kJ/kg K and $R = 0.287$. Assume that each person occupies around 0.05 m^3. Take pressure as 1.013 bar and $T = 20°C$. *Ans:* 5.22 °C

2.22 Helium having a density of 0.1785 kg/m^3 at standard temperature and pressure is filled in a balloon of 12 m diameter. The balloon is designed to carry three persons weighing 60 kg each. Assume that the air density at STP is 1.2495 kg/m^3. Take the mass of ropes and the case and the balloon fabric to be 15% of the total mass of the persons. Assume the value of $g = 9.8$ m/s^2. Calculate the gravitational force acting on helium, person and balloon and also estimate the acceleration of the balloon during take off. *Ans:* 3615 N; 20.3 m/s^2

2.23 A mass of 10 kg is dropped from a height of 150 m above the ground level. Estimate the maximum velocity the mass can attain. Determine the kinetic energy of the mass of 50 and 100 m above the ground. Assume the value of $g = 9.8$ m/s^2. *Ans:* 54.22 m/s; 9799 J; 4898.5 J

2.24 A gas is contained in a cylinder fitted with a piston loaded with a number of small weights as shown in Fig.2.37. The initial pressure is 1.5 bar and the initial volume is 0.05 m^3. The gas is now heated until the volume of the gas increases to 0.1 m^3. Calculate the work done by the gas in the following process:

(i) isobaric process, (ii) isothermal process and (iii) $pV^{1.3} = C$ during the process.

Explain how such processes could be achieved. Show the processes on a *p*-*V* diagram. *Ans:* 7.5 kJ; 5.19 kJ; 4.7 kJ

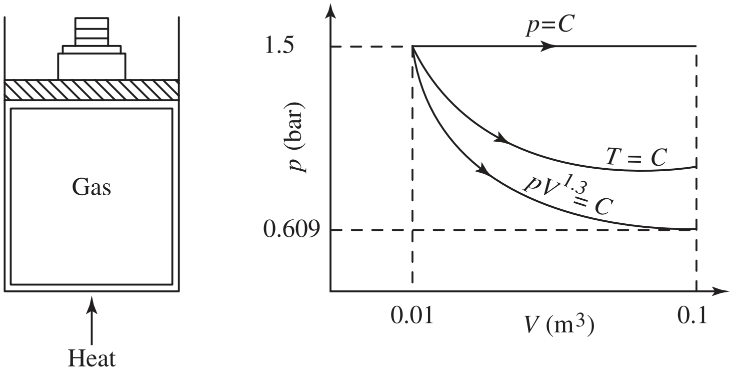


Fig. 2.37

2.25 The heat transfer between electric motor and its surroundings changes with time given by $Q = -0.2\left[1 - e^{-0.65t}\right]$ where t is in seconds and Q is in kW. The shaft of the motor rotates at a constant speed of 100 rad/s and the torque applied is 18 Nm to an external load. The motor draws a constant electrical power input of 2 kW. Obtain the expression for the time rate of change of energy of the motor.

Ans: $\dfrac{dE}{dt} = 0.2 \times \left[1 - e^{-0.65t}\right] + 1.8 - 2 = -0.2 \times e^{-0.65t}$

Multiple Choice Questions (choose the most appropriate answer)

1. The displacement work of a system can be expressed as $\int pdv$. The above expression is valid for

(a) any process
(b) a non-quasi-state process
(c) a reversible process
(d) an isotropic process

2. In a free expansion, work done is

(a) zero
(b) negative
(c) positive
(d) maximum

3. Consider a current flowing through a resistor as a system across a potential difference. The energy flowing into the system is

(a) heat transfer
(b) thermodynamic work transfer
(c) electricity
(d) heat and work transfer

4. In a quasi-static displacement work, the integrating factor is

(a) $\frac{1}{T}$
(b) $\frac{1}{V}$
(c) $\frac{p}{V}$
(d) $\frac{1}{p}$

5. The cyclic integral of which of the following is zero?

(a) work transfer
(b) temperature
(c) latent heat
(d) heat transfer

6. For the reversible heat transfer, the integrating factor is

(a) $\frac{1}{p}$
(b) $\frac{1}{V}$
(c) $\frac{1}{T}$
(d) $\frac{p}{T}$

7. Identify, which of the following is not a boundary phenomenon.

(a) change of temperature
(b) heat transfer
(c) work transfer
(d) mass transfer

8. If the value of n is infinitely large in a polytropic process, $pV^n = C$, the process is called

   (a) constant pressure process
   (b) constant enthalpy process
   (c) constant temperature process
   (d) constant volume process

9. $\int pdv$ expression for obtaining work applicable to

   (a) throttling process
   (b) reversible process
   (c) steady flow reversible process
   (d) irreversible adiabatic process

10. The difference in the amount of work done on a system and the amount of heat supplied from a system to bring back the system to its initial state is

    (a) less than zero
    (b) greater than zero
    (c) zero
    (d) maximum

11. Temperature is defined by

    (a) zeroth law
    (b) first law
    (c) second law
    (d) law of entropy

12. Heat transferred to a closed stationary system at constant volume is equal to

    (a) increase in internal energy
    (b) work transfer
    (c) increase in enthalpy
    (d) increase in entropy

13. Which of the following does not change during the throttling process?

    (a) volume
    (b) enthalpy
    (c) entropy
    (d) pressure

14. Reversible steady flow work interaction is equal to

    (a) $\int pdV$
    (b) $u_1 - u_2$
    (c) $-\int Vdp$
    (d) $p_1V_1 - p_2V_2$

15. Reversible work transfer in a closed system is equal to

    (a) $-\int_1^2 Vdp$
    (b) $h_1 - h_2$
    (c) $p_1V_1 - p_2V_2$
    (d) $\int_1^2 pdV$

16. Heat transferred at constant pressure is equal to

(a) work transfer
(b) change in entropy
(c) change in enthalpy
(d) change in internal energy

17. In which of the following the mass and energy remain constant always

(a) closed system
(b) open system
(c) universe
(d) non-isolated system

18. Work output of a system is

(a) always positive
(b) always negative
(c) zero
(d) it can be both positive and negative

19. If the heat flows into the system from the surroundings, then it is taken as

(a) positive
(b) negative
(c) zero
(d) can be both negative and positive

20. Area under the line of processes platted as a p-V diagram represents

(a) work
(b) heat
(c) energy
(d) entropy

21. Indicator diagram obtained from an engine represents

(a) heat developed by the engine in one cycle
(b) energy dissipation in one cycle
(c) work done in one cycle
(d) all of the above

22. Flow work represents
(a) heat transferred across the boundary
(b) work transferred across the system boundary
(c) energy transferred across the system boundary
(d) none of the above

23. Heat flow takes place when there is a difference in

(a) pressure
(b) volume
(c) entropy
(d) temperature

24. As work is a path function, the change in work is given by

    (a) inexact differential
    (b) exact differential
    (c) partial differential
    (d) total differential

25. As thermodynamic properties are point function their change is given by

    (a) inexact differential
    (b) exact differential
    (c) partial differential
    (d) total differential

Ans: 1. – (c) 2. – (a) 3. – (b) 4. – (d) 5. – (b)
6. – (c) 7. – (a) 8. – (d) 9. – (b) 10. – (c)
11. – (a) 12. – (a) 13. – (b) 14. – (c) 15. – (a)
16. – (c) 17. – (c) 18. – (a) 19. – (a) 20. – (a)
21. – (c) 22. – (b) 23. – (d) 24. – (a) 25. – (b)

3

FIRST LAW OF THERMODYNAMICS AND CLOSED SYSTEMS

3.1 INTRODUCTION

Until 1800, people were using caloric theory for describing heat. Caloric theory considered heat as an invisible fluid. It was believed that heat flows from a body of higher calorie to a body of lower calorie. Around 1797, Benjamin Thompson showed that mechanical action can generate large amounts of heat. This discovery started challenging the caloric theory which was widely accepted at that time. It was Joule, who, by means of experiments, established that heat is a form of *energy.* This laid the foundation for the first law of thermodynamics.

Historically, first established thermodynamic principle was formulated by Sadi Carnot in 1824. This eventually became the second law of thermodynamics. By 1860, two established principles of thermodynamics were evolved which were called the first and second principles. Later they were called as thermodynamic laws. Then, the modern third law was getting adopted. Already we have considered zeroth law of thermodynamics in Chapter 1 in connection with the thermodynamic property, viz temperature. On the whole, there are four laws of thermodynamics. They are:

(i) Zeroth law of thermodynamics
(ii) First law of thermodynamics
(iii) Second law of thermodynamics
(iv) Third law of thermodynamics

In this chapter, we will discuss in detail the first law of thermodynamics and its application to closed systems.

3.2 CONSERVATION OF ENERGY

In Chapter 2, we considered various forms of energy such as heat (Q), work (W) and total energy (E) separately. We did not make any attempt to relate them to each other when a thermodynamic process takes place. Actually, this is done by *conservation of energy principle.* It provides a sound basis for studying the relationships among the various forms of energy and energy interactions. According to the principle of conservation of energy,

Energy can neither be created nor be destroyed during a process; it can only change forms.

That is to say that energy is conserved. Now, let us illustrate the energy conservation principle by means of an example.

Consider a reasonably big stone placed at some height, h, from a datum as shown in Fig.3.1. It will possess a certain amount of potential energy given by mgh, where m is the mass of the stone and h is the height at which the stone is placed. Let this value be 18 kJ. When the stone is made to fall (refer Fig.3.1), part of this potential energy is getting converted to kinetic energy. Experimental evidence shows that the decrease in potential energy is exactly equal to the increase in kinetic energy assuming the air resistance is negligible. As shown in Fig.3.1, at a height $h/2$ from the top, if we measure, it will have potential energy of 9 kJ and kinetic energy of 9 kJ. This means 50% of the potential energy has converted into kinetic energy. When it reaches the datum (ground), all its potential energy would have been converted to kinetic energy. Thereby, $PE_{datum} = 0$ kJ and $KE_{datum} = 18$ kJ. This example confirms the principle of conservation of energy, i.e. transformation of potential energy to kinetic energy. The first law of thermodynamics is nothing but the application of this principle. Therefore, the first law can be looked as *the law of energy conservation.*

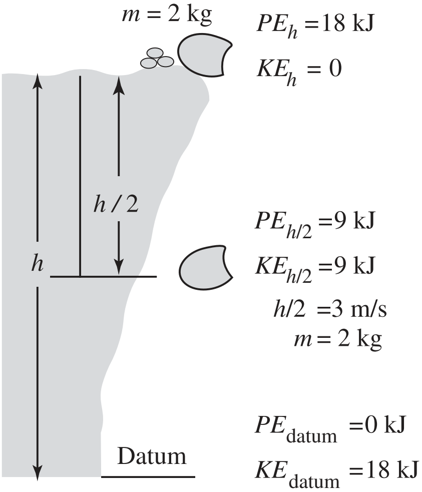


Fig. 3.1 Illustration of conservation of energy principle

3.2.1 The Concept of Total Energy

Consider a system undergoing a series of *adiabatic* processes from a specified state 1 to another specified state 2. As the processes are adiabatic, obviously there is no heat transfer. However, they may involve several kinds of work interactions. Careful measurements during the experiments will bring us to the conclusion that *for all adiabatic processes between two specified states of a closed system, the net work done is the same. It does not depend upon the nature of the closed system or the details of the process.*

What the above statement means is that there are an infinite number of ways to perform work interaction under adiabatic conditions. Note that the above statement is *very powerful* and has far-reaching implications. Also note that the above statement is largely based on the experiments of Joule (see section 3.2.5) and cannot be derived from any known physical principle. Therefore, it is considered as the fundamental principle and this principle is called the *first law of thermodynamics* or just the *first law.*

The important point to understand in the principle is to appreciate the existence of the property called *total energy*, E. Note that the first law makes no reference to the value of the total energy of a closed system at a state. It simply states that during an *adiabatic process*, the *change in the total energy* must be equal to the net work done for a closed system. Therefore, to serve as a reference point, any convenient arbitrary value can be assigned to *total energy* at a specified state.

To put it in a nutshell, the conservation of energy principle(first law) implies, the existence of the property, called, *total energy.* However, the first law is often viewed as a statement of the *conservation of energy.* Now, let us consider some practical examples.

3.2.2 Practical Examples

Let us try to understand the *the principle of conservation of energy* with the help of some familiar examples using intuitive arguments. First, let us consider a device that involves heat transfer but no work interactions and then another example that involves work transfer but no heat transfer.

Example 1:

Consider a steam power plant shown in Fig.3.2, which uses a fuel either gas, oil or coal. Fuel will be burnt in the boiler and certain amount of heat energy, Q_{in}, will be transferred to heat the water. This will cause heat transfer from the burning fuel to water. The water starts boiling and will form high pressure steam in the boiler.

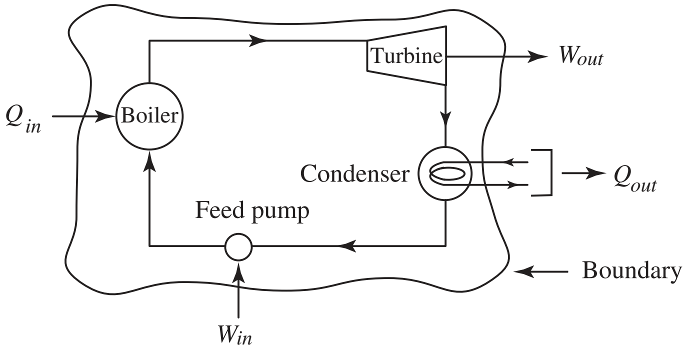


Fig. 3.2 Schematic diagram of a steam power plant

The steam from the boiler can be expanded in a turbine to produce *shaft work*, W_{out}. The expanded steam gets condensed in the condenser liberating certain amount of heat, Q_{out}. Then, the condensed water is pumped back through a feed pump to the boiler. There is a work input, W_{in}, to the feed pump and the cycle repeats. If there is no heat transfer in the boiler then there will be no steam generation and there will be no work output from the turbine. *This makes us to conclude that production of work is subject to heat transfer in the boiler.*

Example 2:

Imagine a bicycle pump for inflating the tyre. It will be observed that the pump becomes hotter during the pumping action. This phenomenon of heating of pump is obviously not from heat transfer. It is due to the work done during the pumping action. *This makes us to conclude that production of heat is subject to work transfer.*

The above two examples indicate clearly that some effects can be caused equivalently by heat or work. Then, it is obvious for us to think that there must be a quantitative relationship between heat and work. Note that no logical relation can be derived between heat and work. However, this can be established through experiments.

3.2.3 Relation between Heat and Work

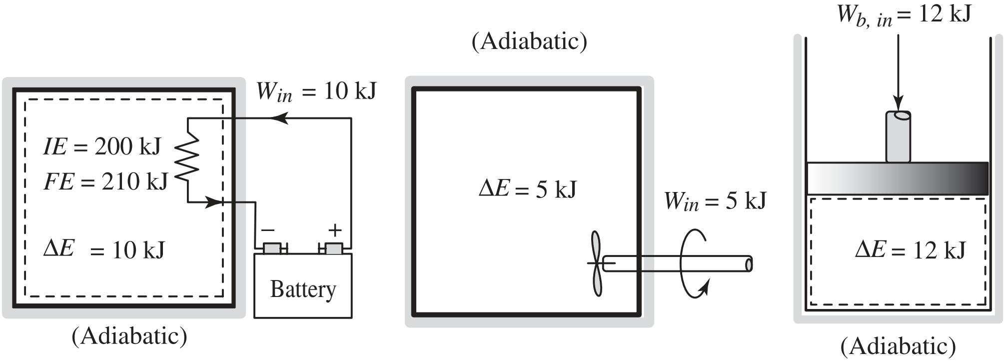
It was Benjamin Thompson who discovered the equivalence of work and heat while manufacturing canon by boring solid metal submerged in water. He was surprised by the boiling of water, even though no heat had been added to the water. He was astonished that mechanical work of boring could generate so much of heat. It was James Prescott *Joule*, an English scientist, with help from Lord *Kelvin* showed conclusively by means of large number of experiments that mechanical work and heat are *equivalent*. Further, experiments, later brought into light, more evidence until some 50 years after the above experiment.

At this point, you should understand that a basic knowledge of *quantitative relation* between the two forms of energy, heat and work, is essential. This will help to carry out a *performance analysis* of the energy conversion devices. In this, the first law confines to the former, viz *quantitative relation,* while the second law deals with the latter, viz *performance analysis.*

By now it should be clear, that the first law of thermodynamics introduces a very important property of a system called *energy* which has many forms. Let us consider some practical devices involving conservation of energy principle.

3.2.4 Practical Devices Involving Conservation of Energy Principle

Consider a well-insulated (adiabatic) system, example, a room, as shown in Fig.3.3(a). Let it be heated by an electric heater. As a result of electrical work done, the energy of the system will increase. Since, the system is adiabatic and cannot have any heat transfer from or to the surroundings (i.e. $Q = 0$). The conservation of energy principle dictates that the electrical work done on the system must equal the energy increase of the system.



(a) Electrical work done on the system

(b) Mechanical work done on the system

(c) Boundary work done on the system

Fig. 3.3 Illustration of total energy principle

Let us illustrate this with some numerical values. Let us assume that initially the system has 200 kJ of energy. Let the energy input through the heater be 10 kJ. Now, the system will have an energy of 210 kJ since it is assumed as adiabatic. Change in the energy, ΔE, is 10 kJ. Therefore, the electrical work done on the system is equal to the change in the energy of the system. Refer Fig.3.3(a).

Let us now replace the electric heater with a paddle wheel as shown in Fig.3.3(b). Due to the stirring process, which causes mechanical work input, the energy of the system will increase.

Further, there is no heat interaction between the system and its surroundings ($Q = 0$). The shaft work done on the system must show up as an increase in the energy of the system, viz the room.

Most of us would have probably noticed the increase in temperature of air when it is compressed [Fig.3.3(c)]. This is because there is energy transfer to the air in the form of boundary work. In the absence of any heat transfer ($Q = 0$), the entire boundary work will be stored in the air as part of its total energy. The conservation of energy principle again requires that the increase in the energy of the system be equal to the boundary work done on the system.

We can extend these discussions to systems that involve heat and work interactions simultaneously. Consider the system shown in Fig.3.4 with numerical values given. Let the initial energy (IE) in the system be 15 kJ. As can be seen that the shaft work input is 6 kJ. There is a heat input of 15 kJ whereas 3 kJ heat energy goes out. Now, the change in energy $\Delta E = (15 - 3) + 6 = 18$ kJ. That is to say that the system gains 18 kJ of energy during the process. That is, the change in the energy of a system during a process is simply equal to the net energy transfer to (or from) the system.

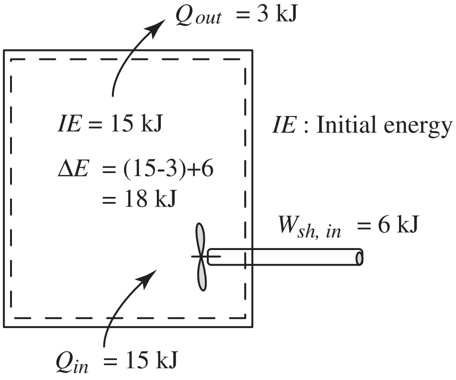
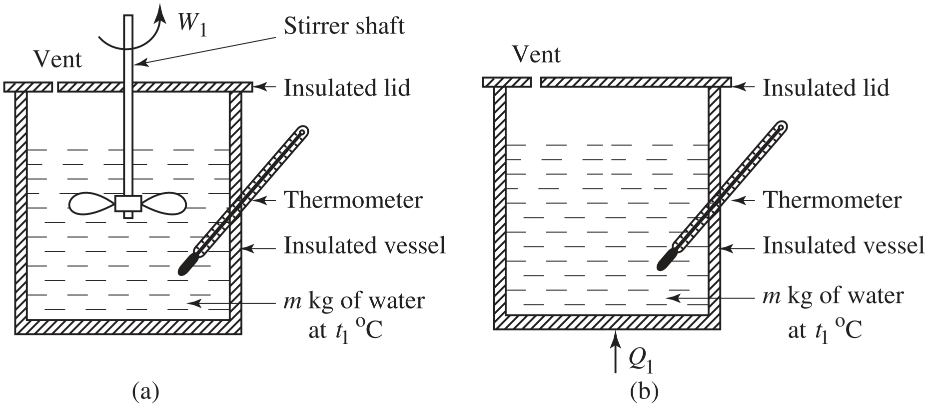


Fig. 3.4 Heat and work interaction

3.2.5 Joule's Experiment for Non-cyclic Process

In the previous section, we discussed the details regarding the conservation of energy principle. According to the conservation of energy principle, Q and W are *equivalent* but not equal. In order to understand the relation between heat, Q and work, W let us illustrate with another example. Consider a closed system which permits both work and heat interaction as in the case of stirring of a fluid in a container [Fig.3.5(a)]. Let us assume that the vessel is insulated and contains m kg of water at t_1 °C. Now, let us start stirring the water. As a result of stirring it could be seen that the temperature of water going up and let this temperature be t_2 °C. The work done by the stirrer can be calculated in terms of torque on the shaft, speed of the shaft and the churning time of the stirrer. Let the magnitude of the work be W [Nm].

Now, let us consider another similar vessel, as shown in Fig.3.5(b). Let it be insulated all around except the bottom. Let us assume as before that it contains m kg of water at t_1 °C. Let us now heat the water to the same final temperature, t_2 °C. Let the magnitude of heat transfer required be Q. By doing this experiment, you will find that the temperature change in both cases is the same. In the first case, the change is brought about by work interaction W, whereas in the second case, it was brought about by heat interaction Q.



(a) (b)

Fig. 3.5 Heating of water by work interaction

Similarly, a number of experiments can be conducted in which the state change of a system is brought about either by work interaction or by heat interaction. Let us represent them by $(W_1, Q_1), (W_2, Q_2), (W_3, Q_3), (W_4, Q_4)$, etc. The experimental results would confirm that the ratio W/Q turns out to be the same and will be equal to 4.189 N m/cal. This is to say that,

$$\frac{W_1}{Q_1} = \frac{W_2}{Q_2} = \frac{W_3}{Q_3} = \frac{W_4}{Q_4} = 4.189 \approx 4.2 \text{ Nm/cal}$$

where cal refers to calorie. Since Joule was the first person to do the experiment and established the relation between W and Q, it is called Joules experiment.

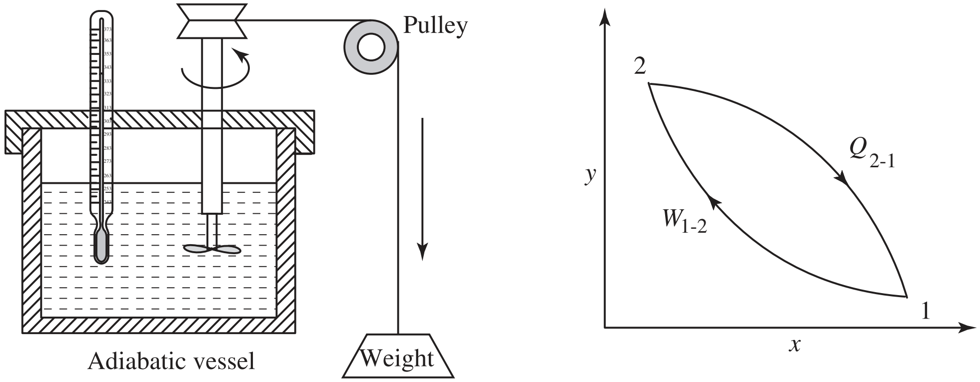
3.2.6 Mechanical Equivalent of Heat

It is now clear that work and heat interactions can bring about identical changes in a system and the ratio is a constant. *This ratio is called the mechanical equivalent of heat.* It is to be noted that 4.2 Nm of work can bring about the same effect as 1 calorie of heat and vice versa.

Let us now introduce a new unit of heat transfer, viz Joule, denoted by J whose magnitude is equal to 1/4.189 of a calorie. By this, the mechanical equivalent of heat turns out to be 4.189 Nm/4.189 J. *This concept helps us to have a common unit for both heat and work interaction.* Therefore, work and heat can be represented either by 'Nm' or by 'J', *since one 'Nm' and one 'J' can bring about identical changes.* In the SI system of units, both heat and work are measured in the derived unit of energy, the Joule. The constant of proportionality, J, is therefore, *unity*, i.e. 1 Nm/J. The discussions of Joule's experiment (section 3.2.5) deals with only non-cyclical processes, which begins with an initial state and ends in a final state. Now, let us see a second representation and historically, the first applied to a cyclical process.

3.2.7 Cyclic Processes

Let us consider a closed system shown in Fig.3.6(a) consisting of a known mass of water, a paddle wheel and a thermometer. The vessel is adiabatic since it is well insulated. Water in the vessel can be churned by the paddle wheel by making the weight to fall. Let the work done by the paddle wheel on the system be denoted by W_{1-2}. Let the initial temperature of the system be t_1 °C, which is the atmospheric temperature. After the work transfer, let the temperature rise to t_2 °C. Assume the pressure to be atmospheric.



(a) A closed system with adiabatic work transfer (b) Closed system undergoing a cycle

Fig. 3.6 Cyclic process of a closed system with adiabatic work transfer

The process 1–2 undergone by the system is shown in Fig.3.6(b) in generalised thermodynamic coordinates, x and y. Let us now remove the insulation. The system will start interacting with the surroundings. There will be heat transfer from the system to the surroundings since $t_2 > t_1$. The heat transfer will continue until the thermal equilibrium $(t_2 = t_1)$ is reached. The amount of heat transfer Q_{2-1} from the system during the process 2–1 can be easily estimated. As can be seen from Fig.3.6(b), the system has executed a cycle. The cycle consists of a definite amount of work transfer W_{1-2} to the system followed by heat transfer Q_{2-1} from the system. Again, as in the previous examples, it will be seen that W_{1-2} is always proportional to Q_{2-1}. We know that the constant of proportionality is called the *mechanical equivalent of heat.* In the example discussed, there are only two energy transfer quantities as the system performs a thermodynamic cycle. Even if the cycle involves many more heat and work quantities, the same result will be obtained. This can be represented *mathematically* as

$$\left(\sum W\right)_{cycle} = J\left(\sum Q\right)_{cycle} \tag{3.1}$$

where J is the Joule's equivalent of heat. This can also be expressed in the integral form as

$$\oint \delta W = J \oint \delta Q \tag{3.2}$$

where $\oint$ is the cyclic integral for the closed path. The above expression is the mathematical form of the first law of thermodynamics for a closed system undergoing a cycle. As $J = 1$, Eq.3.2 can be rearranged as

$$\oint (\delta Q - \delta W) = 0 \tag{3.3}$$

It should be understood that the quantity $(\delta Q - \delta W)$ is a thermodynamic property. What Eq.3.3 means is that when a system executes a cyclic process, the net heat transfer during the process minus the net work transfer during the process is *always equal to zero*, provided the unit for heat and work interaction is the same.

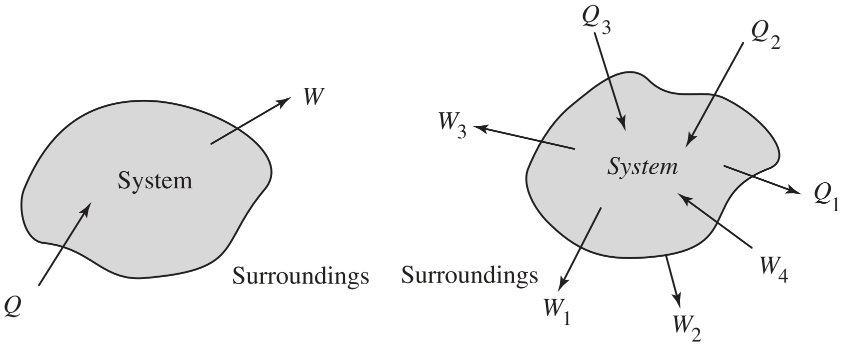
3.2.8 Closed System Undergoing a Change of State

In the previous section, we have seen a cyclic process. Mathematically, it was represented as

$$\left(\sum W\right)_{cycle} = \left(\sum Q\right)_{cycle} \tag{3.4}$$

when W and Q are represented in the same unit (i.e. J =1).

In such a system that undergoes a change of state where both work and heat transfer are involved, the net energy transfer gets accumulated or stored in the system. Let us examine a system shown in Fig.3.7. As can be seen in Fig.3.7(a), Q is the amount of heat transferred to the system and W is the amount of work transferred out of the system. The net energy stored in the system will be equal to $Q - W$.



(a) Single source (b) Multiple source

Fig. 3.7 Heat and work interactions

At this juncture, you should understand one very important point that this stored energy in the system is *neither heat nor work*. It is called the change in the stored energy of the system and is represented by ΔE. *It is called the change in stored energy because of the fact that before the heat and work interaction, the system possessed certain amount of energy stored in it.* Due to heat (Q) and work (W) interaction, this stored energy has changed. Now,

$$Q - W = \Delta E \tag{3.5}$$

or

$$Q = \Delta E + W \tag{3.6}$$

where ΔE is the change in the stored energy of the system. Note that Q, W and ΔE must be expressed in the same unit. Now, consider Fig.3.7(b), where multiple interaction of heat and work transfer is involved. Now, similar to Eq.3.6 and taking care of the sign convention, we can write:

$$(Q_2 + Q_3 - Q_1) = \Delta E + (W_1 + W_2 + W_3 - W_4) \tag{3.7}$$

From the above equation, we can conclude that energy is thus conserved in the operation. Equation 3.7 may also be considered as the definition of energy. Note that this definition does not give an absolute value of energy, E but only the change of energy ΔE for the process. It can, however, be shown that the energy has a definite value at every state of a system and is, therefore, a property of the system.

3.3 ENERGY - A PROPERTY OF THE SYSTEM

Consider the system shown in Fig.3.8. It represents a cycle followed by the system. Let us assume that initially the system changes its state following the path C from state 1 to state 2 and returns to state 1 via path B. Now, as per Eq.3.6, we can write:

For path C, $$Q_C = \Delta E_C + W_C \tag{3.8}$$

and for path B, $$Q_B = \Delta E_B + W_B \tag{3.9}$$

The processes C and B put together constitute a cycle. Therefore, we can write (refer Eq.3.4):

$$\left(\sum W\right)_{cycle} = \left(\sum Q\right)_{cycle} \tag{3.10}$$

$$W_C + W_B = Q_C + Q_B$$

or $$W_C - Q_C = Q_B - W_B \tag{3.11}$$

From Eqs 3.8 and 3.9 we can write, $$-\Delta E_C = \Delta E_B \tag{3.12}$$

or $$\Delta E_C = -\Delta E_B \tag{3.13}$$

Similarly, if the system from state 2 to state 1 following path A instead of path B, then

$$\Delta E_C = -\Delta E_A \tag{3.14}$$

From Eqs 3.13 and 3.14, we get, $$\Delta E_B = \Delta E_A \tag{3.15}$$

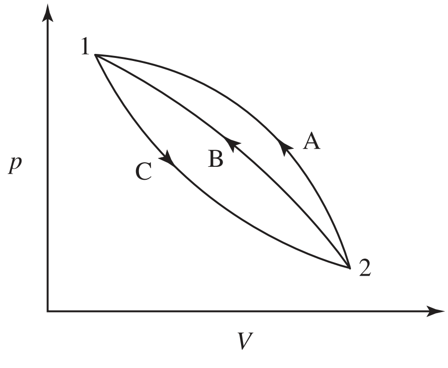


Fig. 3.8 Illustration of energy as property of a system

It is now clear that whatever path the system follows in undergoing the change of state, the *change in energy* between the two states is the same. If we can assign any arbitrary value for energy at state say 2, then the energy at 1 is *fixed* irrespective of the path the system follows. It means that, the energy has a definite value for every state of the system. *Therefore, it is a point function and thereby it is the property of the system.* The following points are to be kept in the mind:

(a) Energy E is an extensive property.

(b) Specific energy, $e = E/m$, is an intensive property.

(c) The cyclic integral of any property is zero, since the final state is exactly identical to the initial state.

3.4 STORED ENERGY

In Chapter 2, we have gone through certain fundamental aspects on energy. In this chapter, we have mentioned the term *stored energy* many times. Let us see what we mean by stored energy. It should be noted that stored energy in a system can be in two forms: (i) macroscopic and (ii) microscopic.

3.4.1 Macroscopic Form of Energy

The macroscopic form includes macroscopic kinetic and potential energy of a system. Let us consider a fluid element of mass m as shown in Fig.3.9. As can be seen, the fluid mass under consideration may contain a number of molecules. The average velocity of the fluid mass, considered macroscopically, has an average centre of mass velocity which we will call as $\vec{\mathbb{V}}$. When the fluid is in motion, the macroscopic kinetic energy, E_{KE}, is given by:

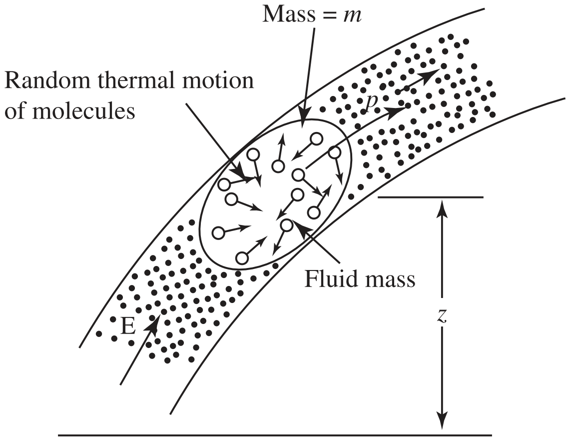


Fig. 3.9 Macroscopic and microscopic energy

$$E_{KE} \quad = \quad \frac{m(\vec{\mathbb{V}})^2}{2} \tag{3.16}$$

As indicated in Fig.3.9, if the elevation of the fluid element from an arbitrary datum is z, then the macroscopic potential energy of the fluid element is given by

$$E_{PE} \quad = \quad mgz \tag{3.17}$$

3.4.2 Microscopic Form of Energy

Energy stored in molecular, atomic and subatomic level is called *microscopic energy.* It is called *molecular internal energy* or simply *internal energy.* It is denoted by the symbol U. We know that matter is composed of molecules. They are in random thermal motion (for gas) with an average velocity, $\vec{\mathbb{V}}_m$. Molecules constantly collide with each other and also with the walls of the container. This causes rotation and vibration of molecules. Further, they may have translational kinetic energy, rotational kinetic energy, vibrational energy, electrical, chemical and nuclear energy (Fig.3.10). If e represents the stored energy or In other words, the internal energy of one molecule, then e can be written as:

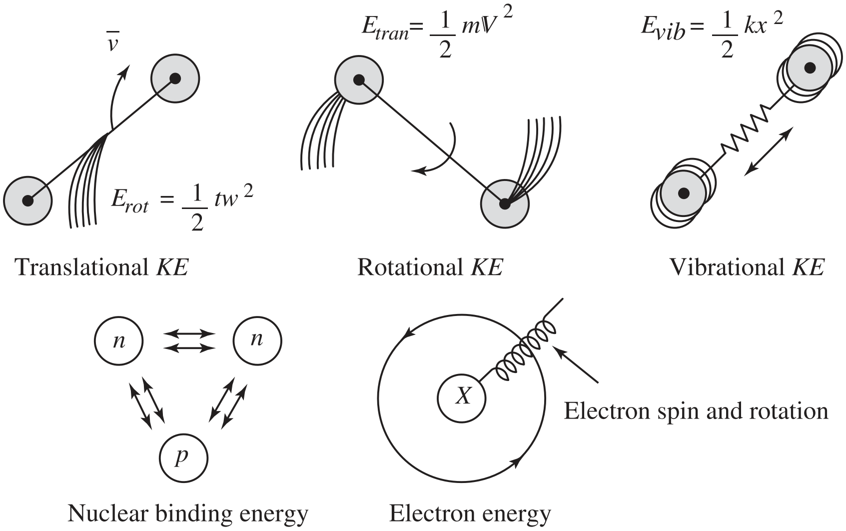


Fig. 3.10 Various components of internal energy stored in a molecule

$$e = e_{tran} + e_{rotat} + e_{vib} + e_{chem} + e_{elec} + e_{neuc} \tag{3.18}$$

If there are N molecules in the system then the total internal energy U is given by

$$U = Ne \tag{3.19}$$

For an ideal gas, the internal energy depends only on temperature since there is no inter molecular attraction or repulsion. Therefore, for an ideal gas, we can write, $U = f(T)$. If we neglect other forms of energy, we can write the total energy, E, of the system as

$$E = \underbrace{E_{KE} + E_{PE}}_{macro} + \underbrace{U}_{micro} \tag{3.20}$$

where E_{KE}, E_{PE} and U are kinetic energy, potential energy and internal energy, respectively. Further, in the absence of motion and gravity,

$$E_{KE} = 0; \qquad E_{PE} = 0; \qquad E = U$$

and Eq.3.5 becomes $\qquad Q - W = \Delta U \qquad (3.21)$

For unit mass flow rate, Eq.3.21 becomes,

$$q - w = \Delta u \tag{3.22}$$

In differential form,

$$\delta Q - \delta W = dU \tag{3.23}$$

$$\delta W = \delta W_{pdV} + \delta W_{shaft} + \delta W_{electrical} + \dots \tag{3.24}$$

where Eq.3.24 takes into account different forms of possible work transfer which may be present. In the absence of all the other work transfer except pdV work, Eq.3.23 becomes

$$\delta Q = dU + \int pdV \tag{3.25}$$

In integral form,

$$Q = \Delta U + \int pdV \tag{3.26}$$

3.5 FIRST LAW OF THERMODYNAMICS

From the above discussions and analysis, by now you should be aware of the following *three principles*:

(i) Law of conservation of energy.

(ii) The flow of heat is a form of energy transfer.

(iii) Performing work is also a form of energy transfer.

All the above three principles are embedded in the first law of thermodynamics. Combining these three principles, we can come to the conclusion that the first law of thermodynamics is nothing but the application of conservation of energy principle to heat and thermodynamic processes. Now, we can arrive at the following statements for a non-cyclic and cyclic thermodynamic processes.

3.5.1 Non-cyclic Thermodynamic Processes

Heat and work are mutually convertible. It implies that as energy can neither be created nor destroyed, the total energy associated with an energy conversion remains constant.

or

No machine can produce energy without incurring the corresponding expenditure of energy. What it means is that it is impossible to construct a perpetual motion machine of first kind.

3.5.2 Cyclic Thermodynamic Processes

When a system undergoes a thermodynamic cycle, then

(i) the net heat supplied to the system from the surroundings is equal to net work done by the system on its surroundings.

(ii) the net heat rejected by the system to the surroundings is equal to net work done on the system by its surroundings.

The first explicit statement of the first law of thermodynamics was made by Rudolf Clausius in 1850. His statement referred to cyclic thermodynamic processes which is a little complex to comprehend. Verbatim his statement is as follows:

In all cases in which work is produced by the agency of heat, a quantity of heat is consumed which is proportional to the work done; and conversely, by the expenditure of an equal quantity of work an equal quantity of heat is produced.

However, Clausius also stated the law in another form (much easier to understand) referring to the existence of a function of state of the system, the internal energy and expressed it in terms of an equation. This equation may described as follows:

In a thermodynamic process involving a closed system, the change in the internal energy is equal to the difference between the heat accumulated by the system and the work done by it.

Because of its definition in terms of change, the value of the internal energy of a system is not uniquely defined. The internal energy is customarily stated relative to a conventionally chosen standard reference state of the system. Thus, the first law makes use of the key concepts of internal energy, heat and system work. It is used extensively in the discussion of heat engines. The standard unit for all these quantities would be the joule.

3.6 ENERGY OF AN ISOLATED SYSTEM

An isolated system is one in which there is no interaction of the system with the surroundings. For an isolated system, $\delta Q = 0$ and $\delta W = 0$. Therefore, for an isolated system, the first law of thermodynamics can be written as:

$$dE = 0 \quad \text{or} \quad E = \text{constant} \tag{3.27}$$

The energy of an isolated system is always constant.

3.7 REAL, IDEAL AND PERFECT GASES

We will discuss in detail the real and ideal gas in Chapter 10. However, we will make a cursory look on the terms, viz real, perfect and ideal gas here because it will be helpful in understanding the applications of laws of thermodynamics. Gases can be classified as real, ideal and perfect. In practise, all gases are only real gases. Real gases are non-hypothetical gases whose molecules occupy space and have interactions with each other. To understand the behaviour of real gases, the following must be taken into account:

(i) compressibility effects,

(ii) variable specific heat capacity,

(iii) van der Waals forces,

(iv) non-equilibrium thermodynamic effects, and

(v) issues with molecular dissociation and elementary reactions with variable composition.

When you take all the above into account, they are not amenable to easy mathematical analysis. For most applications, such a detailed analysis is unnecessary and the ideal/perfect gas approximation can be used with reasonable accuracy. Now, let us see what is meant by an ideal gas? A ideal gas is a working substance which obeys the following:

(i) Boyle's law

(ii) Charles's law

(iii) Joule's law of internal energy

(iv) Dalton's law of partial pressures

(v) the constant value of specific heat

To obey completely all these laws, the substance would not be able to change its state even at absolute zero and the molecules would need to be so far apart that there are no intermolecular forces and no collisions. At normal temperature and pressure, the ideal gases closely obey these laws. For example, oxygen, nitrogen, helium, etc. are the gases which obey these laws and therefore, they are called *ideal* or *semi-perfect.*

Now, the question is what is meant by a perfect gas? A perfect gas is again a theoretical gas composed of many randomly moving point particles. They do not interact except when they collide elastically. The perfect gas concept is useful because it obeys the ideal gas law. Ideal gas law is nothing but a simplified equation of state connecting pressure, volume and temperature. It is amenable to easy mathematical analysis.

At normal conditions such as standard temperature and pressure, most real gases behave qualitatively like an ideal gas. One mole of an ideal gas has a volume of 22.7 ℓ at STP. Many gases such as nitrogen, oxygen, hydrogen, noble gases and some heavier gases like carbon dioxide can be treated like ideal gases within reasonable tolerances. Generally, a gas behaves more like an ideal gas at higher temperatures and lower pressures. Note that the terms *perfect gas* and *ideal gas* are sometimes used interchangeably depending on the particular field of physics and engineering.

3.7.1 The Characteristic Equation of State of an Ideal Gas

At temperatures that are considerably in excess of critical temperature of a fluid and also at very low pressure, the vapour of a fluid tends to obey the equation:

$$\frac{pv}{T} = \text{constant} = R \tag{3.28}$$

Note that in practice, no gas obeys this law rigidly but many gases *tend* towards it.

An imaginary gas which obeys the above law can be called an *ideal gas.* The equation $\frac{pv}{T} = R$ is called the *characteristic equation of a state of an ideal gas.* The constant R is called the *gas constant.* Each ideal gas has a different gas constant. The unit of gas constant R are Nm/kg K or kJ/kg K. Usually, the characteristic equation is written as

$$pv = RT \tag{3.29}$$

or for m kg, occupying V m^3

$$pV = mRT \tag{3.30}$$

The characteristic equation in *another form*, can be derived by using kilogram-mole as a unit.

One *kilogram-mole* is defined as a quantity of a gas equivalent to M kg of the gas, where M is the molecular weight of the gas (e.g. since the molecular weight of oxygen is 32, then 1 kg mole of oxygen is equivalent to 32 kg of oxygen). Now, for m kg of the gas, we have,

$$m = nM \tag{3.31}$$

where n = number of moles.

Note: Since the standard unit of mass is kg, kilogram-mole will be written simply as mole. Substituting for m from Eq.3.31 in Eq.3.30 gives

$$pV = nMRT \tag{3.32}$$

or

$$MR = \frac{pV}{nT} \tag{3.33}$$

According to *Avogadro's hypothesis,* the volume of 1 mole of any gas is same as the volume of 1 mole of any other gas when the gases are at same temperature and pressure. Therefore, $\frac{V}{n}$ is the same for all gases at the same value of p and T, i.e. the quantity $\frac{pV}{nT}$ is a constant for all gases. This constant is called *universal gas constant* and denoted by R_u, i.e.

$$MR = R_u = \frac{pV}{nT} \tag{3.34}$$

or

$$pV = nR_uT \quad (\because MR = R_u) \tag{3.35}$$

Therefore,

$$R = \frac{R_u}{M} \tag{3.36}$$

It has been found experimentally that the volume of 1 mole of any ideal gas at 1 bar and 0 °C is approximately 22.71 m^3. Therefore, from Eq. 3.35,

$$R_u = \frac{pV}{nT} = \frac{1 \times 10^5 \times 22.71}{1 \times 273.15} = 8314.3 \text{ Nm/mole K} \tag{3.37}$$

Using Eq.3.36, the gas constant for any gas can be found when the molecular weight is known. *Example*: For oxygen, which has a molecular weight of 32, the gas constant

$$R = \frac{R_u}{M} = \frac{8314}{32} = 259.8 \text{ Nm/kg K} = 259.8 \text{ J/kg K} \tag{3.38}$$

3.7.2 Specific Heats

We are aware that to raise the temperature of same mass of different substances by one degree it takes different amounts of energy. To give an example, we need about 4.5 kJ of energy to raise the temperature of 1 kg of iron from 20 to 30 °C. However, it needs 41.8 kJ to be exact to raise the temperature of 1 kg of liquid water by the same amount (Fig.3.11). Therefore, it becomes necessary to introduce a property that will enable us to compare the energy-storing capabilities of various substances. This property is what we call the *specific heat.*

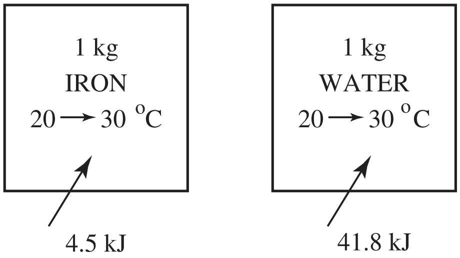


Fig. 3.11 Energy input requirements for different substances

The *specific heat* of a solid or liquid is usually defined as the *heat required to raise unit mass through one degree temperature rise.* For small quantities, we have,

$$dQ = m\, c\, dT \tag{3.39}$$

where m is mass, c is specific heat and dT is temperature rise.

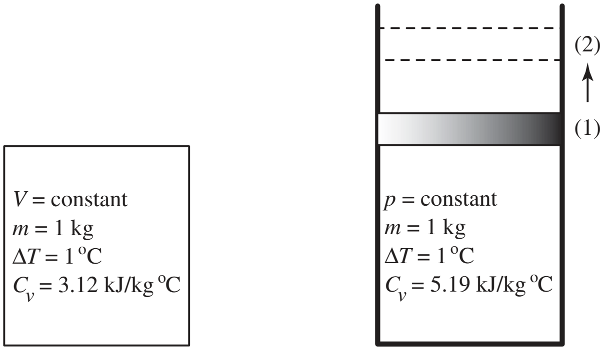
For a gas, there are an infinite number of ways in which heat may be added between any two temperatures and hence, a gas could have an *infinite number of specific heats.* However, only two specific heats for gases are considered:

(i) Specific heat at constant volume, c_V and (ii) Specific heat at constant pressure, c_p

3.7.3 Specific Heat at Constant Volume and Constant Pressure

Energy required to raise the temperature of the unit mass of a substance by one degree keeping the volume constant, is the *specific heat at constant volume*, c_V.

Let us now express the specific heats in terms of other thermodynamic properties. Consider, Fig.3.12(a), which shows a closed system undergoing a constant volume process.



(a) Helium at constant volume (b) Helium at constant pressure

Fig. 3.12 Specific heat at constant volume and constant pressure

The first law relation for such a process can be expressed in its differential form as,

$$\delta q - \delta w = du \tag{3.40}$$

In Eq.3.40, the left-hand side $(\delta q - \delta w)$ represents the amount of energy transfer in the form of heat and/or work to the system. From the definition of c_V, this energy should be equal to $c_V dT$, where dT is the differential change in temperature. From Eq.3.40, we can write for a constant volume process:

$$c_V dT = du \tag{3.41}$$

$$c_V = \left(\frac{\partial u}{\partial T}\right)_v \tag{3.42}$$

Similarly, the energy required to raise the temperature of the unit mass of a substance by one degree keeping the pressure constant is the specific heat at constant pressure c_p. This is illustrated in Fig.3.12(b). The specific heat at constant pressure c_p, is always greater than c_V, because at constant pressure, the system is allowed to expand and the energy for this expansion work must also be supplied to the system. Similarly, an expression for the specific heat at constant pressure c_p can be obtained by considering a constant-pressure expansion or compression process. Before that let us now see another intensive property, viz enthalpy.

3.8 ENTHALPY

Enthalpy is a combination property. The credit for the widespread use of the property enthalpy goes to Professor Richard Mollier. He recognised the importance of the group, $u + pV$, in the analysis of steam turbines. Mollier referred the group $u + pV$, as heat content or total heat. These terms were not quite consistent with the modern thermodynamics terminology. They were replaced by the term *enthalpy* in 1930. It originates from the Greek word *enthalpien* meaning *to heat.* The enthalpy of a substance, h, is defined as

$$h = u + pV \tag{3.43}$$

As per ideal gas law,

$$pV = RT \tag{3.44}$$

Combining the above two relation, we have,

$$h = u + RT = c_V T + RT = (c_V + R)T = c_p T \quad (\text{since } c_p = c_V + R)$$

that is,

$$h = c_p T \quad \text{and} \quad H = m c_p T \tag{3.45}$$

(Note that, it is assumed as $u = 0$ at $T = 0$, then $h = 0$ at $T = 0$).

As R is a constant and $u = u(T)$, it follows that the enthalpy of an ideal gas is also a function of temperature. Thus,

$$h = h(T) \tag{3.46}$$

3.8.1 Specific Heat at Constant Pressure

From Eq. 3.46, it is clear that the enthalpy is a function of temperature. Therefore, for a constant pressure process, we can write,

$$dh = c_p dT \tag{3.47}$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \tag{3.48}$$

As can be seen that both c_V and c_p are expressed in terms of other properties. Therefore, they must be properties themselves. Being a property, c_V and c_p depend on the state. Hence, they are specified by two independent intensive properties. The relations $c_V = \left(\frac{\partial u}{\partial T}\right)_v$ and $c_p = \left(\frac{\partial h}{\partial T}\right)_p$ are the property relations. Therefore, they are independent of the type of process. They are valid for any substance undergoing any process.

The specific heat at constant volume, c_V, may be recognised as the change in the specific internal energy of a substance per unit change in the temperature when the volume is kept constant. In other words, c_V is a measure of the variation of internal energy of a substance with temperature.

The specific heat at constant pressure, c_p, may be recognised as the change in the specific internal enthalpy of a substance per unit change in the temperature when the pressure is kept constant. In other words, c_p is a measure of the variation of enthalpy of a substance with temperature.

By the transfer of energy in any form, both internal energy and enthalpy of a substance can be changed. Probably, the term, specific energy, may be more appropriate than specific heat. However, specific heat is normally used. The unit for specific heat is kJ/kg K. We have,

$$\delta Q = mc_p dT \text{ For a reversible non-flow process at constant pressure and} \tag{3.49}$$

$$\delta Q = mc_V dT \text{ For a reversible non-flow process at constant volume} \tag{3.50}$$

The values of c_p and c_V for a perfect gas are constant at all pressures and temperatures. Hence, integrating Eqs 3.49 and 3.50, we have,

Flow of heat in a reversible constant pressure process is given by $mc_p(T_2 - T_1)$.

Flow of heat in a reversible constant volume process is given by $mc_V(T_2 - T_1)$.

In case of real gases, c_p and c_V vary with temperature but a suitable average value may be used for most practical purposes.

3.8.2 Joule's Law

Joule's law is stated as follows : *The internal energy of an ideal gas is a function of the absolute temperature only, i.e.*

$$u = f(T) \tag{3.51}$$

which means that internal energy varies linearly with absolute temperature. To evaluate this function, let 1 kg of an ideal gas be heated at constant volume. According to non-flow energy equation,

$$\delta Q = du + \delta W \tag{3.52}$$

$$\delta W = 0, \text{ since volume remains constant} \tag{3.53}$$

Therefore,

$$\delta Q = du \tag{3.54}$$

At constant volume for a perfect gas, from Eq.3.50, for 1 kg,

$$\delta Q = c_V dT \tag{3.55}$$

$$\delta Q = du = c_V dT \tag{3.56}$$

and integrating

$$u = c_V T + K, \tag{3.57}$$

K being constant. Internal energy can be made zero at any arbitrary reference temperature. For an ideal gas, it can be assumed that $u = 0$ when $T = 0$, Hence, constant K is zero.

Internal energy for an ideal gas

$$u = c_V T \tag{3.58}$$

or for mass m, of an ideal gas,

$$U = mc_V T \tag{3.59}$$

For an ideal gas, in any process between states 1 and 2, we have from Eq.3.59 gain in internal energy is given by,

$$U_2 - U_1 = mc_V(T_2 - T_1) \tag{3.60}$$

Equation 3.60 gives the gains of internal energy for an ideal gas between two states for any process, reversible or irreversible.

3.8.3 Relationship between Two Specific Heats

Consider an ideal gas being heated at constant pressure from T_1 to T_2. According to non-flow equation,

$$Q = (U_2 - U_1) + W \tag{3.61}$$

For ideal gas,

$$U_2 - U_1 = mc_V(T_2 - T_1) \tag{3.62}$$

$$Q = mc_V(T_2 - T_1) + W \tag{3.63}$$

In a constant pressure process, the work done by the fluid, $W = p(V_2 - V_1)$

Since,

$$p_1V_1 = mRT_1 \quad \text{and} \quad p_2V_2 = mRT_2 \tag{3.64}$$

$$p_1 = p_2 = p \quad \text{(in this case)} \tag{3.65}$$

Therefore,

$$W = mR(T_2 - T_1) \tag{3.66}$$

On substituting,

$$Q = mc_V(T_2 - T_1) + mR(T_2 - T_1) = m(c_V + R)(T_2 - T_1) \tag{3.67}$$

But for a constant pressure process, $Q = mc_p(T_2 - T_1)$

By equating the two expressions, we have,

$$m(c_V + R)(T_2 - T_1) = mc_p(T_2 - T_1) \tag{3.68}$$

Therefore,

$$c_V + R = c_p$$

or

$$c_p - c_V = R \tag{3.69}$$

Dividing both sides by c_V, we get, $\dfrac{c_p}{c_V} - 1 = \dfrac{R}{c_V}$

Therefore,

$$c_V = \frac{R}{\gamma - 1}$$

where $\gamma = \dfrac{c_p}{c_V}$. Similarly, dividing both sides by c_p, we get $c_p = \dfrac{\gamma R}{\gamma - 1}$.

3.8.4 Ratio of Specific Heats

The ratio of specific heat at constant pressure to the specific heat at constant volume is given the symbol γ (gamma), i.e.

$$\gamma = \frac{c_p}{c_V} \tag{3.70}$$

Since $c_p = c_V + R$, it is clear that c_p must be *greater* than c_V for any ideal gas. It follows, therefore, that the ratio, $\dfrac{c_p}{c_V} = \gamma$, is always greater than unity. In general, the approximate values of γ are as follows:

For *monoatomic* gases such as argon, helium, $\gamma = 1.6$.

For *diatomic* gases such as carbon monoxide, hydrogen, nitrogen and oxygen, $\gamma = 1.4$.

For *triatomic* gases such as carbon dioxide and sulphur dioxide, $\gamma = 1.3$.

For some hydrocarbons the value of γ is quite low (e.g. for ethane $\gamma = 1.22$ and for isobutane $\gamma = 1.11$)

3.9 APPLICATION OF FIRST LAW OF THERMODYNAMICS TO CLOSED SYSTEM

We have gone through the details of the principle of energy conservation and the corresponding statement of first law of thermodynamics. In the following section, we will apply first law of thermodynamics for closed systems undergoing various processes and its implications.

3.9.1 Reversible Constant Volume (or Isochoric) Process (v = constant)

When a working substance is contained in a rigid vessel and undergoes a process then it is called a constant volume process. The boundaries of the system are *immovable* in this case. No work is done by or on the system, other than paddle-wheel work input. Usually, *constant volume* implies zero work unless stated otherwise. Figures 3.13(a) and (b) show the system and states before and after the heat addition at constant volume, respectively.

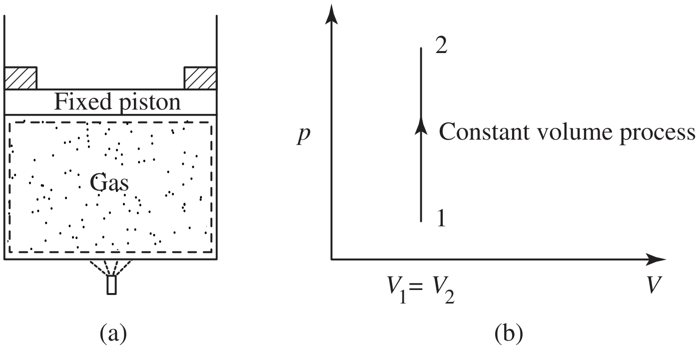


Fig. 3.13 Reversible constant volume process

Assuming *unit mass* and applying the first law of thermodynamics,

$$Q - W = u_2 - u_1 \tag{3.71}$$

The work done during the constant volume precess is zero. Mathematically,

$$W = \int p dv = 0 \tag{3.72}$$

since, $dv = 0$

$$Q = u_2 - u_1 = c_V(T_2 - T_1) \tag{3.73}$$

where c_V is the specific heat at constant volume. Instead of unit mass if there is m kg of the fluid in the container then,

$$Q = U_2 - U_1 = mc_V(T_2 - T_1) = mc_V \Delta T \tag{3.74}$$

3.9.2 Reversible Constant Pressure or Isobaric Process (p = constant)

In the case of a constant volume process, the boundary of the system cannot move. Therefore, at constant volume, the pressure rises when heat is supplied [Fig.3.13(b)]. However, in a constant pressure process, the boundary must move against an external resistance when heat is supplied. The constant pressure process is illustrated in Fig.3.14(a). When the heat is supplied the *piston is pushed through a certain distance* by the force exerted by the heated gas to maintain the constant pressure. Here, the work is done by the gas on its surroundings. Figure 3.14(b) illustrates the states before and after the heat addition at constant pressure. Considering unit mass of working substance and applying first law to the process:

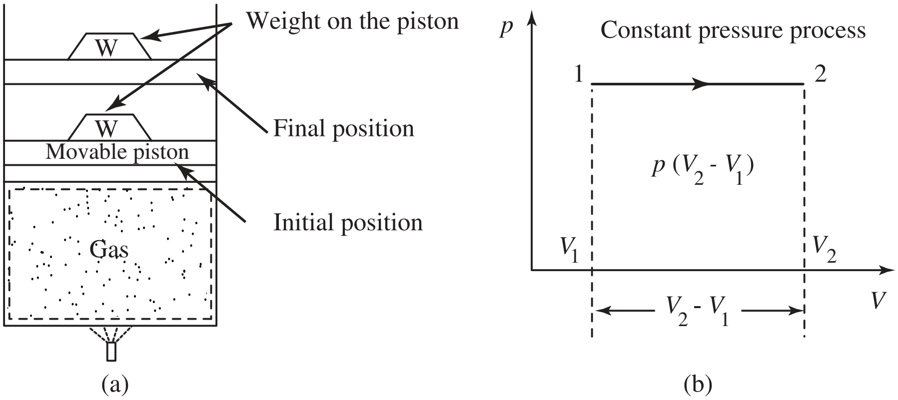


Fig. 3.14 Reversible constant pressure process

$$Q - W = u_2 - u_1 \tag{3.75}$$

The work done by m kg of mass is given by,

$$W = \int pdV = p(V_2 - V_1) \tag{3.76}$$

$$\begin{aligned} Q &= U_2 - U_1 + p(V_2 - V_1) = U_2 - U_1 + pV_2 - pV_1 \\ &= (u_2 + pV_2) - (u_1 + pV_1) = H_2 - H_1 \end{aligned} \tag{3.77}$$

As $H = mc_pT$ (refer Eq.3.45), $\quad Q = H_2 - H_1 = mc_p(T_2 - T_1) = mc_p\Delta T$

3.9.3 Constant Temperature or Isothermal Process (T = constant)

A process is said to be isothermal if it takes place at a constant temperature. For example, when a working substance in a cylinder behind a piston expands from a high pressure to a low pressure, the temperature will fall. If the expansion is to take place under constant temperature condition heat must be added continuously to keep the temperature at the initial value. Similarly, in an isothermal compression, heat must be removed from the working substance continuously during the process. Figure 3.15 shows the system and states before and after the heat addition at constant temperature. Considering unit mass of working substance and applying first law to the process.

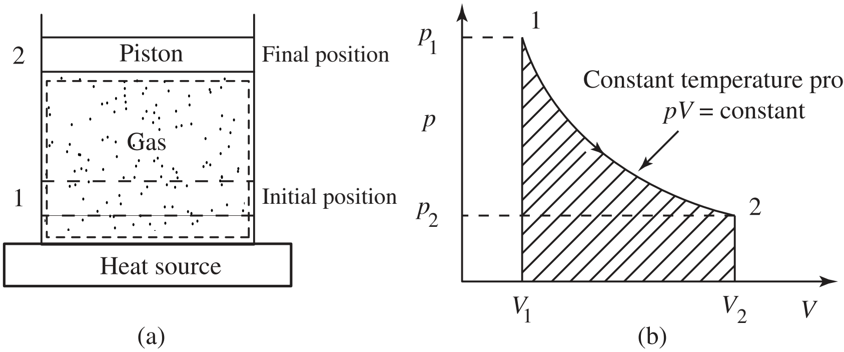


Fig. 3.15 Constant temperature (or isothermal) process

$$Q - W = u_2 - u_1 \tag{3.78}$$

$$Q = c_V(T_2 - T_1) + W = 0 + W \tag{3.79}$$

The work done during this process is given by $W = \int_1^2 pdv$. In this case, pv = constant or $p = C/v$ (where C = constant).

$$W = \int_{v_1}^{v_2} C\frac{dv}{v} = C[\log_e]_{v_1}^{v_2} = C\log_e\frac{v_2}{v_1} \tag{3.80}$$

Now, for a unit mass working fluid,

$$p_1v_1 = p_2v_2 = C \tag{3.81}$$

$$W = p_1v_1\log_e\frac{v_2}{v_1} \quad \text{or} \quad = p_2v_2\log_e\frac{v_2}{v_1} \tag{3.82}$$

$$Q = W = p_1v_1\log_e\frac{v_2}{v_1} \tag{3.83}$$

Instead of unit mass if the mass of the working substance is m, then

$$Q = p_1V_1\log_e\frac{v_2}{v_1} = p_1V_1\log_e\frac{p_1}{p_2} \quad \text{since} \left[\frac{v_2}{v_1} = \frac{p_1}{p_2}\right] \tag{3.84}$$

3.9.4 Reversible Adiabatic Process (pv^γ = constant)

During a process if no heat is transferred to or from the fluid then it is called an adiabatic process. Such a process can be reversible or irreversible. Here, we will consider only a reversible adiabatic non-flow process. Considering unit mass of working substance and applying first law to the process,

$$Q - W = u_2 - u_1 \tag{3.85}$$

As $Q = 0$, we can write,

$$-W = u_2 - u_1 \tag{3.86}$$

$$W = u_1 - u_2 \tag{3.87}$$

Equation 3.87 is true for any adiabatic process whether it is reversible or not. It is to be noted that the work done W by the fluid, in an adiabatic expansion, is at the expense of a reduction in the internal energy of the fluid. Similarly, in an adiabatic compression process all the work done on the fluid goes to increase the internal energy of the fluid. There *must be perfect thermal insulation system for the process to be adiabatic.*

3.9.5 To Derive the Law pv^γ = constant

To obtain a law relating p and v for a reversible adiabatic process, let us consider a unit mass in the non-flow energy equation in differential form,

$$\delta Q = du + \delta W \tag{3.88}$$

For a reversible process,

$$\delta W = pdV \tag{3.89}$$

$$\delta Q = du + pdV = 0 \tag{3.90}$$

Since $\delta Q = 0$ for an adiabatic process and there is no heat transfer and neglecting all other works except pdv and considering unit mass,

$$du + pdv = 0 \tag{3.91}$$

For a perfect gas,

$$pv = RT \quad \text{or} \quad p = \frac{RT}{v} \tag{3.92}$$

Hence, we have,

$$du + \frac{RTdv}{v} = 0 \tag{3.93}$$

Also,

$$u = c_V T \quad \text{or} \quad du = c_V dT \tag{3.94}$$

Therefore,

$$c_V dT + \frac{RTdv}{v} = 0 \tag{3.95}$$

Dividing both sides by T, we will get,

$$c_V \frac{dT}{T} + \frac{Rdv}{v} = 0 \tag{3.96}$$

Integrating,

$$c_V \log_e T + R \log_e v = \text{constant} \tag{3.97}$$

Substituting $T = \dfrac{pv}{R}$

$$c_V \log_e \frac{pv}{R} + R \log_e v = \text{constant} \tag{3.98}$$

Dividing both sides by c_V,

$$\log_e \frac{pv}{R} + \frac{R}{c_V} \log_e v = \text{constant} \tag{3.99}$$

$$c_V = \frac{R}{\gamma - 1} \quad \text{or} \quad \frac{R}{c_V} = \gamma - 1 \tag{3.100}$$

Hence, by substituting the value c_V in Eq.3.99,

$$\log_e \frac{pv}{R} + (\gamma - 1)\log_e v = \text{constant} \quad (3.101)$$

$$\log_e \frac{pv}{R} + \log_e v^{\gamma-1} = \text{constant} \quad (3.102)$$

$$\log_e \frac{pvv^{\gamma-1}}{R} = \text{constant} \quad (3.103)$$

$$\log_e \frac{pv^{\gamma}}{R} = \text{constant} \quad (3.104)$$

$$\frac{pv^{\gamma}}{R} = e^{\text{constant}} \quad (3.105)$$

$$pv^{\gamma} = \text{constant} \quad (3.106)$$

3.9.6 Expression for Work, W

The schematic of a reversible adiabatic process for an ideal gas is shown in Fig.3.16(a) and the compression p-V diagram in Fig.3.16(b). The work done is given by the shaded area and this area can be evaluated by integration. Considering unit mass of the gas,

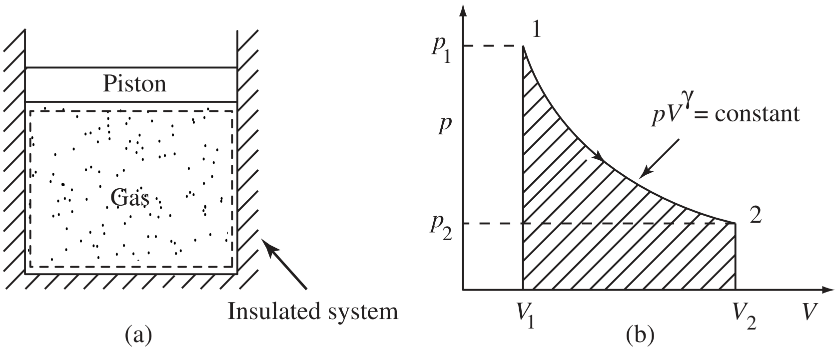


Fig. 3.16 Reversible adiabatic process

$$W = \int_{v_1}^{v_2} p\,dv \quad (3.107)$$

$$\because pv^{\gamma} = C, \quad W = \int_{v_1}^{v_2} C\frac{dv}{v^{\gamma}} \quad (3.108)$$

$$\because p = \frac{C}{v^{\gamma}}, \quad W = C\int_{v_1}^{v_2} \frac{dv}{v^{\gamma}} = C\left|\frac{v^{-\gamma+1}}{-\gamma+1}\right|_{v_1}^{v_2} \quad (3.109)$$

$$= C\left(\frac{v_2^{-\gamma+1} - v_1^{-\gamma+1}}{1-\gamma}\right) = C\left(\frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma-1}\right) \quad (3.110)$$

The constant in the above equation can be written either as $p_1 v_1^{\gamma}$ or $p_2 v_2^{\gamma}$.

$$W = \frac{p_1 v_1^{\gamma} v_1^{-\gamma+1} - p_2 v_2^{\gamma} v_2^{-\gamma+1}}{\gamma - 1} = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} = \frac{R(T_1 - T_2)}{\gamma - 1} \qquad (3.111)$$

3.9.7 Relationship between T and v; and T and p

By using equation $pv = RT$, the relationship between T and v and T and p can be derived as

$$pv = RT \quad \text{or} \quad p = \frac{RT}{v} \qquad (3.112)$$

Substituting this value in the equation $pv^{\gamma} =$ constant,

$$\frac{RT}{v} v^{\gamma} = \text{constant} \quad \text{or} \quad T v^{\gamma - 1} = \text{constant} \qquad (3.113)$$

Also $v = RT/p$, Hence, substituting in equation $pv^{\gamma} =$ constant,

$$p \left(\frac{RT}{p} \right)^{\gamma} = \text{constant} \qquad (3.114)$$

$$\frac{T^{\gamma}}{p^{\gamma - 1}} = \text{constant} \qquad (3.115)$$

$$\frac{T}{p^{\frac{\gamma - 1}{\gamma}}} = \text{constant} \qquad (3.116)$$

Therefore, for a reversible adiabatic process for an ideal gas between states 1 and 2,

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2} \right)^{\gamma} \qquad (3.117)$$

From Eq.3.113
$$T_1 v_1^{\gamma - 1} = T_2 v_2^{\gamma - 1} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2} \right)^{\gamma - 1} \qquad (3.118)$$

From Eq.3.116
$$\frac{T_1}{(p_1)^{\frac{\gamma - 1}{\gamma}}} = \frac{T_2}{(p_2)^{\frac{\gamma - 1}{\gamma}}} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1} \right)^{\frac{\gamma - 1}{\gamma}} \qquad (3.119)$$

The work done in an adiabatic process per unit mass of gas is given by $W = (u_2 - u_1)$. The gain in internal energy of a perfect gas is given by the equation,

$$u_2 - u_1 = c_V (T_2 - T_1) \qquad (3.120)$$

Therefore,
$$W = c_V (T_2 - T_1) \qquad (3.121)$$

Also we know that,
$$c_V = \frac{R}{\gamma - 1} \qquad (3.122)$$

Hence, by substituting
$$W = \frac{R(T_1 - T_2)}{\gamma - 1} \qquad (3.123)$$

Using equation $pv = RT$,
$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} \qquad (3.124)$$

3.9.8 Reversible Polytropic Process (pv^n = constant)

In actual practice, many processes approximate to a reversible law of form pv^n = constant, where n is a constant. In many non-flow processes, both vapours and perfect gases obey this type of law closely. Such processes are *internally reversible.* For any reversible process,

$$W = \int_{v_1}^{v_2} pdv \tag{3.125}$$

Since, $pV^n = C$,
$$W = \int_{v_1}^{v_2} C\frac{dv}{v^n} \tag{3.126}$$

Since, $p = \dfrac{C}{v^n}$
$$\begin{aligned} W = {} & C\int_{v_1}^{v_2} \frac{dv}{v^n} = C\left|\frac{v^{-n+1}}{-n+1}\right|_{v_1}^{v_2} \\ = {} & C\left(\frac{v_2^{-n+1} - v_1^{-n+1}}{1-n}\right) = C\left(\frac{v_1^{-n+1} - v_2^{-n+1}}{n-1}\right) \end{aligned} \tag{3.127}$$

The constant in the above equation can be written either as $p_1v_1^n$ or $p_2v_2^n$.

$$W = \frac{p_1v_1^nv_1^{-n+1} - p_2v_2^nv_2^{-n+1}}{n-1} = \frac{p_1v_1 - p_2v_2}{n-1} \tag{3.128}$$

or
$$W = \frac{R(T_1 - T_2)}{n-1} \tag{3.129}$$

Equation 3.128 is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process, we can write,

$$p_1v_1^n = p_2v_2^n \quad \text{or} \quad \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^n \tag{3.130}$$

$$T_1v_1^{n-1} = T_2v_2^{n-1} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} \tag{3.131}$$

Similar to Eq.3.119
$$\frac{T_1}{(p_1)^{\frac{n-1}{n}}} = \frac{T_2}{(p_2)^{\frac{n-1}{n}}} \quad \text{or} \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \tag{3.132}$$

3.9.9 Heat Transfer During a Polytropic Process

Using non-flow energy equation, the heat flow/transfer during the process can be found, considering unit mass of the gas

$$Q - W = u_2 - u_1 \tag{3.133}$$

$$Q = (u_2 - u_1) + W = c_V(T_2 - T_1) + \frac{R(T_1 - T_2)}{n-1} = \frac{R(T_1 - T_2)}{n-1} - c_V(T_1 - T_2)$$

$$c_V = \frac{R}{\gamma - 1} \tag{3.134}$$

$$Q = \frac{R(T_1 - T_2)}{n-1} - \frac{R}{\gamma - 1}(T_1 - T_2) \tag{3.135}$$

$$Q = R(T_1 - T_2)\left(\frac{1}{n-1} - \frac{1}{\gamma-1}\right) = \frac{R(T_1 - T_2)(\gamma - 1 - n + 1)}{(\gamma-1)(n-1)}$$

$$= \frac{R(T_1 - T_2)(\gamma - n)}{(\gamma-1)(n-1)} = \left(\frac{\gamma - n}{\gamma - 1}\right)\left(\frac{R(T_1 - T_2)}{n-1}\right) = \left(\frac{\gamma - n}{\gamma - 1}\right)W \tag{3.136}$$

since, $W = \dfrac{R(T_1 - T_2)}{n-1}$.

In a polytropic process, the index n depends only on the heat and work quantities during the process. The various processes considered earlier can be taken as special case of polytropic process for a perfect gas. For example,

When $n = 0$ $pv^\circ =$ constant, i.e. $p =$ constant (constant pressure process)

When $n = 1$ $pv =$ constant, , i.e. $T =$ constant (isothermal process)

When $n = \gamma$ $pv^\gamma =$ constant, , i.e. reversible adiabatic (isentropic process)

When $n = \infty$ $pv =$ constant, or $p^{\frac{1}{\infty}}v = C$, $v =$ constant (constant volume process)

3.9.10 Free Expansion

Consider two vessels 1 and 2 interconnected by a short pipe with a valve A and thermally insulated perfectly (Fig.3.17). Initially, let the vessel 1 be filled with a fluid at a certain pressure and let vessel 2 be completely evacuated. When the valve A is opened, the fluid in 1 will expand rapidly to fill both vessels 1 and 2. The pressure finally will be lower than the initial pressure in vessel 1. This is known as *free or unresisted expansion.* The process is *highly irreversible*, since during the process, there will be continuous eddy formation. Now, applying first law of thermodynamics (or nonflow energy equation) between the initial and final states,

$$Q = (u_2 - u_1) + W$$

In this process, no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well insulated. The process is therefore, *adiabatic but irreversible*,

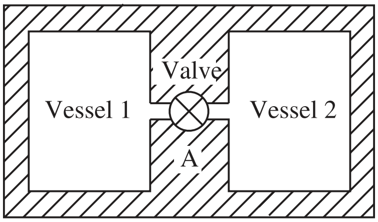


Fig. 3.17 Free expansion

$$u_2 - u_1 = 0 \quad \text{or} \quad u_2 = u_1 \tag{3.137}$$

In a free expansion, the final internal energy will be equal to initial energy. For a perfect gas, $u = c_V dT$.

For a free expansion of a perfect gas,

$$c_V T_1 \quad = \quad c_V T_2, \quad \text{, i.e. } T_1 = T_2 \tag{3.138}$$

That is, for a perfect gas undergoing a free expansion, the initial temperature is equal to the final temperature.

Table 3.1 Summary of processes for perfect gas (unit mass)

Process	Index n	Heat added	$\int_1^2 pdv$	p, v, T relations	Specific heat, c
Constant pressure	$n = 0$	$c_p(T_2 - T_1)$	$p(v_2 - v_1)$	$\frac{T_2}{T_1} = \frac{v_2}{v_1}$	c_p
Constant volume	$n = \infty$	$c_V(T_2 - T_1)$	0	$\frac{T_1}{T_2} = \frac{p_1}{p_2}$	c_V
Constant temperature	$n = 1$	$p_1 v_1 \log_e \frac{v_2}{v_1}$	$p_1 v_1 \log_e \frac{v_2}{v_1}$	$p_1 v_1 = p_2 v_2$	∞
Reversible adiabatic	$n = \gamma$	0	$\dfrac{p_1 v_1 - p_2 v_2}{\gamma - 1}$	$p_1 v_1^\gamma = p_2 v_2^\gamma$	0
				$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma-1}$	
				$= \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$	
Polytropic	$n = n$	$c_n(T_2 - T_1)$	$\dfrac{p_1 v_1 - p_2 v_2}{n - 1}$	$p_1 v_1^n = p_2 v_2^n$	$c_n = c_V\left(\frac{\gamma-n}{1-n}\right)$
		$= c_V\left(\frac{\gamma-n}{1-n}\right) \times (T_2 - T_1)$		$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}$	
		$= \left(\frac{\gamma-n}{\gamma-1}\right) \times$ Work done (non-flow)		$= \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$	

Note : Equations must be used keeping dimensional consistency.

Worked-Out Examples

3.1 A 10 kg stone is at rest at a height of 100 m above a flat ground. When it is dropped from this height using the conservation of energy principle analyse the various energy associated with the mass when it strikes the ground and when it is at half the height. What will be the velocity at $h = 0$ and $h = 50$ m?

Solution

When the stone is at 100 m height, at rest it has no kinetic energy. But it will have only potential energy given by,

$$PE = mgh = 10 \times 9.81 \times 100 = 9810 \text{ kJ}$$

When it strikes the ground, all its PE will be converted to kinetic energy (KE). Then,

$$KE = 9810 \text{ kJ}$$

$$KE = = \frac{1}{2} m \mathbb{V}^2$$

$$\mathbb{V} = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2 \times 9810}{10}} = 44.294 \text{ m/s}$$

When it is at half the height its $PE = mgh = 10 \times 9.81 \times 50 = 4905$ kJ

As potential energy has reduced from 9810 kJ to 4905 kJ, it would have been converted to KE as per the conservation of energy principle.

$$\mathbb{V}_{50} = \sqrt{\frac{2 \times 4905}{10}} = 31.32 \text{ m/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

From the conservation of energy principle,

At 100 m height,	$KE = 0kJ$;	$PE = 9810$ kJ;	$\mathbb{V}_{100} = 0$ m/s
At 50m height,	$KE = 4905$ kJ;	$PE = 4905$ kJ;	$\mathbb{V}_{50} = 31.32$ m/s
At ground level (0 m height),	$KE = 9810kJ$;	$PE = 0kJ$;	$\mathbb{V}_0 = 44.294$ m/s

3.2 100 kJ of heat is supplied to a system at a constant volume. The system rejects 90 kJ of heat at constant pressure and 20 kJ of work is done on it. The system is brought to original state by adiabatic process. Determine (i) the adiabatic work and (ii) the values of internal energy at all end states if initial value is 105 kJ.

Solution Consider the constant volume process 1-2,

$$W_{1-2} = 0 \text{ kJ}; \quad Q_{1-2} = 100 \text{ kJ}$$

$$\Delta U = U_2 - U_1 = Q_{1-2} - W_{1-2} = 100 - 0 = 100 \text{ kJ}$$

$$U_2 = 100 + U_1 = 100 + 105 = 205 \text{ kJ}$$

Consider the constant pressure process 2-3,

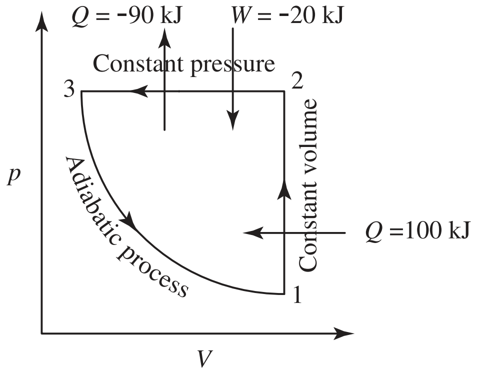


Fig. 3.18

$$U_3 - U_2 = Q_{2-3} - W_{2-3} = -90 - (-20) = -70 \text{ kJ}$$

$$U_3 = -70 + U_2 = -70 + 205 = 135 \text{ kJ}$$

Consider the adiabatic process 3-1,

$$Q_{3-1} = 0$$

$$\oint \delta Q = 100 - 90 = 10 \text{ kJ} \quad \text{and} \quad \oint \delta W = -20 + W_{3-1} = 10$$

$$W_{3-1} = 10 + 20 = 30 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.3 A fluid is contained in a cylinder by a spring-loaded, frictionless piston so that the pressure in the fluid is a linear function of the volume $p = a + bV$. The internal energy of the fluid is given by the equation, $U = 45 + 3.5pV$, where U is in kJ, p in kPa and V in cubic metre. If the fluid changes from an initial state of 200 kPa, 0.05 m^3 to a final state of 420 kPa, 0.09 m^3, with no work other than that done on the piston, find the direction and magnitude of the work and heat transfer.

Solution

$$U = 45 + 3.5pV; \quad U_1 = 45 + 3.5p_1V_1; \quad U_2 = 45 + 3.5p_2V_2$$

$$\Delta U = U_2 - U_1 = (45 + 3.5p_2V_2) - (45 + 3.5p_1V_1) = 3.5 \times (p_2V_2 - p_1V_1)$$

$$= 3.5 \times \left(4.2 \times 10^5 \times 0.09 - 2 \times 10^5 \times 0.05\right) = 97300 \text{ J} = 97.3 \text{ kJ}$$

As p is given as,

$$p = a + bV; \quad p_1 = a + bV_1; \quad p_2 = a + bV_2$$

$$200 = a + b \times 0.05 \qquad (i)$$

$$420 = a + b \times 0.09 \qquad (ii)$$

Subtracting (i) from (ii) we get,

$$220 = 0.04 \times b$$

$$b = \frac{220}{0.04} = 5500 \text{ kN/m}^2$$

Substituting b in (i) we get,

$$200 = a + 5500 \times 0.05$$

$$a = 200 - 5500 \times 0.05 = -75$$

Work transfer during the process,

$$W_{1-2} = \int_{V_1}^{V_2} p dV = \int_{V_1}^{V_2} (a + bV)dV = a(V_2 - V_1) + b\left(\frac{V_2^2 - V_1^2}{2}\right)$$

$$= -75 \times (0.09 - 0.05) + 5500 \times \left(\frac{0.09^2 - 0.05^2}{2}\right) = 12.4 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Heat transfer during the process,

$$Q_{1-2} = \Delta U + W_{1-2} = (U_2 - U_1) + W_{1-2} = 97.3 + 12.4 = 109.7 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.4 Calculate the heat transfer and its direction for a system in which an ideal gas having molecular weight of 16 is compressed from 101 kPa, 27°C to a pressure of 500 kPa following the law $pv^{1.3}$= constant. Take specific heat at constant pressure of gas as 1.82 kJ/kg.K.

Solution

$$\begin{aligned}
\text{Gas constant, } R \quad &= \quad \frac{\text{Universal gas constant}}{\text{Molecular weight}} = \frac{R_u}{m} = \frac{8.314}{16} = 0.52 \text{ kJ/kg K} \\
c_p - c_V \quad &= \quad R \\
c_V \quad &= \quad c_p - R = 1.82 - 0.52 = 1.3 \\
\gamma \quad &= \quad \frac{c_p}{c_V} = \frac{1.82}{1.3} = 1.4
\end{aligned}$$

For polytropic process,

$$\begin{aligned}
T_2 \quad &= \quad T_1 \left(\frac{p_2}{p_1}\right)^{(n-1)/n} = 300 \times \left(\frac{500}{101}\right)^{(1.3-1)/1.3} = 433.9 \\
W \quad &= \quad \frac{R(T_2 - T_1)}{n - 1} = \frac{0.52 \times (433.9 - 300)}{1.3 - 1} = 232 \text{ kJ/kg} \\
Q \quad &= \quad \left(\frac{\gamma - n}{\gamma - 1}\right) \times W = \left(\frac{1.4 - 1.3}{1.4 - 1}\right) \times 232 = 58 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

3.5 One mol of air at 0.45 MPa and 450 K, initially undergoes following processes, sequentially (i) heating at constant pressure till the volume gets doubled and (ii) expansion at constant temperature till the volume is five times of initial volume. Determine the work done by air.

Solution

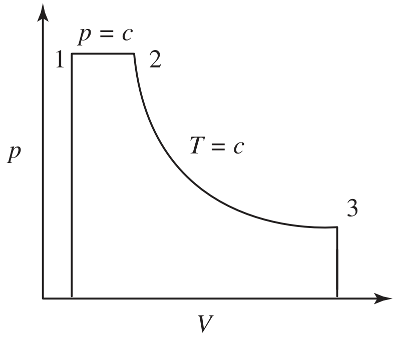


Fig. 3.19

For the constant pressure heating from state 1 to state 2, the work done is given by:

$$W = \int_1^2 pdV = p_1(V_2 - V_1)$$

$$V_2 = 2V_1; \quad V_1 = 0.5V_2$$

$$W_{1-2} = p_1(2V_1 - V_1) = p_1V_1$$

$$W_{1-2} = p_1V_1 = RT_1$$

Consider the subsequent expansion at constant temperature from state 2 to state 3,

$$\frac{V_3}{V_1} = 5 \quad \text{and} \quad \frac{V_3}{0.5 \times V_2} = 5 \quad \text{or} \quad \frac{V_3}{V_2} = 2.5$$

$$\text{Work, } W_b = \int_1^2 pdV = \int_2^3 \frac{RT}{V} dV = RT_2 \ln\left(\frac{V_3}{V_2}\right) = RT_2 \ln(2.5)$$

Temperature at state 2 can be given by,

$$\frac{T_2}{T_1} = \frac{V_2}{V_1} = 2$$

$$T_2 = 2T_1$$

$$\text{Total work done} = W_{1-2} + W_{2-3} = RT_1 + RT_2 \ln(2.5)$$

$$= RT_1 + 2RT_1 \ln(2.5) = RT_1[1 + 2\ln(2.5)]$$

$$= 8.314 \times 450 \times [1 + 2\ln(2.5)] = 10597.5 \text{ kJ} \quad \xLeftarrow{\text{Ans}}$$

3.6 A compressed air bottle of 0.3 m^3 volume contains air at 35 bar, 40°C. This air is used to drive a turbo generator supplying power to a device which consumes 5 W. Calculate the time for which the device can be operated if the actual output of the turbo generator is 60% of the maximum theoretical output. The ambient pressure to which the tank pressure has fallen is 1 bar. For air, $c_p/c_V = 1.4$. State any assumptions made.

Solution

The turbogenerator is run by the air from the bottle, thereby the pressure inside the bottle reduces from 35 bar to 1 bar. *Assume that the process is isentropic.* Let p_1, V_1 and T_1 be the initial pressure, volume and temperature of the bottle, respectively and let the corresponding values be p_2, V_2 and T_2 at the end of the process. The process is transient as filling takes place over a time. Now,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 313 \times \left(\frac{1}{35}\right)^{\frac{0.4}{1.4}} = 113.34 \text{ K} \overset{\textbf{Ans}}{\Longleftarrow}$$

Initial mass of air in the bottle is

$$m_1 = \frac{p_1V_1}{RT_1} = \frac{35 \times 10^2 \times 0.3}{0.287 \times 313} = 11.69 \text{ kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Mass of air at the end of process,

$$m_2 = \frac{p_2V_2}{RT_2} = \frac{1 \times 10^2 \times 0.3}{0.287 \times 113.34} = 0.922 \text{ kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Assume $c_V = 0.717$ kJ/kg K and $c_p = 1.005$ kJ/kg K.

$$\begin{aligned} W &= m_1u_1 - m_2u_2 - (m_1 - m_2)c_pT_2 \\ &= m_1c_VT_1 - m_2c_VT_2 - (m_1 - m_2)c_pT_2 \\ &= (11.69 \times 0.717 \times 313) - (0.922 \times 0.717 \times 113.34) \\ &\quad -[(11.69 - 0.922) \times 1.005 \times 113.34] = 1322 \text{ kJ} \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

This is the maximum work that is realisable by emptying the air bottle between the given pressure limits.

$$\text{Actual turbogenerator output} = 5 \text{ kJ/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Input to the turbogenerator} = \frac{5}{0.6} = 8.33 \text{ kJ/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Duration for which the turbogenerator can be run,

$$\Delta t = \frac{1322}{8.33} \approx 159 \text{ s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.7 A paddle wheel is used to stir a tank containing air as shown in Fig.3.20. The work input to the paddle wheel is 10 MJ and the heat transferred to the surroundings from the tank is 3 MJ. Determine (i) work done and (ii) change in internal energy of the system.

Solution

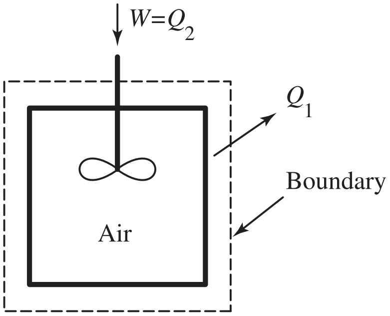


Fig. 3.20

Applying the first law of thermodynamics, $Q - W = U_2 - U_1$

Work input to the tank by means of paddle work can be considered as the input energy in the form of heat input:

$$Q = Q_1 - Q_2 = 3 - 10 = -7 \text{ MJ}$$

Since the process is at constant volume, $W = 0$.

$$U_2 - U_1 = -7 - 0 = -7 \text{ MJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Change in internal energy $= -7000$ kJ.

3.8 Consider a system, executing a non-flow process. The work and heat per degree change of temperature are given by $\frac{\delta W}{dT} = 200$ Ws/°C and $\frac{\delta Q}{dT} = 175$ J/°C. Calculate the change of internal energy of the system when its temperature changes from $T_1 = 50$°C to $T_2 = 90$°C.

Solution

$$\frac{\delta W}{dT} = 200 \text{ J/°C}; \qquad \delta W = 200dT$$

Integrating between the initial and final temperatures,

$$W = \int_{T_1}^{T_2} 200dT = 200\; T|_{T_1}^{T_2} = 200 \times (T_2 - T_1) = 200 \times (90 - 50) = 8000 \text{ J} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\frac{\delta Q}{dT} = 175 \text{ J/°C}$$

$$Q = \int_{T_1}^{T_2} 175dT = 175\; T|_{T_1}^{T_2} = 175 \times (T_2 - T_1) = 175 \times (90 - 50) = 7000 \text{ J} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Applying the first law of thermodynamics,

$$\Delta U \quad = \quad Q - W = 7000 - 8000 = -1000 \text{ J} = -1 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.9 A fluid system contained in a piston and cylinder arrangement. It passes through a complete cycle of four processes. The sum of all heat transferred during a cycle is −400 kJ. The system completes 200 cycles per min. Complete the following table showing the method for each item and compute the net rate of work output in kW.

Process	Q (kJ/min)	W (kJ/min)	E (kJ/min)
1–2	0	4300	–
2–3	40000	0	0
3–4	−4000	–	– 73000
4–1	–	–	–

Solution

Process 1–2
$$\Delta E = Q - W = 0 - 4300 = -4300 \text{ kJ/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Process 2–3
$$\Delta E = Q - W = 40000 - 0 = 40000 \text{ kJ/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Process 3–4
$$\Delta E = Q - W$$

$$-73000 = -4000 - W$$

$$W = 69000 \text{ kJ/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Given that $\sum Q = -400$ kJ

$$Q_{1-2} + Q_{2-3} + Q_{3-4} + Q_{4-1} = -400 \times 200 = -80000$$

$$0 + 40000 + (-4000) + Q_{4-1} = -80000$$

$$Q_{4-1} = -116000 \text{ kJ/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

As the cyclic integral of any property is zero, $\int \Delta E = 0$

$$\Delta E_{1-2} + \Delta E_{2-3} + \Delta E_{3-4} + \Delta E_{4-1} = 0$$

$$-4300 + 40000 - 73000 + \Delta E_{4-1} = 0$$

$$\Delta E_{4-1} = 37300 \text{ kJ/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_{4-1} = Q_{4-1} - \Delta E_{4-1} = -116000 - 37300 = -153300 \text{ kJ/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

The completed table is as follows:

Process	Q (kJ/min)	W (kJ/min)	ΔE (kJ/min)
1–2	0	4300	−4300
2–3	40000	0	4000
3–4	−4000	69000	− 73000
4–1	−116000	−153300	37300

$$\sum_{cycle} Q = \sum_{cycle} W$$

$$\text{Rate of work output} = -80000 \text{ kJ/min} = \frac{-80000}{60} = 1333.33\text{kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.10 A steam power plant is shown in Fig.3.21. The power developed by a turbine is 1250 kW. The heat supplied to the steam in the boiler is 3350 kJ/kg, the heat rejected by the system to cooling water in the condenser is 2525 kJ/kg and the feed pump work required to pump the condensate back into the boiler is 5 kW. Calculate the steam flow round the cycle in kg/s.

Solution

Figure 3.21 shows the system along with the boundary. It is to be noted that the boundary encompasses the entire power plant. However, it is to be assumed that the boundary is encompassing the working fluid only. Then,

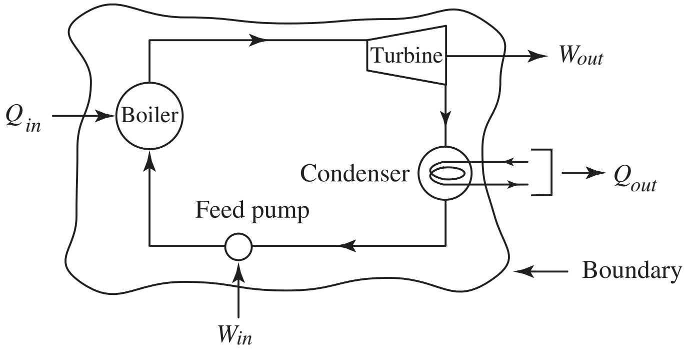


Fig. 3.21 Steam power plant

$$\oint \delta Q \quad = \quad 3350 - 2525 = 825 \text{ kJ/kg}$$

Let $\dot{m}$ be the steam flow in kg/s, medskip

$$\oint \delta Q \quad = \quad \dot{m} \times 825 \text{ kJ/s}$$

$$\int \delta W \quad = \quad 1250 - 5 = 1245$$

But, $\oint \delta Q = \oint \delta W$

$$\begin{aligned} \dot{m} \times 825 &= 1245 \\ \dot{m} &= \frac{1245}{825} = 1.51 \text{ kg/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

3.11 A constant volume closed system experiences a temperature rise of 30 °C when a certain process takes place. The heat transferred in the process is 30 kJ. The specific heat at constant volume for the pure substance comprising the system is 1.2 kJ/kg °C and the system contains 2 kg of this substance. Determine (i) the change in internal energy and (ii) the work done (iii) comment on your answer.

Solution

Change in internal energy,

$$\begin{aligned} \Delta U &= m \int_{T_1}^{T_2} c_V dT = m \int_{T_1}^{T_2} 1.2 dT \\ &= 2 \times 1.2\, T|_{T_1}^{T_2} = 2.4 \times (T_2 - T_1) = 2.4 \times 30 = 72 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

As per first law of thermodynamics,

$$\begin{aligned} \Delta U &= Q - W \\ W &= Q - \Delta U = 30 - 72 = -42 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

What is important to note is that even though the volume is constant, work is not zero, indicating that the process is irreversible.

3.12 A system receives 60 kJ of heat while expanding with volume change of 0.15 m^3 against an atmospheric pressure of $1.2 \times 10^5 \text{ N/m}^2$. A mass of 100 kg in the surroundings is also lifted through a distance of 5 metres.

(i) Find the change in energy of the system.
(ii) If the system returns to its initial volume through an adiabatic process which requires 100 kJ of work. Find the change in energy of the system.
(iii) For the combined processes of (i) and (ii), determine the change in energy of the system.

Solution

From first law of thermodynamics,

$$\Delta E = Q - W \qquad \text{(i)}$$

$$W = p\Delta V + W_{net} = \left(\frac{1.2 \times 10^5 \times 0.15}{1000}\right) + \left(\frac{100 \times 9.8 \times 5}{1000}\right) = 22.9 \text{ kJ}$$

(i) Now, the change in energy of the system,

$$\Delta E = 60 - 22.9 = 37.1 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

(ii) Since the process is adiabatic, $Q = 0$:

$$\Delta E = -W = -(-100) = 100 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

(iii) Change in energy for combined process:

$$\Delta E = Q - W = 60 - [-100 + 22.9] = 60 + 77.1 = 137.1 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.13 One kg of gaseous CO_2 contained in a closed system undergoes a reversible process at constant pressure. During this process the change in internal energy is -40 kJ. Determine the work done during the process. Take $c_p = 850$ J/kg °C and $c_V = 600$ J/kg °C.

Solution

$$\begin{aligned}
\Delta U &= mc_V \Delta T \\
-40000 &= 1 \times 600 \times (T_2 - T_1) \\
T_2 - T_1 &= \frac{-40000}{1 \times 600} = -66.67^\circ \text{ C} \\
Q &= mc_p(T_2 - T_1) = 1 \times 850 \times (-66.67) = -56669.5 \text{ J} = -56.67 \text{ kJ}
\end{aligned}$$

Now, applying the first law of thermodynamics for the process,

$$\begin{aligned}
\Delta U &= Q - W \\
W &= Q - \Delta U = -56.67 - (-40) = -16.67 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

3.14 Consider a frictionless piston which closes a fully insulated cylinder on one side as shown in Fig.3.22. The piston offers a constant resistance during its motion. A paddle work is done on the system. Prove that the paddle work is equal to change in enthalpy.

Solution

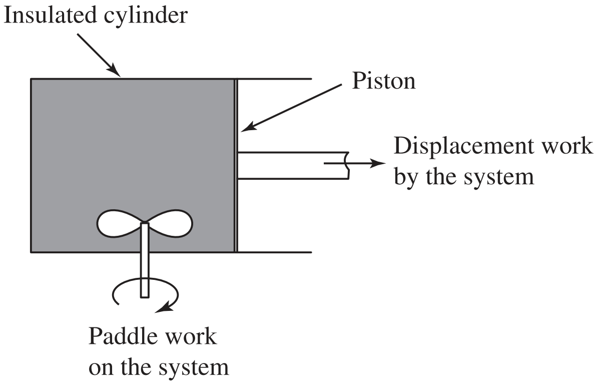


Fig. 3.22

$$Q = W_{\text{paddle}} = \Delta U + p\Delta V = \Delta U + \Delta(pV) = \Delta(U + pV) = \Delta H \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

The paddle work is equal to change in enthalpy.

3.15 0.3 $\text{m}^3$of air at 4.5 bar and 140 °C is contained in a system. A reversible adiabatic expansion takes place until the pressure falls to 1.02 bar. The gas is then heated at constant pressure till enthalpy increases by 75 kJ. Calculate (i) the work done and (ii) the index of expansion, if the above processes are replaced by a single reversible polytropic process giving the same work between the same initial and final states. Take $c_p = 1$ kJ/kg K, $c_V = 0.714$ kJ/kg K.

Solution

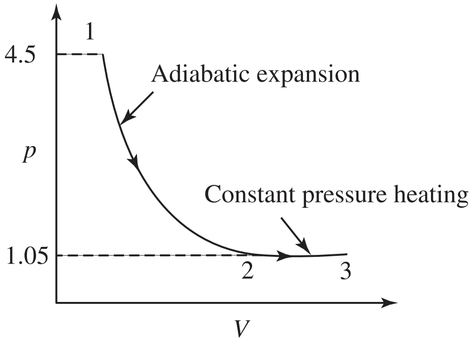


Fig. 3.23

Refer Fig.3.23. Assume the law governing the process 1−2 is given by $p_1V_1^\gamma = p_2V_2^\gamma$,

$$\gamma = \frac{c_p}{c_V} = \frac{1}{0.714} = 1.4$$

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = 0.3 \times \left(\frac{4.5}{1.02}\right)^{\frac{1}{1.4}} = 0.866 \text{ m}^3$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = (273+140) \times \left(\frac{1.02}{4.5}\right)^{\frac{0.4}{1.4}} = 270.26 \text{ K}$$

$$R = c_p - c_V = 1 - 0.714 = 0.286 \text{ kJ/kg K} = 286 \text{ J/kg K}$$

$$m = \frac{p_1V_1}{RT_1} = \frac{4.5 \times 10^5 \times 0.3}{286 \times 413} = 1.143 \text{ kg}$$

$$Q_{2-3} = mc_p\Delta T = mc_p(T_3 - T_2)$$

$$75 = 1.143 \times 1 \times (T_3 - 270.26)$$

$$T_3 = \frac{75}{1.143} + 270.26 = 335.88 \text{ K}$$

$$\frac{V_2}{T_2} = \frac{V_3}{T_3}$$

Process 2–3 is constant pressure process,

$$V_3 = V_2 \times \left(\frac{T_3}{T_2}\right) = 0.866 \times \left(\frac{335.88}{270.26}\right) = 1.08 \text{ m}^3$$

Work done during the process 1–2–3,

$$W_{1-2-3} = W_{1-2} + W_{2-3} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} + p_2(V_3 - V_2)$$

$$= \frac{\left(4.5 \times 10^5 \times 0.3\right) - \left(1.02 \times 10^5 \times 0.866\right)}{1.4 - 1} + 1.02 \times 10^5 \times (1.08 - 0.866)$$

$$= 138498 \text{ J}$$

When the work done by the polytropic process is the same, then

$$W_{1-2-3} = W_{1-3} = \frac{p_1V_1 - p_2V_3}{n - 1}$$

$$138498 = \frac{\left(4.5 \times 10^5 \times 0.3\right) - \left(1.02 \times 10^5 \times 1.08\right)}{n - 1} = \frac{24840}{n - 1}$$

$$n = \frac{24840}{138498} + 1 = 1.18 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.16 A 25 cm diameter vertical cylinder which is closed by a piston is filled with a combustible mixture at a temperature of 37 °C. The piston is free to move and its weight is such that the mixture pressure is 3.7 bar. Upper surface of the piston is exposed to the atmosphere. When the mixture is ignited, the reaction makes the piston to move slowly upwards and heat is transferred to the surroundings. When the reaction is complete, the contents are brought back to the initial temperature of 37 °C. During this process, the piston moves upwards a distance of 10 cm and the magnitude of heat transfer is 5 kJ. Evaluate (i) the work and (ii) decrease in internal energy of the system.

Solution

Work done by the system,

$$W = \int p dV = p \int dV = pV$$

Since $p = 3.7 \times 10^5$, we can write,

$$V = \frac{\pi}{4}D^2L = \frac{\pi}{4} \times 0.25^2 \times 0.1 = 4.9 \times 10^{-3} \text{ m}^3$$

$$W = 3.7 \times 10^5 V = 3.7 \times 10^5 \times 4.9 \times 10^{-3} = 1816.23 \text{ Nm} = 1.816 \text{ kJ}$$

From the first law of thermodynamics,

$$\Delta U = Q - W = -5 - 1.816 = -6.186 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Decrease in internal energy = -6.186 kJ

3.17 0.45 kg of air at 187 °C expands adiabatically to three times its original volume and during the process, there is a fall in temperature of 160 °C. The work done during the process is 53 kJ. Calculate c_p and c_V.

Solution

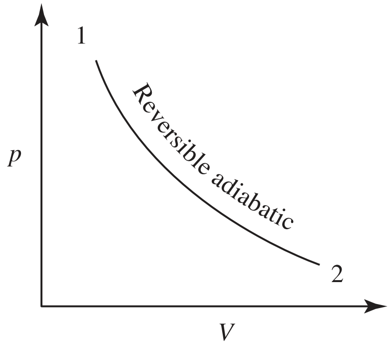


Fig. 3.24

As the process is adiabatic,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

$$\frac{300}{460} = \left(\frac{1}{3}\right)^{\gamma-1}$$

$$\ln\left(\frac{300}{460}\right) = (\gamma-1)\ln\left(\frac{1}{3}\right)$$

$$\gamma - 1 = \frac{\ln\left(\frac{300}{460}\right)}{\ln\left(\frac{1}{3}\right)} = 0.389$$

$$\gamma = 1 + 0.389 = 1.389$$

Work done during the adiabatic process,

$$W_{1-2} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$53 = \frac{0.45 \times R \times (460 - 300)}{1.389 - 1}$$

$$R = \frac{53 \times 0.389}{0.45 \times 160} = 0.286$$

$$c_p - c_V = R; \quad \frac{c_p}{c_V} = \gamma$$

$$c_p = \gamma c_V$$

$$\gamma c_V - c_V = R \quad \text{or} \quad c_V(\gamma - 1) = R$$

$$c_V = \frac{R}{\gamma - 1} = \frac{0.286}{0.389} = 0.735 \text{ kJ/kg K}$$

$$c_p = 1.389 \times 0.735 = 1.02 \text{ kJ/kg K}$$

Ans ⟸

3.18 100 liters of an ideal gas at 300 K and 1 bar is compressed adiabatically to 10 bar. It is then cooled at constant volume and further, expanded isothermally so as to reach the condition from where it started. Draw the p-V diagram and calculate:

(i) pressure at the end of constant volume cooling.

(ii) change in internal energy during constant volume process.

(iii) net work done and heat transferred during the cycle.

Assume $c_p = 14.3$ kJ/kg K and $c_V = 10.2$ kJ/kg K.

Solution

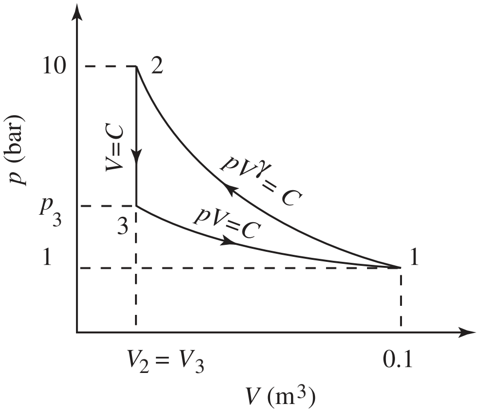


Fig. 3.25

p-V diagram is shown in Fig.3.25.

$$\gamma = \frac{c_p}{c_V} = \frac{14.3}{10.2} = 1.4$$

$$R = c_p - c_V = 14.3 - 10.2 = 4.1 \text{ kJ/kg K}$$

Process 1−2 is given by $p_1 V_1^\gamma = p_2 V_2^\gamma$,

$$V_2 = V_1\left(\frac{p_1}{p_2}\right)^{\frac{1}{\gamma}} = \frac{100}{1000} \times \left(\frac{1}{10}\right)^{\frac{1}{1.4}} = 0.0193 \text{ m}^3$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{10}{1}\right)^{\frac{0.4}{1.4}} = 1.93$$

$$T_2 = T_1 \times 1.93 = 300 \times 1.93 = 579 \text{ K}$$

Process 3−1 is given by $p_3V_3 = p_1V_1$

$$p_3 = \frac{p_1V_1}{V_3} = \frac{1 \times 0.1}{0.0193} = 5.18 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

(ii) To calculate the change in internal energy at constant volume process 2−3:

$$m = \frac{p_1V_1}{RT_1} = \frac{1 \times 10^5 \times 0.1}{4100 \times 300} = 0.00813 \text{ kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\Delta U = U_3 - U_2 = mc_V(T_3 - T_2)$$

$$= 0.00813 \times 10.2 \times (300 - 579) = -23.14 \text{ kJ}$$

The minus sign indicates the decrease in internal energy. This decrease in internal energy must be equal to heat flow to surrounding since work done is zero.

$$W_{1-2} = \frac{p_1V_1 - p_2V_2}{\gamma - 1} = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{0.00813 \times 4.1 \times (300 - 579)}{1.4 - 1} = -23.25 \text{ kJ}$$

$$W_{2-3} = 0 \text{ (since there is no change in volume)}$$

$$W_{3-1} = p_3V_3 \ln\left(\frac{V_1}{V_3}\right) = p_1V_1 \ln\left(\frac{p_3}{p_1}\right)$$

$$= 1 \times 10^5 \times 0.1 \times \ln\left(\frac{5.18}{1}\right) = 16448 \text{ J} = 16.45 \text{ kJ}$$

$$\text{Net workdone, } W = W_{1-2} + W_{2-3} + W_{3-1} = -23.25 + 0 + 16.45 = -6.8 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

For a cyclic process, $\oint \delta Q = \oint \delta W$.

Heat transfer during the cycle = −6.8 kJ. Negative sign indicates the heat rejection.

3.19 A system consisting of 1.2 kg of an ideal gas at 4.8 bar pressure and 0.02 m^3 volume executes a cyclic process comprising the following three distinct operations :

(i) Reversible expansion to 0.08 m^3volume, 1.6 bar pressure, presuming pressure to be a linear function of volume ($p = a + bV$),

(ii) Reversible cooling at constant pressure and

(iii) Reversible hyperbolic compression according to law pV = constant. This brings the gas back to initial conditions.

(a) Sketch the cycle on $p - V$ diagram.

(b) Calculate the work done in each process starting whether it is done on or by the system and evaluate the net cyclic work and heat transfer.

Solution

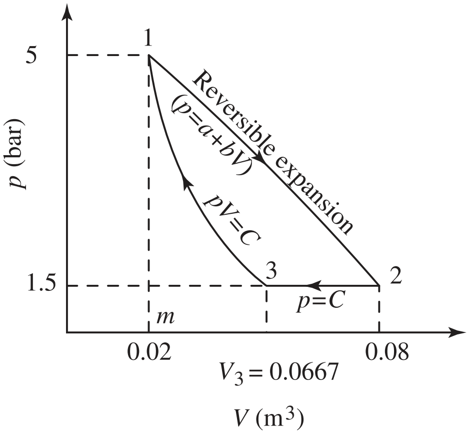


Fig. 3.26

$p - V$ diagram is shown in Fig.3.26. Process 1–2 is the linear expression according to the law, $p = a + bV$. The values for a and b should be determined first from the pressure and volume values at state points 1 and 2:

$$4.8 = a + b \times 0.02$$

$$1.6 = a + b \times 0.08$$

Solving (i) and (ii) we get $b = -53.33$ and $a = 5.87$

$$\begin{aligned} W_{1-2} &= \int_1^2 pdV = \int_1^2 (a + bV)dV = \int_1^2 (5.87 - 53.33V)dV \\ &= 10^5 \times \left| 5.87 \times V - 53.33 \times \frac{V^2}{2} \right|_{0.02}^{0.08} \\ &= 10^5 \times \left[5.87 \times (0.08 - 0.02) - \frac{53.33}{2} \times \left(0.08^2 - 0.02^2\right) \right] \\ &= 19221 \text{ J} = 19.22 \text{ kJ} \end{aligned}$$

$\xLeftarrow{\textbf{Ans}}$

Conisder the constant pressure process 2–3, $p_3 = U_2$, Volume V_3 can be calculated using the hyperbolic compression process, 3−1 from $p_1V_1 = p_3V_3$,

$$V_3 = \frac{p_1V_1}{p_3} = \frac{4.8 \times 0.02}{1.6} = 0.06 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_{2-3} = p_2(V_3 - V_2) = 1.6 \times 10^5 \times (0.06 - 0.08) = -3200 \text{ J} = -3.2 \text{ kJ}$$

$$W_{3-1} = p_3V_3 \ln\left(\frac{V_1}{V_3}\right) = 1.6 \times 10^5 \times 0.06 \times \ln\left(\frac{0.02}{0.06}\right)$$

$$= -10546 \text{ J} = -10.55 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

This is the work done on the system,

$$W_{net} = W_{1-2} + W_{2-3} + W_{3-1} = 19.22 + (-3.2) + (-10.55) = 5.47 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.20 Figure 3.27 shows a cylinder of 10 cm inside diameter having a piston loaded with a spring (stiffness = 160 N/cm of compression). The initial pressure, volume and temperature of air in the cylinder are 3×10^5 N/m^2, 0.000045 m^{3}and 27 °C, respectively. Determine the amount of heat added to the system so that piston moves by 4 cm. Assume $c_V = 0.71$ kJ/kg K and $R = 0.287$ kJ/kg K.

Solution

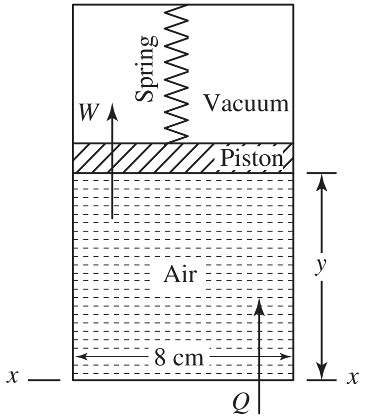


Fig. 3.27

Refer Fig.3.27. Let $x - x$ be the arbitrary datum from which the position of the lower face is measured.

Let y be the height of lower face of the piston from datum, $y = y_0$ be the free length of the spring and p be the pressure of air within the cylinder when $y = y_0$.

Now, analyse the force balance,

$$A_p = S(y - y_0) \qquad (i)$$

where A is the area of the piston and S is the stiffness of the spring.

Let the pressure inside the cylinder increase by dp forcing the piston to move upward by a distance of dy.

The force balance for the piston is given by,

$$A(p+dp) = S(y+dy-y_0) \tag{ii}$$

From Eq.(i) and (ii), $$Adp = Sdy$$

The increase in volume dV of the gas for the piston displacement is given by,

$$dV = Ady$$

$$dp = \frac{S}{A}dy$$

Integerating, $$p = \frac{S}{A^2}V + c' \tag{iii}$$

$$V = \frac{A^2}{S}p - c$$

$$dV = \frac{A^2}{S}dp$$

The above relation between p and V shows that it is linear. The constant c can be evaluated from the knowledge of pressure and volume at any point. Now, substituting, p, V and A in Eq.(iii), we get,

$$p = \frac{160}{\left(\frac{\pi}{4}\times 10^2\right)^2}V + c = 0.026V + c \tag{iv}$$

where p is in N/cm^2 and V is in cc.

$$30 = 0.025V + c = 0.025 \times 50 + c$$

$$c = 28.75$$

$$p = 0.025V + 28.75$$

During this process, the piston moves by a distance of 4 cm. Increase in the volume of air:

$$4 \times A^2 = 4 \times \left(\frac{\pi}{4} \times 10^2\right) = 314 \text{ cc}$$

Final volume of air, $V_2 = 50 + 314 = 364$ cc

Substituting this value in Eq.(iv), we get,

$$p = p_2 = 0.026 \times 364 + 28.7 = 38.16 \text{ N/cm}^2$$

The work during the process is given by,

$$W = \int_1^2 pdV = \int_{p_1}^{p_2} \frac{A^2}{S} pdp$$

$$= \frac{A^2}{S}\frac{(p_2^2 - p_1^2)}{2} = \frac{A^2}{S}\frac{(p_2 + p_1)}{2}(p_2 - p_1) = \frac{A^2}{S}\frac{(p_2 + p_1)}{2}\frac{S}{A^2}(V_2 - V_1)$$

$$W = \frac{(p_2 + p_1)}{2}(V_2 - V_1) = \text{Mean pressure } \times \text{Change in volume}$$

$$= \frac{(38.16 + 30)}{2} \times (364 - 50) = 10701 \text{ N/cm} = 107 \text{ N/m} = 0.107 \text{ kJ}$$

It is to be noted that if we consider the spring and cylinder as a system then, the work done does not cross the boundary. We can find T_2 using the relation

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = \frac{38.16 \times 364 \times 300}{30 \times 50} = 2778 \text{ K}$$

$$m = \frac{p_1 V_1}{R T_1} = \frac{30 \times 50}{287 \times 300} \times \frac{1}{100} = 0.000174 \text{ kg}$$

$$\Delta U = m c_V (T_2 - T_1) = 0.000174 \times 0.71 \times (2778 - 300) = 0.306 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

According to first law, $\Delta U = Q - W$

$$Q = \Delta U + W = 0.306 + 0.107 = 0.413 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.21 A wind mill coupled to a generator develops 5 kW of electric power as on average. Power produced by the generator charges a battery. Heat transfer from the battery is found to be 0.6 kW. Estimate the total amount of energy stored in the battery.

Solution

$$Q - W = \Delta E$$

Work input to the battery,

$$\dot{W} = -5 \times 8 = -40 \text{ kW h}$$

$$\dot{Q} = -0.6 \times 8 = -4.8 \text{ kW h}$$

$$\Delta E = \dot{Q} - \dot{W} = -4.8 + 40 = 35.2 \text{ kWh} = 35.2 \times 3600 = 1.27 \times 10^5 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.22 A mass of 15 kg of air in a piston cylinder device is heated from 20°C to 90 °C by means of a heater inside cylinder. During the heating process, the pressure in the cylinder is held constant at 300 kPa. Heat losses are estimates as 60 kJ. Determine the electrical energy supplied in kW-h and change in internal energy.

Solution

$$\text{Work done, } W = mR(T_2 - T_1) = 15 \times 0.287 \times (363 - 293) = 301.35 \text{ kJ}$$

$$\text{Work done in kW h} = \frac{301.35}{3600} = 0.0837 \text{ kw h}$$

Change in internal energy,

$$\Delta U = Q - W = -60 - 301.35 = -361.35 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

3.23 A mass of 0.35 kg of air at a pressure of 1.05 bar occupies a volume of 0.33 m^3 for an isothermal expansion to 0.99 m^3 find (i) the initial temperature (ii) the final temperature, (iii) external work done, (iv) heat absorbed by the air and (v) change in internal energy. Assume $R = 0.287$ kJ/kg K.

Solution

From ideal gas relation, $p_1V_1 = mRT_1$

$$T_1 = \frac{p_1V_1}{mR} = \frac{1.05 \times 100 \times 0.33}{0.35 \times 0.287} = 345 \text{ K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

For isothermal process, $T_2 = T_1 = 345$ K

$$\text{Work done, } W_1 = p_1V_1 \ln\left(\frac{V_2}{V_1}\right) = 1.05 \times 100 \times 0.33 \ln\left(\frac{0.99}{0.33}\right) = 38.07 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Heat absorbed, } Q = W = 38.07 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Change in internal energy, $\Delta U = 0$ $\qquad \overset{\textbf{Ans}}{\Longleftarrow}$

3.24 A fluid is kept at a constant pressure of 0.15 MPa in which a slow chemical reaction is taking place. Initially, the system is perfectly insulated from the surroundings during the chemical reaction which begins at state 1 and ends at state 2. Then, the insulation is removed and 100 kJ of heat flows into the surrounding space and the fluid goes to state 3. The following observations are made for the fluid at states 1, 2 and 3:

State	V (m^3)	t (°C)
1	0.004	20
2	0.404	350
3	0.080	20

For the fluid as a system, estimate E_2 and E_3 assuming $E_1 = 0$.

Solution

Let us apply the first law for the process 1-2,

$$Q_{1-2} = E_2 - E_1 + W_{1-2}$$

As the system is perfectly insulated during the process 1-2,

$$0 = E_2 - 0 + p(V_2 - V_1)$$

$$E_2 = p(V_1 - V_2) = 150 \times (0.004 - 0.404) = -60 \text{ kJ}$$

For the process 2-3, $Q_{2-3} = E_3 - E_2 + W_{2-3}$

$$E_3 = Q_{2-3} + E_2 - W_{2-3}$$

Substituting the values,

$$E_3 = -100 - 60 - 150 \times (0.08 - 0.404) = -111.4 \text{ kJ}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

3.25 A hypothetical engine receives heat and does work on a slowly moving piston at such rates that the cycle of operation of 1 kg of working fluid can be represented by a circle of 12 cm in diameter on a *p*-*V* diagram on which 1 cm=250 kPa pressure and 1 cm=0.1 m^3/kg specific volume. Find:

(i) How much work is done by each kg of working fluid for each cycle of operation?

(ii) The thermal efficiency defined as the ratio of work output to heat input in a cycle.

If the heat rejected by the engine in a cycle is 1000 kJ/kg of the working fluid, what will be the thermal efficiency?

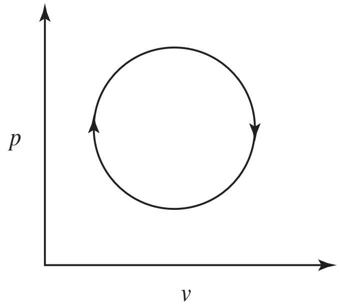


Fig. 3.28

Solution

The *p*-*V* diagram is shown in Fig.3.28. In this 1 cm^2 area of the circle will represent $250 \times 0.1 = 25$ kNm/kg of work. Let this be denoted as X,

$$W = \oint pdV = \pi \times r^2 \times X = \pi \times 6^2 \times X = 113.1 \times 25 = 2827.4 \text{ kJ/kg}$$

$$Q_2 = 1000 \text{ kJ/kg}$$

$$Q_1 - Q_2 = W_{out} = 2827.4 \text{ kJ/kg}$$

$$Q_1 = 2827.4 + 1000 = 3827.4 \text{ kJ/kg}$$

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{2827.4}{3827.4} = 0.739 = 73.9\%$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

Review Questions

3.1 What is caloric theory? What was its basic concept?

3.2 State the principle of conservation of energy.

3.3 Explain by means of a suitable example the principle of conservation energy and explain the concept of total energy with practical example.

3.4 Explain with suitable sketches some practical devices involving conservation of energy.

3.5 Briefly explain with neat sketch the Joules experiment for non-cyclic process.

3.6 What do you understand by mechanical equivalent of heat?

3.7 Prove that energy is the property of a system.

3.8 Explain the difference between microscopic and macroscopic form of energy.

3.9 What are the three basic principles involved in the first law of thermodynamics?

3.10 State the first law of thermodynamics.

3.11 Explain the meaning of non-cyclic thermodynamic process.

3.12 What do you understand by cyclic thermodynamic process? Explain.

3.13 Show that the energy of an isolated system is constant.

3.14 Explain what do you understand by real, ideal and perfect gases.

3.15 Show that $pV = mRT$ *for an ideal gas.*

3.16 Define specific heat.

3.17 Explain what do you understand by specific heat at constant volume.

3.18 What is specific heat at constant pressure?

3.19 What is enthalpy? Explain.

3.20 Show that enthalpy $h = mc_pT$.

3.21 State and explain Joules law.

3.22 Derive the relation between two specific heats.

3.23 Apply the first law of thermodynamics to the following process and develop expression for W *and* Q*: (i) constant volume process, (ii) constant pressure process and (iii) constant temperature process.*

3.24 Derive the expression for heat transfer during a polytropic process.

3.25 Show that for a free expansion process change in internal energy is zero.

Exercise

3.1 A domestic refrigerator is loaded with food and the door is closed. During a certain period, the machine consumes 2 kW h of energy and the internal energy of the system drops by 1000 kJ. Evaluate the net heat transfer for the system. *Ans:* -17.2 kJ

3.2 Two kinds of liquids having a constant specific heat of 2.5 kJ/kg are stirred in a chamber which is well insulated. The temperature rises because of the stirring at 17 °C.

(i) Estimate the change in internal energy and work interaction for the process.

(ii) The same liquid is stirred in a conducting chamber. During the process, if 2 kJ of heat is transferred from the liquid to the surroundings, while the temperature of the liquid rises by 17 °C. Find ΔU and W for the process.*Ans:* (i) -85 kJ (ii) -87 kJ

3.3 The properties of a certain fluids are related by the following equation:

$$u = 200 + 0.717t \quad \text{and} \quad pv = 0.288 \times (t + 273)$$

where u is the specific internal energy (kJ/kg), t is the temperature in °C, p is the pressure in kN/m^2 and v is the specific volume in m^3/kg.

(i) Calculate the c_p and c_V of the fluid.

(ii) If the fluid in the above example having a mass of 2.5 kg expands in a frictionless piston-cylinder arrangement with an initial state of 1000 kPa and 100 °C to the final temperature of 30 °C. Find the net work transfer. Neglect heat transfer if any.

(iii) If all the work in the expansion of the above example is carried out on the moving piston, show that the equation representing the path of the expansion in the pV plane is given by $pV^{1.4} = C$.

Ans: (i) 1.005 kJ/kg K; (ii) 125.475 kJ/kg K; (iii) 1.4

3.4 A stationary system containing 3 kg of fluid expands in an adiabatic process according to $pV^{1.2} = C$. Initial calculations are 1.2 MPa and 200 °C and final pressure is 0.12 MPa. Find the work done and change in internal energy for the process. Explain why work is hot equal to $\int pdV$.

Ans: 270.67 kJ; -270.67; but when we calculate $\int pdV = W'$, we got 540.81. Note that the actual work transfer is 270.67 kJ which is less than the frictionless quasistatic work of 540.81.
The difference of 270.14 kJ of work is lost in friction.

3.5 A mixture of gases expands at constant pressure of 1.2 MPa and volume of 0.04 m^3 to 0.08 m^3 with 85 kJ positive heat transfer. Assuming no work other than that done on a piston, find change in internal energy for the mixture. If the same mixture expands through the same state path while a stirring device does 20 kJ of work on the system, find $\Delta U, Q$ and W for the process. *Ans:* 65 kJ

3.6 A gas of mass 2 kg undergoes a quasi-static expansion according to the law, $p = a + bV$, where a and b are constants. The initial and final pressures are 1 MPa and 200 kPa, respectively. Take the corresponding volume to be 0.5 m^3 and 1.5 m^3. The specific internal energy of the gas is given by $u = 1.5pv - 85$ kJ/kg, where p is in kilo Pascal and v is in m^3/kg. Find the net heat transfer and the maximum internal energy of the gas attained during expansion. *Ans:* -400 kJ; -150 kJ

3.7 Heat capacity at constant pressure of a certain system is expressed by the equation $c_p = 2.1 + \dfrac{41.9}{t+100}$ J/°C, where t is the temperature of the system in °C. The system is heated at a constant pressure of 1 atm until its volume increases from 2000 cc to 2400 cc and its temperature increases from −15°C to 100°C. Find the magnitude of heat interaction and the change in internal energy. *Ans:* 198.47

3.8 An engine engages two-work interaction in one cycle. The magnitude of work interactions is given as 20 kJ to the fluid and 49 kJ from the fluid. There are three heat interactions, two of which are known, viz 80 kJ to the fluid and 45 kJ from the fluid. Calculate the magnitude and direction of the third heat transfer. *Ans:* 6 kJ

3.9 A gas undergoes a thermodynamics cycle, it consists of the following processes:

(i) Process 1-2 : Compression $pV = C, p_2 = 2.0$ bar, $U_2 = 690$ kJ

(ii) Process 2-3: $W_{2-3} = 0; Q_{2-3} = -150$ kJ

(iii) Process 3-1: $W_{3-1} = +50$ kJ

Neglect KE and PE. Take initial state as $p_1 = 1$ bar; $V_1 = 1.5$ m^3 and $U_1 = 510$ kJ. *Ans:* 20 kJ

(i) Sketch the cycle on a p-V diagram.

(ii) Determine the heat interaction Q_{1-2} and Q_{3-1}

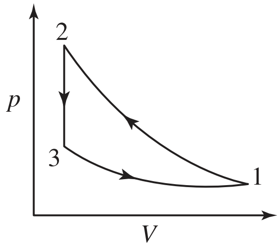


Fig. 3.29

3.10 When a system is taken from state a to state b as shown in Fig.3.30 along the path $a \to c \to b$ 80 kJ of heat flow into the system and the system does 30 kJ work.

(i) Calculate the heat flow into the system along $a \to d \to b$, if the work done is 10 kJ.

(ii) When the system returned from b to a along the curved path, the work done on the system is 20 kJ. Does the system absorb or liberate heat and how much of the heat is absorbed or liberated?

(iii) If $U_a = 0$ and $U_d = 40$ kJ, find the heat absorbed in the processes ad and db.

Ans: −70 kJ; 50 kJ; 10 kJ

3.11 A piston cylinder arrangement contains a fluid system which passes through four processes to complete a cycle. During the cycle, the sum of all heat transfer is −15 MJ. The system completes 120 cycles per minute. Complete the following table showing the procedure adopted for each of the item. Compute the net rate of work output in kW.

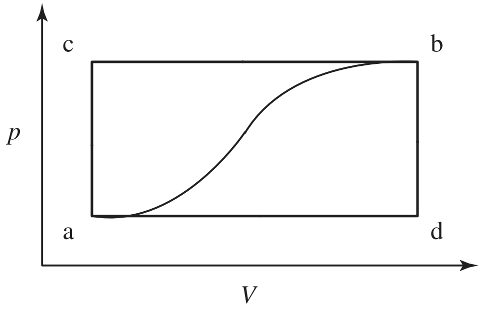


Fig. 3.30

Process	Q (kJ/min)	W (kJ/min)	ΔU (kJ/min)
a-b	0	2200	–
b-c	20000	0	–
c-d	−2000	–	−36000
d-a	–	–	–

Ans: −2200 kJ/min; 20000 kJ/min; 34000 kJ/min; −37000 kJ/min; 11800 kJ/min; −48800 kJ/min; −250 kW

3.12 The internal energy of a certain substance is given by $u = 3.5pv + 80$ where u is given in kJ/kg, p is in kPa and v is in m^3/kg. A system consists of one kg of this substance expands from an initial pressure of 0.5 MPa and a volume of 0.2 m^3 to a fluid pressure of 0.1 MPa. The pressure volume relation may be taken as $pv^{1.2} = C$.

(i) Assuming the expansion as quasistatic, find Q, W and Δu for the process.

(ii) If you consider an heat transfer of 30 kJ/kg, find the work transfer.

(iii) Explain the reason for the difference in work transfer in parts (i) and (ii).

Ans: (i) −82.25 kJ/kg; 117.5; 35.25 kJ/kg (ii) 112.25 kJ
(iii) The work in (ii) is not equal to $\int pdV$, since the process is not quasistatic.

3.13 Assume a person is inflating a cycle tyre. During the process, the person expands 50 kJ of work, how much heat will be transferred to pump?

Ans: Because of the pumping action (mechanical work), the pump gets heated. As per the conservation of energy principle, $Q = W = 50$ kJ

3.14 An air compressor requires shaft work of 200 kJ/kg and the compression of air increases the enthalpy of air by 100 kJ/kg of air. Cooling water required for cooling the compressor picks up heat of 100 kJ/kg of air. Is there any heat transfer from compressor to atmosphere? If so, how much? *Ans:* No; 0

3.15 In an internal combustion engine, during the compression stroke, the heat rejected to the cooling water is 60 kJ/kg and the work input is 120 kJ/kg. Calculate the change in internal energy of the working fluid indicating whether it is a gain or loss. *Ans:* 60 kJ/kg; Gain

3.16 Figure 3.31 shows a closed insulated container having frictionless and smooth moving insulated partition. The total volume of 1.2 m^3 is equally divided. Assume that both the gases are initially at a pressure of 0.6 MPa and at ambient temperature of 27 °C. Subsequently, the nitrogen is heated using electrical heating element such that volume of N_2 becomes 3/4 of the total volume of the container. Determine (i) final pressure of hydrogen, (ii) work done by partition, (iii) work done by N_2 and H_2 and (iv) heat added to N_2 by electric heater. Take c_2 of N_2 = 1.04 kJ/kg K, c_p of H_2 = 14.31 kJ/kg K, R_{N_2} = 0.297 kJ/kg. K, R_{H_2} = 4.12 kJ/kg K.

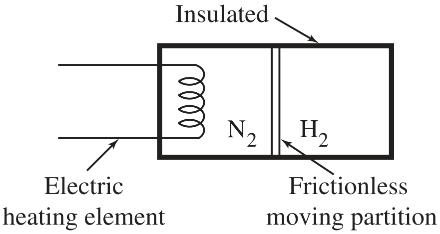


Fig. 3.31

Ans: (i) 1.59 MPa; (ii) 0; (iii) $+2.88 \times 10^5$ J; 2.88×10^5 J; (iv) 2967

3.17 A rigid and insulated tank of 1.5 m^3 volume is divided by partition into two equal halves having air at 0.6 MPa, 37 °C and 1 MPa, 620 K. Determine final pressure and temperature if the partition is removed. *Ans:* 450.91 K; 0.8 MPa

3.18 Three kilograms of air at 1.5 bar pressure and 77 °C temperature at state 1 is compressed polytropically to state 2 at pressure 7.5 bar, index of compression being 1.2. It is further, compressed at constant pressure to state 3 and then cooled at constant temperature to its original state 1. Find the net work done and heat transferred. [U.P.S.C. 1992]
Ans: 71.31 kJ; 71.31 kJ

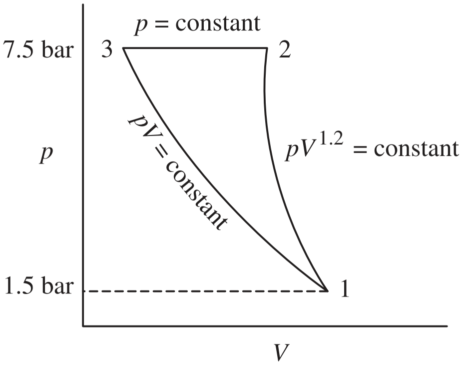


Fig. 3.32

3.19 Five kilograms of steam is contained in a cylinder-piston arrangement. The steam having an internal energy of 2709.9 kJ/kg expands to a state where the internal energy is 2659.6 kJ/kg. There is a heat transfer of 80 kJ during the process. Further, there is a paddle wheel work transfer of 18.5 kJ. Determine the amount of energy transfer by work from the steam to the piston. Also estimate the decrease in system energy. Neglect *KE* and *PE*. *Ans:* 350 kJ; 251.5 kJ

3.20 In a vessel 10 kg of O_2 is heated in a reversible non-flow constant volume process, so that the pressure of O_2 is increased two times that of initial value. The initial temperature is 30 °C. Calculate the final temperature, change in internal energy, change in enthalpy and heat transfer. Take $R = 0.259$ J/kg K and $c_V = 0.625$ for O_2.
Ans: 606 K; 1893.75 kJ; 2678.52 kJ; 1893.75

3.21 A fluid system undergoes a non-flow frictionless process following the pressure-volume relation as $p = \frac{5}{V} + 1.5$, where p is in bar and V is in m^3. During the process, the volume changes from 0.24 m^3 to 0.04 m^3 and the system rejects 50 kJ of heat. Determine (i) change in internal energy and (ii) change in enthalpy. *Ans:* 876 kJ; 846 kJ

3.22 0.3 kg of nitrogen gas at 120 kPa and 37 °C is contained in a cylinder. The piston is moved compressing Nitrogen until the pressure became 1 MPa and temperature became 15 °C. The work done during the process was 30 kJ. Determine the heat transferred from the nitrogen to the surroundings. c_V for nitrogen = 0.75 kJ/kg K. *Ans:* 13 kJ

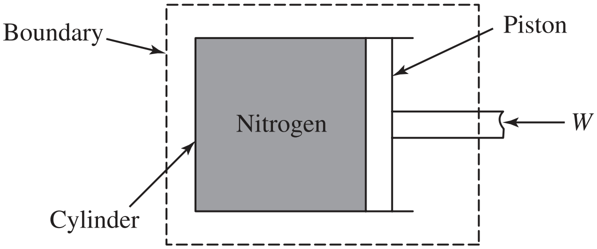


Fig. 3.33

3.23 A stationary mass of gas was compressed without friction at constant pressure from its initial state of 0.5 and 0.125 MPa to change to final state of 0.25 $m^3$and 0.125 MPa. There was a transfer of 40 kJ of heat from the gas during the process. Determine the change in internal energy of the gas. *Ans:* 31.21 kJ

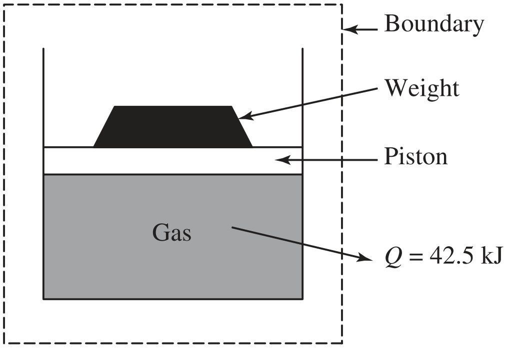


Fig. 3.34

3.24 A cylinder contains 1 m^3 of gas at 100 °C. The gas is compressed polytropically to a volume of 0.25 m^3. The final pressure is 600 kPa. Determine (a) mass of the gas (b) the value of index n for compression (c) change in internal energy of the gas and (d) heat

transferred by the gas during compression. Assume $R = 0.287$ kJ/kg K and $\gamma = 1.4$.
Ans: (i) 0.93 kg; 1.29; (iii) 123.53 kJ; (iv) 46.8 kJ

3.25 An ideal gas of molecular weight 30 kg and specific heat ratio of 1.4 is compressed according to the law, $pV^{1.25} = C$ from 1 bar absolute and 27°C to a pressure of 16 bar. Calculate the temperature at the end of compression, the heat received or rejected, work done on the gas during the process and change in enthalpy. Assume the mass of the gas as 1 kg. *Ans:* 522.33 K; 92.378 kJ; 246.34 kJ; 233.49 kJ

Multiple Choice Questions (choose the most appropriate answer)

1. When the variables of a stream are independent of time, it is called
   (a) steady flow
   (b) unsteady flow
   (c) uniform flow
   (d) closed flow

2. Control volume means
   (a) a specified mass
   (b) an isolated system
   (c) a fixed region in space
   (d) a reversible process only

3. Internal energy of a perfect gas depends on
   (a) temperature, specific heats and pressure
   (b) temperature only
   (c) temperature, specific heats and enthalpy
   (d) temperature, specific heats and entropy

4. In a reversible polytropic process
   (a) the internal energy remains constant
   (b) the entropy remains constant
   (c) the enthalpy remains constant
   (d) true heat transfer occurs

5. An isentropic process is always
   (a) irreversible and adiabatic
   (b) reversible and adiabatic
   (c) frictionless and irreversible
   (d) reversible and isothermal

6. The net work done per kg of gas in a polytropic process is equal to
   (a) $p_1 v_1 \log_e \dfrac{v_2}{v_1}$
   (b) $p_1(v_1 - v_2)$
   (c) $\dfrac{p_1 v_1 - p_2 v_2}{n - 1}$
   (d) $\dfrac{p_2 v_2 - p_1 v_1}{n - 1}$

7. A reversible process requires that
   - (a) Newtons law of viscosity be satisfied
   - (b) temperature of system and surroundings be equal
   - (c) there be no viscous or Coloumb friction in the system
   - (d) heat transfer occurs from surroundings to system only

8. The characteristic equation of gases $pV = mRT$ holds good for
   - (a) diatomic gas
   - (b) real gases
   - (c) ideal gases
   - (d) mixture of gases

9. A gas which obeys kinetic theory perfectly is known as
   - (a) monoatomic gas
   - (b) diatomic gas
   - (c) pure gas
   - (d) perfect gas

10. Work done in a free expansion process is
    - (a) maximum
    - (b) zero
    - (c) positive
    - (d) negative

11. In the polytropic process equation, pV^n = constant, if $n = 0$, the process is termed as
    - (a) adiabatic
    - (b) constant volume
    - (c) constant pressure
    - (d) constant temperature

12. In the polytropic process equation, pV^n = constant, if n is infinitely large, the process is termed as
    - (a) constant volume
    - (b) constant pressure
    - (c) constant temperature
    - (d) adiabatic

13. A process or system that do not involve heat transfer is called
    - (a) isothermal processes
    - (b) thermal processes
    - (c) adiabatic processes
    - (d) steady processes

14. In isothermal process
    - (a) temperature increases gradually
    - (b) pressure remains constant
    - (c) enthalpy change is maximum
    - (d) change in internal energy is zero

15. In a reversible adiabatic process, the ratio (T_1/T_2) is equal to
    - (a) $\left(\dfrac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$
    - (b) $\left(\dfrac{V_1}{V_2}\right)^{\frac{\gamma-1}{\gamma}}$
    - (c) $(V_1 V_2)^{\frac{\gamma-1}{\gamma}}$
    - (d) $\left(\dfrac{V_1}{V_2}\right)^{\frac{1}{\gamma}}$

16. During throttling process
    - (a) enthalpy does not change
    - (b) pressure does not change
    - (c) entropy does not change
    - (d) internal energy does not change

17. When a gas is to be stored, the type of compression that would be ideal is
    - (a) polytropic
    - (b) adiabatic
    - (c) isothermal
    - (d) constant volume

18. If a process can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states, it is known as
    - (a) adiabatic process
    - (b) ideal process
    - (c) isothermal process
    - (d) frictionless process

19. The state of a substance whose evaporation from its liquid state is complete is known as
    - (a) vapour
    - (b) perfect gas
    - (c) air
    - (d) steam

20. In SI units, the value of the universal gas constant is
    - (a) 0.8314 J/mole/K
    - (b) 8.314 J/mole/K
    - (c) 83.14 J/mole/K
    - (d) 8314 J/mole/K

21. When the gas is heated at constant pressure, the heat supplied
    - (a) increases the internal energy of the gas
    - (b) increases the temperature of the gas
    - (c) does some external work during expansion
    - (d) both (a) and (b)

22. The gas constant (R) is equal to the
    - (a) sum of two specific heats
    - (b) product of two specific heats
    - (c) difference of two specific heats
    - (d) ratio of two specific heats

23. The heat absorbed or rejected during a polytropic process is
    - (a) $\left(\frac{\gamma - n}{\gamma - 1}\right) \times$ work done
    - (b) $\left(\frac{\gamma - n}{\gamma - 1}\right)^2 \times$ work done
    - (c) $\left(\frac{\gamma - n}{\gamma - 1}\right)^3 \times$ work done
    - (d) $\left(\frac{\gamma - n}{\gamma - 1}\right)^{\frac{1}{2}} \times$ work done

24. Internal energy is defined by
    - (a) zeroth law of thermodynamics
    - (b) first law of thermodynamics
    - (c) second law of thermodynamics
    - (d) third law of thermodynamics

25. A perpetual motion machine of the first kind is one
    - (a) which perpetually moves without any cessation
    - (b) fictitious machine which violates the first law of thermodynamics
    - (c) in which there is no friction
    - (d) which violates Kelvin-Plank statement

Ans: 1. – (a) 2. – (c) 3. – (b) 4. – (d) 5. – (b)
6. – (c) 7. – (c) 8. – (b) 9. – (d) 10. – (b)
11. – (b) 12. – (a) 13. – (c) 14. – (d) 15. – (a)
16. – (a) 17. – (c) 18. – (b) 19. – (a) 20. – (d)
21. – (d) 22. – (c) 23. – (a) 24. – (b) 25. – (b)

4

FIRST LAW OF THERMODYNAMICS FOR OPEN SYSTEMS

4.1 INTRODUCTION

In the previous chapter, we dealt with the details of the basis of the first law of thermodynamics and also the definition of total energy. Further, we applied the first law for a closed system undergoing different types of processes. In this chapter, we will extend the first law to open systems (control volumes) operating under both steady and unsteady conditions. We will also analyse several flow processes of practical interest.

4.2 APPLICATION OF FIRST LAW TO OPEN SYSTEMS (CONTROL VOLUME)

Let us first recollect what is meant by open system or control volume. When there is a mass transfer across the system boundary, it is called an open system or control volume. In engineering applications, mostly we come across with open systems involving flow of fluid through them. Typical examples are diffusers, heat exchangers, nozzles, rotary compressors, throttling valves, turbines, etc.

The question now is how do we apply first law of thermodynamics to such systems and analyse them. There are two possibilities:

(i) Concentrate upon a certain quantity of fluid which is moving or

(ii) Concentrate upon a certain fixed region in space called control volume through which the moving substance flows.

The second approach is more convenient to analyse. Consider a typical system shown in Fig.4.1. The broken line represents the surface of the control volume and therefore, is called control surface. This is also the system boundary of the open system. The method of analysis is to inspect the control surface and account for all energy transfer through this surface. Further, we should also carry out a mass balance along with the energy balance, since there is both mass transfer and energy transfer across the control surface.

4.2.1 Principle of Conservation of Mass for a General System

Principle of conservation of mass for a control volume can be expressed as follows: *The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.* That is to say

$$m_{in} - m_{out} = \Delta m_{cv} \tag{4.1}$$

where Δm_{cv} is the change in the mass of the control volume during the process. In rate form, it can be expressed as

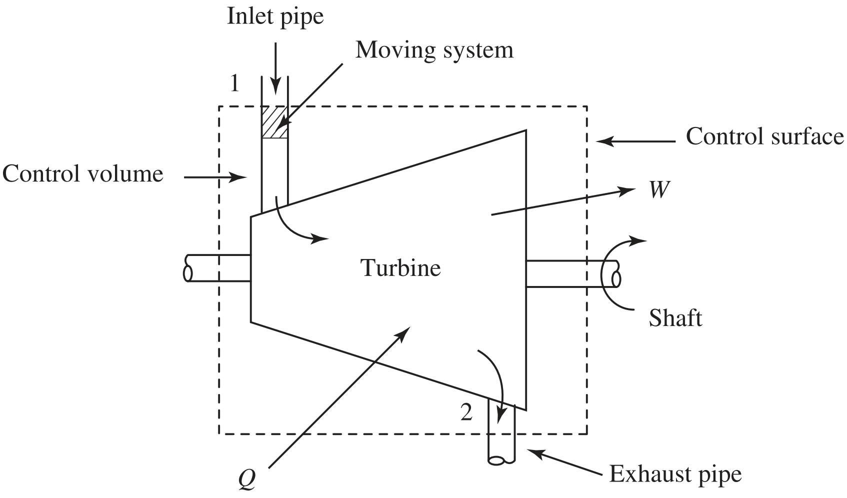


Fig. 4.1 Illustration of the conservation of mass principle

$$\dot{m}_{in} - \dot{m}_{out} = \frac{dm_{cv}}{dt} \tag{4.2}$$

where $\dot{m}_{in}$ and $\dot{m}_{out}$ are the total mass flow rate into and out of the control volume and $\frac{dm_{cv}}{dt}$ is the time rate of change of mass within the control volume boundaries. Equations 4.1 and 4.2 are often referred to as the mass balance and are applicable to any control volume undergoing any kind of process.

4.2.2 Mass Balance for Steady Flow Processes

For steady-flow processes, there is no change in the mass within the control volume with time (m_{cv} = constant). Then, the conservation of mass principle requires that the total amount of mass entering a control volume should be equal the total amount of mass leaving it. While, dealing with steady-flow processes, we are not interested in the amount of mass that flows in or out of a device over time. Instead, we are interested in the amount of mass flowing per unit time, that is, the mass flow rate $\dot{m}$. The *conservation of mass principle* for a general steady-flow system with multiple inlets and outlets can be expressed in rate form as shown in Fig.4.2.

Now, let us consider a general steady flow system. As already stated, a system is said to be a steady flow system when the rate of flow of working fluid is constant. In such a system, there is a conservation of mass. What it means is that the amount of working fluid entering the system is exactly the same as amount of fluid leaving the system after doing work. There is no accumulation of mass within the system. Such types of systems are also called *open systems.*

For steady flows,

$$\sum_{in} \dot{m} = \sum_{out} \dot{m} \tag{4.3}$$

That is to say that the total rate of mass entering a control volume is equal to the total rate of mass leaving it.

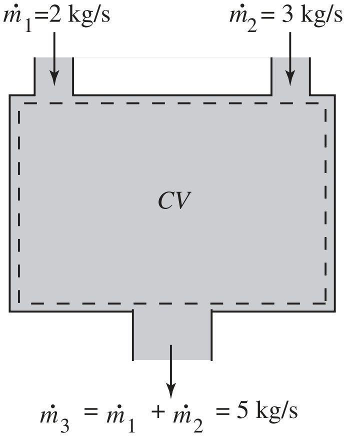


Fig. 4.2 Flow process involving multiple inlet

Most of the engineering equipments such as compressors, diffusers, nozzles, turbines and pumps involve a single stream (only one inlet and one outlet). For these cases, we denote the inlet state by the subscript 1 and the outlet state by the subscript 2 and drop the summation signs. Then, Eq.4.3 reduces, for single-stream steady-flow systems to,

$$\dot{m}_1 = \dot{m}_2; \qquad \rho_1 \overline{\mathbb{V}}_1 A_1 = \rho_2 \overline{\mathbb{V}}_2 A_2 \tag{4.4}$$

$$\sum \rho_1 \overline{\mathbb{V}}_1 A_1 \quad = \quad \sum \rho_2 \overline{\mathbb{V}}_2 A_2 \tag{4.5}$$

where A is the flow area, $\overline{\mathbb{V}}$ is the average velocity and ρ is the density.

4.2.3 Mass Balance for Incompressible Flow

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids. Cancelling the density from both sides of the general steady-flow relation gives $\dot{V}$

$$\sum \overline{\mathbb{V}}_1 A_1 \quad = \quad \sum \overline{\mathbb{V}}_2 A_2 \tag{4.6}$$

For steady incompressible flows,

$$\sum_{in} \dot{V} \quad = \quad \sum_{out} \dot{V} \tag{4.7}$$

For a single-stream steady-flow systems, it becomes

$$\dot{V}_1 = \dot{V}_2; \qquad \mathbb{V}_1 A_1 = \mathbb{V}_2 A_2 \tag{4.8}$$

where $\dot{V}$ represents volume flow rate.

It should be emphasised here that there is no such thing as a 'conservation of volume' principle. Therefore, the volume flow rates into and out of a steady-flow device may vary. For example, the volume flow rate at the outlet of an air compressor is much less than that at the inlet even though the mass flow rate of air through the compressor is constant This is due to the higher density of air at the compressor exit due to increase in pressure. For steady flow of liquids, however, the volume flow rates, as well as the mass flow rates, remain constant. It is due to the fact liquids are essentially incompressible (constant-density) substances.

4.2.4 Principle of Conservation of Energy for Open Systems

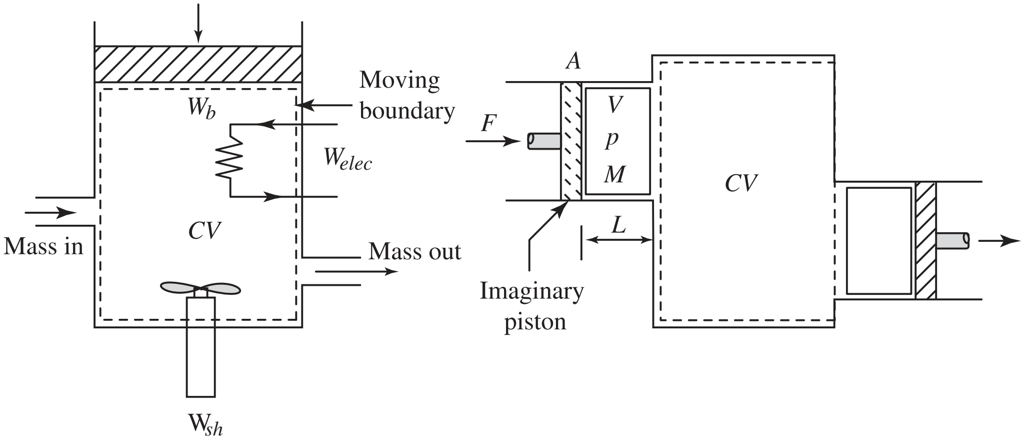
The principle of conservation of energy for an open system, viz control volume for any process can be expressed as

$$\begin{array}{c} \text{Total energy crossing} \\ \text{the CV as heat} \\ \text{and work} \end{array} + \begin{array}{c} \text{Total energy} \\ \text{of the mass entering} \\ \text{the CV} \end{array} - \begin{array}{c} \text{Total energy} \\ \text{of the mass} \\ \text{leaving the CV} \end{array} = \begin{array}{c} \text{Net change} \\ \text{in the energy} \\ \text{of the CV} \end{array}$$

Mathematically it can be written as:

$$Q - W + \sum E_{in} - \sum E_{out} = \Delta E_{CV} \tag{4.9}$$

Consider the typical control volume system shown in Fig.4.3(a). As can be seen, only one heat and work interaction is involved. Note that more than one form of work interaction is possible. The system involves the flow work or flow energy which is the energy required to push fluid mass into and out of the system. This is the energy transported during the process under consideration. Let us now introduce a term called ‘flow work’.



(a) Typical control volume system (b) Flow work

Fig. 4.3 Control volume system and flow work

4.2.5 Flow Work

Figure 4.3(b) illustrates a volume V of the fluid element at a pressure, p, ready to enter the control volume. Assume that there is an imaginary piston trying to push the fluid element inside. The force on the imaginary piston will be $F = pA$, where A is area of cross-section of the piston. To push the entire mass of the fluid element into the control volume, the force F must act through a distance L as shown in Fig.4.3(b). This is called *flow work* and written as

$$W_{flow} = FL = pAL = pV \tag{4.10}$$

where AL is the volume of the fluid element. The flow work per unit mass can be obtained by dividing both sides of Eq.4.10 by the mass of the fluid element. Then, the equation becomes

$$w_{flow} \quad = \quad pv \tag{4.11}$$

It is to be noted that unlike other form of work, flow work is expressed in terms of the properties, especially the combination of properties, viz p and V. It is also called *flow energy* or *convected energy* or *transport energy.*

4.2.6 Total Energy of a Moving Fluid

Total energy of a simple compressible system can be accounted in three parts:

(i) Potential energy, (ii) Kinetic energy, (iii) Internal energy

It can be expressed mathematically on a unit mass basis as

$$e = e_{PE} + e_{KE} + e_{IE} \quad = \quad gz + \left(\frac{\mathbb{V}^2}{2}\right) + u \tag{4.12}$$

where z is the elevation from the datum, $\mathbb{V}$ is the velocity and u is the internal energy, respectively. Further, the fluid entering or leaving the control volume possesses an additional form of energy called the flow energy, pv. If we denote the total energy per unit mass flow as e_{tot}, then

$$e_{tot} \quad = \quad gz + \left(\frac{\mathbb{V}^2}{2}\right) + u + pv \tag{4.13}$$

As we know $u + pv$ is called the specific enthalpy, h. Now, the total energy can be written as:

$$e_{tot} \quad = \quad gz + \left(\frac{\mathbb{V}^2}{2}\right) + h \tag{4.14}$$

Equation 4.14 is the general form of the total energy equation. It represents the energy of the fluid stream flowing into or out of the control volume. Here, afterwards, we will not make any reference to flow work or flow energy. Note that the work term, W, in Eq.4.9 will represent all forms of work, viz boundary work, electrical work, flow work, shaft work, etc.

4.2.7 Conservation of Energy for a Steady Flow Process

Equation 4.14 gives the expression for the total energy of the general system for unit mass flow rate. When we apply this to the steady flow process with multiple inlets and exits, it can be stated as

$$\begin{array}{ccccc} \text{Total energy crossing} & & \text{Total energy transported} & & \text{Total energy transported} \\ \text{the boundary as heat} & = & \text{out of control volume} & - & \text{into the control volume} \\ \text{and work per unit time} & & \text{with mass per unit time} & & \text{with mass per unit time} \end{array}$$

$$\dot{Q} - \dot{W} \quad = \quad \dot{m}_e \dot{e}_{tote} - \dot{m}_i \dot{e}_{toti} \tag{4.15}$$

where $\dot{e}_{tot}$ is the total energy of the flowing fluid including the flow work. Combining Eq.4.13 and 4.14, the steady flow energy equation can be written as:

$$\dot{Q} - \dot{W} \quad = \quad \sum \dot{m}_e \left(gz_e + \frac{\mathbb{V}_e^2}{2} + h_e \right) - \sum \dot{m}_i \left(gz_i + \frac{\mathbb{V}_i^2}{2} + h_i \right) \tag{4.16}$$

It is also the normal practice to represent inlet as 1 and exit as 2.

4.2.8 Single Stream Steady Flow System

In case of single-stream steady flow system, the mass flow rate through the entire control volume remains constant ($\dot{m}_1 = \dot{m}_2$). For this condition, the steady flow energy equation becomes

$$\dot{Q} - \dot{W} \;=\; \dot{m}\left[g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1)\right] \qquad (4.17)$$

where the subscripts 1 and 2 represent inlet and exit conditions, respectively. For unit mass flow rate, the above equation becomes

$$q - w \;=\; g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \qquad (4.18)$$

For a system in which PE and KE changes are negligible in a single-stream steady flow system, the above equation further reduces to

$$q - w \;=\; \Delta h \qquad (4.19)$$

Equation 4.17 is the simplest equation of the first-law relation for control volumes. This equation resembles the first law equation for the closed system, viz $Q - W = \Delta u$ except that Δu is replaced by Δh in this case.

4.3 APPLICATION OF STEADY STATE STEADY FLOW ENERGY EQUATION

In this section, we will discuss the application of steady state steady flow energy equation (SS SFEE) to various engineering devices. We will derive the corresponding equation for the various systems. Although it is not strictly correct, in most textbooks, SS SFEE, is referred as SFEE. We will follow the same in this book also.

4.3.1 Air Compressors

Air compressor is a device which compresses air to a higher pressure compared to the inlet pressure. Compressors can be classified as reciprocating compressors and rotary compressors. First, let us apply first law to *reciprocating compressors.*

Figure 4.4 shows the anatomy of a reciprocating compressor. In this type of compressors like reciprocating internal combustion engines, a piston reciprocates in the cylinder for developing a positive higher pressure of air. The quantity of air displaced depends upon the cubic capacity of the cylinder. Therefore, the amount of air delivered will be comparatively low compared to rotary compressor. It can produce much higher pressures due to mechanical compression by a piston compared to a rotary compressor in which aerodynamic compression takes place. The reciprocating compressors are called positive displacement compressors. The point to note is that it has a larger area of contact with the surroundings and therefore, compared to rotary compressors the heat transfer is more. In these compressors, the change in potential and kinetic energies are almost negligible. According to first law for unit mass flow rate, SFEE is given by

$$Q - W \;=\; g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \qquad (4.20)$$

Note that on the LHS of Eq.4.20, $Q = \frac{Q}{1}$ and $W = \frac{W}{1}$ because of unit mass flow rate, whereas on the RHS h refer to specific enthapies. With the above assumptions, the first law reduces to

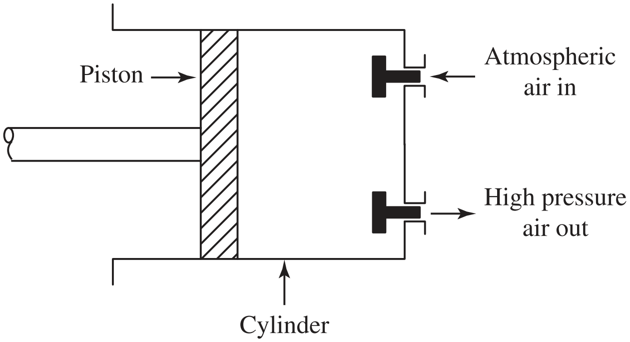


Fig. 4.4 Anatomy of a reciprocating compressor

$$Q - W = h_2 - h_1 = \Delta h \tag{4.21}$$

Applying the sign convention Q is negative, since the heat is rejected and W is also negative since work is done on the system through some external source for making the reciprocating motion of the piston which in turn compresses the air. Therefore, the above equation becomes

$$-Q - (-W) = h_2 - h_1 \tag{4.22}$$

Therefore,

$$W = Q + (h_2 - h_1) \tag{4.23}$$

Now, let us consider a rotary compressor shown in Fig.4.5. Fans, compressors and pumps are devices used to increase the pressure of a fluid as it passes through them. Rotary air compressors are used in jet engines. They spin at very high speed (>60000 rpm). Work is supplied to these devices from an external source through a rotating shaft. Therefore, they involve work inputs. Heat transfer will be negligible since the mass flow rate through these devices is very high and area of heat transfer is quite small. The velocity change between inlet and exit is usually quite small to cause any significant change in the kinetic energy. Further, there will be no change in potential energy. First law for open system with unit mass flow rate is now given by:

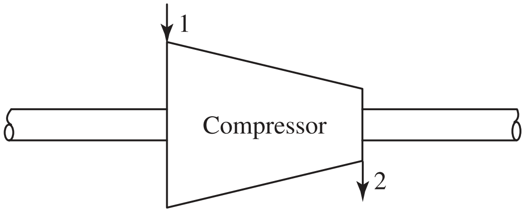


Fig. 4.5 Rotary compressor

$$Q - W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.24}$$

With the above assumptions, the first law reduces to:

$$W = h_2 - h_1 \tag{4.25}$$

4.3.2 Boiler

A boiler is a device which generates high pressure steam by supplying heat to the water. It is basically nothing but a large heat exchanger. Heat originating from combustion gases or other sources are transferred to the water essentially at constant pressure. A typical steam power plant in which boiler is employed is shown in Fig.4.6. The enlarged view of the boiler is given in Fig.4.7. Water enters the *boiler* as a compressed liquid at state 2 and leaves as a superheated vapour at state 3. In this system, heat energy is stored in the steam. Therefore, we have to take into account the internal energy. There is also the flow energy due to the movement of water stream. In boiler, there is no work done by the steam. The change in potential as well as the kinetic energy can be assumed to be negligible. Looking at the boiler in Fig.4.7 and taking 1 as inlet and 2 as outlet, let us apply the first law to analyse the boiler.

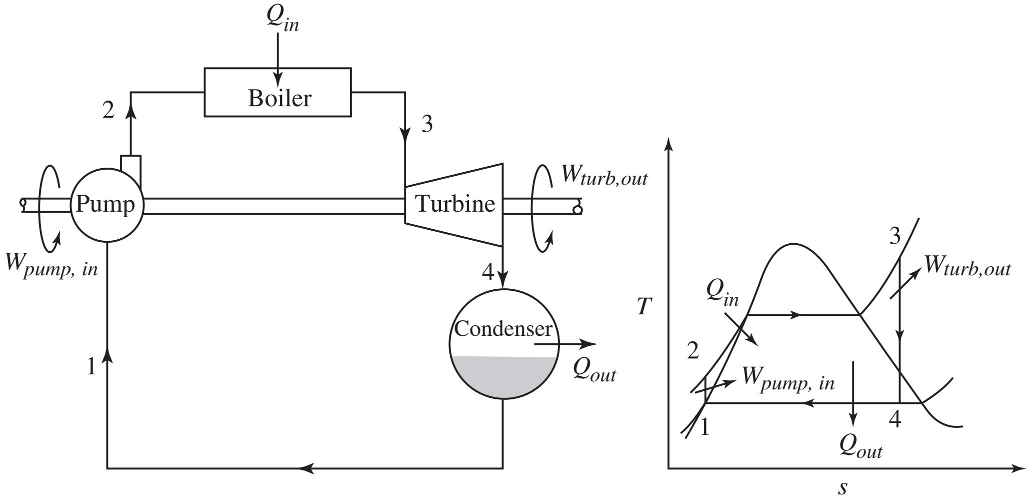


Fig. 4.6 A typical steam power plant

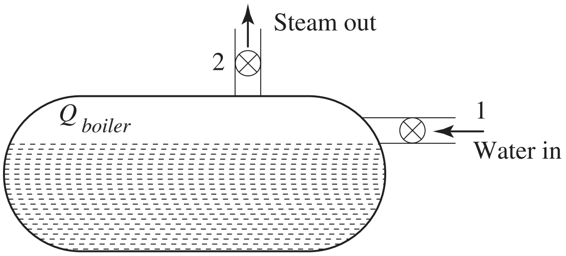


Fig. 4.7 Boiler

According to first law for unit mass flow rate,

$$Q - W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.26}$$

With the above assumptions, the first law reduces to,

$$Q = h_2 - h_1 \tag{4.27}$$

4.3.3 Centrifugal Water Pump

A centrifugal water pump draws water from a lower level and pumps to higher level as shown in Fig.4.8. A typical example is pumping water from the sump to the over head tanks in houses and apartments. There must be work input to run the pump and this may be supplied from an external source such as an electric motor or a diesel engine. Here, $Q = 0$ and $\Delta u = 0$ as there is no change in temperature of water; $v_1 = v_2 = v$. The sign of W *is negative because work is done on the system (or work enters the boundary).*

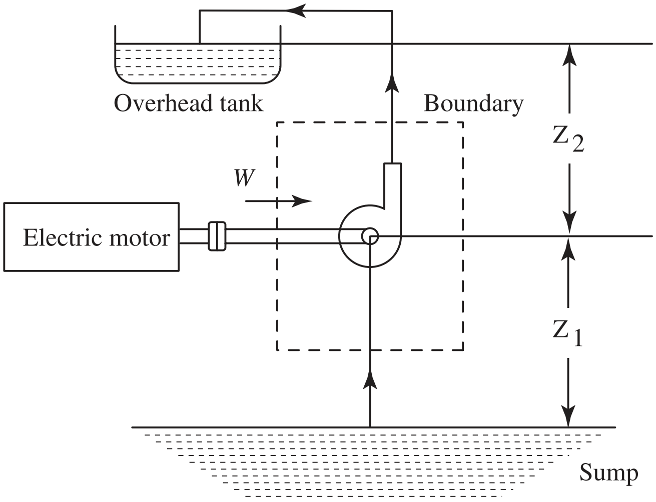


Fig. 4.8 Centrifugal water pump

According to first law for unit mass flow rate,

$$Q - (-W) = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (p_2 v_2 - p_1 v_1) \tag{4.28}$$

As $Q = 0$, the above equation reduces to,

$$W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (p_2 v_2 - p_1 v_1) \tag{4.29}$$

4.3.4 Combustion Chamber

Combustion chambers are commonly used in gas turbine power plants. Combustion takes place under constant pressure conditions. Fuel is injected at high pressure into the combustion chamber. Air is fed into the combustion chamber from a compressor outlet (Fig.4.9). Mixing of air and fuel takes place in the combustion chamber. The fuel is ignited initially by an igniting mechanism. The whole process of combustion takes place at constant pressure. Therefore, it is a two input and one output system. Hence, we shall consider both mass balance and energy balance.

As per mass balance,

$$\dot{m}_p = \dot{m}_a + \dot{m}_f \tag{4.30}$$

where $\dot{m}_a, \dot{m}_f$ and $\dot{m}_p$ are mass flow rates of air, fuel and products of combustion, respectively as shown in Fig.4.9. Now, applying the first law,

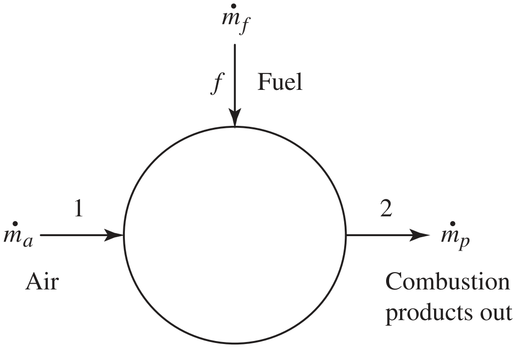


Fig. 4.9 Combustion chamber

$$\dot{Q} - \dot{W} = \dot{m}\left[g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1)\right] \tag{4.31}$$

Now, $\Delta KE = 0, \Delta PE = 0$, and $\dot{W} = 0$. Note that $\dot{m}_1 h_1 = \dot{m}_a h_a + \dot{m}_f h_f$, $\dot{m}_2 h_2 = \dot{m}_p h_p$ and $\dot{Q} = \dot{m}_f \times CV$, where CV is the calorific value of the fuel. With the above assumptions SFEE can be written as:

$$\dot{Q} = \dot{m}_p h_p - (\dot{m}_a h_a + \dot{m}_f h_f) \tag{4.32}$$

$$\dot{m}_f CV + (\dot{m}_a h_a + \dot{m}_f h_f) = \dot{m}_p h_p \tag{4.33}$$

4.3.5 Condenser

Consider the steam power plant is shown in Fig.4.6. As can be seen, a condenser is employed in the steam power plant. It is used to condense the hot steam into water. Natural water is used as a cooling medium for condensation. Therefore, it is nothing but a heat exchanger. A typical condenser is shown in Fig.4.10.

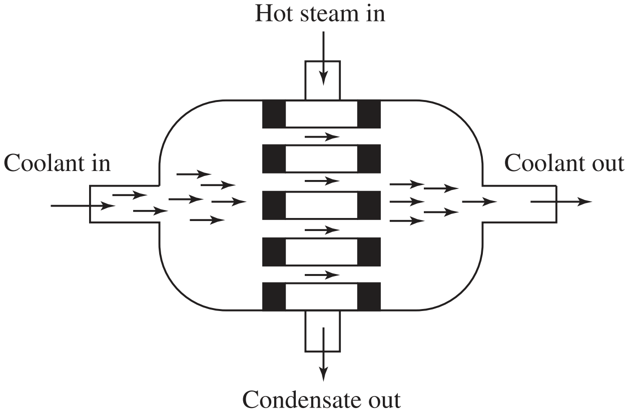


Fig. 4.10 Condenser

The main purpose to have a condenser in the steam power plant is to transfer heat from steam to condenser. In this system, there is no heat input. Change in potential and kinetic energy are negligible. Considering unit mass flow rate and as per the first law,

$$Q - W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.34}$$

With the above assumptions, viz $\Delta PE = 0; \Delta KE = 0$ and $W = 0$, the first law reduces to,

$$Q = h_2 - h_1 \tag{4.35}$$

4.3.6 Diffuser

Diffuser is mainly used in turbomachinery to increase the static pressure. It is a device which increases the static pressure of a fluid stream by decelerating the flow. In other words, it converts the kinetic energy into potential energy. There are different types of diffusers. A typical subsonic conical diffuser is shown in Fig.4.11.

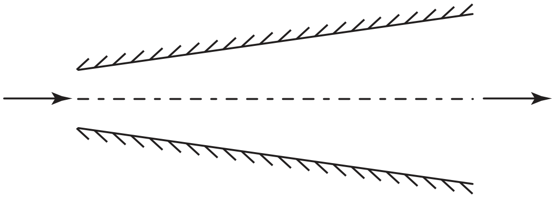


Fig. 4.11 Subsonic diffuser

In this system, the heat transfer is almost negligible since they are usually well insulated. Fluid entering the diffuser has high velocity and passes through the diffuser quite quickly without any significant heat transfer taking place. Therefore, the flow through the diffuser can be taken to be adiabatic. The work transfer, W, is also zero since the static pressure rise is not due to any mechanical work input but only due to ram effect.[1] The change in potential energy is zero as normally there is no change in elevation when the fluid flows through the diffuser. However, there will be a significant change in the kinetic energy since diffusers are mainly used to decelerate the flow.

Now, applying first law for unit mass flow rate,

$$Q - W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.36}$$

With the above assumptions, the first law reduces to,

$$\frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) = 0 \tag{4.37}$$

$$h_1 + \frac{\mathbb{V}_1^2}{2} = h_2 + \frac{\mathbb{V}_2^2}{2} \tag{4.38}$$

or

$$\frac{\mathbb{V}_2^2}{2} - \frac{\mathbb{V}_1^2}{2} = h_1 - h_2 \tag{4.39}$$

---

[1] The compression of air in an inlet duct arising from forward motion is called ram pressure or ram effect.

4.3.7 Evaporator

An evaporator is used in a refrigeration plant. It carries away the heat from the refrigerator to maintain low temperature. In this the refrigerant goes in as liquid and comes out as vapor. For vaporisation, the liquid refrigerant absorbs its latent heat from the surroundings of the evaporator. Figure 4.12 shows a typical evaporating system.

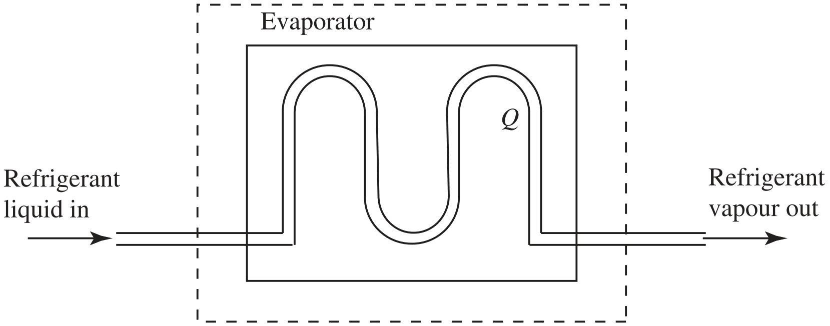


Fig. 4.12 A typical evaporating system

As can be seen from the figure, the change in potential and kinetic energy are almost zero. No work is done on the system or by the system. Therefore, $\Delta KE = 0, \Delta PE = 0$, and $W = 0$.

Applying the first law to the system for unit mass flow rate,

$$Q - W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.40}$$

With the above assumptions, the first law reduces to,

$$Q = h_2 - h_1 \tag{4.41}$$

Q is taken as positive because heat flows *from the surroundings to the system* as the temperature in the system is lower than the surroundings.

4.3.8 Heat Exchangers

Heat exchangers are devices where there are two moving fluid streams. Important point to note is that they exchange heat without mixing. Heat exchangers have wide industrial applications. The simplest form of a heat exchanger is a *double-tube* (also called *tube and shell) heat exchanger.* The details are shown in Fig.4.13.

As can be seen, it is composed of two concentric pipes of different diameters. One fluid flows in the inner pipe and the other in the annular space between the two pipes. Heat is transferred from the hot fluid to the cold one through the wall separating them.

The conservation of mass principle for a heat exchanger under steady operating conditions stipulates the sum of the mass flow rates at inlet to be equal to the sum of the mass flow rates at the outlet. This principle can also be expressed as follows:

Under steady operation, the mass flow rate of each fluid stream flowing through a heat exchanger remains constant.

Heat exchangers do not have work interactions ($W = 0$) and negligible kinetic and potential energy changes ($\Delta KE = 0, \Delta PE = 0$) for each fluid stream. The heat transfer rate associated

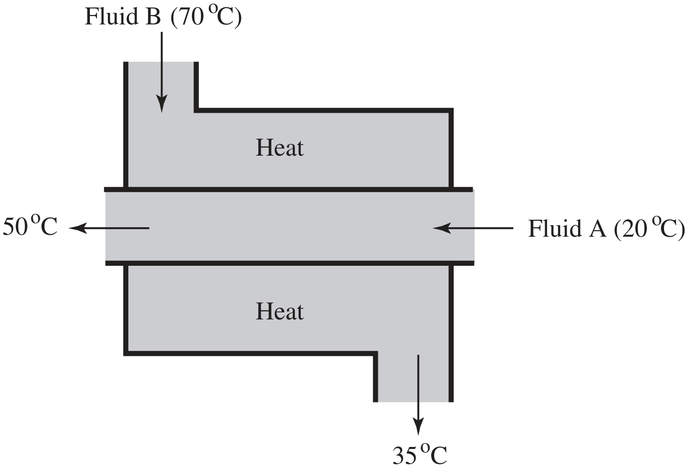
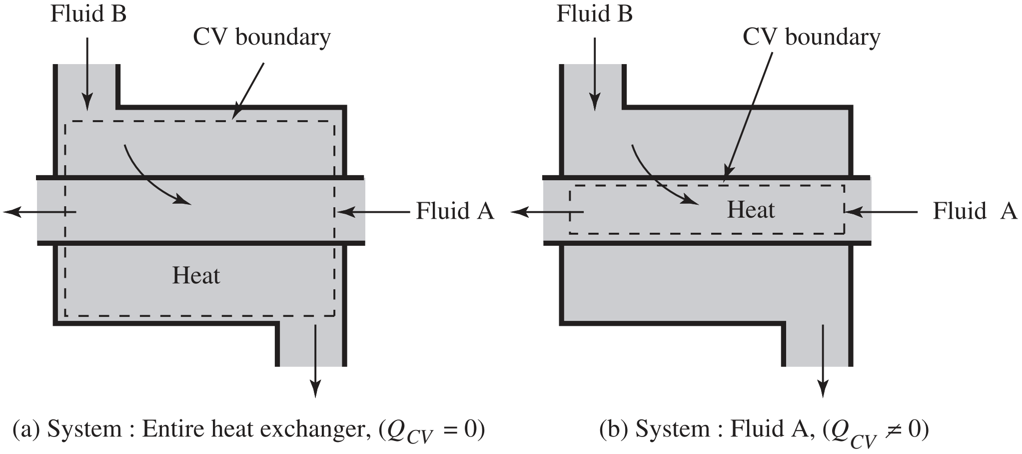


Fig. 4.13 A typical heat exchanger

with heat exchangers depends on the selection of control volume. Heat exchangers are intended for heat transfer between two fluids *within* the device. The outer shell is normally well insulated to prevent any heat loss to the surrounding medium. When the entire heat exchanger is selected as the control volume, $\dot{Q}$, becomes zero, since the boundary for this case lies just beneath the insulation and no heat crosses the boundary (Fig.4.14). If, however, only one of the fluids is selected as the control volume, then heat will cross this boundary as it flows from one fluid to the other and $\dot{Q}$ will not be zero. In fact, $\dot{Q}$ in this case will be the rate of heat transfer between the two fluids.



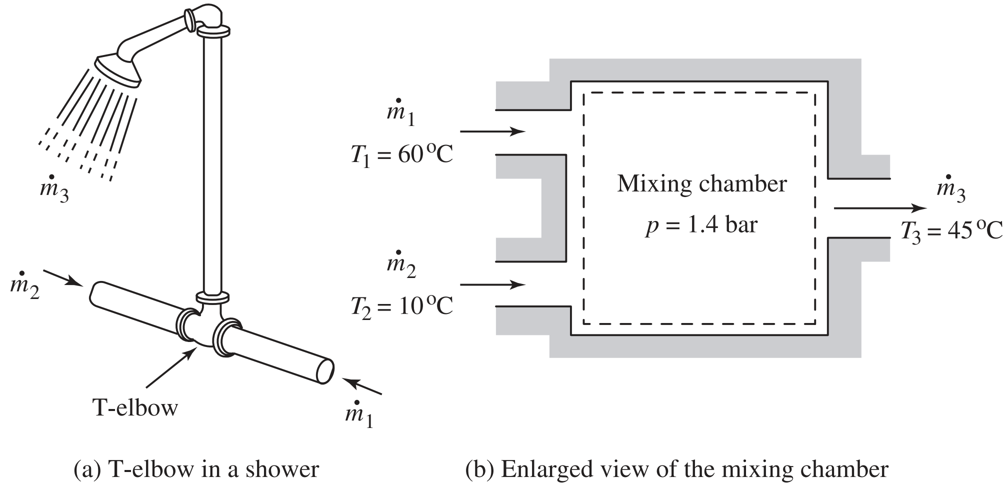
(a) System : Entire heat exchanger, ($Q_{CV} = 0$) (b) System : Fluid A, ($Q_{CV} \neq 0$)

Fig. 4.14 A heat exchanger

4.3.9 Mixing Chambers

In engineering applications, mixing two streams of fluids is quite common. The place where the mixing process takes place is usually referred to as a mixing chamber. Note that the mixing chamber must have a distinct chamber. A typical example is an ordinary T-elbow in a shower, that serves as the mixing chamber for the cold- and hot-water streams (Fig.4.15).

Let $\dot{m}_1$ be the mass flow rate of hot water at a temperature T_1 and $\dot{m}_2$ be the mass flow rate of cold water at a temperature T_2. Both mix at the mixing chamber and come out at a temperature T_3. We should take care of two things: (i) the conservation of mass and (ii) the conservation of energy.



(a) T-elbow in a shower (b) Enlarged view of the mixing chamber

Fig. 4.15 T-elbow as the mixing chamber

The conservation of mass principle for a mixing chamber requires that the sum of the incoming mass flow rates equal the mass flow rate of the outgoing mixture.

Mixing chambers are usually well insulated ($Q \approx 0$). Further, they do not involve any kind of work interaction ($W \approx 0$). Also, the kinetic and potential energy changes of the fluid streams are usually negligible ($\Delta KE \approx 0, \Delta PE \approx 0$). Then, what is left in the energy equation is only the total energies of the incoming streams and the outgoing mixture.

The conservation of energy principle requires that these two are equal to each other. Therefore, the conservation of energy equation becomes analogous to the conservation of mass equation for this case.

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \tag{4.42}$$

Combining the mass and energy balances,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3 \tag{4.43}$$

where $\dot{m}_3 = \dot{m}_1 + \dot{m}_2$.

4.3.10 Steam Nozzle

In many engineering applications, we come across a convergent-divergent nozzle. A typical convergent-divergent nozzle is shown in Fig.4.16. In this velocity of the fluid increases as the fluid pass through the device. In other words, the kinetic energy of the fluid increases. However, this is at the expense of the pressure drop. If this device is used in a steam turbine, then the nozzle converts the part of the energy of steam into kinetic energy before supplying to the

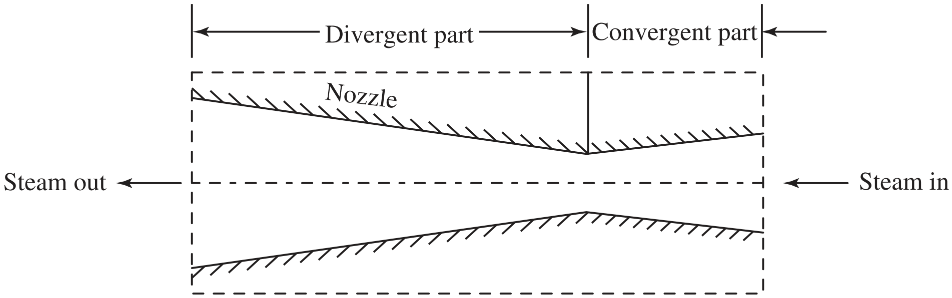


Fig. 4.16 Steam nozzle

turbine. This causes a decrease in the enthalpy of the fluid. For this system, $\Delta PE = 0, W = 0$ and $Q = 0$. Applying the SFEE to the system,

$$Q - W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.44}$$

With the above assumptions, the first law reduces to,

$$\frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) = 0 \tag{4.45}$$

$$h_1 + \frac{\mathbb{V}_1^2}{2} = h_2 + \frac{\mathbb{V}_2^2}{2} \tag{4.46}$$

$$\frac{\mathbb{V}_2^2}{2} - \frac{\mathbb{V}_1^2}{2} = h_1 - h_2 \tag{4.47}$$

$$\mathbb{V}_2^2 - \mathbb{V}_1^2 = 2(h_1 - h_2) \tag{4.48}$$

$$\mathbb{V}_2 = \sqrt{\mathbb{V}_1^2 + 2(h_1 - h_2)} \tag{4.49}$$

where velocity is in m/s and enthalpy is in joules. If the inlet velocity, $\mathbb{V}_1$, is extremely small compared to $\mathbb{V}_2$, i.e. $\mathbb{V}_1 \ll \mathbb{V}_2$, then

$$\mathbb{V}_2 = \sqrt{2(h_1 - h_2)} \tag{4.50}$$

4.3.11 Pipe and Duct Flow

In many engineering equipments, duct flows are involved for the transport of liquids or gases. Most of the time, flow through a pipe or a duct satisfies the steady-flow conditions. Therefore, they can be analysed as a steady-flow process. The control volume can be selected to coincide with the interior surfaces of the portion of the pipe or the duct. The amount of heat gained or lost by the fluid may be very significant during normal operating conditions (Fig.4.17). Sometimes, the sole purpose of the flow is to effect heat transfer. Examples are:

(i) Water flow through the pipes in the furnace of a power plant,

(ii) Flow of refrigerant in a freezer and

(iii) Flow in heat exchangers.

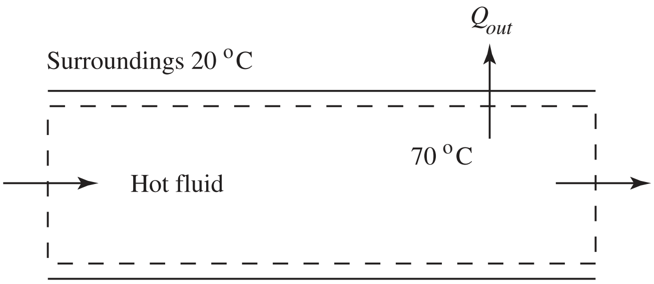


Fig. 4.17 Pipe or duct flow without insulation

Many a times, heat transfer is undesirable. Then, the pipes or ducts are insulated to prevent any heat loss or gain. Particularly if the temperature difference between the flowing fluid and the surroundings is large, insulation is a must to make heat transfer negligible. If the control volume involves a heating section (electric wires), a fan or a pump (shaft), the work interactions should be considered (Fig.4.18). Of these, fan work is comparatively small and often neglected in energy analysis. The velocities involved in pipe and duct flow are relatively low. Therefore, the kinetic energy changes are usually extremely small. This is particularly true when the pipe or duct diameter is constant and the heating effects are negligible.

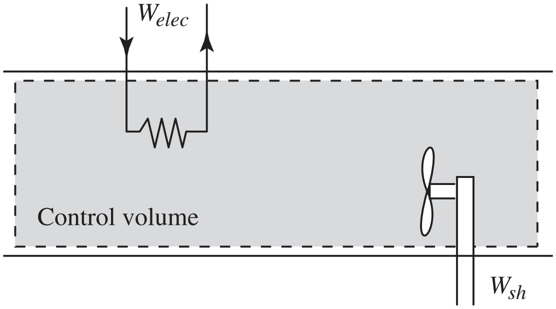


Fig. 4.18 Pipe or duct flow with more than one form of work input

Kinetic energy changes may be significant in some cases. For example, for gas flow in ducts with variable cross-sectional areas, especially when the compressibility effects are significant. The potential energy term may also be significant when the pipe is long and installed vertically.

4.3.12 Steam De-Super Heater

A typical steam de-superheater is shown in Fig.4.19. In this device, the temperature of the superheated steam is reduced by spraying water. As can be seen from Fig.4.19, m_1 kg of water and m_2 kg of superheated steam enters the system. The water is sprayed on the steam. After mixing, m_3 kg of de-superheated steam leaves the device. Let the corresponding enthalpies be h_1, h_2 and h_3, respectively. The change in KE and PE can be neglected. Applying SFEE,

The mass balance $$m_3 = m_1 + m_2 \qquad (4.51)$$

The energy balance $$m_1h_1 + m_2h_2 = m_3h_3 \qquad (4.52)$$

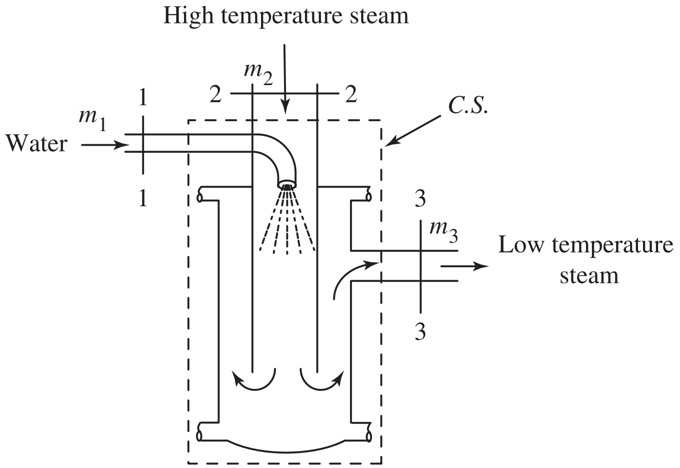


Fig. 4.19 Steam de-superheater

4.3.13 Throttling Valve

Throttling means flow of a fluid through some restricted passage. A typical throttle system is shown in Fig.4.20. The restricted opening can be caused by a valve. Therefore, any kind of flow-restricting device that causes a significant pressure drop in the fluid without involving any work is called throttling valve. Typical examples are:

(i) adjustable valve, (ii) capillary tube and (iii) porous plugs.

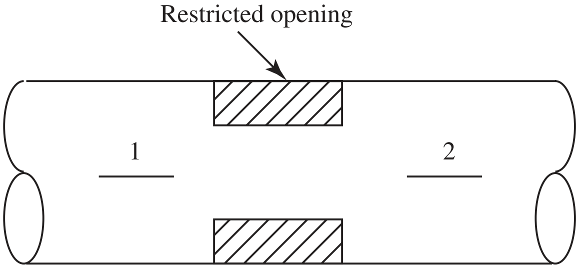


Fig. 4.20 Throttling process

In these devices, considerable temperature drop takes place. Therefore, such devices are commonly employed in refrigeration and air-conditioning equipments.

Throttling valves are usually very small and the flow through them may be assumed to be adiabatic ($Q = 0$). Further, the wok transfer is also zero ($W = 0$). The change in potential energy is zero and in most of the cases, the change in kinetic energy can also be considered zero. With the above assumptions SFEE becomes

$$h_2 = h_1 \tag{4.53}$$

That is to say that the exit enthalpy is same as the inlet enthalpy. For this reason, they are called isenthalpic devices. The above equation can be written as:

$$u_1 + p_1V_1 = u_2 + p_2V_2 \tag{4.54}$$

What the above equation implies is that *internal energy + flow energy = constant.*

If $p_2V_2 < p_1V_1$, then the internal energy has to decrease. Then, there will be a drop in temperature. (i.e. $T_2 \leq T_1$). For an ideal gas, $h = h(T)$ and thus the temperature T has to remain constant during a throttling process.

4.3.14 Turbine

The basic principle involved in a turbine is the conversion of potential energy of the working fluid into mechanical work. There are two types of turbines, viz gas turbine and steam turbine depending upon the working fluid. If steam is used as the working fluid then it is called the steam turbine. If gas (by burning fuel and air) is used then it is called the gas turbine. The turbines are usually well insulated and therefore, the heat transfer $Q = 0$. A typical turbine configuration is shown in Fig.4.21. In the ideal turbine, the expansion of working fluid is treated as isentropic (reversible adiabatic). The general form of SFEE is:

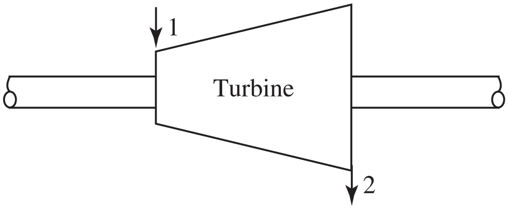


Fig. 4.21 A typical turbine configuration

$$Q - W = \Delta PE + \Delta KE + \Delta h \tag{4.55}$$

If change in both PE and KE is negligible then SFEE becomes $-W = \Delta h$, since $Q = 0$.

With the above assumptions we have:

$$h_2 - h_1 = -W \tag{4.56}$$

$$W = h_1 - h_2 \tag{4.57}$$

However, if the kinetic energy and heat transfer cannot be neglected then the SFEE becomes

$$-Q - (+W) = \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.58}$$

$$-W - Q = \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1) \tag{4.59}$$

$$h_1 + \frac{\mathbb{V}_1^2}{2} - Q = h_2 + \frac{\mathbb{V}_2^2}{2} + W \tag{4.60}$$

4.3.15 Water Turbine

Water turbines are used for generating electricity. Water is stored at certain height so that it posses good amount of PE. The basic principle involved in a turbine is the conversion of potential energy of the water into kinetic energy. The kinetic energy is imparted to the blades of the turbine, the part of which is converted into useful work. The turbine will be

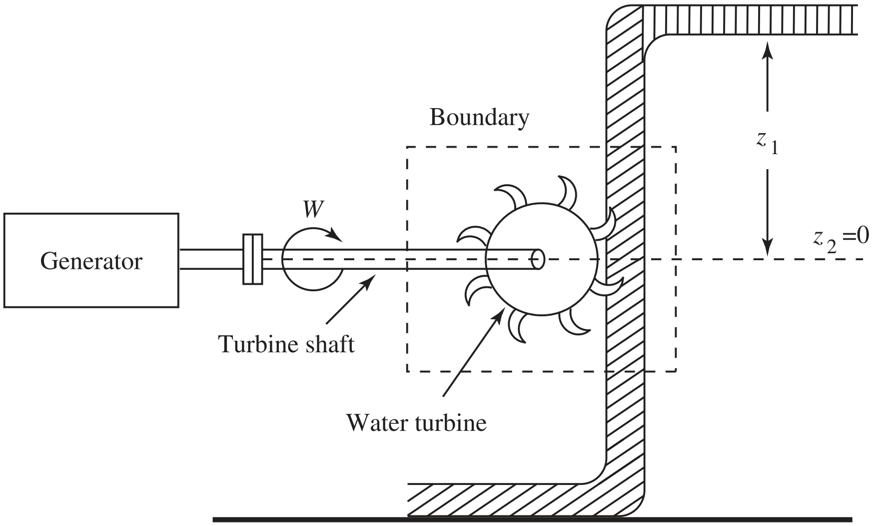


Fig. 4.22 Water turbine

connected to a generator which generates electricity. The schematic arrangement of a typical water turbine power plant is illustrated in Fig.4.22. The general form of SFEE is:

$$Q - W = \Delta PE + \Delta KE + \Delta h \tag{4.61}$$

Writing Δh as $\Delta(u + pV)$ we have,

$$Q - W = \Delta PE + \Delta KE + \Delta(u + pV) \tag{4.62}$$

Considering centre of turbine as datum as shown in Fig.4.22 we can assume the following:

$$Q = 0 \tag{4.63}$$

$$\Delta u = u_2 - u_1 = 0 \tag{4.64}$$

$$\overrightarrow{\mathbb{V}}_1 = \overrightarrow{\mathbb{V}}_2 = \overrightarrow{\mathbb{V}} \tag{4.65}$$

$$z_2 = 0 \tag{4.66}$$

Q is zero since there will be no significant change in temperature of water. Similarly, as internal energy is also a function of temperature the change in internal energy will also be zero. W will be positive as there is a work output from the system. With the above assumptions SFEE becomes

$$0 - (+W) = (0 - gz_1) + \frac{\overrightarrow{\mathbb{V}}_2^2 - \overrightarrow{\mathbb{V}}_1^2}{2} + V(p_2 - p_1) \tag{4.67}$$

Rearranging
$$W = gz_1 + \frac{\overrightarrow{\mathbb{V}}_1^2 - \overrightarrow{\mathbb{V}}_2^2}{2} + V(p_1 - p_2) \tag{4.68}$$

where $\overrightarrow{\mathbb{V}}$ is average velocity and V is volume.

4.4 PERPETUAL MOTION MACHINE OF THE FIRST ORDER (PMM-I)

Perpetual motion machine of the first order (PMM-I) is a hypothetical device based on violation of first law of thermodynamics. Let us think of a system which can create energy as shown in Fig.4.23.

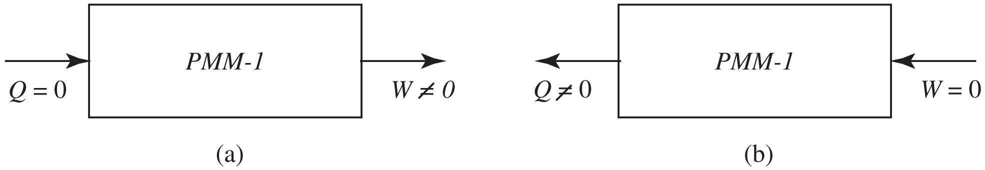


Fig. 4.23 PMM-I based on violation of first law of thermodynamics

Here, a device which is continuously producing work without any other form of energy supplied to it is shown in Fig.4.23(a), which is not feasible. Similarly, a device which is continuously emitting heat without any other form of energy supplied to it is shown in Fig.4.23(b), which is again not feasible. above two imaginary machines are called perpetual motion machines of first kind.

It is the second law which assigns a quality to different forms of energy. Further, it also indicates the direction of any spontaneous process. We will discuss the details of second law of thermodynamics in the next chapter.

4.5 UNSTEADY FLOW SYSTEMS AND THEIR ANALYSIS

In earlier discussions, for a steady flow system, it has been assumed that the properties do not change with time. However, in engineering practice, the unsteady flow processes are as common as the steady flow processes. *The rate of energy and mass transfer into and out of the control volume are not same in the case of unsteady flow process.* Unsteady flow processes are also known as transient flow processes or variable flow processes.

Typical examples of unsteady-flow processes are:

(i) charging of rigid vessels from supply lines,

(ii) discharging a fluid from a pressurised vessel,

(iii) driving a gas turbine with pressurised air stored in a large container,

(iv) inflating tires or balloons and

(v) cooking with an ordinary pressure cooker.

Unlike steady-flow processes, unsteady-flow processes start and end over some finite time period instead of continuing indefinitely. Therefore, in this section, we deal with changes that occur over *some time interval,* t, instead of with the rate of changes (*changes per unit time*). An unsteady-flow system, in some respects, is similar to a closed system, except that the mass within the system boundaries does not remain constant during a process. Another difference between steady- and unsteady-flow systems is that steady-flow systems are fixed in space, size and shape. Unsteady-flow systems, however, need not be (Fig.4.24). They are usually stationary. They are fixed in space but they may involve moving boundaries and thus involve boundary work. In this section, we will explain only the first two example, viz filling up of a

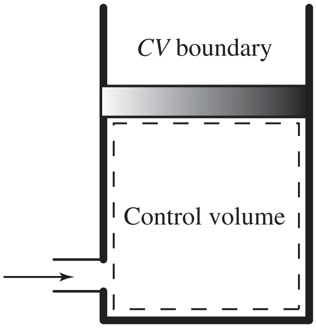


Fig. 4.24 Illustration of an unsteady flow process

rigid vessel or emptying the vessel. As the process proceeds, the properties change continuously in such cases. Such systems cannot be analysed with the steady state assumptions. When an unsteady-flow process is analysed, it is important to keep track of the mass and energy contents of the control volume as well as the energy interactions across the boundary.

Let us take an example of filling up of the bottle.The bottle is filled up gradually, therefore, it is a case of an unsteady system. By conservation of mass, the unsteady process over a period of time, dt, can be expressed as following in generic form:

$$\begin{array}{ccccc}\text{Mass entering} & & \text{Mass leaving} & & \text{Net change in mass}\\ \text{the control volume} & - & \text{the control volume} & = & \text{in control volume}\\ \text{in time } dt & & \text{in time } dt & & \text{in time } dt\end{array}$$

If the mass flow rate at inlet and exit are denoted by m_i and m_e, respectively, then

$$\frac{dm_i}{dt} - \frac{dm_e}{dt} = \frac{dm_{CV}}{dt} \tag{4.69}$$

and also

$$\sum m_i - \sum m_e = \left(m_{\text{final}} - m_{\text{initial}}\right)_{CV} \tag{4.70}$$

By the conservation of energy principle applied on control volume for time dt, energy balance yields,

$$\begin{array}{ccccccc}\text{Net energy} & & \text{Energy} & & \text{Energy} & & \text{Change in}\\ \text{interaction across} & + & \text{entering into} & - & \text{leaving out of} & = & \text{energy}\\ \text{the boundary} & & \text{control volume} & & \text{control volume} & & \text{control volume}\\ \text{in time } dt & & \text{in time } dt & & \text{in time } dt & & \text{in time } dt\end{array}$$

Mathematically, it can be written as:

$$(Q - W) + \sum E_i - \sum E_e = \Delta E_{CV} \tag{4.71}$$

where

$$E_i = \int_0^t m_i\left(h_i + \frac{\vec{\mathbb{V}}_i^2}{2} + gz_i\right)dt \tag{4.72}$$

$$E_e = \int_0^t m_e\left(h_e + \frac{\vec{\mathbb{V}}_e^2}{2} + gz_e\right)dt \tag{4.73}$$

Thus, the above mass balance and energy balance can be used for analysing the unsteady flow systems with suitable assumptions. It may be assumed that:

(i) the control volume state is uniform,

(ii) fluid properties are uniform and

(iii) steady at inlet and exit.

Simplified form of energy balance written above can be written as:

$$Q - W + \sum m_i \left(h_i + \frac{\vec{\mathbb{V}}_i^2}{2} + gz_i \right) - \sum m_e \left(h_e + \frac{\vec{\mathbb{V}}_e^2}{2} + gz_e \right)$$

$$= \left(m_{\text{final}} u_{\text{final}} - m_{\text{initial}} u_{\text{initial}} \right)_{CV} \tag{4.74}$$

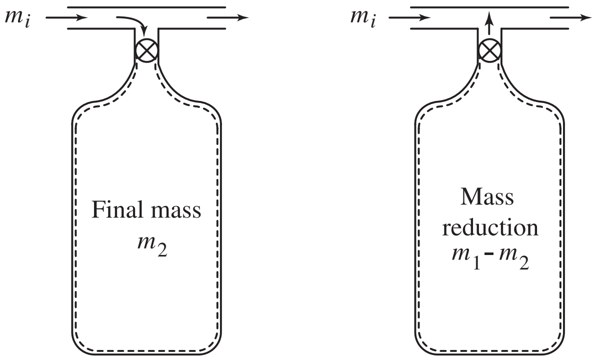
If we assume that kinetic energy and potential energy changes are negligible, then energy balance gets further simplified to,

$$Q - W + \sum m_i h_i - \sum m_e h_e = \left(m_{\text{final}} u_{\text{final}} - m_{\text{initial}} u_{\text{initial}} \right)_{CV} \tag{4.75}$$

Now, let us consider two cases for illustration.

Case 1 : Charging of a rigid vessel from supply line

Let us now use the energy and mass balance to the unsteady flow process of filling up a bottle as shown in Fig.4.25(a). Assume that the rigid vessel which is to be filled is initially empty and connected to a pipeline through a valve. Let us denote initial state of system by subscript 1 and final state by 2. Initially, as bottle is empty, so $m_i = 0$. From mass balance,



(a) Filling of bottle (b) Emptying of bottle

Fig. 4.25 (a) Charging of a rigid vessel from supply line and (b) Discharging a fluid from a pressurised vessel

$$\sum m_e - \sum m_i = (m_e - 0)_{CV} \tag{4.76}$$

As there is no exit from the vessel, so $m_e = 0$. Hence,

$$\sum m_i = m_e \tag{4.77}$$

or
$$m_i = m_2 \tag{4.78}$$

Mass entered into bottle = Final mass inside the bottle

Now, apply the energy balance. Assuming change in kinetic and potential energy to be negligible, treating bottle filling process to be occurring in insulated environment and no work interaction, we can write $Q = 0, W = 0, \Delta KE = 0, \Delta PE = 0$.

Initial internal energy in bottle = 0

Mass leaving = 0

$$0 = -\sum m_i h_i + (m_2 u_2)_{CV} \tag{4.79}$$

or
$$m_i h_i = m_2 u_2 \tag{4.80}$$

Also
$$h_i = u_2 \text{ as } m_i = m_2 \tag{4.81}$$

Enthalpy of fluid entering bottle = Final internal energy of fluid in bottle

If fluid is ideal gas, then
$$c_p T_i = c_V T_2 \tag{4.82}$$

or
$$T_2 = \gamma T_i \tag{4.83}$$

where
$$\frac{c_p}{c_V} = \gamma \tag{4.84}$$

Case 2 : Discharging a fluid from a pressurised vessel

Let us now take a case of emptying a vessel. The arrangement is shown in Fig.4.25(b). Initially, bottle has mass m_1 and finally as a result of emptying, say mass left is m_2 after some time. Applying mass balance, (as mass entering is zero) or

$$0 - \sum m_e = (m_2 - m_1)_{CV} \tag{4.85}$$

or
$$\sum m_e = (m_1 - m_2)_{CV} \tag{4.86}$$

or
$$m_e = (m_1 - m_2)_{CV} \tag{4.87}$$

Total mass leaving the bottle = Reduction of mass in the bottle

Applying the energy balance, with the assumptions, $Q = 0, W = 0, \Delta KE = 0$ and $\Delta PE = 0$

$$-\sum m_e h_e = (m_2 u_2 - m_1 u_1)_{CV} \tag{4.88}$$

$$-m_e h_e = (m_2 u_2 - m_1 u_1)_{CV} \tag{4.89}$$

Substituting of m_e, we get
$$(m_2 - m_1)_{CV} h_e = (m_2 u_2 - m_1 u_1)_{CV} \tag{4.90}$$

In case of complete emptying, $m_2 = 0$ and so $h_e = u_1$.

4.6 LIMITATIONS OF FIRST LAW OF THERMODYNAMICS

From the discussion in the last chapter and this chapter, it must be clear that first law of thermodynamics is based on the principle of energy conservation. No doubt that it is a powerful tool for thermodynamic analysis. The first law is concerned only with amount of energy transfer between the system and surroundings. It also deals with the changes in energy stored in the system. It treats work and heat interactions as equivalent forms of energy in transit. It does not take into account the possibility of a spontaneous process proceeding in a certain direction. Over the period of time, when it was applied to some real systems, it was observed that theoretically, first law stands valid for the processes which are not realisable in practice. It was then thought that there exist certain flaws in first law of thermodynamics and it has certain limitations. Let us illustrate with a typical example.

Consider bicycle wheel and peddle to rotate. Apply the brake suddenly. As a result of braking wheel comes to rest due to its contact with brake shoe. Stopping of wheel is accompanied by heating of brake shoe.

Now, let us examine the situation from first law of thermodynamics point of view. We notice that rotational energy of the wheel has been transformed into heat energy in the shoe causing rise in its temperature. The interesting question now is that if we introduce the same quantity of heat into brake shoe and wish to restore wheel motion will it be possible? We are sure that it is simply impossible. However, according to the first law, conversion of heat to work (rotation of wheel in this case) is possible Therefore, it is obvious that first law of thermodynamics has limitations, which are listed below.

(i) First law of thermodynamics does not differentiate between heat and work. It assures full convertibility of one into other whereas full conversion of work into heat is possible but the vice-versa is not possible.

(ii) First law of thermodynamics does not explain the direction of a process. Theoretically it permits even heat transfer from low temperature body to high temperature body which is not practically feasible. Spontaneity of the process is not taken care of by the first law of thermodynamics.

Worked-Out Examples

4.1 An engine running at 1500 rpm was tested using a water brake. The measured torque is 10^4 Nm. The water consumption of the brake is $0.1 \text{m}^3/\text{s}$. The inlet temperature of water is 20 °C. Find the outlet temperature of the water. Assume that the entire power developed by the engine is ultimately absorbed by the cooling water of the brake.

Solution

$$\text{Engine output power, } bp = \frac{2 \times \pi \times N \times \tau}{60000} = \frac{2 \times \pi \times 1500 \times 10^4}{60000} = 1570.8 \text{ kW}$$

$$\text{Mass flow rate of water, } m = 0.1 \times 1000 = 100 \text{ kg/s}$$

Let t be the outlet temperature of water, then

$$m \times c_p \times \Delta t = Q$$

$$100 \times 4.187 \times (t - 20) = 1570.8$$

$$t - 20 = \frac{1570.8}{100 \times 4.187} = 3.75°\text{C}$$

$$t = 20 + 3.75 = 23.75°\text{C}$$ Ans $\Longleftarrow$

4.2 A blower delivers 1.2 kg/s of air at 20°C with a power consumption of 12 kW. The inlet and outlet velocities of air are 120 m/s and 180 m/s, respectively. Assuming the adiabatic conditions, estimate the exit temperature of air. Take $c_p = 1.005$ kJ/kg K.

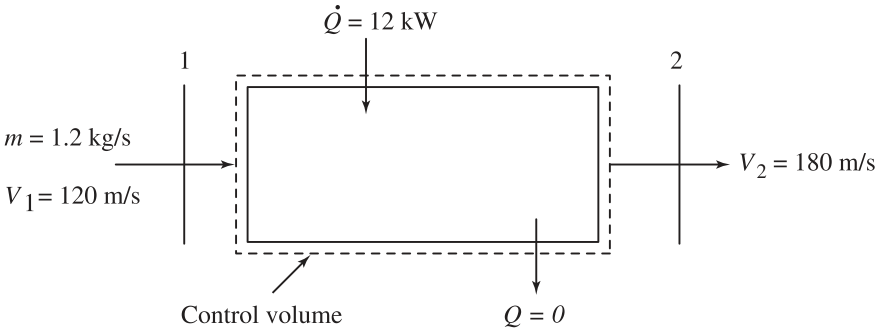


Fig. 4.26

Solution

This a single stream steady flow problem. The steady flow energy equation (SFEE) can be written as:

$$\dot{Q} - \dot{W} = \dot{m}\left[g(z_2 - z_1) + \left(\frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2}\right) + (h_2 - h_1)\right]$$

As we have find the exit temperature, we should rearrange the above equation in terms of change in enthalpy. Accordingly:

$$\dot{m}(h_2 - h_1) = \dot{Q} - \dot{W} - \dot{m}\left[g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2}\right]$$

Here, $\dot{Q} = 0; z_2 = z_1; \dot{W} = -12$ kW; $h = mc_p\Delta t$. Substituting the values we get:

$$1.2 \times (h_2 - h_1) = 0 + 12 - 1.2 \times \left[0 + \frac{180^2 - 120^2}{2} \times 10^{-3}\right]$$

$$h_2 - h_1 = \frac{12}{1.2} - \frac{180^2 - 120^2}{2} \times 10^{-3} = 1$$

$$c_p(t_2 - t_1) = 1$$

$$t_2 = \frac{1}{1.005} + 20 = 20.99°\text{C}$$ Ans $\Longleftarrow$

4.3 The following observations are made at a steam turbine power plant.

		Inlet conditions	*Exit conditions*
Steam pressure	:	1.5 MPa	50 kPa
Enthalpy	:	2800 kJ/kg	2550 kJ/kg
Steam velocity	:	30 m/s	100 m/s
Elevation	:	3 m	0 m
Temperature	:	187°C	

Take the heat loss to the surrounding as 0.3 kJ/s. If the steam flow rate is 0.5 kg/s. Calculate the power output of the turbine in kW.

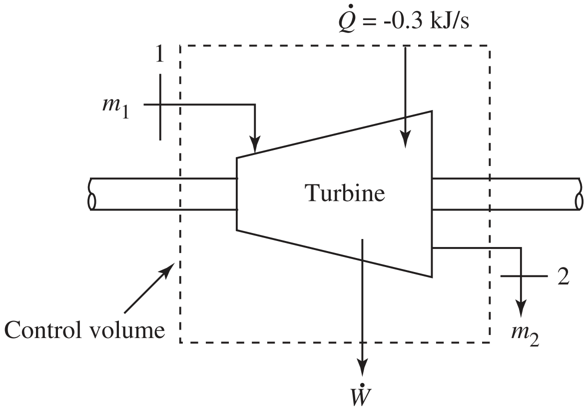


Fig. 4.27

Solution

This is a single stream steady flow problem and applying SFEE:

$$\dot{Q} - \dot{w} = \left[g(z_2 - z_1) + \left(\frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2}\right) + (h_2 - h_1)\right]$$

$$\begin{aligned}
\dot{w} &= \dot{Q} - \left[g(z_2 - z_1) + \left(\frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2}\right) + (h_2 - h_1)\right] \\
&= \frac{0.30}{0.50} - \left[9.81 \times (3 - 0) + \frac{100^2 - 30^2}{2} \times 10^{-3} + (2550 - 2800)\right] \\
&= 216.6 \text{ kJ/kg} \\
\dot{W} &= 216.6 \times 0.5 = 108.3 \text{ kW}
\end{aligned}$$

$\overset{\text{Ans}}{\Longleftarrow}$

4.4 Consider a nozzle which is used to increase the velocity of a steady flowing stream. At the inlet to the nozzle, the enthalpy of fluid is 3000 kJ/kg and the velocity is 50 m/s. At the exit of the nozzle, the enthalpy is 2700 kJ/kg. The nozzle is kept horizontal and is well insulated. Find (i) the velocity at the exit of the nozzle and the mass flow rate and (ii) if the inlet area is 0.12 m^2 and the specific volume of the fluid at the inlet 0.19 m^3/kg,

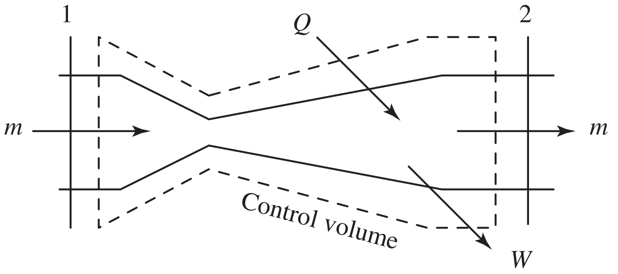


Fig. 4.28

find the exit area of the nozzle; if the specific volume of the fluid at the exit is 0.5 m^3/kg.

Solution

This is a single-stream steady flow system. Applying SFEE, we have,

$$\dot{Q} - \dot{w} = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (h_2 - h_1)$$

Here, $\dot{Q} = 0; \dot{w} = 0; h_1 = 3000$ kJ/kg; $h_2 = 2700$ kJ/kg; $z_1 = z_2$. Substituting the values,

$$0 - 0 = \dot{m}\left[0 + \left(\frac{\mathbb{V}_2^2 - 50^2}{2}\right) + (2700 - 3000)\right]$$

$$\frac{\mathbb{V}_2^2 - 50^2}{2} = 3000 - 2700 = 300 \text{ kJ/kg} = 300 \times 1000 \text{ J/kg}$$

$$\mathbb{V}_2^2 = 2 \times 300 \times 1000 + 50^2 = 600000 + 2500 = 602500$$

$$\mathbb{V}_2 = \sqrt{602500} = 776.2 \text{ m/s}$$

$$m_1 = m_2 = \frac{A\mathbb{V}_1}{V_1} = \frac{0.12 \times 50}{0.19} = 31.6 \text{ kg}$$

$$m_2 = \frac{A_2\mathbb{V}_2}{V_2}$$

$$31.6 = \frac{A_2 \times 776.2}{0.5}$$

$$A = \frac{31.6 \times 0.5}{776.2} = 0.0203 \text{ m}^2$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

4.5 Carbon dioxide passing through a heat exchanger at a rate of 60 kg/hr is to be cooled down from 850 °C to 85°C. Calculate the rate of removal of heat assuming flow to be steady and at constant pressure. Take $c_p = 1.09$ kJ/kg K. State the assumptions made.

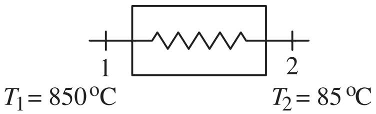


Fig. 4.29

Solution

As nothing is mentioned about the kinetic and potential energy, let us assume that they are negligible. During the steady flow, the work interaction can be considered as zero. Further, it is a constant pressure process.

Applying SFEE,

$$Q - W = \Delta h + \Delta PE + \Delta KE$$

As $W, \Delta PE$ and ΔKE are zero,

$$\begin{aligned} Q &= \Delta h = mc_p(T_2 - T_1) = 60 \times 1.09 \times (850 - 85) \\ &= 50031 \text{ kJ/h} \approx 50 \text{ MJ/h} \end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

4.6 In an auditorium with seating capacity of 1000 persons, the comfort conditions are created by circulating hot water through pipes in winter season. Hot water enters the pipe with an enthalpy of 400 kJ/kg and leaves the pipe with an enthalpy of 200 kJ/kg. The elevation difference between the inlet and exit pipe is 10 m. Exit pipe is at a higher position than the inlet pipe. Assume that the heat requirement per person is 60 kJ/h Estimate the amount of water to be circulated per minute. Neglect changes in velocity in the hot water pipe.

Solution

For comfort condition, the total heat to be supplied in the auditorium:

$$\begin{aligned} Q_{\text{tot}} &= \text{Heat requirement per person} \times \text{Number of persons} \\ &= 60 \times 1000 = 60000 \text{ kJ/h} \end{aligned}$$

The above amount of heat has to come from hot water.

Let us apply SFEE to the hot water flow noting that kinetic energy change is negligible and the work interaction is zero. Then,

$$Q - W = \Delta h + \Delta PE + \Delta KE$$

With above assumptions,

$$Q = \Delta h + \Delta PE = -60000 \text{ kJ}$$

(Negative sign, since heat is to be given out by water.)

$$\begin{aligned} -60000 &= \dot{m}(h_2 - h_1) + \dot{m}(gz_2 - gz_1) = \dot{m}(200 - 400) + \dot{m} \times 9.81 \times 10 \\ &= \dot{m}(-200 + 98.1) = -101.9 \\ \dot{m} &= \frac{-60000}{-101.9} = 588.81 \text{ kJ/h} = 9.81 \text{ kg/min} \end{aligned}$$ $\overset{\text{Ans}}{\Longleftarrow}$

4.7 A cylinder of volume 2.5 m^3 has air at 0.5 MPa and temperature of 350 K. Air from the cylinder is released to atmosphere through a valve on cylinder so as to run a frictionless turbine. Determine the amount of work available from turbine assuming no heat loss and complete kinetic energy being used for running turbine. Take $c_{p,\text{air}} = 1.003$ kJ/kg K, $c_{V,\text{air}} = 0.716$ kJ/kg K, $R_{\text{air}} = 0.287$ kJ/kg K.

Solution

Let initial and final states of air inside cylinder be denoted by m_1, p_1, V_1, T_1 and m_2, p_2, V_2, T_2, respectively. It is a case of emptying a cylinder.

$$\text{Initial mass, } m_1 = \frac{p_1 V_1}{R_{air} T_1} = \frac{0.5 \times 10^3 \times 2.5}{0.287 \times 350} = 12.44 \text{ kg}$$ $\overset{\text{Ans}}{\Longleftarrow}$

Let the expansion through the valve is adiabatic. Then,

$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma - 1}{\gamma}} = 350 \times \left(\frac{1.013 \times 10^5}{0.5 \times 10^6}\right)^{\frac{0.4}{1.4}} = 221.8 \text{ K}$$ $\overset{\text{Ans}}{\Longleftarrow}$

Final mass of air left in the tank,

$$m_2 = \frac{p_2 V_2}{RT_2} = \frac{1.013 \times 10^5 \times 2.5}{0.287 \times 10^3 \times 221.8} = 3.98 \text{ kg}$$ $\overset{\text{Ans}}{\Longleftarrow}$

Writing down the energy equation for unsteady flow,

$$\begin{aligned} (m_1 - m_2)\left(h_2 + \frac{\mathbb{V}_2}{2}\right) &= m_1 u_1 - m_2 u_2 \\ \frac{1}{2}(m_1 - m_2)\mathbb{V}_2 &= m_1 u_1 - m_2 u_2 - (m_1 - m_2) h_2 \\ &= m_1 c_V T_1 - m_2 c_V T_2 - (m_1 - m_2) c_p T_2 \\ &= \big(12.44 \times 0.716 \times 10^3 \times 350\big) - \big(3.98 \times 0.716 \times 10^3 \times 221.8\big) \\ &\quad - \big[(12.24 - 3.98) \times 1.003 \times 10^3 \times 221.8\big] \\ &= 647.84 \times 10^3 \text{ J} \end{aligned}$$ $\overset{\text{Ans}}{\Longleftarrow}$

Kinetic energy available for running the turbine = 647.84×10^3 J.
Amount of work available from the turbine = 647.84 kJ.

4.8 A stone of 25 kg mass and a tank containing 250 kg water comprise of a system. The stone is 25 m above the water level to start with. The stone and water are at the same temperature initially. If the stone falls into water, then determine $\Delta U, \Delta PE, \Delta KE, Q$ and W, when

(i) the stone is about to enter the water,

(ii) the stone has come to rest in the tank, and

(iii) the amount of heat to be transferred to the surroundings so that the stone and water come to their initial temperature.

Solution

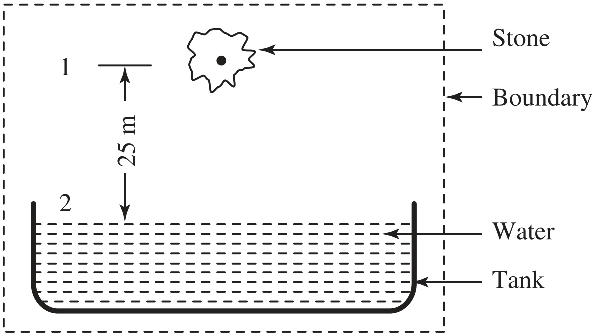


Fig. 4.30

Applying the first law of thermodynamics,

$$Q - W = \Delta KE + \Delta PE + \Delta U \qquad (i)$$

$$= \frac{m\left(c_2^2 - c_1^2\right)}{2} + mg(z_2 - z_1) + (U_2 - U_1)$$

(i) When the stone is about to enter the work, $Q = 0, W = 0$ and $\Delta U = 0$.

$$\Delta PE + \Delta KE = 0$$

$$\Delta KE = -\Delta PE$$

$$\Delta KE = mg(z_2 - z_1) = 25 \times 9.81 \times (0 - 25) = -6131.25 \text{ J}$$

$$\Delta KE = 6131.25 \text{ J} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\Delta PE = -6131.25 \text{ J} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

(ii) When the stone comes to rest in the tank, $Q = 0, W = 0$ and $\Delta KE = 0$.

Substituting values in Eq.(i),

$$\begin{aligned} 0 - 0 &= 0 + \Delta PE - \Delta U \\ \Delta U &= -\Delta PE = -(-6131.25) = 6131.25 \end{aligned} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

ΔU is positive. There will be a slight increase in the temperature of the system.

(iii) When the water and stone come to their initial temperature, $W = 0$ and $\Delta KE = 0$.

$$Q = -\Delta U = -6131.25 \qquad \overset{\text{Ans}}{\Longleftarrow}$$

Negative sign indicates that the heat is lost from the system to the surroundings.

4.9 In an oil cooler, oil flows steadily through a bundle of metal tubes submerged in a steady stream of cooling water under steady flow conditions, the oil enters at 100 °C and leaves at 40 °C. While water enters at 30 °C and leaves at 75 °C. The enthalpy of oil at any temperature is given by $h = 1.7t + 10.5 \times 10^{-4}t^2$ kJ/kg, where t is in °C. Estimate the cooling water flow required for cooling 3 kg/s of oil.

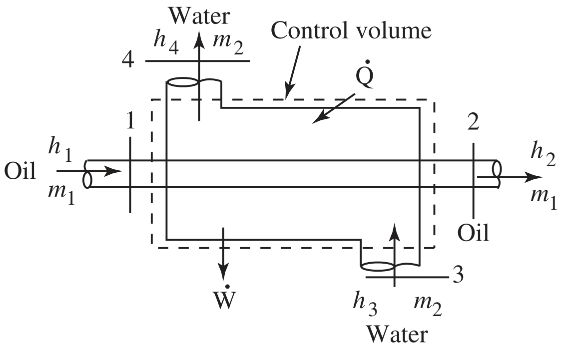


Fig. 4.31

Solution

This is a steady flow system with two steams. Now, referring to Fig.4.31,

$$\begin{aligned} \dot{m}_1 h_1 + \dot{m}_2 h_3 &= \dot{m}_1 h_2 + \dot{m}_2 h_4 \\ \text{or} \qquad \dot{m}_1(h_1 - h_2) &= \dot{m}_2(h_4 - h_3) \end{aligned}$$

where $\dot{m}_1$ represents the oil flow and $\dot{m}_2$ represents water flow.

$$\begin{aligned} h_1 &= 1.7 \times t_1 + 10.5 \times 10^{-4} \times t_1^2 \\ &= 1.7 \times 100 + 10.5 \times 10^{-4} \times 100^2 = 180.5 \\ \text{Similarly,} \qquad h_2 &= 1.7 \times 40 + 10.5 \times 10^{-4} \times 40^2 = 69.7 \end{aligned}$$

$$\left(\overset{\text{Ans}}{\Longleftarrow}\right)$$

$$
\begin{aligned}
\text{Therefore,}\quad 3 \times (180.5 - 69.7) &= m_2 \times c_p \Delta T = m_2 \times 4.187 \times (75 - 40)\\
3 \times 110.8 &= m_2 \times 146.55\\
m_2 &= \frac{3 \times 110.8}{146.55} = 2.27 \text{ kg/s} && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

4.10 A thermoelectric generator consists of a series of semiconductor elements (Fig.4.32), heated on one side and cooled on the other. Electric current flow is produced as a result of energy transfer as heat. In a particular experiment, the current was measured to be 0.6 amp and the electrostatic potential at (a) was 0.9 V above that at (b). Energy transfer as heat to the hot side of the generator was taking place at a rate of 6 watts. Determine the rate of energy transfer as heat from the cold side and the energy conversion efficiency.

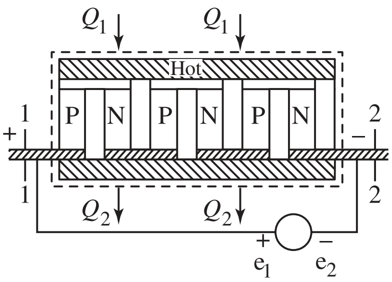


Fig. 4.32

Solution

By energy balance (refer Fig.4.32),

$$
\begin{aligned}
\dot{Q}_1 - \dot{Q}_2 &= \dot{W}\\
\dot{Q}_1 &= 6 \ \text{ watts}\\
\dot{W} &= \Delta V \times I = 0.9 \times 0.6 = 0.54 \text{ W}\\
\dot{Q}_2 &= \dot{Q}_1 - \dot{W} = 6 - 0.54 = 5.46 \text{ W} && \overset{\textbf{Ans}}{\Longleftarrow}\\
\eta &= 1 - \frac{Q_2}{Q_1} = \left(1 - \frac{5.46}{6}\right) \times 100 = 9\% && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

4.11 A turbo compressor delivers 2.5 m^3/s of air at 0.28 MPa, 47 °C which is heated at this pressure to 427 °C and finally expanded in a turbine which delivers 1900 kW. During the expansion, there is a heat transfer of 0.1 MJ/s to the surroundings. Calculate the turbine exhaust temperature if changes in kinetic and potential energy are negligible.

Solution

Considering the turbine as control volume and neglecting changes in KE and PE and referring

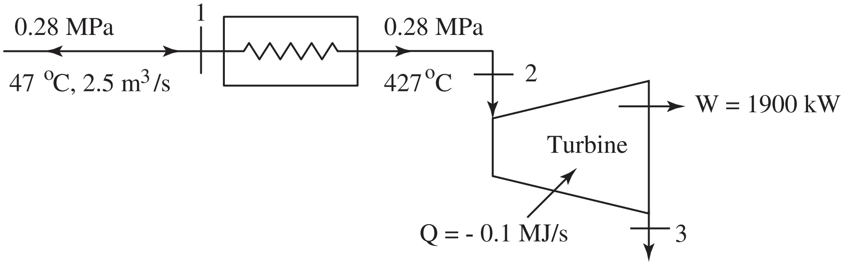


Fig. 4.33

to Fig.4.33,

$$\begin{aligned} mh_2 + Q &= mh_3 + W \\ m(h_2 - h_3) &= W - Q \qquad (i) \end{aligned}$$

$$\dot{m} = \frac{p\dot{V}}{RT} = \frac{0.28 \times 10^3 \times 2.5}{0.287 \times 320} = 7.62 \text{ kg/s}$$

Substituting values in Eq.(i),

$$\begin{aligned} 7.62 \times 1.005 \times (427 - t_3) &= (1900 + 100) \\ 427 - t_3 &= \frac{2000}{7.62 \times 1.005} = 261.16 \\ t_3 &= 427 - 261.16 = 165.83 \text{ °C} \end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

4.12 A reciprocating air compressor takes in 3 m^3/min at 0.12 MPa, 17 °C which it delivers at 1.5 MPa, 112 °C to an aftercooler where the air is cooled at constant pressure to 27 °C. The power absorbed by the compressor is 4 kW. Determine the heat transfer in the compressor and the cooler. State your assumptions.

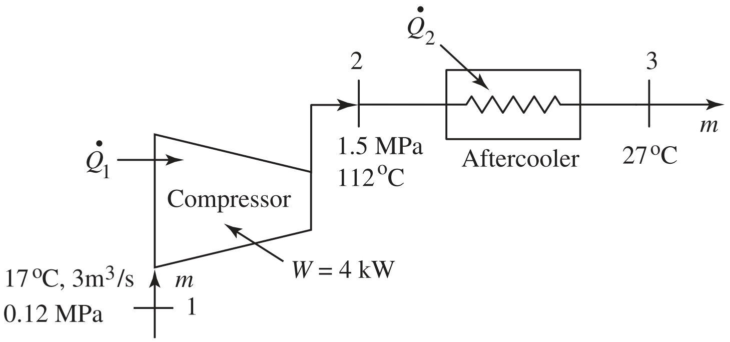


Fig. 4.34

Solution

$$\text{Mass flow rate of air, } \dot{m} = \frac{p\dot{V}}{RT} = \frac{120 \times 3}{0.287 \times 290} = 4.325 \text{ kg/min} = 0.072 \text{ kg/s}$$

$$\dot{m}h_1 + \dot{Q}_1 = \dot{m}h_2 + \dot{W}$$

Considering compressor as control volume,

$$\dot{Q}_1 = mc_p(T_2 - T_1) = 0.072 \times 1.005 \times (112 - 22) - 4 = 2.51 \text{ kJ/s} \quad \overset{\text{Ans}}{\Longleftarrow}$$

Considering after cooler as control volume,

$$\dot{Q}_1 = \dot{m}(h_2 - h_1) + \dot{W}$$

$$\dot{m}h_2 + \dot{Q}_2 = \dot{m}h_3$$

$$\dot{Q}_2 = \dot{m}c_p(T_3 - T_2) = 0.072 \times 1.005 \times (27 - 112) = 6.15 \text{ kJ/s} = 6.15 \text{ kW} \overset{\text{Ans}}{\Longleftarrow}$$

Negative sign indicates the heat rejection. Assumptions: (i) KE and PE are neglected (ii) Air is considered as ideal gas (iii) A constant c_p of 1.005 is taken.

4.13 In a cooling tower, air enters at a height of 1 m above the ground level and leaves at a height of 6 m. The inlet and outlet velocities are 20 m/s and 30 m/s, respectively. Water enters at a height of 8 m and leaves at a height of 0.8 m. The velocity of water at entry and exit are 3 m/s and 1 m/s, respectively. Water temperatures are 77 °C and 47 °C at the entry and exit, respectively. Air temperatures are 30 °C and 70 °C at the entry and exit, respectively. The cooling tower is well insulated and a fan of 2.5 kw drives the air through the cooler. Find the amount of air per second required 1 kg/s of water flow. The values of c_p of air and water are 1.005 and 4.187 kJ/kg K, respectively.

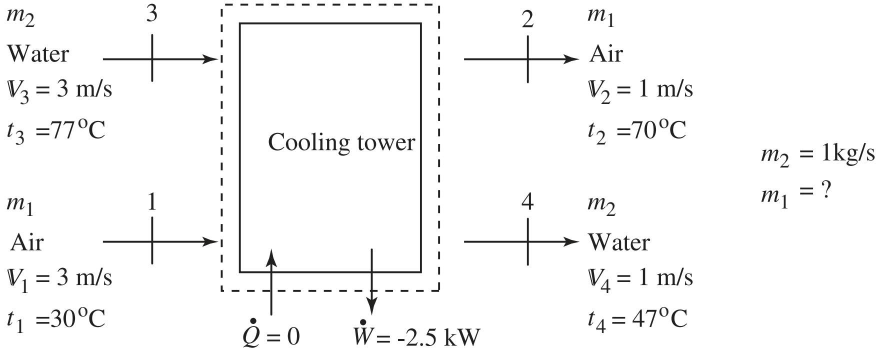


Fig. 4.35

Solution

Writing the SFEE for cooling tower,

$$m_1\left(h_1 + \frac{\mathbb{V}_1^2}{2} + gz_1\right) + m_2\left(h_3 + \frac{\mathbb{V}_3^2}{2} + gz_3\right) + \dot{Q}$$

$$= m_1\left(h_2 + \frac{\mathbb{V}_2^2}{2} + gz_2\right) + m_2\left(h_4 + \frac{\mathbb{V}_4^2}{2} + gz_4\right) + \dot{W}$$

$$m_1\left[(h_1 - h_2) + \frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2} + g(z_1 - z_2)\right] + \dot{Q}$$

$$= m_2\left[(h_4 - h_3) + \frac{\mathbb{V}_4^2 - \mathbb{V}_3^2}{2} + g(z_4 - z_3)\right] + \dot{W}$$

$$\dot{m}_1\left[1.005 \times (30 - 70) + \frac{20^2 - 30^2}{2} \times 10^{-3} + 9.81 \times (1 - 6) \times 10^{-3}\right] + 0$$

$$= 1 \times \left[4.187 \times (47 - 77) + \frac{1^2 - 3^2}{2} \times 10^{-3} + 9.81 \times (0.8 - 8) \times 10^{-3}\right] - 2.5$$

$$-40.50 \times m_1 = -128.2$$

$$m_1 = \frac{-128.2}{-40.50} = 3.16 \text{ kg/s}$$

4.14 Air at 101 kPa, 22 °C is taken into a gas turbine power plant at a velocity of 150 m/s through an opening of 0.15 m^2 cross-sectional area. The air is compressed, heated, expanded through a turbine and exhausted at 0.15 MPa, 150 °C through an opening of 0.10 m^2 cross-sectional area. The power output is 400 kW. Calculate the net amount of heat added to the air in kJ/kg. Assume that air obeys the law $pv = 0.287(t + 273)$, where p is the pressure in kPa, v is the specific volume in m^3/kg and t is the temperature in °C. Take c_p =1.005 kJ/kg K.

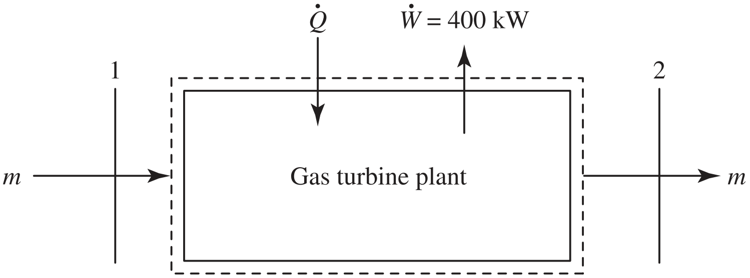


Fig. 4.36

Solution

Applying SFEE,

$$Q - W = \Delta PE + \Delta KE + \Delta h$$

Assuming air behaves like an ideal gas,

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 295}{1.01 \times 100} = 0.838 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{A_1 \mathbb{V}_1}{v_1} = \frac{0.15 \times 150}{0.838} = 26.84 \text{ kg/s}$$

$$v_2 = \frac{0.287 \times (150 + 273)}{150} = 0.809 \text{ m}^3/\text{kg}$$

$$\mathbb{V}_2 = \frac{\dot{m} v_2}{A_2} = \frac{26.84 \times 0.809}{0.1} = 217.14 \text{ m/s}$$

$$\Delta PE = 0$$

$$\Delta h = c_p \times (T_2 - T_1) = 1.005 \times (423 - 295) = 128.64 \text{ kJ}$$

$$\Delta KE = \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} = \frac{217.4^2 - 150^2}{2} \times 10^{-3} = 12.38 \text{ kJ}$$

Substituting the values in SFEE,

$$Q = \frac{400}{26.84} + 12.38 + 128.64 = 155.92 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

4.15 A gas flows steadily through a rotary compressor. The gas enters the compressor at a temperature of 17 °C, a pressure of 100 kPa and an enthalpy of 390 kJ/kg. The gas leaves the compressor at a temperature of 247 °C, a pressure of 0.6 MPa and an enthalpy of 535 kJ/kg. There is no heat transfer to or from the gas as it flows through compressor.

(i) Evaluate the external work done per unit mass of gas assuming the velocities at entry and exit to be negligible.

(ii) Evaluate the external work done per unit mass of gas when the gas velocity at entry is 80 m/s and that at exit is 160 m/s.

Solution

According to SFEE, $Q - W = \Delta PE + \Delta KE + \Delta h$,

(i) As there is no height difference and velocity at entry and exit are negligible, $\Delta PE = 0$ and $\Delta KE = 0$.

$$\Delta h = h_2 - h_1 = 535 - 390 = 145 \text{ kJ/kg}$$

$$Q = 0$$

$$W = -145 \text{ kJ/kg}$$

(ii)
$$\Delta KE = \frac{1}{2}\left(\mathbb{V}_2^2 - \mathbb{V}_1^2\right) \times 10^{-3} = \frac{1}{2}\left(160^2 - 80^2\right) \times 10^{-3} = 9.6$$

Now, $Q - W = \Delta KE$

$$W = Q - \Delta KE = -145 - 9.6 = 154.6 \text{ kJ/kg} \quad \overset{\text{Ans}}{\Longleftarrow}$$

4.16 The steam supply to an engine comprises of two streams which mix before entering the engine. One stream is supplied at the rate of 0.01 kg/s with an enthalpy of 2950 kJ/kg and a velocity of 20 m/s. The other stream is supplied at the rate of 0.1 kg/s with an enthalpy of 2600 kJ/kg and a velocity of 120 m/s. At the exit from the engine, the fluid leaves as two streams, one of water at the rate of 0.001 kg/s with an enthalpy of 410 kJ/kg and the other of steam; the fluid velocities at the exit are negligible. The engine develops a shaft power of 25 kW. The heat transfer is negligible. Evaluate the enthalpy of the second exit stream.

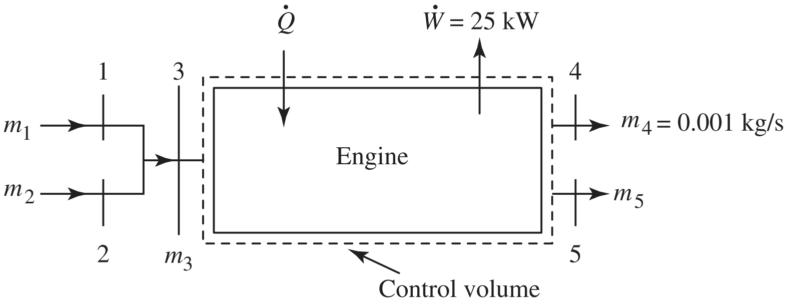


Fig. 4.37

Solution

By enthalpy balance,

$$m_1\left(h_1 + \frac{\mathbb{V}_1^2}{2}\right) + m_2\left(h_2 + \frac{\mathbb{V}_2^2}{2}\right) = m_3 h_3$$

$$0.01 \times \left(2950 + \frac{20^2}{2} \times 10^{-3}\right) + 0.1 \times \left(2600 + \frac{120^2}{2} \times 10^{-3}\right) = 0.11 \times h_3$$

$$29.502 + 260.72 = 0.11 \times h_3$$

$$h_3 = \frac{29.502 + 260.72}{0.11} = 2638.38 \text{ kJ}$$

$$m_4 + m_5 = 0.11 \text{ kg}$$

$$m_5 = 0.11 - 0.001 = 0.109$$

By energy balance,

$$m_3 h_3 + \dot{Q} = m_4 h_4 + m_5 h_5 + \dot{W}$$

$$0.11 \times 2638.38 + 0 = 0.001 \times 410 + 0.109 \times h_5 + 25$$

$$0.109 \times h_5 = 0.11 \times 2638.38 - 0.001 \times 410 - 25 = 264.81$$

$$h_5 = \frac{264.81}{0.109} = 2429.45 \text{ kJ/kg} \quad \overset{\text{Ans}}{\Longleftarrow}$$

4.17 The stream of air and gasoline vapour, in the ratio of 15:1 by mass, enters a gasoline engine at a temperature of 27 °C and leaves as combustion products at a temperature of 660 °C. The engine has a specific fuel consumption of 0.3 kg/kWh. The net heat transfer rate from the fuel-air stream to the jacket cooling water and to the surroundings is 35 kW. The shaft power delivered by the engine is 25 kW. Compute the increase in the specific enthalpy of the fuel-air stream, assuming the changes in kinetic energy and in elevation to be negligible.

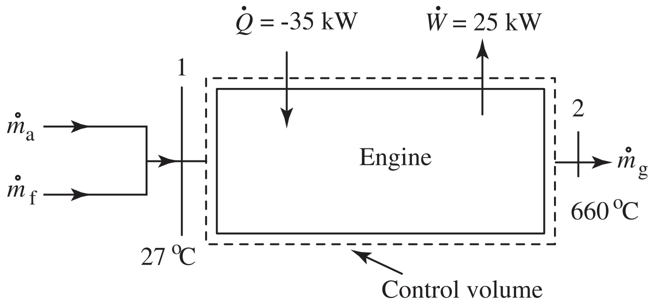


Fig. 4.38

Solution

$$\text{Specific fuel consumption, } sfc = \frac{\text{Fuel consumption}(\dot{m}_f)}{\text{Power output}(P)}$$

$$\dot{m}_f = sfc \times P = 0.3 \times 25 = 7.5 \text{ kg/h}$$

$$\dot{m}_a = 15 \times 7.5 = 112.5 \text{ kg/h}$$

$$\dot{m}_{mix} = \dot{m}_f + \dot{m}_a = 7.5 + 112.5 = 120 \text{ kg/h}$$

According to steady flow energy equation,

$$Q - W = \Delta PE + \Delta KE + \Delta h$$

ΔPE and ΔKE can be neglected.

$$\Delta h = Q - W$$

$$\dot{m}_{mix}(h_2 - h_1) = Q - W$$

$$h_2 - h_1 = \frac{Q - W}{\dot{m}_{mix}} = \frac{-35 - 25}{(120/3600)} = -2100 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Negative sign indicates energy given out by the system.

4.18 An air turbine forms part of an aircraft refrigerating plant. Air at a pressure of 300 kPa and a temperature of 57 °C flows steadily into the turbine with a velocity of 45 m/s. The air leaves the turbine at a pressure of 120 kPa, a temperature of 2 °C and a velocity of 150 m/s. The shaft work delivered by the turbine is 55 kJ/kg of air. Neglecting changes

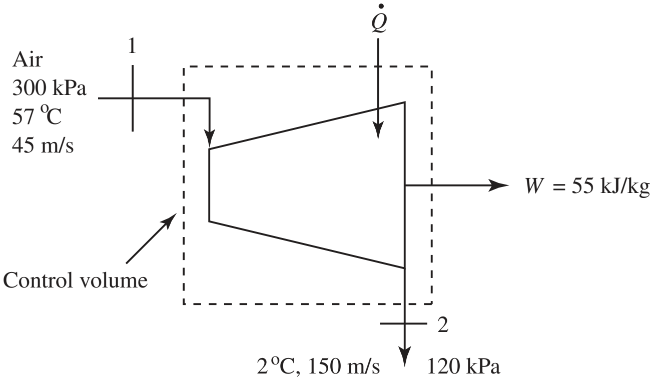


Fig. 4.39

in elevation, determine the magnitude and sign of the heat transfer per unit mass of air flowing. For air, take $c_p = 1.005$ kJ/kg K and the enthalpy $h = c_pt$.

Solution

According to SFEE,

$$Q - W = \Delta PE + \Delta KE + \Delta h$$

$\Delta PE = 0$, as there is no differential height between inlet and exit.

$$\Delta KE = \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} \times 10^{-3} = \frac{150^2 - 45^2}{2000} \times 10^{-3} = 10.24 \text{ kJ/kg}$$

$$\Delta h = c_p(t_2 - t_1) = 1.005 \times (2 - 57) = -55.275$$

$$W = 55 \text{ kJ/kg}$$

Substituting the values,

$$Q = 0 + 10.24 - 55.275 + 55 = 9.965 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

4.19 In a turbomachine, handling an incompressible fluid with a density of 1000 kg/m^3, the conditions of the fluid at the rotor entry and exit are as given below:

Variables	Inlet	Exit
Pressure	1.15 MPa	0.05 MPa
Velocity	30 m/s	15.0 m/s
Height above datum	10 m	2 m

If the volume flow rate of the fluid is 50 m^3/s, estimate the net energy transfer from the fluid as work.

Solution

Applying SFEE for the turbo machine,

$$Q - W = \Delta PE + \Delta KE + \Delta h = \Delta PE + \Delta KE + \Delta(u + pv)$$

Assume no temperature change between inlet and exit $Q = 0$, since $v_1 = v_2$. Since the fluid is water which is incompressible.

$$\Delta PE = g(z_2 - z_1) = 9.81 \times (2 - 10) \times 10^{-3} = -0.078 \text{ kJ/kg}$$

$$\Delta KE = \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} \times 10^{-3} = \frac{15^2 - 30^2}{2} \times 10^{-3} = -0.337 \text{ kJ/kg}$$

$$\Delta u = 0$$

$$\Delta pv = v(p_2 - p_1) = \frac{50 - 1150}{1000} = -1.1 \text{ kJ}$$

$$Q - W = -0.078 - 0.337 + (-1.1) = -1.515$$

$$\text{As } Q = 0; \quad W = 1.515 \text{ kJ/kg}$$ **Ans** ⟸

4.20 A room for four persons has two fans, each consuming 0.14 kW power and three 100 W lamps. Ventilation air at the rate of 80 kg/h enters with an enthalpy of 85 kJ/kg and leaves with an enthalpy of 60 kJ/kg. If each person puts out heat at the rate of 630 kJ/h, determine the rate at which heat is to be removed by a room cooler, so that a steady state is maintained in the room.

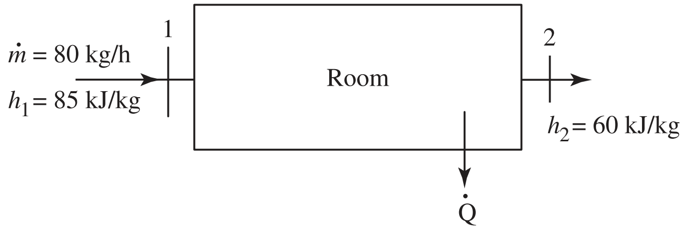


Fig. 4.40

Solution

Total heat load of the room:

$$\text{Fan power} = 2 \times 0.14 = 0.28 \text{ kW}$$

$$\text{Light power} = \frac{3 \times 100}{1000} = 0.30 \text{ kW}$$

$$\text{Heat emanated by persons} = 4 \times \frac{630}{3600} = 0.7 \text{ kW}$$

$$\text{Load from ventilation air} = \frac{80 \times (85 - 60)}{3600} = 0.556 \text{ kW}$$

$$\text{Rate of heat removed from cooler} = 0.28 + 0.30 + 0.7 + 0.556 = 1.836 \text{ kW}$$ **Ans** ⟸

4.21 Air flows steadily at the rate of 0.4 kg/s through an air compressor, entering at 6 m/s with a pressure of 1 bar and a specific volume of 0.87 m^3/kg and leaving at 5 m/s with a pressure of 7 bar and a specific volume of 0.16 m^3/kg. The internal energy of the air leaving is 90 kJ/kg greater than that of the air entering. Cooling water in a jacket

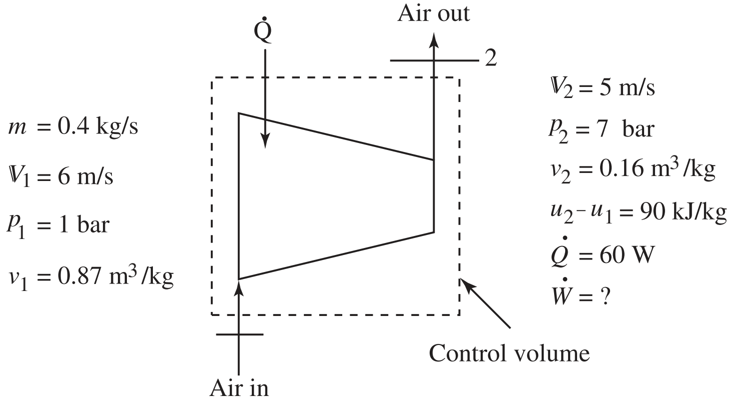


Fig. 4.41

surrounding the cylinder absorbs heat from the air at the rate of 60 W. Calculate the power required to drive the compressor and the inlet and outlet cross-sectional areas.

Solution

Applying SFEE for the compressor taken as control volume CV,

$$Q - W = \Delta PE + \Delta KE + \Delta h = \Delta PE + \Delta KE + \Delta(u + pv)$$

As there is no height difference between inlet and exit, $\Delta PE = 0$.

$$\Delta KE = \frac{1}{2}\left(\mathbb{V}_2^2 - \mathbb{V}_1^2\right) = \frac{1}{2} \times \left(5^2 - 6^2\right) \times 10^{-3} = -5.5 \times 10^{-3}$$

$$\Delta u = u_2 - u_1 = 90 \text{ kJ/kg}$$

$$\Delta(pv) = p_2 v_2 - p_1 v_1 = 7 \times 100 \times 0.16 - 1 \times 100 \times 0.87 = 25 \text{ kJ/kg}$$

$$Q = \frac{60}{0.4} \times 10^{-3} = 0.15 \text{ kJ/kg}$$

Substituting the values in SFEE,

$$0.15 - W = 0 - 5.5 \times 10^{-3} + 90 + 25$$

$$W = 5.5 \times 10^{-3} - 90 - 25 + 0.15 = -114.85 \text{ kJ/kg}$$

$$\dot{W} = \text{Power required to drive the compressor} = 114.84 \times 0.4 = 45.94 \text{ kW}$$

$$\dot{m} = \frac{A_1 \mathbb{V}_1}{v_1} = \frac{A_2 \mathbb{V}_2}{v_2} = 0.4$$

$$A_1 = \frac{0.4 \times 0.87}{6} = 0.058 \text{ m}^2 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$A_2 = \frac{0.4 \times 0.16}{5} = 0.013 \text{ m}^2 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

4.22 The internal energy of air is given, at ordinary temperatures, by $u = u_0 + 0.718t$, where u is in kJ/kg, u_0 is any arbitrary value of u at 0 °C, kJ/kg and t is temperature in °C. Also for air, $pv = 0.287(t + 273)$ where p is in kPa and v is in m^3/kg.

(i) An evacuated bottle is fitted with a valve through which air from the atmosphere, at 760 mm Hg and 27 °C, is allowed to flow slowly to fill the bottle. If no heat is transferred to or from the air in the bottle, what will its temperature be when the pressure in the bottle reaches 760 mm Hg?

(ii) If the bottle initially contains 0.05 m^3 of air at 400 mm Hg and 27 °C, what will the temperature be when the pressure in the bottle reaches 760 mm Hg?

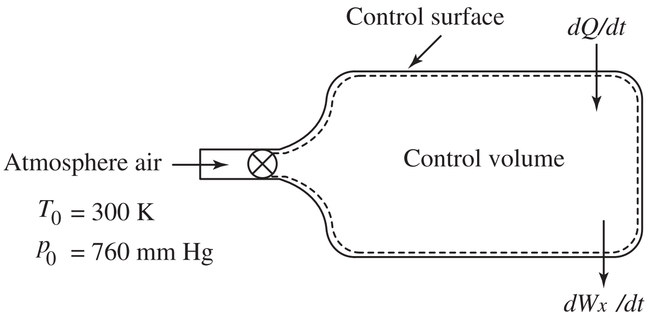


Fig. 4.42

Solution

Now, writing the energy equation for the CV shown in Fig.4.42,

$$\dot{E} - \dot{W} = \Delta PE + \Delta KE + \Delta h$$

(i) Here, $\dot{Q} = 0; m_2 = 0; v_1 = v_2 = 0; z_1 = z_2 = 0; \dot{W} = 0; m_1 = m.$

$$\Delta E = mh_1 \text{ or } (m_f u_f - m_i u_i) = (m_f - m_i)h_1 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

where m_f is final mass of air in the bottle and m_i is initial mass of air in the bottle. Since $m_i = 0; m_f u_f = m_f h_1$.

$$U_f = h_1 = \text{Enthalpy of atmospheric air}$$

$$c_V T = c_p T_0$$

$$T = \gamma T_0 = 1.4 \times 300 = 420 \text{ K}$$

$$\text{Final temperature of air} = 420 - 273 = 147 \text{ K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$(\overset{\textbf{Ans}}{\Longleftarrow})$$

Now
$$u = u_0 + 0.718t$$

$$c_V = \left(\frac{\partial u}{\partial T}\right) = 0.718$$

$$h = u + pv = (u_0 + 0.718t) + (0.287t + 0.287 \times 273)$$

$$c_p = \left(\frac{\partial h}{\partial T}\right) = 0.287 + 0.718 = 1.005$$

$$\gamma = \left(\frac{c_p}{c_V}\right) = \frac{1.005}{0.718} = 1.4$$

$$R = c_p - c_V = 1.005 - 0.718 = 0.287 \text{ kJ/kg}$$

$$p_1 V_1 = m_1 R T_1$$

$$\text{Initial mass of air, } m_1 = m_i = \frac{400}{760} \times \frac{101.325 \times 0.05}{0.287 \times 300} = 0.031 \text{ kg}$$

$$p_2 V_2 = m R T_2$$

$$m_2 = m_f = 101.325 \times 0.05 \times \frac{1}{0.287 \times T_2} = \frac{17.65}{T_2}$$

On substitution,

$$\frac{17.65}{T_2} c_V T_2 - 0.031 c_V \times 300 = \left(\frac{17.65}{T_2} - 0.031\right) \times \frac{c_p}{c_V} \times 300$$

Dividing both sides by c_V,

$$17.65 - 9.3 = \frac{7413}{T_2} - 13.02$$

$$\frac{7413}{T_2} = 21.37$$

$$T_2 = \frac{7413}{21.37} = 346.9 \text{ K}$$

Ans ⟸

4.23 A pressure cylinder of volume V contains air at pressure p_0 and temperature t_0. It is to be filled from a compressed air line maintained at constant pressure p_1 and temperature T_1. Show that the temperature of the air in the cylinder after it has been charged to the pressure of the line is given by

$$T = \frac{\gamma \times T_1}{1 + \frac{p_0}{p_1}\left(\gamma \times \frac{T_1}{T_0} - 1\right)}$$

If p_0=400 mm hg, $p_1 = 760$ m Hg, $T_0 = T_1 = 300$ K and $\gamma = 1.4$, what is the value of T?

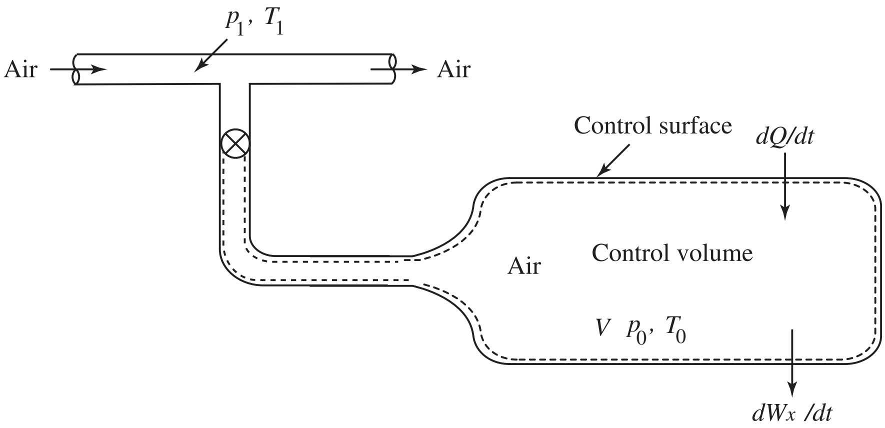


Fig. 4.43

Solution

The energy equation for the control volume is given by:

$$\dot{E}_v = \dot{Q} - \dot{W} + \Delta PE + \Delta KE + \Delta h$$

Here, $\dot{Q} = 0; \mathbb{V}_1 = \mathbb{V}_2 = 0; z_1 = z_2 = 0, m_2 = 0$ and $\dot{W} = 0$.

$$\begin{aligned} \dot{E}_v &= \dot{m}_1 h_1 = h_1 \times \frac{dm}{dt} \\ \Delta E_v &= h_1(m_f - m_i) \\ m_f u_f - m_i u_i &= h_1(m_f - m_i) \end{aligned}$$

where i represents initial state and f represents final state. Assume that air behaves as the ideal gas and writing $u = c_V T$ and $h = c_p T$,

$$\frac{p_1 V_1}{RT} \times c_V T - \frac{p_0 V}{RT_0} \times c_V T_0 = c_p T_1 \left[\frac{p_1 V_1}{RT} - \frac{p_0 V}{RT_0} \right]$$

On simplification,

$$p_1 - p_0 = \gamma T_1 \left(\frac{p_1}{T} - \frac{p_0}{T_0} \right)$$

$$\begin{aligned} \frac{p_1}{T} &= \frac{p_1 - p_0}{\gamma T_1} + \frac{p_0}{T_0} = \frac{p_1 T_0 - p_0 T_0 + \gamma T_1 p_0}{\gamma T_0 T_1} \\ T = \frac{\gamma T_0 T_1 p_1}{p_1 T_0 - p_0 + \gamma T_1 p_0} &= \frac{\gamma T_1}{1 - \dfrac{p_0}{p_1} + \dfrac{\gamma T_1 p_0}{T_0 p_1}} = \frac{\gamma T_1}{1 + \dfrac{p_0}{p_1}\left(\gamma \dfrac{T_1}{T_0} - 1 \right)} \quad \text{(Proved)} \end{aligned}$$

Substituting the given values in the above equation,

$$T \quad = \quad \frac{1.4 \times 300}{1 + \dfrac{400}{760} \times (1.4 - 1)} = 346.95 \text{ K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

4.24 A small reciprocating vacuum pump having the rate of volume displacement, V_d is used to evacuate a large vessel of volume, V. The air in the vessel is maintained at a constant temperature, T, by energy transfer as heat. If the initial and final pressures are p_1 and p_2, respectively, find the time taken for the pressure drop and the necessary energy transfer as heat during evacuation. Assume that for air, $pV = mRT$, where m is the mass and R is a constant and u is a function of T only. If $p_1 = 760$ mm of Hg and $p_2 = 400$ mm of Hg, $v = 0.03$ m^3/s and $v_d = 0.001$ m^3/s, find the required time for the pressure to drop from p_1 to p_2 and the associated heat transfer.

Solution

Volume of air removed from the vessel in time $dt = V_d dt$ where V_d is rate of volume displacement.

Mass air removed in time dt,
$$dm = \frac{p(V_d dt)}{RT} \qquad (i)$$

As air removed from the vessel, its pressure decreases. Therefore,

$$dm = \frac{V dp}{RT} \qquad (ii)$$

Equating Eqs (i) and (ii),
$$-\frac{pV_d dt}{RT} = \frac{V d_p}{RT}$$

$$\int_0^t dt = -\int_{p_1}^{p_2} \frac{V}{V_d} \times \frac{dp}{p}$$

Time taken for the pressure to drop, since V and V_d are constant.

$$t = \frac{V}{V_d} \times \ln\left(\frac{p_1}{p_2}\right)$$

Now, $\Delta Q = dU + \Delta W$. Since temperature is kept constant, $dU = 0$.

$$\Delta Q = \Delta W = V dp$$

or
$$Q = (p_1 - p_2)V \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Time taken,
$$t = \frac{V}{V_d} \ln\left(\frac{p_1}{p_2}\right) = \frac{0.03}{0.001} \ln\left(\frac{760}{400}\right) = 19.26 \text{ s}$$

Associated heat transfer,
$$Q = (p_1 - p_2)V$$

$$= \frac{760 - 400}{760} \times 101.325 \times 0.03 = 1.44 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

4.25 A rigid tank of volume 0.5 m^3 is initially evacuated. A tiny hole develops in the wall and air from the surroundings at 1 bar and 20 °C leaks in. Eventually, the pressure in tank reaches 1 bar the process occurs slowly enough that heat transfer between the tank and the surroundings keeps the temperature of the air inside the tank constant at 20 °C. Determine the amount of heat transfer.

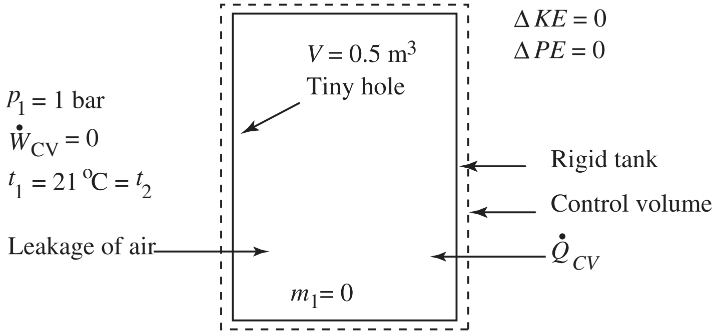


Fig. 4.44

Solution

As per energy equation, $\frac{dU_{CV}}{dt} = \dot{Q}_{CV} - \dot{m}h.$

In certain time,

$$\begin{aligned}
\Delta U_{CV} &= Q_{CV} + h\Delta m = m_2u_2 - m_1u_1 \\
Q_{CV} + h_1(m_2 - m_1) &= m_2u_2 - m_1u_1 \\
h_1 &= u_1 + pv_1 = u_1 + RT_1 \text{ and } m_1 = 0
\end{aligned}$$

$$\begin{aligned}
Q_{CV} &= m_2[u_2 - u_1 - RT_1] = -m_2RT_1 = -p_2V \\
&= -10^5 \times 0.5 \times 10^{-3} = -50 \text{ kJ} \qquad \xLeftarrow{\text{Ans}}
\end{aligned}$$

4.26 An insulated piston-cylinder assembly has an initial volume of V_1 and contains air at p_1 and T_1. Air is supplied to the cylinder from a pipeline, maintained at constant p_p and T_p, through a valve fitted into the cylinder. The piston is restrained in such a manner that the pressure of the air in the cylinder remains constant at p_1 during the process of filling. The filling process is terminated when the final volume V_2 is twice the initial volume V_1. Show that the final temperature of the air T_2 in the cylinder is given by

$$T_2 = \frac{2}{(1/T_1) - (1/T_p)}$$

If p_1 =400 kPa, $T_1 = 200$ °C, $T_p = 150$ °C and $V_1 = 1.2$ m^3, estimate the mass of air that has entered the cylinder through the valve. Assume that air behaves as an ideal gas and use $pV = mRT, u = c_VT$ and $h = c_pT$, where $c_V = 0.718$ kJ/kg K and $R = 0.287$ kJ/kg K.

Solution

Amount of air that has entered the cylinder $= m_2 - m_1$

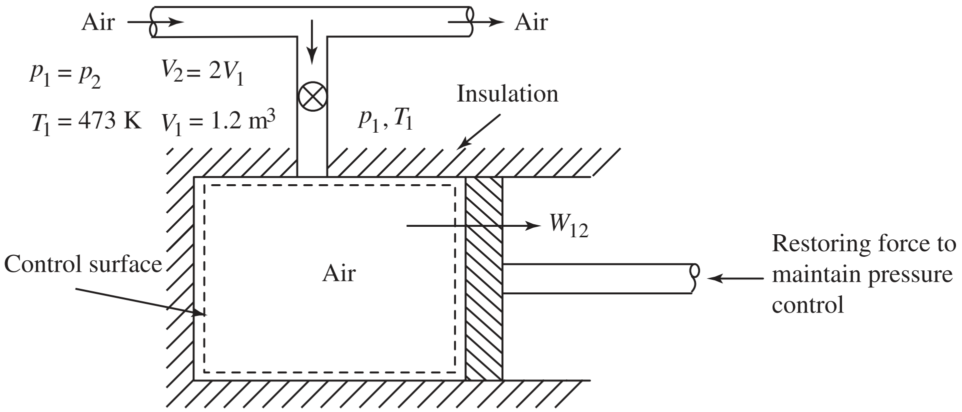


Fig. 4.45

Energy equation can be written as:,

$$h_p(m_2 - m_1) = m_2u_2 - m_1u_1 + W_{1-2}$$

Using $u = c_VT; h = c_pT$ and $m = \dfrac{pV}{RT}$ and since $W = p_1(V_2 - V_1)$,

$$c_pT_p\left(\frac{p_2V_2}{RT_2} - \frac{p_1V_1}{RT_1}\right) = \frac{p_2V_2}{RT_2}c_VT_2 - \frac{p_1V_1}{RT_1}c_VT_1 + p_1(V_2 - V_1)$$

Since $p_1 = p_2 = p$,

$$V_2 - V_1 - \frac{c_pT_p}{R}\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) + \frac{c_V}{R}(V_2 - V_1) = 0$$

$$(V_2 - V_1)\left(1 + \frac{c_V}{R}\right) - \frac{c_pT_p}{R}\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) = 0$$

$$(V_2 - V_1)\frac{c_p}{R} - \frac{c_pT_p}{R}\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) = 0$$

Since $\dfrac{c_p}{R}$ cannot be zero,

$$(V_2 - V_1) - T_p\left(\frac{V_2}{T_2} - \frac{V_1}{T_1}\right) = 0$$

$$\therefore T_p\frac{V_2}{T_2} = V_2 - V_1 + T_p\frac{V_1}{T_1}$$

$$T_2 = \frac{T_pV_2}{V_2 - V_1 + \dfrac{T_pV_1}{T_1}} = \frac{V_2}{\dfrac{V_1}{T_1} + \dfrac{V_2 - V_1}{T_p}}$$

Since $V_2 = 2V_1$,

$$T_2 = \frac{2V_1}{\frac{V_1}{T_1}+\frac{V_1}{T_p}} = \frac{2}{\frac{1}{T_1}+\frac{1}{T_p}} \quad \text{(Proved)} \qquad \xLeftarrow{\text{Ans}}$$

Given: $T_1 = 473$ K and $T_p = 150 + 273 = 423$ K

$$\therefore T_2 = \frac{2}{\frac{1}{473}+\frac{1}{423}} = 446.6 \text{ K}$$

The mass of air that has entered,

$$m_p = m_2 - m_1 = \frac{p_2V_2}{RT_2} - \frac{pV_1}{RT_1} = \frac{p}{R}\left(\frac{2V_1}{T_2} - \frac{V_1}{T_1}\right)$$

$$m_p = \frac{pV_1}{R}\left(\frac{2}{T_2} - \frac{1}{T_1}\right) = \frac{400 \times 1.2}{0.287} \times \left(\frac{2}{446.6} - \frac{1}{473}\right)$$

$$= 3.95 \text{ kg} \qquad \xLeftarrow{\text{Ans}}$$

Review Questions

4.1 Explain the difference between closed system and open system.

4.2 Why the closed system is analysed using control mass principle and the open system is analysed by control volume principle?

4.3 What are the two possibilities of analysing an open system using first law of thermodynamics?

4.4 Explain the principle of conservation of mass.

4.5 With a suitable figure explain the principle of conservation of energy for an open system.

4.6 What do you understand by flow work?

4.7 Derive an expression for conservation of energy for a steady flow process.

4.8 By applying SFEE, show that $W = Q + (h_2 - h_1)$ *for a reciprocating compressor, what will be the equation for a rotary compressor?*

4.9 For a boiler show that SFEE is given by $Q = h_2 - h_1$.

4.10 Derive using appropriate figure the expression for W *for a centrifugal pump applying SFEE.*

4.11 Apply SFEE for a compression chamber and analyse the system.

4.12 What is a condenser? Applying SFEE show that $Q = \Delta h$.

4.13 Show that for a diffuser the change in kinetic energy manifests itself in change in enthalpy.

4.14 With a suitable sketch explain an evaporator. Derive the expression for Q.

4.15 What is the purpose of a heat exchanger and where it is used? Explain a simple heat exchanger.

4.16 What is the purpose of a mixing chamber? With appropriate notation, show that $\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$.

4.17 Explain with a sketch the steam de-superheater.

4.18 What is the purpose of throttling valve? Explain.

4.19 Derive the SFEE for a gas turbine.

4.20 With a sketch explain a water turbine. Apply SFEE and analyse the system.

4.21 What is meant by PMM1? Explain.

4.22 What is the difference between study flow system and unsteady flow system? Mention some typical examples of unsteady flow system?

4.23 Explain SFEE applications in charging of a right vessel from a supply line.

4.24 Explain SFEE in the case of discharging of a fluid from a pressurised vessel.

4.25 What are the limitation of the first law of thermodynamics?

Exercise

4.1 A blower handles 1 kg/s of air at 27 °C and consumes a power of 18 kW. The inlet and outlet velocities of air are 100 m/s and 150 m/s, respectively. Find the exit air temperature, assuming adiabatic conditions. Take c_p, of air is 1.005 kJ/kg K. *Ans:* 40.2 °C

4.2 A turbine operates under steady flow conditions, receiving steam at the following condition pressure 1.5 MPa, temperature 200 °C, enthalpy 3000 kJ/kg, velocity 33.3 m/s and elevation 3 m. The steam leaves the turbine at the following state: Pressure 25 kPa, enthalpy 2500 kJ/kg, velocity 100 m/s and elevation 0 m. Heat is lost to the surroundings at the rate of 0.3 kJ/s. If the rate of steam flow through the turbine is 0.6 kg/s, what is the power output of the turbine in kW. *Ans:* 297.4 kW

4.3 The internal energy of a certain substance is given by $u = 3.6pV + 100$, where u is kJ/kg, p is in kPa and v is in m^3/kg. A system consisting of 3 kg of this substance expands from an initial pressure of 500 kPa and a volume of 0.3 m^3 to a final pressure 100 kPa in a process in which pressure and volume are related by $pV^{1.28}$ = constant.

(i) If the expansion is quasi-static, find Q, W and ΔU for the process.

(ii) If the same system expands according to the same pressure-volume relationship as well as the same initial state to the same final state as in part (i) but the heat transfer in this case is 30 kJ. Find the work transfer for this process.

(iii) Explain the difference in work transfer between the above two cases.

Ans: (i) 8.13 J; 311.7 kJ; −303.57 kJ (ii) −281.7 kJ
(iii) The work transfer in (ii) is not equal to $\int pdV$ since the process is not quasi-static.

4.4 In an oil cooler, oil flows steadily through a bundle of metal tubes submerged in a steady stream of cooling water. Under steady flow conditions, the oil enters at 90 °C and leaves at 30 °C, while the water enters at 27 °C and leaves at 72 °C.The enthalpy of oil at t °C is given by $h = 1.68t + 10.5 \times 10^{-4}t^2$ kJ/kg. What is the cooling water flow required for cooling 2.75 kg/s of oil? *Ans:* 1.581 kg/s

4.5 An air compressor requires shaft work of 200 kJ/kg and the compression of air increases the enthalpy of air by 100 kJ/kg of air. Cooling water required for cooling the compressor picks up heat of 100 kJ/kg of air. Is there any heat transfer from compressor to atmosphere? If so how much?

Ans: Heat transferred to atmosphere is zero.
There is no heat transfer from the compressor to atmosphere.

4.6 A mass of 2 kg is contained in a closed system. During the process, the gas undergoes there is a heat transfer of 25 kJ from the system to surrounding. The amount of work done on the system is 100 kJ. There is decrease of 15 kJ of specific internal energy and the elevation of the system increases by 1 km. Assume $g = 9.6$. Determine the KE of the system. *Ans:* −19.095 kJ

4.7 A turbine operates under steady flow conditions, receiving steam at the following condition pressure 1.5 MPa, temperature 200 °C, enthalpy 3000 kJ/kg, velocity 33.3 m/s and elevation 3 m. The steam leaves the turbine at the following state: Pressure 25 kPa, enthalpy 2500 kJ/kg, velocity 100 m/s and elevation 0 m. Heat is lost to the surroundings at the rate of 0.3 kJ/s. If the rate of steam flow through the turbine is 0.6 kg/s, what is the power output of the turbine in KW. *Ans:* 297.64 kW

4.8 In a steady flow process, 120 kJ/kg of work is done by an working fluid. The pressure, specific volume and velocity of the fluid at inlet are 5 bar, 15 m/s and 0.42 m^3/kg. The inlet of the pipe is 30 m above the ground and the exhaust pipe is at ground level. The pressure, specific volume and velocity at the exit are 1 bar, 0.63 m^3/s. The total heat loss between inlet and outlet is 10 kJ/kg of fluid. When the fluid flows through the system estimate the change in internal energy. *Ans:* −17.44 kJ/kg

4.9 In a steady flow system, 54 kg/min of air enters the control volume at 2 bar and 100 °C. The elevation from the datum is 100 m. The same mass leaves the control volume at an elevation of 150 m. The exit conditions are 10 bar and 300 °C. Take the entrance and exit velocity as 2400 m/s and 1200 m/s, respectively. During the process, 54×10^3 kJ/h of heat is transferred to the control volume. The rise in enthalpy is 10 kJ/kg. Calculate the power developed. *Ans:* 4.59 kW

4.10 Steam flows steadily through a pipeline of 0.4 m diameter from the boiler to turbine, in a steam power plant. The steam from the boiler exhausts at a pressure of 4 bar and temperature of 400 °C. Take the enthalpy of steam as 3220 kJ/kg. The specific volume may be taken as 0.08 m^3/kg. The steam condition at the exit of the turbine are: pressure 3.1 bar; temperature 380 °C; enthalpy 3200 kJ/kg specific volume 0.088 m^3/kg. Total heat loss is estimated as 8.5 kJ/kg. Calculate the mass flow rate of the steam. *Ans:* 331 m/s

4.11 Refer Fig.4.46 which shows a steam turbine which operates at steady flow state condition. Superheated steam at 200 kPa and 400 °C enters the turbine with an enthalpy of 3280 kJ/kg. The entry velocity is 80 m/s. The inlet is at an elevation of 10 m above the datum. The exit steam is at 100 kPa and has a enthalpy of 2465 kJ/kg. The exit port is at a height of 3 m. The energy loss to the surrounding is 25000 kJ/h. Calculate the power output of the turbine. Assume a steam flow rate of 3600 kg/h. *Ans:* 800.079 kW

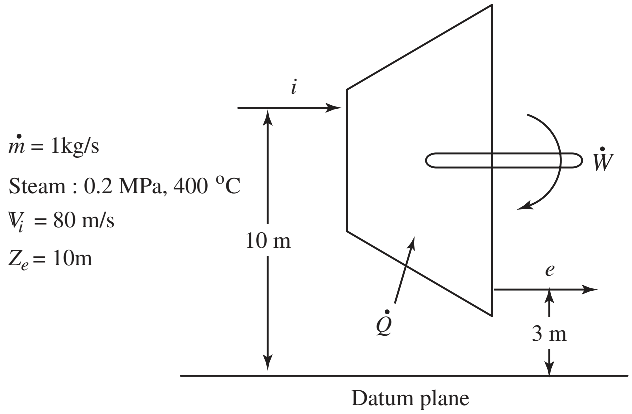


Fig. 4.46

4.12 An ideal gas with $c_p = 7R/2$ and $c_V = 5R/2$ is flowing in a pipeline at 15 bar and 900 K. An insulated tank containing a piston and spring is connected to the line as shown in Fig.4.47. Initially, the tank is empty and the spring is in its natural position and exerts no force on the piston. When the valve is opened the gas enters the tank and the force exerted by the spring is proportional to the distance moved. Determine the temperature of the gas inside the tank when the pressure reached the value of 15 bar. *Ans:* 1050 K

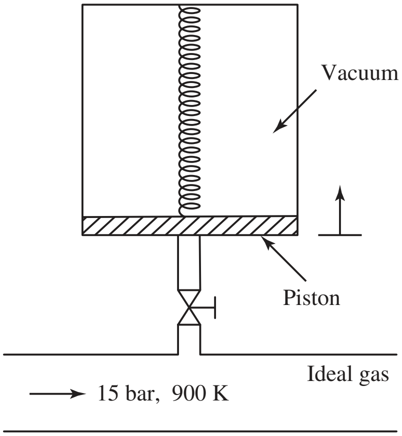


Fig. 4.47

4.13 An insulated gas cylinder of volume 0.1 m^3 contains air at 60 bar and 300 K. The valve of the cylinder is opened allowing the air to escape till the air pressure in the cylinder reduces to 30 bar. Determine (i) temperature of the air that left the cylinder; (ii) mass of air that has escaped from the cylinder. *Ans:* 246.1 K; 2.72 kg

4.14 Consider a tank in which there is one inlet and one exit. A fluid mass enters the tank at the rate of 2.5 kg/s. At the exit, the fluid mass leaves at the rate of $\dot{m}_e = 2.5\big(1 - e^{-0.0251}\big)$ where $\dot{m}_e$ is in kg/s and t is in s. If the tank initially contains 100 kg of mass, estimate the amount of fluid mass in the tank at 100 s. *Ans:* 191.79 kg

4.15 Air under steady state conditions at a pressure of 10 bar and 600 K enters a device at the rate of 2 kg/s. The velocity of air at the entry is 2 m/s. Air leaves the device with a velocity of 20 m/s. The pressure and temperature of the air at exit is 2 bar and 400 K. Determine the power output of the device assuming the device to be well insulated. *Ans:* 401.604 kW

4.16 Assume that a compressor is operating steady state. The fluid enters at p_1, V_1 and leaves at p_2, V_2. Draw the p-V diagram to represent the work done on the compressor and compare it with the work done if the compression is carried out in a piston-cylinder arrangement under non-flow condition. *Ans:* $-\int V dp = \int p dV + (p_2V_2 + p_1V_1)$

4.17 Air enters a well-insulated compressor operating steady state at 100 kPa and 27 °C and exit at 600 kPa and 170 °C. If the flow rate through the compressor is 5 kg/s, calculate the required power to run the compressor. *Ans:* 718.6 kW

4.18 Referring to Fig.4.48, find the power output of the turbine. Also estimate how much error will be introduced if KE and PE terms are neglected.

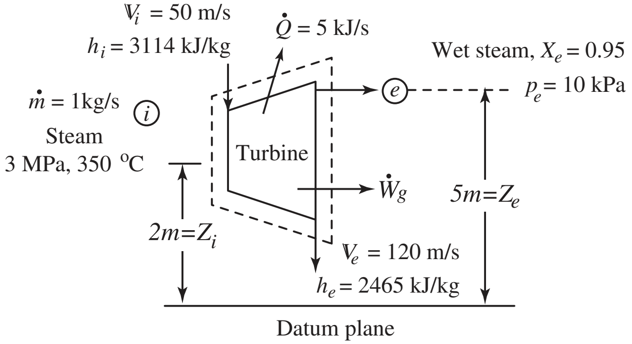


Fig. 4.48

Ans: 638.08 kW; 0.93%; 0.0047%

4.19 Assume air with $\gamma = 1.4$ to be an ideal gas and it expands reversibly and adiabatically through a nozzle from an inlet condition of 0.3 MPa and 300 °C to the exit pressure of 0.1 MPa. Calculate the exit velocity and the mass flow rate if the exit area is 10^{-3}m^2. *Ans:* 556.87 m/s; 0.4628 kg/s

4.20 In a small adiabatic gas turbine operating under steady state condition, air enters at 400 kPa and 400 K. The turbine discharges air at 1 bar and 300 °C. Neglecting the changes in KE, estimate the flow rate of air, if the power output is 10 kW. *Ans:* 0.1 kg/s

4.21 A 1.5 m^3 tank contains air which is considered as an ideal gas with $\gamma = 1.4$ at 100 kPa and 350 K. Attached to this tank is a line in which air flows at 1000 kPa and 700 K.

Air from this line enters the tank until the pressure reaches 1 MPa. If there is no energy transfer as heat from the tank, calculate the final temperature of the air in the tank and the quantity of air that enters the tank. *Ans:* 830.5 K; 3.2 kg

4.22 An insulated cylinder of volume 1 m^3 contains nitrogen at 150 kPa and 300 K. The cylinder is connected to a line carrying nitrogen at 1000 kPa and 300 K through a valve. The valve is opened and held open till the nitrogen pressure in the cylinder reaches 1000 kPa. Determine the final temperature and mass of nitrogen in the cylinder at the end of the filling operation. Attached to this tank is a line in which air flows at 1000 kPa and 700 K. Air from this line enters the tank until the pressure reaches 1 MPa. If there is no energy transfer as heat from the tank, calculate the final temperature of the air in the tank and the quantity of air that enters the tank. *Ans:* 396.23 K; 303.51 kg

4.23 Air at 15°C and 150 kPa enters a diffuser of a jet engine steadily with a velocity of 150 m/s. The inlet area of the diffuser is 0.5 m^2. The air leaves the diffuser with a velocity of 15 m/s. Determine (i) the mass flow rate (ii) the temperature of the air leaving the diffuser. State the assumptions made. Take inlet enthalpy of air to be 285 kJ/kg and density at 0.985 kg/m^3. *Ans:* 73.9 kg/s; 294.67 °C

4.24 Refrigerant 134a enters the capillary tube of a refrigerator as saturated liquid at 800 kPa and throttled to a pressure of 120 kPa. Determine the temperature drop during this process. The properties of the refrigerant are given below: *Ans:* −53.63°C

At 800 kPa, $T_{sat} = 31.3$°C and $h_f = 95.47$ kJ/kg.

At 120 kPa, $T_{sat} = -22.32$°C; $h_f = 22.49$ kJ/kg and $h_g = 236.97$ kJ/kg.

4.25 Consider a 15 kJ/s electric heating system. Air enters the heating section at 1 bar and 17 °C with a volume flow rate of 2.5 m^3/s. Take heat loss from the air in the duct to the surrounding as 200 W. Determine the exit temperature of air. *Ans:* 21.9 °C

Multiple Choice Questions (choose the most appropriate answer)

1. If all the variables are constant with respect to time is called
   (a) steady flow (c) uniform flow
   (b) unsteady flow (d) closed flow

2. Control volume means
   (a) a specified mass (c) an isolated system
   (b) a fixed region in space (d) a closed system

3. Internal energy of a perfect gas mainly depends on
   (a) temperature, specific heats and pressure
   (b) temperature only
   (c) temperature, specific heats and enthalpy
   (d) temperature, specific heats and entropy

4. In a reversible polytropic process,
   (a) enthalpy remains constant (c) internal energy remains constant
   (b) true heat transfer occurs (d) temperature remains constant

5. A process is said to be isentropic when it is
   (a) irreversible and adiabatic
   (b) reversible and isothermal
   (c) frictionless and irreversible
   (d) reversible and adiabatic

6. Steady flow occurs when
   (a) conditions are the same at adjacent points at any instant
   (b) conditions do not change with time at any point
   (c) conditions change steadily with the time
   (d) rate of change of volume is constant

7. A process is said to be reversible process when
   (a) there will be no viscous or Coloumb friction in the system
   (b) there is little heat transfer
   (c) Newtons law of viscosity is satisfied
   (d) temperature of system and surroundings be equal
   (e) heat transfer occurs from surroundings to system only

8. The first law of thermodynamics for steady flow accounts for
   (a) all energy entering and leaving a control volume
   (b) an energy balance for a specified mass of fluid
   (c) the conservation of linear momentum
   (d) heat transfer
   (e) its application to perfect gases

9. The net work done per unit mass of a gas in a polytropic process is equal to
   (a) $p_1 v_1 \log_e \dfrac{v_2}{v_1}$
   (b) $p_1(v_1 - v_2)$
   (c) $\dfrac{p_1 v_1 - p_2 v_2}{n-1}$
   (d) $\dfrac{p_2 v_1 - p_2 v_2}{n-1}$

10. The characteristic equation of gases $pV = mRT$ holds good for
    (a) monoatomic gases
    (b) real gases
    (c) diatomic gas
    (d) ideal gases

11. A gas that obeys kinetic theory of gases perfectly is known as
    (a) monoatomic gas
    (b) perfect gas
    (c) diatomic gas
    (d) real gas

12. Work done in a free expansion process is
    (a) zero
    (b) minimum
    (c) maximum
    (d) positive

13. Which of the following is not a property of the system ?

(a) temperature
(b) pressure
(c) specific volume
(d) heat
(e) none of the above

14. In the polytropic process, equation $pv^n =$ constant, if $n = 0$, the process is called

(a) constant pressure
(b) constant volume
(c) constant temperature
(d) adiabatic

15. In the polytropic process, equation $pv^n =$ constant, if $n = \infty$, the process is called

(a) constant volume
(b) constant pressure
(c) constant temperature
(d) adiabatic
(e) isothermal

16. The processes or systems that do not involve heat transfer are called

(a) isothermal processes
(b) equilibrium processes
(c) adiabatic processes
(d) thermal processes

17. In a reversible adiabatic process the ratio (T1/T2) is equal to

(a) $\left(\frac{p_1}{p_2}\right)^{\frac{\gamma-1}{\gamma}}$

(b) $\left(\frac{v_1}{v_2}\right)^{\frac{\gamma-1}{\gamma}}$

(c) $(v_1 v_2)^{\frac{\gamma-1}{2\gamma}}$

(d) $\left(\frac{v_2}{v_1}\right)^{\gamma}$

18. A process is said to be isothermal if

(a) temperature remains constant
(b) volume remains constant
(c) change in internal energy is zero
(d) pressure remains constant
(e) enthalpy change is maximum

19. In a throttling process

(a) internal energy does not change
(b) pressure does not change
(c) enthalpy does not change
(d) entropy does not change
(e) volume change is negligible

20. If a gas has to be stored, the type of compression that would be ideal is

(a) isothermal
(b) adiabatic

(c) polytropic
(d) constant volume
(e) none of the above

21. If a process can be stopped at any stage and reversed so that the system and surroundings are exactly restored to their initial states, it is called
    (a) adiabatic process
    (b) isothermal process
    (c) ideal process
    (d) frictionless process
    (e) energyless process

22. The state of a substance whose evaporation from its liquid state is complete, it is called
    (a) vapour
    (b) air
    (c) steam
    (d) perfect gas

23. The value of the universal gas constant in SI units is given by
    (a) 0.8314 J/mole/K
    (b) 8.314 J/mole/K
    (c) 83.14 J/mole/K
    (d) 831.4 J/mole/K
    (e) 8314 J/mole/K

24. In a constant pressure heating of a gas, the heat supplied
    (a) increases the internal energy of the gas
    (b) increases the temperature of the gas
    (c) both (b) and (c)
    (d) does some external work during expansion
    (e) none of the above

25. The gas constant (R) is equal to the
    (a) sum of two specific heats, c_p and c_V
    (b) difference of two specific heats, c_p and c_V
    (c) product of two specific heats, c_p and c_V
    (d) ratio of two specific heats, c_p and c_V

Ans: 1. – (a) 2. – (b) 3. – (b) 4. – (b) 5. – (d)
6. – (b) 7. – (a) 8. – (a) 9. – (c) 10. – (d)
11. – (b) 12. – (a) 13. – (d) 14. – (a) 15. – (a)
16. – (c) 17. – (a) 18. – (a) 19. – (c) 20. – (a)
21. – (c) 22. – (d) 23. – (e) 24. – (c) 25. – (b)

5

SECOND LAW OF THERMODYNAMICS

5.1 INTRODUCTION

In chapters 3 and 4, we focused our attention on the first law of thermodynamics and its applications. *According to the first law, all form of energy are equivalent* and one form of energy can be converted into another form. The first law of thermodynamics *establishes equivalence between the quantity of heat used and the mechanical work. However, it does not specify the conditions under which conversion of heat into work is possible.* Further, the direction in which heat transfer can take place is also not specified. This is the main deficiency of the first law. The gap is bridged by the second law of thermodynamics.

In this chapter, we will introduce the *second law of thermodynamics.* Note that second law of thermodynamics cannot be proved like any mathematical theorem. It is a principle that is accepted as true without proof. It is based on experimental evidence only. So far no experiments have disproved the validity of the law. According to *second law*, processes can occur only in a certain direction and that energy has both *quality as well as quantity.* At this point what you should understand is that a process cannot take place unless it satisfies both first as well as second law of thermodynamics. Therefore, second law can be considered as an embodiment of real happenings while retaining the basic nature of first law of thermodynamics.

Feasibility and direction of a process as well as grades of energy such as low and high are the potential answers provided by second law. Second law is capable of indicating the maximum possible efficiencies of heat engines, coefficient of performance of heat pumps and refrigerators, defining a temperature scale independent of physical properties of the fluid etc.

Let us introduce in this chapter the thermal energy reservoirs, reversible and irreversible processes, heat engines, refrigerators and heat pumps. Various statements of the second law will be presented, followed by a discussion of perpetual motion machines and the thermodynamic temperature scale. The Carnot cycle is introduced next and the Carnot principles are discussed. Finally, the idealised Carnot heat engines and heat pumps are examined.

5.2 WHY THE SECOND LAW?

We have already clarified in the previous section ”Why the Second Law?”. Now, let us explain it by means of three simple day to day examples.

Example 1

Consider a cup of hot coffee (Fig.5.1) left in an air-conditioned room. It is our common experience that over a period of time the hot coffee becomes cold. This process satisfies the first law of thermodynamics since the amount of energy lost by the coffee is equal to the amount gained by the surrounding air. Now let us ask the question whether reverse process is possible? That is to ask can the *hot coffee* getting even *hotter* in a *cooler room* as a result of heat transfer from the room air. *We all know that this process can never take place.* Even if it happens it would not violate the first law as long as the amount of energy lost by the air is equal to the amount gained by the coffee.

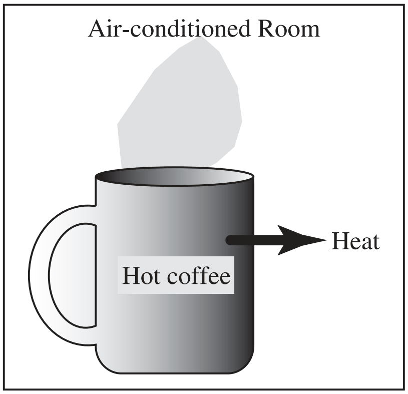
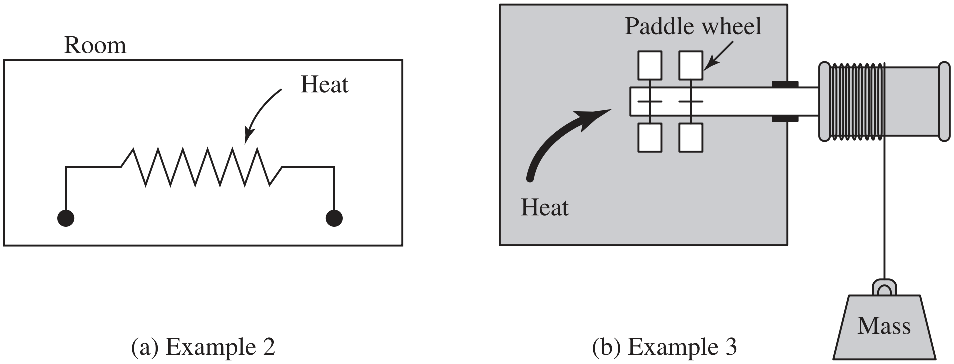


Fig. 5.1 Example 1

Example 2

Let us consider second familiar example, viz the heating of a room by the passage of electric current through a resistor as shown in Fig.5.2(a). Again, as per the first law the amount of electric energy supplied to the resistor will be equal to the amount of energy transferred to the room air as heat. Now let us attempt to reverse this process. It means transferring some heat from the room to the wires. You will not be able to find equivalent amount of electric energy generation in the wires. Note that the first law never forbids it.



(a) Example 2

(b) Example 3

Fig. 5.2 Examples

Example 3

Finally, let us consider a paddle-wheel mechanism operated by the fall of a mass as shown in Fig.5.2(b). As the mass falls, the paddle wheel start rotating and churns the fluid within the insulated container. As a result, the potential energy of the mass decreases. This causes the internal energy of the fluid in the enclosure to increase in accordance with the conservation of energy principle. However, the reverse process is not true. By transferring heat from the fluid to the paddle wheel the mass will not be raised even though it does not violate the first law of thermodynamics.

From these examples it is clear that processes should proceed in a *certain direction.* You are aware that the first law places no restriction on the direction of a process. *Therefore, satisfying the first law does not ensure that the process can really occur.* This deficiency of the first law to identify whether a process can really take place is taken care of by introducing another general principle. It is called the *second law of thermodynamics.* We will show later in this chapter that the reverse processes discussed above violate the second law of thermodynamics. This violation can be easily detected with the help of a property, called *entropy*, which we will take up in Chapter 6. Therefore, it must be clear that a process cannot occur unless it satisfies *both* the first and the second law of thermodynamics which is pictorially illustrated in Fig.5.3.

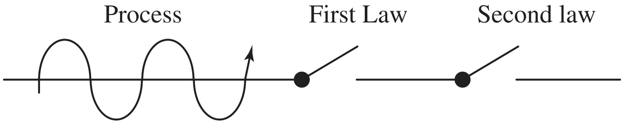


Fig. 5.3 Illustration of a process to happen

There are many valid statements of the second law of thermodynamics. Two such statements will be presented and discussed later in this chapter. We will consider application of second law to some engineering devices that operate on cycles. It may be noted that the use of the second law of thermodynamics is not limited to identifying the direction of processes alone. As already stated, the second law also asserts that energy has *quality* as well as *quantity.* Note that the first law is concerned with the quantity of energy and its transformation from one form to another with no regard to its quality. It is the a major concern of engineers regarding the quality of energy. The second law provides the necessary means to determine the quality as well as the degree of degradation of energy during a process.

It will be shown later in this chapter that, it is the high-temperature energy that will be more useful for conversion to work. It is because, it has a higher quality than the same amount of energy at a lower temperature. The second law of thermodynamics can be used to determine the theoretical limits for the performance of commonly used engineering systems, such as heat engines and refrigerators, as well as predicting the degree of completion of chemical reactions.

5.3 THERMAL ENERGY RESERVOIRS

In order to understand the second law of thermodynamics more precisely, let us take up some basic details and a few practical devices that convert heat to work and work to heat.

To start with, let us understand the basic principle involved in the second law of thermodynamics. Towards this end let us consider a *hypothetical body.* Assume that this body has a relatively large *thermal energy capacity* (mass × specific heat). Let us also assume that it can supply or absorb finite amounts of heat without undergoing any perceptible change in temperature. Such a body is usually called a *thermal energy reservoir* or just a reservoir. Typical examples are oceans, lakes and rivers as well as the atmospheric air. It is because it has large thermal energy storage capabilities.

For example, the *atmosphere* does not warm up as a result of heat losses from residential buildings in winter. Similarly, mega joules of waste energy dumped in large rivers by power plants do not cause any significant change in water temperature. At the same time it should be noted that a body need not be actually very large to be considered a reservoir. Any physical body whose thermal energy capacity is large relative to the amount of energy it supplies or absorbs can be considered as the thermal reservoir. A typical practical example is the air in a larger room where a TV is on. Room can be treated as a reservoir in the analysis of the heat dissipation from a TV set in the room. The amount of heat transfer from the TV set to the room air is not large enough to have any *noticeable effect* on the room air temperature.

A reservoir that supplies energy in the form of heat is called a source and one that absorbs energy in the form of heat is called a *sink* (Fig.5.4). Thermal energy reservoirs are often referred to as *heat reservoirs* since they supply or absorb energy in the form of heat.

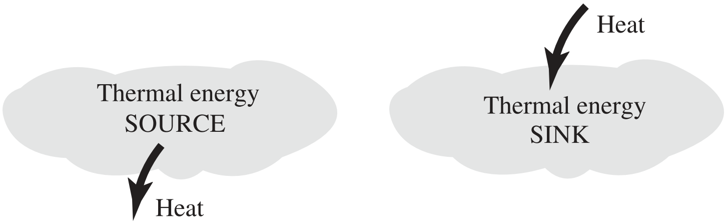


Fig. 5.4 Source and sink

Now-a-days environmentalists as well as engineers are worried about the heat transfer from industrial sources to the environment. Unscientific management of waste energy can significantly increase *thermal pollution.* If it is not properly monitored, thermal pollution can seriously disrupt marine life in lakes and rivers. Therefore, careful design and management of each and every equipment is a must. In this direction proper understanding and application of the laws of thermodynamics is a must. It can considerably reduce dumping of the waste energy into large bodies of water. This will considerably improve the quality of marine life. *Therefore, it will not be an over statement if we say that understanding the laws of thermodynamics is a must for all engineers.*

5.4 HEAT ENGINES

By this time you must be aware that work can easily be converted to other forms of energy. On the other hand, converting other forms of energy to work is not that easy. To illustrate this point let us consider two systems shown in Fig.5.5. The mechanical work done by the shaft shown in Fig.5.5(a), is first converted to the internal energy of the water. This energy may then leave the water as heat. We know from experience that any attempt to reverse this process will not succeed. That is, transferring heat to the water Fig.5.5(b), does not cause the shaft to rotate. From this and other observations, we can conclude that work can be converted to heat directly and completely but converting heat to work [Fig.5.5(b)] requires the use of some special devices. These devices are called *heat engines.*

Heat engines differ from one another considerably but all can be characterised by the following (Fig.5.6):

(i) They receive heat from a high-temperature source (solar energy, oil furnace, nuclear reactor, etc.).

(ii) They convert part of this heat to work (usually in the form of rotating a shaft).

(iii) They reject the remaining (waste) heat to a low-temperature sink (atmosphere, rivers, etc.).

(iv) They operate on a cycle.

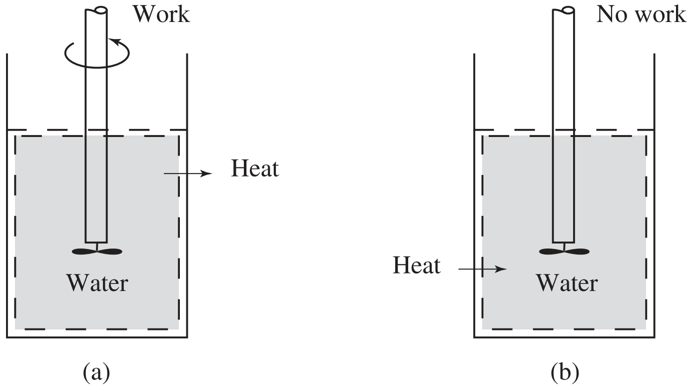


Fig. 5.5 Understanding work to heat and heat to work conversion

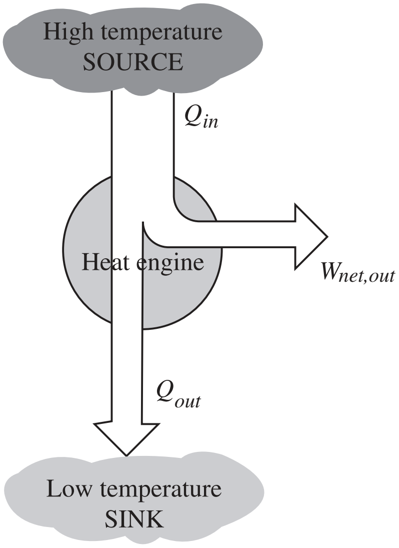


Fig. 5.6 Illustration of heat flow in a heat engine

While undergoing a cycle heat engines and other cyclic devices usually involve heat transfer to and from a fluid. This fluid is called the *working fluid*. It should be emphasised here that the term heat engine is often used in a broader sense to include many work producing devices that do not operate in a thermodynamic cycle. Engines that involve internal combustion such as automobile engines and gas turbines fall into this category. These devices operate in a mechanical cycle. Note that they do not operate in a thermodynamic cycle. It is due to the fact that the working fluid (the products of combustion) does not undergo a complete cycle. Instead of being cooled to the initial temperature, the exhaust gases are released at much higher temperature. They are replaced by fresh air-fuel mixture at the end of the cycle. The work-producing device that will best fit into the definition of a heat engine is the *steam power plant* which is an external-combustion engine. That is, combustion takes place outside the engine and the thermal energy released during this process is transferred to the steam as heat. A simplified schematic diagram of a basic steam power plant is shown in Fig.5.7.

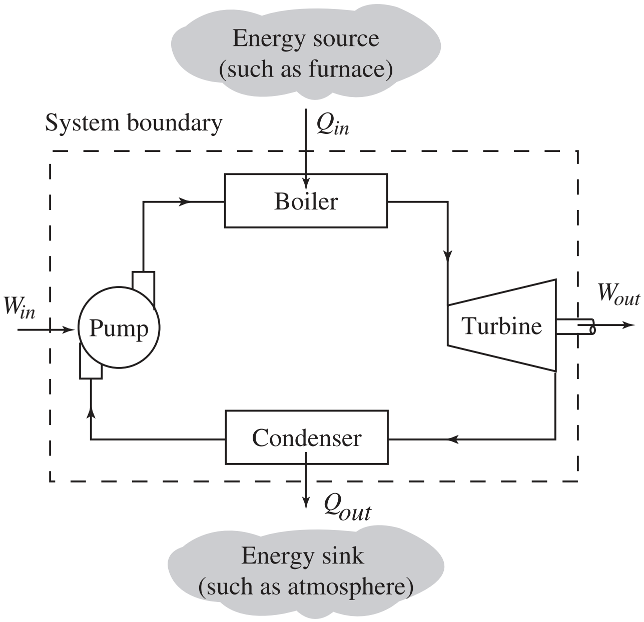


Fig. 5.7 Schematic of a steam power plant

The various quantities shown on this figure are as follows:

(i) Q_{in} : amount of heat supplied to steam in boiler from a high-temperature source (furnace)

(ii) Q_{out} : amount of heat rejected from steam in condenser to a low temperature sink (the atmosphere, a river, etc.)

(iii) W_{out} : amount of work delivered by steam as it expands in turbine

(iv) W_{in} : amount of work required to pressurise water to boiler pressure

Notice that the directions of the heat and work interactions are indicated by the subscripts *in* and *out*. Therefore, all four quantities described are always *positive*. The net work output of this power plant is simply the difference between the total work output of the plant and the total work input (Fig.5.8).

$$W_{net,out} = W_{out} - W_{in} \quad \text{kJ} \tag{5.1}$$

Even from heat transfer data the net work can be calculated. The four components of the steam power plant involve mass flow in and out. Therefore, they should be treated as open systems. These components, together with the connecting pipes, however, always contain the same mass of fluid, assuming the system to be leak proof. No mass enters or leaves the system, enclosed by the broken lines in Fig.5.7. Therefore, it can be analysed as a closed system. Recall that for a closed system undergoing a cycle, the change in internal energy, ΔU is zero and therefore, the net work output of the system is also equal to the net heat transfer to the system:

$$W_{net,out} = Q_{in} - Q_{out} \text{ kJ} \tag{5.2}$$

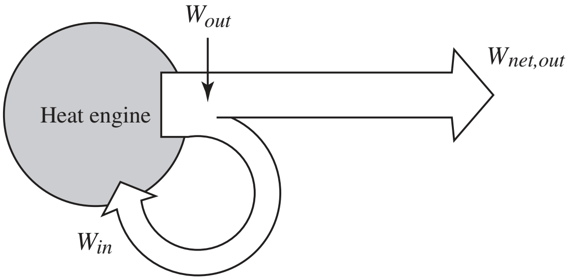


Fig. 5.8 Schematic of work transfer in a heat engine

5.5 THERMAL EFFICIENCY

In Eq.5.2, Q_{out} represents the magnitude of the energy rejected or wasted in order to complete the cycle. It is to be noted that Q_{out} can never be zero. Therefore, the net work output of a heat engine is always less than the amount of heat input (Eq.5.2). Only part of the heat transferred to the heat engine is converted to work. The fraction of the heat input that is converted to net work output is considered as a measure of the performance of a heat engine. This performance parameter is called the *thermal efficiency*, η_{th} (Fig.5.9). For heat engines, the desired output is the net work output. The required input is the amount of heat supplied to the working fluid.

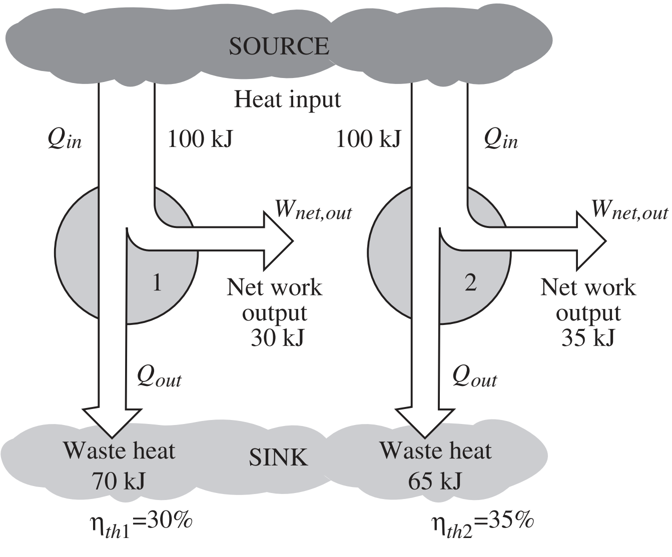


Fig. 5.9 Efficiency of a heat engine

The thermal efficiency of a heat engine can be expressed as

$$\text{Thermal efficiency} = \frac{\text{Net work output}}{\text{Total heat input}} \tag{5.3}$$

or

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} \tag{5.4}$$

It can also be expressed as

$$\eta_{th} = 1 - \frac{Q_{out}}{Q_{in}} \tag{5.5}$$

Practical devices such as heat engines, refrigerators and heat pumps operate between two reservoirs one at a high-temperature T_H and another at a low-temperature T_L. In order to bring uniformity in the treatment of heat engines, refrigerators and heat pumps, let us define two quantities, viz Q_H and Q_L. Here, Q_H and Q_L are magnitude of heat transfer between the cyclic device at high temperature, T_H and low temperature, T_L, respectively. The direction of Q_H and Q_L can be easily determined by inspection. Then, the net work output and thermal efficiency relations for any heat engine (shown in Fig.5.10) can also be expressed as

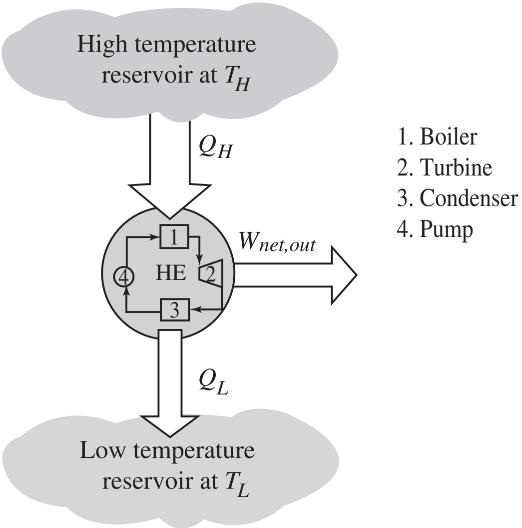


Fig. 5.10 Schematic of a heat engine working between T_H and T_L

$$W_{net,out} = Q_H - Q_L \tag{5.6}$$

and

$$\eta_{th} = \frac{W_{net,out}}{Q_H} \tag{5.7}$$

or

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} \tag{5.8}$$

Since both Q_L and Q_H are defined as positive quantities the thermal efficiency of a heat engine is always less than 100 per cent. Thermal efficiency is a measure of how effectively a heat engine converts the heat input to work output. Now it must be clear that heat engines are built for the purpose of converting heat to work. Engineers are constantly trying to improve the efficiencies of these devices since increased efficiency means less fuel consumption as well as pollution. The thermal efficiencies of work-producing devices are relatively low. Modern spark-ignition automobile engines have a thermal efficiency of about 35 per cent. That is, an automobile engine converts about 35 per cent of the chemical energy of the gasoline to mechanical work. This number is as high as 40 per cent for diesel engines and large gas-turbine plants and is around 50 per cent for large combined gas-steam power plants. Thus, even with the most efficient heat engines available today, almost 50% of the energy supplied is rejected in the rivers, lakes or the atmosphere as waste or useless energy (Fig.5.11). This is alarming to environmentalists.

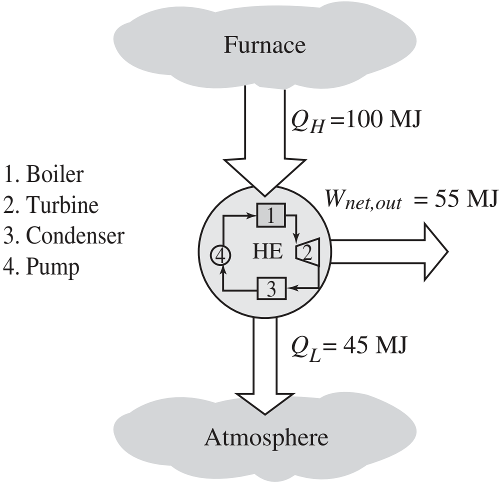


Fig. 5.11 Illustration of heat rejection

5.5.1 A Note on Heat Rejection

In the previous section we saw that even the most efficient heat engines dumps more than 50% of heat input into the atmosphere. Now, let us consider again a steam power plant shown in Fig.5.7. In this, the condenser is the device where large quantities of waste heat are rejected to rivers, lakes or the atmosphere. The interesting question now is, can we not just take the condenser out of the plant and save all that waste energy? The answer to this question is, unfortunately, a firm NO. It is because without a heat rejection process in a condenser, the cycle cannot be completed. Cyclic devices like steam power plants cannot run continuously unless the cycle is completed. Similarly, every heat engine must waste some amount of energy by transferring it to a low-temperature reservoir in order to complete the cycle. The most important requirement of a heat engine is to exchange heat with at least two reservoirs for continuous operation. This forms the basis for the Kelvin-Planck statement of the second law of thermodynamics.

5.6 REFRIGERATORS AND HEAT PUMPS

We all know from experience that heat transfer takes place from a higher temperature source to lower temperature sink. This process of heat transfer occurs in nature without any interference. However, it is to be noted that the reverse process, cannot occur by itself. If we have to make it occur we require some special device. Such a device is called *refrigerator*. Again, refrigerators, like heat engines, are cyclic devices. The working fluid used in the refrigeration cycle is called a *refrigerant*. The most frequently used refrigeration cycle is the *vapour-compression* *refrigeration cycle*, which involves four main components as shown in Fig.5.12 with typical pressure and temperature values.

(i) a compressor,

(ii) a condenser,

(iii) an expansion valve and

(iv) an evaporator,

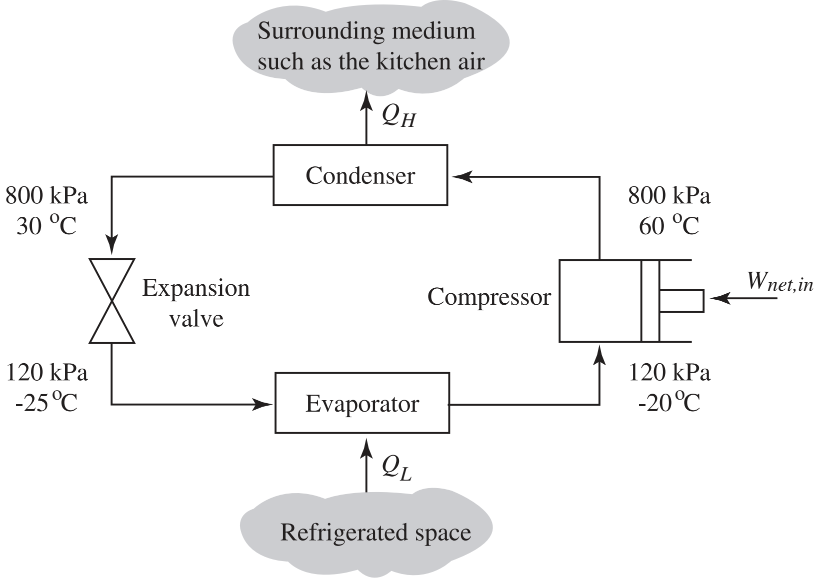


Fig. 5.12 A typical vapour-compression refrigeration cycle

5.6.1 Working Principle

The refrigerant enters the compressor as a vapour. It gets compressed to the condenser pressure and leaves the compressor at a relatively high temperature. It cools down by rejecting heat to the surrounding medium. Because of this, the refrigerant condenses as it flows through the coils of the condenser. Condensed vapour is called condensate. The condensate then enters an expansion value, viz a capillary tube. Its pressure and temperature drop drastically due to the throttling effect. The low-temperature refrigerant then enters the evaporator. It evaporates by absorbing heat from the refrigerated space. The cycle is completed as the evaporated refrigerant leaves the evaporator and reenters the compressor.

In a household refrigerator, the freezer compartment where heat is absorbed by the refrigerant serves as the evaporator. The coils are usually behind the refrigerator. The heat is dissipated to the space where it is kept. Therefore, this serves as the condenser. A typical layout of a refrigerator is shown schematically in Fig.5.13. Here, Q_L is the magnitude of the heat removed from the refrigerated space at temperature T_L and Q_H is the magnitude of the heat rejected to the warm environment at temperature T_H. W_{net} is the net work input to the refrigerator. As discussed before, Q_L *and* Q_H *represent magnitudes and thus are positive quantities.*

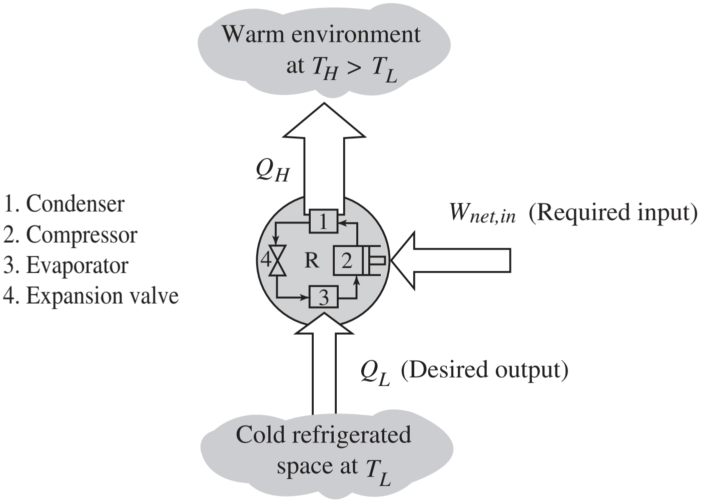


Fig. 5.13 Working principle of a refrigerator

5.6.2 Coefficient of Performance

The efficiency of a refrigerator is expressed in terms of the *coefficient of performance* (COP), denoted by COP_{ref}. The main objective of a refrigerator is to remove heat (Q_L) from the refrigerated space. To accomplish this goal, it requires a work input of $W_{net,in}$. Then, the COP of a refrigerator can be expressed as

$$COP_{ref} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{W_{net,in}} \tag{5.9}$$

This relation can also be expressed in rate form by replacing Q_L by $\dot{Q}_L$ and $W_{net,in}$ by $\dot{W}_{net,in}$.

The conservation of energy principle for a cyclic device requires that

$$W_{net,in} = Q_H - Q_L \text{ (kJ)} \tag{5.10}$$

Then, the COP relation becomes

$$COP_{ref} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1} \tag{5.11}$$

Note that the value of COP_{ref} can be *greater than unity.* That is, the amount of heat removed from the refrigerated space can be greater than the amount of work input. This is in contrast to the thermal efficiency, which can never be greater than 1. In fact, this is one of the reason for expressing the efficiency of a refrigerator by another term – the coefficient of performance COP. It is to avoid the odd feeling of having efficiencies greater than one.

5.7 HEAT PUMPS

Heat pump is another device that transfers heat from a low-temperature medium to a high-temperature one. The details are shown schematically in Fig.5.14. Refrigerators and heat pumps operate on the same cycle but their objectives differ.

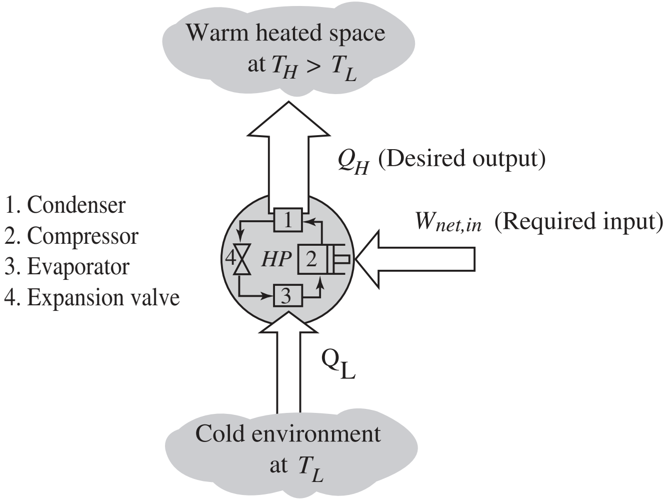


Fig. 5.14 Schematic of a heat pump

The objective of a refrigerator is to maintain the refrigerated space at a low temperature. This is done by removing heat from the refrigerated space. Discharging this heat to a higher-temperature medium is merely a necessity in the part of its operation and not the purpose. The objective of a heat pump, however, is to maintain a heated space at a higher temperature. This is accomplished by absorbing heat from a low-temperature source, such as well water or cold outside air in winter and supplying this heat to the high-temperature medium such as a house (Fig.5.15).

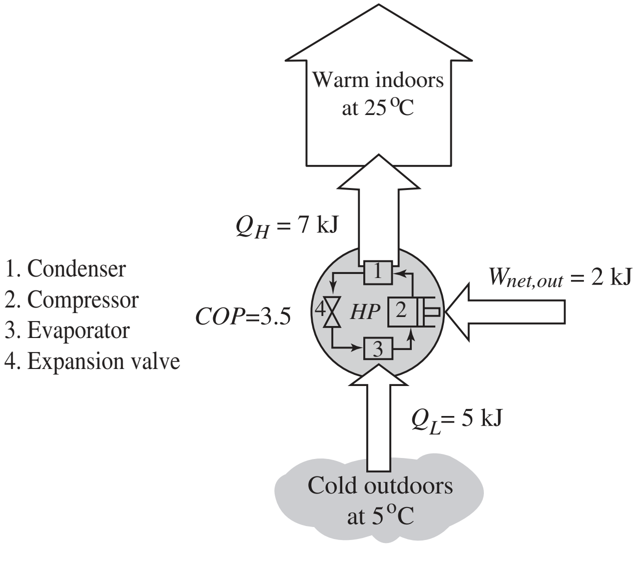


Fig. 5.15 Working principle of a heat pump

You can make a domestic refrigerator to work as a heat pump. For this you should place it in the window of a house with its door open to the cold outside air in winter. The reason is that it will try to cool the outside air by absorbing heat from it and rejecting this heat into the house through the coils behind it. The measure of performance of a heat pump is also expressed in terms of the *coefficient of performance* COP_{HP}, defined as

$$COP_{HP} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_H}{W_{net,in}} \tag{5.12}$$

which can also be expressed as

$$COP_{HP} = \frac{Q_H}{Q_H - Q_L} = \frac{1}{1 - Q_L/Q_H} \tag{5.13}$$

A comparison of Eqs 5.11 and 5.13 reveals that

$$COP_{HP} = COP_{ref} + 1 \tag{5.14}$$

for fixed values of Q_L and Q_H. What the above expression means is that the coefficient of performance of a heat pump is always greater than unity. It is because COP_{ref} is a positive quantity. In the worst case, a heat pump will function as a resistance heater, supplying as much energy to the house as it consumes. In reality, however, part of Q_H is lost to the outside air through piping and other devices. Because of this, COP_{HP} may drop below unity when the outside air temperature is too low. When this happens, the system usually switches to a resistance heating mode. Most heat pumps in operation today have a seasonally averaged COP of 2 to 3.

5.8 CLASSICAL STATEMENT OF SECOND LAW OF THERMODYNAMICS

In previous sections, we went through the practical devices such as heat engines, refrigerators and heat pumps. We have also defined their performance parameters. Now let us see how to analyse these devices by means of second law of thermodynamics. In this connection, there are two classical statements of the second law of thermodynamics. They are

(i) the Kelvin-Planck statement, which is related to heat engines and

(ii) the Clausius statement, which is related to refrigerators or heat pumps

5.8.1 Kelvin-Planck Statement

We have already explained that even under ideal conditions, a heat engine must reject some heat to a low-temperature reservoir. This only will make the cycle to complete. This limitation makes the heat engines to be less efficient. The thermal efficiency of heat engines forms the basis for the Kelvin-Planck statement. There are different ways by which Kelvin-Planck statement can be expressed. Usually it is stated as follows:

It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce an equivalent amount of work. In other words, no process is possible whose sole result is the absorption of heat from a single reservoir and the conversion of this heat into equivalent amount of work.

What the statement means is that there must be two reservoirs for a heat engine to operate. That is to say that a heat engine must exchange heat with a low-temperature sink (environment) as well as a high-temperature source (furnace) to keep operating. That is, no heat engine can convert the entire heat input to useful work without rejecting some heat. Pictorially the Kelvin-Planck statement is shown in Fig.5.16.

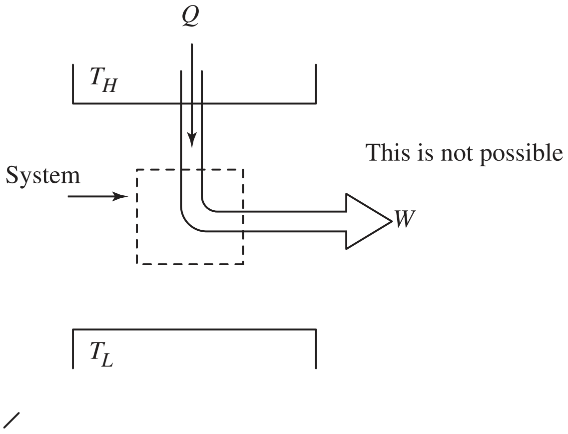


Fig. 5.16 Pictorial representation of Kelvin-Planck statement

Equivalently:

- *For a power plant to operate, the working fluid* **must** *exchange heat with the environment as well as the furnace.*
- *no heat engine can have a thermal efficiency of 100%.*

Note that the impossibility of 100% efficiency of a heat engine is not due to friction or other dissipative effects. It is a limitation that applies to both the idealised and the actual heat engines. We will develop a relation for the maximum thermal efficiency of a heat engine later in this chapter. We will also demonstrate that this maximum value depends on the reservoir temperatures only.

5.8.2 Clausius Statement

Another classical statement of second law of thermodynamics is by Clausius which is given below:

It is impossible to construct a device that operates in a thermodynamic cycle and produces no effect other than the transfer of heat from a lower-temperature body to a higher-temperature body. What it means is *”no process is possible whose sole result is the transfer of heat from a cooler to a hotter body”*. Pictorially the Clausius statement is shown in Fig.5.17.

We all know that heat does not flow, of its own, from a cold medium to a warmer one. *You should not misunderstand that the Clausius statement does not imply that it is impossible to construct a cyclic device that transfers heat from a cold medium to a warmer one.* In fact, this is precisely what a common household refrigerator does. The statement simply means that a refrigerator cannot operate unless its compressor is driven by an external power source, such as an electric motor. This way, the net effect on the surroundings involves the consumption of some energy in the form of work. Further, there is transfer of heat from a colder body to a warmer one. Therefore, a household refrigerator is in complete compliance with the Clausius statement of the second law.

Unfortunately, both the Kelvin-Planck and the Clausius statements of the second law are *negative statements* and it is extremely difficult to prove a negative statement. Like any other physical law, the second law of thermodynamics is based on experimental observations only.

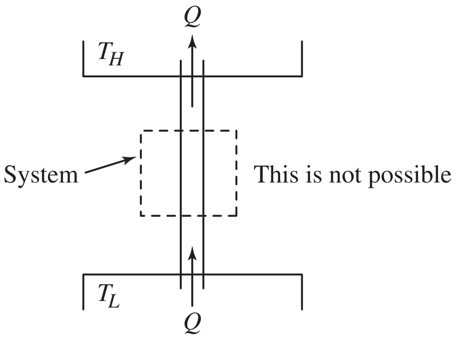


Fig. 5.17 Pictorial representation of Clausius statement

Until now, no experiment has been conducted that contradicts the second law and this should be taken as sufficient proof of its validity.

5.8.3 Equivalence of the Two Statements

At first sight, Kelvin-Plank statement and Clausius statement of the second law of thermodynamics may look as if they are not connected. By proper analysis it can be shown that they are virtually two parallel statements and are equivalent.

A simple way of proving the equivalence of the two statements is as follows:

First, let us consider a device which violates the Clausius statement and prove that it violates Kelvin-Plank statement also. Then, we consider another device which violates the Kelvin-Plank statement and prove that it violates the Clausius statement also.

According to the Clausius statement, *it is impossible for any device operating in a thermodynamic cycle to transfer heat from a low-temperature medium to a high-temperature medium without the help of work interaction from surrounding.* If it does then it violates Clausius statement.

Consider a hypothetical heat pump (HP) shown in Fig.5.18. It clearly *violates* the Clausius statement as $W = 0$. This means that there is no work input. Because of the violation it should transfer heat from a low-temperature reservoir at temperature T_L to a high-temperature reservoir at temperature T_H without any work input. Let the heat interaction with reservoir at T_L be Q_L and with reservoir at T_H be Q_H. Let us assume that the heat pump is working in a cycle. According to first law

$$\sum Q = \sum W \tag{5.15}$$

According to our assumption, $W = 0$.

Therefore,
$$Q_H - Q_L = 0 \tag{5.16}$$

In other words,
$$Q_H = Q_L \tag{5.17}$$

Assume now that a direct heat engine, HE, which works in accordance with the Kelvin-Plank statement. The engine is connected between the two reservoirs as shown in Fig.5.18 delivering some work, W.

According to Kelvin and Plank *No heat engine can have a thermal efficiency of 100 per cent.*

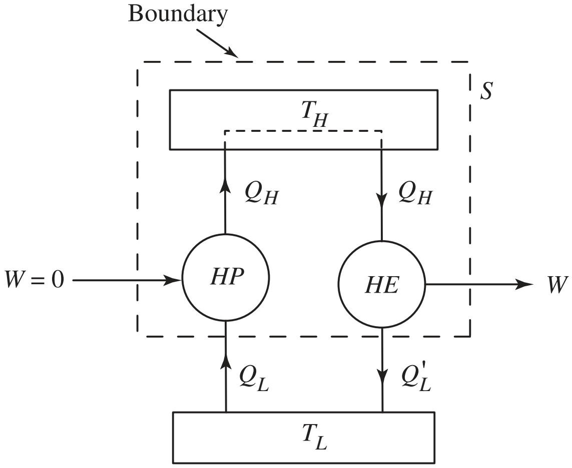


Fig. 5.18 Violation of Clausius statement

As can be seen from Fig.5.18, the engine produces work, W, while rejecting heat Q_L'. Note that Q_H is equal to that is discharged by the heat pump, HP. This means that there is no need for the hot reservoir and the heat Q_H discharged by the pump can be directly fed to the heat engine. Enclose the whole system by a boundary by dashed lines as shown in Fig.5.18. Now we can see that the heat pump, HP and the heat engine, HE, acting together constitute a heat engine. Since the hot reservoir can be removed the heat engine is operating in a cycle. It is clear that heat engine, HE, produces a net work, W. Applying the first law to heat engine,

$$W_{net} = Q_H - Q_L' \tag{5.18}$$

As can be noted that the heat transfer to the system is only from the low temperature reservoir. It is given by $Q_L - Q_L'$. We have already shown (refer Eq.5.17) that $Q_H = Q_L$. Hence, the net heat transfer,

$$Q_{net} = Q_H - Q_L' \tag{5.19}$$

From Eq.5.18 and 5.19,

$$\eta = \frac{W_{net}}{Q_{net}} = \frac{Q_H - Q_L'}{Q_H - Q_L'} = 100\% \tag{5.20}$$

This violates the Kelvin-Plank statement. Therefore, it can be concluded that a violation of Clausius statement leads to the violation of the Kelvin-Plank statement.

Now let us complete the exercise by assuming a heat engine which violates Kelvin-Plank statement and prove that such a device will violate the Clausius statement also. Let us consider a device given in Fig.5.19. Let the heat transfer from the high temperature reservoir to the heat engine be Q_H. As the engine violates the Kelvin-Plank statement no heat is rejected to the low-temperature reservoir by this engine. Now according to the first law of thermodynamics the work output of the engine is given by

$$W = Q_H$$

Let this work drive a heat pump, HP. As shown in Fig.5.19, heat pump receives Q_L from the low temperature reservoir and transfers $Q_H + Q_L$ to the high temperature reservoir. As can be seen the heat pump is operating in accordance with Clausius statement. According to the first law of thermodynamics,

$$Q_{net} \quad = \quad W + Q_L \quad = \quad Q_H + Q_L$$

The net heat transfer to the high temperature reservoir is given by $Q_L + Q_H$ (refer Fig.5.19).

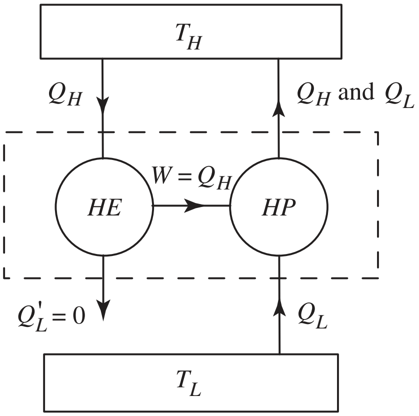


Fig. 5.19 Violation of Kelvin-Planck statement

Now, consider the heat pump and the heat engine together as a system. Enclose the whole system by a boundary by dashed lines as shown in Fig.5.19. As can be seen, the heat engine draws Q_H from the hot reservoir. The heat pump rejects Q_H+Q_L to the hot reservoir. It means that there is no need for the hot reservoir. The system is undergoing a thermodynamic cycle during which, Q_L is transferred from the low-temperature reservoir to the high temperature reservoir without any work from outside the system. This is clearly the violation of Clausius statement. Therefore, it can be concluded that a violation of Kelvin-Plank statement leads to the violation of the Clausius statement. The two analyses we have made, prove that the violation of one statement leads to the violation of the other.

5.9 PERPETUAL-MOTION MACHINES

By this time you should have understood that a process cannot take place unless it satisfies both the first and second laws of thermodynamics. Any device that violates either law is called a *perpetual-motion machine.* Until now nobody has invented any perpetual-motion machine. But this has not stopped inventors from trying to create new ones.

A device that violates the first law of thermodynamics (by creating energy) is called a *perpetual-motion machine of the first kind (PMM1)*, and a device that violates the second law of thermodynamics is called a *perpetual-motion machine of the second kind (PMM2).*

Consider the steam power plant shown in Fig.5.20. It is proposed to raise the steam by resistance heaters placed inside the boiler. Electricity generated by power plant is divided into two parts.

(i) Generated by the steam power plant to be used to supply energy to the resistors as well as the pump.

(ii) The rest to be supplied to the electric net work as the net work output.

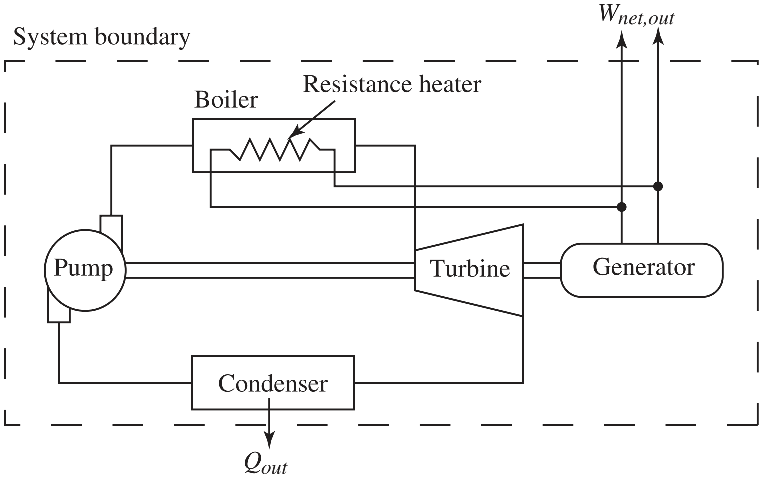


Fig. 5.20 A perpetual-motion machine of first kind (PMM1)

It is claimed that once the system starts working, this power plant will produce electricity indefinitely without requiring any energy input from the outside. A careful examination will reveal that the system enclosed within the dashed lines is continuously supplying energy to the outside at a rate of $Q_{out} + \dot{W}_{net,out}$ without receiving any energy. This means that the system is creating energy at a rate of $\dot{Q}_{out} + \dot{W}_{net,out}$, which is clearly a violation of the first law. Therefore, this device is nothing more than a $PMM1$. It does not warrant any further consideration.

Now let us consider another interesting idea by an unknown inventor. Accepting that energy cannot be created, the inventor suggests the following modification. This he assures will greatly improve the thermal efficiency of that power plant without violating the first law. Already we are aware that more than 50% of the heat transferred to the steam in the furnace is discarded in the condenser which in turn to the environment. The inventor proposes to get rid of this wasteful energy. He suggests sending the steam to the pump as soon as it leaves the turbine, as shown in Fig.5.21. This way, he argues that all the heat transferred to the steam in the boiler will be converted to work. Thus, the power plant will have a theoretical efficiency of 100%. The inventor realises that some heat losses and friction between the moving components are inevitable. He agrees that these effects will hurt the efficiency somewhat. But still expects the efficiency to be above 80% as opposed to 40% in most actual power plants for a carefully designed system. Well, the possibility of doubling the efficiency would certainly an attractive proposition to all of us. Probably most of us like to give this idea a chance, since intuitively we see nothing wrong with it. *But those have studied thermodynamics and have understood second law of thermodynamics will immediately recognise that this device to be a PMM2.* It is because the system works on a cycle and does a net amount of work while exchanging heat with a single reservoir (the furnace) only. It satisfies the first law but violates the second law and therefore, it will not work.

5.10 REVERSIBLE AND IRREVERSIBLE PROCESSES

It is the cherished wish of all of us, especially the engineers, to convert the entire heat completely into work. But, those of us who have understood the laws of thermodynamics can

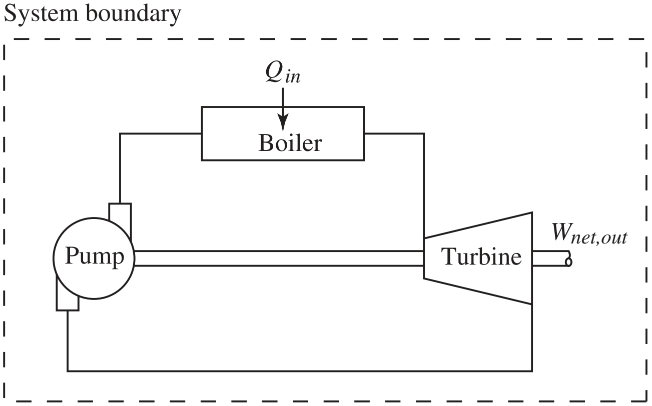


Fig. 5.21 A perpetual-motion machine of second kind (PMM2)

boldly say that it is unattainable. Note that laws of thermodynamics can never be floated since they are the laws of nature. As already discussed, no heat engine can have an efficiency of 100%. Then, the logical questions that will arise in an inquisitive engineer's mind are:

What is the highest efficiency that a heat engine can possibly have? Can the limit of conversion be predicted? If the answer is "yes" can the predicted limit of conversion be realised in practice? If not why? What are the precautions to be taken in the design of an actual engine so that its efficiency tends towards an ideal engine? What is the maximum percentage of conversion of heat to work possible?

Similarly, what is the maximum possible COP of a heat pump or a refrigerator? Can this COP be predicted? If the answer is "yes" can the maximum predicted COP be realised in practice? If not why?

What are the precautions to be taken in the design of a heat pump or a refrigerator so that the performance of an actual heat pump tends towards an ideal reversed heat engine? What is the minimum work required to do the job in an actual practice? We will try to answer *some of these* question in the following sections. Next chapter on entropy will try to answer the remaining ones.

Before we can answer the above questions, first we will define an idealised process, which is called the *reversible process.* The processes that we discussed in the previous sections occurred in a certain direction. Once having taken place, these processes cannot reverse themselves spontaneously and restore the system to its initial state. For this reason, they are classified as *irreversible processes.*

For example, once a cup of hot coffee cools, it will not heat up by retrieving the heat from the surroundings which it has lost. If this could happen, the surroundings, as well as the system (coffee), would be restored to their original condition. Then, such a process can be called a *reversible process.*

Refer Fig.5.22. Let the state of a system be represented by A and let the system be taken to state B by following the path A-B. If the system and also the surroundings are restored to their initial states and no change in the universe is produced, then the process A-B will be a reversible process. In the reverse process, the system has to be taken from state B to A by following the same path B-A. A reversible process should not leave any trace or history to show that the process had ever occurred.

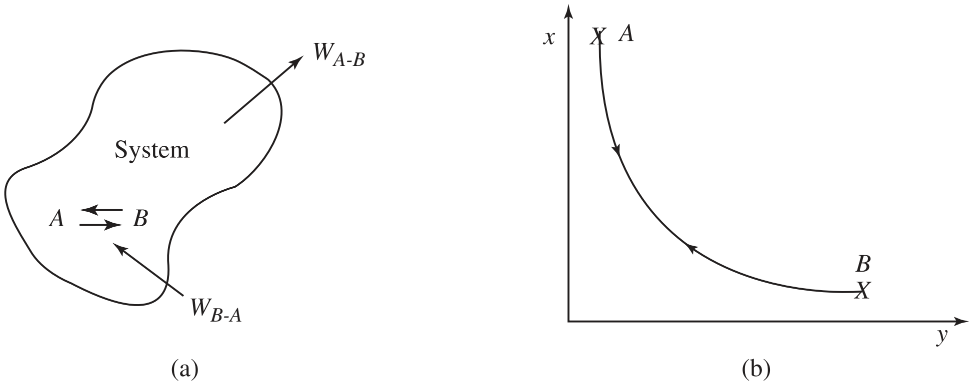


Fig. 5.22 Reversible process

Therefore, a *reversible process* is defined as *a process that can be reversed without leaving any trace on the surroundings.* That is, both the system and the surroundings should return to their initial states at the end of the process. This is possible only if the net heat and net work exchange between the system and the surroundings is zero, for the combined (original and reverse) process. Processes that are not reversible are called *irreversible processes.*

It should be noted that a system can be restored to its initial state following a process, regardless of whether the process is reversible or irreversible. But for a reversible process, this restoration is made without leaving any net change on the surroundings. In an irreversible process, the surroundings usually do some work on the system and therefore, does not return to their original state.

Note that the reversible processes actually do not occur in nature. Therefore, are called *idealised processes.* It is to be pointed out here that reversible processes can only be approximated but they can never be achieved by any actual devices. That is, all the processes occurring in nature are irreversible.

Now you may start wondering, *why* then we should bother with such fictitious processes. There are two reasons.

(i) they are easy to analyse, since a system passes through a series of equilibrium states during a reversible process.

(ii) they serve as idealised models to which actual processes can be compared.

Engineers will definitely be interested in reversible processes. It is because they would like to design the maximum work-producing devices. Typical examples are automobile engines, gas or steam turbines and least work-consuming devices such as compressors, fans and pumps. Reversible processes can be viewed as theoretical limits for the corresponding irreversible ones. Some processes are more irreversible than others. You must be clear that we may never be able to achieve a reversible process but we can certainly approach it. The more closely we approach a reversible process, the more work will be delivered by a work-producing device. Similarly, the less work will be consumed by a work-absorbing device. The concept of reversible processes leads to the definition of the *second law efficiency* for actual processes. This will reveal the degree of approximation to the corresponding reversible processes. Further, this enables us to compare the performance of different devices that are designed to do the same task on the basis of their efficiencies. The better is the design where there is lower irreversibility and higher second-law efficiency. The reasons for a process to be irreversible is due to the irreversibilities inherent in the process. There can be two types of irreversibilities in a process. They are: (i) internal irreversibility and (ii) external irreversibility.

5.11 INTERNALLY AND EXTERNALLY REVERSIBLE PROCESSES

A process is called *internally reversible* if no irreversibilities occur within the boundaries of the system during the process. During an internally reversible process, a system proceeds through a series of equilibrium states. When the process is reversed, the system passes through exactly the same equilibrium states while returning to its initial state. That is, the paths of the forward and reverse processes coincide for an internally reversible process. The quasi-equilibrium process is an example of an internally reversible process.

A process is called *externally reversible* if no irreversibilities occur outside the system boundaries during the process. Heat transfer between a reservoir and a system is an externally reversible process if the outer surface of the system is at the temperature of the reservoir.

A process is called *totally reversible*, or simply *reversible*, if it involves no irreversibilities within the system or its surroundings (Fig.5.23). A totally reversible process involves no heat transfer through a finite temperature difference, no nonquasi-equilibrium changes and no friction or other dissipative effects.

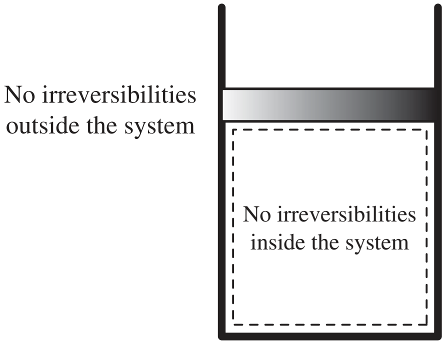


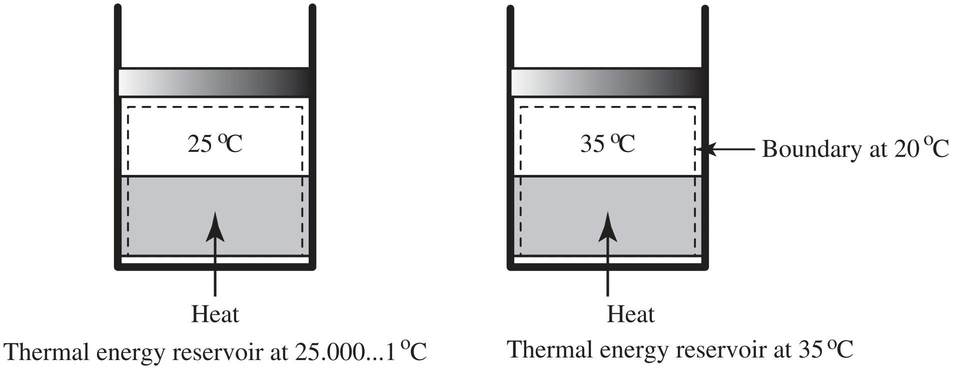
Fig. 5.23 A reversible process involving no internal and external irreversibilities

As an example, consider the transfer of heat to two identical systems that are undergoing a constant-pressure (thus constant-temperature) phase-change process, as shown in Fig.5.24. Both processes are internally reversible. It is because both take place isothermally and both pass through exactly the same equilibrium states. The first process shown is externally reversible also. It is due to the fact that heat transfer for this process takes place through an infinitesimally small temperature difference dT. The second process, however, is externally irreversible, since it involves heat transfer through a finite temperature difference ΔT.

Internal irreversibility is due to internal factors whereas external irreversibility is caused by external factors at the system-surrounding interface. Generic types of irreversibilities are due to

(i) Electrical resistance
(ii) Free expansion
(iii) Friction
(iv) Inelastic solid deformation
(v) Heat transfer through a finite temperature difference
(vi) Non-equilibrium during the process

Difference between reversible and irreversible processes are given in Table 5.1.



(a) Totally reversible (b) Internally reversible

Fig. 5.24 Illustration of reversible process

5.12 CARNOT CYCLE

The efficiency of a heat-engine cycle greatly depends on how the individual processes of the cycle are executed. By suitably analysing this, the net work, thus the cycle efficiency, can be maximised. In order to achieve this we should use processes that require the minimum amount of work input and can deliver the maximum amount of work output. This is possible only by using reversible processes. Therefore, it is no surprise that the most efficient cycles are reversible cycles, that is, cycles that consist entirely of reversible processes.

Table 5.1 Difference between reversible and irreversible processes

	Reversible process		Irreversible process
(i)	Reversible process in practice are not realisable	(i)	All practical processes occurring are in practice irreversible.
(ii)	The process can be carried out in the reverse direction following the same path.	(ii)	When carried out in reverse direction follows a different path from that in forward direction
(iii)	Leaves no traces of occurrence.	(iii)	Leaves traces of occurrence.
(iv)	It can occur in either directions without violating second law of thermodynamics.	(iv)	Occurrence of irreversible processes in either direction in practice is not possible.
(v)	A system undergoing reversible processes has maximum efficiency.	(v)	System having irreversible processes do not have maximum efficiency
(vi)	Reversible process occurs at infinitesimal rate	(vi)	Irreversible processes occur at finite rate
(vii)	System remains throughout in thermodynamic equilibrium during occurrence.	(vii)	System does not remain in thermodynamic equilibrium during occurrence.

Probably the best known reversible cycle is the Carnot cycle. The cycle was first suggested by a French engineer Sadi Carnot in 1824. Any theoretical heat engine that operates on the Carnot cycle is called the Carnot heat engine. The Carnot cycle is composed of four reversible processes two isothermal and two adiabatic. It can be executed either in a closed or a steady-flow system.

Any fluid may be used to operate the Carnot cycle. The schematic is given in Fig.5.25. The cylinder head of the engine is supposed to be perfect conductor or a perfect insulator of heat alternatively. Heat transfer into the cylinder is supposed to take place by the application of high temperature source at temperature T_H *to the cylinder* head during expansion. Heat transfer *from the cylinder* at temperature T_L is by the application of a lower temperature sink to the head during compression.

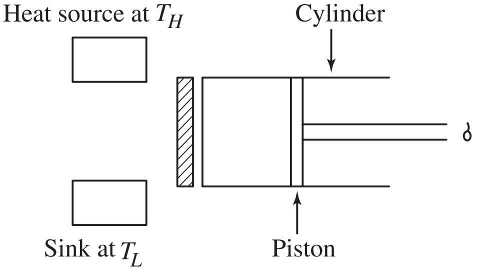


Fig. 5.25 Schematic of a Carnot engine

Now refer Fig.5.26, which consists of a piston cylinder arrangement. Assume that the piston is in the left extreme at position 1. Let there be a certain amount of mass of gas at a temperature T_H and pressure p_1.

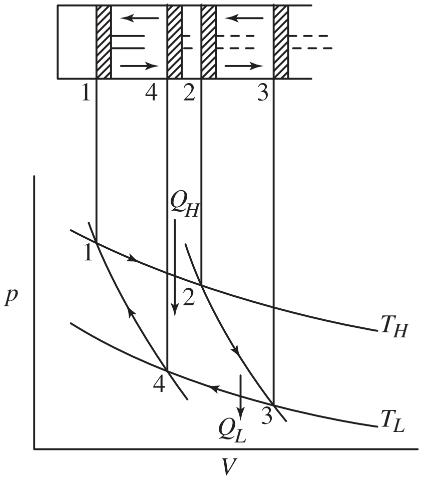


Fig. 5.26 Carnot heat engine (Carnot cycle)

This is the state 1 of the gas as shown in the *p*-*V* diagram. Now let a high temperature source at T_h (T_h is infinitesimally higher than T_H so that for all practical purposes it can be assumed that $T_h = T_H$) be brought in contact with the cylinder head. Assume that heat is be transferred at a very slow rate from the heat source to the gas in the cylinder. This is due to infinitesimally small temperature difference. The gas will start expanding due to the heat input. This will make the piston to move slowly towards right such that the work done by the gas equals heat transfer to the gas. This will keep the gas at the same temperature. This can be construed as the reversible isothermal expansion with temperature remaining constant at T_H. Let this expansion process continue until the gas reach a predetermined state 2. Let the heat transfer to the gas during this expansion process be Q_H. When the gas reaches the state 2 let the high temperature source be removed and in its place an insulating material be brought instantaneously. The expansion of the gas will continue with no heat transfer and consequently the pressure and temperature of the gas will decrease. This expansion process can be construed as reversible and adiabatic. Let the expansion continue until the piston reaches the right extreme position 3. Now the gas is at state 3 with temperature T_3 and pressure p_3. Let a low temperature sink at temperature T_L (where T_L is infinitesimally lower than T_3 so that for all practical purposes it can be assumed that $T_L = T_3$) be brought in contact with the cylinder head instantaneously after removing the insulation material.

Now the gas will start transferring heat very slowly to the sink and the piston will also start moving slowly towards left. Because of the movement of piston towards left the gas gets compressed at the same temperature T_L. This process can be construed as reversible and isothermal. Let this compression process continue until the gas reaches the predetermined state 4. Let the heat rejected during this process be Q_L. When the gas reaches the state 4 let the low temperature sink be removed and in its place an insulating material be brought instantaneously. The compression of the gas will continue with no heat transfer and consequently the pressure and temperature of the gas will increase. This compression process can be construed as reversible and adiabatic. The cycle is completed when the gas reaches its initial state 1 and the piston is at its original left position. The gas is now ready to perform the next cycle in the same sequence. Thus, the Carnot cycle consists of two reversible isothermal processes ($1-2$ expansion at temperature T_H and $3-4$ compression at temperature T_L) and two reversible adiabatic processes ($2-3$ expansion process at temperature T_H and the compression $4-1$ at temperature T_L). The engine operating on this cycle will have the highest efficiency since all the processes are reversible and hence the cycle is reversible. The work delivered from the system during the cycle is represented by the enclosed area of the cycle $1-2-3-4$ [Fig.5.27(a)]. Again for a closed cycle, according to first law of the thermodynamics the work obtained is equal to the difference between the heat supplied by the source (Q_H) and the heat rejected to the sink (Q_L).

$$W = Q_H - Q_L \tag{5.21}$$

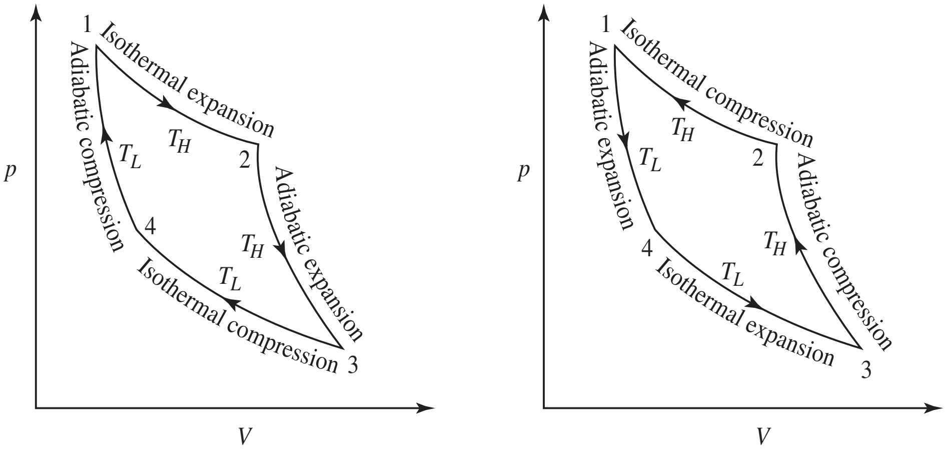
Also thermal efficiency,

$$\eta_{th} = \frac{\text{Work done}}{\text{Heat supplied by the source}} = \frac{Q_H - Q_L}{Q_H} \tag{5.22}$$

$$= 1 - \frac{Q_L}{Q_H} = \left(1 - \frac{T_L}{T_H}\right) \tag{5.23}$$

Now, $Q_H = mc_pT_H$ and $Q_L = mc_pT_L$, where m is mass of fluid. Since, all the processes are reversible it can operate in the reverse direction and will operate as a *heat pump*. Q_L is being taken in at the lower temperature T_L during the isothermal expansion (process $4-3$) and heat Q_H is being rejected at the upper temperature T_H(process $2-1$) [Fig.5.27(b)]. Work, W will be needed to drive the pump. Again, the enclosed area represents this work which is exactly equal to that flowing from it when used as engine.

Carnot engine is only a hypothetical engine. It should be the endeavour of all engineers to design engines which tend towards Carnot engine. The Carnot cycle cannot be put into practice because of the following reasons:



(a) Carnot engine cycle

(b) Carnot heat pump cycle

Fig. 5.27 Carnot cycles

(i) It is impossible to perform a frictionless process.

(ii) It is impossible to transfer the heat without temperature potential.

(iii) Isothermal process can be achieved only if the piston moves very slowly to allow heat transfer so that the temperature remains constant.

(iv) Adiabatic process can be achieved only if the piston moves extremely fast so that the heat transfer is negligible due to very short time available.

(v) The isothermal and adiabatic processes taking place during the same stroke where the piston has to move very slowly for part of the stroke and it has to move very fast during remaining stroke. This variation of motion of the piston during the same stroke is almost impossible to achieve.

5.13 CARNOT THEOREM AND ITS COROLLARIES

Carnot theorem states that: *No engine can have efficiency more than that of reversible engine operating between same temperature limits.*

Referring Fig.5.28, let HE_A and HE_B be the two engines operating between the given source at temperature T_H and the given sink at temperature T_L. Let HE_A be *any* heat engine and HE_B be *any reversible* heat engine. We should analyse and prove that the efficiency of HE_B is higher than that of HE_A.

To start with let us assume that $\eta_A > \eta_B$. Let us also assume that the heat input to both the engines are the same. Mathematically,

$$Q_{HA} = Q_{HB} = Q_H \tag{5.24}$$

Since $\eta_A > \eta_B$

$$\frac{W_A}{Q_{HA}} > \frac{W_B}{Q_{HB}} \tag{5.25}$$

$$W_A > W_B \tag{5.26}$$

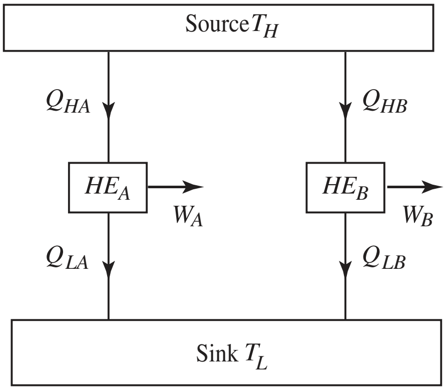
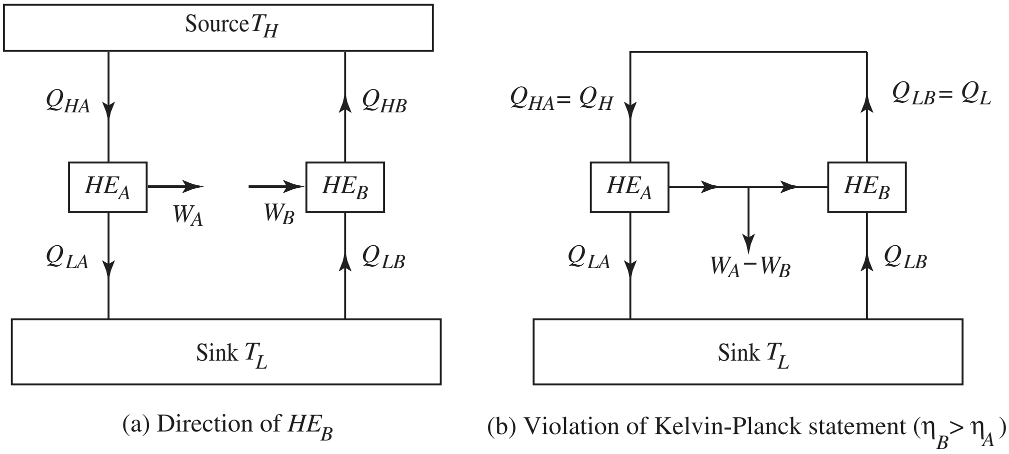


Fig. 5.28 Illustration of Carnot theorem

Now, let us consider the engine, HE_B. Since HE_B is a reversible heat engine, the magnitudes of heat and work transfer quantities will remain the same. However, their directions will be reversed, as shown in Fig.5.29(a). Since $W_A > W_B$, some part of W_A (equal to W_B) may be fed to drive the reversed heat engine, HE_B. Since $Q_{1A} = Q_{1B} = Q_1$, the heat discharged by reversed heat engine HE_B may be supplied to HE_A. The source may, therefore, be eliminated [Fig.5.29(b)]. The net result is that HE_A and reversed heat engine HE_B together constitute a heat engine which, operating in a cycle produces net work $W_A - W_B$ while exchanging heat with a single reservoir at T_L. This violates the Kelvin-Planck statement of the second law. Hence the assumption that $\eta_A > \eta_B$ is wrong.



(a) Direction of HE_B

(b) Violation of Kelvin-Planck statement ($\eta_B > \eta_A$)

Fig. 5.29 Illustration of violation of Kelvin-Planck statement

5.14 COROLLARY OF CARNOT'S THEOREM

The efficiency of all reversible heat engines operating between the same temperature levels is the same. Refer Fig. 5.28. Let both the heat engines HE_A and HE_B be *reversible*. Let us assume $\eta_A > \eta_B$. Similar to the procedure outlined in the previous section, if HE_B is reversed to run say, as a heat pump using some part of the work output (W_A) of heat engine, HE_A, we see that the combined system of heat pump, HE_B and heat engine, HE_A, becomes a PMM2. So η_A cannot be greater than η_B. Similarly, if we assume $\eta_B > \eta_A$ and reverse the engine HE_A, we observe that η_B cannot be greater than η_A. Therefore,

$$\eta_A = \eta_B$$

Since the efficiencies of all reversible engines operating between the same heat reservoirs are the same, *the efficiency of a reversible engine is independent of the nature or amount of the working substance undergoing the cycle.*

5.15 EFFICIENCY OF THE REVERSIBLE HEAT ENGINE

The efficiency of a reversible heat engine in which heat is received solely at T_H is found to be

$$\eta_{rev} = \eta_{max} = 1 - \left(\frac{Q_L}{Q_H}\right)_{rev} = 1 - \frac{T_L}{T_H} \tag{5.27}$$

or

$$\eta_{rev} = \frac{T_H - T_L}{T_H} \tag{5.28}$$

From the above expression, it may be noted that with decrease in T_L and increase in T_H, the efficiency of the reversible cycle increases. Since η is always less than unity, T_L is always greater than zero.

The COP of a refrigerator is given by:

$$COP_{ref} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{\frac{Q_H}{Q_L} - 1} \tag{5.29}$$

For a reversible refrigerator, using $\dfrac{Q_H}{Q_L} = \dfrac{T_H}{T_L}$

$$COP_{rev} = \frac{1}{\dfrac{T_H}{T_L} - 1} \tag{5.30}$$

$$(COP_{ref})_{rev} = \frac{T_L}{T_H - T_L} \tag{5.31}$$

5.16 THERMODYNAMIC TEMPERATURE SCALE

A temperature scale that is independent of the properties of the substances that are used to measure temperature is called a *thermodynamic temperature scale.* In thermodynamic calculations, such a temperature scale provides great conveniences. Derivation of thermodynamic temperature scale can be done using the principle of reversible heat engines.

5.16.1 The Necessity for Thermodynamic Temperature Scale

By now we are aware that it is only a reversible engine can provide maximum work output from the given heat input. Therefore, the efficiency of a reversible engine is the maximum. But the question is that quantitatively what is the maximum conversion possible? Thermodynamic temperature scale concept provides answer for this important question. Further, in the Celsius temperature scale lacunae is removed.

Note that the thermometers with Celsius temperature scale are not calibrated at each 'degree' marking on them. Generally they are calibrated at ice point and steam point. It is because they are reproducible temperatures in the required range. What the thermometer reads in between the calibrated range depends upon the properties of the thermometric substance used. Further, its mode of operation plays an important role. Therefore, it is unlikely that two thermometers of different type give the exactly same reading if inserted into a system at a particular temperature. For that matter, the same type may also not show the same reading. Therefore, there is a necessity for developing a temperature scale by means of a device whose performance depends neither on the properties of the working medium nor on its mode of operation.

In this connection, the Carnot engine can provide the answer. As already explained, the performance of a Carnot engine neither depends on the properties of the working medium nor on its mode of operation. It depends only on the temperature limits between which it operates. You must be aware that under identical conditions a reversible engine operating with air as the working fluid or steam as the working fluid will have the same efficiency. Since, energy reservoirs are characterised by their temperatures, the thermal efficiency of reversible heat engines is a function of the reservoir temperatures only. That is,

$$\eta_{th,rev} = g(T_H, T_L) \tag{5.32}$$

$$\frac{Q_H}{Q_L} = f(T_H, T_L) \tag{5.33}$$

since $\eta_{th} = 1 - \dfrac{Q_L}{Q_H}$. Here, T_H and T_L are the temperatures of the high- and low-temperature reservoirs, respectively. With the help of the three reversible heat engines shown in Fig.5.30, the functional form of $f(T_H, T_L)$ can be developed.

The arrangement of heat engines for the thermodynamic scale development is shown in Fig.5.30. Assume that engines A and C are supplied with the same amount of heat, Q_H, from the high-temperature reservoir at T_H. Engine C rejects Q_L to the low-temperature reservoir at T_L. Engine B receives the heat, Q'_L, rejected by engine A at temperature T'_L and rejects an amount of heat, Q_L, to a reservoir at T_L. Note that the amounts of heat rejected by engines B and C must be the same. It is because engines A and B can be combined into one reversible engine operating between the same two reservoirs as engine C. Thus, the combined engine will have the same efficiency as engine C. Since the heat input to engine C is the same as the heat input to the combined engines A and B, both systems must reject the same amount of heat.

Applying Eq.5.33 to all three engines separately, we get,

For engine A, $$\frac{Q_H}{Q'_L} = f(T_H, T'_L) \tag{5.34}$$

For engine B, $$\frac{Q'_L}{Q_L} = f(T'_L, T_L) \tag{5.35}$$

For engine C, $$\frac{Q_H}{Q_L} = f(T_H, T_L) \tag{5.36}$$

Now consider the identity which corresponds to,

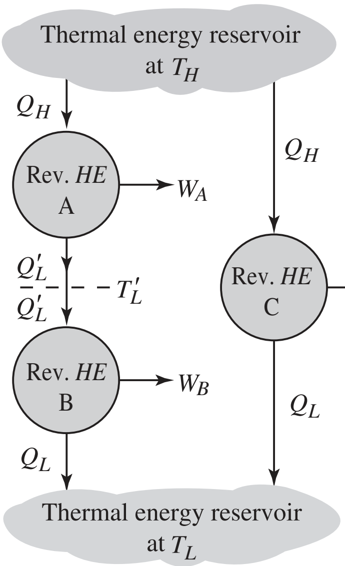


Fig. 5.30 The arrangement of heat engines for the thermodynamic temperature scale

$$\frac{Q_H}{Q_L} = \frac{Q_H}{Q'_L}\frac{Q'_L}{Q_L} \tag{5.37}$$

$$f(T_H, T_L) = f(T_H, T'_L) f(T'_L, T_L) \tag{5.38}$$

$$f(T_H, T'_L) = \frac{\phi(T_H)}{\phi(T'_L)} \tag{5.39}$$

$$f(T'_L, T_L) = \frac{\phi(T'_L)}{\phi(T_L)} \tag{5.40}$$

Substituting Eqs 5.39 and 5.40 in Eq.5.37,

$$\frac{Q_H}{Q_L} = f(T_H, T_L) = \frac{\phi(T_H)}{\phi(T_L)} \tag{5.41}$$

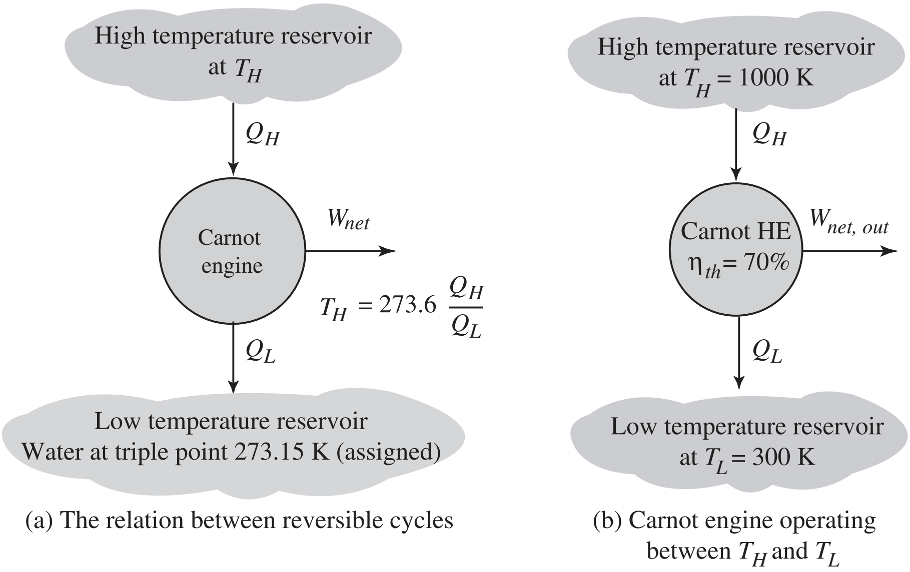
Equation 5.41 is more specific than Eq.5.36,

$$\left(\frac{Q_H}{Q_L}\right)_{rev} = \frac{\phi(T_H)}{\phi(T_L)} \tag{5.42}$$

This is the only requirement that the second law places on the ratio of heat transfers to and from the reversible heat engines. Several functions $\phi(T_H)$ can satisfy this equation. Therefore, the choice is completely arbitrary. It was Lord Kelvin who first proposed taking $\phi(T_H) = T$ to define a thermodynamic temperature scale as (Fig.5.31)(a),

$$\left(\frac{Q_H}{Q_L}\right)_{rev} = \frac{T_H}{T_L} \tag{5.43}$$

This temperature scale is called *Kelvin scale.* The temperatures on this scale are called as *absolute temperatures.* On the Kelvin scale, the temperature ratios depend on the ratios of heat transfer between a reversible heat engine and the reservoirs. They are independent of the physical properties of any substance. On this scale, temperatures vary between zero and infinity.



(a) The relation between reversible cycles

(b) Carnot engine operating between T_H and T_L

Fig. 5.31 A conceptual experimental setup to determine thermodynamic temperature

The thermodynamic temperature scale is not completely defined by Eq.5.43, since it gives us only a ratio of absolute temperatures. We also need to know the magnitude of a kelvin. At the International Conference on Weights and Measures held in 1954, the triple point of water (the state at which all three phases of water exist in equilibrium) was assigned the value 273.15 K [Fig.5.31(a)]. The *magnitude of a kelvin* is defined as 1/273.15 of the temperature interval between absolute zero and the triple-point temperature of water. The magnitudes of temperature units on the Kelvin and Celsius scales are identical (1 K = 1°C). The temperatures on these two scales differ by a constant 273.15.

As can be seen, we have defined the thermodynamic temperature scale with the help of reversible heat engines. Note that it is neither possible nor practical to actually operate such an engine to determine numerical values on absolute temperature scale. Absolute temperatures can be measured accurately by other means, such as constant-volume ideal gas thermometer together with extrapolation techniques. The validity of Eq.5.43 can be demonstrated from physical considerations for a reversible cycle using an ideal gas as the working fluid.

5.17 THE CARNOT HEAT ENGINE

Carnot heat engine is a hypothetical heat engine. It operates on the reversible Carnot cycle. The thermal efficiency of any heat engine whether reversible or irreversible, is given by,

$$\eta_{th} = 1 - \frac{Q_L}{Q_H} \tag{5.44}$$

where Q_H is heat transferred to the heat engine from a high-temperature reservoir at T_H and Q_L is heat rejected to a low-temperature reservoir at T_L. For reversible heat engines, the heat transfer ratio in the above relation can be replaced by the ratio of the absolute temperatures of the two reservoirs, as given by $1 - \frac{T_L}{T_H}$. Then, the efficiency of a Carnot engine or any reversible heat engine, becomes,

$$\eta_{th,rev} \quad = \quad 1 - \frac{T_L}{T_H} \tag{5.45}$$

The above relation is often known as the *Carnot efficiency*, since the Carnot heat engine is the best known reversible engine. *This is the highest efficiency a heat engine operating between the two thermal energy reservoirs at temperatures T_L and T_H can have.* Therefore, all actual heat engines which are irreversible, operating between these temperature limits (T_L and T_H), will have lower efficiencies. It is all due to the irreversibilities associated with the actual cycle. *Note that T_L and T_H in Eq.5.45 are absolute temperatures. Using °C for temperatures in this relation will give wrong results.* The thermal efficiencies of actual and reversible heat engines operating between the same temperature limits compare as follows (Fig.5.32):

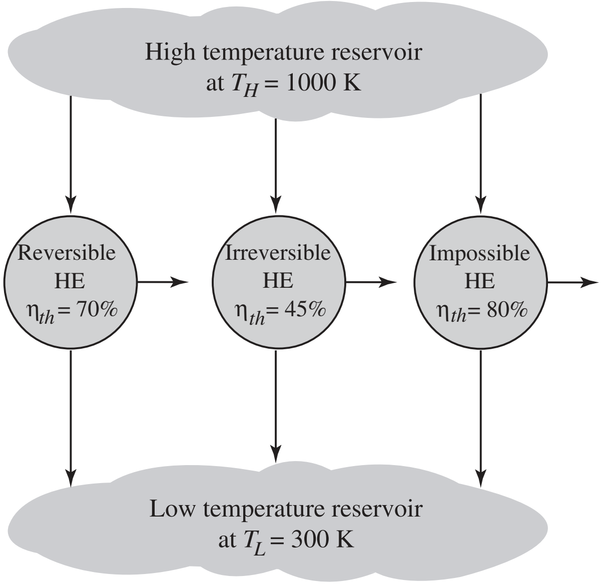


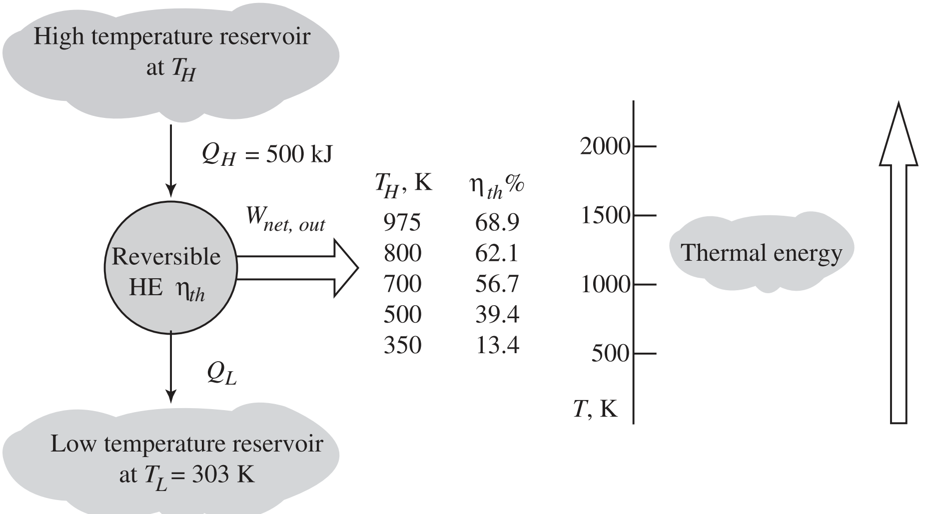
Fig. 5.32 Comparison of efficiency for various engines

Most of the work-producing devices such as heat engines in operation today have efficiencies less than 40%. This is very much lower than 100%. But what you should understand is that it is meaningless to compare efficiencies with 100% Instead, we should compare the value with the efficiency of a reversible heat engine operating between the same temperature limits. It is because this is the true theoretical upper limit for the efficiency and *not 100%.*

Now consider a typical example. The maximum efficiency of a steam power plant operating between T_L = 300 K and T_H = 1000 K. You will find that the efficiency will be equal to 70% $\left(\eta_{th} = \dfrac{T_H - T_L}{T_H}\right)$. Compared with this value, an actual efficiency of 40% is not that bad. There is still lot of scope for improvement. It is obvious from Eq.5.45 that the efficiency of a Carnot heat engine increases as T_H is increased or as T_L is decreased. This is to be expected since as T_L decreases, so does the amount of heat rejection and as T_L approaches zero, the Carnot efficiency approaches unity. This is also true for actual heat engines. *The thermal efficiency of actual heat engines can be maximised by supplying heat to the engine at the highest possible temperature (limited by material strength) and rejecting heat from the engine at the lowest possible temperature (limited by the temperature of the cooling medium such as rivers, lakes or the atmosphere).*

5.18 THE QUALITY OF ENERGY

We are aware that energy has quantity. You should note that it has quality also. Let us explain the quality of energy by an example. Assume a Carnot heat engine shown in Fig.5.33(b) receiving heat from a source at 975 K and converts 68.9% of it to work while rejecting the rest (31.1%) to a sink at 303 K. Now let us examine how the thermal efficiency varies with the source temperature when the sink temperature is held constant. The thermal efficiency of a Carnot heat engine that rejects heat to a sink at (30 °C) 303 K is evaluated at various source temperatures using Eq.5.45 and is listed in Fig.5.33(a). Clearly the thermal efficiency decreases as the source temperature is lowered. When heat is supplied to the heat engine at 500 K instead of 975 K, the thermal efficiency drops from 68.9 to 39.4%. That is, the fraction of heat that can be converted to work drops to 39.4% when the temperature of the source drops to 500 K. When the source temperature is 350 K, this fraction becomes a mere 13.4%.



(a) Effect of source temperature T_H on the efficiency of a Carnot engine

(b) Illustration of quality of energy

Fig. 5.33 Effect of source temperature T_H on the efficiency of a Carnot engine

These efficiency values clearly show that energy has *quality* as well as quantity. It is clear from the thermal efficiency values in Fig.5.33(a) that *more amount of the high-temperature thermal energy can be converted to work. It is obvious that higher the temperature, the higher the quality of the energy* [Fig.5.33(b)].

Large quantities of solar energy, for example, can be stored in large bodies of water called *solar ponds* at about 350 K. This stored energy can then be supplied to a heat engine to produce work (electricity). However, the efficiency of solar pond power plants is very low (under 5%). This is due to the low quality of the energy stored in the source. Further, the construction and maintenance costs are relatively high. Therefore, they are not competitive even though the energy supply to (solar energy) such plants is free. The temperature (and thus the quality) of the solar energy stored could be raised by utilising concentrating collectors. However, the equipment cost in that case becomes very high. Work is a more valuable form of energy than heat since 100% of work can be converted to heat but only a fraction of heat can be converted to work. When heat is transferred from a high-temperature body to a lower temperature one, it is degraded since less of it now can be converted to work. For example, if 100 kJ of heat is transferred from a body at 1000 K to a body at 300 K, at the end we will have 100 kJ of thermal energy stored at 300 K, which has no practical value. But if this conversion is made through a heat engine, up to 1–300/1000=70% of it could be converted to work, which is a more valuable form of energy. Thus, 70 kJ of work potential is wasted as a result of this heat transfer and energy is degraded. To emphasise once again, understand that higher the temperature of the thermal energy, higher is quality.

Worked-Out Examples

5.1 An engine has a heat input of 500 kJ at 437 °C. It rejects 200 kJ at 82 °C. The engine develops 250 kJ of work. Check whether such an engine is possible.

Solution

The maximum efficiency that can be obtained from a reversible engine is given by,

$$\eta\text{max} = 1 - \frac{T_L}{T_H} = 1 - \frac{273+82}{273+437} = 1 - \frac{355}{710} = 0.5 = 50\%$$

For the engine under consideration from the point of view of Q_S and Q_R:

$$Q_S = Q_R + W = 200 + 250 = 450 \text{ kJ}$$

But $Q_S = 500$ kJ and $Q_S \neq Q_R + W$. Therefore, it violates the first law.

The efficiency of the given engine is given by,

$$\eta = \frac{Q_S - Q_R}{Q_S} = \frac{500 - 200}{500} = 0.6 = 60\%$$

As $\eta > \eta\text{max}$, it violates the second law also. Therefore, such an engine is not possible. $\overset{\textbf{Ans}}{\Longleftarrow}$

5.2 A cyclic machine transfers heat from a reservoir at 1100 K to a reservoir at 500 K. It takes 350 kJ of heat from the high-temperature reservoir and delivers 150 kJ of heat to the low-temperature reservoir. The machine develops 200 kJ of work. Check whether such a machine is reversible, irreversible or impossible.

Solution

$$Q_S = Q_R + W = 150 + 200 = 350$$

The given value of Q_S is also 350 kJ. Therefore, it satisfies the first law.

$$\eta_{\text{Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{500}{1100} = 0.5454$$

The efficiency of the cyclic machine is given by $\eta = \frac{W}{Q_S} = \frac{200}{350} = 0.571$

As $\eta > \eta_{\text{Carnot}}$, such machine is not possible. $\xLeftarrow{\textbf{Ans}}$

5.3 Two reversible heat engines A and B are arranged in series, engine A rejecting heat directly to engine B. Engine A receives 180 kJ at a temperature of 422 °C from a hot source, while engine B is in communication with a cold sink at a temperature of 5.5 °C. If the work output of A is twice that of B, find (i) the intermediate temperature between A and B, (ii) the efficiency of each engine and (iii) heat rejected to the cold sink.

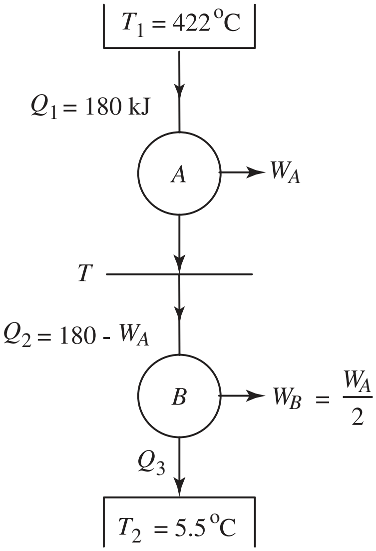


Fig. 5.34

Solution

$$\eta_A = \frac{\text{Work output of } A}{\text{Heat input}} = \frac{W_A}{180} \qquad (i)$$

Efficiency of engine A in terms of intermediate temperature, T,

$$\eta_A = \frac{T_1 - T}{T_1} = 1 - \frac{T}{T_1} = 1 - \frac{T}{422 + 273} \qquad (ii)$$

Equating Eqs (i) and (ii),

$$1 - \frac{T}{695} = \frac{W_A}{180}$$

$$\frac{T}{695} = 1 - \frac{W_A}{180} = \frac{180 - W_A}{180}$$

$$T = \frac{695}{180} \times (180 - W_A) \qquad (iii)$$

Similarly,

$$1 - \frac{278.5}{T} = \frac{W_B}{Q_2} = \frac{W_A}{2 \times (180 - W_A)}$$

$$\frac{278.5}{T} = 1 - \frac{W_A}{360 - 2W_A} = \frac{360 - 2W_A - W_A}{360 - 2W_A}$$

$$T = \frac{278.5 \times (360 - 2W_A)}{360 - 3W_A} \qquad (iv)$$

Equating Eqs (iii) and (iv),

$$\frac{695}{180} \times (180 - W_A) = \frac{278.5 \times 2 \times (180 - W_A)}{360 - 3W_A}$$

Cancelling $(180 - W_A)$ on both sides,

$$3.86 = \frac{557}{360 - 3W_A} = \frac{557}{3 \times (120 - W_A)}$$

$$W_A = 120 - \frac{557}{3 \times 3.86} = 71.9 \text{ kJ}$$

Substituting W_A in Eq.(iii),

$$T = \frac{695}{180} \times (180 - 71.9) = 417.4 \text{ K} = 144.4^\circ \text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_A = \frac{695 - 417.4}{695} = 0.3994 \approx 0.40 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_B = \frac{417.4 - 278.5}{417.4} = 0.333 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$Q_3 = Q_2 - W_B = 180 - W_A - \frac{W_A}{2} = 180 - 1.5W_A$$

$$= 180 - 1.5 \times 71.9 = 72.15 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

5.4 A heat engine operates between the maximum and minimum temperatures of 677 °C and 57 °C, respectively, with an efficiency of 50% of appropriate Carnot efficiency. It drives a heat pump which uses river water at 5 °C to heat a block of flats in which the temperature is to be maintained at 22 °C. Assuming that a temperature difference of

10 °C exists between the working fluid and the river water and the heat pump to operate on the reversed Carnot cycle but with a *COP* of 50% of the ideal COP, find the heat input to the engine per unit heat output from the heat pump. Why is direct heating thermodynamically more wasteful?

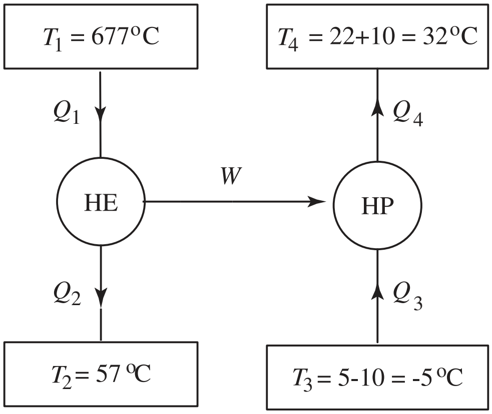


Fig. 5.35

Solution

Consider heat pump (HP)
$T_3 = 5 - 10 = -5°\text{C} = 273 - 5 = 268$ K
$T_4 = 22 + 10 = 32$ °C $= 273 + 32 = 305$ K

Consider heat engine
$T_1 = 677 + 273 = 950$ K
$T_2 = 57 + 273 = 330$ K

$$\eta_{Carnot} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_1}{T_2} = 1 - \frac{330}{950} = 0.653$$

$$\eta_{HE} = 0.5 \times 0.653 = 0.3265$$

$$\eta_{HE} = \frac{W}{Q_1} = 0.3265$$

$$W = 0.3265 \times Q_1 \qquad (i)$$

$$COP_{Carnot} = \frac{T_4}{T_4 - T_3} = \frac{305}{305 - 268} = 8.24$$

$$COP_{HP} = 0.5 \times 8.24 = 4.12$$

$$COP_{HP} = \frac{Q_4}{W}$$

$$W = \frac{Q_4}{COP_{HP}} = \frac{Q_4}{4.12} \qquad (ii)$$

Equating Eqs (i) and (ii),

$$0.3265 \times Q_1 = \frac{Q_4}{4.12}$$

$$\frac{Q_1}{Q_4} = \frac{1}{4.12 \times 0.3265} = 0.7434 \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Heat input per kJ from the heat pump is 0.7434. If it is directly heated by electrical resistance it would require 1 kW. Hence, it is wasteful. $\overset{\textbf{Ans}}{\Longleftarrow}$

5.5 An ice-making plant produces ice at atmospheric pressure and at 0 °C from water. The mean temperature of the cooling water circulating through the condenser of the refrigerating machine is 17 °C. Evaluate the minimum electrical work in kWh required to produce 1 tonne of ice. Enthalpy of fusion of ice at atmospheric pressure is 333.5 kJ/kg.

Solution

$$T_1 = 273 + 17 = 290 \text{ K}$$

$$T_2 = 273 + 0 = 273 \text{ K}$$

$$COP_{Carnot} = \frac{T_2}{T_1 - T_2} = \frac{273}{290 - 273} = 16.06$$

$$COP_{Carnot} = \frac{Q_2}{W} = 16.06$$

$$W = \frac{1000}{3600} \times \frac{333.5}{16.06} = 5.8 \text{ kWh} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

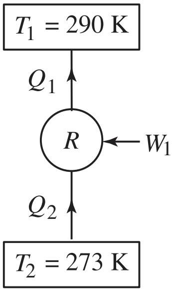


Fig. 5.36

5.6 A reversible engine works between three thermal reservoirs, A, B and C. The engine absorbs an equal amount of heat from the reservoirs A and B kept at temperatures T_A and T_B, respectively and rejects heat to the thermal reservoir C kept at temperature T_C. The efficiency of the engine is α times the efficiency of the reversible engine, which works between two reservoirs A and C. Prove that:

$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha)\frac{T_A}{T_C}$$

Solution

For the reversible engine, $\oint \frac{\delta Q}{T} = 0$. (refer Fig.5.37)

$$-\frac{Q}{T_A} - \frac{Q}{T_B} + \frac{2Q - W}{T_C} = 0$$

$$-Q\left(\frac{1}{T_A} + \frac{1}{T_B}\right) = -\frac{2Q - W}{T_C} \quad (i)$$

$$Q\left(\frac{T_A + T_B}{T_A T_B}\right) = \frac{2Q - W}{T_C}$$

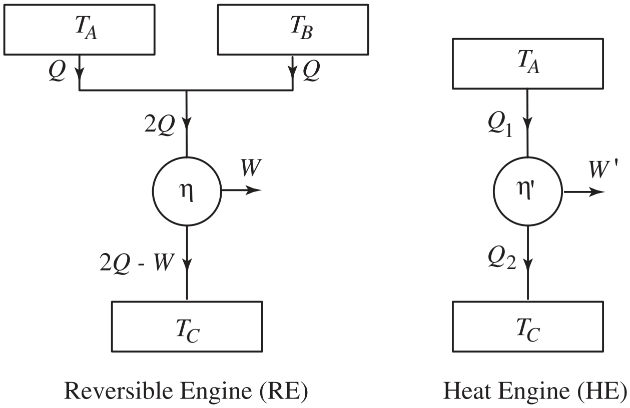


Fig. 5.37

$$\frac{2Q - W}{T_C} = Q\left(\frac{1}{T_A} + \frac{1}{T_B}\right) \qquad (ii)$$

Efficiency of reversible engine, $$\eta' = 1 - \frac{T_C}{T_A}$$

Efficiency of heat engine, $$\eta = 1 - \frac{2Q - W}{2Q} = \alpha\left(1 - \frac{T_C}{T_A}\right)$$

$$1 - \alpha\left(1 - \frac{T_C}{T_A}\right) = \frac{2Q - W}{2Q} = \frac{1}{2}\left(\frac{2Q - W}{Q}\right) \qquad (iii)$$

Substituting Eq.(ii) on the RHS of Eq.(iii),

$$1 - \alpha\left(1 - \frac{T_C}{T_A}\right) = \frac{1}{2} \times T_C\left(\frac{1}{T_A} + \frac{1}{T_B}\right)$$

$$2\left[1 - \alpha\left(1 - \frac{T_C}{T_A}\right)\right] = T_C\left(\frac{1}{T_A} + \frac{1}{T_B}\right)$$

$$2(1 - \alpha) + 2\alpha\frac{T_C}{T_A} = T_C\left(\frac{1}{T_A} + \frac{1}{T_B}\right)$$

Multiplying both sides by $\frac{T_A}{T_C}$,

$$2(1 - \alpha)\frac{T_A}{T_C} + 2\alpha = T_A\left(\frac{1}{T_A} + \frac{1}{T_B}\right) = 1 + \frac{T_A}{T_B}$$

$$\frac{T_A}{T_B} = (2\alpha - 1) + 2(1 - \alpha) \times \frac{T_A}{T_C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

5.7 A reversible engine operates between temperatures T_1 and T, where $T_1 > T$. The energy rejected from this engine is received by a second reversible engine at the same temperature T. The second engine rejects energy at temperature T_2, where $T_2 < T_1$. Show that:

(i) Temperature T is the arithmetic mean of temperatures T_1 and T_2 if the engines produce the same amount of work output and

(ii) Temperature T is the geometric mean of temperatures T_1 and T_2 if the engines have same cycle efficiencies.

Solution

Two reversible engines E_1 and E_2 are in series as shown in Fig.5.38.

$$\eta_1 = 1 - \frac{T}{T_1} = \frac{W_1}{Q_1}$$

$$W_1 = Q_1\left(1 - \frac{T}{T_1}\right) \qquad (i)$$

$$\eta_2 = 1 - \frac{T_2}{T} = \frac{W_2}{Q}$$

$$W_2 = Q\left(1 - \frac{T_2}{T}\right) \qquad (ii)$$

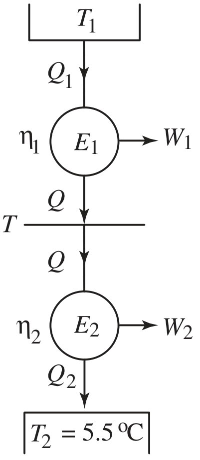


Fig. 5.38

For $W_1 = W_2$, equate Eqs (i) and (ii):

$$Q_1\left(1 - \frac{T}{T_1}\right) = Q\left(1 - \frac{T_2}{T}\right)$$

$$\frac{Q_1}{Q}\left(1 - \frac{T}{T_1}\right) = \left(1 - \frac{T_2}{T}\right)$$

Further, $\dfrac{Q_1}{Q} = \dfrac{T_1}{T}$. Therefore, $\dfrac{T_1}{T}\left(1 - \dfrac{T}{T_1}\right) = 1 - \dfrac{T_2}{T}$

$$\frac{T_1}{T} + \frac{T_2}{T} = 2 \quad \text{or} \quad \frac{T_1 + T_2}{T} = 2$$

$$T = \frac{T_1 + T_2}{2}$$

where the engines produce same amount of output, the intermediate temperature T is the arithmetic mean of temperature T_1 and T_2.

(ii) If $\eta_1 = \eta_2$,

$$1 - \frac{T}{T_1} = 1 - \frac{T_2}{T}$$

$$\frac{T}{T_1} = \frac{T_2}{T} \quad T^2 = T_1 T_2$$

$$T = \sqrt{T_1 T_2}$$

When the engine efficiencies are the same, the intermediate temperature T is the geometric mean of the temperature T_1 and T_2. $\overset{\text{Ans}}{\Longleftarrow}$

5.8 Two Carnot engines A and B are connected in series between two thermal reservoirs maintained at $T_1 = 1000$ K and $T_2 = 100$ K, respectively. Engine A receives 1700 kJ of heat from the high-temperature reservoir and rejects heat to the Carnot engine B. Engine B takes in heat rejected by engine A and rejects heat to the low-temperature reservoir. If engines A and B have equal thermal efficiencies, determine

(i) The heat rejected by the engine B.
(ii) The temperature at which heat is rejected by engine A and
(iii) The work done during the process by engines, A and B, respectively.

If the engines A and B deliver equal work, determine

(iv) The amount of heat taken in by engine B and
(v) The efficiencies of engines A and B.

Solution

$$\begin{aligned}
Q &= Q_1 - W_A = 1700 - 1162.8 = 537.2 \text{ kJ} \\
\eta_B &= 1 - \frac{100}{316.2} = 0.684 = \frac{W_B}{Q} \quad (\because \eta_B = \eta_B) \\
W_B &= 0.684 \times Q = 0.684 \times 537.2 = 367.44 \text{ kJ} \\
Q_2 &= Q - W_B = 537.2 - 367.44 = 169.76 \\
W_A &= Q_1\left(1 - \frac{T}{T_1}\right) \\
W_B &= Q\left(1 - \frac{T_2}{T_1}\right)
\end{aligned}$$

When $W_A = W_B, T = \dfrac{T_1 + T_2}{2}$,

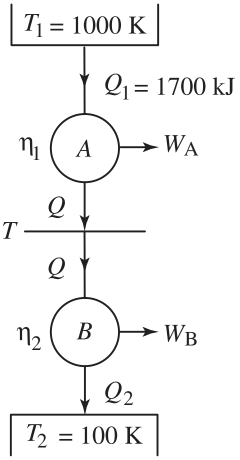


Fig. 5.39

$$\eta_A = 1 - \frac{T}{T_1} \quad \text{and} \quad \eta_B = 1 - \frac{T_2}{T}$$

When $\eta_A = \eta_B$, (refer Example 5.7).

$$\begin{aligned}
T &= \sqrt{T_1 T_2} = \sqrt{1000 \times 100} = 316.22 \text{ K} \\
\frac{Q_1}{T_1} &= \frac{Q_2}{T_2} \\
Q_2 &= \frac{T_2}{T_1} \times Q_1 = \frac{100}{1000} \times 1700 = 170 \text{ kJ} \\
\eta_A &= 1 - \frac{316.22}{1000} = 0.684 = \frac{W_A}{Q_1} \\
W_A &= 0.684 \times Q_1 = 0.684 \times 1700 = 1162.8 \text{ kJ}
\end{aligned}$$

$$T = \frac{1000 + 100}{2} = 550 \text{ K} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

Further, $\dfrac{Q_1}{T_1} = \dfrac{Q}{T}$.

$$Q = \frac{T}{T_1} \times Q_1 = \frac{550}{1000} \times 1700 = 935 \text{ kJ} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\eta_A = 1 - \frac{T}{T_1} = 1 - \frac{550}{1000} = 0.45 \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\eta_B = 1 - \frac{T_2}{T} = 1 - \frac{100}{550} = 0.8182 \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$W_A = \eta_A \times Q_1 = 0.45 \times 1700 = 765 \text{ kJ} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$W_B = Q \times \eta_B = 935 \times 0.8182 = 765 \text{ kJ} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

5.9 A heat pump is to be used to heat a house in winter and then reversed to cool the house in summer. The interior temperature is to be maintained at 17 °C. Heat transfer through the walls and roof is estimated to be 0.5 kJ/s per degree temperature difference between the inside and outside.

(i) If the outside temperature in winter is 5 °C, what is the minimum power required to drive the heat pump?

(ii) If the power output is same as in part (i), what is the maximum outer temperature for which the inside can be maintained at 17 °C?

Solution

Refer Fig.5.40. For the heat pump,

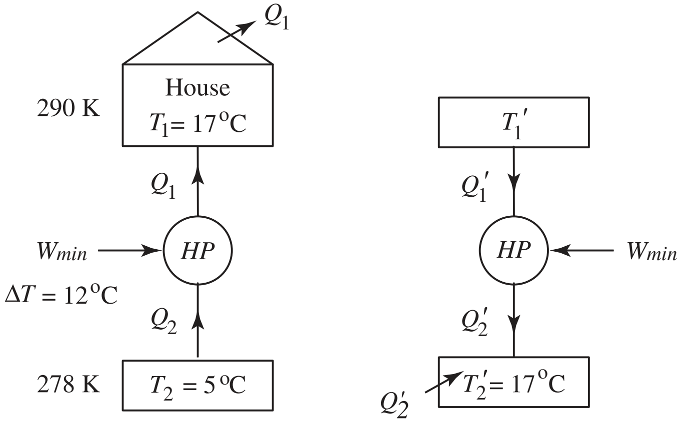


Fig. 5.40

$$Q_1 = 0.5 \times \Delta T = 0.5 \times 12 = 6 \text{ kW}$$

$$COP_{max} = \frac{Q_1}{W_{min}} = \frac{T_1}{T_1 - T_2} = \frac{290}{(290 - 278)} = 24.17$$

$$W_{min} = \frac{Q_1}{COP_{max}} = \frac{6}{24.17} = 0.248 \quad \overset{\text{Ans}}{\Longleftarrow}$$

Let T be the outside temperature, then,

$$Q_2 = 0.5 \times (T - 290) \text{ kW}$$

$$\frac{Q_{2'}}{W} = \frac{T_{2'}}{T_1' - T_{2'}} = \frac{290}{T - 290}$$

$$\frac{0.5 \times (T - 290)}{0.248} = \frac{290}{T - 290}$$

$$(T - 290)^2 = \frac{290 \times 0.248}{0.5} = 143.8$$

$$T - 290 = \sqrt{143.8} \approx 12$$

$$T = 290 + 12 = 302 \text{ K}$$

$$\text{Maximum temperature, } T_1' = 302 - 273 = 29^\circ\text{C} \quad \overset{\text{Ans}}{\Longleftarrow}$$

5.10 A house is to be maintained at a temperature of 17 °C by means of a heat pump pumping heat from the atmosphere. Heat losses through the walls of the house are estimated at 0.66 kW per unit temperature difference between the inside of the house and the atmosphere.

(i) If the atmospheric temperature is −13°C, what is the minimum power required to drive the pump?

(ii) It is proposed to use the same heat pump to cool the house in summer. For the same room temperature, the same heat losses and the same power input to the pump, what is the maximum permissible atmospheric temperature?

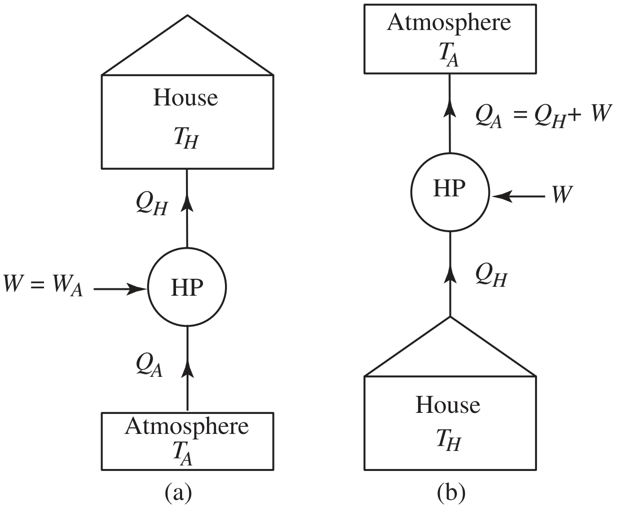


Fig. 5.41

Solution

Refer Fig.5.41(a).

(*i*)
$$\frac{Q_H}{Q_A} = \frac{T_H}{T_A} = \frac{290}{260} = 1.12$$

$$Q_H = 0.66 \times (T_H - T_A) = 0.66 \times (290 - 260) = 19.8 \text{ kW}$$

$$Q_A = \frac{260}{290} \times 19.8 = 17.75 \approx 17.8 \text{ kW}$$

$$-W = Q_A - Q_H \quad \text{or} \quad W = Q_H - Q_A = 19.8 - 17.8 = 2 \text{ kW}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

(*ii*)
$$\frac{Q_H}{Q_A} = \frac{T_H}{T_A} = \frac{290}{T_A}$$

$$Q_H = 0.66 \times (T_A - 290)$$

$$Q_A = Q_H + W = 0.66 \times (T_A - 290) + W$$

$$\frac{Q_A}{Q_H} = \frac{T_A}{T_H} = \frac{0.66 \times (T_A - 290) + W}{0.66 \times (T_A - 290)}$$

$$\frac{T_A}{290} = \frac{0.66 \times (T_A - 290) + 2}{0.66 \times (T_A - 290)}$$

$$\frac{2}{0.66 \times (T_A - 290)} = \frac{T_A}{290} - 1 = \frac{T_A - 290}{290}$$

$$(T_A - 290)^2 = \frac{290 \times 2}{0.66} = 878.78$$

$$T_A - 290 = \sqrt{878.78} = 29.64$$

$$T_A = 290 + 29.64 = 319.64 \approx 320 \text{ K} = 47°\text{C}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

5.11 It takes 10 kW to keep the interior of a certain house at 17 °C when the outside temperature is 7 °C. This heat flow is usually obtained directly by burning gas or oil. Calculate the power required if the 10 kW heat flow were supplied by operating a reversible engine with the house as the upper reservoir and the outside surroundings as the lower reservoir, so that the power were used only to perform work needed to operate the engine.

Solution

For the heat pump,

$$\begin{aligned} \frac{Q_1}{Q_2} &= \frac{T_1}{T_2} \\ \frac{10}{Q_2} &= \frac{290}{280} \\ Q_2 &= \frac{10 \times 280}{290} = 9.655 \text{ kW} \\ W &= Q_1 - Q_2 = 10 - 9.655 = 0.345 \text{ kW} \end{aligned}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

5.12 Prove that the COP of a reversible refrigerator operating between two given temperatures is the maximum.

Solution

Let A and B be the two refrigerators operating between given temperatures T_1 and T_2 [Fig.5.42(a)]. Let B be a reversible refrigerator. Let A be any refrigerator reversible or not. We should prove that, $(COP)_B \geq (COP)_A$.

Let us assume that $(COP)_A \geq (COP)_B$. Then,

$$\frac{Q_{2A}}{W_A} \geq \frac{Q_{2B}}{W_B} \tag{1}$$

Let the two refrigerators work in such a way that they draw the same amount of heat from reservoir 2, i.e. $Q_{2A} = Q_{2B}$. Then, from Eq.(1) $W_B \geq W_A$. Since, B is a reversible refrigerator, we can operate it as a heat engine, by reversing the directions of energy interaction as shown in Fig.5.42(b). As $W_B \geq W_A$, part of W_B can be use to drive the refrigerator A and the reminder $W_B - W_A$ can be delivered as the net work. Since Q_{2A} and Q_{2B} are equal, the sink T_2 can be eliminated [Fig.5.42(c)]. It is seen that A and B together operating in cycles, are producing net work $W_B - W_A$ by exchanging heat with only one reservoir at T_1. This violates Kelvin-Planck statement. Therefore,

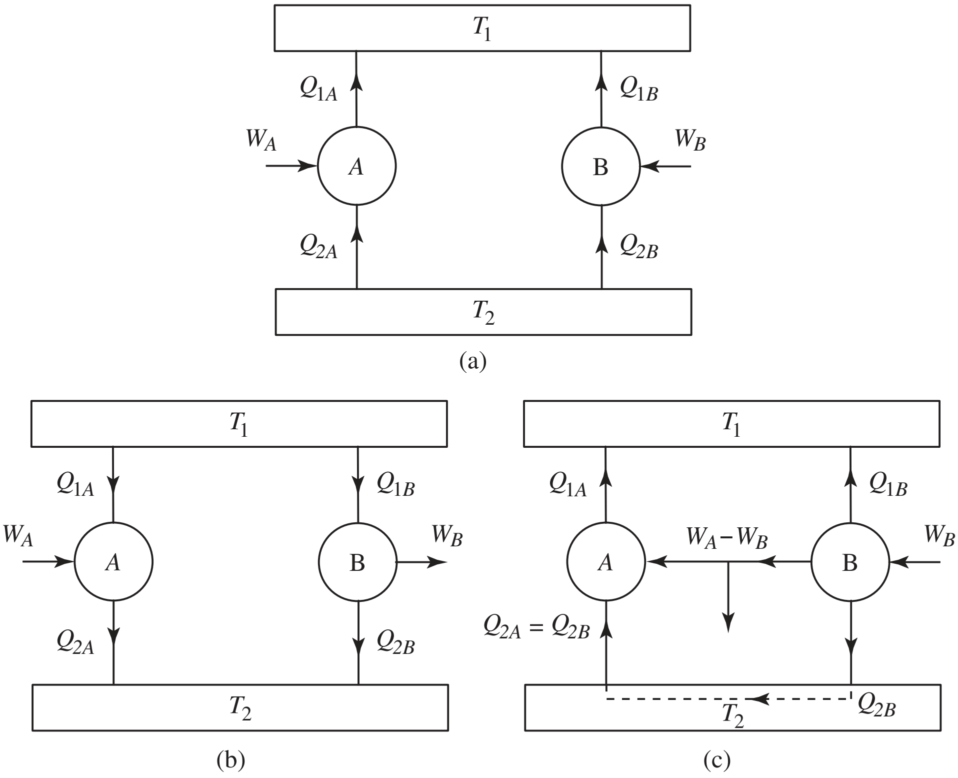


Fig. 5.42

the assumption $(COP)_A \geq (COP)_B$ is wrong. The COP of refrigerator A cannot be greater than COP of the reversible refrigerator B and $(COP)_A = (COP)_B$ only when the refrigerator A is also reversible. $\xLeftarrow{\textbf{Ans}}$

5.13 A heat engine operating between two reservoirs at 727 °C and 27 °C is used to drive a heat pump which extracts heat from the reservoir at 27 °C at a rate twice that at which the engine rejects heat to it. If the efficiency of the engine is 38% of the maximum possible and the COP of the heat pump is 50% of the maximum possible, what is the temperature of the reservoir to which the heat pump rejects heat? What is the rate of heat rejection from the heat pump if the rate of heat supply to the engine is 60 kW?

Solution

For heat engine,
$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2} = \frac{1000}{300}$$

$$Q_2 = \frac{300}{1000} \times Q_1 = 0.3 \times Q_1$$

$$\eta_{max} = 1 - \frac{300}{1000} = 0.7$$

$$\eta = 0.38 \times \eta_{max} = 0.38 \times 0.7 = 0.266$$

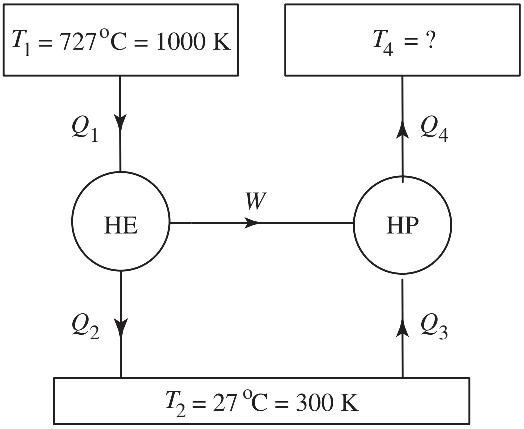


Fig. 5.43

$$\eta = \frac{W}{Q_1} = \frac{W}{W + Q_2}$$

$$\frac{W + Q_2}{W} = \frac{1}{0.266} = 3.76$$

$$1 + \frac{Q_2}{W} = 3.76$$

$$\frac{Q_2}{W} = 2.76$$

For heat pump, $$COP = \frac{2Q_2 + W}{W} = \frac{2Q_2}{W} + 1 = 2 \times 2.76 + 1 = 6.52$$

$$COP = 0.5 \times \frac{T_4}{T_4 - 300}$$

$$6.52 = \frac{0.5 \times T_4}{T_4 - 300}$$

$$T_4 - 300 = \frac{0.5}{6.52} \times T_4 = 0.077 \times T_4$$

$$T_4 = \frac{300}{0.923} = 325 \text{ K} \quad \text{and} \quad Q_1 = 60 \text{ kW}$$

$$W = 0.266 \times 60 = 15.96 \approx 16 \text{ kW and } \; Q_2 = 60 - 16 = 44 \text{ kW}$$

$$Q_4 = 2 \times Q_2 + W = 2 \times 44 + 16 = 104 \text{ kW}$$

Ans ⇐

5.14 A solar-powered heat pump receives heat from a solar collector at T_H, rejects heat to the atmosphere at T_A and pumps heat from a cold space at T_C. The three heat transfer rates are Q_H, Q_A and Q_C, respectively. Derive an expression for the minimum ratio Q_H/Q_C, in terms of the three temperatures. If T_H= 500 K, T_A = 400 K, T_C = 300 K,

$Q_C = 10$ kW, what is the minimum Q_H? If the collector captures 0.2 kW/m^2, what is the minimum collector area required?

Solution

Assume that the heat engine and heat pump are reversible.

$$\eta = \frac{W}{Q_H} = \frac{T_H - T_A}{T_H}$$

$$W = \frac{T_H - T_A}{T_H} \times Q_H$$

$$\frac{Q_C}{W} = \frac{T_C}{T_A - T_C}$$

$$Q_C = \frac{T_C}{T_A - T_C} \times W$$

$$= \frac{T_C}{T_A - T_C} \times \frac{T_H - T_A}{T_H} \times Q_H$$

$$\frac{Q_H}{Q_C} = \frac{(T_A - T_C)T_H}{T_C(T_H - T_A)}$$

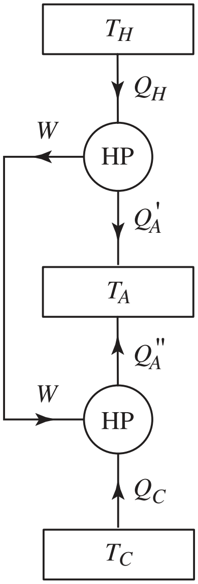


Fig. 5.44

Substituting the values,

$$Q_{H_{min}} = \frac{Q_C}{T_C} \times \frac{(T_A - T_C)}{(T_H - T_A)} \times T_H$$

$$= \frac{10}{300} \times \frac{(400-300)}{(500-400)} \times 500 = 16.7 \text{ kW} \overset{\textbf{Ans}}{\Longleftarrow}$$

Minimum collector area required, $\frac{16.7}{0.2} = 83.5$ m^2. $\overset{\textbf{Ans}}{\Longleftarrow}$

5.15 A reversible power cycle is used to drive a reversible heat pump cycle. The power cycle takes in Q_1 heat units at T_1 and rejects Q_2 at T_2. The heat pump abstracts Q_4 from the sink at T_4 and discharges Q_3 at T_3. Develop an expression for the ratio Q_4/Q_1 in terms of the four temperatures.

Solution

Refer Fig.5.45.

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$$

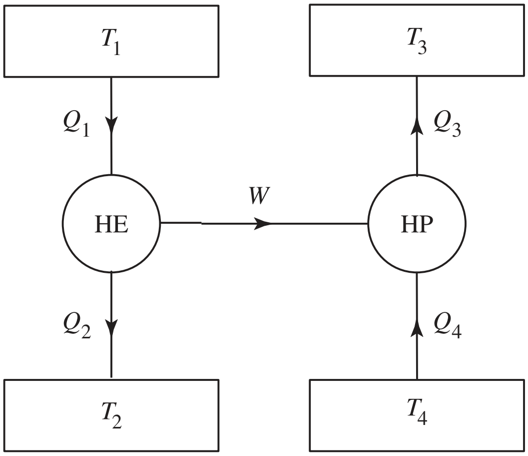


Fig. 5.45

$$W = (T_1 - T_2)\frac{Q_2}{T_2} = (T_1 - T_2)\frac{Q_1}{T_1} \qquad (i)$$

Again, $$\frac{Q_3}{Q_4} = \frac{T_3}{T_4}$$

$$\frac{Q_3 - Q_4}{Q_4} = \frac{(T_3 - T_4)}{T_4}$$

$$W = (T_3 - T_4) \times \frac{Q_4}{T_4} \qquad (ii)$$

From Eqs (i) and (ii), $$(T_1 - T_2) \times \frac{Q_1}{T_1} = (T_3 - T_4) \times \frac{Q_4}{T_4}$$

$$\frac{Q_4}{Q_1} = \frac{T_4}{T_1} \times \frac{(T_1 - T_2)}{(T_3 - T_4)}$$ **Ans** ⟸

5.16 A reversible heat engine in a satellite operates between a hot reservoir at $T_1 = 1250$ K and a radiating panel at T_2. Radiation from the panel is proportional to its area and to T_2^4. For a given work output and T_1 show that area of the panel is minimum when $\frac{T_2}{T_1} = 0.75$. Estimate the minimum area required for an output of 6 kW if the constant of proportionality is 5.55×10^{-8} W/m^2 K^4.

Solution

$$\eta_{max} = \frac{W}{Q_1} = \frac{T_1 - T_2}{T_1} \qquad (i)$$

$$\frac{W}{T_1 - T_2} = \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{kAT_2^4}{T_2} = kAT_2^3$$

where A is the area of the panel

$$A = \frac{W}{kT_2^3(T_1 - T_2)} = \frac{W}{k(T_1T_2^3 - T_2^4)} \qquad (ii)$$

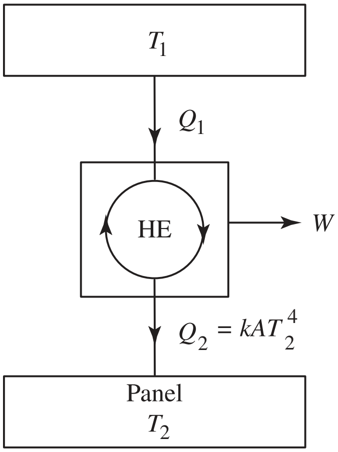


Fig. 5.46

For a given W and T_1 [Eqs (i) and (ii)], A will be the minimum, when

$$\frac{dA}{dT_2} = \frac{-W}{k}\left(3T_1T_2^2 - 4T_2^3\right)\left(T_1T_2^3 - T_2^4\right)^{-2} = 0$$

Since $\left(T_1T_2^3 - T_4^4\right)^{-2} \neq 0$

$$3T_1T_2^2 = 4T_2^3$$

$$\frac{T_2}{T_1} = \frac{3}{4} = 0.75$$

When we substitute $T_2 = 0.75T_1$ in Eq.(ii),

$$A_{min} = \frac{W}{k}\frac{1}{T_1 \times 0.75^3 \times T_1^3 - 0.75^4 \times T_1^4}$$

Now, substituting the given values of W, k and T_1,

$$= \frac{6000}{5.55 \times 10^{-8}} \times \left(\frac{1}{1250 \times 0.75^3 \times 1250^3 - 0.75^4 \times 1250^4}\right)$$

$$= 0.41985 \text{ m}^2$$

Ans ⟸

5.17 A heat engine operating between two reservoirs at temperature 627 °C and 47 °C. It drives a refrigerator operation between reservoirs at temperatures of 47 °C and −13 °C. The heat transfer to the heat engine is 2500 kJ and the net work output of the combined engine and refrigerator plant is 400 kJ. The efficiency of the heat engine and the *COP* of the refrigerator are each 40% of their maximum possible values. (a) Estimate the heat transfer to the refrigerant and the net heat transfer to the reservoir at 47 °C. Reestimate (a) if the efficiency of the heat engine and *COP* of the refrigerator are each 40% of their maximum value.

Solution

The typical layout is given in Fig.5.47,

$$\eta_{max} = \frac{T_1 - T_2}{T_1} = \frac{900 - 320}{900} = 0.644$$

$$\eta_{max} = \frac{W_1}{Q_1} = 0.644$$

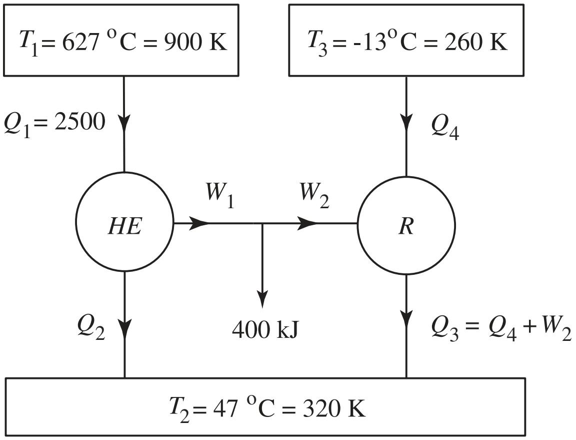


Fig. 5.47

$$W_1 = 0.644 \times 2500 = 1610 \text{ kJ}$$

Maximum COP of the refrigeration cycle,

$$COP_{max} = \frac{T_3}{T_2 - T_3} = \frac{260}{320 - 260} = 4.33$$

$$COP_{max} = \frac{Q_4}{W_2} = 4.33$$

Given $W_1 - W_2 = W = 400$ kJ,

$$W_2 = W_1 - W = 1610 - 400 = 1210 \text{ kJ}$$

$$Q_4 = 4.33 \times W_2 = 4.33 \times 1210 = 5239.3 \text{ kJ}$$

$$Q_3 = Q_4 + W_2 = 5239.3 + 1210 = 6449.3 \text{ kJ}$$

$$Q_2 = Q_1 - W_1 = 2500 - 1610 = 890 \text{ kJ}$$

Heat rejection at 40 °C reservoir,

$$Q_2 + Q_3 = 890 + 6449.3 = 7339.3 \text{ kJ}$$ **Ans** ⟸

(b) Efficiency of the actual heat engine cycle,

$$\eta = 0.4 \times \eta_{max} = 0.4 \times 0.644 = 0.2576$$

$$W_1 = \eta \times Q_1 = 0.2576 \times 2500 = 644 \text{ kJ}$$

$$W_2 = W_1 - 400 = 644 - 400 = 244 \text{ kJ}$$

COP of the actual refrigeration cycle,

$$\begin{aligned} COP &= \frac{Q_4}{W_2} = 0.4 \times 4.33 = 1.732 \\ Q_4 &= W_2 \times COP = 244 \times 1.732 = 422.6 \text{ kJ} \\ Q_3 &= Q_4 + W_2 = 422.6 + 244 = 666.6 \text{ kJ} \\ Q_2 &= Q_1 - W_1 = 2500 - 644 = 1856 \text{ kJ} \end{aligned}$$

Heat rejection to the 40 °C reservoir,

$$Q_2 + Q_3 = 1856 + 666.6 = 2522.6 \text{ kJ} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

5.18 A domestic food freezer maintains a temperature of −20°C. The ambient air is at 33 °C. If heat leaks into the freezer at a continuous rate of 1.8 J/s, what is the minimum power necessary to pump the heat out continuously?

Solution

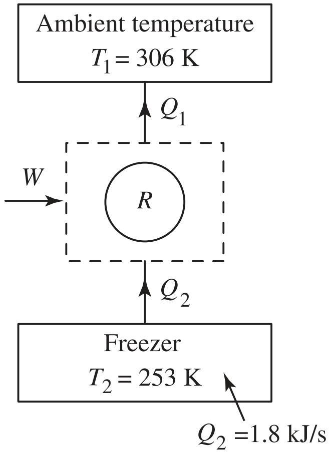


Fig. 5.48

For the freezer (Fig.5.48)

$$\frac{Q_2}{T_2} = \frac{Q_1}{T_1}$$

$$Q_1 = Q_2 \frac{T_1}{T_2} = 1.8 \times \frac{306}{253} = 2.177 \text{ kW}$$

$$W = Q_1 - Q_2 = 2.177 - 1.8 = 0.377 \text{ kW}$$

This is the minimum power required. $\overset{\text{Ans}}{\Longleftarrow}$

5.19 Prove by means of a suitable analysis the effective way to increase the efficiency of a Carnot engine, is to decrease T_L keeping T_H constant.

Solution

The efficiency of Carnot engine is given by,

$$\eta = 1 - \frac{T_L}{T_H + \Delta T} \qquad (i)$$

where L is the sink temperature and H is the source temperature.

As can be seen from Eq.(i), η can be increased either by decreasing T_L keeping T_H constant or by increasing T_H keeping T_L constant.

The question is which is advantageous? (i) to reduce T_L or (ii) to increase T_H. Let us analyse.

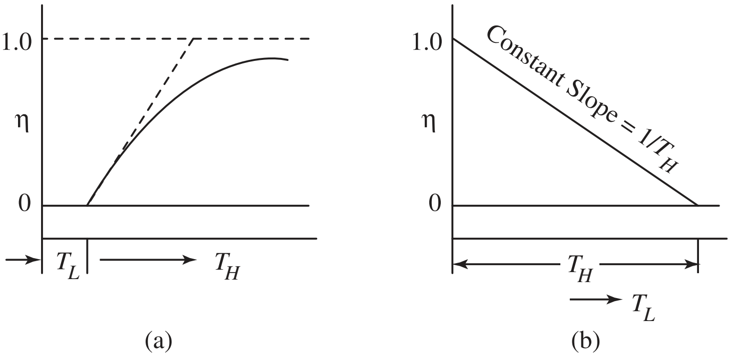


Fig. 5.49

If T_L is kept constant then,

$$\left(\frac{\partial\eta}{\partial T_H}\right)_{T_L} = \frac{T_L}{T_H^2} \tag{ii}$$

Now, if we increase T_H keeping T_L constant from Eq.(ii), it is clear that the slope (rate of increase of η with respect to T_H) decreases. This is clearly seen in Fig.5.49(a). On the other hand, if T_H is kept constant, then,

$$\left(\frac{\partial\eta}{\partial T_L}\right)_{T_H} = -\frac{1}{T_H} \tag{iii}$$

As we decrease T_L, as stated earlier η increases but the slope (the rate of increase of η with respect to T_L) remains constant. This clearly seen in Fig.5.48(b). Further,

$$\left(\frac{\partial\eta}{\partial T_H}\right)_{T_H} = -\frac{T_L}{T_H^2}$$

and
$$\left(\frac{\partial\eta}{\partial T_L}\right)_{T_H} = -\frac{1}{T_H} = \frac{T_H}{T_H^2}$$

Therefore, the better way to increase the efficiency of a Carnot engine is to decrease the T_L (sink temperature) keeping T_H constant (source temperature). $\overset{\textbf{Ans}}{\Longleftarrow}$

5.20 Three engines are in operation. Each of them is supplied with 1000 kJ/s of heat. The source and sink temperatures are 577 °C and 152 °C. The heat rejection of each is given as (i) 800 kW; (ii) 500 kW; and (iii) 200 kW. Determine whether each case represents the reversible, irreversible or impossible heat engines.

Solution

$$\eta_{max} = \frac{T_H - T_C}{T_H} = \frac{850-425}{850} = \frac{425}{850} \times 100 = 50\%$$

Case 1:

$$\eta_1 \quad = \quad \frac{Q_s - Q_r}{Q_s} = \frac{1000 - 800}{1000} = 20\%$$

As $\eta_1 < \eta_{max}$, it is an irreversible engine. $\overset{\textbf{Ans}}{\Longleftarrow}$

Case 2:

$$\eta_2 \quad = \quad \frac{Q_s - Q_r}{Q_s} = \frac{1000 - 500}{1000} = 50\%$$

As $\eta_2 = \eta_{\text{max}}$, it is a reversible engine, since by the second law, all reversible engines have the same efficiency. $\overset{\textbf{Ans}}{\Longleftarrow}$

Case 3:

$$\eta_3 \quad = \quad \frac{Q_s - Q_r}{Q_s} = \frac{1000 - 200}{1000} = 80\%$$

As $\eta_2 > \eta_{\text{max}}$. This engine is impossible. $\overset{\textbf{Ans}}{\Longleftarrow}$

5.21 An inventor claims to have developed a refrigerating machine which can maintain the refrigerated space at −6 °C while the room temperature is 27 °C and has a *COP* of 8.5. Find whether this claim is correct or not.

Solution

The *COP* of the refrigerator working under reversed Carnot cycle is given by,

$$COP \quad = \quad \frac{T_L}{T_H - T_L} = \frac{-6 + 273}{(27 + 273) - (-6 + 273)} = \frac{267}{33} = 8.09$$

As *COP* of the inventor (8.5) is greater than the *COP* of Carnot, his claim is not correct. $\overset{\textbf{Ans}}{\Longleftarrow}$

5.22 Assume that there are two sources of energy supply. Source 1 can supply 12600 kJ/min at 327 °C and source 2 can supply 1 lakh kJ/min more than source 1 at 77 °C. As an engineer which source would you choose to supply energy for an ideal reversible cycle heat engine, given the ambient temperature is 37 °C.

Solution

Consider source 1.

Rate of energy supply = 12600 kJ/min

$$T_{H_1} \quad = \quad 327 + 273 = 600 \text{ K}$$

Consider source 2.

$$\begin{aligned}
\text{Rate of energy supply} &= 100000 + 12600 = 112600 \text{ kJ/min} \\
T_{H_2} &= 77 + 273 = 350 \text{ K} \\
T_L &= 37 + 273 = 310 \text{ K}
\end{aligned}$$

Now, consider a Carnot engine working on the two source temperature and the same sink temperature. The efficiency is given by,

$$\begin{aligned}
\eta_{c_1} &= 1 - \frac{T_L}{T_{H_1}} = 1 - \frac{310}{600} = 0.483 = 48.3\% \\
\eta_{c_2} &= 1 - \frac{T_{H_2}}{T_L} = 1 - \frac{310}{350} = 0.114 = 11.4\%
\end{aligned}$$

Work delivered by the two sources are:

$$\begin{aligned}
W_1 &= 12600 \times 0.483 = 6085 \text{ kJ/min} \\
&= \frac{6085}{60} = 101.4 \text{ kJ/s} = 101.4 \text{ kW} \\
W_2 &= 112600 \times 0.114 = 12836.4 \text{ kJ/min} = 213.9 \text{ kJ/s} = 213.9 \text{ kW}
\end{aligned}$$

Selected power plant is 2. $\stackrel{\textbf{Ans}}{\Longleftarrow}$

Note: Even through the efficiency of the second plant is much lower than the first, it has been chosen because it produces much larger power output. $\stackrel{\textbf{Ans}}{\Longleftarrow}$

5.23 A heat engine is used to drive a heat pump. The heat transfer from the heat engine and from the heat pump is used to heat the water circulating through the radiators around the building. The efficiency of the heat engine is 27% and the COP of the heat pump is 4. (i) Show the arrangement by means of a neat diagram. (ii) Evaluate the ratio of heat transfer to the circulating water to the heat transfer to the heat engine.

Solution

The diagram showing the arrangement is given in Fig.5.50. As can be seen the heat engine is working between the source and sink temperature of T_2 and T_1, respectively. Assume that Q_{S_1} is the heat supplied to the engine from the source at temperature, T_2. The engine rejects an amount of heat Q_{R_1} to the sink at temperature T_1. It supplies work W, to the heat pump as shown in the figure. The heat pump receives a heat of Q_{S_2} from a source at temperature T_3 K and rejects an amount of heat Q_{R_2} to the sink at temperature T_1 it receives an amount of work $W = Q_{S_1} - Q_{R_1}$ from the engine. It is required to find the ratio of $\dfrac{Q_{R_1} + Q_{R_2}}{Q_{S_1}}$.

Efficiency of the heat engine,

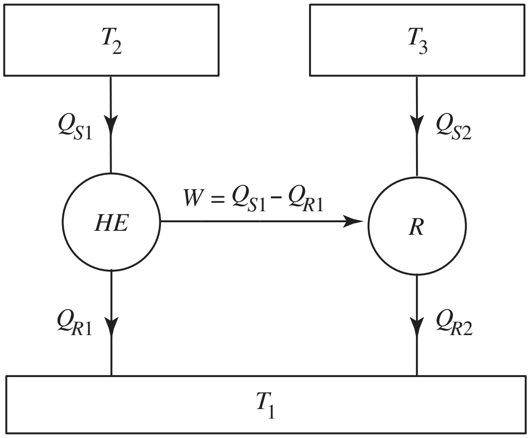


Fig. 5.50

$$\eta_{HE} = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{Q_{S_1} - Q_{R_1}}{Q_{S_1}} = 1 - \frac{Q_{R_1}}{Q_{S_1}}$$

Given that $\eta_{HE} = 0.27$, we have $1 - \dfrac{Q_{R_1}}{Q_{S_1}} = 0.27$; $\dfrac{Q_{R_1}}{Q_{S_1}} = 0.73$; $Q_{R_1} = 0.73 Q_{S_1}$.

For the heat pump coefficient of performance,

$$COP_{HP} = \frac{\text{Heat rejected}}{\text{Work input}} = \frac{Q_{R_2}}{Q_{S_1} - Q_{R_1}} = 4; \quad \frac{Q_{R_2}}{Q_{S_1} - 0.73 Q_{S_1}} = 4$$

$$\frac{Q_{R_2}}{0.27 \times Q_{S_1}} = 4$$

$$Q_{R_2} = 1.08 \times Q_{S_1}$$

Total heat rejected to water,

$$Q = Q_{R_1} + Q_{R_2}$$

$$Q = 0.73 \times Q_{S_1} + 1.08 \times Q_{S_1} = 1.81 \times Q_{S_1}$$

$$\frac{Q}{Q_{S_1}} = \frac{1.81 \times Q_{S_1}}{Q_{S_1}} = 1.81 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

5.24 A Carnot heat engine takes heat from an infinite reservoir at 550 °C and rejects it to a sink at 275 °C. Half of the work delivered by the engine is used to run a generator and the other half is used to run heat pump which takes heat at 275 °C and rejects it at 440 °C. Express the heat rejected at 440 °C by the heat pump as a percentage of heat supplied to the engine at 550 °C. If the operation of the generator requires 500 kW, find the heat rejected per hour by the heat pump at 440 °C.

Solution

For Carnot engine, $\dfrac{Q_{S_1}}{T_1} = \dfrac{Q_{R_1}}{T_2}$

$$Q_{S_1} = \frac{T_1}{T_2} \times Q_{R_1} = \frac{550 + 273}{275 + 273} \times Q_{R_1} = \frac{823}{548} \times Q_{R_1} = 1.5 \times Q_{R_1}$$

Work output of the heat engine,

$$W = Q_{S_1} - Q_{R_1} = 1.5 \times Q_{R_1} - Q_{R_1} = 0.5 \times Q_{R_1} = \frac{Q_{R_1}}{2} \qquad (i)$$

Generator input,

$$W_g = 0.5 \times \text{Work output of heat engine}$$

$$W_g = \frac{0.5 \times Q_{R_1}}{2} = 0.25 \times Q_{R_1}$$

Work input to the heat pump $W_{HP} = 0.25 \times Q_{R_1}$

Heat rejected by the heat pump

$$Q_{R_2} = Q_{S_2} + W_{HP} = Q_{S_2} + 0.25 \times Q_{R_1} \qquad (ii)$$

For reverse heat pump, $\dfrac{Q_{S_2}}{T_2} = \dfrac{Q_{R_2}}{T_4}$

$$Q_{S_2} = \frac{T_2}{T_4} \times Q_{R_2} = \frac{548}{713} \times Q_{R_2} \approx 0.768 = 0.77 \times Q_{R_2}$$

$$Q_{S_2} = 0.77 \times Q_{R_2}$$

Substituting Q_{S_2} in Eq.(ii),

$$Q_{R_2} = 0.77 \times Q_{R_2} + 0.25 \times Q_{R_1} = 0.77 \times Q_{R_2} + 0.25 \times \frac{1}{1.5} \times Q_{S_1}$$

$$(1 - 0.77) \times Q_{R_2} = \frac{0.25}{1.5} \times Q_{S_1}$$

$$0.23 \times Q_{R_2} = 0.166 \times Q_{S_1}$$

$$Q_{R_2} = \frac{0.166}{0.23} = 0.722 \times Q_{S_1}$$

$$\frac{Q_{R_2}}{Q_{S_1}} = 0.722 = 72.2\%$$

The generator power input, $W_g = 500$ kW

$$0.25 \times Q_{R_1} = 500$$

$$Q_{R_1} = \frac{500}{0.25} = 2000 \text{ kW}$$

$$Q_{S_1} = 1.5 \times Q_{R_1} = 1.5 \times 2000 = 3000 \text{ kW}$$

$$Q_{R_2} = 0.722 \times 3000 = 2166 \text{ kW} = \frac{2166 \times 3600}{1000} = 7797.6 \text{ MJ/h} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

5.25 A heat engine operates between a source temperature of 600 °C and a sink at 60 °C. Determine the least rate of heat rejection (kJ/s) per kW of net output of the engine.

Solution

Carnot efficiency of the heat engine,

$$\eta_{HE} = \frac{T_H - T_L}{T_H} \times 100 = \frac{873 - 333}{873} \times 100 = 61.86\%$$

$$\eta_{HE} = \frac{W}{Q_S} = \frac{1}{Q_S}$$

Taking work output as 1 kW,

$$Q_S = \frac{1}{0.6186} = 1.616$$

$$W = Q_S - Q_R$$

$$Q_R = Q_S - W = 1.616 - 1$$

$$= 0.616 \text{ kJ/s/kW of power output}$$

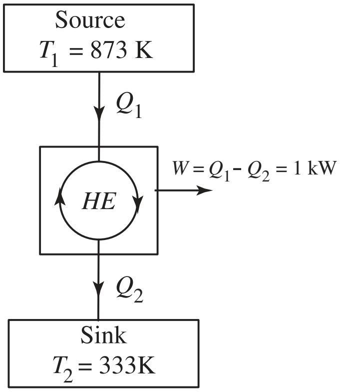


Fig. 5.51

Review Questions

5.1 *What does the first law establish and what is its deficiency?*

5.2 *What is the necessity for second law?*

5.3 *Explain by means of suitable example why the second law is required for meaningful analysis.*

5.4 *What is the basic principle involved in the second law?*

5.5 *With a neat figure bring out the concept of source and sink.*

5.6 *Work can be converted to other form of energy but other form of energy cannot be easily converted to work. Establish this statement.*

5.7 *What is a heat engine? Explain.*

5.8 *What is meant by coefficient of performance? Explain.*

5.9 *Explain by means of a suitable sketch the working principle of a heat pump.*

5.10 *Show that COP of a heat pump is greater than COP of a refrigerator.*

5.11 *What are the two classical statement of second law of thermodynamics?*

5.12 *State Kelvin-Planck statement and explain by means of suitable sketch the meaning of Kelvin-Planck statement.*

5.13 *State Clausius statement and explain by means of suitable sketch the meaning of Clausius statement.*

5.14 *By suitable arguments establish the equivalence of both statements.*

5.15 *Does the refrigerator used in households violate the clausis statement of the second law of thermodynamics. Explain.*

5.16 *What is a perpetual motion machine? Explain PMM2.*

5.17 *Explain reversible and irreversible process from the point of view of second law of thermodynamics.*

5.18 *State the differences between reversible and irreversible processes.*

5.19 *Explain the Carnot cycle engine with a suitable sketch.*

5.20 *State Carnot theorem and explain.*

5.21 *Derive the expression for the efficiency of a reversible heat engine.*

5.22 *What is the necessity for a thermodynamic temperature scale? Explain the principle.*

5.23 *Explain how temperature is measured using a thermodynamic temperature scale.*

5.24 *Explain Carnot heat engine.*

5.25 *What is meant by quality of energy? Explain.*

Exercise

5.1 A cyclic heat engine is working between a source temperature of 900 °C and sink temperature of 300 K. What is the heat rejection rate per kW net output of the engine? Note: All temperatures must be in Kelvin. *Ans:* 0.344 kW

5.2 An engineer has made a claim that his engine working between 900°C and 90°C is able to work with an efficiency of 80%. Explain whether his claim is right or not with justification. *Ans:* Wrong

5.3 Consider Fig.5.52. A boiler provides heat Q_1 at the temperature T_1. This heat is absorbed by the heat engine, which delivers work W and rejects the waste heat Q_2 into the house at temperature T_2. Work delivered by the engine is used to operate a mechanical refrigerator or heat pump which extracts Q_3, from outdoors at temperature T_3 and rejects $Q_{2'}$ (where $Q_{2'} = Q_3 + W$) into the house. Due to this cycle of operation, a total quantity of heat $Q_2 + Q_{2'}$ is liberated in the house against Q_1 which would be provided directly by the ordinary combustion of the fuel. Thus, the ratio $Q_2 + Q_{2'}$ represents the heat multiplication factor of this system. Determine this multiplication factor if $T_1 = 470$ K, $T_2 = 290$ K and $T_3 = 273$ K. *Ans:* 7.15

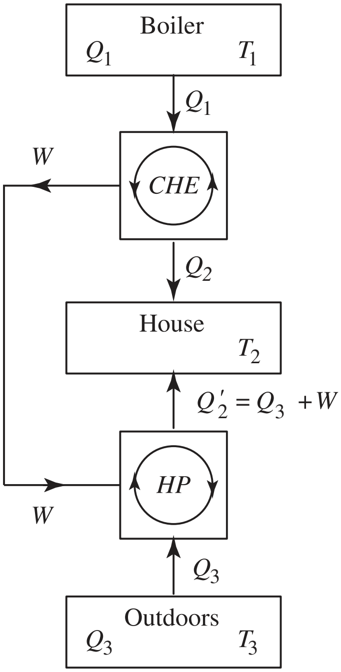


Fig. 5.52

5.4 The inventor claims that he has developed an engine which develops a power of 50 kW with a fuel consumption of 4 kg/hr. The calorific value of the fuel is 45 MJ/kg. Check whether his claim is right. *Ans:* Wrong

5.5 Assume that an ideal gas cycle is represented by a rectangle on a p-V diagram. If p_L and p_H are lower and higher pressures and V_L and V_H are lower and higher volumes, respectively, calculate (i) the work done per cycle, (ii) indicate which part of the cycle involve heat flow and (iii) derive an expression for efficiency. Assume c_p and c_V are constant.

Ans: (i) Work done $= (p_H - p_L) \times (V_H - V_L)$;

(ii) $Q = \dfrac{p_H V_L}{R}\left[c_V \times \dfrac{p_H - p_L}{p_H} + c_p \times \dfrac{V_H - V_L}{V_L}\right]$

(iii) $\eta = \dfrac{\gamma - 1}{\dfrac{V_L}{(V_H - V_L)} + \dfrac{\gamma p_H}{(p_H - p_L)}}$

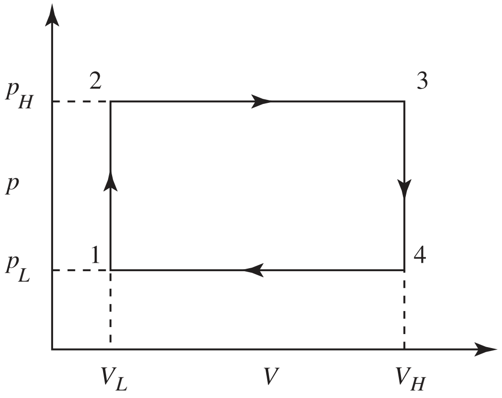


Fig. 5.53

5.6 A reversible heat engine in a satellite operates between a hot reservoir T_H and radiating panel T_L. Radiation from the panel is proportional to its area and to T_L^4. For a given work output and T_H that the area of the panel will be minimum when $\dfrac{T_L}{T_H} = 0.75$. Determine the minimum area of the panel for an output of 1 kW if the constant of proportionality is 5.67×10^{-8} W/m^{2}k^4 and $T_H = 1000$ K. *Ans:* 0.1672 m^2

5.7 A heat engine is provided with a heat input of 1800 kJ/min and the out from the engine is 9 kW. Calculate (i) thermal efficiency and (ii) rate of heat rejection. *Ans:* (i) 30%; (ii) 21 kJ/s

5.8 During a process, a system receives 33 kJ of heat from a reservoir and does 66 kJ of work. Is it possible to reach initial stage by an adiabatic process? *Ans:* Yes

5.9 A mansion requires 200 MJ/h of heating in winter. Heat pump is employed to absorb heat from cold air in winter and send heat to the mansion. Work required to operate the heat pump is 30 MJ/h. Calculate (i) heat abstracted from outside and (ii) coefficient of performance. *Ans:* (i) 170 MJ/h; (ii) 6.66

5.10 In the above example, if the heat requirement of the mansion is the same. Calculate the *COP* of the refrigerator if the same amount of heat had to be abstracted from the house and rejected out for cooling the mansion in summer. *Ans:* 5.66

5.11 Estimate the highest possible theoretical efficiency of a heat engine operating with a source temperature of 2000 °C when the cooling water available is at 17 °C. *Ans:* 87.2%

5.12 A Carnot cycle is operating between the source temperature of 300 °C and the sink temperature of −23 °C. If the system receives 100 kJ from the source, calculate (i) efficiency of the system; (ii) the net work transfer; and (iii) heat rejected to the sink. *Ans:* (i) 56.4%; (ii) 56.4 kJ; (iii) 43.6 kJ

5.13 An inventor claims that his engine has the following specifications:

Source temperature	747 °C
Sink temperature	27 °C
Power developed	80 kW
Fuel burnt	8 kg/h
Heating value	43 MJ/kg

Find whether such engine is possible. *Ans:* No

5.14 A chilling plant requires 50 tonnes of refrigeration. The freezer temperature is −33 °C. If the performance of the plant is 25% of the theoretical reversible Carnot plant performance with in the same temperature limits, find the input power requirement of the plant. Take 1 tonne of refrigeration is 210 kJ/min. Take ambient temperature as 27 °C. *Ans:* 175 kW

5.15 A reversible heat engine operates between two reservoirs at temperature 750 °C and 50 °C. The engine drives a reversible refrigerator which operates at 50 °C and −20 °C. The heat transfer to the engine is 2700 kJ and the net work output of the combined engine refrigerator plant is 440 kJ.

(i) Find the heat transfer to the refrigerator and the net heat transfer to the reservoir at 50 °C.

(ii) Reconsider (i) given that the efficiency of the heat engine and *COP* of the refrigerator are each 40% their maximum values.

Ans: (i) 7338.5 kJ; (ii) 2691.3 kJ

5.16 (i) A reversible heat pump is designed to maintain a temperature of −1 °C in a refrigerator. It rejects heat to surroundings at 27 °C. If the heat removal rate is 1500 kJ/m. Calculate the *COP* of the machine and the input work required.

(ii) If the required input to run the pump is developed by a reversible engine which receives heat at 377 °C and rejects heat to atmosphere. Determine the overall *COP* of the system. *Ans:* (i) 10.71; 2.574 kW (ii) 6.647

5.17 An ice-making plant is working on a reversed Carnot cycle heat pump. It produces 15 tonnes of ice in 24 hours. The ice formation is from water at 0 °C and maintained at the same temperature. The heat rejection is at 27 °C to the atmosphere. The heat pump runs the ice plant coupled to a Carnot engine which absorbs heat from a source which is maintained at 222 °C by burning fuel having a calorific value of 45 MJ/kg and rejects heat to the atmospheric. Determine (i) power developed by the engine and (ii) fuel consumption per hour. Take enthalpy of fusion of ice as 335 kJ/kg. *Ans:* (i) 5.75 kW; (ii) 1.17 kg/h

5.18 Two Carnot engines work in series between the temperature limits of 600 K and 400 K. If both engines develop equal power, determine the intermediate temperature. *Ans:* 500 K

5.19 A Carnot heat engine draws heat from a reservoir at temperature T_1 and rejects it to another reservoir at temperature T_2. The Carnot forward cycle engine drives the Carnot refrigerator which absorbs heat from reservoir at temperature T_2 and rejects heat to a reservoir at temperature T_3. Take $T_1 = 700$ K and $T_2 = 350$ K. Calculate

(i) the temperature T_3 such that the heat supplied to engine Q_1 is equal to the heat absorbed by refrigerator Q_2.

(ii) the efficiency of the Carnot engine and *COP* of Carnot refrigerator.

Ans: (i) 466.66 K; (ii) 33.33%; (iii) 3

5.20 A heat pump working as a reversed Carnot cycle takes in energy from a reservoir maintained at 7 °C and delivers it to another reservoir where the temperature is 77 °C. The heat pump derives power for its operation from a reversible engine operating with in the temperature limit of 1077 °C and 77 °C. For 100 kJ/kg of energy supplied to the reservoir at 77 °C. Calculate the energy taken from the reservoir. *Ans:* 25.25 kJ

5.21 In total, 330 kJ of heat supplied at a constant fixed temperature of 300 °C to a heat engine. The heat rejection takes place at 10 °C. The following results were obtained: (i) 245 kJ/s are rejected (ii) 163 kJ/s are rejected (iii) 105 kJ/s are rejected. Based on the above data and Clausius inequality, classify the engines as reversible, irreversible and impossible. *Ans:* (i) Irreversible; (ii) Reversible; (iii) Impossible

5.22 In an operating power plant, the boiler temperature is 160 °C and the condenser temperature is 50 °C. Water entering and steam leaving the boiler are at saturated conditions. Analyse whether the operation of the power plant satisfies the Clausius inequality. Take necessary values from steam tables. *Ans:* Yes satisfies Clausius inequality

5.23 Consider Fig.5.54 shown as the power plant works in the temperature range of 165 °C and 52 °C. The higher temperature denotes boiler and the lower temperature denotes condenser. All other process in the steady flow cycle is adiabatic. The specific enthalpies are as shown in Fig.5.54. Check whether the power plant satisfies Clausius inequality. *Ans:* Satisfies Clausius inequality

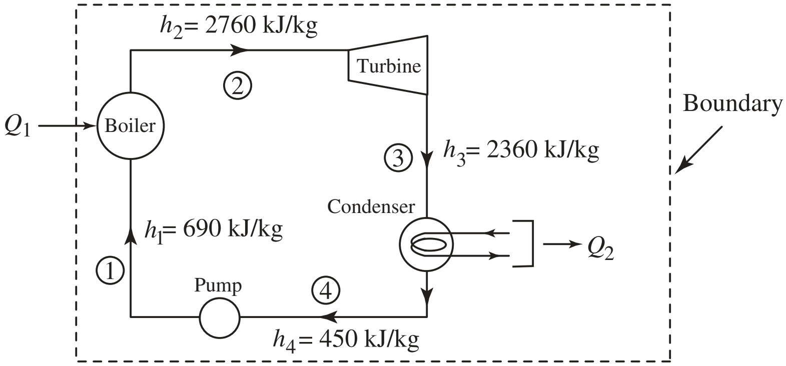


Fig. 5.54

5.24 If the internal efficiency of a reversible heat engine operating between two thermal reservoirs is η_{HE}, determine the *COP* of a reversible refrigerator, operating between the same reservoirs in terms of η_{HE}. *Ans:* $COP_R = \dfrac{1-\eta_{HE}}{\eta_{HE}}$

5.25 Consider two Carnot engines *A* and *B* connected in series. They operate between a high-temperature reservoir at 1500 K and a low-temperature reservoir at 300 K. Assume both engines have same efficiency. Engine *A* absorbs energy from high-temperature reservoir and rejects energy as heat to engine *B*. Engine *B* rejects energy as heat to

the low-temperature reservoir at 300 K. If engine A absorbs 1000 kJ of energy heat from the high-temperature reservoir, calculate the work done by the engines A and B.
Ans: 247.21 kJ

Multiple Choice Questions (choose the most appropriate answer)

1. A cyclic heat engine operates between a source temperature of 927 °C and a sink temperature of 27 °C. What will be the maximum efficiency of the heat engine?
   (a) 75 %
   (b) 100 %
   (c) 80 %
   (d) 70 %

2. Heat engine cycle efficiency is defined as the ratio of
   (a) net work output of the cycle (W_{net}) to total heat input to the cycle (Q_{in})
   (b) total heat input to the cycle (Q_{in}) to net work output of the cycle (W_{net})
   (c) net work output of the cycle (W_{net}) to heat rejected from the system (Q_{out})
   (d) none of the above

3. Perpetual motion machine of the second kind (PMM2) is the violation of
   (a) Kelvin-Planck statement
   (b) Clausius statement
   (c) both (a) and (b)
   (d) none of the above

4. If a process becomes irreversible because of heat interaction between system and surrounding at the boundary due to finite temperature gradient, then the irreversibility is called
   (a) internal irreversibility
   (b) mechanical irreversibility
   (c) external irreversibility
   (d) chemical irreversibility

5. Friction causes irreversibility in the system. It is an example of
   (a) chemical irreversibility
   (b) external irreversibility
   (c) frictional irreversibility
   (d) internal irreversibility

6. A reversible heat engine is operating between a source at T_1 and a sink at T_2. With decrease in T_2, the efficiency of the heat engine
   (a) increases

(b) decreases
(c) remains constant
(d) none of the above

7. Reasonably attainable value of absolute temperature is
(a) always less than zero
(b) always greater than zero
(c) always equals to zero
(d) none of the above

8. When a system undergoes a reversible isothermal process without heat transfer that temperature is called as
(a) critical temperature
(b) Kelvin temperature
(c) reversible temperature
(d) absolute zero temperature

9. To achieve absolute zero temperature on the Kelvin scale, one has to violate
(a) first law of thermodynamics
(b) second law of thermodynamics
(c) both first law and second law of thermodynamics
(d) no law has to be violated

10. In the Kelvin scale, the thermometric property is nothing but
(a) heat energy (Q)
(b) temperature (T)
(c) work energy (W)
(d) all of the above

11. The value of triple point is taken arbitrarily as
(a) 0 K
(b) 273.16 K
(c) 273.16 °C
(d) none of the above

12. Which point is taken as standard reference point to define Kelvin temperature scale?
(a) boiling point of water
(b) freezing point of water
(c) boiling point of mercury
(d) triple point of water

13. Carnot cycle efficiency is the function of
(a) net work done (W_{net})

(b) temperatures (T_1, T_2) between which the Carnot cycle operates
(c) heat supplied (Q_1) and heat rejected (Q_2)
(d) all of the above

14. Assume that two reversible heat engines A and B having different capacity operate between the temperature levels T_1 and T_2, then the efficiency of reversible heat engine B is
(a) equal to the efficiency of the reversible heat engine A
(b) greater than efficiency of the reversible heat engine A
(c) less than the efficiency of the reversible heat engine A
(d) cannot say

15. According to the Carnot's theorem, the efficiency of a reversible heat engine operating between a same given constant temperature source and a given constant temperature sink is
(a) less than any other irreversible heat engine
(b) higher than any other irreversible heat engine
(c) equal to any other irreversible heat engine
(d) none of the above

16. Which of the following is a reversed heat engine?
(a) heat pump
(b) refrigerator
(c) both (a) and (b)
(d) none of the above

17. Carnot cycle is
(a) an irreversible cycle
(b) practical cycle
(c) a reversible cycle
(d) none of the above

18. Application of breaks in a vehicle is
(a) a reversible process
(b) a quasi-static process
(c) an irreversible process
(d) none of the above

19. A process in which work is done without producing an equivalent increase in the kinetic or potential energy of any system is
(a) a reversible process
(b) a quasi-static process
(c) an irreversible process

(d) none of the above

20. Free expansion of gas within a system is
    (a) a reversible process
    (b) a quasi-static process
    (c) an irreversible process
    (d) none of the above

21. If temperature difference between two bodies in which heat transfer takes place increases, the irreversibility of the process
    (a) decreases
    (b) increases
    (c) does not change
    (d) approaches toward reversibility

22. Heat transfer is an irreversible process if
    (a) the temperature difference between the two bodies is infinitesimally small
    (b) the temperature difference between the two bodies is finite
    (c) (a) and (b)
    (d) none of the above

23. In a process, if every state passes by the system is an equilibrium, then it is called
    (a) quasi-static process
    (b) reversible process
    (c) (a) and (b)
    (d) none of the above

24. COP of a heat pump is
    (a) always less than 1 ($COP < 1$)
    (b) always equals to 1 ($COP = 1$)
    (c) always less than infinity ($COP < \infty$)
    (d) always equals to infinity ($COP = \infty$)

25. Choose the correct relation between COP of heat pump and COP of refrigerator?
    (a) $(COP)_{HP} = 1 - (COP)_{ref}$
    (b) $(COP)_{HP} = 1 + (COP)_{ref}$
    (c) $(COP)_{HP} = (COP)_{ref}$
    (d) none of the above

Ans: 1. – (a) 2. – (b) 3. – (c) 4. – (c) 5. – (d)
6. – (a) 7. – (b) 8. – (d) 9. – (b) 10. – (a)
11. – (b) 12. – (d) 13. – (b) 14. – (a) 15. – (b)
16. – (c) 17. – (c) 18. – (c) 19. – (c) 20. – (c)
21. – (b) 22. – (b) 23. – (c) 24. – (c) 25. – (b)

6

ENTROPY

6.1 INTRODUCTION

We have already gone through the details of zeroth, first and second law of thermodynamics in previous chapters. Zeroth law of thermodynamics provides a basis for temperature measurement. The first law of thermodynamics defines a very useful property called internal energy. In other words, the first law deals with the conservation of energy. However, there are some deficiencies in the first law. For overcoming the deficiencies of first law, the second law of thermodynamics was introduced. In Chapter 5, we dealt with the second law in detail. There, we considered only thermodynamic cycles. Even though it is a useful approach, we as scientists and engineers, are more concerned with processes rather than cycles. Therefore, we need some mathematical parameters to arrive at the conclusion regarding the nature of the process. Further, we are interested in its feasibility as well as the irreversibility associated with such a process. In this direction, the second law leads to the definition of a new property called *'entropy'*. Let us recall the statement we made in Chapter 1, i.e. thermodynamics is described as the science of energy and entropy.

The concept of entropy began with the work of French mathematician Lazare Carnot that in any machine, the accelerations and shocks generated by the moving parts represent losses of moment of activity. The term 'entropy' was first used by Clausius. It is taken from the Greek word 'tropee' which means transformation.

In any natural process, there exists an inherent tendency towards the dissipation of useful energy. *Hence, it can be viewed as a measure of the energy that is dissipated away which is* **not available for work** *during a thermodynamic process.* Entropy can therefore, be defined as an index of unavailability or degradation of energy. Naturally, energy in the form of heat always flows from hot to cold bodies. As it flows, only lesser and lesser value becomes available. This unavailability of energy is measured by the term called *entropy.* It is an important thermodynamic property of the working substance representing the unavailability of a system's thermal energy for conversion into mechanical work.

To put it in a nut shell, entropy envisages gradual decline of order into disorder. It is somewhat an abstract property. Giving an exact physical description is rather difficult without considering the microscopic state of the system. However, it has immense significance in the analysis of thermodynamic processes.

Now, let us first try to understand and appreciate the concept of *entropy* by studying its uses in commonly encountered engineering processes. To start with, we will see how to replace a single reversible process by a number of reversible processes. In this chapter, we will discuss Clausius inequality, which forms the basis for the definition of entropy. Unlike energy, entropy is a non-conserved property. *There is no such thing as conservation of entropy.* We will discuss the entropy changes that take place during processes for pure substances, incompressible substances and ideal gases. A special class of idealised processes, called *isentropic processes*, will be examined. The reversible steady-flow work and the isentropic efficiencies of various engineering devices such as turbines and compressors are considered.

6.2 REPLACEMENT OF A REVERSIBLE PROCESS BY EQUIVALENT PROCESSES

Let us consider a reversible process $1-2$ as shown on p-V diagram (Fig.6.1). Let us also have another set of reversible processes consisting of a reversible adiabatic $1-1'$ followed by reversible isothermal $1'-2'$ followed by reversible adiabatic $2'-2$, as an approximation to the original reversible process $1-2$. The second set of processes that we have described is such that area under $1-2$ equals to that under $1-1'-2'-2$.

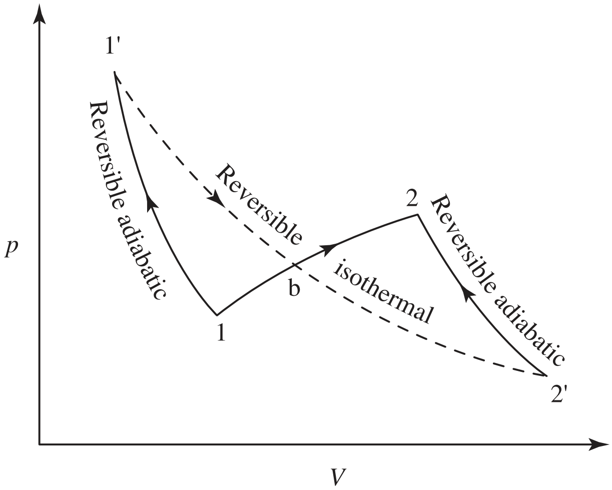


Fig. 6.1 Replacement of a Reversible Process by Equivalent Processes

By the first law of thermodynamics for the process by $1-2$ we can write,

$$Q_{1-2} \quad = \quad (U_2 - U_1) + W_{1-2} \tag{6.1}$$

Let us now apply the first law to the process $1-1'-2'-2$. Remember that heat and work are path functions and internal energy is a point function. Now,

$$Q_{1-1'-2'-2} \quad = \quad (U_2 - U_1) + W_{1-1'-2'-2} \tag{6.2}$$

We have assumed that area under $1-2$ equals to that under $1-1'-2'-2$. You should recollect at this point that the area under a p-V diagram represents work done. Therefore,

$$W_{1-2} = W_{1-1'-2'-2}$$

Now, $$Q_{1-1'-2'-2} = (U_2 - U_1) + W_{1-2} \quad = \quad Q_{1-2}$$

Therefore, $$Q_{1-1'-2'-2} = Q_{1-2}$$

Now, consider the path $1-1'-2'-2$. During adiabatic processes $1-1'$ and $2-2'$, there is no heat interaction. Therefore, the total heat interaction is only during isothermal process $1'-2'$ only.

It should be clear by now that it is always possible to replace any reversible process by a series of reversible processes such as reversible adiabatic and reversible isothermal. However, the heat interaction and work involved should remain the same. If the number of reversible adiabatic and reversible isothermal processes is quite large then the series of such processes will closely represent the original reversible process $(1-2)$ itself. *Therefore, we can conclude that it is possible to replace a reversible process by means of equivalent reversible processes.*

6.3 CLAUSIUS INEQUALITY

Now, let us consider this kind of substitution described in Section 6.2 for the processes in a reversible cycle shown in Fig.6.2. Let $a-b-a$ be a reversible cycle. Let us replace the original cycle by two Carnot cycles $1-2-3-4$ and $5-6-7-8$. Similarly, we can introduce a number of Carnot cycles appearing together in place of original cycle as shown in Fig.6.3.

For illustration, only two Carnot cycles $1-2-3-4$ and $5-6-7-8$, as shown in Fig.6.2, are considered. Magnified view of first Carnot cycle is shown separately as can be seen in Fig.6.2 where heat supplied at high temperature T_{1-2} is Q_{1-2} and heat rejected at low temperature T_{3-4} is Q_{3-4}.

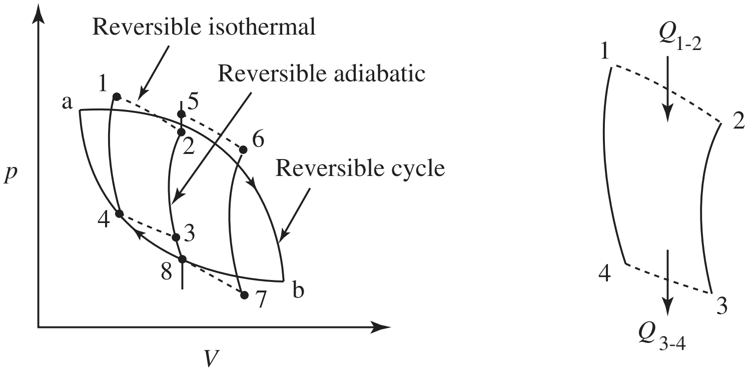


Fig. 6.2 A reversible cycle replaced by reversible adiabatic and reversible isotherms

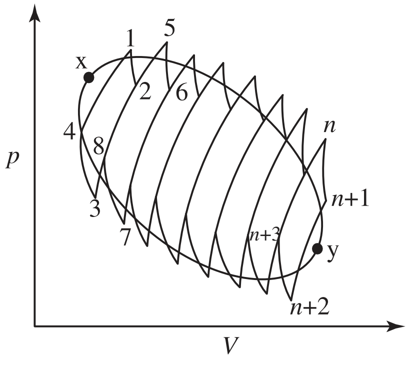


Fig. 6.3 Reversible cycle replaced by multiple Carnot cycles

From thermodynamic temperature scale, for Carnot cycle $1-2-3-4$:

$$\frac{Q_{1-2}}{Q_{3-4}} = \frac{T_{1-2}}{T_{3-4}} \quad \text{and} \quad \frac{Q_{1-2}}{T_{1-2}} = \frac{Q_{3-4}}{T_{3-4}} \tag{6.3}$$

Similarly, for Carnot cycle $5-6-7-8$,

$$\frac{Q_{5-6}}{T_{5-6}} = \frac{Q_{7-8}}{T_{7-8}} \tag{6.4}$$

Taking the sign conventions for heat addition as positive and heat rejection as negative, Eq.6.3 can be written as:

$$\frac{Q_{1-2}}{T_{1-2}} - \left(-\frac{Q_{3-4}}{T_{3-4}}\right) = 0$$

$$\frac{Q_{1-2}}{T_{1-2}} + \frac{Q_{3-4}}{T_{3-4}} = 0$$

Similarly, $$\frac{Q_{5-6}}{T_{5-6}} + \frac{Q_{7-8}}{T_{7-8}} = 0$$

Hence, if there are n number of Carnot cycles replacing the original reversible cycle, then,

$$\left(\frac{Q_{1-2}}{T_{1-2}} + \frac{Q_{3-4}}{T_{3-4}}\right) + \left(\frac{Q_{5-6}}{T_{5-6}} + \frac{Q_{7-8}}{T_{7-8}}\right) + \ldots = 0 \tag{6.5}$$

or, it can be given as summation of the ratio of heat interaction (Q) to the temperature (T) at which it occurs, being equal to zero.

$$\sum \frac{Q}{T} \quad = \quad 0 \tag{6.6}$$

If number of Carnot cycles is very large, then the zig-zag path formed due to adiabatics and isotherms shall reach very close to original cycle. In such situation, the cyclic integral of $\frac{Q}{T}$ will be more appropriate in place of summation. That is to say,

$$\oint \left(\frac{\delta Q}{T}\right)_{\text{rev}} \quad = \quad 0 \tag{6.7}$$

At this juncture, you should understand and appreciate that $\frac{\delta Q}{T}$ can be considered as some thermodynamic property. Above expression is developed for a reversible heat engine cycle. It is also valid for internally reversible engines. In case of internally reversible engines, T shall be temperature of working fluid at the time of heat interaction. Therefore, it can be written as:

$$\oint \left(\frac{\delta Q}{T}\right)_{int,rev} \quad = \quad 0$$

So far, we have discussed a reversible heat engine cycle. Let us now try to find out what happens to $\oint \left(\frac{\delta Q}{T}\right)$ when we have an irreversible engine cycle. Let us consider a reversible as well as an irreversible heat engine operating between same temperature limits, such that heat supplied to them is the same. Let us denote heat supplied by Q_S and heat rejected by

Q_R. From Carnots theorem for both reversible and irreversible heat engine cycles,

$$\eta_{\text{rev}} > \eta_{\text{irrev}}$$

or $$\left(1 - \frac{Q_R}{Q_S}\right)_{\text{rev}} > \left(1 - \frac{Q_R}{Q_S}\right)_{\text{irrev}}$$

This means that the ratio $\left(\frac{Q_R}{Q_S}\right)$ for a irreversible engine is greater than the reversible engine.

Therefore, $$\left(\frac{Q_R}{Q_S}\right)_{\text{irrev}} > \left(\frac{Q_R}{Q_S}\right)_{\text{rev}}$$

For same amount of heat addition,

$$Q_{S,\text{rev}} = Q_{S,\text{irrev}} \quad = \quad Q_S$$

$$Q_{R,\text{irrev}} > Q_{R,\text{rev}}$$

or $$\frac{Q_{R,\text{irrev}}}{Q_{R,\text{rev}}} > 1$$

In absolute thermodynamic temperature scale for a reversible cycle,

$$\left(\frac{Q_R}{Q_S}\right)_{\text{rev}} = \frac{T_R}{T_S}$$

For a irreversible cycle engine, keeping in mind Q_S is same for both, we can write,

$$\frac{Q_{R,\text{irrev}}}{Q_S} > \frac{T_R}{T_S}$$

or $$\frac{Q_{R,\text{irrev}}}{T_R} > \frac{Q_S}{T_S}$$

Considering the sign convention, viz Q_S is positive and Q_R as negative, we can write,

$$\frac{Q_S}{T_S} + \frac{Q_{R,\text{irrev}}}{T_R} < 0$$

or $$\frac{Q_{S,\text{irrev}}}{T_S} + \frac{Q_{R,\text{irrev}}}{T_R} < 0 \quad (\because Q_S \text{ is same for both})$$

If it is written in the form of cyclic integral,

$$\oint \left(\frac{\delta Q}{T}\right)_{\text{irrev}} < 0 \tag{6.8}$$

Now, combining for reversible and irreversible paths (Eqs 6.7 and 6.8) it can be written as:

$$\oint \left(\frac{\delta Q}{T}\right) \leq 0 \tag{6.9}$$

This is called *Clausius inequality.*

6.4 ENTROPY : A THERMODYNAMIC PROPERTY

To develop a relation for the definition of entropy, let us examine the equation $\oint \left(\frac{\delta Q}{T}\right)_{\text{rev}} = 0$ more closely. Here, we have a quantity whose cyclic integral is zero. Let us think for a moment what kind of quantities can have this characteristic. We know that the cyclic integral of work is not zero. Fortunately, it is not. Otherwise, heat engines that work on a cycle such as steam power plants would produce zero net work. Similarly, the cyclic integral of heat is not zero. Now, consider the volume occupied by a gas in a piston-cylinder device undergoing a cycle, as shown in Fig.6.4. When the piston returns to its initial position at the end of a cycle, the volume of the gas also returns to its initial value. Thus, the net change in volume during a cycle is zero. This can be expressed as $\oint dV = 0$. That is to say, the cyclic integral of volume (or any other property) is zero. *Conversely, a quantity whose cyclic integral is zero depends on the state only and not the process path and thus it is a property.* We can easily show that the quantity $\left(\frac{\delta Q}{T}\right)_{\text{int, rev}}$ is also a property in the differential form. For this, consider Fig.6.5.

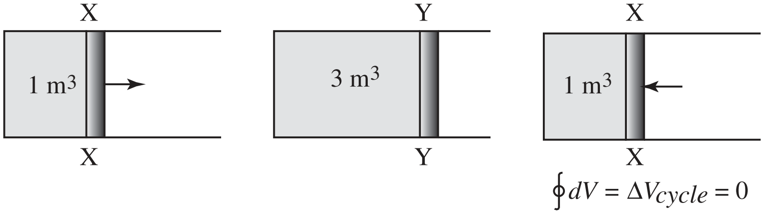


Fig. 6.4 The net change in volume (a property) during a cycle is always zero

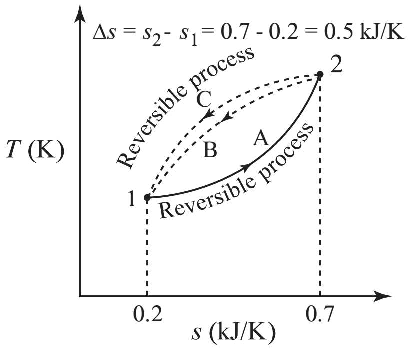


Fig. 6.5 Illustration of entropy change between two specified states

Let the thermodynamic system undergoes a change of state as shown in Fig.6.5 from 1 to 2 by a reversible process 1–A–2 and returns to its original state 1 by another reversible process

2–B–1 and completing a cycle 1–2–1. For this cyclic reversible process, the entropy equation is

$$\oint_{\text{rev}} \frac{\delta Q}{T} = \int_{1A}^{2A} \frac{\delta Q}{T} + \int_{1B}^{2B} \frac{\delta Q}{T} \tag{6.10}$$

Now, let us consider the cycle 1–2–1 is completed by yet another reversible process 2–C–1 instead of 2–B–1. Then,

$$\oint_{\text{rev}} \frac{\delta Q}{T} = 0 = \int_{1A}^{2A} \frac{\delta Q}{T} + \int_{1C}^{2C} \frac{\delta Q}{T} \tag{6.11}$$

Equation 6.11 is from 6.10, since LHS is the same,

$$\int_{1B}^{2B} \frac{\delta Q}{T} = \int_{1C}^{2C} \frac{\delta Q}{T} \tag{6.12}$$

From Eq.6.12, we conclude that $\int \frac{\delta Q}{T}$ is the same for all reversible paths between states 1 and 2, i.e. it is independent of the path and is a function of end states only. Hence, it can be concluded that entropy is a property of a system.

Clausius realised in 1865 that he had discovered a new thermodynamic property and he chose to name this property as *entropy.* It is designated as S and is defined as

$$dS = \left(\frac{\delta Q}{T}\right)_{\text{int, rev}} \tag{6.13}$$

Note that entropy is an extensive property of a system and sometimes is referred to as *total entropy, S,* and has the unit kJ/kg K. Entropy per unit mass, designated s, is an intensive property. The term, entropy, is generally used to refer to both total entropy and entropy per unit mass since the context usually clarifies which one is meant. *Accordingly S or s can be used depending upon the context.* The entropy change of a system during a process can now be determined. The entropy, S, of a system can be defined as the property of a system whose change between the two states 1 and 2 is given by,

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int, rev}} \tag{6.14}$$

In differential form,

$$dS = \left(\frac{\delta Q}{T}\right) \tag{6.15}$$

Notice that we have actually defined the *change in entropy* instead of entropy itself. This is similar to what we defined the *change in energy* instead of the energy itself when we developed the first-law relation. Entropy, as obvious from definition, is defined for change in entropy value. Therefore, absolute value of entropy *cannot* be defined. Entropy is always given as a change, i.e. only relative values only are expressed.

Absolute values of entropy are determined on the basis of the third law of thermodynamics, which is discussed later in this chapter. Engineers are usually concerned only with the changes in entropy. Therefore, the entropy of a substance can be assigned a zero value at some arbitrarily selected reference state. The entropy values at other states can be determined from

Eq.6.14 by choosing state 1 to be the reference state ($S = 0$) and state 2 to be the state at which entropy is to be determined. To perform the integration in Eq.6.14, one needs to know the relation between Q and T during a process. This relation is often not available and the integral in Eq.6.14 can be performed for a few cases only. For the majority of cases, we have to rely on tabulated data for entropy.

Now, it must be clear that entropy is a property. Like all other properties, it has fixed values at fixed states. Therefore, the entropy change ΔS between two specified states is the same, no matter what path (reversible or irreversible) is followed during a process (Fig.6.5). Again note that the integral of $\delta Q/T$ gives us the value of entropy change only if the integration is carried out along an *internally reversible path* between the two states.

Note that the integral of $\delta Q/T$ along an *irreversible path* is not a property and in general, different values will be obtained when the integration is carried out along different irreversible paths. However, you should understand that even for irreversible processes, the entropy change can be determined by carrying out this integration along some *convenient imaginary internally reversible path* between the specified states which is elaborated in Section 6.7. Having established that entropy is a property, let us now make three important statements.

6.5 THREE IMPORTANT STATEMENT ABOUT ENTROPY

Statement 1:

For a system executing a cyclic process, the cycle integral of $\frac{\delta Q}{T}$ is less than or equal to zero. This is called Clausiuss inequality statement. Mathematically,

$$\oint \frac{\delta Q}{T} \quad \leq \quad 0 \tag{6.16}$$

Statement 2:

For a system undergoing an internally reversible cycle, the cyclic integral of $\left(\frac{\delta Q_{\text{int, rev}}}{T}\right)$ is always equal to zero. Mathematically,

$$\oint \left(\frac{\delta Q_{\text{int, rev}}}{T}\right) \quad = \quad 0 \tag{6.17}$$

Statement 3 :

For a system undergoing a change of state, the value of the integral $\left(\frac{\delta Q_{\text{int, rev}}}{T}\right)$ is independent of the path between the given two states. Suppose a system changes from state 1 to 2 through several reversible processes A, B, C, etc. the value of $\int \left(\frac{\delta Q}{T}\right)_{\text{int, rev}}$ between states 1 and 2 is the same for each reversible path. That is,

$$\int_A \left(\frac{\delta Q}{T}\right)_{\text{int, rev}} = \int_B \left(\frac{\delta Q}{T}\right)_{\text{int, rev}} = \int_C \left(\frac{\delta Q}{T}\right)_{\text{int, rev}} \ldots \text{ etc.} \tag{6.18}$$

6.6 SIMILARITIES BETWEEN ENERGY AND ENTROPY

The first law of thermodynamics pertains to systems undergoing cycles only. By using the concepts of energy, it is extended to non-cyclic processes. Similarly, the second law of thermodynamics pertains to systems undergoing cyclic processes only. By using the concepts of entropy, it can be extended to non-cyclic processes.

To put it in a nutshell, the concept of energy is needed to extend the first law to non-cyclic processes, while the concept of entropy is required to extend the second law to non-cyclic processes.

In every day practice, the term, ‘energy’, is used freely. Note that the usage of the term, ‘energy’, may not have the same meaning in science and engineering. Probably everyday use of the term, ‘energy’, makes one to feel that energy is something easy to understand, though in reality it need not necessarily be so. On the other hand, entropy is an unfamiliar word. It is not used as frequently as energy in everyday life. This may probably lead to an uneasy feeling that it is something difficult to understand. But in reality it need not necessarily be so.

Similar to energy, entropy is also not a physical entity. It is also equally abstract and is only a calculable property.

The change of energy is equal to the difference between the magnitudes of two interactions, viz *net heat and net work*. The mathematics involved is quite simple to understand. However, a higher level concept is required, to understand entropy since it involves integration to define entropy. Perhaps this might make you fear that entropy is something very difficult to comprehend. In reality, it is not so. It is as easy or as difficult as understanding energy.

Instead of struggling to appreciate the concept of entropy, let us focus our attention on how it can be applied to practical systems. The two properties, viz *energy and entropy* are very important and highly essential to understand and analyse several practical devices and things in science and technology. Again let us recollect what is stated in the first paragraph of this book, viz *thermodynamics is the science of energy and entropy.*

6.7 ENTROPY AS A COORDINATE

We have established that entropy is a property. Therefore, we can use it as one of the coordinates in representing state diagrams. It shall be seen in later chapters that T-S and h-S diagrams are widely used in analysing several processes of interest in thermodynamics. As can be seen in the state diagram, in Fig.6.6(a), temperature is marked on the y axis, entropy is marked on the x axis. This is known as the T-S diagram. Assume that a reversible process is undergone by a system between an initial state, 1 and a final state 2, as indicated in Fig.6.6(a). Let 1–a and 2–b be the ordinates at the initial and final states, respectively. Let us consider the shaded area 1–2–b–a–l, bound by the process 1–2, the extreme ordinates and the x axis. This area 1–2–b–a–l can be represented by $\int ydx$. In Fig.6.6(a), the temperature is represented on the y axis and entropy on the x axis. Therefore, $\int ydx = \int TdS$.

$$\text{Area } 1 - 2 - b - a - 1 = \int ydx = \int TdS$$

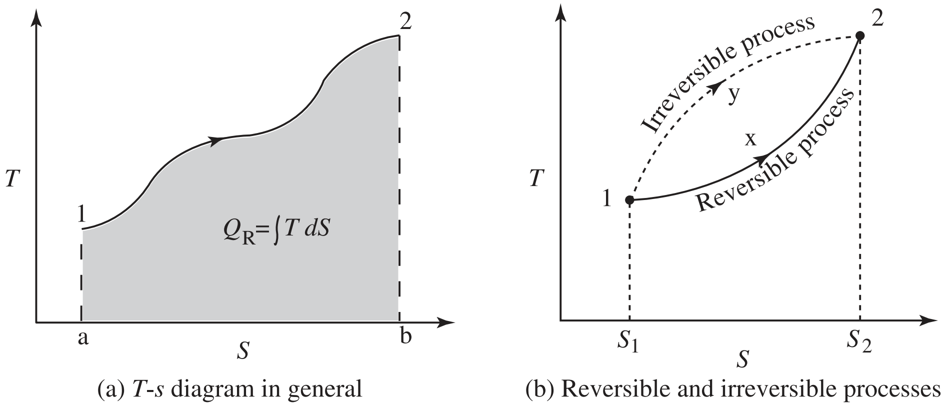
By the definition of entropy given in Eq.6.15, $TdS = \delta Q$, where Q is the net heat interaction.

Now, $\int TdS = \int \delta Q = Q =$ net heat interaction = area 1–2–b–a–l. Q is the net heat interaction of a reversible process. Therefore, on a T-S diagram, the area bound by a reversible or a quasi-static process, the two extreme ordinates and the x axis, is equal to the net heat interaction of a system during the process.

Again let us reiterate that $dS = \dfrac{\delta Q}{T}$ only for internally as reversible processes and that dS is *not equal* to $\dfrac{\delta Q}{T}$ in general. Let us clarify this point in more detail.

Consider a reversible process 1–x–2 on the T-S diagram in Fig.6.6(b). The area below 1–x–2 is equal to the net heat transfer during the reversible process. Further, it also indicates the entropy change, ΔS. Hence, during this process the change in entropy is given by

$$S_2 - S_1 \quad = \quad \int_1^2 \frac{\delta Q}{T} \tag{6.19}$$



(a) *T*-*s* diagram in general (b) Reversible and irreversible processes

Fig. 6.6 T-S diagram

Now, consider an irreversible process, 1–y–2 between the same two states 1 and 2. From Fig.6.6(b) it can be seen as the change in entropy in this process is also equal to $(S_2 - S_1)$. However, understand that it cannot be evaluated from $\dfrac{\delta Q}{T}$ for the irreversible process y. This is to say that $dS \neq \dfrac{\delta Q}{T}$ in the case of an irreversible process. Therefore, we must evolve a method by which entropy change in any irreversible process can be evaluated.

For this, assume any convenient reversible process connecting the same end states as the irreversible process you consider [refer Fig.6.6(b)]. Evaluate $\displaystyle\int \frac{\delta Q_R}{T}$ along this assumed reversible process. Let this be equal to ΔS. The entropy change in the irreversible process should also be equal to the same ΔS. This is always true, as entropy is a property and its change between two states is independent of the path between the two states. Thus, entropy change during an irreversible process can also be calculated by employing a suitable reversible process between the end states.

6.7.1 Entropy as a Quantitative Test

From Clausius inequality statement (Eq.6.9), we should note that entropy can be used as a quantitative test for irreversibility. Let us explain a methodology to assess quantitatively whether a process is irreversible, reversible or impossible, using entropy as a quantitative test.

Consider the thermodynamic cycle 1–y–2–x–1 shown on the T-s diagram of Fig.6.6(b). This cycle on the whole is irreversible because of process 1–y–2, which is irreversible.

According to the Clausius inequality, for this irreversible cycle:

$$\oint \frac{\delta Q}{T} < 0$$

Let us split the cyclic integral to linear integrals corresponding to the two processes. Then,

$$\int_{1(y)}^{2} \frac{\delta Q}{T} + \int_{2(x)}^{1} \frac{\delta Q}{T} \quad < \quad 0 \tag{6.20}$$

Note that process x is reversible. The second term on the right-hand side of Eq.6.20, by the definition of entropy, has to be equal to $(S_1 - S_2)$. Note that for this, process 2 is the initial state and 1 is the end state. Now, Eq.6.20 can be written as:,

$$\int_{1(y)}^{2} \frac{\delta Q}{T} + (S_1 - S_2) \quad < \quad 0 \tag{6.21}$$

$$S_2 - S_1 \quad > \quad \int_{1(y)}^{2} \frac{\delta Q}{T} \tag{6.22}$$

Process y is one among various irreversible processes that can occur between states 1 and 2. Thus, Eq.6.20 is valid for all irreversible processes between states 1 and 2. It can be generalised and stated that for all irreversible processes between two given states, 1 and 2:

$$S_2 - S_1 \quad > \quad \int_{1}^{2} \frac{\delta Q}{T} \tag{6.23}$$

As per the definition of entropy for any reversible process:

$$S_2 - S_1 \quad = \quad \int_{1}^{2} \frac{\delta Q}{T} \tag{6.24}$$

Equations 6.23 and 6.24 can be combined and written as:

$$S_2 - S_1 \quad \geq \quad \int_{1}^{2} \frac{\delta Q}{T} \tag{6.25}$$

where the equality sign holds good for a reversible process and the inequality sign for any irreversible process. For system undergoing *non-cyclic processes*, Eq.6.25 is the mathematical representation of the second law of thermodynamics. This can be stated as follows:

When a system executes a non-cyclic process between two given state points the change in entropy during the process is either equal to or greater than $\int \frac{\delta Q}{T}$.

The differential form of Eq.6.24 for a reversible process is:

$$dS \quad \geq \quad \frac{\delta Q}{T} \tag{6.26}$$

Whether a process is reversible or irreversible can be examined from Eqs 6.25 and 6.26. If during a process, the value of ΔS, the change in entropy, is equal to $\int \frac{\delta Q}{T}$, such a process is reversible. If the change in entropy during a process is greater than $\int \frac{\delta Q}{T}$, such a process is irreversible.

A hypothetical process can be reversible or irreversible or impossible. If $S_2 - S_1 < \int_1^2 \frac{\delta Q}{T}$, then that process is impossible. Thus, entropy can be used as a quantitative test for the irreversibility or otherwise of a process.

6.7.2 Reversible and Irreversible Adiabatic Processes

Where there are no heat interactions, that is, $Q = 0$, the process is called an adiabatic process. Substituting 0 for δQ in Eq.6.25:

$$S_2 - S_1 \quad \geq \quad 0 \tag{6.27}$$

As already explained, the equality holds good for a reversible process and the inequality for an irreversible process. Therefore, a reversible adiabatic process is one where both heat interaction and change in entropy during the process is equal to zero. Hence, a reversible adiabatic process is called an isentropic (constant entropy) process.

On the contrary, in an irreversible adiabatic process, the heat interaction is zero but the change in entropy is not equal to zero, it is greater than zero. In other words, in an adiabatic process, entropy of a system can never decrease. Thus, in general,

$$Q_{\text{adiabatic}} = 0 \text{ (process is reversible) and } \Delta s_{\text{adiabatic}} \geq 0 \text{ (process is irreversible)} \tag{6.28}$$

This fact is embodied in the principle of increase of entropy which is explained in the following section.

6.8 PRINCIPLE OF ENTROPY INCREASE

For a system undergoing any process, the net change in entropy of the system and its surroundings is always greater than or equal to zero. This is called *principle of entropy increase.*

Alternatively, it may also be stated that the entropy of the universe is always on the increase. The fact that $dS \geq 0$ during an adiabatic process can be used to find whether a given process is reversible or irreversible. Note that the quantitative assessment is not restricted to adiabatic processes alone. It can also be applied to all processes, reversible or irreversible, adiabatic or otherwise.

One of the important questions comes to our mind is how to quantitatively decide the feasibility or otherwise of a process. Although Eq.6.25 can provide the required answer, it is not in a convenient form. It is because it may not always be possible to evaluate the right-hand side of this equation.

Whenever Eq.6.25 cannot be used, for want of sufficient data, Eqs 6.23 − 6.26 can conveniently be used to provide the answer to this question. Thus, to reiterate which we have already said, we can conclusively state:

(a) A process is irreversible and possible if:

$$S_2 - S_1 \quad > \quad \int \frac{\delta Q}{T} \tag{6.29}$$

$$\Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \quad > \quad 0 \tag{6.30}$$

Universe comprises of system plus surrounding. Therefore, Eq.6.30 can be written as:

$$\Delta S_{\text{universe}} \quad > \quad 0 \tag{6.31}$$

which means that entropy of universe increases continuously. It is due to the fact that almost all processes occuring in universe are irreversible processes. In other words, it can be stated that the entropy of universe keeps on increasing. This is called the *principle of entropy increase.*

(b) A process is reversible and possible if $S_2 - S_1 = \int \frac{\delta Q}{T}$, or

$$\Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \quad = \quad 0 \tag{6.32}$$

(c) A hypothetical process is impossible if $S_2 - S_1 < \displaystyle\int \frac{\delta Q}{T}$ or

$$\Delta S_{\text{system}} + \Delta S_{\text{surroundings}} \quad < \quad 0 \tag{6.33}$$

6.8.1 Entropy Increase – Indication of Order to Disorder

Based on the concept of second law of thermodynamics, we have shown that entropy is a property. It is derived from the thermodynamics involved in heat engines. A number of definitions are available for entropy. To get a little more insight about entropy, let us consider certain amount of gas in a closed vessel and heat it. On heating the motion of gas molecules inside the vessel increases. State of molecular motion inside vessel depends upon the amount of heat supplied. If we measure the new kinetic energy of gas molecules, it will be found that it is higher. Also, the rate of intermolecular collision and randomness in molecular motion gets increased. In other words, we can say that heating has caused increase in energy level of gas molecules and thus resulting in increased disorderliness inside the vessel. Higher is the energy with molecules, higher shall be the degree of disorderliness.

Entropy can be defined using the degree of disorderliness. It is said that greater is the molecular disorderliness in the system greater shall be entropy. Mathematically, it can be supported by greater entropy value due to large heat supplied ($\delta Q/T$). Thus, entropy can be defined as a parameter for quantifying the degree of molecular disorderliness in the system. Entropy is a measure of driving potential available for a process to occur. Entropy is also an indicator of the direction of occurrence of any thermodynamic process. We have already shown that entropy of an isolated system always increases. Therefore, a process shall always occur in such a direction in which either entropy does not change or increases. In general, almost all real processes are of irreversible in nature and therefore, entropy tends to increase. Entropy cannot be defined as an absolute value and the change in entropy should always have a positive or zero value.

6.9 ENTROPY TRANSFER MECHANISMS

Entropy transfer can take place in two forms: (i) *heat transfer* and (ii) *mass flow*. As entropy can cross the boundary, entropy transfer is recognised at the system boundary. It represents the entropy gained or lost by a system during a process. However, for a closed system (fixed mass), the only form of entropy interaction is through heat transfer. Therefore, the entropy transfer for an adiabatic closed system is zero. Let us go into more detail.

6.9.1 Heat Transfer

Refer Eq.6.15, which is given by:

$$dS \quad = \quad \left(\frac{\delta Q}{T}\right)$$

In the above equation, δQ is taken as positive when heat is added to the system. This will cause entropy to increase. When heat is removed from the system, dQ is negative and the entropy of the system decreases. When heat is transferred to the system of fixed mass the internal energy of the system increases. As a result the molecules (of a gas) move with higher kinetic energy and they collide more frequently. This will make the disorder in the system to increase.

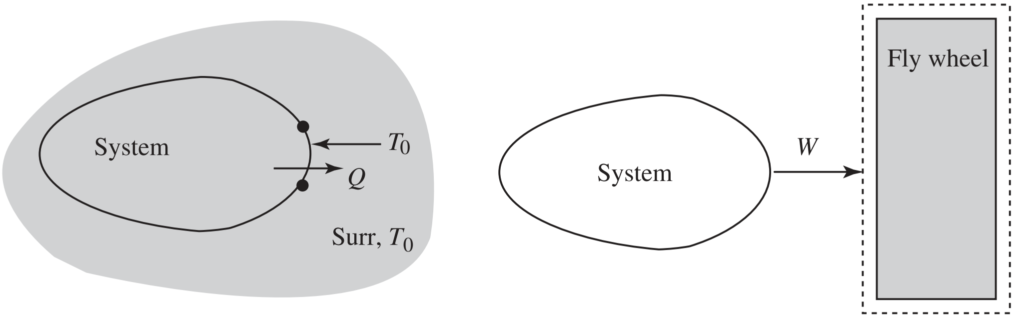
Heat is thus regarded as disorganised or disordered energy transfer which increases molecular chaos. If heat Q flows reversibly from the system to the surroundings at T_0 [Fig.6.7(a)], the entropy increase of the surroundings is:

$$\Delta S_{\text{surr}} = \frac{Q}{T_0} \tag{6.34}$$

The entropy of the system reduces and is given by:

$$\Delta S_{\text{sys}} = -\frac{Q}{T_0} \tag{6.35}$$

However, the temperature of the boundary where heat transfer occurs will remain at constant temperature T_0. It can be viewed as the loss of entropy to the system and gain of entropy to the surroundings. Therefore, there is *entropy transfer* from the system to the surroundings along with heat flow. Since the heat inflow increases the molecular disorder, it may be considered as flow of disorder along with heat. The sign of entropy transfer is exactly the same as the sign of heat flow. *Positive*, if into the system and *negative*, if out of the system.



(a) Entropy transfer along with heat flow (b) No entropy transfer along with work transfer

Fig. 6.7 Entropy transfer

We have discussed the entropy transfer along with heat transfer. Now, let us analyse what happens when there is work transfer. Consider Fig.6.7(b), the system delivers work to a flywheel, where energy is stored in a fully recoverable form. Note that this is the function of a flywheel. The flywheel molecules are simply put into rotation around the axis in a perfectly organised manner. There is no dissipation and hence, no entropy increase of the flywheel. Similar examples are work transfer in the compression of a spring or in the raising of a weight by a certain height. There is thus no entropy transfer along with work. If work is dissipated adiabatically, there by increasing the internal energy of the system, there is no entropy increase in the system. Important point to note is that work is thus *entropy-free* and there is no entropy transfer with work. *Energy is transferred with both heat and work, whereas entropy is transferred only with heat.*

The first law of thermodynamics makes no distinction between heat transfer and work. It considers them as *equals*. The distinction between heat transfer and work is brought about by the second law: *an energy interaction which is accompanied by entropy transfer is heat transfer and an energy interaction which is not accompanied by entropy transfer is work. To reiterate once again only energy is exchanged during work interaction, whereas both energy and entropy are exchanged during heat transfer.*

6.9.2 Mass Flow

Mass contains entropy as well as energy. The entropy and energy of the system are proportional to the mass. When the mass of a system is doubled, so are the entropy and energy of the system. Both entropy and energy are carried into or out of the system by streams of matter. The rates of energy transport into or out of a system are proportional to the mass flow rate. Closed systems do not involve any mass flow and therefore, there is no entropy transport. When an amount of mass m enters or leaves the system there is a entropy change, given by $m \times s$.

6.10 ENTROPY GENERATION IN A CLOSED SYSTEM

In section 6.9, we saw the details of entropy transfer. In this section, we are going to see the details of entropy generation for the closed systems.

6.10.1 Closed System

The increase in entropy of any closed system can be achieved by:

(i) By heat interaction in which there is entropy transfer.

(ii) Due to irreversibilities or dissipative effects.

Assume that an infinitesimal amount of heat, δQ, is transferred to the system through its boundary at temperature, T be small, which is same as that of surroundings. Then, the entropy increase dS of the system can be expressed as:

$$dS = dS_e + dS_i = \frac{\delta Q}{T} + dS_i \tag{6.36}$$

where dS_e is the entropy increase due to the external heat interaction and dS_i is the entropy increase due to internal irreversibility. From Eq.6.36,

$$dS \geq \frac{\delta Q}{T} \tag{6.37}$$

Therefore,

$$dS_i \geq 0 \tag{6.38}$$

The entropy increase due to the internal irreversibilities is also called entropy production or entropy generation, S_{gen}.

In other words, the entropy change of the system during a process is greater than the entropy transfer, $\left(\frac{\delta Q}{T}\right)$ by an amount equal to the entropy generated during the process within the system (dS_i) so that the entropy balance gives:

$$\text{Entropy change} = \text{Entropy transfer} + \text{Entropy generation} \tag{6.39}$$

$$\Delta S_{\text{system}} = \Delta S_{\text{transfer}} + \Delta S_{\text{gen}} \tag{6.40}$$

The details are illustrated in Fig.6.8.

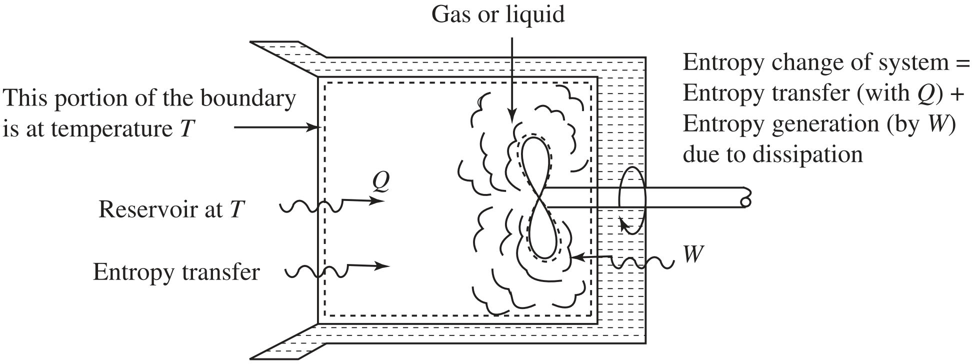


Fig. 6.8 Illustration of the entropy transfer and the entropy production concepts

Now, consider the expansion of a hot fluid in a turbine. There is an entropy decrease of the system due to the heat loss to the surroundings $\left(-\int \frac{\delta Q}{T}\right)$. This is equal to the entropy increase of the system due to internal irreversibilities such as friction, etc. $\left(\int dS_i\right)$. Because of this, the entropy of the system before and after the process will remain the same $\left(\int dS = 0\right)$. From the above discussion, note the following points:

(i) An isentropic process need not be adiabatic or reversible.

(ii) If an isentropic process is reversible then it must be adiabatic.

(iii) If an isentropic process is adiabatic, then it must be reversible.

(iv) An adiabatic process need not be isentropic. Since entropy can increase due to friction.

(v) If a process is adiabatic and reversible, then it must be isentropic.

For an infinitesimal reversible process by a closed system:

$$\delta Q_R = U_R + pdV$$

If the process is irreversible, $\delta Q_I = U_I + \delta W$

Since U is a property, $dU_R = dU_I$

Therefore, $\delta Q_R - pdV = \delta Q_I - \delta W$

Dividing by T on both sides and rearranging:

$$\left(\frac{\delta Q}{T}\right)_R = \left(\frac{\delta Q}{T}\right)_I + \frac{pdV - \delta W}{T} \tag{6.41}$$

The difference $(pdV - \delta W)$ indicate the work that is lost due to irreversibility and it is called the *lost work*. It will approach zero as the process approaches reversibility as the limit. Equation 6.41 can be expressed in the form:

$$dS \quad = \quad dS_e + dS_i$$

Thus, the entropy of a closed system increases due to heat addition (dS_e) and internal dissipation (dS_i).

In any process executed by a system, energy is always conserved but entropy is produced internally. For any process between equilibrium states 1 and 2 (Fig.6.9), the first law can be written as:

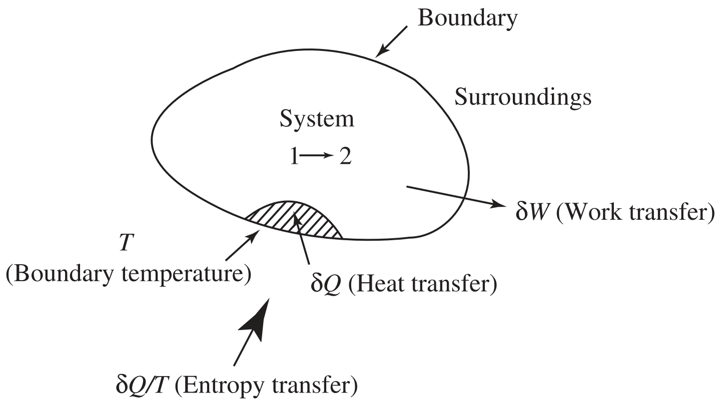


Fig. 6.9 Schematic of a closed system interacting with its surroundings

$$\underbrace{\int_1^2 \delta Q - \int_1^2 \delta W}_{\text{Energy transfer}} = \underbrace{E_2 - E_1}_{\text{Energy change}}$$

$$Q_{1-2} = E_2 - E_1 + W_{1-2} \tag{6.42}$$

By the second law,
$$S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T} \tag{6.43}$$

It is only the transfer of energy as heat accompanied by entropy transfer. Both of these transfers occur at the boundary where the temperature is T. The entropy change of the system ($S_2 - S_1$) exceeds the entropy transfer, $\int_1^2 \frac{\delta Q}{T}$. The difference is produced internally due to irreversibility. The amount of entropy generation S_{gen} is given by:

$$\underset{\substack{\text{Entropy}\\\text{change}}}{S_2 - S_1} - \underset{\substack{\text{Entropy}\\\text{transfer}}}{\int_1^2 \frac{\delta Q}{T}} = \underset{\substack{\text{Entropy}\\\text{production}}}{S_{\text{gen}}} \tag{6.44}$$

Therefore, $S_{\text{gen}} \geq 0$. The second law states that, in general, any thermodynamic process is accompanied by entropy generation. Now, consider Fig.6.10.

Process 1-2, which does not generate any entropy ($S_{\text{gen}} = 0$), is a reversible process. Paths for which $S_{\text{gen}} > 0$ are considered irreversible. Like heat transfer and work transfer during the process 1-2, *the entropy generation also depends on the path, the system follows.* S_{gen}

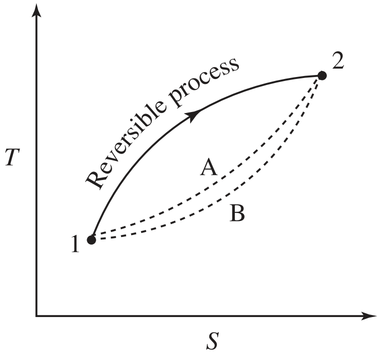


Fig. 6.10 Entropy generation depends on the path

is, therefore, not a thermodynamic property and δS_{gen} is an inexact differential, although $(S_2 - S_1)$ depends only on the end states. In the differential form, Eq.6.44 can be written as:

$$dS_{\text{gen}} = dS - \frac{\delta Q}{T} \tag{6.45}$$

The amount of entropy generation quantifies the intrinsic irreversibility of the process. If the path A causes more entropy generation than path B (Fig.6.10), i.e.

$$S_{\text{gen}_A} > S_{\text{gen}_B} \tag{6.46}$$

the path A is more irreversible than path B and involves more *lost work*.

If heat transfer occurs at several locations on the boundary of a system, the entropy transfer term can be expressed as a sum, so Eq.6.44 takes the form

$$S_2 - S_1 = \sum_j \frac{Q_j}{T_j} + S_{\text{gen}} \tag{6.47}$$

where $\dfrac{Q_j}{T_j}$ is the amount of entropy transferred through the portion of the boundary at temperature T_j.

On a time rate basis, the entropy balance can be written as:

$$\frac{dS}{d\tau} = \sum_j \frac{\dot{Q}_j}{T_j} + \dot{S}_{\text{gen}} \tag{6.48}$$

where $\dfrac{dS}{d\tau}$ is the rate of change of entropy of the system, $\dfrac{\dot{Q}_j}{T_j}$ is the rate of entropy transfer through the portion of the boundary whose instantaneous temperature is T_j and $\dot{S}_{\text{gen}}$ is the rate of entropy generation due to irreversibilities within the system.

6.11 ENTROPY GENERATION OF AN OPEN SYSTEM

In an open system, there is transfer of three quantities: energy, entropy and mass. The control surface can have one or more openings for mass transfer (Fig.6.11). It is rigid and there is shaft work transfer across it.

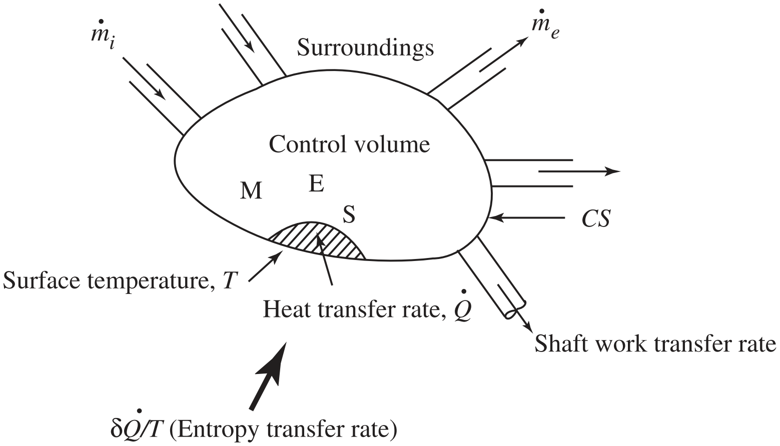


Fig. 6.11 Schematic of an open system and its interaction with surroundings

The continuity equation gives:

$$\underbrace{\sum_i \dot{m}_i - \sum_e \dot{m}_e}_{\text{net mass transfer rate}} = \underbrace{\frac{\partial M}{\partial \tau}}_{\substack{\text{rate of mass}\\ \text{accumulation in the CV}}} \tag{6.49}$$

The energy equation gives:

$$\underbrace{\sum_i \dot{m}_i\left(h+\frac{\mathbb{V}^2}{2}+gZ\right)_i - \sum_e \dot{m}_e\left(h+\frac{\mathbb{V}^2}{2}+gZ\right)_e + \dot{Q} - \dot{W}_{sh}}_{\text{net rate of energy transfer}} = \underbrace{\frac{\partial E}{\partial \tau}}_{\substack{\text{rate of energy}\\ \text{accumulation in the CV}}} \tag{6.50}$$

The *second law inequality* or the entropy principle gives:

$$\underbrace{\sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \frac{\dot{Q}}{T}}_{\substack{\text{net rate of}\\ \text{entropy transfer}}} \leq \underbrace{\frac{\partial S}{\partial \tau}}_{\substack{\text{rate of increase of}\\ \text{entropy of the CV}}} \tag{6.51}$$

Here, $\dot{Q}$ represents the rate of heat transfer at the location of the boundary where the instantaneous temperature is T. The ratio, $\dot{Q}/T$ accounts for the entropy transfer along with heat. The terms $\dot{m}_i s_i$ and $\dot{m}_e s_e$ account, respectively, for rates of entropy transfer into and out of the CV accompanying mass flow. The rate of entropy increase of the control volume exceeds or is equal to, the net rate of entropy transfer into it. The difference is the entropy generated within the control volume due to irreversibility. Hence, the rate of entropy generation is given by:

$$\dot{S}_{\text{gen}} = \frac{\partial S}{\partial \tau} - \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e - \frac{\dot{Q}}{T} \tag{6.52}$$

By the second law, $\dot{S}_{\text{gen}} \geq 0$ (6.53)

If the process is reversible, $\dot{S}_{\text{gen}} = 0$. For an irreversible process, $\dot{S}_{\text{gen}} > 0$. The magnitude of $\dot{S}_{\text{gen}}$ quantifies the irreversibility of the process. If systems A and B operate in such a way that $\left(\dot{S}_{\text{gen}}\right)_A > \left(\dot{S}_{\text{gen}}\right)_B$ then the irreversibility in system A is higher than the irreversibility in system B. The unit of $\dot{S}_{\text{gen}}$ is $\frac{W}{K}$. At steady state, the continuity equation gives:

$$\sum_i \dot{m}_i = \sum_e \dot{m}_e \qquad (6.54)$$

The energy equation becomes:

$$\dot{Q} - \dot{W}_{sh} + \sum_i \dot{m}_i \left(h + \frac{\mathbb{V}^2}{2} + gZ\right)_i - \sum_e \dot{m}_e \left(h + \frac{\mathbb{V}^2}{2} + gZ\right)_e = 0 \qquad (6.55)$$

and the entropy equation reduces to:

$$\frac{\dot{Q}}{T} + \sum_i \dot{m}_i s_i - \sum_e \dot{m}_e s_e + \dot{S}_{\text{gen}} = 0 \qquad (6.56)$$

These equations often must be solved simultaneously, together with appropriate property relations.

Mass and energy are conserved quantities but entropy is not generally conserved. The rate at which entropy is transferred out must exceed the rate at which entropy enters the CV, the difference being the rate of entropy generated within the CV owing to irreversibilities. For one-inlet and one-exit control volumes, the entropy equation becomes:

$$\frac{\dot{Q}}{T} + \dot{m}(s_1 - s_2) + \dot{S}_{\text{gen}} = 0 \qquad (6.57)$$

6.12 EVALUATION OF ENTROPY CHANGE BY Tds RELATION

As per the definition, entropy is written as:

$$\Delta S = S_2 - S_1 > \int_1^2 \frac{\delta Q}{T} \qquad (6.58)$$

This equation is not that easy to calculate entropy change for each and every process. This equation can only be used for either a reversible isothermal process or any other reversible process during which the magnitude of heat transfer is given by a function of temperature. Hence, the utility of this relation is limited. From the two-property rule for a pure substance, it must be possible to calculate the entropy at any state. This can be done by taking any two convenient independent properties. The following paragraphs explain such useful relations among properties.

Shown on the T-s plane in Fig.6.12 is a reversible process undergone by a pure substance between states 1 and 2. Applying the first law in differential form to this process,

$$\delta Q - \delta W = dU \qquad (6.59)$$

As the process is reversible, $\delta Q = \delta Q_R$ and $\delta W = pdV$ (assuming displacement work is the only possible reversible work) and therefore,

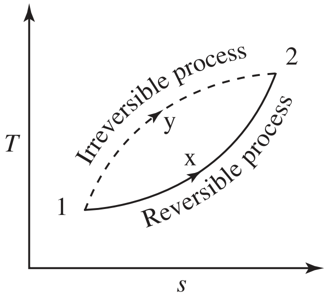


Fig. 6.12 Reversible and irreversible processes on a T-s diagram

$$\delta Q_R - pdV = dU \tag{6.60}$$

By the definition of entropy, $\delta Q_R = TdS$ and hence,

$$TdS = dU + pdV \tag{6.61}$$

This relation, when written in terms of specific properties takes the form:

$$Tds = du + pdv \tag{6.62}$$

or
$$du = Tds - pdv \tag{6.63}$$

By the definition of enthalpy, $h = u + pv$ and hence,

$$dh = du + pdv + vdp \tag{6.64}$$

or
$$du = dh - (pdv + vdp) \tag{6.65}$$

Substituting for du in Eq.6.62,

$$Tds = dh - pdv - vdp + pdv \tag{6.66}$$

or
$$Tds = dh - vdp \tag{6.67}$$

or
$$dh = Tds + vdp \tag{6.68}$$

Equations 6.62 and 6.67 are called the Tds relations and are very important and useful. They are very frequently used in calculating the entropy changes in majority of the cases. For the reversible process 1-2 in Fig.6.12, using Eq. 6.62:

$$\Delta s = \int ds = s_2 - s_1 = \int \frac{du}{T} + \int \left(\frac{p}{T}\right) dv \tag{6.69}$$

Similarly, using 6.67,
$$\Delta s = \int ds = s_2 - s_1 = \int \frac{dh}{T} - \int \left(\frac{v}{T}\right) dp \tag{6.70}$$

Now, consider the irreversible process shown in Fig.6.12. This process also occurs between the same two end states as that of the reversible process. Hence, $s_2 - s_1$ has to be the same for both processes. Although Eqs 6.62 to 6.70 are derived for a reversible process, these relations hold good for calculating entropy changes in irreversible processes also. At this point, it is worth noting that there are no factors in Eqs 6.62 – 6.70 which are path-dependent. T, p, v, s, u and h are all properties of the substance. Hence, the Tds relations can be used to find the entropy change between two states in reversible as well as irreversible processes. However, to do so, the following two informations have to be available:

(i) p-V-T relation for the system under consideration and

(ii) u and/or h as a function of temperature.

6.12.1 Thermodynamic Property Relationship

Having gone through various properties such as p, V, T, U, H, S, etc. they can be related to one another using the combination of mathematical forms of first law, second law of thermodynamics and definitions of properties. Here, specific values of properties are related. Consider Eq.6.68, $dh = Tds + vdp$. The above relations may be used for getting the variation of one property with the other.

Consider a constant pressure process. The above equation reduces to

$$dh = Tds \tag{6.71}$$

or

$$\left(\frac{dh}{ds}\right)_{p=\text{const}} = T \tag{6.72}$$

which means slope of constant pressure line on enthalpy-entropy diagram $(h - s)$ is given by temperature. Also from Eq.6.62,

$$Tds = c_V dT + pdv \quad (\text{ as } du = c_V dT) \tag{6.73}$$

For a constant volume process, Eq.6.73 reduces to

$$\left(\frac{dT}{ds}\right)_{v=\text{const}} = \frac{T}{c_V} \tag{6.74}$$

It gives the slope of constant volume line on a T-s diagram.

Similarly, substituting for $dh = cpdT$ in Eq.6.68, we get,

$$c_p dT = Tds + vdp \tag{6.75}$$

or

$$Tds = c_p dT - vdp \tag{6.76}$$

For a constant pressure process, Eq.6.75 reduces to

$$\left(\frac{dT}{ds}\right)_{p=\text{const}} = \frac{T}{c_p} \tag{6.77}$$

It gives the slope of constant pressure line on T-s diagram. It can be concluded from the above mathematical explanations for slope on a T-s plane that the constant volume lines are steeper than the constant pressure line as $c_V < c_p$. This is illustrated in Fig.6.13.

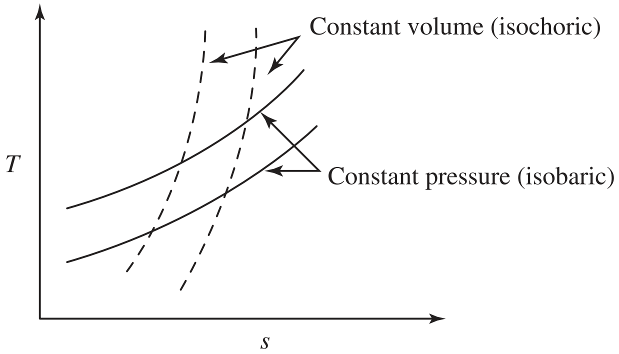


Fig. 6.13 T-s diagram showing isobaric and isochoric process

6.13 ENTROPY RELATIONS FOR IDEAL GAS

Consider heating of an ideal gas (refer Chapter 10) from state 1 to state 2. Let the process be as shown in Fig.6.14 where temperature increases from T_1 and T_2. During heating, there will

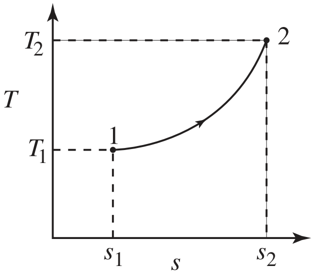


Fig. 6.14 T-s diagram of a typical process

be some change in entropy of the gas. Let us assume that there is heat transfer, dQ, to the gas at an absolute temperature T, then change in entropy is written as

$$dS = \frac{\delta Q}{T} \tag{6.78}$$

Now, let us derive the expressions for change in entropy of an ideal gas in terms of p-V-T.

6.13.1 Expression for dS in terms of Volume and Temperature

As per the law of conservation of energy or first law of thermodynamics and considering only pdV work,

$$\delta Q = \delta W + dU; \quad dU = mc_V dt; \quad \delta W = pdV \tag{6.79}$$

$$\delta Q = pdV + mc_V dT \tag{6.80}$$

Dividing Eq.6.80 throughout by T,

$$\frac{\delta Q}{T} = dS = \frac{p}{T}dV + mc_V\frac{dT}{T} \qquad \left(\because \frac{\delta Q}{T} = dS\right)$$

$$dS = mR\frac{dV}{V} + mc_V\frac{dT}{T} \qquad (\because pV = mRT)$$

Integrating the above equation from 1 to 2,

$$\begin{aligned}
\int_1^2 dS &= mR\int_1^2 \frac{dV}{V} + mc_V\int_1^2 \frac{dT}{T}\\
[S]_1^2 &= mR[\ln V]_1^2 + mc_V[\ln T]_1^2\\
S_2 - S_1 &= mR(\ln V_2 - \ln V_1) + mc_V(\ln T_2 - \ln T_1) \qquad (6.81)\\
\text{Change in entropy, } dS &= mR\ln\frac{V_2}{V_1} + mc_V\ln\frac{T_2}{T_1} \qquad (6.82)
\end{aligned}$$

6.13.2 Expression for dS in terms of Temperature and Pressure

As per ideal gas equation,

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2} \quad \text{and} \quad \frac{V_2}{V_1} = \frac{p_1T_2}{p_2T_1} \qquad (6.83)$$

Substituting Eq.6.83 in Eq.6.82, change in entropy,

$$\begin{aligned}
dS &= mR\ln\left(\frac{p_1}{p_2}\frac{T_2}{T_1}\right) + mc_V\ln\left(\frac{T_2}{T_1}\right) \qquad (6.84)\\
&= mR\ln\left(\frac{p_1}{p_2}\right) + mR\ln\left(\frac{T_2}{T_1}\right) + mc_V\ln\left(\frac{T_2}{T_1}\right) \qquad (6.85)\\
&= mR\ln\left(\frac{p_1}{p_2}\right) + m(c_p - c_V)\ln\left(\frac{T_2}{T_1}\right) + mc_V\ln\left(\frac{T_2}{T_1}\right) \qquad (6.86)\\
&= mR\ln\left(\frac{p_1}{p_2}\right) + mc_p\ln\left(\frac{T_2}{T_1}\right) - mc_V\ln\left(\frac{T_2}{T_1}\right) + mc_V\ln\left(\frac{T_2}{T_1}\right) \qquad (6.87)\\
dS &= mR\ln\left(\frac{p_1}{p_2}\right) + mc_p\ln\left(\frac{T_2}{T_1}\right) \qquad (6.88)\\
&= mc_p\ln\left(\frac{T_2}{T_1}\right) - mR\ln\left(\frac{p_2}{p_1}\right) \qquad (6.89)
\end{aligned}$$

6.13.3 Expression for dS in terms of Pressure and Volume

As per ideal gas equation,

$$\begin{aligned}
\frac{p_1V_1}{T_1} &= \frac{p_2V_2}{T_2} \qquad (6.90)\\
\frac{T_2}{T_1} &= \frac{p_2V_2}{p_1V_1} \qquad (6.91)
\end{aligned}$$

Substituting Eq.6.91 in Eq.6.82,

$$dS = mR\ln\left(\frac{V_2}{V_1}\right) + mc_V\ln\left(\frac{p_2V_2}{p_1V_1}\right) \tag{6.92}$$

$$= m(c_p - c_V)\ln\left(\frac{V_2}{V_1}\right) + mc_V\ln\left(\frac{p_2}{p_1}\right) + mc_V\ln\left(\frac{V_2}{V_1}\right) \tag{6.93}$$

$$= mc_p\ln\left(\frac{V_2}{V_1}\right) - mc_V\ln\left(\frac{V_2}{V_1}\right) + mc_V\ln\left(\frac{p_2}{p_1}\right) + mc_V\ln\left(\frac{V_2}{V_1}\right) \tag{6.94}$$

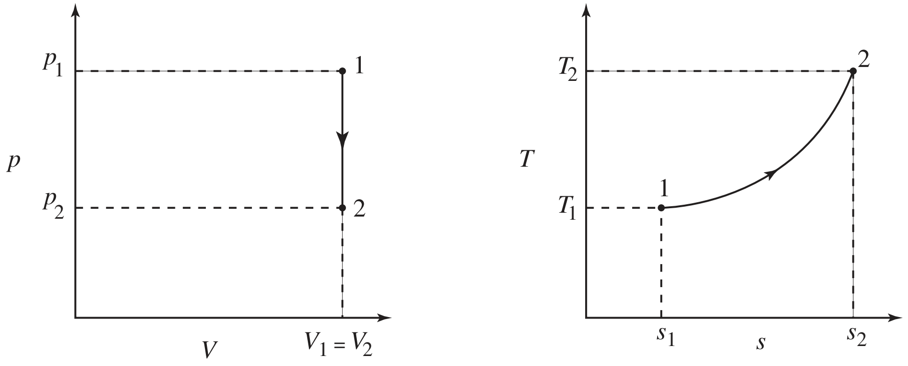
$$dS = mc_V\ln\left(\frac{p_2}{p_1}\right) + mc_p\ln\left(\frac{V_2}{V_1}\right) \tag{6.95}$$

6.14 CHANGE IN ENTROPY FOR VARIOUS PROCESSES

In engineering applications, we come across many different processes. Now, let us derive the equations for change in entropy for various thermodynamic processes.

6.14.1 Constant Volume Process

A system undergoing a change of state from 1 to 2 at a constant volume process is shown in p-V and T-s diagram of Fig.6.15. From Eq.6.82, change in entropy in terms of volume and temperature is given by,



(a) p-V diagram (b) T-s diagram

Fig. 6.15 A system undergoing Constant Volume Process

$$dS = mR\ln\left(\frac{V_2}{V_1}\right) + mc_V\ln\left(\frac{T_2}{T_1}\right) \tag{6.96}$$

But in constant volume process, $V_1 = V_2$ and $\ln\left(\frac{V_2}{V_1}\right) = 0$. Therefore,

$$dS = mc_V\ln\left(\frac{T_2}{T_1}\right) \tag{6.97}$$

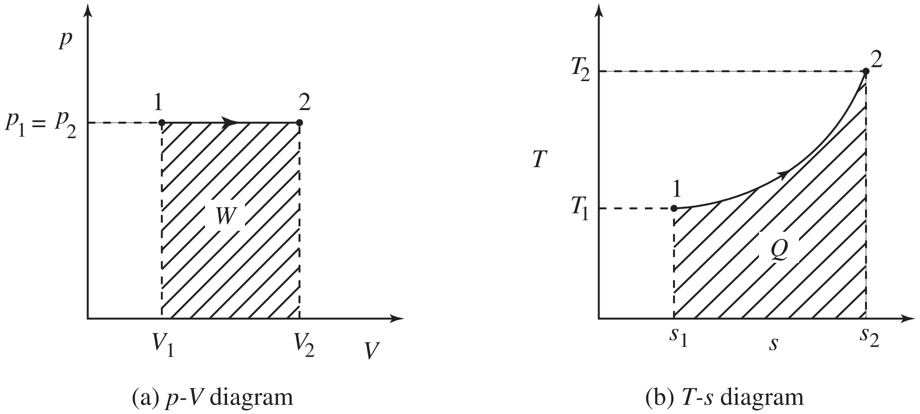
For constant volume process, $\frac{T_2}{T_1} = \frac{p_2}{p_1}$

Therefore, in terms of pressure, we can write,

$$dS \quad = \quad mc_V \ln\left(\frac{p_2}{p_1}\right) \tag{6.98}$$

6.14.2 Constant Pressure Process

A system undergoing a change of state from 1 to 2 at a constant pressure process is shown in p-V and T-s diagram in Fig.6.16. From Eq.6.89, change in entropy in terms of temperature and pressure is given by,



(a) p-V diagram (b) T-s diagram

Fig. 6.16 A system undergoing Constant Pressure Process

$$dS \quad = \quad mc_p \ln\left(\frac{T_2}{T_1}\right) - mR \ln\left(\frac{p_2}{p_1}\right) \tag{6.99}$$

For constant pressure process, $\dfrac{T_2}{T_1} = \dfrac{V_2}{V_1}$. Taking into account, $p_2 = p_1$, Eq.6.99 can be written as

$$dS \quad = \quad mc_p \ln\left(\frac{V_2}{V_1}\right)$$

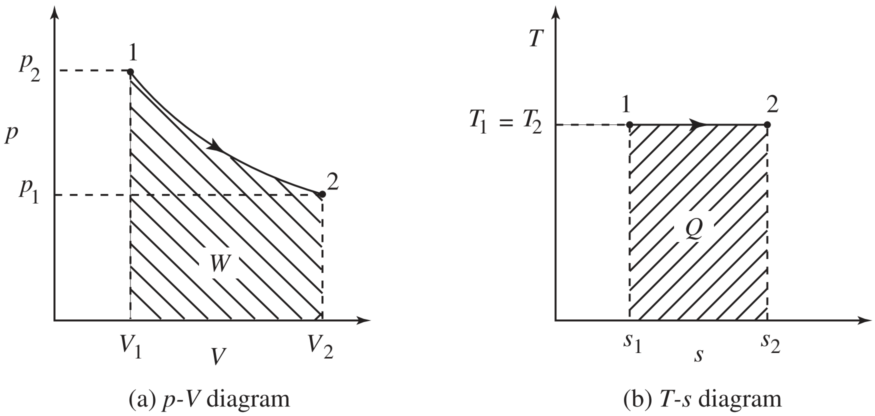
$$dS \quad = \quad mc_p \ln\left(\frac{T_2}{T_1}\right) \tag{6.100}$$

6.14.3 Constant Temperature Process

A system undergoing a change of state from 1 to 2 at constant temperature process is shown in p-V and T-s diagram of Fig.6.17. From Eq.6.82, dS in terms of volume and temperature is given by,

$$dS \quad = \quad mR \ln\left(\frac{V_2}{V_1}\right) + mc_V \ln\left(\frac{T_2}{T_1}\right) \tag{6.101}$$

As $T_1 = T_2$ and $\ln\left(\dfrac{T_2}{T_1}\right) = 0$,



(a) *p-V* diagram (b) *T-s* diagram

Fig. 6.17

$$dS = mR\ln\left(\frac{V_2}{V_1}\right) \tag{6.102}$$

Multiplying and dividing by T on right-hand side of Eq.6.102,

$$dS = \frac{mRT\ln\left(\frac{V_2}{V_1}\right)}{T} \tag{6.103}$$

$$dS = \frac{Q}{T} \qquad (\therefore Q = mRT\ln\left(\frac{V_2}{V_1}\right)) \tag{6.104}$$

6.14.4 Reversible Adiabatic or Isentropic Process

Adiabatic process of a system is one in which there is no heat transfer. The p-V and T-s diagrams are shown in Fig.6.18.

Note : There is no area under the T-s diagram because there is no heat transfer during the process $dS = \frac{\delta Q}{dT}$ and $\delta Q = 0$. Therefore, change in entropy is zero.

$$dS = 0 \tag{6.105}$$

6.14.5 Polytropic Process

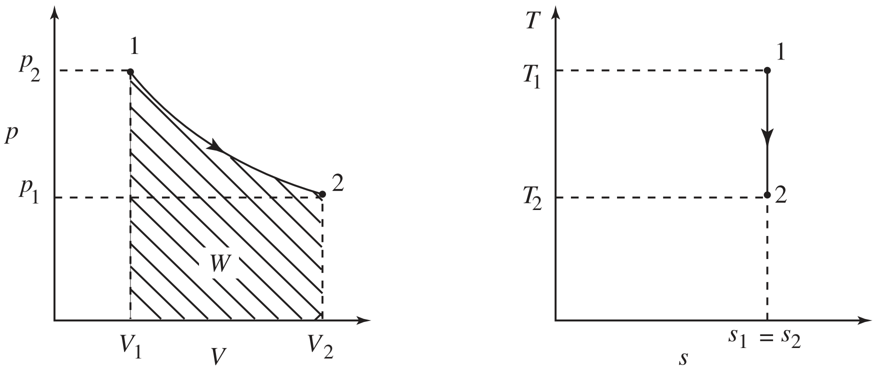
During the polytropic process, there is heat transfer between a system and its surrounding. Further, there will be change in entropy. The p-V and T-s diagrams are shown in Fig.6.19.

Change in entropy can be calculated using either Eqs 6.82, 6.88 or 6.95. As per Eq.6.82,

$$dS = mR\ \ln\left(\frac{V_2}{V_1}\right) + mc_V\ \ln\left(\frac{T_2}{T_1}\right) \tag{6.106}$$

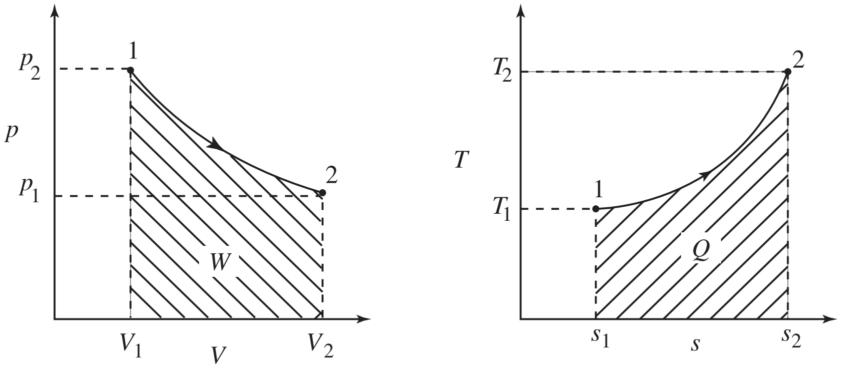
Similarly, other two relations can also be used for change in entropy.

Another expression can be derived in terms of γ, n and volume.



(a) p-V diagram (b) T-s diagram

Fig. 6.18 System undergoing a reversible adiabatic or isentropic process



(a) p-V diagram (b) T-s diagram

Fig. 6.19 Polytropic Process

Heat transfer during polytropic process is given by,

$$\delta Q = \left(\frac{\gamma - n}{\gamma - 1}\right) \times W \tag{6.107}$$

γ is adiabatic index; n is polytropic index and W is work done.

$$\delta Q = \left(\frac{\gamma - n}{\gamma - 1}\right) pdV \qquad (\because W = pdV) \tag{6.108}$$

Dividing both side by T,

$$\frac{\delta Q}{T} = \left(\frac{\gamma - n}{\gamma - 1}\right) \frac{p}{T} dV \tag{6.109}$$

$$dS = \left(\frac{\gamma - n}{\gamma - 1}\right) \frac{mR}{V} dV \left(\because pV = mRT \text{ and } \frac{\delta Q}{T} = dS\right) \tag{6.110}$$

Integrating both sides from initial to final stage,

$$\int_1^2 dS = \left(\frac{\gamma - n}{\gamma - 1}\right) mR \int_1^2 \frac{dV}{V} \tag{6.111}$$

$$S_2 - S_1 = \left(\frac{\gamma - n}{\gamma - 1}\right) mR \ln [V]_1^2 \tag{6.112}$$

$$dS = \left(\frac{\gamma - n}{\gamma - 1}\right) mR \ln \left(\frac{V_2}{V_1}\right) \tag{6.113}$$

6.15 APPLICATIONS OF ENTROPY PRINCIPLE

Entropy principle is the quantitative approach of the second law of thermodynamics. It is due to irreversibility of the process which increases with increase in irreversibility. The following applications are considered:

(i) Transfer of heat through a finite temperature difference.

(ii) Mixing of two fluids.

(iii) Maximum work obtainable from two finite bodies of temperature T_1 and T_2.

(iv) Maximum work obtainable from a finite body and a TER.

6.15.1 Transfer of Heat through a Finite Temperature Difference

Consider two small reservoirs of 1 at T_1 and 2 at T_2 connected by a rod as shown in Fig.6.20. Assume that there is a heat exchange Q from reservoir 1 to reservoir 2.

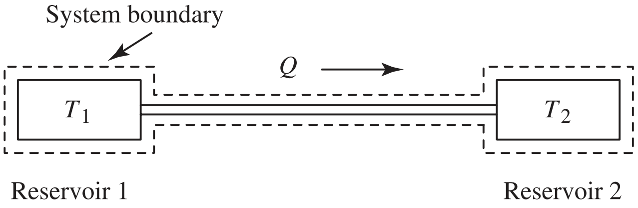


Fig. 6.20 Transfer of Heat through a Finite Temperature Difference

Consider reservoir 1.

Change in entropy, $\Delta S_1 = \frac{-Q}{T_1}$ (−ve sign due to heat is going out of the reservoir 1).

Consider the reservoir 2.

Change in entropy, $\Delta S_2 = \frac{+Q}{T_2}$ (+ve sign is due to heat flows into the reservoir 2).

Assume that heat transfer from reservoir 1 and 2 to the surrounding is negligible. For the isolated system under consideration (2 reservoirs + rod), the change in entropy is additive. Change in entropy of the universe is given by,

$$S = S_1 + S_2 \tag{6.114}$$

Therefore, the total entropy (or) entropy of universe,

$$\Delta S_{\text{univ}} = \Delta S_1 + \Delta S_2 = \frac{-Q}{T_1} + \frac{Q}{T_2} = Q\left(\frac{T_1 - T_2}{T_1 T_2}\right) \tag{6.115}$$

(i) If $\Delta S_{\text{univ}} > 0$ (or) $T_1 > T_2$ the process is irreversible and possible.

(ii) If $\Delta S_{\text{univ}} = 0$ (or) $T_1 = T_2$ the process is reversible.

(iii) If $\Delta S_{\text{univ}} < 0$ (or) $T_1 < T_2$ the process is impossible.

6.15.2 Mixing of Two Fluids

Consider a system with two subsystems as shown in Fig.6.21. Subsystem 1 contains fluid of mass m_1, specific heat at constant pressure, c_1 and temperature, T_1, where $T_1 > T_2$. Subsystem 2, mass m_2 has a specific heat at constant pressure, c_2 and temperature, T_2. Let us assume that these two subsystems are separated by a partition in an adiabatic enclosure. When the partition is removed, the two fluids start mixing with each other. After mixing, it will attain the thermal equilibrium with some final temperature, T_f. Note that T_f will be lesser than T_1 but greater than T_2 ($T_2 < T_f < T_1$).

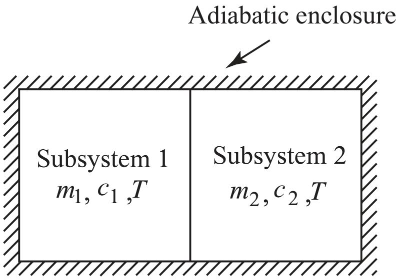


Fig. 6.21 Mixing of Two Fluids

By energy balance,

$$m_1c_1(T_1 - T_f) = m_2c_2(T_f - T_2) \tag{6.116}$$

$$T_f = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2} \tag{6.117}$$

Change in entropy of fluid contained in the subsystem 1,

$$\Delta S_1 = \int_{T_1}^{T_f} \frac{dQ}{T} = \int_{T_1}^{T_f} \frac{m_1c_1dT}{T} = m_1c_1 \ln \frac{T_f}{T_1} \tag{6.118}$$

Similarly, change in entropy of fluid contained in the subsystem 2,

$$\Delta S_2 = \int_{T_2}^{T_f} \frac{dQ}{T} = \int_{T_2}^{T_f} m_2c_2\frac{dT}{T} = m_2c_2 \ln \frac{T_f}{T_2} \tag{6.119}$$

Total entropy,

$$\Delta S_{\text{univ}} = \Delta S_1 + \Delta S_2 = m_1c_1 \ln \frac{T_f}{T_1} + m_2c_2 \ln \frac{T_f}{T_2} \tag{6.120}$$

Note that ΔS_{univ} will be definitely positive since the mixing process is irreversible. In order to evaluate the entropy change for subsystems, we should assume that the irreversible path is replaced by a reversible path on which the integration has been performed. To simplify further, assume $m_1 = m_2 = m$ and $c_1 = c_2 = C$, Now,

$$\Delta S_{\text{univ}} = mC \ln \frac{T_f}{T_1} \times \frac{T_f}{T_2} = mC \ln \frac{T_f^2}{T_1 T_2} \tag{6.121}$$

and also,
$$T_f = \frac{T_1 + T_2}{2} \tag{6.122}$$

Therefore,
$$\Delta S_{\text{univ}} = mC \ln \left[\frac{(T_1 + T_2)^2}{4 T_1 T_2} \right] = mC \ln \left(\frac{T_1 + T_2}{2 \times \sqrt{T_1 T_2}} \right)^2 \tag{6.123}$$

$$\Delta S_{\text{univ}} = 2mC \ln \frac{(T_1 + T_2)/2}{\sqrt{T_1 T_2}} \tag{6.124}$$

This is always positive because the arithmetic mean of any two numbers is always greater that the geometric mean.

6.15.3 Maximum Work from Two Finite Bodies at Temperatures T_H and T_L

Consider two identical finite bodies having same constant heat capacity. Let them be at temperatures T_H and T_L, respectively, $T_H > T_L$. If the two bodies are merely brought together into thermal contact, delivering no work, the final temperature, T_f, reached would be maximum and is given by $T_f = \dfrac{T_H + T_L}{2}$.

If a heat engine is made to operate between these two temperatures of T_H and T_L, some amount of heat (Q_S) from the body 1 will be supplied to the heat engine to convert into work (W). The remaining heat is rejected to the body 2. The temperature of body 1 decreases while the temperature of body 2 will increase. After sometime, both the bodies will reach thermal equilibrium at a temperature of T_f. At thermal equilibrium temperature, the heat engine stops its working. Now, let us analyse the system:

$$\text{Heat supplied to heat engine, } Q_S = c_p(T_H - T_f) \tag{6.125}$$

$$\text{Heat rejected to heat engine, } Q_R = c_p(T_f - T_L) \tag{6.126}$$

$$\text{Work delivered, } W = Q_S - Q_R \tag{6.127}$$

$$W = c_p(T_H - T_f - T_f + T_L) = c_p(T_H + T_L - 2T_f) \tag{6.128}$$

For given values of c_p, T_H and T_L, the work, W, depends on T_f. Work delivered will be maximum when T_f is minimum.

Change in entropy of body 1 is given by,

$$\Delta S_1 = \int_{T_H}^{T_f} c_p \frac{dT}{T} = c_p \ln \frac{T_f}{T_H} \tag{6.129}$$

Similarly, change in entropy of body 2 is given by,

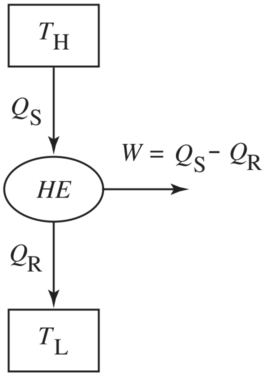


Fig. 6.22 Maximum work from two finite bodies at temperatures T_H and T_L

$$\Delta S_2 = \int_{T_L}^{T_f} c_p \frac{dT}{T} = c_p \ln \frac{T_f}{T_L} \quad (6.130)$$

Applying entropy principle, entropy of universe, ΔS_{univ} is given by,

$$c_p \ln \frac{T_f}{T_H} + c_p \ln \frac{T_f}{T_L} \geq 0 \quad (6.131)$$

$$c_p \ln \frac{T_f^2}{T_H T_L} \geq 0 \quad (6.132)$$

For T_f to be minimum, $c_p \ln \frac{T_f^2}{T_H T_L}$ should be equal to zero.

Therefore,

$$c_p \ln \frac{T_f^2}{T_H T_L} = 0 \quad (6.133)$$

$$\ln \frac{T_f^2}{T_H T_L} = 0 \text{ as } c_p \neq 0 \quad (6.134)$$

$$\ln \frac{T_f^2}{T_H T_L} = \ln 1 \quad (6.135)$$

$$\frac{T_f^2}{T_H T_L} = 1 \quad (6.136)$$

If we denote $T_H = T_1$ and $T_L = T_2$, then,

$$T_f = \sqrt{T_1 T_2} \quad (6.137)$$

Therefore, maximum work,

$$W_{\max} = c_p\left(T_1 + T_2 - 2\sqrt{T_1T_2}\right) \tag{6.138}$$

$$= c_p\left[\left(\sqrt{T_1}\right)^2 + \left(\sqrt{T_2}\right)^2 - 2\sqrt{T_1}\sqrt{T_2}\right] \tag{6.139}$$

$$W_{\max} = c_p\left(\sqrt{T_1} - \sqrt{T_2}\right)^2 \tag{6.140}$$

6.16 ABSOLUTE ENTROPY

By now it should be clear that entropy is defined only in terms of entropy change in a system from initial to final state. It is not measured in terms of absolute entropy. To provide an absolute value for entropy, a zero value of entropy of the system should be chosen arbitrarily as a standard state. Then, the entropy change is calculated with respect to this standard state.

For example, consider a system being at state 1. The absolute entropy, $\Delta S = S_1 - S_o$, where S_1 is entropy of the system at state 1 and S_o is entropy of the system at standard state denoted by o.

In this connection, the entropy of a pure crystalline substance is taken as zero at absolute temperature of zero. This statement is called the third law of thermodynamics. This law provides an absolute reference point is called as absolute entropy. It is more useful in thermodynamic analysis of chemical reactions.

6.17 THIRD LAW OF THERMODYNAMICS

Third law of thermodynamics, is an independent principle, initially recognised by Nernst and developed by Planck. It states that the entropy of a pure substance approaches zero at absolute zero temperature. This fact can also be corroborated by the definition of entropy. Entropy can be looked as a measure of molecular disorderliness. At absolute zero temperature, molecules of the substances get frozen and there is no activity. Therefore, it may be assigned zero entropy value at crystalline state.

The attainment of absolute zero temperature is impossible practically. However, at absolute zero temperature, it can be used for defining absolute entropy value theoretically with respect to zero entropy. By the second law of thermodynamics, it can be shown that absolute zero temperature cannot be reached.

From thermodynamic temperature scale, when heat rejection approaches zero, the temperature of heat rejection also tends to zero as a limiting case. But in such a situation, when heat rejection is zero, the heat engine takes form of a perpetual motion machine of second kind, where work is produced with only heat supplied to it. Thus, it leads to violation of Kelvin-Planck statement.

Also it can be said that 'it is impossible to attain absolute zero temperature in finite number of operations.' There exists absolute zero temperature on thermodynamic temperature scale but cannot be attained without violation of second law of thermodynamics. This fact is popularly explained by third law of thermodynamics. Third law of thermodynamics is of high theoretical significance for the sake of absolute property definitions and has found great utility in thermodynamics.

The third law of thermodynamics is stated as follows:

(i) *The entropy of all perfect crystalline solids is zero at absolute zero temperature.*

(ii) The third law of thermodynamics, often referred to as Nernst law, provides the basis for the calculation of absolute entropies of substances.

According to this law, if entropy is zero at $T = 0$, the absolute entropy s_{abs} of a substance at any temperature T and pressure p is expressed by the expression:

$$s_{abs} \;=\; \int_0^{T_s} = T_{f1} c_{ps} \frac{dT}{T} + \frac{h_{sf}}{T_s} + \int_{T_s}^{T_{f2}=T_g} c_{pf} \frac{dT}{T} + \frac{h_{fg}}{T_g} \int_{T_g}^{T} c_{pg} \frac{dT}{T} \qquad (6.141)$$

where $T_s = T_{f1} = T_{sf} = T_{sat}$ for fusion and $T_{f2} = T_g = T_{fg} = T_{sat}$ for vaporisation; c_{ps}, c_{pf}, c_{pg} = constant pressure specific heats for solids, liquids and gas, respectively and h_{sf}, h_{fg} = latent heats of fusion and vaporisation. Thus, by substituting $s = 0$ at $T = 0$, one may integrate zero kelvin and standard state of 273.15 K and 1 atm and find the entropy difference. Further, it can be shown that the entropy of a crystalline substance at $T = 0$ is not a function of pressure, viz $\left(\frac{\partial s}{\partial p}\right)_{T=0}$.

Worked-Out Examples

6.1 A body of constant heat capacity and initial temperature T_1 is placed in contact with a heat reservoir at temperature at T_2 and comes to thermal equilibrium with it. If $T_2 > T_1$, calculate the entropy change of the universe and show that it is always positive.

Solution

$$\begin{aligned}
(\Delta S)_{\text{body}} &= mc_p \int_{T_1}^{T_2} \frac{dT}{T} = mc_p \ln\left(\frac{T_2}{T_1}\right) \\
(\Delta S)_{\text{res}} &= \frac{-mc_p(T_2 - T_1)}{T_2} \\
(\Delta S)_{\text{univ}} &= (\Delta S)_{\text{body}} + (\Delta S)_{\text{res}} \\
&= mc_p \ln\left(\frac{T_2}{T_1}\right) - \frac{mc_p(T_2 - T_1)}{T_2} \\
&= mc_p \left[\ln\left(\frac{T_2}{T_1}\right) - \left(1 - \frac{T_1}{T_2}\right)\right]
\end{aligned}$$

Let $1 - \frac{T_1}{T_2} = x$. Then, $\frac{T_2}{T_1} = \frac{1}{1-x}$.

Since $\frac{T_2}{T_1} > 1$, x is positive.

$$\begin{aligned}
\frac{(\Delta S)_{\text{univ}}}{mc_p} &= -\ln(1-x) - x = \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots \\
(\Delta S)_{\text{univ}} &> 0 \quad \text{(Proved)}
\end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

6.2 In a temperature range between 0 °C and 100 °C, a particular system is maintained at constant volume. It has a heat capacity given by $c_V = A + 2BT$, where $A = 0.014$ J/K and $B = 4.2 \times 10^{-4}$ J/K^2. A heat reservoir at 0 °C and a reversible work source are available. What is the maximum amount of work that can be transferred to the reversible work source as the system is cooled from 100 °C to the temperature of the reservoir.

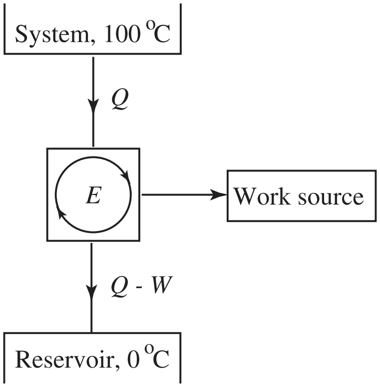


Fig. 6.23

Solution

The engine will stop working as the system temperature drops from 100 °C to 0 °C.

$$\text{Heat withdrawn, } Q = \int_{273}^{373} c_V dT = \int_{273}^{373} (A + 2BT) dT = A[T]_{273}^{373} + 2B\left[\frac{T^2}{2}\right]_{273}^{373}$$

$$= 0.014 \times (373 - 273) + 4.2 \times \left(3.73^2 - 2.73^2\right)$$

$$= 1.4 + 4.2 \times (13.913 - 7.453) = 28.532 \text{ J}$$

$$\Delta S_{system} = \int_{273}^{373} \frac{c_V dT}{T} = \int_{273}^{373} \frac{(A + 2BT)}{T} dT$$

$$= 0.014 \times \ln\left(\frac{273}{373}\right) + 2 \times 4.2 \times 10^{-4} \times (273 - 373)$$

$$= -0.0884 \text{ J/K}$$

$$\Delta S_{reservoir} = \frac{Q - W}{273}$$

By entropy principle,
$$0.0884 + \frac{28.532 - W}{273} \geq 0$$

$$28.532 - W = 24.1332$$

$$W \leq 4.399$$

$$W_{max} = 4.399 \text{ J}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

6.3 A reversible engine, as shown in Fig.6.24, during a cycle of operation draws 6 MJ from 450 K reservoir and does 800 kJ work. Calculate the amount and direction of heat interaction with other reservoir.

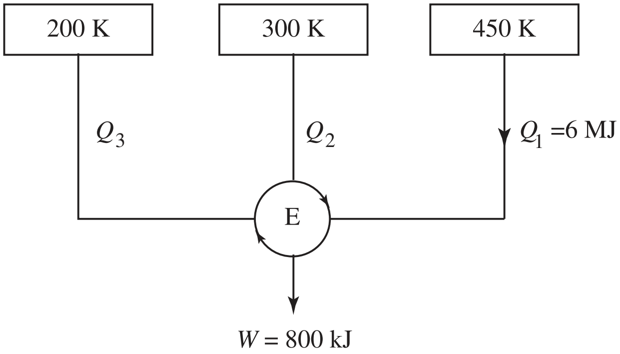


Fig. 6.24

Solution

Let Q_2 and Q_3 be both heat supplied to the reservoirs. By the entropy principle,

$$\frac{-6}{450} + \frac{Q_2}{300} + \frac{Q_3}{200} = 0$$

$$\frac{Q_2}{300} + \frac{Q_3}{200} = \frac{6}{450}$$

$$\frac{Q_2}{3} + \frac{Q_3}{2} = \frac{2}{1.5} = \frac{4}{3}$$

$$2Q_2 + 3Q_3 = 8 \tag{1}$$

$$W = Q_1 - (Q_2 - Q_3) = 0.80 \text{ MJ}$$

$$Q_2 + Q_3 = 8 - 0.8 = 7.2 \tag{2}$$

From Eqs (1) and (2),

$$Q_2 + 1.5Q_3 = 4 \tag{3}$$

$$Q_2 + Q_3 = 7.2 \tag{4}$$

$$0.5Q_3 = -3.2$$

$$Q_3 = -6.4 \text{ MJ}$$

$$Q_2 = 7.2 - Q_3 = 7.2 - 6.4 = 0.8$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

6.4 A fluid for which $\dfrac{pv}{T}$ is a constant quantity equal to R_1 shows that the change in specific entropy between states 1 and 2 is given by, $S_2 - S_1 = \int_{T_1}^{T_2} \left(\frac{c_p}{T}\right) dT = R \ln\left(\frac{p_2}{p_1}\right)$. Now, consider a fluid for which R is a constant and equal to 0.287 kJ/kg K flows steadily through an adiabatic machine entering and leaving through two adiabatic pipes. In one of these pipes, the pressure and temperature are 5 bar and 500 K and in the other pipe the pressure and temperature are 1 bar and 300 K, respectively. Determine which pressure and temperature refer to the inlet pipe. For a given temperature range, c_p is given by $c_p = a \ln T + b$, where T is the absolute temperature and $a = 0.026$ kJ/kg K; $b = 0.86$ kJ/kg K, which are for the inlet pipe.

Solution

For pipe 1 let $p = 5$ bar and $T = 500$ K. Then, pipe 2 will have $p = 1$ bar and $T = 300$ K.

$$\begin{aligned}
Tds &= dh - vdp = c_p dT - vdp \\
\int_1^2 ds &= \int_1^2 c_p \frac{dT}{T} - \int_1^2 \frac{RT}{p}\frac{dp}{T} \\
s_2 - s_1 &= \int_{T_1}^{T_2} \left(\frac{c_p}{T}\right) dT - R \ln\left(\frac{p_2}{p_1}\right) \\
s_2 - s_1 &= \int_{T_1}^{T_2} \frac{a \ln(T) + b}{T} dT - R \ln\left(\frac{p_2}{p_1}\right) \\
&= \int_{500}^{300} \left[a \ln T d(\ln T) + b\frac{dT}{T}\right] - R \ln\left(\frac{p_2}{p_1}\right) \\
&= \frac{0.026 \times (\ln(300))^2 - (\ln(500))^2}{2} + 0.86 \times \ln\left(\frac{300}{500}\right) - 0.287 \times \ln\left(\frac{1}{5}\right) \\
&= 0.013 \times (32.533 - 38.621) - 0.439 + 0.462 = -0.0561
\end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

Since $s_2 - s_1$ is negative, the flow is from 2 to 1, therefore, 2 is the inlet pipe. $\overset{\textbf{Ans}}{\Longleftarrow}$

6.5 Two vessels, 1 and 2 each of volume 2 m³ are connected by a tube of negligible volume. Vessel 1 contains air at 0.8 MPa and 90 °C, while vessel 2 contains air at 0.4 MPa and 200 °C. Find the change in entropy when pipe 1 is connected to pipe 2, by working from the first principles and assuming the mixing to be complete and adiabatic. For air, assume that $c_p = 1.005$ kJ/kg K; $h = c_p T$ and $\frac{v}{T} = \frac{0.287}{p}$ where p, v and T are pressure in kPa, volume in m³/kg and temperature in K, respectively.

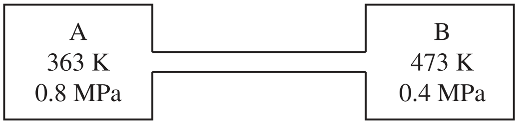


Fig. 6.25

Solution

Given that pipe 1 and pipe 2 are connected by a tube of negligible volume,

$$v_1 = \frac{0.287 \times 363}{800} = 0.130 \text{ m}^3/\text{kg}$$

$$m_1 = \frac{2}{0.13} = 15.38 \text{ kg}$$

$$v_2 = \frac{0.287 \times 473}{400} = 0.339 \text{ m}^3/\text{kg}$$

$$m_2 = \frac{2}{0.339} = 5.89 \text{ kg}$$

After mixing, by energy balance,

$$15.38 \times 1.005(T_f - 363) = 5.89 \times 1.005 \times (473 - T_f)$$

$$T_f - 363 = \frac{5.89}{15.38} \times (473 - T_f) = 0.38 \times (473 - T_f)$$

$$T_f + 0.38 \times T_f = 0.38 \times 473 + 363$$

$$1.38 \times T_f = 542.74 \text{ K}$$

$$T_f = 393.28 \text{ K}$$

$$v_f = \frac{2+2}{5.89 + 15.38} = 0.188 \text{ m}^3/\text{kg}$$

$$p_f = \frac{0.287 \times 393.28}{0.188} = 600.38 \text{ kPa}$$

$$\Delta S_1 = m_1 \left[c_p \ln\left(\frac{T_f}{T_1}\right) - R \ln\left(\frac{p_f}{p_1}\right) \right]$$

$$= 15.38 \times \left[1.005 \times \ln\left(\frac{393.28}{363}\right) - 0.287 \times \ln\left(\frac{600.38}{800}\right) \right]$$

$$= 15.38 \times (0.08105 + 0.0823) = 2.50 \text{ kJ/K}$$

$$\Delta S_2 = m_2 \left[c_p \ln\left(\frac{T_f}{T_2}\right) - R \ln\left(\frac{p_f}{p_2}\right) \right]$$

$$= 5.89 \times \left[1.005 \times \ln\left(\frac{393.28}{473}\right) - 0.287 \times \ln\left(\frac{600.38}{400}\right) \right]$$

$$= 5.89 \times (-0.1854 - 0.1164) = -1.78 \text{ kJ/K}$$

$$\Delta S_{total} = \Delta S_1 + \Delta S_2 = 2.5 - 1.78 = 0.72 \text{ kJ/K} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

6.6 An aluminum block of $c_p = 400$ J/kg K with a mass of 8 kg is initially at 37 °C in a room air at 17 °C. It is cooled reversibly by transferring heat to a completely reversible cyclic heat engine until the block reaches 17 °C. The 17 °C room air serves as a constant temperature sink for the engine. Calculate (i) the change in entropy for the block, (ii) the change in entropy for the room air and (iii) the work done by the engine.

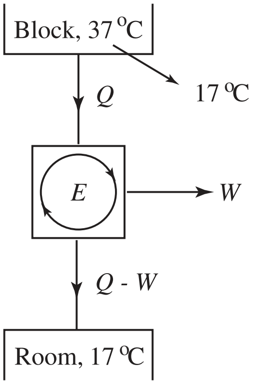


Fig. 6.26

Solution

Referring Fig.6.26,

$$\Delta s_{block} = mc_p \ln\left(\frac{T_2}{T_1}\right) = 8 \times 400 \times \ln\left(\frac{290}{310}\right) = -213.41 \text{ J/K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$Q = 8 \times 400 \times 17 = 54400 \text{ J}$$

$$W_{max} = mc_p\left[(T - T_0) - T_0 \ln\left(\frac{T}{T_0}\right)\right]$$

$$= 8 \times 400 \times \left[(310 - 290)20 - 290 \times \ln\left(\frac{310}{290}\right)\right] = 2110.4 \text{ J} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$Q - W_{max} = 54400 - 2110.4 = 52289.6 \text{ J}$$

$$\Delta S_{room} = \frac{52289.6}{290} = 180.30 \text{ J/K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.7 In the above example, if the aluminum block is allowed to cool by natural convection to room air, compute (i) the change in entropy; (ii) change in entropy for the room air; and (iii) the net change in entropy for the universe.

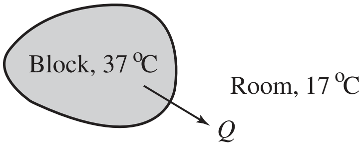


Fig. 6.27

Solution

$$\Delta S_{block} = 8 \times 400 \times \ln\left(\frac{290}{310}\right) = -213.41 \text{ J/K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

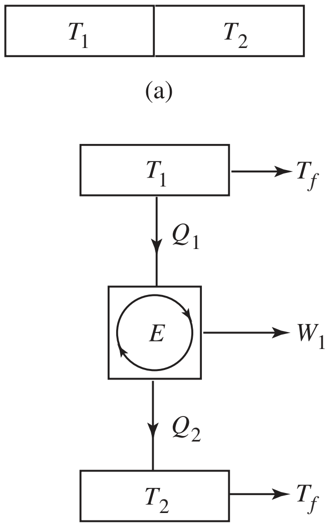
$$Q = 54400 \text{ J} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\Delta S_{room} = \frac{54400}{290} = 187.59 \text{ J/K}$$

$$\Delta S_{universe} = -213.41 + 187.59 = -25.82 \text{ J/K} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

6.8 Two bodies of equal heat capacities C and temperatures T_1 and T_2 from an adiabatically closed system. What will be the final temperature if one lets this system come to equilibrium under the following conditions: (i) freely; (ii) reversibly; and (iii) what is the maximum possible work which can be obtained from this system?

Solution



(a)

(b)

Fig. 6.28

(i) If the two bodies came to equilibrium freely [Fig.6.28(a)], the final temperature would be,

$$T_f = \frac{T_1 + T_2}{2}$$

(ii) If the two bodies came to equilibrium reversibly with the delivery of maximum work W [Fig.6.28(b)],

$$Q_1 = C(T_1 - T_f)$$

$$Q_2 = C(T_f - T_2)$$

$$W = Q_1 - Q_2 = C(T_1 + T_2 - 2T_f) \qquad (1)$$

where T_f is the final temperature (minimum) of the two bodies with the engine stop working. By entropy principle,

$$\Delta S_1 + \Delta S_2 \geq 0$$

$$C \ln\left(\frac{T_f}{T_1}\right) + C \ln\left(\frac{T_f}{T_2}\right) \geq 0$$

$$C \ln\left(\frac{T_f^2}{T_1 T_2}\right) \geq 0$$

For minimum T_f,
$$C \ln\left(\frac{T_f^2}{T_1 T_2}\right) = 0$$

As $C \neq 0$,
$$\ln\left(\frac{T_f^2}{T_1 T_2}\right) = 0 = \ln(1)$$

$$[T_f]_{min} = \sqrt{T_1 T_2}$$

From Eq.(1) W is maximum when T_f is minimum.

$$W_{max} = C\left(T_1 + T_2 - 2\sqrt{T_1 T_2}\right) = C\left(\sqrt{T}_1 - \sqrt{T}_2\right)^2$$

Ans ⟸

6.9 A resistor of 40 ohms is maintained at 27 °C while a current of 10 amperes is allowed to flow for one second. Determine the entropy change of the resistor and the universe.

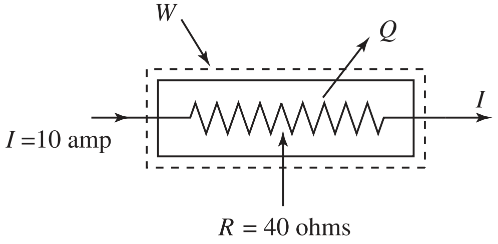


Fig. 6.29

Solution

Referring Fig.6.29,

$$W \quad = \quad I^2 Rt = 10^2 \times 40 \times 1 = 4000 \text{ Ws} = 4000 \text{ J}$$

Since the temperature of the resistor is constant, there is no change in its internal energy. Therefore, by the first law,

$$\begin{aligned}
-Q &= U_2 - U_1 - W = -W \\
Q &= W = 4000 \text{ J} \\
\Delta s_{Resistor} &= 0 \\
\Delta s_{atm} &= \frac{Q}{T_0} = \frac{4000}{300} = 13.33 \text{ J/K} && \overset{\textbf{Ans}}{\Longleftarrow} \\
\Delta s_{universe} &= 13.33 \text{ J/K} && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

6.10 In the above example if the resistor initially at 27 °C is now insulated and the same current is passed for the same time, determine the entropy change of the resistor and the universe. The specific heat of the resistor is 1 kJ/kg K and the mass of the resistor is 10 g.

Solution

Since $Q = 0$, the work will be dissipated to increase the internal energy of the resistor.

$$\begin{aligned}
Q &= U_2 - U_1 - W \\
0 &= U_2 - U_1 - W \\
U_2 - U_1 &= W = 4000\ J \\
mC(T_2 - T_1) &= 4000
\end{aligned}$$

$$\begin{aligned}
T_2 &= \frac{4000}{0.01 \times 1000} + 300 = 700 \text{ K} \\
\Delta s_{Resistor} &= mC \ln\left(\frac{T_2}{T_1}\right) = 10 \times 1 \times \ln\left(\frac{700}{300}\right) = 8.473 \text{ J/K} && \overset{\textbf{Ans}}{\Longleftarrow} \\
\Delta s_{surrounding} &= 0 && \overset{\textbf{Ans}}{\Longleftarrow} \\
\Delta s_{universe} &= 8.473 \text{ J/K} && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

6.11 An adiabatic vessel containing 2 kg of water at 37 °C. By paddle wheel work transfer, the temperature of water is increased to 40 °C. If the specific heat of water is 4.187 kJ/kg K, find the entropy change of the universe.

Solution

The state change from 1 to 2 due to the paddle wheel work transfer is shown in Fig.6.30(a). The entropy change would be $S_2 - S_1$. Now, connect states 1 and 2 by a reversible isobaric path (involving heat transfer) on which integration has to be performed to find the entropy change.

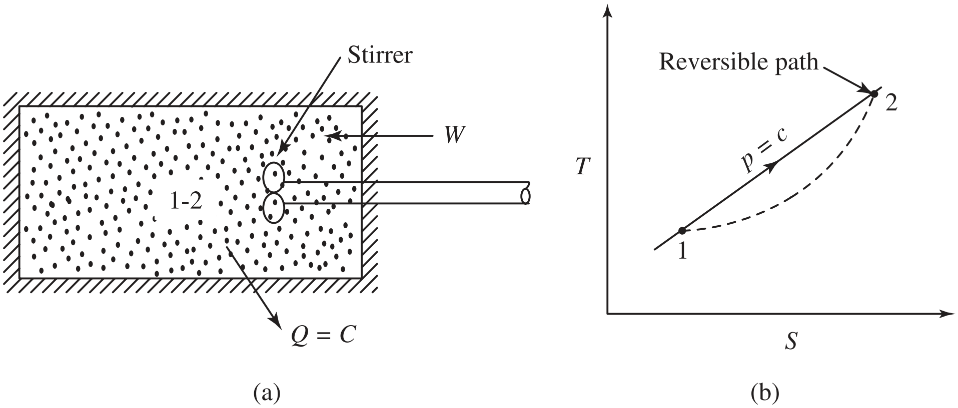


Fig. 6.30

$$S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} = \int_1^2 mc_p \frac{dT}{T} = mc_p \ln\left(\frac{T_2}{T_1}\right)$$

$$= 2 \times 4.187 \times \ln\left(\frac{313}{310}\right) = 0.0806$$

$$\Delta S_{universe} = \Delta S_{system} + \Delta S_{sum} = 0.0806 + 0 = 0.0806 \text{ kJ/K}$$

6.12 A copper rod is of length 1 m and diameter 0.01 m. One end is kept at 120 °C and the other end at 20 °C. The rod is perfectly insulated along the length. Take the thermal conductivity of copper as 380 W/mK. Calculate the rate of heat transfer along the rod and the rate of entropy production due to the irreversibility of this heat transfer.

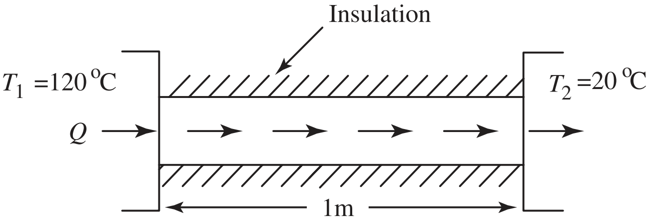


Fig. 6.31

Solution

$$Q = KA\left(\frac{T_1 - T_2}{l}\right) = 380 \times \frac{\pi}{4}(0.01)^2 \times \frac{100}{1} = 2.9845 \text{ W}$$

$$\text{Rate of entropy production} = \frac{-Q}{T_1} + \frac{Q}{T_2} = \frac{-2.9845}{393} + \frac{2.9845}{293}$$

$$= -0.0076 + 0.0102 = 0.0026 \text{ W/K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.13 A body of constant heat capacity, c_p, and at a temperature, T_i, is put in contact with a reservoir at a higher temperature, T_f. The pressure remains constant while the body comes to equilibrium with the reservoir. Show that the entropy change of the universe is equal to

$$c_p\left[\frac{T_i - T_f}{T_f} - \ln\left(1 + \frac{T_i - T_f}{T_f}\right)\right]$$

Prove that this entropy change is positive. Given:

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$

where $x < 1$.

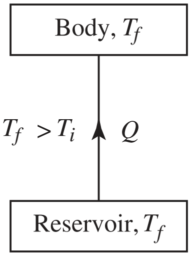


Fig. 6.32

Solution

$$\text{Heat transfer, } Q = c_p(T_f - T_i)$$

$$\Delta S_{body} = c_p \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S_{Reservoir} = -\frac{c_p(T_f - T_i)}{T_f}$$

$$\begin{aligned}\Delta S_{universe} &= c_p\left[\ln\left(\frac{T_f}{T_i}\right) - \frac{T_f - T_i}{T_f}\right] \\ &= c_p\left[\frac{T_i - T_f}{T_f} - \ln\left(1 + \frac{T_i - T_f}{T_f}\right)\right]\end{aligned}$$

Hence, proved.

Let $\dfrac{T_i - T_f}{T_f} = x$ and $x < 1$.

$$\begin{aligned}\Delta S_{universe} &= c_p[x - \ln(1-x)] \\ &= c_p\left[x - \left(x - \frac{x^2}{2} + \frac{x^3}{3} - \dots\right)\right] = c_p\left(\frac{x^2}{2} - \frac{x^3}{3}\right)\end{aligned}$$

Since $x < 1, \Delta S_{universe} > 0$. Hence, proved. $\overset{\textbf{Ans}}{\Longleftarrow}$

6.14 One kilogram of an insulated copper calorimeter can hold 0.1 kg of water and is in equilibrium at a temperature of 27 °C. An experiment now places 0.05 kg of ice at 0 °C in the calorimeter and encloses the later with a heat insulating shield.

(i) When all the ice melts and equilibrium is reached, calculate the temperature of water and the can. Take the specific heat of copper as 0.418 kJ.kg K and the latent heat of fusion of ice as 333 kJ/kg.

(ii) Compute the entropy increase of the universe resulting from the process.

(iii) What will be the minimum work needed by a stirrer to bring back the temperature of water to 27 °C.

Solution

Let t be the final temperature of the water and can. Then, by energy balance,

$$\begin{aligned} 0.1 \times 4.18 \times (27 - t) + 1 \times 0.418 \times (27 - t) &= 0.05 \times 333 + 0.05 \times 4.18 \times (t - 0) \\ 11.286 - 0.418t + 11.286 - 0.418t &= 16.65 + 0.209t \\ 22.572 - 16.65 &= 1.045t \\ t &= 5.67\ ^\circ\text{C} \end{aligned}$$

$$\begin{aligned} \Delta s_{total} = (0.1 \times 4.18 + 1 \times 0.418) \times \ln\left(\frac{273 + 5.67}{300}\right) + 0.05 \times \left[\frac{333}{273} + 4.18 \times \ln\left(\frac{278.67}{273}\right)\right] \\ = 0.00363 \text{ kJ/K} \end{aligned}$$

Heat transfer into the system to bring back the temperature of water and can to 27 °C.

$$= (0.1 + 0.05) \times 4.18 \times (27 - 5.67) + 1 \times 0.418 \times (27 - 5.67) = 22.29 \text{ kJ}$$

This should be the increase in internal energy of the whole system. By first law,

$$Q - W = \Delta U$$

Since $Q = 0$, as the calorimeter is insulated, the work imparted by the stirrer:

$$W = -\Delta U = -22.29 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.15 Show that if two bodies of thermal capacitors C_H and C_L at temperature T_H and T_L are brought to the same temperature T_F by means of a reversible heat engine, then

$$\ln(T) = \frac{C_H \ln(T_H) + C_L \ln(T_L)}{C_H + C_L}$$

Solution

As can be seen from Fig.6.33, $Q_H = C_H(T_H - T_F)$ and $Q_L = C_L(T_F - T_L)$, where T_F is the final temperature which both the bodies will attain before the engine comes into a step:

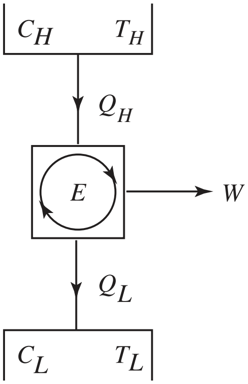


Fig. 6.33

$$\begin{aligned} W &= Q_H - Q_L = C_H T_H - C_H T_F - C_L T_F + C_L T_L \\ &= C_H T_H + C_L T_L - T_F(C_H + C_L) \end{aligned}$$

W will be maximum when T is minimum, other quantities being given.

By entropy principle, $C_H \ln\left(\frac{T_F}{T_H}\right) + 0 + C_L \ln\left(\frac{T_F}{T_L}\right) \geq 0.$

For minimum value of T,

$$\begin{aligned} C_H \ln\left(\frac{T_F}{T_H}\right) + C_L \ln\left(\frac{T_F}{T_L}\right) &= 0 \\ C_H \ln(T_F) - C_H \ln(T_H) + C_L \ln(T_F) - C_L \ln(T_L) &= 0 \end{aligned}$$

$$\ln(T_F) = \frac{C_H \ln(T_H) + C_L \ln(T_L)}{C_H + C_L} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.16 Two blocks of metal each having a mass of 8 kg and a specific heat of 0.50 kJ/kg K are at temperature 40 °C. A reversible refrigerator receives heat from an block and rejects heat to the other. Calculate the work required to cause a temperature difference of 110 °C between the two blocks.

Solution

Let T_{f1} be the final temperature of block 1 and T_{f2} be that of block 2:

$$T_{f1} - T_{f2} = 110^\circ\text{C}$$

$$Q_1 = C(T_{f1} - 313)$$

$$Q_2 = C(313 - T_{f2})$$

where C is the heat capacity of each block.

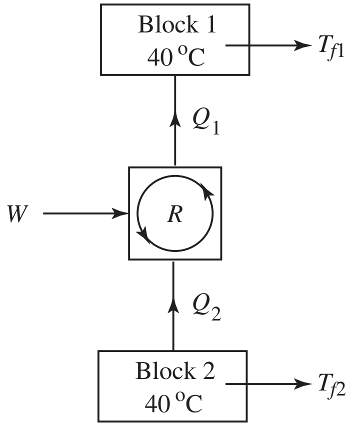


Fig. 6.34

$$W = Q_1 - Q_2 = C(T_{f1} + T_{f2} - 626) \qquad (1)$$

W will be minimum when T_{f1} and T_{f2} are minimam. By entropy principle,

$$C\ln\left(\frac{T_{f1}}{313}\right) + 0 + C\ln\left(\frac{T_{f2}}{313}\right) > 0$$

$$C\ln\left(\frac{T_{f1}\times T_{f2}}{313^2}\right) = 0$$

$$T_{f1}\times T_{f2} = 313^2 = 97969$$

or
$$T_{f1} = \frac{97969}{T_{f2}}$$

Since
$$T_{f1} - T_{f2} = 110$$

$$\frac{97969}{T_{f2}} - T_{f2} = 110$$

or
$$T_{f2}^2 + 110T_{f2} - 97969 = 0$$

$$T_{f2} = \frac{-110 \pm \sqrt{12100 + 4\times 97969}}{2} = 262.8 \text{ K}$$

$$T_{f1} = 372.8 \text{ K}$$

From Eq.(1),
$$W = 8\times 0.5\times(262.8 + 372.8 - 626) = 38.4 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.17 A body of finite mass is originally at a temperature T_1 which is higher than that of a heat reservoir at a temperature T_2. An engine operates in infinitesimal cycle between the body and the reservoir until it leaves the temperature of the body from T_1 to T_2.

In this process, there is a heat flow, Q, out of the body. Prove that the maximum work obtainable from the engine is $Q + T_2(S_1 - S_2)$, where $S_1 - S_2$ is the decrease in entropy of the body.

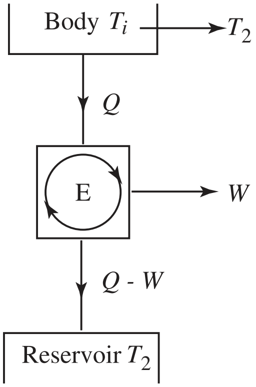


Fig. 6.35

Solution

Since $T_1 > T_2$, as heat Q is withdrawn from the body, the entropy would decrease from S_1 to S_2 (refer Fig.6.35):

$$\Delta S_{body} = S_1 - S_2$$

$$\Delta S_{engine} = 0$$

$$\Delta S_{Reservoir} = \frac{Q - W}{T_2}$$

By entropy principle:

$$S_1 - S_2 + \frac{Q - W}{T_2} \geq 0$$

$$T_2(S_1 - S_2) \geq W - Q$$

$$W - Q \leq T_2(S_1 - S_2)$$

$$W \leq Q + T_2(S_1 - S_2)$$

$$W_{max} = Q + T_2(S_1 - S_2) \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.18 35 grams of water at 30 °C is converted into steam at 250 °C at constant atmospheric pressure. The specific heat of water is assumed constant at 4.2 J/g K and the latent heat of vaporisation at 100 °C is 2260 J/g. For water vapour, assume $pv = mRT$, where $R = 0.4619\ kJ/kgK$ and $\frac{c_p}{R} = a + bT + CT^2$, where $a = 3.634$; $b = 1.195 \times 10^{-3}\ K^{-1}$ and $C = 0.135 \times 10^{-6}\ K^{-2}$. Calculate the entropy change of the system.

Solution

From Fig.6.36,

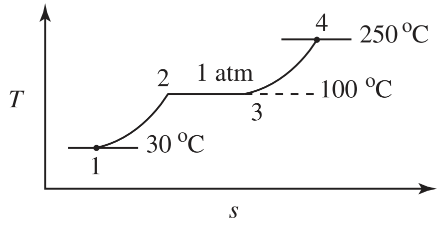


Fig. 6.36

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) = 4.2 \times \ln\left(\frac{373}{303}\right) = 0.873 \text{ kJ/kg K}$$

$$s_3 - s_2 = \frac{h_3 - h_2}{373} = \frac{2260}{373} = 6.059 \text{ kJ/kg K}$$

$$s_4 - s_3 = \int_{T_3}^{T_4} \frac{dT}{T} = \int_{T_3}^{T_4} (a + bT + cT^2)R\frac{dT}{T} = aR\ln\left(\frac{T_4}{T_3}\right) + bR(T_4 - T_3) + cR\frac{(T_4^2 - T_3^2)}{2}$$

$$= 0.4619 \times \left[3.634 \times \ln\left(\frac{523}{373}\right) + 1.195 \times 10^{-3} \times 150 + 0.135 \times 10^{-6} \times \frac{523^2 - 373^2}{2}\right]$$

$$= 0.4619[1.2283 + 0.1793 + 0.0091] = 0.6544 \text{ kJ/kg K}$$

$$s_4 - s_1 = 0.873 + 6.059 + 0.6544 = 7.5864 \text{ kJ/kg K} = 7.5864 \text{ kJ/kg K}$$

Total change in entropy $= m \times (s_4 - s_1) = 35 \times 7.5864 = 265.524$ J/K $\overset{\textbf{Ans}}{\Longleftarrow}$

6.19 A 100 ohm resistor carrying a constant current of 1 amp is kept at a constant temperature of 30 °C by a steam of cooling water. In a time interval of one second,

(i) What is the change in entropy of the resistor?

(ii) What is the change in entropy of the universe?

Solution

$$W = I^2RT = 1^2 \times 100 \times 1 = 100 \text{ J}$$

$$W = U_2 - U_2 + Q = 0 - Q$$

$$-100 = 0 - Q$$

$$Q = 100 \text{ J}$$

$$\Delta S_{Resistor} = 0 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\Delta S_{coolingwater} = \frac{100}{303}$$

$$\Delta S_{universe} = 0.33 \text{ J/K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.20 A lump of ice with a mass of 2 kg at an initial temperature of 250 K melts at a pressure of 1 bar due to heat transfer from the environment. After some time has elapsed, the resulting water attains the temperature of the environment, 293 K. Find the entropy production associated with this process. The latent heat of fusion of ice is 333.3 kJ/kg. The specific heat of ice and water are 2.07 and 4.2 kJ/kg K, respectively. Take that ice melts at 273.15 K.

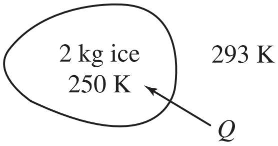


Fig. 6.37

Solution

Entropy change of the system (refer Fig.6.37),

$$\Delta S_{system} = 2 \times \left(2.07 \times \ln\left(\frac{273}{250}\right) + \frac{333.3}{273} + 4.2 \times \ln\left(\frac{293}{273}\right)\right) = 3.4 \text{ kJ/K}$$

$$\Delta S_{environment} = \frac{-2 \times (2.07 \times 23 + 333.3 + 4.2 \times 20)}{293} = -3.173 \text{ kJ/K}$$

$$\Delta S_{universe} = 3.4 - 3.173 = 0.227 \text{ kJ/K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.21 An ideal gas is compressed reversibly and adiabatically from states a and b. It is then heated reversibly at constant volume to state c. After expanding reversibly and adiabatically to state d such that $T_b = T_d$, the gas is again reversibly heated at constant pressure to state c such that $T_e = T_c$. Heat is then rejected reversibly from the gas at constant volume till it returns to state a. Express T_a in terms of T_b and T_c. If $T_b = 550$ K and T_C is 830 K, estimate T_a. Take $\gamma = 1.4$.

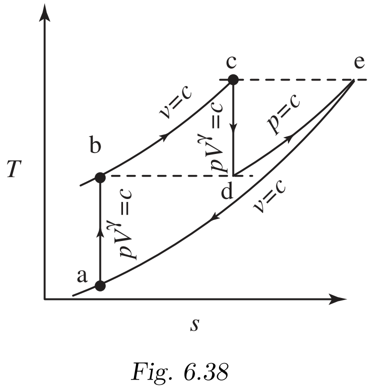


Fig. 6.38

Solution

From Fig.6.38,

$$T_b = T_d \quad \text{and} \quad T_c = T_e$$

$$\frac{T_a}{T_b} = \left(\frac{v_b}{v_a}\right)^{\gamma-1}$$

$$\frac{T_c}{T_d} = \left(\frac{v_d}{v_c}\right)^{\gamma-1} = \left(\frac{v_d}{v_b}\right)^{\gamma-1} \quad (\because v_b = v_c)$$

$$\frac{P_d v_d}{T_d} = \frac{P_e v_e}{T_e}$$

As $a - e$ is a constant volume process, $v_e = v_a$

$$\frac{v_d}{v_a} = \frac{T_d}{T_e} = \frac{T_d}{T_c} = \frac{T_b}{T_c}$$

$$\frac{v_d}{v_a} = \frac{v_b}{v_a} \times \frac{v_d}{v_b} = \frac{v_b}{v_a} \times \frac{v_d}{v_c} = \frac{T_b}{T_c} = \left(\frac{T_a}{T_b}\right)^{\frac{1}{\gamma-1}} \times \left(\frac{T_c}{T_b}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{T_b}{T_c} = \left(\frac{T_a T_c}{T_b}\right)^{\frac{1}{\gamma-1}}$$

$$\frac{T_a T_c}{T_b^2} = \left(\frac{T_b}{T_c}\right)^{\gamma-1}$$

$$T_a = \frac{T_b^{\gamma+1}}{T_c^{\gamma}}$$

Substituting given values,

$$T_a = \frac{550^{2.4}}{830^{1.4}} = 309.14 \text{ K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

6.22 Nitrogen gas at 6 bar, 20 °C enters an insulated control volume operating at a steady state for which $W_{cv} = 0$. Half of the nitrogen exits the device at 1 bar, 82 °C and the other half exits at 1 bar, −40 °C. The effect of KE and PE are negligible. Employing the ideal gas model, decide whether the device can operate as described.

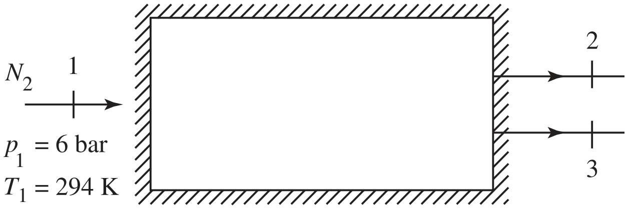


Fig. 6.39

Solution

By energy balance, $\dot{Q}_{cv} - \dot{w}_{cv} + \dot{m}_1 h_1 - \dot{m}_2 h_2 - \dot{m}_3 h_3 = 0$

$$\dot{Q}_{cv} = 0 \quad \text{and} \quad \dot{W}_{cv} = 0$$

$$h_1 - \frac{1}{2}h_2 - \frac{1}{2}h_3 = 0$$

Take $h = f(T)$ only, as nitrogen is assumed as an ideal gas:

$$T_1 = \frac{T_2}{2} + \frac{T_3}{2} = \frac{355 + 233}{2} = 294 \text{ K}$$

Thus, conservation of energy principle is satisfied.

By entropy principle, $\Sigma_j \dfrac{Q_J}{T_j} + \dot{m}_1 s_1 - \dot{m}_2 s_2 - \dot{m}_3 s_3 + \sigma_{cv} = 0$

$$\frac{Q_j}{T_j} = 0$$

$$\dot{m}_2 = \dot{m}_3 = \frac{\dot{m}_1}{2}$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_1} = \frac{1}{2}s_2 + \frac{1}{2}s_3 - s_1 = \frac{1}{2}(s_2 - s_1) + \frac{1}{2}(s_3 - s_1)$$

$$= \frac{1}{2}\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right)\right] + \frac{1}{2}\left[c_p \ln\left(\frac{T_3}{T_1}\right) - R\ln\left(\frac{p_3}{p_1}\right)\right]$$

Since, $\dfrac{p_2}{p_1} = \dfrac{p_3}{p_1}$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_1} = \frac{1}{2}c_p \ln\left(\frac{T_2 T_3}{T_1^2}\right) - R\ln\left(\frac{p_2}{p_1}\right)$$

For nitrogen,

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4}{0.4} = \frac{8.314}{28} = 1.04 \text{ kJ/kg K}$$

$$\frac{\dot{\sigma}_{cv}}{\dot{m}_1} = \frac{1}{2} \times 1.04 \times \ln\left(\frac{355 \times 233}{294^2}\right) - \frac{8.314}{28} \times \ln\left(\frac{1}{6}\right)$$

$$= 0.509 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Thus, second law is also satisfied. The device can thus able to operate as described.

6.23 A rigid tank contains an ideal gas at 37 °C that is being stirred by a paddle wheel. The paddle wheel does 200 kJ of work an ideal gas. It is observed that the temperature of the ideal gas remains constant during the process as a result of heat transfer between the system and the surroundings at 27 °C. Determine the change in entropy of the ideal gas and the total entropy generation.

Solution

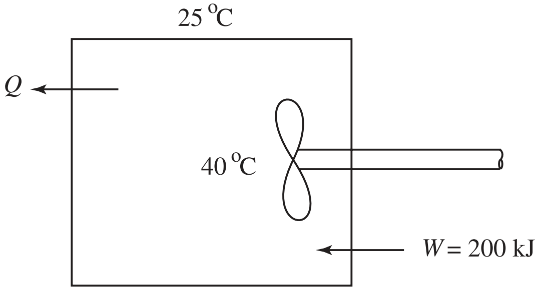


Fig. 6.40

$T_0 = 27 + 273 = 300$ K.

Work transfer, $W = 200$ kJ to the ideal gas. There is no entropy transfer since the gas temperature remains constant. Therefore, the entropy change of ideal gas is zero. By energy balance, $W = Q = 200$ kJ.

$$\Delta S_{surr} = \frac{Q}{T_0} = \frac{200}{300} = 0.667 \text{ kJ/K}$$

$$\Delta S_{total} = \Delta S_{gas} + \Delta S_{surr} = 0.667 \text{ kJ/K}$$ $\overset{\text{Ans}}{\Longleftarrow}$

6.24 Show that minimum theoretical work input required by a refrigeration cycle to bring two finite bodies from the same initial temperature to the final temperature of T_1 and T_2, where $T_2 < T_1$ is given by:

$$W_{min} = mC\left[2(T_1T_2)^{\frac{1}{2}} - T_1 - T_2\right]$$

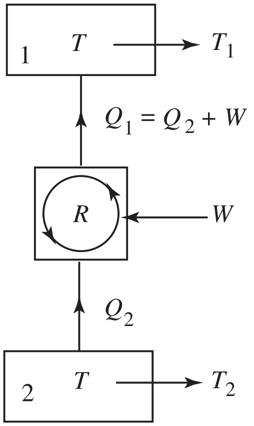


Fig. 6.41

Solution

Assume that the two bodies 1 and 2 are of the same mass m and specific heat C. Both the bodies are initially at the same temperature, T. A refrigeration cycle is operated which takes out energy Q_2 from body 2 and discharges Q_1 to body. The energy transfer takes place until the body 2 temperature falls to T_2. The body 1 rises to temperature T_1. Now,

$$Q_2 = mC(T - T_2)$$

$$Q_1 = mC(T_1 - T)$$

$$W = Q_1 - Q_2 = m \times C(T_1 - T - T + T_2) = mC(T_1 + T_2 - 2T) \qquad (1)$$

For given values of T_1 and T_2, W will be minimum when T is maximum. By entropy principle,

$$\int_T^{T_2} mC\frac{dT}{T} + \int_T^{T_1} mC\frac{dT}{T} + 0 \geq 0$$

$$mC\ln\left(\frac{T_2}{T}\right) + mC\ln\left(\frac{T_1}{T}\right) \geq 0$$

$$mC\ln\left(\frac{T_1T_2}{T^2}\right) \geq 0$$

$$mC\ln\left(\frac{T^2}{T_1T_2}\right) \leq 0 \leq \ln(1)$$

For maximum T,

$$mC\ln\left(\frac{T^2}{T_1T_2}\right) = 0$$

$$\ln\left(\frac{T^2}{T_1T_2}\right) = 0 \quad = \quad \ln(1) \quad (\because m \times c \neq 0)$$

$$T_{max} = \sqrt{T_1T_2}$$

From Eq.(1),

$$W_{min} = mC\left(T_1 + T_2 - 2\sqrt{T_1T_2}\right)$$

If input work is considered as negative,

$$W_{min} = -W_{min} = mC\left(2\sqrt{T_1T_2} - T_1 - T_2\right) \qquad (\xLeftarrow{\textbf{Ans}})$$

6.25 Air is flowing through a horizontal insulated duct. Experiments were carried out by a set of students in the duct. One group of students measure the pressure, temperature and velocity at a location in the duct as 0.95 bar 67 °C and 75 m/s, respectively. In another location, the respective values are found to be 0.8 bar, 22 °C and 310 m/s, respectively. The group forgot to note the direction of flow. Can you find the direction with the help of available data?

Solution

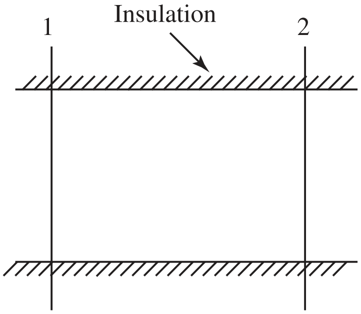


Fig. 6.42

From property relation,

$$Tds = dh - vdp$$

$$ds = \frac{dh}{T} - \frac{vdp}{T} = c_p\frac{dT}{T} - \frac{vdp}{T}$$

For the air flowing through the insulated duct,

$$ds_{universe} = ds_{air} + ds_{surr} - ds_{duct}$$

$$\Delta S_{air} = \int c_p\frac{dT}{T} - \int_1^2 R\frac{dp}{p}$$

$$\Delta S_{air} = \int_1^2 c_p\frac{dT}{T} - R\int_1^2 \frac{dp}{p}$$

$$S_2 - S_1 = 1.005 \times \ln\left(\frac{273+22}{273+67}\right) - 0.287 \times \ln\left(\frac{0.8}{0.95}\right)$$

$$= -0.146 + 0.049 = -0.097 \text{ kJ/kg K} = \Delta S_{universe} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Since, the entropy change of the universe cannot be negative, the direction of flow of air must be from right to left.

Review Questions

6.1 Who originated the concept of entropy? Describe in your own words your understanding of entropy.

6.2 Explain with an appropriate figure the possibility of a reversible process by means of equivalent reversible processes.

6.3 With neat diagrams explain the concept of Clausius inequality and derive the equation that can describe Clausius inequality.

6.4 What are the three important statements about entropy?

6.5 *Prove with appropriate diagram and equation that entropy is a property.*

6.6 *What do you understand by reversible and irreversible process from the point of view of entropy?*

6.7 *Describe concept of increasing entropy. How does entropy increase can be viewed as order to disorder?*

6.8 *Explain the entropy transfer mechanism by means of heat and mass transfer.*

6.9 *Describe the entropy generation for a closed system.*

6.10 *Describe entropy generation of an open system.*

6.11 *How can you evaluate entropy change by means of Tds relation?*

6.12 *Explain why constant volume lines are steeper in a Tds diagram.*

6.13 *Derive the change in entropy relation in terms of temperature and volume.*

6.14 *Derive the change in entropy relation in terms of pressure and temperature.*

6.15 *Derive the change in entropy relation in terms of pressure and volume.*

6.16 *Derive expression for change in entropy for a constant volume process.*

6.17 *Derive expression for change in entropy for a constant pressure process.*

6.18 *Derive expression for change in entropy for a constant temperature process.*

6.19 *Derive expression for change in entropy for a constant entropy process.*

6.20 *Derive expression for change in entropy for a polytropic process.*

6.21 *Explain entropy principle in transfer of heat through finite temperature difference?*

6.22 *Explain the entropy principle in mixing of two fluids and derive appropriate expressions?*

6.23 *Derive an expression for maximum work in terms of T_1 and T_2.*

6.24 *Explain the concept of absolute entropy.*

6.25 *What is third law of thermodynamics? Explain.*

Exercise

6.1 In a hydraulic turbine, when water flows through, it is found that temperature increases from 37 ° to 40 °. Calculate the change in entropy, if the heat transfer is negligible. State the assumptions made. *Ans:* 0.04 kJ/K Assumptions: (i) water is incompressible; (ii) the process is assumed as constant volume.

6.2 Ten kilograms of water at 275 K is brought into contact with a heat reservoir at 373 K.

(i) Find the entropy change of water, heat reservoirs and the universe.

(ii) If the water is heated from 275 to 373 K by first bringing it in contact with the reservoir at 323 K and then with a reservoir at 373 K, what will be the entropy change of the universe.

(iii) Explain how water might be heated from 275 to 373 K with almost no entropy change of the universe. *Ans:* (i) 12.76 kJ/K; 1.76 kJ/K; (ii) 0.9 kJ/K;

(iii) The entropy change in the universe will be less and less, if the water is heated in more and more stages; by bringing water in contact successively with more and more heat reservoirs. Each succeeding reservoirs will be at a higher and higher temperature than the preceding one.

If we heat water in infinite steps by bringing it in contact with an infinite number of reservoirs in succession so that at any instant, the temperature difference between the water and reservoir in contact is infinitesimally small then the net entropy change of the universe would be zero. Then, the water would be reversibly heated.

6.3 One kilogram of ice at −3°C is exposed to the atmosphere which is 27°C. The ice melts and comes into thermal equilibrium with the atmosphere.

(i) Find the entropy increase of the universe.

(ii) What is the minimum amount of work necessary to convert the water bath into ice at −3°C in a refrigerator system.

Take c_p of ice = 2.093 kJ/Kg K and latent heat of fusion of ice = 333.3 kJ/kg. *Ans:* (i) 1.638;(ii) 37.87 kJ

6.4 At STP, 8.5 liters of oxygen and 15 liters of hydrogen mix with each other completely in an isolated chamber. Calculate the entropy change for the process assuming both gases behave as ideal gases. *Ans:* 5.71 J/K

6.5 Consider a large constant temperature reservoir at 600 K. It is continuously stirred by a paddle wheel driven by an electric motor. Estimate the entropy change of the reservoir if the paddle wheel is operated for 3 hours by a 300 W motor. *Ans:* 0.54 kJ/K

6.6 One mole of air contained in a rigid vessel is heated from 300 K to 400 K by placing the container on a hot plate. Determine the entropy change of air assuming it to be an ideal gas. *Ans:* 5.98 J/mol K

6.7 A block of steel weighing 50 kg at a temperature of 527 °C is dropped in 250 kg of oil at 27 °C. Take specific heat of the steel and oil to be 0.50 kJ/ kgK and 2.5 kJ/kgK, respectively. Estimate the change in entropy of steel, oil and system (oil + block of steel). *Ans:* 15.83 kJ/K

6.8 A reversible power cycle R and an irreversible power cycle I operates between the same two reservoirs. Each receives Q_H from hot reservoir. The reversible cycle develops work, W_R, while the irreversible cycle develops work, W_I. Calculate:

(i) rate of entropy generation σ for the cycle I in terms of W_I, W_R and the temperature T_C of the cold reservoir and

(ii) demonstrate that σ is positive, where $\sigma = \oint \frac{dQ}{T}$.

Ans: (i) $\sigma = \dfrac{W_R - W_C}{T_C}$; (ii) σ is positive

6.9 Two identical bodies of constant heat capacity are at the same initial temperature T_i. A refrigerator operates between these two bodies until one body is cooled to temperature T_2. If the bodies remain at constant pressure and undergo no change of phase, show that the minimum amount of work needed to do this is:

$$W_{\text{min}} = c_p\left(\frac{T_i^2}{T_2} + T_2 - 2T_i\right)$$

6.10 The latent heat of fusion of water at 0 °C is 335 kJ/kg. How much does the entropy of 1 kg of ice changes as it melts into water in each of the following ways:

(i) Heat is supplied reversibly to a mixture of ice and water at 0 °C.

(ii) A mixture of ice and water at 0 °C is stirred by a paddle wheel.

Ans: (i) 1.2271 kJ/kg K; (ii) 1.2271 kJ/kg K

6.11 Three kilogram of water at 80 °C are mixed adiabatically with 2 kg of water at 30 °C in a constant pressure process of 1 atm. Calculate the increase in the entropy of the total mass of water due to the mixing process. Take c_p of water as 4.187 kJ/kg. *Ans:* 0.058 kJ/K

6.12 In a Carnot cycle, heat is supplied at 357 °C and rejected at 27 °C. Consider the working fluid as water while receiving heat, water evaporates from liquid at 27 °C to steam at 357 °C. The associated entropy change is 1.54 kJ/kg K.

(i) If the cycle operates on a stationary mass of 1 kg of water, how much work is done per cycle and how much is the heat supplied?

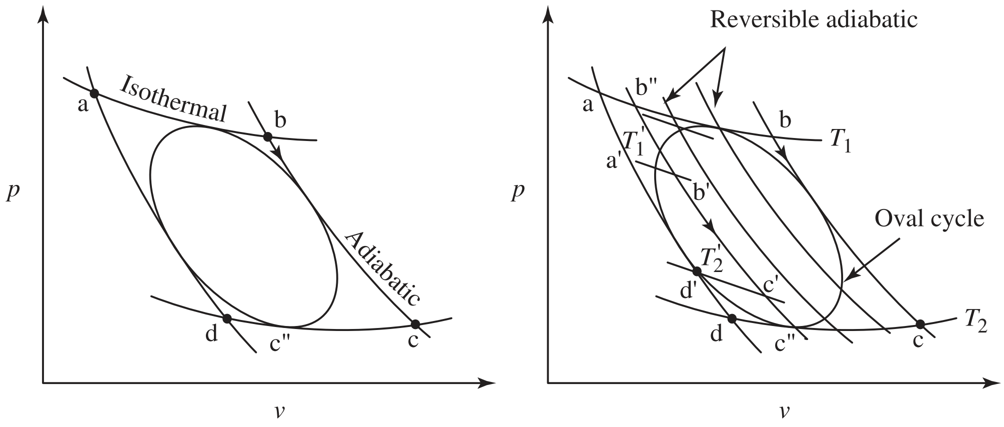
(ii) If the cycle operates in steady flow with power output of 25 kW, calculate the steam flow rate. *Ans:* (i) 500 kJ/kg; (ii) 0.05 kg/s

6.13 A heat engine receives 400 kJ/cycle of heat reversibly from a source at 347 °C and rejects heat reversibly at 37 °C. There is no heat transfer. For each of the hypothetical amount of heat rejected in (i), (ii) and (iii), below compute the cyclic integral $\dfrac{\delta Q}{T}$. (i) 200 kJ/cycle rejection; (ii) 100 kJ/cycle rejection; and (iii) 300 kJ/cycle rejection

From these results, show which case is reversible, irreversible and impossible.

Ans: (i) 0; As $\int \frac{\delta Q}{T} = 0$, the engine operates at a reversible cycle.
(ii) 0.3226; As $\int \frac{\delta Q}{T} > 0$, the cycle (b) is impossible.
(iii) -0.322; As $\displaystyle\int \frac{\delta Q}{T} < 0$, the cycle is irreversible and possible.

6.14 Water is heated at a constant pressure of 70 kPa. The boiling point at this pressure may be taken as 165 °C. Assume that the initial temperature of water is 0 °C. The latent heat of vaporisation may be taken as 2066 kJ/kg. Calculate the increase in entropy of water, if the final state is steam. If the ambient temperature is 27 °C, what is entropy increase of the universe. Take c_p of water 4.187 kJ/kg K. *Ans:* 6.7 kJ/kg K



(a) (b)

Fig. 6.43

6.15 In Fig.6.43, (a), (b), (c) and (d) represent a Carnot cycle at temperatures T_H and $T_L[T_H > T_L]$. The oval cycle bounded by Carnot cycle is reversible, where heat absorption is at temperature $\leq T_H$ and rejects is at temperature $\geq T_L$. Show that the efficiency of the oval cycle is less than that of the Carnot cycle.

6.16 One kilogram of air initially at 0.8 MPa and 17 °C changes to 0.4 MPa and 51 °C by three reversible non-flow process 1-a-2 consists of constant pressure expansion followed by constant volume cooling. Now, consider the process $1 - b - 2$ which is an isothermal expansion followed by a constant pressure expansion process and 1-c-2 which is an adiabatic process followed by constant volume heating. Determine the change of internal energy, enthalpy and entropy for each process. Also find work transfer and heat transfer. Take $c_p = 1.005$ kJ/kg K; $c_V = 0.718$ kJ/kg K. Assume constant specific heat c_p and c_V. Take $pv = 0.287T$ for air, where the pressure p is in kPa; v is the specific volume in m^3/kg and temperature T is in K.
Ans: 24.412 kJ/kg; 34.170 kJ/kg; 0.311 kJ/kg K; 56.29 kJ/kg; 81.75 kJ/kg

6.17 Twenty grams of water at 20 °C is converted into ice at −20 °C at constant atmospheric pressure. Assuming specific heat of liquid water to remaining constant at 4.2 J/K and that of ice to 50% of this value and taking the latent heat of fusion of ice at 0 °C to 335 J/g, find the total entropy change of the system. *Ans:* 2.21 J/K

6.18 Calculate the entropy change of the universe as a result of the following processes.

(i) Copper block of 0.5 kg mass with $c_p = 1.50$ J/K at 100 °C is placed in a lake at 10 °C.

(ii) The same block at 10 °C is dropped from a height of 100 m into the lake.

(iii) Two such blocks at 100 and 0 °C are joined together.

Ans: (i) 490.5 J; (ii) 323 K; (iii) 3.637 J/K

6.19 A system maintained at constant volume is initially at temperature T_1 and a heat reservoir at the lower temperature T_0 is available. Show that the maximum work recoverable as system is cooled to T_0 is:

$$W = c_V \left[(T_1 - T_0) - T_0 \ln \left(\frac{T_1}{T_0} \right) \right]$$

6.20 A body of finite mass is originally at temperature T_1 which is higher than that of a reservoir at temperature T_2. Suppose an engine operates in a cycle between the body and the reservoir until it lowers the temperature of the body from T_1 to T_2 thus extracting heat Q from the body. If the engine does work W, then it will reject heat $Q - W$ to the reservoir at T_2. Apply the entropy principle and prove that the maximum work obtainable from the engine is:

$$W_{max} = Q - T(S_1 - S_2)$$

where $S_1 - S_2$ is the entropy decrease of the body. If the body is maintained at constant volume having constant volume heat capacity $c_p = 8.4$ J/K which is independent of temperature and $T_1 = 375$ K and $T_2 = 305$ K, determine the maximum work obtainable. *Ans:* 58.65 kJ

6.21 Each of the three identical bodies satisfies the equation $U = CT$, where C is the heat capacity of each of the bodies. Their initial temperatures are 250, 300 and 500 K. If C is 8.4 kJ/K, what is the maximum amount of work that can be extracted in a process in which these bodies are brought to a final common temperature. *Ans:* 385.56 kJ/kmol

6.22 Liquid water of mass 8 kg and temperature 27 °C is mixed with 2 kg of ice at −3 °C till equilibrium is reached at 1 atm pressure. Find the entropy change of the system. Given c_p of water=4.18 kJ/kg K, c_p of ice = 2.09 kJ/kg K and latent heat of fusion of ice = 333.3 kJ/kg. *Ans:* 0.168 kJ/K

6.23 A thermally insulated 50 ohm resistor connected a current of 1.5 A for 1 sec. The initial temperature of the resistor is 17 °C. Its mass is 5 g and its specific heat is 0.85 J/g K. Calculate (i) change in entropy of the resistor and (ii) change in entropy of the universe. *Ans:* (i) 0.483 J/K; (ii) 0.483 J/K

6.24 An iron block of unknown mass at 87 °C is dropped into an insulated tank that contains 0.1 m^3 of water at 17 °C. Simultaneously, a paddle wheel driven by 200 W motor is activated to stir the water. Thermal equilibrium is established at 30 min when the final temperature is 27 °C. Determine the mass of the iron block and the entropy generated during the process. Take c_p of the iron block as 0.45 kJ/kg K. *Ans:* 141.48 kg; 4.36 kJ/K

6.25 Assume that a particular body has the equation of state $U = CT$, where $C = 8.5$ J/K and assume that this equation of state is valid throughout the temperature range from 0.6 K to room temperature. How much work must be expanded to cool this body from room temperature of 27 °C to 0.6 K. Consider the ambient atmosphere as the hot reservoir. *Ans:* 13.3 kJ

Multiple Choice Questions (choose the most appropriate answer)

1. The total energy (E) of the universe
   (a) is always constant
   (b) is always increasing
   (c) is always decreasing
   (d) either increases or decreases

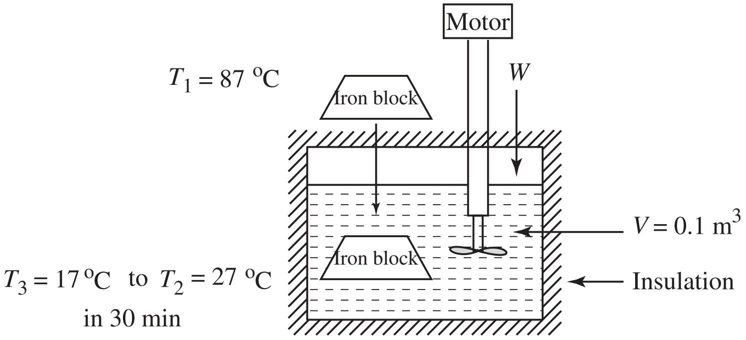


Fig. 6.44

2. In a Carnot cycle, there are two reversible adiabatic process and
   (a) two reversible isentropic processes (c) two reversible isothermal processes
   (b) two reversible isobaric processes (d) two reversible isochoric processes

3. If two parts 1 and 2 of a system with entropies S_1 and S_2 are in equilibrium, the equation for entropy is given by
   (a) $S = S_1 - S_2$ (c) $S = S_1 + S_2$
   (b) $S = (S_1 + S_2)/2$ (d) $S = \sqrt{S_1 S_2}$

4. The entropy at the triple point of water is chosen as
   (a) less than zero (c) more than zero
   (b) zero (d) none of the above

5. The absolute zero entropy value is not attainable because
   (a) absolute zero temperature is not attainable in finite number of operations
   (b) theoretically absolute zero temperature has negative value of entropy and it is not possible
   (c) both (a) and (b)
   (d) none of the above

6. For a pure crystalline substance at the state of perfect order ($W = 1$), the zero entropy defined as the
   (a) entropy at absolute zero temperature
   (b) entropy at 0 °C temperature
   (c) entropy at the freezing point of the substance
   (d) none of the above

7. For a system having two parts A and B with thermodynamic probabilities W_A and W_B, are in equilibrium, the thermodynamic probability of the whole system is given by
   (a) $W = W_A + W_B$ (c) $W = \sqrt{W_A - W_B}$
   (b) $W = (W_A + W_B)/2$ (d) $W = W_A \times W_B$

8. At the most probable state of a system the entropy of the system is
   (a) constant (c) maximum
   (b) minimum (d) none of the above

9. Thermodynamic probability (W)
   (a) decreases with entropy (c) does not change with change in with entropy
   (b) increases with entropy (d) none of the above

10. The degree of disorder of a mixture of two gases
    (a) is always less than the degrees of disorder of individual gases
    (b) is always equals the degrees of disorder of individual gases
    (c) is always greater than the degrees of disorder of individual gases
    (d) none of the above

11. The process that decreases the entropy of the universe is called a
    (a) reversible process (c) impossible process
    (b) irreversible process (d) none of the above

12. All the spontaneous processes in nature
    (a) decreases the entropy on the universe
    (b) increases the entropy on the universe
    (c) do not produce any change in the entropy on the universe
    (d) none of the above

13. If the potential gradient in a process is infinitesimal or zero, the change in entropy of the universe?
    (a) increases (c) decreases
    (b) no change in entropy (d) increases first and then decreases

14. The thermodynamic process that cannot be plotted accurately on thermodynamic coordinates is
    (a) reversible process (c) both reversible and irreversible processes
    (b) irreversible process (d) none of the above

15. If there are irreversibilities in a process of a system, then the entropy generation of the system is
    (a) greater than zero (c) equals to zero
    (b) less than zero (d) none of the above

16. The generation of entropy
    (a) does not depend upon path followed by a system
    (b) depends upon path followed by a system
    (c) takes place in reversible process
    (d) none of the above

17. The entropy of a closed system increases
    (a) by interaction of heat
    (b) by internal irreversibilities
    (c) by dissipative effect in which work is dissipated to increase internal energy of the closed system
    (d) all of the above

18. Entropy generation is nothing but
    (a) increase in the entropy of a system due to internal irreversibility
    (b) no change in entropy of a system
    (c) decrease in the entropy of a system due to internal irreversibility
    (d) none of the above

19. The gain in entropy of a system from the surrounding will cause
    (a) no change on molecular disorder in the system
    (b) molecular disorder increase
    (c) molecular disorder decrease
    (d) none of the above

20. The entropy transfer associated with work is a
    (a) positive entropy transfer (c) negative entropy transfer
    (b) no entropy transfer (d) all of the above

21. Heat transfer causes
    (a) the entropy transfer to take place along with heat flow in the direction of heat transfer
    (b) the entropy transfer to take place along with heat flow in the opposite direction of heat transfer
    (c) no entropy transfer along with heat transfer
    (d) none of the above

22. An electric current, I is passing through a resistor in contact with a reservoir, having constant internal energy. If heat generated is absorbed by surrounding at temperature T_0 entropy of the universe
    (a) remains constant (c) decreases
    (b) increases (d) cannot say

23. A body is at a temperature T_1 with certain heat capacity and a heat engine is operated between the body and a thermal heat reservoir at temperature (T_0). When heat engine stops working the final temperature of the body will be
    (a) T_0 (c) $(T_0 + T_1)/2$
    (b) T_1 (d) $T_f = \sqrt{T_1 T_2}$

24. Two bodies, initially at temperatures T_1 and T_2 ($T_1 > T_2$) have same heat capacity. A heat engine operates between these two bodies delivering maximum work. When the engine stops working, the final temperature, (T_f), of the two bodies will be

    (a) $T_f = (T_1 - T_2)$
    (b) $T_f = (T_1 - T_2)/2$
    (c) $T_f = (T_1 + T_2)/2$
    (d) $T_f = \sqrt{T_1 T_2}$

25. When two bodies, having same heat capacity and initially at temperatures T_1 and T_2 ($T_1 > T_2$) are brought together in thermal contact, without delivering any work, the final temperature will be

    (a) $T_f = (T_1 - T_2)$
    (b) $T_f = (T_1 - T_2)/2$
    (c) $T_f = (T_1 + T_2)/2$
    (d) $T_f = \sqrt{T_1 T_2}$

Ans: 1. – (a) 2. – (c) 3. – (c) 4. – (b) 5. – (a)
6. – (a) 7. – (d) 8. – (b) 9. – (a) 10. – (c)
11. – (c) 12. – (b) 13. – (b) 14. – (b) 15. – (a)
16. – (b) 17. – (d) 18. – (c) 19. – (b) 20. – (b)
21. – (a) 22. – (b) 23. – (a) 24. – (d) 25. – (c)

7

AVAILABLE AND UNAVAILABLE ENERGY

7.1 INTRODUCTION

Over the past few decades, there is a tremendous increase in the use of energy in all walks of life. In order to conserve energy, proper utilisation is a must. For this, efforts are required for identification and elimination of the sources of inefficiency during its use. This obviously requires in a further indepth study and analysis of the laws of thermodynamics. A look into the laws of thermodynamics will reveal the following.

First law of thermodynamics demonstrates numerical equivalence of converting work into heat in both direction (bidirectional). The second law of thermodynamics exhibits a unidirectional equivalence between work and heat. Second law clearly establishes that for a given amount of heat, the equivalent amount of work cannot be obtained whereas vice-versa is possible. This brings the concept of quality of energy. Note that work is considered as high-grade of energy and heat as low-grade of energy.

Engineers are often using the principle of energy conservation. For this, they rely on the concept of energy conservation embedded in the first law of thermodynamics. From the principle of energy conservation, it is evident that energy cannot be destroyed. In practice, one is interested to drive machines, produce products and develop processes using energy. This gives birth to the concept of available and unavailable energy or the concept of maximum work.

This concept is very important in phenomenon, oriented thermodynamics. Lots of research have been carried out pertaining to the effect of ambient temperature on maximum work realisation. In this connection, there is a law of losses. The researchers have also analysed the various losses inhibiting the achievable maximum work. The *law of the loss* for achieving maximum work states that the work obtained in real-life situation is always less than the maximum obtainable work due to the *irreversibility* in thermal processes.

Available energy or the concept of availability came out of these propositions. It was originally proposed by J.W. Gibbs. He indicated that environment plays an important role in evaluating the available energy or availability. Using availability analysis, we can quantitatively define the following:

(i) quality of energy,

(ii) its convertibility into other forms and

(iii) capability to perform work.

In this connection, a new term *exergy* was introduced by Z. Rant in 1956, so as to differentiate it from energy. Exergy analysis or availability analysis has capability to identify and quantify the causes of thermodynamic imperfections in a thermodynamic process. This will give us an idea regarding the possibilities of improving the processes. It is found to be better than energy analysis. Note that energy analysis cannot detect majority of thermodynamic imperfections. Typical examples are the irreversible heat transfer, throttling and adiabatic combustion. These processes do not have any energy loss but make the quality of energy inferior. Energy entering with fuel, electricity, flowing streams of matter and so on can be accounted for in products, since energy cannot be destroyed. The idea that something can be destroyed may be useful but you should be careful not to apply to energy. However, it could be applied to the other variable which we have just mentioned, viz exergy.

Be aware that, it is exergy and not energy that properly gauges the quality (utility). For example, 1 kJ of electricity generated by a power plant has a higher quality and the greater economic value of energy than 1 kJ cooling water stream in a plant. These phenomena can be evaluated by second law analysis easily. Exergy analysis could be integrated with principles of engineering economics to determine the potential for cost-effective improvement of existing systems. Exergy and costing principles can also be used at initial design stage to develop systems that are optimised in annualised cost, sparing in use of fossil fuels and environmentally friendly.

7.2 GRADES OF ENERGY

From what we have discussed in the previous paragraphs, it is obvious that energy can be conveniently categorised as low-grade energy and high-grade energy. Typical examples of high-grade and low-grade energy are given in Table 7.1.

Table 7.1 High- and low-grade energy

High-grade energy	Low-grade energy
Electrical energy	Heat or thermal energy
Kinetic energy of a jet	Heat derived from combustion of fossil fuels
Mechanical work	Heat derived from nuclear fission or fusion
Tidal power and water power	Heat derived from burning wood
Wind power	Heat derived from solar energy

Also, the second law of thermodynamics prohibits the *complete conversion* of low-grade energy into high-grade energy. The portion of low-grade energy that can be converted to high-grade is called *available energy* or *exergy* or *availability.* The portion of energy not available for conversion is called *unavailable energy* or *anergy.* Mathematically,

$$\text{Anergy} \quad = \quad \text{Energy} - \text{Exergy} \tag{7.1}$$

7.3 AVAILABLE AND UNAVAILABLE ENERGY

From the discussion in the previous section, it is clear that energy can exist in many forms. Note that all these forms cannot be converted completely into work even under ideal conditions. This indicates that energy has two parts: *available part* and *unavailable part. Available energy is the maximum portion of energy which could be converted into useful work by ideal processes. After the conversion the system reduces to a dead state.* A dead state is one which is in equilibrium with the earth and its atmosphere. Understand that there can be only one value for maximum work. This could be achieved only by the system while descending to its dead state. This means that available energy is a property.

For example, if a system has a pressure difference from that of surroundings then work can be obtained through expansion process. If it has a temperature difference, heat can be transferred to a energy conversion device and work can be obtained. But when the temperature and pressure becomes equal to that of the earth (atmosphere), transfer of energy ceases. However, the system may contain internal energy and this energy is called unavailable energy. In short, available energy denote, the capability of energy to do work. In this sense, it can be applied to energy in the system or in the surroundings.

The theoretical maximum work that can be obtained from a system at any state p_1 *and* T_1 *while operating with a reservoir at the constant pressure and temperature* p_0 *and* T_0 *(say atmosphere) is called availability.*

From the above discussion, it must be clear that the high-grade energy in the form of electrical energy or mechanical work can be obtained from sources of low-grade energy. However, the complete conversion of low-grade energy, viz heat into high-grade energy, viz shaft work is not possible. Therefore, that part of low-grade energy which is available for conversion is referred to as available energy (AE). That part which according to the second law must be rejected is known as unavailable energy (UE).

Now, let us discuss a little more in detail regarding AE and UE by means of two examples. Let us consider a simple situation shown in Fig.7.1(a). Assume that there is a very large energy source Q, in the form of heat. Since, the source is very large, it will be a constant temperature reservoir at temperature T. The question is how much of this energy in the reservoir can be converted to useful work? To answer this question, let us imagine an ideal cyclic heat engine attached to the reservoir as shown in Fig.7.1(b). Assume that this ideal engine is capable of converting the maximum fraction of Q into work (W_{max}). For this, we would require a completely reversible engine, i.e. a Carnot cycle engine. Further, the lower temperature reservoir should be at the lowest temperature possible but not necessarily at the ambient temperature.

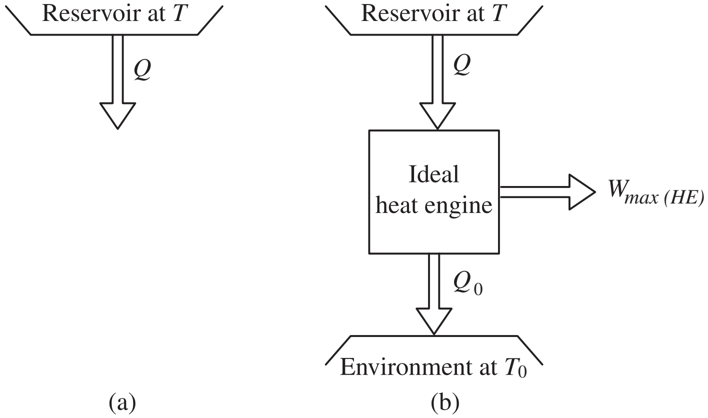


Fig. 7.1 Heat transfer from a constant temperature energy source

From the first and second laws for Carnot cycle and with usual sign convention,

$$W_{max} = Q - Q_0 \tag{7.2}$$

where $Q = AE + UE$ and $Q_0 = UE$. Refer Fig.7.1(b). Since, $\dfrac{Q}{T} = \dfrac{Q_0}{T_0}$, the above equation can be written as:

$$Q = T\left(\frac{Q_0}{T_0}\right) \tag{7.3}$$

$$W_{max} = Q\left(1 - \frac{T_0}{T}\right) \tag{7.4}$$

Here, you should note one important point, viz W_{max} can never be equal to 1. If it has to become 1, then T_0 should become 0 K, i.e. $-273°$C. It is not practicable. Therefore, it is only the fraction given by the quantity in the bracket, i.e. $\left[\left(1 - \frac{T_0}{T}\right)\right]$ multiplied by Q on the right side of the equation is the available portion of the total quantity of energy, Q.

As a first example, examine the situation shown on the T-s diagram in Fig.7.2. The total shaded area (both dark and light) in the diagram is Q. The portion of Q that is below the environment temperature, T_0, cannot be converted into work by any heat engine and must be thrown away. This portion is therefore, the unavailable portion of energy Q, which is called *unavailable energy*. The portion lying between the two temperatures T and T_0 is the *available energy*.

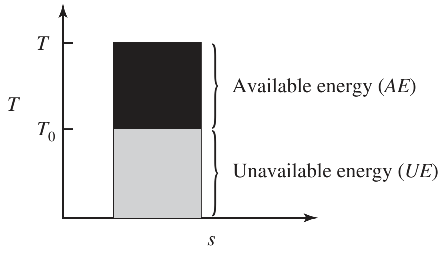


Fig. 7.2 T-s diagram for an energy source at constant temperature

As a second example, let us consider the same situation except that the heat transfer Q is available from a constant pressure source. Let it be a simple heat exchanger as shown in Fig.7.3(a). Now, let us replace the Carnot cycle by a sequence for such an engine by a constant pressure process. The T-s diagram then will be as shown in Fig.7.3(b).

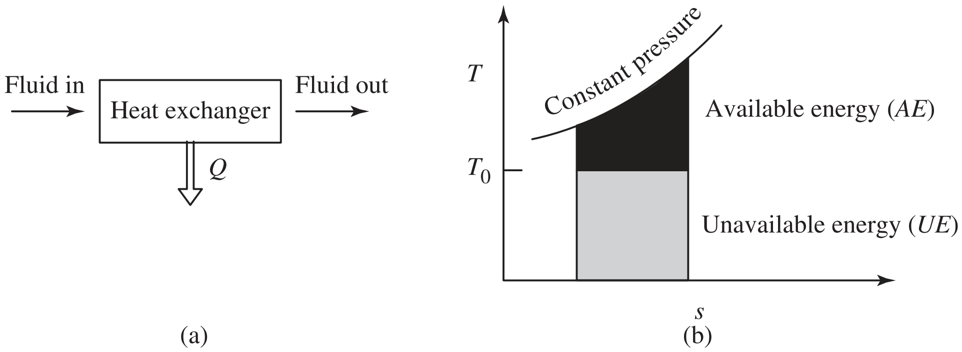


Fig. 7.3 T-s Diagram for an energy source at changing temperatures

The only difference between the first and the second example is that the second includes an integral, which corresponds to ΔS.

$$\Delta S = \int \left(\frac{\delta Q_{rev}}{T} \right) \tag{7.5}$$

$$Q_{rev} = Q - T_0 \times \Delta S \tag{7.6}$$

Note that this ΔS quantity does not include the standard sign convention. It corresponds to the change of entropy. The equation specifies the available portion of the quantity Q. the portion unavailable for producing work in this circumstance lies below T_0. Thus, the unavailable energy is the product of lowest temperature of heat rejection and the change of entropy of the system during the process of supplying heat.

7.4 QUALITY OF ENERGY

The available energy or exergy of a fluid at a higher temperature T_1 is more than that at a lower temperature T_2 and decreases as the temperature decreases. When the fluid reaches the ambient temperature, its exergy is zero. The second law, therefore, affixes a quality to energy of a system at any state. Therefore, the quality of energy of a gas at, say, 1000 °C is superior to that at, say, 100 °C, since the gas at 1000 °C has the capacity of doing more work than the gas at 100 °C, under the same environmental conditions. An awareness of this energy quality as of energy quantity is essential for the efficient use of our energy resources and for energy conservation. The concept of *available energy or exergy provides a useful measure of this energy quality.*

7.5 LAW OF DEGRADATION OF ENERGY

The available energy of a system decreases as its temperature or pressure decreases and approaches that of the surroundings. When heat is transferred from a system, its temperature decreases and hence, the quality of its energy deteriorates. The degradation is more for energy loss at a higher temperature than that at a lower temperature. Quantity-wise the energy loss may be the same but quality-wise the losses are different. While *the first law states that energy is always conserved quantity-wise, the second law emphasises that energy always degrades quality-wise.* When a gas is throttled adiabatically from a high to a low pressure, the enthalpy (or energy per unit mass) remains the same but there is a degradation of energy or available work. The same holds good for pressure drop due to friction of a fluid flowing through an insulated pipe. If the first law is the law of conservation of energy, the second law is called the *law of degradation of energy.* Energy is always conserved but its quality is always degraded.

It is known that energy gets degraded by thermal irreversibility and produces less useful work can be explained in a little different way. Let two bodies 1 and 2 of constant heat capacities c_1 and c_2 be at temperatures T_1 and T_2 ($T_1 > T_2$). These are connected by a rod and a small quantity of heat Q flows from 1 to 2. The total change of entropy is

$$\Delta S = \Delta S_1 + \Delta S_2 = Q\left(\frac{1}{T_2} - \frac{1}{T_1}\right) > 0 \ (\because T_1 > T_2) \tag{7.7}$$

The entropy will continue to increase till thermal equilibrium is reached.

Let us now suppose that instead of allowing heat Q to flow from 1 to 2, we used it to operate a Carnot engine and obtain mechanical work, with T_0 as the sink temperature. The maximum work obtainable is

$$W_{max} \quad = \quad Q\left[1 - \frac{T_0}{T_1}\right] \tag{7.8}$$

If, however, we first allow Q to flow from 1 to 2, i.e. from temperature T_1 to T_2 and then use it to operate the Carnot engine, we obtain,

$$W_2 \quad = \quad Q\left(1 - \frac{T_0}{T_2}\right) < W_1 \tag{7.9}$$

Thus, in the course of the irreversible heat conduction, the energy has become degraded to the extent that the useful work has been decreased by

$$\Delta W \quad = \quad W_{max} - W_2 = T_0 \Delta S \tag{7.10}$$

The increase in entropy in an irreversible change is thus a measure of the extent to which energy becomes degraded in that change. Conversely, in order to extract the maximum work from a system, changes must be performed in a reversible manner so that total entropy $(\Delta S_{sys} + \Delta S_{surr})$ is conserved.

It is worth pointing that if the two bodies were allowed to reach thermal equilibrium (i) by heat conduction and (ii) by operating a Carnot engine between them and extracting work, the final equilibrium temperatures would be different in the two cases. In the first, $U_1 + U_2$ is conserved and the final temperature is

$$T_f^{(U)} \quad = \quad \frac{c_1 T_1 + c_2 T_2}{c_1 + c_2} \tag{7.11}$$

In the second case, $S_1 + S_2$ is conserved and,

$$W \quad = \quad -\Delta U(-\Delta U_1 + U_2)$$

Therefore, $\delta W = \delta Q - dU, \; < T ds - dU$ so that $\delta W_{max} = -dU$. In the isentropic process, the final temperature is given by:

$$T_f^{(s)} \quad = \quad T_1^{\frac{c_1}{(c_1 + c_2)}} \, T_2^{\frac{c_2}{(c_1 + c_2)}} < T_f^{(U)} \tag{7.12}$$

If $c_1 = c_2 = C$, then

$$T_f^{(U)} \qquad \frac{T_1 + T_2}{2} \quad \text{and} \quad T_f^{(s)} = (T_1 T_2)^{\frac{1}{2}} \tag{7.13}$$

The difference in final temperature is due to the lower value of the total internal energy which results from work having been done at the expense of internal energy. Similarly, it can be shown that due to mechanical irreversibility also, energy gets degraded so that the degradation of energy quality is a universal principle.

7.6 USEFUL WORK

All of the actual work, W_{act}, of the system with a flexible boundary would not be available for delivery. A certain portion of it would be spent in pushing out the atmosphere (Fig.7.4). The useful work is defined as the actual work delivered by a system less the work performed on the atmosphere. If V_1 and V_2 are the initial and final volume of the system and p_0 is the atmospheric pressure, then the work done on the atmosphere is $p_0(V_2 - V_1)$. Therefore, the useful work W_u becomes

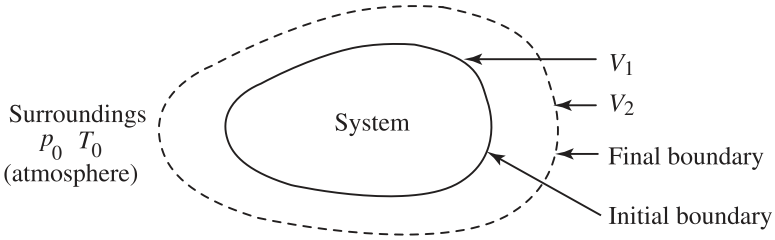


Fig. 7.4 Work done by a closed system in pushing out the atmosphere

$$W_u = W_{act} - p_0(V_2 - V_1) \tag{7.14}$$

Similarly, the maximum useful work will be

$$(W_u)_{max} = W_{max} - p_0(V_2 - V_1) \tag{7.15}$$

In differential form,

$$(\delta W_u)_{max} = \delta W_{max} - p_0 \delta V \tag{7.16}$$

In a steady flow system, the volume of the system does not change. Hence, the maximum useful work would remain the same, i.e. no work is done on the atmosphere or,

$$(\delta W_u) = \delta W_{max} \tag{7.17}$$

But in the case of an unsteady-flow open or closed system, the volume of the system changes. Hence, when a system exchanges heat only with the atmosphere, the maximum useful work is given by Eq.7.16.

If KE and PE changes are neglected. It can be shown that

$$(W_u)_{max} = U_1 - U_2 + p_0(V_1 - V_2) - T_0(S_1 - S_2) \tag{7.18}$$

This can also be written in the following form:

$$(W_u)_{max} = (U_1 + p_0V_1 - T_0S_1) - (U_2 + p_0V_2 - T_0S_2) = \phi_1 - \phi_2 \tag{7.19}$$

where ϕ is called the availability function for a closed system given by:

$$\phi = U + p_0V - T_0S \tag{7.20}$$

The useful work per unit mass becomes

$$(W_u)_{max} = (u_1 + p_0v_1 - T_0s_1) - (u_2 + p_0v_2 - T_0s_2) \tag{7.21}$$

7.7 EFFECTIVENESS

Effectiveness is defined as the ratio of actual useful work to the maximum useful work. The useful output of a system is given by the increase of availability of the surroundings.

$$\text{Effectiveness, } \epsilon = \frac{\text{Increase of availability of surroundings}}{\text{Loss of availability of the system}} \tag{7.22}$$

For a compression or heating process, the effectiveness is given by:

$$\epsilon = \frac{\text{Increase of availability of system}}{\text{Loss of availability of the surroundings}} \tag{7.23}$$

$$\epsilon = \frac{W_{\text{useful}}}{W_{\text{max useful}}} \tag{7.24}$$

The *effectiveness of an actual process is always less than unity.* Thus, effectiveness of a process is the measure of the extent to which advantage can be taken to obtain useful work.

7.8 AVAILABILITY ANALYSIS FOR CLOSED AND OPEN SYSTEMS

In engineering, we come across both closed and open systems. In the following sections, we will make an availability analysis of closed as well as open systems.

- **Closed System**

7.8.1 Constant Volume Process

Consider, a certain amount of air is heated from the temperature of T_1 to T_2 at constant volume along with heat transfer to the surrounding at T_0.

Total heat energy supplied,

$$Q = mc_V(T_2 - T_1) \tag{7.25}$$

$$AE = Q - T_0\Delta S \tag{7.26}$$

where $\Delta S = mc_V \ln\left(\frac{T_2}{T_1}\right)$ (refer Eq.6.96)

$$UE = Q - AE = mc_V(T_2 - T_1) - [mc_V(T_2 - T_1) - T_0\Delta S] = T_0\Delta S \tag{7.27}$$

7.8.2 Constant Pressure Process

Let us assume that if air is heated from T_1 to T_2 with the surroundings at the temperature, T_0, at constant pressure.

$$\text{Heat energy supplied, } Q = mc_p(T_2 - T_1) \tag{7.28}$$

$$AE = Q - T_0\Delta S \tag{7.29}$$

where $\Delta S = mc_p \ln\left(\frac{T_2}{T_1}\right)$ (refer Eq.6.98)

$$UE = Q - AE = T_0\Delta S \tag{7.30}$$

7.8.3 Constant Temperature or Isothermal Process

In this process, let us assume that air is heated at constant temperature along with heat transfer to the surroundings,

$$\begin{aligned} \text{Heat energy supplied, } Q &= p_1V_1 \ln\left(\frac{p_2}{p_1}\right) \\ AE &= Q - T_0\Delta S \\ \Delta S &= \frac{Q}{T_1} \quad \text{or} \quad \frac{Q}{T_2} && (7.31) \\ UE &= T_0\Delta S && (7.32) \end{aligned}$$

7.8.4 Polytropic Process

In this process, the air will be heated according to the law, $pV^n = C$ along with heat transfer,

$$\begin{aligned} \text{Heat energy supplied, } Q &= (\gamma - n)\left(\frac{p_1V_1 - p_2V_2}{n-1}\right) = (\gamma - n)\left(\frac{mR[T_1 - T_2]}{n-1}\right) && (7.33) \\ AE &= Q - T_0\Delta S \end{aligned}$$

where for polytropic process (refer Eq.6.89 and 6.82)

$$\begin{aligned} \Delta S &= c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{p_1}\right) = c_p \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right) \\ UE &= Q - AE = T_0\Delta S && (7.34) \\ & && (7.35) \end{aligned}$$

- *Open Systems*

7.8.5 Compressor/Pump

The following assumptions are made such as change in potential and kinetic heads are negligible, so SFEE reduces to

$$\begin{aligned} mh_1 + Q &= mh_2 + W && (7.36) \\ W &= m(h_1 - h_2) + Q && (7.37) \\ \text{Actual work input, } W_{\text{act}} &= m(h_2 - h_1) = Q && (7.38) \end{aligned}$$

Compressor/pumps are work absorbing devices.

$$\begin{aligned} \text{Maximum work input, } W_{max} &= m(h_2 - h_1) - T_0(s_2 - s_1) && (7.39) \\ UE \text{ or Irreversibility, } I &= T_0\Delta S && (7.40) \end{aligned}$$

where $\Delta s = s_2 - s_1$.

7.8.6 Heat Exchanger

A simplified diagram of a heat exchanger is shown in Fig.7.5. Consider m_1 kg of hot fluid with a specific heat of c_1 exchanges heat with a cold fluid of m_2 kg with specific heat of c_2. Therefore, heat gained by cold fluid = heat given out by cold fluid.

$$m_2 c_2 (T_4 - T_3) = m_1 c_1 (T_1 - T_2) \tag{7.41}$$

$$AE \text{ or availability of cold fluid, } B_c = m_2 c_2 (T_4 - T_3) - T_0 (s_4 - s_3) \tag{7.42}$$

$$AE \text{ or availability of hot fluid, } B_h = m_1 c_1 (T_1 - T_2) - T_0 (s_4 - s_3) \tag{7.43}$$

$$UE \text{ or irreversibility, } I = B_h - B_c \tag{7.44}$$

$$\tag{7.45}$$

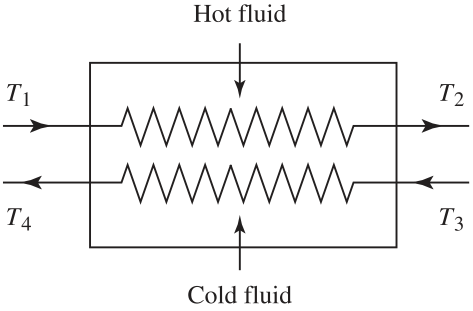


Fig. 7.5 Heat exchanger

7.8.7 Throttling Process

For throttling process,

$$\text{Enthalpy before throttling} = \text{Enthalpy after throttling} \tag{7.46}$$

$$h_1 = h_2 \tag{7.47}$$

Work output and heat transfer are zero.

$$\tag{7.48}$$

7.9 FIRST LAW AND SECOND LAW RECALLED

(i) As per the first law of thermodynamics, the energy in the form of heat can be converted into work without losses. Note that work is a high-grade energy whereas heat is a low-grade energy. When $Q = W,\ \eta_I = 100\%$.

(ii) The second law of thermodynamics says that all the energy absorbed as heat by an engine cannot be completely converted into work. Some part of the energy must be rejected to sink:

$$Q_1 = W + Q_2 \tag{7.49}$$

where W is useful work, $Q_1$and Q_2 are heat absorption and heat rejection, respectively. Losses do occur due to friction and other reasons which causes irreversibility. Hence, according to second law, the efficiency $\eta_{II} < 100\%$. This rises the question of availability or available energy for conversion from heat to work.

7.10 FIRST AND SECOND LAW EFFICIENCY

A normal measure on energy efficiency is the first law efficiency, η_I. It is the *ratio of the output energy to the input energy of the device.* The first law is concerned only with the quantities of energy. It disregards the forms in which the energy exists. It also does not takes into account the available energies with respect to temperature. It is the second law of thermodynamics that provides a means of assigning a quality index to energy. A useful measure of energy quality is evolved from the concept of available energy or exergy (Sec.7.4).

Using this concept, we can analyse the possible methods by which the consumption of available energy to perform a given process can be minimised. Further, it will indicate the possibility of the most efficient conversion of energy for the required task. The second law efficiency, η_{II}, of a process is defined as *the ratio of the minimum available energy (or exergy) which must be consumed to do a task divided by the actual amount of available energy (or exergy) consumed in performing the task.*

$$\eta_{II} = \frac{\text{minimum exergy intake to perform the given task}}{\text{actual exergy intake to perform the same task}} = \frac{A_{min}}{A} \tag{7.50}$$

where A is the availability or exergy. A power plant converts a fraction of available exergy A or W_{max} to useful work W. For the desired output of W, we should aim at A_{min}, where $A_{min} = W$ and $A = W_{max}$. Here,

$$UE \text{ or Irreversibility,} \quad I = W_{max} - W \quad \text{and} \quad \eta_{II} = \frac{W}{W_{max}} \tag{7.51}$$

$$\text{Now,} \qquad \eta_I = \frac{W}{Q_1} = \frac{W}{W_{max}} \cdot \frac{W_{max}}{Q_1} = \eta_{II} \cdot \eta_{Carnot} \tag{7.52}$$

$$\text{Therefore,} \qquad \eta_{II} = \frac{\eta_I}{\eta_{Carnot}} \tag{7.53}$$

Since $W_{max} = Q_1\left(1 - \dfrac{T_0}{T}\right)$, Eq.7.53 can also be obtained directly as follows:

$$\eta_{II} = \frac{W}{Q_1\left(1 - \dfrac{T_0}{T}\right)} = \frac{\eta_I}{\eta_{Carnot}}$$

If work is involved, $A_{min} = W$ (desired) and if heat is involved, $A_{min} = Q\left(1 - \dfrac{T_0}{T}\right)$.

For example, if solar energy Q_r is available at a reservoir storage temperature T_r and if quantity of heat Q_a is transferred by the solar collector at temperature T_a, then,

$$\eta_I = \frac{Q_a}{Q_r}$$

and
$$\eta_{II} = \frac{\text{Exergy output}}{\text{exergy input}} = \frac{Q_1\left(1 - \dfrac{T_0}{T_a}\right)}{Q_r\left(1 - \dfrac{T_0}{T_r}\right)} = \eta_I \frac{1 - \dfrac{T_0}{T_a}}{1 - \dfrac{T_0}{T_r}}$$

In the case of a heat pump, the task is to add heat Q_a to a reservoir to be maintained at temperature, T_a and the input shaft work is W_i.

$$COP = \frac{Q_a}{W_i} = \eta_I, \qquad \text{(say)}$$

$$(COP)_{max} = \frac{T_a}{T_a - T_0} = \frac{Q_a}{W_i} = \frac{Q_a}{A_{min}}$$

Therefore,
$$A_{min} = Q_a\left(1 - \frac{T_0}{T_a}\right)$$

and
$$\eta_{II} = \frac{A_{min}}{A} = \frac{Q_a\left(1 - \frac{T_0}{T_a}\right)}{W_i}$$

$$\eta_{II} = \eta_I\left(1 - \frac{T_0}{T_a}\right) \tag{7.54}$$

Similarly, expressions of η_I and η_{II} can be obtained for many thermal equipments. Some of which are given in Table 7.2.

7.10.1 Further, Illustrations of Second Law Efficiencies from the Point of View of Available Energy

Second law efficiency of different equipments can be expressed in different forms. It is derived by using the exergy balance rate given below:

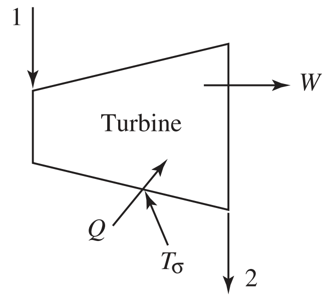


Fig. 7.6 Exergy balance of a turbine

(a) Turbines

Table 7.2 Availabilities, η_I and η_{II} expressions for common thermal tasks

	$T_r > T_a > T_0 > T_c$	
Task	Energy input	
	Input shaft work, W_i	Q for reservoir at T_r
Produce work, W_0	$A = W_i$	$A = Q_r\left(1 - \frac{T_0}{T_r}\right)$
	$A_{min} = W_0$	$A_{min} = W_0$
	$\eta_I = \frac{W_0}{W_i}$	$\eta_I = \frac{W_0}{Q_r}$
	$\eta_{II} = \frac{A_{min}}{A}$	$\eta_{II} = \eta_I \frac{1}{1 - \frac{T_0}{T_r}}$
	$\eta_{II} = \eta_I$ (electric motor)	(heat engine)
Add heat, Q_a to reservoir at T_a	$A = W_i$	$A = Q_r\left(1 - \frac{T_0}{T_r}\right)$
	$A_{min} = Q_a\left(1 - \frac{T_0}{T_a}\right)$	$A_{min} = Q_a\left(1 - \frac{T_0}{T_a}\right)$
	$^*\eta_I = \frac{Q_a}{W_i}$	$\eta_I = \frac{Q_a}{Q_r}$
	$^*\eta_{II} = \eta_I\left(1 - \frac{T_0}{T_a}\right)$	$\eta_{II} = \eta_I \frac{1 - \frac{T_0}{T_a}}{1 - \frac{T_0}{T_r}}$
	(heat pump)	(solar water heater)
Extract heat, Q_c from cold reservoir at T_c (below ambient)	$A = W_i$	$A = Q_r\left(1 - \frac{T_0}{T_r}\right)$
	$A_{min} = Q_c\left(\frac{T_0}{T_c} - 1\right)$	$A_{min} = Q_c\left(\frac{T_0}{T_c} - 1\right)$
	$^*\eta_I = \frac{Q_c}{W_i}$	$^*\eta_I = \frac{Q_c}{Q_r}$
	$^*\eta_{II} = \eta_I\left(\frac{T_0}{T_c} - 1\right)$	$^*\eta_{II} = \eta_I \frac{\frac{T_0}{T_c} - 1}{1 - \frac{T_0}{T_r}}$
	(refrigerator electric motor driven)	(refrigerator heat operated)

* Strictly speaking, it is *COP*.

where T_a Solar collector temperature
T_0 Atmospheric temperature
T_c Cold reservoir temperature
T_r Reservoir storage temperature

The steady state exergy balance (Fig.7.6) gives:

$$\frac{\dot{Q}}{\dot{m}}\left(1-\frac{T_0}{T_\sigma}\right)+a_{f_1} = \frac{\dot{W}}{\dot{m}}+a_{f_2}+\frac{\dot{I}}{\dot{m}} \tag{7.55}$$

where $a_{f_1} = h_1 - T_0 s_1$ and $a_{f_2} = h_2 - T_0 s_2$.

If there is no heat loss,

$$a_{f_1} - a_{f_2} = \frac{\dot{W}}{\dot{m}}+\frac{\dot{I}}{\dot{m}} \tag{7.56}$$

The second law efficiency,

$$\eta_{II} = \frac{\dfrac{\dot{W}}{\dot{m}}}{a_{f_1} - a_{f_2}} \tag{7.57}$$

(b) Compressor and Pump

Similarly, for a compressor or a pump,

$$\frac{\dot{W}}{\dot{m}} = a_{f_2} - a_{f_1} + \frac{\dot{I}}{\dot{m}}$$

and

$$\eta_{II} = \frac{a_{f_2} - a_{f_1}}{-\dfrac{\dot{W}}{\dot{m}}} \tag{7.58}$$

(c) Heat Exchanger

Writing the exergy balance for the heat exchanger (Fig.7.7):

$$\sum\left[1-\frac{T_0}{T_i}\right]\dot{Q}_i - \dot{W}_{CV} + [\dot{m}_h a_{f_1} + \dot{m}_c a_{f_3}] - (\dot{m}_h a_{f_2} + \dot{m}_c a_{f_4}) - \dot{I}_{CV} = 0 \tag{7.59}$$

where Q_i is heat transfer at temperature T_1. If there is no heat transfer and work transfer,

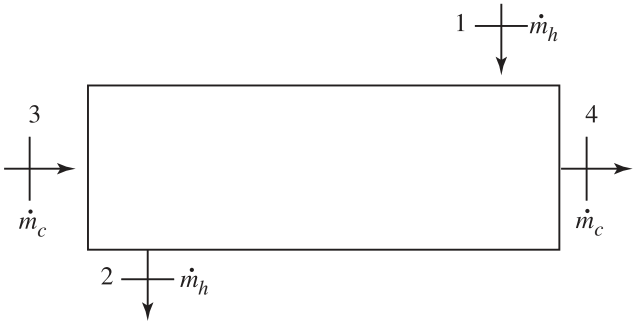


Fig. 7.7 Exergy balance of a heat exchanger

$$\dot{m}_h[a_{f_1} - a_{f_2}] = \dot{m}_c[a_{f_4} - a_{f_3}] + \dot{I}_{CV} \tag{7.60}$$

$$\eta_{II} = \frac{\dot{m}_c[a_{f_4} - a_{f_3}]}{\dot{m}_h[a_{f_1} - a_{f_2}]} \tag{7.61}$$

(d) Mixing of Two fluids

Exergy balance for the mixer (Fig.7.8) gives:

$$\left(1 - \frac{T_0}{T_\sigma}\right)\dot{Q} + \dot{m}_1 a_{f_1} + \dot{m}_2 a_{f_2} = \dot{W}_{CV} + \dot{m}_3 a_{f_3} + I_{CV}$$

If the mixing is adiabatic and since,

$$\dot{W}_{CV} = 0 \quad \text{and} \quad \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_1[a_{f_1} - a_{f_3}] = \dot{m}_2[a_{f_3} - a_{f_2}] + \dot{I} \tag{7.62}$$

$$\eta_{II} = \frac{\dot{m}_2[a_{f_3} - a_{f_2}]}{\dot{m}_1[a_{f_1} - a_{f_3}]} \tag{7.63}$$

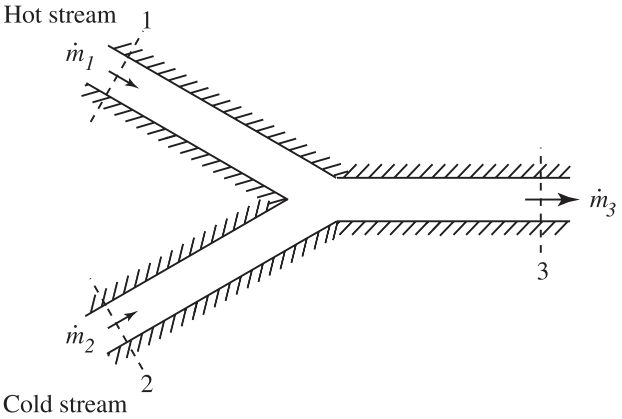


Fig. 7.8 Energy loss due to mixing

7.11 COMMENTS ON EXERGY

Like mass, the energy of the universe is constant. Nowadays, quite often we are reminded of 'conserve' energy. As engineers, we are aware that energy is always conserved. What is not conserved is the exergy, which is nothing but the useful work potential of the energy. Once the exergy is wasted, it can never be recovered. When we use energy (electricity) to heat our homes, we are not destroying any energy. What we should understand is that we are merely converting it to a less useful form, which has less exergy value. The maximum useful work potential of a system at the specified state is called exergy. It is a composite property depending on the state of the system and the surroundings. A system which is in equilibrium with its surroundings is said to be at the dead state having zero exergy.

The first law efficiency alone is not a realistic measure of performance for engineering devices. Consider two heat engines, having, for example, a thermal efficiency of, say, 30%.

One of the engines (A) is supplied with heat Q from a source at 600 K and the other engine (B) is supplied with the same amount of heat Q from a source at 1000 K. Both the engines reject heat to the surroundings at 300 K.

$$(W_A)_{rev} = Q\left(1 - \frac{300}{600}\right) = 0.5 \times Q, while (W_A)_{act} = 0.3 \times Q$$

Similarly, $$(W_B)_{rev} = Q\left(1 - \frac{300}{1000}\right) = 0.7 \times Q \quad \text{and} \quad (W_B)_{act} = 0.3 \times Q$$

At first glance, both engines seem to convert the same fraction of heat, that they receive, to work, thus performing equally well from the viewpoint of the first law. However, in the light of second law, the engine B has a greater work potential ($0.7Q$) available to it and thus should do a lot better than engine A. Therefore, it can be said that engine B is performing poorly relative to engine A, even though both have the same thermal efficiency.

To overcome the deficiency of the first law efficiency, a second law efficiency η_{II} can be defined as the ratio of actual thermal efficiency to the maximum possible thermal efficiency under the same conditions.

$$\eta_{II} = \frac{\eta_I}{\eta_{rev}} \tag{7.64}$$

So, for engine A, $\eta_{II} = \dfrac{0.3}{0.5} = 0.60$

and for engine B, $\eta_{II} = \dfrac{0.3}{0.7} = 0.43.$

Therefore, the engine A is converting 60% of the available work potential (exergy) to useful work. This is only 43% for the engine B. Therefore,

For heat engines and other work producing devices,

$$\eta_{II} = \frac{\eta_{act}}{\eta_{rev}} = \frac{W_u}{W_{rev}}$$

For refrigerators, heat pumps and other work absorbing devices,

$$\eta_{II} = \frac{COP}{COP_{rev}} = \frac{W_{rev}}{W_u}$$

where W_u is the useful work. The exergies of a closed system (ϕ) and a flowing fluid system(ψ) are given on unit mass basis.

$$\phi = (u - u_0) - T_0(s - s_0) + p_0(v - v_0) \text{ kJ/kg} \tag{7.65}$$

$$\Psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \text{ kJ/kg} \tag{7.66}$$

7.12 CARNOT CYCLE ON THE T-s DIAGRAM

In Fig.7.9(a) and (b), the Carnot cycle is shown on the p-V and T-s diagrams, respectively. The p-V diagram has already been explained in Section 5.12. Corresponding state points on

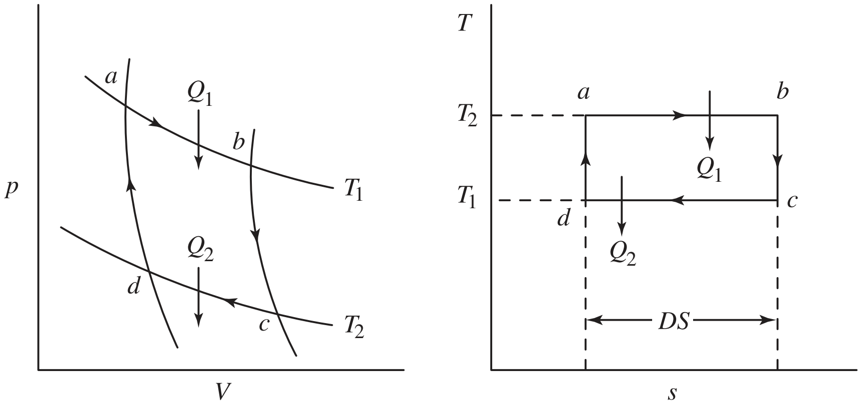


Fig. 7.9 Carnot Cycle on the p-V and T-s diagrams

both the diagrams are indicated by the same alphabets. In the following analysis, only the T-s diagram is considered.

During a—b, the system undergoes a reversible isothermal expansion process. Hence, a—b is along a horizontal line with $T_a = T_1 = T_b$. During this process, net heat transfer to the system at a constant temperature of T_1 is Q_1. Therefore,

$$s_b - s_a = \int_a^b \frac{\delta Q_A}{T} = \frac{Q_1}{T_1} \tag{7.67}$$

or

$$Q_1 = T_1(s_b - s_a) = T_1 \Delta S = \text{Area abfea} \tag{7.68}$$

As process b—c is reversible and adiabatic, $s_b = s_c$ and hence, b and c must lie on a vertical line. c—d is the reversible isothermal compression or heat rejection process during which the system transfers out Q_2 at a steady temperature of T_2, such that $T_d = T_2 = T_c$. Hence,

$$s_d - s_c = \int \frac{\delta Q_R}{T} = \frac{Q_2}{T_2} \tag{7.69}$$

$$Q_2 = T_2(s_d - s_c) = \text{Area cdefc} \tag{7.70}$$

In Eq.7.70, the entropy change is negative as heat is transferred out of the system. Thus, the magnitude of Q_2 is given by,

$$Q_2 = T_2(s_c - s_d) = T_2 \Delta S \tag{7.71}$$

Process d—a is reversible and adiabatic and hence, it is along a vertical line and $s_d = s_c$. Thus, the Carnot cycle on a T-s diagram is bound by two horizontal and two vertical lines and is always a rectangle. From Eqs 7.68 and 7.70, net work W for the cycle is given by:

$$W = Q_1 - Q_2 = (T_1 \Delta S - T_2 \Delta S) = (\text{Area abfea} - \text{Area cdefc }) \tag{7.72}$$

$$= \text{Area of the diagram abcda} \tag{7.73}$$

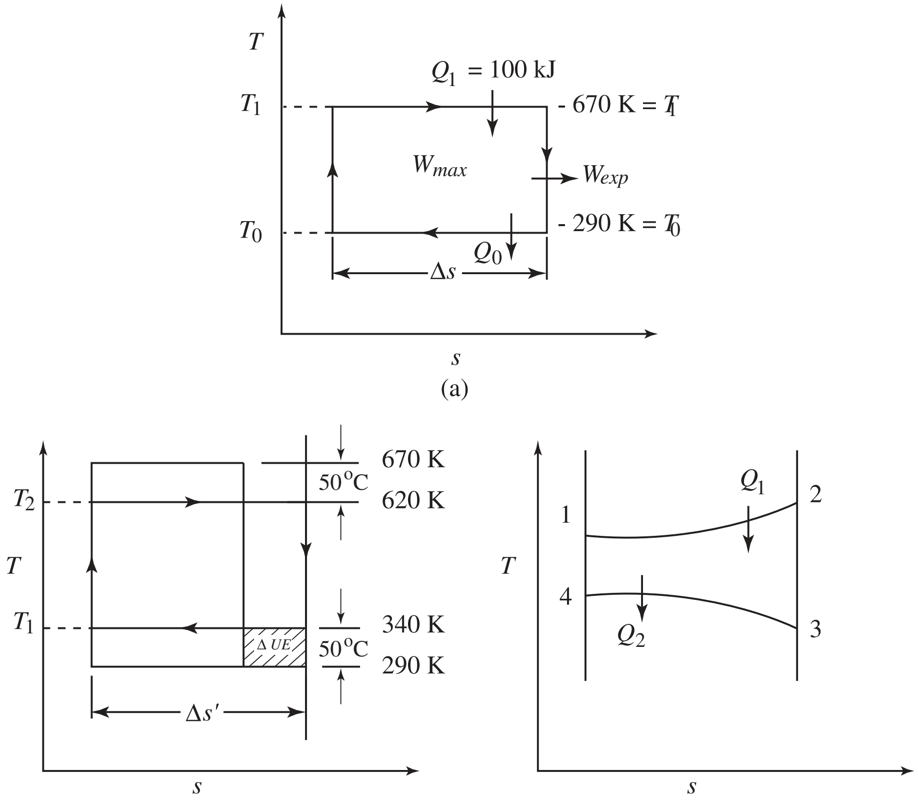
$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 \Delta S - T_2 \Delta S}{T_1 \Delta S} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \tag{7.74}$$

Worked-Out Examples

7.1 What is the maximum useful work which can be obtained when 100 kJ are abstracted from a heat reservoir at 670 K in an environment at 290 K. Estimate the loss of useful work if:

(i) a temperature drop of 50 °C is introduced between the heat source and the heat engine, on the one hand and the heat engine and the heat sink on the other hand.

(ii) the source temperature drops by 50 °C and the sink temperature rises by 50 °C during the heat transfer process according to the linear law $\frac{\delta Q}{dT} = \pm C$.



(b) (c)

Fig. 7.10

Solution

$$\Delta S = \frac{Q_1}{T_1} = \frac{100}{670} = 0.149 \text{ kJ/K}$$

Unavailable energy, $$UE = Q_0 = T_0 \Delta S = 290 \times 0.149 = 43.21 \text{ kJ}$$

$$W_{max} = \text{ available energy, } AE = Q_1 - UE = 100 - 43.21 = 56.79 \text{ kJ}$$

From Fig.7.10(a), $$\Delta S' = \frac{100}{620} = 0.161$$

$$UE = T_0 \Delta S' = 340 \times 0.161 = 54.74 \text{ kJ}$$

$$\text{Loss of useful work } = \Delta UE = 54.74 - 43.21 = 11.53 \text{ kJ}$$

$$\frac{\delta Q}{dT} = \pm C$$

$$CdT = \pm \delta Q = \pm T ds$$

$$\frac{dT}{ds} = \pm \frac{T}{C}$$

Since C is constant, $\frac{dT}{ds}$ change with T. Refer Fig.7.10(b). Given $T_1 = 620$ K; $T_2 = 670$ K; $T_3 = 290$ K; and $T_4 = 340$ K. C is the heat capacity of the source or sink.

$$Q_1 = \int_{T_1}^{T_2} c_{p1}(670 - 620)$$

$$100 = c_{p1} 50$$

$$c_{p1} = 2 \text{ kJ/K}$$

Again, $$Q_2 = \int_{T_3}^{T_4} c_{p2} dT = c_{p2}(T_4 - T_3) \qquad \text{(i)}$$

By entropy principle, $$\Delta S_{source} + \Delta S_{HE} + \Delta S_{sink} \geq 0$$

$$c_{p1} \ln \frac{T_2}{T_1} + 0 + c_{p2} \ln \frac{T_2}{T_1} \geq 0$$

$$2 \times \ln \frac{620}{670} + c_{p2} \times \ln \frac{340}{290} \geq 0$$

$$-0.155 + c_{p2} \times 0.159 \geq 0$$

$$c_{p2} \geq \frac{0.155}{0.159} \geq 0.975$$

W will be maximum when Q_2 and c_{p2} are minimum.

$$\left(c_{p2}\right)_{min} = 0.975 \text{ kJ/K}$$

From Eq.(i), $$UE = Q_2 = 0.975 \times 50 = 48.75 \text{ kJ}$$

$$\text{Loss of useful work} = 48.75 - 43.21 = 5.54 \text{ kJ}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

7.2 In a steam generator, water is evaporated at 250 °C, while the combustion gas (c_p = 1.1 kJ/kg K) is cooled from 1300 °C to 300 °C. Determine the loss in available energy due to the above heat transfer per kg of water evaporated. Take latent heat of vaporisation of water at 250 °C is 1660 kJ/kg. Take the surrounding temperature as 30 °C.

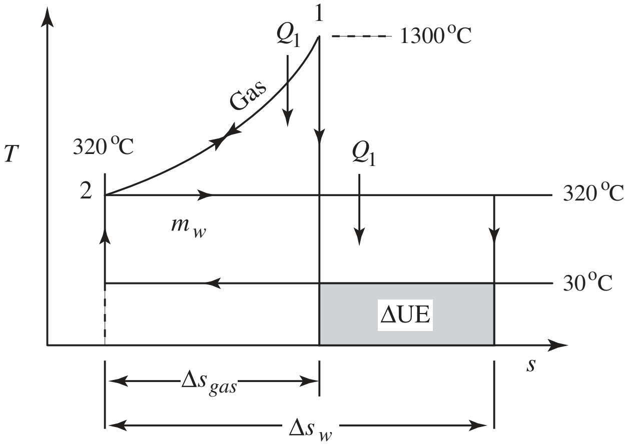


Fig. 7.11

Solution

From Fig.7.11, by energy balance:

$$
\begin{aligned}
m_g c_{pg}(T_1 - T_2) &= m_w h_{fg} \\
m_g c_{pg} &= \frac{1 \times 1660}{1300 - 300} = 1.66 \text{ kJ/K} \\
\Delta s_g &= m_g c_{pg} \ln \frac{T_2}{T_1} = 1.66 \times \ln \frac{573}{1573} = -1.68 \text{ kJ/kg water K} \\
UE &= T_0 \Delta s_g = 303 \times 1.68 = 509 \text{ kJ/kg of water} \\
\Delta S_{water} &= \frac{m_w h_{fg}}{250 + 273} = \frac{1 \times 1660}{523} = 3.174 \text{ kJ/kg K} \\
UE' &= T_0 \Delta S_w = 303 \times 3.174 = 961.72 \text{ kJ/kg}
\end{aligned}
$$

Loss of available energy:

$$
\Delta UE = UE' - UE = 961.72 - 509 = 452.72 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
$$

7.3 Exhaust gases leave an internal combustion engine at 750 °C and 1 atm after having done 1060 kJ of work per kilogram of gas in the engine (c_p of gas 1.1 kJ/kg K). Temperature of the surroundings is 27 °C.

(i) How much available energy per kilogram of gas is lost by throwing away the exhaust gases?

(ii) What is the ratio of lost available energy to the engine work?

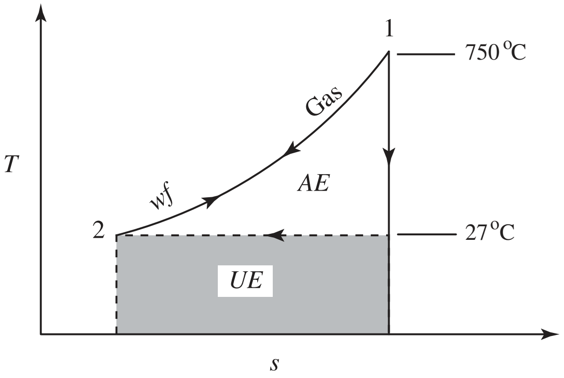


Fig. 7.12

Solution

From Fig.7.12, $\Delta S = m_g c_{pg} \ln \dfrac{T_1}{T_2} = 1 \times 1.1 \times \ln \dfrac{1023}{300} = 1.35$ kJ/kg K

$$AE \text{ lost} = UE = T_0 \Delta S = 300 \times 1.35 = 405 \text{ kJ}$$

$$\frac{UE}{W} = \frac{405}{1060} = 0.382$$ **Ans** ⟸

7.4 A hot spring sprinkles water at a temperature of 57 °C. The sprinkled water flows into a large lake, with a mean temperature of 17 °C at a rate of 0.1 m^3 of water per min. What is the rate of working of an ideal heat engine which uses all the available energy? Take c_p of water as 4.187 kJ/kg K.

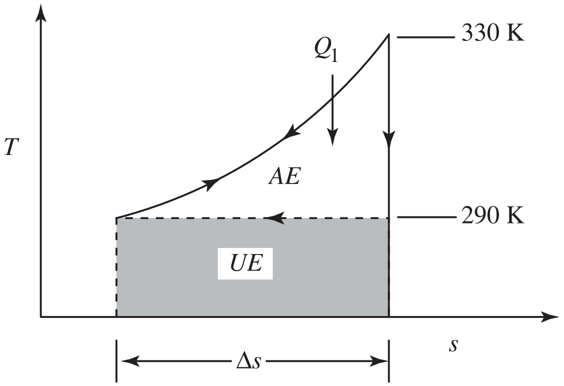


Fig. 7.13

Solution

From Fig.7.13,

$$\Delta S \quad = \quad c_p \ln\left(\frac{T_1}{T_0}\right) = 4.187 \times \ln\left(\frac{330}{290}\right) = 0.541 \text{ kJ/kg K}$$

$$UE = \dot{m}T_0\Delta s = \frac{0.1 \times 1000}{60} \times 290 \times 0.541 = 261.48 \text{ kW}$$

$$\dot{Q}_1 = \text{Rate of heat supply} = \frac{0.1 \times 1000}{60} \times 4.187 \times (330 - 290) = 279.13 \text{ kW}$$

$$\dot{W} = \text{Rate of working of an ideal heat engine}$$

$$= \dot{Q} - UE = 279.13 - 261.48 = 17.65 \text{ kW}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

7.5 0.1 kg of air at 300 °C is heated reversibly at constant pressure to 2060 K. Find the available and unavailable energies of heat addition. Take $T_0 = 27$ °C and $c_p = 1.005$ kJ/kg K.

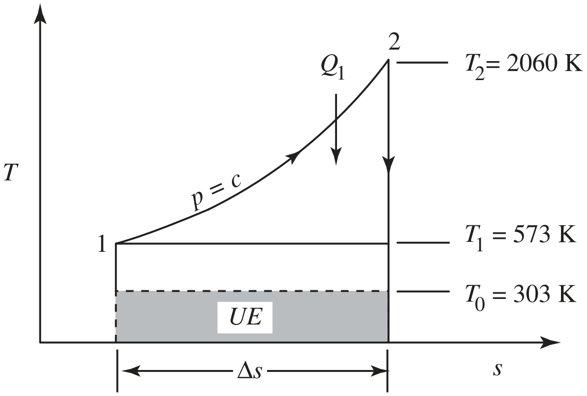


Fig. 7.14

Solution

From Fig.7.14,

$$UE = T_0\Delta S = T_0 m_a c_{pa} \ln\left(\frac{T_2}{T_1}\right) = 290 \times 0.1 \times 1.005 \times \ln\left(\frac{2060}{573}\right) = 37.29 \text{ kJ}$$

$$Q_1 = m_a c_{pa}(T_2 - T_1) = 0.1 \times 1.005 \times (2060 - 573) = 149.44 \text{ kJ}$$

$$AE = 149.44 - 37.29 = 112.15 \text{ kJ}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

7.6 Hundred kilogram of water at 100 °C is mixed into 50 kg of water at 50 °C while the temperature of the surrounding is 17 °C. Determine the decrease in available energy due to mixing. Take c_p of water as 4.186 kJ/kg K.

Solution

$$AE = mc_p\left[(T - T_0) - T_0 \ln\left(\frac{T}{T_0}\right)\right]$$

$$\text{Before mixing, } AE_{100} = 100 \times c_p\left[(373 - 290) - 290 \times \ln\left(\frac{373}{290}\right)\right]$$

$$= 1000.77 \times c_p$$

$$AE_{50} = 50 \times c_p\left[(323-290) - 290 \times \ln\left(\frac{323}{290}\right)\right] = 87.31 \times c_p$$

$$\text{Total } AE = (1000.77 + 87.31) \times c_p = 1088.08 \times c_p$$

After mixing, by energy balance, $100 \times c_p \times (100 - t) = 50 \times c_p(t - 50)$, where t is temperature after mixing:

$$2 \times (100 - t) = (t - 50)$$

$$3t = 250$$

$$t = 83.33\ ^\circ\text{C}$$

$$T = 273 + 83.33 = 356.33$$

$$AE_{150} = 150 \times c_p\left(356.33 - 290 - 290 \times \ln\frac{356.33}{290}\right) = 989.53 \text{ kJ}$$

Decrease in available energy due to mixing:

$$1088.08 \times c_p - 989.53 \times c_p = 98.55 \times c_p = 98.55 \times 4.186 = 412.53 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.7 A lead storage battery used in an automobile is able to deliver 5.2 MJ of electrical energy. This energy is used for starting a car. Let us consider compressed air for doing an equivalent amount of work in starting the car. The compressed air is to be stored at 7.5 MPa, 27 °C. What is the volume of tank that would be required to let the compressed air have an availability of 5.5 MJ. Take for air $pv = 0.287T$, where T is in K, p is in kPa and v is in m^3/kg. Take atmospheric pressure as 1 bar.

Solution

Availability of compressed air per unit mass,

$$\phi = (u - u_0) - T_0(s - s_0) + p_0(v - v_0)$$

Since T is constant, $T_0 = 300 \text{ K} = T; (u - u_0) = 0$

$$\phi = -T_0 R \ln\frac{p_0}{p} + p_0\left(\frac{RT}{p} - \frac{RT_0}{p_0}\right)$$

$$= -T_0 R \ln\frac{p_0}{p} + RT_0\left(\frac{p_0}{p} - 1\right) \quad (\because T = T_0)$$

$$= -300 \times 0.287 \times \ln\frac{0.1}{7.5} + 0.287 \times 300 \times \left(\frac{0.1}{7.5} - 1\right)$$

$$= 300 \times 0.287 \times \left(\ln\frac{7.5}{0.1} + \frac{0.1}{7.5} - 1\right) = 286.78 \text{ kJ/kg}$$

$$m \times 286.78 = 5500$$

$$m = 19.17 \text{ kg}$$

Volume of the tank required,

$$V = \frac{mRT}{p} = \frac{19.17 \times 0.287 \times 300}{7500} = 0.22 \text{ m}^3 \qquad \underset{\Longleftarrow}{\textbf{Ans}}$$

7.8 Ice is to be made from water supplied at 17 °C by the process shown in Fig.7.15. The final temperature of ice is −10 °C. The final temperature of water that is used as cooling water in the condenser is 27 °C. Determine minimum work required to produce 1 tonne of ice. Take c_p of water = 4.187 kJ/kg K and c_p of ice = 2.093 kJ/kg K. Latent heat of fusion of ice = 334 kJ kg.

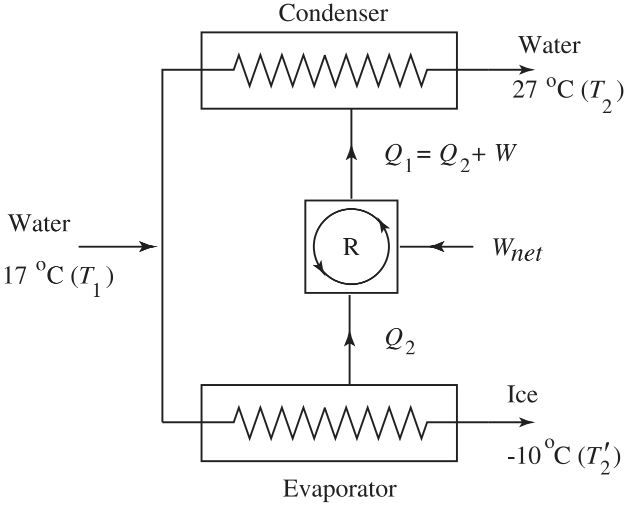


Fig. 7.15

Solution

Let m be the mass of water flowing through the condenser. For minimum work, $\Delta S_{universe} = 0$ or $\Delta S_{cond} + \Delta S_{refri} + \Delta s_{evop} = 0$

$$\Delta S_{cond} = mc_p \ln \frac{T_2}{T_1} = m \times 4.187 \times \ln \frac{300}{290} = m \times 0.142 \text{ kJ/K}$$

$$\Delta S_{refri} = \oint dS = 0$$

$$\Delta S_{evop} = m_i \left[c_{pw} \ln \frac{T}{T_1} + \frac{-\ell}{T} + c_{pi} \ln \frac{T_{2'}}{T} \right]$$

where ℓ is latent heat of fusion.

$$= -m \left[c_p \ln \frac{T}{T_0} + \frac{\ell}{T} - c_{pi} \ln \frac{T}{T_{2'}} \right]$$

$$= -1000 \times \left[4.187 \times \ln \frac{290}{273} + \frac{334}{273} + 2.093 \times \ln \frac{273}{263} \right] = -1554.5 \text{ kJ/K}$$

$$\begin{aligned}
\Delta S_{univ} &= m \times 0.141 - 1554.5 = 0 \\
m &= \frac{1554.5}{0.141} = 11024.7 \text{ kg} \\
Q_1 &= 11024.7 \times 4.187 \times (27 - 17) = 4615.75 \text{ kJ} = 461.6 \text{ MJ} \\
Q_2 &= 1000 \times (4.187 \times 17 + 334 + 2.093 \times 10) = 426109 \text{ kJ} \approx 426 \text{ MJ} \\
W_{net} &= Q_1 - Q_2 = 600 - 461.6 = 35.6 \text{ MJ} \qquad \xleftarrow{\textbf{Ans}}
\end{aligned}$$

7.9 A pressure vessel has a volume of 1 m^3 and holds air at 1.5 MPa and 177 °C. The air is cooled to 27 °C by heat transfer to the surroundings at 27 °C. Calculate the availability in the initial and final states and the irreversibility of this process, Take $P_0 = 100$ kPa.

Solution

$$\begin{aligned}
\text{Mass of air, } m &= \frac{pv}{RT} = \frac{1500 \times 1}{0.287 \times (273 + 177)} = 11.61 \text{ kg} \\
\text{Initial availability} &= a_1 = u_1 - u_0 + p_0(v_1 - v_0) - T_0(s_1 - s_0) \\
&= c_V(T_1 - T_0) + p_0\left[\frac{RT_1}{p_1} - \frac{RT_0}{p_0}\right] - T_0\left[c_p \ln\frac{T_1}{T_0} - R\ln\frac{p_1}{p_0}\right] \\
&= 0.718 \times (450 - 300) + 100 \times 0.287 \times \left[\frac{450}{1500} - \frac{300}{100}\right] \\
&= -300 \times \left[1.005 \times \ln\frac{450}{300} - 0.287 \times \ln\frac{1500}{100}\right] \\
&= 107.7 - 77.49 + 110.91 = 141.1 \text{ kJ/kg} \\
\text{Final availability, } a_2 &= u_2 - u_0 + p_0(v_2 - v_0) - T_0(s_2 - s_0) \\
&= c_V(T_2 - T_0) + p_0\left[\frac{RT_2}{p_2} - \frac{RT_0}{p_0}\right] - T_0\left(c_p \ln\frac{T_2}{T_0} - R\ln\frac{p_2}{p_0}\right)
\end{aligned}$$

Since $T_2 = T_0$,

$$\begin{aligned}
a_2 &= RT_0\left(\frac{p_0}{p_2} - 1\right) + RT_0 \ln\frac{p_2}{p_0} \\
\frac{p_1 v_1}{T_1} &= \frac{p_2 v_2}{T_2} \\
p_2 &= \frac{T_2}{T_1} p_1 = \frac{300}{450} \times 1500 = 1000 \text{ kPa} \\
a_2 &= RT_0\left[\frac{p_0}{p_2} - 1 + \ln\frac{p_2}{p_0}\right] \\
&= 0.287 \times 300 \times \left(\frac{100}{1000} - 1 + \ln\frac{1000}{100}\right) = 120.76 \text{ kJ/kg}
\end{aligned}$$

$$\text{Irreversibility, } I \quad = \quad m(a_1 - a_2) = 11.61 \times (141.1 - 120.76) = 236.15 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.10 Air flows through an adiabatic compressor at 2 kg/s. The inlet conditions are 1 bar and 320 K and the exit conditions are 7 bar and 570 K. Compute the net rate of availability transfer and the irreversibility. Take $T_0 = 300$ K.

Solution

$$\begin{aligned} W_{rev} &= c_p(T_1 - T_2) - T_0(s_1 - s_2) \\ &= 1.005 \times (320 - 570) + 300 \times \left[1.005 \times \ln \frac{570}{320} - 0.287 \times \ln \frac{7}{1}\right] \\ &= -251.25 + 300 \times (0.580 - 0.558) = 244.65 \text{ kg} \end{aligned}$$

Rate of irreversibility transfer = 2 × 244.65 = 489.3kW

$$\begin{aligned} W_{actual} &= m(h_2 - h_1) = 2 \times c_p(T_2 - T_1) \\ &= 2 \times 1.005 \times (570 - 320) = 502.5 \text{ kJ} \end{aligned}$$

$$\text{Irreversibility, } I \quad = \quad 502.5 - 489.3 = 13.2 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.11 An adiabatic turbine receives a gas with $c_p = 1.09$ and $c_V = 0.838 \, kJ/kgK$ at 7.5 bar and 1275 K and discharging at 1.5 bar and 950 K. Determine the second law and isentropic efficiencies of the turbine. Take $T_0 = 300$ K.

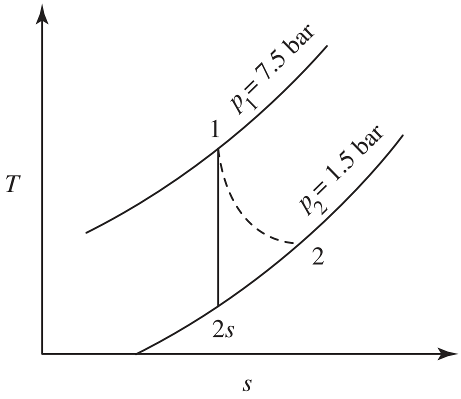


Fig. 7.16

Solution

$$\begin{aligned} W_{rev} &= c_p(T_1 - T_2) - T_0(s_1 - s_2) \\ &= c_p(T_1 - T_2) - T_0\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{p_1}{p_2}\right)\right] \\ &= 1.09 \times (1275 - 950) - 300 \times \left[1.005 \times \ln \frac{950}{1275} - 0.252 \times \ln \frac{1.5}{7.5}\right] \end{aligned}$$

$$= 354.25 + 300 \times (-0.296 + 0.406) = 387.25 \text{ kJ/kg}$$

$$W_{act} = h_1 - h_2 = c_p(T_1 - T_2) = 1.09 \times (1275 - 950) = 354.25 \text{ kJ/kg}$$

$$\eta_T = \frac{W_{act}}{W_{rev}} = \frac{354.25}{387.25} = 0.915$$

$$\gamma = \frac{c_p}{c_V} = \frac{1.09}{0.838} = 1.3$$

From Fig.7.16:

$$\frac{T_{2s}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1.5}{7.5}\right)^{\frac{0.3}{1.3}} = 0.69$$

$$T_{2s} = 0.69 \times 1275 = 878.7$$

$$\eta_{isen} = \frac{T_1 - T_2}{T_1 - T_2s} = \frac{1275 - 950}{1275 - 878.7} = 0.82 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.12 An air pre-heater is used to heat up the air used for combustion by cooling the outgoing products of combustion from a furnace. The rate of flow of the products is 10 kg/s and the products are cooled from 350 °C to 250 °C. c_p of air may be taken as 1.09 kJ/kg K. The rate of air flow is 10 kg/s. The initial temperature is 37 °C and for air $c_p = 1.005$ kJ/kg K. Take $T_0 = 300$ K. Estimate,

(i) the initial and final availability of the products.

(ii) the irreversibility of their process.

(iii) if the heat transfer from the products were to take place reversibly through heat engines what would be the final temperature of air.

(iv) the power developed by the heat engines.

Solution

Initial availability of the products of combustion:

$$= h_1 - h_0 - T_0(s_1 - s_0) = c_p(T_1 - T_0) - T_0(s_1 - s_0)$$

$$= 1.09\left[623 - 300 - 300 \times \ln\frac{623}{300}\right] = 113.11 \text{ kJ/kg} \overset{\textbf{Ans}}{\Longleftarrow}$$

Final availability of products $= 1.09\left[423 - 300 - 300 \times \ln\frac{423}{300}\right] = 21.71 \text{ kJ/kg} \overset{\textbf{Ans}}{\Longleftarrow}$

Rise in air temperature $= \dfrac{10 \times 1.09 \times (350 - 250)}{10 \times 1.005} = 108.46$

Final air temperature, $t_{a2} = 108.46 + 37 = 145.46$ °C

$$\Delta S_{air} = 10 \times 1.005 \times \ln \frac{273 + 145.46}{310} = 3.01 \text{ kW/K}$$

$$\Delta \dot{S}_{gas} = 10 \times 1.09 \times \ln \frac{523}{623} = -1.90 \text{ kW/K}$$

$$\Delta \dot{S}_{univ} = 3.01 - 1.90 = 1.11 \text{ kW/K}$$

∴ Rate of irreversibility or exergy destruction,

$$I = T_0 \Delta S_{univ} = 300 \times 1.11 = 330 \text{ kW}$$

For reversible heat transfer between gas and air, the entropy generation of the universe will be zero:

$$\therefore \Delta \dot{S}_{univ} = \Delta \dot{S}_{gas} + \Delta \dot{S}_{air} = 0$$

$$\Delta \dot{S}_{air} = -\Delta \dot{S}_{gas}$$

$$\dot{m}_a c_p \ln \frac{T_{a2}}{T_{a1}} = m_g c_{pg} \ln \frac{T_{g1}}{T_{g2}}$$

$$10 \times 1.005 \times \ln \frac{T_{a2}}{313} = 1.90$$

$$\ln \frac{T_{a2}}{313} = \frac{1.90}{10 \times 1.005} = 0.189$$

$$\frac{T_{a2}}{313} = e^{0.189} = 1.20$$

$$T_{a2} = 375.6 \text{ K}$$

Final temperature, $= 375.6 - 273 = 102.6\ ^\circ\text{C}$ $\overset{\textbf{Ans}}{\Longleftarrow}$

Heat input to heat engines,

$$Q_1 = 10 \times 1.09 \times 100 = 1090 \text{ kW}$$

Heat rejected by heat engine,

$$Q_2 = 10 \times 1.005 \times (102.6 - 37) = 659.3 \text{ kJ}$$

Power developed, $P = 1090 - 659.3 = 430.7$ kW $\overset{\textbf{Ans}}{\Longleftarrow}$

7.13 A mass of 3 kg of air in a vessel expands from 3 bar, 70 °C to 1 bar 40 °C. While receiving 1.1 kJ of heat from a reservoir at 120 °C. The environment is at 0.99 bar at 27 °C. Calculate the maximum work and the work done on the atmosphere.

Solution

$$\text{Maximum work, } W_{max} = U_1 - U_2 - T_0(S_1 - S_2) + Q\left(1 - \frac{T_0}{T}\right)$$

$$U_1 - U_2 = mc_V(T_1 - T_2) = 3 \times 0.718 \times (70 - 40) = 64.62 \text{ kJ}$$

$$S_2 - S_1 = m\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{T_1}\right)\right]$$

$$= 3 \times \left[1.005 \times \ln\frac{313}{343} - 0.287\ln\frac{1}{3}\right] = 0.67 \text{ kJ/kg}$$

$$\therefore W_{max} = 64.62 - 300 \times \left[-0.67 + 1.1 \times \left(1 - \frac{300}{393}\right)\right] = 187.528 \text{ kJ}$$

Work done on the atmosphere, $p_0(V_2 - V_1) = p_0 mR\left(\frac{T_2}{p_2} - \frac{T_1}{p_1}\right)$

$$= 0.99 \times 100 \times 3 \times 0.287 \times \left(\frac{313}{100} - \frac{343}{300}\right) = 169.34 \text{ kJ}$$ **Ans** ⟸

7.14 Air enters the compressor of a gas turbine at 1 bar, 27 °C and leaves at 5 bar. The compressor has an efficiency of 85%. Calculate for unit mass of air: (i) the work of compression; (ii) the reversible work of compression and (iii) the irreversibility.

For air, use the relation, $\frac{T_{2'}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$

where $T_{2'}$ is the temperature of air after isentropic compression and $\gamma = 1.4$.

The compressor efficiency is given by:

$$\eta_c = \frac{T_{2'} - T_1}{T_2 - T_1}$$

where T_2 is the actual temperature of air after compression.

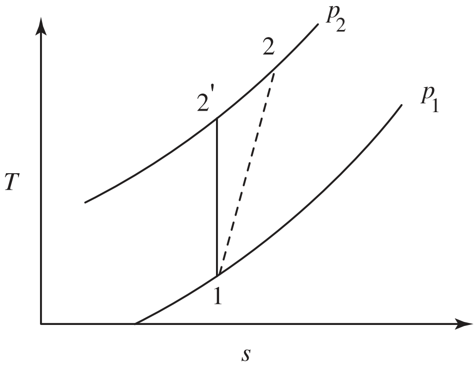


Fig. 7.17

Solution

$$\frac{T_{2'}}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}$$

$$T_{2'} = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 300 \times (5)^{\frac{0.4}{1.4}} = 475.15 \text{ K}$$

$$\eta_C = \frac{T_{2'} - T_1}{T_2 - T_1} = 0.85$$

$$T_2 - T_1 = \frac{T_{2'} - T_1}{0.85} = \frac{475.15 - 300}{0.85} = 206 \text{ K}$$

$$T_2 = T_1 + 206 = 300 + 206 = 506 \text{ K}$$

Using SFEE for compressor, $W_C + h_1 = Q + h_2$. Assume $Q = 0$:

$$\therefore W_x = h_2 - h_1 = 1.005 \times (506 - 300) = 207 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_{rev} = c_p(T_1 - T_2) - T_0(s_2 - s_1)$$

$$= c_p(T_1 - T_2) - T_0\left[c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\left(\frac{p_2}{T_1}\right)\right]$$

$$= 1.005 \times (300 - 506) + 300 \times \left[1.005 \times \ln\frac{506}{300} - 0.287 \times \ln\frac{5}{1}\right]$$

$$= -188 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Irreversibility, } I = W_{rev} - W_{act} = 207 - 188 = 19 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.15 A mass of 7 kg of air is in a vessel at 200 kPa and 300 K. Heat is transferred to the air from 1000 K until the temperature of air rises to 600 K. The environment is at 100 kPa and 290 K. Determine

(i) the initial and final availability of air

(ii) the maximum useful work associated with the process.

Solution

$$v_0 = \frac{RT_0}{p_0} = \frac{0.287 \times 290}{100} = 0.832 \text{ m}^3 \text{ kg}$$

$$s_1 - s_0 = c_p \ln\frac{T_1}{T_0} - R\ln\frac{p_1}{p_0} = 1.005 \times \ln\frac{300}{290} - 0.287 \times \ln\frac{200}{100}$$

$$= 0.0341 - 0.1989 = -0.165 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{200} = 0.43 \text{ m}^3\text{/kg}$$

Initial availability of air:

$$\begin{aligned}
\phi_1 &= m[(u_1-u_0)-p_0(v_1-v_0)-T_0(s_1-s_0)] \\
&= 7[0.718\times(300-290)+100\times(0.43-0.832)-290\times(-0.165)] \\
&= 7\times(0.718-40.2+47.85) = 103.81 \text{ kJ} \\
s_2-s_0 &= c_p\ln\frac{T_2}{T_0}-R\ln\frac{p_2}{p_0} \\
&= 1.005\times\ln\frac{600}{290}-0.287\times\ln\frac{400}{100} = 0.333 \text{ kJ/kg K} \\
\frac{p_1v_1}{T_1} &= \frac{p_2v_2}{T_2} \\
p_2 &= \frac{T_2}{T_1}p_1 = \frac{600}{300}\times 200 = 400 \text{ kPa}(\because v_2=v_1)
\end{aligned}$$

Final availability of air:

$$\begin{aligned}
\phi_2 &= m[(u_2-u_0)+p_0(v_2-v_0)-T_0(s_2-s_0)] \\
&= 7\times[0.718\times(600-290)+100\times(0.43-0.832)-290\times 0.333] \\
&= 7\times(222.6-40.2-96.37) = 600.8 \text{ kJ} \\
W_{max} &= \phi_1-\phi_2 = 103.81-600.8 = -497 \text{ kJ} \\
\text{Heat transfer, } Q &= \Delta U+W = m(u_2-u_1)+0 \\
&= 7\times 0.718\times(600-300) = 1507.8 \text{ kJ}
\end{aligned}$$

$$\begin{aligned}
(W_{useful})_{max} &= (\phi_1-\phi_2)+\phi_R\left(1-\frac{T_0}{T_R}\right) \\
&= -497+1507.8\times\left[1-\frac{290}{1000}\right] = 573.54 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

7.16 Air enters a compressor in steady flow at 150 kPa, 17 °C and 80 m/s and leaves at 350 kPa and 127 °C and 120 m/s. The environment is at 100 kPa and 7 °C. Calculate for unit mass of air

(i) the actual amount of work required.

(ii) the minimum work required.

(iii) the irreversibility of the process.

Solution

Assume the compressor to be adiabatic then $Q = 0$. By SFEE,

$$
\begin{aligned}
h_1 + W + \frac{\mathbb{V}_1^2}{2} &= h_2 + \frac{\mathbb{V}_2^2}{2} + Q \\
\therefore W_{act} &= (h_2 - h_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} \\
&= 1.005 \times (127 - 17) + \frac{120^2 - 80^2}{2} \times 10^{-3} \\
&= 110.55 + 4 = 114.55 \text{ kJ/kg} \\
W_{min} &= h_2 - h_1 + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} - T_0(s_2 - s_1) \\
&= 114.55 - 280 \times \left[1.005 \ln \frac{400}{290} - 0.287 \times \ln \frac{350}{150}\right] = 92.15 \text{ kJ/kg} \\
I &= W_{act} - W_{min} = 114.55 - 92.15 = 22.4 \text{ kJ/kg} \qquad \overset{\text{Ans}}{\Longleftarrow}
\end{aligned}
$$

7.17 Air expands in a turbine adiabatically from 600 kPa, 400 K and 150 m/s to 120 kPa 300 K and 75 m/s. The environment is at 100 kPa and 17 °C. Calculate per kg of air:

(i) the actual amount of work required;

(ii) the minimum work required; and

(iii) the irreversibility of the process.

Solution

$$
\begin{aligned}
W_{act} &= h_1 - h_2 - \frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2} \\
&= 1.005 \times (400 - 300) + \frac{150^2 - 75^2}{2} \times 10^{-3} 108.94 \qquad \overset{\text{Ans}}{\Longleftarrow} \\
W_{max} &= (h_1 - h_2) + \frac{\mathbb{V}_1^2 - \mathbb{V}_2^2}{2} - T_0(s_1 - s_2)
\end{aligned}
$$

$$
\begin{aligned}
&= W_{act} - T_0(s_1 - s_2) \\
&= 108.94 - 290 \times \left(1.005 \times \ln \frac{300}{400} - 0.287 \times \ln \frac{6}{1}\right) = 341.91 \text{ kJ/kg} \qquad \overset{\text{Ans}}{\Longleftarrow} \\
\text{Irreversibility} &= W_{max} - W_{act} = 341.91 - 108.94 = 232.97 \qquad \overset{\text{Ans}}{\Longleftarrow}
\end{aligned}
$$

7.18 Calculate the specific exergy of air for a state of 3 bar, 390 K when the surroundings are at 1 bar and 290 K. Take $c_p = 1$ kJ/kg K and $R = 0.287$ kJ/kg K.

Solution

$$\begin{aligned}
\text{Specific energy of air, } \phi \quad &= \quad u - u_0 - p_0(v - v_0) - T_0(s - s_0) \\
&= \quad c_V(T - T_0) - p_0 R\left(\frac{T}{p} - \frac{T_0}{p_0}\right) - T_0\left[c_p \ln\frac{T}{T_0} - R\ln\frac{p}{p_0}\right] \\
&= \quad 0.713 \times (390 - 290) + 100 \times 0.287\left[\frac{390}{300} - \frac{290}{100}\right] - \\
& \qquad 290 \times \left(1 \times \ln\frac{390}{290} - 0.287 \times \ln\frac{3}{1}\right) \\
&= \quad 71.3 + 28.7 \times \left(\frac{390}{300} - \frac{290}{100}\right) - 290 \times (0.296 - 0.315) \\
&= \quad 71.3 - 45.92 + 5.51 = 30.91 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

7.19 Calculate the specific exergy of CO_2 ($c_p = 0.866$ and $R = 0.189$ kJ/kg K) for a state at 0.7 bar, 270 K and environment at 1 bar and 290 K.

Solution

$$\begin{aligned}
R \quad &= \quad c_p - c_V; \quad c_V = c_p - R = 0.866 - 0.189 = 0.677 \text{ kJ/kg K} \\
\text{Specific exergy, } e \quad &= \quad c_V(T - T_0) - p_0 R\left(\frac{T}{p} - \frac{T_0}{p_0}\right) - T_0\left[c_p \ln\frac{T}{T_0} - R\ln\frac{p}{p_0}\right] \\
&= \quad 0.677 \times (270 - 290) + 100 \times 0.189 \times \left(\frac{270}{70} - \frac{290}{100}\right) - \\
& \qquad 290 \times \left[0.866 \times \ln\frac{270}{290} - 0.189 \times \ln\frac{0.7}{1}\right] \\
&= \quad -13.54 + 18.9 \times (3.857 - 2.9) - 290 \times (-0.062 + 0.0674) \\
&= \quad 2.9813 \text{ J/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

7.20 A pipe carries a stream of salt solution with a mass flow of 6 kg/s. Because of inadequate thermal insulation the temperature of the salt solution increases from 250 K at the pipe inlet to 255 K at the exit. Neglecting pressure losses, calculate the irreversibility rate(or rate of energy degradation) associated with the heat leakage. Assume T_0 =295 K and c_p=2.85 kJ/kg K.

Solution

Refer Fig.7.18. Rate of heat transfer to the salt solution:

$$\dot{Q} \quad = \quad mc_p\Delta T = mc_p(T_2 - T_1) = 6 \times 2.85 \times (255 - 250) = 85.5 \text{ kW}$$

Rate of entropy increase of salt solution:

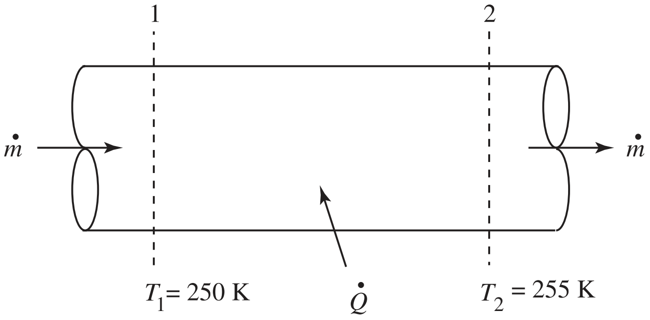


Fig. 7.18

$$\Delta\dot{S}_{sys} = \dot{m}c_p \ln\frac{T_2}{T_1} = 6 \times 2.85 \times \ln\frac{255}{250} = 0.338 \text{ kW/K}$$

Rate of entropy decrease of the surroundings:

$$\Delta S_{surr} = -\frac{Q}{T_0} = -\frac{85.5}{295} = -0.29 \text{ kW/K}$$

Rate of entropy increase of the universe:

$$\Delta S_{univ} = 0.338 - 0.29 = 0.048 \text{ kW/K}$$

$\therefore$ Irreversibility rate,

$$\dot{I} = T_0\Delta\dot{S}_{univ} = 295 \times 0.048 = 14.16 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.21 In an adiabatic throttling process, energy per unit mass of enthalpy remains the same. However, there is loss of exergy. An ideal gas flowing at the rate of $\dot{m}$ is throttled from pressure p_1 to pressure p_2 when the environment is at temperature T_0. What is the rate of exergy loss due to throttling?

Solution

From property relation, $Tds = dh - vdp$ and in an adiabatic throttling process, $dh = 0$

$$Tds = -vdp$$

$$\text{For an ideal gas, } ds = -R\frac{dp}{p}$$

$$s_2 - s_1 = -R\ln\frac{p_2}{p_1}$$

$$\dot{S}_{gen} = -\dot{m}R\ln\frac{p_2}{p_1} = \dot{m}R\ln\frac{p_1}{p_2}$$

$$\text{Rate of energy loss due to throttling, } \dot{I} = T_0\dot{S}_{gen} = \dot{m}RT_0\ln\frac{p_1}{p_2} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.22 A Carnot cycle engine receives and rejects heat with a 20 °C temperature differential between itself and the thermal energy reservoirs. The expansion and compression processes have a pressure ratio of 50. For unit mass of air as the working substance with the cycle temperature limits of 1000 K and 300 K and $T_0 = 280$ K, determine the second law efficiency.

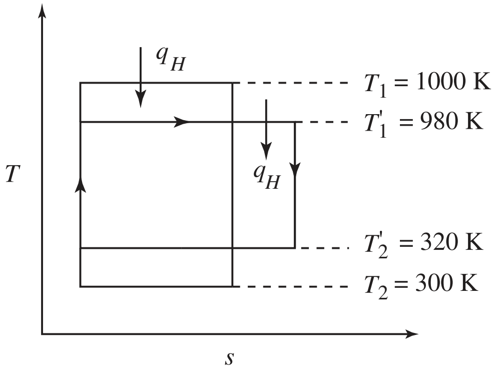


Fig. 7.19

Solution

$$\text{Actual available energy, } (AE)_{act} = \left(1 - \frac{T_2}{T_1}\right) q_H$$

Minimum available energy required

$$(AE)_{min} = W_{net}$$

$$\text{Second law efficiency, } \eta_{II} = \frac{(AE)_{min}}{(AE)_{act}} = \frac{W_{net}}{\left(1 - \dfrac{T_2}{T_1}\right)} q_H = \frac{\left(1 - \dfrac{T_{2'}}{T_1'}\right) q_H}{\left(1 - \dfrac{T_2}{T_1}\right) q_H}$$

$$= \frac{1 - \dfrac{320}{980}}{1 - \dfrac{300}{1000}} = \frac{0.673}{0.7} = 0.962 = 96.2\% \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.23 Energy is received by a solar collector at the rate of 300 kW from a source temperature of 2500 K. If 50 kW of this energy is lost to the surroundings at steady state and if the user temperature remains constant at 500 K, determine the first and second law efficiencies. Take $T_0 = 300$ K.

Solution

Let $\dot{Q}_r$ = incident radiation; $\dot{Q}_a$ = energy used; $\dot{Q}_l$ = energy lost; T_r = source temperature and T_a = user temperature. By energy balance (Fig.7.20):

$$\dot{Q}_r = \dot{Q}_a + \dot{Q}_l$$

By availability balance:

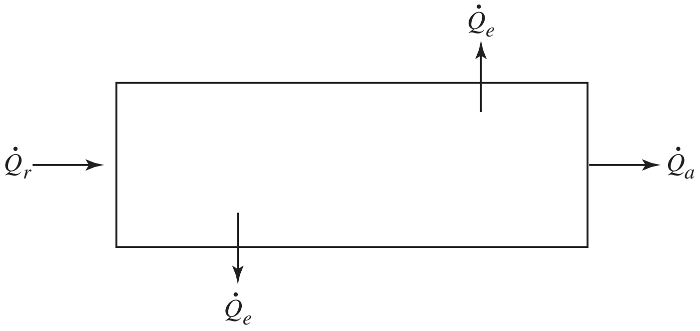


Fig. 7.20

$$\dot{Q}_r\left(1-\frac{T_0}{T_r}\right) \quad = \quad \dot{Q}_a\left(1-\frac{T_0}{T_a}\right)+\dot{Q}_l\left(1-\frac{T_0}{T_l}\right)+\dot{I}$$

where $\dot{I}$ is irreversibility rate.

$$\text{First law efficiency, } \eta_I \quad = \quad \frac{\dot{Q}_a}{\dot{Q}_r}=\frac{\dot{Q}_r-\dot{Q}_l}{\dot{Q}_r}=\frac{300-50}{300}=0.833$$

$$\text{Second law efficiency, } \eta_{II} \quad = \quad \eta_I\times\left(\frac{1-\dfrac{T_0}{T_a}}{1-\dfrac{T_0}{T_r}}\right)=0.833\times\left(\frac{1-\dfrac{300}{500}}{1-\dfrac{300}{2500}}\right)=0.378\overset{\textbf{Ans}}{\Longleftarrow}$$

7.24 For flow of an ideal gas through an insulated pipeline, the pressure drops from 95 bar 90 bar. If the gas flows at the rate of 2 kg/s. Take $c_p = 1.005$ kJ/kg K and $c_V = 0.718$ kJ/kg K and $T_0 = 300$ K. Find the rate of entropy generation and rate of loss of exergy.

Solution

Since the pipe is insulated $Q = 0$ and therefore, $dh = 0$. From properties relation:

$$Tds \quad = \quad dh - vdp$$

$$Tds \quad = \quad = -vdp \quad \text{as } dh = 0$$

For an ideal gas, $ds = -R\frac{dp}{p}$

$$s_2 - s_1 \quad = \quad -R\ln\frac{p_2}{p_1}$$

$$\therefore \dot{S}_{gen} \quad = \quad -\dot{m}R\ln\frac{p_2}{p_1}=\dot{m}R\ln\frac{p_1}{p_2}=2\times(1.005-0.718)\times\ln\frac{95}{90}=0.031\text{ kW/K}$$

Rate of energy loss due to pressure drops:

$$\dot{I} \quad = \quad T_0\dot{S}_{gen}=300\times0.031=9.3\text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

7.25 The cylinder of an internal combustion engine contains gases at 2400 °C and 60 bar. Expansion takes place through a volume ratio of 9 according to $pv^{1.38} = C$. The surrounding is at 20 °C and 1.2 bar. Determine the loss of availability the work transfer and the heat transfer per unit mass. Consider the gas to be ideal and take $R = 0.26$ kJ/kg K and $c_V = 0.82$ kJ/kg K.

Solution

Expansion is $pv^{1.38} = C$. Then,

$$
\begin{aligned}
\frac{T_2}{T_1} &= \left(\frac{v_1}{v_2}\right)^{n-1} \\
T_2 &= T_1 \times \left(\frac{v_1}{v_2}\right)^{0.38} \\
T_2 &= 2673 \times \left(\frac{1}{9}\right)^{0.38} = 1159.80 \text{ K} \\
s_2 - s_1 &= c_V \ln\frac{T_2}{T_1} + R\ln\frac{v_2}{v_1} = 0.82 \times \ln\frac{1159.8}{2673} + 0.26 \times \ln 9 \\
&= -0.1134 \text{ kJ/kg K} \\
v_1 &= \frac{RT_1}{p_1} = \frac{0.26 \times 2673}{6000} = 0.1158 \text{ m}^3\text{/kg} \\
v_1 - v_2 &= -8 \times 0.1158 = -0.9264 \text{ m}^3\text{/kg} \\
\text{Loss of energy} &= \phi_1 - \phi_2 \\
&= (u_1 - u_2) + p_0(v_1 - v_2) - T_0(s_1 - s_2) \\
&= 0.82 \times (2673 - 1159.8) + 120 \times (-0.9264) - 293 \times (0.1134) \\
&= 1096.43 \text{ kJ/kg}
\end{aligned}
$$

$$
\begin{aligned}
\text{Work transfer, } W &= \frac{p_1v_1 - p_2v_2}{n-1} = \frac{R(T_1 - T_2)}{n-1} \\
&= \frac{0.26 \times (2673 - 1159.8)}{0.38} = 1035.35 \text{ kJ/kg} \\
\text{Heat transfer, } Q &= u_2 - u_1 + W = 0.82 \times (T_2 - T_1) + W \\
&= 0.82 \times (1159.8 - 2673) + 1035.35 = -205.47 \text{ kJ/kg}
\end{aligned}
$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

Review Questions

7.1 What does the first law of thermodynamics demonstrate?

7.2 What does the second law of thermodynamics exhibit?

7.3 Who gave birth to the concept of available energy and maximum work?

7.4 What are the three propositions which laid the foundation for the concept of availability?

7.5 What is exergy and who was responsible for introducing the term in thermodynamics?

7.6 What are high-grade and low-grade energies, explain?

7.7 Define available energy.

7.8 Explain the concept of available energy by suitable examples.

7.9 What do you understand by the law of degradation of energy?

7.10 Show that if two bodies are allowed to reach thermal equilibrium by heat conduction, the final temperature is given by:

$$\frac{c_1T_1^2 + c_2T_2^2}{c_1 + c_2}$$

where T_1 *and* T_2 *are the temperatures of bodies 1 and 2, respectively.* c_1 *and* c_2 *are specific heats.*

7.11 If two bodies are allowed to reach thermal equilibrium operating on Carnot engine then the final temperature $T_f = (T_1T_2)^{1/2}$*, where* T_1 *and* T_2 *are the temperatures of source and sink, respectively.*

7.12 What is effectiveness?

7.13 What are first law and second law efficiencies?

7.14 Analyse the concept of availability for a closed system for the following process: (a) constant volume process; (b) constant pressure process; and (c) constant temperature process.

7.15 Carry out availability analysis of the following open systems: (a) turbine; (b) compressor/pump; and (c) heat exchanger.

7.16 Draw the Carnot cycle on the $T - S$ *diagram and explain.*

7.17 What is the deficiency of the first law efficiency? How does the second law efficiency?

7.18 How does exergy value provide a useful measure of the quality of energy?

7.19 What are the availability functions for a closed system and steady flow system?

7.20 What do you understand by useful work?

7.21 Derive the expression for useful work per unit mass for a closed system.

7.22 Energy is always conserved but its quality is always degraded. Explain.

7.23 What do you understand by dead state?

7.24 Give the exergy balance for a steady flow system.

7.25 What is entropy generation number?

Exercise

7.1 In a counter flow heat exchanger, oil with $c_p = 2.1$ kJ/kg K is cooled from 440 to 320 K while water with $c_p = 4.2$ kJ/kg K is heated from 290 to temperature T. Neglect all losses. Take mass flow rate of oil and water as 800 and 3200 kg/h, respectively. Calculate

(i) the temperature, T,

(ii) the rate of exergy destruction, and

(iii) the second law efficiency.

Take $T_0 = 17$ °C and $p_0 = 1$ atm. *Ans:* 10.9%

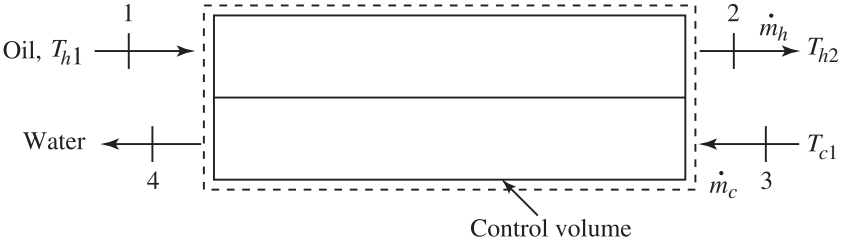


Fig. 7.21

7.2 Oxygen enters a nozzle which is operating at steady state at 4.0 MPa and 387 °C and 10 m/s. At the nozzle exit, the conditions are 160 kPa, 37 °C and 750 m/s. Determine

(i) the heat transfer per kg and

(ii) irreversibility.

Assume oxygen as an ideal gas, Take $T_0 = 20$ °C, $p_0 = 1$ atm. *Ans:* 82 kJ/kg

7.3 Argon gas has an adiabatic expansion in a turbine from 2 MPa, 1000 °C and 300 kPa. The mass flow rate is 0.5 kg/s and the turbine develops power at the rate of 100 kW. Determine

(i) the temperature of argon at the turbine exit,

(ii) the irreversibility rate,

(iii) the second law efficiency.

Neglect KE and PE effects and assume $T_0 = 20$ °C, $p_0 = 1$ atm and $\gamma = 1.67$ for argon. *Ans:* 76.7%

7.4 In the boiler of a power plant, water flows through the tubes and it is brought from 0.8 MPa, 150 °C ($h = 632.6$ kJ/kg and $s = 1.8418$ kJ/kg K) to 0.8 MPa, 250 °C ($h = 2950$ kJ/kg and $s = 7.0384$ kJ/kg K) combustion gases passing over the tubes cool from 1067 °C to 547 °C. These gases may considered for all practical purposes, as air (ideal gas) having $c_p = 1.005$ kJ/kg K. Assuming steady state and neglecting any heat loss and KE and PE effects, determine (i) mass flow rate of combustion gases per kg of steam, (ii) the loss of exergy per kg of steam and (iii) the second law efficiency. Take $T_0 = 25$ °C; $p_0 = 1$ atm. *Ans:* (i) 4.434; (ii) 896.2 kJ/kg of steam (iii) 46.2%

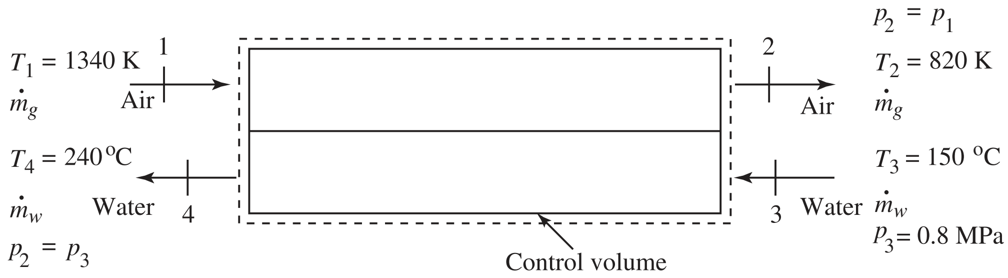


Fig. 7.22

7.5 Air enters a hair dryer at 22 °C and 1 bar with a velocity of 4 m/s and exit at 83 °C and 1 bar with a velocity of 9 m/s through an area of 18.9 cm^2. Neglecting heat losses and PE effect. Taking $T_0 = 22$ °C,

(i) evaluate the power required in kW.

(ii) device and evaluate the second law efficiency. *Ans:* 9.1%

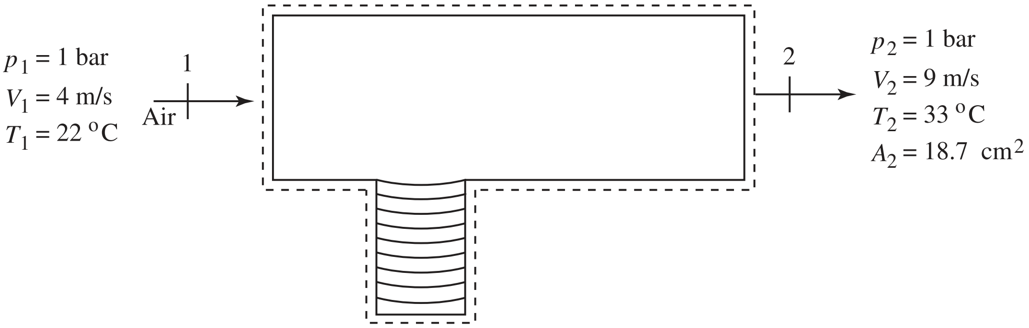


Fig. 7.23

7.6 An isolated system consists of two solid blocks. One block has a mass of 5 kg and initially at 302 °C. The other block has a mass of 10 kg and initially at −53 °C. The blocks are allowed to come to thermal equilibrium. Assume the blocks are incompressible with constant specific heats of 1 and 0.4 kJ/kg K, respectively. Determine (i) the final temperature and (ii) the irreversibility. *Ans:* 417.22 K; 286.8 kJ

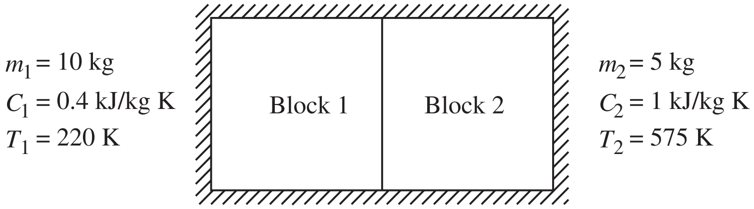


Fig. 7.24

7.7 Consider a vapour, while condenses at 427 °C transfers heat to water evaporates at 247 °C. The resulting steam is used in a power cycle which rejects heat at 37 °C. Calculate the fraction of the available energy in the heat transferred from the process vapour at 427 °C that is lost due to the irreversible heat transfer at 247 °C. *Ans:* 0.275

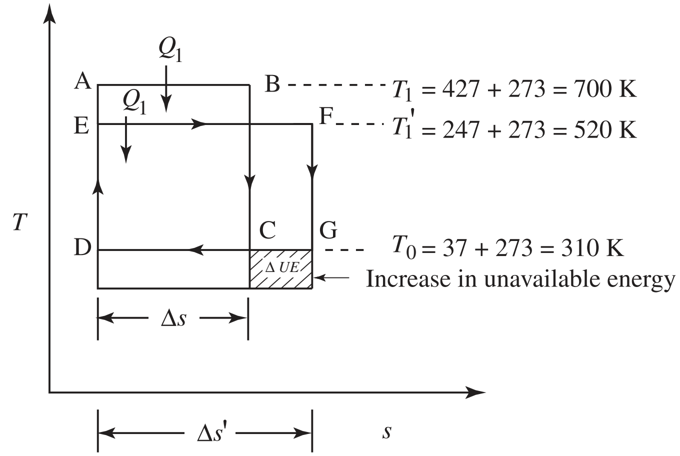


Fig. 7.25

7.8 In a steam boiler, heat is transferred to water which vaporises at constant temperature. In one experiment, the gases are cooled from 1000 K to 500 K while the water evaporates at 200 °C. Specific heat of gases is 1.005 kJ/kg K and the latent heat of vaporisation may be taken at 1860 kJ/kg at 200 °C. Assume that entire heat from gases to the water. Calculate the increase in total entropy of the combined system of gas and water because of the irreversible heat transfer in entropy as the result on the basis of 1 kg of water evaporated. If the temperature of the surroundings is 27 °C, find the increase in unavailable energy due to irreversible heat transfer. *Ans:* 513 kJ

7.9 Calculate the available energy of 50 kg of water at 75 °C with respect to the surroundings at 5 °C, the pressure of water is atmospheric. *Ans:* 13020 kJ

7.10 Calculate the decrease in available energy when 30 kg of water at 95 °C mix with 40 kg water at 35 °C the pressure assumed to be constant and the temperature of the surroundings be 15 °C (Take c_p of water=4.2 kJ/kg K). *Ans:* 330.98 kJ

7.11 The moment of inertia of a fly wheel is 0.54 kg m^2 and rotates at a speed of 3000 rpm in a large heat insulated system, the temperature being 15 °C. If the kinetic energy of the fly wheel is dissipated as frictional heat at the shaft bearing which have a water equivalent of 3 kg, find the rise in the temperature of the bearings when the fly wheel has come to rest. Calculate the greatest possible amount of this heat which may be returned to the fly wheel as high-grade energy showing how much of the original kinetic energy is now unavailable. What would be the final rpm of the fly wheel, if it is set in motion with their available energy. *Ans:* 181 rpm

7.12 Three kilogram of air 500 kPa and 80 °C expands adiabatically in a closed system until its volume is doubled and its temperature becomes equal to that of the surroundings which its volume becomes equal to that of the surroundings which is at 100 kPa, 5 °C. For this process, find (i) the maximum work; (ii) the change in availability; and (iii) the irreversibility.

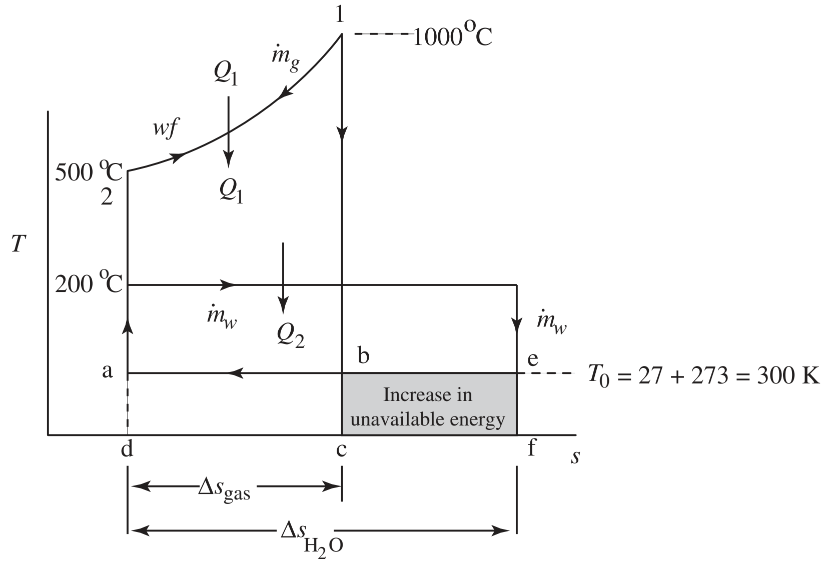


Fig. 7.26

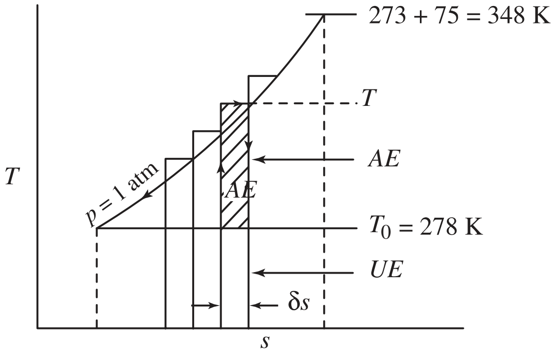


Fig. 7.27

For air, take $c_c = 0.718$ kJ/kg K as constant volume and $pv = mRT$ where p is pressure in kPa, R a constant equal to 0.287 kJ/kg K and T is temperature in K.
Ans: (i) 184.44 kJ; (ii) 123.65 kJ; (iii) 22.8 kJ

7.13 Air expands through a turbine from 500 kPa, 520 °C to 100 kPa and 300 °C. During expansion, 10 kJ/kg of heat is lost to the surroundings which is at 98 kPa, 20 °C. Neglecting KE and PE changes determine per kg of air:

(i) decrease in availability,
(ii) the maximum work, and
(iii) the irreversibility.

For air, take $c_p = 1.005$ and $h = c_pT$ where c_p is constant and p, V, T relation is given by $pV = mRT$. *Ans:* (i) 260.7 kJ/kg; (ii) 260.7 kJ/kg; (iii) 49.6 kJ/kg

7.14 An air pre-heater is used to cool the product of combustion from a furnace while heating the air to be used for combustion. The rate of flow of products is 12 kg/s and the products are cooled from 300 °C to 200 °C and the c_p of the products in this temperature range $= 1.09$ kJ/kg K. The rate of air flow is 11 kg/s. The initial air temperature is 40 °C and for the air $c_p = 1.005$ kJ/kg K. Estimate

(i) initial and final availability of the products.
(ii) irreversibility of the process.
(iii) if the heat transfer from the products occurs reversibly through heat engines, what will be the final temperature of air?

What will be the power developed by the heat engine?

Ans: (i) 85.97 kJ/kg; 39.68 kJ/kg; (ii) 311 kW; (iii) 392.815 K (iv) 425.65 kW

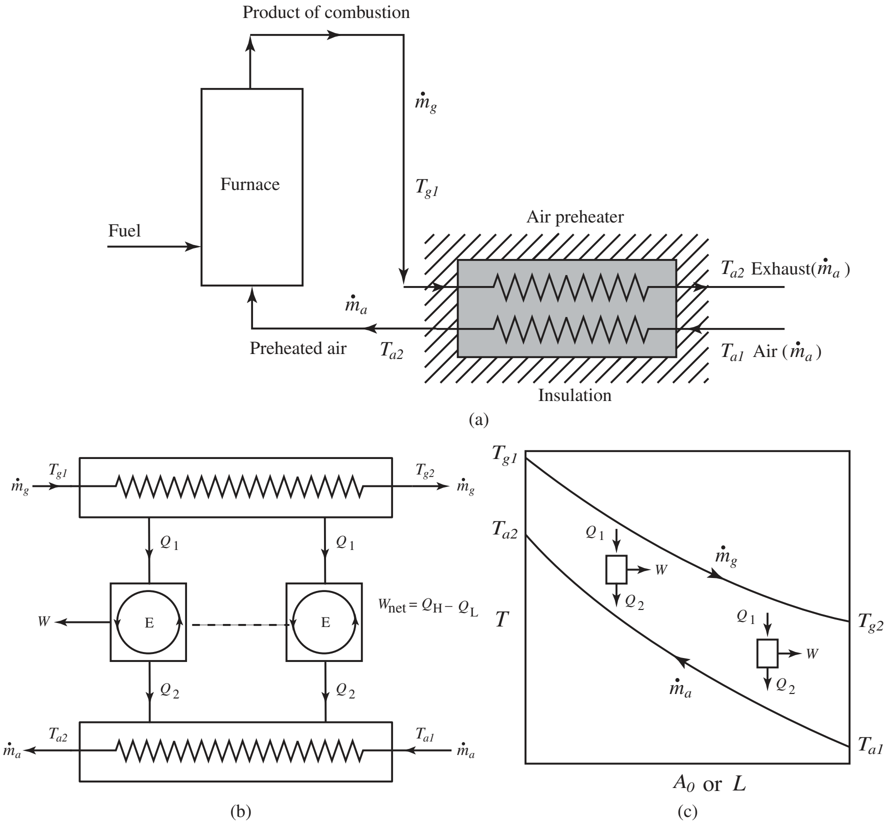


Fig. 7.28

7.15 A gas flowing through a pipe at the rate of 1 kg/s. Because of inadequate insulation, the gas temperature decreases from 800 °C to 790 °C between two sections in the pipe. Neglecting pressure losses, calculate the irreversibility rate (or rate of energy degradation) due to this heat loss. Take $T_0 = 300$ K and $c_p = 1.1$ kJ/kg K. For the same temperature, drop of 10 °C when the gas cools from 80 °C to 70 °C due to heat loss what is the rate of energy degradation? Take the same value of T_a and c_p. What the inference you can draw from this example? *Ans:* 1.52 kW

7.16 An ideal gas is flowing through an insulator pipe at the rate of 5 kg/s. There is 12% pressure drop from inlet to exit of the pipe. What is the rate of exergy loss because of the pressure drop due to friction? Take $R = 0.287$ kJ/kg K ans $T_0 = 300$ K. *Ans:* 51.56 kW

7.17 Water at 90 °C flowing at the rate of 3 kg/s mixes adiabatically with another steam of water at 30 °C flowing at the rate of 2 kg/s. Estimate the entropy generation rate and the rate of exergy loss due to mixing. Take $T_0 = 300$ K. *Ans:* 24 kW

7.18 By burning a fuel, the rate of heat release is found to be 500 kW at 2000 K. Calculate the first and second law efficiencies, if

(i) energy is absorbed in a metallurgical furnace at the rate of 480 kW at 1000 K,

(ii) energy is absorbed at the rate of 450 kW for generation of steam at 500 K,

(iii) energy is absorbed by a chemical process at the rate of 300 kW at 320 K. Take $T_0 = 300$ K, and

(iv) had the energy absorption rate been equal to 450 kW in all three cases what would have been the second law efficiencies. What is the inference you can derive from this example? *Ans:* (i) 79%; (ii) 42.3%; (iii) 4.41%; (iv) 6.61%

It is seen that as the energy loss $\left(\dot{Q}_r - \dot{Q}_a\right)$ increases, the first law efficiency decreases. For the same heat loss, however, as the temperature difference between the source and the use temperature increase, the second law efficiencies decreases or In other words, the energy loss increases.

7.19 A system, as shown in Fig.7.29 undergoes a power cycle while receiving energy Q_1 at temperature T_1 and discharging energy Q_2 at temperature T_2. There are no other heat transfers. Show that the thermal efficiency of the cycle can be expressed as:

(i) $\eta = 1 - \dfrac{T_2}{T_1} = \dfrac{T_2 I}{T_0 Q_1}$, where T_0 is the ambient temperature and I is the irreversibility of the cycle.

(ii) Obtain an expression for the maximum theoretical value for the thermal efficiency. and

(iii) Derive an expression for the irreversibility for which no net work is developed by the cycle. What conclusion do you infer from it?

Ans: (ii) $\eta_{max} = 1 - \frac{T_2}{T_1}$; (iii) $T_0 S_{gen}$

7.20 A compressor operating at steady state takes in 1 kg/s of air at 1 bar and 27 °C and compresses to 10 bar and 167 °C. Heat transfer from the compressor to its surroundings occurs at the rate of 100 kW.

(i) Find the power input in kW.

(ii) Determine the second law efficiency of the compressor.

Ans: (i) −240.7 kW; (ii) 92.9%

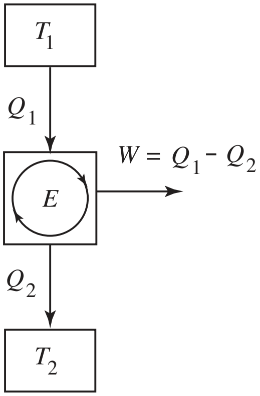


Fig. 7.29

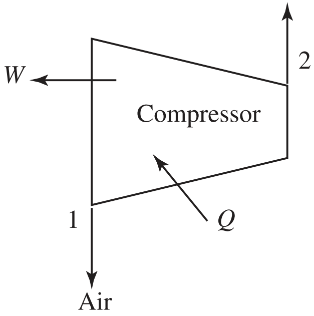


Fig. 7.30

7.21 Calculate the exergy of 1 m^3 of complete vacuum. *Ans:* 100 kJ

7.22 A mass of 1 tonne of fish initially at 1 bar, 300 K is to be cooled to −23 °C. The freezing point of fish is −3 °C and the specific heats of fish below and above the freezing point are 1.7 and 3.2 kJ/kg K, respectively. The latent heat of fusion for the fish can be taken as 235 kJ/kg. Calculate the exergy produced in the chilling process. Take $T_0 = 300$ K and $p_0 = 1$ bar. *Ans:* 9.86 kW h

7.23 A quantity of air initially at 1 bar, 300 K undergoes two types of interactions:

(i) It is brought to a final temperature of 500 K adiabatically by paddle-wheel work transfer.

(ii) The same temperature rise is brought about by heat transfer from a thermal reservoir at 500 K. Take $T_0 = 300$ K, $p_0 = 1$ atm. Determine the irreversibility in (kJ/kg) in each of the cases and comment on the results.

Ans: (i) 110.1 kJ/kg; (ii) 3831 kJ/kg
The irreversibility in case (ii) is always less than in case (i) and the two values will approach each other only at high reservoir temperature, i.e.
$I_a \to I_b$ as $T \to \infty$.

7.24 Steam enters at 30 bar 400 °C (h = 3230 kJ/kg s=6.9212 kJ/kg and with a velocity of 160 m/s. Steam leaves as saturated vapour at 100 °C (h = 2676.1 kJ/kg, s =

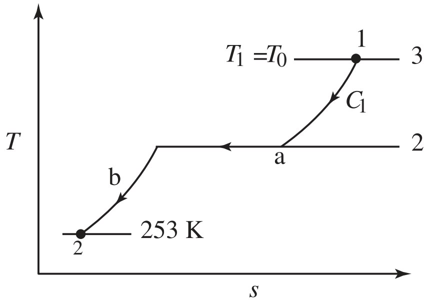
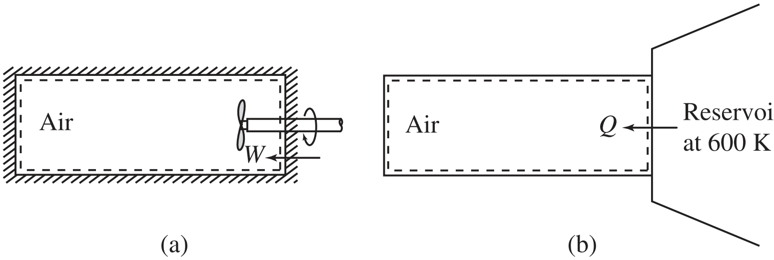


Fig. 7.31



(a) (b)

Fig. 7.32

7.3549 kJ/kg K) with a velocity of 100 m/s. At steady state, the turbine develops work at the rate of 540 kJ/kg. Heat transfer between the turbine and its surroundings occurs at an average outer surface temperature of 500 K. Calculate the irreversibility per unit mass. Give an energy balance and estimate the second law efficiency of the turbine. Take $p_0 = 1$ atm; $T_0 = 298$ K and neglect PE effect. *Ans:* 142.71 kJ/kg; 78%

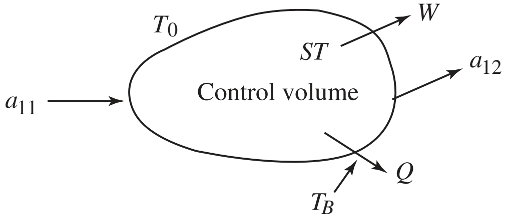


Fig. 7.33

7.25 A furnace is heated by an electric resistor. At steady state, electric power is supplied to the resistor at the rate of 8 kW per meter length to maintain it at 1500 K. When the furnace walls are 500 K. Assume $T_0 = 300$ K.

(i) For the resistor as the system, determine the rate of availability transfer accompanying heat and the irreversibility rate.

(ii) For the space between the resistor and the walls as the system, evaluate irreversibility rate.

Ans: (i) 1.6 kW; 6.4 kW; (ii) 3.2 kW

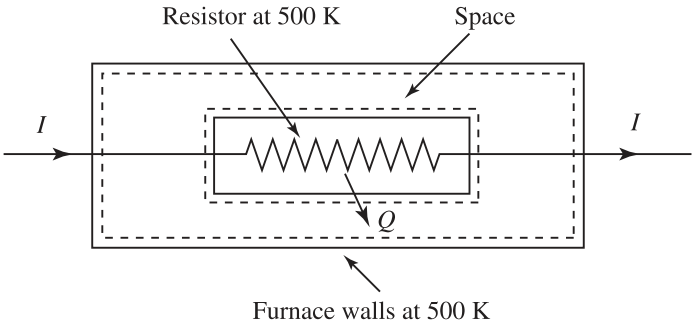


Fig. 7.34

Multiple Choice Questions (choose the most appropriate answer)

1. Which of the following is not a high-grade energy?
   (a) electrical energy
   (b) thermal energy
   (c) kinetic energy
   (d) wind energy

2. Which of the following is not a low-grade energy?
   (a) thermal energy
   (b) heat derived from nuclear fusion
   (c) heat derived from combustion of fossil fuels
   (d) mechanical work

3. The exergy of an isolated system in a process
   (a) can never increase
   (b) can never decrease
   (c) always remains constant
   (d) is always positive

4. At chemical equilibrium Gibbs function is
   (a) maximum
   (b) minimum
   (c) zero
   (d) constant

5. Availability function is expressed as
   (a) $a = (u + p_0 v - T_0 s)$
   (b) $a = (u + p_0 v - T_0 ds)$
   (c) $a = (du + p_0 dv - T_0 ds)$
   (d) $a = (u + p_0 dv - T_0 ds)$

6. Gibbs function is expressed as
   (a) $(u + pv - Ts)$
   (b) $(u + pv + Ts)$
   (c) $(u + pdv - Tds)$
   (d) $(u + pv - Tds)$

7. To increase the work capacity of energy transfer through heat transfer from high-temperature to low-temperature
   (a) higher temperature should be increased keeping temperature difference same
   (b) temperature difference should be increase
   (c) lower temperature should be lowered keeping temperature difference same
   (d) temperature difference should be decreased

8. Helmholtz function is expressed as

   (a) $u - Ts$ (c) $-sdT + vdp$
   (b) $h - Ts$ (d) $u + pv$

9. If a heat source at temperature T_1 transfers heat to a system at temperature T_2 $(T_1 > T_2)$, choose which of the following statement is not true

   (a) Δ_{system} decreases (c) $(\Delta_{source} + \Delta_{system})$ decreases
   (b) Δ_{source} decreases (d) $(\Delta_{source} + \Delta_{system})$ increases

10. Which of the following relations is/are correct?

    (a) $Tds = du + pdv$ (c) $Tds = c_p dT - vdp$
    (b) $dh = Tds + vdp$ (d) All of these

11. On T-s diagram, the slope of constant pressure line (mp) and the slope of constant volume line (mv) can be related as,

    (a) $mp = mv$ (c) $mp > mv$
    (b) $mv > mp$ (d) $mp - mv = 1$

12. Exergy cannot be less than zero as the maximum work interaction cannot be

    (a) less than zero (c) zero
    (b) greater than zero (d) none of these

13. Law of degradation of energy says that the unavailable energy is gradually increasing due to

    (a) increase in reversible processes (c) increase in unavailable processes
    (b) increase in irreversible processes (d) none of these

14. Which of the following is/are correct? I = Irreversibility, A = Availability, $W_{c,max}$ = maximum work, T_0 = dead state temperature, S_c = combined system entropy change.

    (a) $I = T_0 \Delta Sc$ (c) $A = W_{c,max}$
    (b) $W_c = A - I$ (d) All of these

15. The efficiency of reversible and irreversible heat engine cycles can be related as

    (a) $\eta_{rev} = \eta_{irrev}$ (c) $\eta_{rev} > \eta_{irrev}$
    (b) $\eta_{rev} < \eta_{irrev}$ (d) none of these

16. Which of the following is/are correct?
    (a) Entropy is used as a convenient coordinate in thermodynamics.
    (b) Entropy is used to device whether a process is reversible, irreversible or impossible.
    (c) Entropy is used to determine the heat transfer in reversible process.
    (d) All of these.

17. Equation $\Delta G = \Delta H - T\Delta S$ tells us that an exothermic reaction will be associated with which of the following:

    (i) $-$ve ΔH (iv) spontaneous reaction
    (ii) $+$ve ΔH (v) more ordered $-$ve Δs
    (iii) more disordered, $+$ve Δs

Possible answers:

(a) (i) and (v)
(b) (ii) and (v)
(c) (ii) and (iii)
(d) (iii) and (iv)

18. Availability of a closed system at state 1 and dead states referred with subscript 0 is given by
    (a) $(E_1 - E_0) + p_0(V_1 - V_0) - T_0(S_1 - S_0)$
    (b) $(E_1 - E_0) + p_0(V_1 - V_2) - T_0(S_1 - S_2)$
    (c) $(h_1 - h_0) + g(Z_1 - Z_0) - T_0(S_1 - S_0) + \frac{V_1^2}{2}$
    (d) none of these

19. The availability of steady flowing fluid, also called stream availability is given by (the current state is 1 and dead state is given by 0).
    (a) $(E_1 - E_0) + p_0(V_1 - V_0) - T_0(S_1 - S_0)$
    (b) $(h_1 - h_2) + g(Z_1 - Z_2) - T_0(S_1 - S_2) + \frac{(V_1^2 - V_2^2)}{2}$
    (c) $(h_1 - h_0) + g(Z_1 - Z_0) - T_0(S_1 - S_0) + \frac{V_1^2}{2}$
    (d) none of these

20. Gibbs function is expressed as
    (a) $(u + pv - Ts)$
    (b) $(u + pv - Tds)$
    (c) $(u + pdv - Tds)$
    (d) $(u + pv - sdT)$

21. Availability function is expressed as
    (a) $a = (u + p_0 v - T_0 s)$
    (b) $a = (u + p_0 v - T_0 ds)$
    (c) $a = (du + p_0 dv - T_0 ds)$
    (d) $a = (u + p_0 v - T_0 s)$

22. To increase work capacity of energy transferred by heat transfer from high temperature to low temperature
    (a) Lower temperature should be lowered keeping temperature difference same
    (b) Higher temperature should be increased keeping temperature difference same
    (c) Temperature difference should be increased
    (d) Temperature difference should be decreased

23. Helmholtz function is expressed as
    (a) $(u - Ts)$
    (b) $(h - Ts)$
    (c) $(-sdT - vdp)$
    (d) $(u + pv)$

24. If a heat source at temperature T_1 transfers heat to a system at temperature $T_2 (T_1 > T_2)$, state which of the following statements is not true?
    (a) Δ_{system} decreases
    (b) $(\Delta_{system} + \Delta_{source})$ decreases
    (c) Δ_{source} decreases
    (d) $(\Delta_{system} + \Delta_{source})$ increases

25. Consider the following two processes

    (i) A heat source at 1200 K loses 2500 kJ of heat to a sink at 800 K

(ii) A heat source at 800 K loses 2000 kJ of heat to a sink at 500 K

Which of the following statement is true?

(a) Process I is more irreversible than process II

(b) Process II is more irreversible than process I

(c) Irreversibility associated in both the processes are equal

(d) Both the processes are reversible

Ans: 1. – (b) 2. – (d) 3. – (??) 4. – (c) 5. – (a)
6. – (d) 7. – (c) 8. – (a) 9. – (d) 10. – (d)
11. – (b) 12. – (a) 13. – (b) 14. – (d) 15. – (c)
16. – (d) 17. – (d) 18. – (a) 19. – (c) 20. – (a)
21. – (a) 22. – (d) 23. – (a) 24. – (d) 25. – (b)

8

PROPERTIES OF PURE SUBSTANCES

8.1 INTRODUCTION

In engineering, we quite often use the word 'substance'. It refers to either an element or a compound, having a definite composition always. Now, the question is what is meant by *pure substance? A pure substance is one which has a fixed chemical composition throughout.* Typical examples are: carbon dioxide, helium, nitrogen and water.

Note that a pure substance need not have to be of a single element or compound. However, a mixture of various chemical elements or compounds can also be a pure substance as long as the mixture is homogeneous. Further, a mixture of two or more phases of a pure substance is still a pure substance. Only condition is that the chemical composition of all phases must be the same. For example, a mixture of ice and liquid water is a pure substance. It is because both phases have the same chemical composition.

Now, the question that might come to our mind is 'what about air'? Note that air is a mixture of several gases. It is *most often considered as a pure substance* because it has a uniform chemical composition. A mixture of liquid air and gaseous air is not a pure substance since the mixture is no longer chemically homogeneous. The composition of liquid air is different from the composition of gaseous air. Similarly, a mixture of oil and water cannot be called a pure substance because oil cannot mix with water.

Many engineering systems require some substance to act as working fluid for energy and mass interactions. A number of working fluids are available in practice. Typical example is steam which has favourable properties for engineering as well as domestic applications. It is therefore, essential for an engineer to know the details of formation and properties of steam. As steam is produced from water and water is a pure substance, steam is also a pure substance. The knowledge we gain by studying properties of water can be generalised and the methodology can be extended to other pure substances as well. Water, as a pure substance, is capable of retaining its chemical composition in all its three phases, viz ice (solid), water (liquid) and also steam (vapour). As of now, it is almost freely available! It is the gift of nature.

8.2 PHASES OF PURE SUBSTANCES

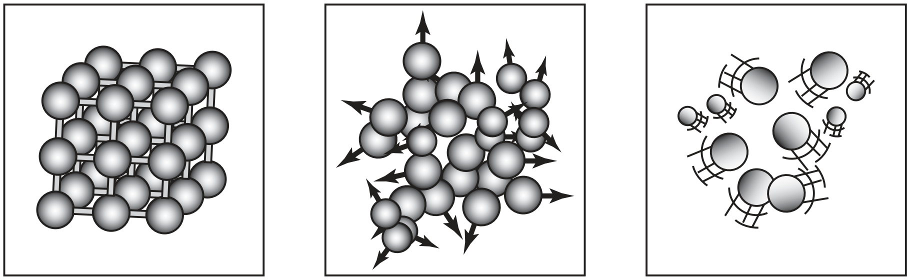
First of all, let us see what is meant by phase? *A phase is defined as having a distinct molecular arrangement that is homogenous throughout and separated from others (if any) by easily identifiable boundary surfaces.* A substance may have several phases within a principal phase. Each phase can have different molecular structures. For example, carbon may exist as graphite or diamond in the solid phase. Ice can exist in seven different phases at high pressures. Molecular bonds are the strongest in solids and the weakest in gases.

We are aware that substances exist in distinctly different physical phases depending upon the pressure and temperature. For example, at room temperature and pressure, copper is a solid, mercury is a liquid and nitrogen is a gas. This is true of pure substances also.

As can be seen in Fig.8.1(a), the molecules in a *solid* are arranged in a three-dimensional pattern (lattice) that is repeated throughout. In other words, the molecules are at relatively in a fixed position.

The molecular spacing in the *liquid* phase is not much different from that of the solid phase. In liquids, molecules are no longer at fixed positions relative to each other [Fig.8.1(b)]. They can rotate and translate freely. In a liquid, the intermolecular forces are weaker compared to solids but still relatively strong compared to gases. Groups of molecules move about each other in the liquid phase.

In the *gas* phase [Fig.8.1(c)], the molecules are far apart from each other and there is no order in molecular spacing. Gas molecules move randomly. They collide continually with each other and the walls of the container in which they are contained. Particularly at low densities, the intermolecular forces are very small. Molecules in the gas phase have considerably higher energy level than they are in the liquid or solid phases. Therefore, the gas must release a large amount of its energy before it can condense or freeze.



(a) Molecules are relatively fixed positions in a solid

(b) Grouups of molecules move about each other in the liquid phase

(c) Molecules move about at random in the gas phase

Fig. 8.1 The arrangement of atoms in different phases

8.2.1 Phase Rule

The number of independent variables associated with a multiphase multicomponent, system is given by the phase rule. This rule is usually called as Gibbs phase rule. It is expressed by the equation as

$$n = C - \phi + 2 \tag{8.1}$$

where n is the number of independent variables; C, the number of components; ϕ, the number of phases present in equilibrium.

For the single-component ($C = 1$) two-phase system ($\phi = 2$), the number of independent variable needed is

$$n = 1 - 2 + 2 = 1 \tag{8.2}$$

For example, for water, $C = 1$. At the triple point, it has three phases ($\phi = 3$)and therefore, $n = 0$. That is, none of the properties of a pure substance at the triple point can be varied. From this rule, we can know that a pure substance which exists in a single phase ($\phi = 1$) will have two independent variables ($n = 2$). It means there are two independent intensive properties required to be specified to fix up the state of the system at equilibrium.

8.3 PHASE-CHANGE PROCESS

When we add heat or extract heat from a pure substance phase-change takes place. Consider a process where a pure substance, say ice, starts as a solid. When it is heated up at constant pressure, it will become liquid (water) and further heating will make it to become gas (steam). As stated earlier, steam is extensively used in many engineering applications and therefore, in this chapter, we will study the details of phase-change of the pure substance, viz water when heat is added.

To put it in a nutshell, heat addition or heat extraction causes phase-change in a pure substance. There are many practical situations where two phases of a pure substance coexist in equilibrium. Typical example are:

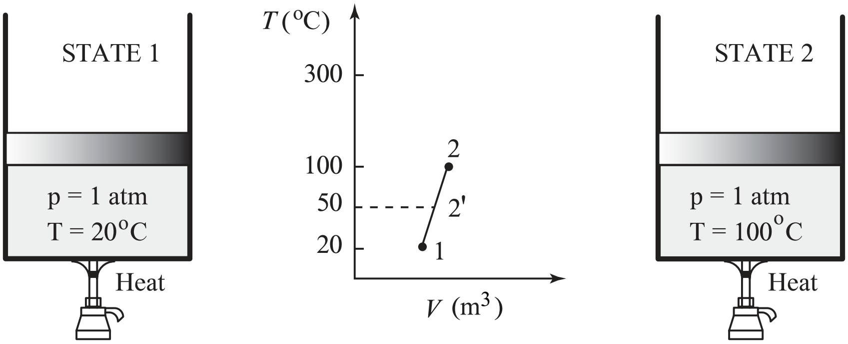
(i) Water exists as a mixture of liquid and vapour in the boiler and the condenser of a steam power plant.

(ii) Refrigerant changes its phase from liquid to vapour in the freezer of a refrigerator.

Remember that all pure substances do exhibit the same general behaviour. As a familiar substance, we will consider water. First we will go through the details of phase-change for water at constant pressure when heat is added and then at various pressures.

8.4 PHASE CHANGE PROCESS OF LIQUID WATER AT CONSTANT PRESSURE

Consider a piston-cylinder arrangement shown in Fig.8.2(a). Assume that liquid water is at 20 °C and 1 atm pressure. Consider this as state 1, water exists in liquid phase under these conditions. This state is called *compressed liquid* or *subcooled liquid* state.



(a) At 1 atm and 20° C, water exists in the liquid phase (compressed liquid)

(b) T-V diagram for the heating process of water at constant pressure

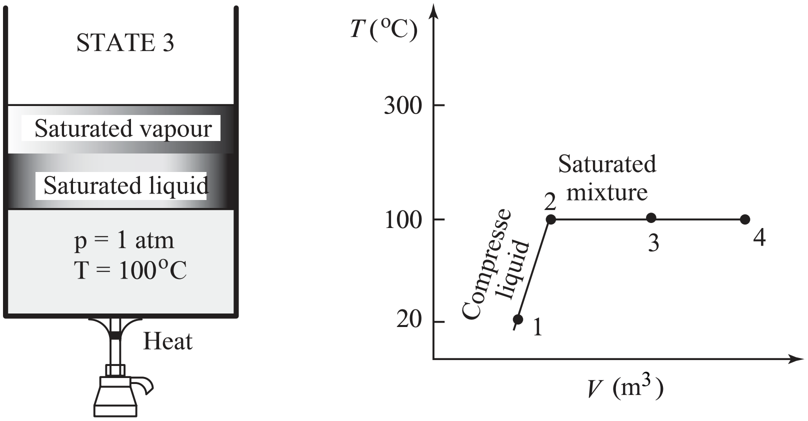
(c) At 1 atm pressure and 100 °C, water exists as a liquid that is ready to vaporize (saturated liquid)

Fig. 8.2 Phase-change process of liquid water at constant pressure and 50 °C

Compressed liquid implies that the pressure is greater than the saturation pressure at that temperature. *Subcooled liquid* implies that the temperature is lower than the saturation temperature at that pressure. In simple terms, it means that water will remain in liquid state and not going to vaporise.

Now, let us transfer heat to the water by means of a burner shown in Fig.8.2(a), until its temperature rises to, say, 50°C. Because of the temperature rise, the liquid water expands slightly. So its volume also increases. To accommodate this increase in volume, V, the piston will move up slightly. However, the pressure in the cylinder remains constant at 1 atm during this process. It is because pressure depends only on the outside barometric pressure and the weight of the piston. Note that both of them are held constant. Water is still a compressed liquid at this state, since it has not started to vaporise. This heating process is depicted graphically as process 1-2′ in Fig.8.2(b). On further heating, the temperature keeps rising until it reaches 100 °C [state 2, Fig.8.2(b)]. Even at this point, water is still a liquid. At this condition, if we add even a small amount of heat, some of the liquid will just start vaporising. That is to say, a phase-change process from liquid to vapour starts taking place. *A liquid that is about to vaporise is called a saturated liquid.* Therefore, state 2 is a *saturated liquid state.* This process 1-2 is shown in Fig.8.2(c).

Once the saturated liquid water starts boiling, there will not be any temperature rise. This will go on until the entire liquid completely vaporises. If we keep the pressure constant, the temperature will remain constant during this phase-change process. This can be verified easily by placing a thermometer into boiling pure water. At sea level ($p = 1$ atm), the thermometer will always read 100 °C. During the boiling process, the only change we will observe is a large increase in the volume and a steady decrease in the liquid level. This is due to more liquid turning to vapour. Half way about the vaporisation line (state 3, Fig.8.3), the cylinder will contain equal amounts of liquid and vapour. If we continue to heat, the vaporisation process will continue until the last drop of liquid is vaporised (state 4, Fig.8.4). At this point, the entire cylinder is filled with vapour. This is called the borderline of liquid phase. Any heat loss from this vapour will cause some of the vapour to condense (phase-change from vapour to liquid). A vapour that is *about to condense* is called a *saturated vapour.* Therefore, state 4 is a saturated vapour state.

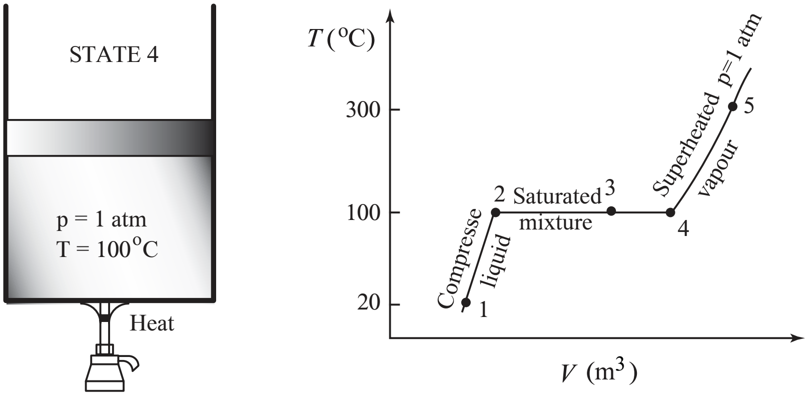


(a) As more heat is transferred, part of the saturated liquid vaporizes (saturated liquid–vapor mixture)

(b) T-V diagram for the heating process of water at constant pressure

Fig. 8.3 Phase-change process of liquid water at constant pressure and temperature $\leq 100°C$

A substance at states between 2 and 4 is referred to as a *saturated liquid-vapour mixture* since the *liquid and vapour* phases coexist in equilibrium at these states. Once the phase-



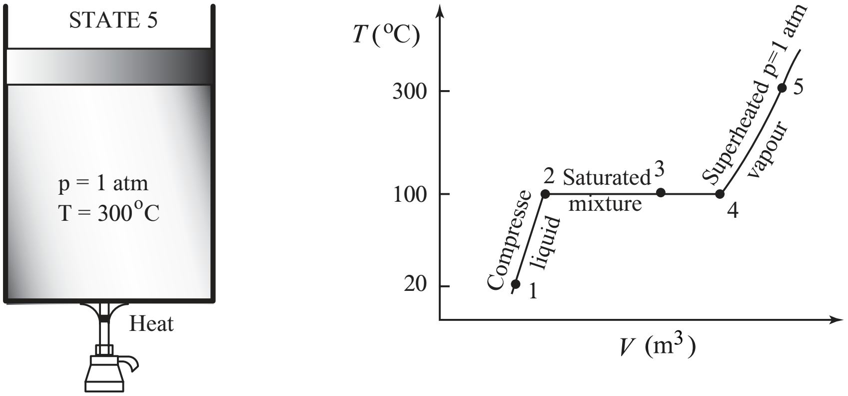
(a) At 1 atm pressure, the temperature remains constant at 100^{0} C until the last drop of liquid is vaporized (saturated vapor)

(b) T-V diagram for the heating process of water at constant pressure

Fig. 8.4 Phase change process of liquid water at constant pressure and temperature $\geq 100^\circ C$

change process is completed, we are back to a single-phase region again. Note that now we are in the vapour phase.

Any further transfer of heat increases both the temperature and the specific volume (Fig.8.5). At state 5, the temperature of the vapour is, let us say, 300 °C; and if we transfer some heat from the vapour, the temperature may drop somewhat without any condensation. This will hold good as long as the temperature remains above 100°C (for $p = 1$ atm).



(a) As more heat is transferred, the temperature of the vapor starts to rise (superheated vapor)

(b) T-V diagram for the heating process of water at constant pressure

Fig. 8.5 A complete phase-change process

A vapour that is *not about to condense (i.e. not a saturated vapour)* is called a *superheated vapour*. Therefore, water at state 5 is a superheated vapour. This constant-pressure phase-change process is clearly shown on a T-V diagram in Fig.8.5(b). The entire process will reverse if we start cooling the superheated steam at state 5 while maintaining the pressure at the same value. The water will go back to state 1, retracing the same path. In doing so, the amount of heat released during the cooling process will exactly be the same as during the heating process. In our daily life, water implies liquid water and steam implies water vapour. However, in thermodynamics, both water and steam usually mean only one thing: H_2O.

It is only our perception that water boils at 100°C. *Strictly speaking, the perception that water boils at 100 °C is not correct.* The correct way to put it is *water boils at 100 °C at 1 atm. pressure.* The only reason water started boiling at 100 °C is due to the pressure being held constant at 1 atm (101.325 kPa). If we raise the pressure to 500 kPa by adding weights on top of the piston [Fig.8.2(c)], water would start boiling at 151.8 °C. Therefore, it should be clear that *the temperature at which water starts boiling depends on the pressure. If we fix the pressure, indirectly we are fixing the boiling temperature.*

At a given pressure, the temperature at which a pure substance changes phase is called the *saturation temperature,* T_{sat}. Likewise, at a given temperature, the pressure at which a pure substance changes phase is called the *saturation pressure*, p_{sat}. Saturation tables list the saturation pressure against the temperature or the saturation temperature against the pressure, such tables are available for almost all substances.

Note that it takes a large amount of energy to melt a solid or vaporise a liquid. The amount of energy absorbed or released during a phase-change process is called the *latent heat.* More specifically, the amount of energy absorbed during melting is called the *latent heat of fusion* and is equivalent to the amount of energy released during freezing. Similarly, the amount of energy absorbed during vaporisation is called the *latent heat of vaporisation* and is equivalent to the energy released during condensation. The magnitudes of the latent heats depend on the temperature or pressure at which the phase-change occurs. At 1 atmospheric pressure, the latent heat of fusion of water is 333.7 kJ/kg and the latent heat of vaporisation is 2256.5 kJ/kg.

By this time, you must have understood that during a phase-change process, pressure and temperature are obviously dependent properties. There is a definite relation between them A typical plot of T_{sat} versus p_{sat}, for water is shown in Fig.8.6. It is called a *liquid-vapour saturation curve.* The curve is similar for all pure substances. Note that a substance at higher pressure boils at higher temperature.

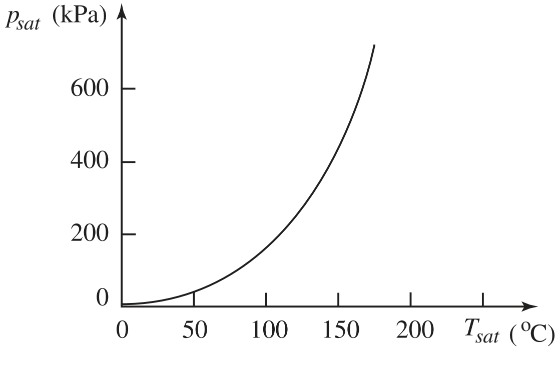


Fig. 8.6 Liquid-vapour saturation curve of a pure substance (numerical values are for water)

To give day-to-day example, consider cooking in a pressure cooker. Higher boiling temperatures mean faster cooking and energy savings. For example, it may take, say, one hour to cook rice in a regular vessel that operates at 1 atm pressure. But only 20 min in a pressure cooker operating at 3 atm absolute pressure (corresponding boiling temperature: 134 °C). Another point to note is about the atmospheric pressure. You are aware that atmospheric pressure decreases with altitude and, thereby, the boiling temperature of water also decreases with altitude. Therefore, it takes longer to cook at higher altitudes than it does at sea level. The liquid-vapour saturation curve of a pure substance (numerical values are for water) is given in Fig.8.6. Table 8.1 gives the saturation (boiling) pressure of water at various temperatures.

Table 8.1 Saturation (boiling) pressure of water at various temperatures

T,°C	P_{sat}, kPa	T,°C	P_{sat}, kPa
−10	0.26	30	4.25
−5	0.40	40	7.39
0	0.61	50	12.35
5	0.87	100	101.4
10	1.23	150	476.2
15	1.71	200	1555
20	2.34	250	3976
25	3.17	300	8588

8.5 PHASE-CHANGE PROCESS OF WATER AT DIFFERENT PRESSURES

In Section 8.4, we have gone through the phase-change process of water from its liquid state to vapour state at constant pressure. To understand a little more about the phase-change, let us go through the details again the phase-change process of water from its solid phase to vapour phase at different pressures. For this, consider a transparent cylinder piston arrangement as shown in Fig.8.7. Also note that it is equipped with instruments for measuring pressure, volume and temperature. Also assume that the cylinder piston arrangement has provision for loading the piston to any desired pressure. We will study the phase-change process for 5 different pressures, from very low [(0.1 kPa) to very high (25000 kPa). To start with, let us assume that water in the cylinder is at −20°C (ice).

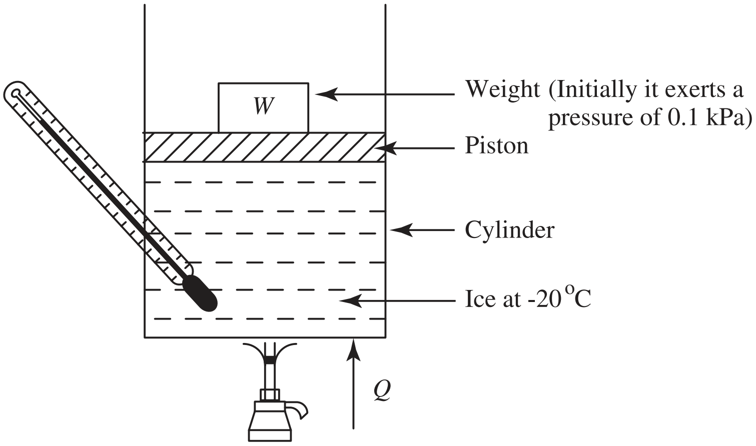


Fig. 8.7 Heating of water at different pressures

Initially, let the piston be loaded with predetermined weight so that it exerts a very low pressure of 0.1 kPa [0.001 bar] as shown in Fig.8.7. Now, start heating the ice slowly by means of a burner as shown in the figure. The volume and temperature start increasing steadily until the temperature reaches close to 0°C. It will be interesting to note that any further heating will not increase the temperature but the volume keeps increasing. If you watch through the transparent cylinder you will notice the ice getting converted directly to water vapour (steam) instead of liquid water. If we continue heating, we will see that the quantity of ice starts decreasing whereas the quantity of steam starts increasing as shown in variation (i) in Fig.8.8. At one point of time, all the ice gets converted to steam.

On further heating, you will notice that both temperature and volume will increase. The details of the variation of temperature with respect to time (curve i) are as shown in Fig.8.8. We have taken time as the x-axis, since the heating is steady and therefore, the quantity of heat added will be proportional to time. You should note that the water has changed its phase from solid to vapour directly at this low pressure. The process of getting converted from the solid phase directly to the vapour phase is called *sublimation.*

Now, let us study the phase-change at other four pressures. All these five pressures we have chosen will cover fairly wide range of pressures that are normally used in practice and hence, the observations and results recorded can be generalised. Let us consider the pressure of 100 kPa (1 bar). The details of the constant pressure heating process at this pressure are shown by curve (ii). As already explained in the previous section, initially with an increase in heat input by the burner, the temperature and volume of ice keep increasing. When the temperature is very close to 0°C, any further heating will not increase the temperature. However, the volume keeps decreasing at the same temperature as ice start melting. A look into the transparent cylinder, will reveal that ice is getting converted to liquid water at 0°C, unlike the previous low pressure at which ice has directly converted to water vapour. Until the last part of ice melting process, the temperature remains constant at 0°C. At this juncture, the following points are worth noting:

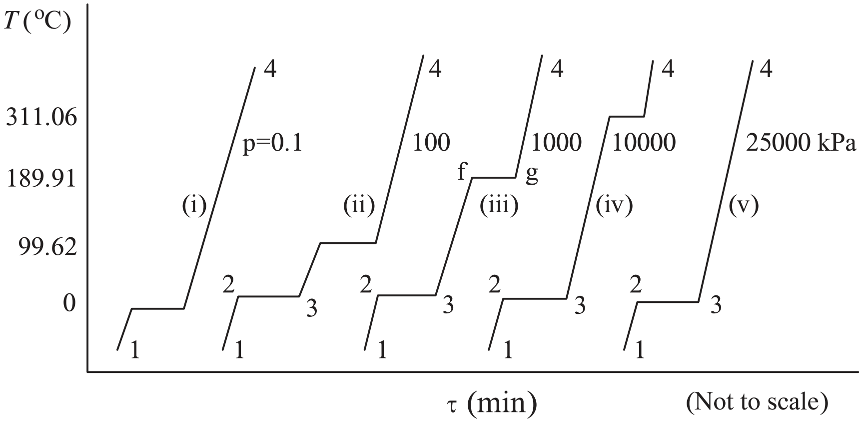


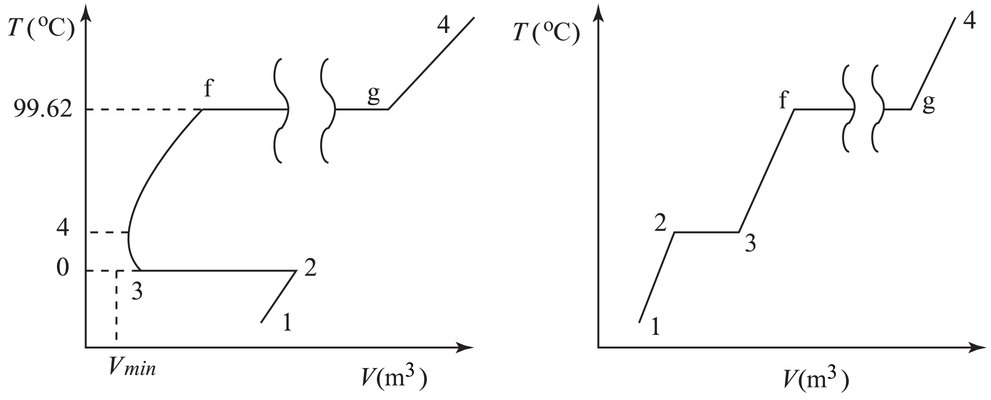
Fig. 8.8 Temperature time diagram for five arbitrary constant pressure processes

(i) At very low pressure of 0.1 kPa, sublimation takes place.

(ii) When the phase-change takes place at constant pressure, the temperature remains constant.

(iii) At the pressure of 1 bar during the melting process and between 0 and 4 °C, there is a decrease in volume. This variation is enlarged and shown on T-V coordinates in 8.9(a).

This is unique to water. Figure 8.9(b) shows T-V variation for other pure substances. It may be noticed that the volume keeps increasing continuously all through the constant pressure heating.



(a) Change in volume of water during constant pressure heating

(b) Change in volume of a pure substance, other than water, during constant pressure heating

Fig. 8.9 Change in volume

When all the ice melts, further heating increases both the temperature and volume of water [Fig.8.9(a)]. Again you will notice that when the temperature reaches 99.62 °C, there will be no increase in temperature even with steady and continuous heating. A look into the transparent cylinder again will reveal that the water has started boiling at this temperature.

Further, you will notice that water is slowly getting converted to steam. During the process of vaporisation, there will be a large volume change. Until the last drop of water evaporates to form steam, there will be no temperature rise with heating. When the last drop of water evaporates, the temperature will start rising forever. This has already been explained in detail in the previous section. This variation is shown by curve (ii) in Fig.8.8. Both the curves (i) and (ii) are seen to start at point 1 and ends at point 4. However, there is a basic difference between the two. Curve (i) has only one horizontal step whereas curve (ii), marked as 123fg4, has two horizontal steps.

Now, consider the pressure 1000 kPa [10 bar]. The path traced during heating is represented in curve (iii).You will notice that for this pressure also, there are two horizontal steps. However, the second horizontal step at which the boiling occurs is at 179.91 °C. Further, the width of this horizontal step is less. This implies that vaporisation is faster at higher pressures. Hence, the magnitude of heat transfer required to change the phase from liquid to vapour is comparatively small at higher pressure.

Let us consider the pressure 10000 kPa [100 bar]. The trend of the variation is similar to curve (ii) and (iii) except that the second horizontal step has very small width. Boiling occurs at an elevated temperature of 311.06 °C. Now, consider the pressure of 25000 kPa [250 bar]. The variation reveals that the second horizontal step has totally vanished. From the above discussion, the following points are to be noted:

(i) At very low [0.001 bar] as well as very high [250 bar] pressures, there is only one horizontal step indicating only one phase-change.

(ii) At other pressures, there are two horizontal steps. The first one represents the phase-change from solid to liquid while the second one represents from liquid to vapour. It can be generally stated that during phase-changes, there is no change in temperature at a given pressure.

(iii) With increase in pressure, the second horizontal step occurs at higher temperatures. This clearly shows that the boiling temperature increases with increase in pressure.

(iv) The width of the second step decreases with increase in pressure. This means the heat required to bring about a change of phase from liquid to vapour decreases with increase in pressure.

From the temperature-time (T-τ) diagram in Fig.8.8, we can make a cross plot of temperature-pressure (T-p) diagram. For this plot, the temperature at which various horizontal steps occurred on the temperature-time (T-τ) diagrams should be plotted on temperature-pressure (T-p) plane. The cross plot will be as shown in Fig.8.10. Note that the figure is only representative in nature and not drawn to scale. There are two curves in this diagram. One is a steeply rising curve called liquid-vapour saturation line which is the locus of boiling temperature. The other one is more or less a horizontal straight line called the solid-liquid saturation line in the locus of melting temperatures. The point of intersection of these two curves is labelled TP, meaning triple point. The point where the top curve suddenly vanishes is labelled as CP, meaning critical point. The extended portion of the liquid-vapour saturation line below TP is called the *solid-vapour saturation line.*

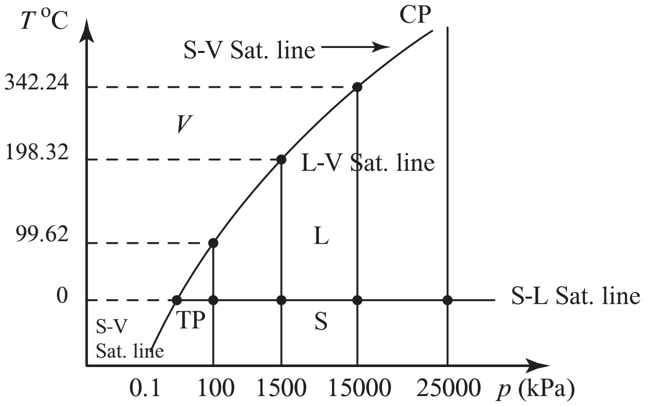


Fig. 8.10 T-p diagram for water

A careful study of Fig.8.10 brings out the following points, viz various phases are separated by saturation lines:

(i) S-L saturation line separates solid and liquid regions,

(ii) L-V saturation line separates liquid and vapour regions, and

(iii) S-V saturation line separates the solid and vapour regions.

Heating at pressures between TP and CP makes the water change gradually from solid (ice) to liquid (water) to vapour (steam). Further, we should note the following two points:

(i) The change of phase from solid to liquid is called *melting.* The melting temperature is more or less independent of pressure.

(ii) The change of phase from liquid to vapour is called *boiling* or *vaporisation.* Boiling temperature increases with an increase in pressure.

8.6 DRYNESS FRACTION OR QUALITY OF STEAM

Consider a rigid vessel containing m kg of wet steam at a pressure p. This wet steam is called a mixture. It consists of both liquid water (wet part) and steam (dry vapour). The two phases are in equilibrium with each other. In other words, both phases are at same pressure and temperature. Let m_f be the mass of liquid water and m_g be the mass of gaseous water (steam). Then, total mass m is

$$m = m_f + m_g \tag{8.3}$$

The dryness fraction of this wet steam or mixture is usually denoted by x. It is defined as the ratio of the mass of the dry steam (m_g) actually present to the mass of the total steam. It is denoted by x.

$$\text{Dryness fraction, } x = \frac{m_g}{m_f + m_g} \tag{8.4}$$

where m_g is mass of dry steam in kg and m_f is the mass of water vapour in suspension. This term presents only for wet steam. For dry steam, $m_f = 0$. Therefore, $x = 1$. The dryness fraction when expressed in percentage (i.e. 100 x) is called the *quality of steam.* The value of x varies between 0 and 1. For saturated water, i.e. when water just starts boiling, $m_g = 0$ and therefore, $x = 0$ and for saturated vapour, when vaporisation is complete, $m_f = 0$ and therefore, $x = 1$. At this condition, the vapour is said to be *dry saturated.*

Points f in Fig.8.11 (a), (b) and (c) indicate the saturated liquid states with $x = 0$ and points g indicate the saturated vapour states with $x = 1$, the lines fg indicating the transition from liquid to vapour.

Points a, b and c at various pressures indicate the situations when the masses of vapour are 25, 50 and 75% of the total mass. For example, at points a, the mass of vapour (m_g) is 25% and the mass of liquid (m_f) is 75% of the total mass (m); at points b, the mixture consists of 50% liquid and 50% vapour by mass; and at points c, the mixture consists of 75% vapour and 25% liquid by mass. The lines passing through points a, b and c are the constant quality lines of 0.25, 0.50 and 0.75, respectively. Constant quality lines start from the critical point.

Let V be the total volume of a liquid vapour mixture of quality x, V_f is the volume of the saturated liquid and V_g is the volume of the saturated vapour, the corresponding masses being m, m_f and m_g, respectively. Now,

$$m = m_f + m_g \quad \text{and} \quad V = V_f + V_g \tag{8.5}$$

$$mv = m_f v_f + m_g v_g = (m - m_g)v_f + m_g v_g$$

$$v = \left(1 - \frac{m_g}{m}\right)v_f + \frac{m_g}{m}v_g = (1 - x)v_f + x v_g \tag{8.6}$$

where $x = \dfrac{m_g}{m}, v_f$ is specific volume of saturated liquid; v_g, the specific volume of saturated vapour; and v, the specific volume of the mixture of quality x. Similarly,

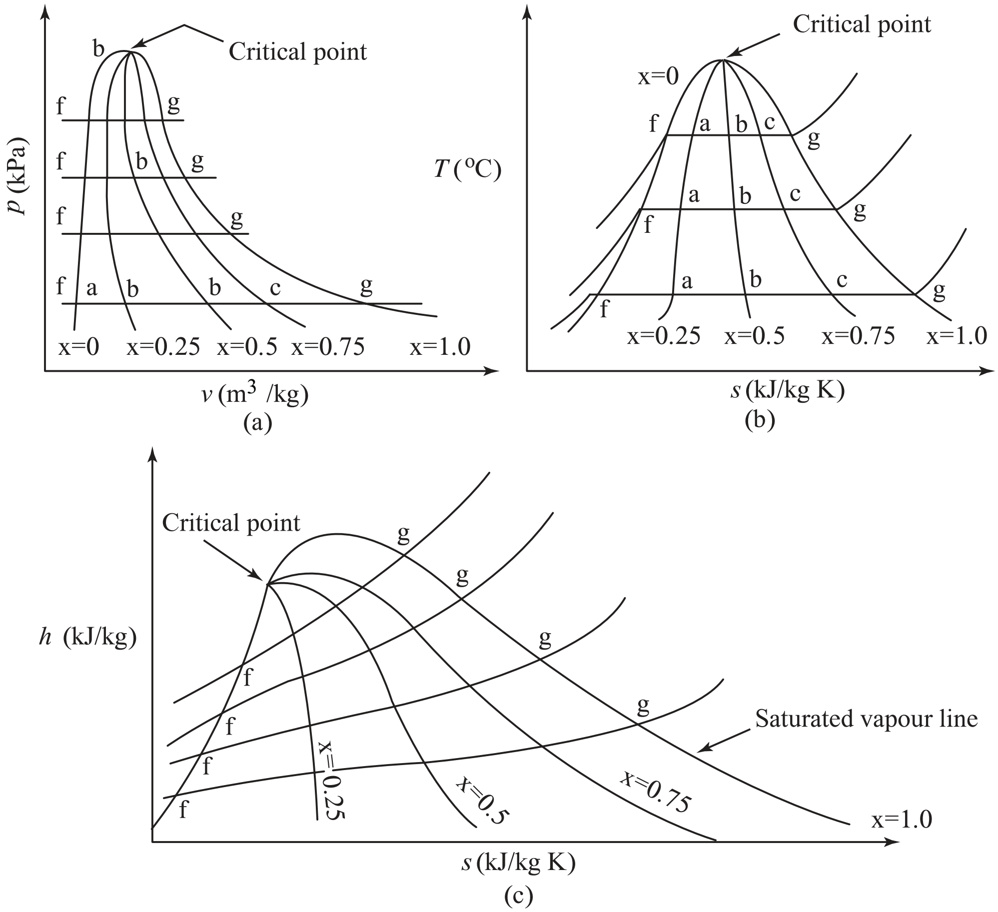


Fig. 8.11 Constant quality lines on p-v, T-s and h-s diagrams

$$s = (1-x)s_f + xs_g \tag{8.7}$$

$$h = (1-x)h_f + xh_g \tag{8.8}$$

$$u = (1-x)u_f + xu_g \tag{8.9}$$

where s, h and u refer to the mixture of quality x, suffixes f and g indicate the conditions of saturated liquid and saturated vapour, respectively. From Eq.8.6,

$$v = (1-x)v_f + xv_g = v_f + x(v_g - v_f) = v_f + xv_{fg} \tag{8.10}$$

Similarly, $$h = h_f + xh_{fg} \tag{8.11}$$

$$s = s_f + xs_{fg} \tag{8.12}$$

$$u = u_f + xu_{fg} \tag{8.13}$$

8.7 WETNESS FRACTION

It is defined as the ratio of the mass of water vapour in suspension to the total steam:

$$\begin{aligned}\text{Wetness fraction} &= \frac{m_f}{m_f+m_g}+1-1=1-\left(1-\frac{m_f}{m_f+m_g}\right)\\ &= 1-\frac{m_g}{m}=1-x \quad (\because m_f+m_g=m) \qquad (8.14)\end{aligned}$$

The wetness fraction expressed in percentage, i.e. $100 \times (1-x)$ is called *priming*.

8.7.1 Triple Point

The point TP on the T-p diagram (Fig.8.10) illustrates the *triple point*. It is that state at which all the three phases of a pure substance coexist in equilibrium. For water, the triple point pressure and temperature are 0.006117 bar and 0.01 °C, respectively. We will elaborate this in Section 8.9.4.

8.7.2 Critical Point

A *critical point* is that state beyond which it is difficult to distinguish between liquid and vapour phases. It can also be defined as that state at which liquid can be converted to vapour completely without any heat transfer. For water, the critical point pressure and temperature are 220.9 bar and 374.14 °C, respectively.

8.7.3 Enthalpy Change During Constant-Pressure Heating

When water is heated at constant pressure, from the first law of thermodynamics:

$$Q-W = U_{\text{final}}-U_{\text{initial}} \qquad (8.15)$$

Q and W are the heat and work transfers during the heating period. The initial and final states could be selected arbitrarily. The work involved here is only displacement work at constant pressure and hence:

$$W = \int p dV = p\int dV = p(V_{\text{final}}-V_{\text{initial}}) \qquad (8.16)$$

Substituting for W from Eq.8.16 in Eq.8.15:

$$\begin{aligned}Q &= p(V_{\text{final}}-V_{\text{initial}})+(U_{\text{final}}-U_{\text{initial}})\\ &= (pV_{\text{final}}+U_{\text{final}})-(pV_{\text{initial}}+U_{\text{initial}}) \qquad (8.17)\end{aligned}$$

For a constant pressure process, $p=P_{\text{final}}=P_{\text{initial}}$ and hence,

$$Q = (p_{\text{final}}V_{\text{final}}+U_{\text{final}})-(p_{\text{initial}}V_{\text{initial}}+U_{\text{initial}}) \qquad (8.18)$$

By definition, enthalpy,

$$H = U+pV \qquad (8.19)$$

and therefore, $Q=H_{\text{final}}-H_{\text{initial}}=\Delta H$

Between any two state points, the magnitude of heat transfer during a constant pressure process is equal to the change in enthalpy of the substance being heated. And per kilogram of substance, it is equal to the change in specific enthalpy, h. Therefore,

$$q = \Delta h = h_{\text{final}} - h_{\text{initial}} \tag{8.20}$$

8.8 SOME IMPORTANT TERMS AND THEIR DEFINITIONS

By now you are conversant with the meaning of *pure substance*. For thermodynamic analysis of pure substances, we quite often use the following terms. The definition of various terms in thermodynamic analysis are:

(i) *Sensible heating*: It means heating of the substance in single phase which causes rise in temperature of substance. In case of cooling in above conditions, it shall be called sensible cooling.

(ii) *Latent heating* : It means heating of a substance which causes phase-change at constant temperature. If heat is extracted for causing phase-change without any change in its temperature it will be called latent cooling.

(iii) *Normal boiling point*: The temperature at which vapour pressure equals to atmospheric pressure is called normal boiling point. At this temperature, phase-change takes place from liquid to gas.

(iv) *Melting point*: The temperature at which phase-change takes place from solid to liquid by supplying latent heat is called melting point.

(v) *Saturation states*: It refers to the state at which its phase transformation takes place without any change in pressure and temperature. It can be (a) saturated solid state, (b) saturated liquid state or (c) saturated vapour state. For example, saturated vapour state refers to the state of water, which changes from liquid to vapour at constant pressure and temperature.

(vi) *Saturation pressure*: It refers to the pressure at which a substance changes its phase for any given temperature. For example, at any given temperature, water shall get converted into steam only at a definite pressure. This pressure is called saturation pressure corresponding to given temperature. For water at 100 °C, the saturation pressure is 1 atm.

(vii) *Saturation temperature*: The temperature at which a substance changes its phase for any given pressure. For water at 1 atm pressure the saturation temperature is 100 °C.

(viii) *Triple point*: Triple point of a substance refers to the state at which substance can coexist in solid, liquid and gaseous phase in equilibrium. For water, it is 0.01 °C. At this temperature, ice, water and steam can coexist in equilibrium. Table 8.2 gives the triple point data for number of substances.

Table 8.2 Triple point data

Acetylene, C_2H_2	192.4	962
Ammonia, NH_3	195.42	45.58
Argon, A	83.78	515.7
Carbon dioxide, CO_2	216.55	3885.1
Carbon monoxide, CO	68.14	115.14
Ethane, C_2H_6	89.88	0.006
Ethylene, C_2H_4	104.00	0.9
Hydrogen, H_2	13.84	52.8
Methane. CH_2	90.67	87.7
Nitrogen, N_2	63.15	94.01
Oxygen, O_2	54.35	1.14
Water, H_2O	273.16	4.587

8.9 PROPERTY DIAGRAMS FOR PHASE-CHANGE PROCESSES

Property diagrams help us in understanding the variations of properties such as pressure, volume and temperature during phase-change processes. We will discuss the details by means of T-v, p-v, p-T and p-v-T surface diagrams for the pure substance, viz water.

8.9.1 T-v Diagram

The phase-change process of water at a constant pressure of 1 atm is explained in Section 8.4 and the corresponding T-V diagram is given in Fig.8.5. The same is reproduced here for unit mass in Fig.8.12. If we repeat this process for different pressures (refer Fig.8.8), we can develop the T-v diagram for different pressures. Now, consider a pressure of 1 MPa. At this pressure, water will have a smaller specific volume than it has at 1 atm. (i.e. at 0.1 MPa) pressure. If we transfer heat to the water at this higher pressure, the process will follow a similar path. It will be very much like the process path for 1 atm. pressure, as shown in Fig.8.13. However, there will be two major noticeable differences:

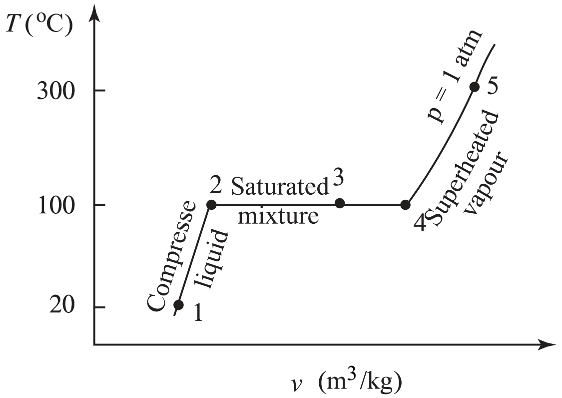


Fig. 8.12 T-v diagram for the heating process of water at constant pressure

(i) Water starts boiling at a much higher temperature (179.91 °C) at this pressure.

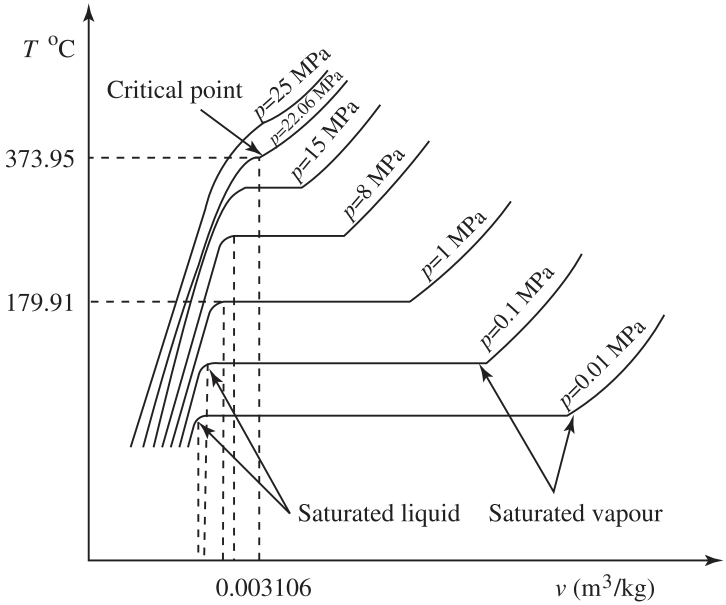


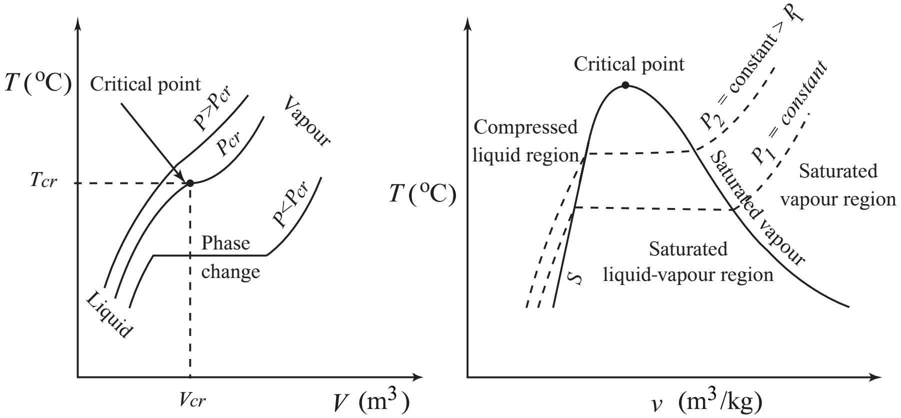
Fig. 8.13 T-v diagram of constant-pressure phase-change processes of a pure substance at various pressures (numerical values are for water)

(ii) The specific volume of the saturated liquid will be larger and the specific volume of the saturated vapour will be smaller than the corresponding values at 1 atm pressure.

This means the horizontal line that connects the saturated liquid and saturated vapour states is much shorter compared to 1 atm pressure. With further increase in pressure, this saturation line continues to become shorter, as shown in Fig.8.13. It becomes a point when the pressure reaches 22.06 MPa for water. This point is called the *critical point* and it is defined as *the point at which the saturated liquid and saturated vapour states are identical.* The temperature, pressure and specific volume of a substance at the critical point are called the *critical temperature* T_{cr}, *critical pressure* p_{cr} and *critical specific volume,* v_{cr}, respectively. At pressures above the critical pressure, distinct phase-change cannot be identified, Fig.8.14(a). However, the specific volume of the substance increases. At all times, there is only one phase present and it resembles a vapour. It is not possible to tell when the change has occurred. Above the critical state, there is no line that separates the compressed liquid region and the superheated vapour region. However, it is customary to refer to the substance as superheated vapour at temperatures above the critical temperature. Below it is referred as compressed liquid.

The saturated liquid states in Fig.8.13 can be connected by a line called the *saturated liquid line.* Similarly, saturated vapour states in the same figure can be connected by another line, called the *saturated vapour line.* These two lines meet at the critical point, forming a dome as shown in Fig.8.14(b).

All the compressed liquid states are located in the region to the left of the saturated liquid line, called the *compressed liquid region.* All the superheated vapour states are located to the right of the saturated vapour line, called the superheated vapour region. In these two regions, the substance exists in a single phase, either a liquid or a vapour. All the states that involve



(a) At supercritical pressures (p-p$_{cr}$), there is no distinct phase-change (boiling) process

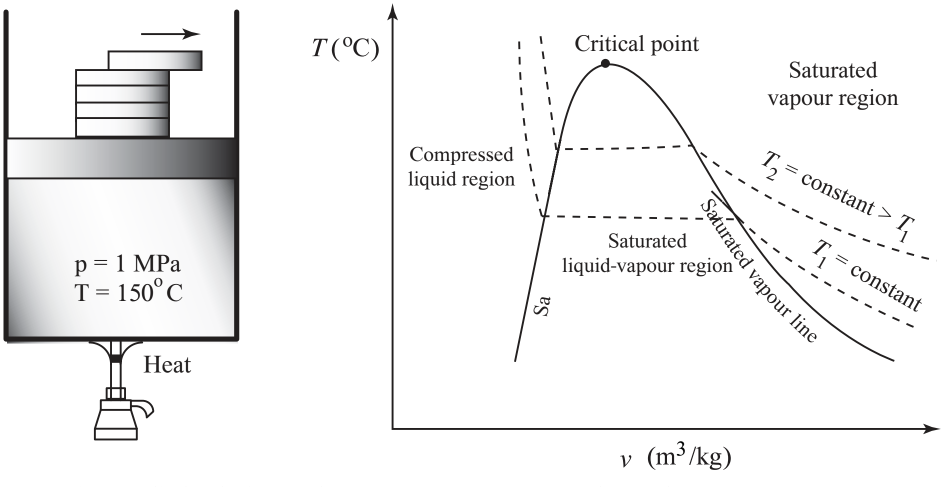
(b) T-v diagram of a pure substance

Fig. 8.14 T-v variation during phase-change process

both phases in equilibrium are located under the dome, called the *saturated liquid-vapour mixture region or the wet region.*

8.9.2 The *p*-*v* Diagram

The general shape of the *p*-*v* and *T*-*v* diagram are almost identical. However, the constant *T* lines on this diagram have a downward trend, as shown in Fig.8.15(b). Consider again a



(a) Pressure in a piston-cylinder device can be reduced by reducing the weight of the piston

(b) p-v diagram

Fig. 8.15 p-v variation during phase-change process

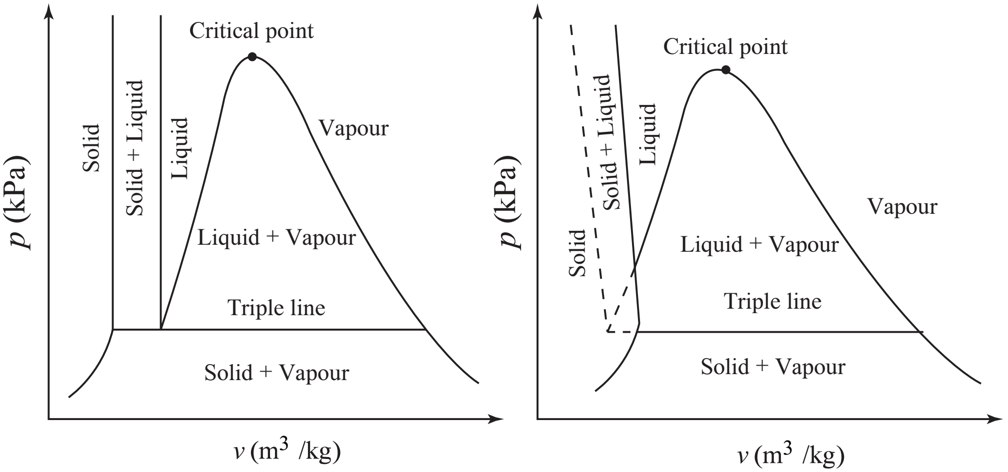
piston-cylinder device that contains liquid water at 1 MPa and 150 °C. Water at this state exists as a compressed liquid. If the weights on top of the piston are removed one by one so that the pressure inside the cylinder decreases gradually [Fig.8.15(b)]. Assume that the water is allowed to exchange heat with the surroundings so its temperature remains constant.

Note that the volume of the water increases slightly with decrease in pressure. At the specified temperature, when the pressure reaches the saturation-pressure value (0.4762 MPa), the water starts boiling. During this vaporisation process, both the temperature and the pressure remain constant. However, the specific volume increases. Once the last drop of liquid is vaporised, further reduction in pressure results a further increase in specific volume. Note that during the phase-change process, we did not remove any weights. Removing the weights would cause the pressure as well as the temperature to drop [since $T_{sat} = f(p_{sat})$]. The process would no longer be isothermal. When the process is repeated for other temperatures, similar paths are obtained for the phase-change processes. Connecting the saturated liquid and the saturated vapour states by a curve, we obtain the *p*-*v* diagram of a pure substance obtained which is shown in Fig.8.15(b).

8.9.3 Inclusion of Solid Phase

The *T*-*v* and *p*-*v* diagrams developed so far represent the equilibrium states involving the liquid and the vapour phases only. However, these diagrams can easily be extended to include (i) the solid phase; (ii) the solid-liquid saturation regions; and (iii) the solid-vapour saturation regions.

The principle involved with the liquid-vapour phase-change process apply equally to the solid-liquid and solid-vapour phase-change processes also. Most substances contract during a solidification (freezing) process. However, some substances like water expand as they freeze. The *p*-*v* diagrams for both groups of substances are given in Figs.8.16(a) and (b), respectively. These two diagrams differ only in the solid-liquid saturation region. The *T*-*v* diagrams look very much like the *p*-*v* diagrams, especially for substances that contract on freezing.



(a) *p*-*v* diagram of a substance that contracts on freezing

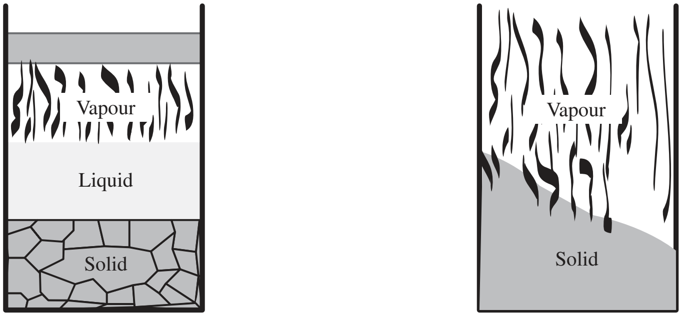
(b) *p*-*v* diagram of a substance that expands on freezing (such as water)

Fig. 8.16 p-v diagram of a pure substance

These figures are similar to what we have discussed in section 8.5 [Figs.8.9(a) and (b)]. You must appreciate that the nature has purposely made water to expand while freezing. Further, you should admire the nature's creation. If water contracted while freezing, like most other substances, then the ice formed would be heavier than the liquid water. It would settle to the bottom of rivers, lakes and oceans, instead of floating at the top. Sun rays would never reach these ice layers. Then, the bottoms of many rivers, lakes and oceans would be covered with ice which would seriously disrupt marine life.

8.9.4 Triple Line and Triple Point

By this time, it should have become clear that two phases can be in equilibrium. However, under some conditions, all three phases of a pure substance can coexist in equilibrium (Fig.8.17). On p-v or T-v diagrams, these triple-phase states form a line called the *triple line*. The states on the triple line of a substance have the same pressure and temperature but different specific volumes. The triple line appears as a point on the p-T diagrams and therefore, is often called the *triple point*. The triple-point temperatures and pressures of various substances are given in Table 8.3.



(a) At triple-point pressure and temperature, a substance exists in three phases in equilibrium

(b) At low pressures (below the triple point value), solids evaporate without melting first (sublimation)

Fig. 8.17 Triple-point

For water, the triple-point temperature and pressure are 0.01 °C and 0.6117 kPa, respectively. That is, all three phases of water coexist in equilibrium only if the temperature and pressure have *precisely these values*. No substance can exist in the liquid phase in stable equilibrium at pressures below the triple-point pressure. As already stated, passing from the solid phase directly into the vapour phase is called *sublimation* [Fig.8.17(b)]. For substances that have a triple-point pressure above the atmospheric pressure such as solid CO_2 (dry ice), sublimation is the only way to change from the solid to vapour phase at atmospheric conditions.

8.9.5 The p-T Diagram

Figure 8.18 shows the p-T diagram of a pure substance. This diagram is often called the *phase diagram* , since all three phases are separated from each other by three lines. The sublimation line separates the solid and vapour regions, The vaporisation line separates the liquid and vapour regions. The melting (or fusion) line separates the solid and liquid regions. These three lines meet at the triple point, where all three phases coexist in equilibrium (Fig.8.18). The vaporisation line ends at the critical point because no distinction can be made between

Table 8.3 Triple-point temperatures and pressures of various substances

Substance	Formula	T_{tp}, K	P_{tp}, kPa
Acetylene	C_2H_2	192.4	120
Ammonia	NH_3	195.40	6.076
Argon	A	83.81	68.9
Carbon (graphite)	C	3900	10, 100
Carbon dioxide	CO_2	216.55	517
Carbon monoxide	CO	68.10	15.37
Deuterium	D_2	18.63	17.1
Ethane	C_2H_6	89.89	8×10^{-4}
Ethylene	C_2H_4	104.0	0.12
Helium 4 (l point)	He	2.19	5.1
Hydrogen	H_2	13.84	7.04
Hydrogen chloride	HCl	158.96	13.9
Mercury	Hg	234.2	1.65×10^{-7}
Methane	CH_4	90.68	11.7
Neon	Ne	24.57	43.2
Nitric oxide	NO	109.50	21.92
Nitrogen	N_2	63.18	12.6
Nitrous oxide	N_2O	182.34	87.85
Oxygen	O_2	54.36	0.152
Palladium	Pd	1825	3.5×10^{-3}
Platinum	Pt	2045	2.0×10^{-4}
Sulfur dioxide	SO_2	197.69	1.67
Titanium	Ti	1941	5.3×10^{-3}
Uranium hexafluoride	UF_6	337.17	151.7
Water	H_2O	273.16	0.61
Xenon	Xe	161.3	81.5
Zinc	Zn	692.65	0.065

Source: Data from National Bureau of Standards (U.S.) Circ., 500 (1952).

liquid and vapour phases above the critical point. Substances that expand and contract on freezing differ only in the melting line on the p-T diagram.

8.9.6 The p-v-T Surface

Any two independent intensive properties can fix the state of a simple compressible substance. Once the two appropriate properties are fixed, all the other properties become dependent properties. Any equation with two independent variables in the form $z = z(x, y)$ represents a surface in space. Therefore, we can represent the p-v-T behaviour of a substance as a surface in space. The details are shown in Fig.8.19(a) and (b). Here, T and v can be considered as the independent variable (the base) and p as the dependent variable (the height). All the points on the surface will represent equilibrium states. All states along the path of a quasi-equilibrium process lie on the p-v-T surface. It is because such a process must pass through equilibrium states. The single-phase regions will appear as curved surfaces on the p-v-T surface. The two-phase regions will appear as surfaces perpendicular to the p-T plane. This is expected since the projections of two-phase regions on the p-T plane are lines. All the two-dimensional

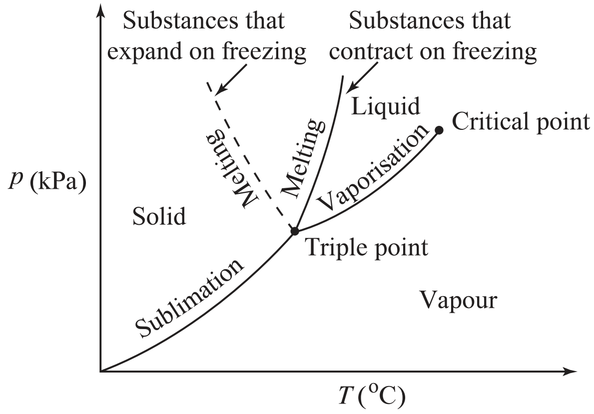
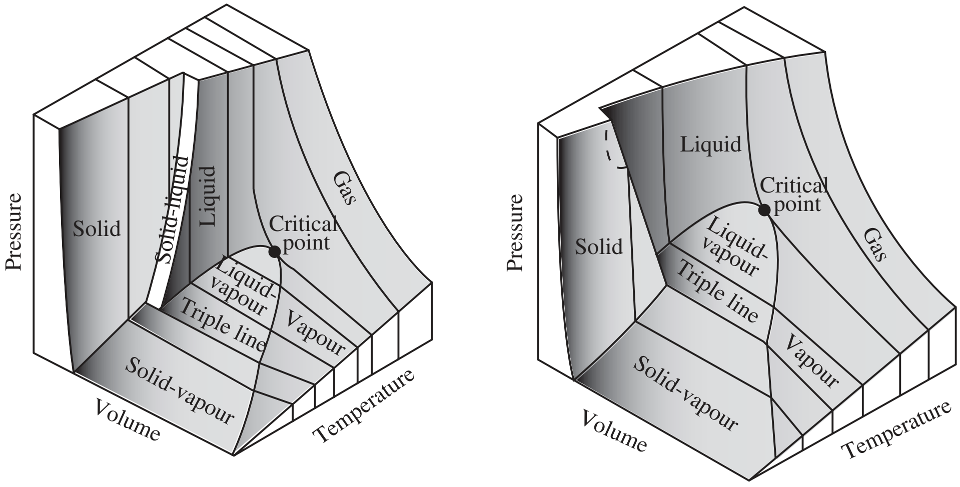


Fig. 8.18 p-T diagram of pure substances

diagrams we have discussed so far are merely projections of this three-dimensional surface onto the appropriate planes. A *p*-*v* diagram is just a projection of the *p*-*v*-*T* surface on the *p*-*v* plane. Similarly, a *T*-*v* diagram is nothing more than the birds-eye view of this surface. Note that the *p*-*v*-*T* surfaces provide a good amount of information. However, in a thermodynamic analysis, it is comparatively easier to work with two-dimensional diagrams, such as the *p*-*v* and *T*-*v* diagrams.



(a) *p*-*v*-*T* surface of a substance that contracts on freezing

(b) *p*-*v*-*T* surface of a substance that expands on freezing (like water)

Fig. 8.19 p-v-T surface of a substance

8.10 STEAM TABLES AND MOLLIER DIAGRAM

Steam table is nothing but the tabular presentation of properties of pure substances, viz steam. It has its unique and constant properties at different pressures and temperatures. Therefore, its thermodynamic properties can be estimated once and tabulated for future use.

Properties such as specific enthalpy, entropy, internal energy and specific volume at different saturation pressures and temperatures can be presented in the form of a table. Such a table may be presented either on pressure basis or on temperature basis. The table on pressure basis has continuous variation of pressure and corresponding to it : saturation temperature (T_{sat}), enthalpy of saturated liquid (h_f), enthalpy of saturated vapour (h_g), entropy of saturated liquid (s_f), entropy of saturated vapour (s_g), specific volume of saturated liquid (v_f), specific volume of saturated vapour (v_g), internal energy of saturated liquid (u_f), internal energy of saturated vapour (u_g) are given on unit mass basis, i.e. as shown in Table 8.4. Similar to above the temperature-based table, which gives continuous variation of temperature and corresponding to it saturation pressure and other properties as $h_f, h_g, h_{fg}, s_f, s_g, s_{fg}, v_f, v_g, u_f, u_g$ and u_{fg} are given. Similarly, steam properties for superheated steam are also estimated and tabulated at some discrete pressures for varying degree of superheat. Superheated steam table are available for getting enthalpy, entropy, specific volume and internal energy separately. A typical example of superheated steam table for enthalpy is given in Table 8.6.

Table 8.4 Pressure-based steam table

p	**Sat. Temp.**	**Enthalpy**			**Entropy**			**Specific volume**		**Internal energy**		
	T_{sat} °C	h_f kJ/kg	h_g kJ/kg	h_{fg} kJ/kg	s_f kJ/kg K	s_g kJ/kg K	s_{fg} kJ/kg K	v_f m^3/kg	v_g m^3/kg	u_f kJ/kg	u_g kJ/kg	u_{fg} kJ/kg

Table 8.5 Temperature-based steam table

T	**Sat. Temp.**	**Enthalpy**			**Entropy**			**Specific volume**		**Internal energy**		
°C	kPa	h_f kJ/kg	h_g kJ/kg	h_{fg} kJ/kg	s_f kJ/kg K	s_g kJ/kg K	s_{fg} kJ/kg K	v_f m^3/kg	v_g m^3/kg	u_f kJ/kg	u_g kJ/kg	u_{fg} kJ/kg

Table 8.6 Superheated steam table for enthalpy

Pressure	Sat. temp.°C	Enthalpy values for varying degree of superheat (kJ/kg)						
kPa	(T_{sat})	T_1	T_2	T_3	T_4	T_5	T_6	T_7

Here, $T_1, T_2, T_3, T_4, T_5, T_6, T_7, \ldots$ are more than T_{sat} and note that the degree of superheat is increasing. Steam tables as discussed above are available in Appendix A in this book. *Mollier diagram* is nothing but the enthalpy-entropy (*h-s*) diagram for steam. This diagram is obtained on the basis of following equation depending upon the phase transformation as discussed earlier. $Tds = dh - vdp$ (first and second law combined). For constant pressure:

$$\left(\frac{dh}{ds}\right)_p = T \tag{8.21}$$

Enthalpy entropy diagram obtained for all phases of water is shown in Fig.8.20. For engineering systems, liquid and vapour region is only of interest. Therefore, only those portion of *h-s* diagram is shown in Fig.8.21. It is popularly known as mollier diagram or mollier chart.

Various significant lines shown in Mollier diagram are: (i) saturated liquid line, (ii) saturated vapour line, (iii) isobaric lines, (iv) isothermal lines, (v) constant specific volume lines, (vi) constant dryness fraction lines.

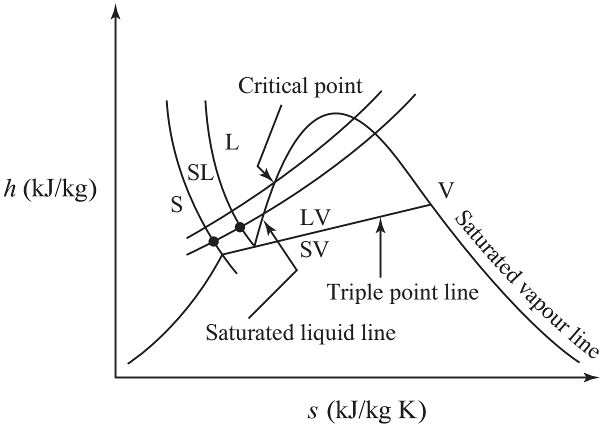


Fig. 8.20 Enthalpy-entropy diagram for all phases

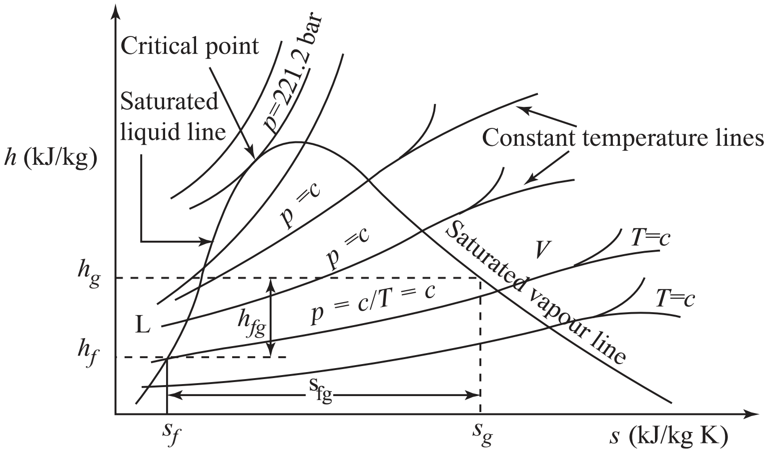


Fig. 8.21 h-s diagram (Mollier diagram)

Nature of variation of different lines can be explained from the real behaviour of substance. Mathematical expression for such lines are based on combination of first and second law. The question that comes to our inquisitive mind is why isobaric lines diverge from one another? The answer is the increase in saturation temperature with increase in pressure. Slope of isobar is equal to saturation temperature. Therefore, it will also increase with increasing pressure. Another question that may come to our mind is why isothermal lines are not visible in wet region? It is because constant temperature lines and constant pressure lines coincide with each other in wet region. For every pressure, there shall be definite saturation temperature which remains constant in wet region. Mollier chart is available at the end of this book.

8.11 DRYNESS FRACTION MEASUREMENT

Dryness fraction is the basic parameter required for knowing the state of substance in liquid-vapour mixture region (wet region). As already stated, for any pressure, the dryness fraction varies from 0 to 1 in the wet region, i.e. from saturated liquid to saturated vapour. Dryness fraction being ratio of mass of vapour and total mass of substance can be conveniently estimated if these two mass values are known. It may also be termed as *quality* of steam or *dryness fraction* depending upon how we express it. (either in % or in decimal):

$$\text{Dryness fraction} = \frac{\text{Mass of vapour}}{\text{Total mass, i.e. (mass of vapour + mass of liquid)}} \tag{8.22}$$

There are standard methods available for dryness fraction measurement:

(i) Throttling calorimeter

(ii) Separating calorimeter

(iii) Separating and throttling calorimeter

(iv) Electrical calorimeter

8.11.1 Throttling Calorimeter

In this calorimeter for obtaining dryness fraction, the principle of throttling action is used. We are aware that if a mixture is throttled, its enthalpy remains constant. Now, let us examine this process on the *h-s* diagram (Fig.8.22). Let us assume that wet mixture is at state 1 initially. Let it attain a new state 2 because of throttling to pressure p_2. This state at the end of throttling lies in the superheated region such that, $h_1 = h_2$.

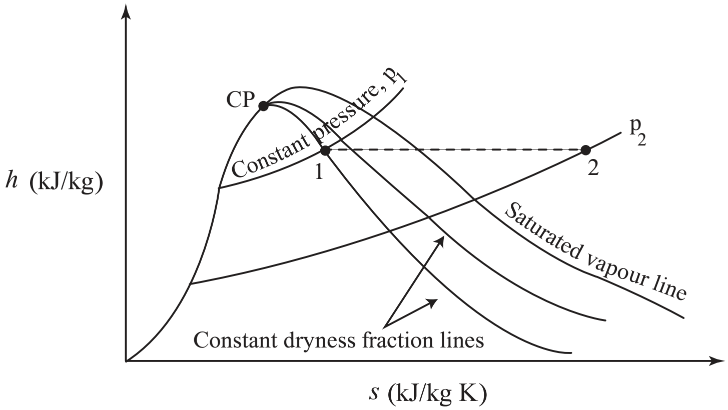


Fig. 8.22 Throttling process on h-s diagram

Let the dryness fraction at state 1 be x, then enthalpy at this point can be given as:

$$h_1 = h_{f \text{ at } p_1} + x \times h_{fg \text{ at } p_1} \tag{8.23}$$

In the above expression, $h_{f \text{ at } p_1}$ and $h_{fg \text{ at } p_1}$ can be taken from steam table if pressure of wet steam is known. Also the enthalpy at state 2 (end of throttling) can be seen from superheated steam table if pressure and temperature at 2 are known. Substituting in, $h_1 = h_2$

$$h_{f \text{ at } p_1} + x \times h_{fg \text{ at } p_1} = h_2 \tag{8.24}$$

Here, $h_2, h_{f \text{ at } p_1}, h_{fg \text{ at } p_1}$ are all known. Therefore,

$$x = \frac{h_2 - h_f \text{ at } _{p_1}}{h_{fg} \text{ at } _{p_1}} \tag{8.25}$$

Now, the provisions are to be made for the following:

(i) measurement of pressure of wet steam in the beginning,

(ii) throttling of wet mixture such that state at the end of throttling lies in superheated region and

(iii) measurement of pressure, temperature of throttled steam.

Details of the arrangement that is normally used in throttling calorimeter is shown in Fig.8.23.

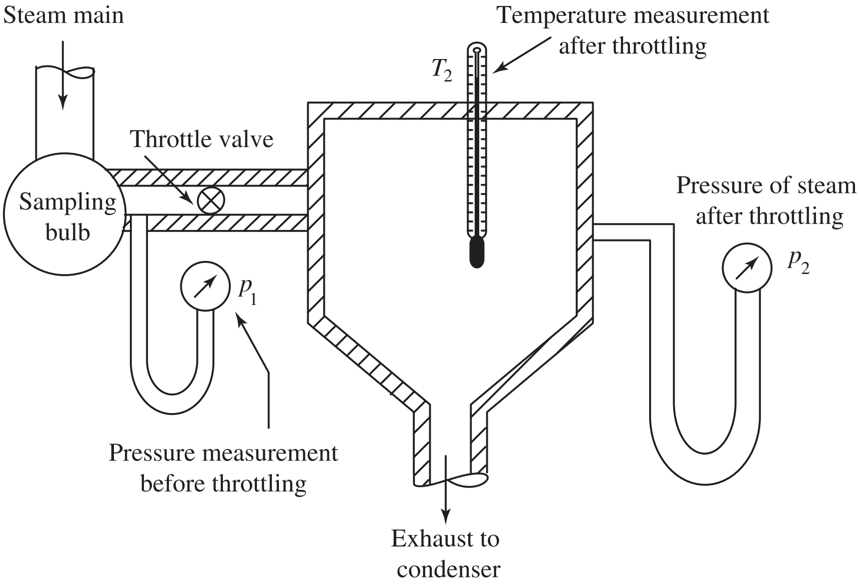


Fig. 8.23 Throttling calorimeter

8.11.2 Separating Calorimeter

In this calorimeter, a known mass of wet mixture is collected initially through a sampling bulb. Then, it is sent to a separating chamber. Separating chamber is provided with a series of obstacles and zig-zag path inside as shown Fig.8.24. This makes the liquid particles to separate when mixture passes through them. This is due to sudden change in direction of flow and gravity. Liquid thus separated out is collected in a collection tank and can be easily measured. Thus, by knowing the two masses dryness fraction can be estimated as

$$\text{Dryness fraction} = \frac{\text{Total mass} - \text{ Mass of liquid}}{\text{Total mass}} \tag{8.26}$$

Layout of separating calorimeter is given in Fig.8.24.

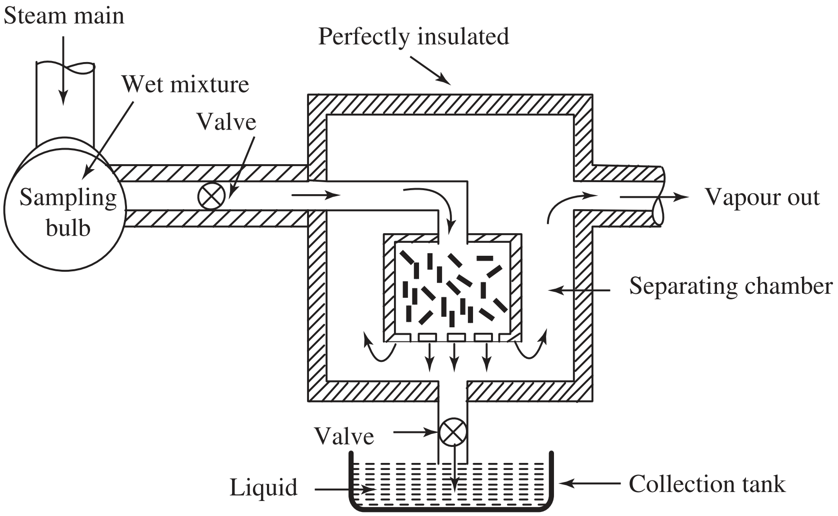


Fig. 8.24 Separating calorimeter

8.11.3 Separating and Throttling Calorimeter

At times, it may happen that the wet mixture is too wet. Therefore, during throttling process, it is difficult to make it superheated. In such a situation, mixture is first passed through separating calorimeter. This reduces liquid fraction in it. Subsequently, the mixture which is less wet is passed through throttling calorimeter. A typical arrangement of such a calorimeter is shown Fig.8.25. Here, excessively wet steam is first sent to separating calorimeter. The wetness is reduced by separating out some liquid fraction, say mass m_{f1}. The less wet steam is sent to throttling calorimeter. Let the dryness fraction be x_2. Then, the throttled steam (superheated) is made to pass through a condenser. Mass of condensate is measured from

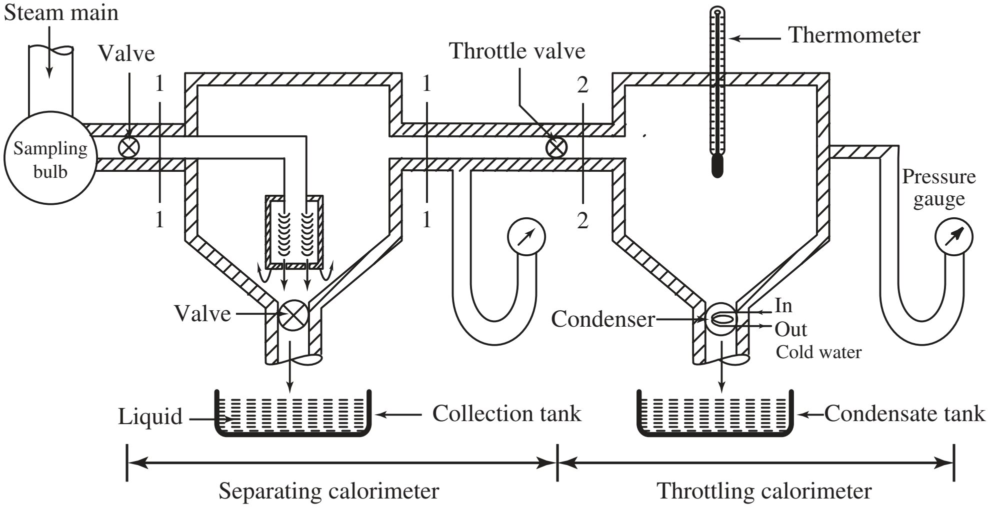


Fig. 8.25 Separating and throttling calorimeter

that collected in condensate tank. Let it be m_2. Thus, m_2 is total mass of steam sent from separating calorimeter to throttling calorimeter. As mass of liquid collected in collection tank of separating calorimeter is m_{f1}, then total mass of wet steam under consideration is $(m_{f1} + m_2)$. Dryness fraction at section 1-1 shall be

$$x_1 = \frac{\text{Mass of vapour at 1-1}}{\text{Total mass}} \tag{8.27}$$

Mass of vapour at 1-1 shall be similar to mass of vapour entering at 2-2:

$$\text{Mass of vapour at 2-2} = x_2 \times m_2 \tag{8.28}$$

Hence, dryness fraction at 1-1:

$$x_1 = \frac{x_2 m_2}{m_{f1} + m_2} \tag{8.29}$$

Separating and throttling processes are shown in *h-s* diagram in Fig.8.26.

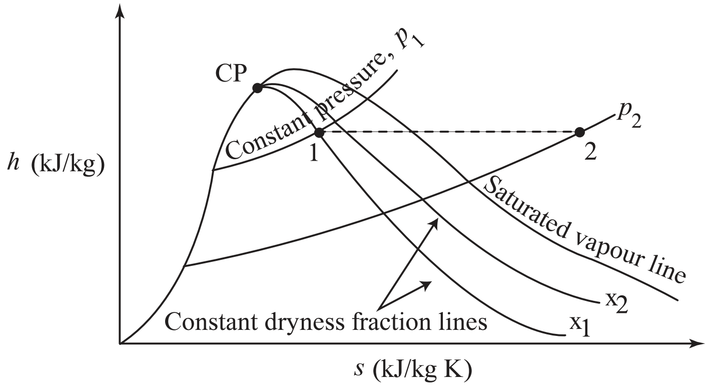


Fig. 8.26 Separating and throttling together on h-s diagram

8.11.4 Electrical Calorimeter

Electrical calorimeter also employs the principle which is similar to that of throttling calorimeter. In this calorimeter, wet mixture is brought to the superheated state by heating and not by throttling. A typical electrical calorimeter is shown in Fig.8.27. In this, the amount of heat added is known and the final enthalpy for superheated steam is also known. Therefore, we can find the initial enthalpy. For mass m of mixture, let heat added by heater be Q_{add} and the enthalpies before and after heating being h_1, h_2. Then, from steady-flow energy equation, we have:

$$mh_1 + Q_{add} = mh_2 \tag{8.30}$$

For electrical heater $Q_{add} = V \cdot I$, where V and I are voltage and current, respectively. h_2 is known, as mixture is brought to superheated state. Pressure and temperature are measured. Now, locate enthalpy from superheated steam table. Also,

$$h_1 = h_{f \text{ at } p_1} + x h_{fg \text{ at } p_1} \tag{8.31}$$

Here, $h_{f \text{ at } p_1}$ and $h_{fg \text{ at } p_1}$ can be taken from steam table. Now, h_1 being known, for known m, h_2, Q_{add}, dryness fraction x can be easily obtained.

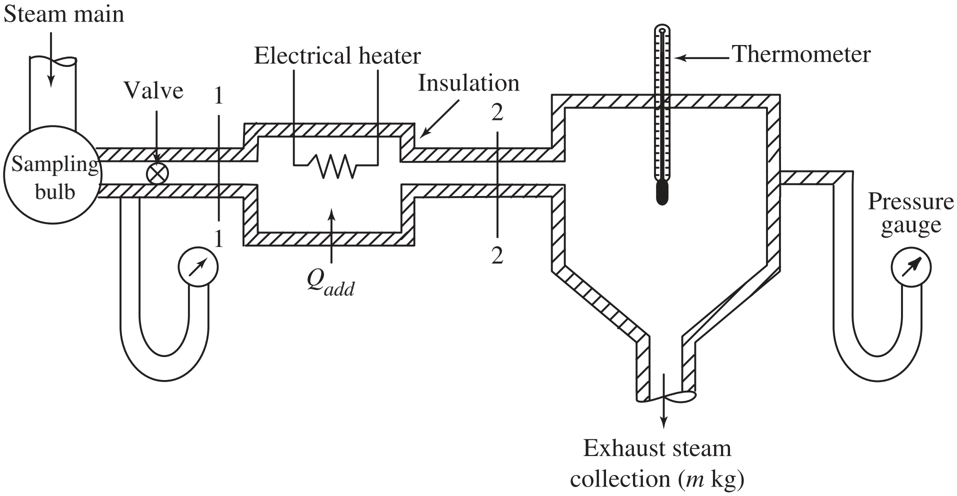


Fig. 8.27 Electrical calorimeter

8.12 THERMODYNAMIC PROPERTIES OF STEAM

We are aware that steam can exist in the following three conditions:

(i) wet, (ii) dry and (iii) superheated.

We will discuss three thermodynamic properties of steam at various conditions listed above.

(i) *Enthalpy of steam* (h)

It is the amount of heat added to the water from freezing point to till the water becomes wet or dry or superheated steam.

For wet steam, $h_{wet} = h_1 + xh_{fg}$ kJ/kg.

For dry steam, $h_{dry} = h_g = h_1 + h_{fg}$ kJ/kg.

For superheated steam, $h_{sup} = h_g + c_p(T_{sup} - T_{sat})$ kJ/kg.

where $(T_{sup} - T_{sat})$ is called as *degree of superheat.*

(ii) *Specific volume of steam (v)*

It is defined as the volume occupied by the unit mass of the steam at the given pressure and temperature.

For wet steam, $v_{wet} = xv_g$ m^3/kg

For dry steam, $v_{dry} = v_g$ m^3/kg

for superheated steam $v_{sup} = \dfrac{v_g T_{sup}}{T_{sup}}$

(iii) *Density of steam (ρ)*

It is defined as the ratio of mass to the unit volume of the steam at given pressure and temperature. Its value for wet, dry and superheated steam is the reciprocal of the specific volume of the steam.

(iv) *Internal energy of steam (u)*

Internal energy of steam is defined as the actual heat energy stored in the steam above the freezing point of water at the given conditions. It is the difference between enthalpy of steam and the external work done:

$$h = W + \Delta u \tag{8.32}$$

$$\Delta u = h - W \tag{8.33}$$

For wet steam, $u_{wet} = (h_f + xh_{fg}) - (100pxv_g)$ kJ/kg.

For dry steam, $u_{dry} = (h_f + h_{fg}) - (100pv_g)$ kJ/kg.

For superheated steam, $u_{sup} = h_{sup} - (100pv_{sup})$kJ/kg.

(v) *Entropy of steam (s)*

It is the property of the steam which increases with increase in temperature and vice versa.

For wet steam, $s_{wet} = s_f + xs_{fg}$ kJ/kg K.

For dry steam, $s_{dry} = s_f + s_{fg}$ kJ/kg K.

For superheated steam, $s_{sup} = s_g + c_{ps} \log_e \left(\frac{T_{sup}}{T_s} \right)$ kJ/kg K.

8.13 WORK DONE AND HEAT TRANSFER IN NON-FLOW PROCESSES

We have already discussed, various non-flow processes in detail as applicable to gases. In this chapter, the calculation of work done, heat transfer, enthalpy, entropy and internal energy for vapours are discussed. The aforesaid properties for vapours can be calculated with the use of basic first and second law of thermodynamics equations with suitable modifications.

The basic equation for non-flow process is

$$Q = W + \Delta U \tag{8.34}$$

$$dQ = pdV + dU \tag{8.35}$$

In this section, various non-flow processes are discussed.

(i) *Constant volume process*

Consider the process 1-2 shown in Fig.8.28. At state 1, the steam is wet with properties x_1, p_1, v_{g1}, T_1. Assume that it is heated to superheated state 2 with p_2, V_{sup2}, T_{sup2} at constant volume. Fig.8.28 shows *p*-*v* and *T*-*s* diagram of this process. Since the mass of steam before and after the heating process remains same,

$$m = \frac{V}{x_1 v_{g1}} = \frac{V}{v_{sup2}} \tag{8.36}$$

Therefore,
$$x_1 v_{g1} = v_{sup2} = v_{g2} \frac{T_{sup2}}{T_{s2}} \tag{8.37}$$

$$T_{sup2} = \frac{T_{s2} x_1 v_{g1}}{v_{g2}} \tag{8.38}$$

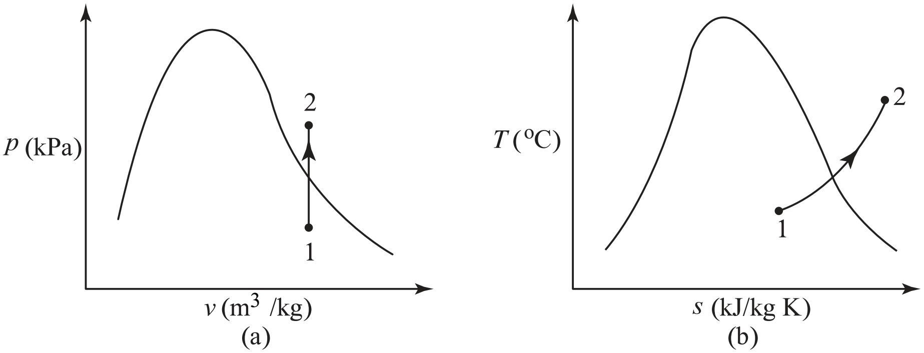


Fig. 8.28 Constant volume process

From the above equation, T_{sup2} can be calculated. The values of T_{s2} and V_{s2} can be obtained from steam tables corresponding to p_2. From first law of thermodynamics,

$$Q = W + \Delta u \tag{8.39}$$

Since for constant volume process, $W = 0$,

$$\begin{aligned} Q &= \Delta u = u_2 - u_1 \\ &= (h_2 - p_2 V_{sup2}) - (h_1 - p_1 x_1 v_{g1}) \quad (\because h = u + pv) \qquad (8.40) \\ Q &= (h_2 - h_1) - (p_2 V_{sup2} - p_1 x_1 v_{g1}) \qquad (8.41) \end{aligned}$$

If the steam is in wet condition at at 2 with x_2 after heating,

$$x_1 v_{g1} = x_2 v_{g2} \tag{8.42}$$

$$x_2 = \frac{x_1 v_{g1}}{v_{g2}} \tag{8.43}$$

The value of v_{g1} and v_{g2} can be obtained from steam table corresponding to p_1 and p_2.

$$\text{Heat transfer, } Q = u_2 - u_1 = (h_2 - h_1) - (p_2 x_2 v_{g2} - p_1 x_1 v_{g1}) \tag{8.44}$$

(ii) *Constant pressure process*

Consider the processes 1-2 shown in Fig.8.29. At state 1, the steam is wet with properties x_1, p_1, v_{g1}, T_1. Assume that it is heated to superheated state 2 with p_2, V_{sup2}, T_{sup2} at constant pressure. Figure 8.29 shows p-v and T-s diagram of this process.

Typical example is production of steam in boiler which is at constant pressure heating process. For constant pressure process, work done is given by the equation:

$$W = p_2 v_2 - p_1 v_1 = p(v_2 - v_1) \qquad (p_2 = p_1 = p) \tag{8.45}$$

where $v_1 = x_1 v_{g1}$ and $v_2 = v_{sup2} = v_{g2} \dfrac{T_{sup2}}{T_{s2}}$

From first law of thermodynamics:

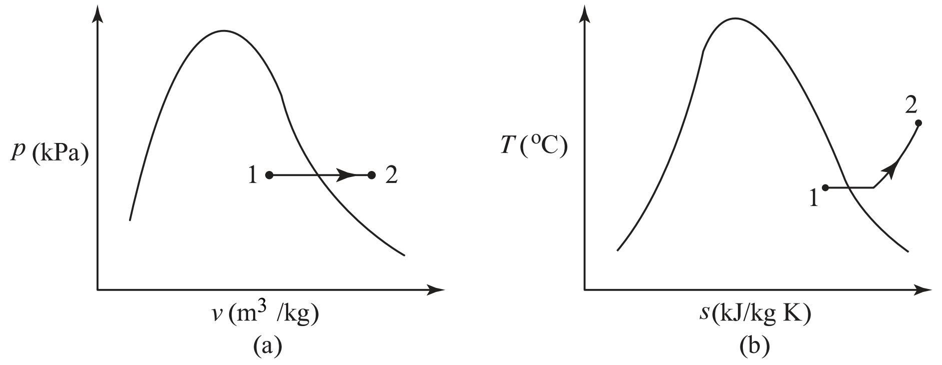


Fig. 8.29 Constant pressure process

$$Q = W + du = (p_2v_2 - p_1v_1) + (u_2 - u_1) = (u_2 + p_2v_2) - (u_1 + p_1v_1) \quad (8.46)$$

$$Q = h_2 - h_1 (\because h = u + pv) \quad (8.47)$$

If the steam is in wet condition at x_2 after heating, then substitute $v_2 = x_2v_{g2}$.

(iii) *Constant temperature process*

Consider the process 1-2 shown in Fig.8.29. At state 1, the steam is wet with properties x_1, p_1, v_{g1}, T_1. Assume that it is heated at constant temperature to dry saturated state 2 with p_2, V_{sat2}, T_{sat2}. Figure 8.30 shows p-v and T-s diagram of this process. *In wet region, constant temperature process is same as constant pressure process.* In the superheated region, steam behaves like a gas and the constant pressure process becomes hyperbolic process. Therefore, constant temperature process is limited to wet region only. For constant pressure process, work done is given by the equation:

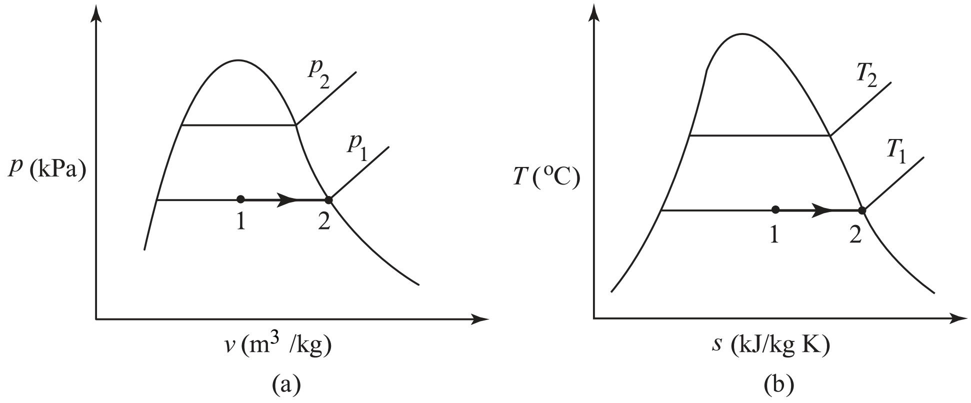


Fig. 8.30 Constant temperature process

$$W = p_2v_2 - p_1v_1 = p(v_2 - v_1) \qquad (p_2 = p_1 = p) \quad (8.48)$$

$$Q = h_2 - h_1 (\because h = u + pv) \quad (8.49)$$

(iv) *Hyperbolic process*

Note that, this process is applicable to superheated region only. This process follows the law pv = constant. Understand that for pure substances like steam, pv = constant is not a constant temperature process

$$W = p_1v_1 \log_e\left(\frac{v_2}{v_1}\right) \tag{8.50}$$

From first law of thermodynamics,

$$Q = W + \Delta U \tag{8.51}$$

$$Q = p_1v_1 \log_e\left(\frac{v_2}{v_1}\right) + u_2 - u_1 = p_1v_1 \log_e\left(\frac{v_2}{v_1}\right) + (h_2 - p_2v_2) - (h_1 - p_1v_1)$$

$$= p_1v_1 \log_e\left(\frac{v_2}{v_1}\right) + (h_2 - h_1) \qquad (\because p_1v_1 = p_2v_2) \tag{8.52}$$

(v) *Reversible adiabatic or isentropic process*: During this process, entropy remains constant ($s_2 = s_1$). It is represented by a vertical line in T-s diagram as shown in Fig.8.31. For adiabatic process, heat transfer, $Q = 0$ and from first law of thermodynamics,

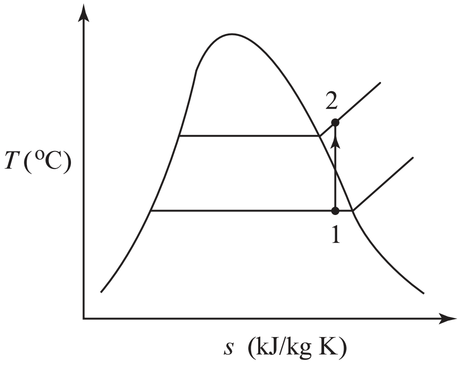


Fig. 8.31 Reversible adiabatic or isentropic process

$Q = W + \Delta u$. As $Q = 0$ for this process, work done, $W = \Delta u = u_1 - u_2$.

(vi) *Polytropic process*

Polytropic process follows the law pv^n = constant. Figure 8.32 shows p-v and T-s diagrams of polytropic expansion process:

$$W = \frac{p_1v_1 - p_2v_2}{n-1} \tag{8.53}$$

From first law of thermodynamics, $Q = W + \Delta U$

$$W = \frac{p_1v_1 - p_2v_2}{n-1} + (u_2 - u_1) = \frac{p_1v_1 - p_2v_2}{n-1} + (h_2 - p_2v_2) - (h_1 - p_1v_1) \tag{8.54}$$

$$Q = \frac{n}{n-1}(p_1v_1 - p_2v_2) + (h_2 - h_1) \tag{8.55}$$

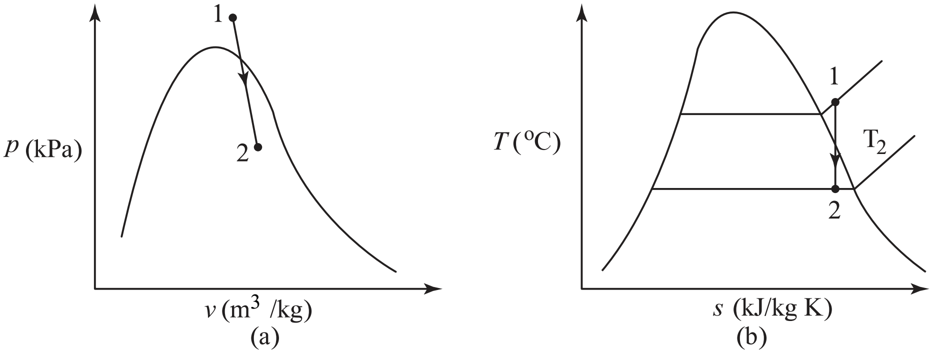


Fig. 8.32 Polytropic process

Irreversible adiabatic process also follows a polytropic condition with an index n. In case of steam, the value of index n varies from 1.13 (for wet steam) and 1.3 (for superheated steam).

(vii) *Throttling process*

During throttling process, enthalpy remains constant ($h_1 = h_2$). Figure 8.33 shows this process in T-s and h-s diagrams. Since the total enthalpy remains constant during the process, $h_1 = h_2$.

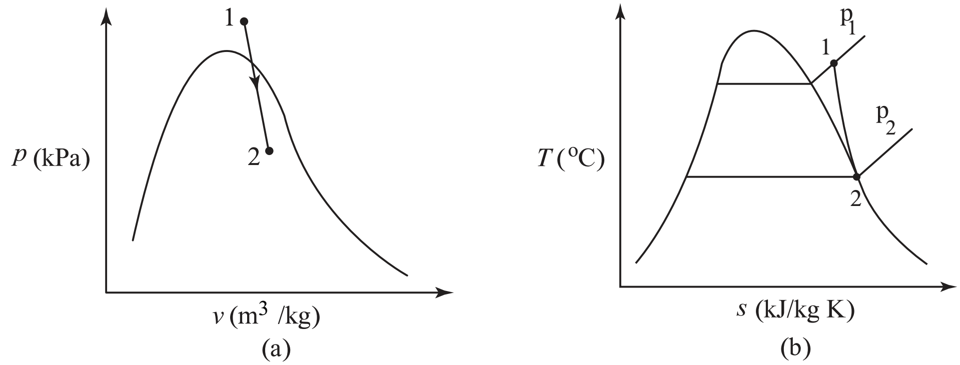


Fig. 8.33 Throttling process

$$h_{f1} + x_1 h_{fg1} \quad = \quad h_{f2} + x_2 h_{fg2} \tag{8.56}$$

The above equation is applicable only if the state of steam before and after the throttling process is in wet condition. If the steam is in superheated condition, then the above equation becomes

$$h_{f1} + x_1 h_{fg1} \quad = \quad h_{g2} + c_{ps}(T_{sup2} - T_{s2}) \tag{8.57}$$

Table 8.7 gives the various property relations in a nutshell. Table 8.8 provides the corresponding equations for work and heat transfer for non-flow processes and Table 8.9 provides details for flow processes.

Table 8.7 For properties of steam

Properties	Wet steam	Dry steam	Superheated steam
Enthalpy (h) (kJ/kg)	$h_{wet} = h_f + xh_{fg}$	$h_g = h_f + h_{fg}$	$h_{sup} = h_g + c_p(T_{sup} - T_s)$
Specific volume (v) (m^3/kg)	$v_{wet} = xv_g$	$v_{dry} = v_g$	$v_{sup} = v_g T_{sup}/T_s$
Density (ρ) (kg/m^3)	$\rho_{wet} = \dfrac{1}{v_{wet}}$	$\rho_g = \dfrac{1}{v_g}$	$\rho_{sup} = \dfrac{1}{v_{sup}}$
Workdone (W) (kJ/kg)	$W_{wet} = 100\ pv_{wet}$	$W_g = 100\ pv_g$	$W_{sup} = 100\ pv_{sup}$
Internal energy (U) (kJ/kg)	$U_{wet} = h_{wet} - W_{wet}$	$U_g = h_g - W_g$	$U_{sup} = h_{sup} - W_{sup}$
Specific entropy (s) (kJ/kg K)	$s_{wet} = s_f + xs_{fg}$	$s_{dry} = s_f + s_{fg}$	$s_{sup} = s_g + c_{ps} \log_e (T_{sup}/T_s)$

Table 8.8 For non-flow processes

Processes	Workdone	Heat transfer
Constant volume	$W = 0$	$Q = (h_2 - h_1) - (p_2 v_{sup2} - p_1 x_1 v_{g1})$
Constant pressure	$W = p(v_2 - v_1)$	$Q = h_2 - h_1$
Constant temperature	$W = p(v_2 - v_1)$	$Q = h_2 - h_1$
Hyperbolic	$W = p_1 v_1 \log_e (v_2/v_1)$	$Q = p_1 v_1 \log_e (v_2/v_1) + (h_2 - h_1)$
Isentropic	$W = u_1 - u_2$	$Q = 0$
Polytropic	$W = \dfrac{p_1 v_1 - p_2 v_2}{n - 1}$	$Q = \dfrac{n}{n-1} (p_1 v_1 - p_2 v_2) + (h_2 - h_1)$

Table 8.9 For flow processes

Systems	Workdone	Heat transfer
Boiler	$W = 0$	$Q = h_2 - h_1$
Turbine	$W = h_2 - h_1$	$Q = 0$
Condenser	$W = 0$	$Q = h_2 - h_1$
Nozzle	$W = 0$	$Q = 0$

Worked-Out Examples

8.1 Using steam tables, find the following at a pressure of 1.2 MPa: (i) saturation temperature; (ii) change in specific volume; (iii) change in entropy during evaporation; and (iv) latent heat of vaporisation of steam.

Solution

Refer steam table.

Saturation temperature, t_{sat} = 187.99 °C $\overset{\text{Ans}}{\Longleftarrow}$

Specific volume of saturated liquid, v_f = 0.001139 m^3/kg

Specific volume of saturated vapour, v_g = 0.16333 m^3/kg

Change in specific volume = $v_g - v_f = 0.16333 - 0.001139$

= 0.162191 m^3/kg $\overset{\text{Ans}}{\Longleftarrow}$

Entropy of saturated liquid, s_f = 2.2165

Entropy of saturated vapour, s_g = 6.5233

Change in entropy during evaporation = $s_g - s_f$

= $6.5233 - 2.2165 = 4.3068$ kJ/kg K $\overset{\text{Ans}}{\Longleftarrow}$

Latent heat of vaporisation = 1986.2 $\overset{\text{Ans}}{\Longleftarrow}$

8.2 Saturated steam has an entropy of 6.0018 kJ/kg K. What are its pressure, temperature, specific volume and enthalpy?

Solution

From steam table, when $s_g = 6.0018$, $p = 4.6886$ MPa $\overset{\text{Ans}}{\Longleftarrow}$

t = 260 °C $\overset{\text{Ans}}{\Longleftarrow}$

v_g = 0.04220 m^3/kg $\overset{\text{Ans}}{\Longleftarrow}$

h_g = 2796.9 kJ/kg $\overset{\text{Ans}}{\Longleftarrow}$

8.3 Find the enthalpy and entropy of steam when the pressure is 2 MPa and specific volume is 0.09 m^3/kg.

Solution

In steam table for p is 2 MPa, v_f=0.001177 m^3/kg and v_g = 0.9963 m^3/kg. Since the given volume lies between v_f and v_g, the steam will be a mixture of liquid and vapour. The state will be within the vapour dome. When in the two-phase region, the composition of the mixture or its quality should be evaluated first. For this,

$$v = v_f = xV_{fg}$$

$$0.09 = 0.001177 + x(0.09963 - 0.001177)$$

$$x = 0.902 \quad \text{or} \quad 90.2$$

At 2 MPa, $h_f = 908.79$ kJ/kg $h_{fg} = 1890.7$ kJ/kg
$s_f = 2.4474$ kJ/kg K $s_{fg} = 3.8935$ kJ/kg K

$$h = h_f + xh_{fg} = 908.79 + 0.902 \times 1890.7 = 2614.20 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$s = s_f + xs_{fg} = 2.4474 + 0.902 \times 3.8935 = 5.9594 \text{ kJ/kg K} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.4 Find the specific volume, enthalpy and entropy of steam at 1.4 MPa and 300 °C.
Solution

At 1.4 MPa, from steam take $t_{sat} = 195.07$ °C. Therefore, the state of the steam will be superheated steam. Now, the properties of the superheated steam can be seen from steam table. At 1.4 MPa and 300 °C,

$$v = 0.18228 \text{ m}^3/\text{kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$h = 3040.4 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$s = 6.9533 \text{ kJ/kg J} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.5 A vessel of volume 0.05 m^3 contains a mixture of saturated water and saturated steam at temperature 250 °C. The mass of the liquid present is 10 kg. Find the pressure, mass, specific volume, enthalpy, entropy and internal energy.
Solution

From steam table, at 250 °C, $p_{sat} = 3.973\ MPa$ and

$v_f = 0.0012512$ m^3/kg $h_f = 1085.36$ kJ/kg $s_g = 2.7927\ kJ/$ kg K
$v_g = 0.05013$ m^3/kg $h_{fg} = 1716.2$ kJ/kg $s_{fg} = 3.2802$ kJ/kg K

Volume of liquid, $V_f = mv_f = 10 \times 0.0012512 = 0.012512 \text{ m}^3$

Volume of vapour V_g = Volume of vessel − Volume of liquid

$$V_g = 0.05 - 0.012512 = 0.037488 \text{ m}^3$$

$$\text{Mass of vapour, } m_g = \frac{V_g}{v_g} = \frac{0.037488}{0.05013} = 0.74782 \text{ kg}$$

Total mass of mixture, $m = m_f + m_g = 10 + 0.74782 = 10.74782$ kg

Quality of mixture $x = \dfrac{m_g}{m} = \dfrac{0.74782}{10.74782} = 0.06958$

$$\begin{aligned} v = V_f + xV_g &= 0.012512 + 0.06958 \times (0.05013 - 0.012512) \\ &= 0.0151 \text{ m}^3/\text{kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\ h = h_f + xh_{fg} &= 1085.36 + 0.074782 \times 1716.2 = 1213.7 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\ s = s_f + xs_{fg} &= 2.7927 + 0.06958 \times 3.2802 = 3.0209 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\ u = h - pv &= 1213.7 - 3.973 \times 10^3 \times 0.0151 = 1153.71 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

8.6 Consider a power plant in which steam is at a pressure of 0.3 MPa and temperature is 250 °C. It is cooled at constant volume.

(i) Find at what temperature, the steam becomes a saturated vapour.
(ii) What will be its quality at 80 °C?
(iii) What is the heat transfer per kg of steam by cooling it from 250 °C to 80 °C?

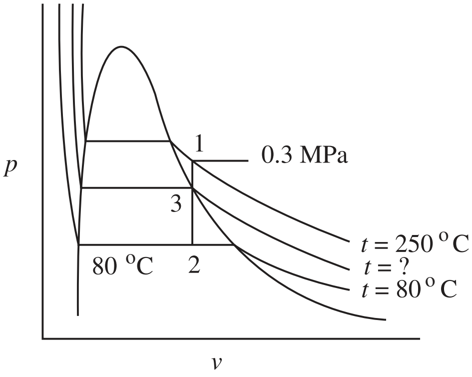


Fig. 8.34

Solution

From steam tables, we find that at 0.3 MPa, $t_{sat} = 133.55$ °C. Since $t > t_{sat}$, the state would be in superheated state (Fig.8.34). From steam tables, the properties of superheated steam at 0.3 MPa and 250 °C.

$$\begin{aligned} v_1 &= 0.7964 \text{ m}^3/\text{kg} \quad \text{and } h_1 = 2967.6 \text{ kJ/kg} \\ v_1 &= v_2 = v_3 = 0.7964 \text{ m}^3/\text{kg (Fig.8.34)} \end{aligned}$$

In steam table, when $v_g = 0.8919$ m^3/kg, $t_{sat} = 120$ °C and when $v_g = 0.7706$ m^3/kg, $t_{sat} = 125$ °C. Therefore, when $v_g = 0.7964, t_{sat}$ can be calculated by interpolation. This would be 123.9 °C. Steam would become saturated vapour at $t = 123.9$ °C. Therefore,

at 80 °C,

$v_f = 0.001029$ m^3/kg $\quad h_f = 334.91$ kJ/kg $\quad p_{sat} = 47.39$ kPa

$v_g = 3.407$ m^3/kg $\quad h_{fg} = 2308.8$ kJ/kg $\quad v_1 = v_2 = 0.7964$ m^3/kg

$$\begin{aligned} v_2 &= v_{f_{80^\circ\mathrm{C}}} + x_2 v_{fg_{80^\circ\mathrm{C}}} = 0.7964\ \mathrm{m^3/kg} \\ &= 0.001029 + x_2(3.407 - 0.001029) \\ x_2 &= \frac{0.79537}{3.40597} = 0.234 \end{aligned}$$

$$\begin{aligned} h_2 &= 334.91 + 0.234 \times 2308.8 = 875.2\ \mathrm{kJ/kg} \\ p_2 &= 47.39\ \mathrm{kPa} \end{aligned}$$

From the first law of thermodynamics, $\delta Q = du + pdv$:

$$\begin{aligned} \therefore (\delta Q)_v &= du \quad (\because dv = 0) \\ Q_{1-2} &= u_2 - u_1 = (h_2 - p_2 v_2) - (h_1 - p_1 v_1) \\ &= (h_2 - h_1) + v(p_1 - p_2) \\ &= (875.2 - 2967.6) + 0.7964 \times (300 - 47.39) \\ &= -2091.4 + 201.2 = -1890.2\ \mathrm{kJ/kg} \end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

8.7 Consider a steam turbine shown in Fig.8.35. The entry temperature of steam is 300 °C at 1.5 MPa. It expands isentropically to 40 °C. Calculate the ideal work output of the turbine per kg of steam.

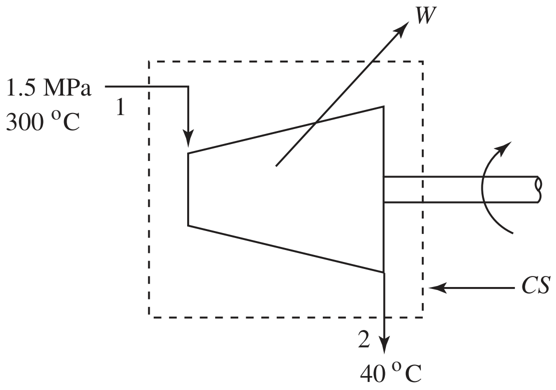


Fig. 8.35

Solution

The control volume is shown in the figure with dotted lines. The steady flow equation can be written as $W = h_1 - h_2$.

The work done by the steam is due to the fall in its enthalpy value. Assume that the process is reversible and adiabatic. The process is shown in T-s and h-s diagram.

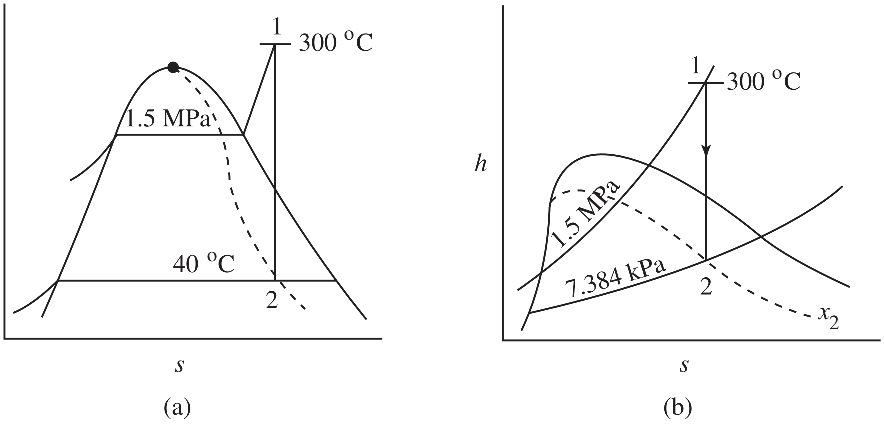


Fig. 8.36

From steam table, at 40 °C,

$s_f = 0.5725$ kJ/kg K $h_f = 167.57$ kJ/kg $p_{sat} = 7.384\ kPa$

$s_{fg} = 7.6845$ kJ/kg K $h_{fg} = 2406.7$ kJ/kg

At $p = 1.5$ MPa, $t = 300$ °C from tabulated properties of superheated steam from the steam table, we get $s_1 = 6.9189$ kJ/kg K and $h_1 = 3037.6$ kJ/kg:

$$s_1 = s_2$$

$$6.9189 = s_f + x_2 \times s_{fg40\ ^\circ\text{C}} = 0.5725 + x_2 \times 7.6845$$

$$x_2 = \frac{6.3464}{7.6845} = 0.826$$

$$h_2 = h_{f_{40^\circ\text{C}}} + x_2 h_{fg_{40^\circ\text{C}}} = 167.57 + 0.826 \times 2406.7 = 2155.50 \text{ kJ/kg}$$

$$W = h_1 - h_2 = 3037.6 - 2155.50 = 882.1 \text{ kJ/kg} \qquad \xLeftarrow{\textbf{Ans}}$$

8.8 Consider Fig.8.37, two steams are mixed adiabatically in pipe 1 the steam is flowing at the rate of 1 kg/s at 0.8 MPa and 250 °C. In pipe 2, wet steam at 0.8 MPa and 0.95 dry is flowing at the rate pf 1.3 kg/s. Evaluate the condition of steam after mixing. The mixture now expands in a frictionless nozzle isentropically to a pressure of 0.4 MPa. Calculate the exit velocity of steam from the nozzle. Neglect the velocity of steam in the pipe.

Solution

The energy equation for the adiabatic mixing of the two steams gives:

$$m_1h_1 + m_2h_2 = m_3h_3 \qquad (1)$$

at 0.8 MPa and 250 °C,

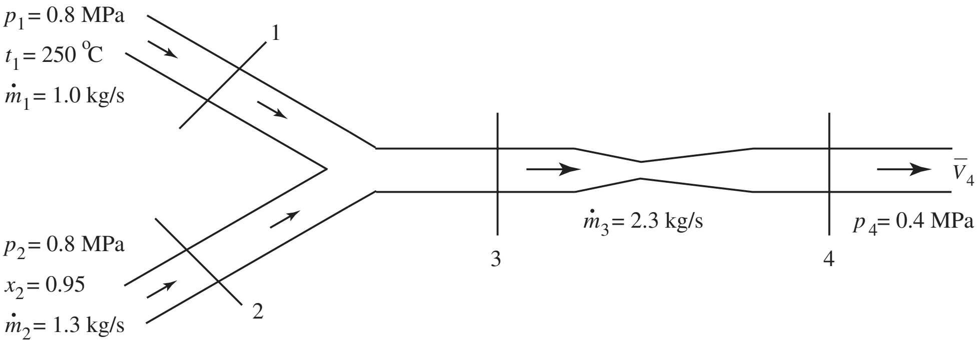
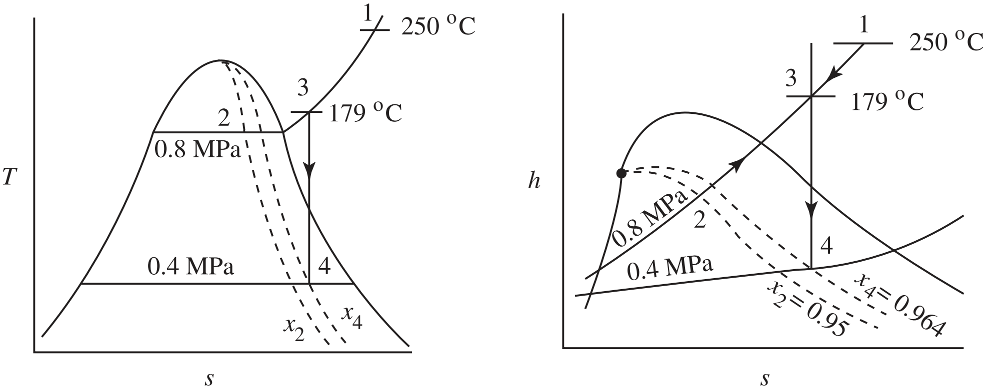


Fig. 8.37



(a) (b)

Fig. 8.38

$$h_1 = 2950.0 \text{ kJ/kg}$$

$$h_2 = 721.11 + 0.95 \times 2048 = 2666.71 \text{ kJ/kg}$$

Therefore, from Eq.(1)

$$1 \times 2950 + 1.3 \times 2666.71 = 2.3 \times h_3$$

$$h_3 = 2790 \text{ kJ/kg}$$

Since $(h_g)_{0.8 \text{ MPa}} = 2769.1$ kJ/kg and $h_3 > h_{g\ steam}$ is in a superheated region.

From steam tables for $p = 0.8$ MPa and $t = 200$ °C, $h = 2839.3$ kJ/kg.

When $p = 0.8$ MPa, $t_{sat} = 170.43$ °C and $h_g = 2769.1$ kJ/kg.

By linear interpolation, $t_3 = 179$ °C. Therefore, degree of superheat $= 179 - 170.43 = 8.57$ °C.

Condition of steam after mixing = 0.8 MPa and 179 °C

Energy equation for nozzle is given by, $\dfrac{v_4^2}{2} + h_4 = h_3$.

Since, v_3 is assumed to be zero, steam expands isentropically in the nozzle to 0.4 MPa. By interpolation:

$$\begin{aligned} s_3 &= 6.7087\ kJ/kg\ K = s_4 \\ 6.7087 &= 1.7763 + x_4 \times 5.1193 \\ x_4 &= 0.964 \end{aligned}$$

At 0.4 MPa, from steam tables, $h_{f4} = 604.74$ kJ/kg and $h_{fg} = 2133.8$ kJ/kg.

$$\begin{aligned} h_4 &= 604.74 + 0.964 \times 2133.8 = 2661.72 \text{ kJ/kg} \\ v_4^2 \times 10^{-3} &= 2(h_3 - h_4) = 2 \times (2790 - 2661.72) = 256.6 \\ v_4 &= \sqrt{25.66 \times 10^4} = \sqrt{25.66} \times 100 = 506.6 \text{ m/s} \end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

The processes are shown on the *h*-*s* and *T*-*s* diagram in Fig.8.38.

8.9 In a pipeline, steam flows at 1.5 MPa. It is made to expand by means of a throttling calorimeter to a pressure of 0.1 MPa and temperature of 120 °C. Estimate the quality of steam in the pipeline. What is the maximum moisture at 1.5 MPa that can be determined with this set up if at least 5 °C of superheat is expected after throttling for accurate readings.

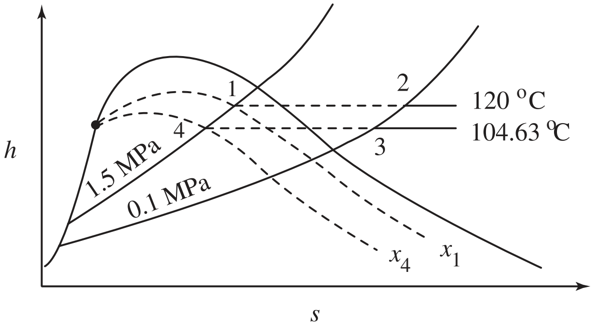


Fig. 8.39

Solution

Refer Fig.8.39. At state 2, when $p = 0.1$ MPa and $t = 120$ °C by interpolation, $h_2 = 2716.2$ kJ/kg, $p = 1.5$ MPa, $h_f = 844.9$ kJ/kg and $h_{fg} = 1947.3$ kJ/kg.

As $h_2 = h_1 = 2716.2$ kJ/kg, now,

$$h_1 = h_2 = h_{f_{1.5\text{MPa}}} + x_1 h_{fg_{1.5\text{MPa}}} = 844.9 + x_1 \times 1947.3$$

$$x_1 = \frac{1871.3}{1947.3} = 0.960$$

When $p = 0.1$ MPa and $t = 99.63 + 5 = 104.63$ °C,

$$h_3 = 2685.5 \text{ kJ/kg}$$

Since $h_3 = h_4$, $\quad 2685.5 = 844.89 + x_4 \times 1947.3$

$$x_4 = \frac{1840.6}{1947.3} = 0.945 \qquad (\overset{\textbf{Ans}}{\Longleftarrow})$$

The maximum moisture that can be determine with this set up is only 5.5%. $\overset{\textbf{Ans}}{\Longleftarrow}$

8.10 The following data were measured while doing an experiment with a separating and throttling calorimeter:

Pipeline pressure	1.5 Mpa
Conditions after throttling	0.1 MPa, 110 °C
During 5 min, moisture collected in the separator	0.150 litre at 70 °C
Steam condenser after throttling during 5 min of operation	3.24 kg

Estimate the quality of steam in the pipeline.

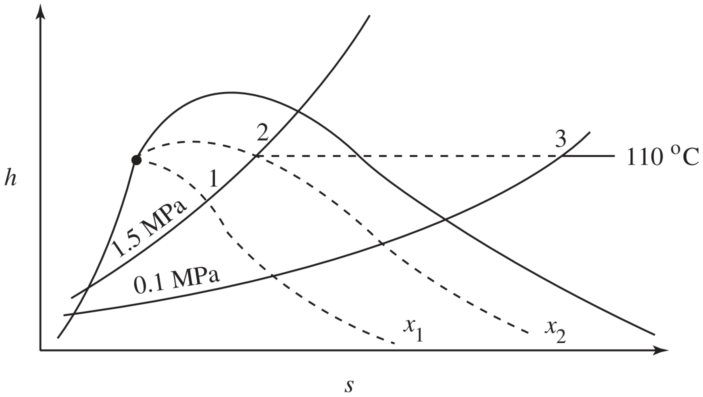


Fig. 8.40

Solution

Refer Fig.8.40. At 0.1 MPa and 110 °C, $h_3 = 2696.2$ kJ/kg.

$$h_3 = h_2 = h_{f1.5\ MPa} + x_2 h_{fg1.5\ MPa}$$

$$2696.2 = 844.89 + x_2 \times 1947.3$$

$$x_2 = \frac{1851.31}{1947.3} = 0.951$$

If m_1 is the mass of moisture collected in separator in 5 min and m_2 is the mass of steam condensed after throttling in 5 min, then

$$x_1 = \frac{x_2 m_2}{m_1 + m_2}$$

At 70 °C, $V_f = 0.001023 \text{ m}^3/\text{kg}$.

$$m_1 = \frac{150 \times 10^{-6}}{1023 \times 10^{-6}} = 0.1462 \text{ kg}$$

$$m_2 = 3.24 \text{ kg}$$

$$x_1 = \frac{0.955 \times 3.24}{0.1462 + 3.24} = \frac{3.1}{3.3862} = 0.915 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.11 Five cubic meter of steam and equal volume of water is contained in a steam boiler initially. Steam is withdrawn from the boiler at a constant pressure until 4 m^3 of water is left. Calculate the amount of heat transferred during the process.

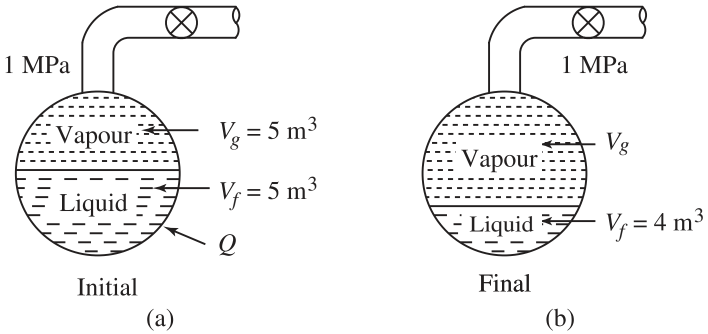


Fig. 8.41

Solution

At 1 MPa,

$v_f = 0.001127$ m^3/kg	$u_f = 761.68$ kg	$u_{fg} = 1822$ kJ/kg
$v_g = 0.1944$ m^3/kg	$h_g = 2778.1$ kJ/kg	$u_g = 2583.6$ kJ/kg

The initial mass of saturated water an steam in the boiler (refer Fig.8.41):

$$\frac{V_f}{v_f} + \frac{V_g}{v_g} = \frac{5}{0.001127} + \frac{5}{0.1944} = \left(4.44 \times 10^3 + 25.72\right) \text{ kg}$$

where suffix f refers to saturated water and suffix g refers to saturated vapour. Final mass of saturated water and steam,

$$\frac{4}{0.001127} + \frac{6}{0.1944} = \left(3.55 \times 10^3 + 30.86\right) \text{ kg}$$

Mass of steam taken out of the boiler,

$$m_s = \left(4.44 \times 10^3 + 25.72\right) - \left(3.55 \times 10^3 + 30.86\right) = 0.89 \times 10^3 - 5.14 = 884.86 \text{ kg}$$

Making an energy balance, we have, initial energy stored in saturated water and steam + heat transferred from the external source = final energy stored in saturated water and steam + energy leaving with the steam:

$$U_1 + Q = U_f + m_s h_g$$

Let us assume that the steam taken out is dry, $x = 1$:

$m_f u_f + m_g u_g + Q = m_{rf} u_f + m_{mix} u_g + m_s h_g$

where m_{rf} is mass remaining fluid (water) and m_{mix} is mass of water and steam:

$4.44 \times 10^3 \times 761.68 + 25.72 \times 2583.6 + Q = 3.55 \times 10^3 \times 761.68 + 30.86 \times 2583.6 + 884.86 \times 2778.1$

$$Q = 1793614 \text{ kJ} = 1793.614 \text{ MJ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.12 A 280-mm diameter cylinder is fitted with a frictionless leak proof piston, contains 0.02 kg of steam at a pressure of 0.6 MPa and temperature of 200 °C. As the piston moves slowly outwards through a distance of 305 mm, the steam undergoes a fully resisted expansion during which the steam pressure p and steam volume V are related by pV^n = constant where n is the index of expansion. The final pressure of steam is 0.12 MPa. Find

(i) the value of n

(ii) the work done by the steam

(iii) the magnitude and sign of the heat transfer

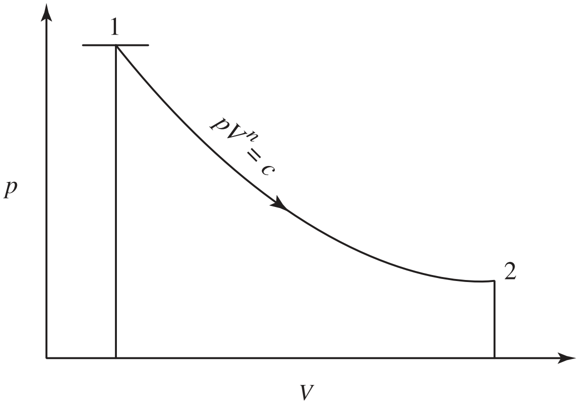


Fig. 8.42

Solution

As the expansion follows the equation, $pV^n = C; p_1 V_1^n = p_2 V_2^n$.

Taking log on both sides, $n = \dfrac{\log \frac{p_1}{p_2}}{\log \frac{V_2}{V_1}}$

Now, at 0.6 MPa and 200 °C from steam stable, $v_1 = 0.352 \ \ \text{m}^3/\text{kg}$ and $h_1 = 2850.1$ kJ/kg

Volume, V_1 at state 1 $= 0.352 \times 0.02 = 0.00704 \text{ m}^3$

$$\text{Displaced volume} = \frac{\pi}{4} d^2 \ell = \frac{\pi}{4} \times (0.28)^2 \times 0.305 = 0.0188 \text{ m}^3$$

Therefore, total volume V_2 after expansion $= 0.0188 + 0.00704 = 0.02584 \text{ m}^3$

$$n = \frac{\log\left(\dfrac{0.6}{0.12}\right)}{\log\left(\dfrac{0.02584}{0.00704}\right)} = 1.24$$

Work done by steam in the expansion process,

$$W_{1-2} = \int_{V_1}^{V_2} pdV = \frac{p_1V_1 - p_2V_2}{n-1}$$

$$= \frac{6 \times 10^5 \times 0.00704 - 1.2 \times 10^5 \times 0.02584}{1.24 - 1} = \frac{4224 - 3100.8}{0.24}$$

$$= 4680 \text{ Nm} = 4.68 \text{ kJ}$$

$$V_2 = 0.02584 \text{ m}^3$$

$$v_2 = \frac{0.02584}{0.02} = 1.292 \text{ m}^3/\text{kg}$$

$$v_2 = v_{f_{0.12\ MPa}} + x_2 v_{fg_{0.12\ MPa}}$$

$$1.292 = 0.0010476 + x_2 \times 1.4271$$

$$x_2 = \frac{1.291}{1.427} = 0.906$$

At 0.12 MPa, $u_f = 439.3$ kJ/kg and $u_g = 2512.0$ kJ/kg:

$$u_2 = u_f + x_2(u_g - u_f) = 439.4 + 0.906 \times (2512 - 439.3) = 2317.3 \text{ kJ/kg}$$

$$h_1 = 2850.1 \text{ kJ/kg}$$

$$u_1 = h_1 - p_1V_1 = 2850.1 - \frac{0.6 \times 10^3 \times 0.00704}{0.02}$$

$$= 2638.9 \text{ kJ/kg}$$

By the first law,

$$Q_{1-2} = U_2 - U_1 + W_{1-2} = m(u_2 - u_1) + W_{1-2}$$

$$= 0.02 \times (2317.3 - 2638.5) + 4.68 = -6.424 + 4.68$$

$$= -1.749 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.13 Consider Figure shown in Fig.8.43. There are two chambers in an insulated vessel. The left portion contains 5 kg of dry saturated steam at 0.2 MPa. The right portion contains 10 kg of steam at 0.5 MPa with 0.8 quality. If the portion is suddenly removed and the steam is mixed thoroughly and allowed to settle, find the final pressure, steam quality and entropy change in the process.

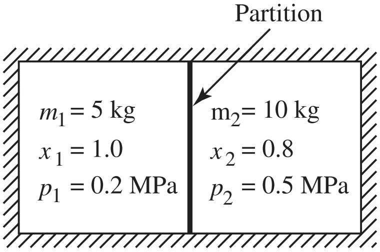


Fig. 8.43

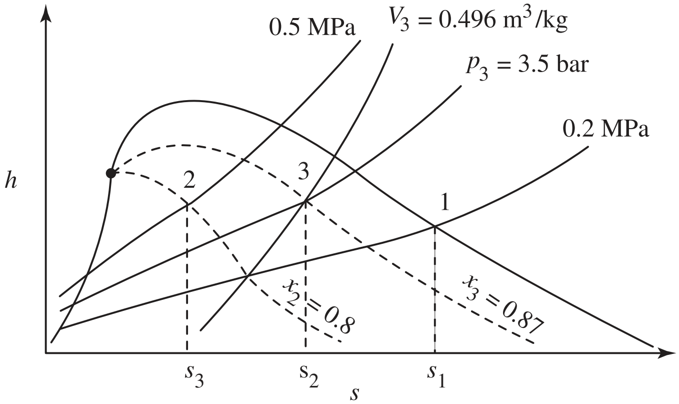


Fig. 8.44

Solution

At 20 MPa,

$$v_g = v_1 = 0.8857 \ \text{m}^3/\text{kg}$$

$$V_1 = m_1 v_1 = 5 \times 0.8857 = 4.4285 \text{ m}^3$$

At 0.5 MPa,

$$v_2 = v_f + x_2 v_{fg} = 0.001093 + 0.8 \times 0.3749 = 0.30101 \text{ m}^3/\text{kg}$$

$$\therefore V_2 = m_2 v_2 = 10 \times 0.30101 = 3.0101 \text{ m}^3$$

$$\therefore \text{Total volume, } V_m = V_1 + V_2 = 4.4285 + 3.0101 = 7.4386 \text{ m}^3 \text{ (of mixture)}$$

Total mass of the mixture,

$$m_m = m_1 + m_2 = 5 + 10 = 15 \text{ kg}$$

Specific volume of the mixture,

$$v_m = \frac{V}{m_m} = \frac{7.4386}{15} = 0.496 \text{ m}^3/\text{kg}$$

By energy balance, $m_1 u_1 + m_2 u_2 = m_3 u_3$

At 0.2 MPa, $h_g = h_1 = 2706.7$ kJ/kg,

$$u_1 = h_1 - p_1V_1 \approx 2706.7 \text{ kJ/kg}$$

At 0.5 MPa,

$$h_2 = h_f + x_2h_{fg} = 640.23 + 0.8 \times 2108.5 = 2327.03 \text{ kJ/kg}$$

$$u_2 = h_2 - p_2V_2 \approx h_2 = 2327.03 \text{ kJ/kg}$$

$$h_3 = h_m = \frac{5 \times 2706.7 + 10 \times 2327.03}{15} = 2453.6 \text{ kJ/kg} \approx u_3$$

Now, for mixture $h_3 = 2453.6$ kJ/kg $= u_3$ and $v_3 = 0.496$ m^3/kg.

From the Mollier chart with the given values of h and v, point 3 after mixing is fixed (Fig.8.44):

$$x_3 = 0.870$$

$$s_3 = 6.298 \text{ kJ/kg K}$$

$$p_3 = 3.5 \text{ bar}$$

$$s_4 = s_{g_{0.2\ MPa}} = 7.1271 \text{ kJ/kg K}$$

$$\begin{aligned} s_2 &= s_{f_{0.5 \text{ MPa}}} + 0.8 \times s_{fg_{0.5 \text{ MPa}}} \\ &= 1.8607 + 0.8 \times 4.9606 = 5.8292 \text{ kJ/kg K} \end{aligned}$$

Therefore, entropy change during the process:

$$\begin{aligned} &= m_3s_3 - (m_1s_1 + m_2s_2) \\ &= 15 \times 6.298 - (5 \times 7.1271 + 10 \times 5.8292) = 0.54 \text{ kJ/kg} \end{aligned}$$

$\xleftarrow{\textbf{Ans}}$

8.14 A steam turbine is supplied with steam at a pressure of 6 MPa and 400 °C through a throttle valve which reduces the pressure to 5 MPa. Steam expands in the turbine to 0.2 MPa and assume that expansion is adiabatic. The isentropic efficiencies of the turbine is 82%. Determine the availability of steam before and after the throttle valve and the turbine exhaust. Neglecting KE and PE, calculate the specific work output from the turbine. Take $T_0 = 20$ °C.

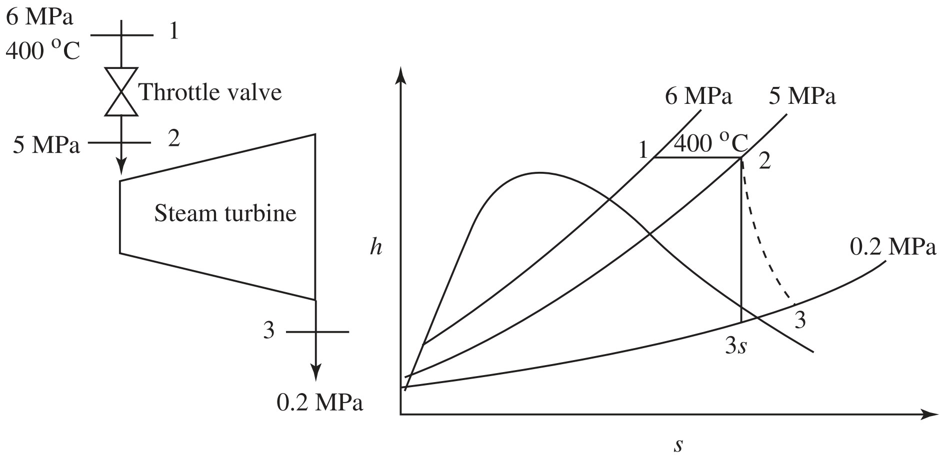
Solution

Steady flow availability ψ is given by,

$$\psi = (h - h_0) - T_0(s - s_0) + PE + KE$$

Subscript 0 refers to the surroundings. Since PE and KE changes are negligible.

ψ_1 is availability of steam before throttling:



(a) (b)

Fig. 8.45

$$\psi_1 = (h_1 - h_0) - T_0(s_1 - s_0)$$

At 6 MPa, 400 °C (Fig.8.45) $h_1 = 3177.2$ kJ/kg and $s_1 = 6.5408$ kJ/kg K.

At 20 °C, $h_0 = 83.96$ kJ/kg and $s_0 = 0.2966$ kJ/kg K:

$$\psi = (3177.2 - 83.96) - 293 \times (6.5408 - 0.2966) = 1263.7 \text{ kJ/kg}$$

For throttling process, $h_1 = h_2$.

At $h = 3177.2$ kJ/kg and $p = 5$ MPa from the superheated steam table:

$$t_2 = 390\ ^\circ\ \text{C} \quad \text{and} \quad s_2 = 6.63\ \ \text{kJ kg K}$$

ψ_2 = availability of steam after throttling:

$$\begin{aligned}\psi_2 &= (h_2 - h_0) - T_0(s_2 - s_0)\\ &= (3177.2 - 83.96) - 293 \times (6.63 - 0.2966) = 1237.55 \text{ kJ/kg}\end{aligned}$$

Decrease in availability due to throttling:

$$\begin{aligned}\psi_1 - \psi_2 &= 1263.7 - 1237.55 = 26.15 \text{ kJ/kg}\\ s_2 &= s_{3s} = 6.63 = 1.5301 + x_{3s}(7.1271 - 1.5301)\\ x_{3s} &= \frac{5.10}{5.5970} = 0.9112\\ h_{3s} &= h_4 + x_{3s}h_{fg3s}\end{aligned}$$

From steam table, $h_4 = 504.7$ kJ/kg and $h_{fg3s} = 22091$ kJ/kg:

$$
\begin{aligned}
h_{3s} &= 504.7 + 0.9112 \times 2209 = 2517.5 \text{ kJ/kg} \\
h_2 - h_{3s} &= 3177.2 - 2517.5 = 659.7 \text{ kJ/kg} \\
h_2 - h_3 &= \eta_{is} \times (h_1 - h_{3s}) = 0.82 \times (3177.2 - 2517.5) = 540.95 \text{ kJ/kg}
\end{aligned}
$$

$$
\begin{aligned}
h_3 &= h_2 - 540.95 = 3177.2 - 540.95 = 2636.25 \text{ kJ/kg} \\
h_3 &= h_4 + x_3 h_{fg} \\
h_3 &= 2636.25 \text{ kJ/kg} = 504.7 + x_3 \times 2201.7 \\
x_3 &= \frac{2636.25}{2201.7} = 0.968 \\
s_3 &= 1.5301 + 0.968 \times 5.597 = 6.95
\end{aligned}
$$

ψ_3 = availability of steam at turbine exhaust:

$$
\begin{aligned}
\psi_3 &= (h_3 - h_0) - T_0(s_3 - s_0) \\
&= (2636.25 - 83.96) - 293 \times (6.95 - 0.2966) = 602.84 \text{ kJ/kg}
\end{aligned}
$$

Specific work output from the turbine, $W = h_2 - h_3$:

$$
h_2 - h_3 = 3177.2 - 2636.25 = 540.95 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
$$

The work done is less than the loss of availability of steam between states 2 and 3 because of the irreversibility accounted for, by the isentropic efficiencies.

8.15 A steam turbine is supplied 600 kg/h of steam at 25 bar, 350 °C. Steam is extracted at the rate of 150 kg/h at 200 °C at a certain stage. The remaining steam is let out at 0.2 bar, 0.92 dry. During the expansion process, there is heat transfer from the turbine to the surroundings at the rate of 10 kJ/s. Evaluate per kg of steam entering the turbine:

(i) the availability of steam entering and leaving the turbine,

(ii) the maximum work, and

(iii) the irreversibility.

The atmosphere is at 30 °C.

Solution

At 25 bar and 350 °C, $h_1 = 3125.87$ kJ/kg and $s_1 = 6.8486$ kJ/kg K.

At 30 °C, $h_0 = 125.79$ kJ/kg and $s_0 = s_{f_{30^\circ\text{C}}} = 0.4369$ kJ/kg K.

At 3 bar and 200 °C, $h_2 = 2865.5$ kJ/kg and $s_2 = 7.3115$ kJ/kg K.

At 0.2 bar (0.92 dry).

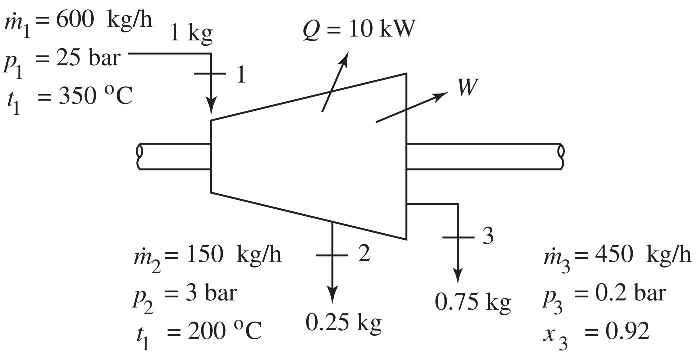


Fig. 8.46

$h_f = 251.4$ kJ/kg $\quad s_f = 0.8320$ kJ/kg K

$h_{fg} = -2358.3$ kJ/kg K $\quad s_g = 7.9085$ kJ/kg K

$$h_3 = 251.4 + 0.92 \times 2358.3 = 2421.04 \text{ kJ/kg}$$

$$s_3 = 0.8320 + 0.92 \times 7.0765 = 7.3424 \text{ kJ/kg K}$$

The states of steam are as shown in Fig.8.46.

(i) Availability of steam entering the turbine:

$$\psi_1 = (h_1 - h_0) - T_0(s_1 - s_0)$$
$$= (3125.87 - 125.79) - 303 \times (6.8481 - 0.4369) = 1057.48 \text{ kJ/kg}$$

Availability of steam leaving the turbine at state 2:

$$\psi_2 = (h_2 - h_0) - T_0(s_2 - s_0)$$
$$= (2865.5 - 125.79) - 303 \times (7.3115 - 0.4369) = 656.71 \text{ kJ/kg}$$

Availability of steam leaving the turbine at state 3:

$$\psi_3 = (h_3 - h_0) - T_0(s_3 - s_0)$$
$$= (2421.04 - 125.79) - 303 \times (7.3524 - 0.4369) = 199.85 \text{ kJ/kg}$$

(ii) Maximum work per kg of steam entering the turbine:

$$W_{rev} = \psi_1 - \frac{m_2}{m_1}\psi_2 - \frac{m_3}{m_1}\psi_3$$
$$= 1057.48 - 0.25 \times 656.71 - 0.75 \times 199.85 = 743.41 \text{ kJ/kg}$$

(iii) Irreversibility:

$$\begin{aligned} I &= T_0(m_2s_2 + m_3s_3 - m_1s_1) - Q \\ &= 303 \times (150 \times 7.3115 + 450 \times 7.3424 - 600 \times 6.8486) - (-10 \times 3600) \\ &= 124368.44 \text{ kJ/h} = 124.368 \text{ MJ/h} = \frac{124.461 \times 10^3}{3600} = 34.57 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

8.16 Determine the exergy of:

(i) 2 kg of water at 1 bar and 90 °C
(ii) 0.3 kg of steam at 4 MPa and 500 °C
(iii) 0.5 kg of wet steam at 0.1 bar and 0.85 quality
(iv) 6 kg of ice at 1 bar and 10 °C

Assume a dead state of 1 bar and 300 K.

Solution

At the dead state of 1 bar and 300 K:

$u_0 = 113.1$ kJ/kg $\qquad h_0 = 113.2$ kJ/kg K
$v_0 = 0.00105$ m^3/kg $\qquad s_0 = 0.395$ kJ/kg

Exergy of the system:

$$\begin{aligned} \phi &= m[(u + p_0v - T_0s) - (u_0 + p_0v_0 - T_0s_0)] \\ u_0 + p_0v_0 - T_0s_0 &= h_0 - T_0s_0 = 113.2 - 300 \times 0.395 = -5.3 \text{ kJ/kg} \end{aligned}$$

(i) For water at 1 bar and 90 °C:

$u = 376.9$ kJ/kg $\qquad h = 377$ kJ/kg
$v = 0.001035$ m^3/kg $\qquad s = 1.193$ kJ/kg K

since $p = p_0$:

$$\begin{aligned} u + p_0v - T_0s &= u + pv - T_0s = h - T_0s \\ &= 377 - 300 \times 1.193 = 19.1 \text{ kJ/kg} \\ \phi &= 2 \times [19.1 - (-5.3)] = 2 \times 24.4 = 48.8 \text{ kJ} \end{aligned}$$

(ii) At $p = 4$ MPa and $t = 500\ ^\circ$ C:

$u = 3099.8$ kJ/kg $\qquad h = 3446.3$ kJ/kg
$v = 0.08637$ m^3/kg $\qquad s = 7.090$ kJ/kg K

$$\begin{aligned} u + p_0v - T_0s &= 3099.8 + 100 \times 0.08637 - 300 \times 7.090 = 981.4 \text{ kJ/kg} \\ \phi &= 0.3 \times [981.4 - (-5.3)] = 296 \text{ kJ} \end{aligned}$$

(iii) At 0.1 bar, 0.85 quality:

$$\begin{aligned}
u &= 192 + 0.85 \times 2245 = 2100.25 \text{ kJ/kg} \\
h &= 192 + 0.85 \times 2392 = 2225.2 \text{ kJ/kg} \\
s &= 0.649 + 0.85 \times 7.499 = 7.023 \text{ kJ/kg K} \\
v &= 0.001010 + 0.85 \times 14.67 = 12.47 \;\; \text{m}^3\text{/kg} \\
u + p_0 v - T_0 s &= 2100.25 + 100 \times 12.47 - 300 \times 7.023 = 1240.4 \text{ kJ/kg} \\
\phi &= 0.5 \times (1240.4 - (-5.3)) = 622.85 \text{ kJ}
\end{aligned}$$

(iv) Since $p = p_0$:

$$\begin{aligned}
\phi &= U - U_0 + p_0(V - V_0) - T_0(S - S_0) \\
&= H - H_0 - V(p - p_0) - T_0(S - S_0) \\
&= m[(h - h_0) - T_0(s - s_0)]
\end{aligned}$$

At 100 kPa and −10 °C,

$$h = -354.1 \;\; \text{kJ/kg} \quad \text{and} \quad s = -1.298 \;\; \text{kJ/kg K}$$

$$\phi = 6 \times [-354.1 - 113.2 - 300 \times (-1.289 - 0.0395)] = -412.5 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.17 A steam of hot water is used for heating a relatively cold water. The hot water is at 90 °C and cold water is at 25 °C. The heated cold water temperature is raised to 50 °C. Assume that rate of cold water flow is 1 kg/s. The whole process is taking place in a parallel flow heat exchanger. It is to be seen that the exit water temperature of the hot water stream should never be less than 60 °C. However, in a counter flow operation, the exit temperature of hot water can be as low as 35 °C. Evaluate the second law efficiency and the rate of exergy destination in two modes of operation. Take $T_0 = 300$ K.

Solution

Refer Fig.8.47. Given:

$t_{h1} = 90\ ^\circ\ \text{C}$ $\quad$ $t_{c1} = 25\ ^\circ\ \text{C}$ $\quad$ $t_{c2} = 60\ ^\circ\ \text{C}$

$T_0 = 300\ \ \text{K}$ $\quad$ $\dot{m}_c = 1\ \ \text{kg/s}$

For parallel mode $t_{h2} = 60$ °C. Neglecting any heat loss:

$$\begin{aligned}
\dot{m}_h c_h (t_{h1} - t_{h2}) &= \dot{m}_c c_c (t_{c2} - t_{c1}) \\
\dot{m}_h \times (90 - 60) &= 1 \times (50 - 25) \\
\dot{m}_h &= 0.833 \text{ kg/s}
\end{aligned}$$

In counter flow mode, $t_{h2} = 35$ °C:

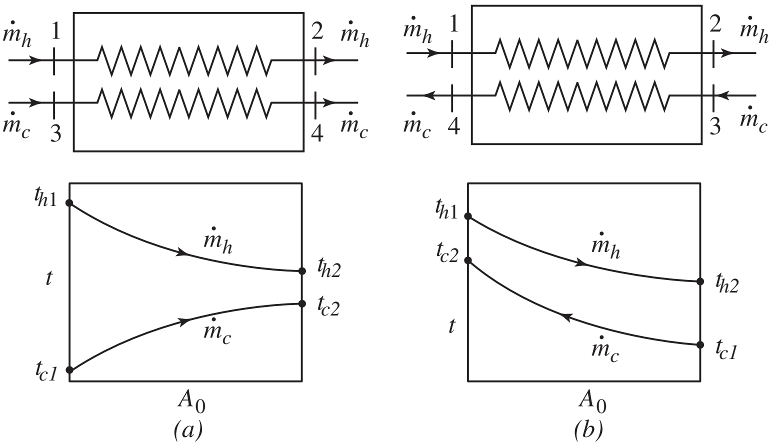


Fig. 8.47

$$\dot{m}_h \times (90 - 35) = 1 \times (50 - 25)$$

$$\dot{m}_h = \frac{25}{55} = 0.454 \text{ kg/s}$$

Thus, the counter flow arrangement uses significantly less amount of hot water. Assuming that the hot water stream on exit from the heat exchanger is simply developed into the drain, the energy flow rate of the hot water steam at entry is considered as the exergy input rate to the process:

$$a_{f1} = \dot{m}_h[(h_1 - h_0) - T_0(s_1 - s_0)]$$

At 300 K or 27 °C, $h_0 = 113.2$ kJ/kg and $s_0 = 0.395$ kJ/kg K.

At 90 °C, $h_1 = 376.92$ kJ/kg and $s_1 = 1.1925$ kJ/kg K:

$$a_{f1} = 0.833 \times [(376.92 - 113.2) - 300 \times (1.1925 - 0.395)] = 20.38 \text{ kW}$$

Parallel flow, at 60 °C, $h_2 = 251.13$ kJ/kg and $s_2 = 0.8312$ kJ/kg K.

At 25 °C, $h_3 = 104.89$ kJ/kg and $s_3 = 0.3674$ kJ/kg K.

At 50 °C, $h_4 = 209.33$ kJ/kg and $s_4 = 0.7038$ kJ/kg K.

Rate of exergy gain $= \dot{m}_c[(h_4 - h_3) - T_0(s_4 - s_3)]$:

$$= 1 \times (209.33 - 104.89) - 300 \times (0.7038 - 0.3674) = 3.52 \text{ kW}$$

$$(\eta_{II})_{parallel} = \frac{3.52}{20.38} = 0.172 = 17.2\%$$

Rate of exergy loss by hot water $= \dot{m}_h[(h_1 - h_2) - T_0(s_1 - s_2)]$

$$= \quad 0.833 \times [(376.92 - 251.13) - 300 \times (1.1925 - 0.8312)]$$

$$= \quad 0.833(125.79 - 108.39) = 14.494 \text{ kW}$$

Rate if irreversibility or exergy destruction as:

$$w \quad = \quad 14.494 - 3.52 = 10.974 \text{ kW}$$

If hot water steam is not dumped to the drain:

$$(\eta_{II})_{parallel} \quad = \quad \frac{3.52}{14.494} = 0.243 = 24.3\%$$

$$\text{Counter flow, } h_2 \quad = \quad 146.68 \;\; \text{kJ/kg} \quad \text{and} \quad s_2 = 0.5053 \;\; \text{kJ/kg K}$$

Rate of exergy gain of cold water $\dot{m}_c[(h_4 - h_3) - T_0(s_4 - s_3)]$

$$= \quad 1 \times [(209.33 - 104.89) - 300 \times (0.708 - 0.367)]$$

$$= \quad 2.14 \text{ kW (same as parallel flow)}$$

Rate of exergy input (if the existing hot water is dumped to the surroundings)

$$= \quad 0.454 \times (263.72 - 239.25) = 11.11 \text{ kW}$$

$$(\eta_{II})_{counter} \quad = \quad \frac{3.52}{11.11} = 0.3168 = 31.68\%$$

Rate of exergy loss of hot water

$$= \quad \dot{m}_h[(h_1 - h_2) - T_0(s_1 - s_2)]$$

$$= \quad 0.454 \times [(376.92 - 146.68) - 300 \times (1.1925 - 0.5053)]$$

$$= \quad 0.454 \times (230.24 - 206.16) = 10.94 \text{ kW}$$

$$(\eta_{II})_{counter} \quad = \quad \frac{3.52}{10.94} = 0.3217 = 32.17\%$$

Rate of irreversibility or exergy destruction = (10.94 – 3.52) = 7.42 kW $\xLeftarrow{\textbf{Ans}}$

The second law efficiency of counter flow arrangement is significantly higher and the rate irreversibility is substantially lower compared to the parallel flow arrangement.

8.18 Saturated steam vapour at 150 °C is coming out of a geothermal well in a desert at the rate of 50 kg/h. The environment temperature is 45 °C. This steam is contemplated for cooling purpose of homes to a temperature of 23 °C. The steam will emerge from this system as saturated liquid at 1 atm. Calculate the maximum cooling rate that could be provided by such a system.

Solution

The energy balance of the control volume as shown in Fig.8.48 gives:

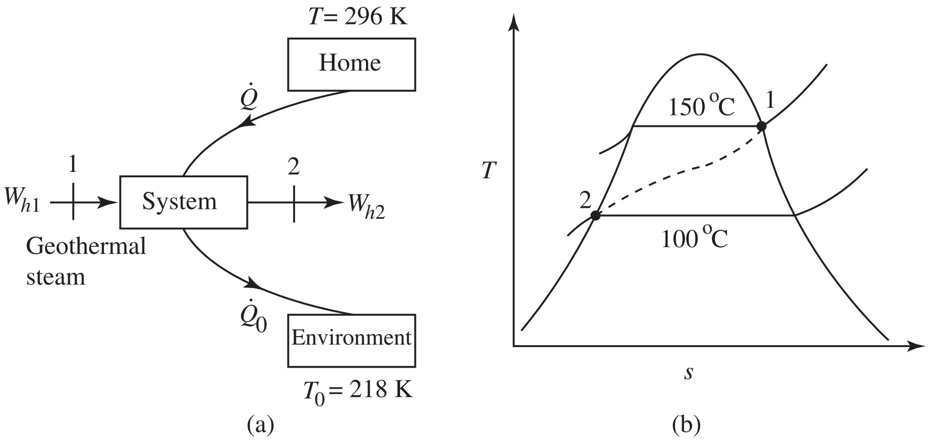


Fig. 8.48

$$\dot{Q} + \dot{m}h_1 = \dot{Q}_0 + \dot{m}h_2$$

The entropy balance is,

$$\dot{S}_{gen} = \left(\frac{\dot{Q}_0}{T_0} + \dot{m}s_2\right) - \left(\frac{\dot{Q}}{T} + \dot{m}s_1\right)$$

where T be the temperature maintained by the homes. Solving for $\dot{Q}$

$$\dot{Q} = \frac{\dot{m}[(h_1 - T_0s_1) - (h_2 - T_0s_2)] - T_0\dot{S}_{gen}}{\left(\frac{T_0}{T}\right) - 1}$$

By second law, $\dot{S}_{gen} > 0$. Therefore, for a given discharge state 2 the maximum $\dot{Q}$ would be

$$\dot{Q}_{max} = \frac{\dot{m}(b_1 - b_2)}{\left(\frac{T_0}{T}\right) - 1}$$

State 1, $T_1 = 150\ °\text{C} = 423\ \text{K}$ (saturated vapour), $h_1 = 2746.4\ \text{kJ/kg}$ and $s_1 = 6.8387\ \text{kJ/kg K}$.

State 2, $T_2 - 100\ °\text{C} = 373\ \text{K}$ (saturated liquid), $h_2 = 419.0\ \text{kJ/kg}$; $s_2 = 1.3071\ \text{kJ/kg}$ and $T_0 = 318K$:

$$b_1 = h_1 - T_0s_1 = 2746.4 - 318 \times 6.8387 = 589.7\ \text{kJ/kg}$$

$$b_2 = h_2 - T_0s_2 = 419.0 - 318 \times 1.3071 = 3.3\ \text{kJ/kg}$$

$$\dot{Q}_{max} = \frac{50 \times (589.7 - 3.3)}{\left(\frac{318}{296}\right) - 1} = 3.94 \times 10^5\ \text{kJ/h} = 109\ \text{kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.19 Calculate the dryness fraction or the quality of steam which has 2 kg of water in suspension with 60 kg of steam.

Solution

$$\text{Dryness fraction or quality of steam} = \frac{\text{Mass of dry steam}}{\text{Mass of dry steam} + \text{Mass of water in suspension}}$$

$$= \frac{60}{60+2} = 0.968$$

8.20 Consider a vessel containing liquid water and water vapour in equilibrium. The volume of the vessel is 0.5 m^3. The amount of liquid water and water vapour is 3 kg. The pressure is 0.5 MPa. Calculate (i) mass and volume of liquid and (ii) mass and volume of vapour.

Solution

Volume of the vessel, $V = 0.5\ \text{m}^3$

Mass of liquid water and water vapour, $m = 5$ kg

$$p = 0.5\ MPa = 5\ \text{bar}$$

Specific volume, $v = \dfrac{V}{m} = \dfrac{0.5}{5} = 0.1\ \text{m}^3/\text{kg}$

At 0.5 MPa from steam table,

$$v_{fg} = v_g - v_f = 0.375 - 0.00109 = 0.3739\ \text{m}^3/\text{kg}$$

We have, $v = v_g - (1-x)v_{fg}$

where x is the quality of steam (vapour).

$$0.1 = 0.375 - (1-x) \times 0.3739$$

$$1 - x = \frac{0.375 - 0.1}{0.3739} = 0.7355$$

$$x = 1 - 0.7355 = 0.2645$$

Mass of liquid, $m_l = m(1-x) = 5 \times (1 - 0.2645) = 3.6775$ kg

Volume of liquid, $V_l = m_l \times v_f = 3.6775 \times 0.00109 = 0.0040\ \text{m}^3$

Mass of vapour, $m_{vap} = 5 \times 0.2645 = 1.3225$ kg

Volume of vapour, $V_{vap} = m_{vap} \times v_g = 1.3225 \times 0.375 = 0.4959\ \text{m}^3$

8.21 A cylindrical vessel with a volume of 0.05 m^3 contains a mixture of saturated water and saturated steam at 250 °C. The mass of liquid present is 6 kg. Estimate the following parameters:

(i) pressure
(ii) mass
(iii) specific volume
(iv) specific enthalpy
(v) specific entropy
(vi) specific internal energy

Solution

From the steam table, corresponding to 250 °C:

$v_f = 0.00125 \text{ m}^3/\text{kg}$ $\quad h_f = 1085.30 \text{ kJ/kg}$ $\quad s_f = 2.7927 \text{ kJ/kg K}$

$v_g = 0.05013 \text{ m}^3/\text{kg}$ $\quad h_{fg} = 1716.2 \text{ kJ/kg}$ $\quad s_{fg} = 3.2802 \text{ kJ/kg K}$

$p_{sat} = 3.973 \text{ MPa}$

(i) Pressure, $p = 3.973 \text{ MPa} = 39.73 \text{ bar}.$ $\overset{\textbf{Ans}}{\Longleftarrow}$

(ii) Volume of liquid, $V_f = m_f v_f == 6 \times 0.00125 = 0.0075 \text{ m}^3$

Volume of vapour, $\quad V_g = 0.05 - 0.0075 = 0.0425 \text{ m}^3$

Mass of vapour, $\quad m_g = \dfrac{V_g}{v_g} = \dfrac{0.0425}{0.05013} = 0.8478$

Total mass of mixture, $m = m_f + m_g = 6 + 0.8478 = 6.8478$ $\overset{\textbf{Ans}}{\Longleftarrow}$

Quality of mixture, $\quad x = \dfrac{m_g}{m_g + m_f} = \dfrac{0.8478}{0.8478 + 6} = 0.1238$

Specific volume,

$$
\begin{aligned}
v &= v_f + x(v_g - v_f) \\
&= 0.00125 + 0.1238 \times (0.05013 - 0.00125) \\
&= 0.0073 \text{ m}^3/\text{kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

Specific enthalpy,

$$
\begin{aligned}
h &= h_f + x h_{fg} = 1085.30 + 0.1238 \times 1716.2 \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\
&= 1297.76 \text{ kJ/kg}
\end{aligned}
$$

Specific entropy,

$$
\begin{aligned}
s &= s_f + x s_{fg} = 2.7927 + 0.1238 \times 3.2802 \\
&= 3.199 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

Specific internal energy,

$$
\begin{aligned}
u &= h - pv = 1297.76 - 3.973 \times 10^3 \times 0.0073 \\
&= 1268.76 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

8.22 Calculate the required amount of heat to be supplied to 5 kg of water at 30 °C to convert into steam at 0.5 MPa and 0.85 dry.

Solution

At 0.5 MPa, from steam table:

$$h_f = 640.21 \ \text{kJ/kg} \quad \text{and} \quad h_{fg} = 2108.5 \ \text{kJ/kg}$$

Enthalpy of 1 kg of steam (above 0 °C):

$$h = h_f + xh_{fg} = 640.2 + 0.85 \times 2108.5 = 2432.425$$

Sensible heat associated with 1 kg of water:

$$= m_w + c_{pw} \times (t_w - 0) = 1 \times 4.18 \times (30 - 0) = 125.4 \text{ kJ}$$

Net quantity of heat to be supplied per kg of water = Enthalpy of steam − Sensible heat of water:

$$= 2432.425 - 125.4 = 2307 \text{ kJ}$$

Total amount of heat to be supplied:

$$= 5 \times 2307 = 11535 \text{ kJ} = 11.535 \text{ MJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.23 Estimate the amount of heat that is required to produce 4 kg of steam at a pressure of 0.6 MPa and temperature of 250 °C from water at 25 °C. Assume the specific heat of superheated steam at 2.2 kJ/kg.

Solution

From the steam table for superheated steam, at 0.6 MPa and 250 °C:

$$t_s = 158.85\ ^\circ\text{C}$$

For the saturation temperature of 158.85 °C from steam table:

$$h_f = 670.4 \ \text{kJ/kg} \quad \text{and} \quad h_{fg} = 2085 \ \text{kJ/kg}$$

Enthalpy of 1 kg of superheated steam reckoned from 0 °C:

$$h_{vap} = h_f + h_{fg} + c_{ps}(T_{sup} - T_{sat}) = 670.4 + 2085 + 2.2 \times (250 - 158.85) = 2956 \text{ kJ}$$

Sensible heat of 1 kg of water

$$= 1 \times 4.18 \times (25 - 0) = 104.5 \text{ kJ}$$

Net amount of heat required to be supplied per kg

$$= 2956 - 104.5 = 2851.5 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Total amount of heat required

$$= 4 \times 2851.5 = 11406 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

8.24 Calculate the mass of 0.15 m^3 of wet steam at a pressure of 4 bar and dryness fraction 0.8. Estimate the heat of 1 m^3 of steam.

Solution

Volume of wet steam, $V = 0.15 \text{ m}^3$

Pressure of wet steam, $p = 4$ bar

Dryness fraction, $x = 0.8$

From steam tables at 4 bar, $v_g = 0.462 \text{ m}^3/\text{kg}$ $h_f = 604.7 \text{ kJ/kg K}$ $h_{fg} = 2133 \text{ kJ/kg}$

$$\text{Density} = \frac{1}{xv_g} = \frac{1}{0.8 \times 0.462} = 2.7056 \text{ kg/m}^3$$

Mass of 0.15 m^3 of steam $= 0.15 \times 2.7056 = 0.4058$ kg

Total heat of 1 m^3 steam which has a mass of 2.7056 kg $= 2.7056h$, where h is the total heat of 1 kg of steam

$$= 2.7056 \times (h_f + xh_{fg})$$

$$= 2.7056 \times (604.7 + 0.8 \times 2133) = 6252.9 \text{ kJ}$$

8.25 A boiler is capable of generating 500 kg of steam at 1.6 MPa and with a quality of 0.9. The steam is allowed to pass through a superheater via a stop valve. The temperature of steam is raised to 380 °C in the superheater. If the feedwater temperature is 30 °C, calculate

(i) the total heat supplied to feedwater per hour to produce wet steam.
(ii) the total heat absorbed per hour in the superheated.

Take specific heat of superheated steam as 2.2 kJ/kg K.

Solution

At 1.6 MPa, from steam table,

$$t_s = 201.4\ {}^\circ\text{C} = 474.4\ K; \quad h_f = 858.6 \text{ kJ/kg} \quad h_{fg} = 1933.2 \text{ kJ/kg}$$

Heat supplied to feedwater per hour to produce wet steam is given by:

$$H = m[(h_f + xh_{fg}) - 1 \times 4.18 \times (30 - 0)] = 500 \times [858.6 + 0.9 \times 1933.2 - 4.18 \times 30]$$
$$= 500 \times [858.6 + 1739.88 - 125.4] = 1236.54 \times 10^3 \text{ kJ}$$

Heat absorbed by superheated per hour

$$= m[(1 - x)h_{fg} + c_{ps}(T_{sup} - T_S)]$$

$$= 500 \times [(1 - 0.9) \times 1933.2 + 2.2 \times (653 - 474.4)]$$

$$= 500 \times (193.32 + 392.92) = 293.12 \times 10^3 \text{ kJ} \qquad \xLeftarrow{\textbf{Ans}}$$

Review Questions

8.1 *What do you understand by pure substance? and what are the different phases of a pure substance?*

8.2 *Explain the phase-change process of water with appropriate figures.*

8.3 *Explain the temperature-pressure (T-p) diagram for water. What are the most important points to be noted.*

8.4 *What do you understand by triple point and critical point?*

8.5 *What is sensible heating and latent heating? Explain.*

8.6 *Define normal boiling point and melting point.*

8.7 *Define saturation pressure and saturation temperature.*

8.8 *Explain with appropriate figures water as compressed and saturated liquid.*

8.9 *Explain the process the phase-change of liquid water into saturated and superheated vapour.*

8.10 *Explain the T-v diagram for the heating process of water at constant pressure.*

8.11 *Explain saturation temperature and saturation pressure in connection with water.*

8.12 *Explain T-v diagram of constant-pressure phase-change processes of a pure substance at various pressures.*

8.13 *Explain the p-V diagram for water by including solid phase of water.*

8.14 *Explain triple line and triple point.*

8.15 *What is p-T diagram? What is its use?*

8.16 *Explain with figures the usefulness of a p-V-T diagram for water.*

8.17 *What do you mean by the following: (i) internal latent heat; (ii) internal energy of steam; (iii) external work of evaporation; (iv) entropy of evaporation; (v) entropy of wet steam; and (vi) entropy of superheated steam.*

8.18 *Draw a neat sketch of throttling calorimeter and explain how dryness fraction of steam is determined. Explain its limitations.*

8.19 *Describe with a neat sketch a separating and throttling calorimeter for measuring the dryness fraction of steam.*

8.20 *Why do the isobars on Mollier diagram diverge from one another?*

8.21 *Why do isotherms on Mollier diagram become horizontal in the superheated region at low pressures?*

8.22 *What do you understand by the degree ofsuperheat and the degree of subcooling?*

8.23 *What is quality of steam? What are the different methods of measurement of quality?*

8.24 Why cannot a throttling calorimeter measure the quality if the steam is very wet? How is the quality measured then?

8.25 What is the principle of operation of an electrical calorimeter?

Exercise

8.1 Using steam table, determine the mass specific heat for superheated steam:

(i) at 1 bar between 100 °C and 150 °C

(ii) at 0.5 bar between 300 °C and 500 °C*Ans:* (i) 2.004 kJ/kg K; (ii) 2.066 kJ/kg K

8.2 A pressure cooker contains 2 kg of saturated steam at 0.5 MPa. Estimate the quality of heat to be rejected so as to bring down the quality to 60%. Find the pressure and temperature of steam at the new state. *Ans:* −1625.66 kJ; 2.9 bar; 132.4 °C

8.3 Consider a spherical vessel containing steam at 8 bar and with a quality of 0.9. Certain amount of steam is blown off so as to reduce the pressure to 4 bar. The valve is closed at this point and the steam is allowed to cool until the pressure is 3 bar. Assuming constant enthalpy process during blown off period. Take the volume of the vessel as 0.9 m^3, Find

(i) the mass of steam blown off.

(ii) the dryness fraction of steam in the vessel after cooling.

(iii) the heat lost by steam per kg during cooling.

Ans: (i) 2.045 kg; (ii) 0.699; (iii) 913.6 kJ

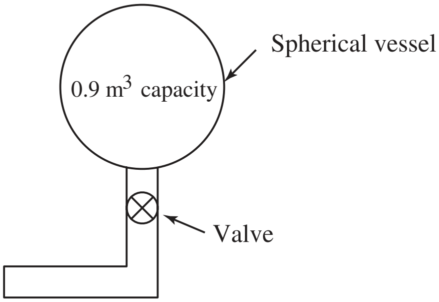


Fig. 8.49

8.4 If steam is produced at 1 MPa with dryness fraction of 0.9 calculate, (i) external work done during evaporation and (ii) internal latent heat of steam.

Ans: (i) 174.6 kJ; (ii) 1436.3 kJ

8.5 A quality of steam at 1 MPa and 0.8 dry occupies 0.16 m^3. Determine the amount of heat to be supplied in order to raise the temperature of steam to 300 °C at constant pressure. Also find the percentage of this heat which appears as external work. Assume the specific heat of superheated steam as 2.2 kJ/kg K. *Ans:* 687.61 kJ; 13.5%

8.6 Find the specific volume, enthalpy and internal energy of wet steam at 20 bar and dryness fraction 0.8. *Ans:* 0.0796 m^3/kg; 2419.48 kJ/kg; 2261.524 kJ/kg

8.7 Estimate the dryness fraction, specific volume and internal energy of steam at 7 bar and enthalpy 2550 kJ/kg. *Ans:* 0.897; 2379.67 kJ/kg

8.8 Steam at 12 MPa has a specific volume of 0.0172 m^3/kg. Estimate the temperature, enthalpy and the internal energy. *Ans:* 350 °C; 2847.7 kJ/kg; 2641.18 kJ/kg

8.9 Steam at 14 MPa has an enthalpy of 3002 kJ/kg. Find the temperature, specific volume and internal energy. *Ans:* 400 °C; 0.01722 m^3/kg; 2760.92 kJ/kg

8.10 Find the internal energy per kg of superheated steam at a pressure of 1 MPa and temperature of 30 °C. Also, find the change of internal energy if this steam is expanded to 0.14 MPa and dryness fraction 0.8. *Ans:* 2806.2 kJ/kg; −674.28 kJ/kg

8.11 Find the internal energy of 1 kg of steam at 20 bar when it is

(i) superheated to a temperature of 400 °C

(ii) wet with dryness fraction is 0.9

Assume that superheated steam behaves like a perfect gas from the start of superheating obeying Charles law. Specific heat of steam is 2.3 kJ/kg.

Ans: (i) 2592.88 kJ/kg; 2429.24 kJ/kg

8.12 A common main is connected to two pipes as shown in Fig.8.50. Two boilers, B1 and B2 are connected to the two pipes. Boiler 1 (B1) is having a superheater whereas boiler 2 (B2) has no superheater. Assume the boiler pressure and the main pipe pressure is 2 MPa. The temperature of the steam from B1 is 350 °C and in the main pipe is 250 °C. Calculate the quality of steam supplied by B2. Take c_p of steam as 2.25 kJ/kg. *Ans:* 0.925

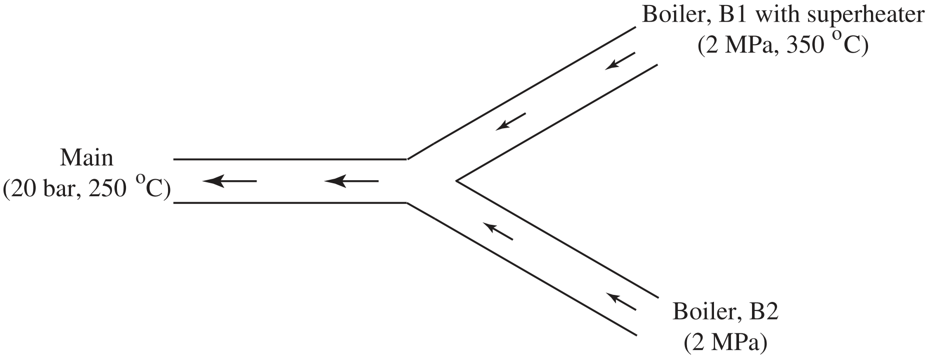


Fig. 8.50

8.13 Determine the entropy of 1 kg of wet steam at a pressure of 0.6 MPa and 0.8 dry. Calculate from freezing point of 0 °C. *Ans:* 5.7865 kJ/kg K

8.14 Steam enters a steam engine at a pressure of 1 MPa absolute and 400 °C. It is exhausted at 0.02 MPa with a quality of 0.9. Calculate (i) drop in enthalpy and (ii) change in entropy. *Ans:* (i) 889.9 kJ/kg; (ii) 0.2634 kJ/kg K (decrease)

8.15 Estimate the entropy of 2 kg of superheated steam at a pressure of 1.2 MP and a temperature of 250 °C. Take the specific heat of superheated steam as 2.2 kJ/kg K *Ans:* 13.548 kJ/kg

8.16 Three kilogram of wet steam at 1.4 bar is contained in piston cylinder arrangement. The initial volume is 2025 m^3. The steam is heated until its temperature reaches 400 °C. The piston is free to move up or down unless it reaches the stops at the top. When the piston is up against the stops the cylinder volume is 4.65 m^3. Determine the amount of work and heat transfer to or from steam. *Ans:* 4390.3 kJ; 614.5 kJ

8.17 Using appropriate calculation fill up the following table of properties for unit mass of water (liquid vapour or mixture).

	P (bar)	t (°C)	v (m^3/kg)	x (%)	Superheat (°C)	h (kJ/kg)	s (kJ/kg K)
(a)	-	35.0	25.22	-	-	–	–
(b)	-	–	0.0010	-	-	419.04	–
(c)	-	212.42	–	90	-	–	–
(d)	1	–	–	-	-	–	6.104
(e)	10	320.0	–	-	-	–	–
(f)	5	–	0.4646	-	-	–	–
(g)	4	–	0.4400	-	-	–	–
(h)	-	500.0	–	-	-	3445.3	–
(i)	20	–	–	-	50	–	–
(j)	15	–	–	-	-	–	7.2690

Ans:

	P (bar)	t (°C)	v (m^3/kg)	x (%)	Superheat (°C)	h (kJ/kg)	s (kJ/kg K)
(a)	0.05628	35	25.22	100	0	2564.4	8.3547
(b)	1.0133	100	0.0010	0	–	419.4	1.3069
(c)	20	212.42	0.0888	90	–	2610.42	5.9516
(d)	1	99.63	1.3431	0.7927	–	2207.44	6.104
(e)	10	320320.0	0.2677	–	140.16	3093.8	7.1942
(f)	5	250.0	0.4646	–	98.2	2960.7	7.2709
(g)	4	143.6	0.4400	0.951	–	2633.98	6.645
(h)	40	500	0.0864	–	249.69	3445.3	7.0901
(i)	20	262.42	0.1149	–	50	2932.75	6.6006
(j)	15	400.0	0.204	–	201.68	3256.6	7.2690

8.18 Consider a tank containing water into which steam at a pressure of 0.5 MPa is passed. The temperature and mass of the tank before the admission of steam is 20 °C and 50 kg, respectively. Find the dryness fraction of the steam as it enters the tank. Three kilogram of steam gets condensed and resulting temperature of the mixture is 40 °C. Take water equivalent of, tank as 1.5 bar. *Ans:* 0.456

8.19 A tank contains 90 kg of water at 25 °C steam with a quality of 0.95 and at a pressure of 1.1 bar is passed into the tank. The mass of the tank is 12.5 kg and specific heat of metal is 0.42 kJ/kg K. If the temperature of the water rises to 40 °C after the steam passage, calculate the mass of steam condensed. Neglect all losses. *Ans:* 2.488 kg

8.20 A stream of steam is flowing through a pipe at pressure of 0.8 MPa. A throttling calorimeter is used to measure the dryness fraction of this steam. The steam after flowing through the calorimeter has a pressure of 0.1 MPa and a temperature of 115 °C. Estimate the dryness fraction of the steam in the pipe. Take $c_{ps} = 2.1$ kJ/kg K. *Ans:* 0.97

8.21 In order to estimate the quality of steam, a separating and throttling calorimeter is arranged in series. The following observations were made:

Water separated	2 kg
Steam discharged	20 kg
Temperature of steam after throttling	110 °C
Initial pressure	0.12 MPa
Barometer reading	760 mm of Hg
Final pressure	5 mm of Hg

Calculate the quality of steam supplied. *Ans:* 0.87

8.22 A pipeline carries 5 tonnes of steam at 5 bar an 300 °C per hour water at 40 °C is continuously sprayed into the pipeline. At a section downstream where the pressure is 3 bar, the quality of steam is 0.95. Find the rate of water spray in kg/h. *Ans:* 542.05 kJ

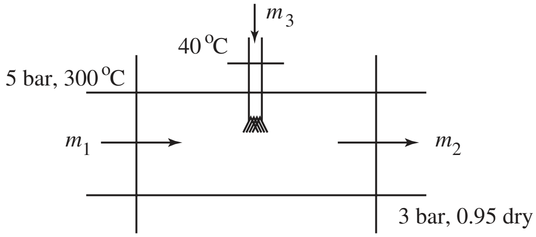


Fig. 8.51

8.23 The following observations were made while conducting a test using combined separating and throttling calorimeter:

The pressure of steam at the sample point	15 bar
Exit steam pressure	1 bar
Temperature of exit steam	150 °C
Discharge from separating calorimeter	0.5 kg/min
Discharge from throttling calorimeter	10 kg/min

Calculate the dryness fraction of the sample steam. *Ans:* 0.946

8.24 (a) A rigid vessel contains 1 kg of steam and has a volume of 0.8 m^3. The steam pressure is 2 bar. Determine, (i) specific volume; (ii) temperature; (iii) dryness fraction; (iv) internal energy; (v) enthalpy; and (vi) entropy.

(b) The steam is heated to a temperature of 150 °C. Show the process as a sketch of the *p-v* diagram and evaluate the effect of pressure on the increase in enthalpy, increase in internal energy, increase in entropy and the heat transfer. Evaluate also the pressure at which the steam becomes dry and saturated.

Ans: (i) 0.8 m^3; (ii) 120.23 °C; (iii) 0.903; (iv) 2333 m^3/kg; (v) 2493 kJ/kg; (vi) 6.585 kJ/kg K (b) (i) 273.5 kJ/kg K; (ii) 249.5 kJ/kg; (iii) 0.625 kJ/kg K; (iv) 249.5 kJ/kg

8.25 Twenty kilogram of water at 45 °C is heated at a constant pressure of 10 bar until it becomes superheated vapour at 300 °C. Find the change in volume, enthalpy, internal energy and entropy. *Ans:* 5.1378 m^3; 28627.5 kJ; 129.684 kJ/K; 56741.22 kJ

Multiple Choice Questions (choose the most appropriate answer)

1. A pure substance is
   (a) a homogeneous mixture of two substances of same composition
   (b) a substance with constant chemical composition throughout its mass
   (c) both (a) and (b)
   (d) none of the above

2. A pure substance exists in
   (a) solid phase (c) gaseous phase
   (b) liquid phase (d) all of the above

3. Which one in the examples is not a pure substance?
   (a) baking soda (c) sea water
   (b) diamond (d) table salt

4. A substance from which a phase-change occurs without a change of pressure or temperature is called
   (a) pure state (c) critical state
   (b) phase state (d) saturation state

5. The state at which saturated liquid line with respect to vaporisation and saturated vapour line on $p-V$ diagram of pure substance, meet is called
   (a) critical state (c) vaporisation state
   (b) saturation state (d) superheated vapour state

6. Saturated solid state can be defined as
   (a) a state at which solid can change into liquid at constant pressure but changing temperature
   (b) a state at which solid can change into liquid at constant pressure and temperature
   (c) a state at which solid can change into liquid at constant temperature but change in pressure
   (d) none of the above

7. The temperature of a substance at which the vapour pressure is equal to 760 mm Hg is called as
   (a) normal vapour point (c) normal boiling point
   (b) normal pressure point (d) none of the above

8. The temperature at which a pure liquid transforms into vapour at constant pressure is called as
   (a) saturation temperature (c) vaporisation temperature
   (b) normal temperature (d) none of the above

9. Choose the correct statement.
   (a) Specific volume of water decreases on freezing
   (b) Boiling point of water decreases with increasing pressure
   (c) Freezing temperature of water decreases with increasing pressure
   (d) Specific volume of water increases on freezing

10. Which one of this statement is correct?
    (a) The slope of vaporisation curve is always negative
    (b) The slope of vaporisation curve is always positive
    (c) The slope of sublimation curve in negative for all pure substances
    (d) The slope of fusion curve is positive for all pure substances

11. Which one of the following statement is correct?
    (a) The process of passing from liquid to vapour is condensation
    (b) Pressure and temperature are independent during phase-change
    (c) An isothermal line is also a constant pressure line during wet region
    (d) The term, dryness fraction, is used to describe the fraction by mass of liquid in the mixture of liquid water and water vapour

12. The latent heat of vaporisation at critical point is
    (a) less than zero
    (b) greater than zero
    (c) equal to zero
    (d) none of the above

13. Choose the most appropriate statement.
    (a) Critical point involves equilibrium of solid and vapour phases
    (b) Critical point involves equilibrium of solid and liquid phases
    (c) Critical point involves equilibrium of solid, liquid and vapour phases
    (d) Triple point involves equilibrium of solid, liquid and vapour phases

14. With the increase in pressure, boiling point of water
    (a) increases and enthalpy of evaporation increases
    (b) increases and enthalpy of evaporation decreases
    (c) decreases and enthalpy of evaporation increases
    (d) none of the above

15. With increase in pressure, enthalpy of dry saturated steam
    (a) increases
    (b) first increases and then decreases
    (c) remains same
    (d) decreases

16. Dryness fraction of steam is defined as
    (a) $\dfrac{\text{Mass of water vapour in suspension}}{\text{Mass of water vapour in suspension} + \text{Mass of dry steam}}$

    (b) $\dfrac{\text{Mass of dry steam}}{\text{Mass of water vapour in suspension}}$

    (c) $\dfrac{\text{Mass of dry steam}}{\text{Mass of dry steam} + \text{Mass of water vapour in suspension}}$

    (d) $\dfrac{\text{Mass of water vapour in suspension}}{\text{Mass of dry steam}}$

17. The specific volume of water when heated at 0 °.
    (a) first increases and then decreases
    (b) first decreases and then increases
    (c) increases steadily
    (d) decreases steadily

18. Only throttling calorimeter is used for measuring
    (a) very low dryness fraction upto 0.7
    (b) very high dryness fraction upto 0.98
    (c) dryness fraction of only low pressure steam
    (d) dryness fraction of only high pressure steam

19. At a pressure of 4 Mpa, the temperature at which liquid water boils is
    (a) 29.0 °C (c) 143.6 °C
    (b) 100 °C (d) 250.4 °C

20. A system contains water at 2.00 MPa, 220 °C. The phase of this water is
    (a) Liquid (c) Vapour
    (b) Liquid-vapour mixture (d) Solid

21. Steam at 1 MPa, 250 °C is contained in a rigid vessel. It is now cooled to 25 °. The final quality (if applicable) of the vessel contents is
    (a) 0.54% (c) 27.8%
    (b) 2.63% (d) none of the above

22. Heat of superheated steam is given by
    (a) $h_{sup} = h_f + h_{fg} + c_{ps} \log_e \frac{T_{sup}}{T_s}$
    (b) $h_{sup} = h_f + xh_{fg}$
    (c) $h_{sup} = h_f + h_{fg}$
    (d) $h_{sup} = h_f + xh_{fg} + c_{ps} \log_e \frac{T_s}{273}$

23. Volume of wet steam (per kg) with dryness fraction x is given by
    (a) x^3V_g (c) $x^2(V_g - V_f)$
    (b) xV_f (d) none of the above

24. Internal latent heat is given by
    - (a) $h_{fg} - \frac{pV_g}{J}$
    - (b) $h_g - \frac{pV_g}{J}$
    - (c) $h_{sup} - \frac{pV_f}{J}$
    - (d) $h_{fg} + \frac{pV_g}{J}$
25. Entropy of 1 kg of water at T K is given by
    - (a) $c_{pw} \log_e \frac{T}{273}$
    - (b) $c_{pw} \log_e \frac{T_2}{T_1}$
    - (c) $c_{pw} \log_{10} \frac{T}{273}$
    - (d) $c_{pw} \log_e \frac{T_2}{273}$

Ans: 1. – (b) 2. – (d) 3. – (c) 4. – (d) 5. – (a)
6. – (b) 7. – (c) 8. – (a) 9. – (c) 10. – (a)
11. – (c) 12. – (c) 13. – (d) 14. – (b) 15. – (d)
16. – (c) 17. – (b) 18. – (b) 19. – (d) 20. – (c)
21. – (a) 22. – (a) 23. – (d) 24. – (a) 25. – (a)

9

PSYCHROMETRY

9.1 INTRODUCTION

Psychrometry is a general term used to describe physical and thermodynamic properties of gas-vapour mixtures. The term is derived from the Greek words *psuchron* ($\psi\upsilon\chi\rho\nu$) meaning 'cold' and *metron* ($\mu\tau\rho o\nu$) meaning *means of measurement.* In particular, *psychrometry* is the study of the properties of mixtures of air and water vapour The art of measuring the moisture content in air is termed *psychrometry.* Water vapour and its effect on human activities is in fact altogether disproportionate to its relative weights. It is nevertheless one of most important factors in human comfort and has significant effects on many materials. Other names for psychrometry is *hygrometry* or *psychometrics.*

Psychrometry, in short, is the science that goes into the details of the following:

(i) properties of air,

(ii) method of controlling its temperature and moisture content or humidity and

(iii) its effect on various materials and human beings.

The principle of psychrometry is used in the field of air-conditioning. The term 'air-conditioning' means treating of air or conditioning the air to change its temperature or the moisture as per the requirements of various applications. Air conditioners are used throughout the world for a number of applications. Due to climate change, we are reaching a stage that one just can't imagine the life without them. They are the devices or machines that condition or alter the state of the air by changing its temperature and the humidity level.

Studying psychrometry helps in understanding different constituents of air and how they affect each other, which in turn unravels various mysteries of the atmosphere and the nature. Some of the psychrometric properties of air that we are going to study in this chapter are dry-bulb temperature, wet-bulb temperature, dew point temperature, relative humidity, etc.

9.2 COMPOSITION OF ATMOSPHERIC AIR

Atmospheric air comprises mixture of various gases and water vapour or moisture. The air without any water vapour is called *dry air.* As such the air always contains some amount of water vapour, so the pure dry air doesn't really exist. However, its concept is very important in understanding the properties of the air and how various changes occur in the air-conditioning process. The dry air and water vapour mixture is merely a physical one, as there is no chemical reaction between the two.

The dry air is composed of various gases, mainly nitrogen (78%) and oxygen (21%). The remaining 1% of the gases includes carbon dioxide and very small quantities of inert gases like argon, hydrogen, helium and neon. The water vapour also is a small part of the air included among remaining 1% of the gases.

The amount of moisture in air by its mass keeps on varying from place to place. Further, it depends on the atmospheric conditions at a particular place. The places located close to the sea areas contain more moisture while the desert areas contain less moisture. Similarly, during rainy season, the moisture content of the air is high while during summers and winters, it is low. The atmospheric air contains usually 1 to 3% of moisture by mass. As stated earlier, atmospheric air is a mixture of many gases plus water vapour and a number of pollutants. The amount of water vapour and pollutants vary from place to place. The concentration of water vapour and pollutants decrease with altitude and above an altitude of about 10 km, atmospheric air consists of only dry air. At sea level conditions, the pollutants have to be filtered out before processing the air. Hence, what we process is essentially a mixture of various gases that constitute air and water vapour and is known as *moist air*.

9.2.1 Dry Air

Dry air is never found in practice. As already stated, it always contains some moisture. Hence, when we say 'air' *usually* it means *moist air*. However, in 1949, a standard composition of dry air was fixed by the International Joint Committee on Psychrometric data and is given in Table 9.1.

Table 9.1 Composition of standard air

Constituent	Molecular weight	Mol fraction
Oxygen	32.000	0.2095
Nitrogen	28.016	0.7809
Argon	39.944	0.0093
Carbon dioxide	44.010	0.0003

Based on the above composition, the molecular weight of dry air is found to be 28.966 and the gas constant, R, is 287.035 J/kg K. It is the normal practice to take these values as 29 and 287, respectively. Similarly, for calculation purposes, the molecular weight of water vapour is taken as 18.015 and its gas constant is 461.52 J/kg K.

At a given temperature and pressure, the dry air can only hold a certain maximum amount of moisture. The term *dry air* is used to indicate the *water-free contents* of air having no degree of moisture.

9.2.2 Saturated Air

When the moisture content is maximum, the air is known as *saturated air*. In other words, moist air is said to be saturated when its condition is such that it can coexist in natural equilibrium with an associated condensed moisture phase. For *a given temperature, only a given quantity of air can be saturated with a fixed quantity of moisture.* At higher temperatures, a larger quantity of moisture is required to saturate it. At saturation, vapour pressure of moisture in air corresponds to the saturation pressure. This can be taken from in steam tables corresponding to the given temperature of air.

9.3 PSYCHROMETRIC PARAMETERS

As already stated, the moist air is a mixture of dry air and moisture. For all practical purposes, the composition of dry air can be considered as constant. Air to be processed in air-conditioning systems is a mixture of dry air and water vapour. While the composition of dry air is constant, the amount of water vapour present in the air may vary from zero to a maximum depending upon the temperature and pressure of the mixture. Before proceeding further, let us go through some important definitions associated with psychrometry.

9.3.1 Dry-Bulb Temperature (t_{db})

It is the temperature of the moist air registered by a standard thermometer or any other temperature measuring instruments. It is abbreviated as DBT.

9.3.2 Wet-Bulb Temperature (t_{wb})

It is the temperature registered by a thermometer when the thermometer bulb is covered by a wetted wick and is exposed to a current of rapidly moving air. It is abbreviated as WBT.

9.3.3 Saturated Vapour Pressure (p_{sat})

It is the saturated partial pressure of water vapour at the corresponding dry-bulb temperature. This is readily available in thermodynamic tables and charts. ASHRAE suggests the following regression equation for saturated vapour pressure of water, which is valid for 0 – 100 °C:

$$\ln(p_{sat}) = \frac{c_1}{T} + c_2 + c_3 T + c_4 T^2 + c_5 T^3 + c_6 \ln(T) \tag{9.1}$$

where p_{sat} = saturated vapour pressure of water in kilo Pascals, T = temperature in K.

The regression coefficients c_1 to c_6 are given by:

$c_1 = -5.80022006$ E+03 $\quad c_2 = -5.516256$ E+00 $\quad c_3 = -4.8640239$ E−02

$c_4 = 4.1764768$ E−05 $\quad c_5 = -1.4452093$E−08 $\quad c_6 = 6.5459673$E+00

9.3.4 Adiabatic Saturation Temperature (t_{wb})

It is the temperature at which the water or ice can saturate air by adiabatic evaporation into it. It is numerically equivalent to the measured thermodynamic wet-bulb temperature. Therefore, it is designated as (t_{wb}). If necessary, correction can be applied to take into account the radiation and conduction.

9.3.5 Wet-Bulb Depression ($t_{db} - t_{wb}$)

The difference between dry-bulb and wet-bulb temperature is called wet-bulb depression.

9.3.6 Dew Point Temperature (t_{dp})

If unsaturated moist air is cooled at constant pressure, then the temperature at which the moisture in the air begins to condense is known as *dew point.* It is equal to steam table saturation temperature corresponding to the actual partial pressure of water vapour in the air. It is abbreviated as DPT.

An approximate equation for dew point temperature is given by

$$DPT = \frac{4030 \times (DBT + 235)}{4030 - (DBT + 235)\ln\phi} - 235 \tag{9.2}$$

where ϕ is the relative humidity (in fraction). DBT and DPT are in °C. Since from its definition, the dew point temperature is the saturation temperature corresponding to the vapour pressure of water vapour, it can be obtained either from steam tables or using Eq.9.1.

9.3.7 Dew Point Depression $(t_{db} - t_{dp})$

It is the difference between the dry-bulb and dew point temperatures.

9.3.8 Degree of Saturation, μ

The degree of saturation is the ratio of the humidity ratio W to the humidity ratio of a saturated mixture W_s at the same temperature and pressure, i.e.

$$\mu = \frac{W}{W_s} \tag{9.3}$$

9.3.9 Sensible Heat

It is the heat that changes the temperature of a substance when added to or removed from it.

9.3.10 Latent Heat

It is the heat that does not affect the temperature but changes the state of substance when added to or removed from it.

9.3.11 Enthalpy (h)

It is the combination energy which represents the sum of internal and flow energy in a steady flow process. It is determined from an arbitrary datum point for the air mixture and is expressed as kJ per kg of dry air.

The enthalpy of moist air is the sum of the enthalpy of the dry air and the enthalpy of the water vapour. Enthalpy values are always based on some reference value. In moist air, the enthalpy of dry air is given a zero value at 0 °C. For water vapour, the enthalpy of saturated water is taken as zero at 0 °C.

The enthalpy of moist air is given by

$$h = h_a + Wh_g = c_p t + W\left(h_{fg} + c_{pw} t\right) \tag{9.4}$$

where c_p = specific heat of dry air at constant pressure, kJ/kg K; c_{pw} = specific heat of water vapour, kJ/kg K; t = dry-bulb temperature of air-vapour mixture, °C; W = humidity ratio; h_a = enthalpy of dry air at temperature t, kJ/kg h_g = enthalpy of water vapour at temperature t, kJ/kg h_{fg} = latent heat of vaporisation at 0 °C, kJ/kg.

The unit of h is kJ/kg of dry air. Substituting the appropriate values of c_p and h_g, we get an expression for h as

$$h = 1.005t + W\left(2501 + 1.88t\right) \tag{9.5}$$

9.3.12 Humid Specific Heat

From the equation for enthalpy of moist air, the humid specific heat of moist air can be written as:

$$c_{pm} = c_p + Wc_{pw} \tag{9.6}$$

where c_{pm} = humid specific heat, kJ/kg K; c_p = specific heat of dry air, kJ/kg K; c_{pw} = specific heat of water vapour, kJ/kg K; W = humidity ratio, kg of water vapour/kg of dry air

Since the second term in the above equation (Wc_{pw}) is very small compared to the first term, for all practical purposes, the humid specific heat of moist air, c_{pm} can be taken as 1.0216 kJ/kg dry air K.

9.3.13 Specific Volume

The specific volume is defined as the number of cubic meters of moist air per kilogram of dry air. From perfect gas equation, we know that the volumes occupied by the individual substances are the same. Then, the specific volume is equal to the number of cubic meters of moist air per kilogram of dry air is given by

$$v = \frac{R_a T}{p_a} = \frac{R_a T}{p_t - p_v} \text{ m}^3/\text{kg dry air} \tag{9.7}$$

Note: When air is saturated DBT, WBT and DPT are equal.

9.4 HUMIDITY

It is the amount of water vapour in the air. Water vapour is the gaseous state of water and is invisible. Water vapour in the air is due to condensation and evaporation of moisture. There are three types of humidity and are measured as (i) absolute humidity, (ii) specific humidity and (iii) relative humidity.

9.4.1 Absolute Humidity

Absolute humidity is the measure of water vapour (moisture) in the air, regardless of temperature. It is expressed as grams of moisture per cubic meter of air (g/m^3). The maximum absolute humidity of warm air at 30 °C is approximately 30g of water vapour (30 g/m^3). The maximum absolute humidity of cold air at 0 °C is approximately 5 g of water vapour (5 g/m^3).

9.4.2 Specific Humidity or Humidity Ratio, (W)

It is defined as the ratio of the amount of water vapour in the air to the amount of dry air in the area.

It is a ratio of the water vapour content of the mixture to the total air content on a mass basis. It can also be expressed as ratio of the mass of water vapour per unit mass of dry air in the mixture of vapour and air. It is usually expressed as grams of water per kg of dry air.

For a given barometric pressure, it is a function of dew point temperature alone. Assuming both water vapour and dry air to be perfect gases, the humidity ratio is given by

$$W = \frac{\text{kilogram of water vapour}}{\text{kilogram of dry air}} \tag{9.8}$$

9.4.3 Relative Humidity (ϕ)

It is the ratio of the partial pressure of water vapour in the mixture to the saturated partial pressure at the dry-bulb temperature, expressed as percentage. It can also be defined as the ratio of the mole fraction of water vapour in moist air to mole fraction of water vapour in saturated air at the same temperature and pressure. It is abbreviated as RH and designated as ϕ.

Using perfect gas equation, we can show that

$$\phi = \frac{\text{Partial pressure of water vapour}}{\text{Saturation partial pressure of pure water vapour at same temperature}} \tag{9.9}$$

Relative humidity is normally expressed as a percentage. When ϕ is 100 percent, the air is saturated.

9.5 PSYCHROMETRIC RELATIONS

Pressure: We are aware that atmospheric air is a mixture of dry air and moisture. In order to determine the pressure of a mixture of gases, Gibbs-Dalton law of partial pressure is employed. This is an additive law.

According to the Gibbs-Dalton law for a mixture of perfect gases, the total pressure exerted by the mixture is equal to the sum of partial pressures of the constituent gases. According to this law, for a homogeneous perfect gas mixture occupying a volume V and at temperature T, each constituent gas behaves as though the other gases are not present (i.e. there is no interaction between the gases). Each gas obeys perfect gas equation. Hence, the partial pressures exerted by each gas, $p_1, p_2, p_3 \ldots$ and the total pressure p_t are given by

$$p_1 = \frac{n_1 R_u T}{V}, \quad p_2 = \frac{n_2 R_u T}{V}, \quad p_3 = \frac{n_3 R_u T}{V}, \quad \ldots \quad p_t = p_1 + p_2 + p_3 \ldots \tag{9.10}$$

where $n_1, n_2, n_3, \ldots$ are the number of moles of gases $1, 2, 3, \ldots$ and R_u is the universal gas constant. Applying this equation to moist air:

$$p \quad = \quad p_t = p_a + p_v \tag{9.11}$$

where $p = p_t =$ total barometric pressure; $p_a =$ partial pressure of dry air; $p_v =$ partial pressure of water vapour.

It may be noted that this law as well as Boyle's and Charle's law are only approximately correct. Modern psychrometric tables of atmospheric air properties are based on the correct versions. For calculating partial pressure of water vapour in the air, many equations have been proposed. Probably Carrier's equation is most widely used which is given by

$$p_v \quad = \quad (p_{vs})_{sb} - \frac{[pt - (p_{vs})_{wb}]\,(t_{db} - t_{wb})}{1527.4 - 1.3t_{wb}} \tag{9.12}$$

where $p_v =$ partial pressure of water vapour, $p_{vs} =$ partial pressure of water vapour when air is fully saturated, $p_t =$ total pressure of moist air, $t_{db} =$ dry-bulb temperature (°C) and t_{wb} = wet-bulb temperature (°C).

9.5.1 Specific Humidity (W)

As already defined, specific humidity is the ratio of mass of water vapour to the mass of dry air and designated by letter W:

$$\text{Specific humidity} = \frac{\text{Mass of water vapour}}{\text{Mass of dry air}}$$

or

$$W = \frac{m_v}{m_a}$$

Also,

$$m_a = \frac{p_a V}{R_a T} \tag{9.13}$$

$$m_v = \frac{p_v V}{R_v T} \tag{9.14}$$

where $p_a =$ partial pressure of dry air, $p_v =$ partial pressure of water vapour, $V =$ volume of mixture, $R_a =$ characteristic gas constant for dry air and $R_v =$ characteristic gas constant for water vapour.

From Eqs 9.13 and 9.14:

$$W = \frac{p_v \times V}{R_v \times T} \times \frac{R_a \times T}{p_a \times V} = \frac{R_a}{R_v} \times \frac{p_v}{p_a} \tag{9.15}$$

But,

$$R_a = \frac{R_u}{M_a} \quad \left(= \frac{8.3143}{28.97} = 0.287 \text{ kJ/kg K in SI units}\right) \tag{9.16}$$

$$R_v = \frac{R_u}{M_v} \quad \left(= \frac{8.3143}{18} = 0.462 \text{ kJ/kg K in SI units}\right) \tag{9.17}$$

where R_0 = universal gas constant, M_a = molecular weight of air and M_v = molecular weight of water vapour.

$$W = \frac{0.287}{0.462}\frac{p_v}{p_a} = 0.622\frac{p_v}{p_t - p_v} \tag{9.18}$$

The masses of air and water vapour in terms of specific volumes are given by expression as:

$$m_a = \frac{V}{v_a} \quad \text{and} \quad m_v = \frac{V}{v_v} \tag{9.19}$$

where v_a is specific volume of dry air and v_v is specific volume of water vapour.

$$W = \frac{v_a}{v_v} \tag{9.20}$$

9.5.2 Degree of Saturation (μ)

$$\text{Degree of saturation, } \mu = \frac{\begin{array}{c}\text{Mass of water vapour associated}\\ \text{with unit mass of dry air}\end{array}}{\begin{array}{c}\text{Mass of water vapour associated with}\\ \text{saturated unit mass of dry saturated air}\end{array}} = \frac{W}{W_s} \tag{9.21}$$

where W_s is specific humidity of air when air is fully saturated.

$$\mu = \frac{0.622 \times \left(\dfrac{p_v}{p_t - p_v}\right)}{0.622 \times \left(\dfrac{p_{vs}}{p_t - p_{vs}}\right)} = \frac{p_v\,(p_t - p_{vs})}{p_{vs}\,(p_t - p_v)} = \frac{p_v}{p_{vs}}\left[\frac{\left(1 - \dfrac{p_{vs}}{p_t}\right)}{\left(1 - \dfrac{p_v}{p_t}\right)}\right] \tag{9.22}$$

where p_{vs} is partial pressure of water vapour when air is fully saturated (p_{vs} can be calculated from steam tables corresponding to the dry-bulb temperature of the air).

9.5.3 Relative Humidity (RH), ϕ

$$\text{Relative humidity, } \phi = \frac{\text{Mass of water vapour in a given volume}}{\begin{array}{c}\text{Mass of water vapour in the same}\\ \text{volume if saturated at the same temperature}\end{array}}$$

$$\phi = \frac{m}{m_{vs}} = \frac{\dfrac{p_v T}{R_v T}}{\dfrac{p_{vs} T}{R_v T}} = \frac{p_v}{p_{vs}} \tag{9.23}$$

Inserting the value of Eq.9.23 into Eq.9.22, we get:

$$\mu = \phi\left(\frac{1-\dfrac{p_{vs}}{p_v}}{1-\dfrac{p_v}{p_t}}\right) = \phi\left(\frac{1-\dfrac{p_{vs}}{p_t}}{1-\dfrac{\phi p_{vs}}{p_t}}\right) = \phi\left(\frac{p_t-p_{vs}}{p_t-\phi p_{vs}}\right) \tag{9.24}$$

Therefore, $\phi\,(p_t - p_{vs}) = \mu\,(p_t - \phi p_{vs})$

$$\phi\,(p_t - p_{vs} + \mu p_{vs}) = \mu p_t \tag{9.25}$$

$$\phi = \frac{\mu p_t}{p_t - p_{vs} + \mu p_{vs}} = \frac{\mu}{1-(1-\mu)\dfrac{p_{vs}}{p_t}} \tag{9.26}$$

Since $p_{vs} \ll p_t$, therefore, $\phi \approx \mu$. Inserting the value of p_v from Eq.9.18 into Eq.9.23, we get:

$$\phi = \frac{p_a W}{0.622} \times \frac{1}{p_{vs}} = 1.6 \times W \times \frac{p_a}{p_{vs}} \tag{9.27}$$

Note:

(i) Relative humidity as compared to specific humidity plays a vital role in comfort air-conditioning and industrial air-conditioning. *Relative humidity signifies the absorption capacity of air.* If initial relative humidity of air is less it will absorb more moisture.

(ii) W and ϕ cannot be conveniently measured as they require measurement of p_v and p_{vs}. The value of p_v can be obtained from the measurement of the wet-bulb temperature and the value of p_{vs} can be calculated from steam tables corresponding to given air temperature.

9.5.4 Enthalpy of Moist Air

It is the sum of enthalpy of dry air and enthalpy of water vapour associated with dry air. It is expressed in kJ/kg of dry air:

$$h = h_{air} + W h_{vapour} = c_p t_{db} + W h_{vapour} \tag{9.28}$$

where h is enthalpy of mixture/kg of dry air, h_{air} is enthalpy of 1 kg of dry air, h_{vapour} is enthalpy of 1 kg of vapour obtained from steam tables, W is specific humidity in kg/kg of dry air, and c_p is specific heat of dry air normally assumed as 1.005 kJ/kg K. Also,

$$h_{vapour} = h_g + c_{ps}\,(t_{db} - t_{dp}) \tag{9.29}$$

where h_g is enthalpy of saturated steam at dew point temperature and $c_{ps} = 1.88$ kJ/kg K

$$h = c_p t_{db} + W\,[h_g + c_{ps}\,(t_{db} - t_{dp})] \tag{9.30}$$

$$= (c_p + c_{ps} W)\,t_{db} + W\,(h_g + c_{ps} t_{dp}) = c_{pm} t_{db} + W\,(h_g + c_{ps} t_{dp}) \tag{9.31}$$

where $c_{pm} = (c_p + c_{ps} W)$ is the specific heat of humid air or humid specific heat.

The value of c_{pm} is taken as 1.021 kJ/kg dry air per K. It is the heat capacity of $(1+W)$ kg of moisture per kg of dry air. $h_{vapour} = h_g$ at dry-bulb temperature. So,

$$h = c_p t_{db} + W h_g \tag{9.32}$$

However, a better approximation is given by the following relationship :

$$h_{vapour} = 2500 + 1.88 t_{db} \text{ kJ/kg of water vapour} \tag{9.33}$$

where t_{db} is dry-bulb temperature in °C and the datum state is liquid water at 0 °C.

$$h = 1.005 t_{db} + W\left(2500 + 1.88 t_{db}\right) \text{ kJ/kg dry air} \tag{9.34}$$

9.6 ADIABATIC SATURATION PROCESS

Consider an insulated chamber where unsaturated air flows over a long sheet of water as shown in Fig.9.1. Because of the flow of air, the water evaporates and the specific humidity of the air increases. Both the air and water gets cooled due to evaporation. This process will continue until the energy transferred from the air to the water is equal to the energy required to vaporise the water. When such a situation is reached, thermal equilibrium is achieved with respect to water, air and water vapour. The equilibrium temperature is called the adiabatic saturation temperature or the thermodynamic wet-bulb temperature. To keep the water-level constant, make-up water is added at this temperature. The adiabatic cooling process is shown in Fig.9.2 for the vapour in the air-vapour mixture. Note that the total pressure of the mixture is constant. However, the partial pressure of the vapour increases and the saturated state corresponds to the adiabatic saturation temperature. The vapour is initially at dry-bulb temperature t_{db1} and is cooled adiabatically to t_{db2} which is equal to the adiabatic saturation t_{wb2}. For all practical purposes, the adiabatic saturation temperature and wet-bulb temperatures are taken to be equal. The wet-bulb temperature will be in between the dry-bulb temperature and dew point temperature.

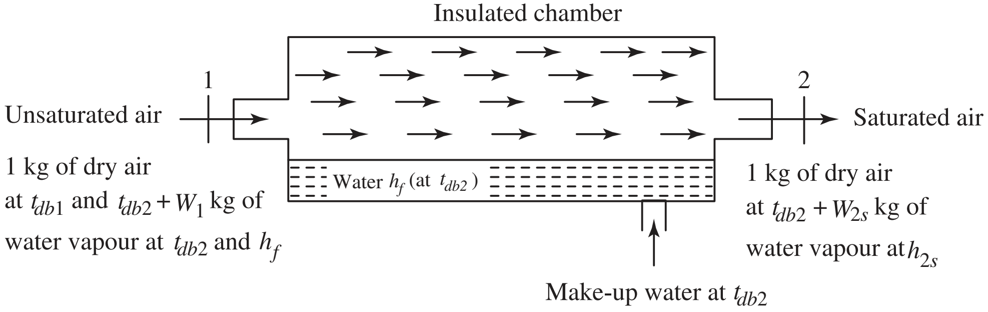


Fig. 9.1 Adiabatic Saturation Process

9.6.1 Analysis of Adiabatic Cooling Process

Applying the first law to the entire process and assuming a steady flow process and neglecting changes in kinetic and potential energies, we can write

$$h_1 + \left(W_{2s} - W_1\right) h_{f2} = h_{2s} \tag{9.35}$$

The quantities W_{2s}, h_{2s} and h_{f2} are the functions of temperature t_{db2}. Since, $\left(W_{2s} - W_1\right) h_{f2}$

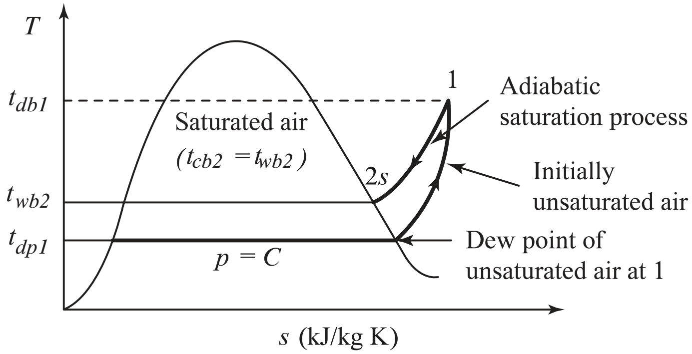


Fig. 9.2 Adiabatic Cooling Process

is very small compared to h_1, this term may be safely neglected. Now, it may be noted that the enthalpy remains constant in adiabatic saturation. Therefore, Eq.9.35 can be rewritten as:

$$h_1 - W_1 h_{f2} = h_{2s} - W_{2s} h_{f2} \tag{9.36}$$

If we generalise the inlet terms then the expression can be written as:

$$\sum = h_{2s} - W_{2s} h_{f2} = h_1 - W_1 h_{f2} = h_x - W_x h_{f2} \tag{9.37}$$

From the above equation, it may be concluded that sigma function ($\sum$) is constant for any wet-bulb temperature. Also,

$$h_1 = h_{2s} \left(W_{2s} - W_1 \right) h_{f2} \tag{9.38}$$

Equation 9.38 indicates that the enthalpy of an air-water vapour mixture is equal to the enthalpy of saturated air at the same wet-bulb temperature along with a small correction term, $(W_{2s} - W_1)\, h_{f2}$. This correction term is called *enthalpy deviation.*

$$h_1 = h_{air(1)} + W_1 h_{vapour(1)} \tag{9.39}$$

$$h_{2s} = h_{air(2)} + W_{2s} h_{vapour(2)} \tag{9.40}$$

$$h_{air(1)} + W_1 h_{g1} + \left(W_{2s} - W_1 \right) h_{f2} = h_{air(2)} + W_{2s} h_{g2} \tag{9.41}$$

$$c_p t_{db1} + W_1 h_{g1} + \left(W_{2s} - W_1 \right) h_{f2} = c_p t_{db2} + W_{2s} h_{g2} \tag{9.42}$$

After rearranging we get, $W_1 = \dfrac{c_p \left(t_{db2} - t_{db1} \right) + W_{2s} \left(h_{g2} - h_{f2} \right)}{h_{g1} - h_{f2}}$

Since $W_{2s} \times h_{f2}$ is negligibly small

$$W_1 = \frac{c_p \left(t_{db2} - t_{db1} \right) + W_{2s} h_{g2}}{h_{g1} - h_{f2}} \tag{9.43}$$

Note: The wet-bulb temperature is influenced by heat and mass transfer rates and therefore, it is not a property of moist air. Hence, in psychrometric equations and psychrometric charts where the wet-bulb temperature appears, it is to be taken as the thermodynamic wet-bulb temperature.

9.7 PSYCHROMETERS

A psychrometer is a device which is used for measuring dry-bulb and wet-bulb temperatures simultaneously and to estimate the amount of humidity in the atmosphere. Another name for psychrometer is hygrometer The psychrometers are usually classified as follows :

(i) Laboratory hygrometer or wet- and dry-bulb psychrometer

(ii) Aspiration psychrometer

(iii) Sling psychrometer

9.7.1 Laboratory Hygrometer or Wet- and Dry-Bulb Psychrometer

A typical wet- and dry-bulb psychrometer is shown in Fig.9.3. It is used to calculate the water vapour content (relative humidity) of air at a given temperature.

Wet- and dry-bulb psychrometer is the simplest of all hygrometers. It is common in laboratories and used for measuring humidity. As can be seen from the figure, it has two identical mercury thermometers. One has a wet-bulb and the other one has a dry-bulb. The wet-bulb thermometer has it is bulb wrapped in a tight fitting muslin cloth material called wick which is soaked in water. Evaporation of water from the wick causes its temperature reading to drop. Therefore, it shows a lower temperature than the dry-bulb (refer Fig.9.3). The function of the wick on the wet-bulb thermometer is to provide a thin film of water on the sensing bulb. To prevent errors, there should be a continuous film of water on the wick. The wick should be replaced frequently and only distilled water should be used for wetting it. The wick should extend beyond the bulb by 1 or 2 cm to minimise the heat conduction effects along the stem.

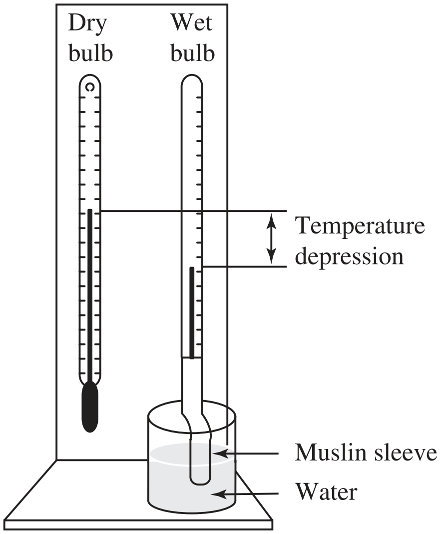


Fig. 9.3 Wet and dry-bulb psychrometer

Relative humidity is calculated by comparing the readings using the Table 9.2 that compares the ambient temperature (the temperature given by the dry-bulb) to the difference in temperatures between the two thermometers, viz depression.

9.7.2 Aspiration Psychrometer

The aspirator psychrometer is used for measuring the air temperature and the humidity. Two parallel mounted, identical mercury thermometers are used (Fig.9.3). The bulb of one thermometer (wet-bulb thermometer) is covered by a wick. It must be frequently moistened for a measurement. The bulb of the other thermometer remains without moistened wick and therefore, it is called dry-bulb. Both bulbs are encased by two radiation tubes. In order to achieve an effective radiation shield, the surface of the instrument is polished. Both thermometer bulbs are ventilated during measurements by fan. The ventilating or aspirating speed at the bulbs averages 3 m/s. Depending on the amount of water vapour in the ambient air, the water evaporates from the wick of the wet-bulb thermometer. The evaporative cooling causes the mercury column of the wet-bulb thermometer to drop. The dry-bulb thermometer indicates true air temperature. From the difference in temperatures of both the thermometers, the psychrometric depression can be calculated. From this, the relative humidity, the dew point temperature and the vapour pressure of the air may be computed or determined from Table 9.2.

9.7.3 Sling Psychrometer

Sling psychrometer is used to measure both dry- and wet-bulb temperatures at the same time. From these measurement of temperatures, humidity of air can be determined from Table 9.2. The sling psychrometer is used to measure relative humidity by measuring DBT and WBT. A typical sling psychrometer is shown in Fig.9.4. The wet-bulb exists below the dry bulb. This is done purposely to see that wick can be dipped in water without wetting the dry bulb.

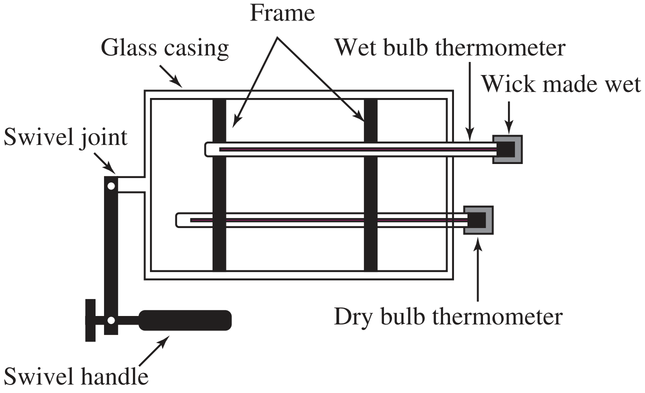


Fig. 9.4 Wet- and dry-bulb psychrometer

The main parts of the instrument are:

(i) The instrument frame which holds the thermometers.

(ii) One mercury in glass thermometer which is called as the dry-bulb thermometer.

(iii) Another mercury in glass thermometer whose sensing bulb is covered with a cotton or muslin wick. It is made wet with pure water and is called the wet-bulb thermometer.

(iv) The instrument frame carrying the thermometer is covered by a glass casing.

(v) A swivel handle is attached to frame for rotation.

Table 9.2 Relative humidity (per cent)

Dry-bulb (air) temperature	Dry bulb °C	Depression of wet-bulb temperature (Dry-bulb temperature minus wet-bulb temperature = Depression of wet-bulb)																					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
	−20	28																					
	−18	40																					
	−16	48	0																				
	−14	55	11																				
	−12	61	23																				
	−10	66	33	0																			
	−8	71	41	13																			
	−6	73	48	20	0																		
	−4	77	54	32	11																		
	−2	79	58	37	20	1																	
	0	81	63	45	28	11																	
	2	84	67	51	36	20	6																
	4	85	70	56	42	27	14																
	6	86	72	59	46	35	22	10	0														
	8	87	74	62	51	39	28	17	6														
	10	88	76	65	54	43	38	24	13	4													
	12	88	78	67	57	48	38	28	19	10	2												
	14	89	79	69	60	50	41	33	25	16	8	1											
	16	90	80	77	62	54	45	37	29	21	14	7	1										
	18	91	81	72	64	56	48	40	33	26	19	12	6	0									
	20	91	82	74	66	58	51	44	36	30	23	17	11	5									
	22	92	83	75	68	60	53	46	40	33	27	21	15	10	4	0							
	24	92	84	76	69	62	55	49	42	36	30	25	20	14	9	4	0						
	26	92	85	77	70	64	57	51	45	39	34	28	23	18	13	9	5						
	28	93	86	78	71	65	59	53	45	42	36	31	26	21	17	12	8	4					
	30	93	86	79	72	66	61	55	49	44	39	34	29	25	20	16	12	8	4				
	32	93	86	80	73	68	62	56	51	46	41	36	32	27	22	19	14	11	8	4			
	34	93	86	81	74	69	63	58	52	48	43	38	34	30	26	22	18	14	11	8	5		
	36	94	87	81	75	69	64	59	54	50	44	40	36	32	28	24	21	17	13	10	7	4	
	38	94	87	82	76	70	66	60	55	51	46	42	38	34	30	26	23	20	16	13	10	7	5
	40	94	89	82	76	71	67	61	57	52	48	44	40	36	33	29	25	22	19	16	13	10	7

To determine the relative humidity, find the air (dry bulb) temperature on the vertical axis (far left) and the depression of the wet-bulb on the horizontal axis (top) where the two meet. For example, when *DBT* is 20 °C and a *WBT* is 14 °C, then the depression of the wet-bulb is 6 °C (20 °C – 14 °C). From the table, the relative humidity is 51 per cent and the dew point is 10 °C.

In order to measure the dry-bulb and the wet-bulb temperature, the psychrometer frame is rotated at a reasonable speed to get the necessary air motion. An important point to note is that for correct and accurate measurement of wet-bulb temperature air movement at the wet-bulb should be around 5–10 m/s. The thermometer whose bulb is bare will indicate the dry-bulb temperature. At the same time, the thermometer whose bulb is covered with the wet wick will indicate the wet-bulb temperature which will naturally be lesser than the dry-bulb temperature. When dry- and wet-bulb temperatures are known the other psychrometric properties like relative humidity, dew point temperature, degree of saturation, humidity ratio and volume of air-vapour mixture per kg of dry air are determined by calculations.

If the psychrometer is rotated only for a short period, say less than a minute, then the wet-bulb temperature recorded will not be accurate. On the other hand, if the psychrometer is rotated for a long period, say more than 3 min, the wick will get dried soon and the wet-bulb temperature will not be at its minimum value. Therefore, it will be reasonable if it is rotated for 2 min.

9.7.4 Application

(i) It is used for checking humidity level in air-conditioned rooms and installations.

(ii) It is used to set and check hair hygrometer.

(iii) It is used in the measurement range of 0 to 100% RH.

(iv) It is used for measuring wet-bulb temperature between 0 °C and 180 °C.

9.7.5 Limitation

(i) The measured medium is disturbed due to the act of rotation.

(ii) The evaporation process at the wet-bulb will add moisture to the air.

(iii) It cannot be used in automated measurements.

(iv) It cannot be used for continuous recording purpose.

9.7.6 Continuous Recording Psychrometer/Data Logger

Continuous recordings of psychrometers are available for measurement of dry-bulb and relative humidity of the air using various types of sensors and transducers. The values of the air quality will be recorded and it is possible to get digital output of data. A typical psychrometer shown in Fig.9.5 is battery powered but requires some manual operation. There are automatic ways of determining and recording temperatures and relative humidity. The hygro thermograph (shown in Fig.9.5) measures and records both temperature and relative humidity on a continuous recording graph using appropriate sensors. This instrument is utilised by placing it in a specially designed field instrument shelter. Certain standards have been established to ensure uniformity of measurements. For example, the shelter will be positioned 1.5 m above the ground in a non-shaded open area that has good air ventilation. Such shelters and weather instruments are found at every weather station.

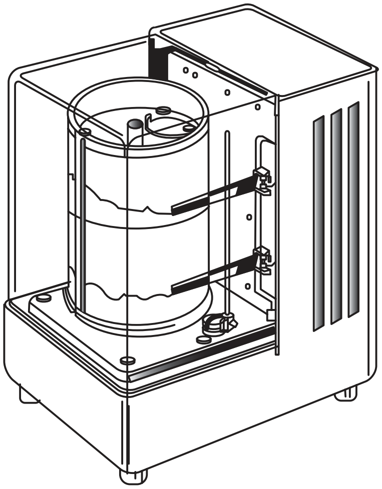


Fig. 9.5 Continuous recording hygrometer

9.8 PSYCHROMETRIC CHARTS

Figure 9.6 shows the outer boundary of a psychrometric chart with typical values. In this chart, x-axis represents the dry-bulb temperature whereas y-axis on the right-side represents specific humidity. The outer curved line at the left of the chart represents 100% relative humidity. The magnitude of the dry-bulb temperature from −10° to +55°C is marked on the x-axis. The magnitude of specific humidity from 0 to 0.03 is marked on the y-axis.

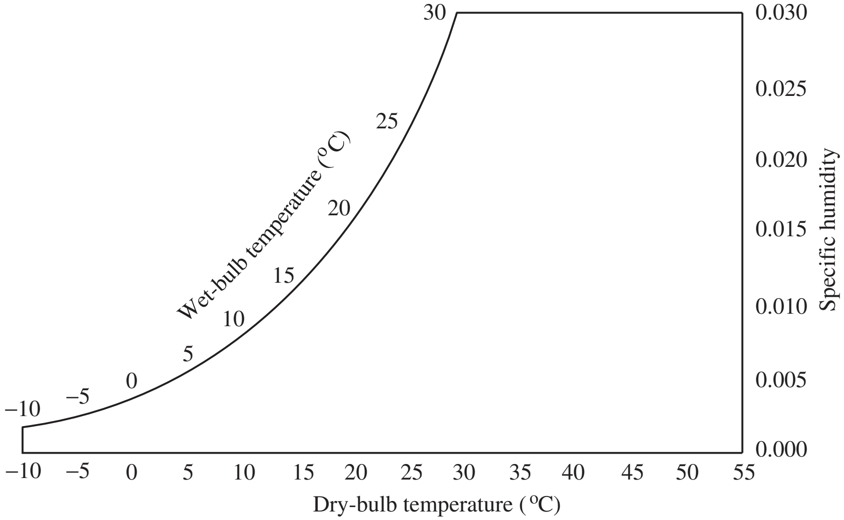


Fig. 9.6 Outer boundary of a psychrometric chart

As shown in Fig.9.7, if we draw vertical lines from the x-axis (drawn as dotted lines), it will represent constant dry-bulb lines. Where it meets this 100% RH curve, it denotes the WBT. The point to note is that when RH is 100% the DBT and WBT are the same.

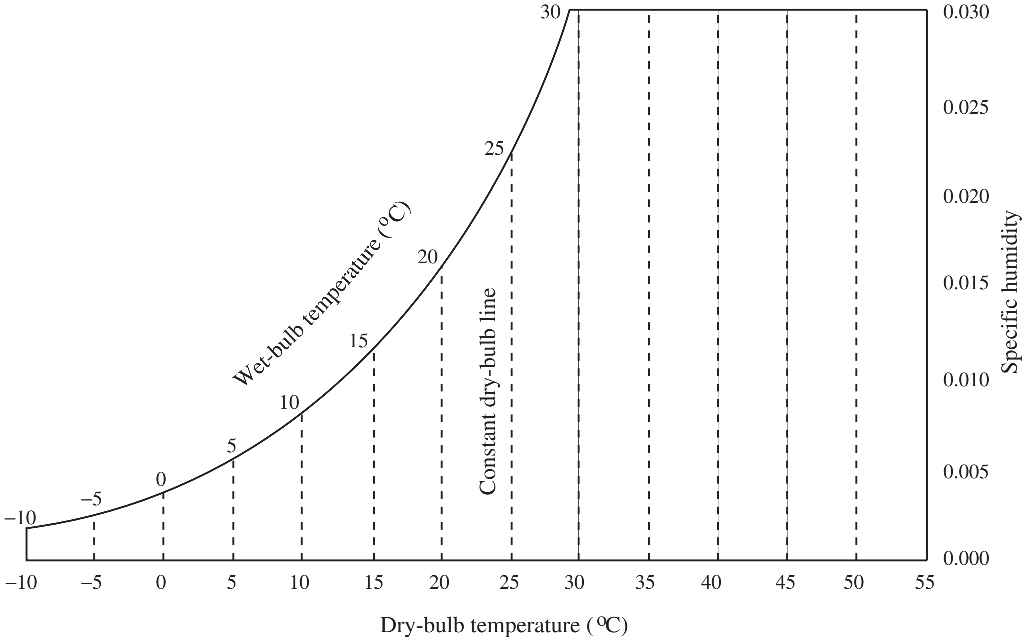


Fig. 9.7 Vertical lines shown on a typical psychrometric chart

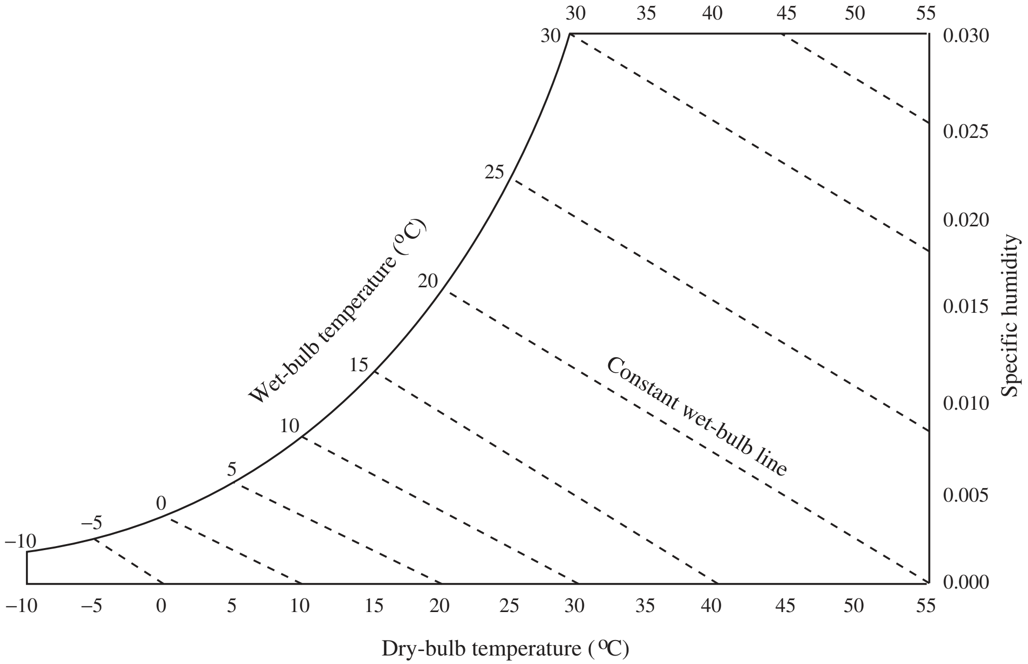


Fig. 9.8 Constant wet-bulb lines of a typical psychrometric chart

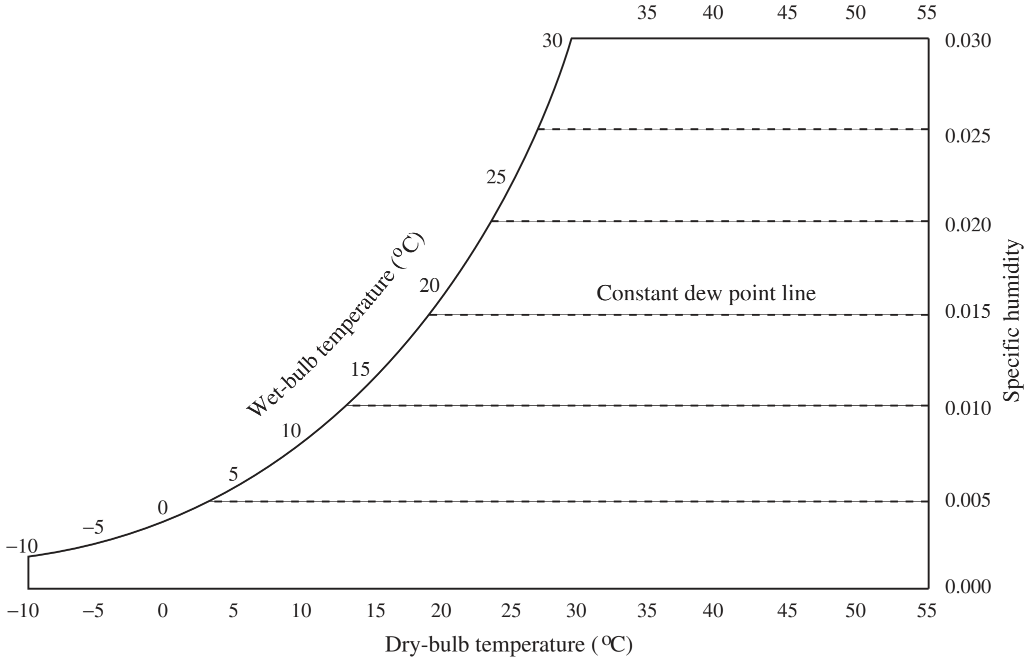


Fig. 9.9 Dew point temperatures and specific humidity lines of a typical psychrometric chart

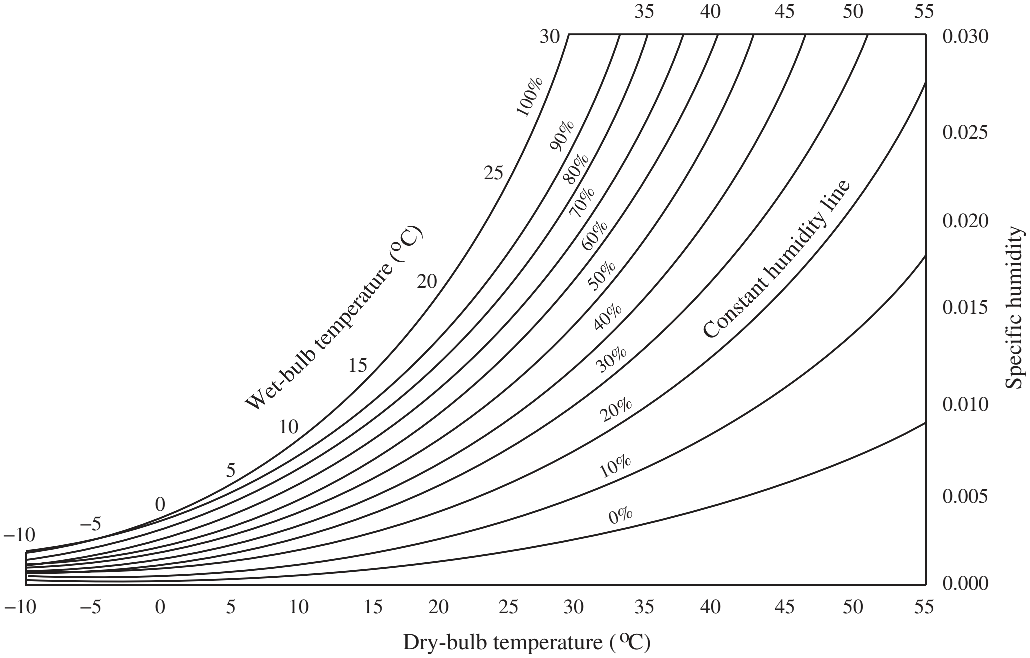


Fig. 9.10 Constant relative humidity lines of a typical psychrometric chart

Now consider Fig.9.8, the wet-bulb temperatures are shown along the outer curved line at the left of the chart. The constant wet-bulb lines run downward at an angle of approximately 30° from the horizontal. The constant wet-bulb lines (drawn as dotted lines) extend from left margin to the right margin. Also you should note that at RH 100%, the wet-bulb temperature and the dew point temperature are the same. Recall that the temperature at which moisture completely saturates the air is called dew point. Only at 100% RH, the air is completely saturated.

Now consider Fig.9.9, the constant dew point lines run horizontally from the left curved line to the right. Any point on any one constant dew point line will correspond to the dew point temperature given on the curved line. Similarly, the constant specific humidity lines are also horizontal and will coincide with the constant dew point lines. This indicates that the amount of water vapour in the air is dependent upon the dew point of air.

Figure 9.10 shows the curved constant humidity lines extending upwards from left to right. As can be seen, the magnitude of these relative humidity lines decreases as we move away from the saturation lines to the right and downward. The values for these RH lines are indicated as percentage in Fig.9.10.

Figure 9.11 shows the constant specific volume lines. These lines are approximately at an angle of 60° from the horizontal. The values are in increasing order from left to right as can be seen from the figure. On the chart, the spacing between the lines will represent a change in specific volume. Note that the values for these specific volume lines are given every fifth line. Therefore, any point falling between these lines will be only estimated values. If one wants to find the density of air at any particular condition, one merely take the inverse of the specific volume. In air-conditioning calculations, we deal with the mass of air rather than volume. Therefore, it is appropriate to use specific volume (m^3/kg) rather than density (kg/m^3).

Figure 9.12 shows the constant enthalpy lines. It may be noticed that these are almost the mere extension of the wet-bulb lines. It is due to the fact that the total heat of the air is mainly dependent on the wet-bulb temperature. The scale at the far left of the chart gives the total heat of air in J/g of dry air and increases from left to right as shown in the figure.

Let us now look at the complete chart in Fig.9.13. In this chart, all the constant value lines are superimposed in one chart and all of them have the same relative position on this chart.

9.8.1 Locating Parameters on the Chart

Dry-bulb temperature: These lines are drawn straight, not always parallel to each other and slightly inclined from the vertical position. This is the x-axis, the abscissa (horizontal) axis. Each line represents a constant temperature.

Dew point temperature: From the state point follow the horizontal line of constant humidity ratio to the intercept of 100% RH, also known as the saturation curve. The dew point temperature is equal to the fully saturated dry-bulb or wet-bulb temperatures.

Wet-bulb temperature: These lines are oblique lines that differ slightly from the enthalpy lines. They are identically straight but are not exactly parallel to each other. These intersect the saturation curve at DBT point.

Relative humidity: These hyperbolic lines are shown in intervals of 10%. The saturation curve is at 100% RH, while dry air is at 0% RH.

Humidity ratio: These are the horizontal lines on the chart. Humidity ratio is usually expressed as mass of moisture per mass of dry air (kilogram of moisture per kilogram of dry air, respectively). The range is from 0 for dry air up to 0.03 (kg_w/kg_a) on the right hand W-axis, the ordinate or vertical axis of the chart.

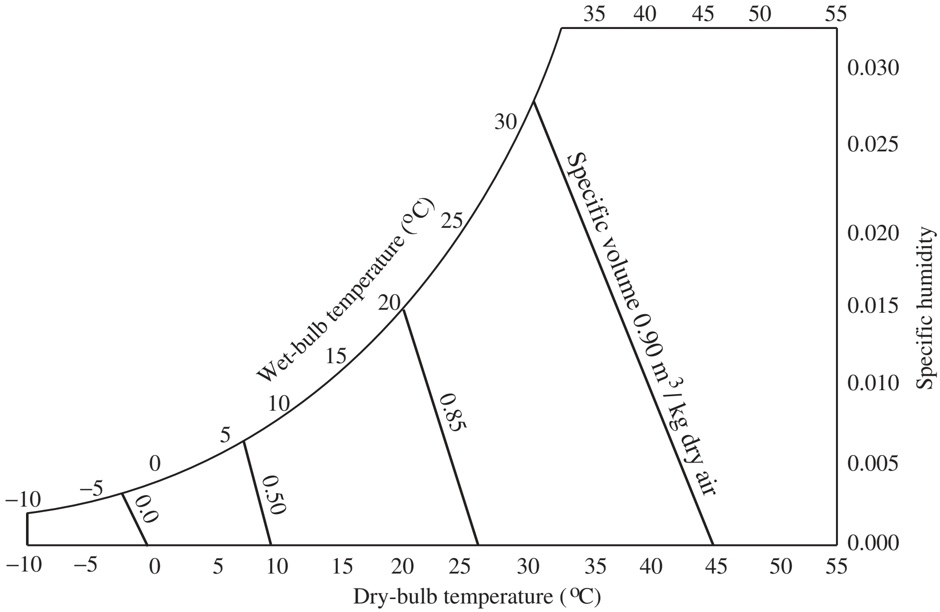


Fig. 9.11 Constant specific volume lines of a typical psychrometric chart

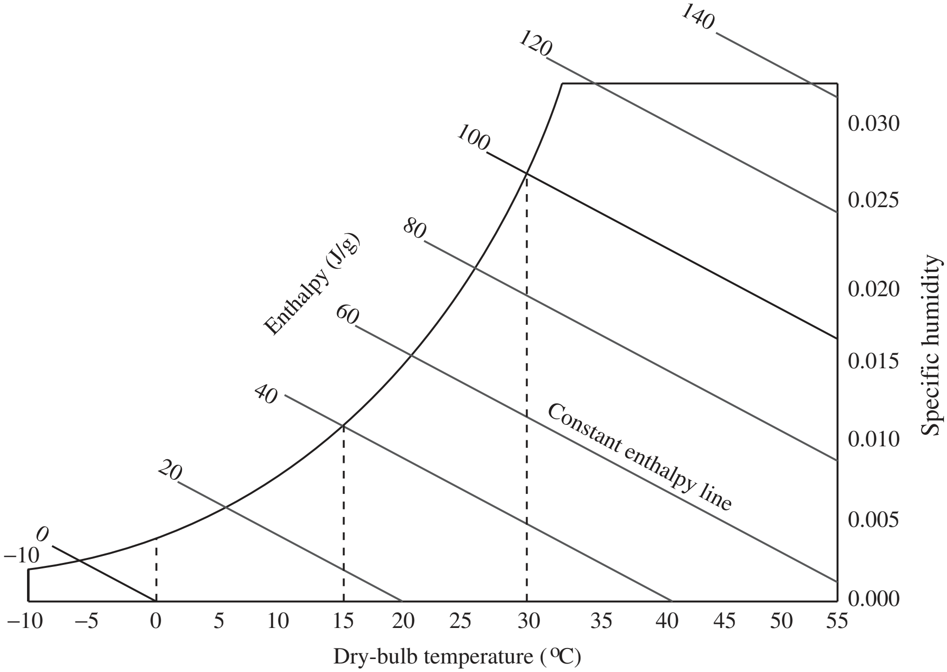


Fig. 9.12 Constant enthalpy lines of a typical psychrometric chart

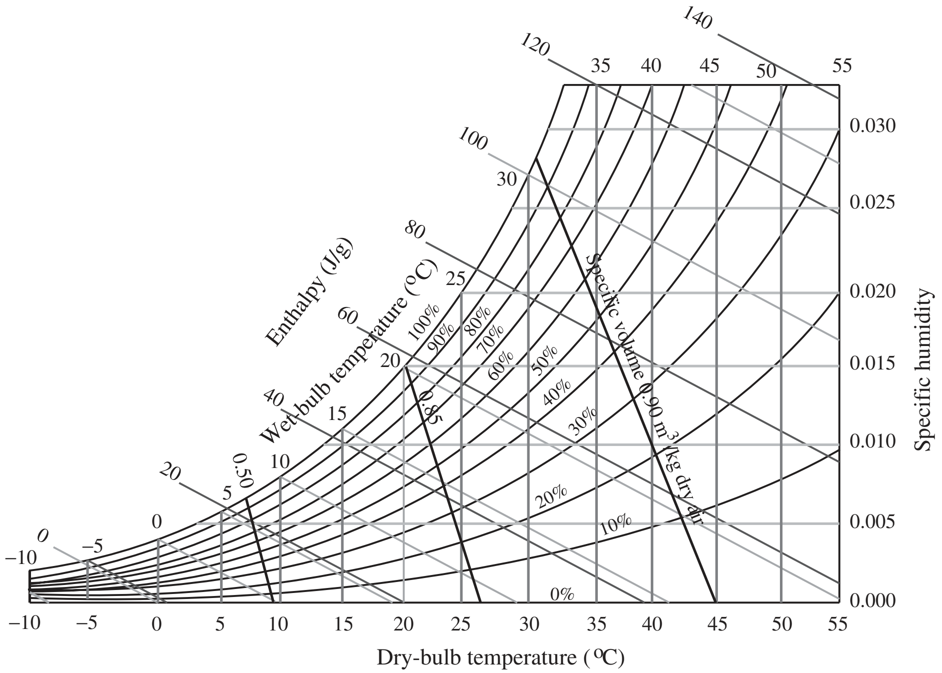


Fig. 9.13 Constant dry-bulb lines of a typical psychrometric chart

Specific enthalpy: These are oblique lines drawn diagonally downward from left to right across the chart that are parallel to each other. These are not parallel to wet-bulb temperature lines.

Specific volume: These are a family of equally spaced straight lines that are nearly parallel.

The region above the saturation curve is a two-phase region that represents a mixture of saturated moist air and liquid water, in thermal equilibrium.

9.8.2 How to Read the Chart : Fundamental Examples

We now have on the composite psychrometric chart with number of lines criss-crossing. If we have to place a point on a constant dry-bulb line, that point will also correspond to different values on the other constant lines for wet-bulb temperature, dew point, relative humidity, specific volume and enthalpy. Any given two of these constant lines will cross at only one point on the chart. Therefore, we can spot this point exactly, if we know any two of the properties of the air. From this point, we can then move along the respective constant lines for the other properties of air. Then, we can read them from their respective scales without going for any calculation. Even though this method may not be very accurate compared to calculation method, it is much faster. The degree of accuracy is acceptable for any engineering calculations. Let us explain with some typical examples.

(i) Relative humidity: The percentage relative humidity can be located at the intersection of the vertical dry-bulb and diagonally down sloping wet-bulb temperature lines. Consider a dry-bulb of 25 °C and a wet-bulb of 20 °C. The relative humidity can be read as 67.5%. In this example, the humidity ratio is 0.0126 kg water per kg dry air.

(ii) Effect of temperature change on relative humidity: For air of a fixed water composition or moisture ratio, find the starting relative humidity from the intersection of the wet- and dry-bulb temperature lines. Using the conditions from the previous example, the relative humidity at a different dry-bulb temperatures can be found along the horizontal humidity ratio line of 0.0126, either in kilogram water per kilogram dry air.

A common variation of this problem is determining the final humidity of air leaving an air-conditioner evaporator coil heated to a higher temperature. Assume that the temperature leaving the coil is 10 °C and is heated to room temperature (not mixed with room air), which is found by following the horizontal humidity ratio from the dew point or saturation line to the room dry-bulb temperature line and reading the relative humidity. In typical practice, the conditioned air is mixed with room air that is being infiltrated with outside air.

(iii) Determining the amount of water to be removed or added in lowering or raising relative humidity: This is the difference in humidity ratio between the initial and final conditions multiplied by the weight of dry air.

9.9 PSYCHROMETRIC PROCESSES

For human comfort and for the optimum control of an industrial process, certain processes are to be carried out on the atmospheric air available. The processes that affect the psychrometric properties of air are called *psychrometric processes.* These involve the following important psychrometric processes:

(i) Adiabatic mixing of air streams
(ii) Cooling and dehumidification
(iii) Cooling and humidification
(iv) Heating and dehumidification
(v) Heating and humidification
(vi) Sensible cooling
(vii) Sensible heating

9.9.1 Adiabatic Mixing of Air Streams

In air-conditioning process, mixing of several air streams becomes necessary. Usually this mixing takes place without the addition or rejection of either heat or moisture. Therefore, it is called adiabatic mixing at constant total moisture content. Typical diagram showing mixing of two streams of air is shown in Figs.9.14(a) and the corresponding processes are shown in Fig.9.14(b).

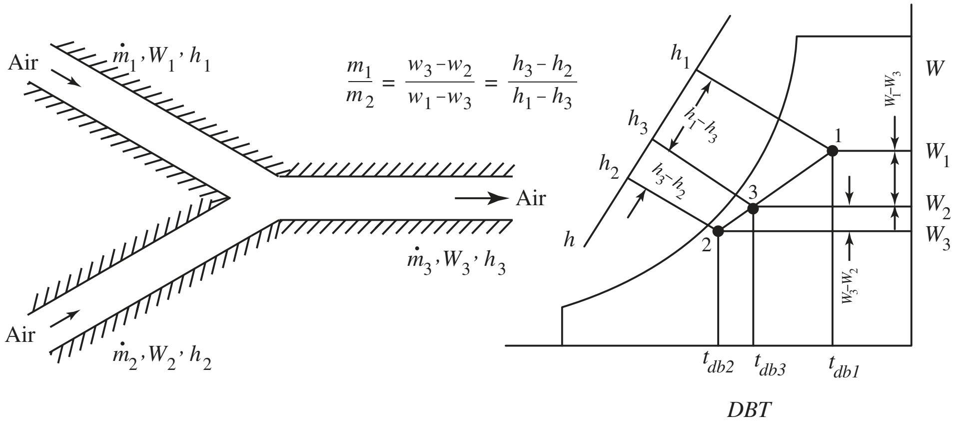
From Fig.9.14, we can write the following equations:

$$m_1 + m_2 = m_3 \tag{9.44}$$

$$m_1 W_1 + m_2 W_2 = m_3 W_3 = (m_1 + m_2) W_3 \tag{9.45}$$

$$m_1 h_1 + m_2 h_2 = m_3 h_3 \tag{9.46}$$

By rearranging Eq.9.45, we can write:



(a) (b)

Fig. 9.14 Mixing of air streams

$$m_1 (W_1 - W_3) = m_2 (W_3 - W_2) \tag{9.47}$$

Similarly, from Eq.9.46:

$$m_1 (h_1 - h_3) = m_2 (h_3 - h_2) \tag{9.48}$$

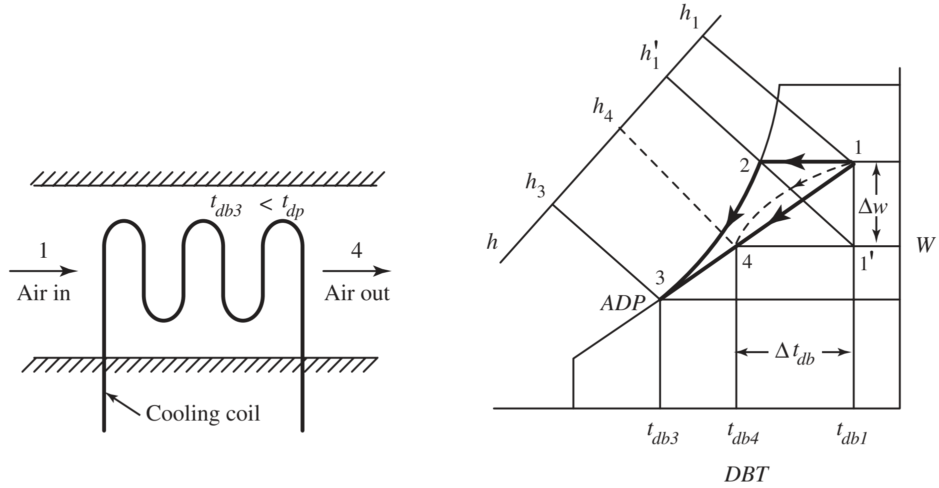
Now,

$$\frac{m_1}{m_2} = \frac{W_3 - W_2}{W_1 - W_3} = \frac{h_3 - h_2}{h_1 - h_3} \tag{9.49}$$

On the psychrometric chart, the specific humidity and enthalpy scales for all practical purposes are linear. Therefore, the final state 3 lies on a straight line connecting the initial states of streams 1 and 2 before mixing. The final state 3 divides this line into two parts. It is evident that it will be in the same ratio of the two masses of air before mixing. If the air quantities are in terms of volume instead of mass, it can be considered sufficiently accurate to use units of m^3 or m^3/min. in the mixing equations. The inaccuracy introduced, if any, is due to the difference in specific volume at two initial states. This difference can be considered small for most of the comfort air-conditioning problems.

9.9.2 Cooling and Dehumidification

Consider Fig.9.15, where air is made to pass over a cooling coil. Assume that the surface temperature is at a temperature less than the dew point temperature of the air. Condensation of some of the water vapour in air will occur simultaneously with the sensible cooling process. As air that comes into sufficient contact with the cooling surface, the temperature gets reduced to the mean surface temperature. The process path 1-2-3 is shown in Fig.9.15(b). Note that there will be condensation and therefore, dehumidification will take place between points 2 and 3. Some air may not come into contact with the cooling surface. However, it will be finally cooled by mixing with the portion that came into contact with the cooling surface. The final state point will somewhere on the straight line connecting points 1 and 3. Note that the actual path of air will not be a straight line. It will be something similar to the dashed curved line 1–4. It is due to continuous mixing of air that is in contact with a particular portion of the coil and the air which is bypassing it. It is convenient and easier to analyse the problem using the straight line. Assume that the final air results from the mixing of air that has completely by passed the coil with air that has been cooled to the mean effective surface temperature. If there is enough contact between air and surface then the entire air will reach the mean surface temperature. This process is one of zero by pass. In any practical system, complete saturation is not obtained and final state will be a point such as 4 in Fig.9.15(b) with an equivalent by pass factor equal to $\frac{\text{length } 3-4}{\text{length } 3-1}$.



(a) (b)

Fig. 9.15 Cooling and Dehumidification

For processes involving condensation, the effective surface temperature, viz t_{db3}, in Fig.9.15 is called *apparatus dew point* (ADP). The final state point of air will be in mixture condition. It is because it passes through a cooling and dehumidifying apparatus. The final mixing condition is due to mixing that fraction of the air having equivalent bypass factor (BF) and is at initial state point. The remaining fraction which is equal to one minus by pass factor $(1 - BF)$ and is saturated at the ADP.

Total heat removed from the air is given by:

$$Q_t = (h_1 - h_4) = (h_1 - h_1') + (h_1' - h_4) = Q_L + Q_S \tag{9.50}$$

where Q_L is latent heat removed, viz $(h_1 - h_1')$ and Q_S is sensible heat removed, viz $(h_1' - h_4)$.

The ratio $\frac{Q_S}{Q_L}$ is called sensible heat factor (SHF) or sensible heat ratio (SHR). Therefore,

$$SHF = \frac{Q_S}{Q_L + Q_S} \tag{9.51}$$

The ratio fixes the slope of the line 1–4 on the chart shown in Fig.9.15(b). SHF slope lines are given on the psychrometric chart. If the initial condition and SHF are known for the given process, then the process line can be drawn. This line is drawn from the given initial condition at a slope given by SHF on the psychrometric chart. The capacity of the cooling coil in tonnes of refrigeration (TR) is given by:

$$\text{Capacity in } TR \quad = \quad \frac{\dot{m}_a\,(h_1 - h_4) \times 60}{14000} \tag{9.52}$$

where $\dot{m}_a$ is mass of air, kg/min and h is enthalpy in kJ/kg of air.

9.9.3 Cooling and Humidification

Consider unsaturated air passing through a spray of continuously recirculated water. Now, the specific humidity will increase while the dry-bulb temperature will decrease. This process is called *adiabatic saturation* or *evaporative cooling process.* For all practical purposes, it is one of the constant wet-bulb temperature process. The process is shown as path 1-2 on Fig.9.16. The wet-bulb temperature of air is point 3. This is also equilibrium temperature of the recirculated water. If there is sufficient contact between air and spray, the air will leave at a condition very close to that of point 3. The concept of equivalent bypass can be applied to this process also. However, more often it is used to describe the performance of a humidifying apparatus. The *saturating* or *humidifying efficiency* is defined as the ratio of dry-bulb temperature decrease to the entering wet-bulb depression usually expressed as percentage. Thus, from Fig.9.16, the saturating efficiency can be written as:

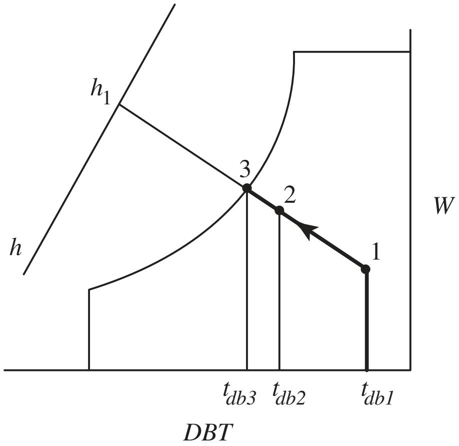


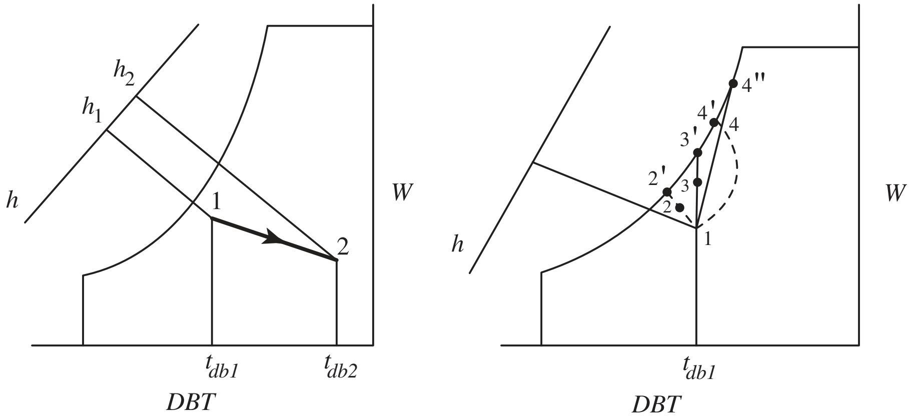
Fig. 9.16 Cooling and humidification

$$\eta_{sat} \quad = \quad \left(\frac{t_{db1} - t_{db2}}{t_{db1} - t_{db3}}\right) \times 100 \tag{9.53}$$

As a fraction, it is equal to one minus the by pass factor for the process. This adiabatic process, for all practical purposes, is line of constant enthalpy. The moisture added can be obtained from the increase in specific humidity.

9.9.4 Heating and Dehumidification

If air is passed over a solid absorbent surface or through a liquid absorbent spray simultaneous heating and dehumidification will be accomplished. In either case, the dehumidification results from adsorbent[1] or absorbent. It is because it has a lower water vapour pressure than air. To start with moisture gets condensed from the air. Consequently, the latent heat of condensation is liberated, causing sensible heating of air. If we consider only the energy, the process would be the inverse of the adiabatic saturation process. However, an additional energy absorbtion or liberation by the active material. It is usually called as the heat of adsorption or absorption. For the solid adsorbents used commercially, such as silica gel or activated alumina. Similarly, common liquid absorbent, such as solutions of organic salts or inorganic compounds like ethylene, glycol will also involve suitable heating and results in additional sensible heating. Thus, the path lies above a constant wet-bulb line on the psychrometric chart such as path 1-2 in Fig.9.17(a).



(a) Heating and dehumidification (b) Heating and humidification

Fig. 9.17 Heating, dehumidification and humidification

9.9.5 Heating and Humidification

If air is passed through a humidifier which has heated water sprays instead of simply recirculated spray, the air is humidified. It may be heated, cooled or temperature may remain constant. In such a process, specific humidity and the enthalpy of air increase. Further, the dry-bulb temperature may increase or decrease according to the initial temperature of the air and that of the spray. If sufficient water is supplied relative to the mass flow of air, the air will approach saturation at water temperature. Examples of such processes are shown on Fig.9.17(b).

Process 1-2: It denotes the cases in which the temperature of the heated spray water is less than the air DBT.

Process 1-3: It denotes the cases in which the temperature is equal to the air DBT.

Process 1-4: It denotes the cases in which a spray temperature is greater than air DBT. As in the case of adiabatic saturation, the degree to which the process approaches saturation can be expressed in terms of the bypass factor or a saturating efficiency. If the water rate relative to the air quantity is smaller, the water temperature will drop significantly during the

---

[1] A material that has the ability to extract certain substances from gases, liquids or solids by causing them to adhere to its surface without changing the physical properties of the adsorbent. Activated carbon, silica gel and activated alumina are materials frequently used for this application.

process. The resultant process will be a curved line such as the dashed 1-4 where 4 represents the leaving water temperature.

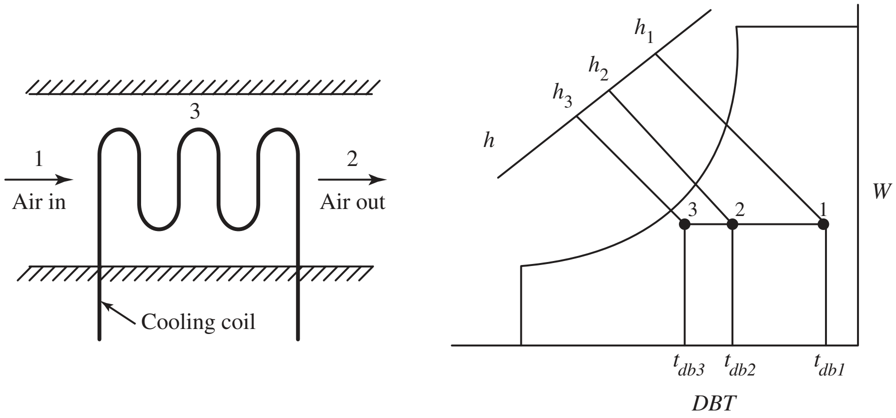
Note. It is possible to accomplish heating and humidification by evaporation from an open pan of heated water or by direct injection of heated water or steam. The latter is more common. The process line for it is of little value because the process is essentially an instantaneous mixing of steam and the air. The final state point of the air can be found, however, by making a humidity and enthalpy balance for the process. The solution of such a problem usually involves cut-and-try procedure.

9.9.6 Sensible Cooling

Refer Fig.9.18(a). Air undergoes sensible cooling whenever it passes over a surface that is at a temperature lower than the dry-bulb temperature of the air but greater than the dew point temperature. Thus, sensible cooling can be achieved by passing the air over cooling coil like evaporating coil in the refrigerator or secondary brine coil. During the process, the specific humidity remains constant. However, dry-bulb temperature decreases, approaching the mean effective surface temperature. On a psychrometric chart, the process will appear as a horizonal line 1–2 (Fig.9.18(b), where point 3 represents the effective surface temperature. For this process:

$$BF = \frac{t_{db2} - t_{db3}}{t_{db1} - t_{db3}} \tag{9.54}$$

The heat removed from air can be obtained from the enthalpy difference $(h_1 - h_2)$ or from humid specific heat multiplied by the temperature difference $(t_{db1} - t_{db2})$.



(a) (b)

Fig. 9.18 Sensible Cooling

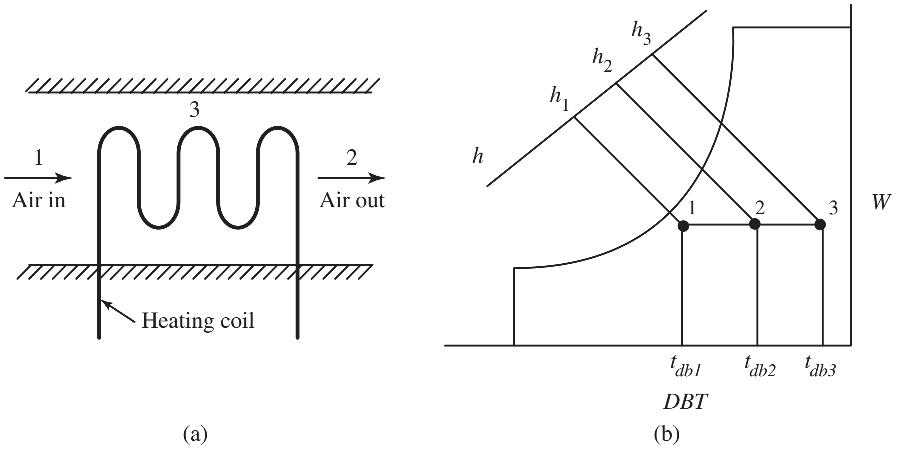
9.9.7 Sensible Heating

When air passes over a dry surface which is at a temperature higher than its dry-bulb temperature of the flowing air, it is considered as sensible heating process. Thus, heating of air can be achieved by passing the air over heating coil like steam or electric resistance heating coils. During such a process, the specific humidity remains constant. However, the dry-bulb temperature rises and approaches that of the surface. The extent to which it approaches the mean effective surface temperature of the coil is conveniently expressed in terms of the equivalent pass factor.

The bypass factor (BF) for the process is defined as the ratio of the difference between the mean surface temperature of the coil and leaving air temperature to the difference between the mean surface temperature and the entering air temperature. Thus, on Fig.9.19(a), air at temperature t_{db1}, passes over a heating coil with a mean surface temperature t_{db3} and the air leaves at temperature t_{db2}. The bypass factor is expressed as follows:

$$BF \quad = \frac{t_{db3} - t_{db2}}{t_{db3} - t_{db1}} \tag{9.55}$$

or in terms of lengths on the chart [Fig.9.19(b)] it is $\frac{\text{length 2-3}}{\text{length 1-3}}$. The value of the bypass factor is a function of coil design and velocity of air. The heat added to the air can be obtained directly from the entering and leaving enthalpies ($h_2 - h_1$) or it can be obtained from the humid specific heat multiplied by the temperature difference ($t_{db2} - t_{db1}$). In a complete air-conditioning system, the preheating and reheating of air are among the familiar examples of sensible heating. Bypass factor can be viewed as the fraction of air which does not come into contact with coil surface.



(a) (b)

Fig. 9.19 Sensible Heating

9.9.8 Applications

The principles of psychrometry can be applied to any physical system consisting of gas-vapour mixtures. However, the most common system of interest is the mixture of water vapour and air. The important application are in heating, ventilating and air-conditioning and meteorology. In industrial drying applications, such as drying paper, manufacturers usually try to achieve higher drying rate and reduce energy usage, which decreases as exhaust relative humidity increases. In many industrial applications. It is important to avoid condensation that would ruin product or cause corrosion. Molds and fungi can be controlled by keeping relative humidity low. Wood destroying fungi generally do not grow at relative humidity below 75%.

In subsurface environmental applications, engineers have a particular interest in psychrometry for two reasons. First, if we are to comprehend fully the thermodynamic processes that occur in ventilation circuits then variations in humidity must be taken into account. For example, strata heat may be emitted into a wet airway without there being a corresponding increase in air temperature. This could occur if all the added heat were utilised in exciting some of the water molecules until their kinetic energy exceeded the attractive forces of other molecules in the liquid water. They would then escape through the liquid/air surface and become airborne as a gas. The process of evaporation increases the energy content of the air/vapour mixture. This may be termed a *latent* (or hidden) rise in the heat content of the air as there is no commensurate increase in temperature and hence, no indication on an ordinary thermometer. Alternatively, if there were no liquid water present, then the strata heat would be directed immediately to the airstream, causing a temperature rise of the air that would be sensed by a thermometer. This is an increase in the sensible heat of the air. These examples illustrate that if we are to predict quantitatively the climatic effects of strata heat, water inflows, machines or air coolers, then we need to have methods of analysis that take humidity into account. The second reason for the study of psychrometry is the effect of heat and humidity on the human body. However, for the time being, we will concentrate on developing means of quantifying the psychrometric relationships that enable predictions to be made of temperature and other climatic variables in the environment.

Worked-Out Examples

9.1 Atmospheric air at 1 atm has a dry-bulb temperature of 30 °C and a wet-bulb temperature of 25 °C. Calculate the following:

- (i) the partial pressure of water vapour
- (ii) the specific humidity
- (iii) the dew point temperature
- (iv) the relative humidity
- (v) the degree of saturation
- (vi) the density of the air in the mixture
- (vii) the density of vapour in the mixture
- (viii) the enthalpy of the mixture

Take specific heat of steam $c_{ps} = 1.88$ kJ/kg K.

Solution

For the given conditions, viz $DBT = 30°\text{C}$ and $WBT = 25$ °C. The state of air is indicated in Fig.9.20. The line 1-2 represents the constant WBT and enthalpy of air, which holds good approximately for an adiabatic saturation process. The specific humidity at state 2 is given by Eq.9.18. If we consider saturation pressure at WBT, Eq.9.18 can be written by:

$$W_2 = 0.622 \times \frac{p_s}{p - p_s}$$

where p_s is saturation pressure and the p is the actual pressure = 1.0132 bar. The saturation pressure p_s at the WBT of 25 °C from steam table is read as 0.03169:

$$W_2 = \frac{0.622 \times 0.03169}{1.0132 - 0.03169} = 0.0201 \text{ kg vap/kg dry air}$$

Referring Eq.9.43 and taking notation from Fig.9.1:

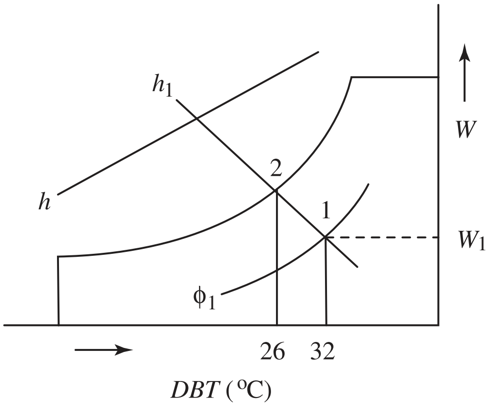


Fig. 9.20

$$W_1 = \frac{c_{pa}(T_2 - T_1) + W_2 \times h_{fg2}}{h_{w1} - h_{f2}}$$

From steam table, at 25 °C, $h_{fg2} = 2442.3$ and $h_{f2} = 104.87$ and At 30 °C, $h_{w1} = h_g = 2556.2$.

$$W_1 = \frac{1.005 \times (25 - 30) + 0.0201 \times 2442.3}{2556.2 - 104.87}$$

$$= 0.018 \text{ kg vap/kg dry air}$$

$$W_1 = 0.622 \times \frac{p_v}{p - p_v} = 0.018$$

$$\frac{p - p_v}{p_v} = \frac{0.622}{0.018} = 34.56$$

$$\frac{p}{p_v} - 1 = 34.56$$

$$p_v = \frac{1.0132}{35.56} = 0.028 \text{ bar}$$

By interpolation, saturation temperature at 0.028 bar = 22.88 °C.

$$\text{Relative humidity, } \phi = \frac{p_v}{p_{sat}}$$

At 30 °C, $p_{sat} = 0.0425$ bar

$$\phi = \frac{0.028}{0.0425} = 0.659 = 65.9\%$$

$$\text{Partial pressure of dry air, } p_a = p - p_v = 1.0132 - 0.028 = 0.9852 \text{ bar}$$

$$\text{Density of dry air, } \rho_a = \frac{p_a}{RTdb} = \frac{0.9852 \times 100}{0.287 \times (273 + 30)} = 1.13 \text{ kg/m}^3 \text{ dryair}$$

$$\text{Density of water vapour, } \rho_w = 0.018 \times 1.13 = 0.0204 \text{ kg vap/m}^3 \text{ dry air}$$

$$\begin{aligned} \text{Enthalpy of mixture, } h &= h_a + Wh_w = c_p t_a + W[h_g + c_{ps} \times (t_{db} - t_{DBT})] \\ &= 1.005 \times 30 + 0.018 \times [2556.2 + 1.88 \times (30 - 22.88)] \\ &= 76.4 \text{ kJ/kg} \qquad \overset{\text{Ans}}{\Longleftarrow} \end{aligned}$$

9.2 In an adiabatic saturator, an air-water vapour mixture enters at 35 °C and leaves at 25 °C which is the adiabatic saturation temperature. The pressure remains at 1 bar. Calculate the relative humidity and the humidity ratio of the inlet mixture.

Solution

$$\text{Specific humidity at exit, } W_2 = 0.622 \times \frac{p_s}{p - p_s}$$

Saturation pressure at 25 °C from steam table 3.169 kPa:

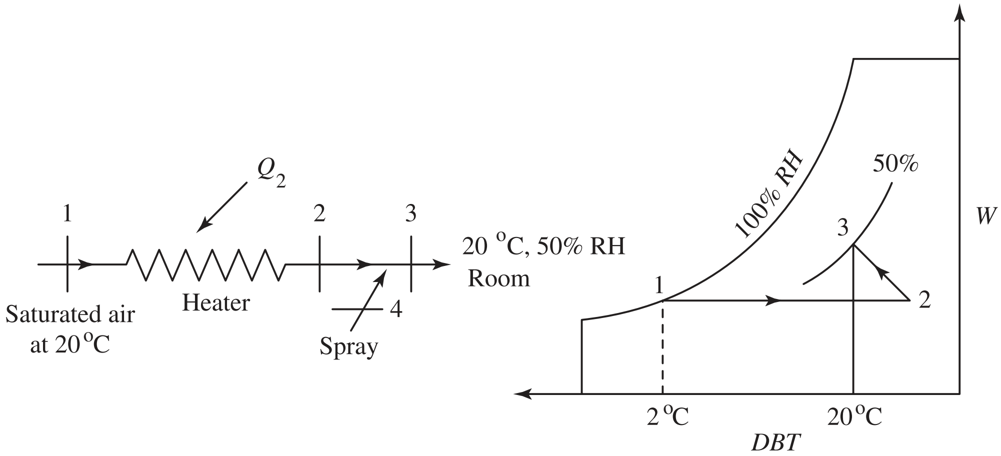
$$\begin{aligned} W_2 &= 0.622 \times \frac{3.169}{100 - 3.169} = 0.020 \text{ kg vap/kg dry air} \\ \text{Specific humidity at inlet, } W_1 &= \frac{c_{pa}(T_2 - T_1) + W_2 \times h_{fg2}}{h_{w1} - h_{f2}} \\ &= \frac{1.005 \times (25 - 35) + 0.020 \times 2442.3}{2565.3 - 104.87} \\ &= \frac{-10.05 + 48.846}{2460.43} = 0.0157 \text{ kg vap/kg dry air} \\ W_1 &= 0.0157 = 0.622 \times \left[\frac{p_{v1}}{100 - p_{v1}}\right] \end{aligned}$$

where p_{v1} represents unsaturated pressure:

$$\begin{aligned} \frac{100 - p_{v1}}{p_{v1}} &= \frac{0.622}{0.0157} = 39.62 \\ \frac{100}{p_{v1}} - 1 &= 39.62 \\ p_{v1} &= \frac{100}{40.62} = 2.46 \text{ kPa} \\ \phi &= \frac{p_{v1}}{p_{s1}} = \frac{2.46}{5.628} = 0.437 = 43.7\% \qquad \overset{\text{Ans}}{\Longleftarrow} \end{aligned}$$

where 5.628 kPa is the saturation pressure at 35 °C taken from the steam table.

9.3 A meeting hall required saturated air at 2 °C so as to maintain a temperature of 20 °C with a relative humidity of 50%. First air is heated and then water is sprinkled at 10 °C to provide the required relative humidity. At room conditions, determine the amount of water to be sprinkled per m^3 of air and the temperature to which air must be heated. Assume a constant total pressure of 1.013 bar.



(a) (b)

Fig. 9.21

Solution

$$\phi_3 = 0.5 = \frac{p_{v3}}{(P_{sat})_{t3}} = \frac{p_{v3}}{2.339}$$

$$p_{v3} = 1.17 \text{ kPa}$$

$$p_{a3} = 101.3 - 1.17 = 100.13 \text{ kPa}$$

$$W_3 = 0.622 \times \frac{p_{v3}}{p_{a3}} = 0.622 \times \frac{1.17}{100.13} = 0.00727$$

$$\phi_3 = \frac{p_{v1}}{(P_{sat})_{2^\circ\text{C}}} = 1$$

At 2 °C, $p_{sat} = 0.7156$ kPa and $p_{v1} = 0.7156$ kPa

$$p_{a1} = 101.3 - 0.7156 = 100.5844 \text{ kPa}$$

$$W_1 = 0.622 \times \frac{0.7156}{100.5844} = 0.00442 \text{ kg vap/kg dry air}$$

$$W_3 - W_1 = 0.00727 - 0.00442 = 0.00285 \text{ kg vap/kg dry air}$$

$$v_{a3} = \frac{R_a T_3}{p_{a3}} = \frac{0.287 \times 293}{100.13} = 0.84 \text{ m}^3\text{/kg of dry air}$$

$$\text{Amount of sprinkled water} = \frac{0.00285}{0.84} = 0.00339 \text{ kg mixture/m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$m_2 h_2 + m_{w4} h_4 = m_3 h_3$$

$$h_2 + (W_3 - W_2) h_4 = h_3$$

$$h_{a2} + W_2 h_{w2} + (W_3 - W_2)h_4 = h_{a3} + W_3 h_{w3}$$

$$c_p(t_3 - t_2) + W_3 h_{w3} - W_2 h_{w2} - (W_3 - W_2)h_4 = 0$$

From steam table at $p_w = 1.17$ kPa, $h_g = 2518$ kJ/kg and $t_{sat} = 9.65°\text{C}$

$$1.005 \times (20 - t_2) + 0.00727 \times [2518 + 1.884 \times (20 - 9.65)] -$$

$$0.00442 \times [2518 + 1.884 \times (t_2 - 9.65)] - 0.00285 \times 10 = 0$$

Solving the above equation, $t_2 = 27.2°$ C $\overset{\textbf{Ans}}{\Longleftarrow}$

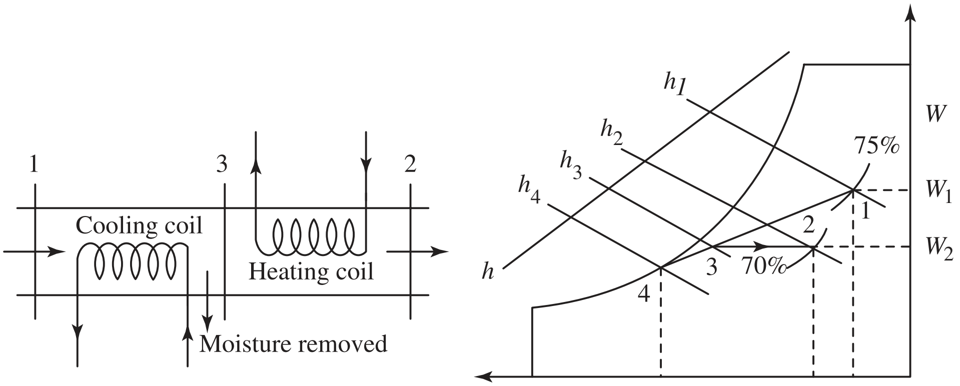
9.4 A typical air-conditioning system has to satisfy the following parameters.

Conditions:

Outdoor temperature	30 °C *DBT*
Outdoor *RH*	0.75
Required indoor conditions	22 °C *DBT* 70%*RH*
Free air circulator	3.33 m^3
Coil dew point temperature	14 °C

The required condition is achieved by cooling and dehumidification and then by heating. Determine

(i) capacity of cooling coil in tonnes,
(ii) capacity of heating coil in kW and
(iii) the amount of water vapour removed in kg/s.



(a) (b)

Fig. 9.22

Solution

The processes are shown in Fig.9.22. The property values are obtained from psychometric chart.

$h_1 = 82$ $h_2 = 52;$ $h_3 = 47$ $h_4 = 40$ kJ/kg dry air
$W_1 = 0.020$ $W_2 = W_3 = 0.0115$ kg vap/kg dry air $v_1 = 0.887\ \text{m}^3$/kg dry air

$$\text{Mass of dry air, } M = \frac{3.33}{0.887} = 3.754 \text{ kg dry air/s}$$

$$\text{Cooling coil capacity} = M(h_1 - h_3) = 3.754 \times (82 - 47)$$

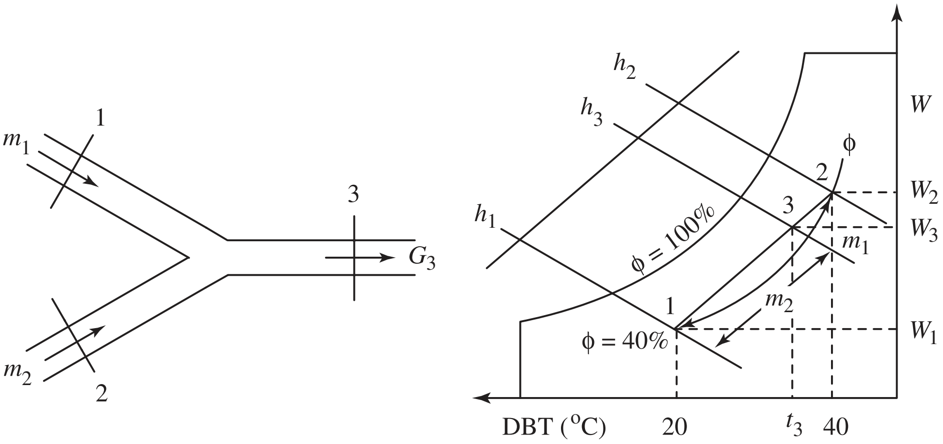
$$= \frac{3.754 \times 35 \times 3600}{14000} = 33.8 \text{ tonnes}$$

$$\text{Capacity of heating coil} = M(h_2 - h_3) = 3.754 \times (52 - 47) \text{ kJ/s} = 18.8 \text{ kW}$$

$$\text{Rate of water removed} = M(W_1 - W_3) = 3.754 \times (0.02 - 0.0115)$$

$$= 0.032 \text{ kg/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.5 One kilogram of air at 20 °C with 0.4 relative humidity is mixed adiabatically with 2 kg of dry air at 40 °C and 0.4 relative humidity. Estimate the final condition of air.



(a) (b)

Fig. 9.23

Solution

Figure 9.23 gives the details of mixing process of two air stream. Let m_1, m_2, m_3 be the amount of dry air:

$$m_1 + m_2 = m_3 \qquad (i)$$

$$m_1 W_1 + m_2 W_2 = m_3 W_3 \qquad (ii)$$

$$m_1 h_1 + m_2 h_2 = m_3 h_3 \qquad (iii)$$

Manipulating the above three equations we get:

$$\frac{W_2 - W_3}{W_3 - W_1} = \frac{h_2 - h_3}{h_3 - h_1} = \frac{m_1}{m_2}$$

From the psychometric chart,

$W_1 = 0.0058$kg vap/kg dry air; $h_1 = 35$ kJ/kg dry air;
$W_2 = 0.0187$ kg vap/kg dry air; $h_2 = 90$ kJ/kg dry air

$$\frac{0.0187 - W_3}{W_3 - 0.0058} = \frac{m_1}{m_2} = \frac{1}{2}$$

$$W_3 = \frac{2}{3} \times 0.187 + \frac{1}{3} \times 0.0058 = 0.1266 \text{ kg vap/kg dry air}$$

Again,

$$\frac{h_2 - h_1}{h_3 - h_1} = \frac{m_1}{m_2} = \frac{1}{2}$$

$$h_3 = \frac{2}{3}h_2 + \frac{1}{3}h_1 = \frac{2}{3} \times 90 + \frac{1}{3} \times 35 = 71.67 \text{ kJ /kg dry air}$$

$\therefore$ Final condition of air is given by:

$$W_3 = 0.014 \text{ kg vap/kg dry air} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$h_3 = 71.7 \text{ kJ /kg dry air} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

9.6 Air at 21 °C which is saturated passes through a drier so that its final RH is 20%. The stick gel absorbent is used in the drier without change in specific humidity the air passes through a cooler to reach its final temperature of 21 °C. Find out

(i) the temperature of air at the end of the drying process,
(ii) the heat rejected in kJ/kg of dry air during the cooling process,
(iii) the relative humidity at the end of the cooling process,
(iv) the dew point temperature at the end of drying process, and
(v) the moisture removed during the drying process in kg vap/kg dry air.

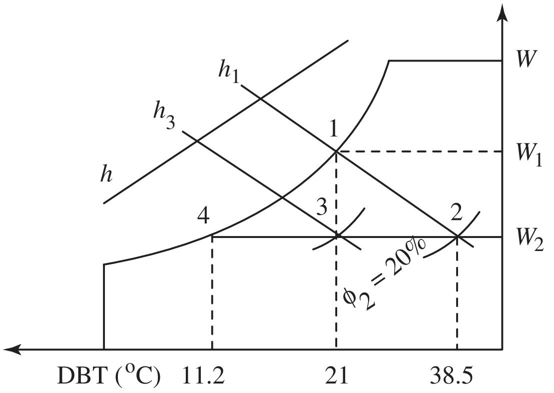


Fig. 9.24

Solution

From the psychometric chart, $T_2 = 38.5$ °C

$$h_1 - h_3 = 60.5 - 42 = 18.5 \text{ kJ/kg dry air} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\phi_3 = 53\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$t_4 = 11.2^\circ\text{C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_1 - W_2 = 0.0153 - 0.0083 = 0.007 \text{ kg vap/kg dry air} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.7 An auditorium to be air-conditioned for the following conditions,

Outdoor dry-bulb temperature	40 °C
Outdoor wet-bulb temperature	20 °C
Expected comfort condition	20 °DBT 60%RH
Seating capacity of the auditorium	1500 persons
Air supplied from outdoor	0.3 m^3/min per person

If the required condition is achieved first by adiabatic humidification and then by cooling, determine (i) the capacity of the cooling coil in tonnes and (ii) the capacity of humidifier in kg/h.

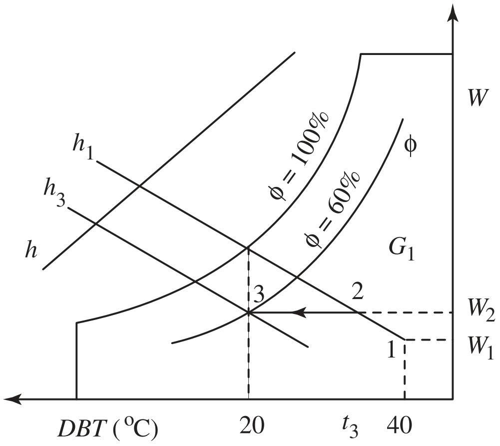


Fig. 9.25

Solution

From the psychometric chart (refer Fig.9.25),

$h_1 = h_2 = 57.0$ kJ/kg dry air $\quad h_3 = 42.0$ kJ/kg dry air

$W_1 = 0.0065$ kg vap/kg dry air $\quad W_2 = W_3 = 0.0088$ kg vap/kg dry air

$t_2 = 34.5^\circ$C $\quad v_1 = 0.896\ \text{m}^3$/kg dry air

$$\text{Amount of dry air supplied, } m = \frac{1500 \times 0.3}{0.896} = 502 \text{ kg/min}$$

$$\text{Capacity of cooling coil, } m(h_2 - h_3) = 502 \times (57 - 42) \text{ kJ/min}$$

$$= \frac{502 \times 15 \times 60}{14000} = 32.27 \text{ tonnes}$$

$$\text{Capacity of humidifier, } m(W_2 - W_3) = 502 \times (0.0088 - 0.0065) = 69.3 \text{ kg/h} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.8 Water at the rate of 1.5 kg per kg of air flows into a cooling tower at 30 °C. Air enters the tower at a DBT of 20 °C with a RH of 60% and leaves at 28 °DBT with 90% RH Make-up water is supplied at 20 °C. Calculate

(i) the temperature of water leaving the tower
(ii) the fraction of water evaporated
(iii) approach and range of cooling tower

Solution

From psychometric chart,

$t_{wb1} = 15.2$ °C $\qquad t_{wb2} = 26.7$ °C

$h_1 = 43$ kJ/kg K of dry air $\qquad h_2 = 83.5$ kJ/kg of dry air

$W_1 = 0.0088$ kg water vapour/kg dry air $\qquad W_2 = 0.0213$ kg water vapour/kg dry air

Enthalpies of water entering the tower and the make-up water are $h_{w3} = 125.8$ kJ/kg and $h_m = 84$ kJ/kg, respectively

From the energy balance:

$$h_{w3} - h_{w4} = \frac{m}{m_w}[(h_2 - h_1) - (W_2 - W_1)h_w]$$

$$= \frac{1}{1.5} \times [(83.5 - 43) - (0.0213 - 0.0088) \times 84] = 26.3 \text{ kJ/kg}$$

Temperature drop of water:

$$t_{w3} - t_{w4} = \frac{26.22}{4.19} = 30 - t_{w4}$$

$$t_{w4} = 23.7$$

$$\text{Approach} = t_{w4} - t_{wb1} = 23.7 - 15.2 = 8.5°\text{C} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Range} = t_{w3} - t_{w4} = 30 - 23.7 = 6.3°\text{C} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Fraction of water evaporated:

$$x = m(W_2 - W_1) = 1 \times (0.0213 - 0.0088) = 0.0125 \text{ kg/kg of dry air} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.9 A cooling system uses water as the cooling medium. This water is to be cooled in a cooling tower at the rate of 2.78 kg/s. The entry temperature of water to the tower is 65 °C and water leaves the collectivity tank at the base at 30 °C. Air enters the base at 15 °C, 0.1 Mpa and 55% *RH* and leaves at the top of the tower at 35 °C, 0.1 Mpa saturated. Make-up water enters the collecting tank at 14 °C. Estimate the air flow rate into the tower in m^3/s and the make-up water flow rate.

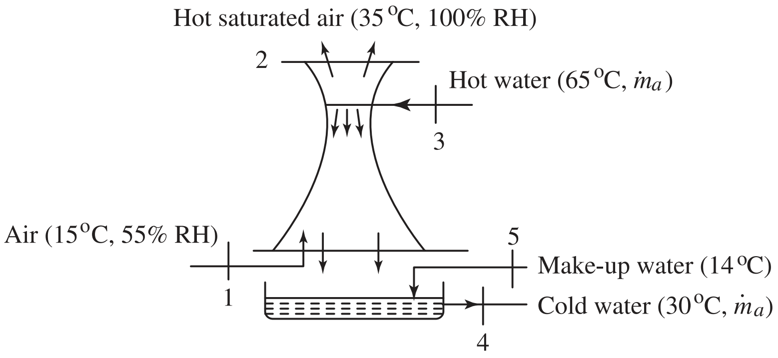


Fig. 9.26

Solution

Figure 9.26 shows the flow diagram of the cooling tower. From steam tables,

at 15 °C, $p_{sat} = 0.01705$ bar and $h_g = 2528.9$ kJ/kg.

at 35 °C, $p_{sat} = 0.05628$ bar and $h_g = 2565.3$ kJ/kg.

$$\phi = \frac{p_w}{(p_{sat})_{15°\mathrm{C}}} = 0.55$$

$$p_{w1} = 0.55 \times 0.01705 = 0.938 \times 10^{-2} \text{ bar}$$

$$\phi_2 = \frac{p_w}{(p_{sat})_{35°\mathrm{C}}} = 1.00$$

$$p_{w2} = 0.05628 \text{ bar}$$

$$W_1 = 0.622 \times \frac{p_w}{p - p_w} = \frac{0.622 \times 0.938 \times 10^{-2}}{1.00 - 0.00938}$$

$$= 0.00589 \text{ kg vap/kg dry air}$$

$$W_2 = 0.622 \times \frac{0.05628}{1 - 0.05628} = 0.0371 \text{ kg vap/kg dry air}$$

$$\text{Make-up water} = W_2 - W_1 = 0.0371 - 0.00589$$

$$= 0.03121 \text{ kg vap/kg dry air}$$

Energy balance gives $H_2 + H_4 - H_1 - H_3 - H_5 = 0$

For 1 kg of dry air:

$$c_{pa}(t_2 - t_1) + W_2h_2 - W_1h_1 + \dot{m}_w(h_4 - h_3) - (W_2 - W_1)h_5 = 0$$

$$1.005 \times (35 - 15) + 0.0371 \times 2565.3 - 0.00589 \times 2528.9 + \dot{m}_w(-35) \times 4.187 - 0.03121 \times 4.187 \times 14 = 0$$

$$146.55\dot{m}_w = 98.54$$

$$\dot{m}_w = 0.672 \text{ kg water/kg dry air}$$

Since water flow rate is 2.78 kg/s:

$$\text{Rate of dry air flow} = \frac{2.78}{0.672} = 4.137 \text{ kg/s}$$

$$\text{Make-up water flow rate} = 0.03121 \times 4.137 = 0.129 \text{ kg/s}$$

$$\text{Rate of dry air flow} = 4.137 \text{ kg/s}$$

$$\text{Rate of wet air flow} = 4.137 \times (1 + W_1) = 4.137 \times 1.00589 = 4.16 \text{ kg/s}$$

$$\text{Volume flow rate of air} = \frac{\dot{m}_a RT}{p} = \frac{4.16 \times 0.287 \times 288}{100} = 3.44 \text{ m}^3\text{/s}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

9.10 A big room in a building is supplied with air at 20 °C in winter and it has 60% *RH*. If the barometric pressure is 1.01325 bar, calculate (i) the specific humidity and (ii) the dew point for the given conditions.

Solution

Corresponding to 20 °C from steam tables:

$$p_{vs} = 0.0234 \text{ bar}$$

$$\text{Relative humidity, } \phi = \frac{p_v}{p_{vs}}$$

$$0.6 = \frac{p_v}{0.0234}$$

$$p_v = 0.0140 \text{ bar}$$

$$\text{Specific humidity, } W = \frac{0.622p_v}{p - p_v} = \frac{0.622 \times 0.0140}{1.01325 - 0.014} = 0.0087 \text{ kg/kg of dry air}$$

As the air is cooled at constant pressure, the vapour will begin to condense at the saturation temperature corresponding to 0.014 bar. From steam table for 0.014 bar, $t_{dp} = 12°\text{C}$ $\overset{\textbf{Ans}}{\Longleftarrow}$

9.11 0.005 kg of water vapour per kg of air is removed. After removing the water vapour, the temperature becomes 15 °C. Calculate (i) relative humidity and (ii) dew point temperature. The atmospheric conditions are 30 °C and 55% *RH* and the atmospheric pressure is 1.0132 bar.

Solution

Corresponding to 30 °C from steam table:

$$p_{vs} = 0.0425 \text{ bar}$$

$$\text{Relative humidity, } RH\phi = \frac{p_v}{p_{vs}}$$

$$0.55 = \frac{p_v}{0.0425}$$

$$p_v = 0.02337 \text{ bar}$$

$$\text{Specific humidity, } W = 0.622 \times \frac{p_v}{p - p_v} = \frac{0.622 \times 0.02337}{1.1032 - 0.02337}$$

$$= 0.0134 \text{ kg/kg of dry air}$$

The specific humidity after removing 0.005 kg of water vapour becomes, $0.0134 - 0.005 = 0.0084$ and the temperature, $DBT = 15$ °C.

Partial pressure of water vapour, p_v, is given by:

$$W = \frac{0.622 p_v}{p_t - p_v}$$

$$0.0084 = \frac{0.622 p_v}{1.0132 - p_v}$$

$$0.0084 \times (1.0132 - p_v) = 0.622 p_v$$

$$0.622 p_v + 0.0084 p_v = 0.00851$$

$$p_v = 0.0134 \text{ bar}$$

Corresponding to 15 °C from steam table:

$$p_{vs} = 0.0170$$

$$\phi = \frac{p}{p_{vs}} = \frac{0.0155}{0.0170} = 91.1\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.12 In a test conducted at a laboratory, the following recordings were made using sling psychrometer:

Dry-bulb temperature, $DBT = 37$ °C and wet-bulb temperature, $WBT = 27$ °C.

Find (i) specific humidity; (ii) relative humidity; (iii) vapour density of air; (iv) dew point temperature; and (v) enthalpy of mixture per kg of dry air.

Solution

Partial pressure of vapour can be calculated using Carriers equation is given by:

$$p_v = (p_{vs})_{WBT} - \frac{[p_t - (p_{vs})_{WBT}][t_{DBT} - t_{WBT}]}{1527.4 - 1.3 t_{WBT}}$$

Corresponding to 27 °C (from steam table), $(p_{vs})_{WBT} = 0.0357$ bar. Substituting the values:

$$p_v = 0.0357 - \frac{(1.0132 - 0.0357) \times (37 - 27)}{1527.4 - 1.3 \times 27}$$

$$= 0.0357 - 0.00655 = 0.0292 \text{ bar}$$

$$W = \frac{0.622 p_v}{p_t - p_v} = \frac{0.622 \times 0.0292}{1.0312 - 0.0292} = 0.0181 \text{ kg/kg of dry air} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Relative humidity, $\phi = \dfrac{p_v}{p_{vs}}$

where $p_{vs} = 0.0628$ from steam table corresponding to 37 °C:

$$\phi = \frac{0.0292}{0.0628} = 0.465 = 46.5\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

From characteristic gas equation:

$$p_v V_v = m_v R T_v$$

$$p_v = \frac{m_v}{V_v} R T_v = \rho_v R_v T_v \qquad (i)$$

where ρ_v is the vapour density and

$$R_v = \frac{\text{Universal gas constant}}{\text{Molecular weight of H}_2\text{O}} = \frac{8314.3}{18} = 461.9$$

Substituting in Eq.(i):

$$0.0292 \times 10^5 = \rho_v \times 461.9 \times (273 + 37)$$

$$\rho_v = \frac{0.0292 \times 10^5}{461.9 \times 310} = 0.020 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Dew point temperature t_{dpt} corresponding to 0.0292 bar from steam table, $t_{dp} \approx 24$°C.

Enthalpy of mixture per kg of dry air:

$$h = c_p t_{DBT} + W h_{vapour}$$

$$= (1.005 \times 37) + 0.0181[h_g - 1.88(t_{DBT} - t_{dpt})]$$

h_g corresponding to 37 °C = 2568.9 kJ/kg

$$h = 1.005 \times 37 + 0.0181[2568.9 + 1.88(37 - 24)]$$

$$= 84.12 \text{ kJ/kg of dry air} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

9.13 Two kilogram of air at 22 °C *DBT* and 13 °C dew point temperature is mixed with 1 kg of air at 37 °C *DBT* and 60 *RH*. Determine the specific humidity of the mixture.

Solution

Consider air at 37 °C *DBT* and 60 *RH*. Corresponding to 37 °C temperature from steam table, $p_{vs} = 0.0628$ bar:

$$\begin{aligned}
\text{Relative humidity, } \phi &= \frac{p_v}{p_{vs}} \\
p_v &= \phi p_{vs} = 0.6 \times 0.0628 = 0.0377 \\
W &= \frac{0.622 p_v}{p_t - p_v} = \frac{0.622 \times 0.0377}{1.0132 - 0.0377} = 0.0240 \text{ kg/kg of dry air}
\end{aligned}$$

Corresponding to 0.0377 bar from steam tables, $t_{dpt} \approx 28$ °C:

$$\begin{aligned}
h = c_p t_{DBT} + W h_{vapour} &= 1.005 \times 37 + 0.024 \times [2568.9 + 1.88 \times (37 - 28)] \\
&= 99.33 \text{ kJ/kg} \qquad \overset{\text{Ans}}{\Longleftarrow}
\end{aligned}$$

Consider air at 22 °C *DBT* and 13 °C. Let p_v be the vapour pressure corresponding to saturation pressure of steam at 13 °C $p_v = 0.0150$ bar:

$$\begin{aligned}
W &= \frac{0.622 \times p_v}{p_t - p_v} = \frac{0.622 \times 0.015}{1.0132 - 0.015} = 0.00935 \text{ kg/kg of dry air} \\
\text{Enthalpy, } h &= c_p t_{DBT} + W h_{vapour} \\
&= 1.005 \times 22 + 0.00935[h_g + 1.88(t_{DBT} - t_{dpt})] \\
&= 22.11 + 0.00935[2541.7 + 1.88(22 - 13)] \\
&= 46.03 \text{ kJ/kg of dry air} \\
\text{Enthalpy per kg of air} &= \frac{1}{3} \times \left[\frac{99.24}{1.0240} + \frac{46.03 \times 2}{1.00935}\right] = 0.333 \times [96.91 + 91.21] \\
&= 62.70 \text{ kJ/kg of moist air}
\end{aligned}$$

$$\text{Mass of vapour/kg of moist air} = \frac{1}{3} \times \left[\frac{0.0240}{1.0240} + \frac{0.00935 \times 2}{1.00935}\right] = 0.014$$

$$\text{Specific humidity of mixture} = \frac{0.014}{1 - 0.014} = 0.0142 \text{ kg/kg of dry air} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

9.14 100 m^3 of air with 75% RH and at 20 °C DBT is heated in a vessel until its temperature reaches 30 °C. Determine (i) RH of heated air and (ii) heat addition per minute on the air.

Solution

For air at 20 °C and 75% $RH, p_{vs} = 0.0234$ bar (from steam table)

$$p_v = \phi \times p_{vs} = 0.75 \times 0.0234 = 0.01755 \text{ bar}$$

$$t_{dpt} = 15 + (16 - 15)\left(\frac{0.01755 - 0.017}{0.0182 - 0.017}\right) = 15.458°\text{C}$$

$$W_1 = 0.622 \times \frac{p_v}{p_t - p_v} = \frac{0.622 \times 0.01755}{1.0132 - 0.01755} = 0.0109 \text{ kg/kg of dry air}$$

$$\begin{aligned} \text{Enthalpy, } h_1 &= c_p t_{DBT} + W h_{vapour} \\ &= 1.005 \times 20 + 0.0109 \times [h_g + 1.88 \times (t_{DBT} - t_{WBT})] \\ &= 1.005 \times 20 + 0.0109 \times [2538.1 + 1.88 \times (20 - 15.5)] \\ &= 47.85 \text{ kJ/kg of dry air} \end{aligned}$$

For air at 30 °C DBT,

Note that saturation pressure of water vapour at 30 °C is higher than the saturation pressure of water vapour at 20 °C. Therefore, it is sensible heating, where p_v is same after heating:

$$\text{Relative humidity, } \phi = \frac{p_v}{p_{vs}} = \frac{0.01755}{0.0425} = 0.412 = 41.2\%$$

$$p_{vs} = 0.0425 \ \text{ bar corresponding to } 30°\text{C}$$

i.e. Relative humidity of heated air is 41.2%. $\overset{\textbf{Ans}}{\Longleftarrow}$

$$\begin{aligned} \text{Enthalpy, } h_g &= c_p t_{DBT} + W h_{vapour} \\ &= 1.005 \times 30 + 0.0109 \times [h_g + 1.88(t_{DBT} - t_{dpt})] \\ &= 1.005 \times 30 + 0.0109[2556.3 + 1.88 \times (30 - 15.5)] \\ &= 58.31 \text{ kJ/kg of dry air} \end{aligned}$$

Mass of air in 100 m^3 of air supplied:

$$\begin{aligned} m_a &= \frac{pV}{RT} = \frac{(p_1 - p_v)V}{RT} = \frac{(1.0132 - 0.01753) \times 10^5 \times 100}{287 \times 293} \\ &= 118.40 \text{ kg/min} \end{aligned}$$

Amount of heat added per mixture:

$$118.4 \times (h_2 - h_1) = 118.4 \times (58.31 - 47.85) = 1238.46 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.15 50 m^3 of air at 35 °C *DBT* and with relative humidity of 0.5 is cooled to 25 °C *DBT* maintaining its specific humidity constant. Determine

(i) Relative humidity (*RH*) of cooled air.

(ii) Heat removed from air.

Solution

Air at 35 °C *DBT* and 50% *RH*, $p_{vs} = 0.0563$ bar (from steam table):

$$\phi = \frac{p_v}{p_{vs}}$$

$$p_v = \phi p_{vs} = 0.5 \times 0.0563 = 0.02815 \text{ bar}$$

$$W_1 = \frac{0.622 p_v}{p_t - p_v} = \frac{0.622 \times 0.02815}{1.0132 - 0.02815} = 0.0177 \text{ kg/kg of dry air}$$

$$h_1 = c_p t_{DBT} + W_1(h_{g1} + 1.88(t_{DBT1} - t_{dpt1}))$$

Corresponding to 0.02815 bar, $t_{DBT1} = 23$ °C:

$$h_1 = 1.005 \times 35 + 0.0177 \times [2526.3 + 1.88(35 - 23)]$$

$$= 80.289 \text{ kJ/kg of dry air}$$

Consider air at 25 °C:

Since the specific humidity remains constant, the vapour pressure in the air remains constant:

$$\phi = \frac{p_v}{p_{vs}} = \frac{0.02815}{0.0317} = 0.888 = 88.8\%$$

Relative humidity of air = 88.8%

Heat removed from the air:

$$h_2 = c_p t_{DBT2} + W_2[h_{g2} + 1.88(t_{DBT2} - t_{dpt2})]$$

$$= 1.005 \times 25 + 0.0177 \times (2547.2 + 1.88(25 - 23))$$

$$= 70.27 \text{ kJ/kg of dry air}$$

To find the mass of dry air (m_a), we can use the relation:

$$p_a v_a = m_a R_a T_a$$

$$W_1 = W_2 = 0.0177 \text{ kg/kg of dry air}$$

$t_{dpt2} = t_{dpt1} = 23$°C since p_v is constant.

$$m_a = \frac{p_a v_a}{RT_a} = \frac{(1.0132 - 0.02815) \times 10^5 \times 50}{287 \times (273 + 35)} = 55.71 \text{ kg}$$

$$\text{Heat removed} = m_a(h_1 - h_2) = 55.71 \times (80.98 - 70.27) = 596.65 \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.16 100 m^3 of air per minute at 35 °C *DBT* and 50% *RH* is cooled to 20 °C *DBT* by passing through a cooling coil. Calculate

(i) *RH* of the outgoing air and its *WBT*.
(ii) Capacity of cooling coil in tonnes of refrigerator.
(iii) Amount of water vapour removed per hour.

Solution

At 35 °C *DBT* and 50 *RH* $p_{vs} = 0.0563$ bar (from steam table)

$$p_v = \phi\ p_{vs} = 0.5 \times 0.0563 = 0.02815 \text{ bar}$$

$$W_1 = 0.622 \times \frac{p_v}{p_t - p_v} = \frac{0.622 \times 0.02815}{1.0132 - 0.02815} = 0.0177 \text{ kg/kg of dry air}$$

$$h_1 = c_p t_{DBT1} + W_1(h_{g1} + 1.88 \times (t_{DBT1} - t_{dpt1}))$$

$$t_{dpt1} = 23°\text{C (corresponding to 0.02815 bar)}$$

$$h_1 = (1.005 \times 35) + 0.0177 \times [2565.3 + 1.88 \times (35 - 23)]$$

$$= 80.98 \text{ kJ/kg of dry air}$$

Saturation pressure at 20 °C is 00234 bar. This is less than the vapour pressure 0.02815 bar at 35 °C. Therefore, condensation takes place and air gets saturated at 20 °C.

Therefore, *RH* of exit air is 100%.

Since the air is saturated, wet-bulb and dry-bulb temperature will be the same which in equal to 20 °C:

$$p_v = p_{vs} = 0.0234 \text{ bar}$$

$$W_2 = \frac{0.622 \times p_v}{p_t - p_v} = \frac{0.622 \times 0.0234}{1.0132 - 0.0234} = 0.0147 \text{ kg/kg of dry air}$$

$$h_2 = c_p t_{bdt2} + W_2[h_{g2} + 1.88 \times (t_{DBT} - t_{dpt2})]$$

$$= 1.005 \times 20 + 0.147 \times [2538.1 + 1.88 \times (20 - 20)] = 393.2 \text{ kg/kg of dry air}$$

The weight of water vapour removed:

$$= 0.177 - 0.0147 = 0.1623 \text{ kg/kg of dry air}$$

Heat removed per kg of dry air:

$$h_1 - h_2 \quad = \quad 80.98 - 57.41 = 23.57 \text{ kJ/kg of dry air}$$

Mass of dry air passing through coil per minute:

$$m_a \quad = \quad \frac{p_a V_a}{RT_a} = \frac{(1.0132 - 0.02815) \times 10^5 \times 100}{287 \times (35 + 273)} = 111.41$$

Capacity of the cooling coil in tonnes of refrigeration

$$= \quad \frac{m_a(h_1 - h_2)}{14000} = \frac{111.41 \times 23.57 \times 60}{14000} = 11.25 \text{ TR}$$

Amount of water removed per hour:

$$= \quad m_a(W_1 - W_2) \times 60 = 111.41 \times (0.0177 - 0.0147) \times 60 = 20.05 \text{ kg h} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.17 200 m^3 of air per minute flows through an adiabatic humidifier. Inlet condition of air is 35 °C *DBT* and 20% *RH* and the exit condition of air is 20 °C *DBT* and 15 °C *WBT*. Estimate the following:

- (i) dew point temperature,
- (ii) relative humidity of exit air and
- (iii) the amount of water vapour added to air per minute.

Solution

From steam tables at 35 °C $p_{vs} = 0.0563$ bar:

$$\begin{aligned} p_v &= \phi p_{vs} = 0.2 \times 0.0563 = 0.1126 \text{ bar} \\ W_1 &= \frac{0.622 \times p_v}{p_t - p_v} = \frac{0.622 \times 0.1126}{1.0132 - 0.01126} \\ &= 0.07 \text{ kg/kg of air} \end{aligned}$$

The dew point temperature of which is nothing but the saturation temperature of steam corresponding to the pressure of 0.01126 bar by interpolation is:

$$8 + (9 - 8) \times \frac{0.01126 - 0.01072}{0.01150 - 0.01072} \quad = \quad 8.7°\text{C}$$

Dew point temperature = 8.7 °C

For air at 20 °C *DBT* and 15 °C *WBT*:

$$p_v = (p_{vs})_{WBT} - \frac{[p_t - (p_{vs})_{WBT}] - (t_{DBT} - t_{WBT})}{1527.4 - 1.3t_{WBT}}$$

$$= 0.0170 - \frac{(1.0132 - 0.0170)(20 - 15)}{1527.4 - 1.3 \times 15}$$

$$= 0.0137 \text{ bar}$$

$$W_2 = \frac{0.622p_v}{p_t - p_v} = \frac{0.622 \times 0.0137}{1.0132 - 0.0137} = 0.00852 \text{ kg/kg of dry air}$$

$$\text{Relative humidity} = \frac{p_v}{p_{vs}} = \frac{0.0137}{0.0234} = 0.585 = 58.5\%$$

The dew point temperature of air which is the saturation temperature of steam corresponding to 0.0137 bar is 11 °C (from steam table).

The amount of water vapour per kg of dry air:

$$W_2 - W_1 = 0.00852 - 0.007 = 0.00152 \text{ kg}$$

Mass of dry air in 200 m^3 of mixture:

$$m_a = \frac{p_a V_a}{R_a T_a} = \frac{(1.0132 - 0.1126) \times 10^5 \times 200}{287 \times (35 + 273)} = 203.76 \text{ kg}$$

The amount of water vapour added to air per minute:

$$m_a(W_2 - W_1) = 203.76 \times 0.00152 = 0.3097 \text{ kg/min}$$ **Ans** $\Longleftarrow$

9.18 A mixture containing air and water vapour passes through an adiabatic saturation chamber at constant pressure of 1 bar. The adiabatic saturation temperature at inlet is 28 °C and at the exit the mixture temperature is 18 °C. Find the relative humidity and the humidity ratio of the inlet mixture.

Solution

The specific humidity at the exit:

$$W_{2s} = \frac{0.622p_s}{p_t - p_s} = \frac{0.622 \times 0.0206}{1 - 0.0206} = 0.0131 \text{ kg/kg of dry air}$$

The specific humidity at the inlet:

$$W_1 = \frac{c_p(t_{WBT2} - t_{DBT1}) + W_{2s}(h_{g2} - h_{f2})}{h_{g1} - h_{f2}}$$

$$= \frac{1.005 \times (18 - 28) + 0.01308 \times (2534.4 - 75.6)}{2552.6 - 75.6}$$

$$= 0.00892 \text{ kg/kg of dry air}$$

$$W_t = \frac{0.622 \times p_{v1}}{p_t - p_{v1}}$$

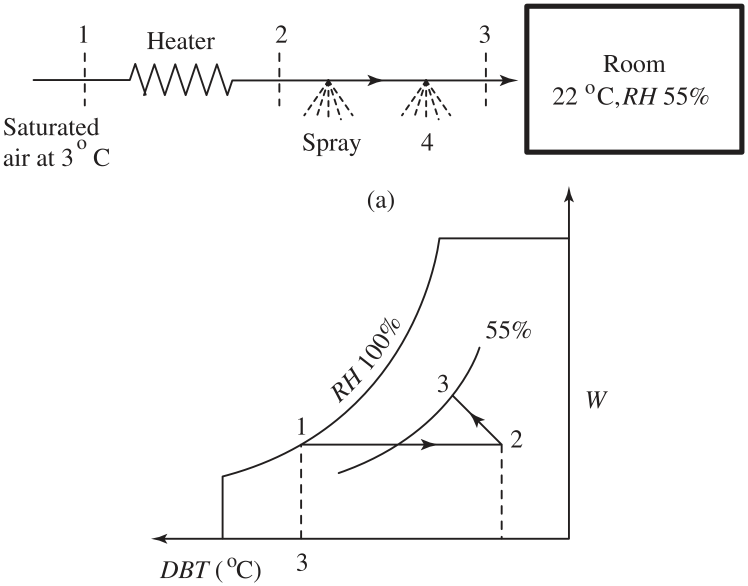
$$
\begin{aligned}
0.00892 &= \frac{0.622 \times p_{v1}}{1 - p_{v1}} \\
0.00892 \times (1 - p_{v1}) &= 0.622 \times p_{v1} \\
0.63092 \times p_{v1} &= 0.00892 \\
p_{v1} &= \frac{0.00892}{0.63092} = 0.0141 \text{ bar} \\
\text{Relative humidity} &= \frac{p_{v1}}{p_{s1}} = \frac{0.0141}{0.03782} = 0.3728 = 37.28\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

9.19 A room is designed to have temperature of 22 °C with $RH = 55\%$. For this, saturated air is supplied at 3 °C. Initially, the air is heated and the water at 10 °C is sprayed to provide the required humidity calculate

(i) the amount of water to be sprayed per m^3 of air at room conditions and

(ii) the temperature to which the air must be heated.

Neglect the fan power. Assume that the total pressure is constant at 1.0132 bar.



(a)

(b)

Fig. 9.27

Solution

From steam tables at 22 °C $p_{vs} = 0.0264$ bar:

$$\phi_3 = \frac{p_{v3}}{p_v s_2} = \frac{p_{vs3}}{0.0264} = 0.55$$

$$p_{v3} = 0.55 \times 0.0264 = 0.01452 \text{ bar}$$

$$W_3 = \frac{0.622 \times 0.01452}{1.0132 - 0.01452} = 0.00904 \text{ kg/kg of dry air}$$

From steam tables, at 3 °C $p_{vs} = 0.0076$ bar:

$$\phi_1 = \frac{p_{v1}}{p_{vs1}} = 1.00$$

$$p_{v1} = p_{vs1} = 0.0076 \text{ bar}$$

$$W_1 = \frac{0.622 \times 0.0076}{1.0132 - 0.0076} = 0.0047 \text{ kg/kg of dry air}$$

$$W_3 - W_1 = 0.00904 - 0.0047 = 0.00434 \text{ kg/kg of dry air}$$

$$v_{a3} = \frac{R_a T_3}{p_{a3}} = \frac{287 \times (273 + 32)}{(1.0132 - 0.01452) \times 10^5} = 0.8765 \text{ m}^3\text{/kg of dry air}$$

$$\text{Water to be sprayed} = \frac{0.00434}{0.8765} = 0.005 \text{ kg moisture/m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$h_3 = h_2 + (W_3 - W_2)h_4 = [c_p t_{DBT2} + W_2 h_{vap2}] + (W_3 - W_2)h_2$$

where $h_3 = c_p t_{DBT3} + W_3 h_{vapour3}$. Now

$$c_p(t_{DBT3} - t_{DBT2}) + W_3 h_{vap3} - W_2 h_{vap2} - (W_3 - W_2)h_4 = 0$$

From steam tables at $p_v = 0.01452$ bar; $h_g = 2524$ kJ/kg and $t_{dpt} = t_{sat} = 12.5°$ C:

$$1.005 \times (22 - t_{DBT2}) + 0.00904 \times [2524 + 1.88 \times (22 - 12.5)] -$$
$$0.0047 \times (2524 + 1.88 \times (t_{DBT2} - 12.5)) - 0.00434 \times 4.187 \times 10 = 0$$

On solving, $t_{DBT2} = 32.7°$ C $\qquad \overset{\textbf{Ans}}{\Longleftarrow}$

9.20 A cooling tower is designed to cool 5.5 liters of water per second. The inlet temperature of water is 44 °C. A fan is used to circulate 9 m^3/s of air through the tower and power input to the fan is 4.75 kW. The air entering temperature and *RH* are 18 °C and 60%, respectively. The exit conditions of air are 26 °C and 100% *RH*. Find

(i) the amount of makeup cooling water required and

(ii) the final temperature of the water.

Assume constant pressure throughout.

Solution

From steam table 18 °C, $p_{vs} = 0.0206$ bar:

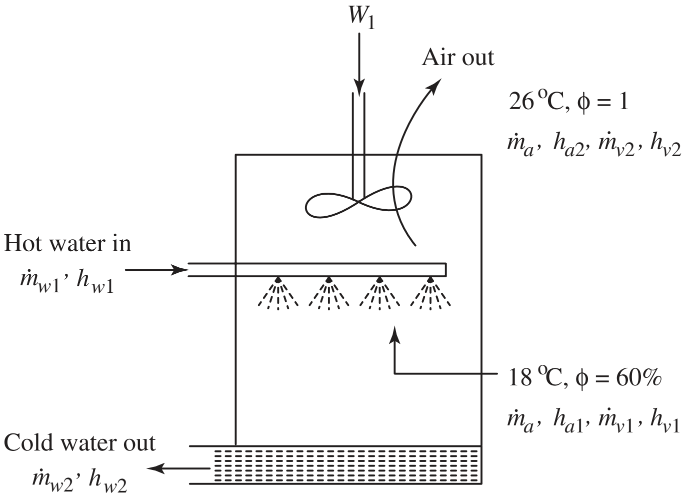


Fig. 9.28

$$p_v = \phi \times p_{vs} = 0.6 \times 0.0206 = 0.1236 \text{ bar}$$

$$p_{a1} = 1.013 - 0.01236 = 1.00064 \text{ bar}$$

$$\dot{m}_a = \frac{10^5 \times 1.00064 \times 9}{0.287 \times 10^3 \times (18 + 273)} = 10.7832 \text{ kg/s}$$

$$\dot{m}_{v1} = \frac{10^5 \times 0.01236 \times 9}{0.4618 \times 10^3 \times (18 + 273)} = 0.8278 \text{ kg/s}$$

where suffix v denotes vapour and a denotes air. At 26 °C at the exit from steam tables, $p_{vs} = 0.0336$ and $\phi = 1$.

$$p_v = p_{vs} = 0.0336 \text{ bar}$$

$$W_a = \frac{0.622 p_v}{p_t - p_v} = \frac{0.622 \times 0.0336}{1.012 - 0.0336} = 0.02133 \text{ kg}$$

$$W = \frac{m_v}{m_a}$$

$$\dot{m}_{v2} = 0.02133 \times 10.76 = 0.23 \text{ kg/s}$$

$$\text{Make-up water required} = 0.23 - 0.0828 = 0.1472 \text{ kg/s}$$

$$\dot{m}_{w1} = 5.5 \times 1 = 5.5 \text{ kg/s}$$

$$\dot{m}_{w2} = \dot{m}_{w1} - \text{make-up water} = 5.5 - 0.1472$$

$$= 5.353 \text{ kg/s}$$

Applying SFEE by neglecting KE and PE:

$$W_1 + \dot{m}_{w1}h_{w1} + \dot{m}_{a1}h_{a1} + \dot{m}_{v1}h_{v1} = \dot{m}_{a2}h_{a2} + \dot{m}_{v2}h_{v2} + \dot{m}_{w2}h_{w2}$$

Now, work input = 4.75 kJ/s

Assume datum as 0 °C and evaluate enthalpies:

$$h_{w1} = h_f \text{ at } 44°\text{C} = 184.3 \text{ kJ/kg}$$

$$h_{a1} = 1.005 \times (18 - 0) = 18.09 \text{ kJ/kg}$$

$$h_{v1} = 2519.7 + 1.88 \times (18 - 10) = 2534.74 \text{ kJ/kg}$$

Corresponding to $p_v = 0.01236$ bar,

$$t_s = t_{DBT} = 10°\text{C}$$

The vapour is superheated,

$$h_{v1} = h_g \text{ at } 26°\text{C} = 2549 \text{ kJ/kg}$$

$$h_{a1} = 1.005 \times (26 - 0) = 26.13 \text{ kJ/kg}$$

By substituting the various values we get:

$$4.75 + 5.5 \times 184.3 + 10.78 \times 18.09 + 0.0828 \times 2534.74 = 10.78 \times 26.13 + 0.23 \times 2549 + 5.353 \times h_{w2}$$

On solving, $h_{w2} = 103.7$ kJ/kg

By interpolation, $h_f = 103.74$ kJ/kg at 26.7°C

Hence, the final temperature of water = 26.7 °C. $\xLeftarrow{\textbf{Ans}}$

9.21 Ten big fans are installed in a very big cooling tower in a power plant. It circulates 1 tonne of water minute. Water inlet temperature is 35 °C and exit temperature is 30 °C. Take atmospheric conditions as 35 °C *DBT* and 25 °C *WBT*. Air leaves the tower at 30 °C with 90% *RH*. Calculate the amount of air handled per fan per hour and the quantity of make-up water required per hour.

Solution

Refer Fig.9.29.

Heat to be absorbed from the cooling tower:

$$= \dot{m}_w c_{pw} \times \Delta T = 1000 \times 4.18 \times (35 - 30) \times 60 = 1.254 \times 10^6 \text{ kJ/h}$$

From psynchrometric chart, we have at 35 °C *DBT* and 25 °C *WBT*:

$$h_1 = 76.5 \text{ kJ/kg}$$

$$w_1 = 0.016 \text{ kg/kg of dry air}$$

At 30 °C and 90% *RH*, $h_2 = 92.5$ kJ/kg and $w_2 = 0.0246$ kg/kg of air:

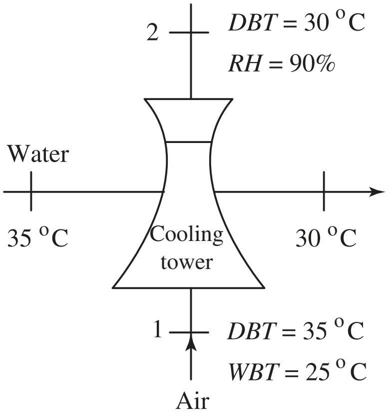


Fig. 9.29

Heat gained by air = Heat lost by water

$$\dot{m}_a(h_2 - h_1) \quad = \quad 1.256 \times 10^6 \text{ kJ/h}$$

$$\therefore \text{Mass of air} \quad = \quad \frac{1.256 \times 10^6}{(h_2 - h_1)} = \frac{1.256 \times 10^6}{(92.5 - 76.5)} = 78.5 \times 10^3 \text{ kg/h}$$

$$\text{Quantity of air handled per fan} = \frac{78.5 \times 10^3}{10} = 7850 \text{ kg/h}$$

Quantity of makeup water $= \dot{m}_a(W_2 - W_1) = 7850 \times (0.0246 - 0.016) = 67.51$ kg/h $\overset{\textbf{Ans}}{\Longleftarrow}$

9.22 The following data pertain to an air-conditioning system

DBT of unconditioned space	30 °C
WBT of unconditioned space	22 °C
Surface temperature of cold air duct	14 °C

Determine (i) dew point temperature and (ii) explain whether condensation will occur in the duct.

Solution

Refer Fig.9.30.

(i) To determine the dew point temperature for the given conditions, find the intersection of 30 °C DBT and 22 °C WBT and move horizontally as shown by the arrow to find the dew point temperature. The *dpt* = 18.6 °C.

(ii) Since the duct temperature (14 °C) is less than t_{dpt}(18.6 °C), there will be moisture condensation on the duct surface.

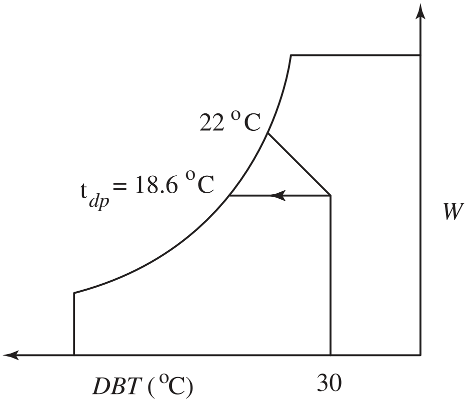


Fig. 9.30

9.23 Consider air at 15 °C *DBT* with relative humidity of 0.75. It flows through a pipe at the rate of 250 m^3/min and is heated to 25 °C. Estimate (i) *RH* of heated air; (ii) *WBT* of heated air; and (iii) heat added to air per minute.

Solution

Refer Fig.9.31 and follow the following steps:

(i) Locate point 1, on the chart which is the intersection of 15 °C *DBT* and 75% *RH* lines.

(ii) Through point1 draw a horizontal line to cut 25 °C *DBT* line to get point 2.

(iii) Read the following values from the chart, $h_1 = 35.4$ kJ/kg; $h_2 = 45.2$ kJ/kg; and $v_{s1} = 0.8267$ m^3/kg.

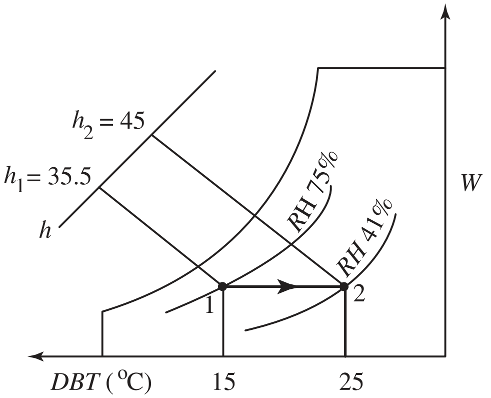


Fig. 9.31

$$RH \text{ of heated air from chart} = 41\% \quad \xleftarrow{\text{Ans}}$$

$$WBT \text{ of heated air from chart} = 16°\text{C} \quad \xleftarrow{\text{Ans}}$$

$$\text{Mass of air circulated per minute} = \frac{250}{0.8267} = 302.4 \text{ kg}$$

$$\therefore \text{ Heat added to air/minute} = m_a(h_2 - h_1) = 302.4 \times (45 - 35.5)$$

$$= 2872.8 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

9.24 It is required to design an air-conditioning system for a meeting hall for the following conditions. Outdoor condition: 14 °C DBT and 10 °C WBT

Condition to be maintained: 20 °C DBT and 60% RH Amount of air circulated = 0.33 m^3/min/person, seating capacity = 100 persons. The expected conditions to be achieved first by heating and then by adiabatic humidification. For the above conditions, estimate

(i) heating capacity of the coil in kW and the surface temperature required if the by pass factor of the coil is 0.35 and

(ii) the capacity of the humidifier use psychrometric chart for solution.

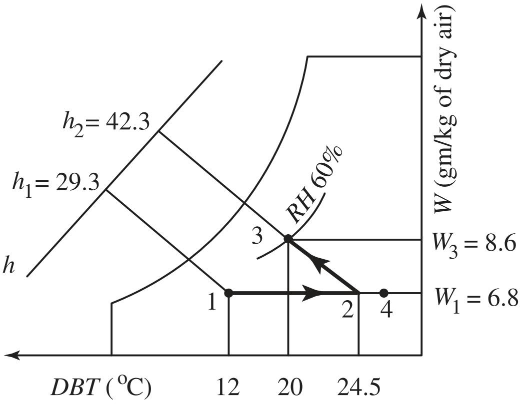


Fig. 9.32

Solution

Refer Fig.9.32. Locate points 1 and 3 on the psychrometric chart.

Draw constant enthalpy line through 3 and constant specific humidity line through 1. From chart $h_1 = 29.3$ kJ/kg; $h_2 = h_3 = 42.3$ kJ/kg; $t_{DBT2} = 24.5°$ C and $v_{s1} = 0.817$ m^3/kg.

The mass of air circulated per minute,

$$m_a = \frac{0.33 \times 100}{0.817} = 40.33 \text{ kg/min}$$

$$\text{Heating coil capacity} = \dot{m}_a(h_2 - h_1) = 40.33 \times (42.3 - 29.3)$$

$$= 524.3 \text{ kJ/min} = \frac{524.3}{60} = 8.74 \text{ kJ/s} = 8.74 \text{ kW}$$

The BF of heating coil is given by,

$$
\begin{aligned}
BF &= \frac{t_{DBT4} - t_{DBT2}}{t_{DBT4} - t_{DBT1}} \\
0.35 &= \frac{t_{DBT4} - 24.5}{t_{DBT4} - 12} \\
0.35 \times (t_{DBT4} - 12) &= t_{DBT4} - 24.5 \\
0.35 \times t_{DBT4} - 4.2 &= t_{DBT4} - 24.5 \\
0.65 t_{DBT4} &= 20.3 \\
DBT4 &= 31.23°\text{C} \\
\text{Capacity of the humidifier} &= \frac{\dot{m}_a(W_3 - W_1)}{1000} \times 100 \\
&= \frac{40.33(8.6 - 6.8)}{1000} \times 100 = 7.26 \text{ kg/h}
\end{aligned}
$$

Ans ⟸

9.25 An air-conditioning system to be designed for the following conditions:

Outdoor condition	32 °C *DBT* and 65% *RH*
Inlet air-condition required	25 °C *DBT* and 60% *RH*
Amount of air to be circulated	333 m^3/min
Dew point temperature of the coil	13 °C

The required condition is to be achieved by first cooling and dehumidifying and then by heating. Find

(i) the cooling capacity of the cooling coil.

(ii) heating capacity of the heating coil in kW and the surface temperature of the heating coil if the bypass factor is 0.30.

(iii) the mass of water vapour removed per hour.

Use psychrometric chart.

Solution

Refer Fig.9.33. Locate points 1, 5 and 3 as shown join lines 1–5.

Draw constant specific humidity line through 3 which cuts the lines 1–5 at point 2. Thus, point 2 is located.

From psychrometric chart, $h_1 = 82.5$ kJ/kg; $h_2 = 47.5$ kJ/kg; $h_3 = 55.7$ kJ/kg; $h_5 = 36.6$ kJ/kg; $W_1 = 19.6$ gm/kg; $W_3 = 11.8$ gm/kg; $t_{DBT2} = 17.6°\text{C}$ and $v_{s1} = 0.892 \text{ m}^3/\text{kg}$

The mass of air circulated,

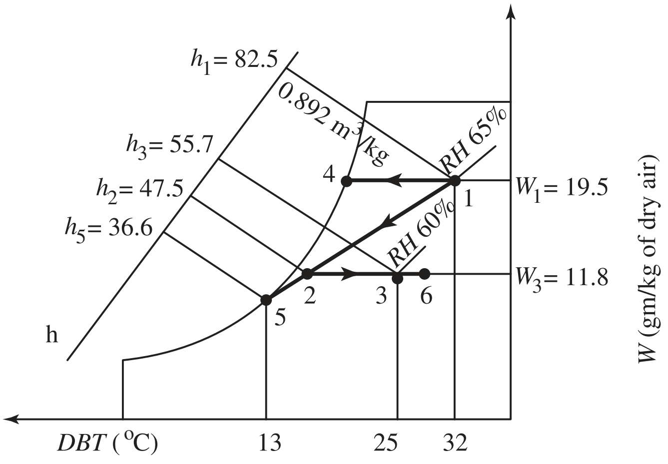


Fig. 9.33

$$m_a = \frac{333}{0.892} = 373.32 \text{ kg/min}$$

$$\text{Capacity of cooling coil} = \frac{m_a(h_1 - h_2) \times 60}{14000}$$

$$= \frac{373.32 \times (82.5 - 47.5) \times 60}{14000} = 56 \text{ TR}$$

The bypass factor of cooling coil,

$$BF = \frac{h_2 - h_5}{h_1 - h_5} = \frac{47.5 - 36.6}{82.5 - 36.6} = 0.237$$

Heating capacity of the heating coil,

$$m_a(h_3 - h_2) = 373.32(55.7 - 47.5)$$

$$= 3061.22 \text{ kJ/min} = 51 \text{ kJ/s} = 51 \text{ kW}$$

The bypass factor of the heating coil,

$$BF = \frac{t_{DBT6} - t_{DBT3}}{t_{DBT6} - t_{DBT2}}$$

$$0.3 = \frac{t_{DBT6} - 25}{t_{DBT6} - 17.6}$$

On solving, $t_{DBT6} = 28.2°$ C.

The surface temperature of heating coil = 28.2 °C **Ans** $\Longleftarrow$

The mass of water vapour removed per hour

$$= \frac{\dot{m}_a \times (W_1 - W_3) \times 60}{1000}$$

$$= \frac{373.32 \times (19.6 - 11.8) \times 60}{1000} = 174.71 \text{ kg/h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Review Questions

9.1 *What is psychrometry? Where is it used?*

9.2 *What are the composition of atmospheric air? Explain.*

9.3 *What is dry air and what are its composition? What is its molecular weight?*

9.4 *What do you understand by saturated air?*

9.5 *Define dry-bulb and wet-bulb temperature.*

9.6 *What is saturated vapour pressure? Explain.*

9.7 *What is wet-bulb depression and dew point depression?*

9.8 *What is dew point temperature?*

9.9 *What is humidity? What are various types of humidity?*

9.10 *What do you understand by humidity ratio?*

9.11 *Define relative humidity.*

9.12 *What is degree of saturation?*

9.13 *What is the difference between sensible heat and latent heat?*

9.14 *Define specific humidity. Show that specific humidity can be derived as:*

$$W = \frac{0.287}{0.462}\frac{p_v}{p_a} = 0.622\ \frac{p_v}{p_t - p_v}$$

9.15 *Show that degree of saturation can be expressed as:*

$$\mu \frac{p_v}{p_s}\left[\frac{\left(1 - \frac{p_{vs}}{p_t}\right)}{\left(1 - \frac{p_v}{p_t}\right)}\right]$$

9.16 *Show that the relative humidity can be derived as:*

$$\phi = \frac{p_a W}{0.622} \times \frac{1}{p_{vs}} = 1.6 \times W \times \frac{p_a}{p_{vs}}$$

9.17 *Derive an expression for the enthalpy of moist air.*

9.18 *Explain by means of a sketch the adiabatic saturation process.*

9.19 *Analyse by means of a T-s diagram the adiabatic cooling process.*

9.20 *What is a psychrometer? Explain with a sketch the wet- and dry-bulb psychrometer.*

9.21 *With a suitable sketch explain the sling psychrometer. Bring out its application and limitations.*

9.22 *Explain the working principle of (i) continuous recording psychrometer and (ii) aspiration psychrometer.*

9.23 *What is a psychrometric chart. Explain its use in the field of air-conditioning.*

9.24 *With an example explain how to read a psychrometric chart.*

9.25 *Make short notes on the following:*

(i) adiabatic mixing of air streams
(ii) heating and dehumidification
(iii) heating and humidification
(iv) cooling and humidification
(v) sensible heating and sensible cooling

Exercise

9.1 A sling psychrometor is used to measure the dry-bulb and wet-bulb temperature. The measured values are 40 °C DBT and 36 °C WBT. Determine the following:

(i) humidity ratio
(ii) relative humidity
(iii) dew point temperature
(iv) specific volume
(v) enthalpy of air

Ans: (i) 0.03 kg of water vapour/kg of dry air; (ii) 81%; (iii) 36 ° C; (iv) 0.9271 m^3/kg; (v) 140.63 kJ/kg dry air

9.2 The initial and final conditions of air are 35 °C, 0.7RH and 25 °C, 0.6 RH, respectively. Estimate the amount of heat removed per kg of air to reach the final condition from initial air. *Ans:* 45 kJ/kg of dry air

9.3 Calculate the specific humidity, dew point, WBT, mass of dry air and mass of water vapour when the air water vapour mixture is at 1 kPa, 30 °C with $RH = 0.8$. Take the volume of the mixture as 50 m^3. If the mixture is cooled at constant pressure to 5 °C, estimate the amount water vapour condensed.

Ans: 0.022 kg water vap/kg of dry air; 0.888 m^3/kg of dry air; 27 °C; 56.3 kg dry air; 1.23 kg; 0.0156 kg water vapour/kg of dry air

9.4 There is an adiabatic mixing of two streams of air with following conditions to get 0.3 kg/s of dry air at 30 °C.

First stream : $DBT = 25°\text{C}$ $RH = 50\%$ Second stream : $DBT = 25°\text{C}$ $RH = 60\%$

Calculate the amount of air drawn from the streams and the humidity ratio of mixed air. *Ans:* 0.0113 kg of water vapour/kg of dry air; 0.0107 kg of water vapour/kg of dry air

9.5 Air at 40 °C DBT and 27 °C WBT is to be cooled and dehumidified by passing it over a refrigerant filled coil to give a final condition of 15 °C and 90% RH. Find the amount heat and moisture removed per kg of dry air.*Ans:* 0.0072 kg water vapour/kg dry air

9.6 In a heater cum humidifier, air-water vapour enters at 5 °C and 1 bar with 0.5 RH. The flow rate of dry air is 0.1 kg/s. Liquid water at the rate of 0.002 kg/s is sprayed into the mixture at 10 °C. The exit temperature of the mixture is 30 °C and 1 bar. Calculate the relative humidity at the exit and the rate of heat transfer to the unit.
Ans: 2.261 kW; 80%

9.7 A research room with a volume of 470 m^3 is to be maintained at 20 °C and 52.5% RH. The complete air change takes place once every hour. The fresh air at a pressure of 1.05 bar, 32 °C and 86% RH is drawn from the atmosphere. This air is passed through a cooler to reduce the temperature and it causes condensation. The condensate is drowned at 8 °C. The resulting saturated air is heated to room condition. Assume that the total pressure is constant throughout. Find the (i) temperature of air leaving the cooler; (ii) rate of condensation; (iii) heat transfer in the cooler; and (iv) heat transfer in the heater.
Ans: (i) 10 ° C; (ii) 2.87×10^{-3} kg/s; (iii) 11.181 kW; (iv) 1.47 kW

9.8 A cooling coil is used in an air-conditioning system to cool and dehumidify the air. The given conditions are :

Initial condition of air at inlet to the cooling coil:

Dry-bulb temperature, DBT	25 °C
Partial pressure of water vapour	0.019 bar
Absolute total pressure	1.02 bar
Final condition of air at exit of the cooling coil:	
Dry-bulb temperature, DBT	15 °C
RH	90%
Absolute total pressure	1.02 bar
Other relevant data are:	
R_{air}	287 J/kg K
R_{wv}	461.5 J/kg K
Saturation pressure for water at 15 °C	0.017 bar
Enthalpy of dry air	1.005 t kJ/kg
Enthalpy of water vapour	$(2500 + 1.88t)$ kJ/kg where t is in °C.

Find (i) moisture removed from air and (ii) heat removed by the cooling coil per kg of dry air.
Ans: 15.83 kJ/kg of dry air

9.9 Air at 30 °C, 80% RH is cooled by spraying water at 12 °C. This causes saturation followed by condensation. The mixing is assumed to take place adiabatically and the condensate is drowned off at 16.7 °C. The resulting saturated mixture is then heated to produce the required conditions of 60% RH at 25 °C. The total pressure is constant at 10 kPa. Determine the mass of water supplied to the provide 10 m^3/h of conditioned air. What is the heater power required?
Ans: 11.63 kg of dry air/h; 0.029 kW

9.10 Air-water mixture in a room is at atmospheric pressure of 1 atm. It has a dry-bulb temperature of 44 °C and wet-bulb temperature of 29 °C. Calculate the specific humidity, relative humidity and enthalpy of mixture. Compare the calculated results using adiabatic saturation relation from the values obtained from the psychrometric chart.
Ans: 0.021; 0.364; 98.435 kJ/kg of dry air

9.11 In a laboratory, the dry-bulb and wet-bulb temperatures were measured using sling psychrometric. The measured values are, $DBT = 30°\text{C}$; $WBT = 20°\text{C}$. Find the specific humidity, the relative humidity and enthalpy of the air-water vapour mixture using psychrometric chart. *Ans:* 0.0106 kg H_2O/kg dry air; 0.4; 57.5 kJ/kg of dry air

9.12 On a normal day, the dry-bulb and wet-bulb temperatures were measured. The corresponding values are 35 °C and 25 °C, respectively. Determine the following without using psychrometric chart: (i) specific humidity; (ii) the relative humidity; and (iii) enthalpy of moisture air. *Ans:* (i) 0.0159 kg of water/kg of dry air; (ii) 0.449; (iii) 76 kJ/kg

9.13 A mixture of nitrogen and water vapour at 100 °C and 1 bar has the molar analysis 90% nitrogen and 10% water vapour. If the mixture is cooled at constant pressure, calculate the temperature at which water starts to condense. *Ans:* 45.83 °C

9.14 The DBT and WBT in a auditorium is measured as 30 °C and 20 °C, respectively. If the air inside the auditorium is 0.95 bar, calculate the specific and relative humidity of the air. *Ans:* (i) 0.0115 kg of water vapour/kg of dry air; 0.406

9.15 The DBT and WBT of the humid air in a room reads 35 °C and 20 °C. Using psychrometric chart determine the dew point temperature. *Ans:* 14.5 °C

9.16 The following data are recorded for a mixture of air-water-vapour at $DBT = 40°\text{C}$, $\phi = 50\%$. It is cooled to 25 °C at a constant pressure of 1 atm. Using psychrometric chart, calculate (i) the relative humidity; (ii) the dew point temperature in the first state; and (iii) the quality of heat removed from the mixture per kg of dry air. *Ans:* (i) 1; (ii) 25 °C; (iii) 24.16 kJ

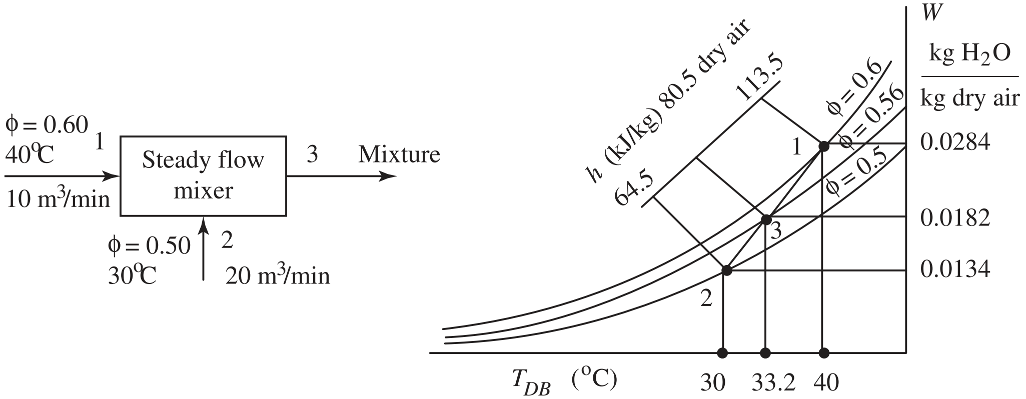
9.17 An adiabatic mixing device is operating at the steady state. It receives 30000 liters per minute of air at 40 °C with a RH of 60%. A second stream of air enters the mixing device at 20 °C with RH of 10%. The volume rate of second stream is 10000 liters per minute. The pressure at the mixing device is 1 atm. Calculate the RH and the temperature of the steam at the exit. *Ans:* 0.0211 kg of water vapour/kg of dry air; 35 °C

9.18 Two streams of moist air at 1 atm pressure are mixed adiabatically in a steady flow mixture as shown in Fig.9.34(a). Determine the specific humidity, relative humidity, the temperature and enthalpy of resulting mixture using psychrometric chart. *Ans:* 0.0182 kg of water vapour per kg of dry air; 0.56; 80.5 kJ/kg of dry air

9.19 On a hot summer day, the atmospheric temperature of air is 40 °C and has a RH of 30%. Air at this condition enters an evaporative cooler at a rate of 3 m^3/s. It leaves the cooler with 80% RH. Calculate the temperature and specific humidity of the air leaving the cooler. Also estimate the water required to operate the cooler of 12 h. *Ans:* 28 °C; 0.019 kg of water vapour/kg of dry air; 743.04 kg

9.20 A library hall is to be cooled by supplying cold air at the rate of 2 m^3/s. The condition of air at entry is 20 °C with 40% relative humidity. The atmospheric air available is at 35 °C with 70% relative humidity. Design an appropriate air-conditioning system and estimate the rate of cooling and heating required. *Ans:* 28.86 kW

9.21 Air enters a heating unit at steady flow condition at a pressure of 1 atm and 10 °C. It has a RH of 40%. This air is first humidified, by rejecting saturated steam again at 1 atm.



(a) Mixing chamber

(b) Psycrometric chart showing the process of adibatic mixing of stream

Fig. 9.34

Air leaves the heating unit at 27 °C and 50% *RH*. If the dry air flow rate is 1 kg/s. Determine (i) the amount of steam required for one hour and (ii) the power input to the heating unit. *Ans:* (i) 29.52 kg; (ii) 12 kW

9.22 A forced draft cooling tower is used in a large thermal power plant for providing cold water at 30 °C to the condensers. The water enters the cooling tower at 37 °C at the rate of one tonne/s. The atmospheric air at 27 °C and $RH = 40\%$ enters the cooling tower and leaves at 33 °C with $RH = 80\%$. Determine the make-up water and air flow rate requirement. *Ans:* 9.91 kg/s; 619.88 kg/s

9.23 A gas mixture has the following composition by volume:

Constituent	CO_2	He	N_2	O_2	H_2O
Volume %	30	10	20	30	10

The mixture is available at 1 bar and 400 K. It is proposed to remove the water vapour by cooling the mixture. One kmol of the mixture is cooled at constant pressure. Determine the temperature at which water beginning to change. If the temperature of the mixture is reduced to 15 °C below the condensation temperature, find the amount of water that is condensed. *Ans:* 1.04 kg

9.24 In a drier operating at steady-state atmospheric air is at 30 °C and 1 atm with a *RH* of 40%. It is first heated to 110 °C at constant pressure. The heated air is then allowed to pass over the mixture to be dried and the air leaves the drier at 70 °C with a relative humidity of 0.5. It is required to remove 1 kg/s of moisture from the material. Determine the mass flow rate of dry air required in kg/min and the rate of heat transfer to the air as it passes through the heating unit. *Ans:* 585.94 kg of dry air/min; 797.17 kW

9.25 In an air-conditioning plant saturated air at 10 °C is heated at constant pressure to 25 °C. Determine the specific humidity and the relative humidity of the air leaving the unit. Also determine the energy required per kg of dry air for heating. *Ans:* 0.0075 kg H_2O/kg of dry air; 0.39; 15 kJ

Multiple Choice Questions (choose the most appropriate answer)

1. Dry air does NOT contain

   (a) krypton
   (b) argon
   (c) neon
   (d) none of the above

2. The ratio of partial pressure of water vapour in a mixture to the saturation pressure of water at the same temperature of the mixture is called as

   (a) humidity
   (b) partial humidity
   (c) specific humidity
   (d) relative humidity

3. What is the specific humidity?

   (a) the ratio of the mass of water vapour to the mass of the total mixture of air and water vapour.

   (b) the ratio of the mass of dry air to the mass of the total mixture of air and water vapour.

   (c) the ratio of the mass of dry air to the mass of water vapour in a mixture of air and water vapour.

   (d) the ratio of the mass of water vapour to the mass of dry air in a mixture of air and water vapour.

4. What is the relative humidity for a saturated air?

   (a) 0%
   (b) 50%
   (c) 100%
   (d) cannot say

5. What is the degree of saturation?

   (a) The ratio of the actual relative humidity to the saturated specific humidity at the same temperature

   (b) The ratio of the actual specific humidity to the saturated specific humidity at the same temperature

   (c) The ratio of the saturated specific humidity to the actual specific humidity at the same temperature

   (d) None of the above

6. The degree of saturation varies between

   (a) 1 to infinity
   (b) 0 to infinity
   (c) 0 to 1
   (d) none of the above

7. At any point on the saturation curve in psychometric chart, the dry-bulb temperature is always

   (a) less than the corresponding wet-bulb temperature

   (b) more than the corresponding wet-bulb temperature

   (c) equal to the corresponding wet-bulb temperature

   (d) cannot predict

8. The moisture content lines in psychrometric chart are also called as

(a) relative humidity lines
(c) both (a) and (b)
(b) specific humidity lines
(d) none of the above

9. The horizontal line in psychrometric chart joining the change of state of air represents

(a) humidification
(c) sensible cooling or heating with humidification
sensible cooling or heating
(d) sensible cooling or heating with dehumidification

10. Air at dry-bulb temperature T_{d1} is passed through the heating coil and some amount of air is by passed unaffected and leaves with dry-bulb temperature T_{d2} and the temperature of heated air is T_{d3} (where $T_{d2} < T_{d3}$), what is the correct formula for by pass factor (The amount of air by passed unaffected)?

(a) Bypass factor $= \dfrac{(T_{d3} - T_{d2})}{(T_{d3} - T_{d1})}$

(b) Bypass factor $= \dfrac{(T_{d3} - T_{d1})}{(T_{d3} - T_{d2})}$

(c) Bypass factor $= \dfrac{(T_{d2} - T_{d1})}{(T_{d2} - T_{d3})}$

(d) Bypass factor $= \dfrac{(T_{d2} - T_{d3})}{(T_{d2} - T_{d1})}$

11. Humidification is the process of addition moisture in air at

(a) constant wet-bulb temperature
(c) constant latent heat
(b) constant dry-bulb temperature
(d) none of the above

12. What is the perfect condition for dehumidification of air?

(a) air is heated above its dew point temperature
(b) air is cooled up to its dew point temperature
(c) air is heated below its dew point temperature
(d) air is cooled below its dew point temperature

13. At 100% relative humidity, the wet-bulb temperature is

(a) lower than the dew point temperature
(b) higher than the dew point temperature
(c) equal to the dew point temperature
(d) none of the above

14. The dew point temperature is less than the wet-bulb temperature for

(a) saturated air
(c) both saturated and unsaturated air
(b) unsaturated air
(d) none of the above

15. When, H_1 = Total heat of air entering the coil (heating or cooling) H_2 = Total heat of air leaving the coil (heating or cooling) H_3 = Total heat of air at the end of the process (humidification or dehumidification) then, the sensible heat factor $\left(\dfrac{H_2 - H_1}{H_3 - H_1}\right)$ represents the process of

    (a) cooling and humidification
    (b) cooling and dehumidification
    (c) heating and humidification
    (d) heating and dehumidification

16. When the rate of evaporation of water is zero, the relative humidity of the air is

    (a) 0%
    (b) 100%
    (c) 50%
    (d) unpredictable

17. What is the temperature at which the water vapour in the mixture of water vapour in air starts condensing called?

    (a) condensation temperature
    (b) dew point temperature
    (c) vaporisation temperature
    (d) all of the above

18. The temperature of air recorded by thermometer when the bulb is covered by a cotton wick saturated by water is called as

    (a) DBT
    (b) WBT
    (c) stream temperature
    (d) psychrometric temperature

19. Which temperature can be measured by an instrument called psychrometer?

    (a) DBT
    (b) WBT
    (c) both (a) and (b)
    (d) none of the above

20. When DBT and WBT are measured, greater the difference between DBT and WBT,

    (a) greater the amount of water vapour held in the mixture

    (b) smaller the amount of water vapour held in the mixture

    (c) same the amount of water vapour held in the mixture

    (d) none of the above

21. What is sensible cooling of air?

    (a) Te cooling in which sensible heat of air is removed in order to reduce temperature

    (b) The cooling in which temperature of air is reduced without changing in its moisture content

    (c) Both (a) and (b)

    (d) None of the above

22. When the humidity ratio of air increases the the air is said to be

    (a) dehumidified
    (b) humidified
    (c) heated
    (d) cooled

23. The temperature at which the air cannot hold all the water vapour mixed in it and some vapour starts condensing is called as
    (a) humidification temperature
    (b) dehumidification temperature
    (c) dew point temperature
    (d) none of the above

24. The formation of fog starts when,
    (a) air temperature is equal to the dew point temperature
    (b) air temperature is greater than the dew point temperature
    (c) both (a) and (b)
    (d) none of the above

25. When the dew point temperature is equal to the air temperature then the relative humidity is
    (a) 0%
    (b) 50%
    (c) 100%
    (d) unpredictable

Ans: 1. – (d) 2. – (d) 3. – (d) 4. – (c) 5. – (b)
6. – (c) 7. – (c) 8. – (b) 9. – (b) 10. – (d)
11. – (b) 12. – (d) 13. – (c) 14. – (b) 15. – (c)
16. – (b) 17. – (b) 18. – (b) 19. – (c) 20. – (b)
21. – (c) 22. – (b) 23. – (c) 24. – (a) 25. – (c)

10

PROPERTIES OF GASES AND GAS MIXTURES

10.1 INTRODUCTION

In general, matter can be divided into three states, namely solid, liquid and gas, They are recognised through their key molecular arrangement. Solids have strong composition of molecular attraction. It gives them definite shape and mass. Liquids take the form of their container since the molecules are moving with respect to one another. On the other hand, gases, unlike solids and liquids, have indefinite shape and indefinite volume. As a result, they can be subjected to pressure changes, volume changes and temperature changes. Gases diffuse easily in air since the molecules are moving freely. The characteristics of gases are very distinct. There are gases that are strong enough to react with other matter. There are gases with very strong odour. Some of them can be dissolved in water. From the thermodynamic point of view, gases can be broadly divided into two categories:

(i) Ideal gas (ii) Real gas.

10.2 IDEAL AND REAL GASES

To start with, let us look at ideal gases, since their behaviour is simpler and comparatively easier to understand. Probably then, it might become easier to understand the real gas. Further, in many important thermodynamics applications, it requires homogenous mixture of several gases rather than a single gas. Therefore, it is important to develop an understanding of mixture of gases and learn how to use them for engineering applications. In this chapter, we deal with non-reacting gases and their mixtures. A non-reacting gas mixture can be treated as a pure substance since it is a homogeneous mixture of different gases. The thermodynamic behaviour of a mixture of gases depends upon the individual properties of its constituent gases. Therefore, wide variation is possible in the properties of gaseous mixtures.

10.2.1 Ideal Gas

As already stated, in Chapter 3, Section 3.7.1, an ideal gas is an imaginary substance that obeys the equation of state at all pressures and temperatures. The equation of state provides the relation between pressure, volume and temperature and is given by:

$$pV = mRT \quad \text{or} \quad pv = RT \tag{10.1}$$

where p is the pressure of the gas in kPa, v is the specific volume in m^3/kg, V is the total volume of the gas in m^3 and R is the universal gas constant which is equal to 8.314 kJ/kg mol K. In term of mass, $R = 0.287$ kJ/kg K.

The equation of state is also known as ideal gas equation. At low pressures and high temperatures, the density of a gas decreases and at that time, the gas behaves as an ideal gas. In practice, there is no ideal gas. All the gases we come across are real gases only. In engineering analysis, we assume that the gases tends towards ideal gas behaviour in order to simplify the calculation.

An idea gas obeys the following laws:

(i) Boyle's law
(ii) Charles's law
(iii) Joule's law
(iv) Regnault's law
(v) Avaogaoro's law

10.2.2 Boyle's Law

Boyle's law states *the volume of given mass of a gas varies inversely with its absolute pressure when the temperature remains constant.* Mathematically,

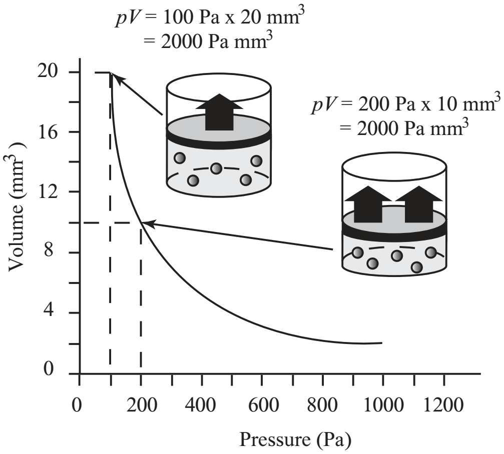


Fig. 10.1 Boyle's Law

$V \alpha \frac{1}{P}$ or pv = constant as long as temperature is constant. For example, for the ideal gas at state 1 can be written as:

$$p_1 v_1 = C \tag{10.2}$$

At state 2,

$$p_2 v_2 = C \tag{10.3}$$

which means,

$$p_1 v_1 = p_2 v_2 \tag{10.4}$$

This formula is applicable for isothermal process.

10.2.3 Charles's Law

Charles's law states, *the volume of given mass of a gas varies directly with its absolute temperature when the pressure remains constant.* Mathematically, $\frac{v}{T}$ = Constant.

At state point 1,

$$\frac{v_1}{T_1} = C \tag{10.5}$$

At state point 2,

$$\frac{v_2}{T_2} = C \tag{10.6}$$

Therefore,
$$\frac{v_1}{T_1} = \frac{v_2}{T_2} \tag{10.7}$$

or
$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \tag{10.8}$$

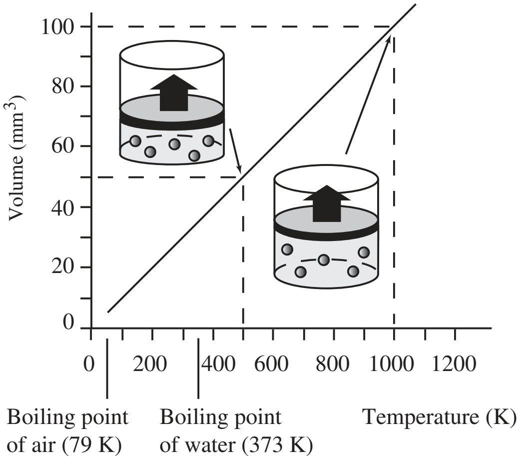


Fig. 10.2 Charle's Law

The above equation is applicable to all constant pressure process. Charles's law also states *the pressure of given mass of a gas varies directly with its absolute temperature when the volume remains constant.* $p\alpha T$ or $\frac{p}{T} = C$.

At state point 1,
$$\frac{p_1}{T_1} = C \tag{10.9}$$

At state point 2,
$$\frac{p_2}{T_2} = C \tag{10.10}$$

Therefore,
$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \tag{10.11}$$

or
$$\frac{p_1}{v_2} = \frac{T_1}{T_2} \tag{10.12}$$

At state 1,
$$p_1v_1 = C \tag{10.13}$$

At state 2,
$$p_2v_2 = C \tag{10.14}$$

which means,
$$p_1v_1 = p_2v_2 \tag{10.15}$$

10.2.4 Joule's Law

Joule's law states, *the internal energy of a given quantity of a gas depends only on the temperature.* It is independent of pressure and volume:

$$\Delta U \quad = \quad C_V \Delta T \tag{10.16}$$

10.2.5 Regnault's Law

Regnault's law states, *the two specific heats, viz C_p and C_V, of a gas do not change with the change of temperature and pressure.* In other words, C_p and C_V of a gas always remain constant.

The above law is only an approximate one, because the specific heats of a gas vary with temperature even though it has very little effect on pressure.

10.2.6 Avogadro's Law

Avogardro's law states, *equal volumes of different perfect gases at the same temperature and pressure contain equal number of molecules.* Molecules will repulse each other.

It can also be stated as *the volume of one gram mol of all gases at the pressure of 760 mm Hg and temperature of 0° C is the same and equal to 22.4 litres.*

One gram mole of gas = 22.4 liters = 22.4 $\times 10^{-3}$ m^3. Therefore, one kilogram mole of the gas = 22.4 m^3.

In other words, it can be stated as *all gases whose mass is equal to its mole will occupy the same volume at normal temperature and pressure*:

Pressure at NTP (p) = 1.013 bar,
temperature at NTP (T) = 0°C,
volume at NTP (v) = 22.4kg/m^3,
As per Avaogaoro's law Mv = Constant.

where M is the molecular weight of the gas and v is the specific volume. So, the equation of state when multiplied by M on both sides:

$$Mpv = MRT \tag{10.17}$$

where Mv is the molar volume where $MR = \overline{R}$ is the universal gas constant. Now, Eq.10.17 becomes

$$p\overline{V} = \overline{R}T \tag{10.18}$$

At NTP,

$$\overline{R} = \frac{p\overline{V}}{T} = \frac{1.013 \times 10^3 \times 22.4}{273} = 8314.3\frac{\text{Nm}}{\text{kg}}\text{mole K} \tag{10.19}$$

An ideal gas has the following properties:

(i) An ideal gas is considered to be a point mass. A point mass is nothing but a particle which is so small and its mass is very nearly zero. This means an ideal gas particle has virtually no volume.

(ii) Collisions between ideal gases are elastic. It means that no attractive or repulsive forces are involved during collisions. Also, the kinetic energy of the gas molecules remains constant since the inter-particle forces are lacking.

10.3 MIXTURE OF IDEAL GASES

When the two ideal gases are mixed together, the state of components before mixing is same as the states of both the mixture of individual components after mixing. For example, let there be three gases, A, B and C at different pressures temperatures which are mixed adiabatically in a closed chamber. To do this, initially, these three gases should be in different tanks which are connected by pipelines. Then, these pipelines are made to meet at one point. This adiabatic mixing can be done in another way also. Initially, three different gases are filled of fixed volume (in each compartment). The compartments are made by partitions. After few minutes, the partitions are removed or ruptured to form the mixture. The following assumptions are made while mixing of gases as follows:

(i) Mixing of gases is adiabatic,

(ii) There is no work done.

According to mass balance, the total mass of gas after mixing = the sum of masses of different gases before mixing:

$$m_m = m_A + m_B + m_C \tag{10.20}$$

where m_m is total mass of gas mixture and m_A, m_B, m_C is masses of gas A, B and C, respectively. Similarly, the total volume of gas mixture after mixing is the sum of volume of different gases before mixing.

$$V_m = V_A + V_B + V_C \tag{10.21}$$

where V_A, V_B, V_C is volume of gas A, B and C, respectively.

According to the first law of thermodynamics, the change in internal energy becomes zero,

$$\Delta U = 0 \qquad (\because Q \text{ and } W \text{ are zero }) \tag{10.22}$$

Similarly, the total change in internal energy of gas mixture is given by:

$$\begin{aligned} \Delta U_m &= \Delta U_A + \Delta U_B + \Delta U_C \\ &= \Delta m_A u_A + \Delta m_B u_B + \Delta m_C u_C \qquad (\because U = mu) \end{aligned} \tag{10.23}$$

We know that:

$$\Delta U_m = m c_V \Delta T \, (\because \Delta T = \text{initial temperature} - \text{final temperature}) \tag{10.24}$$

In this case, the temperature T_m corresponds to the temperature of the mixture after thorough mixing:

$$\Delta U_m = m_A c_{V_A} (T_m - T_A) + m_B c_{V_B} (T_m - T_B) + m_C c_{V_C} (T_m - T_C) \tag{10.25}$$

Since $\Delta U_m = 0$,

$$m_A c_{V_A} (T_m - T_A) + m_B c_{V_B} (T_m - T_B) + m_C c_{V_C} (T_m - T_C) = 0 \tag{10.26}$$

$$T_m (m_A c_{V_A} + m_B c_{V_B} + m_C c_{V_C}) = (m_A c_{V_A} T_A + m_B c_{V_B} T_B + m_C c_{V_C} T_C) \tag{10.27}$$

$$T_m = \frac{(m_A c_{V_A} T_A + m_B c_{V_B} T_B + m_C c_{V_C} T_C)}{(m_A c_{V_A} + m_B c_{V_B} + m_C c_{V_C})} \tag{10.28}$$

The pressure of the mixture is calculated by using the equation of state as

$$p_m = \frac{m_m R_m T_m}{V_m} \tag{10.29}$$

where m_m is the mass of the mixture; R_m is the gas constant of the mixture determined by experiments; T_m is the temperature of the mixture; V_m is the volume of the mixture. Similarly, the change in enthalpy can be written:

$$\Delta H_m = \Delta H_A + \Delta H_B + \Delta H_C \tag{10.30}$$

Now, $\Delta H_m = mc_p \Delta T$. Now, Eq.10.30 becomes

$$\Delta H_m = \Delta m_A h_A + \Delta m_B h_B + \Delta m_A h_C \tag{10.31}$$

Since $H = mh, \quad \Delta h_m = mc_p \Delta T$, where $\Delta T =$ initial temperature $-$ final temperature.

In this case, the temperature T_m corresponds to the temperature of the mixture after through mixing:

$$m_A c_{P_A} (T_m - T_A) + m_B c_{P_B} (T_m - T_B) + m_C c_{P_C} (T_m - T_C) = 0$$

$$T_m (m_A c_{P_A} + m_B c_{P_B} + m_C c_{P_C}) = (m_A c_{P_A} T_A + m_B c_{P_B} T_B + m_C c_{P_C} T_C)$$

$$T_m = \frac{(m_A c_{P_A} T_A + m_B c_{P_B} T_B + m_C c_{P_C} T_C)}{(m_A c_{P_A} + m_B c_{P_B} + m_C c_{P_C})} \tag{10.32}$$

Since the mixing of gases is done adiabatically and irreversibly the entropy of system will increase. Total entropy of gas mixture:

$$\Delta S_m = \Delta S_A + \Delta S_B + \Delta S_C \tag{10.33}$$

For irreversible process, $\Delta s > 0$:

$$\Delta S_m = \Delta S_A + \Delta S_B + \Delta S_C > 0 \tag{10.34}$$

10.3.1 Real Gas

At higher pressures, the gases start to deviate from ideal gas behaviour. So, that deviation should be accounted for. For accounting this deviation, a factor called compressibility is introduced. So, the state equation for real gases for unit mass is given by

$$pv = ZRT \tag{10.35}$$

where Z is compressibility factor (refer Section 10.8.2). Therefore, the compressibility factor can be wrtitten as

$$Z = \frac{pv}{RT} \tag{10.36}$$

It can also be expressed as

$$X = \frac{v_{actual}}{v_{ideal}} \tag{10.37}$$

10.3.2 Properties of a Real Gas

(i) The specific heats varies with pressure and temperature.

(ii) The enthalpy and internal energy are the function of pressure and temperature.

(iii) The entropy equation pertaining to ideal gas will not be valid for the real gas.

10.4 INTERMOLECULAR FORCES

When the pressure is quite low, there will be an intermolecular attractive force. First, let us assume that the molecules start attracted towards each other at a distance, say r_o (refer Fig.10.3). Less than this distance r_o, the repulsive forces come into action. The repulsive forces increases with decrease in the distance between the molecules (see Fig.10.3). More than the distance r_o, the attractive forces increases (see the arrow in Fig.10.3) until it reaches a distance of d, where d is the diameter of the molecules and then start decreasing. When the distance reaches $4d$ or more the attractive force almost reaches zero.

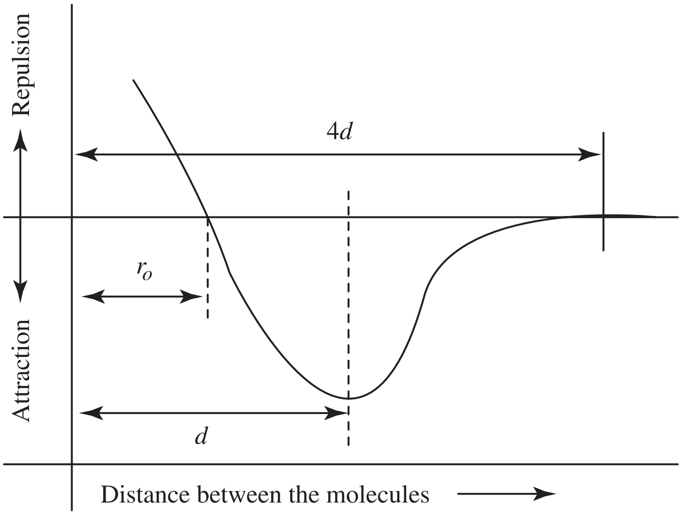


Fig. 10.3 Attractive and repulsive forces

Let us consider a vessel containing real gases. In these gases, let us assume that the molecule is exactly at the middle. If that molecule is away from the wall but surrounded by other molecules quite tightly, the force attraction will be equal in all directions. Under these conditions, the resultant attractive force is zero. In case, the resultant force has some value, the molecule has large probability to hit the wall. This attractive force is directly proportional to the number of molecules per unit volume of the gas.

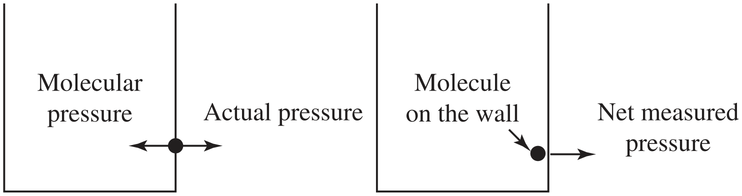


Fig. 10.4 Pressure exerted by the molecules

Now, the force of attraction is proportional to $\dfrac{1}{v^2}$ and the actual pressure of the gas exerted will increase to $\left(p + \dfrac{a}{v^2}\right)$, where p is pressure; v is specific volume of gas and a the constant.

10.4.1 Shape Factor

A correction factor is usually calculated by considering the volume occupied by molecules. For this, the molecules are assumed as rigid sphere. The space between molecules may either compress or expand. At low pressures, the volume occupied by molecules is extremely small. So, it can be neglected. However, at high pressures, the volume cannot be neglected.

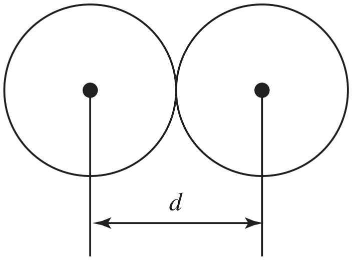


Fig. 10.5 Two molecules at the time of collision

Let us take the diameter of each molecule to be d. The distance between two molecules from centre to centre is also d (see Fig.10.5). The molecules cannot come closer than the distance d. The volume of sphere is $V = \frac{4}{3}\pi\frac{d^3}{8}$.

Under these conditions, each molecule can be surrounded by eight molecules. The effective volume of the molecules will be given by $V = \frac{4}{3}\pi\frac{d^3}{8}$. It is considered for four molecules around each molecule. This is known as *shape factor*. Generally, it is denoted by b. So, the effective volume of the gas per unit mass is $v-b$. To compensate the behaviour of the real gas molecules, these two corrections for pressure and volume are carried out.

10.5 OTHER EQUATIONS OF STATE

Ideal gas equations can be derived from the kinetic theory of gases. In ideal gas equation, $pv = RT$. There are two important assumptions: (i) there is no intermolecular attraction and (ii) the volume occupied by the molecules are negligibly small. However, these assumptions are good for a real gas. Therefore, a number of other equations were postulated by different people. We will discuss some of them in the following sections.

10.5.1 van der Waals Equation

Scientist van der Waals considered two corrections to analyse the behaviour of real gases during 1873. According to him, the equation of state for real gases is given by

$$\left(p + \frac{a}{\overline{v}^2}\right)\ (\overline{v} - b) \quad = \quad \overline{R}T \tag{10.38}$$

when a and b are zero, then it becomes ideal gas equation.

For real gases, the values of a and b are dependent upon the type of fluid or the gases. The units are Pa for pressure, p, K for temperature, T. $\overline{v}$ is the molar volume and $\overline{R}$ is the universal gas constant = 8.314 Nm/kg mol K. a is (Pa/kg mol)^2 and b is in m^3/kg mol. The value of a and b are usually taken as:

$$a = \frac{27R^2(T_c)^2}{64p_c} \tag{10.39}$$

$$b = \frac{RT_c}{p_c} \tag{10.40}$$

where suffix c denoted critical point. van der Waals equation has the following limitations:

(i) The study had not been made under actual conditions and its validity fails in many occasions.

(ii) The values of a and b are taken as constant but they will vary with temperature. It is usually determined experimentally.

(iii) At critical point, the van der Waals equation:

$$\frac{p_C V_C}{RT_C} = \frac{3}{8} \quad \text{and} \quad \frac{p_C V_C}{RT_C} = 1 \tag{10.41}$$

Whereas experimentally determined values for $\frac{p_C V_C}{RT_C}$ are between 0.2 and 0.3. Thus, it fails quantitatively in many particulars.

10.6 BEATTIE-BRIDGEMAN EQUATION OF STATE

This equation is based on experimentally determined constants and was proposed in 1928. There are five of them. The equation is of the form

$$p = \frac{RuT}{\overline{v}^2}\left(1 - \frac{c}{\overline{v}T^3}\right)(\overline{v} + B) - \frac{A}{\overline{v}^2} \tag{10.42}$$

where $A = A_0\left(1 - \frac{a}{\overline{v}}\right)$ and $B = B_0\left(1 - \frac{b}{\overline{v}}\right)$ p is in kPa. $\overline{v}$ is in m^3/k mol; T is in Kelvin and $Ru = 8.314$ kPa m^3/kmol K.

The Beattie-Bridgeman equation is known to be reasonably accurate for densities upto $0.8\rho_{cr}$, where ρ_{cr} is the density of the substance at critical point.

The five constants of air is given as $A_0 = 131.8441; B_0 = 0.04611; a = 0.01931; b = -0.0011$ and $c = 43.4$.

10.7 BENEDICT-WEBB-RUBIN EQUATION OF STATE

In 1940, Benedict, Webb and Rubin experimentally determined and increased the number of constants to eight in the Beattie-Bridgeman equation.

This equation of state is expressed in the form

$$p = \frac{R_uT}{\overline{v}} + \left(B_0R_uT - A_0 - \frac{c_0}{T^2}\right)\frac{1}{\overline{v}^2} + \frac{bR_uT - a}{\overline{v}^3} + \frac{a\alpha}{\overline{v}^6} + \frac{c}{\overline{v}^3T^2}\left(1 + \frac{\gamma}{\overline{v}^2}\right)e^{\left(\frac{-\gamma}{\overline{v}^2}\right)} \tag{10.43}$$

where Ru is the universal gas constant. This equation is accurate up to $2.5\rho_{cr}$.

The eight constants for N_2 are given as $A_0 = 106.73; B_0 = 0.04074; a = 2.54; b = 0.082328;$ $c = 7.370 \times 10^{-4}$; CO $= 8.164 \times 10^{-4}$; $\alpha = 12.72 \times 10^5$ and $\gamma = 0.0053$.

10.8 VIRIAL EQUATION OF STATE

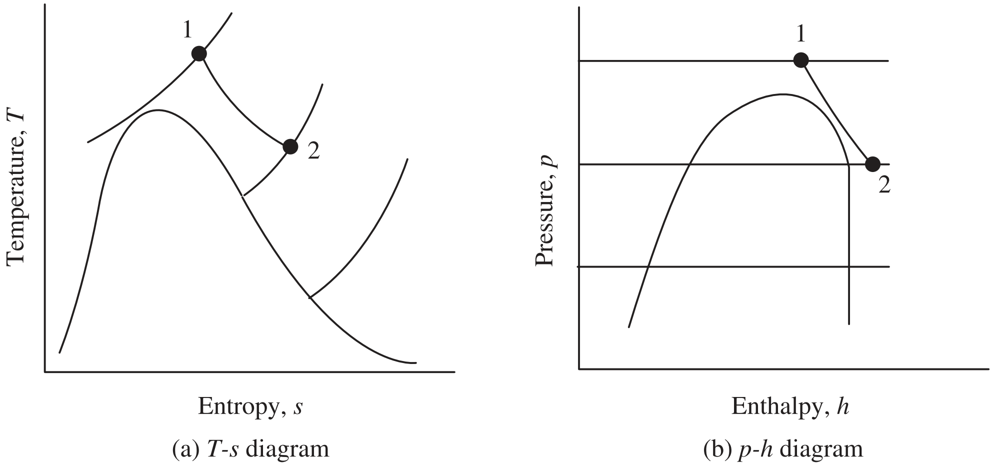
Virial equation of state or virial expansions are only applicable to gases of low and medium densities. The virial equation of state of a substance is given by

$$p = \frac{RT}{V} + \frac{a(T)}{V^2} + \frac{b(T)}{V^3} + \frac{c(T)}{V^4} + \frac{d(T)}{V^5} + \ldots \tag{10.44}$$

The coefficient of $a(T), b(T), c(T), d(T)$ are virial coefficients. The virial coefficient will vanish when the pressure becomes zero. Then, the equation of state reduces to the ideal gas equation.

10.8.1 Enthalpy and Entropy of Real Gases

The generalised compressibility factors for an ideal gas can be approximated to enthalpy and entropy changes, Δh and Δs, respectively.



(a) T-s diagram

(b) p-h diagram

Fig. 10.6 Entropy and enthalpy diagram

Δh and Δs can be written as:

$$\Delta h = \int c_p dT - \int \left(T \left(\frac{\partial V}{\partial T} \right)_p - v \right) dp \tag{10.45}$$

Entropy of any real gas is given by:

$$\Delta s = \int c_p \frac{dT}{T} - \int \left(\frac{\partial V}{\partial T} \right)_p dp \tag{10.46}$$

10.8.2 Compressibility Factor

The perfect gas equation is given by

$$pv = RT \tag{10.47}$$

As already explained, for real gas, a correction factor has to be introduced in the ideal gas equation to take into account the deviation of the real gas from the ideal gas equation. This factor is known as compressibility factor (Z) and is given by:

$$Z = \frac{pv}{RT} \tag{10.48}$$

The general compressibility chart is plotted with compressibility factor (Z) versus reduced pressure p_r for various values of reduced temperature (T_r). This is done by plotting the known data of one or more gases and can be used for any gas. The equation of state for real gas at any state becomes $pv = ZRT$. Similarly, the equation of state for the same real gas at critical point becomes

$$p_c v_c = Z_c R T_c \tag{10.49}$$

The general compressibility chart is plotted with compressibility factor (Z) versus reduced pressured, p_r for various values of reduced temperature, T_r. This is constructed by plotting the known data of one or more gases and can be used for any gas as shown in Fig.10.7.

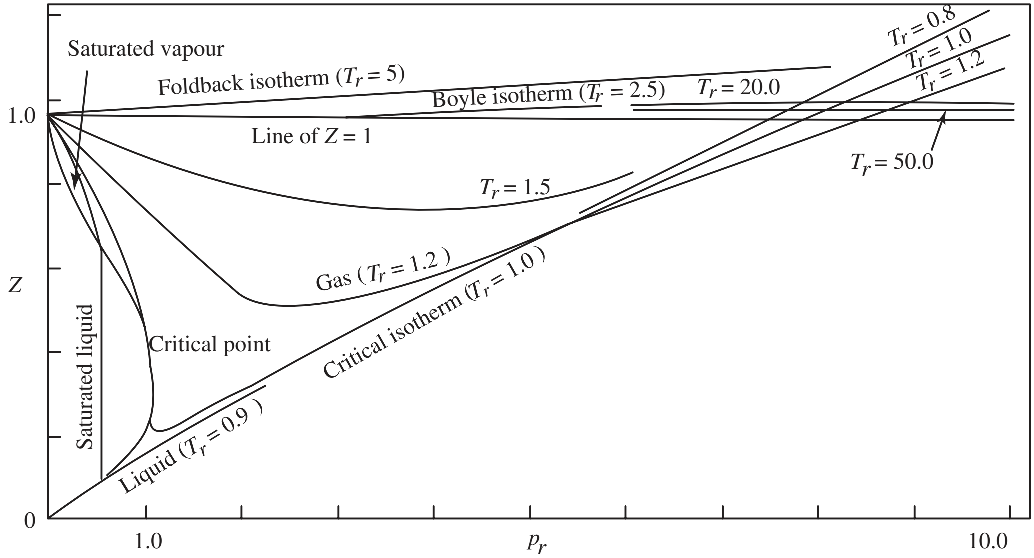


Fig. 10.7 Compressibility chart

The following observations can be made from the generalised compressibility chart:

(i) At very low pressures ($pr \ll 1$), the gases behave as ideal gas.

(ii) At high temperatures ($T_r > 2$), ideal gas behaviour can be assumed with good accuracy of pressures.

(iii) The deviation of a gas from ideal gas behaviour is the greatest in the vicinity of critical point.

For different gases, the behaviour changes on the basis of thermal properties like reduced pressure and reduced temperature can be studied.

10.9 GAS MIXTURES

In many engineering applications, it becomes important to know the composition of the mixture as well as the properties of the individual components to determine the properties of mixture. In general, there are two ways to describe the composition of a mixture of gases.

10.9.1 Mass Fraction

If a gas mixture consists of gases say 1, 2, 3 and so on, the mass of the mixture is the sum of the masses of the component gases. Mathematically,

$$m_m = m_1 + m_2 + m_3 + \ldots m_i = \sum_{i=1}^{n} m_i \qquad (10.50)$$

The *mass fraction* of any component is defined as the ratio of mass of a component to the mass of the mixture mathematically:

$$x_i = \frac{m_i}{m_m} \qquad (10.51)$$

where m_i is the mass of individual component and m_m is the mass of the mixture.

10.9.2 Molar Fraction

It is the ratio of the mole number of a component to the mole number of the mixture. The total number of moles of a mixture is the sum of the number of its components.

$$N_m = N_1 + N_2 + N_3 + \ldots N_i = \sum_{i=1}^{K} N_i \qquad (10.52)$$

Then, the mole fraction is given by:

$$y_i = \frac{N_i}{N_m} \qquad (10.53)$$

where N_i is the mole number of individual component and N_m is the mole number of the mixture.

In molar analysis, moles of each component are specified. The number of moles N, the mass m and the molar mass M of a component and mixture is related by

$$m_i = N_i M_i \qquad (10.54)$$

$$m_m = N_m M_m \qquad (10.55)$$

From Eqs 10.50, 10.54 and 10.55,

$$m_m = \sum m_i = \sum N_i m_i \qquad (10.56)$$

and

$$M_m = \frac{m_m}{N_m} = \frac{\sum N_i M_i}{N_m} = \sum y_i M_i \qquad (10.57)$$

$$y_i = x_i \frac{M_m}{M_i} \qquad (10.58)$$

10.10 PARTIAL PRESSURE AND PARTIAL VOLUME

If the sum of the partial pressures of the components of a gas mixture is equal to the mixture pressure, the partial pressure, p_i of a component in a gas mixture is given by

$$p_i = y_i p_m \tag{10.59}$$

where y_i = mole fraction p_m = mixture pressure

$$\sum p_i = \sum y_i p_m = p_m \sum y_i = p_m \tag{10.60}$$

This relation applies to any gas mixtures, whether it is ideal or not.

The sum of *partial volumes* of the components of a gas mixture is equal to the volume of the mixture. The partial volume V_i of a component i in a gas mixture is given by:

$$V_i = y_i V_m \tag{10.61}$$

where V_m = volume of mixture fraction

$$\sum V_i = \sum y_i V_m = V_m \sum y_i = V_m \tag{10.62}$$

The partial volume is not the actual volume of a component as it exists in the mixture because each component fills the entire volume of the vessel that holds the mixture.

10.10.1 Dalton's Law of Partial Pressure

According to Dalton's law of partial pressure, the total pressure of a non-reacting gas mixture is equal to the sum of pressures of its each components. This law is also called *Dalton's law of additive pressure*. This empirical law was postulated by John Dalton in 1801 and is related to ideal gas laws.

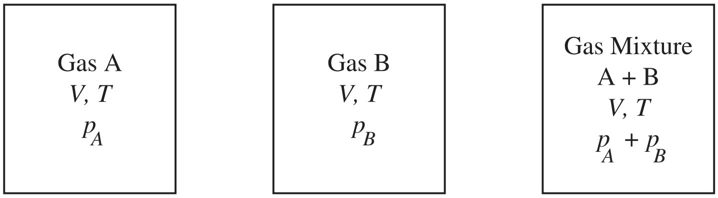


Fig. 10.8 Daltons law of partial pressure

$$p_m = p_1 + p_2 + p_3 + \cdots + p_k = \sum_{i=1}^{k} p_i \tag{10.63}$$

where p_m is pressure of the mixture and $p_1, p_2, p_3 \ldots p_k$ are pressures of each of the components.

If there are N_A moles of gas A, N_B moles of gas B and N_C moles of gas C in the mixture, the gas equation is given by:

$$p_m V_m = (N_A + N_B + N_C)\,\overline{R} T_m \tag{10.64}$$

where $\overline{R} = 8.314$ kJ/kg mole K.

$$\begin{aligned} p_m &= \frac{N_A\overline{R}T_m}{V_m} + \frac{N_B\overline{R}T_m}{V_m} + \frac{N_C\overline{R}T_m}{V_m} \\ &= p_A\left(N_A, T_m, V_m\right) + p_B\left(N_B, T_m, V_m\right) + p_C\left(N_C, T_m, V_m\right) \qquad (10.65) \end{aligned}$$

where $p_A\left(N_A, T_m, V_m\right)$ is the pressure of N_A moles of component A at the temperature T_m and volume V_m.

For ideal gas, p_i and V_i can be related to y_i by using the ideal gas relation for both the components and gas mixture:

$$\frac{p_i\left(T_m, V_m\right)}{p_m} = \frac{\dfrac{N_i\overline{R}T_m}{V_m}}{\dfrac{N_m\overline{R}T_m}{V_m}} = \frac{N_i}{N_m} = y_i \qquad (10.66)$$

For a real gas,

$$p_m V_m = Z_m N_m \overline{R} T_m \qquad (10.67)$$

where Z_m is the compressibility factor for the mixture. Z_m can be expressed in terms of compressibility factors for the individual gases:

$$Z_m = \sum_{i=1}^{k} y_i Z_i \qquad (10.68)$$

where Z_i is taken at T_m and V_m.

10.11 AMAGAT'S LAW OF PARTIAL VOLUME

According to Amagat's law of partial volumes, the volume of a gas mixture is equal to the sum of the volumes of each gas.

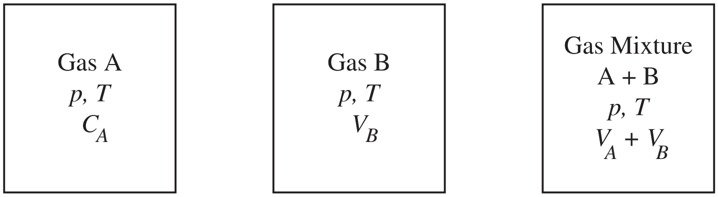


Fig. 10.9 Amagat's law of partial volume

$$V_m = V_1 + V + V_3 + \cdots + V_i = \sum_{i=1}^{k} V_i \qquad (10.69)$$

where V_m is volume of mixture and $V_1, V_2, \ldots V_i$ is volume of each component in mixture.

If there are N_A, N_B and N_C moles of gases A, B and C, respectively in the mixture, the gas equation can be written as:

$$p_m V_m = (N_A + N_B + N_C)\,\overline{R}T_m \tag{10.70}$$

$$V_m = \frac{N_A\overline{R}T_m}{p_m} + \frac{N_B\overline{R}T_m}{p_m} + \frac{N_C\overline{R}T_m}{p_m} \tag{10.71}$$

$$V_m \quad V_A(N_A, T_m, p_m) + V_B(N_B, T_m, p_m) + V_C(N_C, T_m, p_m) \tag{10.72}$$

where $V_A(N_A, T_m, p_m)$ is the volume of N_A moles of component A as the temperature, T_m and pressure, p_m.

10.12 PROPERTIES OF GAS MIXTURES

In this section, we will discuss the various properties of gas mixtures such as pressure, volume, temperature, internal energy, enthalpy, entropy and special heats.

10.12.1 Pressure

According to Dalton's law of partial pressure, the pressure of a mixture of gases is the sum of the pressures of its each component at the temperature and volume of the mixture.

10.12.2 Volume

Using Amagat's law of partial volumes, the volume of a mixture is the sum of volume of its each component at the temperature-pressure of the mixture.

10.12.3 Temperature

For any uniform mixture, the temperature is the same for component of the mixture:

$$T_m = T_A = T_B = T_C \tag{10.73}$$

10.12.4 Internal Energy

For a mixture of gases, the internal energy is given by

$$\begin{aligned} U_m &= U_A(N_A, T_m) + U_B(N_B, T_m) + U_C(N_C, T_m) \\ &= \sum_{i=1}^{k} U_i = \sum_{i=1}^{k} m_i u_i = \sum_{i=1}^{k} N_i \overline{u}_i \end{aligned} \tag{10.74}$$

For unit mass,

$$\begin{aligned} u_m &= \frac{U_m}{m_m} = \frac{U_A + U_B + U_C}{m_m} = \frac{m_A u_A + m_B u_B + m_C u_C}{m_m} \\ &= \frac{m_A}{m_m}u_A + \frac{m_B}{m_m}u_B + \frac{m_C}{m_m}u_C = x_A u_A + x_b u_B + x_C u_C \end{aligned} \tag{10.75}$$

$$U_m = \sum_{i=1}^{k} x_i u_i \tag{10.76}$$

where mass fraction, $x_i = \dfrac{m_i}{m_m}$.

10.12.5 Enthalpy

For a mixture of ideal gases, enthalpy is given by

$$\begin{aligned} H_m &= H_A\left(N_A, T_m\right) + H_B\left(N_B, T_m\right) + H_C\left(N_C, T_m\right) \\ &= \sum_{i=1}^{k} H_i = \sum_{i=1}^{k} m_i h_i = \sum_{i=1}^{k} N_i \overline{h}_i \qquad (10.77) \end{aligned}$$

$$\begin{aligned} h_m &= \frac{H_m}{m_m} = \frac{H_A + H_B + H_C}{m_m} = \frac{m_A h_A + m_B h_B + m_C h_C}{m_m} \\ &= \frac{m_A}{m_m} h_A + \frac{m_B}{m_m} h_B + \frac{m_C}{m_m} h_C = x_A h_A + x_b h_B + x_C h_C \qquad (10.78) \end{aligned}$$

$$h_m = \sum_{i=1}^{k} x_i h_i \qquad (10.79)$$

10.12.6 Entropy

According to Gibbs theorem, the total entropy of gases is the sum of the partial entropies:

$$\begin{aligned} S_m &= S_A\left(N_A, T_m, p_A\right) + S_B\left(N_B, T_m, p_B\right) + S_C\left(N_C, T_m, p_C\right) \\ &= \sum_{i=1}^{k} S_i = \sum_{i=1}^{k} m_i \overline{s}_i = \sum_{i=1}^{k} N_i \overline{s}_i \qquad (10.80) \end{aligned}$$

Entropy per unit mass is given by:

$$\begin{aligned} s_m &= \frac{S_m}{m_m} = \frac{S_A + S_B + \quad S_C}{m_m} = \frac{m_A s_A + m_B s_B + m_C s_C}{m_m} \qquad (10.81) \\ &= \frac{m_A}{m_m} s_A + \frac{m_B}{m_m} s_B + \frac{m_C}{m_m} s_C = x_A s_A + x_b s_B + x_C s_C \qquad (10.82) \end{aligned}$$

$$s_m = \sum_{i=1}^{k} x_i s_i \qquad (10.83)$$

10.12.7 Specific Heats

For a mixture of ideal gases, specific heats are given by

$$\begin{aligned} c_{V_m} &= \frac{m_A c_{V_A} + m_B c_{V_B} + m_C c_{V_C}}{m_m} \\ &= \frac{m_A}{m_m} c_{V_A} + \frac{m_B}{m_m} c_{V_b} + \frac{m_C}{m_m} c_{V_c} = x_A c_{V_A} + x_b c_{V_B} + x_C c_{V_C} \end{aligned}$$

$$c_{V_m} = \sum_{i=1}^{k} x_i c_{V_i} \qquad (10.84)$$

Similarly,

$$\begin{aligned} c_{P_m} &= \frac{m_A c_{P_A} + m_B c_{P_B} + m_C c_{P_C}}{m_m} \\ &= \frac{m_A}{m_m} c_{P_A} + \frac{m_B}{m_m} c_{P_B} + \frac{m_C}{m_m} c_{P_c} = x_A c_{P_A} + x_b c_{P_B} + x_C c_{P_C} \\ c_{P_m} &= \sum_{i=1}^{k} x_i c_{P_i} \qquad (10.85) \end{aligned}$$

10.12.8 Gas Constant

$$\begin{aligned} R_m &= \frac{m_A}{m_m} R_A + \frac{m_B}{m_m} R_B + \frac{m_C}{m_m} R_C \\ &= x_A c_{R_A} + x_b c_{R_B} + x_C c_{R_C} \qquad (10.86) \\ c_{P_m} &= \sum_{i=1}^{k} x_i R_i \qquad (10.87) \end{aligned}$$

Gas constant of the mixture can also be obtained from the equation:

$$R_m = \frac{\overline{R}}{M_m} \qquad (10.88)$$

where M_m is the molar mass of the mixture

10.13 MIXTURES OF REAL GASES

To analyse mixture of real gases, pVT relationships can be used. According to Dalton's law, the pressure of mixture of gases is equal to the sum of the pressures of individual gases. For example, if three gases are mixed, the mixture pressure becomes

$$p_m = p_A + p_B + p_C \qquad (10.89)$$

But pressure is the function of mole fraction, temperature and volume of gases. Therefore, Eq.10.89 can be written as:

$$p_m = p_A(N_A, T_m, p_m) + p_B(N_B, T_m, p_m) + p_C(N_C, T_m, p_m) \qquad (10.90)$$

where T_m and V_m are temperature and volume of mixture gas, respectively. N_A, N_B and N_C are the mole fraction of gas A, B and C, respectively. Similarly, volume can be written as:

$$V_m = V_A(N_A, T_m, p_m) + V_B(N_B, T_m, p_m) + V_C(N_C, T_m, p_m) \qquad (10.91)$$

Here, volume is the function of molar fraction, pressure and temperature. This is the second method based on Amagat law.

$$\text{Mole fraction, } y_i = \frac{\text{Molecular weight of individual gas}}{\text{Molecular weight of mixture}} = \frac{N_i}{N_m} \qquad (10.92)$$

Therefore,

$$\begin{aligned}\overline{V}_m &= \frac{V_m}{N_m} = \frac{V_A\left(N_m, p_m, T_m\right)}{N_m} + \ldots\ldots \\ &= \left(\frac{y_A V_A\left(N_m p_m, T_m\right)}{N_m}\right) + \ldots (\because U = mu) \\ &= y_A \overline{v}_A\left(p_m, T_m\right) + \ldots \left(\because \frac{v_A}{V_A} = \overline{v}_A\right) \qquad (10.93)\end{aligned}$$

We can use compressibility factor to analyse the mixture of real gases. It is given by:

$$p_m V_m = Z_m N_m \overline{R} T_m \qquad (10.94)$$

Determining Z_m is very difficult task. Therefore, it is approximated as

$$Z_m = y_a Z_A + y_A Z_B \qquad (10.95)$$

Similarly, enthalpy and entropy of a real gas mixture is calculated as

$$\text{Enthalpy, } \Delta\overline{h}_m = y_A \Delta\overline{h}_A + y_b \Delta\overline{h}_B + \ldots \qquad (10.96)$$

$$\text{Entropy, } \Delta\overline{S}_m = y_A \Delta\overline{S}_A + y_b \Delta\overline{S}_B + \ldots \qquad (10.97)$$

Worked-Out Examples

10.1 A room has the dimensions of 5 m×10× 4 m. If the pressure is 1 bar and temperature 27 °C, find the mass of air in the room.

Solution

$$\begin{aligned}\text{Volume of the room, } V &= 5 \times 10 \times 4 = 200 \text{ m}^3 \\ \text{Mass of air, } m &= \frac{pV}{RT} = \frac{101.325 \times 200}{0.287 \times 300} = 235.36 \text{ kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}\end{aligned}$$

10.2 Cooking gas is predominantly methane. Assume that it is contained in cylinder of 30 cm diameter and 75 cm in height. It is changed to 10 MPa at room temperature of 300 K.

(i) Find the mass of gas in the cylinder assuming it to be an ideal gas.

(ii) Explain how the actual cylinder can contain 15 kg of gas.

(iii) If the cylinder is to be protected against excessive pressure by means of a fusible plug, at what temperature, the plug should melt to limit the maximum pressure to 12.5 MPa.

Solution

Gas constant, R for CH_4:

$$R_{\mathrm{CH_4}} = \frac{8.314}{16} = 0.5196 \text{ kJ/kg K}$$

$$V = \frac{\pi}{4}d^2 \times h = \frac{\pi}{4} \times (0.3)^2 \times 0.75 = 0.053 \text{ m}^3$$

$$m = \frac{pV}{RT} = \frac{10000 \times 0.053}{0.5196 \times 300} = 3.4 \text{ kg}$$

Only 3.4 kg is in gaseous state. Remaining 11.6 kg will be in liquefied state.

$$T = \frac{pV}{mR} = \frac{12500 \times 0.053}{3.4 \times 0.5196} = 375 \text{ K} = 102\ ^\circ\text{C} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

10.3 A certain gas has $c_p = 0.91$ and $c_V = 0.65$ kJ/kg K. Find the molecular weight and the gas constant.

Solution

$$\text{Gas constant, } R = c_p - c_V = 0.91 - 0.65 = 0.26 \text{ kJ/kg K}$$

$$\text{Molecular weight, } M = \frac{\overline{R}}{R} = \frac{8.314}{0.26} = 31.98 \text{ kg/kmol} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

10.4 The specific heat ratio of acetylene (C_2H_2) was measured experimentally to be 1.25. Find c_p and c_V.

Solution

$$\gamma = 1.25; \quad \text{Molecular weight of } C_2H_2 = (2 \times 12) + 2 = 26 \text{ kg}$$

$$R = \frac{8.314}{0.26} = 0.3197$$

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.25 \times 0.3197}{1.25 - 1} = 1.5988 \text{ kJ/kg K} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$c_V = c_p - R = 1.5985 - 0.3197 = 1.28 \text{ kJ/kg K} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

10.5 Find the molar specific heats of monatomic, diatomic and polyatomic gases, if the ratio of their specific heats are $\frac{5}{3}; \frac{7}{5}$ and $\frac{4}{3}$, respectively.

Solution

$$\text{Monoatomic, } \gamma = \frac{5}{3} = 1.667$$

$$\overline{C}_p = \frac{\gamma \overline{R}}{\gamma - 1} = \frac{1.667 \times 8.3143}{1.667 - 1} = 20.778 \text{ kJ/kg K}$$

$$\overline{C}_V = \frac{\overline{R}}{\gamma - 1} = \frac{8.314}{1.667 - 1} = 12.464 \text{ kJ/kg K}$$

$$\text{Diatomic gas, } \gamma = \frac{7}{5} = 1.4$$

$$\overline{C}_p = \frac{1.4 \times 8.314}{0.4} = 29.1 \text{ kJ/kg mol K}$$

$$\overline{C}_V = \frac{8.3143}{0.4} = 20.786 \text{ kJ/kg mol K}$$

$$\text{Polyatomic gas, } \gamma = \frac{4}{3} = 1.33$$

$$\overline{C}_p = \frac{1.33 \times 8.314}{0.33} = 33.51 \text{ kJ/kg mol K}$$

$$\overline{C}_V = \frac{8.314}{0.33} = 25.19 \text{ kJ/kg mol K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.6 Natural gas at a pressure of 1.1 bar and −150 °C is pumped to height of 800 m above the storage level where the pressure is 1.2 bar and temperature is 15 °C. The pumping rate is 1000 m^3/h. Neglect KE. Assume gas properties of methane (CH_4) and consider it to be ideal gas. Take $\gamma = 1.33$ and $g = 9.75 \text{ m/s}^2$.

Solution

From the given data,

$p_1 = 110 \text{ kN/m}^2$ $\qquad p_2 = 120 \text{ kN/m}^2$

$T_1 = -150 + 273 = 123 \text{ K}$ $\qquad T_2 = 15 + 273 = 288 \text{ K}$

$Z_1 = 0$ $\qquad Z_2 = 800 \text{ m}$

$g = 9.7 \text{ m/s}^2$

$$v_2 = \frac{RT_2}{p_2} = \frac{8.314 \times 288}{16 \times 120} = 1.247 \text{ m}^3\text{/kg}$$

$$\dot{m} = \frac{1000}{1.247 \times 3600} = 0.223 \text{ kg/s}$$

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.33 \times 8.314}{16 \times 0.33} = 2.094 \text{ kJ/kg K}$$

Using SFEE,

$$\dot{m}h_1 + \dot{m}gZ_1 + \dot{Q} = \dot{m}h_2 + \dot{m}gZ_2 + \dot{W}$$

$$\dot{Q} = \dot{m}(h_2 - h_1) + \dot{m}g(Z_2 - Z_1) + \dot{W}$$

$$= 0.223 \times 2.094 \times (288 - 125) + 0.223 \times 9.75 \times 800 \times 10^{-3} - 15$$

$$= 62.855 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.7 A constant volume chamber of 0.5 m^3 capacity contains 1 kg of air at 7 °C. Heat is transferred to the air to raise the temperature to 100 °C. Find the work done, the heat transferred and the change in internal energy, enthalpy and entropy.

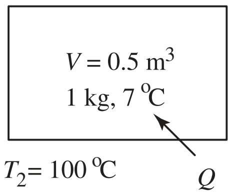


Fig. 10.10

Solution

$$Q - W = \Delta U$$

As $W = 0$,

$$\begin{aligned}
Q &= \Delta U = mc_V(T_2 - T_1) = 1 \times 0.718 \times (100 - 7) = 66.8 \text{ kJ} \\
\Delta U &= 66.8 \text{ kJ} \\
\Delta H &= mc_p(T_2 - T_1) = 1 + 1.005 \times 93 = 93.5 \text{ kJ} \\
\Delta S &= S_2 - S_1 = mc_V \ln\left(\frac{T_2}{T_1}\right) = 1 \times 0.718 \times \ln\left(\frac{373}{280}\right) = 0.206 \text{ kJ/K} \quad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

10.8 Two kilogram of air in a closed system, having initial volume and temperature of 0.5 m^3 and 7 °C, respectively, undergoes a constant pressure heating process to 100 °C. There is no work other than *pdv* work. Find (i) the work done during the process, (ii) the heat transferred and (iii) the entropy change.

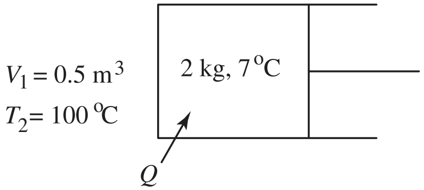


Fig. 10.11

Solution

$$\begin{aligned}
p_1 &= \frac{mRT_1}{V_1} = \frac{2 \times 0.287 \times 280}{0.5} = 321.44 \text{ kN/m}^2 = p_2 \\
V_2 &= \frac{mRT_2}{p_2} = \frac{2 \times 0.287 \times 373}{321.44} = 0.67 \text{ m}^3 \\
W_{1-2} &= pdv = p(V_2 - V_1) = 321.44(0.67 - 0.5) = 54.65 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow} \\
Q_{1-2} &= mc_p(T_2 - T_1) = 2 \times 1.005 \times 93 = 186.93 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow} \\
S_2 - S_1 &= mc_p \ln\left(\frac{T_2}{T_1}\right) = 2 \times 1.005 \times \ln\left(\frac{373}{280}\right) = 0.576 \text{ kJ/K} \quad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

10.9 0.2 m^3 of hydrogen initially at 2.4 MPa and 200 °C undergoes a reversible constant temperature expansion to 0.2 MPa. Find (i) the work done during the process; (ii) heat transferred and (iii) entropy change.

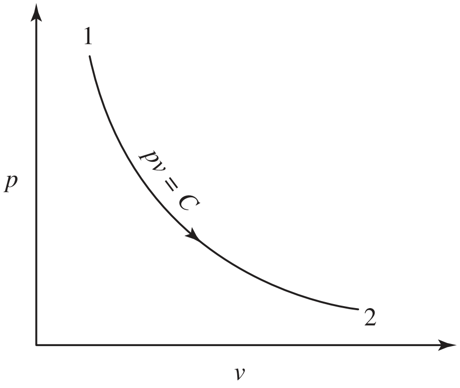


Fig. 10.12

Solution

The process undergoes a reversible isothermal expansion:

$$R = \frac{8.314}{2} = 4.157 \text{ kJ/kg K}$$

$$m = \frac{p_1V_1}{RT_1} = \frac{2400 \times 0.2}{4.157 \times 473} = 0.244 \text{ kg}$$

$$W_{1-2} = \int_1^2 pdV = \int_1^2 \frac{mRT}{V}dV = mRT\ln\left(\frac{V_2}{V_1}\right) = mRT\ln\left(\frac{p_1}{p_2}\right)$$

$$= 0.244 \times 4.157 \times 473 \times \ln\left(\frac{2.4}{0.2}\right) = 1192 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\delta Q = dU - \delta w$$

$$dU = mc_V(T_2 - T_1) = 0$$

$$Q_{1-2} = W_{1-2} = 1192 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$S_2 - S_1 = mR\ln\left(\frac{p_1}{p_2}\right) = 0.244 \times 4.157 \times \ln\left(\frac{2.4}{0.2}\right) = 2.52 \text{ kJ/K} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.10 Air in closed station systems expands in a reversible adiabatic process from 0.5 MPa, 17 °C to 0.2 MPa. Find the final temperature and also for unit mass of air, calculate the change in enthalpy, the heat transferred and the work done.

Solution

From given data, $p_1 = 0.5$ MPa; $T_1 = 290$ K; $p_2 = 0.2$ MPa:

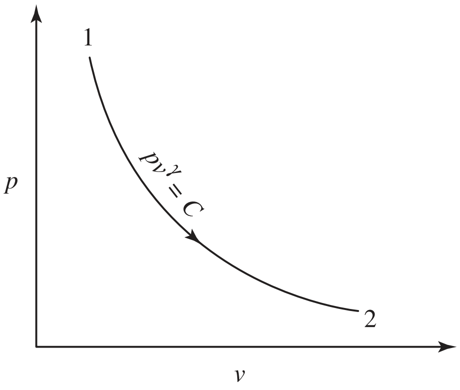


Fig. 10.13

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{0.2}{0.5}\right)^{0.286} = 0.7695$$

$$T_2 = T_1 \times 0.7695 = 290 \times 0.7695 = 223.15 \text{ K} = -49.845°\text{C} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$h_2 - h_1 = c_p(T_2 - T_1) = 1.005 \times (223.15 - 290) = -67.18 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$Q_{1-2} = 0 \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_{1-2} = \frac{p_1v_1 - p_2v_2}{\gamma - 1} = \frac{R(T_1 - T_2)}{\gamma - 1} = \frac{0.287 \times 67.18}{0.4} = 48.20 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.11 Two vessels A and B each having a volume of 5 m^3 is connected together by a tube of negligible volume. Vessel A contains air at 8 bar and 100 °C while B contains air at 4 bar and 200 °C. Find the change of entropy when A is connected to B. Assume perfect mixing under adiabatic condition.

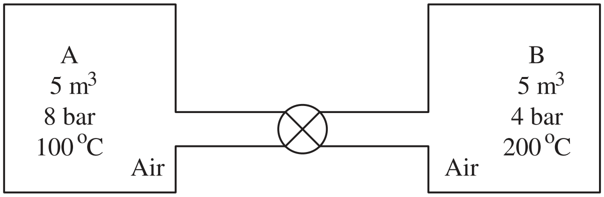


Fig. 10.14

Solution

Refer Fig.10.14.

$$T_A = 100 + 273 = 373$$

$$T_B = 200 + 273 = 473$$

$$m_A = \frac{p_AV_A}{RT_A} = \frac{8 \times 100 \times 5}{0.287 \times 373} = 37.37 \text{ kg}$$

$$m_B = \frac{p_B V_B}{RT_B} = \frac{4 \times 100 \times 5}{0.287 \times 473} = 14.73 \text{ kg}$$

$$m_A c_V (T - T_A) = m_B c_V (T_B - T)$$

$$T = \frac{m_A T_A + m_B T_B}{m_A + m_B} = \frac{37.37 \times 373 + 14.73 \times 473}{37.37 + 14.73} = 401.3 \text{ K}$$

$$\Delta S_A = m_A \left[c_V \times \ln\left(\frac{T}{T_A}\right) + R \ln\left(\frac{V}{V_A}\right)\right]$$

$$= 37.37 \times \left[0.718 \times \ln\left(\frac{401.3}{373}\right) + 0.287 \times \ln 2\right] = 9.4 \text{ kJ/K}$$

$$\Delta S_B = 14.73 \times \left[0.718 \times \ln \frac{401.3}{473} + 0.287 \times \ln 2\right] = 1.19 \text{ kJ/K}$$

Total entropy change:

$$\Delta S = \Delta S_A + \Delta S_B = 9.4 + 1.19 = 10.59 \text{ kJ/K} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.12 An ideal gas at temperature T_1 is heated at constant pressure to T_2 and then expanded reversibly, according to the law, pv^n = constant, until the temperature is once again T_1. What is the required value of n if the changes of entropy during the separate processes are equal?

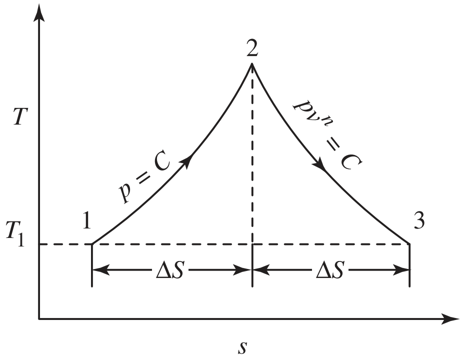


Fig. 10.15

Solution

Refer Fig.10.15.

$$S_2 - S_1 = c_p \ln\left(\frac{T_2}{T_1}\right)$$

$$S_3 - S_2 = \frac{n - \gamma}{(\gamma - 1)(n - 1)} R \ln\left(\frac{T_3}{T_2}\right)$$

$$S_2 - S_1 = S_3 - S_2$$

$$c_p \ln\left(\frac{T_2}{T_1}\right) = \frac{n-\gamma}{(\gamma-1)(n-1)} R \ln\left(\frac{T_3}{T_2}\right) = -\frac{n-\gamma}{(\gamma-1)(n-1)} R \ln\left(\frac{T_2}{T_3}\right)$$

Since $T_3 = T_1$:

$$\frac{c_p}{R} = -\frac{n-\gamma}{(\gamma-1)(n-1)} = \frac{\gamma}{\gamma-1}$$

$$-\frac{n-\gamma}{n-1} = \gamma$$

$$-n+\gamma = \gamma n - \gamma$$

$$2\gamma = (\gamma+1)n$$

$$n = \frac{2\gamma}{\gamma+1} \qquad \xLeftarrow{\textbf{Ans}}$$

10.13 A cylinder contains amount of sulphur dioxide (SO_2). The volume of the cylinder is 0.15 m^3. The pressure and temperature are 23 bar and 17 °C, respectively. A valve is opened momentarily because of which the pressure falls to 7 bar. After sometime, the pressure and temperature were found to be 9 bar and 17 °C, respectively. Calculate the value of γ of SO_2.

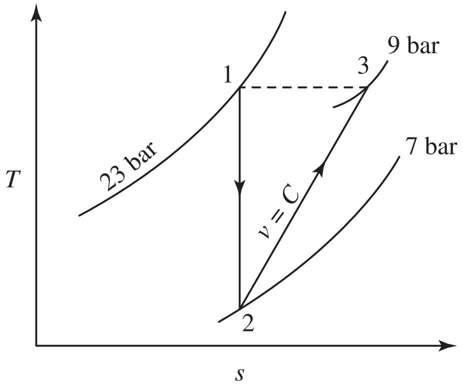


Fig. 10.16

Solution

From given data,

$$T_1 = 17 + 273 = 290 \text{ K}$$

$$v_2 = v_3$$

$$\frac{p_2 v_2}{T_2} = \frac{p_3 v_3}{T_3}$$

$$\therefore \frac{T_2}{T_3} = \frac{p_2}{p_3} = \frac{T_2}{T_1} \quad (\because T_3 = T_1)$$

When the valve is opened momentarily, the pressure drops from 23 bar to 7 bar. Assume the process to be isentropic (1-2). In course of time, the gas comes to equilibrium temperature of 290 K which is a constant volume process (2-3).

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{p_2}{p_3}$$

$$\therefore \frac{\gamma-1}{\gamma} \times \ln\left(\frac{p_2}{p_1}\right) = \ln\left(\frac{p_2}{p_3}\right)$$

$$\left(1-\frac{1}{\gamma}\right) \times \ln\left(\frac{7}{23}\right) = \ln\left(\frac{7}{9}\right)$$

$$\left(1-\frac{1}{\gamma}\right)(-1.19) = -0.25$$

$$1-\frac{1}{\gamma} = \frac{-0.25}{-1.19} = 0.21$$

$$\frac{1}{\gamma} = 1-0.21 = 0.79$$

$$\gamma = \frac{1}{0.79} = 1.27 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.14 A gaseous mixture contains 21% by volume of nitrogen, 50% by volume of hydrogen and 29% by volume of carbon dioxide. Calculate the molecular weight of the mixture, the characteristic gas constant R for the mixture and the value of the reversible adiabatic index γ. Take c_p of nitrogen, hydrogen and carbon monoxide as 1.039, 14.235 and 0.828 kJ/kg K, respectively at 17 °C.

A cylinder contains 0.09 m^3 of the mixture at 1 bar an 17 °C. The gas undergoes a reversible non-flow process during which its volume is reduced to 1/5th of its original volume. Take the law of compression as $pv^{1.2} = c$. Estimate the work and heat transfer in magnitude and sense and the change in entropy.

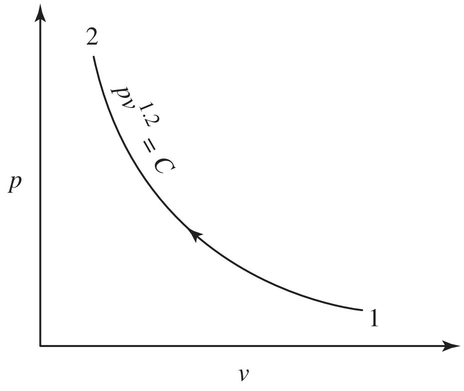


Fig. 10.17

Solution

$$x_{\mathrm{N}_2} = 0.21; \quad x_{H2} = 0.50; \quad x_{\mathrm{CO}_2} = 0.29$$

Molecular weight of the mixture,

$$\begin{aligned} \mu &= x_1\mu_1 + x_2\mu_2 + x_3\mu_3 \\ &= 0.21 \times 28 + 0.5 \times 2 + 0.29 \times 44 = 19.64 \text{ kJ/kg mol} \end{aligned}$$

Gas constant of the mixture,

$$\begin{aligned} R &= \frac{\overline{R}}{\mu} = \frac{8.3143}{19.64} = 0.4233 \text{ kJ/kg K} \\ c_p &= \frac{m_1c_{p1} + m_2c_{p2} + m_3c_{p3}}{m_1 + m_2 + m_3} \\ &= \frac{0.21 \times 28 \times 1.039 + 0.5 \times 2 \times 14.235 + 0.29 \times 0.828 \times 44}{19.64} \\ &= \frac{30.903}{19.64} = 1.5738 \text{ kJ/kg K} \\ c_V &= c_p - R = 1.5738 - 0.4233 = 1.1505 \text{ kJ/kg K} \\ \therefore \gamma &= \frac{1.5738}{1.1505} = 1.368 \\ p_1 &= 100 \text{ kPa}; \quad T_1 = 290 \text{ K} \\ \frac{V_2}{V_1} &= \frac{1}{5} = 0.2 \\ W_{1-2} &= \frac{p_1v_1 - p_2v_2}{n-1} = \frac{p_1V_1 - p_2V_2}{n-1} = p_1V_1\left[\frac{1 - \left(\frac{V_2}{V_1}\right)^{1-n}}{n-1}\right] \\ &= \frac{100 \times 0.09 \times \left(1 - 0.2^{0.2}\right)}{1.2 - 1} = 12.38 \text{ kJ} \end{aligned}$$

The work is done on the system:

$$\begin{aligned} \frac{T_2}{T_1} &= \left(\frac{V_1}{V_2}\right)^{n-1} = 5^{0.2} \\ T_2 &= 400.1 \text{ K} \\ m &= \frac{pV}{RT} = \frac{100 \times 0.09}{0.4233 \times 290} = 0.073 \text{ kg} \\ Q_{1-2} &= U_2 - U_1 + W_{1-2} = mc_V(T_2 - T_1) + W_{1-2} \end{aligned}$$

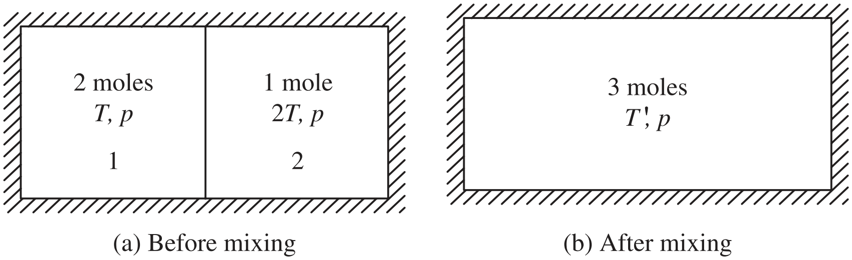
$$= \quad 0.073 \times 1.1505 \times (400.1 - 290) - 12.38 = -7.83 \text{ kJ}$$

Negative sign indicates heat flows out of the system.

$$s_2 - s_1 = -\frac{n-\gamma}{\gamma-1} R \ln\left(\frac{V_2}{V_1}\right) = -\frac{1.2 - 1.368}{1.368 - 1} \times 0.4233 \times \ln(0.2)$$

$$= -0.311 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.15 Two mole of an ideal gas at temperature T and pressure p are contained in a compartment. In an adjacent compartment is one mole of an ideal gas at temperature $2T$ and pressure p. The gases mix adiabatically but do not react chemically when a partition separating the compartment is withdrawn. Show that the entropy increase due to mixing process is given by $R\left[\ln\left(\frac{27}{4}\right) + \frac{\gamma}{\gamma-1}\ln\left(\frac{64}{27}\right)\right]$. Provided that the gases are different and that the ratio of specific heat γ is the same for both gases and remains constant. What would the entropy change be if the mixing gases were of the same species?



(a) Before mixing

(b) After mixing

Fig. 10.18

Solution

Refer Fig.10.18.

After mixing, let the final temperature be T'. The final pressure remains at p. The partial pressures of the gases 1 and 2 in (b) after mixing are:

$$p_1 = x_1 p = \frac{2}{3} p$$

$$p_2 = x_2 p = \frac{1}{3} p$$

By energy balance,

$$2\overline{C}_p(T - T') = 1\overline{C}_p(T' - 2T)$$

$$\therefore 2T - 2T' = T' - 2T$$

$$T' = \frac{4T}{3}$$

The entropy change of gas 1 is:

$$\Delta S_1 = n_1\overline{C}_p \ln\left(\frac{T'}{T}\right) - n_1\overline{R}\ln\left(\frac{p_1}{p}\right) = 2\overline{C}_p \ln\left(\frac{4}{3}\right) - 2\overline{R}\ln\left(\frac{2}{3}\right)$$

The entropy change of gas 2 is:

$$\Delta S_2 = n_2\overline{C}_p \ln\left(\frac{T'}{T}\right) - n_2\overline{R}\ln\left(\frac{p_2}{p}\right) = 1\overline{C}_p \ln\left(\frac{4}{3}\right) - 1\overline{R}\ln\left(\frac{1}{3}\right)$$

Total entropy change is:

$$\begin{aligned}
\Delta S &= \Delta S_1 + \Delta S_2 \\
&= 2\overline{C}_p \ln\left(\frac{4}{3}\right) - 2\overline{R}\ln\left(\frac{2}{3}\right) + c_n \ln\left(\frac{4}{3}\right) - \overline{R}\ln\left(\frac{1}{3}\right) \\
&= \overline{C}_p \ln\left(\frac{16}{9}\times\frac{4}{3}\right) - \overline{R}\ln\left(\frac{4}{9}\times\frac{1}{3}\right) \\
&= \overline{C}_p \ln\left(\frac{64}{27}\right) - \overline{R}\ln\left(\frac{4}{27}\right) \\
&= \overline{R}\left[\ln\left(\frac{27}{4}\right) + \frac{\gamma}{\gamma-1}\times\ln\left(\frac{64}{27}\right)\right] \qquad \xleftarrow{\textbf{Ans}}
\end{aligned}$$

If the gases were of the same species, the entropy change is zero.

10.16 n_1 moles of an ideal gas at pressure p_1 and temperature T are in one component of an insulated container. In an adjoining compartment, separated by a partition, are n_2 moles of an ideal gas at pressure, p_2 and temperature, T. When the partition is removed, find

(i) the final pressure of the mixture,
(ii) the entropy change when the gases are identical and
(iii) the entropy change when the gases are different.

Prove that the entropy change in (iii) is the same as that produced by two independent free expansion.

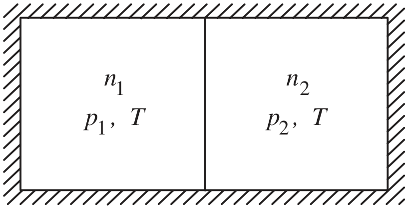


Fig. 10.19

Solution

Refer Fig.10.19.

$$n = n_1 + n_2$$

$$p_1V_1 = n_1\overline{R}T; \quad p_2V_2 = n_2\overline{R}T$$

$$V = V_1 + V_2 = \left(\frac{n_1}{p_1} + \frac{n_2}{p_2}\right)\overline{R}T$$

After mixing, $pV = n\overline{R}T$

$$p\left(\frac{n_1}{p_1} + \frac{n_2}{p_2}\right)\overline{R}T = (n_1 + n_2)\overline{R}T$$

Therefore, final pressure, $p = \dfrac{n_1 + n_2}{\frac{n_1}{p_1} + \frac{n_2}{p_2}}$

$$Tds = dH - Vdp = -Vdp$$

$$ds = -\frac{n\overline{R}dp}{p}$$

$$\therefore S_2 - S_1 = -\overline{R}\left[n_1 \ln\left(\frac{p}{p_1}\right) + n_2 \times \ln\left(\frac{p}{p_2}\right)\right] \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

where p is calculated using the expression given above.

If the gases are of the same species, there is no diffusion and there is no increase in disorder or entropy. Entropy change would be the same as that produced by two independent free expansions from total pressure p to the respective partial pressures p_1 and p_2.

10.17 An adiabatic air turbine is found to expand air as per the law, $pv^{1.25} = c$. If KE and PE are neglected, show that shaft work per unit mass as a function of pressure is given by the following relation:

$$W = 3.5\ p_1v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{3.5}}\right]$$

Take γ for air 1.4.

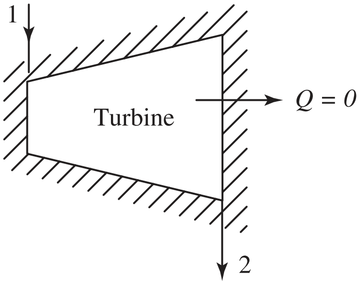


Fig. 10.20

Solution

Refer Fig.10.20. As per SFEE,

$$\begin{aligned}
Q - W_x - \Delta\left[\frac{\mathbb{V}^2}{2} + gz\right] &= h_2 - h_1 \\
-W_x &= c_p(T_2 - T_1) = \frac{\gamma R T_1}{\gamma - 1}\left(\frac{T_2}{T_1} - 1\right) \\
W_x &= \frac{1.4\; p_1 v_1}{0.4}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}}\right] = 3.5\; p_1 v_1\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{1}{3.5}}\right]
\end{aligned}$$

Ans ⟸

10.18 A mass of an ideal gas exists initially at a pressure 250 kPa, temperature 27 °C and specific volume of 0.45 m^3/kg. The value of γ is 1.4. Determine:

(i) Specific heats of the gas.

(ii) What is the change in entropy when the gas is expanded to 150 kPa according to the law $pv^{1.3} = c$?

(iii) Evaluate the entropy change if expansion law is $pv^{1.5} = c$ (by the application of a cooling jacket during the process).

(iv) What is the inference you would draw from this example?

Solution

From the given data,

$p_1 = 250$ kPa $\quad v_1 = 0.45$ m^3/kg

$T_1 = 300$ K $\quad \gamma = 1.4$

$$R = \frac{p_1 v_1}{T_1} = \frac{250 \times 0.45}{300} = 0.375 \text{ kJ/kg K}$$

$$c_p = \frac{\gamma R}{\gamma - 1} = \frac{1.4 \times 0.375}{0.4} = 1.3125 \text{ kJ/kg K}$$

Ans ⟸

$$c_V = \frac{c_p}{\gamma} = \frac{1.3125}{1.4} = 0.9375 \text{ kJ/kg K}$$

Ans ⟸

$$\begin{aligned}
s_2 - s_1 &= \frac{n - \gamma}{n(\gamma - 1)} R \ln\left(\frac{p_2}{p_1}\right) \\
&= \frac{1.3 - 1.4}{1.3 \times 0.4} \times 0.375 \times \ln\left(\frac{150}{250}\right) = 0.03683 \text{ kJ/kg K}
\end{aligned}$$

Ans ⟸

When $n = 1.5$,

$$s_2 - s_1 = \frac{1.5 - 1.4}{1.5 \times 0.4} \times 0.375 \times \ln\left(\frac{150}{250}\right) = -0.031 \text{ kJ/kg K}$$

Ans ⟸

When $n < \gamma$ entropy change is positive and entropy decreases whereas when $n > \gamma$ entropy change is negative and entropy decreases. **Ans** ⟸

10.19 One kilogram of air is at a pressure of 5 atm and at a temperature of 227 °C. It expands reversibly to a pressure of 2 atm following the law, $pV^{1.3} = c$. Assuming air is an ideal gas calculate:

(i) the work done and heat transferred.

(ii) If the system does the same expansion in a steady flow process. What is the work done by the system?

Solution

From the given data:

$$m = 1 \text{ kg} \qquad p_2 = 2 \text{ atm}$$
$$p_1 = 5 \text{ atm} \qquad T_1 = 500 \text{ K}$$

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{2}{5}\right)^{\frac{0.30}{1.30}} = 0.809$$

$$T_2 = 0.809 \times 500 = 404.5 \text{ K}$$

$$W = \frac{mRT_1}{n-1}\left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right] = \frac{1 \times 0.287 \times 500}{1.3 - 1}[1 - 0.809] = 91.36 \text{ kJ}$$

$$Q = U_2 - U_1 + W = 1 \times 0.718 \times (404.5 - 500) + 91.36 = 22.791 \text{ kJ}$$

For a steady flow system:

$$W = \frac{n}{n-1} mRT_1 \left[1 - \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

As we have already calculated, the value of W without n:

$$W_{SF} = nW = 1.3 \times 91.36 = 118.8 \text{ kJ}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

10.20 A cylinder with a piston contains air. It is compressed isentropically, the mass of air in the cylinder is 0.5 kg. The initial pressure is 1 bar and the temperature is 27 °C. The final volume is $\frac{1}{10}$th of the initial volume. Find the work done and heat transfer.

Solution

From the given data:

$$m = 0.5 \text{ kg} \qquad T_1 = 300 \text{ K}$$
$$p_1 = 1 \text{ bar} = 100 \text{ kPa} \qquad \frac{V_2}{V_1} = \frac{1}{10}$$

$$\begin{aligned}
pv^{1.4} &= c \\
\frac{p_2}{p_1} &= \left(\frac{v_1}{v_2}\right)^{\gamma} = 10^{1.4} = 25.12 \\
\frac{T_2}{T_1} &= \left(\frac{v_1}{v_2}\right)^{\gamma-1} = 10^{0.4} = 2.51 \\
\therefore T_2 &= 2.51 \times 300 = 753 \text{ K} \\
W_c &= \frac{mR(T_1 - T_2)}{\gamma - 1} = \frac{0.5 \times 0.287(300 - 753)}{0.4} = -162.5 \text{ kJ}
\end{aligned}$$

As it is a reversible adiabatic process, $Q = 0$.

10.21 Air is contained in a cylinder fitted with frictionless piston. Initially, the cylinder contains 0.5 m^3 of air at 2 bar and 27 °C. The air is then compressed reversibly according to the law, $pv^n = c$, until the final pressure is 8 bar at which point temperature is 137 °C. Calculate (i) the polytropic index n; (ii) the final volume of air; (iii) the work done on air; (iv) the heat transfer; and (v) the change in entropy.

Solution

From the given data:

$p_1 = 2$ bar $\qquad p_2 = 8$ bar

$T_1 = 300$ K $\qquad T_2 = 410$ K

$V_1 = 0.5$ m^3

$$\begin{aligned}
\frac{T_2}{T_1} &= \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} \\
\ln\left(\frac{T_2}{T_1}\right) &= \frac{n-1}{n} \times \ln\left(\frac{p_2}{p_1}\right) \\
\ln\left(\frac{410}{300}\right) &= \left(1 - \frac{1}{n}\right) \times \ln\left(\frac{8}{2}\right) \\
0.312 &= \left(1 - \frac{1}{n}\right) \times 1.386 \\
1 - \frac{1}{n} &= \frac{0.312}{1.386} = 0.225 \\
\frac{1}{n} &= 1 - 0.225 = 0.775 \\
n &= \frac{1}{0.775} = 1.29
\end{aligned}$$

$$\frac{V_2}{V_1} = \left(\frac{p_1}{p_2}\right)^{\frac{1}{n}} = \left(\frac{2}{8}\right)^{\frac{1}{n}} = 0.25^{\frac{1}{1.29}} = 0.341$$

$$V_2 = 0.5 \times 0.341 = 0.1705 \text{ m}^3$$

$$W = \frac{p_1V_1}{n-1}\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}\right] = \frac{200\times 0.5}{0.29}\left[1-4^{\left(\frac{0.29}{1.29}\right)}\right] = -126.1 \text{ kJ/kg}$$

$$Q = mc_V(T_2-T_1)+W$$

$$m = \frac{pV}{RT_1} = \frac{200\times 0.5}{0.287\times 300} = 1.161 \text{ kg}$$

$$Q = 1.161\times 0.717\times(410-300)-126.1 = -34.53 \text{ kJ}$$

$$s_2-s_1 = \frac{n-\gamma}{(\gamma-1)(n-1)}R\ln\left(\frac{T_2}{T_1}\right)$$

$$= \frac{1.29-1.4}{0.4\times 0.29}\times 0.287\times\ln\left(\frac{410}{300}\right) = -0.085$$

$$S_2-S_1 = m(s_2-s_1) = 1.161\times(-0.085) = -0.099 \text{ kJ/K}$$ **Ans** ⟸

10.22 The specific heat at constant pressure for air is given by $c_p = 0.9169 + 2.577\times 10^{-4}T - 3.974\times 10^{-8}T^2$ kJ/kg K. Calculate the change in internal energy and the entropy of air when it undergoes a change of state from 1 atm and 300 K to a temperature of 2000 K at the same pressure.

Solution

$$c_p = a + bT + cT^2$$

where $a = 0.9169; b = 2.577\times 10^{-4}; c = -3.974\times 10^{-8}$

$$h_2-h_1 = \int_{T_1}^{T_2} c_p dT = \int_{T_1}^{T_2}\left(a+bT+cT^2\right)dT$$

$$= a(T_2-T_1)+\frac{b}{2}\left(T_2^2-T_1^2\right)-\frac{c}{3}\left(T_2^3-T_1^3\right)$$

$$h_2-h_1 = 0.9169\times(2000-300)+\frac{2.577\times 10^{-4}}{2}\times\left(2000^2-300^2\right)- \frac{3.974\times 10^{-8}}{3}\times\left(2000^3-300^3\right)$$

$$= 1558.73+503.80-105.61 = 1956.9 \text{ kJ}$$

$$p_2v_2-p_1v_1 = R(T_2-T_1) = 0.287\times(2000-300) = 487.9$$

$$u_2-u_1 = (h_2-h_1)-(p_2v_2-p_1v_1)$$

$$= \quad 1956.92 - 487.9 = 1469 \text{ kJ/kg}$$

$$\int_1^2 ds \quad = \quad \int_1^2 c_p \frac{dT}{T} = \int_1^2 \left(a + bT + cT^2\right)\frac{dT}{T}$$

$$= \quad \int_{T_1}^{T_2} a\frac{dT}{T} + \int_{T_1}^{T_2} adT - \int_{T_1}^{T_2} cTdT$$

$$s_2 - s_1 \quad = \quad a \times \ln\left(\frac{T_2}{T_1}\right) + b(T_2 - T_1) - c\left(\frac{T_2^2 - T_1^2}{2}\right)$$

$$= 0.9169 \times \ln\left(\frac{2000}{300}\right) + 2.577 \times 10^{-4}(2000 - 300) - 3.974 \times 10^{-8}\left(2000^2 - 300^2\right)$$

$$= \quad 1.7395 + 0.4381 - 0.1554 \approx 2.02 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

10.23 Nitrogen is allowed to expand in a closed container from a volume of 0.3 m^3 to 0.9 m^3 along a path $pv^{1.33} = c$. The initial pressure of the gas in the container is 200 kPa and its initial temperature is 100 °C. Draw the p-V diagram and calculate (i) final temperature and pressure of the gas; (ii) work and heat transfer; and (iii) entropy change of nitrogen.

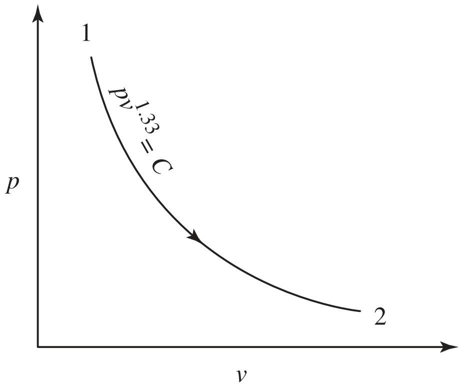


Fig. 10.21

Solution

From the given data:

$p_1 = 200$ kPa $\qquad T_1 = 373$ K
$V_1 = 0.3\ m^3 \qquad V_2 = 0.9\ m^3$

$$\frac{T_2}{T_1} \quad = \quad \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{1}{3}\right)^{0.33} = 0.696$$

$$T_2 \quad = \quad 373 \times 0.696 = 259.6 \text{ K}$$

$$\frac{p_2}{p_1} \quad = \quad \left(\frac{v_1}{v_2}\right)^{n} = \left(\frac{1}{3}\right)^{1.33} = 0.232$$

$$p_2 = 200 \times 0.232 = 46.4 \text{ kPa}$$

$$R = \frac{8.314}{28} = 0.297 \text{ kJ/kg K}$$

$$W = \frac{p_1V_1 - p_2V_2}{n-1} = \frac{R(T_1 - T_2)}{n-1} = \frac{0.297 \times (373 - 259.6)}{0.33} = 102.1 \text{ kJ/kg}$$

$$m = \frac{200 \times 0.3}{0.297 \times 373} = 0.5416 \text{ kg}$$

$$W_{1-2} = 102.1 \times 0.5416 = 55.3 \text{ kJ}$$

$$c_V = \frac{R}{\gamma - 1} = \frac{0.297}{0.4} = 0.7425 \text{ kJ/kg K}$$

$$Q_{1-2} = U_2 - U_1 + W_{1-2} = 0.5416 \times 0.7425 \times (259.6 - 373) + 55.3 = 9.69 \text{ kJ}$$

$$S_2 - S_1 = m\left[c_V \ln\left(\frac{T_2}{T_1}\right) + R\ln\left(\frac{V_2}{V_1}\right)\right]$$

$$= 0.5416 \times \left[0.725 \times \ln\left(\frac{259.6}{373}\right) + 0.297 \times \ln\left(\frac{0.9}{0.3}\right)\right] = 0.0344 \text{ kJ/K} \overset{\textbf{Ans}}{\Longleftarrow}$$

10.24 c_p of methane can be given by the expression, $c_p = a + bT$ kJ/kg mol K, where $a = 17.66$ and $b = 0.06188$. Consider 1 kg of methane being heated at constant volume from 27 °C to 507 °C. If the initial pressure of the gas is 1 atm, calculate the final pressure, the heat transfer, the work done and the change in entropy.

Solution

From the given data, $p_1 = 1$ atm; $T_1 = 300$ K; $T_2 = 780$ K and c_p of methane is:

$$c_p = a + bT = 17.66 + 0.06188T \text{ kJ/kg mol K}$$

$$\frac{p_1v_1}{T_1} = \frac{p_2v_2}{T_2}$$

As it is a constant volume process $v_1 = v_2$:

$$\therefore p_2 = \frac{T_2}{T_1}p_1 = \frac{780}{300} \times 1 = 2.6 \text{ atm}$$

$$\overline{C}_V = \overline{C}_p - R = 17.66 + 0.06188T - 8.314 = 9.3457 + 0.06188T$$

$$Q = \int_{T_1}^{T_2} c_V dT = \int_{T_1}^{T_2} (9.3457 + 0.06188T)dT$$

$$= 9.3457 \times (780 - 300) + 0.06188 \times \frac{780^2 - 300^2}{2}$$

$$= 20525.2 \text{ kJ/kg mol} = \frac{20525.2}{16} = 1282.83 \text{ kJ/kg}$$

$$W = 0$$

$$
\begin{aligned}
s_2 - s_1 &= \int_{T_1}^{T_2} \frac{c_V dT}{T} = \int_{T_1}^{T_2} (a+bT)\frac{dT}{T} \\
&= a \times \ln\left(\frac{T_2}{T_1}\right) + b(T_2 - T_1) \\
&= 9.3457 \times \ln\left(\frac{780}{300}\right) + 0.06188 \times (780 - 300) \\
&= 38.6 \text{ kJ/kg mol K} = \frac{38.6}{16} = 2.41 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

10.25 Air is compressed reversibly according to the law, $pv^{1.3} = C$, from an initial pressure of 1 bar and 1 m^3 to final volume of 0.5 m^3. Determine the final pressure and the change in entropy per kg of air.

Solution

From the given data, $p_1 = 1$ bar; $V_1 = 1$ m^3; $V_2 = 0.5$ m^3:

$$
\begin{aligned}
Pv^{1.3} &= C \\
\frac{p_2}{p_1} &= \left(\frac{V_1}{V_2}\right)^n = \left(\frac{1}{0.5}\right)^{1.3} = 2.46 \\
p_2 &= 1 \times 2.46 = 2.46 \text{ bar} \\
s_2 - s_1 &= \frac{\gamma - n}{\gamma - 1} R \ln\left(\frac{v_2}{v_1}\right) = \frac{1.4 - 1.3}{1.4 - 1} \times 0.287 \times \ln 0.5 = -0.0497 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

Review Questions

10.1 What are the three states of matter?

10.2 Explain how solid, liquid and gas are recognised through their key characteristics.

10.3 How gases are classified?

10.4 What are the two basic properties of an ideal gas?

10.5 Explain what do you understand by mass fraction.

10.6 What is molar fraction? Explain.

10.7 What is partial pressure and partial volume? Explain.

10.8 Define the law of partial pressures.

10.9 Explain with a suitable figure, Dalton's law of partial pressures.

10.10 Explain with a suitable figure, Amagat's law of partial volumes.

10.11 What are the various properties of gas mixture?

10.12 Derive an expression for the internal energy of gas mixtures.

10.13 Derive an expression for the enthalpy of gas mixtures.

10.14 State Gibbs theorem.

10.15 Derive an expression for the entropy of gas mixtures.

10.16 Derive expressions for specific heat at constant pressure and specific heat at constant volume of a gas mixture.

10.17 How the gas constant for a mixture is evaluated?

10.18 When three gases are mixed, derive the expression for T_m *of the mixture.*

10.19 What is a real gas?

10.20 Explain mole fraction and compressibility factor.

10.21 For an ideal gas, show that:

(i) $ds = \dfrac{du}{T} + \dfrac{p}{T} dv$

(ii) $\left(\dfrac{\partial s}{\partial T}\right)_v = \dfrac{1}{T}\left(\dfrac{\partial u}{\partial V}\right)_v$

(iii) $\left(\dfrac{\partial s}{\partial V}\right)_T = \dfrac{1}{T}\left(\dfrac{\partial u}{\partial V}\right)_T dv + p$

(iv) $c_p = \left(\dfrac{\partial h}{\partial T}\right)_p .$

(v) $du = \left(\dfrac{\partial u}{\partial T}\right)_v dT$ *for a real gas.*

10.22 Show that the change in entropy of an ideal gas in terms of temperature and volume is given by $c_V \ln\left(\dfrac{T_2}{T_1}\right) + R \ln\left(\dfrac{v_2}{v_1}\right)$.

10.23 Show that the change in entropy of an ideal gas in terms of volume and pressure is given by $c_p \ln\left(\dfrac{v_2}{v_1}\right) + c_V \ln\left(\dfrac{p_2}{p_1}\right)$.

10.24 State the following laws:

(i) *Boyle's law*
(ii) *Charles's law*
(iii) *Joule's law*
(iv) *Regnault's law*

10.25 Explain the standard equation of state. Explain other equations of state that you know.

Exercise

10.1 A fluid at 250 kPa and 300 °C has a volume of 0.7 m^3, under a frictionless constant volume process, the pressure changes to 100 kPa. Find the final temperature and the amount of heat transfer, if fluid is (i) air and (ii) steam.

Ans: (i) 229.2 K; −262.28 kJ; (ii) 372.6 K; −850.54 kJ

10.2 A fluid expands at constant pressure without any friction from its initial state of 150 °C and having a specific volume of 0.95 m^3/kg to a volume of 1.55 m^3/kg. Find for unit mass of fluid, the work, heat transfer and final temperature if the fluid is (i) air and (ii) steam. *Ans:* (i) 76.7 kJ; 417.16 °C; 268.5 kJ; (ii) 507.7 kJ; 400 °C; 118 kJ

10.3 A fluid is compressed by means of the reversible isothermal process from its initial condition at 250 °C and 300 kPa to 1/16 of its initial volume. Calculate the final pressure and change in internal energy per unit mass of the fluid if (i) the fluid is air and (ii) the fluid is steam. *Ans:* (i) 4800 kPa; 0; (ii) 300 kPa; 175 kJ/kg

10.4 A fluid expands under reversible adiabatic process from 10 bar at 300 °C to 1 bar. Find the specific volume and work done per kg of fluid if it is (i) air and (ii) steam.
Ans: (i) 0.164 m^3/kg; 197.75 kJ/kg; (ii) 1.626 m^3/kg; 370.7 kJ/kg

10.5 A reversible polytropic process is employed to expand a fluid from 10 bar and 200 °C to 1 bar. The polytropic exponent is 1.15. Find the final specific volume, final temperature and the heat transferred, if the fluid is (i) air and (ii) steam.
Ans: (i) 1.0054 m^3/kg; 77.3 °C; 147.35 kJ;
(ii) 0.95598 m^3/kg; 99.62 °C −367.47 kJ/kg

10.6 An ideal gas which is monatomic occupies 1 litre at 270 °C and 1 kPa. The internal energy is assumed to be zero in these conditions. The gas undergoes the following cycle:

- (i) initially the temperature of the gas is raised to 27 °C at constant volume,
- (ii) then it is expanded adiabatically till its temperature is −270 °C, and
- (iii) then it is compressed to its original volume isothermally.

Draw the p-V diagram and calculate:

- (i) the work done and heat transferred in each process,
- (ii) the internal energy at the end of each process, and
- (iii) the thermal efficiency of the cycle.

Ans: (i) −6.9 J; −6.9 J; (ii) 0; (iii)95.4%

10.7 Consider Fig.10.22, where two vessels A and B, both containing nitrogen, are connected by means of a pipe with valve. The valve is opened to allow the constant to mix and achieve an equilibrium temperature of 300 K. Initial conditions of the vessels A and B are as follows:

Vessel A	Vessel B
$p = 1500$ kPa	$p = 600$ kPa
$t = 50$°C	$t = 20$°C
$\eta = 0.5$ kg mol	x $m = 2.5$ kg

Consider the final equilibrium pressure and the amount of heat transferred to the surroundings. Assuming the vessels are perfectly insulated, calculate the final temperature and pressure in the vessel. Take $\gamma = 1.4$. *Ans:* 45.5 °C; 1240.7 kPa

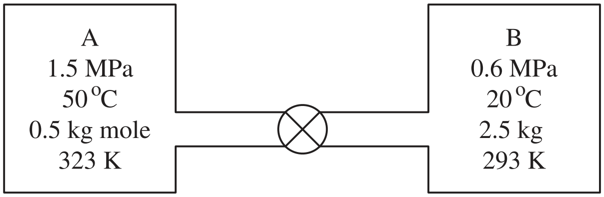


Fig. 10.22

10.8 The c_p and c_V of a certain gas is given as 1.968 kJ/kg K and 1.507 kJ/kg K. Find its gas constant molecular weight. Two kilogram of this gas at 278 K is contained in a chamber of 0.3 m^3. The temperature is raised by heating to 100 °C. Calculate (i) work done; (ii) heat transferred; (iii) change in internal energy; (iv) change in enthalpy; and (v) change in entropy. *Ans:* (i) 0; (ii) 286.33 kJ; (iii) 286.33 kJ; (iv) 373.92 kJ; (v) 0.996 kJ/K

10.9 The specific heats of a gas are given by $c_p = m+RT$ and $c_V = n+RT$, where m, n and R are constants. For an isentropic expansion of this gas, show that $T^n v^{m-n} e^R T$ =constant. 2 kg of this gas at 5600 kPa occupying a volume of 0.06 m^3 expands isentropically to a temperature of 250 °C. If $m = 0.95$ and $n = 0.66$ and $R = 10^{-4}$. Calculate the work done in the expansion. *Ans:* 80.52 kJ

10.10 Show that for an ideal gas, the slope of constant volume line on the T-s diagram is more than that of constant pressure line. *Ans:* $\left(\frac{\partial T}{\partial s}\right)_v > \left(\frac{\partial T}{\partial s}\right)_p$

10.11 One kilogram of air is compressed isentropically from 80 kPa, 60 °C to 400 kPa and expanded at constant pressure to the original volume. Sketch these processes on p-V and T-s planes. Calculate the heat transfer and work transfer for the whole path. *Ans:* 187.2 kJ; 1143.34 kJ

10.12 A certain mass of air has the following initial conditions: temperature, $T_1 = 300$°C; pressure, $p_1 = 800$ kPa; volume, $V_1 = 0.03$ m^3. Initially, it expands to three times its initial volume. Then, a polytropic process with $n = 1.5$ is carried out followed by an isothermal process that all the processes are reversible and adiabatic:

(i) Sketch the cycle on p-V and T-s planes.
(ii) Find the heat received and heat rejected in the cycle.
(iii) Calculate the efficiency of the cycle.

Ans: (ii) 156.41 kJ; 93.19 kJ; (iii) 40%

10.13 A mass of 0.5 kg of an ideal gas has a pressure of 500 kPa and temperature 77 °C and volume 0.07 m^3. The gas undergoes an irreversible adiabatic process to a final pressure of 300 kPa and volume 0.17 m^3, during which the work done on the gas is 50 kJ. Calculate the c_p and c_V of the gas and the increase in entropy of the gas. *Ans:* 0.625 kJ/kg K; 0.825 kJ/kg K; 0.5256 kJ/kg K

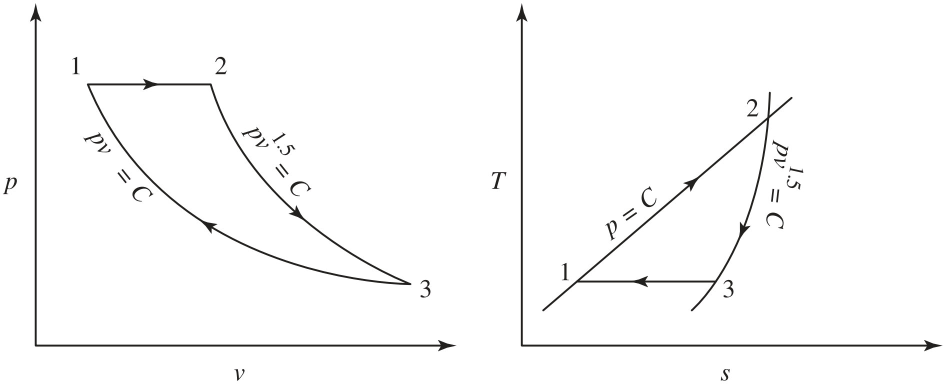


Fig. 10.23

10.14 The pressure, volume and temperature at the end of suction stroke of an internal combustion engine are 1 bar 800 cc and 75 °C, respectively. The air is compressed to 15 bar and 1/8th of its volume reversibly. Heat addition takes place at constant volume until the pressure rests to 50 bar. Find

(i) The index of compression process.
(ii) Change of entropy of air during each process.
(iii) Determine the heat transfer with direction of heat flow if any between air and cylinder walls during compression.

Ans: (i) 1.3; (ii) −0.0001164 kJ/K; 0.000692 kJ/K; (iii) −0.058 kJ/kg

10.15 Consider a mixture of ideal gases consisting of 3 kg of nitrogen and 5 kg carbon dioxide. The pressure and temperature of the mixture are 300 kPa and 20 °C, respectively. Find

(i) the mole fraction of each constituent,
(ii) the equivalent molecular weight of the mixture,
(iii) the partial pressure and partial volumes,
(iv) equivalent gas constant of the mixture,
(v) the volume and density of the mixture, and
(vi) c_p and c_V of the mixture.

Take γ for CO_2 and N_2 as 1.286 and 1.4, respectively.

Ans: (i) N_2 : 0.485; CO_2 : 0.515 (ii) 36.25 kJ/kg mol;
(iii) Partial pressure : N_2 : 145.5 kPa; CO_2 : 154.5 kPa;
Partial volume : N_2 : 0.87 m^3; CO_2 : 0.923 m^3;
(iv) 0.229 kJ/kg K; (v) 1.79 m^3; 4.46 m^3; (vi) 0.92 kJ/kg; 0.69 kJ/kg K

10.16 In the above example, find the change in internal energy, enthalpy and entropy of the mixture, if the mixture is heated to 40 °C (i) at constant volume and (ii) at constant pressure.

Ans: (i) 110.4 kJ; 147.2 kJ; 0.368 kJ/kg K; (ii) 110.4 kJ; 147.2 kJ; 0.486 kJ/kg K

10.17 Calculate the increase in entropy when 2 kg of oxygen at 50 °C is mixed with 6 kg of nitrogen at the same temperature. The initial pressure of each constituent is 103 kPa and assume the same for the mixture. *Ans:* 1.2314 kJ/kg K

10.18 The molecular weight of neon is 20.183 and its critical temperature is 44.5 K at 2.73 MPa and 0.0416 m^3/kg mol. Reading from a compressibility chart for a reduced pressure of 2 and a reduced temperature of 1.3, the compressibility factor, $Z = 0.7$. What are the corresponding specific volume, pressure, temperature and reduced volume? *Ans:* 3.05×10^{-3} m^3/kg; 5.46 MPa; 57.85 K; 1.48

10.19 Consider the Berthelot equation of state given by, $p = \dfrac{RT}{v-b} - \dfrac{a}{Tv^2}$. Show that

(i) $\lim\limits_{\substack{p\to 0\\ T\to\infty}} (RT - pv) = 0$

(ii) $\lim\limits_{T\to\infty} \dfrac{v}{T} = \dfrac{R}{p}$

(iii) Boyle temperature $T_B = \sqrt{\dfrac{a}{bR}}$

(iv) Critical properties, $p_c = \dfrac{1}{12b}\sqrt{\dfrac{2aR}{3b}}; v_c = 3b, T_c = \sqrt{\dfrac{8a}{27bR}}$

(v) Law of corresponding state, $\left(p_v + \dfrac{3}{t_r \times v_r^2}\right)(3v_r - 1) = 8T_r$

10.20 A gaseous mixture consists of 1.5 kg of oxygen and 2.5 kg of nitrogen at pressure of 150 kPa and a temperature of 20 °C. Determine the changes in (i) the internal energy (ii) the enthalpy (iii) the entropy of the mixture when the mixture is heated to a temperature of 100 °C (i) at constant volume (ii) at constant pressure *Ans:* (i) 226.4 kJ; (ii) 257.6 kJ; (iii) 0.683 kJ/K; 0.777 kJ/K

10.21 A diaphragm divides a closed cylinder into two compartments of equal volume of 0.2 m^3 each. Each compartment contains air at a temperature of 27 °C. The pressure in one compartment is 2.5 MPa and in the other is 1.5 MPa. For mixing of air in both the compartments, the diaphragm is ruptured. Because of the rupture, mixing takes place and the pressure becomes uniform throughout the cylinder. Find the net change of entropy for the mixing process. *Ans:* 0.84 kJ/K

10.22 A vessel is divided into three chambers, *A*, *B* and *C*, by two portions. Chamber *A* contains O_2 and has a volume of 0.1 m^3. Chamber *B* contains nitrogen and has a volume of 0.2 m^3. Chamber *C* contains CO2 and has a volume of 0.05 m^3. All three chambers are at 2 bar pressure and 13 °C temperature. When the partitions are removed, three gases mix together. Determine (i) the change in entropy of each component, (ii) the final pressure of the vessel and (iii) the partial pressure of each component. Assume that the vessel is completely isolated from the surroundings.

Ans: (i) 0.0875 kJ/K; 0.0775 kJ/K; 0.0679 kJ/K;
(ii) 0.5714 bar; 1.148 bar; 0.2858 bar;
(iii) 0.5714 bar; 1.148 bar; 0.2858 bar

10.23 An ideal cycle consists of three reversible process in the following sequence:

(i) Constant volume pressure rise.

(ii) Isentropic expansion to r times the initial volume.

(iii) Constant pressure decrease in volume.

Sketch the cycle on the p-V and T-s diagrams. Show that the efficiency of the cycle is $\eta_e = \dfrac{r^\gamma - 1 - \gamma(r-1)}{r^\gamma - 1}$. Find the cycle efficiency when $\gamma = 1.33$ and $r = 10$. *Ans:* 41.3%

10.24 The number of moles, the pressure and the temperature of the three gases A, B and C are given below:

Gas	m (kg mol)	p (kPa)	t (°C)
N_2	1	350	100
CO	3	420	200
O_2	2	700	300

If the containers are connected allowing the gases to mix freely. Find

(i) the pressure and temperature of the resulting mixture at equilibrium and

(ii) the change of entropy of each constituent and that of the mixture.

Ans: (i) 216.67 ° C; 483.11 kPa; (ii) 1.7703 kJ/K

10.25 Find the volume of 2.5 kg moles of steam at 236.4 atm and 776.76 K with the help of compressibility factor versus reduced pressure graph. At this volume and the given pressure, what would be the temperature in K if steam behaves like a van der Waals gas. The critical pressure, volume and temperature of steam are 218.2 atm 57 C^3/g mol and 647.3 K, respectively. *Ans:* 0.546 m^3; 803.4 K

Multiple Choice Questions (choose the most appropriate answer)

1. An ideal gas is one

   (a) which obeys the law $pv = RT$

   (b) which obeys the law $pv = R/T$

   (c) which obeys the law $p = vR/T$

   (d) none of the above

2. The perfect example of an ideal gas is

   (a) air

   (b) hydrogen

   (c) water vapour

   (d) none of the above

3. A real gas obeys closely the ideal gas equation when

   (a) pressure is very small and temperature is very high

   (b) pressure is very high and temperature is very low

   (c) both pressure and temperature are very high

   (d) both pressure and temperature are very low

4. Choose the correct van der Waals equation for an ideal gas from the following: $(a/v_2) =$ force of cohesion and $b =$ coefficient related to volume of molecules.

   (a) $\left(p + \frac{a}{v^2}\right)(v + b) = RT$ (c) $\left(p + \frac{a}{v^2}\right)(v - b) = RT$
   (b) $\left(p - \frac{a}{v^2}\right)(v - b) = RT$ (d) $\left(p - \frac{a}{v^2}\right)(v + b) = RT$

5. In high pressure condition, the real gases conform more closely with
   (a) van der Waals equation (c) both (a) and(b)
   (b) ideal gas equation (d) none of the above

6. Choose the correct statement
   (a) A perfect gas does not obey the law $pv = RT$
   (b) A perfect gas obeys the law $pv = RT$ and has constant specific heat
   (c) A perfect gas obeys the law $pv = RT$ but have variable specific heat capacities
   (d) All of the above

7. Boyles law states that, when temperature is constant, the volume of a given mass of a perfect gas
   (a) varies directly as the absolute pressure
   (b) varies inversely as the absolute pressure
   (c) varies as square of the absolute pressure
   (d) does not vary with the absolute pressure

8. According to Charles law if any gas is heated at constant pressure, its volume
   (a) changes directly as it absolute temperature
   (b) changes inversely as its absolute temperature
   (c) changes as square of the absolute temperature
   (d) does not change with absolute temperature

9. With usual notation, the equation of the state per kg of a perfect gas is given by
   (a) $p^2v = RT$ (c) $pv^2 = RT$
   (b) $pv = RT$ (d) $p^2v^2 = RT$

   where p, v, R and T are the pressure, volume, characteristic gas constant and temperature of the gas, respectively.

10. The equation of state of an ideal gas is a relationship between the variables
    (a) pressure and volume (c) pressure, volume and temperature
    (b) pressure and temperature (d) none of the above

11. Joules law states that the specific internal energy of a gas depends only on
    (a) the pressure of the gas (c) the temperature of the gas
    (b) the volume of the gas (d) none of the above

12. Equation for specific heat at constant pressure of an ideal gas is given by

    (a) $c_p = a + KT + K_1T^2 + K_2T^3$ (c) $c_p = a + KT^2 + K_1T^4 + K_2T$
    (b) $c_p = a + KT^2 + K_1T^3 + K_2T^4$ (d) $c_p = a + KT^2 + K_1T^3 + K_2T^2$

    where a, K, K_1 and K_2 are constants.

13. The order of the rate of diffusion of gases NH_3, SO_2, Cl_2 and CO_2 is

    (a) $NH_3 > SO_2 > Cl_2 > CO_2$ (c) $Cl_2 > SO_2 > CO_2 > NH_3$
    (b) $NH_3 > CO_2 > SO_2 > Cl_2$ (d) None of these

14. Pressure remaining constant, temperature at which the volume of gas doubles with respect to 0 °C is at

    (a) 546 °C (c) 546 K
    (b) 200 °C (d) 273 K

15. Which of the following gases will have the same number of molecules at STP?

    (a) 280 cm^3 of CO_2 and 280 cm^3 of N_2O (c) 44g of CO_2 and 11.2 dm^3 of CO
    (b) 11.2 dm^3 of O_2 and 32 g of O_2 (d) 28g of N_2 and 5.6 dm^3 of oxygen

16. If absolute temperature of a gas is doubled and the pressure is halved, the volume of gas will

    (a) remain unchanged (c) reduce to
    (b) increases four times (d) be double

17. To prevent the volume of a given mass of gas from expanding when its mass is increased

    (a) temperature should be lowered and pressure should be increased

    (b) temperature should be increased and the pressure should be s lowered

    (c) temperature and pressure both should be lowered

    (d) temperature and pressure both should be increased

18. Gases deviate from ideal behaviour at high pressure. Based on this, choose the most appropriate statement for non-ideal conditions?

    (a) At high pressure, the gas molecules move in one direction only

    (b) At high pressure, the collisions between, the gas molecules are increased manifold

    (c) At high pressure, the volume of gas becomes insignificant

    (d) At high pressure, the intermolecular attractions, become significant

19. The deviation of a gas from ideal behaviour is maximum at

    (a) −10°C and 5.0 atm (c) −100°C and 2.0 atm
    (b) −10°C and 2.0 atm (d) 0°C and 2.0 atm

20. At high temperature isotherm moves away from both the axis because of increase in

    (a) pressure (c) number of moles
    (b) volume (d) all the above

21. Values of Charles law constant K depends upon

    (a) mass of gas (c) number of moles of gas
    (b) pressure gas (d) all the above

22. Equal volumes of H_2 and He are inserted in the same vessel. The pressure exerted by H_2 and He are in the ratio:
    (a) 1:1
    (b) 2:1
    (c) 1:2
    (d) all the above

23. Which of the following have same number of molecules at STP?
    (a) 1000 cm^3 of $N2H_4$ and O_2
    (b) 200 cm^3 of CO_2 and N_2O
    (c) 50 cm^3 each of CO and N_2
    (d) all the above

24. If absolute temperature is doubled and the pressure is increased four times. The volume is
    (a) half
    (b) double
    (c) four times
    (d) remains unchanged

25. Density of a gas is usually expressed in
    (a) kg m^3
    (b) kg dm^3
    (c) g dm^{-3}
    (d) g cm^{-3}

Ans: 1. – (a) 2. – (d) 3. – (a) 4. – (c) 5. – (a)
6. – (b) 7. – (b) 8. – (a) 9. – (b) 10. – (c)
11. – (c) 12. – (a) 13. – (b) 14. – (b) 15. – (a)
16. – (b) 17. – (a) 18. – (d) 19. – (a) 20. – (b)
21. – (d) 22. – (a) 23. – (d) 24. – (a) 25. – (d)

11

STEAM POWER CYCLES

11.1 INTRODUCTION

Thermal power plants generate electricity by using fuels like coal, oil or natural gas. Steam turbines are used for the generation of power. A simple power plant consists of a boiler, turbine, condenser and a pump. It uses steam as working fluid. Steam is generated employing water which undergoes a change of phase. It works on Rankine cycle. It is an idealised thermodynamic cycle of a heat engine that converts heat into mechanical work. Rankine cycle is used to predict the performance of the steam power plant. The heat is supplied externally to a closed loop, which usually uses water as the working fluid. It is named after William John Macquorn Rankine, a Scottish polymath and Glasgow University professor.

11.2 WORKING OF A STEAM POWER PLANT

A typical steam power plant is shown in Fig.11.1. It consists of a boiler, superheater, turbine, condenser and a pump. Heat energy in the form of fuel is added both in the boiler and superheater. Initially, water is pumped into the boiler by a pump. Fuel burned in the boiler heats the water to generate steam. The steam is then heated to a superheated state in the superheater. This steam is used to rotate the turbine which is connected to the generator. Electrical energy is generated when the generator windings rotate in a strong magnetic field. After the steam leaves the turbine, it is cooled to its liquid state in the condenser. The liquid is pressurised again by the pump before going back to the boiler and the cycle continues.

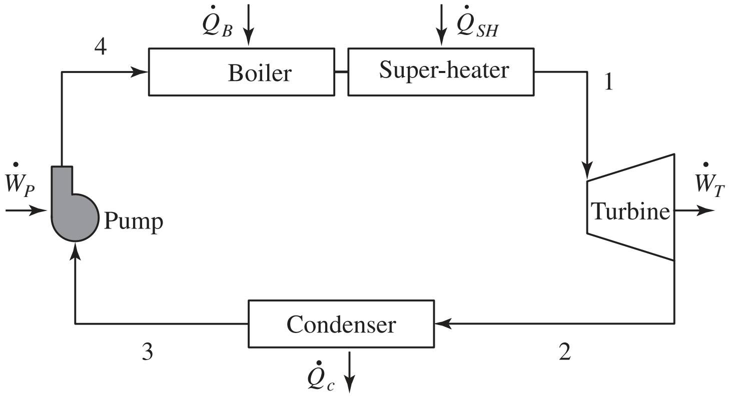


Fig. 11.1 Layout of a typical steam power plant

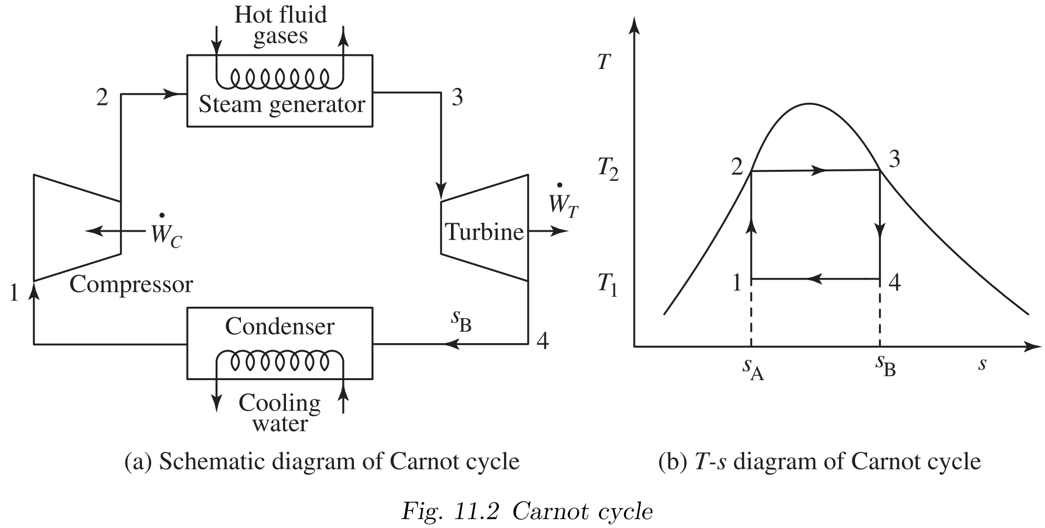
The Rankine cycle is the one which closely describes the processes taking place in steam-operated heat engines. The efficiency of the Rankine cycle is limited because of its high heat of vaporisation of the working fluid, viz water. Therefore, the temperature range in which the cycle can operate becomes quite small. In order to increase the efficiency, the pressure and temperature should be raised to super-critical levels in the steam boiler. Steam turbine entry temperatures are typically around 565°C and steam condenser temperatures are around 30°C. This gives a theoretical maximum Carnot efficiency for the steam turbine alone of about 63%. The actual overall thermal efficiency of the power plant is around 42% for a modern coal-fired power station. The low steam turbine entry temperature (compared to a gas turbine) makes the Rankine (steam) cycle to be often used as a bottoming cycle. This will help to recover waste heat in combined-cycle gas turbine power stations. However, the thermal efficiencies of actual large steam power stations and large modern gas turbine stations are comparable.

11.3 CARNOT CYCLE AS A VAPOUR POWER CYCLE

We are aware that the Carnot cycle consists of only reversible processes. It has the highest possible efficiency of all the real cycles operating between two thermal reservoirs. Therefore, a study of the Carnot cycle employing vapour as a working medium provides a good basis for the study of vapour power cycles. Consider the Carnot cycle shown in Fig.11.2. Wet steam at state 1 is isentropically compressed to the pressure and leaves the steam generator as saturated vapour (state 3). Then, the saturated vapour is allowed to expand isentropically in a turbine and leaves the turbine at state 4. Finally, the turbine effluent is condensed to state 1, thus completing the cycle. Energy is added to the working fluid at constant temperature T_1, while energy is rejected at constant temperature T_2 in the condenser. We know that the efficiency of a Carnot engine is given by:

$$\eta = \frac{\text{Net work done}}{\text{Energy supplied}} = 1 - \frac{T_2}{T_1} \tag{11.1}$$

It is not practical to operate a power plant on a Carnot cycle because of the difficulties associated with isentropic compression of wet steam in state 1. Moreover, the rate of delivery of work by such a plant will be so low that it is not worthwhile to pressurise it. Though the isentropic compression of the wet steam in state 1 is not desirable. However, the energy addition and rejection processes at constant temperature and pressure in the steam generator and the condenser are highly desirable. These desirable processes are retained and the un-



(a) Schematic diagram of Carnot cycle (b) *T*-*s* diagram of Carnot cycle

Fig. 11.2 Carnot cycle

desirable process is replaced by a technically feasible process in devising the modified Carnot cycle which is discussed in the following sections.

11.4 MODIFIED CARNOT CYCLE AS A VAPOUR POWER CYCLE

Figure 11.3 shows a modified Carnot cycle on T-s and p-V diagrams. It consists of

(i) two constant pressure operations (4-1) and (2-3) and

(ii) two frictionless adiabatic operations (1-2) and (3-4).

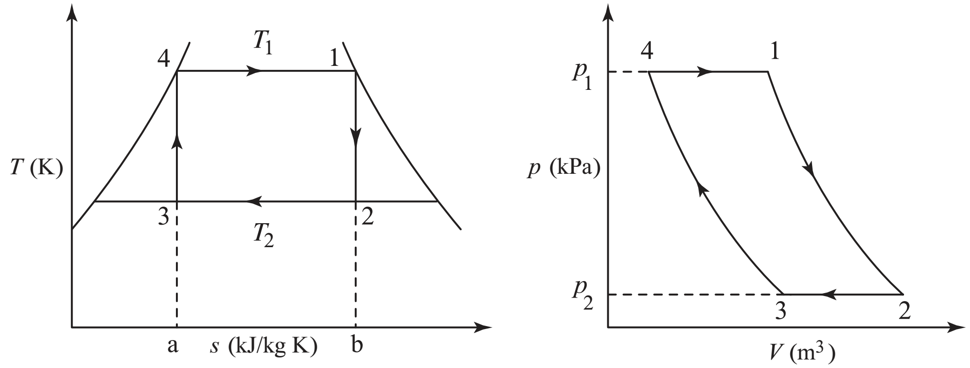
Details are discussed below :

(i) *Operation 4–1.* Let 1 kg of boiling water at temperature T_1 is heated to form wet steam of dryness fraction x_1. Heat is absorbed at constant temperature T_1 and pressure p_1 during this operation.

(ii) *Operation 1–2.* During operation 1–2, steam is expanded isentropically to temperature T_2 and pressure p_2. The point 2 represents the condition of steam after expansion.

(iii) *Operation 2–3.* During operation 2–3, heat is rejected at constant pressure p_2 and temperature T_2. As the steam is exhausted, it becomes wetter and cooled from 2 to 3.

(iv) *Operation 3–4.* In operation 3–4, where the wet steam at 3 is compressed isentropically till the steam regains its original state of temperature T_1 and pressure p_1. Thus, cycle is completed.



(a) *T-s* diagram of Carnot cycle (b) *p-V* diagram of Carnot cycle

Fig. 11.3 Carnot cycle on T-s and p-V diagrams

Referring to Fig.11.3(a), during operation 4–1, heat supplied at constant temperature T_1

$$= \text{Area 4–1–b–a} = T_1(s_1 - s_4) = T_1(s_2 - s_3)$$

During operation 2–3, heat rejected at constant temperature T_2

$$= \text{Area 2–3–a–b} = T_1(s_2 - s_3)$$

As there is no heat transfer during isentropic operations 1–2 and 3–4,

$$\begin{aligned}\text{Net work done} &= \text{Heat supplied} - \text{Heat rejected}\\ &= T_1(s_2 - s_3) - T_2(s_2 - s_3) = (T_1 - T_2)(s_2 - s_3) \qquad (11.2)\\ \text{Carnot cycle, } \eta &= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{(T_1 - T_2)(s_2 - s_3)}{T_1(s_2 - s_3)} = \frac{T_1 - T_2}{T_1} \qquad (11.3)\end{aligned}$$

11.4.1 Limitations

Thermodynamically, Carnot cycle is simple. It has the highest thermal efficiency for given values of T_1 and T_2. However, in practice, it is extremely difficult to make a device to work on this cycle because of the following reasons:

(i) Isentropic compression of wet vapour is extremely difficult to the saturated state which is required by the process 3–4.

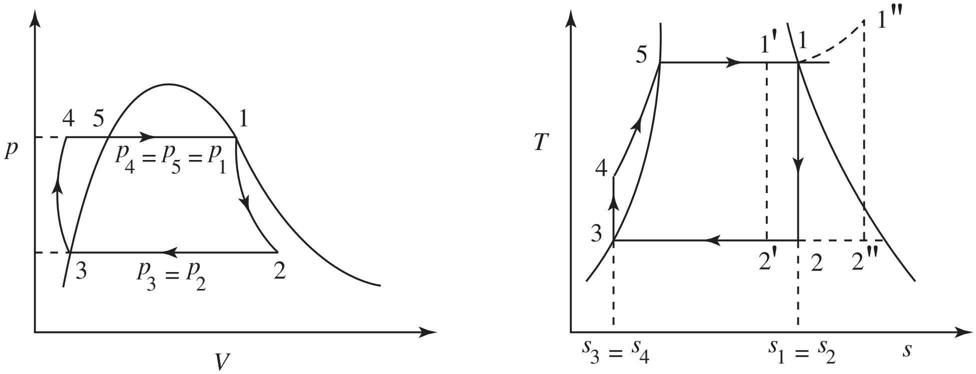
(ii) Control of the quality of the condensate at the exit of the condenser is difficult.

(iii) The efficiency of the Carnot cycle is largely dependant on the temperature T_1. The critical temperature for steam is only 374 °C, therefore, if the cycle is to be operated in the wet region, there is a limitation on maximum possible temperature.

(iv) The cycle is much more difficult to operate in practice with superheated steam. It is due to the fact that supplying the superheat at constant temperature is more difficult than constant pressure. In a practical cycle, limits of pressure and volume are far more easily realised than limits of temperature. Therefore, as to date, no practical engine operates on the Carnot cycle.

11.5 THE IDEAL RANKINE CYCLE

The schematic diagram of the steam turbine power plant is shown in Fig.11.1. Corresponding p-V and T-s diagrams are shown in Fig.11.4. Let us discuss the various processes involved in the cycle.



(a) p-V diagram

(b) T-s diagram

Fig. 11.4 Rankine cycle including pump work

Process 1-2: Let us assume that initially the steam is at state 1 which is dry and saturated. This dry saturated steam from boiler (state 1) is expanded in the turbine isentropically (upto state 2) which produces mechanical work. Because of expansion, the pressure of steam falls from $p_1$to p_2. The temperature at the end of expansion is T_2 which is the saturated temperature at condenser pressure p_2. The steam after expansion is in wet condition with dryness fraction x_2. The work done by the turbine is given by

$$W_T = h_1 - h_2 \tag{11.4}$$

Now, referring to Fig.11.4(b), it may be noted that it is not necessary always for the steam to enter the turbine at dry saturated condition. It can enter at wet conditions $1'$ or $1''$ which is the superheated condition. Then, points $2'$ and $2''$ will be the corresponding outlet condition.

Process 2-3: As the steam expands in the turbine at the end of the expansion, it will be a wet steam. The wet steam gets condensed in the condenser isothermally and isobarically. The wet steam is converted into water in condenser rejecting heat Q_R. This is a heat rejection process in which the heat is rejected from wet steam to atmosphere. Heat rejected in the condenser,

$$Q_R = h_2 - h_3 = h_2 - h_{f2} \qquad (\text{ as } h_3 = h_{f2}) \tag{11.5}$$

Process 3-4: The water from the condenser is pumped isentropically from pressure, p_3 to the boiler pressure, p_4. There may be a very slight rise in temperature from T_3 to T_4. The enthalpy of water increases due to the pump work. The work done input to the pump is given by

$$W_P = h_4 - h_3 = v_3\,(p_1 - p_2) \tag{11.6}$$

$$W_P = v_{f3}\,(p_4 - p_3) = v_{f2}\,(p_1 - p_2) \tag{11.7}$$

(Note that $p_4 = p_3; p_4 = p_2; v_3 = v_{f2}$), where v_{f2} is specific volume at saturated water of pressure p_2.

Process 4-5: The heat is supplied by the boiler to raise the temperature of water to saturated temperature of T_5 at pressure of p_5. Heat supplied (Q_S) during the process 4–5 is given by

$$Q_{S4-5} = h_5 - h_4 \tag{11.8}$$

Process 5-1: The saturated water is then heated in the boiler to the initial dry saturated liquid condition at the pressure p_1. The enthalpy increases by a large value during evaporation. Heat is supplied during 5-1:

$$Q_{S5-1} = h_1 - h_5 \tag{11.9}$$

$$\begin{aligned}
\text{Total heat supplied, } Q_S &= Q_{S4-5} + Q_{S5-1} = h_5 - h_4 + h_1 - h_5 \\
&= h_1 - h_4 = h_1 - h_{f4} \quad (\because h_1 = h_{f4}) \\
&= h_1 - (h_3 + W_P)\,[\because h_4 = h_3 + W_P = \ h_{f_2} + W_P] \\
\text{Net work output, } W &= W_T - W_P = (h_1 - h_2) - W_P = h_1\,(h_{f_2} + W_P) \qquad (11.10) \\
\text{Efficiency of the cycle, } \eta &= \frac{W}{Q_S} = \frac{(h_1 - h_2) - W_P}{(h_1 - h_4)} \qquad (11.11)
\end{aligned}$$

$$\eta = \frac{Q_S - Q_R}{Q_S} = \frac{(h_1 - h_4) - (h_2 - h_3)}{(h_1 - h_4)} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

$$= \frac{(h_1 - h_2) - W_p}{h_1 - (h_{f2} + W_p)}$$

The pump work is usually very small compared to the expansion work. Hence, it may be neglected for the low-pressure operations. However, it should be included for high-pressure operation.

If the pump work is neglected then the efficiency equation reduces to:

$$\eta = \frac{h_1 - h_2}{h_1 - h_{f_2}} \quad (\because h_3 = h_{f_2}) \tag{11.12}$$

The T-s diagram for Rankine cycle without feed pump is shown in Fig.11.5.

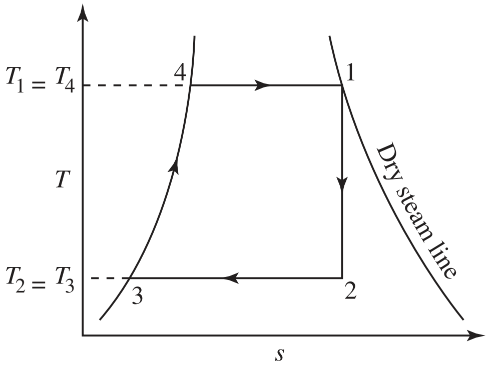


Fig. 11.5 Rankine cycle neglecting pump work

Specific steam consumption (SSC)

It is defined as the mass flow of steam required to develop unit power output given by

$$SSC = \frac{3600}{W} \text{ kg/kW h} \tag{11.13}$$

where W is net work output.

$$W = (h_1 - h_2) - W_P \text{ for cycle with pump work} \tag{11.14}$$

$$W = h_1 - h_2 \text{ for cycle without pump work} \tag{11.15}$$

(ii) Specific steam flow rate (SSF)

It is defined as the steam flow rate in kg/s required to develop unit power output:

$$SSF = \frac{1}{W} \text{ kg/kW} \tag{11.16}$$

(iii) Work ratio

It is defined as the ratio of net work to the gross work.

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Turbine Work}} = \frac{W_T - W_P}{W_T} \tag{11.17}$$

11.5.1 Comparison of Rankine Cycle with Carnot Cycle

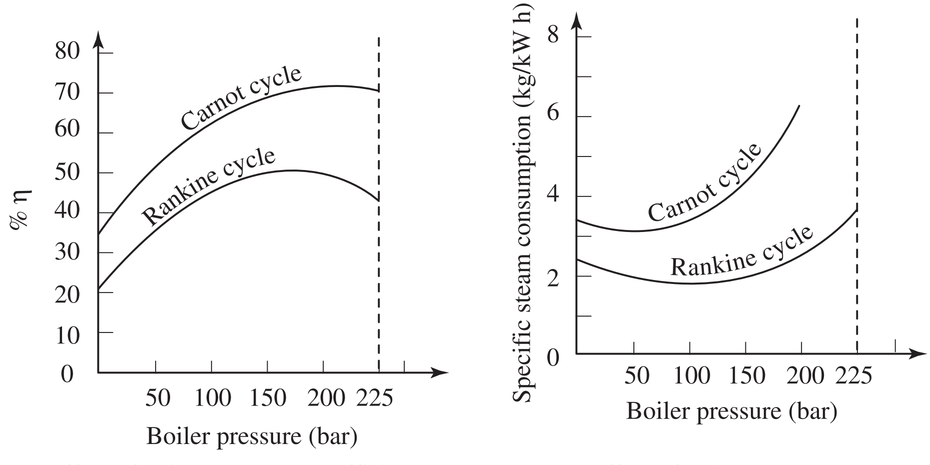
We have gone through both Rankine cycle and Carnot cycle. Now, let us make a comparison of both the cycle.

(i) Within the same temperature limits, Rankine cycle provides a higher specific work output than a Carnot cycle. This makes Rankine cycle to have a comparatively smaller steam flow rate resulting in smaller size plant for a given power output. However, it requires higher rates of heat transfer in boiler and condenser.

(ii) In Rankine cycle, only part of the heat is supplied isothermally at constant higher temperature T_1. Therefore, its efficiency is lower than that of Carnot cycle. The efficiency of the Rankine cycle approaches Carnot cycle more closely when the superheat temperature rise is reduced.

(iii) It is advantageous to feed liquid to the boiler instead of compressing a wet vapour for compression since higher input work is required.

Figure 11.6 shows the variations of efficiency and specific steam consumption against boiler pressure for Carnot and ideal Rankine cycles. The Rankine cycle efficiency can be improved by : (i) increasing the average temperature at which heat is supplied and (ii) decreasing/reducing the average temperature at which heat is rejected.



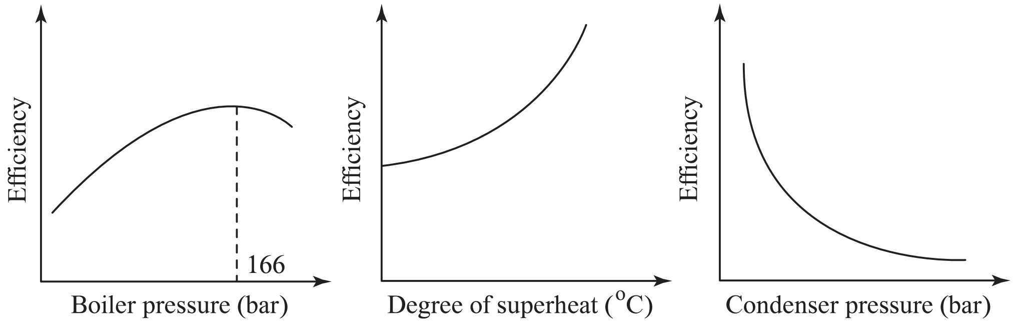
(a) Effect of boiler pressure on efficiency

(b) Effect of boiler pressure on specific steam consumption

Fig. 11.6 Comparison of η and SFC for Carnot and ideal Rankine cycles for various boiler pressure

This is achieved by making changes in the process of steam generation and/or condensation. The details are discussed below:

(i) *Increasing boiler pressure:* As can be seen from Fig.11.7(a) that by increasing the boiler pressure and keeping other factors same, the cycle efficiency increases and reaches a maximum value at a boiler pressure of about 166 bar.



(a) (b) (c)

Fig. 11.7 Effect of operating conditions on the thermal efficiency of the Rankine cycle

(ii) *Superheating:* Keeping other factors same and increasing the degree of superheat increases the efficiency. The superheated steam provides longer turbine blade life because of the absence of erosion from high-velocity suspended water particles in wet steam.

(iii) *Reducing condenser pressure:* The thermal efficiency of the cycle can be improved by reducing the condenser pressure [Fig.11.7(c)]. The temperature at which heat is rejected, especially in higher vacuums. However, it is at the cost of high-quality condensers.

The thermal efficiency of the Rankine cycle is also improved by the following methods:

(i) Regenerative feed heating.

(ii) Reheating of steam.

(iii) Water extraction.

(iv) Using binary vapour.

11.6 ACTUAL RANKINE CYCLE

The reversible adiabatic process is practically not possible because of frictional and other losses in turbine and pump. The T-s diagram for actual cycle is shown in Fig.11.8. The actual expansion is irreversible as shown by line 1-2′, similarly, the actual compression process is irreversible as indicated by line 3-4′. The power output and the efficiency of various components is given by:

$$W = (h_1 - h_2) \text{ for cycle without pump work} \tag{11.18}$$

$$\text{Isentropic efficiency} = \frac{\text{Actual work}}{\text{Isentropic work}} \quad \text{for an expansion process} \tag{11.19}$$

$$\therefore \text{ Turbine efficiency, } \eta_T = \frac{h_1 - h_{2'}}{h_1 - h_2} \tag{11.20}$$

$$\text{Isentropic efficiency} = \frac{\text{Isentropic work input}}{\text{Actual work input}} \quad \text{for a compression process} \tag{11.21}$$

$$\therefore \text{Pump efficiency, } \eta_P = \frac{h_4 - h_3}{h_{4'} - h_3} \tag{11.22}$$

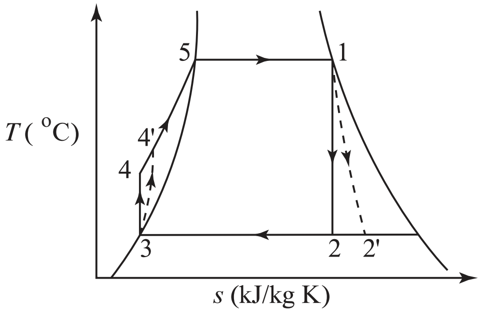


Fig. 11.8 Actual Rankine cycle

11.6.1 Efficiency Ratio

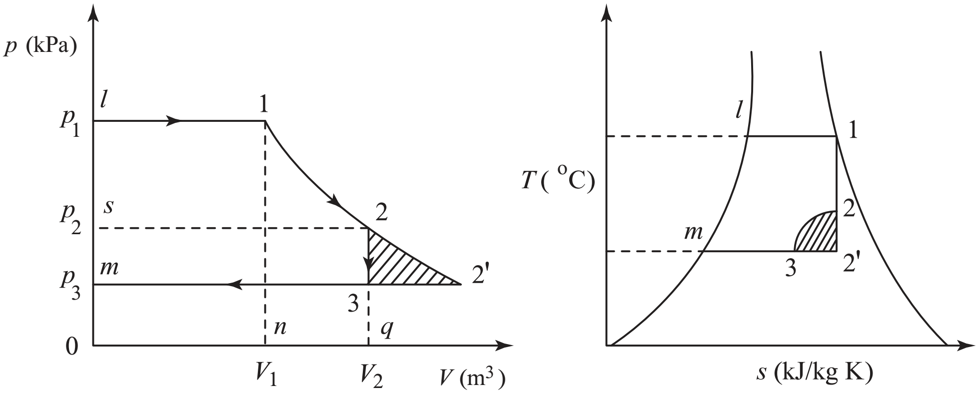
The efficiency ratio of the cycle is the ratio of actual cycle efficiency to the ideal efficiency:

$$\text{Effeciency ratio} = \frac{\text{Actual cycle efficiency}}{\text{Rankine efficiency}} \tag{11.23}$$

where Actual cycle efficiency, $\eta = \dfrac{(h_1 - h_{2'}) - W_P}{h_1 - (h_{f_3} + W_P)}$

11.7 MODIFIED RANKINE CYCLE

Figure 11.9(a) and (b) shows the modified Rankine cycle on p-V and T-s diagrams (neglecting pump work), respectively. It may be seen from p-V diagram that there is very narrow region at the toe, i.e. point $2'$, and the work obtained near toe is very small. In fact, this work is too inadequate to overcome friction due to reciprocating parts. Therefore, the adiabatic is terminated at 2; the pressure decreases suddenly at constant volume. This operation is represented by line 2–3. By doing this, the stroke length is reduced. In other words, the cylinder dimensions reduce but at the expense of small loss of work (area 2–3–2$'$) which, however, is negligibly small.



(a) p-V diagram (b) T-s diagram

Fig. 11.9 p-V and T-s diagrams of modified Rankine cycle

The work done by the modified Rankine cycle can be evaluated in the following manner:

Let p_1, v_1, u_1 and h_1 correspond to initial condition of steam at 1.

Let p_2, v_2, u_2 and h_2 correspond to condition of steam at 2.

Let p_3, h_3 correspond to condition of steam at 3.

Work done during the cycle/unit mass of steam

$$\begin{aligned} &= \text{Area } l-1-2-3-m \\ &= \text{Area } o-l-1-n+\text{Area } 1-2-q-n-\text{Area } o-m-3-q \\ &= p_1v_1+(u_1-u_2)-p_3v_2 \end{aligned}$$

$$\text{Heat supplied} = h_1 - h_{f3}$$

Therefore, the modified Rankine efficiency

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{p_1v_1+(u_1-u_2)-p_3v_2}{h_1-h_{f3}} \tag{1}$$

Another way by which we can evaluate the modified Rankine efficiency is as follows:

Work done during the cycle/kg of steam

$$\begin{aligned} &= \text{Area } l-1-2-3-m = \text{Area } l-1-2-s+\text{Area } s-2-3-m \\ &= (h_1-h_2)+(p_2-p_3)\times v_2 \end{aligned}$$

$$\text{Heat supplied} = h_1 - h_{f3}$$

Modified Rankine efficiency

$$= \frac{\text{Work done}}{\text{Heat supplied}} = \frac{(h_1-h_2)+(p_2-p_3)\times v_2}{h_1-h_{f3}} \tag{11.24}$$

Note: Modified Rankine cycle is mainly for reciprocating steam engines. The stroke length and hence, cylinder size is reduced with the sacrifice of negligible amount of work done in reciprocating steam engine.

11.7.1 Reheat Rankine Cycle

It is the endeavour of engineers to increase the efficiency of the Rankine engine. For this, the average temperature and pressure of the steam at which the heat is supplied to it should be increased. Further, the steam should be kept as dry as possible at the end of expansion. If the pressure increases, the expansion ratio in the turbine will also increase and the steam becomes wet at the end of expansion. Increasing the moisture of the steam will cause the erosion of the turbine blades which will increase the losses.

To overcome this in reheat cycle, the expansion is being carried out in two stages. The steam is initially expanded in the HP turbine to some pressure and then heated with the help of flue gases in the boiler. Then, the steam is expanded in the LP turbine to the condenser pressure. The main purpose of reheating is to increase the dryness fraction of the steam passing through the turbine which should never fall below 0.88. The thermal efficiency increases with reheat cycle and also the specific steam consumption decreases. However, the thermal efficiency of the reheat cycle may decrease at low pressures. The T-s diagram for reheat cycle is shown in Fig.11.10. The process 1–2 represents isentropic expansion in the high-pressure turbine and 3–4 represents isentropic expansion in the low-pressure turbine. The steam is reheated at constant pressure process 2–3. The reheat can be carried out by returning the steam to the boiler and passing it through a heat exchanger placed in the boiler at constant pressure. Other process remains same as in ideal Rankine cycle:

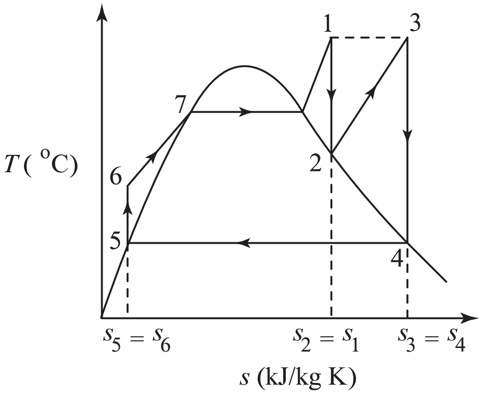


Fig. 11.10 Reheat Rankine cycle

$$\text{Heat supplied, } Q_S = Q_{S_{1-6}} + Q_{S_{2-3}} = (h_1 - h_6) + (h_3 - h_2) \tag{11.25}$$

$$\begin{aligned} \text{Work output } W &= (W_{1-2} + W_{3-4} - W_P) \\ &= (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5) \end{aligned} \tag{11.26}$$

Therefore, the efficiency of the reheat Rankine cycle is:

$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4) - W_P}{h_1 - (h_{f_4} - W_P) + (h_3 - h_2)} \tag{11.27}$$

where $W_P = v_{f_4}(p_1 - p_4)$

If the pump work is neglected:

$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4)}{(h_1 - h_{f_4}) + (h_3 - h_2)} \tag{11.28}$$

where h_1 is the enthalpy of superheated steam $= h_{g_1} + c_{P_g}(T_{sup} - T_{sat})$

and h_2 is the enthalpy of steam at intermediate pressure p_2.

(11.29)

Note: The steam at this stage may be superheated, wet or dry. It should be found by comparing s_1 and s_2].

If $s_2 = s_1$, the steam is in dry condition and $h_2 = h_{g_2}$.

If $s_2 < s_1$, the steam is in superheated condition, then $h_2 = h_{g_2} + c_{P_g}(T_{sup} - T_{sat})$.

If $s_2 > s_1$, the steam is in wet condition, then $h_3 = h_{f_2} + x_2 h_{fg_2}$.

where h_3 is the enthalpy of superheated steam at pressure $p_3 = p_2$. h_4 is the enthalpy of steam at pressure p_4, i.e. is the condenser pressure.

h_6 is the enthalpy of steam at pump outlet:

$$h_6 = h_5 + W_P \tag{11.30}$$

$$W_P = v_5 (p_6 - p_5) = v_{f_4} (p_1 - p_4) \tag{11.31}$$

$$v_5 = v_f \quad \text{at condenser pressure} \tag{11.32}$$

Note that $p_6 = p_1$ and $p_5 = p_4$.

11.8 REGENERATIVE RANKINE CYCLE

In thermodynamic power cycles, an improvement of even 0.5% in overall cycle efficiency is considered as an appreciable gain. To save fuel, efforts are constantly made to improve the efficiency of the cycle. One of the more common ways to improve the efficiency of a steam cycle is to use regeneration. Regeneration is a process where heat is taken from steam between turbine stages and used to heat water as it goes through pump stages. The main purpose is to increase the fluid average temperature during heat addition and decrease the fluid temperature during heat rejection. In this section, we will introduce one such cycle – the ideal regenerative Rankine cycle, which increases the fluid average temperature during the heat addition process.

11.8.1 Ideal Regenerative Rankine Cycle

The Rankine cycle efficiency will be less than that of Carnot cycle efficiency for the same temperature limit. It is because heating of feedwater takes place with a large temperature difference. If the temperature of feedwater is increased to the saturation temperature corresponding to the boiler pressure before it enters into the boiler, the cycle will be as close as Carnot cycle. The schematic arrangement of this cycle is shown in Fig.11.11(a) and corresponding T-s diagram is shown in Fig.11.11(b). As can be seen from Fig.11.11, the working fluid leaving the feed pump circulates around the turbine casing. Here, heat transfer takes

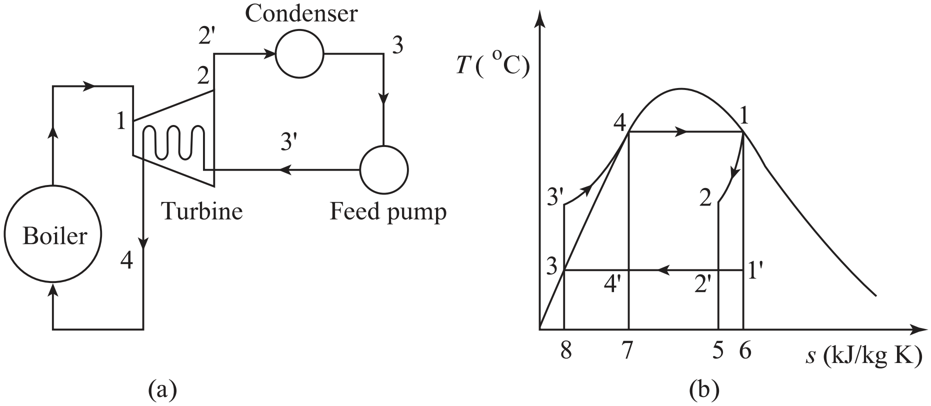


Fig. 11.11 Regenerative Rankine cycle

place between the incoming steam and the fluid in the liquid state inside the casing. This process is represented by 1–2 in Fig.11.11(b). We know that, the area under the T-s diagram represents the heat transfer during the cycle execution. The area under the curve 1–2 (i.e. 2–5–6–1) represents the heat loss experienced by the steam in the turbine as it flows through it. The area under the curve 3–4, i.e. (3–4–7–8–3) represents the heat gained by the working fluid as it flows through the turbine casing. The two areas should be the same since heat transfer is reversible.

The area under the curve 4–1 represents the amount of heat transfer from the boiler to the working fluid. Similarly, the area under the curve 2′–3′ represents the amount of heat transferred from the working fluid to the coolant. Since, the area under the curves 1–2 and 3–4 are equal, the area 2′–3–8–5–2′ will be equal to the area 1′–4′–7–6′–1′. If we consider the Carnot cycle operated between the same temperature limits, the heat transfer is given by the area 1–4–4′–l′–1. The heat rejected to the coolant is represented by the area 1′–4′–7–6–1′. which is equal to the heat rejected by ideal regenerative Rankine cycle. From these, the Carnot efficiency can be calculated.

11.8.2 Single-Stage Regeneration

The cycle discussed in Section 11.8.1 is clearly not a practical proposition because the turbine operates with wet steam having low dryness fraction. It will affect the turbine blades very badly. However, the Rankine efficiency can be improved in practice by bleeding off some of the steam at an intermediate pressure during expansion. This steam is used to heat the feedwater in a separate feedwater heater. This arrangement is shown in Fig.11.12(a) and corresponding T-s diagram in Fig.11.12(b).

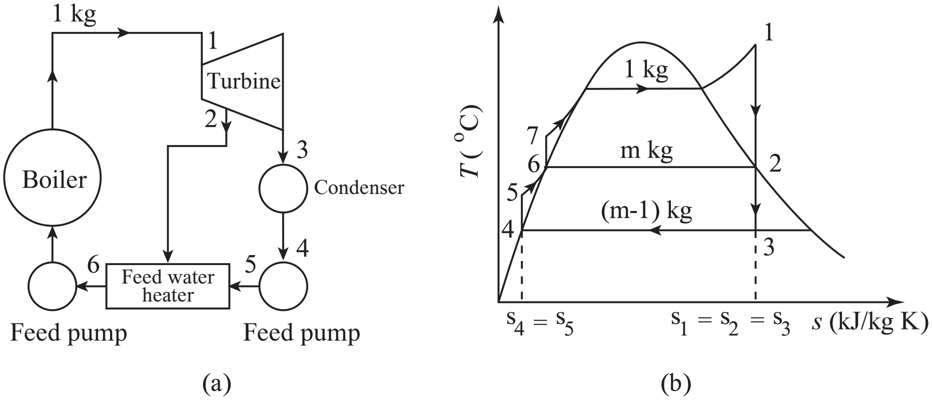


Fig. 11.12 Practical single-stage regenerative Rankine cycle

Let the steam expand from condition 1 through the turbine. At the pressure corresponding to point 2, a quantity of steam, say m kg per kg of steam supplied from boiler, is bled off for heating feedwater. The rest of the steam of $(1 - m)$ kg completes the expansion and is sent to the condenser at pressure p_3. This amount of steam is then condensed and pumped to the same pressure as the bleed steam. The bleed steam of m kg and feedwater are mixed in the feedwater heater under ideal adiabatic conditions. The state of the condensed mass of m kg and the feedwater of $(1 - m)$ kg leaving the heater will be the same. It is represented by state 7. The feedwater is then pumped by the second feed pump of the boiler pressure where it is heated to state 1 and thus the cycle is repeated.

Work done by the turbine per kg of steam:

$$W_T = (h_1 - h_2) + (1 - m)(h_2 - h_3)$$

$$\text{Pump work, } W_P = (1 - m)(h_5 - h_4) + (h_7 - h_6)$$

$$W_P = (1 - m)\, v_4\, (p_5 - p_4) + v_6\, (p_7 - p_6)$$

$$= (1 - m)\, v_{f3}\, (p_2 - p_3) + v_{f_2}\, (p_1 - p_2)$$

$$\text{Net work, } W = W_T - W_P$$

$$\text{Heat supplied in the boiler, } Q_S = h_1 - h_7$$

$$\text{Heat rejected in the condenser, } Q_R = (1 - m)(h_3 - h_4) = (1 - m)(h_3 - h_{f_3})$$

$$\eta_{regen} = \frac{Q_S - Q_R}{Q_S} = \frac{(h_1 - h_7) - (1 - m)(h_3 - h_{f_3})}{h_1 - h_7}$$

Amount of steam extracted (m) is determined from the heat balance in the feedwater heater:

Heat lost by the steam = Heat gained by the water

$$m(h_2 - h_6) = (1 - m)(h_6 - h_5)$$

$$mh_2 - mh_6 = (h_6 - h_5) - mh_6 + mh_5$$

$$mh_2 - mh_5 = h_6 - h_5$$

$$m = \frac{h_6 - h_5}{h_2 - h_5}$$

It can also be found from energy balance equation:

$$mh_2 + (1 - m)\, h_5 = h_6$$

$$mh_2 + h_5 - mh_5 = h_6$$

$$m = \frac{h_6 - h_5}{h_2 - h_5} = \frac{h_{f_2} - h_5}{h_2 - h_5} \quad (\because h_6 = h_{f_2}) \tag{11.33}$$

11.8.3 Two-Stage Regeneration Cycle

If a regenerative cycle employs two feedwater heaters, i.e. steam is bled from two places of the turbine then the *T*-*s* diagram will be shown in Fig.11.13. The efficiency of the cycle can be derived as follows.

Work done per *kg* of steam supplied to the turbine:

$$W_T = 1 \times (h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4)$$

$$W_P = W_{P_1} + W_{P_2} + W_{P_3}$$

$$= (1 - m_1 - m_2)(h_6 - h_5) + (1 - m_1)(h_8 - h_7) + 1 \times (h_{10} - h_9)$$

$$= (1 - m_1 - m_2)\, v_{f_4}\, (p_3 - p_4) + (1 - m_1)\, v_{f_3}\, (p_2 - p_3) + v_{f_2}\, (p_1 - p_2)$$

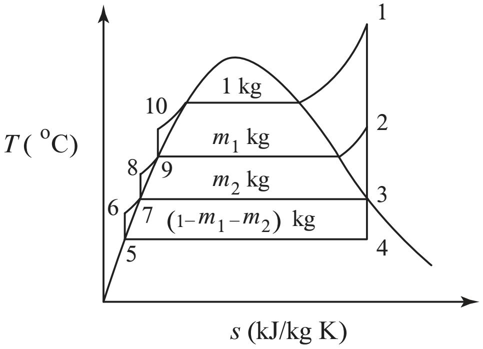


Fig. 11.13 Two-stage regenerative cycle

Heat supplied by the boiler per kg of steam generated:

$$Q_S = (h_1 - h_{10})$$

$$\therefore \eta = \frac{W}{Q_S} = \frac{W_T - W_P}{h_1 - h_{10}}$$

The amount of steam bled from turbine in two stages m_1 and m_2 can be determined from energy balance equations.

For first heater:

$$m_1 h_2 + (1 - m_1)\, h_8 = 1 \times h_9$$

$$m_1 h_2 + h_8 - m_1 h_8 = h_9$$

$$m_1 = \frac{(h_2 - h_8)}{(h_2 - h_8)} = \frac{(h_{f_2} - h_8)}{(h_2 - h_8)}$$

For second heater:

$$m_2 h_3 + (1 - m_1 - m_2)\, h_6 = (1 - m_1)\, h_7$$

$$m_2 h_3 + h_6 - m_1 h_6 - m_2 h_6 = h_7 - m_1 h_7$$

$$m_2 = \frac{(1 - m_1)\,(h_7 - h_6)}{(h_3 - h_6)}$$

The actual vapour power cycle differs from the ideal Rankine cycle because of irreversibilities. These irreversibilities are present in the inherent components which is caused by fluid friction and heat loss to the surroundings. Fluid friction causes pressure drops in the boiler, the condenser and the piping between the components. As a result, the steam leaves the boiler at a lower pressure. Heat loss reduces the net work output. Thus, more heat addition to the steam in the boiler is required to maintain the same level of net work output.

11.9 BINARY VAPOUR CYCLE

The thermal efficiency of Carnot cycle is the highest of all cycles and is given by $\dfrac{T_1 - T_2}{T_1}$. To approach this cycle in any actual engine, it is necessary that the entire heat must be supplied at constant temperature T_1 and rejected at T_2. This can be achieved only by using a vapour in the wet condition but not in the superheated state. The efficiency depends on temperature T_1, since T_2 is fixed by the natural sink to which heat is rejected. This implies that T_1 should be as *large as possible, consistent with the vapour being saturated.*

If we use steam as the working medium, the temperature rise is accompanied by rise in pressure. At critical temperature of 374 °C, the pressure is as high as 225 bar. This will create many problems in the design, operation and control. It would be desirable to use some fluid other than steam which has more desirable thermodynamic properties than water.

An ideal fluid for this purpose should have a *very high critical temperature combined with low pressure.* Only very few fluids like mercury, diphenyl oxide and similar compounds, aluminium bromide and zinc ammonium chloride are fluids that possess the required properties. However, mercury is the only working fluid which has been successfully used in practice. It has high critical temperature (588.4 °C) and correspondingly low critical pressure (21 bar abs.). *The mercury alone cannot be used as its saturation temperature at atmospheric pressure is high (357 °C). Hence, binary vapour cycle is generally used to increase the overall efficiency of the plant. Two fluids (mercury and water) are used in cascade in the binary cycle for power generation.*

Few more properties required for an ideal binary fluid used in high-temperature limit are listed below:

(i) At reasonably low pressure, it should have high critical temperature.

(ii) It should have high heat of vaporisation. This will keep the weight of fluid in the cycle to minimum.

(iii) Freezing temperature should be below room temperature.

(iv) It should have chemical stability throughout the working cycle.

(v) It must be non-corrosive.

(vi) It must have an ability to wet the metal surfaces to increase the heat transfer.

(vii) The vapour pressure should be nearly atmospheric at a desirable condensation temperature.

(viii) After expansion through the turbine the vapour should be almost saturated.

(ix) It must be easily available at reasonable cost.

(x) It should be non-toxic.

Although mercury does not have all the required properties, it is more favourable than any other fluid investigated. It is most stable under all operating conditions. Although, mercury does not cause any corrosion to metals, it is an extremely dangerous substance for human life. Hence, a lot of elaborate precautions must be taken to prevent the escape of vapour. The major disadvantage associated with mercury is that it does not wet the surface of the metal. Due to this, there is a serious resistance to heat flow. This difficulty can be considerably reduced by adding magnesium and titanium (2 parts in 100000 parts) in mercury.

11.9.1 Thermal Properties of Mercury

Mercury has reasonably good number of desirable thermodynamic properties. The thermal properties of mercury are as follows:

(i) At atmospheric pressure, the freezing point of mercury is −3.3 °C and boiling point is −354.4 °C.

(ii) The maximum pressure required at the temperature of vapour (540 °C) is only 12.5 bar, approximately. Therefore, heavy construction is not required to get high initial temperature.

(iii) Liquid saturation curve of mercury is very steep. It means that it is closely approaching the isentropic process of the Carnot cycle, which is vertical.

(iv) Corrosive or erosive effects are almost nil on metals commonly used in practice.

(v) Its critical temperature is very high. Therefore, it will not cause any problem to the metals normally used.

Some undesirable properties of mercury are listed below:

(i) The latent heat of mercury is very low over a wide range of desirable condensation temperatures. Therefore, large amount of mercury should be circulated per kilogram of water evaporated in binary cycle.

(ii) The cost is considerably high. The quantity of mercury required is 8–10 times the quantity of water circulated in binary system.

(iii) Mercury vapour inhaled in larger quantities is poisonous. Therefore, the system must be leak proof.

Figure 11.14 shows the schematic diagram of binary vapour cycle using mercury and water as working fluids. The processes are represented on T-s diagram as shown in Fig.11.15.

11.9.2 Analysis of Binary Vapour Cycle

Now, let us analyse the binary vapour cycle:

h_{hg1} = Heat supplied per kg of Hg (mercury) vapour formed in the mercury boiler
h_{hg2} = Heat lost by one kg of Hg vapour in the mercury condenser
h_s = Heat given per kg of steam generated in mercury condenser or steam boiler
W_{hg1} = Work done per kg of Hg in the cycle
W_s = Work done per kg of steam in the steam cycle
η_s = Thermal efficiency of the steam cycle
η_{hg} = Thermal efficiency of the Hg cycle
m = Mass of Hg in the Hg cycle per kg of steam circulated in the steam cycle

We will neglect the heat losses to the surroundings in this analysis. Steam generated is considered as 1 kg and Hg in the circuit taken as m kg per kg of water in the steam cycle.

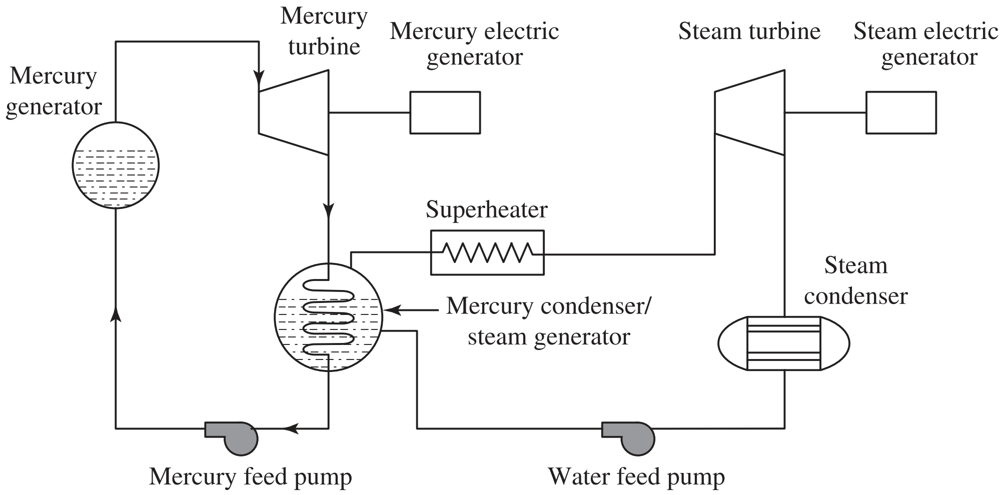


Fig. 11.14 Line diagram of binary vapour cycle

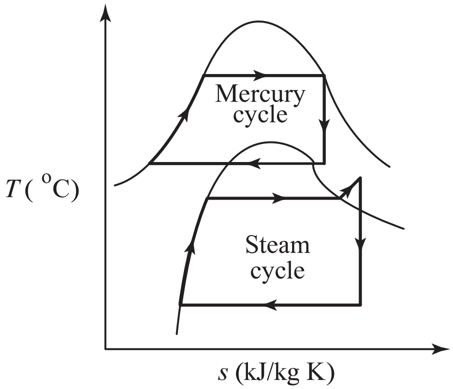


Fig. 11.15 Binary vapour cycle on T-s diagram

Heat supplied in the Hg boiler, $h_t = m \times h_{hg1}$

Work done in the mercury cycle $= mW_{hg}$

Work done in the steam cycle $= 1 \times W_s$

Total work done in binary cycle, $W_t = mW_{hg} + W_s$

Overall efficiency of binary cycle, $\eta = \dfrac{\text{Work done}}{\text{Heat supplied}} = \dfrac{W_t}{h_t}$

$$= \frac{mW_{hg} + W_s}{mh_{hg_1}} \tag{11.34}$$

Thermal efficiency of mercury cycle, $\eta_{hg} = \dfrac{mW_{hg}}{mh_{hg1}} = \dfrac{W_{hg}}{h_{hg1}}$

$$= \frac{h_{hg1} - h_{hg2}}{h_{hg1}} = 1 - \frac{h_{hg2}}{h_{hg1}} \qquad (11.35)$$

$$= \frac{mh_{hg1} - h_s}{mh_{hg1}} = 1 - \frac{1}{m}.\frac{h_s}{h_{hg1}} \qquad (11.36)$$

Heat lost by mercury vapour $=$ Heat gained by steam

$$mh_{hg2} = 1 \times h_s \qquad (11.37)$$

Substituting the value of m from Eq.11.37 into Eq.11.36, we get,

$$\eta_{hg} = 1 - \frac{h_{hg2}}{h_{hg1}} \qquad (11.38)$$

The thermal efficiency of the steam cycle is given by:

$$\eta_s = \frac{W_s}{h_s} = \frac{h_{s1} - h_{s2}}{h_{s1}} = \frac{h_{s1} - h_{s2}}{mh_{hg2}} \qquad (11.39)$$

From Eqs 11.34, 11.36, 11.37, 11.38 and 11.39, we get,

$$\eta = \eta_{hg}(1 - \eta_s) + \eta_s \qquad (11.40)$$

Major constraint in the design of binary cycle is the limit of exhaust pressure of the mercury which will provide maximum work per kilogram of Hg circulated in the system and high thermal efficiency of the cycle. It is not easy to decide, as the number of controlling factors are too many.

Worked-Out Examples

11.1 A steam power plant working on ideal Rankine cycle has the boiler outlet pressure of 1000 kPa and condenser pressure of 100 kPa. Find (i) turbine output (W_T) in kJ/kg; (ii) pump work (W_p) in kJ/kg; (iii) heat input Q in kJ/kg; (iv) cycle efficiency; (v) steam rate in kg/kW h; and (vi) moisture at the end of the turbine process.

Solution

Referring to Fig.11.16, it is seen that the steam is expanding from 1000 kPa (1) to 100 kPa (2), i.e. from 1 MPa to 0.1 MPa. Ideal cycle is represented by 1-2s-3-4s-1. Now let us take down the various properties from the steam table at the two pressures.

$p_1 = 1$ MPa	$p_2 = 0.1$ MPa
$h_1 = h_g = 2778.1$ kJ/kg	$h_3 = h_f = 417.16$ kJ/kg
$s_1 = s_g = 6.5864$ kJ/kg K	$s_3 = s_f = 1.3025$ kJ/kg K
$s_1 = s_{2s} = 6.5864$ kJ/kg K	$h_{fg} = 2258.0$ kJ/kg
–	$s_{fg_{2s}} = 6.0588$ kJ/kg K

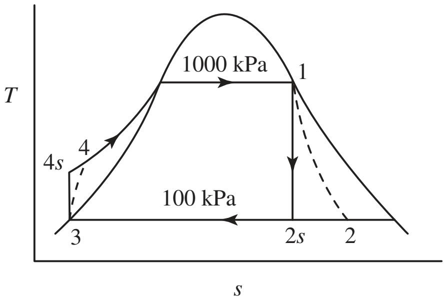


Fig. 11.16

$$s_{2s} = s_3 + x_{2s} \times s_{fg_{2s}}$$

$$6.5864 = 1.3025 + x_{2s} \times 6.0588$$

$$x_{2s} = \frac{(6.5864 - 1.3025)}{6.0588} = 0.8721$$

$$h_{2s} = h_f + x_{2s} \times h_{fg} = 417.16 + 0.8721 \times 2258 = 2386.36 \text{ kJ/kg}$$

$$\text{Work input to the pump, } W_p = h_{4s} - h_3$$

At state 3, the working fluid is a saturated liquid. From the steam table, the specific volume

$$v_f = 0.001 \text{ m}^3\text{/kg}$$

As liquid is incompressible, its specific volume will be constant.

$$\therefore v_3 = (v_f)_{1 \text{ bar}} = 0.001 \text{ m}^3\text{/kg}$$

$$W_p = \int_3^{4s} v dp = v_3(p_{4s} - p_3) = 0.001 \times (1000 - 100) = 0.9 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Turbine work, } W_T = h_1 - h_{2s} = 2778.1 - 2386.36 = 391.74 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_{net} = W_T - W_p = 391.74 - 0.9 = 390.84 \text{ kJ/kg}$$

$$W_p = h_{4s} - h_3 = 0.9$$

$$h_{4s} = h_3 + 0.9 = 417.16 + 0.9 = 418.1 \text{ kJ/kg}$$

$$\text{Heat input, } Q_1 = h_1 - h_{4s} = 2778.1 - 418.1 = 2360 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{390.84}{2360.0} = 0.1656 = 16.56\% \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Steam rate} = \frac{3600}{W_{net}} = \frac{3600}{390.84} = 9.21 \text{ kg/kW h} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Moisture constant} = (1 - x_{2s}) = 1 - 0.8721 = 0.1279 = 12.79\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.2 In the above example 11.1, if W_p is considered negligible, calculate the cycle efficiency and steam flow rate.

Solution

From the previous example, $W_T = 391.74$ kJ/kg. As $W_p = 0; W_{net} = W_T = 391.74$ kJ/kg.

$$\begin{aligned} Q_1 &= h_1 - h_3 \quad (\text{ as } W_p = 0) \\ &= 2778.1 - 417.46 = 2360.94 \text{ kJ/kg} \\ \eta_{cycle} &= \frac{W_{net}}{Q_1} = \frac{391.74}{2360.94} = 0.166 = 16.6\% \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\ \text{Steam flow rate} &= \frac{3600}{W_{net}} = \frac{3600}{391.74} = 9.19 \text{ kJ/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

Note that there is almost no variation in η and steam flow rate. It indicates that W_p is negligible.

11.3 In example 11.1, if the turbine has an η of 75%, calculate the efficiency, moisture content and steam flow rate. Assume pump efficiency as 0.75 or 75%.

Solution

As turbine efficiency has been given, the cycle becomes an actual cycle. The details are shown by dotted lines in Fig.11.16. The expansion process becomes 1-2 and the compression by pump becomes 3-4. The cycle is represented by 1-2-3-4-1.

$$\begin{aligned} \text{Turbine work, } W_T &= h_1 - h_2 = \eta_T(h_1 - h_{2s}) \qquad (1) \\ W_T &= 0.75 \times 391.74 = 293.8 \text{ kJ/kg} \\ \text{Input work to pump} &= \frac{W_p}{\eta_p} = \frac{0.9}{0.75} = 1.2 \text{ kJ/kg} \\ W_{net} &= W_T - W_p = 293.8 - 1.2 = 292.6 \text{ kJ/kg} \\ h_4 &= h_3 + W_p = 417.16 + 1.2 = 418.36 \text{ kJ/kg} \\ Q_1 &= h_1 - h_4 = 2778.1 - 418.36 = 2359.74 \text{ kJ/kg} \\ \eta_{cycle} &= \frac{W_{net}}{Q_1} = \frac{292.6}{2359.74} = 0.124 = 12.4\% \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

From Eq.(1) $h_2 = h_1 - \eta_T(h_1 - h_{2s})$. From example 11.1, $(h_1 - h_{2s}) = 391.74$. Therefore,

$$\begin{aligned} h_2 &= 2778.1 - 0.75 \times 391.74 = 2484.3 \text{ kJ/kg} \\ \text{Now, } h_2 &= h_3 + x_2 \times h_{fg2} \\ 2484.3 &= 417.46 + x_2 \times 2258 \end{aligned}$$

$$x_2 = \frac{(2484.3 - 417.16)}{2258} = 0.9155$$

$$\text{Moisture content} = 1 - x_2 = 1 - 0.9155 = 0.0845 = 8.45\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Steam rate} = \frac{3600}{W_{net}} = \frac{3600}{292.6} = 12.3 \text{ kg/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.4 In example 11.1, if the condenser pressure is 10 kPa and working as ideal Rankine cycle, calculate the efficiency of the cycle, steam flow rate and the moisture in the steam at turbine exit.

Solution As in example, 11.1 the boiler pressure is 1000 kPa whereas the condenser pressure has been reduced to 10 kPa (refer Fig.11.17). Now let us write down the various values from the steam table.

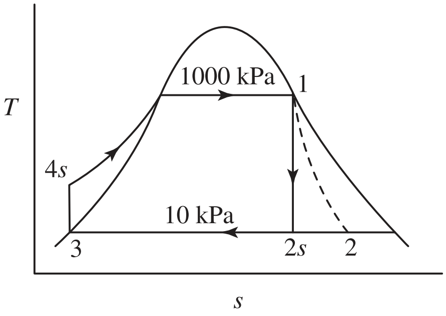


Fig. 11.17

$$\begin{array}{ll}
p_1 = 1 \text{ MPa} & p_2 = 0.01 \text{ MPa} \\
h_1 = h_g = 2778.1 \text{ kJ/kg} & h_3 = h_f = 191.81 \text{ kJ/kg} \\
s_1 = s_g = 6.5864 \text{ kJ/kg K} & s_3 = s_f = 0.6492 \text{ kJ/kg K} \\
s_1 = s_{2s} = 6.5864 \text{ kJ/kg K} & h_{fg} = 2392.8 \text{ kJ/kg} \\
- & s_{fg_{2s}} = 7.5010 \text{ kJ/kg K}
\end{array}$$

$$\begin{aligned}
s_{2s} &= s_1 = s_3 + x_{2s}.s_{fg_{2s}} \\
6.5864 &= 0.6492 + x_{2s} \times 7.5010 \\
x_{2s} &= \frac{(6.5864 - 0.6492)}{7.5010} = 0.7915 \\
h_{2s} &= h_3 + x_{2s} h_{fg_{2s}} = 191.82 + 0.7915 \times 2392.8 = 2085.72 \text{ kJ/kg} \\
W_p &= h_{4s} - h_3 = 0.001 \times (1000 - 10) = 0.99 \approx 1 \text{ kJ/kg} \\
h_{4s} &= h_3 + W_p = 191.82 + 1 = 192.82 \text{ kJ/kg} \\
Q_1 &= h_1 - h_{4s} = 2778.1 - 192.82 = 2585.3 \text{ kJ/kg}
\end{aligned}$$

$$
\begin{aligned}
W_T &= h_1 - h_{2s} = 2778.1 - 2085.72 = 692.38 \text{ kJ/kg} \\
W_{net} &= W_T - W_p = 692.38 - 1 = 691.38 \text{ kJ/kg} \\
\eta_{cycle} &= \frac{W_{net}}{Q_1} = \frac{691.38}{2585.3} = 0.2674 = 26.74\% && \stackrel{\textbf{Ans}}{\Longleftarrow} \\
\text{Steam rate} &= \frac{3600}{691.38} = 5.21 \text{ kg/kW h} && \stackrel{\textbf{Ans}}{\Longleftarrow} \\
\text{Moisture} &= 1 - x_{2s} = 1 - 0.7915 = 0.2085 = 20.85\% && \stackrel{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

11.5 In example 11.1, if the boiler outlet steam is superheated at 1000 kPa and 300 °C with the condenser pressure 10 kPa. Find the cycle efficiency, steam rate and moisture at turbine exit.

Solution

From steam table for superheated at 300 °C and 1000 kPa and for the condenser pressure of 10 kPa, let us take down the values. Refer Fig.11.18:

$p_1 = 1000$ kPa and $T = 300$°C (superheat)
$h_1 = 3051.2$ kJ/kg
$s_1 = 7.1228$ kJ/kg K
$s_{2s} = s_1 = 7.1228$ kJ/kg K
–

$p_2 = 10$ kPa
$h_3 = h_f = 191.81$ kJ/kg
$s_3 = s_f = 0.6492$ kJ/kg K
$h_{fg} = 2392.8$ kJ/kg K
$s_{fg_{2s}} = 7.5010$ kJ/kg K

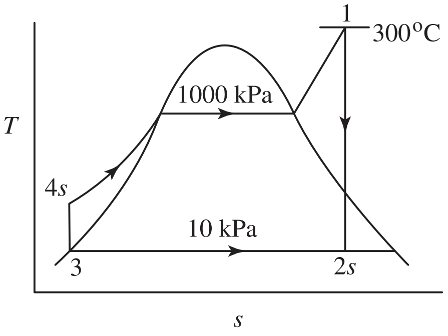


Fig. 11.18 hello

$$
\begin{aligned}
s_{2s} &= s_3 + x_{2s} \times s_{fg_{2s}} \\
7.1228 &= 0.6492 + x_{2s} \times 7.5010 \\
x_{2s} &= \frac{(7.1228 - 0.6492)}{7.5010} = 0.8630 \\
h_{2s} &= h_3 + x_{2s} h_{fg} = 191.81 + 0.8630 \times 2392.8 = 2256.8 \text{ kJ/Kg}
\end{aligned}
$$

$$\begin{aligned}
W_p h_{4s} &= h_{4s} - h_3 = 1 \text{ kJ/kg (from previous example)}\\
h_{4s} &= W_p + h_3 = 1 + 191.86 = 192.86 \text{ kJ/kg}\\
W_T &= h_1 - h_{2s} = 3051.2 - 2256.8 = 794.4 \text{ kJ/kg}\\
W_{net} &= W_T - W_p = 794.4 - 1 = 793.4 \text{ kJ/kg}\\
Q_1 &= h_1 - h_{4s} = 3051.2 - 192.86 = 2858.34 \text{ kJ/kg}\\
\eta_{cycle} &= \frac{W_T}{Q_1} = \frac{794.4}{2858.34} = 0.278 = 27.8\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}\\
\text{Steam rate} &= \frac{3600}{W_{net}} = \frac{3600}{793.4} = 4.537 \text{ kg/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}\\
\text{Moisture content} &= 1 - x_{2s} = 1 - 0.863 = 0.137 = 13.7\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

11.6 In Example 11.1, if the boiler outlet pressure is 150 bar at 600 °C with a condenser pressure of 10 kPa and working on ideal Rankine cycle. Calculate the cycle efficiency, steam flow rate and moisture content.

Solution

Here, the boiler pressure is 15 MPa and condenser pressure is 10 kPa. Further, the steam is superheated to 600 °C. Hence, let us write down the required property values from the steam table.

$p_1 = 15$ MPa, $T_1 = 600°$C
$h_1 = 3582.3$ kJ/kg
$s_1 = 6.6775$ kJ/kg K
$s_{2s} = s_1 = 6.6775$ kJ/kg K
–

$p_2 = 10$ kPa
$h_3 = h_f = 191.81$ kJ/kg
$s_3 = s_f = 0.6492$ kJ/kg K
$h_{fg_{2s}} = 2392.8$ kJ/kg
$s_{fg_{2s}} = 7.5010$ kJ/kg K

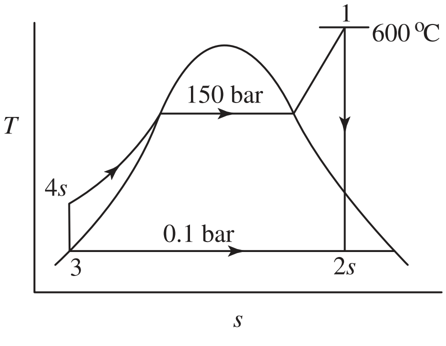


Fig. 11.19

$$\begin{aligned}
s_{2s} &= s_1 = s_3 + s_{fg_{2s}} \times x_{2s}\\
6.6775 &= 0.6492 + 7.5010 \times x_{2s}
\end{aligned}$$

$$
\begin{aligned}
x_{2s} &= \frac{(6.6775 - 0.6492)}{7.5010} = 0.804 \\
h_{2s} &= h_3 + x_{2s} \times h_{fg_{2s}} = 191.81 + 0.804 \times 2392.8 = 2115.62 \text{ kJ/kg} \\
h_{4s} &= W_p + h_3 \\
W_p &= \int_3^{4s} v_f dp = v_f \times [p_{4s} - p_3] = 0.001 \times (15000 - 10) \approx 15 \text{ kJ/kg} \\
h_{4s} &= 15 + 191.81 = 206.81 \text{ kJ/kg} \\
W_T &= h_1 - h_{2s} = 3582.3 - 2115.62 = 1466.8 \text{ kJ/kg} \\
Q_1 &= h_1 - h_{4s} = 3582.3 - 206.81 = 3375.49 \text{ kJ/kg} \\
\eta_{cycle} &= \frac{W_T}{Q_1} = \frac{1466.68}{3375.49} = 0.4345 = 43.45\% \\
\text{Steam rate} &= \frac{3600}{1466.68} = 2.4545 \text{ kg/kW h} && \overset{\textbf{Ans}}{\Longleftarrow} \\
\text{Moisture} &= 1 - x_{2s} = 1 - 0.804 = 0.196 = 19.6\% && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

11.7 In Example 11.6, if the steam is reheated to 600 °C at maximum intermediate pressure to limit the moisture at turbine exit to 15%. Calculate cycle efficiency and steam flow rate.

Solution

The T-s diagram is shown in Fig.11.20. Now for the condition given, let us write down the values from the steam table and put it in the following table.

$p_1 = 15$ MPa, $T_1 = 600°\text{C} = T_3$	$p_2 = 0.1$ bar
$h_1 = 3582.3$ kJ/kg	$h_5 = h_f = 191.81$ kJ/kg
$s_1 = s_{2s} = 6.6775$ kJ/kg K	$s_5 = s_f = 0.6492$ kJ/kg k
	$h_3 = h_1 = 3582.3$ kJ/kg
	$s_{fg_{4s}} = 7.5010$ kJ/kg K
	$h_{fg_{4s}} = 2392.8$ kJ/kg
	$x_{4s} = 0.8$

$$
s_{4s} = s_5 + (1 - x_{4s})s_{fg} = 0.6492 + (1 - 0.15) \times 7.5010 = 7.025 \text{ kJ/kg K} = s_3
$$

Now from the table for superheated steam at 600 °C and for $s_3 = 7.025$ kJ/kg K, it matches well with 8 MPa pressure. Therefore, we can take the intermediate pressure as 8 MPa = 80 bar. The corresponding h_3 value can be taken as 3642.0 kJ/kg.

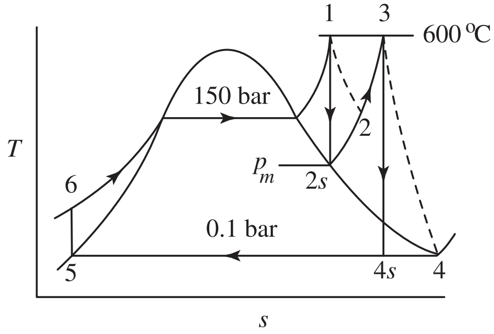


Fig. 11.20

$$h_{4s} = h_5 + x_{4s} \times h_{fg_{4s}} = 191.81 + 0.85 \times 2392.8 = 2225.69 \text{ kJ/kg K}$$

$$W_p = h_6 - h_5 \approx 15 \text{ kJ/kg(From previous problem)}$$

$$h_6 = h_5 + W_p = 191.85 + 15 = 206.85 \text{ kJ/kg}$$

Consider process 1–2s:

$$s_1 = s_{2s} = 6.6775 \text{ kJ/kg K}$$

With p_{rh} = 8 bar and s_{2s} = 6.6775 kJ/kg K, from steam table s_g= 5.7431. As $s_{2s} > s_g$, the steam is superheated condition. In order to find the superheat temperature, we should interpolate values in the steam table. Now looking at the table under 8 bar, we see that s_{2s} lies between 500 °C and 550 °C.

At 8 bar and 500 °C, $s = 6.7239$ kJ/kg K and at 8 bar and 550 °C, $s = 6.8778.$ kJ/kg K. Now making linear interpolation, T_{sup} at $s = 6.775$:

$$T_{sup} = 500 + \frac{550 - 500}{(6.8778 - 6.7239)} \times (6.8778 - 6.775) = 533.4°\text{C}$$

Now let us calculate h_{2s} at 533.4 °C. h at 500 °C and 8 bar = 3398.3 kJ/kg and h at 550 °C and 8 bar = 3521.0 kJ/kg.

Now h_{2s} at 533.4 °C:

$$h_{2s} = 3398.3 \times \frac{(3521 - 3398.3)}{(550 - 500)} \times (533.4 - 500) = 3480.3 \text{ kJ/kg}$$

$$W_T = (h_1 - h_{2s}) + (h_3 - h_{4s})$$

$$= (3582.3 - 3480.3) + (3642.0 - 2225.7) = 1518 \text{ kJ/kg}$$

$$W_{net} = W_T - W_p = 1518 - 15 = 1503 \text{ kJ/kg}$$

$$Q_1 = (h_1 - h_6) + (h_3 - h_{2s})$$

$$= (3582.3 - 206.85) + (3582.3 - 3480.3) = 3477.45 \text{ kJ/kg}$$

$$\eta_{cycle} = \frac{1518}{3477.45} = 0.4365 = 43.65\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Steam rate} = \frac{3600}{W_{net}} = \frac{3600}{1503} = 2.4 \text{ kg/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.8 In Example 11.6, if the turbine efficiency is 85%, calculate the cycle efficiency and steam flow rate.

Solution

As the angle change is turbine efficiency which is now 85%:

$$W_T = 0.85 \times 1518 = 1290.3 \text{ kJ/kg}$$

$$W_p = 15 \text{ kJ/kg}$$

$$W_{net} = W_T - W_p = 1290.3 - 15 = 1275.3 \text{ kJ/kg}$$

$$Q_1 = 3477.45$$

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{1275.3}{3477.45} = 0.367 = 36.7\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Steam rate} = \frac{3600}{W_{net}} = \frac{3600}{1275.3} = 2.823 \text{ kg/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.9 In example 11.1, if the boiler outlet pressure is 1000 kPa and steam is saturated with the condenser pressure of 10 kPa and if the pump process ends on saturated liquid line, calculate the efficiency of the cycle and steam flow rate.

Solution

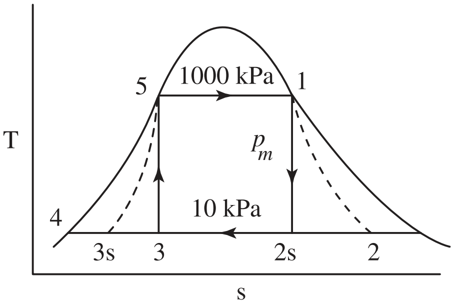


Fig. 11.21

Refer Fig.11.21. From steam table, at 1000 kPa = 1 MPa. For the given condition, let us write down the required values from the steam table.

$$\begin{array}{ll}
p = 1 \text{ MPa}, T = T_{sat} & p = 0.01 \text{ MPa} \\
h_1 = 2778.1 \text{ kJ/kg} & h_4 = 191.81 \text{ kJ/kg} \\
s_1 = 6.5864 \text{ kJ/kg K} & h_{fg} = 2392.8 \text{ kJ/kg} \\
h_5 = 762.79 \text{ kJ/kg} & s_4 = 0.6492 \text{ kJ/kg K} \\
s_5 = 2.1386 \text{ kJ/kg K} & s_{fg} = 7.5010 \text{ kJ/kg K}
\end{array}$$

As can be seen from Fig.11.21, $s_{2s} = s_1$ and $s_3 = s_5$:

$$s_{2s} = s_4 + x_{2s} \times s_{fg_{2s}}$$

As $s_{2s} = s_1$, we have,

$$\begin{aligned}
6.5864 &= 0.6492 + x_{2s} \times 7.5010 \\
x_{2s} &= \frac{(6.5864 - 0.6492)}{7.5010} = 0.7915 \\
h_{2s} &= h_4 + x_{2s} \times h_{fg_{2s}} = 191.81 + 0.7915 \times 2392.8 = 2085.71 \text{ kJ/kg} \\
s_3 &= s_5 = s_4 + x_3 \times s_{fg} \quad (\because s_3 = s_5) \\
2.1386 &= 0.6492 + x_3 \times 7.5010 \\
x_3 &= \frac{(2.1386 - 0.6492)}{7.5010} = 0.1986 \\
h_3 &= h_4 + x_3 \times h_{fg} = 191.81 + 0.1986 \times 2392.8 = 667 \text{ kJ/kg} \\
W_T &= h_1 - h_{2s} = 2778.1 - 2085.71 = 692.4 \text{ kJ/kg} \\
W_p &= h_5 - h_3 = 762.79 - 667 = 95.79 \text{ kJ/kg} \\
W_{net} &= W_T - W_p = 692.4 - 95.79 = 596.61 \text{ kJ/kg} \\
Q_1 &= h_1 - h_5 = 2778.1 - 762.79 = 2015.31 \text{ kJ/kg} \\
\eta_{cycle} &= \frac{W_{net}}{Q_1} = \frac{596.61}{2015.31} = 0.296 = 29.6\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

11.10 In example 11.9, if you assume the machine efficiency as 80%, calculate the cycle efficiency and steam flow rate.

Solution

$$\begin{aligned}
W_T &= \eta_T \times W_T = 0.8 \times 692.4 = 553.92 \text{ kJ/kg} \\
W_p &= \frac{W_p}{\eta_p} = \frac{95.79}{0.8} = 119.73 \text{ kJ/kg} \\
W_{net} &= W_T - W_p = 553.92 - 119.73 = 434.19 \text{ kJ/kg} \\
Q_1 &= h_1 - h_5 = 2778.1 - 762.79 = 2015.31 \text{ kJ/kg}
\end{aligned}$$

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{434.19}{2015.31} = 0.2154 = 21.54\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Steam flow rate} = \frac{3600}{434.19} = 8.29 \text{ kJ/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.11 In example 11.10, if the power plant is working on ideal regeneration cycle, calculate the cycle efficiency and steam flow rate.

Solution

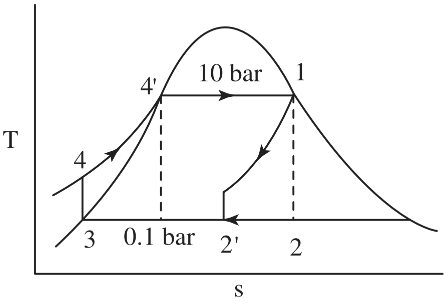


Fig. 11.22

The T-s diagram of the regeneration cycle is shown in Fig.11.22. From steam table let us pick up the values:

$p = 1000 \text{ kPa} = 1 \text{ MPa}$	$p_2 = 0.01 \text{ MPa}$
$h_1 = 2778.1 \text{ kJ/kg}$	$h_3 = 191.81 \text{ kJ/kg}$
$s_1 = 6.5864 \text{ kJ/kg K} = s_2$	$h_{fg} = 2392.8 \text{ kJ/kg}$
$s_{4'} = 2.1386 \text{ kJ/kg K}$	$s_3 = 0.6479 \text{ kJ/kg K}$
$h_{4'} = 762.79 \text{ kJ/kg}$	$s_{fg} = 7.5010 \text{ kJ/kg K}$

$$h_4 - h_3 = W_p = \int vdp = v(p_4 - p_3) = 0.001 \times (1000 - 10) = 1 \text{ kJ/kg}$$

$$h_4 = h_3 + W_p = 191.81 + 1 = 192.81 \text{ kJ/kg K}$$

$$s_2 - s_{2'} = s_{4'} - s_3$$

$$6.5824 - s_{2'} = 2.1386 - 0.6492$$

$$s_{2'} = 6.5824 - 2.1386 + 0.6492 = 5.093 \text{ kJ/kg K}$$

$$s_{2'} = s_3 + x_{2'} s_{fg}$$

$$5.093 = 0.6492 + x_{2'} \times 7.5010$$

$$x_{2'} = \frac{5.093 - 0.6492}{7.5010} = 0.5924$$

$$h_{2'} = h_3 + x_{2'} \times h_{2'} fg = 191.81 + 0.5924 \times 2392.8 = 1609.3 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\begin{aligned}
Q_1 &= h_1 - (h_{4'} - h_4) = 2778.1 - 762.79 + 192.81 = 2208.12 \text{ kJ/kg} \\
W_T &= (h_1 - h_{2'}) - (h_{4'} - h_4) = (2778.1 - 1609.3) - (762.79 - 192.81) \\
&= 598.82 \text{ kJ/kg} \\
W_{net} &= W_T - W_p = 598.82 - 1 = 597.82 \text{ kJ/kg} \\
\eta_{cycle} = \frac{W_{net}}{Q_1} &= \frac{597.82}{2208.12} = 27.1\% \\
\text{Steam rate} &= \frac{3600}{W_{net}} = \frac{3600}{597.82} = 6.022 \text{ kg/kW h}
\end{aligned}$$

Ans ⇐

11.12 In example 11.10, all the other conditions remain the same, if a single open heater at 110 °C is added, find the efficiency of the cycle as well as the steam flow rate.

Solution

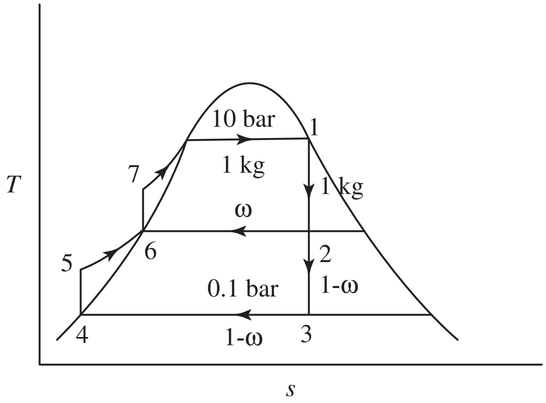


Fig. 11.23

Here, an open heater at 110 °C is added. T-s diagram with solid lines is shown in Fig.11.23. Below are the required values from the steam tables:

$p = 1000$ kPa

$h_1 = 2778.1$ kJ/kg

$s_1 = 6.5864$ kJ/kg K $= s_2$

$s_4 = 2.1386$ kJ/kg K

$p_2 = 10$ kPa

$h_4 = 191.81$ kJ/kg

$h_{fg} = 2392.8$ kJ/kg

$s_{4'} = 0.6492$ kJ/kg K

$s_{fg} = 7.5010$ kJ/kg K

Now from steam table (with respect to temperature) at 110 °C and referring to Fig.11.23:

$h_6 = 461.27$ kJ/kg

$h_{fg6} = 2230.2$ kJ/kg

$s_6 = 1.4184$ kJ/kg K

$s_{fg6} = 5.8202$ kJ/kg K

$$
\begin{aligned}
s_1 &= s_2 = s_6 + x_2 \times s_{fg6} \\
6.5864 &= 1.4184 + x_2 \times 5.8202 \\
x_2 &= \frac{(6.5864 - 1.4181)}{5.8202} = 0.8879 \\
h_2 &= h_6 + x_2 \times h_{fg2} = 461.27 + 0.8879 \times 2230.2 = 2441.46 \text{ kJ/kg} \\
\text{Now } s_3 &= s_1 = h_4 + x_3 \times s_{fg3} \\
6.5861 &= 0.6492 + x_3 \times 7.5010 \\
x_3 &= \frac{6.5861 - 0.6492}{7.5010} = 0.791 \\
h_3 &= h_4 + x_3 \times h_{fg3} = 191.81 + 0.791 \times 2392.8 = 2084.51 \text{ kJ/kg}
\end{aligned}
$$

Assume that $h_5 \approx h_4 = 191.45$ kJ/kg and $h_6 \approx h_7 = 461.27$ kJ/kg. Now by energy balance:

$$
\begin{aligned}
\dot{m}(h_2 - h_6) &= (1 - \dot{m})(h_6 - h_5) \\
\dot{m} \times (2441.46 - 461.27) &= (1 - \dot{m}) \times (461.27 - 191.45) \\
1980.19 \times \dot{m} &= (1 - \dot{m}) \times 269.82 \\
(1980.19 + 269.82) \times \dot{m} &= 269.82 \\
\dot{m} &= \frac{269.82}{1980.19 + 269.82} = 0.12 \text{ kg/s} \\
W_T &= (h_1 - h_2) + (1 - \dot{m})(h_2 - h_3) \\
&= (2778.1 - 2441.46) + (1 - 0.12) \times (2441.46 - 2084.51) \\
&= 650.756 \text{ kJ/kg}
\end{aligned}
$$

Assume that pump input is negligible small:

$$
\begin{aligned}
W_{net} &= W_T = 650.76 \text{ kJ/kg} \\
Q_1 &= h_1 - h_7 = 2778.1 - 461.27 = 2316.34 \text{ kJ/kg} \\
\eta_{cycle} &= \frac{W_{net}}{Q_1} = \frac{650.76}{2316.83} = 0.28 = 28\% \\
\text{Steam rate} &= \frac{3600}{W_{net}} = \frac{3600}{650.76} = 5.532 \text{ kg/kW h}
\end{aligned}
$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

11.13 All the other conditions remain the same, instead of an open heater, if two open heaters at 90 °C and 135 °C are added, calculate the cycle efficiency and the steam flow rate.

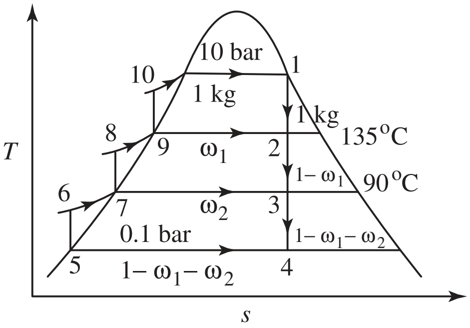


Fig. 11.24

Solution

In this example, there are two open heaters. Now let us write down the various required values from the steam table.

$p_1 = 1000$ kPa
$h_1 = 2778.1$ kJ/kg
$s_1 = 6.5864$ kJ/kg K $= s_2 = s_3 = s_4$

$p_2 = 10$ kPa
$h_5 = 191.81$ kJ/kg
$s_5 = 0.6492$ kJ/kg K
$s_{fg4} = 7.5010$ kJ/kg K
$h_{fg4} = 2392.8$ kJ/kg

135 °C feedwater heater
$s_6 = s_f = 1.6869$ kJ/kg K
$s_{fg2} = 5.2907$ kJ/kg K
$h_f = h_9 = 567.67$ kJ/kg
$h_{fg} = 2159.6$ kJ/kg K

90 °C feedwater heater
$s_7 = s_f = 1.1924$ kJ/kg K
$s_{fg3} = 6.2866$ kJ/kg K
$h_f = h_7 = 376.90$ kJ/kg
$h_{fg} = 2283.2$ kJ/kg K

$$\begin{aligned}
s_2 &= s_9 + x_2 s_{fg2} \\
6.5864 &= 1.6869 + x_2 \times 5.2907 \\
x_2 &= \frac{6.5864 - 1.6869}{5.2907} = 0.9260 \\
h_2 &= h_6 + x_2 h_{fg} = 567.67 + 0.9260 \times 2159.6 = 2567.46 \text{ kJ/kg} \\
s_3 &= s_7 + s_{fg2} x_3 \\
6.5864 &= 1.1924 + 6.2866 \times x_3 \\
x_3 &= \frac{6.5864 - 1.1924}{6.2866} = 0.8580 \\
h_3 &= h_3 + x_3 \times h_{fg3} = 3769 + 0.8580 \times 2283.2 = 2335.88 \text{ kJ/kg}
\end{aligned}$$

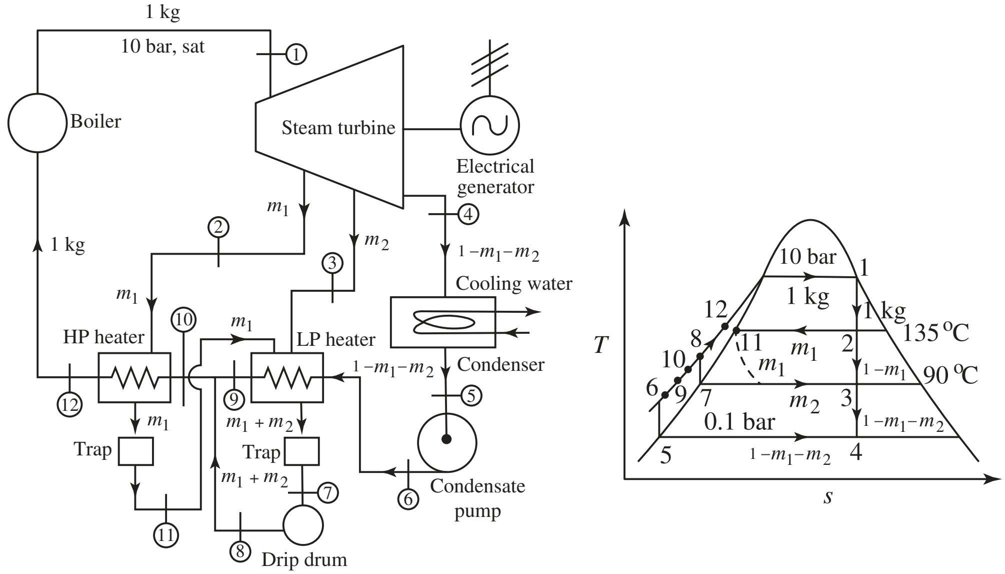
$$\begin{aligned}
s_4 &= s_5 + x_4 \times s_{fg4} \\
6.5864 &= 0.6492 + x_4 \times 7.5010 \\
x_4 &= \frac{6.5864 - 0.6492}{7.5010} = 0.7915 \\
h_4 &= h_5 + x_4 \times h_{fg4} \\
&= 191.81 + 0.7915 \times 2392.8 = 2085.71 \text{ kJ/kg} \\
\dot{m}_1 \times (2567.46 - 567.67) &= (1 - \dot{m}_1) \times (567.67 - 376.90) \\
1999.79 \times \dot{m}_1 &= 190.77 - 190.77 \times \dot{m}_1 \\
2190.56 \times \dot{m}_1 &= 190.77 \\
\dot{m}_1 &= \frac{190.77}{2190.56} = 0.087 \text{ kg/s} \\
\text{Now } \dot{m}_2(h_3 - h_7) &= (1 - \dot{m}_1 - \dot{m}_2)(h_7 - h_5) \\
\dot{m}_2 \times (2335.88 - 376.9) &= (1 - 0.087 - \dot{m}_2) \times (376.9 - 191.81) \\
1958.98 \times \dot{m}_2 &= (0.913 - \dot{m}_2) \times (185.09) \\
1958.98 \times \dot{m}_2 + 185.09 \times \dot{m}_2 &= 0.913 \times 185.09 \\
2144.07 \times \dot{m}_2 &= 168.99 \\
\dot{m}_2 &= \frac{168.99}{2144.07} = 0.079 \text{ kg/s}
\end{aligned}$$

$$W_T = (h_1 - h_2) + (1 - \dot{m}_1)(h_2 - h_3) + (1 - \dot{m}_1 - \dot{m}_2)(h_3 - h_7)$$

$$= (2778.1 - 2567.46) + (1 - 0.087)(2567.46 - 2335.88) + (1 - 0.087 - 0.079)(2335.88 - 2085.71)$$

$$\begin{aligned}
&= 210.64 + 211.43 + 208.64 = 630.71 \text{ kJ/kg} \\
&\cong W_{net} \quad \text{(Neglect pump work)} \\
Q_1 &= h_1 - h_9 = 2778.14 - 567.67 = 2210.47 \text{ kJ/kg} \\
\eta_{cycle} &= \frac{630.71}{2210.47} = 0.285 = 28.5\% \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\
\text{Steam rate} &= \frac{3600}{W_{net}} = \frac{3600}{630.71} = 5.71 \text{ kg/kW h} \\
x_{exhaust} &= x_4 = 0.7915 \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

11.14 Re-do the above problem with all the above conditions remaining the same but the heaters are now closed. Draw a schematic and T-s diagram.



(a) (b)

Fig. 11.25

Solution

From previous problem, $h_1 = 2778.1$ kJ/kg; $h_2 = 2567.46$ kJ/kg; $h_3 = 2335.89$ kJ/kg; $h_4 = 2085.71$ kJ/kg; and $h_{11} = h_{12} = 367.67$ kJ/kg. Take $h_7 \approx h_8 = h_9 \approx h_{10} = 376.90$ kJ/kg and $h_5 \approx h_6 = 191.81$ kJ/kg.

$$\begin{aligned}
\text{By energy balance, } \dot{m}_1(h_2 - h_{11}) &= 1(h_{12} - h_{10}) = h_{11} - h_{10} \\
\dot{m}_2(h_3 - h_7) &= \dot{m}_1(h_{11} - h_7) = 1(h_9 - h_6) \\
(1 - \dot{m}_1 - \dot{m}_2)h_9 + (\dot{m}_1 + \dot{m}_2)h_8 &= 1 \times h_{10} \\
\dot{m}_1(2567.46 - 567.67) &= (567.67 - 376.9) \\
\dot{m}_1 &= 0.0954 \text{ kg/s} \\
\dot{m}_2(2335.88 - 376.9) + 0.0954(567.67 - 376.9) &= 1(376.90 - 191.81) \\
1958.98 \times \dot{m}_2 + 18.2 &= 185.09 \\
\dot{m}_2 &= \frac{166.89}{1958.98} = 0.0852 \text{ kg/s}
\end{aligned}$$

$$\begin{aligned}
W_T &= 1(h_1 - h_2) + (1 - \dot{m}_1)(h_2 - h_3) + (1 - \dot{m}_1 - \dot{m}_2)(h_3 - h_4) \\
&= 1(2778.1 - 2567.46) + (1 - 0.0954)(2567.46 - 2335.88) \\
&\quad +1 - 0.0954 - 0.0852(2335.88 - 2085.71)
\end{aligned}$$

$$
\begin{aligned}
&= 210.64 + 209.49 + 204.99 = 625.12\\
W_p &= 1 \text{ kJ/kg} \quad \text{(from previous example)}\\
W_{net} &= 624.18 \text{ kJ/kg}\\
Q_1 &= h_1 - h_2 = 2778.1 - 567.67 = 2210.43\\
\eta_{cycle} &= \frac{624.12}{2210.43} = 0.2823 = 28.23\%\\
\text{Steam rate} &= \frac{3600}{624.12} = 5.77 \text{ kg/kW h}
\end{aligned}
$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

11.15 Assume that steam is available at 4.5 bar and 175 °C from a geothermal source. The steam leaves the turbine at 100 mm of Hg absolute pressure. Take the isentropic efficiency of the turbine as 0.75. Find the efficiency of the plant. If the unit produces 12.5 MW, what is the steam flow rate?

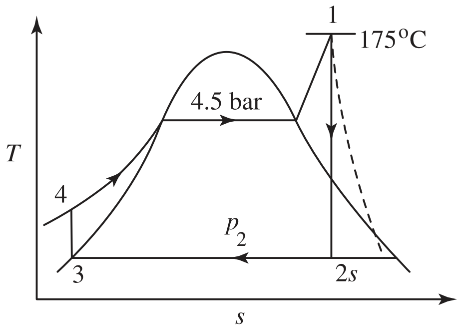


Fig. 11.26

Solution

$$
\begin{aligned}
p_2 &= \frac{100}{760} \times 101.325 = 13.33 \text{ kPa}\\
h_1 &= 2731 \text{ kJ/kg}\\
s_1 &= 7.0 = 0.7539 + 7.2561 \times x_{2s}\\
x_{2s} &= 0.86\\
h_{2s} &= 220 + 0.86 \times 2592 = 2449 \text{ kJ/kg}\\
h_3 &= 220 \text{ kJ/kg}\\
W_{net} &= W_T = \dot{m}_s(h_1 - h_2) = \dot{m}_s\eta_T(h_1 - h_{2s})\\
12.5 \times 10^3 &= \dot{m}_s \times 0.75 \times (2731 - 2449)\\
\dot{m}_s &= 59.1 \text{ kg/s}
\end{aligned}
$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

$$\eta_{cycle} = \frac{h_1 - h_2}{h_1 - h_3} = \frac{0.75 \times (2731 - 2449)}{2731 - 220} = 0.0842 = 8.42\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.16 Solar energy is used as a heat source in a simple steam power plant. Water enters the pump as a saturated liquid at 40 °C and exit pressure of water is 2 bar. The water from the pump enters the boiler and evaporates at the same pressure. Leaving the boiler, steam enters the turbine as a saturated vapour. Steam exits the turbine at 40 °C with 10% moisture. Mass flow rate of steam is 150 kg/h. Calculate (i) the turbine isentropic efficiency; (ii) the net work output; (iii) the cycle efficiency; and (iv) the area of the solar collector needed if the collections pick up 0.6 kW/m^2.

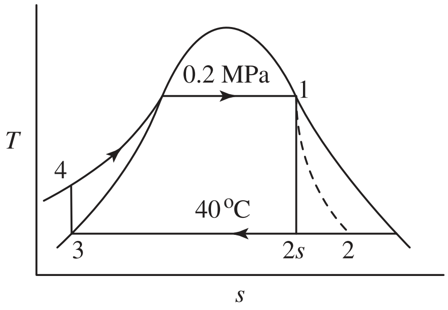


Fig. 11.27

Solution

Refer Fig.11.27 for the cycle operation, $x_2 = 0.90; m_s = 150$ kg/h; $h_1 = 2706.7$ kJ/kg; and $s_1 = 7.1271$ kJ/kg K.

$$h_2 = 167.57 + 0.90 \times 2406.7 = 2333.6 \text{ kJ/kg}$$

$$s_1 = s_{2s} = 7.1271 = 0.5725 + x_{2s} \times 7.6847$$

$$x_{2s} = 0.853$$

$$h_{2s} = 167.57 + 0.853 \times 2406.7 = 2220.34 \text{ kJ/kg}$$

$$h_T = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{2706.7 - 2333.6}{2706.7 - 2220.34} = \frac{373.1}{486.36} = 0.767 = 76.7\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$W_p = v\Delta P = 0.001 \times 0.2 \times 1000 = 0.2 \text{ kJ/kg}$$

$$W_{net} = W_T = 373.1 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\therefore \text{ Net work output} = \frac{h_1 - h_2}{h_1 - h_3} = \frac{373.1}{2539.13} = 0.1467 = 14.7\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$Q_1 = m_s(h_1 - h_3) = \frac{150}{3600} \times 2539.13 = 105.8 \text{ kW}$$

$$\text{Collector area} = \frac{105.8}{0.6} = 176.33 \text{ m}^2 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.17 A reheat cycle is working under the following conditions:

Maximum pressure = 150 bar; maximum temperature = 600 °C; condenser pressure = 0.1 bar and the moisture at the condenser inlet = 5.17%. Assume that steam expands to saturated state in first stage. Consider the cycle to be ideal. Determine (i) the reheat pressure; (ii) the cycle efficiency; and (iii) the steam flow rate. Draw T-s diagram of the cycle.

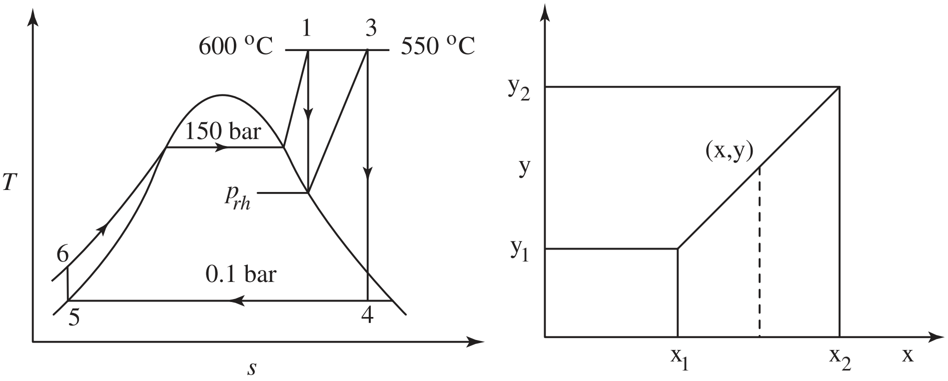


Fig. 11.28

Solution

T-s diagram is shown in Fig.11.28. Now let us take the values from the steam table at 150 bar and 600 °C, $h_1 = 3582.3$ kJ/kg K and $s_1 = 6.6775$ kJ/kg K.

Given that $x_4 = 0.95$ [refer Fig.11.28(a)], as process 3-4 is isentropic $s_4 = s_3$. We can find the value of s_4 by using the formula:

$$s_4 \quad = \quad s_{f5} + x s_{fg4}$$

From steam table at 0.1 bar, $s_{f5} = 0.6492$ kJ/kg K and $s_{fg4} = 7.5010$ kJ/kg K.

$$s_4 \quad = \quad 0.6492 + 0.9628 \times 7.5010 = 7.871 \text{ kJ/kg K}$$

As $s_4 = s_3$ and at 3, the steam is in superheated state. From superheated steam table corresponding to 7.811 kJ/kg K at 600 °C, the reheat pressure can be calculated as 1.4 MPa = 14 bar. Therefore, the reheat pressure = 14 bar. $h_3 = 3694.8$ kJ/kg and

$$h_4 \quad = \quad h_{f5} + x_4 h_{fg4}$$

From steam table at 0.1 bar $h_{f5} = 191.81$ kJ/kg and $h_{fg4} = 2392.8$ kJ/kg.

$$h_4 \quad = \quad 191.81 + 0.9628 \times 2392.8 = 2495.6 \text{ kJ/kg K}$$

$$h_6 \quad = \quad h_5 + W_p$$

W_p from previous example = 15 kJ/kg

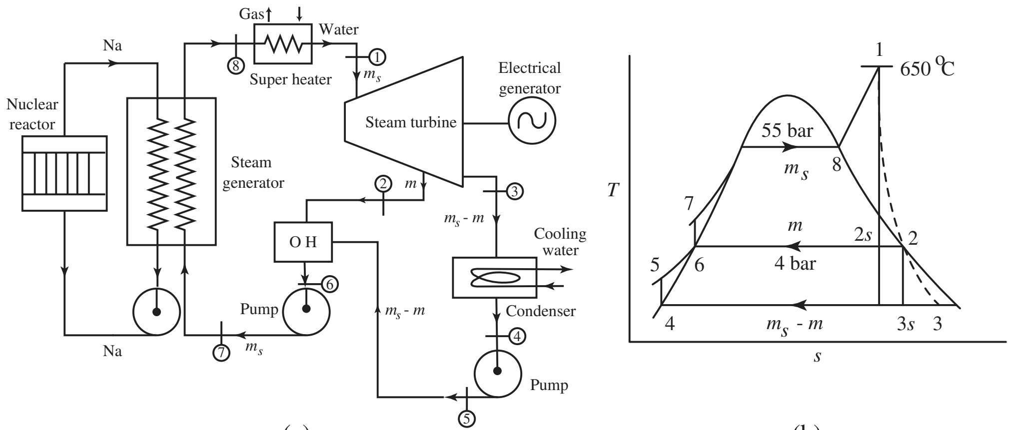
$$h_6 \quad = \quad 191.81 + 15 = 206.81 \text{ kJ/kg}$$

At 150 bar and 600 °C, from steam table, $h_1 = 3582.2$ kJ/kg K and $s_1 = 6.6775$ kJ/kg K.

At 14 bar, $s_g = 6.4692$ kJ/kg K. Let us assume that $h_2 \approx h_g = 2790$ kJ/kg:

$$\begin{aligned}
W_p &= h_6 - h_5 = 150 \times 100 \times 10^{-3} = 15 \text{ kJ/kg} \\
W_T &= h_1 - h_2 + h_3 - h_4 \\
&= (3582.2 - 2790) + (3694.8 - 2495.6) = 1991.4 \text{ kJ/kg} \\
Q &= (h_1 - h_6) + (h_3 - h_2) = (3582.2 - 206.81) + (3694.8 - 2495.6) \\
&= 4574.59 \text{ kJ/kg} \\
W_{net} &= W_T - w_p = 1991.4 - 15 = 1965.4 \text{ kJ/kg} \\
\eta_{cycle} &= \frac{1965.4}{4574.59} = 42.96 \approx 43\% \qquad \overset{\text{Ans}}{\Longleftarrow} \\
\text{Steam flow rate} &= \frac{3600}{1965.4} = 1.832 \text{ kg/kW h} \qquad \overset{\text{Ans}}{\Longleftarrow}
\end{aligned}$$

11.18 Liquid sodium is used in a nuclear power plant to which heat is transferred. The liquid sodium is than pumped into a heat exchanger where heat is transferred to steam. The steam leaves this heat exchanger as saturated vapour at 55 bar and then superheated in an external gas-fired superheater at 650 °C. The steam then enters the turbine, which has one extraction point at 4 bar where the steam flows to an open feedwater heater. The turbine efficiency is 75%. The condenser temperature is 40 °C. Determine the heat transfer in the reactor and in the superheater to produce a power output of 80 MW.



(a) (b)

Fig. 11.29

Solution

Refer Figs.11.29(a) and (b) for schematic and T-s diagram. Let us take the required values from the steam tables corresponding to the input conditions given. Note that if the values at the required conditions are not available in the table, then we may have to interpolate.

From steam table (Appendix A):

Interpolation for h:

At 5 MPa, 600 °C $\quad h_1 = 3666.5$ kJ/kg

At 6 MPa, 600 °C $\quad h_1 = 3658.4$ kJ/kg

At 5.5 MPa, 600 °C $\quad h_1 = \dfrac{3666.5 + 3658.4}{2} = 3662.45$ kJ/kg

At 5 MPa, 700 °C $\quad h_1 = 3900.1$ kJ/kg

At 6 MPa, 700 °C $\quad h_1 = 3894.3$ kJ/kg

At 5.5 MPa, 700 °C $\quad h = \dfrac{3900.1 + 3894.3}{2} = 3897.2$ kJ/kg

At 5.5 MPa, 650 °C $\quad h_1 = \dfrac{3662.45 + 3897.2}{2} = 3779.825$ kJ/kg

Interpolation for s

At 5 MPa, 600 °C $\quad s = 7.2588$ kJ/kg K

At 6 MPa, 600 °C $\quad s = 7.1676$ kJ/kg K

At 5.5 MPa, 600 °C $\quad s = \dfrac{7.2588 + 7.1676}{2} = 7.2132$ kJ/kg K

At 5 MPa, 700 °C $\quad s = 7.5122$ kJ/kg K

At 6 MPa, 700 °C $\quad s = 7.4234$ kJ/kg K

At 5.5 MPa, 700 °C $\quad s = \dfrac{7.5122 + 7.4234}{2} = 7.4678$ kJ/kg K

At 5.5 MPa, 650 °C $\quad s_1 = \dfrac{7.2132 + 7.4678}{2} = 7.3405$ kJ/kg K

As can be seen in Fig.11.29(b), $s_{2s} = s_1 = 7.3405$. From the steam table at 0.4 MPa (400 kPa), $s_g = 6.8958$. As $s_{2s} > s_g$, the steam at $2s$ is in the superheated region. From steam table, for superheated steam at 0.4 MPa and for $s_{2s} = 7.3405$ by interpolation:

$$h_{2s} = 2945.12 \text{ kJ/kg}$$

$$h_1 - h_{2s} = 3779.825 - 2945.12 = 834.705 \text{ kJ/kg}$$

$$h_1 - h_2 = 0.75 \times 834.705 = 626.03 \text{ kJ/kg}$$

$$h_2 = h_1 - 626.03 = 3779.825 - 626.03 = 3153.8 \text{ kJ/kg}$$

At 0.4 MPa and saturated state (from steam table), $s_g = s_2 = 6.8958$ kJ/kg K.

$$s_2 = s_{3s} = s_f + x_{3s} \times s_{fg}$$

From steam tables at the condensate temperature of 40 °C, $s_f = 0.5724$ kJ/kg K and $s_{fg} = 7.6845$ kJ/kg K. Therefore,

$$6.8598 = 0.5724 + x_{3s} \times 7.6845$$

$$x_{3s} = \frac{6.8598 - 0.5724}{7.6845} = 0.8182$$

$$h_{3s} = h_f + x_{3s} \times h_{fg} = 167.54 + 0.8182 \times 2406.7 = 2136.70 \text{ kJ/kg}$$

$$h_2 - h_{3s} = 3153.8 - 2136.70 = 1017.1 \text{ kJ/kg}$$

$$h_2 - h_3 = 0.75 \times 1071.1 = 762.825 \text{ kJ/kg}$$

By energy balance, work output of the turbine:

$$W_T = \dot{m}_s(h_1 - h_2) + (\dot{m}_s - \dot{m})(h_2 - h_3) = 80 \times 10^3 \text{ kJ/kg}$$

$$\dot{m}_s \times 626.03 + (\dot{m}_s - \dot{m}) \times 762.825 = 80 \times 10^3$$

$$626.03 + \left(1 - \frac{\dot{m}}{\dot{m}_s}\right)762.825 = \frac{80 \times 10^3}{\dot{m}_s} \tag{1}$$

Again, $$\dot{m}(h_2 - h_6) = (\dot{m}_s - \dot{m})(h_6 - h_5) \tag{2}$$

From steam table, at 0.4 MPa, $h_f = h_6 = 604.73$ kJ/kg

$$h_5 = h_{f4} + vdp$$

At 40 °C from steam tables, $h_{f4} = 167.54$ kJ/kg and $p_{sat} = 7.3837$ bar

$$vdp = 0.001 \times (400 - 7.3837) = 0.39 \text{ kJ/kg}$$

$$h_5 = 167.54 + 0.39 = 167.93 \text{ kJ/kg}$$

Substituting the values in Eq.(2), we have,

$$\dot{m}(3153.8 - 604.73) = (\dot{m}_s - \dot{m})(604.73 - 167.93)$$

$$\frac{\dot{m}_s - \dot{m}}{\dot{m}} = \frac{(3153.8 - 604.73)}{(604.73 - 167.93)} = 5.84$$

$$\frac{\dot{m}_s}{\dot{m}} - 1 = 5.84 \tag{3}$$

$$\frac{\dot{m}_s}{\dot{m}} = 6.84$$

$$\frac{\dot{m}}{\dot{m}_s} = \frac{1}{6.84} = 0.146$$

From Eq.(1), $626.03 + (1 - 0.146) \times 762.825 = \dfrac{80 \times 10^3}{\dot{m}_s}$

$$\dot{m}_s = 62.62 \text{ kg/s}$$

$$\dot{m} = \frac{\dot{m}_s}{7.84} = \frac{62.62}{6.84} = 9.15 \text{ kg/s}$$

$$h_7 - h_6 = vdp = 0.001 \times (5500 - 400) = 5.1 \text{ kJ/kg}$$

$$h_7 = h_6 + vdp = 604.73 + 5.1 = 609.83 \text{ kJ/kg}$$

Heat transfer in the rector, $HT_R = \dot{m}_s(h_8 - h_7)$

h_8 from the steam table:

$$h_8 = \frac{2794.3 + 2784.3}{2} = 2789.3 \text{ kJ/kg}$$

$$\begin{aligned} HT_R &= 62.62 \times (2789.3 - 609.83) \\ &= 136478 \text{ W} \approx 136.5 \text{ kW} \end{aligned}$$

Heat transfer to the superheater:

$$\begin{aligned} HT_{SH} = \dot{m}_s(h_1 - h_8) &= 61.94 \times (3779.825 - 2789.3) = 61353 \text{ W} \\ &= 61.353 \text{ kW} \end{aligned}$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

11.19 A high-speed turbine working as a reheat cycle is used to expand steam at 500 °C until it is saturated vapour. The steam is reheated to a temperature of 400 °C at constant pressure. Then, it is expanded in a low-pressure turbine to a temperature of 40 °C. The moisture content in the turbine exhaust is limited to 15%. Calculate (i) the reheat pressure; (ii) the pressure of steam at the inlet to the hp turbine; (iii) the net specific work output; (iv) the cycle efficiency; and (v) the steam rate. Assume that all processes are ideal. Had there been no reheat, what would be the quality, the work output and the cycle efficiency. Assume that all other conditions remain the same.

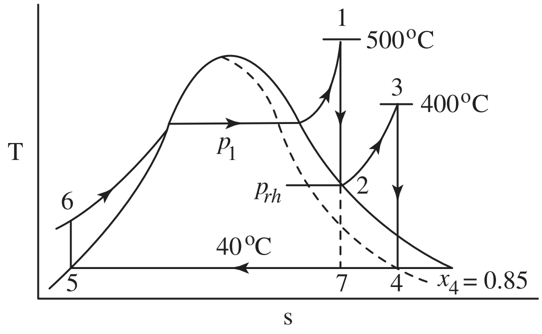


Fig. 11.30

Solution

Refer Fig.11.30. From steam table, h_5 = 167.54 kJ/kg; s_5 = 0.5724 kJ/kg K; h_{fg4} = 2406.7 kJ/kg; and $s_{fg4} = 7.6845 kJ/kg\ K$:

$$\begin{aligned} x_4 &= (1 - 0.15) = 0.85 \\ h_4 &= h_5 + x_4 h_{fg4} = 167.45 + 0.85 \times 2406.7 = 2213.24 \text{ kJ/kg} \\ s_4 &= s_5 + x_4 s_{fg4} = 0.5724 + 0.85 \times 7.6845 = 7.1042 \text{ kJ/kg K} \end{aligned}$$

$$s_4 = s_3 = 7.1042 \text{ kJ/kg K}$$

$$t_3 = 400°\text{C (given)}$$

From steam table, for superheated steam at 3, $s_3 = 7.1042$ kJ/kg K and $t_3 = 400$ °C. The reheat pressure can be read as $p_{rh} \approx 2.0$ MPa = 20 bar; $h_3 = 3247.6$ kJ/kg; $s_2 = s_g = 6.3408$ kJ/kg K; and $h_2 = h_g = 2799.5$ kJ/kg.

From superheated steam table, where $t_1 = 500$ °C and $s_1 = 6.3408$ kJ/kg K; $p_1 \approx 15$ MPa = 150 bar; and $h_1 = 3308.5$ kJ/kg:

$$h_6 - h_5 = v\ dp = 0.001 \times 15000 = 15 \text{ kJ/kg}$$

$$h_6 = 182.54 \text{ kJ/kg}$$

$$W_{net} = W_T - W_p = (h_1 - h_2) + (h_3 - h_4) - (h_6 - h_5)$$

$$= (3308.5 - 2799.5) + (3247.6 - 2213.4) - 15 = 1528.36 \text{ kJ/kg}$$

$$Q_1 = (h_1 - h_6) + (h_3 - h_2) = (3308.5 - 182.5) + (3257.6 - 2799.5)$$

$$= 3584 \text{ kJ/kg}$$

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{1528.36}{3584} = 0.4264 = 42.64\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Steam rate} = \frac{3600}{W_{net}} = \frac{3600}{1528.36} = 2.355 \text{ kg/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Without reheat, } s_1 = s_2 = s_7 = 6.3408 \text{ kJ/kg} = s_1 + x_7 s_{fg7}$$

$$6.3408 = 0.5724 + x_7 \times 7.6845$$

$$x_7 = \frac{6.3408 - 0.5724}{7.6845} = 0.75$$

$$h_7 = h_5 + x_7 h_{fg7}$$

From steam table, $h_5 = 167.54$ and $h_{fg7} = 2406.7$.

$$h_7 = 167.54 + 0.75 \times 2406.7 = 1974.14 \text{ kJ/kg}$$

$$W_T = h_1 - h_7 = 3308.5 - 7974.14 = 1334.36 \text{ kJ/kg}$$

$$W_{net} = 1334.36 - 15 = 1319.36 \text{ kJ/kg}$$

$$Q_1 = h_1 - h_6 = 3308.5 - 182.54 = 3126 \text{ kJ/kg}$$

$$\eta_{cycle} = \frac{W_{net}}{Q_1} = \frac{1319.36}{3126} = 0.422 = 42.2\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

11.20 A steam power plant is working on a regenerative cycle. Steam is supplied at 30 bar and 300 °C. Condenser pressure is 0.08 bar. Steam is extracted at two points for the two heaters at 3.5 and 0.7 bar, respectively. Of the two heaters, one is closed and the other one is open type. Calculate the thermal efficiency of the plant. Neglect pump work.

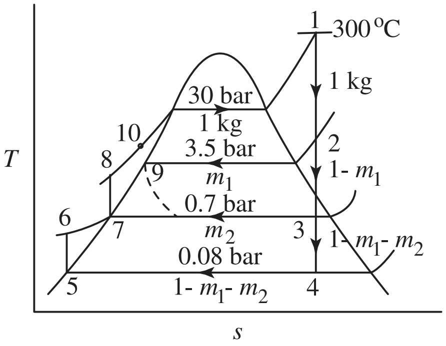


Fig. 11.31

Solution

Refer Fig.11.31. It can be seen that steam is expanding from boiler pressure of 30 bar to the condenser pressure of 0.08 bar. There are two bleedings and two heaters are used. Now let us take down the various properties from the steam table and put it in the following table:

$p_1 = 30$ bar
$h_1 = 2993.5$ kJ/kg
$s_1 = 6.539$ kJ/kg K
$s_1 = s_2 = s_3 = s_4$

$p_2 = 3.5$ bar
$h_9 = h_f = 584.38$ kJ/kg
$h_{fg2} = 2148.1$ kJ/kg
$s_g = 6.9405$ kJ/kg K
$s_9 = s_f = 1.7275$ kJ/kg K
$s_{fg2} = 5.213$ kJ/kg K

$p_3 = 0.7$ bar
$h_7 = h_f = 376.68$ kJ/kg
$h_{fg3} = 2283.47$ kJ/kg
$s_g = 7.4809$ kJ/kg K
$s_7 = s_f = 1.1918$ kJ/kg K
$s_{fg3} = 7.64$ kJ/kg K

$p_4 = 0.08$ bar
$h_5 = h_6 = h_f = 173.49$ kJ/kg (as $W_p = 0$)
$h_7 = h_8 = 376.68$ kJ/kg
$h_{fg} = 2403.4$ kJ/kg
$s_5 = xs_f = 0.592$ kJ/kg K
$s_{fg} = 7.640$ kJ/kg K

$$s_2 = s_1 = s_4 + x_2 \times 4 \times S_{fg_2}$$

$$6.539 = 1.7275 \times x_2 \times 5.213$$

$$x_2 = \frac{(6.539 - 1.7275)}{5.213} = 0.923$$

$$s_3 = s_1 = s_7 + x_3 \times s_{fg_3}$$

$$6.539 = 1.1918 + x_3 \times 6.2891$$

$$x_3 = \frac{(6.539 - 1.1918)}{6.2891} = 0.85$$

$$s_4 = s_1 = s_5 + x_4 \times s_{fg_4}$$

$$6.539 = 0.592 + x_4 \times 7.64$$

$$x_4 = \frac{(6.539 - 0.592)}{7.64} = 0.778$$

$$h_2 = h_9 + x_2 \times h_{fg_2} = 584.38 + 0.923 \times 2148.1 = 2567.0 \text{ kJ/kg}$$

$$h_3 = h_7 + x_3 \times h_{fg_3} = 173.49 + 0.85 \times 2403.4 = 2317.6 \text{ kJ/kg}$$

$$h_4 = h_5 + x_4 \times h_{fg_4} = 173.49 + 0.778 \times 2403.4 = 2043.4 \text{ kJ/kg}$$

By energy balance,

$$m_1(h_2 - h_9) = 1(h_9 - h_7)$$

$$m_1 = \frac{h_4 - h_9}{h_2 - h_9} = \frac{584.39 - 376.68}{2567 - 584.38} = 0.1047 \text{ kg}$$

$m_2(h_3 - h_7) + m_1(h_9 - h_7) = (1 - m_1 - m_2)(h_7 - h_5)$

Substituting various values and solving we get:

$$m_2 = 0.095 \text{ kg}$$

$$\begin{aligned} W_T &= 1(h_1 - h_2) + (1 - m_1)(h_2 - h_3) + (1 - m_1 - m_2)(h_3 - h_4) \\ &= (2993.5 - 2567) + 0.8953 \times (2567 - 2317.6) + \\ &\quad 0.8003 \times (2317.6 - 2043.4) \\ &= 426.5 + 223.3 + 219.4 = 869.2 \text{ kJ/kg} \end{aligned}$$

$$Q_1 = (h_1 - h_9) = 2993.5 - 584.39 = 2409.1 \text{ kJ/kg}$$

$$\eta_{cycle} = \frac{W_T}{Q_1} = \frac{869.2}{2409.2} = 0.36 = 36\%$$

Ans ⟸

11.21 A thermal power plant is working on a reheat generative cycle and produces 100 MW. Steam enters high-pressure turbine (HPT) at 90 bar, 550 °C and expands to 7 bar. Some of the steam is diverted to an open heater. Rest of the steam is reheated to 400 °C. The reheated steam expands to 0.07 bar.

(i) Calculate the steam flow rate to the hp turbine.

(ii) Estimate the pump work.

(iii) Find the cycle efficiency.

(iv) If there is a 10 °C rise in the temperature of the cooling water, calculate the rate of flow of the cooling water in the condenser.

(v) If the velocity of steam flowing from the turbine to the condenser is limited to 130 m/s, find the diameter of the connecting tube.

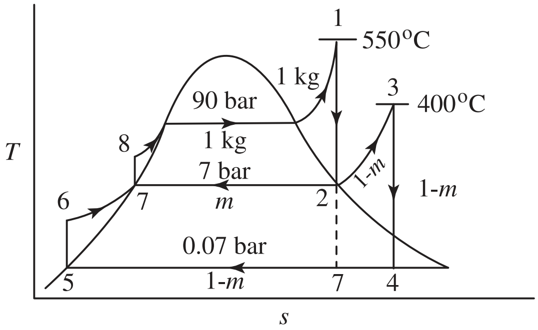


Fig. 11.32

Solution

Refer Fig.11.32. It is a reheat and regenerative cycle. Let us take down the properties at the salient points from the following table.

$p_1 = 90$ bar and $t_1 = 550$°C	$p_2 = 7$ bar
$h_1 = 3511.0$ kJ/kg	$h_2 = h_g = 2763.5$ kJ/kg
$s_1 = 6.8142$ kJ/kg K	$s_g = 6.7080 kJ/kg\ K = s_1 = s_2$ As steam is expanded to saturation condition: let $s_2 = s_g = 6.7080$ kJ/kg K
$p_2 = 7$ bar and $t_1 = 400$ °C	At $p_3 = 0.07$ bar
$h_3 = 3268.9$ kJ/kg	$h_5 = 163.16$ kJ/kg
$s_3 = 7.6372$ kJ/kg K	$s_3 = s_4 = 7.6372$ kJ/kg K
$s_3 = s_4$	$s_5 = s_{f_5} = 0.5582$ kJ/kg K
$h_f = h_7 = 697.0$ kJ/kg	$s_{fg_4} = 7.72$ kJ/kg K
$s_7 = s_f = 1.1918$ kJ/kg K	
$s_{fg3} = 7.64$ kJ/kg K	

$$s_3 = s_4 = s_5 + x_5 \times s_{fg_4}$$

$$7.6372 = 0.5582 + x_5 \times 7.72$$

$$x_5 = \frac{7.6372 - 0.5582}{7.72} = 0.917$$

$$h_4 = h_5 + x_4 \times h_{fg_4}$$

$$= 163.16 + 0.917 \times 2409.54 = 2372.63 \text{ kJ/kg}$$

Neglect pump work then $h_6 = h_5$. By energy balance,

$$(1-m)(h_7 - h_6) = m(h_2 - h_7)$$

$$\frac{1-m}{m} = \frac{h_2 - h_7}{h_7 - h_6} = \frac{2763.5 - 697}{697 - 163.16} = 3.87$$

$$\begin{aligned}
\frac{1}{m} - 1 &= 3.87 \\
m &= \frac{1}{4.87} = 0.2058 \text{ kg} \\
W_T &= (h_1 - h_2) + (1 - m)(h_3 - h_4) \\
&= (3511.0 - 2763.5) + (1 - 0.2058) \times (3268.9 - 2372.63) \\
&= 1459.77 \text{ kJ/kg} \\
W_p &= 8300 \times 0.001 + 700 \times 0.001 = 9 \text{ kJ/kg} \\
W_{net} &= 1459.77 - 9 = 1450.77 \text{ kJ/kg} \\
m_s \times 1450.77 &= 100 \times 1000 \text{ kW} \\
\dot{m}_s &= \frac{10^5}{1450.77} = 68.93 \text{ kg/s} \\
W_p &= 9 \times 68.93 = 620.37 \text{ kW} \\
Q_1 &= (h_1 - h_8) + (1 - m)(h_3 - h_2) \\
&= (3511 - 705.3) + 0.7947 \times (3268.9 - 2763.5) = 1755.87 \text{ kJ/kg} \\
\eta_{cycle} &= \frac{1450.77}{3207.34} = 0.4523 = 45.23\% \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\
Q_2 &= (1 - m)(h_4 - h_5) = 0.7947 \times (2372.63 - 163.16) \\
&= 1755.87 \text{ kJ/kg} \\
\dot{m}_s \times 1755.87 &= \dot{m}_c c_p \times (t_{c_2} - t_{c_1}) \\
68.93 \times 1755.87 &= m_c \times 4.187 \times 10 \\
\dot{m}_c &= \frac{68.93 \times 1755.87}{4.187 \times 10} = 2890.66 \text{ kg/s} \\
v_4 &= v_f + x_4 \times v_{fg} = 0.001 + 0.917 \times 20.6128 = 18.9 \text{ m}^3\text{/kg}
\end{aligned}$$

Volume flow rate of exhaust steam:

$$\begin{aligned}
&= (1 - m)m_s \times v_4 = 0.7947 \times 18.9 \times 68.93 = 1035.42 \text{ m/s} \\
1035.42 &= \frac{\Pi}{4} d^2 \times \mathbb{V} \\
d^2 &= \frac{4 \times 1035.42}{\Pi \times 130} = 10.181 \text{ m}^2 \\
d &= \sqrt{1}0.181 = 3.184 \text{ m}
\end{aligned}$$

11.22 A mercury cycle is superimposed on the steam cycle operating between the boiler outlet conditions of 40 bar, 400 °C and the condenser temperature of 40 °C. The heat released by mercury condensing at 0.2 bar is used to import the latent heat of vaporisation to the water in the steam cycle. Mercury enters the mercury turbine as saturated vapour at 10 bar. Compute:

(i) kilogram of mercury circulated per kilogram of water and

(ii) the efficiency of the combined cycle.

The property values of saturated mercury are given below:

p (bar)	t (°C)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kgK)	s_g (kJ/kgK)	v_f (m^3/kg)	v_g (m^3/kg)
10	515.5	72.23	36.3	0.1478	0.5167	80.9×10^{-6}	0.0333
0.2	277.3	38.35	336.55	0.0967	0.6385	77.4×10^{-6}	1.163

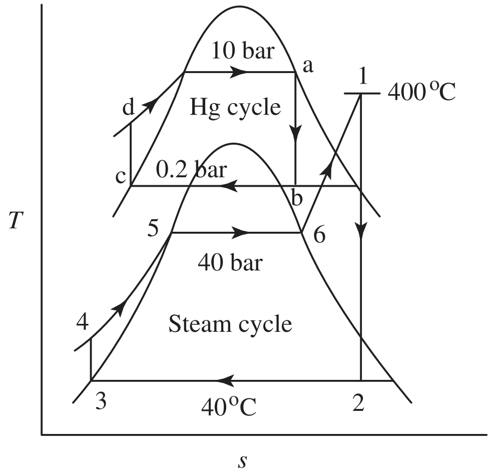


Fig. 11.33

Solution

Refer Fig.11.33 which gives T - s diagrams of the cycle. The property values of the mercury is given. First consider the mercury cycle.

$$h_a = 363 \text{ kJ/kg}; \quad s_a = s_b = 0.5167 \text{ kJ/kg K}$$

$$s_b = s_c + x_b \times s_{fg_b} = s_c + x_b \times (s_g - s_f)_b$$

$$0.5167 = 0.0967 + x_b \times (0.6385 - 0.0967)$$

$$x_b = \frac{0.5167 - 0.0967}{0.6385 - 0.0967} = 0.7753$$

$$h_b = h_c + x_b(h_g - h_f)$$

$$= 38.35 + 0.7753 \times (336.55 - 38.35) = 269.56 \text{ kJ/kg}$$

Neglecting pump work, $h_c = h_d = 38.35$ kJ/kg. Consider the steam cycle. From steam table:

$h_1 = 3213.6$ kJ/kg

$$\begin{aligned}
s_1 &= s_2 = 6.769 \text{ kJ/kg K} \\
s_2 &= s_3 + x_2 s_{fg2} \\
6.769 &= 0.5725 + x_2 \times 7.6845 \\
x_2 &= \frac{6.769 - 0.5725}{7.6845} = 0.8064 \\
h_2 &= h_3 + x_2 h_{fg2}
\end{aligned}$$

Substituting the values from steam table:

$$h_2 = 167.57 + 0.8064 \times 2406.7 = 2108.33 \text{ kJ/kg}$$

Let us assume that for 1 kg of steam, there is m kg of mercury flow. By energy balance:

$$1 \times (h_6 - h_5) = m \times (h_a - h_c)$$

As $h_6 - h_5 = h_{fg}$ of steam at 40 bar:

$$m = \frac{h_{fg}}{h_b - h_c} = \frac{1714.1}{284.16 - 38.35} = 7.4136 \text{ kg}$$

Work output of mercury turbine:

$$(W_T)_{hg} = m(h_a - h_b) = 7.4136 \times (363 - 269.56) = 692.73 \text{ kW}$$

Work output of the steam turbine:

$$\begin{aligned}
(W_T)_{steam} &= h_1 - h_2 = 3213.6 - 2108.33 = 1105.27 \text{ kJ/kg} \\
(W_T)_{total} &= 692.73 + 1105.27 = 1798 \text{ kW} \\
\dot{Q}_1 &= m(h_a - h_d) + (h_1 - h_6) + (h_5 - h_4) \\
&= 7.4136 \times (363 - 38.35) + (3213.6 - 2801.4) + (1087.31 - 167.57) \\
&= 2406.83 + 413.2 + 910.74 = 3738.77 \text{ kJ/kg} \\
&= 3738.77 \text{ kW for 1 kg/s of steam flow} \\
\eta_{cycle} &= \frac{W_{total}}{Q_1} = \frac{1798}{3738.77} = 0.4811 = 48.11\%
\end{aligned}$$

$\underset{\Longleftarrow}{\textbf{Ans}}$

11.23 A power plant generating electricity is working on a binary vapour cycle. Mercury is used in upper cycle and steam in the lower cycle. The ratio of mercury flow rate to steam flow rate is 10:1 on mass basis. At an evaporation of 10^6 kg/h for mercury, its specific enthalpy rises by 356 kJ/kg in passing through the boiler furnace adds 586 kJ to the steam specific enthalpy. The mercury gives up 251.2 kJ/kg during condensation, and the steam gives up 2003 kJ/kg in its condenser. The overall boiler efficiency is 85%. The combined turbine mechanical and generator efficiencies are each 95% for the mercury and steam units. The steam auxiliaries require 5% of the energy generated by the units. Estimate the overall efficiency of the plant.

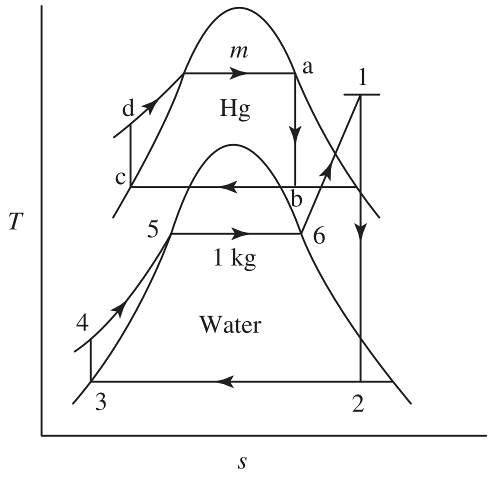


Fig. 11.34

Solution

Refer Fig.11.34. a – b – c – d – a is the mercury cycle; 1 – 2 – 3 – 4 – 5 – 6 – 1 is the steam cycle.

Specific enthalpy rise of mercury $= h_a - h_d = 436 \text{ kJ/kg}$

Specific enthalpy rise of steam $= h_1 - h_6 = 586 \text{ kJ/kg}$

Specific enthalpy drop of mercury while condensation

$$= h_b - h_c = 251.2 \text{ kJ/kg}$$

Specific enthalpy drop of steam $= h_2 - h_3 = 2003 \text{ kJ/kg}$

$$\eta_B = 0.85$$

Turbine efficiency, $(\eta_T)_{Hg} = (\eta_T)_{steam} = 0.95$

$$\eta_{aux} = 0.95$$

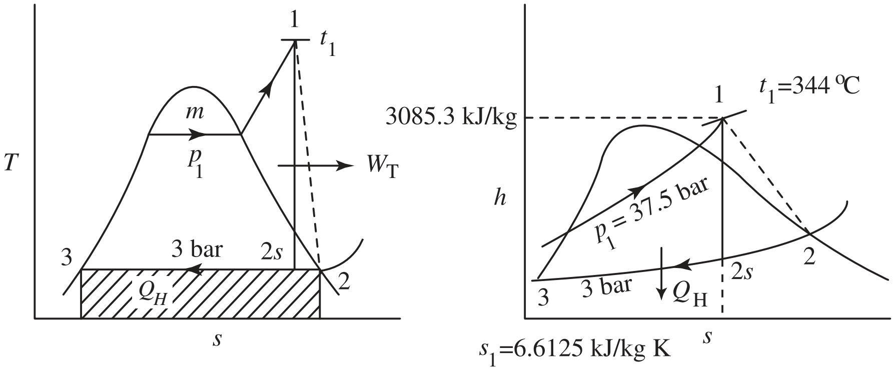
Heat input, $Q_1 = m_{Hg} \times (h_a - h_d) + m_{st} \times (h_1 - h_2)$

$$= 456 \times \frac{10^6}{3600} + \frac{10^6}{10 \times 3600} \times 586$$

$$= 126666.67 + 16277.78 = 142944.45 \text{ kW}$$

$$
\begin{aligned}
Q_2 = \frac{10^6}{10\times 3600}(h_2 - h_3) \quad &= \quad \frac{10^5}{3600}\times 2003 = 55638.89 \text{ kW}\\
\eta_{cycle} \quad &= \quad 1-\frac{Q_2}{Q_1} = 1-\frac{55638.89}{142944.45} = 0.6108 = 61.08\%\\
\text{Overall efficiency, } \eta_{overall} \quad &= \quad \eta_{cycle}\times\eta_{boiler}\times(\eta_T)_{Hg}\times(\eta_T)_{steam}\times\eta_{aux}\\
&= \quad 0.6108\times 0.85\times 0.95\times 0.95\times 0.95\\
&= \quad 0.4686 = 46.86\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

11.24 A processing factory requires 10 tonne/h of saturated steam for process heating at 3 bar. A back pressure turbine of 70% internal efficiency is employed. Estimate steam condition required at the inlet to the turbine. The power requirement is 1 MW.



(a) (b)

Fig. 11.35

Solution

Refer Fig.11.35(a). Let m be the mass flow rate of steam.

$$
\begin{aligned}
m(h_1 - h_2) \quad &= \quad 1000 \text{ kW}\\
h_1 - h_2 \quad &= \quad \frac{1000\times 3600}{10\times 1000} = 360 \text{ kJ/kg}
\end{aligned}
$$

From steam table, at the pressure of 3 bar, $h_g = h_2 = 2725.3$ kJ/kg; $s_f = s_3 = 1.6718$ kJ/kg K and $h_f = h_g = 361.45$ kJ/kg.

$$
\begin{aligned}
h_1 \quad &= \quad h_2 + 360 = 6725.3 + 360 = 3085.3 \text{ kJ/kg}\\
s_{fg_{2s}} \quad &= \quad 3.3201 kJ/kg\ K; \quad h_{fg_{2s}} = 2163.8 \text{ kJ/kg}\\
h_1 - h_{2s} \quad &= \quad \frac{360}{0.7} = 514.286 \text{ kJ/kg}\\
\therefore h_{2s} \quad &= \quad h_1 - 514.286 = 3085.2 - 514.286 = 2571.014 \text{ kJ/kg}
\end{aligned}
$$

$$h_{2s} = h_3 + x_{2s} h_{fg_{2s}}$$

$$2571.016 = 561.45 + x_{2s} \times 2163.8$$

$$x_{2s} = \frac{2571.014 - 561.45}{2163.8} = 0.9287$$

$$s_{2s} = s_3 + x_{2s} \times s_{fg_{2s}} = 1.6718 + 0.9287 \times 5.3201 = 6.6125 \text{ kJ/kg K}$$

$$s_{2s} = s_1$$

Corresponding to $h_1 = 3085.3$ kJ/kg and $s_1 = 6.6125$ kJ/kg K, from Mollier diagram [refer Fig.11.35(a)]:

$p_1 = 37.5$ bar and $t_1 = 344$ °C, which is the state of steam at turbine inlet.

11.25 A 10 MW steam turbine operates with steam at the inlet at 4000 kPa and 400 °C and exhausts at 10 kPa. Ten tonnes per hour of steam at 300 kPa is to be extracted for process work. Isentropic efficiency of turbine is 75% throughout. Calculate the boiler capacity.

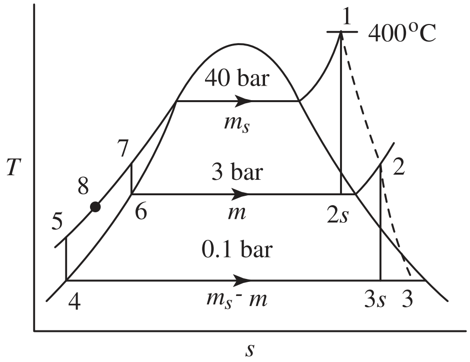


Fig. 11.36

Solution

Refer Fig.11.36. As 10 tonnes/hour of steam is required for process work,

$$m = 10 \text{ tonnes/hour} = 10 \times 10^3 = 10000 \text{ kg/h}$$

The capacity of the power plant = 10 MW = 10000 kW. Referring to Fig.11.36, we can write

$$m_s(h_1 - h_2) + (m_s - m)(h_2 - h_3) = 10000$$

From steam table at 40 bar and 400 °C, $h_1 = 3215.9$ kJ/kg; $s_1 = 6.7743$ kJ/kg K

$$s_1 = s_{2s} = s_6 + x_{2s} s_{fg_{2s}}$$

From steam table, $s_{f6} = s_6 = 1.6714$ and $s_{fg} = s_{fg6} = 5.3198$ kJ/kg K:

$$
\begin{aligned}
6.7743 &= 1.6714 + x_{2s} \times 5.3198 \\
x_{2s} &= \frac{6.7743 - 1.6714}{5.3198} = 0.9592 \\
h_{2s} &= h_6 + x_{2s} \times h_{fg_{2s}}
\end{aligned}
$$

From steam table, at 3 bar, $h_f = h_6 = 561.34$ and $h_{fg_{2s}} = 2163.37$; $h_g = 2725.3$:

$$
\begin{aligned}
h_{2s} &= 561.34 + 0.9592 \times 2163.37 = 2636.5 \text{ kJ/kg} \\
\eta_T &= \frac{h_1 - h_2}{h_1 - h_{2s}} \\
h_1 - h_2 &= \eta_T \times (h_1 - h_{2s}) \\
h_1 - h_2 &= 0.75 \times (3215.9 - 2636.5) = 434.55 \text{ kJ/kg} \\
h_2 &= h_1 - 434.55 = 3215.9 - 434.55 = 2781.35 \text{ kJ/kg}
\end{aligned}
$$

As $h_2 > h_g$, the steam is at superheated state. From superheated steam table, corresponding to $h_2 = 2781.35$, by interpolation $t_2 = 170$ °C; $s_2 = 7.08$ kJ/kg K:

$$
s_2 = s_{3s} = s_4 + x_{3s} s_{fg_{3s}}
$$

Again from steam table at 0.1 bar $s_4 = 0.6479$ and $s_{fg} = 7.5055$ kJ/kg K:

$$
\begin{aligned}
7.08 &= 0.6479 + x_{3s} \times 7.5055 \\
x_{3s} &= \frac{7.08 - 0.6479}{7.5055} = 0.857 \\
h_{3s} &= h_4 + x_{3s} \times h_{fg_{3s}}
\end{aligned}
$$

From steam table, $h_f = h_4 = 191.45$ kJ/kg and $h_{fg_{3s}} = 2393.29$ kJ/kg:

$$
\begin{aligned}
h_{3s} &= 191.45 + 0.857 \times 2393.29 = 2242.46 \text{ kJ/kg} \\
h_1 - h_3 &= 0.75(h_1 - h_{3s}) \\
&= 0.75 \times (2781.35 - 2243.46) = 404.17 \\
h_3 &= 2781.35 - 404.17 = 2377.18 \text{ kJ/kg}
\end{aligned}
$$

$$\dot{m}_s(h_1 - h_2) + (\dot{m}_s - 2.78)(h_3 - h_4) = 10000$$

$$\dot{m}_s(3215.9 - 2781.35) + (\dot{m}_s - 2.78) \times (2781.35 - 2377.18) = 10000$$

$$
\begin{aligned}
\dot{m}_s(434.55 - 404.17) &= 10000 + 1123.59 \\
\dot{m}_s &= 13.263 \text{ kg/s} = 47745.2 \text{ kg/h} \\
&= 47.745 \text{tonne/hour}
\end{aligned}
$$

$\overset{\textbf{Ans}}{\Longleftarrow}$

Review Questions

11.1 Explain the working principle of a steam power plant.

11.2 Draw the schematic as well as the T-s and p-V diagram of a Carnot steam cycle and explain the various processes associated with it. Mention the limitations of the Carnot cycle.

11.3 Draw the T-s and p-V diagram of an ideal Rankine cycle and explain.

11.4 Derive the expression for efficiency based on the T-s diagram.

11.5 Draw the T-s diagram of the Rankine cycle without feed pump.

11.6 Define work ratio and develop an expression for the same.

11.7 Compare Rankine and Carnot cycles.

11.8 Draw the plots of efficiency and specific steam rate with respect to boiler pressure for Rankine and Carnot cycle and explain.

11.9 With appropriate graphs, explain the variation of efficiency of Rankine cycle with respect to (i) boiler pressure (ii) degree of superheat and (iii) condenser pressure.

11.10 Mention the methods by which the efficiency of the Rankine cycle can be improved.

11.11 Draw the actual Rankine cycle on a T-s plane and explain how it is different from ideal Rankine cycle.

11.12 Define efficiency ratio.

11.13 Draw the modified Rankine cycle on the p-V and T-s diagram and explain. Develop an expression for the modified Rankine cycle efficiency.

11.14 With a suitable diagram on a T-s plane, explain the reheat Rankine cycle.

11.15 Develop an expression for efficiency of a reheat Rankine cycle.

11.16 What do you understand by regeneration. How is it different from reheat cycle?

11.17 Draw the schematic of a regenerative cycle and explain.

11.18 With a neat sketch explain the working of a single-stage Rankine cycle.

11.19 Derive an expression for efficiency of a single-stage regenerative cycle.

11.20 Draw the various processes on T-s diagram for a two stage regenerative cycle.

11.21 Explain what do you understand by binary vapour cycle. What are the various desirable properties of a binary fluid?

11.22 Can mercury be used as a binary fluid. What are its various desirable properties?

11.23 What are the undesirable properties of mercury when used as the binary fluid?

11.24 Draw the line diagram of a binary vapour cycle and analyse the same.

11.25 Define the various efficiencies of a steam power plant and explain their importance.

Exercise

11.1 A steam power plant of 50 MW capacity operates with steam at the inlet at 60 bar, 450 °C and the steam exits at 0.1 bar with 80% turbine efficiency. It is proposed to replace the above power plant with a new plant with bodies and a topping turbine efficiency of 85% operating with inlet steam at 180 bar and 500 °C. The exhaust from the topping turbine at 60 bar is reheated to 450 °C and admitted to the old turbine. The flow rate is just sufficient to produce the rated output from the old turbine. Calculate the improvement in efficiency with the new setup. What is the additional power developed.
Ans: 5%; 239.34 kJ/kg; 12.82 MW

11.2 Initial pressure and temperature of a steam power plant is 20 bar and 400 °C. It exhausts to a heating system at 2 bar. The condensate is returned to the boiler at 65 °C from the heating system. For this, the heating system utilises 90% of the energy transferred from the steam it receives. The turbine efficiency is 70%. Calculate

(i) fraction of the energy supplied to the steam plant serving for a useful purpose and

(ii) the fraction of the energy supplied would have served a useful purpose if two separate steam plants had been set up to produce the same useful energy, one generating steam at 2 bar and the other to generate power through a cycle working between 20 bar 400 °C and 0.07 bar. *Ans:* 91.34%; 55.3%

11.3 Saturated steam enters a *hp* turbine at 30 bar and expands isentropically to a pressure at which its quality is 0.841. At this pressure, the steam is passed through a moisture separator which removes all the liquid saturated vapour leaves the separator and expands isentropically to 0.04 bar in law pressure turbine. The saturated liquid leaves the separator and returns via feed pump to the boiler. The condensate leaving the condenser at 0.04 bar is also returned to the boiler via second feed pump. Calculate the cycle efficiency and turbine outlet quality taking into account the feed pump term.
Ans: 35.26%; 0.823

11.4 Steam enters a regenerative-reheat cycle whose power output is 80 MW. Steam enters the *hp* turbine at 80 bar and 500 °C and expands till it becomes saturated vapour. Some of the steam then goes to an open feedwater heater and the balance is reheated to 400 °C. Then, it expands in the *lp* turbine to 0.07 bar. Compute

(i) the reheat pressure,

(ii) the steam flow rate to the *hp* turbine, and

(iii) cycle efficiency.

Neglect pump work. *Ans:* (i) 6.5 bar; (ii) 59.32 kg/s; (iii) 43.2%

11.5 In a combined power and process plant the boiler generates 21000 kg/h of steam at a pressure of 17 bar and temperature 230 °C. A part of the steam goes to a process heater which consumes 132.56 kW, the steam leaving the process heater is 0.957 dry at 17 bar being throttled to 3.5 bar. The remaining steam flows through a *HP* turbine which exhausts at a pressure of 3.5 bar. The exhaust steam mixes with the process steam before entering *LP* turbine which develops 1337.5 kW. At the exhaust, the pressure is 0.3 bar and the steam is 0.912 dry. Neglect pump work. Draw a line diagram of the plant and determine

(i) the steam quality at the exhaust from the *HP* turbine,
(ii) the power developed by the *HP* turbine, and
(iii) the isentropic efficiency of the *HP* turbine.

Ans: (i) 0.956; (ii) 1154.54 kW; (iii) 76.44%;

11.6 There are two loads to be taken care of in a cogeneration power plant. One, the power load of 5600 kW and the other, the heating load of 1163 kW. Steam generation takes place at 40 bar and 500 °C and is expanded isentropically through a turbine to condenser at 0.06 bar. Heating load is supplied by extracting steam at 2 bar which condenses in the process heater to saturated liquid at 2 bar. Then, it is pumped back to the boiler. Calculate

(i) the steam generation capacity of the boiler in tonnes/h,
(ii) heat input to the boiler in *MW*, and
(iii) heat rejected to the condenser in *MW*.

Ans: (i) 16.73 t/h; (ii) 15.111 MW; (iii) 8.367 MW;

11.7 A pass out turbine is employed in a process industry for which dry saturated is supplied at 35 bar and 350 °C. The low-pressure turbine exhausts steam at 0.07 bar and the condition line may be assumed to be straight. (Note that a condition line is nothing but the locus passing through the states of steam leaving various stages of the turbine). If the power requirement in 1000 kW and the maximum process load is 1.4 MW, calculate the maximum steam flow through the higher and low pressure stages. Assume that the steam condenses in the process plant. *Ans:* 1.578 kg/s; 0.926 kg/s

11.8 A continuous supply of steam is obtained from a geothermal energy source at 2 bar and 0.87 dry. The flow rate is 2.7 t/h. This steam is used in a mixed-pressure cycle to argument the superheated exhaust from a high-pressure turbine of 83% internal efficiency, which is supplied with 5.5 t/h of steam at 40 bar and 500 °C. The mixing process is adiabatic and the mixture is expanded to a condenser pressure of 0.10 bar in a low-pressure turbine of 78% internal efficiency. Calculate the power output and the thermal efficiency of the plant. *Ans:* 1716.18 kW; 34.5%

11.9 It is found in a space project that there is a possibility of condensing the working fluid at -40 °C. In this connection, a binary cycle is proposed using R-12 as the low-temperature fluid and water as the high-temperature fluid. Steam generation is at 80 bar and 500 °C and expands in a turbine of 81% isentropic efficiency to 0.06 bar, at which pressure it is condensed by the generation of dry saturated refrigerant vapour at 30 °C from saturated liquid at -40 °C. The isentropic efficiency of the R-12 turbine is 83%. Find the mass ratio of R-12 to water and the efficiency of the cycle. Neglect all losses. *Ans:* 10.89 kg; 44.2%

11.10 A boiler generates steam at 70 bar and 500 °C. It is expanded in a turbine to 30 bar with an isentropic efficiency of 77%. At this condition, it is mixed twice its mass of steam at 30 bar, 400 °C. The mixture then expands with an isentropic efficiency of 80% to 0.06 bar. Steam is bled at 5 bar pressure from the system for feedwater heating in a direct contact heater, which rises the feedwater to the saturated temperature of the bled stem. Calculate the mass of steam bled per kg of high-pressure steam and the cycle efficiency. Assume that *LP* expansion condition line is straight. *Ans:* 0.545 kg; 34.7%

11.11 An ideal steam power plant operates between 70 bar and 550 °C and 0.075 bar. It has 7 feedwater heaters. Find the optimum pressure and temperature at which each of the heaters operate. *Ans:* 30.7 °C; 0.03265 MPa

11.12 Steam at 550 °C expands in an HP turbine under reheat cycle until it becomes a saturated vapour. It is reheated at a constant pressure to 400 °C and then expands in a LP turbine to 40 °C. The moisture constant at the turbine exhaust is 14.67%, find

(i) the reheat pressure,
(ii) heat input to the boiler in mw,
(iii) pressure of steam at inlet to the HP turbine, and
(iv) the cycle efficiency.

Consider all the processes to be ideal.

Ans: (i) 20 bar; (ii) 200 bar; (iii) 1604.5 kJ/kg; (iv) 43.85%

11.13 The maximum temperature in a reheat cycle is limited to 500 °C. The condenser pressure is 0.1 bar and the quality of at turbine exhaust is 0.8778. Without reheat, the possible exhaust quality would be 0.7592. Assuming all processes are ideal, find

(i) the reheat pressure,
(ii) the boiler pressure,
(iii) the cycle efficiency and
(iv) the steam rate. *Ans:* (i) 3 MPa; (ii) 15 MPa; (iii) 42.97%; (iv) 2.274 kg/kW h

11.14 Steam enters the HP stage of a two-stage turbine in a cogeneration plant at 1000 kPa and 200 °C. It leaves the first stage at 300 kPa. The remaining steam expands in the LP stage of the turbine at 40 kPa. The turbine is expected to produce a total power of 1000 kW. The heat exchanger provides a heating rate of 500 kW. Calculate the mass rate of steam into the HP stage of the turbine. Assume steady condition throughout the plant and neglect velocity and gravity terms. Take both turbine stages to be adiabatic with isentropic efficiencies of 0.80. *Ans:* 2.463 kg/s

11.15 Estimate the work required to compress steam isentropically from 1 bar pressure to 10 bar pressure, if initial state of the steam is (i) saturated liquid and (ii) saturated vapour. Neglect changes in KE and PE. Derive you conclusion from the result. *Ans:* −0.9387 kJ/kg; −520 kJ/kg

11.16 Steam expands in a steam turbine to 0.08 bar from 20 bar and 360 °C. After expansion, it enters a condenser where it gets condensed to saturated liquid water. The condensed water is pumped back into the boiler by the feedback pump. Assume that the processes are ideal. Calculate (i) net work per kg of steam; (ii) the cycle efficiency; and (iii) assuming the turbine and pump works with an efficiency of 80% estimate the % reduction in the net work and cycle efficiency. *Ans:* (i) 969.61 kJ/kg; (ii) 32.5%; (iii) 20.1%; 20.1%

11.17 In a steam power plant, the turbine inlet temperature is 360 °C. The exhaust pressure after isentropic expansion in the turbine, the moisture content is limited to 15%. Determine the maximum allowable steam pressure at the turbine inlet and find the Rankine cycle efficiency for the steam conditions. Estimate also the mean temperature of heat addition. *Ans:* 16.832 bar; 31.68%; 187.51 °C

11.18 A steam power plant uses steam at the following conditions: steam at boiler outlet = 150 bar at 550 °C; reheat at 40 bar to 550 °C; condenser pressure = 0.1 bar. Assuming ideal processes and using Mollier chart, find the (i) quality of steam at turbine exhaust; (ii) the cycle efficiency and (iii) steam rate.

Ans: (i) 0.88; (ii) 43.9%; (iii) 2.18 kg/kW h

11.19 Inlet condition of steam at the steam generator is 20 bar and 300 °C. The condenser pressure 0.1 bar. Two feedwater heaters operate at optimum temperature. Neglecting pump work, determine (i) the quality of steam at turbine exhaust; (ii) net work per kg of steam; (iii) the cycle efficiency; and (iv) the steam rate.
Ans: (i) 0.816; (ii) 797.48 kJ/kg; (iii) 33.34%; (iv) 4.51 kg/kW h

11.20 An isentropic expansion takes place in a steam turbine where dry saturated steam expands from a pressure of 40 bar to the condenser pressure of 0.075 bar. For steam generation, hot gases are used at 1 atm pressure and 2000 K. The exit temperature of the hot gases exhausted to atmosphere at 450 K. The atmospheric conditions are 1 atm pressure and 300 K. Heating rate provides by gas stream is 100 MW. Take c_p of hot gases as 1.1 kJ/kg K. Draw the exergy balance of the plant, compare this with energy balance, and find second law efficiency. *Ans:* 2.8 MW; 47.3%

11.21 The condition of steam in a power plant at inlet to the turbine is 80 bar and 500 °C. The condenser pressure is 0.1 bar. The heat source comprises a steam of exhaust gases from a gas turbine discharging at 560 °C and 1 atm pressure. The minimum temperature of the exhaust gases steam acceptable is 450 K. The mass flow rate of the exhaust gases is regularly in such a way that the heat input take to the steam cycle is 100 MW. The ambient condition is given by 300 K and 1 atm. Determine η_I, work ratio and η_{II} for the following cycles:

(i) Basic Rankine cycle without superheat.
(ii) Rankine cycle with superheat.
(iii) Rankine cycle with reheat such that the steam expands in the *HP* turbine until it exits as dry saturated vapour.
(iv) Ideal regenerative cycle, with the exit temperature of the exhaust gas steam taken as 320 °C. Since the saturation temperature of steam at 80 bar is close to 300 °C.

Neglect pump work. *Ans:* (i) 36.5%; 0.991; 61.6%; (ii) 39.4%; 0.994; 61.55%;
(iii) 40.5%; 0.995; 68.3%; (iv) 43.9%; 0.988; 46.4%

11.22 A chemical plant requires steam at the following conditions: temperature of steam = 120 °C; energy flow through steam = 5.83 MJ/s; power from electrical generation = 1000 kW. Both the heat and power requirements are met by back pressure turbine of 80% brake and 85% internal efficiency. Turbine exhausts steam at 120 °C under dry saturated conditions. All the latent heat released during condensation is utilised in the process heater. Find the pressure and temperature of steam at the inlet to the turbine. Assume 90% for the generator. *Ans:* 22.5 bar; 360 ° C

11.23 An average electrical load in a factory is 1500 kW and requires 3.5 mJ/s for heating purpose. For this purpose, a single-extraction pass out steam turbine to operate under the following conditions: internal pressure = 1500 kPa; internal temperature = 300 °C;

and condenser pressure = 10 kPa. steam is extracted between two turbine sections at 3 bar, 0.96 dry and is isobarically cooled without any intercooling in heaters to supply the heating load. The internal efficiency of the turbine (in the *LP* section) is 0.80 and the efficiency of the boiler is 0.85. The calorific value of the fuel used is 44 mJ/kg. If 10% of boiler steam is used for auxiliaries, calculate the fuel consumption per day. Assume that the condensate from the heaters (at 3 bar) and that from the condenser (at 0.1 bar) mix freely in a separate vessel (hot well) before being pumped to the boiler. Neglect losses. *Ans:* 18.16 tonnes/day

11.24 A turbine is supplied with steam at 70 bar and 450 °C. First, it expands in the *HP* stage to 25 bar. Then, it is reheated to 420 °C at constant pressure. Next, it expands in intermediate pressure stages to an appropriate minimum pressure such that part of the steam held at this pressure heats the feedwater to a temperature of 180 °C. The remaining steam expands from this pressure to a condenser pressure of 0.07 bar in the low pressure stage. The isentropic efficiency of the *HP* stage is 78.5% while that of the intermediate and *LP* stage is 83% each. Neglect the pump work. From the above data determine:

(i) the minimum pressure at which bleeding is necessary and sketch a line diagram of the arrangement of the plant,

(ii) sketch on the *T*-*s* diagram all the processes,

(iii) quality of steam bled per kg of flow at the turbine inlet and

(iv) cycle efficiency.

Ans: (i) 10 bar; (ii) *T*-*s* diagram (iii) 0.206 kg/kg steam flow at turbine inlet; (iv) 35.92%

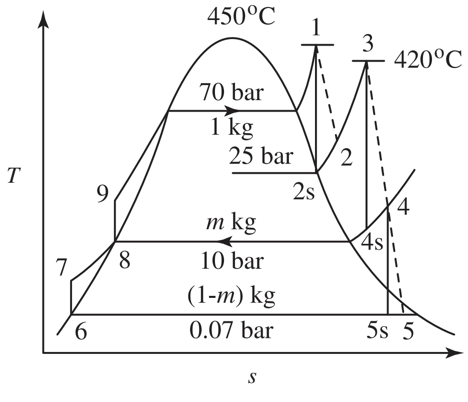


Fig. 11.37 T-s diagram

11.25 Consider a binary vapour cycle operating with mercury and steam. Mercury vapour is supplied at 4.5 to the mercury turbine. The exhaust pressure of the mercury turbine is 0.04 bar. The mercury condenser generates saturated steam at 15 bar. This steam expands in a steam turbine to 0.04 bar.

(i) Find the overall efficiency of the cycle.

(ii) If 50 tonnes of steam flows through the steam turbine per hour, find the flow through the mercury turbine.

(iii) Estimate the useful work done in the binary vapour cycle for the specified steam flow if all the processes are reversible and

(iv) If the steam leaving the mercury condenser is superheated to a temperature of 300 °C in a superheater located in the mercury boiler, and if the internal efficiencies of the mercury and steam turbine are 0.85 and 0.87, respectively.

Calculate the overall efficiency of the cycle. The properties of saturated mercury are as follows:

p (bar)	t (°C)	h_f (kJ/kg)	h_g (kJ/kg)	s_f (kJ/kgK)	s_g (kJ/kgK)	v_f (m^3/kg)	v_g (m^3/kg)
4.5	450	62.93	355.98	0.1352	0.5397	79.9×10^{-6}	0.068
0.04	216.9	29.98	329.85	0.0808	0.6925	76.5×10^{-6}	5.178

Ans: (i) 52.94%; (ii) 59.35 $\times 10^4$ kg; (iii) 28.49 MW; (iv) 46.2%

Multiple Choice Questions (choose the most appropriate answer)

1. The relation between $\eta_{Rankine}$ and η_{Actual} cycle is given by:
   (a) $\eta_{Rankine} > \eta_{Actual}$
   (b) $\eta_{Rankine} = \eta_{Actual}$
   (c) $\eta_{Rankine} < \eta_{Actual}$
   (d) none of the above

2. The losses present in actual vapour power cycle at the exit of the boiler and at the entry of the turbine are
   (a) friction loss
   (b) constant pressure heat loss
   (c) both (a) and (b)
   (d) none of the above

3. Assuming only the heat loss (Q_{loss}) in the expansion process at the turbine, the correct relation among enthalpies of steam entering (h_1) and leaving (h_2), turbine work (W_T) and heat loss (Q_{loss}) is given by
   (a) $h_1 - h_2 = -W_T - Q_{loss}$
   (b) $h_1 - h_2 = W_T - Q_{loss}$
   (c) $h_1 - h_2 = W_T + Q_{loss}$
   (d) none of the above

4. Rankine cycle efficiency of a good steam power plant is in the range of
   (a) 15 – 20%
   (b) 30 – 45%
   (c) 70 – 80%
   (d) 90 – 95%

5. Rankine efficiency of a steam power plant is
   (a) comparatively higher in summer as compared to that in winter
   (b) comparatively higher in winter as compared to that in summer
   (c) climatic conditions have no effect on efficiency
   (d) none of the above

6. A Rankine cycle comprises of
   (a) two isentropic processes and two constant volume processes
   (b) two isothermal processes and two constant pressure processes

(c) two isentropic processes and two constant pressure processes

(d) none of the above

7. Rankine cycle operating between pressure limit of p_1 (low) and p_2 (high)
   (a) has higher thermal efficiency than the Carnot cycle operating between same pressure limits
   (b) has lower thermal efficiency than Carnot cycle operating between same pressure limits
   (c) has same thermal efficiency as Carnot cycle operating between same pressure limits
   (d) may be more or less depending upon the magnitudes of p_1 and p_2

8. In Rankine cycle, the work output from the turbine is given by
   (a) change of internal energy between inlet and outlet
   (b) change of enthalpy between inlet and outlet
   (c) change of entropy between inlet and outlet
   (d) change of temperature between inlet and outlet

9. Bleeding steam to reheat feedwater to boiler (regenerative heating)
   (a) decreases thermal efficiency of the cycle
   (b) increases thermal efficiency of the cycle
   (c) does not affect thermal efficiency of the cycle
   (d) may increase or decrease thermal efficiency of the cycle depending upon the point of extraction of steam

10. Thermal efficiency of a regenerative Rankine cycle is
    (a) is greater than simple Rankine cycle thermal efficiency only when steam is bled at particular pressure
    (b) is same as simple Rankine cycle thermal efficiency
    (c) is always less than simple Rankine cycle thermal efficiency.
    (d) always greater than simple Rankine thermal efficiency

11. In a regenerative feed heating cycle, the optimum value of the fraction of steam extracted for feed heating
    (a) decreases with increase in Rankine cycle efficiency
    (b) increases with increase in Rankine cycle efficiency
    (c) is unaffected by increase in Rankine cycle efficiency
    (d) none of the above

12. In a regenerative feed heating cycle, the economy is affected maximum
    (a) when steam is extracted from only one suitable point of steam turbine
    (b) when steam is extracted only from the last stage of steam turbine
    (c) when steam is extracted from several places in different stages of steam turbine
    (d) when steam is extracted only from the first stage of steam turbine

13. The maximum percentage gain in Regenerative feed heating cycle thermal efficiency
    (a) increases with number of feed heaters increasing
    (b) decreases with number of feed heaters increasing
    (c) remains same unaffected by number of feed heaters
    (d) none of the above

14. The Rankine cycle consists of
    (a) two isothermal and two isochoric processes
    (b) two isentropic and two isobaric processes
    (c) two isentropic and two isothermal processes
    (d) two isothermal and two isobaric processes

15. At ideal condition of vapour power cycle, reversible constant pressure heat rejection is carried out at
    (a) boiler (c) condenser
    (b) turbine (d) feed pump

16. Which ideal process is carried out at the turbine in vapour power cycle?
    (a) reversible adiabatic compression
    (b) reversible constant pressure heat addition
    (c) reversible constant pressure heat rejection
    (d) reversible adiabatic expansion

17. In the T-s diagram of vapour power cycle, the condition of steam at the starting of turbine expansion is
    (a) wet with dryness fraction 0.8 (c) dry saturated
    (b) wet with dryness fraction 0.99 (d) superheated

18. With usual notation the formula for efficiency of Rankine cycle is given by:
    (a) $\eta_{Rankine} = \dfrac{(h_2 - h_3) + (h_1 - h_4)}{h_2 - h_1}$
    (b) $\eta_{Rankine} = \dfrac{(h_2 - h_3) - (h_1 - h_4)}{h_2 - h_1}$
    (c) $\eta_{Rankine} = \dfrac{h_2 - h_3}{h_2 - h_1}$
    (d) none of the above

19. In some occasions, the pump work in vapour power cycle can be neglected because
    (a) the pump work in not considered in efficiency of vapour power cycle
    (b) the pump work is very small compared to the heat addition
    (c) the pump work is very small compared to the turbine work
    (d) none of the above

20. The capacity of vapour power plant is expressed in terms of
    (a) heat rate (c) work output
    (b) steam rate (d) none of the above

21. Heat rate in steam power plant is given by
    (a) the rate of heat input in kJ per heat input in kW
    (b) the rate of heat input in kJ required to produce unit turbine work (1 kW)
    (c) the rate of heat input in kJ required to produce unit net shaft work (1 kW)
    (d) none of the above

22. The area enclosed in a T-s diagram of vapour power cycle indicates
    (a) heat added in cycle
    (b) work produced by turbine
    (c) heat rejected from cycle
    (d) net work produced by cycle

23. To maximise the work output at turbine, the specific volume of working fluid should be
    (a) as large as possible
    (b) as small as possible
    (c) constant throughout the cycle
    (d) none of the above

24. For the same pressure ratio, the relation between work required to compress steam in vapour form and work required to compress steam in liquid form is the
    (a) work required to compress steam in vapour form is equal work required to compress steam in liquid form
    (b) work required to compress steam in vapour form is more than work required to compress steam in liquid form
    (c) work required to compress steam in vapour form less than work required to compress steam in liquid form
    (d) none of the above

25. The factor that affects the turbine work output is more than pump work input in vapour power cycle for the same pressure ratio is
    (a) specific heat added to the working fluid
    (b) specific volume of working fluid
    (c) both (a) and (b)
    (d) none of the above

Ans: 1. – (a) 2. – (c) 3. – (c) 4. – (b) 5. – (b)
6. – (c) 7. – (a) 8. – (b) 9. – (b) 10. – (d)
11. – (b) 12. – (c) 13. – (a) 14. – (b) 15. – (c)
16. – (d) 17. – (c) 18. – (b) 19. – (c) 20. – (b)
21. – (c) 22. – (d) 23. – (a) 24. – (b) 25. – (b)

12

REFRIGERATION CYCLES

12.1 INTRODUCTION

Refrigeration is a science. In simple terms, it is a process of moving heat from one location to another under controlled conditions. Refrigeration can be defined as *producing and maintaining temperatures below that of the surrounding atmosphere.* This implies the removal of heat from a substance that is to be cooled. Heat always passes from a warm body to a cooler one. This happens until both bodies are at the same temperature. Nowadays, many human work spaces in offices and factory buildings are air-conditioned. In all these, a refrigeration unit is the heart of the system. Before the mechanical refrigeration system was invented, water was kept cool by storing it in semiporous jugs like mud pots so that the water could seep through and evaporate. The process of evaporation carries away heat and cools the water.

Even in olden days, Egyptians and Indians were using natural refrigeration system. Natural ice from lakes and rivers was often cut during winter and stored in caves, straw lined pits and later in sawdust-insulated buildings. It was used as and when required. The Romans carried pack trains of snow from Alps to Rome for cooling the Emperors hot drinks. In these methods of cooling, they make use of natural phenomena, in order to maintain a lower temperature in a space or product. This way of keeping products and space at low temperature in olden days may also be called refrigeration. In short, *refrigeration means the cooling of or removal of heat from a system.* The equipment employed to maintain the system at a low temperature is termed as *refrigerating system or refrigerator.* In general, refrigeration can be produced in any one of the following *three ways*:

(i) By melting of a solid.

(ii) By sublimation of a solid.

(iii) By evaporation of a liquid.

Most of the commercial refrigeration systems use, the principle of evaporation of a liquid called *refrigerant. Mechanical refrigeration* depends upon the evaporation of liquid refrigerant. The refrigeration system includes equipments called *evaporator, compressor, condenser and expansion valve.* Refrigeration is mainly used for preservation of food, manufacture of ice and production of solid carbon dioxide. It is also used to control air temperature and humidity in the air-conditioning system. Nowadays refrigeration is used in almost all fields. Some of them are listed below in the alphabetical order:

(i) Chemical and related industries
(ii) Comfort air-conditioning
(iii) Ice making
(iv) Industrial air-conditioning
(v) Oil refining
(vi) Medical and surgical aids
(vii) Processing food products and beverages
(viii) Manufacturing and treatment of metals
(ix) Transportation of foods
(x) Freezing food products

12.1.1 Basic Units of a Refrigeration Systems

There are at least *four basic units* in a refrigeration system. They are:

- (i) A low-temperature thermal 'sink' to which heat will flow from the space to be cooled.
- (ii) Provision for extracting energy from the sink.
- (iii) A receiver to absorb heat from the high temperature high-pressure refrigerant.
- (iv) Provision for reducing pressure and temperature of the refrigerant from the receiver to the 'sink'.

12.1.2 Refrigeration Systems

Refrigeration systems can be classified into:

- (i) Ice refrigeration
- (ii) Air refrigeration system
- (iii) Special refrigeration systems
- (iv) Vapour absorption refrigeration system
- (v) Vapour compression refrigeration system

Special refrigeration can be further divided into:

- (i) Adsorption refrigeration system
- (ii) Cascade refrigeration system
- (iii) Mixed refrigeration system
- (iv) Vortex tube refrigeration system
- (v) Thermoelectric refrigeration
- (vi) Steam jet refrigeration system

12.1.3 Coefficient of Performance

The performance of a refrigeration system is expressed by a term called coefficient of performance, (COP). It is defined as the *ratio of heat absorbed by the refrigerant while passing through the evaporator to the work input required to compress the refrigerant in the compressor.* In simple terms, it may be defined as the *ratio of heat extracted and work done* (in heat units). Then,

$$\begin{aligned} COP &= \frac{R_n}{W} \\ \text{Relative } COP &= \frac{\text{Actual } COP}{\text{Theoretical } COP} \qquad (12.1) \end{aligned}$$

where actual COP is ratio of R_n and W actually measured during a test and theoretical COP is ratio of theoretical values of R_n and W obtained by applying laws of thermodynamics to the refrigeration cycle.

12.1.4 Standard Rating of a Refrigeration Machine

The rating of a refrigeration machine is done by considering refrigerating effect or amount of heat extracted in a given time from a body. The rating of a refrigeration machine is expressed in *standard commercial tonne of refrigeration. It is defined as the refrigerating effect produced by the melting of one tonne of ice at 0 °C in 24 hours.* Since, the latent heat of fusion of ice is 336 kJ/kg, the refrigerating effect of 336×1000 kJ in 24 hours is rated as one tonne, i.e. 1 tonne of refrigeration (TR) $= \dfrac{336 \times 1000}{24} = 14000$ kJ/h.

12.1.5 Tonne of Refrigeration

A tonne of refrigeration is basically an American unit. It is calculated as the rate at which heat is required to be removed to freeze *one tonne of water from 0* °C *to ice at 0* °C. Using American units, this is equal to removal of 200 BTU of heat per minute and MKS unit it is adopted as 50 kcal/min or 3000 kcal/hour. In SI units, its conversion is rounded of to 3.5 kJ/s (kW) or 210 kJ/min. In this book, we will use, 1 tonne of refrigeration = 14000 kJ/h (1 tonne = 0.9 tonne).

12.2 AIR REFRIGERATION SYSTEM

Air cycle refrigeration is one of the oldest methods of cooling. Due to its very low COP and high operating costs, it became obsolete. However, its application is most appropriate to aircraft refrigeration systems. It can utilise a portion of the cabin air because of low equipment weight. The main characteristic feature of *air refrigeration system, is that throughout the cycle the refrigerant remains in gaseous state.* The air refrigeration can be classified into two types of systems:

(i) Closed system (ii) Open system

In a *closed (or dense air) system, the air refrigerant is contained within the piping or components parts of the system at all times and refrigerator with usually pressures above atmospheric pressure.*

Advantages of a closed system:

(i) In a closed system, the compressor suction may be at higher pressure. The sizes of expander and compressor can be kept within reasonable limits by using dense air.

(ii) In a closed system, there is no possibility of moisture freezing.

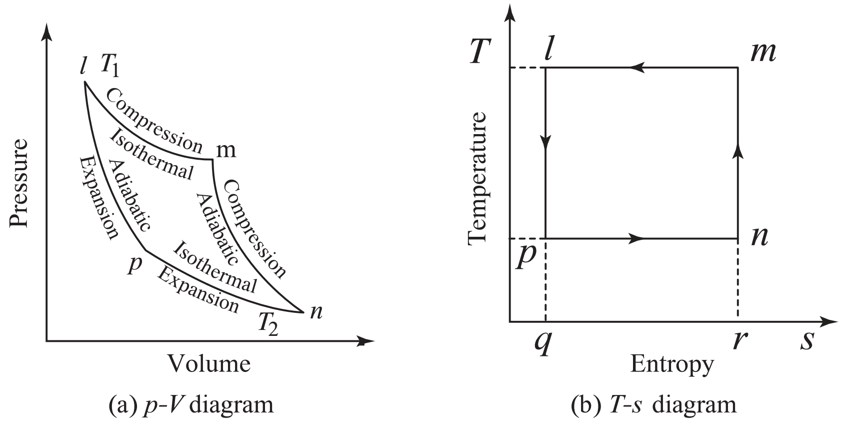
(iii) In a closed system, the expansion of the refrigerant can be carried out to any lower pressure, not necessarily only upto atmospheric pressure.

In the *open system*, the refrigerator is replaced by the *actual space to be cooled with the air expanded to atmospheric pressure, circulated through the cold room and then compressed to the cooler pressure.* The pressure of operation in this system is inherently limited to operation *at atmospheric pressure* in the refrigerator.

12.2.1 Reversed Carnot Cycle

If a machine is made to work on a reversed Carnot cycle, driven from an external source, it will function as a refrigerator. However, such a machine is not possible in practice because*the adiabatic portion of the stroke would need a very high speed while during isothermal portion of stroke, a very low speed will be necessary. This variation of speed during the stroke, however, is not practicable.* However, it has theoretical interest to assess the maximum possible efficiency with which such a machine can work.

p-V and T-s diagrams of reversed Carnot cycle are shown in Figs.12.1(a) and (b). Let us start from point l. At this point, the volume above the piston top in the compressor is clearance volume which is filled with air. This air is then expanded adiabatically to point p during which its temperature falls from T_1 to T_2. At this time, the cylinder is put in contact with a cold body at temperature T_2. The air is then expanded isothermally to the point n. During this period, heat is extracted from the cold body at temperature T_2. Now, the cold body is removed. From n to m, air undergoes adiabatic compression with the assistance of some external power. Due to compression, temperature rises to T_1. Now, let us assume that a hot body at temperature T_1 is put in contact with the cylinder. Finally, the air is compressed isothermally during which process heat is rejected to the hot body.



(a) p-V diagram (b) T-s diagram

Fig. 12.1 Reversed Carnot cycle

Refer Fig.12.1(b).

$$\text{Heat taken from the cold body} = \text{Area npqr} = T_2 \times pn$$

$$\text{Work done per cycle} = \text{Area lpnm} = (T_1 - T_2) \times pn$$

$$\text{Coefficient of performance, } COP = \frac{\text{Heat extracted from the cold body}}{\text{Work done per cycle}}$$

$$= \frac{\text{Area npqr}}{\text{Area lpnm}} = \frac{T_2 \times pn}{(T_1 - T_2) \times pn} = \frac{T_2}{T_1 - T_2} \quad (12.2)$$

COP means the ratio of the desired effect in kJ/kg to the energy supplied in kJ/kg, therefore, COP in case of Carnot cycle runs either as a refrigerating machine or a heat pump or as a heat engine will be as given as follows:

(i) For a reversed Carnot cycle run as *refrigerating machine*:

$$COP_{ref} = \frac{\text{Heat extracted from the cold body/cycle}}{\text{Work done per cycle}}$$

$$\frac{T_2 \times pn}{(T_1 - T_2) \times pn} = \frac{T_2}{T_1 - T_2} \quad (12.3)$$

(ii) For a Carnot cycle heat pump:

$$COP_{heatpump} = \frac{\text{Heat input to the cold body/cycle}}{\text{Work done per cycle}}$$

$$= \frac{T_1 \times lm}{(T_1 - T_2) \times pn} = \frac{T_1 \times pn}{(T_1 - T_2) \times pn} \quad (\because lm = pn)$$

$$= \frac{T_1}{T_1 - T_2} = \frac{T_1 - T_2 + T_2}{T_1 - T_2} = 1 + \frac{T_2}{T_1 - T_2} \tag{12.4}$$

This indicates that COP of heat pump is greater than that of a refrigerator working on reversed Carnot cycle between the same temperature limits T_1 and T_2 by unity.

(iii) For a Carnot cycle *heat engine:*

$$COP_{heatengine} = \frac{\text{Work obtained/cycle}}{\text{Heat supplied / cycle}} = \frac{(T_1 - T_2) \times pn}{T_1 \times lm}$$

$$= \frac{(T_1 - T_2) \times pn}{T_1 \times pn} (\because lm = pn) = \frac{T_1 - T_2}{T_1} \tag{12.5}$$

12.2.2 Reversed Brayton Cycle

Figure 12.2 is a schematic diagram of an air refrigeration system working on reversed Brayton cycle. Elements of this system are:

- (i) Compressor
- (ii) Cooler (heat exchanger)
- (iii) Expander
- (iv) Refrigerator

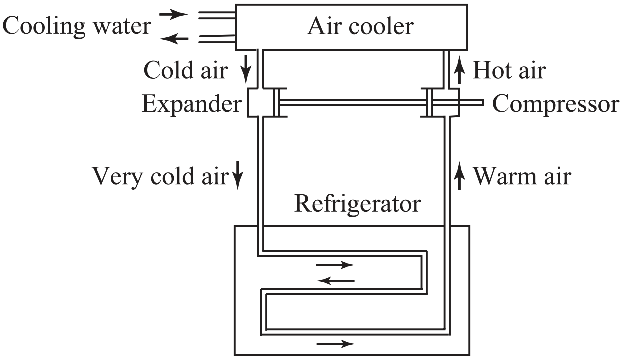


Fig. 12.2 Air refrigeration system

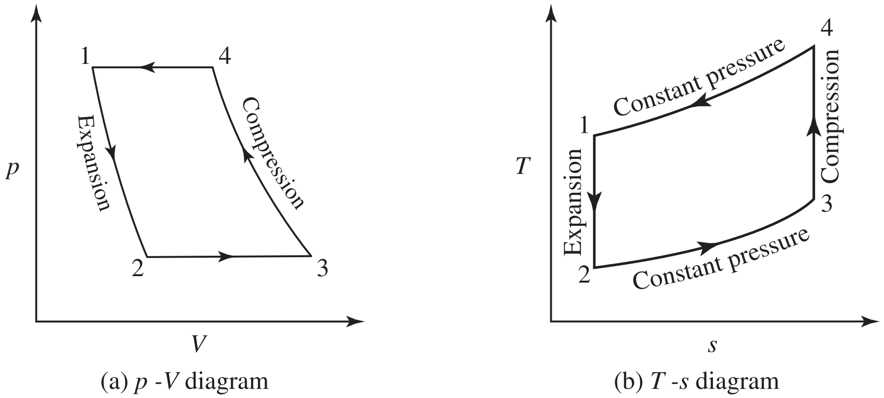
In this system, work output of expander may be used for compressing air. In practice, it may or may not be used. In some aircraft refrigeration systems which employ air refrigeration cycle, the expansion work may be used for driving other devices. Consequently, less external work is needed for operation of the system. This system works on reversed *Brayton cycle.* The working principle is described below:

Figure 12.3(a) and (b) shows p-V and T-s diagrams for a reversed Brayton cycle, respectively. Here, it is assumed that *absorption and rejection of heat are at constant pressure.* Further, compression and expansion are assumed to be *isentropic.*

Considering m kg of air:

$$\text{Heat absorbed in refrigerator, } Q_{added} = m \times c_p \times (T_3 - T_2)$$

$$\text{Heat rejected in cooler, } Q_{rejected} = m \times c_p \times (T_4 - T_1)$$



(a) p -V diagram (b) T -s diagram

Fig. 12.3 p-V and T-s diagrams for a reversed Brayton cycle

If the process is considered to be polytropic, the steady flow work of compression is given by:

$$W_{comp} = \frac{n}{n-1}(p_4V_4 - p_3V_3) \tag{12.6}$$

Similarly, work of expansion is given by,

$$W_{exp} = \frac{n}{n-1}(p_1V_1 - p_2V_2) \tag{12.7}$$

Equations 12.6 and 12.7 may easily be reduced to the theoretical isentropic process shown in Fig.12.3 by substituting $\gamma = n$ and the known relationship.

$$R = c_p\left(\frac{\gamma-1}{\gamma}\right) \text{J} \tag{12.8}$$

The net external work required for operation of the cycle

$$= \text{Steady flow work of compression} - \text{Steady flow work of expansion}$$

$$= W_{comp} - W_{exp} = \left(\frac{n}{n-1}\right)(p_4V_4 - p_3V_3 - p_1V_1 + p_2V_2)$$

Substituting $pV = mRT$:

$$\begin{aligned} W_{net} &= \left(\frac{n}{n-1}\right)mR(T_4 - T_3 - T_1 + T_2) \\ &= \left(\frac{n}{n-1}\right)\frac{mR}{J}(T_4 - T_3 - T_1 + T_2) \end{aligned}$$

where R is given by Eq.12.8. J = 1 in SI units.

$$\therefore W_{comp} - W_{exp} = \left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)mc_p(T_4 - T_3 + T_2 - T_1) \tag{12.9}$$

For *isentropic compression and expansion,*

$$W_{net} = mc_p(T_4 - T_3 + T_2 - T_1) \tag{12.10}$$

Now, according to law of conservation of energy, the net work on the gas must be equivalent to the net heat rejected. Now,

$$COP = \frac{Q_{added}}{Q_{rejected} - Q_{added}} = \frac{Q_{added}}{W_{net}}$$

For the air cycle, assuming polytropic compression and expansion, coefficient of performance is

$$\begin{aligned} COP &= \frac{m \times c_p \times (T_3 - T_2)}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right) m \times c_p \times (T_4 - T_3 + T_2 - T_1)} \\ &= \frac{T_3 - T_2}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)(T_4 - T_3 + T_2 - T_1)} \end{aligned} \tag{12.11}$$

Note: The reversed Brayton cycle is same as the Bell-Coleman cycle. Usually, Bell-Coleman cycle refers to a closed cycle. The expansion and compression takes place in reciprocating expander and compressor, respectively. Heat rejection and heat absorption takes place in condenser and evaporator, respectively. Because of the availability of modern efficient centrifugal compressors and gas turbines, the processes of compression and expansion can be carried out very effectively. Thus, the shortcomings of conventional reciprocating expander and compressor can be overcome. Reversed Brayton cycle finds its application for air-conditioning of aircrafts where air is used as refrigerant.

Merits and Demerits of the System

Merits:

(i) Since air is non-flammable, there is no risk of fire, compared to machines using NH_3 as the refrigerant.

(ii) As to date air is almost free and is easily available as compared to other refrigerants.

(iii) As compared to the other refrigeration systems, the weight of *air refrigeration system per tonne of refrigeration is extremely low. Because of this reason, this system is normally used in aircrafts.*

Demerits:

(i) The *COP* of this system is very low in comparison to other systems.

(ii) The weight of air required to be circulated is more compared to refrigerants used in other systems. This is due to the fact that heat is carried by air in the form of *sensible heat.*

12.3 SIMPLE VAPOUR COMPRESSION SYSTEM

The vapour compression system is most commonly used for *commercial and domestic refrigeration.* It is the most practical form of a refrigeration system. In this, the *working fluid is a vapour.* The refrigerant readily evaporates and condenses. In other words, it changes alternately between the vapour and liquid phases without leaving the refrigerating plant. During evaporation, it absorbs heat from the cold body. This heat is used as its latent heat for converting it from liquid to vapour. During condensing or cooling, it rejects heat to external body. This causes a cooling effect in the working fluid. This refrigeration system thus acts as a latent heat pump. It pumps its latent heat from the cold body and rejects it to the external hot body. The principle upon which the vapour compression system works applies to all the vapours. Thermodynamic properties of different refrigerants are available in the form of a table.

12.3.1 Simple Vapour Compression Cycle

In a simple vapour compression system, the four fundamental processes involved are compression, condensation, expansion and vaporisation. All these four processes are completed in one cycle. The typical layout of such a system is shown in Fig.12.4. Consider state 2 in Fig.12.4.

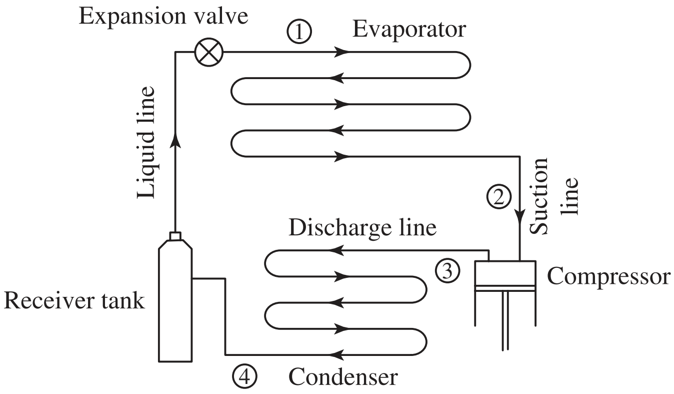


Fig. 12.4 Vapour compression system

At this state, the *vapour at low pressure and temperature enters the compressor.* It is assumed to be compressed isentropically and because of this, its temperature and pressure increase considerably to state 3. This vapour after leaving the compressor enters the condenser. It gets condensed into *high-pressure liquid* to state 4. It is collected in a *receiver tank shown in the figure.* From the receiver tank, it passes through the *expansion valve.* It is *throttled down to a lower pressure* and has a low temperature (state 1). After passing through expansion valve, it finally passes on to *evaporator.* It extracts heat from the surroundings and vaporises to low pressure vapour again to state 2.

12.3.2 Merits and Demerits of Vapour Compression System

Merits :

(i) Higher COP.

(ii) Running cost is only 20% of air refrigeration system.

(iii) For the same refrigerating effect the size of the evaporator is quite small.

(iv) The required temperature of the evaporator can be achieved simply by adjusting the throttle valve.

Demerits :

(i) Initial cost is high.

(ii) The major disadvantages are *inflammability, leakage of the refrigerant* and *toxity.*

Over the period, these have been overcome to a great extent by design improvement.

12.3.3 Temperature-Entropy (T-s) Diagram of Vapour Compression Cycle

In vapour compression cycle, the following three cases are considered:

Case 1. The vapour is dry and saturated after compression.

Consider the T-s diagram in Fig.12.5. Points 1–4 represent the state points in Fig.12.4. At point 2, the vapour enters into the compressor at low temperature (T_2) and low pressure. It gets compressed adiabatically to 3. Its temperature increases to the temperature T_1. It is then condensed in the condenser (refer line 3–4) where it gives up its latent heat to the condensing medium. Then, it undergoes expansion through a throttling valve. Because of expansion, its temperature reduces to T_2. This process is represented by the line 4–1 in the figure. From the T-s diagram, it may be noted that due to this expansion, the liquid partially evaporates, as its dryness fraction is represented by the ratio of length of the line b–1 and b–2. At 1, it enters the evaporator where further evaporation takes place at constant pressure and constant temperature to the point 2 and the cycle is completed.

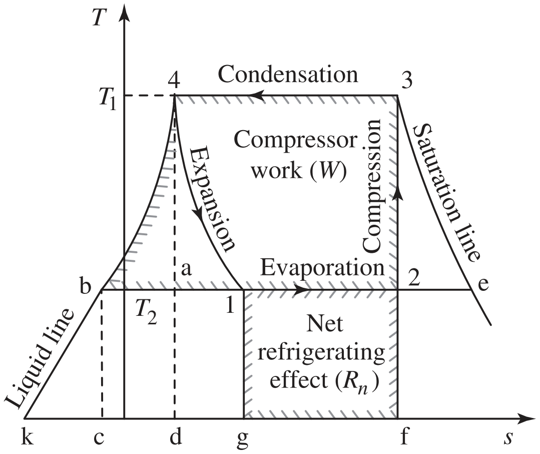


Fig. 12.5 T-s diagram for case 1

$$\text{Work done by the compressor, } W = \text{Area 2–3–4–b–2}$$

$$\text{Heat absorbed} = \text{Area 2–1–g–f–2}$$

$$\therefore COP = \frac{\text{Heat absorbed or refrigerating effect}}{\text{Work done}} = \frac{\text{Area 2–1–g–f–2}}{\text{Area 2–3–4–b–2}}$$

$$COP = \frac{h_2 - h_1}{h_3 - h_2} = \frac{h_2 - h_4}{h_3 - h_2} \qquad (12.12)$$

$h_1 = h_4$, since expansion 4–1 through throttling is a constant enthalpy process.

Case 2. The vapour is superheated after compression.

Consider the T-s diagram shown in Fig.12.6. The compression of the vapour is continued even after it has become dry. This causes the vapour to be superheated. As can be seen, from the figure, the vapour enters the compressor at condition 2. It is compressed to 3 where it reaches the superheated state with temperature T_{sup}. Then, it enters the condenser. Here, superheated vapour cools to temperature T_1 (represented by line 3–3′) and then it condenses at constant temperature along the line 3′–4. It then undergoes expansion through throttling valve. Due to expansion, its temperature reduces to T_2. This process is represented by the line 4–1 in the figure. From the T-s diagram, it may be noted that due to this expansion, the liquid partially evaporates, as its dryness fraction is represented by the ratio $\dfrac{\text{length } b \to 1}{\text{length } b \to 2}$. At 1, it enters the evaporator where it further evaporates at constant pressure and constant temperature to the point 2 and the cycle is completed.

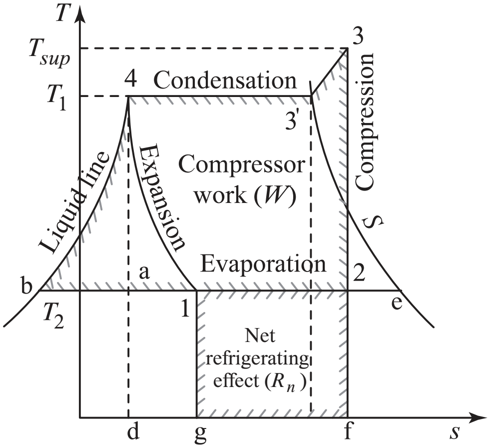


Fig. 12.6 T-s diagram for case 2

Now, Work done = Area 2–3–3′–4–b–2 and Heat extracted/absorbed = Area 2–1–g–f–2.

$$\therefore COP = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{\text{Area 2–1–g–f–2}}{\text{Area 2–3–3}'\text{–4–b–2}} = \frac{h_2 - h_1}{h_3 - h_2} \qquad (12.13)$$

In this case, $h_3 = h_{3'} + c_p(T_{sup} - T_{sat})$ and $h_{3'}$ = total heat of dry and saturated vapour at the point 3′.

Case 3. The vapour is wet after compression.

Refer Fig.12.7.

$$\text{Work done by compressor} = \text{Area 2–3–4–b–2}$$

$$\text{Heat extracted} = \text{Area 2–1–g–f–2}$$

$$\therefore COP = \frac{\text{Heat extracted}}{\text{Work done}} = \frac{\text{Area 2–1–g–f–2}}{\text{Area 2–3–4–b–2}} = \frac{h_2 - h_1}{h_3 - h_2} \qquad (12.14)$$

Note. If the vapour is not superheated after compression, the operation is called *wet compression* and if the vapour is superheated at the end of compression, it is known as *dry compression.* Dry compression, in actual practice, is always preferred as it gives *higher volumetric efficiency.* Further, because of higher *mechanical efficiency*, there are *less chances of compressor damage.*

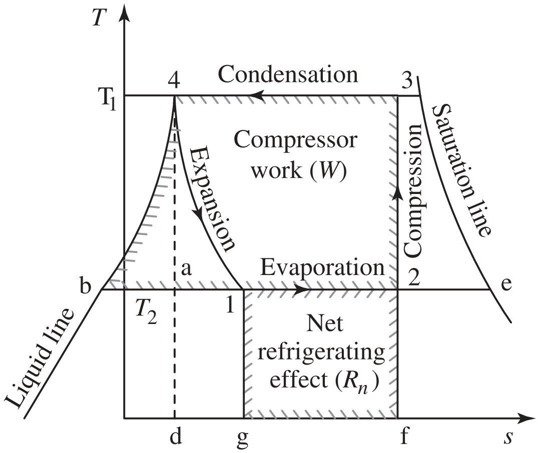


Fig. 12.7 T-s diagram for case 3

12.3.4 Pressure-Enthalpy (p-h) Chart

There are two diagrams that are commonly used in the analysis of the refrigeration cycle. They are:

(i) Pressure-enthalpy (p-h) chart (ii) Temperature-entropy (T-s) chart.

Of the two, the pressure-enthalpy diagram is comparatively more useful. The condition of the refrigerant in any thermodynamic state can be represented as a point on the p-h chart. It may be located if any two properties of the refrigerant for that state are known. The other properties of the refrigerant for that state can be determined directly from the chart. It helps in studying the performance of the machines.

Figure 12.8 shows a typical p-h chart. The horizontal lines extending across the chart are lines of *constant pressure.* The vertical lines are lines of constant enthalpy. As can be seen, the chart is divided into three regions. They are separated from each other by the saturated liquid and saturated vapour lines. The region on the chart to the left of the saturated liquid line is called the sub-cooled region. The lines of *constant temperature* in the sub-cooled region are almost vertical on the chart and parallel to the lines of constant enthalpy. At any point in the sub-cooled region, the refrigerant is in the liquid phase. Its temperature is below the saturation temperature corresponding to its pressure. The region to the *right* of the saturated vapour line is superheated region and the refrigerant is in the form of a *superheated vapour.*

The section between the saturated liquid and saturated vapour lines in the chart is the two-phase region. It represents the change in phase of the refrigerant between liquid and vapour phases. Now, consider the constant pressure line shown in the chart. At any point between two saturation lines, the refrigerant is in the form of a liquid vapour mixture. The distance between the two saturation lines along any constant pressure line represents the latent heat of vaporisation of the refrigerant at that pressure. In the centre section, the refrigerant changes

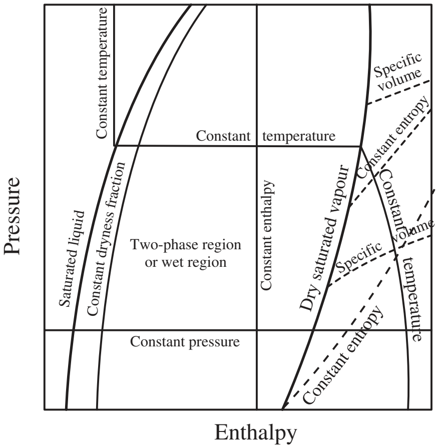


Fig. 12.8 Pressure-enthalpy diagram

state at a constant temperature and pressure. Therefore, the lines of constant temperature are parallel to and coincide with the lines of constant pressure. Refer Fig.12.8.

At the saturated vapour line, the lines of constant temperature change direction again. In the superheated vapour region, they fall of sharply toward the bottom of the chart. The straight lines which extend diagonally and almost vertically across the superheated vapour region are lines of constant entropy. The curved, nearly horizontal lines crossing the superheated vapour region are lines of constant volume. From *p*-*h* chart we can get directly the changes in enthalpy and pressure during a process which is useful for thermodynamic analysis.

12.3.5 Vapour Compression Cycle on *p*-*h* Chart

Figure 12.9 shows a simple vapour compression cycle on a *p*-*h* chart. The points 1–4 correspond to the points marked in Fig.12.4. The compressor draws dry saturated vapour (at state 2) from evaporator at a lower pressure p_1. Then, it is compressed isentropically to a higher pressure p_2. The line 2–3 represents isentropic compression. At the start of compression, the vapour is dry and saturated. It becomes superheated at the end of compression as given by point 3. The process of condensation takes place at constant pressure. It is represented by the line

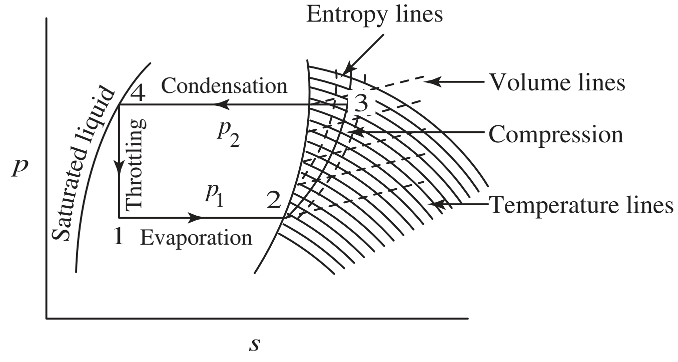


Fig. 12.9 Simple vapour compression cycle on p-h chart

3–4. The vapour now becomes saturated liquid. Then, at point 4, it is throttled through an expansion valve. The process is shown by the line 4–1. At point 1, a mixture of vapour and liquid enters the evaporator. It becomes dry and saturated as shown by the point 2. The cycle is thus completed.

Heat extracted (or refrigerating effect produced),

$$R_n = h_2 - h_1$$

$$\text{Work done, } W = h_3 - h_2$$

$$\therefore COP = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2} \qquad (12.15)$$

The values of h_1–h_3 can be directly read from *p*-*h* chart.

12.4 FACTORS AFFECTING THE PERFORMANCE OF A VAPOUR COMPRESSION SYSTEM

The factors which affect the performance of a vapour compression system are presented and discussed in this section.

12.4.1 Suction Pressure

Figure 12.10 illustrates the effect of decreasing the suction pressure.

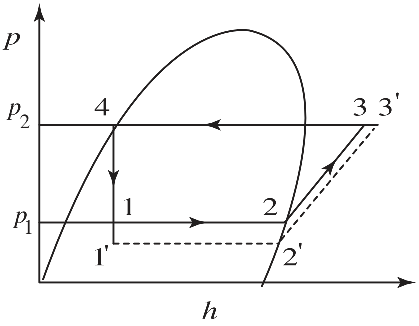


Fig. 12.10 Effect of decrease in suction pressure

The *COP* of the original cycle,

$$COP = \frac{h_2 - h_1}{h_3 - h_2}$$

The *COP* of the cycle when suction pressure is decreased,

$$COP = \frac{h_{2'} - h_{1'}}{h_{3'} - h_{2'}} = \frac{(h_2 - h_1) - (h_2 - h_{2'})}{(h_3 - h_2) + (h_2 - h_{2'}) + (h_{3'} - h_3)} (\because h_1 = h_{1'}) \qquad (12.16)$$

This shows that the refrigerating effect is decreased (see numerator) and work required is increased (see denominator). This clearly shows that the net effect is the reduced refrigerating capacity of the system (with the same amount of refrigerant flow) and also the *COP*.

12.4.2 Delivery Pressure

Figure 12.11 illustrates the effect of *increase in delivery pressure.*

COP of the original cycle,

$$COP = \frac{h_2 - h_1}{h_3 - h_2}$$

COP of the cycle when delivery pressure is increased,

$$COP = \frac{h_2 - h_{1'}}{h_{3'} - h_2} = \frac{(h_2 - h_1) - (h_{1'} - h_1)}{(h_3 - h_2) + (h_{3'} - h_3)} \tag{12.17}$$

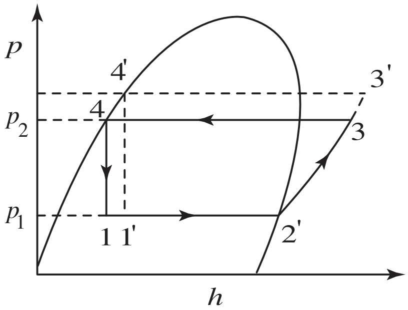


Fig. 12.11 Effect of increase in delivery pressure

The effect of increasing the delivery/discharge pressure is just similar to the effect of decreasing the suction pressure, viz reduction in refrigerating capacity and increase in work input. *The only difference is that the effect of decreasing the suction pressure is more predominant than the effect of increasing the discharge pressure.* The following points are to be noted:

(i) *As the discharge temperature requirement in the summer is more compared to winter, the same machine will give less refrigerating effect at a higher cost.*

(ii) *Discharge pressure increase becomes necessary for high-condensing temperatures. Further, decrease in suction pressure is also necessary to maintain low temperature in the evaporator.*

12.4.3 Superheating

As is evident from Fig.12.12 that the effect of superheating will increase the refrigerating effect. However, this is at the cost of increase in amount of work to attain the desired pressure. *Since the increase in work input is higher compared to increase in refrigerating effect, the overall effect of superheating lowers COP.* The following points are worth noting:

(i) *As the discharge temperature required in the summer is more as compared to winter, the same machine will give less refrigerating effect with increase in cost.*

(ii) *Higher discharge pressure is necessary for higher condensing temperatures. Further, decrease in suction pressure is necessary to maintain low temperature in the evaporator.*

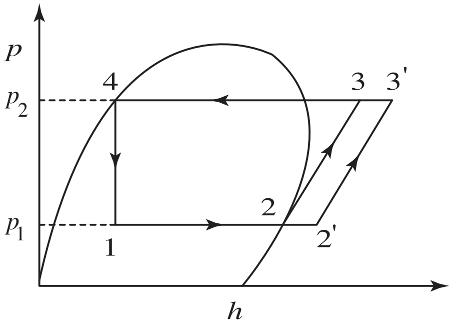


Fig. 12.12 Effect of superheating

12.4.4 Subcooling of Liquid

Subcooling is the process of cooling the liquid refrigerant below the condensing temperature for a given pressure. The process of subcooling is shown by 4–4′ in Fig.12.13. As can be seen, from the figure, the *effect of subcooling is to increase the refrigerating effect. Thus, subcooling increases COP. This is on the assumption that no f'urther energy has to be spent to obtain the extra cooling required for the coolant.* The *subcooling or under-cooling* may be done by any of the following methods:

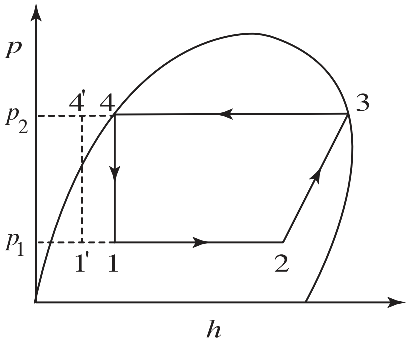


Fig. 12.13 Effect of subcooling of liquid

(i) Inserting a special coil between the condenser and the expansion valve.

(ii) Circulating greater quantity of cooling water through the condenser.

(iii) Using water cooler in main circulating water.

12.4.5 Suction Temperature and Condenser Temperature

The performance of the vapour compression refrigeration system varies considerably with both vaporising and condensing temperatures. Of the two, the *vaporising temperature has comparatively higher significant effect. The performance of the refrigerating system improves as the vaporising temperature increases and as the condensing temperature decreases.* Towards this, the refrigerating system should *always be designed to operate at the highest possible vaporising temperature and lowest possible condensing temperature.*

The basic differences between actual and theoretical vapour compression cycle are:

(i) Mostly the liquid refrigerant is subcooled before it is allowed to enter the expansion valve. Usually the gas leaving the evaporator is superheated a few degrees before it enters the compressor. This superheating occurs by the type of expansion control device used. This may also be mainly due to picking up heat in the suction line between the evaporator and compressor.

(ii) Compression although usually assumed to be isentropic, in practice, it is neither isentropic nor polytropic.

(iii) Both the compressor suction and discharge valves are actuated by pressure difference. This process requires actual suction pressure inside the compressor to be slightly lower than the evaporator. The discharge pressure should be slightly above that of condenser.

(iv) Isentropic compression implies no heat transfer between the refrigerant and the cylinder walls. Actually the cylinder walls are hotter than the incoming gases from the evaporator. Further, the incoming gases are cooler than the compressed gases discharged to the condenser.

(v) There will be considerable pressure drop in long suction and liquid line piping.

12.4.6 Actual Vapour Compression Cycle

Figure 12.14 shows the actual vapour compression cycle on a T-s plane. The various processes involved are explained in the following paragraphs.

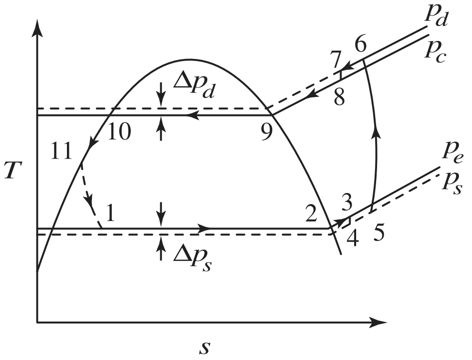


Fig. 12.14 Actual vapour compression cycle (T-s diagram)

Process 1–2–3: This process represents passage of refrigerant through the evaporator. The process *1–2 indicates gain of latent heat of vaporisation* and 2–3, *the gain of superheat before the refrigerant enters the compressor*. Theoretically, both these processes approach very closely to the constant pressure conditions.

Process 3–4–5–6–7–8: This path represents the passage of the vapour refrigerant from entry to the exit of the compressor. Path 3–4 represents the throttling action due to the passage through the suction valves. Path 7–8 represents the throttling of the vapour due to the passage through exhaust valves. Both these actions cause an entropy increase and a small drop in temperature.

Compression of the refrigerant occurs along path 5–6. Note that this is actually neither isentropic nor polytropic. The heat transfer during 4–5 and 6–7 occur essentially at constant pressure.

Process 8–9–10–11: It represents the details as the refrigerant flows through the condenser. The process 8–9 represents removal of superheat, 9–10 removal of latent heat and 10–11 removal of heat from liquid, viz subcooling.

Process 11–1 represents passage of the refrigerant through the expansion valve. Both these processes are theoretically and practically irreversible adiabatic processes.

12.4.7 Volumetric Efficiency

An ideal compressor without any losses would pump on each stroke, a quantity of refrigerant equal to piston displacement. However, actual compressor will not be able to do this. It is due to the fact that it is impossible to construct a compressor without clearance. Further, there will be resistance to the flow during suction and discharge through valves. There will be superheating of the suction gases when it comes into contact with the cylinder walls. Further, there will be leakage of gas past the piston as well as the valves. All these factors affect the pumping capacity of the compressor. Therefore, the gas pumped into the compressor will be much less actually. Hence, a term called volumetric efficiency is introduced to take care of this. *Volumetric efficiency* is defined as the *ratio of actual volume of gas drawn into the compressor* (at evaporator temperature and pressure) on *each stroke to the piston displacement.*

If the effect of clearance is considered, the resulting expression may be termed as clearance volumetric efficiency. The equation taking into consideration all the above factors affecting efficiency will represent the total volumetric efficiency.

Clearance volumetric efficiency: *Clearance volume is the volume of space under the cylinder head and above piston top when the piston is in top dead centre position.* The clearance volume is expressed as a percentage of piston displacement. In Fig.12.15, the piston displacement and clearance volume are shown clearly.

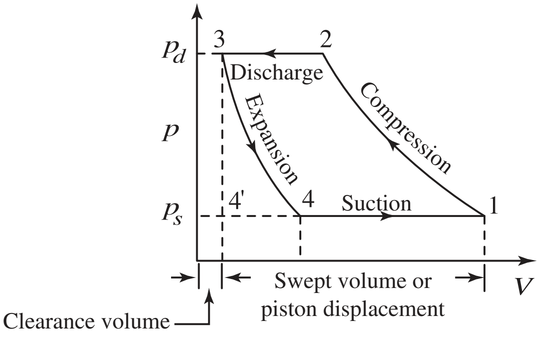


Fig. 12.15 p-V diagram

During the suction stroke $4'$–1, the vapour in the clearance space at a discharge pressure p_d expands along 3–4 until the pressure becomes p_s. The suction valve opens only at this pressure p_s. Therefore, actual volume sucked will be only $(V_1 - V_4)$ while the swept volume is $(V_1 - V_{4'})$. The *ratio of actual volume of vapour sucked to the swept volume is defined as clearance volumetric efficiency.* Thus, clearance volumetric efficiency,

$$\eta_{CV} = \frac{V_1 - V_4}{V_1 - V_{4'}} = \frac{V_1 - V_4}{V_1 - V_3} \quad (\because V_{4'} = V_3)$$

Considering polytropic expansion process 3–4, we have:

$$p_s V_4^n = p_d V_3^n \quad \text{or} \quad \frac{p_d}{p_s} = \left(\frac{V_4}{V_3}\right)^n \quad \text{or} \quad V_4 = V_3\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}$$

Let the clearance ratio, $C = \dfrac{V_3}{V_1 - V_3} = \dfrac{\text{Clearance volume}}{\text{Swept volume}}$, then,

$$\eta_{CV} = \frac{V_1 - V_4}{V_1 - V_3} = \frac{(V_1 - V_{4'}) - (V_4 - V_{4'})}{(V_1 - V_3)} = \frac{(V_1 - V_3) - (V_4 - V_3)}{(V_1 - V_3)} \quad (\because V_{4'} = V_3)$$

$$= 1 - \frac{V_4 - V_3}{V_1 - V_3} = 1 - \frac{V_3\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}} - V_3}{V_1 - V_3} = 1 + \frac{V_3}{V_1 - V_3}\left[1 - \left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}\right]$$

$$= 1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}} \tag{12.18}$$

12.4.8 Total Volumetric Efficiency

The total volumetric efficiency (η_{tv}) is estimated by actual *measurements of the amount of refrigerant compressed and delivered to the condenser.* In most cases, it is very difficult to predict the effects of wire-drawing, cylinder-wall heating. Therefore, total volumetric efficiency can be calculated only approximately, provided, the pressure drop through the suction valves and the temperature of the gases at the end of the suction stroke are known. The volumetric efficiency can be estimated if there is no leakage past the piston during compression and it can be calculated (by modifying Eq.12.18) by using the following equation:

$$\eta_{tv} = \left[1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}\right] \times \frac{p_c}{p_s} \times \frac{T_s}{T_c} \tag{12.19}$$

where the subscript c refers to compressor cylinder and s refers to the evaporator or the suction line just adjacent to the compressor.

12.4.9 Analysis of Vapour Compression Refrigeration Systems

There are about six performance parameters for analysing the performance of a vapour compression system. They are:

(i) Refrigerating effect
(ii) Mass of refrigerant
(iii) Piston displacement
(iv) Power requirement
(v) Heat rejection
(vi) Heat removal at condenser

Let us discuss the effect of these parameters on the performance of the system.

(i) *Refrigerating effect:* Refrigerating effect is the amount of heat absorbed by the refrigerant in its travel through the evaporator. In Fig.12.5, this effect is represented by the expression:

$$Q_{evap} = (h_2 - h_1) \text{ kJ/kg} \tag{12.20}$$

In addition to the latent heat of vaporisation, it may include any heat of superheat absorbed in the evaporator.

(ii) *Mass of refrigerant:* Mass of refrigerant estimated (per second per tonne of refrigeration) may be calculated by *dividing the amount of heat absorbed by the refrigerant.* Therefore, mass of refrigerant circulated,

$$m = \frac{14000}{3600(h_2 - h_1)} \text{ kg/s-tonne} \tag{12.21}$$

Note that one tonne of refrigeration means cooling effect of 14000 kJ/h.

(iii) *Theoretical piston displacement.* Theoretical piston displacement (per tonne of refrigeration per minute) can be found by *multiplying the mass of refrigerant to be circulated (per tonne of refrigeration per second) by the specific volume of the refrigerant gas,* $(V_g)_2$, at its entrance of compressor. Thus,

$$\text{Piston displacement (theoretical)} = \frac{14000}{3600(h_2 - h_1)}(V_g)_2 \text{ m}^3\text{/s-tonne} \tag{12.22}$$

(iv) *Theoretical power required:* It is the power, *theoretically required per tonne of refrigeration to compress the refrigerant.* Note that in this, volumetric and mechanical efficiencies are not taken into account. Power requirement can be calculated as follows:

(a) For isentropic compression:

$$\text{Work of compression} = h_3 - h_2 \tag{12.23}$$

$$\text{Power required} = m(h_3 - h_2) \text{ kW} \tag{12.24}$$

where m is mass of refrigerant circulated in kg/s.

(b) For polytropic compression, $pV^n = \text{constant}$

$$\text{Work of compression} = \frac{n}{n-1}(p_3V_3 - p_2V_2) \text{ Nm/kg} \tag{12.25}$$

$$\text{Power required} = m \times \frac{n}{n-1}(p_3V_3 - p_2V_2) \times \frac{1}{10^3} \text{ kW} \tag{12.26}$$

where p is in N/m^2.

(v) *Heat rejection to cooling water:* If the compressor cylinders are enclosed in a jacket, an appreciable amount of heat may be rejected to the cooling water during compression. If the suction as well as discharge compression conditions are known, the heat rejection can be determined as follows:

Heat rejected to compressor cooling water:

$$= \left[\frac{n}{n-1}\left(\frac{p_3V_3 - p_2V_2}{1000}\right) - (h_3 - h_2)\right] \text{ kJ/kg} \tag{12.27}$$

where p is in N/m^2.

(vi) *Heat removal at condenser:* This includes all heat removed at the condenser, either as latent heat, heat of superheat or heat of liquid. This is nothing but *the heat absorbed in the evaporator plus the work of compression.* Therefore,

Heat removed through condenser

$$= m(h_3 - h_4) \text{ kJ/s} \tag{12.28}$$

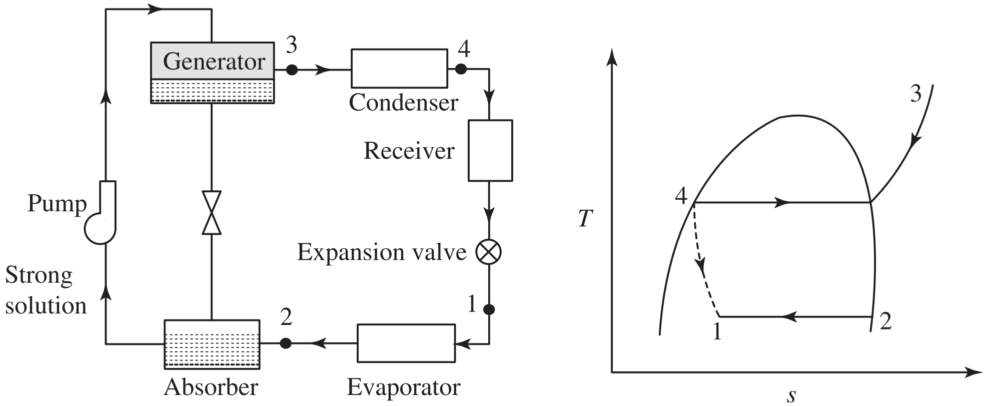
where m is mass of refrigerant circulated in kg/s.

12.5 VAPOUR ABSORPTION SYSTEM

As the refrigerant is absorbed on leaving the evaporator, it is called vapour absorption system. The absorbing medium can be either a liquid or a solid. As the operation should be continuous, it is necessary for the refrigerant to be desorbed from the absorbent. Then, it should be condensed before returning to the evaporator. The separation is accomplished by the application of direct heat is generated by some source. The solubility of the refrigerant and absorbent must be compatible. The plant which uses *ammonia as the refrigerant and water as absorbent* will be discussed in the following section.

12.5.1 Simple Vapour Absorption System

Figure 12.16(a) shows the schematic diagram of a simple vapour absorption system. It uses favourable characteristic of the solubility of ammonia in water at low temperatures and pressures compared to higher temperatures and pressures. The ammonia vapour leaving the evaporator at point 2 is readily absorbed in the low-temperature hot solution in the absorber. This is accompanied with the rejection of heat. The ammonia in water solution is pressurised to a higher pressure and then heated in a generator. *Due to reduced solubility of ammonia in water at the higher pressure and temperature, the vapour can be removed from the solution.* The vapour then gets to the condenser and the low-concentration ammonia in water solution is returned to the absorber. In this system, the *work done on compression is less than in vapour compression cycle*, since pumping a liquid requires much less work than compressing a vapour between the same pressures. However, energy input to the generator is required. The heat energy may be supplied by either steam or gas.



(a) Simple vapour absorption system

(b) *T-s* diagram

Fig. 12.16 Simple vapour absorption system

12.5.2 Practical Vapour Absorption System

A simple vapour absorption system can provide refrigeration. However, *its operating efficiency is low.* A typical layout of a practical vapour absorption system is shown in Fig.12.17. The following parts are fitted to make the system more practical and improve the performance and working:

(i) Heat exchanger (ii) Analyser (iii) Rectifier.

1. Heat exchanger: A heat exchanger is a must and is located between the generator and the absorber. The strong solution which is pumped from the absorber to the generator must be heated. The weak solution sent from the generator to the absorber must be cooled. This can be accomplished by a heat exchanger and, consequently, *cost of heating the generator and cost of cooling the absorber will be less.*

2. Analyser: An analyser consists of a series of trays mounted above the generator. The *main function of the analyser is to remove partly some of the unwanted water particles in the ammonia vapour getting into the condenser. If these water particles are allowed to enter the condenser, they may enter the expansion valve and freeze. This might choke the pipeline.*

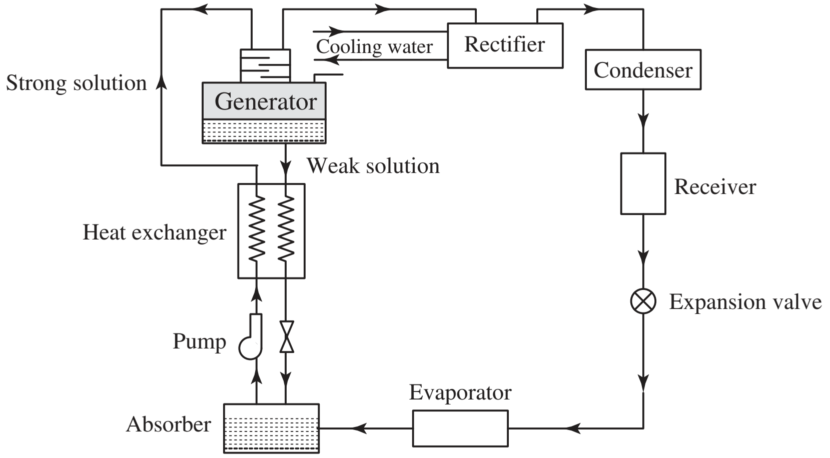


Fig. 12.17 A practical vapour absorption system

3. Rectifier: A rectifier is nothing but a water-cooled heat exchanger *which condenses water vapour and some ammonia and sends back to the generator.* Thus, *final reduction or elimination of the percentage of water vapour takes place in a rectifier.*

The COP of this system is given by:

$$COP = \frac{\text{Heat extracted from the evaporator}}{\text{Heat supplied in the generator} + \text{Work input to the liquid pump}} \tag{12.29}$$

12.5.3 Comparison Between Vapour Compression and Vapour Absorption Systems

Comparison between centrifugal and reciprocating compressors is given in the Table 12.2.

12.6 REFRIGERANTS

Any substance that absorbs heat through expansion or vaporisation and loses it through condensation in a refrigeration system is called refrigerant. In a broader sense, the term *'refrig-*

Table 12.1 Comparison between vapour compression and vapour absorption systems

S. No	Particulars	Vapour compression system	Vapour absorption system
1	Type of energy supplied	Mechanical – a high-grade energy	Mainly heat – a low-grade energy
2	Energy supply	Low	High
3	Wear and tear	More	Less
4	Performance at part loads	Poor	System not affected by variations of loads
5	Suitability	Used where high-grade mechanical energy is available	Can also be used at remote places as it can work even with a simple kerosene lamp (if the capacity is small)
6	Charging of refrigerant	Simple	Difficult
7	Leakage of refrigerant	More chances	No chance as there is no compressor
8	Damage	Liquid traces in suction line may damage the compressor	Liquid traces of refrigerant present in piping at the exit of evaporator does not pose any danger

erant' is also applied to such secondary cooling mediums as cold water or brine solutions. Refrigerants usually means only those working mediums which pass through the cycle of evaporation, recovery, compression, condensation and liquefaction.

These substances absorb heat at one place at low-temperature level. They reject the same at some other place having higher temperature and pressure. The rejection of heat is due to mechanical work done. Note that circulating cold or cooling mediums (such as ice and solid carbon dioxide) are not primary refrigerants. Long back, only four refrigerants, viz air, ammonia (NH_3), carbon dioxide (CO_2), sulphur dioxide (SO_2) were used. It is because they possess chemical, physical and thermodynamic properties for efficient use in refrigeration equipments.

12.6.1 Classification of Refrigerants

The refrigerants are classified as follows:

(i) Primary refrigerants (ii) Secondary refrigerant

1. Primary refrigerants are those working mediums or heat carriers which directly take part in the refrigeration system and cool the substance by the absorption of latent heat, for example, ammonia, carbon dioxide, sulphur dioxide, methyl chloride, methylene chloride, ethyl chloride, freon group, etc.

2. Secondary refrigerants are those circulating substances which are first cooled with the help of the primary refrigerants and are then employed for cooling purposes, for example, ice, solid carbon dioxide, etc. These refrigerants cool substances by absorption of their sensible

Table 12.2 Comparison between centrifugal and reciprocating compressors

S. No	Particulars	Centrifugal compressor	Reciprocating compressor
1	Suitability	Suitable for handling large volumes of air at low pressures	Suitable for low discharges of air at the pressure
2	Operational speeds	Usually high	Low
3	Air supply	Continuous	Pulsating
4	Balancing	Less vibrations	Cyclic vibrations occur
5	Lubrication system	Generally simple lubrication systems are required	Generally complicated Lubrication systems are required
6	Quality of air delivered	Air delivered is relatively more clean	Generally contaminated with oil
7	Air compressor size	Small for given discharge	Large for same discharge
8	Free air handled	2000-3000 m^3/min	250-300 m^3/min
9	Delivery pressure	Normally below 10 bar	500 to 800 bar
10	Usually standard of compression	Isentropic compression	Isothermal compression
11	Action of compressor	Dynamic action	Positive displacement

heat.

Halocarbon Refrigerants: In 1928, Charles Kettening and Dr. Thomas Mighey invented and developed this group of refrigerants. In this group are included refrigerants which contain one or more of three halogens, chlorine and bromine, and they are sold in the market under names as *freon, genetron, isotron* and *areton.* Since the refrigerants belonging to this group have outstanding merits over the other groups refrigerants, therefore, they find wide field of application in domestic, commercial and industrial applications. The list of the halocarbon-refrigerants commonly used is given below: Typical examples are R-12 – Dichloro-difluoro methane (CCl_2F_2) and R-22 – mono chloro difluoro methane ($CHClF_2$).

Hydrocarbons: Most of the refrigerants of this group are organic compounds. Several hydrocarbons are used successfully in commercial and industrial installations. Most of them possess satisfactory thermodynamic properties but are highly inflammable. Typical examples are: R-50 - methane (CH_4), R-170- ethane (C_2H_6) and R-290- propane (C_2H_8).

Inorganic compounds: Before the introduction of hydrocarbon group, these refrigerants were most commonly used for all purposes. Typical examples are: R-717- ammonia (NH_3), R-718- water (H_2O) and R-744- carbon dioxide (CO_2).

12.6.2 Desirable Properties of an Ideal Refrigerant

An ideal refrigerant should possess the following properties:

1. **Thermodynamic Properties:**

(i) Low boiling point

(ii) Low freezing point

(iii) Medium positive pressures

(iv) High saturation temperature

(v) High latent heat of vaporisation

2. **Chemical Properties :**

(i) Non-toxicity

(ii) Non-flammable and non-explosive

(iii) Non-corrosiveness

(iv) Non-irritating and odourless

(v) Chemical stability in reacting

3. **Physical Properties :**

(i) Low specific volume of vapour

(ii) Low specific heat

(iii) High thermal conductivity

(iv) Low viscosity

(v) High electrical insulation

4. **Other Properties :**

(i) Ease of leakage location

(ii) Availability and low cost

(iii) Ease of handling

(iv) High *COP*

(v) Low power consumption per tonne of refrigeration

(vi) Low pressure ratio and pressure difference

Current applications of refrigeration

(i) Nowadays, widespread applications of refrigeration are for air-conditioning of private homes and public buildings and refrigerating foodstuffs in homes, restaurants and large storage warehouses.

(ii) The use of refrigerators in kitchens for storing fruits and vegetables has allowed adding fresh salads to the modern diet year round and storing fish and meats safely for long periods. Optimum temperature range for perishable food storage is 3 – 5 °C.

(iii) Refrigeration is used to liquefy gases such as oxygen, nitrogen, propane and methane.

(iv) In compressed air purification, it is used to condense water vapour to reduce its moisture content.

(v) In oil refineries, chemical plants and petrochemical plants, refrigeration is used to maintain certain processes at their needed low temperatures (e.g. in alkylation of butanes and butane to produce a high-octane gasoline component).

(vi) Metal workers use refrigeration to temper steel and cutlery.

(vii) In transporting temperature-sensitive foodstuffs and other materials by trucks, trains, airplanes and seagoing vessels, refrigeration is a necessity.

(viii) Dairy products are constantly in need of refrigeration.

(ix) It was only discovered in the past few decades, that eggs needed to be refrigerated during shipment rather than waiting to be refrigerated after arrival at the grocery store.

(x) Meats, poultry and fish all must be kept in climate-controlled environments before being sold. Refrigeration also helps keep fruits and vegetables edible longer.

Worked-Out Examples

12.1 A refrigerator operates as the reversed Carnot cycle. The maximum temperature of the refrigerant is 37 °C and the lowest temperature is −13 °C. The capacity of the system is 10 tonnes. Neglecting all losses find,

(i) the coefficient of performance
(ii) heat rejected from the system per hour
(iii) power required to run the refrigeration.

Solution

$$COP = \frac{T_2}{T - T_2} = \frac{260}{310 - 260} = \frac{260}{50} = 5.2 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Heat rejected from the system per hour:

$$\begin{aligned}
COP &= \frac{\text{Refrigerating effect}}{\text{Work input}} \\
\text{Work input} &= \frac{\text{Refrigerating effect}}{COP} = \frac{10 \times 14000}{5.2} = 26923 \text{ kJ/h} \\
\text{Heat rejected per hour} &= \text{Refrigerating effect per hour} + \text{Work input per hour} \\
&= 10 \times 14000 + 26923 = 166923 \text{ kJ/h} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\
\text{Power required} &= \frac{\text{Work input per hour}}{60 \times 60} = \frac{26923}{3600} = 7.48 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

12.2 A cold storage unit is maintained at −3 °C while the surroundings are at 37 °C. The heat leakage into the cold storage from the surroundings is 30 kW. The actual *COP* of the refrigeration plant used is one-third that of an ideal plant working between the same temperatures. Find the power required to drive the plant.

Solution

$$\begin{aligned}
\text{Ideal } COP &= \frac{T_2}{T_1 - T_2} = \frac{270}{310 - 270} = \frac{270}{40} = 6.75 \\
\text{Actual } COP &= \frac{1}{3} \times 6.75 = 2.25 = \frac{R_n}{W}
\end{aligned}$$

where R_N is net refrigerating effect and W is work done.

$$\begin{aligned}
2.25 &= \frac{30}{W} \\
\dot{W} &= \frac{30}{2.5} = 12 \text{ kJ/s}
\end{aligned}$$

Power required to drive the plant = 12 kW $\overset{\textbf{Ans}}{\Longleftarrow}$

12.3 Ice is formed at 0 °C from water at 27 °C. The temperature of the brine is −8°C. Find out kilogram of ice formed per kW h. Assume that the refrigeration cycle used is ideal and reversed Carnot cycle. Take latent heat of ice as 335 kJ/kg.

Solution

$$COP = \frac{T_2}{T_1 - T_2} = \frac{265}{300 - 265} = 7.57$$

Heat to be exhausted per kg of water (to form ice at 0 °C, i.e. 273 K)

$$R_n = 1 \times c_{pw} \times (300 - 273) + \text{ Latent heat of ice}$$

$$= 1 \times 4.18 \times 27 + 335 = 447.86 \text{ kJ/kg}$$

$$1 \text{ kW h} = 1 \times 3600 = 3600 \text{ kJ}$$

$$COP = \frac{R_n}{W} = \frac{\text{Refrigerating effect in kJ/kg}}{\text{Work done in kJ}}$$

$$7.57 = \frac{m_{ice} \times 447.86}{3600}$$

$$\text{Ice formed per kW h, } m_{ice} = \frac{7.57 \times 3600}{447.86} = 60.85 \text{ kg} \quad \xleftarrow{\textbf{Ans}}$$

12.4 Calculate the minimum power required for a perfect reversed heat engine that makes 444 kg of ice per hour at −13 °C from feedwater at 17 °C. Assume specific heat of ice as 2.1 kJ/kg K and latent heat 334 kJ/kg.

Solution

$$COP = \frac{T_2}{T_1 - T_2} = \frac{260}{290 - 260} = \frac{260}{30} = 8.67$$

Heat absorbed per kg of water (to form ice at −13°C, i.e. 260 K)

$$R_n = 1 \times 4.18 \times (290 - 260) + 334 + 1 \times 2.09 \times (273 - 260) = 486.6 \text{ kJ/kg}$$

$$COP = \frac{\text{Net refrigerating effect}}{\text{Work done}} = \frac{R_n}{W}$$

$$8.67 = \frac{486.6 \times 444}{W}$$

$$W = \frac{486.6 \times 444}{8.67} = 24919.3 \text{ kJ/h} = \frac{24919.3}{3600} = 6.922 \text{ kW} \quad \xleftarrow{\textbf{Ans}}$$

Hence, minimum power required is 6.922 kW.

12.5 The capacity of a refrigerator working on a reversed Carnot cycle is 300 tonnes. The operating temperature range is −13°C and 27 °C. Calculate:

(i) Quantity of ice produced within 24 hours when water is supplied at 20 °C.

(ii) Minimum power required in kW.

Solution

Heat extraction per kg of water to form ice at 0 °C $= 4.18 \times 20 + 335 = 418.6$ kJ/kg.

Heat extracting capacity of the refrigerator = 300 tonnes $= 300 \times 14000 = 4200000$ kJ/h.

Quantity of ice produces in 24 hours,

$$m_{ice} = \frac{4200000 \times 24}{418.6 \times 1000} = 240.80 \text{ tonnes}$$

$$COP = \frac{T_2}{T_1 - T_2} = \frac{260}{300 - 260} = 6.5$$

$$COP = \frac{\text{Net refrigerating effect}}{\text{Work done/min}} = \frac{R_n}{W}$$

$$6.5 = \frac{4200000}{W}$$

$$\dot{W} = \frac{4200000}{6.5 \times 3600} = 179.5 \text{ kJ/s}$$

$$\text{Power required} = 179.5 \text{ kW}$$

$\xLeftarrow{\textbf{Ans}}$

12.6 Twenty-five tonnes of fish is to be stored in a cold storage. The temperature of fish when supplied is 27 °C. Fish storage temperature is −13°C. Specific heats of fish above and below freezing point are 2.93 kJ/kg °C and 1.25 kJ/kg °C, respectively. Freezing point of fish is −3°C. Latent heat of fish = 232 kJ/kg. If the cooling is achieved in 10 hours, calculate:

(i) Capacity of the refrigerating plant.
(ii) Carnot cycle *COP* between this temperature range.
(iii) If the actual *COP* is 1/3 of the Carnot *COP*, calculate the power required to run the plant.

Solution

Heat removed in 10 hours from each kg of fish

$$= 1 \times 2.93 \times (27 - (-3)) + 232 + 1 \times 1.25 \times (-3 - (-13))$$

$$= 87.9 + 232 + 12.5 = 332.4 \text{ kJ/kg}$$

$$\text{Heat removed by the plant/min} = \frac{332.4 \times 25 \times 1000}{10} = 831000 \text{ kJ/h}$$

$$\text{Capacity of the refrigerating plant} = \frac{831000}{14000} = 59.36 \text{ tonnes}$$

$$\text{COP of reversed Carnot cycle} = \frac{T_2}{T_1 - T_2} = \frac{260}{300 - 260} = 6.5$$

$$\text{Actual } COP = \frac{1}{3} \times \text{ Carnot } COP = \frac{1}{3} \times 6.5 = 2.17$$

But actual $COP = \dfrac{R_n}{W}$.

$$\dot{W} = \frac{R_n}{COP} \quad = \quad \frac{831000}{2.17} = 382949 \text{ kJ/h} = \frac{382949}{3600} = 106.37 \text{ kJ/s}$$

Power required to run the plant = 106.37 kW $\overset{\textbf{Ans}}{\Longleftarrow}$

12.7 Interior of a house is heated by a heat pump in cold climate. The ambient temperature is −8°C and the desired interior temperature is 27 °C. The compressor of heat pump is driven by a heat engine working between 1000 °C and 27 °C. Consider both cycles as reversible and calculate the ratio in which the heat pump and heat engine share the heat load.

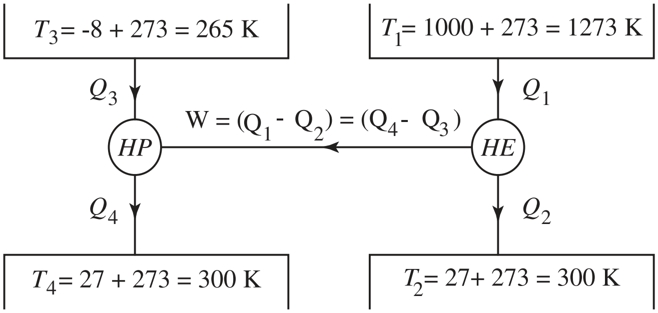


Fig. 12.18

Solution

Refer Fig.12.18. Since both the cycles are reversible:

$$\frac{Q_3}{Q_4} = \frac{T_3}{T_4} \quad \text{and} \quad \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\frac{Q_3}{Q_4} = \frac{265}{300}$$

$$Q_3 = \frac{265}{300} \times Q_4 \quad \text{and} \quad \frac{Q_2}{Q_1} = \frac{300}{1273}$$

As heat engine drives the heat pump:

$$W = (Q_1 - Q_2) = (Q_4 - Q_3)$$

Dividing both sides by Q_1, we get:

$$1 - \frac{Q_2}{Q_1} = \frac{Q_4 - Q_3}{Q_1}$$

$$1 - \frac{300}{1273} = \frac{Q_4 - \frac{265}{300} \times Q_4}{Q_1}$$

$$\frac{973}{1273} = \frac{35}{300}\frac{Q_4}{Q_1}$$

$$\frac{Q_4}{Q_1} = \frac{973}{1273} \times \frac{300}{35} = 6.55 \quad \xleftarrow{\text{Ans}}$$

12.8 A refrigerator is operating on the Bell-Coleman cycle between the limits of 100 kPa and 100 kPa. Air is drawn from the cold chamber at 7 °C, compressed and then it is cooled to 27 °C before entering the expansion cylinder. Expansion and compression follow the law, $pV^{1.3} = C$. Calculate the theoretical *COP* of the system. Take $\gamma = 1.4$ and $c_p = 1.003$ kJ/kg K for air.

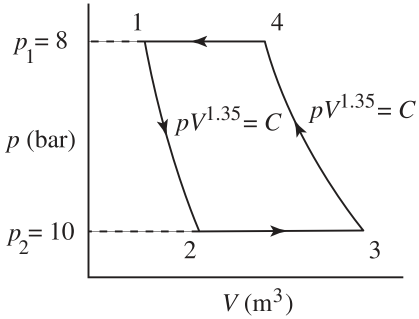


Fig. 12.19

Solution

Refer Fig.12.19. consider the polytropic compression 3–4:

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\frac{n-1}{n}} = \left(\frac{800}{100}\right)^{\frac{1.3-1}{1.3}} = 1.616$$

$$T_4 = T_3 \times 1.616 = 280 \times 1.616 = 452.5 \text{ K}$$

Now, considering the polytropic expansion process 1–2, we can write:

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{1.3-1}{1.3}} = 8^{\frac{0.3}{1.3}} = 1.616$$

$$T_2 = \frac{T_1}{1.616} = \frac{300}{1.616} = 185.6 \text{ K}$$

Heat extracted from cold chamber per kg of air

$$= c_p(T_3 - T_1) = 1.003 \times (280 - 185.6) = 94.7 \text{ kJ/kg}$$

Heat rejected in the cooling chamber per kg of air

$$= c_p(T_4 - T_1) = 1.003 \times (452.5 - 300) = 152.96 \text{ kJ/kg}$$

Note that compression and expansion are not isentropic. Further, difference between heat rejected and heat absorbed is not equal to the work done. There is heat transfer to the surroundings and from the surroundings during compression and expansion. Therefore, to find the work done, the area of the diagram 1–2–3–4 is to be considered:

$$W = \frac{n}{n-1}(p_4V_4 - p_3V_3) - \frac{n}{n-1}(p_1V_1 - p_2V_2) = \frac{n}{n-1}R[(T_4 - T_3) - (T_1 - T_2)]$$

$$\gamma = \frac{c_p}{c_V}$$

$$c_V = \frac{c_p}{\gamma} = \frac{1.003}{1.4} = 0.716$$

$$R = (c_p - c_V) = (1.003 - 0.716) = 0.287$$

$$W = \frac{1.3}{0.3} \times 0.287 \times [(452.5 - 280) - (300 - 185.6)] = 72.25 \text{ kJ/kg}$$

$$COP = \frac{\text{Heat abstracted}}{\text{Work done}} = \frac{94.7}{72.25} = 1.31$$

Ans ⟸

12.9 A machine used for refrigeration is working as Bell-Coleman cycle and has a upper pressure limit of 5.2 bar. The temperature and pressure at the start of compressor are 16 °C and 1 bar. The compressed air that is cooled at the constant pressure enters the cylinder at 41 °C. Assume that expansion and compression processes are adiabatic with $\gamma = 1.4$. Calculate:

(i) *COP*.
(ii) Quantity of air in circulation per minute.
(iii) Piston displacement of compressor and expander.
(iv) Diameter of compressor and expander cylinder. Take that the unit is double acting and runs at a speed of 240 rpm.
(v) Power required to drive the unit.

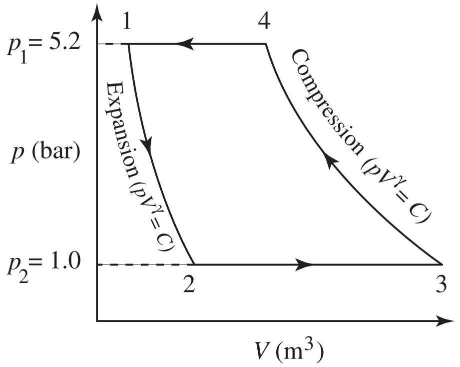


Fig. 12.20

Solution

Refer Fig.12.20. Consider the isentropic compression process 3–4:

$$\frac{T_4}{T_3} = \left(\frac{p_1}{p_2}\right)^{\dfrac{\gamma-1}{\gamma}} = \left(\frac{5.2}{1}\right)^{0.286} = 1.6$$

$$T_4 = 1.6 \times T_3 = 1.6 \times 289 = 462.4$$

Consider the adiabatic expansion process 1–2, we have:

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\dfrac{\gamma-1}{\gamma}}$$

$$\frac{314}{T_2} = \left(\frac{5.2}{1}\right)^{\dfrac{0.4}{1.4}} = 1.6$$

$$T_2 = \frac{314}{1.6} = 196.25$$

Since both compression and expansion processes are isentropic. They are reversible:

$$COP \text{ of machine} = \frac{T_2}{T_1 - T_2} = \frac{196.25}{314 - 196.25} = 1.67 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Refrigerating effect per kg of air $= c_p(T_3 - T_2) = 1.003 \times (289 - 196.23) = 93.03$ kJ/kg

Refrigerating effect produced by the machine $6 \times 14000 = 84000$ kJ/kg

Mass of air circulated $= \dfrac{84000}{93.03 \times 60} = 15.05$ kg/min

Piston displacement of compressor = Volume corresponding to point 3 $= V_3$

$$V_3 = \frac{mRT_3}{p_2} = \frac{15.05 \times 0.287 \times 1000 \times 289}{10 \times 10^5} = 12.48 \text{ m}^3\text{/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Swept volume per stroke} = \frac{12.48}{2 \times 240} = 0.026 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Let d_c be the diameter of the compressor cylinder and l be the stroke.

$$\frac{\pi}{4} \times d_c^2 \times \left(\frac{200}{1000}\right) = 0.026$$

$$d_c = \left(\frac{0.026 \times 1000 \times 4}{\pi \times 200}\right)^{\dfrac{1}{2}} = 0.407 \text{ m} = 407 \text{ mm}$$

Diameter of the compressor cylinder = 407 mm $\qquad \overset{\textbf{Ans}}{\Longleftarrow}$

Piston displacement of the expander = Volume corresponding to point 2 $= V_2$

$$V_2 = \frac{mRT_2}{p_2} = \frac{15.05 \times 0.287 \times 1000 \times 196.25}{1 \times 10^5} = 8.476 \text{ m}^3/\text{min}$$

$$\text{Swept volume per stroke} = \frac{8.476}{2 \times 240} = 0.0176 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

d_e = diameter of the expander, l = length of stroke

$$\frac{\pi}{4} d_e^2 l = 0.0176$$

$$\frac{\pi}{4} d_e^2 \left(\frac{200}{1000}\right) = 0.0176$$

$$d_e = \left[\frac{0.0176 \times 1000 \times 4}{\pi \times 200}\right]^{\frac{1}{2}} = 0.335 \text{ or } 335 \text{ mm} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Diameter of the expander cylinder = 335 mm.

Power required to drive the unit

$$COP = \frac{\text{Refrigerating effect}}{\text{Work input}}$$

$$1.67 = \frac{6 \times 14000}{1.67} = 50299.4 \text{ kJ/h} = 13.97 \text{ kJ/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Power required} = 13.97 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.10 Ice is produced at 0 °C from water at 20 °C. The condenser and evaporator temperatures are 25 °C and −5°C, respectively. Take the relative efficiency as 50%. Freon-12 at the rate of 6 kg per minute is circulated through the system. The dryness fraction of refrigerant entering the compression is 0.6. Specific heat of water is 4.187 kJ/kg K and the latent heat of ice is 335 kJ/kg. Estimate the amount of ice production in 24 h. Properties of Freon-12 are given the following table:

Temperature (°C)	Liquid heat (kJ/kg)	Latent heat (kJ/kg)	Entropy (kJ/kgK)
20	59.7	138	0.2232
−5	31.4	154	0.1251

Solution

Refer Fig.12.21. From the given data h_{f2}= 31.4 kJ/kg; h_{fg2} = 154.0 kJ/kg; h_{f3} = 59.7 kJ/kg; h_{fg3} = 138 kJ/kg; and h_{f4} = 59.7 kJ/kg:

$$h_2 = h_{f2} + x_2 \times h_{fg2} = 31.4 + 0.6 \times 154 = 123.8 \text{ kJ/kg}$$

For isentropic compression 2-3,

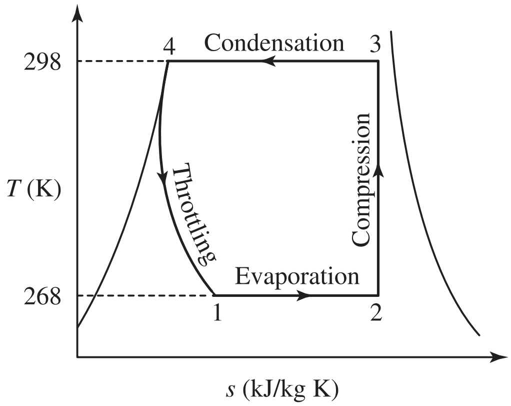


Fig. 12.21

$$s_3 = s_2$$

$$s_{f3} + x_3 \frac{h_{fg3}}{T_3} = s_{f2} + x_2 \frac{h_{fg2}}{T_2}$$

$$0.2232 + x_3 \times \frac{138}{298} = 0.1251 + 0.6 \times \frac{154}{268}$$

$$x_3 = (0.4698 - 0.2232) \times \frac{298}{138} = 0.5325$$

$$h_3 = h_{f3} + x_3 \times h_{fg3} = 59.7 + 0.5325 \times 138 = 133.2 \text{ kJ/kg}$$

$$h_1 = h_{f4} = 59.7 \text{ kJ/kg}$$

$$\text{Theoretical } COP = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{123.8 - 59.7}{133.2 - 123.8} = 6.82$$

$$\text{Actual } COP = \eta_{\text{relative}} \times (COP)_{\text{theoretical}} = 0.5 \times 6.82 = 3.41$$

Heat extracted from 1 kg of water at 20 °C for the formation of 1 kg ice at 0 °C,

$$= 1 \times 4.187 \times (20 - 0) + 335 = 418.74 \text{ kJ/kg}$$

$$m_{ice} = \text{Mass of ice formation in kg/min}$$

$$COP_{actual} = 3.41 = \frac{R_N(actual)}{W} = \frac{m_{ice} \times 418.74}{m(h_3 - h_2)} = \frac{m_{ice} \times 418.74}{6(133.2 - 123.8)}$$

$$m_{ice} = \frac{6(133.2 - 123.8) \times 3.41}{418.74}$$

$$= \frac{0.459 \times 60 \times 24}{1000} \text{ tonnes in 24 hours } = 0.661 \text{ tonne} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.11 An ammonia plant is used to produce 30 tonnes of ice at 0 °C from water at 0 °C. Compressor temperature range is 298–258 K. At the end of compression, the vapour is dry and saturated. For expansion, an expansion value is used. Assuming the actual COP to be 62% of the theoretical COP, calculate the power required to drive the compressor. The properties values of ammonia is given in the following table. Take the latent heat of ice as 335 kJ/kg.

Temperature (K)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
	Liquid	Vapour	Liquid	Vapour
298	100.04	1319.23	0.3473	4.4852
258	−56.56	1304.99	−2.1338	5.0585

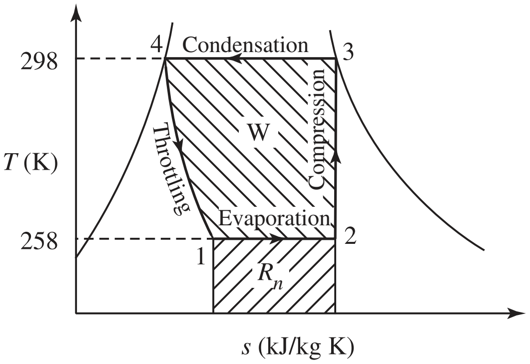


Fig. 12.22

Solution

$$\text{Theoretical } COP = \frac{h_2 - h_1}{h_3 - h_2}$$

$$h_3 = 1319.22$$

$$h_1 = h_4(\text{i.e. } h_{f4})$$

where $h_{f4} = 100.04$ kJ/kg from the given table. Now, we have to find h_2. Let us first find x_2.

$$s_2 = s_3$$

$$s_{f2} + x_2 s_{fg2} = s_{g3}$$

$$-2.1338 + x_2 \times [5.0585 - (-2.1338)] = 4.4852$$

$$x_2 = \frac{4.4852 + 2.1338}{5.0585 + 2.1338} = 0.92$$

$$h_2 = h_{f2} + x_2 h_{fg2} = -54.56 + 0.92 \times [1304.99 - (-54.56)]$$

$$= 1196.23 \text{ kJ/kg}$$

$$COP_{\text{theoretical}} = \frac{1196.23 - 100.04}{1319.22 - 1196.23} = 8.91$$

$$COP_{\text{actual}} = 0.62 \times COP_{\text{theoretical}} = 0.62 \times 8.91 = 5.52$$

$$\text{Actual refrigeration effect/kg} = COP_{\text{actual}} \times \text{Work done} = 5.52 \times (h_3 - h_2)$$

$$= 5.52 \times (1319.22 - 1196.23) = 678.9 \text{ kJ/kg}$$

$$\text{Heat to be extracted per second} = \frac{28 \times 1000 \times 335}{24 \times 3600} = 108.56 \text{ kJ/s}$$

$$\text{Mass of refrigeration circulated per second} = \frac{108.56}{678.9} = 0.1599 \text{ kg}$$

$$\text{Work done by the compressor per second} = 0.1599 \times (h_3 - h_2)$$

$$= 0.1599 \times (1319.22 - 1196.23)$$

$$= 19.67 \text{ kJ/s} = 19.67 \text{ kW}$$

Ans ⟸

12.12 An ammonia refrigerating plant works between the temperature range of −268 K and 298 K. Ammonia has a dryness fraction of 0.62 at the entry to compressor. The relative efficiency of the plant is 55%. Calculate the amount of ice formed during a period of 24 h. The ice is formed at 0 °C from water at 15 °C. Ammonia is circulated at the rate of 6.4 kg/min. Specific heat of water is 4.187 kJ/kg and latent heat of ice is 335 kJ/kg.

Properties of NH_3 (Datum −40°C)

Temperature (K)	Liquid heat (kJ/kg)	Latent heat (kJ/kg)	Entropy (kJ/kg K)
298	298.9	1167.1	1.124
−268	158.2	1280.8	0.630

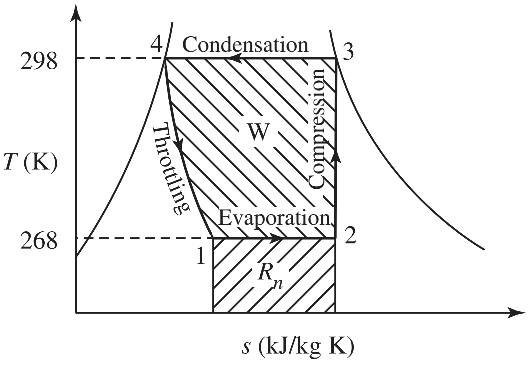


Fig. 12.23

Solution

The T-s diagram of the cycle is shown in Fig.12.23.

$$\text{Enthalpy at point 2, } h_2 = h_{f2} + x_2 h_{fg2} = 158.2 + 0.62 \times 1280.8$$

$$= 952.3 \text{ kJ/kg}$$

$$\text{Enthalpy at point 1, } h_1 = h_{f4} = 298.9 \text{ kJ/kg}$$

$$
\begin{aligned}
\text{Entropy at point 2, } s_2 &= \text{Entropy at point 3, } s_3\\
s_{f2} + x_2 s_{fg2} &= s_{f3} + x_3 s_{fg3}\\
0.630 + 0.62 \times \frac{1280.8}{268} &= 1.124 + x_2 \times \frac{1167.1}{298}\\
x_3 &= 0.63\\
\text{Enthalpy at point 3, } h_3 &= h_{f3} + x_3 h_{fg3} = 298.9 + 0.63 \times 1167.1\\
&= 1034.17 \text{ kJ/kg}\\
COP_{\text{theoretical}} &= \frac{h_2 - h_1}{h_3 - h_2} = \frac{952.3 - 298.9}{1034.17 - 952.5} = \frac{653.4}{81.87} = 7.98\\
COP_{\text{actual}} &= 0.55 \times 7.98 = 4.39\\
\text{Work done per kg of refrigerant} &= h_3 - h_2 = 1034.17 - 952.3 = 81.87 \text{ kJ/kg}\\
\text{Amount of refrigerant circulated} &= 6.4 \text{ kg/min}\\
\text{Work done per second} &= 81.87 \times \frac{6.4}{60} = 8.73 \text{ kJ/s}
\end{aligned}
$$

Heat extracted per kg of ice formed in 24 hours

$$
\begin{aligned}
&= 15 \times 41.187 + 335 = 397.8 \text{ kJ}\\
m_{ice} &= \frac{8.73 \times 3600 \times 24}{397.8} = 1896.1 \text{ kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

12.13 A simple vapour compression plant produces 5 tonnes of refrigeration. The enthalpy values at inlet to compressor, at exit from compressor and at exit from the condenser are 183.19, 209.41 and 74.59 kJ/kg, respectively. Calculate:

(i) the refrigerant flow rate

(ii) the *COP*

(iii) the power required to derive the compressor

(iv) the rate of heat rejection to the condenser

Solution

Total refrigeration effect produced = 5 TR = 5 × 14000 = 70000 kJ/h

Note that 1 TR is 14000 kJ/h. Refer Fig.12.24.

Net refrigerating effect produced per kg:

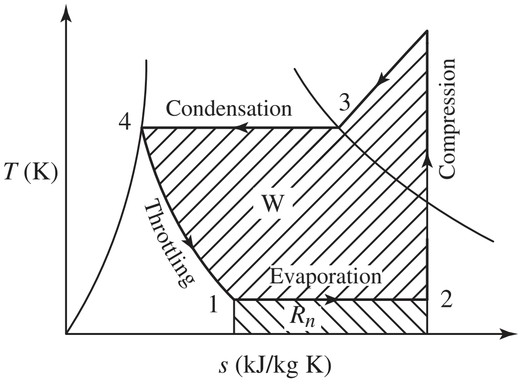


Fig. 12.24

$$h_2 - h_1 = 183.19 - 74.59 = 108.6 \text{ kJ/kg}$$

$$\text{Refrigerating flow rate, } \dot{m} = \frac{19.44}{108.6} = 0.179 \text{ kg/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$COP = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{183.19 - 74.59}{209.41 - 183.19} = 4.142 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Power required to drive the compressor:

$$p = \dot{m}(h_3 - h_2) = 0.179 \times (209.41 - 183.19) = 4.69 \text{ kW}$$

The rate of heat rejection to the condenser:

$$\dot{m}(h_3 - h_4) = 0.179 \times (209.41 - 74.59) = 24.13 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.14 An ice-making machine operates an ideal vapour compression cycle using $R-12$. The refrigerant enters the compressor as dry saturated vapour at $-15°$C and leaves the condenser as saturated liquid at 30 °C. Water enters the machine at 15 °C and leaves as ice at $-5°$C. For an ice production rate of 2400 kg in a day, determine the power required to run the unit. Find also the COP of the machine. Use refrigerant table only to solve the problem. Take the latent heat of fusion for water as 335 kJ/kg.

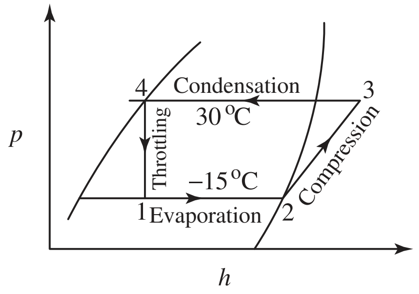


Fig. 12.25

Solution

Refer Fig.12.25. Using property Table of R-12, $h_2 = 344.927$ kJ/kg and $h_4 = h_1 = 228.538$ kJ/kg.

$$\begin{aligned}(c_p)_v &= 0.611 \text{ kJ/kg}^{\circ}\text{C} \\ s_2 &= s_3 \\ 1.56323 &= 1.5434 + 0.611 \times \ln\left[\frac{t_3 + 273}{30 + 273}\right]\end{aligned}$$

On solving, $t_3 = 40^{\circ}$C.

$$\begin{aligned}h_3 &= 363.575 + 0.611(39.995 - 30) = 369.68 \text{ kJ/kg} \\ \frac{R_n}{kg} &= h_2 - h_1 = 344.927 - 228.538 = 116.389 \text{ kJ/kg} \\ \frac{W}{kg} &= h_3 - h_2 = 369.68 - 344.927 = 24.753 \\ COP &= \frac{R_n}{W} = \frac{116.389}{24.753} = 4.72\end{aligned}$$

Ans ⟸

Assuming c_p for ice = 2.0935 kJ/kg °C.

$$\begin{aligned}\text{Heat to be removed} &= \frac{2400}{24 \times 3600} \times [4.187 \times (15 - 0) + 335 + 2.0935 \times (0 - (-5))] \\ &= 11.340 \text{ kJ/s} \\ \text{Power required} &= \frac{11.340}{4.72} = 2.41 \text{ kW}\end{aligned}$$

Ans ⟸

12.15 Freon-12 is used in a refrigerator which operates between −10°C and +30°C. A twin-cylinder, single-acting compressor is used which has the dimension of 20 × 15 cm. The volumetric efficiency is 85%. The compressor speed is 500 rpm. The refrigerant is subcooled and it enters at 22 °C in an expansion valve. The vapour is superheated and enters the compressor at −2°C. Show the process as a *T*-*s* and *p*-*h* diagram and calculate:

(i) the amount of refrigerant circulated per minute
(ii) the tonnes of refrigeration
(iii) the *COP* of the system

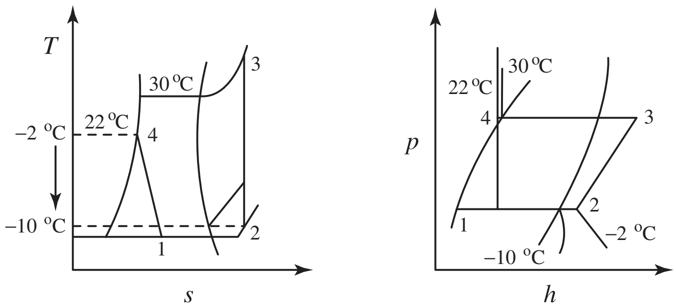
Solution

The *T*-*s* and *p*-*h* diagrams are as shown in Fig.12.26(a) and (b). The values of enthalpy and specific volume from the *p*-*h* chart are $h_2 = 352$ kJ/kg; $h_3 = 374$ kJ/kg and $h_4 = 221$ kJ/kg:

$$v_2 = 0.08 \text{ m}^3\text{/kg}$$

$$\text{Refrigerating effect per kg, } (h_2 - h_1) = 352 - 221 = 131 \text{ kJ/kg}$$

$$\text{Volume of refrigerant admitted per minute} = \frac{\pi}{4} d^2 L \times \text{Speed} \times 2 \times \eta_{vol}$$



(a) (b)

Fig. 12.26

$$= \frac{\pi}{4} \times 0.2^2 \times 0.15 \times 500 \times 2 \times 0.85 = 4 \text{ m}^3/\text{min}$$

$$\text{Mass of refrigerant per minute} = \frac{4}{0.08} = 50 \text{ kg/min} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\text{Cooling capacity} = 50 \times (h_2 - h_1) = 50 \times 131 = 6550 \text{ kJ/min}$$

$$= 393000 \text{ kJ/h} = \frac{393000}{14000} = 28.07 \text{ TR} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\text{Work per kg} = (h_2 - h_1) = 374 - 352 = 22 \text{ kJ/kg}$$

$$COP = \frac{131}{22} = 5.95 \qquad \overset{\text{Ans}}{\Longleftarrow}$$

12.16 In a standard vapour compression refrigeration cycle operating between an evaporator temperature of −10 °C and a condenser temperature of 40 °C, the enthalpy of the refrigerant, Freon-12, at the end of compression is 220 kJ/kg. Show cycle diagram on a T-s plane. Calculate (i) COP of the cycle; (ii) the refrigerating capacity and (iii) the compressor power assuming refrigerant flow rate of 1 kg/min. You may use the extract of Freon-12 property table given below.

Temperature, T (°C)	p (MPa)	h_f (kJ/kg)	h_g (kJ/kg)
−10	0.2191	26.85	183.1
40	0.9607	74.53	203.1

Solution

The T-s diagram of the cycle is shown in Fig.12.27. From the table given, $h_2 = 183.1$ kJ/kg and $h_1 = h_{f4} = 26.85$ kJ/kg.

$$COP = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{183.1 - 74.53}{220 - 183.1} = 2.94 \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\text{Refrigerating capacity} = m(h_2 - h_1) = 1 \times (183.1 - 74.53) = 108.57 \text{ kJ/min}$$

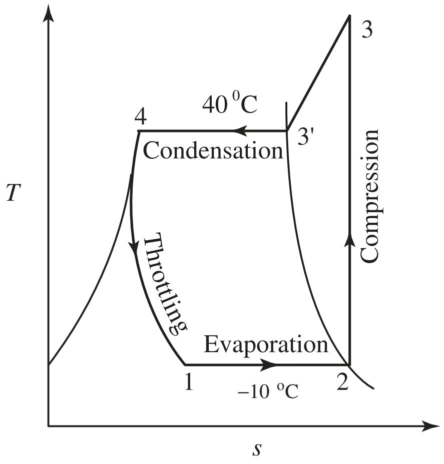


Fig. 12.27

$$\text{Compressor pressure} = m(h_3 - h_2) = 1 \times (220 - 183.1)$$

$$= 36.9 \text{ kJ/min} = 0.615 \text{ kJ/s} = 0.615 \text{ kW} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.17 A Freon-12 refrigeration producing cooling effect of 20 kJ/s operates on a simple cycle with pressure limits of 1.509 bar and 9.607 bar. The vapour leaves the evaporator dry and saturated and there is no under cooling. Determine the power required by the machine. If the compressor operates 300 rpm and has clearance volume of 3% of stroke volume, determine the piston displacement of the compressor. For compressor, assume that the expansion follows the law $pv^{1.3}$ = constant. Take the property values from the following table.

Temperature, T (°C)	p_s (bar)	V_g (m^3)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Specific heat (kJ/kg K)
			h_f	h_g	s_f	s_g	
−20	1.509	0.1088	17.8	178.61	0.073	0.7082	–
40	9.607	–	74.53	203.05	0.2716	0.682	0.747

Solution

Refer Fig.12.28 which gives the T-s diagram. $h_2 = 178.61$ kJ/kg and $h_3 = 203.05$ kJ/kg

$$h_{f4} = 74.53 \text{ kJ/kg} = h_1$$

$$\text{Cooling effect} = \dot{m}(h_2 - h_1)$$

$$20 = \dot{m} \times (178.61 - 74.53)$$

$$\dot{m} = \frac{20}{178.61 - 74.53} = 0.192 \text{ kg/s}$$

$$s_3 = s_2$$

$$s_{3'} + c_p \times \ln\left(\frac{T_3}{T_{3'}}\right) = 0.7082$$

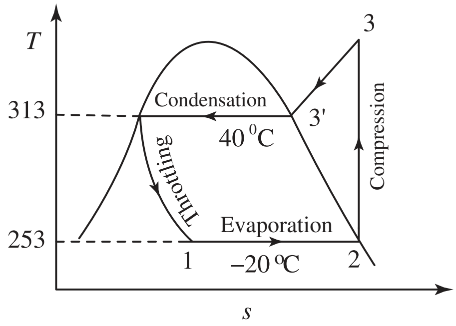


Fig. 12.28

$$0.682 + 0.747 \times \ln\left(\frac{T_3}{313}\right) = 0.7082$$

$$\ln\left(\frac{T_3}{313}\right) = \frac{0.7082 - 0.682}{0.747} = 0.03507$$

$$\frac{T_3}{313} = e^{0.03507} = 1.0357$$

$$T_3 = 313 \times 1.0357 = 324.2 \text{ K}$$

$$h_3 = h_{3'} + c_p \times (324.2 - 303)$$

$$= 203.05 + 0.747 \times (324.2 - 313) = 211.4 \text{ kJ/kg}$$

$$\text{Power required} = \dot{m}(h_3 - h_2) = 0.192 \times (211.4 - 178.61) = 6.29 \text{ kW}$$

$$\eta_V = 1 + C - C \times \left(\frac{p_d}{p_s}\right)^{\frac{1}{n}} = 1 + 0.03 - 0.03 \times \left(\frac{9.607}{1.509}\right)^{\frac{1}{1.13}} = 0.876 = 87.6\%$$

Volume of refrigerant of the intake condition is,

$$\dot{m} \times v_g = 0.192 \times 0.1088 = 0.02089 \text{ m}^3/\text{s}$$

$$\text{Swept volume, } V_s = \frac{0.02089}{\eta_{vol}} = \frac{0.02089}{0.85} = 0.02385 \text{ m}^3/\text{s} \qquad \xLeftarrow{\textbf{Ans}}$$

$$V = \frac{0.02385 \times 60}{300} = 0.00477 \text{ m}^3 \qquad \xLeftarrow{\textbf{Ans}}$$

12.18 A food-storage locker requires a refrigeration capacity of 50 kW. It works between the condenser temperature of 35 °C and an evaporator temperature of −10 °C. The refrigerant is ammonia. It is subcooled by 5 °C before entering the expansion valve by the dry saturated vapour leaving the evaporator. Assuming a single-cylinder single-acting compressor operating at 1000 rpm with stroke equal to 1.2 times the bore, determine (i) the power required and (ii) the cylinder dimensions.

Properties of ammonia are:

T (°C)	p (bar)	Enthalpy (kJ/kg)		Entropy (kJ/kg)		Specific volume (m^3/kg)		Specific heat (kJ/kg K)	
		Liq	Vap	Liq	Vap	Liq	Vap	Liq	Vap
−10	2.92	154.056	1450.22	0.829	5.755	–	0.417	–	2.492
35	13.52	366.072	1488.57	1.566	5.209	1.7023	0.096	4.556	2.903

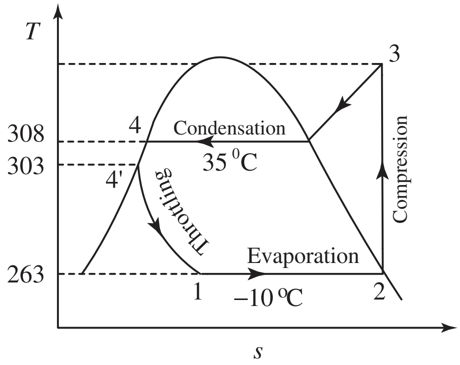


Fig. 12.29

Solution

Refer Fig.12.29. $h_2 = 1450.22$ kJ/kg; $h_{3'} = 1488.57$ kJ/kg; $h_{f4} = 366.072$ kJ/kg.

$$\begin{aligned}
h_{f4'} &= h_1 = h_{f4} - 4.556(308 - 303) \\
&= 366.07 - 4.556(308 - 303) = 343.29 \text{ kJ/kg} \\
s_3 &= s_2 \\
s_{3'} + c_p \times \ln\left(\frac{T_3}{T_{3'}}\right) &= 5.755 \\
5.2086 + 2.903 \times \ln\left(\frac{T_3}{308}\right) &= 5.755 \\
\ln\left(\frac{T_3}{308}\right) &= \frac{5.755 - 5.2086}{2.903} = 0.1882 \\
\frac{T_3}{308} &= e^{0.1882} = 371.8 \text{ K} \\
h_3 &= h_{3'} + c_p(T_3 - T_{3'}) \\
&= 1488.57 + 2.903 \times (371.8 - 308) = 1673.8 \text{ kJ/kg} \\
\text{Mass of refrigerant, } \dot{m} &= \frac{50}{h_2 - h_1} = \frac{50}{1450.22 - 343.29} = 0.04517 \text{ kJ/s}
\end{aligned}$$

$$\text{Power required} = \dot{m}(h_3 - h_2)$$

$$= 0.04517 \times (1673.8 - 1450.22) = 10.1 \text{ kW} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

Cylinder dimensions,

$$\dot{m} = \frac{\pi}{4} D^2 L \frac{N}{60} \times 0.417477 = 0.04517$$

$$\frac{\pi}{4} D^2 \times 1.2D \times \frac{1000}{60} \times 0.417477 = 0.04517 \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$D^3 = \frac{0.04517 \times 4 \times 60}{\pi \times 1.2 \times 1000 \times 0.417477} = 0.006888$$

$$\text{Diameter of cylinder, } d = (0.006888)^{\frac{1}{3}} = 0.19 \text{ m} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$L = 1.2d = 1.2 \times 0.19 = 0.228 \text{ m} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.19 A refrigeration cycle uses Freon-12 as the working fluid. The temperature of the refrigerant in the evaporator is −10°C. The condensing temperature is 40 °C. The cooling load is 150 W and the volumetric efficiency of the compressor is 80%. The speed of the compressor is 720 rpm. Calculate the mass flow rate of the refrigerant and the displacement volume of the compressor.

Properties of Freon-12:

Temperature (°C)	Saturation pressure (MPa)	Enthalpy (kJ/kg)		Specific volume (m^3/kg)
		Liquid	Vapour	
−10	0.22	26.8	183.0	0.08
40	0.96	74.5	203.1	0.02

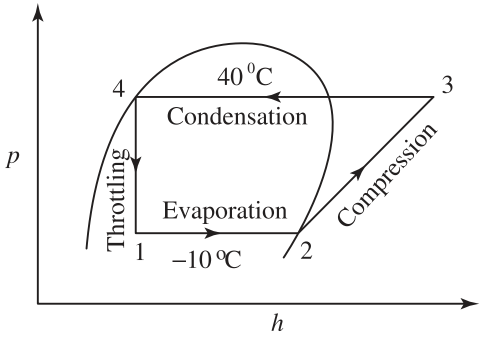


Fig. 12.30

Solution

Refer Fig.12.30.

$$\text{Refrigerating effect} = h_2 - h_1 = 183 - 74.5 = 108.5 \text{ kJ/kg}$$

$$\text{Cooling load} = \dot{m} \times 108.5 \times 1000 = 150 \text{ K}$$

$$\dot{m} = \frac{150}{108.5 \times 1000} = 0.001382 \text{ kg/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Specific volume at entry to compressor, $v_2 = 0.08 \text{ m}^3/\text{kg}$

$$\text{Displacement volume of the compressor} = \frac{\dot{m}v_2}{\eta_{vol}} = \frac{0.001382 \times 0.08}{0.8} = 0.000138 \text{ m}^3/\text{s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.20 In a simple vapour compression cycle, following are the properties of the refrigerant R12 at various points:

Compression inlet	$h_2 = 183.2$ kJ/kg	$v_2 = 0.0767 \text{ m}^3/\text{kg}$
Compression discharge	$h_3 = 222.6$ kJ/kg	$v_3 = 0.0164 \text{ m}^3/\text{kg}$
Compression exit	$h_4 = 84.9$ kJ/kg	$v_4 = 0.00083 \text{ m}^3/\text{kg}$

The piston displacement volume for compression is 1.5 liters per stroke and its volumetric efficiency is 80%. The speed of the compression is 1600 rpm. Find (i) power rating of the compression and (ii) refrigerating effect (kW).

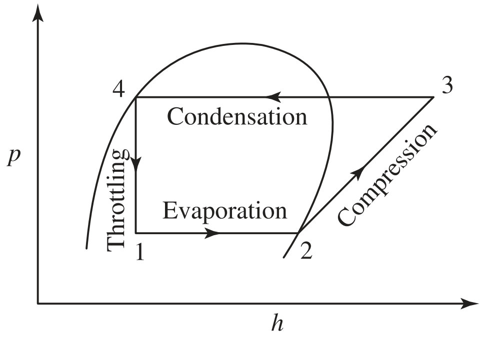


Fig. 12.31

Solution

Refer Fig.12.31.

$$\text{Piston displacement volume} = \frac{\pi}{4}d^2 \times l = 1.5 \text{ l} = 1.5 \times 1000 \times 10^{-6} \text{ m}^3/\text{stroke}$$

$$= 0.0015 \text{ m}^3/\text{revolution}$$

$$\text{Compressor discharge} = 0.0015 \times 1600 \times 0.8 = 1.92 \text{ m}^3/\text{min}$$

$$\dot{m} = \frac{\text{Compressor discharge}}{v_2} = \frac{1.92}{0.0767} = 25.03 \text{ kg/min}$$

$$\begin{aligned}
\text{Power rating of the compressor} &= \dot{m}(h_3 - h_2) \\
&= \frac{25.03}{60} \times (222.6 - 183.2) = 16.44 \text{ kW} && \overset{\textbf{Ans}}{\Longleftarrow} \\
\text{Refrigerating effect} &= \dot{m}(h_2 - h_1) = \dot{m}(h_2 - h_4) \\
&= \frac{25.03}{60} \times (183.2 - 84.9) = 41 \text{ kW} && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

12.21 A refrigerator operating on a standard vapour compression cycle has a *COP* of 6.5 and is driven by a 50 kW compressor. Enthalpies of saturated liquid and saturated vapour refrigerant at the operating condensing temperature of 35 °C are 62.55 kJ/kg and 201.45 kJ/kg, respectively. The saturated refrigerant vapour leaving evaporator has an enthalpy of 187.53 kJ/kg. Find the refrigerant temperature at compressor discharge. The c_p of refrigerant vapour may be taken as 0.6155 kJ/kg °C.

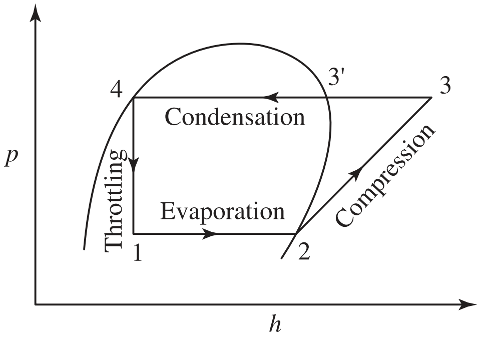


Fig. 12.32

Solution

Refer Fig.12.32.

$$\begin{aligned}
\text{Refrigerating capacity} &= 50 \times COP = 50 \times 6.5 = 325 \text{ kW} \\
\text{Heat extracted per kg of refrigerant} &= 187.53 - 69.55 = 117.98 \text{ kJ/kg} \\
\text{Refrigerant flow rate} &= \frac{325}{117.98} = 2.755 \text{ kg/s} \\
\text{Compressor pressure} &= 50 \text{ kW} \\
\text{Heat input per kg} &= \frac{50}{2.755} = 18.15 \text{ kJ/kg} \\
\text{Enthalpy of vapour after compression} &= h_2 + 18.15 = 187.53 + 18.15 = 205.68 \text{ kJ/kg} \\
\text{Superheat} &= 205.68 - h_{3'} = 205.68 - 201.45 = 4.23 \text{ kJ/kg} \\
4.23 &= mc_p(t_3 - t_{3'}) = 1 \times 0.6155 \times (t_3 - 35) \\
t_3 &= \frac{4.23}{0.6155} + 35 = 41.87 \text{ K} && \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

Note: The compressor rating of 50 kW is assumed to the enthalpy of compression in the absence of any data regarding the efficiency of compressor.

12.22 A vapour compression heat-pump driven by a power cycle having a thermal efficiency of 25%. For the heat pump, refrigerant-12 is compressed from saturated vapour at 2.0 bar to the condenser pressure of 12 bar. The isentropic efficiency of the compressor is 80%. Saturated liquid enters the expansion valve at 12 bar. For the power cycle, 80% of the heat rejected by it is transferred to the heated space which has a total heating requirement of 500 kJ/min. Determine the power input to the heat-pump compressor. The following data for refrigerant 12 may be used. Take vapour specific heat at constant pressure as 0.7 kJ/kg K.

Temperature (°C)	Pressure (bar)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
		Liquid	Vapour	Liquid	Vapour
−12.53	2.0	24.57	182.07	0.0992	0.7035
49.31	12.0	84.21	206.24	0.3015	0.6799

Solution

$$\text{Heat rejected by the cycle} = \frac{500}{0.8} = 625 \text{ kJ/min}$$

Assuming isentropic compression of refrigerant we have:

Enthalpy of dry saturated vapour at 2 bar = Entropy of superheated vapour at 12 bar

$$0.7035 = 0.6799 + c_p \times \ln \frac{T}{(49.31 + 273)}$$

$$= 0.6799 + 0.7 \times \ln \frac{T}{322.31}$$

$$\ln \left[\frac{T}{322.31}\right] = \frac{0.7035 - 0.6799}{0.7} = 0.03371$$

$$T = 322.31e^{0.03371} = 333.4 \text{ K}$$

Enthalpy of superheated vapour at 12 bar = 206.24 + 0.7 × (333.4 − 322.31) = 214 kJ/kg

$$\text{Heat rejected per cycle} = 214 - 84.21 = 129.88 \text{ kJ/kg}$$

$$\text{Mass flow rate of refrigerant} = \frac{625}{129.88} = 4.812 \text{ kg/min}$$

$$\text{Work done on compressor} = 4.812 \times (214 - 182.07)$$

$$= 153.65 \text{ kJ/min} = 2.56 \text{ kJ/s} = 2.56 \text{ kW}$$

$$\text{Actual work of compression} = \frac{2.56}{0.8} = 3.2 \text{ kW}$$

Hence, power input to the heat pump compressor = 3.2 kW.

12.23 A food-storage locker requires a refrigeration system of 2400 kJ/min capacity at an evaporation temperature of 263 K and a condenser temperature of 303 K. The refrigerant used is Freon-12 and is subcooled by 6 °C before entering the expansion value and the vapour is superheated by 7 °C before leaving the evaporated coil. The compression of refrigerant is reversible adiabatic. The refrigeration compression is two-cylinder, single-acting with stroke equal to 1.25 times the bore and operates at 1000 rpm.

Properties of Freon 12:

Saturation temp. (°C)	Absolute pressure (bar)	Specific volume of vapour (m^3/kg)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
			Liquid	Vapour	Liquid	Vapour
263	2.19	0.0767	26.9	183.2	0.1080	0.7020
303	7.45	0.0235	64.6	199.6	0.2399	0.6854

Liquid specific heat = 1.235 kJ/kg K and vapour specific heat = 0.733 kJ/kg K. Determine,

(i) Refrigerating effect per kg

(ii) Mass of refrigerant to be circulated per minute

(iii) Theoretical piston displacement per minute

(iv) Theoretical power required to run the compressor, in kW

(v) Heat removed through condenser per minute

(vi) Theoretical bore and stroke of compression.

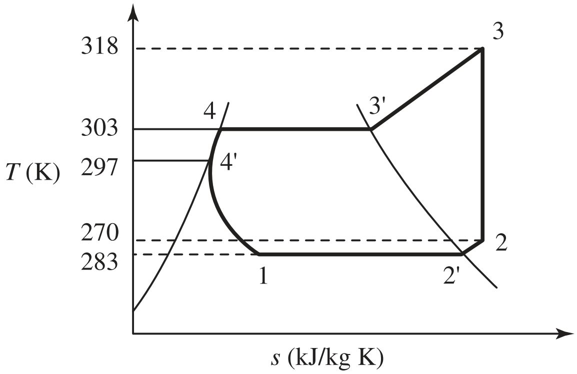


Fig. 12.33

Solution

Refer Fig.12.33, which shows the cycle on a T-s plane.

$$\text{Enthalpy at 2, } h_2 = h_{2'} + c_p(T_2 - T_{2'})$$

Form the given table, $h_{2'} = 183.2$ kJ/kg:

$$T_2 - T_{2'} = \text{Degree of superheat (as vapour enters compressor)}$$

$$= 7°\text{C}$$

$$s_2 = s_{2'} + c_p \times \ln \frac{T_2}{T_{2'}} = 0.7020 + 0.733 \times \ln\left(\frac{270}{263}\right) = 0.7212 \text{ kJ/kg K}$$

For the isentropic process 2–3, Entropy at 2 = Entropy at 3:

$$0.7212 = s_{3'} + c_p \times \ln\left(\frac{T_3}{T_{3'}}\right) = 0.6854 + 0.733 \times \ln\left(\frac{T_3}{303}\right)$$

$$\ln\left(\frac{T_3}{303}\right) = 0.0488$$

$$T_3 = 318 \text{ K}$$

$$\text{Enthalpy at 3, } h_3 = h_{3'} + c_p \times (T_3 - T_{3'})$$

$$= 199.6 + 0.733 \times (318 - 303) = 210.6 \text{ kJ/kg}$$

$$\text{Enthalpy at } 4', h_{f4'} = h_{f4} - (c_p)_{liq}(T_4 - h_{4'})$$

$$= 64.6 - 1.235 \times 6 = 57.19 \text{ kJ/kg}$$

For the process $4' - 1$, Enthalpy at $4'$ = Enthalpy at 1 = 57.19 kJ/kg

For specific volume at 2, $\frac{v_{2'}}{T_{2'}} = \frac{V_2}{T_2}$

$$V_2 = \frac{V_{2'}}{T_{2'}} \times T_2 = 0.0767 \times \frac{270}{263} = 0.07874 \text{ m}^3\text{/kg}$$

$$\text{Refrigerating effect per kg} = h_2 - h_1 = 188.33 - 57.19 = 131.14 \text{ kJ/kg}$$

Mass of refrigerant to be circulated per minute for producing effect of 2400 kJ/min

$$= \frac{2400}{131.14} = 18.3 \text{ kg/min}$$

Theoretical piston displacement per minute = Mass flow/min × Specific volume at suction

$$= 18.3 \times 0.07874 = 1.441 \text{ m}^3\text{/min}$$

Theoretical power required to run the compressor = Mass flow of refrigerant per second × compressor work/kg

$$= \frac{18.3}{60} \times (h_3 - h_2) = \frac{18.3}{60} \times (210.6 - 188.33) \text{ kJ/s} = 6.79 \text{ kJ/s} = 6.79 \text{ kW}$$

Heat removed through the condenser per minute = Mass flow of refrigerant × Heat removed per kg of refrigerant

$$= 18.3 \times (h_3 - h_{f4'}) = 18.3 \times (210.6 - 57.19)$$

$$= 2807.4 \text{ kJ/min}$$

$$\text{Theoretical piston displacement per cylinder} = \frac{\text{Total displacement per minute}}{\text{Number of cylinders}}$$

$$= \frac{1.441}{2} = 0.7205 \text{ m}^3\text{/min}$$

$$\text{Length of stroke} = 1.25\text{dia. of piston} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$0.7205 = \frac{\pi}{4} d^2 \times (1.25d) \times 100$$

$$d = 0.09 \text{ m} = 90 \text{ mm}$$

$$l = 1.25d = 1.25 \times 90 = 112.5 \text{ mm} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.24 For food-storage purpose, a refrigeration plant of 10.5 TR is required at an evaporation temperature of -12 °C and condenser temperature of 27 °C. The refrigerant is ammonia. It is subcooled by 6 °C before entering the expansion valve. The vapour is 0.95 dry as it leaves the evaporator coil. The compression is adiabatic. Using *p*-*h* chart, calculate:

(i) condition of vapour at outlet of the compressor,
(ii) condition of vapour at entrance to evaporator,
(iii) *COP* and
(iv) power required in kW.

Neglect throttling and clearance effect.

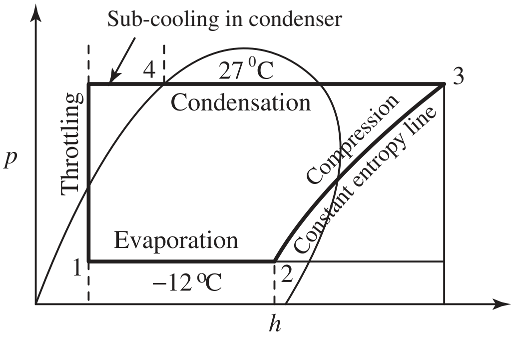


Fig. 12.34

Solution

Refer Fig.12.34. Using *p*-*h* chart for ammonia:

(i) Locate point 2 where -12 °C cuts 0.95 dryness fraction line.

(ii) From point 2 move along constant entropy line and locate point 3, where it cuts the constant pressure line until it cuts +21°C temperature line to get point 4.

(iii) From point 4, drop a vertical line to cut constant pressure line corresponding to −12°C and get the point 5.

The values read from the chart are as as follows: $h_2 = 1597$ kJ/kg; $h_3 = 1790$ kJ/kg; $h_4 = h_1 = 513$ kJ/kg; $t_3 = 58°$C and $x_1 = 0.13$.

Condition of the vapour at the outlet of the compressor = 58 − 27 = 31 °C (superheat) $\overset{\textbf{Ans}}{\Longleftarrow}$

Condition of the vapour at the entrance of the evaporator, $x_1 = 0.13$ $\overset{\textbf{Ans}}{\Longleftarrow}$

$$COP = \frac{h_2 - h_1}{h_3 - h_2} = \frac{1597 - 513}{1790 - 1597} = 5.6 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$COP = \frac{\text{Net refrigerating effect}}{\text{Work done}} = \frac{R_n}{W}$$

$$5.6 = \frac{10.5 \times 1000}{W \times 60}$$

$$W = \frac{10500}{60 \times 5.6} = 437.5 \text{ kJ/min} = 7.29 \text{ kJ/s} = 7.29 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

12.25 The evaporator and condenser temperatures of 20 tonnes capacity freezer are −28°C and 23 °C, respectively. The refrigerant R-22 is subcooled by 3 °C before it enters the expansion value and is superheated to 8 °C before leaving the evaporator. The compression is isentropic. A single-acting, six-cylinder compressor with stroke equal to bore running at 250 rpm is used. Determine:

(i) Refrigerating effect per kg.
(ii) Mass of refrigerant to be circulated per minute.
(iii) Theoretical piston displacement per minute.
(iv) Theoretical power.
(v) *COP*.
(vi) Heat removed through condensed.
(vii) Theoretical bore and stroke of the compressor.

Neglect throttling and clearance effect.

Solution

Refer Fig.12.35. Follow the same procedure as indicated in the previous example and plot 1 to 4 on *p-h* chart Freon-22.

The following values can be obtained $h_2 = 615$ kJ/kg; $h_3 = 664$ kJ/kg; $h_4 = 446$ kJ/kg; and $V_2 = 0.14$ m^3/kg.

$$\text{Refrigerating effect per kg} = h_2 - h_1 = 615 - 446 = 169 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Refrigerant to be circulated per minute,

$$m = \frac{20 \times 14000}{169 \times 60} = 27.6 \text{ kg/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Theoretical piston displacement per minute = Specific volume at suction × Mass of refrigerant used per minute

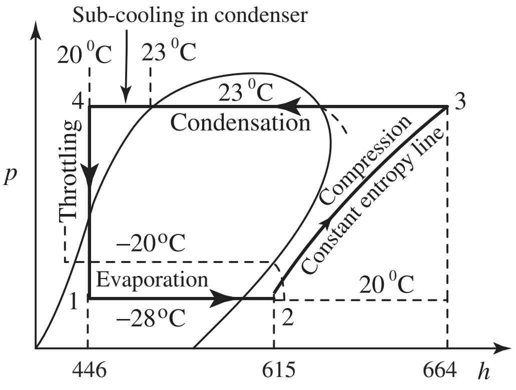


Fig. 12.35

$$= \quad 0.14 \times 27.6 = 3.864 \text{ m}^3/\text{min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Theoretical power} = \dot{m}(h_3 - h_2) \quad = \quad \frac{27.6}{60} \times (664 - 615) = 22.54 \text{ kJ/s} = 22.54 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$COP \quad = \quad \frac{h_2 - h_1}{h_3 - h_2} = \frac{615 - 446}{664 - 615} = 3.45 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Heat removed through the condenser} = m(h_3 - h_4) = 27.6(664 - 446) = 6016.8 \text{ kJ/min} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Theoretical bore } (d) \text{ and stroke } (l) = \frac{\text{Total displacement/min}}{\text{Number of cylinders}} = \frac{3.864}{6} = 0.644 \text{ m}^3/\text{min}$$

$$\frac{\pi}{4}d^2 l \quad = \quad \frac{0.644}{950}$$

$$\frac{\pi}{4}d^2 d \quad = \quad \frac{0.644}{950} \qquad (\because l = d)$$

$$d \quad = \quad 0.0952 \text{ m} = 95.2 \text{ mm} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$l \quad = \quad 95.2 \text{ mm} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Review Questions

12.1 Explain the meaning of the term, ‘refrigeration’.

12.2 What are the different ways by which the effect of refrigeration can be produced?

12.3 What are the various fields in which refrigeration is used?

12.4 What are the four basic units of a refrigeration system and classify the types of refrigeration system.

12.5 Mention different types of special refrigeration system.

12.6 Define coefficient of refrigeration and what is 1 tonne of refrigeration?

12.7 Explain closed and open type of air refrigeration system.

12.8 Explain reversed Carnot cycle for the application of refrigeration.

12.9 Derive the expressions for

- *(i) COP of a reversed Carnot cycle refrigerating machine.*
- *(ii) COP of a Carnot cycle heat pump.*
- *(iii) η_{COP} of a Carnot cycle heat engine.*

12.10 With appropriate figures, explain the reversed Joule's cycle and derive the expression for COP of reversed Joule's cycle.

12.11 Explain the merits of reversed Joule's cycle.

12.12 With a suitable figure, explain the working principle of a simple vapour compression cycle.

12.13 What are the merits and demerits of a simple vapour compression cycle,

12.14 With the help of a h-s diagram, derive the expression for COP of a simple vapour compression cycle.

12.15 Draw the p-h diagram of a simple vapour compression cycle and explain the various processes.

12.16 Explain the effect of superheating on the COP of a simple vapour compression cycle.

12.17 What are the difference between actual and theoretical vapour compression cycle?

12.18 What do you understand by the term, 'volumetric efficiency'? Define the same.

12.19 Derive the expressions for theoretical and total volumetric efficiency of a compressor.

12.20 What are the six parameters for analysing the performance of a vapour compression system? Explain their effects on the performance.

12.21 Explain with a figure the simple vapour absorption system.

12.22 Explain the practical vapour absorption system.

12.23 What is refrigerant? How is it classified?

12.24 Explain the desired properties of a refrigerant.

12.25 Explain the current application of refrigerant.

Exercise

12.1 A refrigerator is supposed to maintain a refrigerated space at $-17\ ^\circ$C and is operating on reversed Carnot cycle. It extracts 600 kJ/min heat from a refrigerated space and rejects heat to the atmosphere at 32 °C. Find the work input required to run the refrigerator in kW. *Ans:* 1.92 kW

12.2 Consider a refrigerator working on a reversed Carnot cycle. The temperature range is -10°C and 30 °C. The capacity of the refrigerator is 1000 tonnes. Assuming that water being supplied at 27 °C, find the rate of ice formation. Also calculate the minimum power required. Take latent heat of ice as 335 kJ/kg. *Ans:* 7.6 kg/s; 532.32 kW

12.3 To maintain a temperature of space at -23°C, a 2.25 kW/tonne of refrigerator is required. The refrigerator used for this purpose is working a Carnot cycle. Determine COP of cycle and temperature of surroundings to which the heat is rejected by the refrigerator. *Ans:* 1.56; 410.26 K

12.4 Pressure ratio of an air refrigerator cycle is 7. Temperature of air at the beginning of compression is -33°C and air is cooled up to 27 °C after compression. The isentropic efficiency of compression and expansion are 85% and 87%, respectively. Determine the refrigeration capacity and COP. Take $c_p = 1.005$ kJ/kg K; $\gamma = 1.4$ and air flow rate of 1 kg/s. *Ans:* 14.76 tonne; 0.52

12.5 In a refrigeration system air leaves the refrigerated space at 10 °C and 1 bar. It is then compressed isentropically to 5 bar. Then, it is cooled to 30 °C at this pressure. Further, it is expanded isentropically up to 1 bar and discharged into refrigerated space. Calculate the COP of the system. *Ans:* 1.711

12.6 A refrigerator is working on Bell Coleman cycle. The air enters compressor at 1 bar and -13°C and compressed to 6 bar. Compressed air is cooled to 27 °C before going into expander for expansion up to 1 bar. Then, it passes through refrigerated space. Determine the refrigeration capacity, power required to run compressor and COP of the system if the air flow rate is 0.9 kg/s. *Ans:* 20.75 tonne; 157.3 kW; 1.5

12.7 An aircraft refrigeration system consists of two cooling turbines, one intercooler and a fan. First cooling turbine is supplied with ram air at 1.25 bar 17 °C and delivers it after expansion to inter cooler at 0.95 bar for cooling the air coming from compressor at 5 bar. The cooling air from intercooler is sucked by fan and discharged to atmosphere. The cooled air from the intercooler is expanded up to 1 bar in second turbine and discharged into passenger cabin. The air is exhausted from cabin at 27 °C and 10 tonne of refrigeration is required in cabin. Index of compression and expansion is 1.45 and 1.3, respectively. Determine the mass flow rate of air in cabin, ram air mass flow rate and COP of the system, if the compressed air is cooled to 50 °C in intercooler and temperature rise in intercooler for ram air is limited to 30 °C. *Ans:* 0.451 kg/s; 3.13 kg/s; 0.495

12.8 An air-cooling system for an aircraft has a refrigeration capacity of 16 tonnes. Atmospheric air available is at 0.92 bar and 7 °C. Due to ramming action, pressure rises up to 1 bar. The ram air leaves main compressor at 4 bar and subsequently 66% of heat of compressor air is removed in air-cooled heat exchanger and then passed through an evaporation for further cooling. Temperature of air leaves by 10 °C in evaporated and this cool air is then expanded in cooling turbine before being supplied, passenger cabin maintained at 1 bar. Air leaves the cabin at 25 °C. Take isentropic efficiency of compressor and turbine as 0.9 and 0.8, respectively. Calculate (i) the power required to run the system and (ii) *COP* of the system. *Ans:* (i) 44.56 kW; (ii) 1.257

12.9 A refrigerator is working on a simple vapour compression cycle. It operates between the temperature of 25 °C and −15 °C. The refrigerant is ammonia, NH_3. Assume that after compression the ammonia is dry. There is no under cooling of liquid refrigerant. Use the following properties and values for ammonia and calculate the *COP* of the system. *Ans:* 5.94

Temperature (°C)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)	
	h_f	h_g	s_f	s_g
−15	−54.51	1303.74	−0.2132	5.0536
25	99.94	1317.95	0.3386	4.4809

12.10 Freon-12 is used in a vapour compression cycle. The condensation occurs at 40 °C and evaporation takes place at −20°C. Refrigeration effect obtained is 2.86 tonne from the cycle. The compressor runs at 1500 rpm and has a clearance volume of 2% of the stroke volume. Take the index for compression as 1.13. The properties of Froen-12 is given in the following table. Calculate (i) the *COP* and (ii) the piston displacement.

Temperature (°C)	Saturation pressure (bar)	Specific volume (m^3/kg)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Specific heat (°C)	
			h_f	h_g	s_f	s_g	c_{pf}	c_{pg}
−20	1.51	0.1088	17.8	178.61	0.0730	0.7082	–	0.605
40	9.61	–	74.53	203.05	0.2716	0.6820	0.976	0.747

Ans: (i) 3.175 (ii) 569.45 cc

12.11 Carbon dioxide is used as a refrigerant in a vapour compression refrigeration cycle. Condenser and evaporator temperatures are 20 °C and −10°C. Temperature after isentropic compression is 40 °C. Condensate leaves at 10 °C before being passed through expansion calve. Find the *COP* of the system and mass flow rate of CO_2, if the refrigeration effect required is 2 kW. Take the properties of CO_2 from the following table:

Temperature (°C)	Saturation pressure (bar)	Specific volume (m^3/kg)	Enthalpy (kJ/kg)		Entropy (kJ/kg K)		Specific heat (°C)	
			h_f	h_g	s_f	s_g	c_{pf}	c_{pg}
20	57.27	–	114.11	299.62	0.523	1.0527	2.889	2.135
−10	26.49	0.014	60.78	322.28	0.238	1.2324	–	–

Ans: 6.51; 0.01216 kg/s

12.12 A cold storage is to be maintained at −15 °C while surroundings are 25 °C. The heat leakage from surroundings is 30 kW. The actual *COP* of the refrigeration plant is 30% that of an ideal plant working between the same temperature. Estimate the power required to run the plant. *Ans:* 15.5 kW

12.13 Refrigerant R-134 a is used in a refrigerator and operates on an ideal vapour compression cycle between 140 kPa and 80 kPa. If the mass flow rate of refrigerant is 0.05 kg/s, calculate:

(i) the rate of heat removal from the refrigerated space

(ii) power required to run the compressor

(iii) rate of heat rejection in the condenser

(iv) the *COP* *Ans:* (i) 7.131 kJ/s; (ii) 1.80 kJ/s; (iii) 8.93 kJ/s; (iv) 3.961

12.14 Five tonnes of refrigeration is to produced by a R-12 plant. The evaporator and condenser temperatures are −10 °C and 40 °C, respectively. Estimate

(i) flow rate of refrigerant in kg/s

(ii) volume flow rate handled by the compressor in m^3/s

(iii) compressor discharge temperature

(iv) the pressure ratio

(v) heat rejected to the condenser

(vi) flash gas percentage after throttling

(vii) the *COP*

(viii) power required to drive the compressor

How does the *COP* compare with that of a Carnot refrigerator operating between 40 and −20 °C? *Ans:* (i) 0.0139 m^3/s; (ii) 48 °C; (iii) 4.39; (iv) 24.27 kW; (v) 30.5%; (vi) 4.14; (vii) 4.72 kW; (viii) 0.787

12.15 An R-12 refrigeration plant producing 12 TR operates between 35 °C and −10 °C of condenser and evaporator temperature, respectively. To subcool the saturated liquid leaving the condenser, a suction line heat exchanger is employed. Saturated vapour leaving the evaporator is superheated so that its discharge temperature is 60 °C. Which is obtained after isentropic compression. Find

(i) subcooling achieved in the heat exchanger

(ii) the flow rate of refrigerant

(iii) cylinder dimensions of the twin cylinder compressor. Assume that the compressor runs at 900 rpm on its stroke to bore ratio is 1.1 volumetric efficiency of the compressor is 80%.

(iv) the *COP* of the plant

(v) the power required to drive the compressor

Ans: (i) 126 kJ/kg; (ii) 0.31 kg/s; (iii) $d = 10.8$ cm; $L = 11.88$ cm; (iv) 4.85; (v) 8.02 kW

12.16 A refrigeration system works on a two-stage vapour compression cycle with a heat exchange (flash chamber) operating with ammonia as the refrigerant. The evaporator and condenser temperatures are −30 and 40 °C, respectively. If the capacity of the plant is 30 TR, estimate the total work of compression and the COP. If the compression is done in a single-stage, what will be the percentage increase in the work of compression? What is the percentage increase in the *COP* owing to the staging of the compression process? *Ans:* (i) 39.08 kW; 2.986; (ii) 15.81%; 15.82%

12.17 A refrigeration plant works on aqua-ammonia absorption system. Heat is supplied to the generator by condensing steam at 200 kPa with 90% quality. The temperature to be maintained in the refrigerator is −10 °C. The ambient temperature is 30 °C. Estimate the *COP* of the refrigerator. If the actual *COP* is 50% of the maximum *COP* and the refrigeration load is 2 TR, what will be the required steam flow rate?
Ans: 0.75; 0.0654 kg/s

12.18 In an aircraft cooling system, air enters the compressor at 100 kPa at 4 °C is compressed to 300 kPa with an isentropic efficiency of 75%. After being cooled to 55 °C at constant pressure in a heat exchanger, air then expands in a turbine to 100 kPa with an isentropic efficiency of 80%. The low temperature absorbs a cooling load of 3 TR at constant before re-entering the compressor which is driven by the turbine. Consider air to be an ideal gas, determine the *COP* of the refrigerator, the driving power required and the mass flow rate of air. *Ans:* 0.268; 43.53 kW; 0.66 kg/s

12.19 A vapour compression heat pump system uses R-12 as the working fluid. The refrigerant enters the compressor at 2.4 bar, 0 °C with a volumetric flow rate of 0.5 m^3/min. Compression is adiabatic to 9 bar at 60 °C and the saturated liquid exits the condenser at 9 bar. Find

(i) the power input to the compressor
(ii) the heating capacity of the system
(iii) the coefficient of performance
(iv) the isentropic compressor efficiency

Ans: (i) 3.6 kW; (ii) 4.99 tonnes; (iii) 4.853; (iv) 79.9%

12.20 In an air-refrigeration system working on reversed Brayton cycle. The temperature of air at entrance to compressor (pressure ratio = 4, efficiency = 0.8) is 275 K and the inlet pressure is 1 bar. The pressure loss in the cooler is 0.1 bar and in the cooled chamber, it is 0.08 bar. The temperature of air at turbine (efficiency = 0.85) inlet is 310 K. Estimate the pressure ratio for the turbine and *COP* of the cycle. *Ans:* 3.611; 0.533

12.21 A refrigeration power plant works as vapour compression cycle with R-12 as refrigerant. It has a refrigeration load of 5 TR. The evaporator and condenser temperature as −20 °C and 40 °C, respectively. Determine

(i) the refrigerant flow rate in kg/s
(ii) the volume flow rate handled by the compressor in m^3/s
(iii) the work input to the compressor in kW
(iv) the heat rejected in the condenser in kW and
(v) the isentropic discharge temperature

If there is 5 °C of superheating of vapour before it enters the compressor and 5 °C subcooling of liquid before it flows through the expansion valve, determine the above quantities. *Ans:* (i) 0.187 kg/s; (ii) 0.02 m^3/s; (iii) 5.495 (iv) 24.96 (v) 52 °C
(i) 0.172 kg/s (ii) 0.0189 m^3/s; (iii) 5.16 kW (iv) 24.51 kW (v) 52 °C

12.22 An R-12 refrigeration plant maintains a cold storage at −15 °C. The capacity of the plant 5 TR. The refrigerant flow rate is 0.133 kg. The vapour leaves the evaporator with 5 °C superheated. Necessary cooling water is available at 25 °C. Before throttling a suction line, heat exchanger subcools the refrigerant. Calculate (i) the compression discharge temperature, (ii) *COP*, (iii) amount of subcooling in °C, and (iv) the cylinder dimensions of the compressor.

The compressor runs at 1000 rpm, stroke to bore ratio is 1.20 and volumetric efficiency is 95%. Allow about 5 °C temperature difference in the evaporator and the condenser.

Ans: (i) 40 °C (ii) 4.50 (iii) 4 °C (iv) $d = 12.282$ cm; $L = 14.74$ cm

12.23 A vapour compression refrigeration system uses R-12 and operates between pressure limits of 0.745 MPa and 0.15 MPa. The vapour entering the compressor has a temperature of −10 °C and the liquid leaving the condenser is required at 28 °C. A refrigerating load of 2 kW. Determine the *COP* and the swept volume of the compressor if it has a volumetric efficiency of 76% and runs at 600 rpm. *Ans:* 4.15; 247.5 cc

12.24 A refrigeration system works with an evaporator temperature of −35 °C and condenser temperature of 25 °C. The capacity of the system is 20 tonnes. The refrigerant is R-12, it is subcooled by 4 °C before entering the expansion valve. The vapour is superheated by 5 °C before leaving the evaporator. A six-cylinder single-acting compressor with bore to stroke ratio of 1 is used. The compressor runs at 1500 rpm. Find

(i) the refrigerant flow rate
(ii) the theoretical piston displacement per second
(iii) the theoretical power required in kW
(iv) the *COP*
(v) the heat removed in the condenser
(vi) bore and stroke of the compressor

Ans: (i) 119 kJ/kg; (ii) 0.654 kg/s; (iii) 0.1373 m^3/s; (iv) 3.132;
(v) 102.68 kW; (vi) $d = L = 10.522$ cm

12.25 An R-12 vapour compression refrigeration system is operating at a condenser pressure of 9.6 bar and an evaporator pressure of 2.19 bar. Refrigeration capacity of the system is 15 TR. Enthalpy values at inlet and outlet of the evaporator are 64.6 and 195.7 kJ/kg. The specific volume at inlet to the reciprocating compressor is 0.082 m^3/kg. The index of compression is 1.13. Determine (i) power input in kW required for the compressor and (ii) the *COP*. Take 1 TR as equivalent to heat removed at the rate of 3.517 kW.

Ans: 12.84 kW; 4.53

Multiple Choice Questions (choose the most appropriate answer)

1. The gas-refrigeration system is
   (a) more heavy and bulky than the vapour compression cycle
   (b) less heavy and bulky than the vapour compression cycle
   (c) same in construction with the vapour compression cycle
   (d) none of the above

2. How is the COP of gas-refrigeration cycle compared with the COP of vapour compression cycle, for the same capacity?
   (a) the COP of gas-refrigeration cycle is lower than the COP of vapour compression cycle, for the same capacity
   (b) the COP of gas-refrigeration cycle is higher than the COP of vapour compression cycle, for the same capacity
   (c) the COP of gas-refrigeration cycle is same as the COP of vapour compression cycle
   (d) none of the above

3. The ideal gas refrigeration cycle is similar to
   (a) Brayton cycle
   (b) Reversed Brayton cycle
   (c) Rankine cycle
   (d) Reversed Rankine cycle

4. How much is the temperature drop of real gas by throttling?
   (a) very big temperature drop
   (b) very small temperature drop
   (c) no temperature drop
   (d) cannot say

5. In gas cycle refrigeration system, the throttle valve of a vapour compression refrigerant system is replaced by
   (a) capillary tube
   (b) expander
   (c) reverse throttle valve
   (d) none of the above

6. In year-round air-conditioning, by reversing refrigerating cycle
   (a) condenser acts like evaporator
   (b) evaporator acts like condenser
   (c) both (a) and (b)
   (d) none of the above

7. Which of the following is NOT a type of heat pump?
   (a) vapour compression type
   (b) vapour absorption type
   (c) both (a) and (b)
   (d) none of the above

8. Heat pump is used
   (a) to transform low-grade rejected heat into high-temperature heat source
   (b) to transform high-grade rejected heat into low-temperature heat sink
   (c) both (a) and (b)
   (d) none of the above

9. For the same capacity of plant, COP of the vapour absorption refrigeration system is
   (a) lower than the COP of the vapour compression refrigeration system
   (b) higher than the COP of the vapour compression refrigeration system
   (c) same as the COP of the vapour compression refrigeration system
   (d) cannot say

10. The vapour absorption refrigeration system is
    (a) more noisy than the vapour compression refrigeration system
    (b) more silent than the vapour compression refrigeration system
    (c) equally noisy as the vapour compression refrigeration system

(d) it depends upon plant capacity

11. Which type of energy is required to expend in vapour absorption refrigeration system?
    (a) high-grade energy
    (b) low-grade energy
    (c) medium-grade energy
    (d) none of the above

12. For a same pressure, the saturation temperature of ammonia is
    (a) higher than the saturation temperature of water
    (b) lower than the saturation temperature of water
    (c) same as the saturation temperature of water
    (d) none of the above

13. What is an advantage using analyser-rectifier in aqua-ammonia absorption system?
    (a) it makes ammonia-water solution strong
    (b) it prevents expansion valve from blocking due to ice formation
    (c) both (a) and (b)
    (d) none of the above

14. What is the purpose of analyser-rectifier in aqua-ammonia absorption system?
    (a) to absorb ammonia into the water
    (b) to remove water vapour out from ammonia after generator
    (c) to exchange heat between weak solution and strong solution
    (d) none of the above

15. In absorber of aqua-ammonia absorption refrigeration system
    (a) strong solution is taken in and weak solution is given out
    (b) weak solution is taken in and strong solution is given out
    (c) the ammonia vapour from strong solution is taken out and made it a weak solution
    (d) none of the above

16. What is the process carried out in generator of vapour absorption refrigeration cycle?
    (a) weak solution of ammonia in water is heated
    (b) strong solution of ammonia in water is heated
    (c) only water is heated and heat is given to the ammonia to form its vapour
    (d) none of the above

17. What is the condition of refrigerant at the exit of evaporator in aqua-ammonia absorption system?
    (a) low-pressure ammonia vapour
    (b) high-pressure ammonia vapour
    (c) low-pressure strong vapour mixture of ammonia and water
    (d) high-pressure strong vapour mixture of ammonia and water

18. The compressor in the vapour compression system is replaced by

    (a) an absorber
    (b) a generator
    (c) an absorber-generator
    (d) none of the above

19. The mechanical work required to run vapour absorption system

    (a) is more than the mechanical work required to run vapour compression system
    (b) is less than the mechanical work required to run vapour compression system
    (c) is similar to the mechanical work required to run vapour compression system
    (d) cannot say

20. What is the disadvantage of ammonia using as a refrigerant?

    (a) ammonia cannot be detected in case of leakage
    (b) ammonia has a bad effect on ozone layer
    (c) ammonia is toxic in nature
    (d) ammonia has higher energy cost

21. Which refrigerant is widely used in refrigeration facilities of food as cooling of fresh vegetables, dairy products, meat and fish and similar process industries?

    (a) sulphur dioxide
    (b) ethyl chloride
    (c) propane
    (d) ammonia

22. What is the desirable characteristic of a refrigerant?

    (a) should not be toxic
    (b) should not be non-corrosive
    (c) should have minimum enthalpy of vaporisation
    (d) all of the above

23. When two refrigerating cycles are used in series with two different refrigerants, the system is known as

    (a) dual refrigeration system
    (b) cascade refrigeration system
    (c) vapour absorption refrigeration system
    (d) none of the above

24. What is used as an intercooler in multistage compression of vapour compression refrigeration system?

    (a) water
    (b) air
    (c) refrigerant itself
    (d) none of the above

25. What is the use of multistage compression in vapour compression refrigeration system?

    (a) to increase volumetric efficiency of compressor
    (b) to allow the system to operate between higher pressures
    (c) to reduce risk of damage to refrigerant
    (d) all of the above

Ans: 1. – (b) 2. – (a) 3. – (b) 4. – (b) 5. – (b)
6. – (c) 7. – (d) 8. – (a) 9. – (a) 10. – (b)
11. – (b) 12. – (b) 13. – (b) 14. – (b) 15. – (b)
16. – (b) 17. – (a) 18. – (c) 19. – (b) 20. – (c)
21. – (d) 22. – (a) 23. – (b) 24. – (c) 25. – (d)

13

GAS POWER CYCLE

13.1 INTRODUCTION

By now, you must be aware that matter, in whatever form it is, possesses internal energy. Historically, it is the endeavour of engineers to continuously convert this internal energy into work. This gave birth to a new subject called thermodynamics. In this connection, the most important objective of thermodynamics is to analyse power cycles. This can bring into light how energy absorbed as heat can be converted into work. This can provide important information regarding the design of new cycles or improving existing cycles.

Another important area of interest is production of low temperature which is called refrigeration. In this, heat is removed from a low-temperature body and rejected to a high-temperature sink. Device employed for this purpose is called refrigerator. We have already dealt with refrigeration cycles. In this chapter, we will concentrate on heat engines and their cycles. The working fluid in a heat engine is a gas and therefore, it is called gas power cycle.

Different types of fluid are employed in power plants. The nature of the working fluid can be classified under two groups, viz vapour and gas. Power cycles are accordingly grouped as:

(i) vapour power cycles and (ii) gas power cycles.

In vapour (steam) power cycles, the working fluid undergoes a phase-change during a cyclic process whereas in gas power cycle the working fluid does not undergo any phase-change. In the thermodynamic analysis of power cycles our main interest is to estimate the efficiency of energy conversion which is called as thermal efficiency. There are several types of power cycles in practice. It is not intended to discuss all of them in this chapter. We will discuss some common power cycles to illustrate the principles. Once you are thorough with the principles and methods of analysis then the thermodynamic analysis of any power cycles can be carried out with ease.

In this chapter, we will review the various cycles and also derive the equations for work output, mean effective pressure, efficiency, etc. Also, comparison will be made between Otto, Dual and Diesel cycles to see which cycle is more efficient under a set of given operating conditions. The operating cycle of an internal combustion (IC) engine can be broken down into a sequence of separate processes, viz intake, compression, combustion, expansion and exhaust. The IC engine does not operate on a thermodynamic cycle as it involves an open system, i.e. the working fluid enters the system at one set of conditions and leaves at another. However, it is often possible to analyse the open cycle as though it were a closed one by imagining one or more processes that would bring the working fluid at the exit conditions back to the condition of the starting point.

The accurate analysis of IC engine processes is very complicated. In order to understand them, it is advantageous to analyse the performance of an idealised closed cycle that closely approximates the real cycle. One such approach is the air-standard cycle, which is based on the following assumptions:

(i) The working medium is assumed to be a perfect gas and follows the relation, $pV = mRT$ or $p = \rho RT$.

(ii) There is no change in the mass of the working medium.

(iii) All the processes that constitute the cycle are reversible.

(iv) Heat is assumed to be supplied from a constant high-temperature source and not from chemical reactions during the cycle.

(v) Some heat is assumed to be rejected to a constant low-temperature sink during the cycle.

(vi) It is assumed that there are no heat losses from the system to the surroundings.

(vii) The working medium has constant specific heats throughout the cycle.

(viii) The physical constants, viz c_p, c_V, γ and M of working medium are the same as those of air at standard atmospheric conditions. For example, in SI units,

$$c_p = 1.005 \text{ kJ/kg K} \qquad M = 29 \text{ kg/kmol}$$

$$c_V = 0.717 \text{ kJ/kg K} \qquad \gamma = 1.4$$

Due to these assumptions, the analysis becomes over simplified and the results do not agree with those of the actual engine. Work output, peak pressure, peak temperature and thermal efficiency based on air-standard cycles will be the maximum that can be attained and will differ considerably from those of the actual engine. It is often used, mainly because of the simplicity in getting approximate answers to the complicated processes in internal combustion engines.

In this chapter, we will review the various cycles and also derive the equations for work output, mean effective pressure, efficiency, etc. Comparison will be made between Otto, Dual and Diesel cycles to see which cycle is more efficient under a set of given operating conditions.

13.2 THE CARNOT CYCLE

Sadi Carnot, a French engineer, proposed a reversible cycle in 1824, in which the working medium receives heat at a higher temperature and rejects heat at a lower temperature. The cycle will consist of two isothermal and two reversible adiabatic processes as shown in Fig.13.1. Carnot cycle is represented as a standard of perfection and engines can be compared with it to judge the degree of perfection. It gives the concept of maximising work output between two temperature limits.

The working of an engine based on the Carnot cycle can be explained referring to Fig.13.2 which shows a cylinder and piston arrangement working without friction. The walls of cylinder are assumed to be perfect insulators. The cylinder head is so arranged that it can be a perfect heat conductor as well as a perfect heat insulator.

First the heat is transferred from a high-temperature source, (T_3), to the working medium in the cylinder and as a result the working medium expands. This is represented by the isothermal process 3→4 in Fig.13.1. Now, the cylinder head is sealed and it acts as a perfect insulator. The working medium in the cylinder is now allowed to expand further from state 4 to state 1 and is represented by reversible adiabatic process 4→1 in p-V and T-s diagrams in Fig.13.1. Now, the system is brought into contact with a constant low-temperature sink, (T_1), as the cylinder head is now made to act as a perfect heat conductor. Some heat is rejected to the sink without altering the temperature of sink and as a result the working medium is compressed from state 1 to 2 which is represented by isothermal line 1→2. Finally,

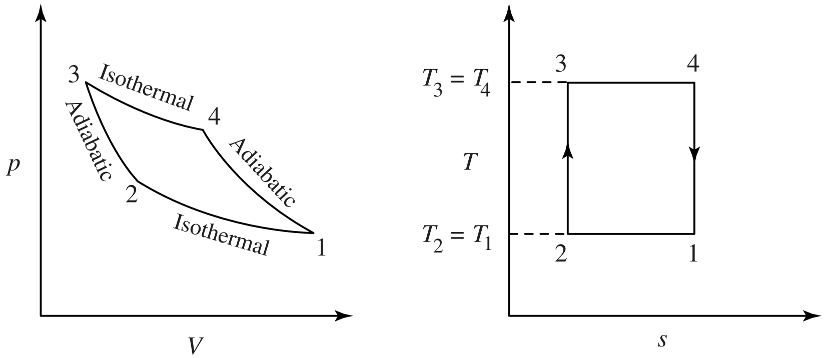


Fig. 13.1 Carnot engine

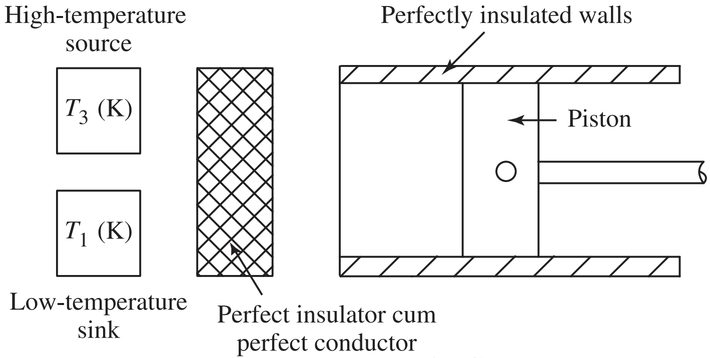


Fig. 13.2 Working principle of a Carnot engine

the cylinder head is made again to act as a perfect insulator and the working medium is compressed adiabatically from state 2 to 3 which is represented by process 2→3. Thus, the cycle is completed.

Analysing the cycle thermodynamically the efficiency of the cycle can be written as:

$$\eta_{Carnot} = \frac{\text{Work done by the system during the cycle, } (W)}{\text{Heat supplied to the system during the cycle, } (Q_S)}$$

According to the first law of thermodynamics,

$$\text{Work done} = \text{Heat supplied} - \text{Heat rejected}$$

$$W = Q_S - Q_R \tag{13.1}$$

Considering the isothermal processes 1→2 and 3→4, we get:

$$Q_R = mRT_1 \log_e \frac{V_1}{V_2} \tag{13.2}$$

$$Q_S = mRT_3 \log_e \frac{V_4}{V_3} \tag{13.3}$$

Considering adiabatic processes 2→3 and 4→1:

$$\frac{V_3}{V_2} = \left(\frac{T_2}{T_3}\right)^{\left(\frac{1}{\gamma-1}\right)} \tag{13.4}$$

and
$$\frac{V_4}{V_1} = \left(\frac{T_1}{T_4}\right)^{\left(\frac{1}{\gamma-1}\right)} \tag{13.5}$$

Since $T_1 = T_2$ and $T_4 = T_3$, we have,

$$\frac{V_4}{V_1} = \frac{V_3}{V_2}$$

or
$$\frac{V_4}{V_3} = \frac{V_1}{V_2} = r \qquad \text{(say)} \tag{13.6}$$

then,
$$\eta_{Carnot} = \frac{mRT_3 \log_e r - mRT_1 \log_e r}{mRT_3 \log_e r} = \frac{T_3 - T_1}{T_3} = 1 - \frac{T_1}{T_3} \tag{13.7}$$

The lower temperature, i.e. sink temperature, T_1, is normally the atmospheric temperature or the cooling water temperature and hence, fixed. So the increase in thermal efficiency can be achieved only by increasing the source temperature. In other words, the upper temperature is required to be maintained as high as possible, to achieve maximum thermal efficiency. Between two fixed temperatures, Carnot cycle (and other reversible cycles) has the maximum possible efficiency compared to other air-standard cycles. In spite of this advantage, Carnot cycle does not provide a suitable basis for the operation of an engine using a gaseous working fluid because the work output from this cycle will be quite low.

Mean effective pressure, p_m, is defined as that hypothetical constant pressure acting on the piston during its expansion stroke producing the same work output as that from the actual cycle. Mathematically:

$$p_m = \frac{\text{Work Output}}{\text{Swept Volume}} \tag{13.8}$$

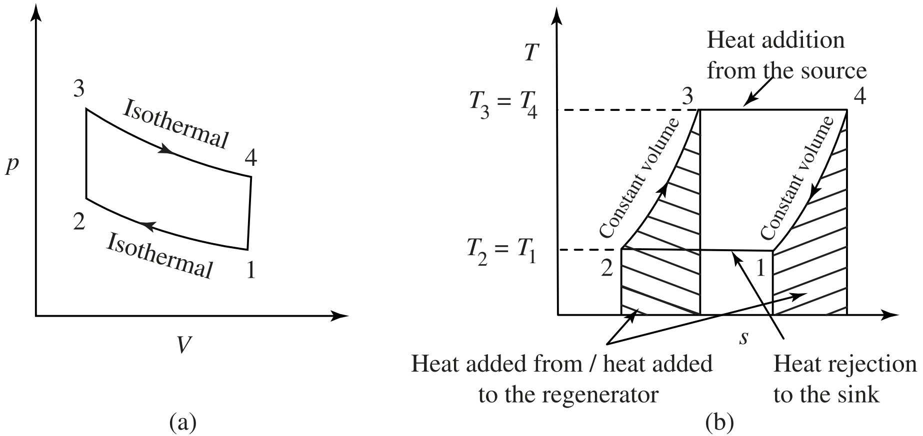
It can be shown as:

$$p_m = \frac{\text{Area of indicator diagram}}{\text{Length of diagram}} \times \text{constant} \tag{13.9}$$

The constant depends on the mechanism used to get the indicator diagram and has the units, bar/m. These formulae are quite often used to calculate the performance of an internal combustion engine. If the work output is the indicated output then it is called indicated mean effective pressure, p_{im}, and if the work output is the brake output then it is called brake mean effective pressure, p_{bm}.

13.3 THE STIRLING CYCLE

The Carnot cycle has a low-mean effective pressure because of its very low work output. Hence, one of the modified forms of the cycle to produce higher mean effective pressure whilst theoretically achieving full Carnot cycle efficiency is the Stirling cycle. It consists of two isothermal and two constant volume processes. The heat rejection and addition take place at constant temperature. The p-V and T-s diagrams for the Stirling cycle are shown in



(a) (b)

Fig. 13.3 Stirling cycle

Figs.13.3(a) and (b), respectively. It is clear from Fig.13.3(b) that the amount of heat addition and rejection during constant volume processes is same. Hence, the efficiency of the cycle is given as:

$$\eta_{Stirling} = \frac{RT_3 \log_e\left(\frac{V_4}{V_3}\right) - RT_1 \log_e\left(\frac{V_1}{V_2}\right)}{RT_3 \log_e\left(\frac{V_4}{V_3}\right)} \tag{13.10}$$

But $V_3 = V_2$ and $V_4 = V_1$:

$$\eta_{Stirling} = \frac{T_3 - T_1}{T_3} \tag{13.11}$$

same as Carnot efficiency.

Stirling cycle was used earlier for hot air engines and became obsolete as Otto and Diesel cycles came into use. The design of Stirling engine involves a major difficulty in the design and construction of heat exchanger to operate continuously at very high temperatures. However, with the development in metallurgy and intensive research in this type of engine, Stirling engine has staged a come back in practical appearance. In practice, the heat exchanger efficiency cannot be 100%. Hence, the Stirling cycle efficiency will be less than Carnot efficiency and can be written as:

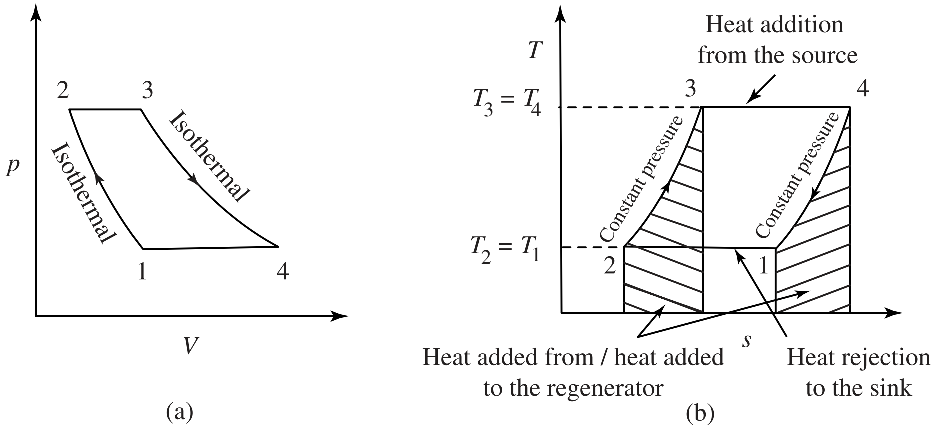
$$\eta = \frac{R(T_3 - T_1)\log_e r}{RT_3 \log_e r + (1-\epsilon)c_V(T_3 - T_1)} \tag{13.12}$$

where ϵ is the heat exchanger effectiveness.

13.4 THE ERICSSON CYCLE

The Ericsson cycle consists of two isothermal and two constant pressure processes. The heat addition and rejection take place at constant pressure as well as isothermal processes. Since the process 2→3 and 3→4 are parallel to each other on the T-s diagram, the net effect is that the heat need be added only at constant temperature $T_3 = T_4$ and rejected at the constant temperature $T_1 = T_2$.

The cycle is shown on p-V and T-s diagrams in Fig.13.4(a) and 13.4(b), respectively. The advantage of Ericsson cycle over the Carnot and Stirling cycles is its smaller pressure ratio for a given ratio of maximum to minimum specific volume with higher mean effective pressure. The Ericsson cycle does not find practical application in piston engines but is approached by a gas turbine employing a large number of stages with heat exchangers, insulators and reheaters.

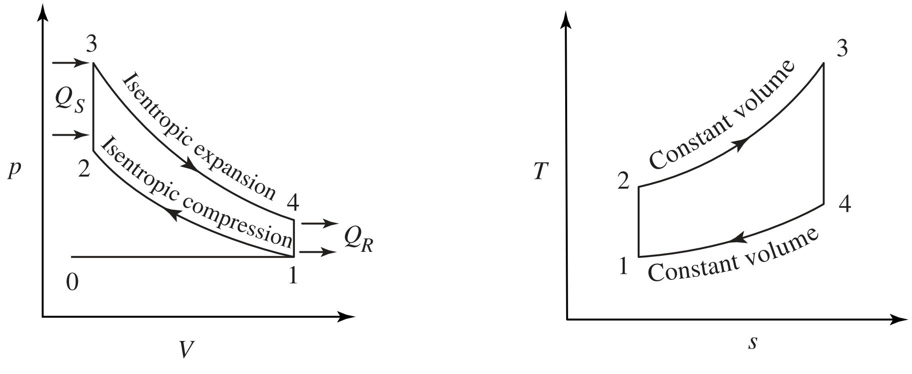


(a) (b)

Fig. 13.4 Ericsson cycle

13.5 THE OTTO CYCLE

The main drawback of the Carnot cycle is its impracticability due to high-pressure and high-volume ratios employed with comparatively low-mean effective pressure. Nicolaus Otto (1876), proposed a constant-volume heat addition cycle which forms the basis for the working of today's spark-ignition engines. The cycle is shown on p-V and T-s diagrams in Figs.13.5(a) and (b), respectively.



(a) (b)

Fig. 13.5 Otto cycle

When the engine is working on full throttle, the processes 0→1 and 1→0 on the p-V diagram represents suction and exhaust processes and their effect is nullified. The process

1→2 represents isentropic compression of the air when the piston moves from bottom dead centre to top dead centre. During the process 2→3, heat is supplied reversibly at constant volume. This process corresponds to spark-ignition and combustion in the actual engine. The processes 3→4 and 4→1 represent isentropic expansion and constant volume heat rejection, respectively.

13.5.1 Thermal Efficiency

The thermal efficiency of Otto cycle can be written as:

$$\eta_{Otto} = \frac{Q_S - Q_R}{Q_S} \tag{13.13}$$

Considering constant volume processes 2→3 and 4→1, the heat supplied and rejected of air can be written as:

$$Q_S = mc_V(T_3 - T_2) \tag{13.14}$$

$$Q_R = mc_V(T_4 - T_1) \tag{13.15}$$

$$\eta_{Otto} = \frac{m(T_3 - T_2) - m(T_4 - T_1)}{m(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} \tag{13.16}$$

Considering isentropic processes 1→2 and 3→4, we have:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} \tag{13.17}$$

and

$$\frac{T_3}{T_4} = \left(\frac{V_4}{V_3}\right)^{(\gamma-1)} \tag{13.18}$$

But the volume ratios V_1/V_2 and V_4/V_3 are equal to the compression ratio, r. Therefore,

$$\frac{V_1}{V_2} = \frac{V_4}{V_3} \quad = \quad r \tag{13.19}$$

therefore,

$$\frac{T_2}{T_1} = \frac{T_3}{T_4} \tag{13.20}$$

From Eq.13.20, it can be easily shown that:

$$\frac{T_4}{T_3} = \frac{T_1}{T_2} \quad = \quad \frac{T_4 - T_1}{T_3 - T_2} \tag{13.21}$$

$$\eta_{Otto} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{\left(\frac{V_1}{V_2}\right)^{(\gamma-1)}} = 1 - \frac{1}{r^{(\gamma-1)}} \tag{13.22}$$

Note that the thermal efficiency of Otto cycle is a function of compression ratio r and the ratio of specific heats, γ. As γ is assumed to be a constant for any working fluid, the efficiency is increased by increasing the compression ratio. Further, the efficiency is independent of heat supplied and pressure ratio. The use of gases with higher γ values would increase efficiency of Otto cycle. Fig.13.6 shows the effect of γ and r on the efficiency.

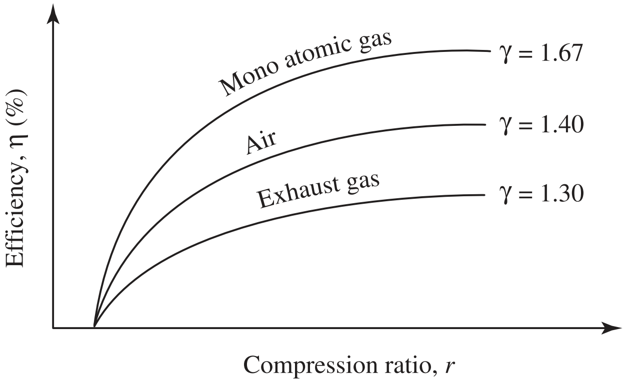


Fig. 13.6 Effect of r and γ on efficiency for Otto cycle

13.5.2 Work Output

The net work output for an Otto cycle can be expressed as:

$$W = \frac{p_3V_3 - p_4V_4}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1} \tag{13.23}$$

Also $$\frac{p_2}{p_1} = \frac{p_3}{p_4} = r^\gamma \quad \text{and} \quad \frac{p_3}{p_2} = \frac{p_4}{p_1} = r_p \quad \text{(say)} \tag{13.24}$$

$$V_1 = rV_2 \qquad \text{and} \qquad V_4 = rV_3$$

therefore, $$W = \frac{p_1V_1}{\gamma - 1}\left(\frac{p_3V_3}{p_1V_1} - \frac{p_4V_4}{p_1V_1} - \frac{p_2V_2}{p_1V_1} + 1\right) = \frac{p_1V_1}{\gamma - 1}\left(\frac{r_pr^\gamma}{r} - r_p - \frac{r^\gamma}{r} + 1\right)$$

$$= \frac{p_1V_1}{\gamma - 1}\big(r_pr^{\gamma-1} - r_p - r^{\gamma-1} + 1\big) = \frac{p_1V_1}{\gamma - 1}(r_p - 1)\big(r^{\gamma-1} - 1\big) \tag{13.25}$$

13.5.3 Mean Effective Pressure

The mean effective pressure of the cycle is given by:

$$p_m = \frac{\text{Work output}}{\text{Swept volume}} \tag{13.26}$$

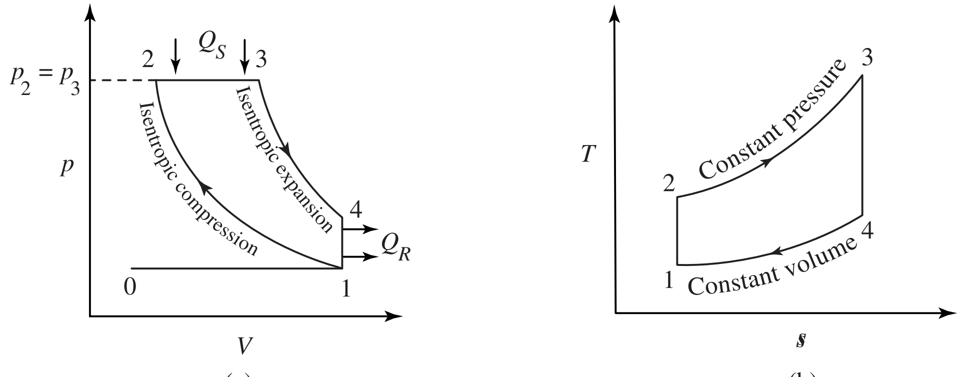
$$\text{Swept volume} = V_1 - V_2 = V_2(r - 1)$$

$$p_m = \frac{\frac{1}{\gamma-1}p_1V_1(r_p - 1)\big(r^{(\gamma-1)} - 1\big)}{V_2(r - 1)} = \frac{p_1r(r_p - 1)\big(r^{(\gamma-1)} - 1\big)}{(\gamma - 1)(r - 1)} \tag{13.27}$$

Thus, it can be seen that the work output is directly proportional to pressure ratio, r_p. The mean effective pressure which is an indication of the internal work output increases with a pressure ratio at a fixed value of compression ratio and ratio of specific heats. For an Otto cycle, an increase in the compression ratio leads to an increase in the mean effective pressure as well as the thermal efficiency.

13.6 THE DIESEL CYCLE

In actual spark-ignition engines, the upper limit of compression ratio is limited by the self-ignition temperature of the fuel. This limitation on the compression ratio can be circumvented if air and fuel are compressed separately and brought together at the time of combustion. In such an arrangement fuel can be injected into the cylinder which contains compressed air at a higher temperature than the self-ignition temperature of the fuel. Hence, the fuel ignites on its own accord and requires no special device like an ignition system in a spark-ignition engine. Such engines work on heavy liquid fuels. These engines are called compression-ignition engines and they work on a ideal cycle known as Diesel cycle. The difference between Otto and Diesel cycles is in the process of heat addition. In Otto cycle, the heat addition takes place at constant volume whereas in the Diesel cycle, it is at constant pressure. For this reason, the Diesel cycle is often referred to as the constant-pressure cycle. It is better to avoid this term as it creates confusion with Joules cycle. The Diesel cycle is shown on p-V and T-s diagrams in Figs.13.7(a) and (b), respectively.



(a) (b)

Fig. 13.7 Diesel cycle

To analyse the diesel cycle the suction and exhaust strokes, represented by 0→1 and 1→0, are neglected as in the case of the Otto cycle. Here, the volume ratio V_1/V_2 is the compression ratio, r. The volume ratio V_3/V_2 is called the cut-off ratio, r_c.

13.6.1 Thermal Efficiency

The thermal efficiency of the Diesel cycle is given by:

$$\eta_{Diesel} = \frac{Q_S - Q_R}{Q_S} = \frac{mc_p(T_3 - T_2) - mc_V(T_4 - T_1)}{mc_p(T_3 - T_2)} \tag{13.28}$$

$$= 1 - \frac{c_V(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{1}{\gamma}\left(\frac{T_4 - T_1}{T_3 - T_2}\right) \tag{13.29}$$

Considering the process 1→2:

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = T_1 r^{(\gamma-1)} \tag{13.30}$$

Considering the constant pressure process 2→3, we have:

$$\frac{V_2}{T_2} = \frac{V_3}{T_3} \quad \text{and} \quad \frac{T_3}{T_2} = \frac{V_3}{V_2} = r_c \quad \text{(say)}$$

$$T_3 = T_2 r_c \tag{13.31}$$

From Eqs 13.30 and 13.31, we have:

$$T_3 = T_1 r^{(\gamma-1)} r_c \tag{13.32}$$

Considering process 3→4:

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{(\gamma-1)} \tag{13.33}$$

$$= T_3\left(\frac{V_3}{V_2} \times \frac{V_2}{V_4}\right)^{(\gamma-1)} = T_3\left(\frac{r_c}{r}\right)^{(\gamma-1)} \tag{13.34}$$

From Eqs 13.32 and 13.33, we have:

$$T_4 = T_1 r^{(\gamma-1)} r_c \left(\frac{r_c}{r}\right)^{(\gamma-1)} = T_1 r_c^{\gamma}$$

$$\eta_{Diesel} = 1 - \frac{1}{\gamma}\left[\frac{T_1(r_c^{\gamma}-1)}{T_1\left(r^{(\gamma-1)} r_c - r^{(\gamma-1)}\right)}\right] = 1 - \frac{1}{\gamma}\left[\frac{(r_c^{\gamma}-1)}{r^{(\gamma-1)} r_c - r^{(\gamma-1)}}\right]$$

$$= 1 - \frac{1}{r^{(\gamma-1)}}\left[\frac{r_c^{\gamma}-1}{\gamma(r_c-1)}\right] \tag{13.35}$$

It may be noted that the efficiency of the Diesel cycle is different from that of the Otto cycle only in the bracketed factor. This factor is always greater than unity. Hence, for a given compression ratio, the Otto cycle is more efficient. In diesel engines, the fuel cut-off ratio, r_c, depends on output, being maximum for maximum output. Therefore, unlike the Otto cycle the air-standard efficiency of the Diesel cycle depends on output. The higher efficiency of the Otto cycle as compared to the Diesel cycle for the same compression ratio is of no practical importance. In practice, the operating compression ratios of diesel engines are much higher compared to spark-ignition engines working on Otto cycle. The normal range of compression ratio for diesel engine is 16–20 whereas for spark-ignition engines it is 6–10. Due to the higher compression ratios used in diesel engines, the efficiency of a diesel engine is more than that of the gasoline engine.

13.6.2 Work Output

The net work output for a Diesel cycle is given by:

$$W = p_2(V_3 - V_2) + \frac{p_3V_3 - p_4V_4}{\gamma-1} - \frac{p_2V_2 - p_1V_1}{\gamma-1} \tag{13.36}$$

$$
\begin{aligned}
&= p_2V_2(r_c-1)+\frac{p_3r_cV_2-p_4rV_2}{\gamma-1}-\frac{p_2V_2-p_1rV_2}{\gamma-1}\\
&= V_2\left[\frac{p_2(r_c-1)(\gamma-1)+p_3r_c-p_4r-(p_2-p_1r)}{\gamma-1}\right]\\
&= V_2\left[\frac{p_2(r_c-1)(\gamma-1)+p_3\left(r_c-\frac{p_4}{p_3}r\right)-p_2\left(1-\frac{p_1}{p_2}r\right)}{\gamma-1}\right]\\
&= p_2V_2\left[\frac{(r_c-1)(\gamma-1)+\left(r_c-r_c^{\gamma}r^{(1-\gamma)}\right)-\left(1-r^{(1-\gamma)}\right)}{\gamma-1}\right]\\
&= \frac{p_1V_1r^{(\gamma-1)}\left[\gamma(r_c-1)-r^{(1-\gamma)}(r_c^{\gamma}-1)\right]}{\gamma-1} \qquad (13.37)
\end{aligned}
$$

13.6.3 Mean Effective Pressure

The expression for mean effective pressure can be shown to be

$$p_m = \frac{p_1V_1\left[r^{(\gamma-1)}\gamma(r_c-1)-(r_c^{\gamma}-1)\right]}{(\gamma-1)V_1\left(\frac{r-1}{r}\right)}=\frac{p_1[\gamma r^{\gamma}(r_c-1)-r(r_c^{\gamma}-1)]}{(\gamma-1)(r-1)} \qquad (13.38)$$

13.7 THE DUAL CYCLE

In the Otto cycle, combustion is assumed at constant volume while in Diesel cycle, combustion is at constant pressure. In practice, they are far from real. Combustion cannot take place at constant volume since some time interval is required for the chemical reactions during combustion process. Similarly, due to rapid uncontrolled combustion in diesel engines, combustion does not occur at constant pressure. Dual cycle is also called a *mixed cycle* or *limited pressure cycle*, which is a compromise between Otto and Diesel cycles. Figures 13.8(a) and (b) show the Dual cycle on p-V and T-s diagrams, respectively. In a Dual cycle, a part of the heat is first supplied to the system at constant volume and then the remaining part at constant pressure.

13.7.1 Thermal Efficiency

The efficiency of the cycle may be written as:

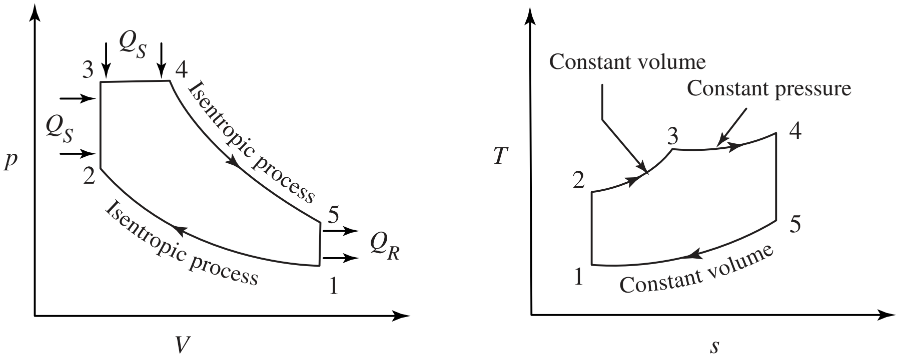
$$
\begin{aligned}
\eta_{Dual} &= \frac{Q_S-Q_R}{Q_S}=\frac{mc_V(T_3-T_2)+mc_p(T_4-T_3)-mc_V(T_5-T_1)}{mc_V(T_3-T_2)+mc_p(T_4-T_3)}\\
&= 1-\frac{T_5-T_1}{(T_3-T_2)+\gamma(T_4-T_3)} \qquad (13.39)
\end{aligned}
$$

Now,

$$T_2=T_1\left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = T_1r^{(\gamma-1)} \qquad (13.40)$$

$$T_3=T_2\left(\frac{p_3}{p_2}\right) = T_1r_pr^{(\gamma-1)} \qquad (13.41)$$

where r_p is the pressure ratio in the constant volume heat addition process and is equal to $\dfrac{p_3}{p_2}$.



(a) (b)

Fig. 13.8 Dual cycle

Cut-off ratio, r_c, is given by $\left(\dfrac{V_4}{V_3}\right)$:

$$T_4 = T_3 \frac{V_4}{V_3} = T_3 r_c$$

Substituting for T_3 from Eq.13.41:

$$T_4 = T_1 r_c r_p r^{(\gamma-1)} \tag{13.42}$$

and
$$T_5 = T_4 \left(\frac{V_4}{V_5}\right)^{(\gamma-1)} = T_1 r_p r_c r^{(\gamma-1)} \left(\frac{V_4}{V_5}\right)^{(\gamma-1)} \tag{13.43}$$

Now
$$\frac{V_4}{V_5} = \frac{V_4}{V_1} \quad = \quad \frac{V_4}{V_3} \times \frac{V_3}{V_1} = \frac{V_4}{V_3} \times \frac{V_2}{V_1} \qquad (\text{since} V_2 = V_3)$$

Therefore,
$$\frac{V_4}{V_5} = \frac{r_c}{r} \tag{13.44}$$

where V_4/V_5 is the expansion ratio. Now,

$$T_5 = T_1 r_p r_c r^{\gamma-1} \left(\frac{r_c}{r}\right)^{\gamma-1} = T_1 r_p r_c^{\gamma} \tag{13.45}$$

Substituting for T_2, T_3, T_4 and T_5 into Eq.13.39 and simplifying:

$$\eta_{dual} = 1 - \frac{1}{r^{(\gamma-1)}} \left[\frac{r_p r_c^{\gamma} - 1}{(r_p - 1) + r_p \gamma (r_c - 1)} \right] \tag{13.46}$$

It can be seen from the above equation that a value of $r_p > 1$ results in an increased efficiency for a given value of r_c and γ. Thus, the efficiency of Dual cycle lies between that of the Otto cycle and the Diesel cycle having same compression ratio.

With $r_c = 1$, it becomes an Otto cycle and with $r_p = 1$, it becomes a Diesel cycle.

13.7.2 Work Output

The net work output of the cycle is given by:

$$\begin{aligned}
W &= p_3(V_4 - V_3) + \frac{p_4V_4 - p_5V_5}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1} \\
&= \frac{p_1V_1}{\gamma - 1}\left[(\gamma - 1)\left(\frac{p_4V_4}{p_1V_1} - \frac{p_3V_3}{p_1V_1}\right) + \frac{p_4V_4}{p_1V_1} - \frac{p_5V_5}{p_1V_1} - \frac{p_2V_2}{p_1V_1} + 1\right] \\
&= \frac{p_1V_1}{\gamma - 1}\big[(\gamma - 1)\big(r_c r_p r^{\gamma-1} - r_p r^{\gamma-1}\big) + r_c r_p r^{\gamma-1} \\
&\qquad - r_p r_c^{\gamma} - r^{\gamma-1} + 1\big] \\
&= \frac{p_1V_1}{\gamma - 1}\big[\gamma r_c r_p r^{\gamma-1} - \gamma r_p r^{\gamma-1} + r_p r^{\gamma-1} - r_p r_c^{\gamma} - r^{\gamma-1} + 1\big] \\
&= \frac{p_1V_1}{\gamma - 1}\big[\gamma r_p r^{\gamma-1}(r_c - 1) + r^{\gamma-1}(r_p - 1) - (r_p r_c^{\gamma} - 1)\big] \qquad (13.47)
\end{aligned}$$

13.7.3 Mean Effective Pressure

The mean effective pressure is given by:

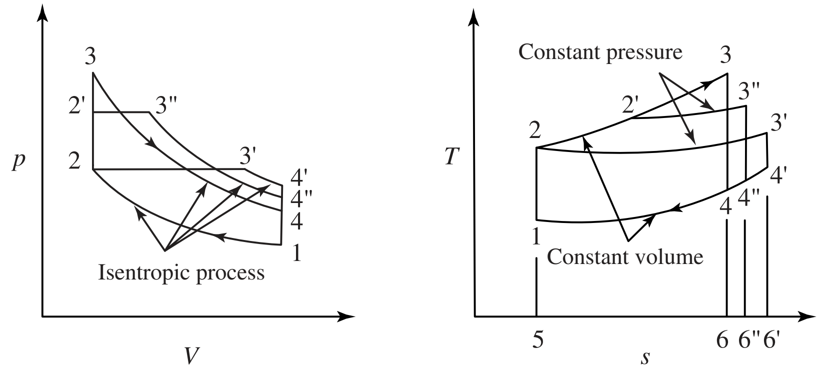
$$\begin{aligned}
p_m &= \frac{\text{Work output}}{\text{Swept volume}} = \frac{W}{V_s} \\
&= \frac{1}{V_1 - V_2}\frac{p_1V_1}{\gamma - 1}\big[\gamma r_p r^{\gamma-1}(r_c - 1) + r^{\gamma-1}(r_p - 1) - (r_p r_c^{\gamma} - 1)\big] \\
&= \frac{1}{\left(1 - \frac{V_2}{V_1}\right)}\frac{p_1}{(\gamma - 1)}\big[\gamma r_p r^{\gamma-1}(r_c - 1) + r^{\gamma-1}(r_p - 1) \\
&\qquad - (r_p r_c^{\gamma} - 1)\big] \\
&= p_1\frac{\big[\gamma r_p r^{\gamma}(r_c - 1) + r^{\gamma}(r_p - 1) - r(r_p r_c^{\gamma} - 1)\big]}{(\gamma - 1)(r - 1)} \qquad (13.48)
\end{aligned}$$

13.8 COMPARISON OF OTTO, DIESEL AND DUAL CYCLES

The important variable factors which are used as the basis for comparison of the cycles are compression ratio, peak pressure, heat addition, heat rejection and the net work. In order to compare the performance of the Otto, Diesel and Dual combustion cycles, some of the variable factors must be fixed. In this section, a comparison of these three cycles is made for the same compression ratio, same heat addition, constant maximum pressure and temperature, same heat rejection and net work output. This analysis will show which cycle is more efficient for a given set of operating conditions.

13.8.1 Same Compression Ratio and Heat Addition

The Otto cycle 1→2→3→4→1, the Diesel cycle 1→2→3′→4′→1 and the dual cycle 1→2→2′ →3′→4′→1 are shown in p-V and T-s diagrams in Fig.13.9(a) and (b), respectively for the same compression ratio and heat input. From the T-s diagram, it can be seen that Area 5236 = Area $523'6'$ = Area $522''3''6''$ as this area represents the heat input which is the same for all cycles.



(a) (b)

Fig. 13.9 Same compression ratio and heat addition

All the cycles start from the same initial state point 1 and the air is compressed from state 1 to 2 as the compression ratio is same. It is seen from the T-s diagram for the same heat input, the heat rejection in Otto cycle (area 5146) is minimum and heat rejection in Diesel cycle ($514'6'$) is maximum. Consequently, Otto cycle has the highest work output and efficiency. Diesel cycle has the least efficiency and Dual cycle having the efficiency between the two. For same compression ratio and heat addition, $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$.

One more observation can be made, i.e. Otto cycle allows the working medium to expand more whereas Diesel cycle is least in this respect. The reason is heat is added before expansion in the case of former (Otto cycle) and the last portion of heat supplied to the fluid has a relatively short expansion in case of the latter (Diesel cycle).

13.8.2 Same Compression Ratio and Heat Rejection

The p-V and T-s diagrams are shown in Figs.13.10(a) and (b). Now,

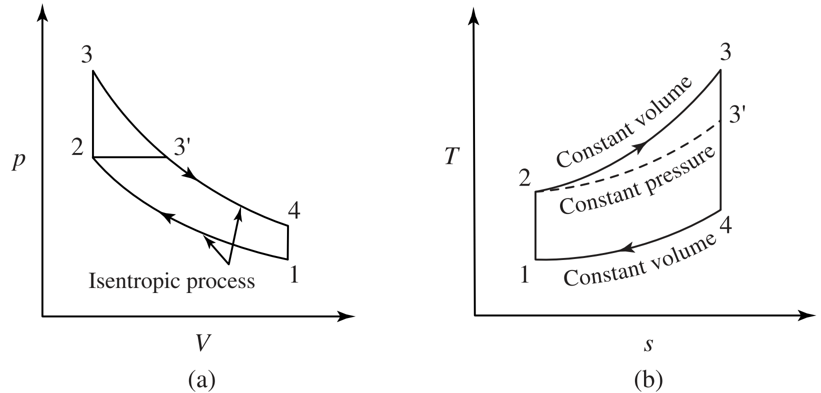
$$\eta_{Otto} \quad = \quad 1 - \frac{Q_R}{Q_S}$$

where Q_S is the heat supplied in the Otto cycle and is equal to the area under the curve 2→3 on the T-s diagram [Fig.13.10(b)]. The efficiency of the Diesel cycle is given by:

$$\eta_{Diesel} \quad = \quad 1 - \frac{Q_R}{Q'_S}$$

where Q'_S is heat supplied in the Diesel cycle and is equal to the area under the curve 2→3′ on the T-s diagram [Fig.13.10(b)].

From the T-s diagram in Fig.13.10, it is clear that $Q_S > Q'_S$, i.e. heat supplied in the Otto cycle is more than that of the Diesel cycle. *Hence, it is evident that, the efficiency of the Otto*



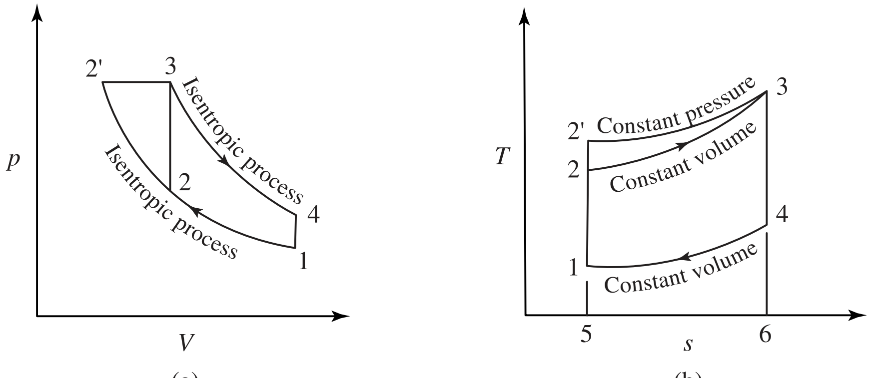
(a) (b)

Fig. 13.10 Same compression ratio and heat rejection

cycle is greater than the efficiency of the Diesel cycle for a given compression ratio and heat rejection. Also, for the same compression ratio and heat rejection, $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$.

13.8.3 Same Peak Pressure, Peak Temperature and Heat Rejection

Figures 13.11(a) and (b) show the Otto cycle 1→2→3→4 and Diesel cycle 1→2′→3→4 on p-V and T-s coordinates, where the peak pressure and temperature and the amount of heat rejected are the same.



(a) (b)

Fig. 13.11 Same peak pressure and temperature

The efficiency of the Otto cycle 1→2→3→4 is given by:

$$\eta_{Otto} = 1 - \frac{Q_R}{Q_S}$$

where Q_S in the area under the curve 2→3 in Fig.13.11(b).
The efficiency of the Diesel cycle, 1→2→3′→3→4 is:

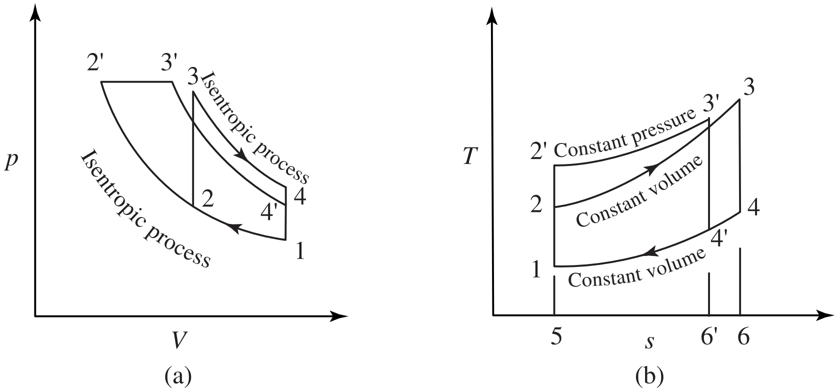
$$\eta_{Diesel} = 1 - \frac{Q_R}{Q'_S}$$

where Q'_S is the area under the curve $2' \rightarrow 3$ in Fig.13.11(b).

It is evident from Fig.13.11 that $Q'_S > Q_S$. Therefore, the Diesel cycle efficiency is greater than the Otto cycle efficiency when both engines are built to withstand the same thermal and mechanical stresses. Also, for the same peak pressure, peak temperature and heat rejection, $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$.

13.8.4 Same Maximum Pressure and Heat Input

For same maximum pressure and same heat input, the Otto cycle (12341) and Diesel cycle ($12'3'4'1$) are shown on *p*-*V* and *T*-*s* diagrams in Figs.13.12(a) and (b), respectively.



(a) (b)

Fig. 13.12 Same maximum pressure and heat input

It is evident from the figure that the heat rejection for Otto cycle (area 1564 on *T*-*s* diagram) is more than the heat rejected in Diesel cycle ($156'4'$). Hence, Diesel cycle is more efficient than Otto cycle for the condition of same maximum pressure and heat input. One can make a note that with these conditions, the Diesel cycle has higher compression ratio $\dfrac{V_1}{V_{2'}}$ than that of Otto cycle $\dfrac{V_1}{V_2}$. One should also note that the cycle which is having higher efficiency allows maximum expansion. The Dual cycle efficiency will be between these two. Hence, for the same maximum pressure and heat input, $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$.

13.8.5 Same Maximum Pressure and Work Output

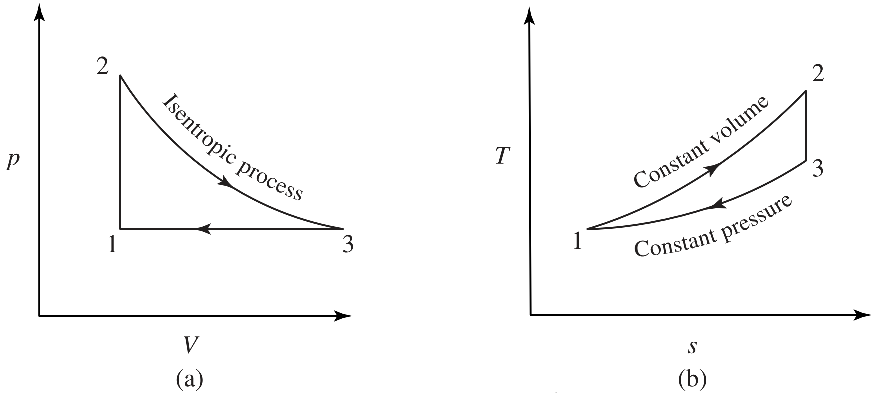
The efficiency, η, can be written as:

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{\text{Work done}}{\text{Work done} + \text{Heat rejected}}$$

Refer to *T*-*s* diagram in Fig.13.12(b). For same work output, the area 1234 (work output of Otto cycle) and area $12'3'4'$ (work output of Diesel cycle) are same. To achieve this, the entropy at 3 should be greater than entropy at $3'$. It is clear that the heat rejection for Otto cycle is more than that of Diesel cycle. Hence, for these conditions the Diesel cycle is more efficient than the Otto cycle. The efficiency of Dual cycle lies between the two cycles. Hence, for the same maximum pressure and work output, $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$.

13.9 THE LENOIR CYCLE

The Lenoir cycle consists of the following processes [see Fig.13.13(a)]. Constant volume heat addition (1→2); isentropic expansion (2→3); constant pressure heat rejection (3→1). The Lenoir cycle is used for pulse jet engines:



(a) (b)

Fig. 13.13 Lenoir cycle

$$\eta_{Lenoir} = \frac{Q_S - Q_F}{Q_S}$$

$$Q_S = mc_V(T_2 - T_1) \tag{13.49}$$

$$Q_R = mc_p(T_3 - T_1) \tag{13.50}$$

$$\eta_{Lenoir} = \frac{mc_V(T_2 - T_1) - mc_p(T_3 - T_1)}{mc_V(T_2 - T_1)} = 1 - \gamma\left(\frac{T_3 - T_1}{T_2 - T_1}\right) \tag{13.51}$$

Taking $p_2/p_1 = r_p$, we have $T_2 = T_1 r_p$ and,

$$\frac{T_3}{T_2} = \left(\frac{p_3}{p_2}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} \tag{13.52}$$

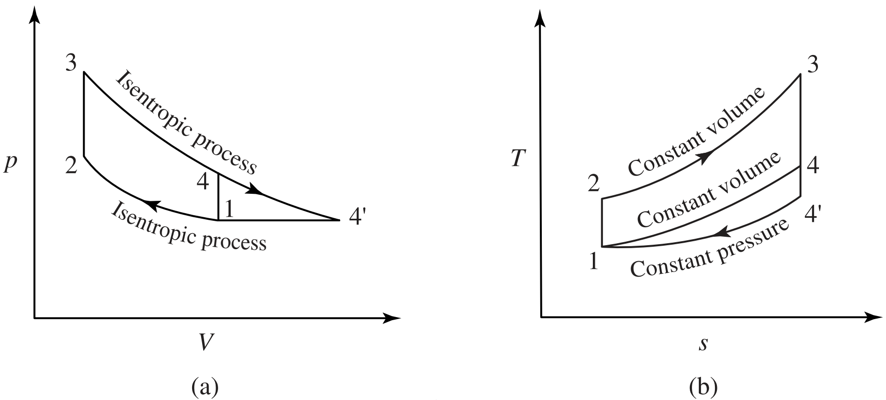
$$T_3 = T_2\left(\frac{1}{r_p}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = T_1 r_p \left(\frac{1}{r_p}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = T_1 r_p^{(1/\gamma)}$$

$$\eta = 1 - \gamma\left(\frac{T_1 r_p^{(1/\gamma)} - T_1}{T_1 r_p - T_1}\right) = 1 - \gamma\left(\frac{r_p^{(1/\gamma)} - 1}{r_p - 1}\right) \tag{13.53}$$

Thus, the efficiency of the Lenoir cycle depends upon the pressure ratio as well as the ratio of specific heats, viz γ.

13.10 THE ATKINSON CYCLE

Atkinson cycle is an ideal cycle for Otto engine exhausting to a gas turbine. In this cycle, the isentropic expansion (3→4) of an Otto cycle (1234) is further allowed to proceed to the lowest cycle pressure so as to increase the work output. With this modification, the cycle is known as Atkinson cycle. The cycle is shown on p-V and T-s diagrams in Fig.13.14(a) and (b), respectively.



(a) (b)

Fig. 13.14 Atkinson cycle

$$\eta_{Atkinson} = \frac{Q_S - Q_R}{Q_S} = \frac{mc_V(T_3 - T_2) - mc_p(T_{4'} - T_1)}{mc_V(T_3 - T_2)} \tag{13.54}$$

$$= 1 - \gamma\left(\frac{T_{4'} - T_1}{T_3 - T_2}\right) \tag{13.55}$$

compression ratio, $r = \frac{V_1}{V_2}$ and the expansion ratio $e = \frac{V_{4'}}{V_3}$. Now,

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = r^{(\gamma-1)} \tag{13.56}$$

$$T_2 = T_1 r^{(\gamma-1)} \tag{13.57}$$

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = \left(\frac{p_3}{p_{4'}} \times \frac{p_{4'}}{p_2}\right) = \left(\frac{p_3}{p_{4'}} \times \frac{p_1}{p_2}\right) \tag{13.58}$$

$$\frac{p_3}{p_{4'}} = \left(\frac{V_{4'}}{V_3}\right)^{\gamma} = e^{\gamma} \tag{13.59}$$

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\gamma} = \frac{1}{r^{\gamma}} \tag{13.60}$$

Substituting Eqs 13.59 and 13.60 in Eq.13.58:

$$\frac{T_3}{T_2} = \frac{e^{\gamma}}{r^{\gamma}} \tag{13.61}$$

$$T_3 = T_2\frac{e^{\gamma}}{r^{\gamma}} = T_1 r^{(\gamma-1)}\frac{e^{\gamma}}{r^{\gamma}} = T_1\frac{e^{\gamma}}{r} \tag{13.62}$$

$$\frac{T_{4'}}{T_3} = \left(\frac{V_3}{V_{4'}}\right)^{(\gamma-1)} = \frac{1}{e^{(\gamma-1)}}$$

$$T_{4'} = T_3\frac{1}{e^{(\gamma-1)}} = T_1\left(\frac{e^\gamma}{r}\right)\left(\frac{1}{e^{(\gamma-1)}}\right)$$

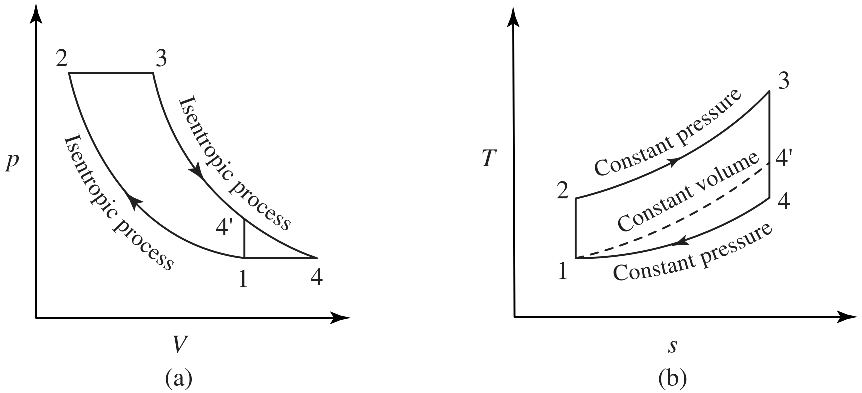
$$T_{4'} = T_1\frac{e}{r} \tag{13.63}$$

Substituting the values of T_2, T_3, $T_{4'}$ in Eq.13.55:

$$\eta_{Atkinson} = 1-\gamma\left[\frac{T_1e/r - T_1}{T_1e^\gamma/r - T_1r^{(\gamma-1)}}\right] = 1-\gamma\left[\frac{e-r}{e^\gamma - r^\gamma}\right] \tag{13.64}$$

13.11 THE BRAYTON CYCLE

Brayton cycle is a theoretical cycle for gas turbines. This cycle consists of two reversible adiabatic or isentropic processes and two constant pressure processes. Figure 13.15 shows the Brayton cycle on p-V and T-s coordinates. The cycle is similar to the Diesel cycle in compression and heat addition. The isentropic expansion of the Diesel cycle is further extended followed by constant pressure heat rejection.



(a) (b)

Fig. 13.15 Brayton cycle

$$\eta_{Brayton} = \frac{Q_S - Q_R}{Q_S} = \frac{mc_p(T_3-T_2) - mc_p(T_4-T_1)}{mc_p(T_3-T_2)} = 1 - \frac{T_4-T_1}{T_3-T_2}$$

If r is compression ratio, i.e. (V_1/V_2) and r_p is the pressure ratio, i.e. (p_2/p_1) then,

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{\gamma-1}{\gamma}\right)} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = r^{(\gamma-1)} \tag{13.65}$$

$$T_4 = \frac{T_3}{r^{(\gamma-1)}} \tag{13.66}$$

$$T_1 = \frac{T_2}{r^{(\gamma-1)}} \tag{13.67}$$

$$\eta_{Brayton} = 1 - \frac{(T_3/r^{(\gamma-1)}) - (T_2/r^{(\gamma-1)})}{T_3 - T_2} = 1 - \frac{1}{r^{(\gamma-1)}} \tag{13.68}$$

$$r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = r_p^{\frac{1}{\gamma}} = 1 - \frac{1}{\left(r_p^{\frac{1}{\gamma}}\right)^{\gamma-1}} = 1 - \frac{1}{r_p^{\left(\frac{\gamma-1}{\gamma}\right)}} \tag{13.69}$$

From Eq.13.69, it is seen that the efficiency of the Brayton cycle depends only on the pressure ratio and the ratio of specific heat, γ.

$$\begin{aligned} \text{Net work output} &= \text{Expansion work} - \text{Compression work} \\ &= c_p(T_3 - T_4) - c_p(T_2 - T_1) \\ &= c_pT_1\left(\frac{T_3}{T_1} - \frac{T_4}{T_1} - \frac{T_2}{T_1} + 1\right) \end{aligned}$$

$$\frac{W}{c_pT_1} = \frac{T_3}{T_1} - \frac{T_4}{T_3}\frac{T_3}{T_1} - \frac{T_2}{T_1} + 1 \tag{13.70}$$

It can be easily seen from Eq.13.70 (work output) that, the work output of the cycle depends on initial temperature, T_1, the ratio of the maximum to minimum temperature, T_3/T_1, pressure ratio, r_p and γ which are used in the calculation of T_2/T_1. Therefore, for the same pressure ratio and initial conditions work output depends on the maximum temperature of the cycle.

Worked-Out Examples

OTTO CYCLE

13.1 An engine working on Otto cycle has the following conditions : Pressure at the beginning of compression is 1 bar and pressure at the end of compression is 11 bar. Calculate the compression ratio and air-standard efficiency of the engine. Assume $\gamma = 1.4$.

Solution

$$r = \frac{V_1}{V_2} = \left(\frac{p_2}{p_1}\right)^{\left(\frac{1}{\gamma}\right)} = 11^{\frac{1}{1.4}} = 5.54 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Air-standard efficiency} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \left(\frac{1}{5.54}\right)^{0.4} = 0.496 = 49.6\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.2 In an engine working on ideal Otto cycle, the temperatures at the beginning and end of compression are 50 °C and 373 °C. Find the compression ratio and the air-standard efficiency of the engine.

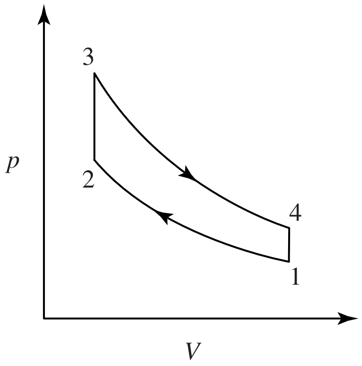
Solution

$$r = \frac{V_1}{V_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}} = \left(\frac{646}{323}\right)^{\frac{1}{0.4}} = 5.66 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{T_1}{T_2} = 1 - \frac{323}{646} = 0.5 = 50\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.3 In an Otto cycle, air at 17 °C and 1 bar is compressed adiabatically until the pressure is 15 bar. Heat is added at constant volume until the pressure rises to 40 bar. Calculate the air-standard efficiency, the compression ratio and the mean effective pressure for the cycle. Assume $c_V = 0.717$ kJ/kg K and $R = 8.314$ kJ/kmol K.

Solution



Consider the process 1 – 2:

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

$$\frac{V_1}{V_2} = r = \left(\frac{p_2}{p_1}\right)^{\dfrac{1}{\gamma}} = \left(\frac{15}{1}\right)^{\dfrac{1}{1.4}} = 6.91$$

$$\eta = 1 - \left(\frac{1}{r}\right)^{\gamma-1} = 1 - \left(\frac{1}{6.91}\right)^{0.4} = 0.539 = 53.9\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$T_2 = \frac{p_2V_2}{p_1V_1}T_1 = \frac{15}{1} \times \frac{1}{6.91} \times 290 = 629.5 \text{ K}$$

Consider the process 2 – 3:

$$T_3 = \frac{p_3T_2}{p_2} = \frac{40}{15} \times 629.5 = 1678.7$$

$$\text{Heat supplied} = c_V(T_3 - T_2) = 0.717 \times (1678.7 - 629.5) = 752.3 \text{ kJ/kg}$$

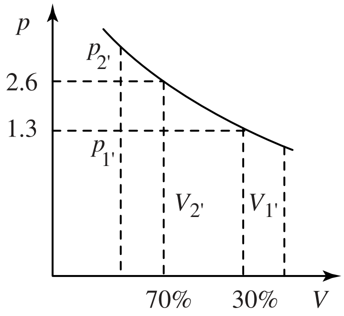
$$\text{Work done} = \eta \times q_s = 0.539 \times 752.3 = 405.5 \text{ kJ/kg}$$

$$v_1 = \frac{V_1}{m} = M\frac{RT_1}{p_1} = \frac{8314 \times 290}{29 \times 1 \times 10^5} = 0.8314 \text{ m}^3\text{/kg}$$

$$v_1 - v_2 = \frac{5.91}{6.91} \times 0.8314 = 0.711 \text{ m}^3\text{/kg}$$

$$p_m = \frac{\text{Work done}}{\text{Swept volume}} = \frac{405.5}{0.711} \times 10^3 = 5.70 \times 10^5 \text{ N/m}^2 = 5.70 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.4 Fuel supplied to an SI engine has a calorific value 42000 kJ/kg. The pressure in the cylinder at 30 and 70% of the compression stroke are 1.3 bar and 2.6 bar, respectively. Assuming that the compression follows the law $pV^{1.3}$ = constant. Find the compression ratio. If the relative efficiency of the engine compared with the air-standard efficiency is 50%, calculate the fuel consumption in kg/kW h.



Solution

$$V_2 = 1$$

$$V_{1'} = 1 + 0.7(r - 1) = 0.7r + 0.3$$

$$V_{2'} = 1 + 0.3(r - 1) = 0.3r + 0.7$$

$$\frac{V_{1'}}{V_{2'}} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{n}} = \left(\frac{2.6}{1.3}\right)^{\frac{1}{1.3}} = 1.7$$

$$\frac{0.7r + 0.3}{0.3r + 0.7} = 1.7$$

$$r = 4.68 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Air-standard efficiency} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{4.68^{0.4}} = 0.46 = 46\%$$

$$\text{Relative efficiency} = \frac{\text{Indicated thermal efficiency}}{\text{Air-standard efficiency}}$$

$$\eta_{ith} = 0.5 \times 0.46 = 0.23$$

$$\eta_{th} = \frac{ip}{CV \times \dot{m}} \quad \text{(where } \dot{m} \text{ is in kg/s)}$$

$$\frac{\dot{m}}{ip} = \frac{1}{42000 \times 0.23} = \; = 1.035 \times 10^{-4} \text{ kg/kW s} = 1.035 \times 10^{-4} \times 3600 \text{ kg/kW h}$$

$$isfc = 0.373 \text{ kg/kW h} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

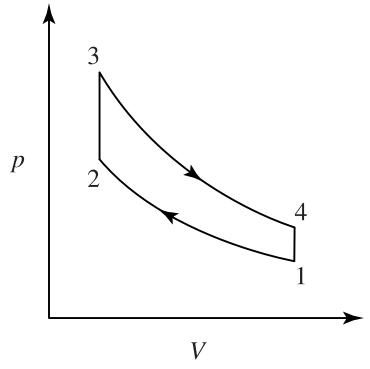
13.5 A gas engine working on the Otto cycle has a cylinder of diameter 200 mm and stroke 250 mm. The clearance volume is 1570 cc. Find the air-standard efficiency. Assume $c_p = 1.004$ kJ/kg K and $c_V = 0.717$ kJ/kg K for air.

Solution

$$\begin{aligned}
\text{Stroke volume, } V_s &= \pi 4 d^2 L = \frac{\pi}{4} \times 20^2 \times 25 = 7853.98 \text{ cc} \\
\text{Compression ratio, } r &= 1 + \frac{V_s}{V_c} = 1 + \frac{7853.98}{1570} = 6.0 \\
\gamma &= \frac{c_p}{c_V} = \frac{1.004}{0.717} = 1.4 \\
\text{Air-standard efficiency} &= 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{6^{(0.4)}} = 0.512 = 51.2\% \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

13.6 In an SI engine working on the ideal Otto cycle, the compression ratio is 5.5. The pressure and temperature at the beginning of compression are 1 bar and 27 °C, respectively. The peak pressure is 30 bar. Determine the pressure and temperatures at the salient points, the air-standard efficiency and the mean effective pressure. Assume ratio of specific heats to be 1.4 for air.

Solution



Since $V_2 = V_3 = V_c$ and $V_1 = rV_2 = rV_c$, consider the process 1–2:

$$\begin{aligned}
\frac{p_2}{p_1} &= r^{\gamma} = 5.5^{1.4} = 10.88 \\
p_2 &= 10.88 \times 1 \times 10^5 = 10.88 \times 10^5 \text{ N/m}^2 \qquad \stackrel{\textbf{Ans}}{\Longleftarrow} \\
\frac{T_2}{T_1} &= r^{\gamma-1} = 5.5^{0.4} = 1.978 \\
T_2 &= 1.978 \times 300 = 593.4 \text{ K} = 320.4\ ^{\circ}\text{ C} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

Consider the process 2–3:

$$
\begin{aligned}
p_3 &= 30 \times 10^5\ \mathrm{N/m^2} \\
\frac{T_3}{T_2} &= \frac{p_3}{p_2} = \frac{30}{10.88} = 2.757 \\
T_3 &= 2.757 \times 593.4 = 1636\ \mathrm{K} = 1363\ ^\circ\ \mathrm{C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

Consider the process 3–4:

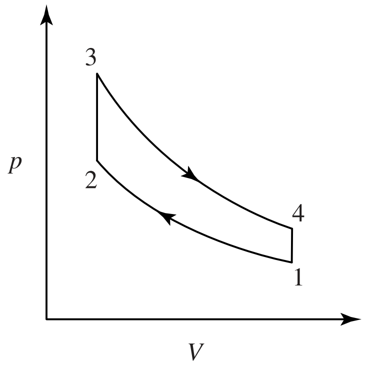
$$
\begin{aligned}
\frac{p_3}{p_4} &= \left(\frac{V_4}{V_3}\right)^\gamma = \left(\frac{V_1}{V_2}\right)^\gamma = r^\gamma = 5.5^{1.4} = 10.88 \\
p_4 &= \frac{p_3}{10.88} = 2.76 \times 10^5\ \mathrm{N/m^2} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\
\frac{T_3}{T_4} &= r^{\gamma-1} = 5.5^{0.4} = 1.978 \\
T_4 &= \frac{T_3}{1.978} = \frac{1636}{1.978} = 827.1\ \mathrm{K} = 554.1\ ^\circ\ \mathrm{C} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\
\eta_{Otto} &= 1 - \frac{1}{r^{(\gamma-1)}} = 1 - \frac{1}{5.5^{0.4}} = 0.4943 = 49.43\% \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\
p_m &= \frac{\text{Indicated work/cycle}}{V_s} = \frac{\text{Area of p-V diagram 1234}}{V_s} \\
\text{Area 1234} &= \text{Area under 3-4 – Area under 2-1} \\
&= \frac{p_3V_3 - p_4V_4}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1} \\
&= \frac{30 \times 10^5 \times V_c - 2.76 \times 10^5 \times 5.5V_c}{0.4} - \frac{10.88 \times 10^5 \times V_c - 1 \times 10^5 \times 5.5V_c}{0.4} \\
&= 23.6 \times 10^5 \times V_c = p_m \times V_s \\
p_m &= \frac{23.6 \times 10^5 \times V_c}{V_s} = \frac{23.6 \times 10^5 \times V_c}{4.5 \times V_c} = 5.24 \times 10^5\ \mathrm{N/m^2} = 5.24\ \text{bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

13.7 A gas engine operating on the ideal Otto cycle has a compression ratio of 6:1. The pressure and temperature at the commencement of compression are 1 bar and 27 °C. Heat added during the constant volume combustion process is 1170 kJ/kg. Determine the peak pressure and temperature, work output per kg of air and air-standard efficiency. Assume $c_V = 0.717$ kJ/kg K and $\gamma = 1.4$ for air.

Solution

Consider the process 1–2:

$$
\begin{aligned}
\frac{p_2}{p_1} &= r^\gamma = 6^{1.4} = 12.28 \\
p_2 &= 12.28 \times 10^5\ \mathrm{N/m^2}
\end{aligned}
$$



$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = r^{\gamma-1} = 6^{0.4} = 2.05$$

$$T_2 = 2.05 \times 300 = 615K = 342^\circ \text{ C}$$

Consider the process 2–3. For unit mass flow:

$$q_s = q_{2-3} = c_V(T_3 - T_2) = 1170 \text{ kJ/kg}$$

$$T_3 - T_2 = \frac{1170}{0.717} = 1631.8$$

$$T_3 = 1631.8 + 615 = 2246.8 \text{ K} = 1973.8\ ^\circ \text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\frac{p_3}{p_2} = \frac{T_3}{T_2} = \frac{2246.8}{615} = 3.65$$

$$\text{Peak pressure,} p_3 = 3.65 \times 12.28 \times 10^5 = 44.82 \times 10^5 \text{ N/m}^2 = 44.82 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Work output = Area of p-V diagram = Area under (3–4) − Area under (2–1)

$$= \frac{p_3V_3 - p_4V_4}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1}$$

$$= \frac{mR}{\gamma - 1}[(T_3 - T_4) - (T_2 - T_1)]$$

$$R = c_p - c_V = 1.004 - 0.717 = 0.287 \text{ kJ/kg K}$$

$$\frac{T_3}{T_4} = \left(\frac{V_3}{V_4}\right)^{\gamma-1} = r^{(\gamma-1)} = 6^{0.4} = 2.048$$

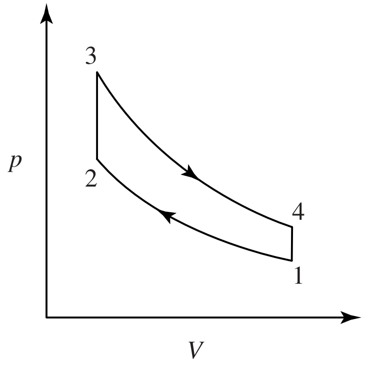
$$T_4 = \frac{T_3}{2.048} = \frac{2246.8}{2.048} = 1097.1 \text{ K}$$

$$\text{Work output/kg} = \frac{0.287}{0.4} \times [(2246.8 - 1097.1) - (615 - 300)] = 598.9 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_{Otto} = 1 - \frac{1}{r^{(\gamma-1)}} = 1 - \frac{1}{6^{0.4}} = 0.5116 = 51.16\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.8 A spark-ignition engine working on ideal Otto cycle has the compression ratio 6. The initial pressure and temperature of air are 1 bar and 37 °C. The maximum pressure in the cycle is 30 bar. For unit mass flow, calculate (i) p, V and T at various salient points of the cycle and (ii) the ratio of heat supplied to the heat rejected. Assume $\gamma = 1.4$ and $R = 8.314$ kJ/kmol K.

Solution



Consider point 1:

$$n = \frac{m}{M} = \frac{1}{29}$$

$$V_1 = \frac{nRT_1}{p_1} = \frac{1 \times 8314 \times 310}{29 \times 10^5} = 0.889 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Consider point 2:

$$p_2 = p_1 \; r^{\gamma} = 10^5 \times 6^{1.4} = 12.3 \times 10^5 \text{ N/m}^2 = 12.3 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$V_2 = \frac{V_1}{6} = \frac{0.889}{6} = 0.148 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$T_2 = \frac{p_2 V_2}{p_1 V_1} T_1 = \frac{12.3 \times 10^5 \times 0.148 \times 310}{1 \times 10^5 \times 0.889} = 634.8 \text{ K} = 361.8 \,^\circ \text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Consider point 3:

$$V_3 = V_2 = 0.148 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$p_3 = 30 \times 10^5 \text{ N/m}^2 = 30 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$T_3 = \frac{30 \times 10^5}{12.3 \times 10^5} \times 634.8 = 1548 \text{ K} = 1275\ ^\circ \text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Consider point 4:

$$p_3 V_3^\gamma = p_4 V_4^\gamma$$

$$p_4 = p_3 \left(\frac{V_3}{V_4}\right)^\gamma = 30 \times 10^5 \left(\frac{1}{6}\right)^{1.4} = 2.44 \times 10^5 \text{ N/m}^2 = 2.44 \text{ bar} \overset{\textbf{Ans}}{\Longleftarrow}$$

$$V_4 = V_1 = 0.889 \text{ m}^3 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$T_4 = T_1 \frac{p_4}{p_1} = 310 \times \frac{2.44 \times 10^5}{1 \times 10^5} = 756.4 \text{ K} = 483.4\ ^\circ \text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$c_V = \frac{R}{M(\gamma - 1)} = \frac{8.314}{29 \times 0.4} = 0.717 \text{ kJ/kg K}$$

For unit mass:

$$\text{Heat supplied} = c_V(T_3 - T_2) = 0.717 \times (1548 - 635.5) = 654.3 \text{ kJ}$$

$$\text{Heat rejected} = c_V(T_4 - T_1) = 0.717 \times (756.4 - 310) = 320.1 \text{ kJ}$$

$$\frac{\text{Heat supplied}}{\text{Heat rejected}} = \frac{654.3}{320.1} = 2.04 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.9 In an Otto engine, pressure and temperature at the beginning of compression are 1 bar and 37 °C, respectively. Calculate the theoretical thermal efficiency of this cycle, if the pressure at the end of the adiabatic compression is 15 bar. Peak temperature during the cycle is 2000 K. Calculate (i) the heat supplied per kg of air (ii) the work done per kg of air and (iii) the pressure at the end of adiabatic expansion. Take $c_V = 0.717$ kJ/kg K and $\gamma = 1.4$.

Solution

Consider the process 1–2:

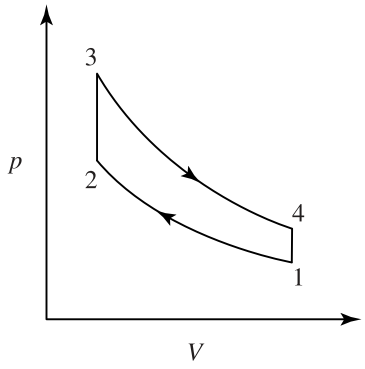
$$T_2 = T_1 \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = 310 \times (15)^{(0.4/1.4)} = 672 \text{ K}$$

$$\eta = 1 - \frac{T_1}{T_2} = 1 - \frac{310}{672} = 0.539 = 53.9\%$$

Consider the process 2–3. For unit mass flow:

$$\text{Heat supplied, } q_s = c_V(T_3 - T_2) = 0.717 \times (2000 - 672) = 952.2 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta = \frac{\text{Work done per kg of air}}{\text{Heat supplied per kg of air}} = \frac{w}{q_s}$$



$$w = \eta q_s = 0.539 \times 952.2 = 513.2 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$p_3 = p_2\left(\frac{T_3}{T_2}\right) = 15 \times \frac{2000}{672} = 44.64 \text{ bar}$$

Consider the process 3–4:

$$\frac{p_3}{p_4} = \left(\frac{V_4}{V_3}\right)^{\gamma} = \left(\frac{V_1}{V_2}\right)^{\gamma} = \frac{p_2}{p_1}$$

$$p_4 = p_3\left(\frac{p_1}{p_2}\right) = \frac{44.64 \times 1}{15} = 2.98 \text{ bar} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.10 Compare the efficiencies of ideal Atkinson cycle and Otto cycle for a compression ratio is 5.5. The pressure and temperature of air at the beginning of compression stroke are 1 bar and 27 °C, respectively. The peak pressure is 25 bar for both cycles. Assume $\gamma = 1.4$ for air.

Solution

$$\eta_{Otto} = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{5.5^{0.4}} = 49.43\%$$

(Refer Fig.13.14)

$$\eta_{Atkinson} = 1 - \frac{\gamma(e-r)}{e^{\gamma} - r^{\gamma}}$$

$$r = 5.5$$

$$e = \frac{V_{4'}}{V_3} = \left(\frac{p_3}{p_{4'}}\right)^{\frac{1}{\gamma}} = \left(\frac{25}{1}\right)^{\frac{1}{1.4}} = 9.966$$

$$e^{\gamma} = 9.966^{1.4} = 25$$

$$r^{\gamma} = 5.5^{1.4} = 10.88$$

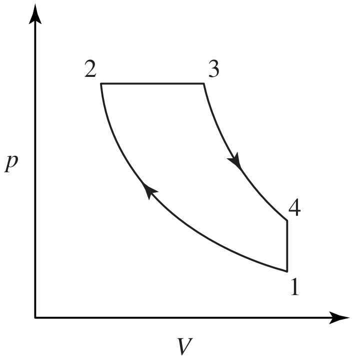
$$\eta_{Atkinson} = 1 - \frac{1.4 \times (9.966 - 5.5)}{25 - 10.88} = 55.72\%$$

$$\frac{\eta_{Atkinson}}{\eta_{Otto}} = \frac{55.72}{49.43} = 1.127 \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

DIESEL CYCLE

13.11 A Diesel engine has a compression ratio of 20 and cut-off takes place at 5% of the stroke. Find the air-standard efficiency. Assume $\gamma = 1.4$.

Solution



$$r = \frac{V_1}{V_2} = 20$$

$$V_1 = 20V_2$$

$$V_s = 20V_2 - V_2 = 19\ V_2$$

$$V_3 = 0.05V_s + V_2 = 0.05 \times 19V_2 + V_2 = 1.95\ V_2$$

$$r_c = \frac{V_3}{V_2} = \frac{1.95\ V_2}{V_2} = 1.95$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\ \frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)} = 1 - \frac{1}{20^{0.4}} \times \left[\frac{1.95^{1.4} - 1}{1.4 \times (1.95 - 1)}\right] = 0.649 = 64.9\% \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.12 Determine the ideal efficiency of the diesel engine having a cylinder with bore 250 mm, stroke 375 mm and a clearance volume of 1500 cc, with fuel cut-off occurring at 5% of the stroke. Assume $\gamma = 1.4$ for air.

Solution

$$V_s = \frac{\pi}{4}\ d^2\ L = \frac{\pi}{4} \times 25^2 \times 37.5 = 18407.8 \text{ cc}$$

$$r = 1 + \frac{V_s}{V_c} = 1 + \frac{18407.8}{1500} = 13.27$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}} \frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}$$

$$r_c = \frac{V_3}{V_2}$$

$$\text{Cut-off volume} = V_3 - V_2 = 0.05V_s = 0.05 \times 12.27V_c$$

$$V_2 = V_c$$

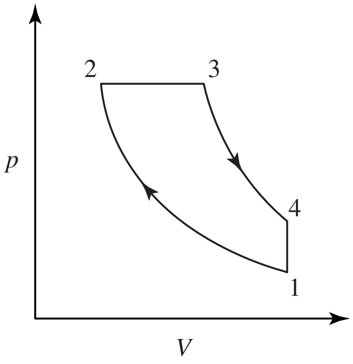
$$V_3 = 1.6135\ V_c$$

$$r_c = \frac{V_3}{V_2} = 1.6135$$

$$\eta = 1 - \frac{1}{13.27^{0.4}} \times \frac{1.6135^{1.4} - 1}{1.4 \times (1.6135 - 1)} = 0.6052 = 60.52\% \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.13 In an engine working on Diesel cycle, inlet pressure and temperature are 1 bar and 17 °C, respectively. Pressure at the end of adiabatic compression is 35 bar. The ratio of expansion, i.e. after constant pressure heat addition is 5. Calculate the heat addition, heat rejection and the efficiency of the cycle. Assume $\gamma = 1.4$, $c_p = 1.004$ kJ/kg K and $c_V = 0.717$ kJ/kg K.

Solution



Consider the process 1–2:

$$\frac{V_1}{V_2} = r = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \left(\frac{35}{1}\right)^{\frac{1}{1.4}} = 12.674$$

$$\text{Cut-off ratio} = \frac{V_3}{V_2} = \frac{V_3}{V_1} \times \frac{V_1}{V_2}$$

$$= \frac{\text{Compression ratio}}{\text{Expansion ratio}} = \frac{12.674}{5} = 2.535$$

$$\begin{aligned}\frac{T_2}{T_1} &= \left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{35}{1}\right)^{0.286} = 2.76\\ T_2 &= 2.76 \times 290 = 801.7 \text{ K}\end{aligned}$$

Consider the process 2–3:

$$\begin{aligned}T_3 &= T_2\frac{V_3}{V_2} = 801.7 \times \frac{V_3}{V_2}\\ &= 801.7 \times 2.535 = 2032.3 \text{ K}\end{aligned}$$

Consider the process 3–4:

$$T_4 = T_3\left(\frac{V_3}{V_4}\right)^{\gamma-1} = 2032.3 \times \left(\frac{1}{5}\right)^{0.4} = 1067.6 \text{ K}$$

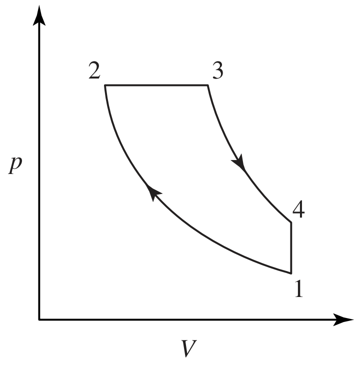
$$\text{Heat added} = c_p(T_3 - T_2) = 1.004 \times (2032.3 - 801.7) = 1235.5 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Heat rejected} = c_V(T_4 - T_1) = 0.717 \times (1067.6 - 290) = 557.5 \text{ kJ/kg} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Efficiency} = \frac{\text{Heat supplied - Heat rejected}}{\text{Heat supplied}} = \frac{1235.5 - 557.5}{1235.5} = 54.9\% \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.14 A Diesel engine is working with a compression ratio of 15 and expansion ratio of 10. Calculate the air-standard efficiency of the cycle. Assume $\gamma = 1.4$.

Solution



$$\begin{aligned}r &= \frac{V_1}{V_2} = 15\\ r_e &= \frac{V_4}{V_3} = 10\end{aligned}$$

$$\eta = 1 - \frac{1}{\gamma}\,\frac{1}{r^{\gamma-1}}\left[\frac{\left(\frac{r}{r_e}\right)^{\gamma}-1}{\left(\frac{r}{r_e}\right)-1}\right]$$

$$= 1 - \frac{1}{1.4}\times\frac{1}{15^{0.4}}\times\left[\frac{\left(\frac{15}{10}\right)^{1.4}-1}{\left(\frac{15}{10}\right)-1}\right] = 0.63 = 63\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

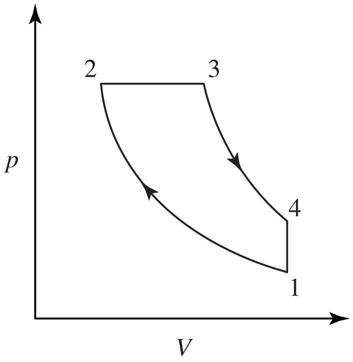
13.15 A Diesel engine works on Diesel cycle with a compression ratio of 15 and cut-off ratio of 1.75. Calculate the air-standard efficiency assuming $\gamma = 1.4$.

Solution

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\frac{1}{\gamma}\left(\frac{r_c^{\gamma}-1}{r-1}\right)$$

$$= 1 - \frac{1}{15^{0.4}}\times\frac{1}{1.4}\times\left(\frac{1.75^{1.4}-1}{1.75-1}\right) = 0.617 = 61.7\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.16 A Diesel cycle operates at a pressure of 1 bar at the beginning of compression and the volume is compressed to $\frac{1}{16}$ of the initial volume. Heat is supplied until the volume is twice that of the clearance volume. Calculate the mean effective pressure of the cycle. Take $\gamma = 1.4$.



Solution

$$V_1 = 16V_2 \quad \text{and} \quad V_3 = 2\,V_2$$

$$V_s = V_1 - V_2 = (r-1)V_2 = 15\,V_2$$

$$V_2 = \frac{V_s}{15}$$

Consider the process 1–2:

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1 \times 16^{1.4} = 48.5 \text{ bar}$$

$$p_2 = p_3 = 48.5 \text{ bar}$$

$$p_4 = p_3\left(\frac{V_3}{V_4}\right)^{1.4} = 48.5 \times \left(\frac{2}{16}\right)^{1.4}$$

$$= \frac{48.5}{8^{1.4}} = 2.64 \text{ bar} \qquad (\text{ Since}, V_4 = V_1)$$

Mean effective pressure, p_m, is given by:

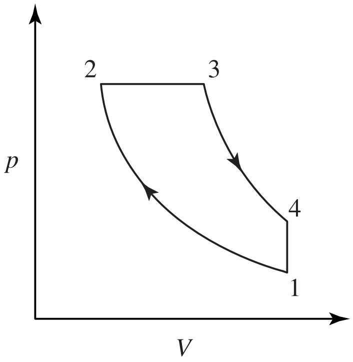
$$p_m = \frac{1}{V_s}\left[p_2(V_3 - V_2) + \frac{p_3V_3 - p_4V_4}{\gamma - 1} - \frac{p_2V_2 - p_1V_1}{\gamma - 1}\right]$$

$$= \frac{V_2}{V_s}\left[p_3\left(\frac{V_3}{V_2} - 1\right) + \frac{\left(p_3\frac{V_3}{V_2} - p_4\frac{V_4}{V_2}\right)}{\gamma - 1} - \frac{\left(p_2 - p_1\frac{V_1}{V_2}\right)}{\gamma - 1}\right]$$

$$= \frac{1}{15} \times \left[48.5 \times (2 - 1) + \frac{48.5 \times 2 - 2.64 \times 16}{1.4 - 1} - \frac{48.5 - 1 \times 16}{1.4 - 1}\right] = 6.94 \text{ bar} \xLeftarrow{\textbf{Ans}}$$

13.17 In an engine working on the Diesel cycle, the ratios of the weights of air and fuel supplied is 50 : 1. The temperature of air at the beginning of the compression is 60 °C and the compression ratio used is 14 : 1. What is the ideal efficiency of the engine. Calorific value of fuel used is 42000 kJ/kg. Assume $c_p = 1.004$ kJ/kg K and $c_V = 0.717$ kJ/kg K for air.

Solution



$$\eta = 1 - \frac{1}{r^{\gamma-1}}\ \frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}$$

$$\gamma = \frac{c_p}{c_V} = \frac{1.004}{0.717} = 1.4$$

$$\text{Cut-off ratio, } r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2}$$

Consider the process 1–2:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)}$$

$$= r^{(\gamma-1)} = 14^{0.4} = 2.874$$

$$T_2 = 2.874 \times 333 = 957.04 \text{ K}$$

Consider the process 2–3:

$$\text{Heat added/kg of air} = c_p\,(T_3 - T_2) = F/A \times CV$$

$$T_3 - T_2 = \frac{F/A \times CV}{c_p} = \frac{42000}{50 \times 1.004} = 836.6$$

$$T_3 = 1793.64 \text{ K}$$

$$r_c = \frac{T_3}{T_2} = \frac{1793.64}{957.04} = 1.874$$

$$\eta = 1 - \frac{1}{1.4 \times 14^{0.4}} \times \left(\frac{1.874^{1.4} - 1}{0.874}\right) = 0.60 = 60\% \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.18 In an ideal Diesel cycle, the pressure and temperature are 1.03 bar and 27 °C, respectively. The maximum pressure in the cycle is 47 bar and the heat supplied during the cycle is 545 kJ/kg. Determine (i) the compression ratio (ii) the temperature at the end of compression (iii) the temperature at the end of constant pressure combustion and (iv) the air-standard efficiency. Assume γ= 1.4 and c_p = 1.004 kJ/kg K for air.

Solution

$$p_2 = p_3 = 47 \times 10^5 \text{ N/m}^2$$

$$\frac{p_2}{p_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = r^{\gamma}$$

$$r = \left(\frac{p_2}{p_1}\right)^{\left(\frac{1}{\gamma}\right)} = \left(\frac{47}{1.03}\right)^{\left(\frac{1}{1.4}\right)} = 15.32 \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{(\gamma-1)} = r^{(\gamma-1)} = 15.32^{0.4} = 2.979$$

$$T_2 = 2.979 \times 300 = 893.7 \text{ K} = 620.7\ ^\circ\text{ C} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Heat supplied/kg} = c_p\,(T_3 - T_2) = 545$$

$$T_3 - T_2 = \frac{545}{1.004} = 542.8$$

$$T_3 = 542.8 + 893.7 = 1436.5 \text{ K} = 1163.5\ ^\circ\text{C} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

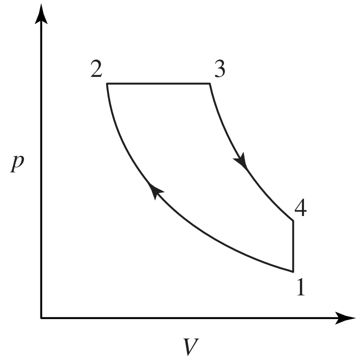
$$\eta = 1 - \frac{1}{r^{\gamma-1}}\ \frac{r_c^{\gamma} - 1}{\gamma(r_c - 1)}$$

$$r_c = \frac{V_3}{V_2} = \frac{T_3}{T_2} = \frac{1436.5}{893.7} = 1.61$$

$$\eta_{Diesel} = 1 - \left[\frac{1}{1.4 \times 15.32^{0.4}} \times \left(\frac{1.61^{1.4} - 1}{0.61}\right)\right] = 0.6275 = 62.75\% \qquad \overset{\text{Ans}}{\Longleftarrow}$$

13.19 A diesel engine operating on the air-standard Diesel cycle has six cylinders of 100 mm bore and 120 mm stroke. The engine speed is 1800 rpm. At the beginning of compression, the pressure and temperature of air are 1.03 bar and 35 °C. If the clearance volume is 1/8th of the stroke volume, calculate (i) the pressure and temperature at the salient points of the cycle, (ii) the compression ratio, (iii) the efficiency of the cycle and (iv) the power output if the air is heated to 1500 °C Assume c_p and c_V of air to be 1.004 and 0.717 kJ/kg K respectively.

Solution



$$r = 1 + \frac{V_s}{V_c} = 1 + 8 = 9 \qquad \overset{\text{Ans}}{\Longleftarrow}$$

Consider the process 1–2:

$$\frac{p_2}{p_1} = r^{\gamma} = 9^{1.4} = 21.67$$

$$p_2 = 21.67 \times 1.03 \times 10^5 = 22.32 \times 10^5 \text{ N/m}^2 = 22.32 \text{ bar} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

$$\frac{T_2}{T_1} = r^{(\gamma-1)} = 9^{0.4} = 2.408$$

$$T_2 = 308 \times 2.408 = 741.6 \text{ K} = 468.6\ ^\circ\text{C} \qquad \overset{\text{Ans}}{\Longleftarrow}$$

Consider the process 2–3:

$$p_3 = p_2 = 22.32 \times 10^5 \text{ N/m}^2 = 22.32 \text{ bar} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$T_3 = 1773 \text{ K} = 1500\ ^\circ \text{ C} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

Consider the process 3–4:

$$\frac{T_3}{T_4} = r_e^{(\gamma-1)}$$

$$r_c = \frac{T_3}{T_2} = \frac{1773}{741.6} = 2.39$$

$$r_e = \frac{r}{r_c} = \frac{9}{2.391} = 3.764$$

$$\frac{T_3}{T_4} = 1.7$$

$$T_4 = \frac{T_3}{1.7} = \frac{1773}{1.7} = 1042.9 \text{ K} = 769.9\ ^\circ \text{ C} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$\frac{p_3}{p_4} = r_e^{\gamma} = 3.764^{1.4} = 6.396$$

$$p_4 = \frac{p_3}{6.396} = \frac{22.32 \times 10^5}{6.396} = 3.49 \times 10^5 \text{ N/m}^2 = 3.49 \text{ bar} \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_{Cycle} = \frac{\text{Work output}}{\text{Heat added}} = 1 - \frac{\text{Heat rejected}}{\text{Heat added}} = 1 - \frac{q_{4-1}}{q_{2-3}}$$

$$q_{4-1} = c_V\,(T_4 - T_1) = 0.717 \times (1042.9 - 308) = 526.9 \text{ kJ/kg}$$

$$q_{2-3} = c_p\,(T_3 - T_2) = 1.004 \times (1773 - 741.6) = 1035.5 \text{ kJ/kg}$$

$$\eta_{Cycle} = 1 - \frac{526.9}{1035.5} = 0.4912 = 49.12\% \qquad \stackrel{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Work output} = q_{2-3} - q_{4-1} = 1035.5 - 526.9 = 508.6 \text{ kJ/kg}$$

$$\text{Power output} = \text{Work output} \times \dot{m}_a$$

$$\dot{m}_a = \frac{p_1 V_1}{R T_1} \times \frac{N}{2}$$

$$R = c_p - c_V = 0.287 \text{ kJ/kg K}$$

$$V_1 = V_s + V_c = \frac{9}{8}\, V_s$$

$$V_s = 6 \times \frac{\pi}{4} d^2\ L = 6 \times \frac{\pi}{4} \times 10^2 \times 12 = 5654.8 \text{ cc} = 5.65 \times 10^{-3} \text{ m}^3$$

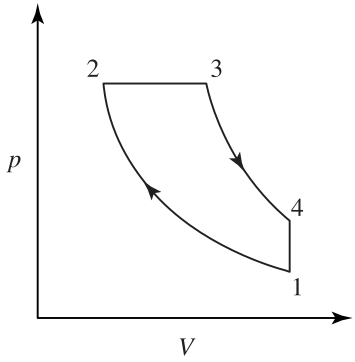
$$V_1 = 5.65 \times 10^{-3} \times \frac{9}{8} = 6.36 \times 10^{-3}\ \text{m}^3$$

$$\dot{m}_a = \frac{1.03 \times 10^5 \times 6.36 \times 10^{-3} \times 30}{287 \times 308 \times 2} = 0.111\ \text{kg/s}$$

$$\text{Power output} = 508.6 \times 0.111 = 56.45\ \text{kW}$$ $\overset{\textbf{Ans}}{\Longleftarrow}$

13.20 The mean effective pressure of an ideal Diesel cycle is 8 bar. If the initial pressure is 1.03 bar and the compression ratio is 12, determine the cut-off ratio and the air-standard efficiency. Assume ratio of specific heats for air to be 1.4.

Solution



$$\text{Work output} = p_m \times V_s = \text{Area 1234}$$

$$= \text{Area under } 2 - 3 + \text{Area under } 3 - 4 - \text{Area under } 2 - 1$$

$$= p_2 (V_3 - V_2) + \frac{p_3 V_3 - p_4 V_4}{\gamma - 1} - \frac{p_2 V_2 - p_1 V_1}{\gamma - 1}$$

$$r = \frac{V_1}{V_2} = 1 + \frac{V_s}{V_c} = 12$$

$$V_s = 11 V_c \quad \text{and} \quad V_2 = V_c$$

$$V_1 = V_4 = 12 V_2 = 12 V_c$$

$$V_3 = r_c V_2 = r_c V_c$$

$$\frac{p_2}{p_1} = r^{\gamma} = 12^{1.4} = 32.42$$

$$p_2 = 32.42 \times 1.03 \times 10^5 = 33.39 \times 10^5\ \text{N/m}^2 = p_3$$

$$\frac{p_3}{p_4} = \left(\frac{r}{r_c}\right)^{1.4} = \frac{12^{1.4}}{r_c^{1.4}} = \frac{32.42}{r_c^{1.4}}$$

$$p_4 = \frac{33.39}{32.42} \times r_c^{1.4} \times 10^5 = 1.03 r_c^{1.4} \times 10^5$$

$$\text{Area 1234} = 33.39\,(r_c V_c - V_c) + \frac{33.39 \times r_c V_c - 1.03 r_c^{1.4} \times 12 V_c}{0.4} - \frac{33.39 \times V_c - 1.03 \times 12 V_c}{0.4} \times 10^5$$

$$p_m \times V_s = 8 \times 11 V_c \times 10^5$$

$$\text{Area 1234} = p_m \times V_s$$

Substituting and simplifying, $0.672 r_c - 0.178 r_c^{1.4} = 1$

Solving by iteration,

$$r_c = 2.38 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta = 1 - \frac{1}{r^{\gamma-1}}\,\frac{r_c^{\gamma}-1}{\gamma(r_c-1)} = 1 - \frac{1}{1.4 \times 12^{0.4}} \times \left(\frac{2.38^{1.4}-1}{1.38}\right)$$

$$= 0.5466 = 54.66\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.21 An air-standard Dual cycle has a compression ratio of 10. The pressure and temperature at the beginning of compression are 1 bar and 27 °C. The maximum pressure reached is 42 bar and the maximum temperature is 1500 °C. Determine (i) the temperature at the end of constant volume heat addition, (ii) cut-off ratio, (iii) work done per kg of air and (iv) the cycle efficiency. Assume $c_p = 1.004$ kJ/kg K and $c_V = 0.717$ kJ/kg K for air.

Solution

$$\frac{V_s}{V_c} = r - 1 = 9$$

$$V_s = 9 V_c$$

$$\gamma = \frac{c_p}{c_V} = \frac{1.004}{0.717} = 1.4$$

$$T_2 = 2.512 \times 300 = 753.6 \text{ K}$$

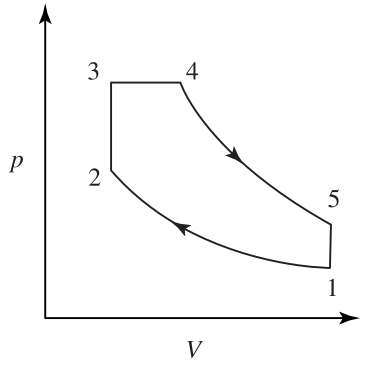
Consider the process 1–2:

$$\frac{T_2}{T_1} = r^{(\gamma-1)} = 10^{0.4} = 2.512$$

$$\frac{p_2}{p_1} = r^{\gamma} = 10^{1.4} = 25.12$$

$$p_2 = 25.12 \times 10^5 \text{ N/m}^2$$

Consider the process 3–4:



$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = \frac{42}{25.12} = 1.672$$

$$T_3 = 1.672 \times 753.6 = 1260 \text{ K} = 987\ ^\circ\text{C} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$r_c = \frac{T_4}{T_3} = \frac{1773}{1260} = 1.407 \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Work done/kg} = \text{Heat supplied - Heat rejected}$$

$$\begin{aligned}\text{Heat supplied/kg} &= c_V\,(T_3 - T_2) + c_p\,(T_4 - T_3)\\ &= 0.717 \times (1260 - 753.6) + 1.004 \times (1773 - 1260)\\ &= 878.1 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}\end{aligned}$$

Consider the process 4–5:

$$\frac{T_4}{T_5} = \left(\frac{V_5}{V_4}\right)^{(\gamma-1)} = \left(\frac{r}{r_c}\right)^{(\gamma-1)} = \left(\frac{10}{1.407}\right)^{0.4} = 2.191$$

$$T_5 = \frac{T_4}{2.191} = 809.2 \text{ K} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

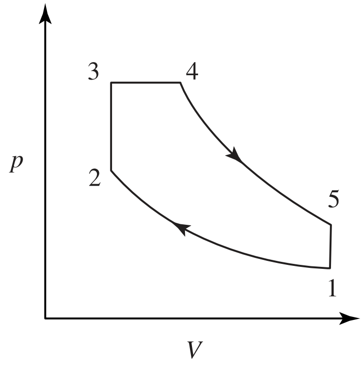
$$\text{Heat rejected/kg} = c_V\,(T_5 - T_1) = 0.717 \times (809.2 - 300) = 365.1 \text{ kJ}$$

$$\text{Work output/kg} = 878.1 - 365.1 = 513 \text{ kJ} \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta_{Dual} = \frac{\text{Work output}}{\text{Heat added}} = \frac{513}{878.1} = 0.5842 = 58.42\% \quad \overset{\textbf{Ans}}{\Longleftarrow}$$

13.22 For an engine working on the ideal Dual cycle, the compression ratio is 10 and the maximum pressure is limited to 70 bar. If the heat supplied is 1680 kJ/kg, find the pressures and temperatures at the various salient points of the cycle and the cycle efficiency. The pressure and temperature of air at the commencement of compression are 1 bar and 100 °C, respectively. Assume $c_p = 1.004$ kJ/kg K and $c_V = 0.717$ kJ/kg K for air.

Solution



$$\frac{V_s}{V_c} = r - 1 = 9$$

$$\gamma = \frac{c_p}{c_V} = \frac{1.004}{0.717} = 1.4$$

Consider the process 1–2:

$$\frac{p_2}{p_1} = r^\gamma = 10^{1.4} = 25.12$$

$$p_2 = 25.12 \times 10^5 \text{ N/m}^2 = 25.12 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\frac{T_2}{T_1} = r^{(\gamma-1)} = 10^{0.4} = 2.512$$

$$T_2 = 2.512 \times 373 = 936.9 \text{ K} = 663.9\ ^\circ \text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Consider the process 2–3 and 3–4:

$$\frac{T_3}{T_2} = \frac{p_3}{p_2} = \frac{70}{25.12} = 2.787$$

$$T_3 = 2.787 \times 936.9 = 2611.1 \text{ K} = 2338\ ^\circ \text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Heat added during constant volume combustion:

$$c_V\,(T_3 - T_2) = 0.717 \times (2611.1 - 936.9) = 1200.4 \text{ kJ/kg}$$

$$\text{Total heat added} = 1680 \text{ kJ/kg}$$

Hence, heat added during constant pressure combustion:

$$
\begin{aligned}
&= 1680 - 1200.4 = 479.6 \text{ kJ/kg} = c_p\,(T_4 - T_3)\\
T_4 - T_3 &= \frac{479.6}{1.004} = 477.7 \text{ K}\\
T_4 &= 477.7 + 2611.1 = 3088.8 \text{ K} = 2815.8\ ^\circ\text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}\\
\text{Cut-off ratio, } r_c &= \frac{V_4}{V_3} = \frac{T_4}{T_3} = \frac{3088.8}{2611.1} = 1.183
\end{aligned}
$$

Consider the process 4–5:

$$
\begin{aligned}
\frac{T_4}{T_5} &= \left(\frac{r}{r_c}\right)^{(\gamma-1)} = 8.453^{0.4} = 2.35\\
T_5 &= \frac{T_4}{2.35} = \frac{3088.8}{2.35} = 1314.4 \text{ K} = 1041.4\ ^\circ\text{ C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}\\
\frac{p_4}{p_5} &= \left(\frac{r}{r_c}\right)^{\gamma} = 19.85\\
p_5 &= \frac{p_4}{19.85} = \frac{70 \times 10^5}{19.85} = 3.53 \times 10^5 \text{ N/m}^2 = 3.53 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}\\
\text{Heat rejected} &= c_V(T_5 - T_1) = 0.717 \times (1314.4 - 373) = 674.98 \text{ kJ/kg}\\
\eta &= \frac{1680 - 674.98}{1680} = 59.82\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}
$$

DUAL CYCLES

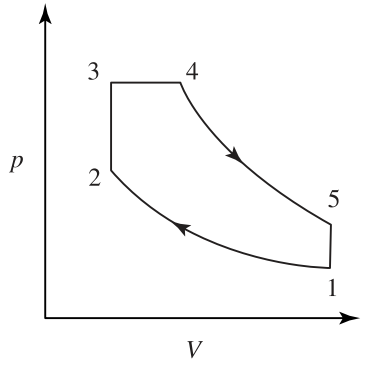
13.23 An oil engine works on the Dual cycle, the heat liberated at constant pressure being twice that liberated at constant volume. The compression ratio of the engine is 8 and the expansion ratio is 5.3. However, the compression and expansion processes follow the law, $pV^{1.3} = \text{C}$. The pressure and temperature at the beginning of compression are 1 bar and 27 °C, respectively. Assuming $c_p = 1.004$ kJ/kg K and $c_V = 0.717$ kJ/kg K for air, find the air-standard efficiency and the mean effective pressure.

Solution

$$
\begin{aligned}
\gamma &= \frac{c_p}{c_V} = \frac{1.004}{0.717} = 1.4\\
\frac{V_s}{V_c} &= r - 1 = 7\\
V_s &= 7V_c\\
r_e &= \frac{r}{r_c} = 5.3\\
r_c &= \frac{8}{5.3} = 1.509
\end{aligned}
$$

Mean effective pressure = Area $12345/V_s$

$\overset{\textbf{Ans}}{\Longleftarrow}$



$$\begin{aligned}
\text{Area } 12345 &= \text{Area under } 3-4+\text{Area under } 4-5-\text{Area under } 2-1 \\
&= p_3\,(V_3-V_4)+\frac{p_4V_4-p_5V_5}{n-1}-\frac{p_2V_2-p_1V_1}{n-1} \\
V_2 &= V_3=V_c \quad \text{and} \quad V_1=V_5=rV_c=8V_c \\
V_4 &= r_cV_3=1.509V_c \\
\frac{T_2}{T_1} &= r^{(n-1)}=8^{0.3}=1.866 \\
T_2 &= 1.866\times 300=559.82\text{ K} \\
\frac{p_2}{p_1} &= r^n=8^{1.3}=14.93 \\
p_2 &= 14.93\times p_1=14.93\times 10^5\text{ N/m}^2
\end{aligned}$$

Heat released during constant pressure combustion
= 2 × Heat released during constant volume combustion

$$\begin{aligned}
c_p\,(T_4-T_3) &= 2c_V\,(T_3-T_2) \\
1.004\times(T_4-T_3) &= 2\times 0.717\times(T_3-T_2) \\
T_4-T_3 &= 1.428\times(T_3-T_2) \\
\frac{T_4}{T_3} &= \frac{V_4}{V_3}=r_c=1.509 \\
T_4 &= 1.509\;T_3 \\
1.509\;T_3-T_3 &= 1.428\times(T_3-559.82)
\end{aligned}$$

$$T_3 = 869.88 \text{ K}$$

$$T_4 = 1312.65 \text{ K}$$

$$\frac{p_3}{p_2} = \frac{T_3}{T_2} = \frac{869.88}{559.82} = 1.554$$

$$p_3 = 1.554 \times 14.93 \times 10^5 = 23.20 \times 10^5 \text{ N/m}^2 = p_4$$

$$\frac{T_4}{T_5} = r_e^{(n-1)} = 5.3^{0.3} = 1.649$$

$$T_5 = \frac{1312.65}{1.649} = 796.03 \text{ K}$$

$$\frac{p_4}{p_5} = r_e^n = 5.3^{1.3} = 8.741$$

$$p_5 = \frac{p_4}{8.741} = \frac{23.2 \times 10^5}{8.741} = 2.654 \times 10^5 \text{ N/m}^2$$

$$\text{Area } 12345 = \left[\frac{23.2 \times 1.509V_c - 2.654 \times 8V_c}{0.3} + 23.2 \times (1.509V_c - V_c) - \frac{14.93 \times V_c - 1 \times 8V_c}{0.3}\right] \times 10^5$$

$$= 34.63 \times V_c \times 10^5 \text{ N/m}^2$$

$$V_c = \frac{V_1}{8}$$

$$\text{Area } 12345 = p_m \times V_s = p_m \times 7 \times V_c$$

$$p_m = \frac{34.63}{7} = 4.95 \times 10^5 \text{ N/m}^2 = 4.95 \text{ bar}$$ **Ans** $\Longleftarrow$

$$\eta = \frac{w}{q_s}$$

$$v_1 = \frac{mRT_1}{p_1} = \frac{1 \times 287 \times 300}{1 \times 10^5} = 0.861 \text{ m}^3\text{/kg}$$

$$w = 34.63 \times 10^5 \times \frac{v_1}{8} = 34.63 \times 10^5 \times \frac{0.861}{8}$$

$$= 3.727 \times 10^5 \text{ J/kg} = 372.7 \text{ kJ/kg}$$

Note : Work done should be calculated only from the area ($\int pdv$) for a polytropic process:

$$q_s = q(T_3 - T_2) + c_p(T_4 - T_3) = 667.1 \text{ kJ/kg}$$

$$\eta = \frac{372.7}{667.1} \times 100 = 55.9\%$$ **Ans** $\Longleftarrow$

COMPARISON OF CYCLES

13.24 A four-cylinder, four-stroke, spark-ignition engine has a displacement volume of 300 cc per cylinder. The compression ratio of the engine is 10 and operates at a speed of 3000 rev/min. The engine is required to develop an output of 40 kW at this speed. Calculate the cycle efficiency, the necessary rate of heat addition, the mean effective pressure and the maximum temperature of the cycle. Assume that the engine operates on the Otto cycle and that the pressure and temperature at the inlet conditions are 1 bar and 27 °C, respectively.

If the above engine is a compression-ignition engine operating on the Diesel cycle and receiving heat at the same rate, calculate efficiency, the maximum temperature of the cycle, the cycle efficiency, the power output and the mean effective pressure. Take $c_V = 0.717$ kJ/kg K and $\gamma = 1.4$.

Solution

Consider the Otto cycle, Fig.13.9(a):

$$\eta = 1 - \frac{1}{r^{\gamma-1}} = 1 - \frac{1}{10^{0.4}} = 0.602 = 60.2\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\eta = \frac{\text{Power output}}{\text{Heat supplied}}$$

$$\text{Heat supplied} = \frac{40}{0.602} = 66.5 \text{ kW} = 66.5 \text{ kJ/s} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Number of cycles/sec} = \frac{3000}{2 \times 60} = 25$$

$$\text{Net work output per cycle from each cylinder} = \frac{40}{4 \times 25} = 0.4 \text{ kJ}$$

$$p_m = \frac{W}{V_s} = \frac{0.4 \times 1000}{300 \times 10^{-6}} = 13.3 \times 10^5 \text{ N/m}^2 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 300 \times 10^{0.4} = 753.6 \text{ K}$$

$$\text{Heat supplied/cylinder/cycle, } (Q_{2-3}) = \frac{66.5}{4 \times 25} = 0.665 \text{ kJ}$$

$$Q_{2-3} = mc_V\,(T_3 - T_2)$$

$$v_1 = \frac{RT_1}{p_1} = \frac{287 \times 300}{1 \times 10^5} = 0.861 \text{ m}^3\text{/kg}$$

This initial volume of air in the cylinder is:

$$V_1 = V_2 + V_s = \left(\frac{V_1}{10}\right) + V_s$$

$$0.9V_1 = V_s$$

$$V_1 = \frac{V_s}{0.9} = \frac{300 \times 10^{-6}}{0.9} = 333 \times 10^{-6}\ \text{m}^3$$

$$m = \frac{V_1}{v_1} = \frac{333 \times 10^{-6}}{0.861} = 0.387 \times 10^{-3}\ \text{kg}$$

The temperature rise resulting from heat addition is

$$T_3 - T_2 = \frac{Q_{2-3}}{m\ c_V} = \frac{0.665}{0.387 \times 10^{-3} \times 0.717} = 2396.6\ \text{K}$$

$$T_3 = T_2 + 2396.6 = 753.6 + 2396.6 = 3150.2\ \text{K} = 2877\ ^\circ\ \text{C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Now let us consider the Diesel cycle. T_2 is same as in the previous case, i.e. $T_2 = 753.6$ K.

Heat supplied per cycle per cylinder is also same, i.e. $Q_{2-3'} = 0.665$ kJ

$$Q_{2-3'} = m\ c_p(T_{3'} - T_2)$$

$$T_{3'} - T_2 = \frac{0.665}{0.387 \times 10^{-3}} \times \frac{1}{1.004} = 1711.5\ \text{K}$$

$$T_{3'} = 1711.5 + 753.6 = 2465.1\ \text{K} = 2192.1\ ^\circ\ \text{C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Cut-off ratio, } r_c = \frac{V_3}{V_2} = \frac{T_{3'}}{T_2} = \frac{2465.1}{753.6} = 3.27$$

$$\text{Air-standard efficiency} = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{r_c^\gamma - 1}{\gamma\,(r_c - 1)}\right]$$

$$= 1 - \frac{1}{10^{0.4}} \times \left[\frac{3.27^{1.4} - 1}{1.4 \times (3.27 - 1)}\right]$$

$$= 1 - 0.398 \times 1.338 = 0.467 = 46.7\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\text{Power output} = \eta \times \text{total rate of heat added}$$

$$= 0.467 \times 66.5 = 31.1\ \text{kW}$$

$$\text{Power output/cylinder} = \frac{31.1}{4} = 7.76\ \text{kW}$$

$$\text{Work done/cylinder/cycle} = \frac{7.76}{25} = 0.3104\ \text{kJ}$$

$$p_m = \frac{W}{V_s} = \frac{0.3104 \times 1000}{300 \times 10^{-6}} = 10.35 \times 10^5\ \text{N/m}^2 \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

As discussed in the text, this problem illustrates that for the same compression ratio and heat input, Otto cycle is more efficient.

13.25 The compression ratio of an engine is 10 and the temperature and pressure at the start of compression is 37 °C and 1 bar. The compression and expansion processes are both isentropic and the heat is rejected at exhaust at constant volume. The amount of heat added during the cycle is 2730 kJ/kg. Determine the mean effective pressure and thermal efficiency of the cycle if (i) the maximum pressure is limited to 70 bar and heat is added at both constant volume and constant pressure and (ii) if all the heat is added at constant volume. In this case, how much additional work per kg of charge would be obtained if it were possible to expand isentropically the exhaust gases to their original pressure of 1 bar. Assume that the charge has the same physical properties as that of air.

Solution

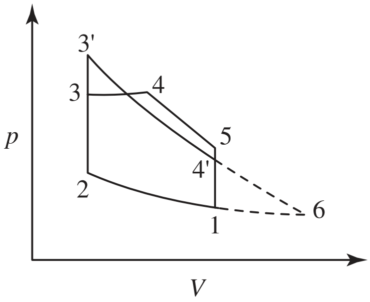


Fig. 13.16 Dual cycle

$$v_1 = \frac{V_1}{m} = \frac{RT_1}{p_1} = \frac{287 \times 310}{1 \times 10^5} = 0.89 \text{ m}^3/\text{kg}$$

Consider the process 1–2:

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1} = 310 \times 10^{0.4} = 778.7 \text{ K}$$

$$p_2 = p_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 25.12 \text{ bar}$$

Consider the limited pressure cycle (123451):

$$T_3 = T_2\frac{p_3}{p_2} = 778.7 \times \frac{70}{25.12} = 2170 \text{ K}$$

Heat supplied at constant volume = 0.717 × (2170 – 778.7) = 997.56 kJ/kg

Heat supplied at constant pressure = 2730 – 997.56 = 1732.44 kJ/kg

$$1732.4 = 1.004 \times (T_4 - 2170)$$

$$T_4 = 2170 + \frac{1732.4}{1.004} = 3895.5 \text{ K}$$

$$v_4 = v_3\frac{T_4}{T_3} = \frac{0.89}{10} \times \frac{3895.5}{2170} = 0.16 \text{ m}^3/\text{kg}$$

$$T_5 = T_4\left(\frac{v_4}{v_5}\right)^{\gamma-1} = 3895.5 \times \left(\frac{0.16}{0.89}\right)^{0.4} = 1961 \text{ K}$$

$$\text{Heat rejected} = c_V\,(T_5 - T_1) = 0.717 \times (1961 - 310) = 1184 \text{ kJ/kg}$$

$$\text{Work done} = \text{Heat supplied} - \text{Heat rejected} = 2730 - 1184 = 1546 \text{ kJ/kg}$$

$$\eta = \frac{w}{q} = \frac{1546}{2730} = 0.57 = 57\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$p_m = \frac{w}{v_1 - v_2} = \frac{w}{v_1\left(1 - \frac{1}{r}\right)} = \frac{1546 \times 10^3}{0.89 \times \left(1 - \frac{1}{10}\right)} = 19.32 \times 10^5 \text{ N/m}^2 = 19.32 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Consider the constant-volume cycle ($123'4'1$):

$$q_{2-3'} = 2730 \text{ kJ/kg}$$

$$2730 = 0.717 \times (T_{3'} - 778)$$

$$T_{3'} = 4586 \text{ K}$$

$$p_{3'} = T_{3'}\left(\frac{p_2}{T_2}\right) = 4586 \times \frac{25.12}{778.7} = 147.9 \text{ bar}$$

$$\frac{T_{3'}}{T_{4'}} = \left(\frac{V_{4'}}{V_{3'}}\right)^{\gamma-1}$$

$$T_{4'} = 4586 \times \left(\frac{1}{10}\right)^{0.4} = 1852.7$$

$$\text{Heat rejected} = 0.717 \times (1825.7 - 310) = 1087 \text{ kJ/kg}$$

$$w = 2730 - 1087 = 1643 \text{ kJ}$$

$$\eta = \frac{1643}{2730} = 0.602 = 60.2\% \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$p_m = \frac{1643 \times 10^3}{\left(1 - \frac{1}{10}\right) \times 0.89} = 20.53 \times 10^5 \text{ N/m}^2 = 20.53 \text{ bar} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

If the gases were expanded isentropically to their original pressure of 1 bar, then the temperature T_6 at the end of expansion would be:

$$T_6 = T_{3'}\left(\frac{p_6}{p_{3'}}\right)^{\dfrac{\gamma - 1}{\gamma}} = 4586 \times \left(\frac{1}{147.9}\right)^{\dfrac{0.4}{1.4}} = 1100.12$$

Heat rejected at constant pressure $= 1.004 \times (1100.12 - 310) = 793.3$ kJ/kg

$$\text{Work increase} = 1087 - 793.3 = 293.7 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Review Questions

13.1 What is the simplest way by which an IC engine cycle can be analysed? Do IC engines operate on a thermodynamic cycle?

13.2 What is the use of air-standard cycle analysis?

13.3 Mention the various assumption made in air-standard cycle analysis.

13.4 What is Carnot cycle and what is its importance? How is this cycle reversible?

13.5 Draw the Carnot cycle on p-V and T-s diagrams. Derive an expression for its efficiency. Comment on the significance of this result as it related to source and sink temperature.

13.6 Define mean effective pressure and comment its application in internal combustion engines.

13.7 Name the cycles which have the same efficiency of Carnot cycle. Are these cycles reversible in the sense the Carnot cycle is?

13.8 Draw the Stirling cycle on p-V and T-s diagrams and show how the cycle is reversible?

13.9 Derive the expression for Stirling cycle efficiency and show that the expression is same as that of Carnot cycle.

13.10 If you include the efficiency of the heat exchanger, show how the expression is modified.

13.11 Draw the p-V and T-s diagram of Ericsson cycle and show how it is made reversible.

13.12 Compare Carnot, Stirling and Ericsson cycles operating between the same source and sink temperatures and with equal changes in specific volume.

13.13 Draw the Otto cycle on p-V and T-s diagrams mark the various processes.

13.14 Derive an expression for the efficiency of Otto cycle and comment on the effect of compression ratio on the efficiency with respect of ratio of specific heats by means of a suitable graph.

13.15 Obtain an expression for mean effective pressure of an Otto cycle.

13.16 What is the basic difference between an Otto cycle and Diesel cycle? Derive the expression for the efficiency and mean effective pressure of the Diesel cycle.

13.17 Show that the efficiency of the Diesel cycle is lower than that of Otto cycle for the same compression ratio. Comment why the higher efficiency of the Otto cycle compared to Diesel cycle for the same compression ratio is only of a academic interest and not practical importance.

13.18 Compare the Otto cycle for the same peak pressure and temperature. Illustrate the cycles on p-V and T-s diagrams.

13.19 Draw the p-V and T-s diagrams of a Dual cycle. Why this cycle is also called limited pressure or mixed cycle?

13.20 Derive the expressions for the efficiency and mean effective pressure of a Dual cycle.

13.21 Compare Otto, Diesel and Dual cycles for the

- *(i) same compression ratio and heat input*
- *(ii) same maximum pressure and heat input*

(iii) same maximum pressure and temperature

(iv) same maximum pressure and work output

13.22 Sketch the Lenoir cycle on p-V and T-s diagrams and obtain an expression for its air-standard efficiency.

13.23 Compare the Otto cycle and Atkinson cycle. Derive the expression for the efficiency of Atkinson cycle.

13.24 Derive an expression for the air-standard efficiency of the Joule cycle in terms of

(i) compression ratio *(ii) pressure ratio.*

13.25 Where do the following cycles have applications?

(i) Atkinson cycle
(ii) Diesel cycle
(iii) Dual cycle
(iv) Ericsson cycle
(v) Joule cycle
(vi) Lenoir cycle
(vii) Otto cycle
(viii) Stirling cycle

Exercise

13.1 Assume working substance for a Carnot cycle to be air with $c_p = 1$ kJ/kg K and $c_V = 0.717$ kJ/kg K. Temperature at which heat is added is 2000 K and temperature at which heat is rejected is 300 K. The amount of heat added per kg of the working substance is 840 kJ/kg. Calculate for the cycle (i) the maximum pressure developed, (ii) the compression ratio assuming adiabatic compression and (iii) the efficiency of the cycle. The pressure at the beginning of isothermal compression is 1 bar.
Ans: (i) 3304.8 bar (ii) 114.75 (iii) 85%

13.2 An engine operates on Otto cycle between pressures 1 bar and 30 bar. The ratio of pressure at constant volume is 4. The temperature at the end of compression is 200 °C and the law of compression and expansion is $PV^{1.3}$ = constant. If the engine now operates on Carnot cycle for the same range of temperature, find the efficiency of the cycle. *Ans:* 84.3%

13.3 An Otto cycle engine having a clearance volume of 250 cc has a compression ratio of 8. The ratio of pressure rise at constant volume is 4. If the initial pressure is 1 bar, find the work done per cycle and the theoretical mean effective pressure. Take $\gamma = 1.4$.
Ans: (i) 1946.1 J/cycle (ii) 11.12 bar

13.4 Find the *mep* for the ideal air-standard Otto cycle having a maximum pressure of 40 bar and minimum pressure of 1 bar. The compression ratio is 5:1. Take $\gamma = 1.4$
Ans: 9.04 bar

13.5 An engine working on ideal Otto cycle, the ratio of temperature at the beginning of compression is 300 K. If the ideal air-standard efficiency = 0.5, calculate the compression ratio of the engine. If the peak temperature of the cycle is 1150 K, calculate the temperature when the piston is at BDC during expansion stroke. *Ans:* (i) 5.66 (ii) 575 K

13.6 A 2.7 litre cubic capacity, six-cylinder, four-stroke Otto engine has a compression ratio of 10. The engine develops 138 kW at 5000 rpm. Calculate (i) air-standard efficiency, (ii) the necessary rate of heat addition, (iii) the mean effective pressure of the cycle and (iv) the peak temperature and pressure of the cycle.
Ans: (i) 0.602 (ii) 229.23 kW (iii) 12.26 bar (iv) 2966 K (v) 98.8 bar

13.7 It is desired to increase the output of an SI engine working on ideal Otto cycle, either by

(i) increasing the compression ratio

(ii) increasing the inlet pressure from 1 to 1.5 bar

Which one will give higher peak pressure in the cycle and what is its value? Assume heat supplied at the constant volume process is the same in both the cases which is 420 kJ. Take $T_1 = 27°, p_1 = 1$ bar, $\gamma = 1.4$. Comment on the result.
Ans: (i) Increase in compression ratio will produce higher peak pressure.
(ii) 44.6 bar

13.8 A petrol engine working on Otto cycle has an maximum pressure of 50 bar. Heat supplied is 1000 kJ/kg. If the pressure ratio during compression 12.286, find the compression ratio and also the ratio of peak temperature to inlet temperature. Take $p_1 = 1$ bar and $T_1 = 27°$ C. *Ans:* (i) 6 (ii) 8.34

13.9 An Otto cycle takes in air at 300 K. The ratio of maximum to minimum temperature is 6. Find out the optimum compression ratio for the maximum work output of the cycle. *Ans:* 9.39

13.10 The pressure and temperature of a Diesel cycle at the start are 1 bar and 20 °C, respectively and the compression ratio is 14. The pressure at the end of expansion is 2.5 bar. Find the percentage of working stroke at which heat is supplied and heat supplied per kg of air. Assume $\gamma = 1.4$ and $c_p = 1.004$ kJ/kg K. *Ans:* (i) 7.11% (ii) 781.46 kJ/kg

13.11 An oil engine works on Diesel cycle, the compression ratio being 15. The temperature at the start of compression is 17 °C and 700 kJ of heat is supplied at constant pressure per kg of air and it attains a temperature of 417 °C at the end of adiabatic expansion. Find the air-standard efficiency of the cycle. What would be the theoretical work done per kg of air. Take $c_V = 0.717$ kJ/kg K and $\gamma = 1.4$. *Ans:* (i) 59.03% (ii) 413.20 kJ

13.12 An internal combustion engine works on Diesel cycle with a compression ratio of 8 and expansion ratio of 5. Calculate the air-standard efficiency. Assume $\gamma = 1.41$. *Ans:* 52.6%

13.13 A Diesel engine works on Diesel cycle with a compression ratio of 16 and cut-off ratio of 1.8. Calculate the thermal efficiency assuming $\gamma = 1.4$. *Ans:* 62.38%

13.14 An internal combustion engine works on Diesel cycle with a compression ratio of 14 and cut-off takes place at 10 % of the stroke. Find the ratio of cut-off and the air-standard efficiency. *Ans:* (i) 2.3 (ii) 57.8%

13.15 An ideal Diesel cycle operates on a pressure of 1 bar and a temperature of 27 °C at the beginning of compression and a pressure of 2 bar at the end of adiabatic expansion. Calculate the amount of heat required to be supplied per kg of air if the ideal thermal efficiency is taken as 60%. Take $c_V = 0.717$ kJ/kg K. *Ans:* 537.75 kJ/kg

13.16 The pressure and temperature of a Diesel cycle at the start are 1 bar and 17 °C. The pressure at the end of compression is 40 bar and that at the end of expansion is 2 bar. Find the air-standard efficiency. Assume $\gamma = 1.4$. *Ans:* 61.14%

13.17 A Diesel cycle operates at a pressure of 1 bar at the beginning of compression and the volume is compressed to $\frac{1}{15}$ of the initial volume. Heat is then supplied until the volume is twice that of the clearance volume. Determine the mean effective pressure. Take $\gamma = 1.4$. *Ans:* 6.69 bar

13.18 A semi-diesel engine works on dual combustion cycle. The pressure and temperature at the beginning of the compression is 1 bar and 27 °C, respectively and the compression ratio being 12. If the maximum pressure is 50 bar and heat received at constant pressure is for $\frac{1}{30}$th of the stroke, find the work done per kg of air and the thermal efficiency. Take $c_V = 0.717$ and $c_p = 1.004$. *Ans:* (i) 476.76 kJ/kg (ii) 61.5%

13.19 A compression-ignition engine has a compression ratio of 10 and $\frac{2}{3}$ of heat of combustion is liberated at constant volume and the remainder at constant pressure. The pressure and temperature at the beginning are 1 bar and 27 °C and the maximum pressure is 40 bar. Find the temperatures at the end of compression and expansion, if it follows the law $pV^{1.35}$ = constant, and $\gamma = 1.4$. *Ans:* (i) 398.6 °C (ii) 380.3 °C

13.20 A compression-ignition engine works on dual combustion cycle. The pressure and temperature at the beginning of compression are 1 bar and 27 °C, respectively and the pressure at the end of compression is 25 bar. If 420 kJ of heat is supplied per kg of air during constant volume heating and the pressure at the end of adiabatic expansion is found to be 3 bar, find the ideal thermal efficiency. Assume $c_p = 1.004$ kJ/kg K and $c_V = 0.717$ kJ/kg K. *Ans:* 41.36%

13.21 The cycle of an internal combustion engine with isochoric heat supply is performed with the compression ratio equal to 8. Find heat supplied to the cycle and the useful work, if the removed heat is 500 kJ/kg and the working fluid is air. *Ans:* (i) 1148.63 kJ/kg (ii) 648.63 kJ/kg

13.22 The initial parameters (at the beginning of compression) of the cycle of an internal combustion engine with isobaric heat supply are 0.1 MPa and 80 °C. The compression ratio is 16 and the heat supplied is 850 kJ/kg. Calculate the parameters at the characteristic points of the cycle and the thermal efficiency, if the working fluid is air. *Ans:* (i) $p_2 = 48.5$ bar; (ii) $p_3 = 48.5$ bar; (iii) $p_4 = 2.26$ bar; (iv) $T_2 = 1070.1$ K; (v) $T_3 = 1916$ K; (vi) $T_4 = 797.73$ K; (vii) $\eta_{th} = 62.5\%$

13.23 The pressure ratio $\lambda = 1.5$ in the process of isochoric heat supply for the cycle of an internal combustion engine with a mixed supply of heat = 1034 kJ/kg and the compression ratio = 13. Find the thermal efficiency and temperature at the characteristic points of the cycle if the initial parameters are 0.09 MPa and 70 °C and the working substance is air. *Ans:* (i) $\eta_{th} = 57.5\%$; (ii) $T_2 = 956.9$ K; (iii) $T_3 = 1435.5$ K; (iv) $T_4 = 2122.86$ K; (v) $T_5 = 890$ K

13.24 The parameters of the initial state of 1 kg of air in the cycle of an internal combustion engine are 0.095 MPa and 65 °C. The compression ratio is 11. Compare the values of the thermal efficiency for isobaric and isochoric heat supply in amounts of 800 kJ, assuming that $k = 1.4$. *Ans:* $\eta_{t_p} = 55.7\%$, $\eta_{t_v} = 61.7\%$

13.25 A gas turbine unit works on an air-standard Brayton cycle. The pressure ratio across the compression is 6. Air enters the compressor at 1 bar and 27 °C. The maximum temperature of the cycle is 850 °C. Calculate the specific output of the cycle. What will be the power developed by the unit for a mass flow rate of 10 kg/s. Would you recommend this cycle for a reciprocating engine? For air $\gamma = 1.4$ and $c_p = 1.005$ kJ/kg K.

Ans: (i) 250.64 kJ/kg; (ii) 2506.4 kW; (iii) The volume of the cylinder will be too large due to high specific volume at state 4, therefore, this cycle is not recommended for reciprocating engine.

Multiple Choice Questions (choose the most appropriate answer)

1. The efficiency of Carnot engine is 0.75. If the cycle is reversed, its coefficient of performance as heat refrigerator is
   (a) 0.25
   (b) 0.33
   (c) 1.33
   (d) 4.0

2. A perfect engine works on the Carnot cycle between 727 °C and 227°C. The efficiency of the engine is
   (a) 0.5
   (b) 2
   (c) $\dfrac{227}{727}$
   (d) $\dfrac{500}{727}$

3. Efficiency of stirling cycle is same as
   (a) Otto cycle
   (b) Diesel cycle
   (c) Carnot cycle
   (d) Ericsson cycle

4. The air-standard efficiency of Otto cycle is
   (a) $\eta = 1 - r^{\gamma - 1}$
   (b) $\eta = 1 - \frac{1}{r^{\gamma - 1}}$
   (c) $\eta = 1 - r^{\frac{\gamma - 1}{\gamma}}$
   (d) $\eta = 1 - \frac{1}{r^{\frac{\gamma - 1}{\gamma}}}$

5. The air-standard Otto cycle consists of
   (a) two constant volume and two isentropic processes
   (b) two constant pressure and two isentropic processes
   (c) two constant pressure and two constant volume processes
   (d) none of the above

6. In air-standard Diesel cycle at fixed r and fixed γ:
   (a) $\eta_{thermal}$ increases with increase in heat addition and cut-off ratio
   (b) $\eta_{thermal}$ decreases with increase in heat addition and cut-off ratio
   (c) $\eta_{thermal}$ remains the same with increase in heat addition and cut-off ratio
   (d) none of the above

7. Mean effective pressure of Otto cycle
   (a) inversely proportional to pressure ratio

(b) directly proportional to pressure ratio

(c) does not depend on pressure ratio

(d) proportional to square root of pressure ratio

8. For a given compression ratio, the work output of Otto cycle is

(a) increases with increase in r
(b) decreases with increase in r
(c) is not affected
(d) none of the above

9. For a given value of r, efficiency of Otto cycle

(a) decreases with compression ratio
(b) increases with compression ratio
(c) is not affected
(d) none of the above

10. For dual combustion cycle, for fixed value of heat addition and compression ratio

(a) *mep* will be greater with increase in r_p and decrease in r_c

(b) *mep* will be greater with decrease in r_p and decrease in r_c

(c) *mep* remain the same with increase in r_p and decrease in r_c

(d) none of the above

11. The normal range of compression ratio for Otto cycle is

(a) 6–10
(b) 2–4
(c) > 10
(d) none of the above

12. The normal range of compression ratio for Diesel cycle is

(a) 4–6
(b) 6–8
(c) 15–20
(d) >25

13. For the same compression ratio and heat addition

(a) $\eta_{Otto} > \eta_{Diesel} > \eta_{Dual}$
(b) $\eta_{Diesel} > \eta_{Otto} > \eta_{Dual}$
(c) $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$
(d) $\eta_{Dual} > \eta_{Diesel} > \eta_{Otto}$

14. For the same compression ratio and heat rejection

(a) $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$
(b) $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$
(c) $\eta_{Dual} > \eta_{Diesel} > \eta_{Otto}$
(d) $\eta_{Dual} > \eta_{Otto} > \eta_{Diesel}$

15. When the engines are built to withstand the same thermal and mechanical stresses

(a) $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$
(b) $\eta_{Dual} > \eta_{Diesel} > \eta_{Otto}$
(c) $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$
(d) $\eta_{Otto} > \eta_{Diesel} > \eta_{Dual}$

16. For the same peak pressure and heat input

(a) $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$
(b) $\eta_{Otto} > \eta_{Diesel} > \eta_{Dual}$
(c) $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$
(d) $\eta_{Diesel} > \eta_{Otto} > \eta_{Dual}$

17. For the same peak pressure and work output

(a) $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$
(b) $\eta_{Otto} > \eta_{Diesel} > \eta_{Dual}$
(c) $\eta_{Diesel} > \eta_{Otto} > \eta_{Dual}$
(d) $\eta_{Diesel} > \eta_{Dual} > \eta_{Otto}$

18. Lenoir cycle is used in

(a) SI engines
(b) CI engines
(c) pulse jet engines
(d) gas turbines

19. A Brayton cycle consists of

(a) two constant volume and two constant pressure processes
(b) two constant volume and two isentropic processes
(c) one constant pressure, one constant volume and two isentropic processes
(d) none of the above

20. Brayton cycle is used in

(a) Ramjet engines
(b) gas turbines
(c) pulse jet engines
(d) CI engines
(e) SI engines

21. Thermal efficiency of a gas turbine plant as compared to Diesel engine plant is

(a) higher
(b) lower
(c) same
(d) may be higher or lower

22. Mechanical efficiency of a gas turbine as compared to internal combustion reciprocating engine is

(a) lower
(b) higher
(c) same
(d) unpredictable

23. For a gas turbine, the pressure ratio may be in the range

(a) 2–3
(b) 3–5
(c) 16–18
(d) 18–22

24. With the increase in pressure ratio, thermal efficiency of a simple gas turbine plant with fixed turbine inlet temperature

(a) decreases
(b) increases
(c) first increases and then decreases
(d) first decreases and then increases

25. Two reversible isothermal processes and two reversible isobaric processes are carried out in

(a) Rankine cycle
(b) Carnot cycle
(c) Stirling cycle
(d) Ericsson cycle

Ans: 1. – (b) 2. – (a) 3. – (c) 4. – (b) 5. – (a)
6. – (b) 7. – (b) 8. – (a) 9. – (b) 10. – (a)
11. – (a) 12. – (c) 13. – (c) 14. – (a) 15. – (a)
16. – (c) 17. – (d) 18. – (c) 19. – (c) 20. – (b)
21. – (b) 22. – (b) 23. – (d) 24. – (b) 25. – (d)

14

THERMODYNAMIC RELATIONS

Thermodynamic properties are important to evaluate engineering problems. We are already aware of various important thermodynamic properties, viz

(i) pressure (p)
(ii) temperature (T)
(iii) volume (v)
(iv) internal energy (U)
(v) enthalpy (h)
(vi) entropy (s)
(vii) Helmholz function (a)
(viii) Gibbs function (g)

Out of the above eight properties only three, viz pressure, volume and temperature are measurable. The other five are derived from the three measurable properties.

In this chapter, we will develop inter-relation between them for a system of constant chemical composition. We will derive relations in terms of change in these properties. They are applicable to both flow and non-flow processes. Before going into the derivation, we should understand three basic mathematical theorems.

14.1 THREE BASIC MATHEMATICAL THEOREMS

Theorem 1: *If a relation exists among the variables say, x, y and z, then z may be expressed as a function of x and y.*

For example, z can be expressed as: $z = f(x, y)$ (14.1)

then according to a partial derivation rule, the differential function of two variables is equal to sum of the partial derivatives, by changing one variable at a time, keeping the other one constant. Now, applying the partial derivation to Eq.14.1:

$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx + \left(\frac{\partial z}{\partial y}\right)_x dy \tag{14.2}$$

Let us denote $\left(\frac{\partial z}{\partial x}\right)_y = M$ and $\left(\frac{\partial z}{\partial y}\right)_x = N$ (14.3)

Now, we can write Eq.14.2 as $dz = M dx + N dy$ (14.4)

Note that M and N are the functions of x and y, respectively. Let us differentiate M partially with respect to y and N with respect to x in Eq.14.3. Now, we can write:

$$\left(\frac{\partial M}{\partial y}\right)_x = \frac{\partial^2 z}{\partial x.\, \partial y} \tag{14.5}$$

and
$$\left(\frac{\partial N}{\partial x}\right)_y = \frac{\partial^2 z}{\partial y .\, \partial x} \tag{14.6}$$

This means
$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \tag{14.7}$$

This is the condition for exact or perfect differential.

Theorem 2: *If a quantity f is a function of the variables say, x, y and z and a relation exists among the variables say, x, y and z, then f is a function of any two of x, y and z. Similarly, x, y and z may be regarded as a function of f. Thus, if*

$$x = x\,(f, y) \tag{14.8}$$

$$dx = \left(\frac{\partial x}{\partial f}\right)_y df \; + \left(\frac{\partial x}{\partial y}\right)_f dy \tag{14.9}$$

Similarly, if
$$y = y\,(f, z) \tag{14.10}$$

$$dy = \left(\frac{\partial y}{\partial f}\right)_x df \; + \left(\frac{\partial y}{\partial z}\right)_f dz \tag{14.11}$$

Substituting the expression of dy in the preceding equation:

$$dx = \left(\frac{\partial x}{\partial f}\right)_y df \; + \left(\frac{\partial x}{\partial y}\right)_f \left[\left(\frac{\partial y}{\partial f}\right)_z df \; + \left(\frac{\partial y}{\partial z}\right)_f dz\right] \tag{14.12}$$

$$= \left[\left(\frac{\partial x}{\partial f}\right)_y + \left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial f}\right)_z\right] df + \left[\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f\right] dz$$

Again,
$$dx = \left(\frac{\partial x}{\partial f}\right)_z df \; + \left(\frac{\partial x}{\partial z}\right)_f dz \tag{14.13}$$

$$\left(\frac{\partial x}{\partial z}\right)_f = \left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \tag{14.14}$$

$$\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1 \tag{14.15}$$

Theorem 3: *Among the variables say, x, y and z any one variable may be considered as a function of the other two. Thus, if*

$$x = x\,(y, z) \tag{14.16}$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy \; + \left(\frac{\partial x}{\partial z}\right)_y dz \tag{14.17}$$

Similarly,
$$dz = \left(\frac{\partial z}{\partial x}\right)_y dx \; + \left(\frac{\partial x}{\partial y}\right)_x dy \tag{14.18}$$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy \;+ \left(\frac{\partial x}{\partial z}\right)_y \left[\left(\frac{\partial z}{\partial x}\right)_y dx \;+ \left(\frac{\partial z}{\partial y}\right)_x dy\right] \tag{14.19}$$

$$= \left[\left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x\right] dy + \left[\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial x}\right)_y\right] dx \tag{14.20}$$

$$dx = \left[\left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x\right] dy + dx \tag{14.21}$$

Cancelling dx on both sides and as $dy \neq 0$, we have

$$\left(\frac{\partial x}{\partial y}\right)_z + \left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x = 0 \tag{14.22}$$

$$\left(\frac{\partial x}{\partial z}\right)_y \left(\frac{\partial z}{\partial y}\right)_x = -\left(\frac{\partial x}{\partial y}\right)_z \tag{14.23}$$

Taking inverse on both sides,

$$\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial x}{\partial y}\right)_z = -1 \tag{14.24}$$

By rearranging

$$\left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial x}{\partial y}\right)_z = -1 \tag{14.25}$$

Coming to the three thermodynamic variables p, V and T, the following relations holds good

$$\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1 \tag{14.26}$$

14.2 MAXWELL RELATION

A pure substance with single phase has only two independent variables. Of the eight properties, we mentioned in the beginning of this chapter any one may be expressed as a function of any of the other two. For a pure substance undergoing an infinitesimal reversible process we have already derived the following equations:

$$dU = TdS - pdV \tag{14.27}$$

$$dH = dU + pdV + Vdp = Tds + Vdp \tag{14.28}$$

$$dF = dU - TdS = -pdV - SdT \tag{14.29}$$

$$dG = dH - Tds = Vdp - SdT \tag{14.30}$$

Since U, H, F and G are thermodynamic properties their exact differential will be of the general form:

$$dz = Mdx + Ndy \tag{14.31}$$

$$\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y \tag{14.32}$$

Applying the above equation we can write:

$$\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial p}{\partial S}\right)_V \tag{14.33}$$

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p \tag{14.34}$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \tag{14.35}$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p \tag{14.36}$$

The above four relations, Eq.14.33–14.36 are known as Maxwell relations. Maxwell relations are quite important, for two reasons:

(i) They show us that derivatives of thermodynamic parameters are not completely independent. This can serve as a consistency check in both experiments and in pen-and-paper analysis.

(ii) They provide a method to express derivatives involving difficult-to-measure quantities in terms of ones that are readily accessible experimentally.

14.3 MNEMONIC DIAGRAM

We have derived the differential expression for the thermodynamic properties U, H, A and G. Further, the above Maxwell relations can be remembered conveniently using a thermodynamic mnemonic diagram. As can be seen in Fig.14.1, the diagram consists of a square with two diagonal arrows pointing upwards. The sides of the square are labelled with the thermodynamic properties A, G, H and U. Starting from the top, the properties are placed in alphabetical order clockwise. The independent variables of the thermodynamic properties are placed in the corners such that each property is flanked by its own independent variables.

The differential expression for the property can be expressed in terms of differentials of its independent variables. The coefficients associated with the independent variables are indicated by the diagonal arrow. The sign associated with the coefficient is positive if the arrow is pointing away from the independent variable. The sign associated with the coefficient is negative if the arrow is taken as pointing towards the independent variable. Thus,

$$dU = (\text{sign}) \times (\text{coeff}) ds + (\text{sign}) \times (\text{coeff}) dV$$

$$dU = (\text{sign}) TdS + (\text{sign}) pdV$$

$$dU = +TdS - pdV \quad \text{or} \quad dU = TdS - pdV \tag{14.37}$$

Similarly, for the property G we can write:

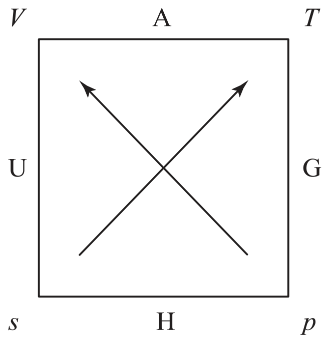


Fig. 14.1 Thermodynamic mnemonic diagram

$$\begin{aligned} dU &= (\text{sign})dT + (\text{sign})dp \\ dg &= (\text{sign}) + (\text{sign})Vdp \\ dg &= -SdT + Vdp \end{aligned} \tag{14.38}$$

while writing the Maxwell relations, we need to concentrate on the direction of the arrows and the independent variables only. By observing Fig.14.1, we can write one of the Maxwell relation as:

$$\left(\frac{\partial V}{\partial S}\right)_p = \left(\frac{\partial T}{\partial p}\right)_S \quad (\text{ refer Eq.14.34}) \tag{14.39}$$

Since both the arrows are pointing in the same direction (upwards), there is no need to change the sign. In case, if one arrow points upwards and the other arrow points downwards, the equation would carry a negative sign. To write the next Maxwell relation, *rotate the diagram clockwise by 90 °C* as shown in Fig.14.2:

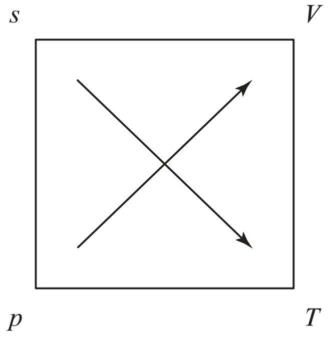


Fig. 14.2 Rotated thermodynamic mnemonic diagram

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p \tag{14.40}$$

Similarly, the other two Maxwell relations also can be written.

14.3.1 Coefficient of Volume Expansion

It is defined as the change in volume with change in temperature per unit volume keeping the pressure constant. It is denoted by β. Mathematically,

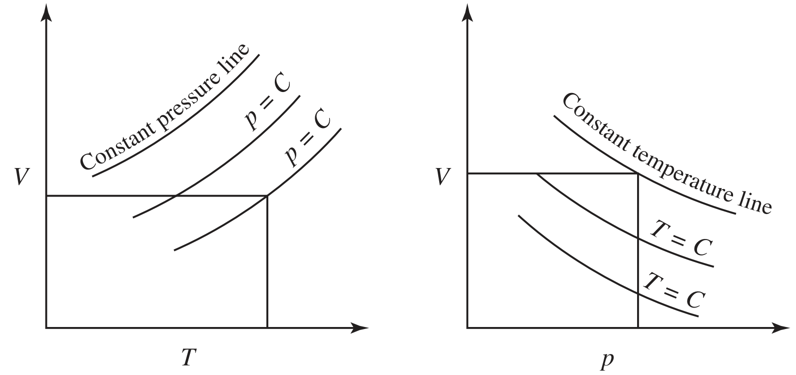
$$\beta = \frac{1}{v}\left(\frac{\partial V}{\partial T}\right)_p \tag{14.41}$$

14.3.2 Isothermal Compressibility

It is defined as the change in volume with the change in pressure per unit volume keeping the temperature constant. It is denoted by K:

$$K = -\frac{1}{v}\left(\frac{\partial V}{\partial p}\right)_T \tag{14.42}$$

The negative sign in the equation indicates that the slope is decreasing.



(a) Coefficient of volume expansion (b) Isothermal compression

Fig. 14.3 Isothermal compressibility

14.3.3 Ratio of Coefficient of Volume Expansion (P) and Isothermal Compressibility (K)

From the coefficient of volume expansion given by Eq.14.41, we can write:

$$\left(\frac{\partial V}{\partial T}\right)_p = \beta v \tag{14.43}$$

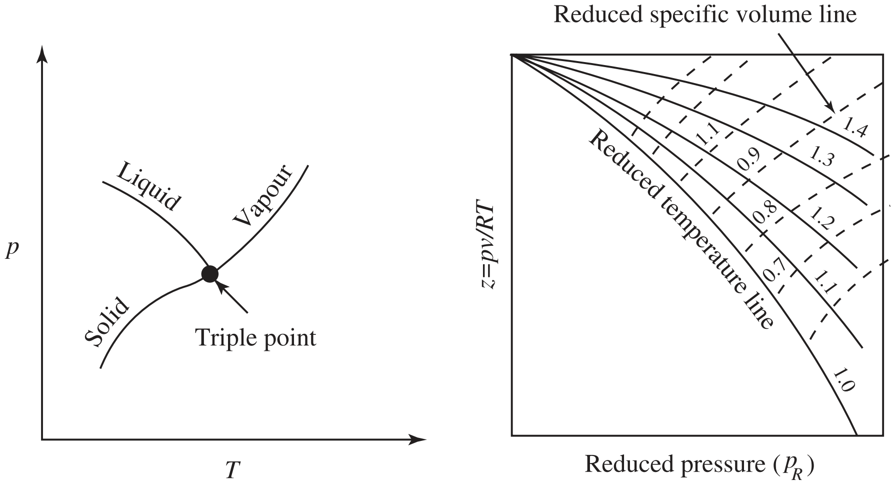
From the isothermal compressibility given by Eq.14.42, we can write:

$$\left(\frac{\partial V}{\partial p}\right)_T = -Kv \tag{14.44}$$

We have already shown that (refer Eq.14.26):

$$\left(\frac{\partial p}{\partial V}\right)_T\left(\frac{\partial V}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_V = -1 \tag{14.45}$$

The inverse is also true which means:



(a) Ratio of coefficient of volume expansion (b) Ratio of isothermal compressibility

Fig. 14.4 Ratio of coefficient of volume expansion (β) *and isothermal compressibility* (K)

$$\left(\frac{\partial V}{\partial p}\right)_T \left(\frac{\partial T}{\partial V}\right)_p \left(\frac{\partial p}{\partial T}\right)_V = -1 \tag{14.46}$$

Substituting from Eqs 14.43 and 14.44:

$$(-Kv)\left(\frac{1}{\beta v}\right)\left(\frac{\partial p}{\partial T}\right)_V = -1 \tag{14.47}$$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{\beta}{K} \tag{14.48}$$

14.4 ENTROPY RELATIONS (Tds EQUATIONS)

We know that entropy is a function of temperature and specific volume. Mathematically, it can be expressed as

$$s = f(T, v) \tag{14.49}$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \tag{14.50}$$

Multiplying by T on both sides,

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_v dT + T\left(\frac{\partial s}{\partial v}\right)_T dv \tag{14.51}$$

Now, c_V can be expressed as

$$c_V = T\left(\frac{\partial s}{\partial T}\right)_V \tag{14.52}$$

From Maxwell equations, we have

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v \tag{14.53}$$

Substituting Eq.14.52 and Eq.14.53 in Tds Eq.14.51

$$Tds = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_T dv \tag{14.54}$$

This is known as first Tds equation.

Now, consider $s = f\,(T, p)$,

$$ds = \left(\frac{\partial s}{\partial T}\right)_p dT + \left(\frac{\partial s}{\partial p}\right)_T dp \tag{14.55}$$

Multiplying by T on both sides,

$$Tds = T\left(\frac{\partial s}{\partial T}\right)_p dT + T\left(\frac{\partial s}{\partial p}\right)_T dp \tag{14.56}$$

Now, c_p can be expressed as

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \tag{14.57}$$

From Maxwell equations, we have

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p \tag{14.58}$$

$$Tds = c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp \tag{14.59}$$

This is known as second Tds equation.

14.5 EQUATION FOR SPECIFIC HEAT CAPACITIES

There are two specific heat capacities, viz one at constant pressure, c_p and other at constant volume, c_V. Mathematically, it is given by

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \tag{14.60}$$

$$c_V = \left(\frac{\partial u}{\partial T}\right)_v \tag{14.61}$$

From Eq.14.60,

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = \left(\frac{\partial h}{\partial s}\right)_p \left(\frac{\partial s}{\partial T}\right)_p \tag{14.62}$$

As $\left(\frac{\partial h}{\partial s}\right)_p = T$
$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \tag{14.63}$$

Now,
$$c_V = \left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial u}{\partial s}\right)_v\left(\frac{\partial s}{\partial T}\right)_v \tag{14.64}$$

As $\left(\frac{\partial u}{\partial s}\right)_p = T$
$$c_V = T\left(\frac{\partial s}{\partial T}\right)_v \tag{14.65}$$

Now,
$$c_p - c_V = T\left[\left(\frac{\partial s}{\partial T}\right)_p - \left(\frac{\partial s}{\partial T}\right)_v\right] \tag{14.66}$$

Now, consider entropy as a function of temperature T and specific volume, v

$$s = f(T, v) \tag{14.67}$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \tag{14.68}$$

Dividing by dT on both sides
$$\left(\frac{\partial s}{\partial T}\right)_p = \left(\frac{\partial s}{\partial T}\right)_v + \left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial v}{\partial T}\right)_p \tag{14.69}$$

$$\left(\frac{\partial s}{\partial T}\right)_p - \left(\frac{\partial s}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T\left(\frac{\partial v}{\partial T}\right)_p \tag{14.70}$$

Substituting Eq.14.70 in Eq.14.69,

$$c_p - c_V = T\left[\left(\frac{\partial s}{\partial v}\right)_T - \left(\frac{\partial v}{\partial T}\right)_p\right] \tag{14.71}$$

From Maxwell relation, we have,
$$\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T \tag{14.72}$$

Now,
$$c_p - c_V = T\left[\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial v}{\partial T}\right)_p\right] \tag{14.73}$$

Also
$$\left(\frac{\partial v}{\partial p}\right)_T\left(\frac{\partial T}{\partial v}\right)_p\left(\frac{\partial p}{\partial T}\right)_V = -1 \tag{14.74}$$

$$c_p - c_V = -T\left(\frac{\partial v}{\partial T}\right)_T^2\left(\frac{\partial p}{\partial v}\right)_T \tag{14.75}$$

Equation 14.75 is a very important equation in thermodynamics and the following facts can be deduced from the same:

(i) Since $\left(\frac{\partial v}{\partial T}\right)_p^2$ is always positive and $\left(\frac{\partial p}{\partial v}\right)$ for any substance is negative, $(c_p - c_V)$ is always positive. This means c_p is always greater than c_V.

(ii) As $T \to 0$ K, $c_p \to c_V$ or at absolute zero $c_p = c_V$.

(iii) When $\left(\dfrac{\partial v}{\partial T}\right) = 0$, for example, for water at 4 °C when density is maximum or specific volume is minimum $c_p = c_V$.

From Eq.14.73 we can determine the difference between the two specific heats c_p and c_V in terms of measurable properties p, v and T. We know that,

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{\beta}{K} \tag{14.76}$$

and

$$\left(\frac{\partial V}{\partial T}\right)_p = \beta v \tag{14.77}$$

Substituting $\left(\dfrac{\partial p}{\partial T}\right)_V$ and $\left(\dfrac{\partial V}{\partial T}\right)_p$ values in Eq.14.73,

$$c_p - c_V = T\left[\left(\frac{\beta}{K}\right)_v \beta v\right] \tag{14.78}$$

$$c_p - c_V = \frac{T\beta^2 v}{K} \tag{14.79}$$

14.6 RATIO OF SPECIFIC HEAT CAPACITIES

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p \tag{14.80}$$

$$c_V = T\left(\frac{\partial s}{\partial T}\right)_v \tag{14.81}$$

Dividing Eq.14.80 by Eq.14.81,

$$\gamma = \frac{c_p}{c_V} = \frac{T\left(\frac{\partial s}{\partial T}\right)_p}{T\left(\frac{\partial s}{\partial T}\right)_v} = \frac{\left(\frac{\partial s}{\partial T}\right)_p}{\left(\frac{\partial s}{\partial T}\right)_v} = \left(\frac{\partial s}{\partial T}\right)_p \left(\frac{\partial T}{\partial S}\right)_v \tag{14.82}$$

14.7 INTERNAL ENERGY RELATIONS

The internal energy is expressed as

$$du = Tds - pdv \tag{14.83}$$

We know that entropy is a function of temperature and specific volume. Mathematically, it can be expressed as

$$s = f\,(T, v) \tag{14.84}$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \tag{14.85}$$

Substituting ds from the above equation in Eq.14.83, c_V can be expressed as

$$c_V = T\left(\frac{\partial s}{\partial T}\right)_v \tag{14.86}$$

From Maxwell equations, we have

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial V}\right)_T \tag{14.87}$$

Sustituting in Eq.14.86, we have

$$du = c_V\ dT +\ T\left(\frac{\partial p}{\partial T}\right)_V dv - pdv \tag{14.88}$$

From Eq.14.88 we can determine the internal energy.

$$du = T\left[\left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv\right] - pdv \tag{14.89}$$

in terms of measurable properties such as p, v, T and C.

14.8 ENTHALPY RELATIONS

From the first law of thermodynamics we have

$$dh = Tds + vdp$$

From entropy relations,

$$ds = c_p\left(\frac{dT}{T}\right) - \left(\frac{\partial v}{\partial T}\right)_p dp$$

Substituting ds in dh equation,

$$dh = T\left[c_p\left(\frac{dT}{T}\right) - \left(\frac{\partial v}{\partial T}\right)_p dp\right] + vdp$$

$$dh = \left[c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp\right] + vdp \tag{14.90}$$

From Eq.14.90, the enthalpy in terms of measurable properties such as p, v and T can be calculated.

14.9 ENERGY EQUATION

For a system undergoing an infinitesimal reversible process between two equilibrium states, the change in internal energy is given by

$$dU = Tds - pdV$$

Substituting the first Tds equation, we get

$$dU = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV - pdV \tag{14.91}$$

$$= c_V dT + T\left[\left(\frac{\partial p}{\partial T}\right)_V - p\right] dV \tag{14.92}$$

If $U = U(T, V)$

$$dU = \left[\left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV\right] \tag{14.93}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p \tag{14.94}$$

The above equation is known as the energy equation. We will discuss two application of this equation:

For an ideal gas, $p = \dfrac{n\overline{R}T}{V}$

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{n\overline{R}}{V} = \frac{p}{T} \tag{14.95}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\frac{p}{T} - p = 0 \tag{14.96}$$

U does not change when V changes at constant temperature

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial U}\right)_T = 1 \tag{14.97}$$

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T - \left(\frac{\partial U}{\partial V}\right)_T = 0 \tag{14.98}$$

Since $\left(\dfrac{\partial p}{\partial V}\right)_T \neq 0$;

$$\left(\frac{\partial U}{\partial p}\right)_T = 0 \tag{14.99}$$

When $\left(\dfrac{\partial U}{\partial p}\right)_T = 0$ in Eq.14.98,

$$\left(\frac{\partial U}{\partial V}\right)_T = 0 \tag{14.100}$$

From Eq. 14.99 and Eq. 14.100, it is clear that U does not change either when p or v changes at $T = C$. This means that U of an ideal gas is a function of temperature only. Another important point to note is that in Eq.14.92, for an ideal gas, $pV = nRT$ and,

$$T\left(\frac{\partial p}{\partial T}\right)_v - p = 0 \tag{14.101}$$

Therefore, $dU = c_V dT$ holds good for an ideal gas in any process (even when the volume changes). But for any other substance $dU = c_V dT$ is true only when the volume is constant, i.e. $dV = 0$. Similarly, $dH = TdS + Vdp$.

$$TdS = c_p dT - T\left(\frac{\partial V}{\partial T}\right)_p dp \tag{14.102}$$

$$dH = c_p dT + \left[V - \ T\left(\frac{\partial V}{\partial T}\right)_p\right] dp \tag{14.103}$$

Expressing $c_p dT = dH$, the above equation reduces to

$$dH = dH + \left[V - \ T\left(\frac{\partial V}{\partial T}\right)_p\right] dp \tag{14.104}$$

$$V - T\left(\frac{\partial V}{\partial T}\right)_p = 0 \tag{14.105}$$

Similar to internal energy, we can prove that enthalpy of an ideal gas is *not* a function of either volume or pressure. This means,

$$\left(\frac{\partial H}{\partial p}\right)_T = 0 \tag{14.106}$$

and

$$\left(\frac{\partial H}{\partial V}\right)_T = 0 \tag{14.107}$$

Since for an ideal gas $pV = nRT$ and $V - T\left(\frac{\partial V}{\partial T}\right)_p = 0$, the relation $dH = c_p dT$ for any process even where there is a pressure change. However, for any other substance, the relation $dH = c_p dT$ holds good only when the pressure remains constant, i.e. $p = C$ or $dp = 0$.

14.10 STEFAN-BOLTZMAN LAW

When an enclosed system is in equilibrium with thermal radiation, its energy content depends only on volume and temperature. The energy density, u, defined as the ratio of energy to volume, is a function of temperature only. Mathematically, it can be written as:

$$u = \frac{U}{V} = f(T) \tag{14.108}$$

As per the electromagnetic theory of radiation, the radiation is equivalent to a photon gas which exerts pressure. The pressure exerted by the black-body radiation in an enclosure is given by,

$$p = \frac{u}{3} \tag{14.109}$$

Black-body is thus specified by pressure, volume and temperature of the radiation.

$$U = uV \quad \text{and} \quad p = \frac{u}{3} \tag{14.110}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = u \quad \text{and} \quad \left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3}\frac{du}{dT} \tag{14.111}$$

By substituting in the energy Eq.14.92:

$$u = \frac{T}{3}\frac{du}{dT} - \frac{u}{3} \tag{14.112}$$

$$\frac{du}{u} = 4\frac{dT}{T} \tag{14.113}$$

$$\ln u = \ln T^4 + \ln b \tag{14.114}$$

$$u = bT^4 \tag{14.115}$$

where b is a constant. The above equation is called *Stefan-Boltzman law.* Since,

$$U = uV = VbT^4 \tag{14.116}$$

$$\left(\frac{\partial U}{\partial T}\right)_V = c_V = 4VbT^3 \tag{14.117}$$

and

$$\left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3}\frac{du}{dT} = \frac{4}{3}VbT^3 \tag{14.118}$$

From the first Tds equation:

$$Tds = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dV \tag{14.119}$$

$$= 4VbT^3 dT + \frac{4}{3}bT^4 dV \tag{14.120}$$

For a reversible isothermal change of volume, the heat to be supplied reversibly to keep temperature constant:

$$Q = \frac{4}{3}bT^4 \Delta V \tag{14.121}$$

For a reversible adiabatic change of volume:

$$\frac{4}{3}bT^4 dV = -4VbT^3 dT \tag{14.122}$$

$$\frac{dV}{V} = -3\frac{dT}{T} \tag{14.123}$$

or

$$VT^3 = \text{Constant} \tag{14.124}$$

If the temperature is one-half the original temperature, the volume of the black-body radiation is to be increased adiabatically eight times its original volume so that the radiation remains in equilibrium with matter at that temperature.

14.11 CLAPEYRON EQUATION

Clapeyron equation is mainly used when multi-phases are involved. It provides the relationship between the saturation pressure, saturation temperature, the enthalpy of evaporation and the specific volume of at least the two phases involved. This equation provides a basis for calculation of properties in a two-phase region. It gives the slope of a curve separating the two phases in the $p - T$ diagram.

Let entropy (s) be a function of temperature (T) and volume (v). Mathematically,

$$s = f(T, v) \tag{14.125}$$

$$ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv \tag{14.126}$$

We know that when a phase-change takes place from saturated liquid to saturated vapour, the temperature remains constant. Therefore, in Eq.14.125, the first term is zero. Therefore,

$$ds = \left(\frac{\partial s}{\partial v}\right)_T dv \tag{14.127}$$

$$\frac{ds}{dv} = \left(\frac{\partial s}{\partial v}\right)_T \tag{14.128}$$

From Maxwell relation, we have

$$\left(\frac{\partial S}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V \tag{14.129}$$

Substituting Eq.14.128 in Eq.14.129,

$$\left(\frac{ds}{dv}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v \tag{14.130}$$

or

$$ds = \left(\frac{\partial p}{\partial T}\right)_v dv \tag{14.131}$$

The term, $\left(\frac{\partial p}{\partial T}\right)$, is nothing but the slope of the of the saturation curve. Integrating the above equation between the saturated liquid (f) and the saturated vapour (g):

$$\int_{s_f}^{s_g} ds = \left(\frac{dp}{dT}\right) \int_{v_f}^{v_g} dv \tag{14.132}$$

$$s_{s_f}^{s_g} = \frac{dp}{dT}[v]_{v_f}^{v_g} \tag{14.133}$$

$$s_g - s_f = \frac{dp}{dT}(v_g - v_f) \tag{14.134}$$

$$\frac{dp}{dT} = \frac{[s_g - s_f]}{[v_g - v_f]} = \frac{s_{fg}}{v_{fg}} \tag{14.135}$$

Since $s_{fg} = [s_g - \ s_f]$ and $v_{fg} = [v_g - v_f]$. From the second law of thermodynamics, we have

$$ds = \frac{dQ}{T} \tag{14.136}$$

For a constant pressure process:

$$dQ = dh \tag{14.137}$$

$$ds = \frac{dh}{T} \tag{14.138}$$

$$s_{fg} = \frac{h_{fg}}{T} \tag{14.139}$$

$$\frac{dp}{dT} = \frac{[s_g - \ s_f]}{[v_g - \ v_f]} = \frac{s_{fg}}{v_{fg}} = \frac{h_{fg}}{Tv_{fg}} \tag{14.140}$$

The above equation is known as *Claperon equation.*

14.12 JOULE-THOMSON COEFFICIENT

Thermodynamically, the Joule-Thomson coefficient is defined as change in temperature in a fluid caused by a unitary pressure drop during an the is enthalpic (constant enthalpy) process. Mathematically,

$$\mu = \left(\frac{\partial T}{\partial p}\right)_H \tag{14.141}$$

Using thermodynamic relationships, alternative expressions can also be written. An interesting observation is that the Joule-Thompson coefficient of an ideal gas is *identically equal to zero.* However, real fluids take positive or negative Joule-Thompson values.

14.12.1 Throttling Process

Throttling process is defined as the process when the fluid expansion takes place through a minute orifice or slightly opened valve. During the throttling process, pressure and velocity are reduced. However, there is no heat transfer and no work done by the system. In this process, *enthalpy remains constant.*

14.12.2 Joule-Thomson Experiment

Figure 14.5 shows a typical porous plug experimental setup. In this experiment, a stream of gas at a pressure p_1 and Temperature T_1 is allowed to flow continuously through a porous plug. The gas comes out from the other side of the porous plug at a pressure p_2 and temperature T_2. The whole apparatus is completely insulated. Therefore, there is no heat transfer, i.e. $Q = 0$. Also the system does not exchange work with the surrounding. Therefore, $W = 0$.

As the flow is steady, we can apply steady flow energy equation given by,

$$gz_1 + \frac{V_1^2}{2} + h_1 + Q = gz_2 + \frac{V_2^2}{2} + h_2 + W \tag{14.142}$$

Since there is not much change of velocity, let $V_1 = V_2$ and $z_1 = z_2$. For the flow under consideration, $Q = 0$ and $W = 0$. The steady flow energy equation reduces to $h_1 = h_2$

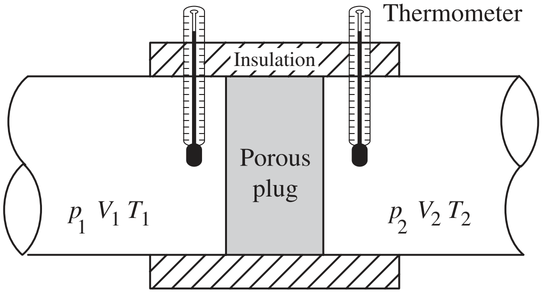


Fig. 14.5 Joule-Thomson experiment

enthalpy at inlet, h_1 = enthalpy at outlet, h_2. It indicates that the enthalpy is constant for throttling process.

By considering a number of experiments on a real gas keeping the initial pressure p_1 and temperature T_1 constant, a number of points can be plotted on a T-p graph. However, the stream pressure is reduced from p_1 to p_2, p_3, p_4, etc. that the downstream temperature also changes. The results from these experiments can be plotted as a constant enthalpy curve on T-p (temperature-pressure) plane as shown in Fig.14.6. The slope of a constant enthalpy is

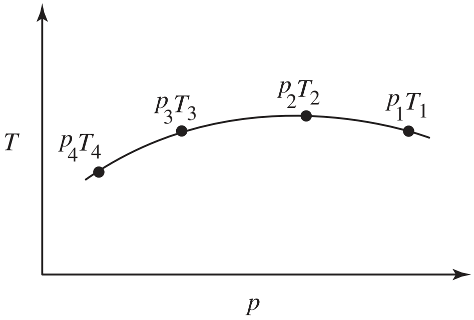


Fig. 14.6 Constant enthalpy line

known as *Joule-Thomson Coefficient.* It is denoted by μ:

$$\mu = \left(\frac{\partial T}{\partial p}\right)_h \tag{14.143}$$

For a real gas, μ, may be either positive, negative or zero depending upon the thermodynamic state of the gas.

Case (a): During a throttling process, there is always pressure drop. So, Δp and temperature change ΔT are negative. Therefore, μ is positive:

$$\mu = \left(\frac{\partial T}{\partial p}\right) \tag{14.144}$$

This type of throttling process produces cooling effect (temperature reduces).

Case (b) : There is always pressure drop in throttling process. So, Δp is negative.

When the temperature change is positive, μ is negative. This throttling process produces heating effect.

$$\mu \quad = \quad \left(\frac{\partial T}{\partial p}\right) (\because \text{ temperature increases}) \tag{14.145}$$

Case (c): When μ is zero, the temperature of the gas remains constant with throttling. The temperature at which $\mu = 0$ is called *inversion temperature* for a given pressure.

14.12.3 Inversion Curve

The maximum point on each curve on T-p plane is called *inversion point* and the locus of the inversion point is called *inversion curve*. This is shown in Fig.14.7.

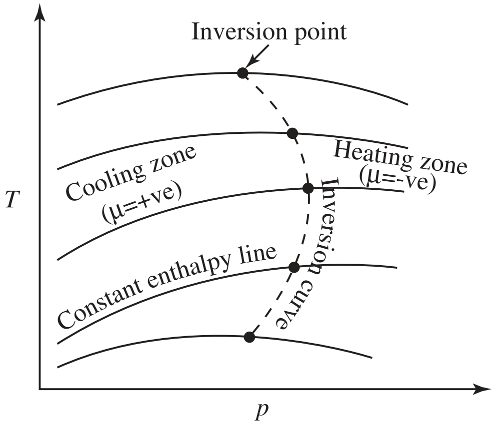


Fig. 14.7 Inversion Curve

A generalised equation of the Joule-Thomson coefficient can be derived by using change of enthalpy equation. We know that, change in enthalpy:

$$dh = \left[c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp\right] + v dp \tag{14.146}$$

$$dh - v dp = c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp \tag{14.147}$$

Now, $$dh - v dp + T\left(\frac{\partial v}{\partial T}\right)_p dp = c_p dT \tag{14.148}$$

Dividing both sides by c_p, $$dT = \frac{dh}{c_p} - \frac{v dp}{c_p} + \frac{T}{c_p}\left(\frac{\partial v}{\partial T}\right)_p dp \tag{14.149}$$

$$dT = \frac{dh}{c_p} - \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] dp \tag{14.150}$$

Differentiating this equation with respect to pressure at constant enthalpy:

$$\left(\frac{dT}{\partial p}\right)_h = 0 - \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \tag{14.151}$$

$$\left(\frac{dT}{\partial p}\right)_h = \mu = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \tag{14.152}$$

From Equation 14.152, we can determine the Joule-Thomson coefficient (μ) in terms of measurable properties such as pressure (p), temperature (T), specific volume (v) and c_p.

14.12.4 Joule-Thomson Coefficient for Ideal Gas

The Joule-Thomson coefficient is defined as the change in temperature with change in pressure, keeping the enthalpy constant. It is denoted by:

$$\left(\frac{dT}{\partial p}\right)_h = \mu = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \tag{14.153}$$

The equation of state is given by $pv = RT$. Differentiating the above equation of state with respect to T by keeping pressure, p constant:

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} = \frac{v}{T} \tag{14.154}$$

$$\mu = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] \tag{14.155}$$

$\mu = 0$ implies that the Joule-Thompson coefficient is zero for ideal gas.

14.12.5 Constant Temperature Coefficient

Let enthalpy be a function of pressure and temperature, i.e. $h = f(p, T)$

$$dh = \left(\frac{\partial h}{\partial p}\right)_T dp + \left(\frac{\partial h}{\partial T}\right)_p dT \tag{14.156}$$

For a throttling process, enthalpy remains constant, i.e. $h = C$ or $dh = 0$. Substituting dh in Eq.14.156:

$$0 = \left(\frac{\partial h}{\partial p}\right)_T dp + \left(\frac{\partial h}{\partial T}\right)_p dT \tag{14.157}$$

Dividing by dT,

$$\left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_p + \left(\frac{\partial h}{\partial T}\right)_p = 0 \tag{14.158}$$

$$\left(\frac{\partial h}{\partial p}\right)_T \left(\frac{\partial p}{\partial T}\right)_p = -\left(\frac{\partial h}{\partial T}\right)_p \tag{14.159}$$

$$\left(\frac{\partial h}{\partial p}\right)_T \frac{1}{\mu} = -\left(\frac{\partial h}{\partial T}\right)_p \tag{14.160}$$

$$\left(\frac{\partial h}{\partial T}\right)_p = -\frac{1}{\mu}\left(\frac{\partial h}{\partial p}\right)_T \tag{14.161}$$

The property $c_p = \left(\frac{\partial h}{\partial p}\right)_T$ is known as *constant temperature coefficient* .

14.13 HELMHOLTZ POTENTIAL

Consider a system that interacts with the surroundings. The work done by the system during a process is given by

$$dW = -dU + dQ$$

Let the surroundings be maintained at a constant temperature T_0. Let dS and dS_0 represent the changes in the entropy of the system and the surroundings during the interaction, respectively. From the second law of thermodynamics,

$$dS + dS_0 \geq 0$$

The change in the entropy of the surroundings is given by,

$$\begin{aligned} dS_0 &= -\frac{dQ}{T_0} \\ dS - \frac{dQ}{T_0} &\geq 0 \\ dQ &\leq T_0 dS \end{aligned}$$

Now, the work done by the system is given by,

$$dW = -dU + dQ \leq T_0 dS - dU$$

or

$$W \leq (U_1 - U_2) - T_0 \times (S_1 - S_2) \tag{14.162}$$

If the initial and final temperatures of the system are equal to the surrounding temperature ($T_1 = T_2 = T_0$), then

$$\begin{aligned} W &\leq (U_1 - U_2) - (T_1 S_1 - T_2 S_2) \\ W &\leq (U_1 - T_1 S_1) - (U_2 - T_2 S_2) \end{aligned}$$

or

$$W \leq A_1 - A_2 \tag{14.163}$$

From the above equation, we can infer the following. The maximum work that can be obtained from a system during a given process, in which the initial and final temperatures of the system are equal to the surrounding temperature, is equal to the decrease in the Helmholtz function. The maximum work is delivered only when the process is reversible. The work delivered is less than the decrease in the Helmholtz function. This is due to any irreversibility present in the process. Since the decrease in the Helmholtz function represents the potential to do work by the system, then it can be considered as a thermodynamic potential.

Now, if work is performed on the system, the minimum work required during a reversible process is equal to the increase in the Helmholtz function. On the other hand, if the process is irreversible, the work required is greater than the increase in the Helmholtz function. Quite often the Helmholtz function is called Helmholtz free energy or Helmholtz potential.

14.14 GIBBS FREE ENERGY

Consider a steady state, steady flow of a material through a control volume to examine how the decrease in the Gibbs free energy is related to the work obtained under the specified conditions. Neglect the changes in the potential energy and kinetic energy terms. Under these conditions, the first law of thermodynamics for a control volume reduces to

$$\dot{m}(h_e - h_i) = \dot{H}_e - \dot{H}_i = \dot{Q} - \dot{W}_s$$

or

$$\dot{W}_s = \dot{Q} - \left(\dot{H}_e - \dot{H}_i\right)$$

If the control volume receives energy in the form of heat from the surroundings at temperature T_0, the change in the entropy of the surroundings is given by,

$$\Delta\dot{S}_0 = -\frac{\dot{Q}}{T_0}$$

According to the second law of thermodynamics, $\Delta\dot{S} + \Delta\dot{S}_0 \geq 0$ and $\Delta\dot{S}_0 - \dfrac{\dot{Q}}{T_0} \geq 0$ or $\dot{Q} \leq T_0\Delta\dot{S}$. Therefore,

$$\dot{W}_s \leq T_0\left(\dot{S}_e - \dot{S}_i\right) - \left(\dot{H}_e - \dot{H}_i\right) \tag{14.164}$$

If the material flows through the control volume such that its temperature is equal to that of the surroundings ($T_i = T_e = T_0$), then the above equation reduces to

$$\dot{W}_s \leq \left(\dot{H}_i - T_i\dot{S}_i\right) - \left(\dot{H}_e - T_e\dot{S}_e\right)$$

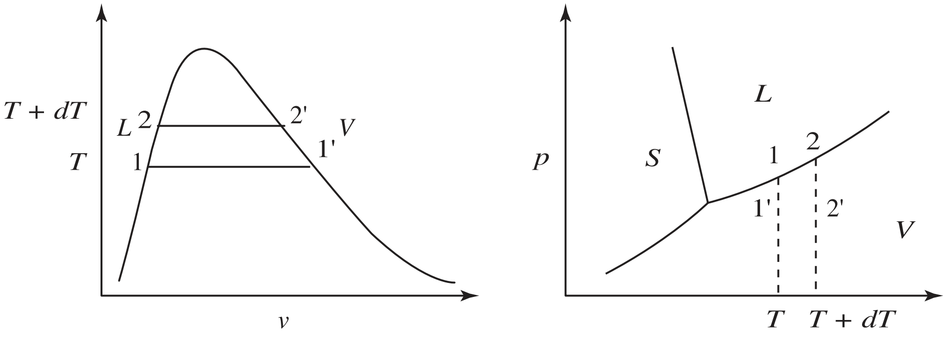
or $\dot{W}_s \leq \dot{G}_i - \dot{G}_e$

The above equation implies that the maximum power that can be obtained from a flow process, in which the initial and final temperatures of the flowing fluid are the same as that of the surroundings, is equal to the rate of decrease in the Gibbs free energy of the flowing fluid. If there is any irreversibility in the process, the power that can be obtained is less than just the decrease in the Gibbs free energy. Since the Gibbs free energy represents the potential to do work, it is also a thermodynamic potential.

14.15 KIRCHHOFF EQUATION

The latent heat of vaporisation and the latent heat of fusion vary with temperature. From the steam tables, it can be seen that the latent heat of vaporisation at 100 °C is 2256.94 kJ/kg whereas at the critical point it is zero. The latent heat of vaporisation decreases as the saturation temperature increases. At the critical point, where the two phases are indistinguishable, it reduces to zero. This is true for all substances. Hence, while dealing with large variation of temperature and pressure, the latent heat of phase transition cannot be assumed to be constant. The derivation of Kirchoff relation which predicts the effect of temperature on the latent heat of phase transition is given below.

Consider the vaporisation of a liquid at constant temperature and pressure as shown in Fig.14.8. The latent heat of vaporisation associated with the phase-change 1 to 1′ is $(h_{g1} - h_{f1})$ at temperature T. When the saturation temperature is raised to $(T + dT)$, the latent heat of vaporisation is $(h_{g2} - h_{f2})$. The change in the latent heat associated with a change in the saturation temperature is given by



(a) *T*-*v* diagram indicating the vaporization process

(b) Phase diagram showing the change of state

Fig. 14.8 Kirchhoff equation

$$dh_{fg} = dh_g - dh_f$$

The variation in the enthalpy associated with the variation in the independent variables T and p is given by,

$$dh = c_p dT + v(1 - \beta T)dp$$

or

$$dh_{fg} = (c_{pg} - c_{pf}) + [(v_g - v_f) - (v_g\beta_g - v_f\beta_f)T]dp$$

or

$$\frac{\partial h_{fg}}{\partial T} = (c_{pg} - c_{pf}) + [(v_g - v_f) - (v_g\beta_g - v_f\beta_f)T]\left(\frac{\partial p}{\partial T}\right)_{sat}$$

Substituting for $\left(\frac{\partial p}{\partial T}\right)_{sat}$ from the Claperon equation (Eq.14.140), we get

$$\frac{\partial h_{fg}}{\partial T} = (c_{pg} - c_{pf}) - \frac{(v_g\beta_g - v_f\beta_f)h_{fg}}{v_g - v_f} + \frac{h_{fg}}{T} \qquad (14.165)$$

This is known as the Kirchoff relation. This can be rearranged as:

$$\frac{\partial\left(\dfrac{h_{fg}}{T}\right)}{\partial T}+\frac{v_g\beta_g-v_f\beta_f}{v_g-v_f}\times\frac{h_{fg}}{T} = \frac{c_{pg}-c_{pf}}{T} \tag{14.166}$$

The Kirchoff relation can be simplified by making some suitable assumptions. For solid to liquid transition, it is a reasonably good approximation to assume that the molar heat capacity and the molar volume are constant in each phase and the coefficient of volume expansion β is negligible for each phase. With these approximations, the Kirchoff equation (Eq.14.166) reduces to

$$\frac{\partial\left(\dfrac{h_{sf}}{T}\right)}{\partial T} = \frac{c_{pf}-c_{ps}}{T}$$

where h_{sf} is the latent heat of fusion.

For the transition from liquid phase to vapour phase, the molar volume of the liquid phase can be neglected compared to the molar volume of the vapour phase ($v_g \gg v_f$) and $\beta_g \gg \beta_f$. Further, the vapour phase may be approximated as an ideal gas. Then, $\beta = \frac{1}{T}$. Since, $\beta_g \gg \beta_f$ and $v_g \gg v_f$, Kirchoff equation (Eq.14.165) reduces to

$$\left(\frac{\partial h_{fg}}{\partial T}\right) = c_{pg}-c_{pf} \tag{14.167}$$

14.16 GIBBS PHASE RULE

As per the state, postulate tells us that the number of thermodynamic properties that can be varied independently is equal to the number of relevant reversible work modes plus one. Hence, for a single-component simple system, the number of variables that can be varied independently is equal to two. This means two independent variables are to be specified for the complete description of the state of such a system. When we deal with a complex system consisting of several phases and several components, the situation is not that simple.

Let us consider a system consisting of C components which are distributed among all the P phases. Since each phase contains C components, each phase is completely specified by the variables, viz temperature, pressure and $(C-1)$ mole fractions of the components (since the sum of all the mole fractions must add to unity). That is, there are $(C+1)$ variables. Then, for the entire system, p, $(C+1)$ variables are needed for complete description. If the system is existing in a state of thermodynamic equilibrium, the criteria for equilibrium tells that the following equalities between the variables must be satisfied.

$$T^1 = T^2 = T^3 = \ldots = T^p \tag{14.168}$$

That is, there are $(p-1)$ equations in temperature:

$$p^1 = p^2 = p^3 = \ldots = p^p \tag{14.169}$$

That is, there are $(p-1)$ equations in pressure:

$$\mu_1^1 = \mu_1^2 = \mu_1^3 = \ldots = \mu_1^p \tag{14.170}$$

$$\mu_2^1 = \mu_2^2 = \mu_2^3 = \ldots = \mu_2^p \tag{14.171}$$

$$\mu_c^1 = \mu_c^2 = \mu_c^3 = \ldots = \mu_c^p \tag{14.172}$$

That is, there are $(p-1)$ equations among each of the chemical potentials μ_i. The superscripts, $1, 2, \ldots, p$ denote the phases and the subscripts $1, 2, \ldots C$ denote the components. Thus, the total number of relations existing between the variables is $(p-1)(C+2)$. The additional variables over and above the number of relationships existing among them would represent the degree of freedom of the system. If F denotes the degrees of freedom of the system (that is F variables can be varied independently), then,

$$F = p(C+1) - (P-1)(C+2) \tag{14.173}$$

or $P + F = C + 2$, which is called the *Gibbs phase rule*.

The phase rule provides a relationship among the degrees of freedom of the system, the number of phases and the number of components. The relationship is very simple and its derivation is based on the criterion of equilibrium which in turn is based on the laws of thermodynamics.

14.17 SINGLE-COMPONENT SYSTEM

Just for illustration, let us apply the phase rule for a single-component system. Then, $F = C + 2 - P = 3 - P$. Therefore, there are three possibilities.

When $P = 3; F = 0$, that is, the system is invariant. Therefore, for a single-component system, the maximum number of phases that can coexist are equal to three. Such a system has no degrees freedom, that is, no property of the system can be varied independently. The system is present at the triple point and the temperature and pressure of the triple point are fixed.

When $P = 2, F = 1$, the system is univariant; that is, it has only one degree of freedom when one system contains two phases in a state of equilibrium. Consider the saturation curve shown on the phase diagram shown in Fig.14.9. When the saturated liquid is in equilibrium with the saturated vapour, identification of either saturation pressure or saturation temperature fixes the other variables. Along the identification, fusion and vaporisation curves, the system is univariant.

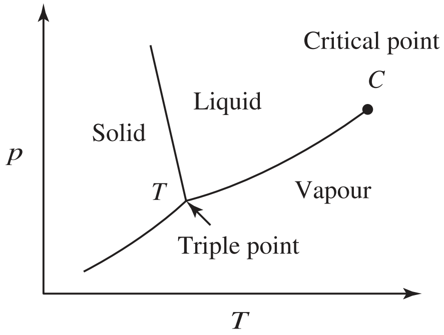


Fig. 14.9 Phase diagram for a single-component system

If $P = 1; F = 2$. When the system is present in only one phase, the degrees of freedom of such a system is equal to two or the system is bivariant. That is, two intensive properties can be varied independently and the system lies in any of the regions indicated as solid, liquid or vapour in the phase diagram. That is, two independent variables P and T are to be specified for a complete description of the system.

Worked-Out Examples

14.1 Prove that for any fluid:

(i) $\left(\dfrac{\partial h}{\partial v}\right)_T = v\left(\dfrac{\partial p}{\partial v}\right)_T + T\left(\dfrac{\partial p}{\partial T}\right)_v$

(ii) $\left(\dfrac{\partial h}{\partial p}\right)_T = v - T\left(\dfrac{\partial v}{\partial T}\right)_p$

(iii) Show that for a fluid obeying van der Waals equation, $p = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$, where R, a and b are constants.

(iv) $h(enthalpy) = \dfrac{RTb}{v-b} - \dfrac{2a}{v} + f(T)$, where $f(T)$ is arbitrary.

Solution

We know that $$ds = \frac{c_V}{T}dT + \left(\frac{\partial p}{\partial T}\right)_v dv$$

Also $$dh = Tds + vdp = T\left[\frac{c_V}{T}dT + \left(\frac{\partial p}{\partial T}\right)_v dv\right] + vdp$$

, i.e. $$dh = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_v + dv + vdp$$

Taking $dT = 0$, we get

$$\left(\frac{\partial h}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v + v\left(\frac{\partial p}{\partial v}\right)_T \qquad \text{(Proved)} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

$$\left(\frac{\partial h}{\partial p}\right)_T = \left(\frac{\partial h}{\partial v}\right)_T\left(\frac{\partial v}{\partial p}\right)_T = \left[T\left(\frac{\partial p}{\partial T}\right)_v + v\left(\frac{\partial p}{\partial v}\right)_T\right]\left(\frac{\partial v}{\partial p}\right)_T$$

, i.e. $$\left(\frac{\partial h}{\partial p}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial v}{\partial p}\right)_T + v \qquad \text{(i)}$$

Also $$\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial v}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}\right)_p$$

Therefore, Eqn.(i) becomes

$$\left(\frac{\partial h}{\partial p}\right)_T = v - T\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{(Proved)} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

Now, $$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

and $$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

$$\therefore \quad \left(\frac{\partial h}{\partial v}\right)_T = v\left[\frac{-RT}{(v-b)^2} + \frac{2a}{v^3}\right] + T\left(\frac{R}{v-b}\right)$$

$$= -\frac{RTv}{(v-b)^2} + \frac{2a}{v^2} + \frac{RT}{v-b} = \frac{-RTv}{(v-b)^2} + \frac{RT}{v-b} + \frac{2a}{v^2}$$

$$= \frac{-RTv + RT(v-b)}{(v-b)^2} + \frac{2a}{v^2} = \frac{-RTv + RTv - RTb}{(v-b)^2} + \frac{2a}{v^2}$$

, i.e. $$\left(\frac{\partial h}{\partial v}\right)_T = \frac{-RTb}{(v-b)^2} + \frac{2a}{v^2}$$

or $$h = \frac{RTb}{v-b} - \frac{2a}{v} + f(T)$$

This shows h depends on T and v. $\overset{\textbf{Ans}}{\Longleftarrow}$

14.2 A cylinder-piston arrangement contains 2 kg of superheated steam at 10 bar and 573 K. It undergoes an adiabatic expansion to 1 bar pressure. Find the work that can be obtained during the expansion process.

Solution

The point to note is that work obtained from a system which undergoes an adiabatic process and equal to the decrease in the thermodynamic potential u. Mathematically, $W = -\Delta U$

From steam table at 10 bar and 573 K (300 °C), $u_1 = 2793.2$ kJ/kg and $s_1 = 7.1228$kJ/kg K

For a reversible adiabatic expansion process $s_2 = s_1$:

$$s_2 = s_1 = x_2 s_g + (1 - x_2) \times s_f$$

From steam table at 1 bar $s_g = 7.3593$ and $s_f = 1.3025$:

$$\therefore 7.1228 = x_2 \times 7.3593 + (1 - x_2) \times 1.3025$$

$$x_2 = \frac{7.1228 - 1.3025}{7.3593 - 1.3025} = 0.961$$

$$h_2 = x_2 h_g + (1 - x_2) \times h_f$$

$$= 0.961 \times 2675.5 + (1 - 0.961) \times 417.44 = 2587.44 \text{ kJ/kg}$$

$$v_2 = x_2 v_g + (1 - x_2) \times v_f$$

$$= 0.961 \times 1.694 + (1 - 0.916) \times 0.001043 = 1.628 \text{ m}^3/\text{kg}$$

$$u_2 = h_2 - p_2 v_2$$

$$\begin{aligned}
u_2 &= \left(2587.44\times 10^3 - 1\times 10^5\times 1.628\right)\times 10^{-3} = 2424.54 \text{ kJ}\\
u_1 - u_2 &= 2793.2 - 2424.54 = 368.66 \text{ kg}\\
W &= 2\times 368.66 = 737.32 \text{ kJ} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

14.3 One kmol of an ideal gas is to be compressed from an initial pressure of 1 bar and 27 °C to final state of 10 bar and 27 °C in a piston cylinder assembly. Determine the minimum amount of work required for this process assuming the surroundings temperature to be 300 K.

Solution

Given $T_1 = 273 + 27 = 300$ K; $T_2 = 273 + 27 = 300$ K and $T_{sur} = 300$ K. As the initial and final state of the system are at 300 K which is also the temperature of the surroundings the minimum work required is equal to the increase in the Helmholtz potential or Helmholtz function:

$$W_{min} = \Delta a = a_2 - a_1 = (u_2 - T_0 s_2) - (u_1 - T_0 s_1)$$

For an ideal gas, u is a function of temperature, $u(T)$ only. Since, $T_1 = T_2; u_1 = u_2$:

$$\begin{aligned}
W_{min} &= -T_0(s_2 - s_1)\\
s_2 - s_1 &= c_p\times \ln\frac{T_2}{T_1} - R\times \ln\frac{p_2}{p_1}\\
&= 0 - 8.314\times \ln\left(\frac{1\times 10^6}{100\times 10^3}\right) = -19.1437 \text{ kJ/K mol K}\\
W_{min} &= -T_0\Delta S = 300\times 19.1437 = 5743.11 \text{ kJ/K mol K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

14.4 Superheated steam enters an isentropic turbine at 800 kPa and 250 °C at the rate of 1 kg/s and leaves at 100 kPa. Calculate the power output of the turbine.

Solution

As steam turbine is a steady flow system, the rate of work done by the turbine is equal to the decrease in enthalpy of steam.

$$\dot{W} = -\Delta H$$

From steam tables at 800 kPa and 250 °C, $h_1 = 2947.95$ kJ/kg and $s_1 = 7.0248$ kJ/kg K

For an isentropic process, $s_2 = s_1$. Therefore $s_2 = 7.0248$ kJ/kg K.

$$\begin{aligned}
\therefore s_2 &= x_2 s_g - (1 - x_2)\times s_f\\
7.0248 &= x_2\times 7.3598 + (1 - x_2)\times 1.3027\\
x_2 &= 0.945\\
h_2 &= 2675.4\times 0.945 + (1 - 0.945)\times 417.54 = 2551.22 \text{ kJ/kg}\\
W &= H_1 - H_2 = 2947.95 - 2551.22 = 396.73 \text{ kJ/s} = 396.73 \text{ kW} \qquad \overset{\textbf{Ans}}{\Longleftarrow}
\end{aligned}$$

14.5 Liquid water at 25 °C is compressed isentropically from 10 kPa to 10000 kPa. Find the temperature rise. Take $c_p = 4.2$ kJ/kg K and $\beta = 2.09 \times 10^{-4}$ K^{-1}.

Solution

The rise in temperature with a rise in pressure due to isentropic compression is given by

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{Tv\beta}{c_p}$$

From steam tables, the specific volume of liquid water at 25 °C is given by,

$$v_f = 0.001003 \text{ m}^3/\text{kg}$$

$$\left(\frac{\partial T}{\partial p}\right)_s = \frac{298 \times 0.001003 \times 2.07 \times 10^{-4}}{4.2 \times 10^3} = 1.473 \times 10^{-8} = \left(\frac{\Delta T}{\Delta p}\right)_s$$

$$\Delta T = 1.473 \times 10^{-8} \times (10000 - 100) \times 10^3 = 0.15 \text{ °C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

14.6 A domestic pressure cooker is set to have a buildup pressure of 200 kPa inside the cooker. Find the temperature at which the water boils at this pressure. Take latent heat of vaporisation of water = 2257 kJ/kg at 100 °C and atmospheric pressure 100 kPa. Assume that no other data are available.

Solution

As no other data such as v_f and v_g are available. Clapeyron equation may be used to find the saturation temperature corresponding to given pressure. The equation is:

$$\ln\frac{p_2}{p_1} = \frac{h_{fg}}{R} \times \left(\frac{1}{T_1} - \frac{1}{T_2}\right)$$

$$\ln\frac{200}{100} = \frac{2257}{\left(\frac{8.314}{18}\right)} \times \left(\frac{1}{373} - \frac{1}{T_2}\right)$$

$$\ln 2 = 4886.46 \times \left(\frac{1}{373} - \frac{1}{T_2}\right)$$

$$0.6931 = 4886.46 \times \left(\frac{1}{373} - \frac{1}{T_2}\right)$$

$$\left(\frac{1}{373} - \frac{1}{T_2}\right) = \frac{0.6931}{4886.46} = 0.14 \times 10^{-3}$$

$$\frac{1}{T_2} = \frac{1}{373} - 0.14 \times 10^{-3}$$

$$= 2.681 \times 10^{-3} - 0.14 \times 10^{-3} = 2.541 \times 10^{-3}$$

$$T_2 = \frac{1}{2.541} \times 10^3 = 0.3935 \times 10^3 = 393.5 \text{ K} = 120.5 \text{ °C} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

14.7 The latent heat of vaporisation of water at 100 °C is 2257 kJ/kg. Calculate the latent heat of vaporisation of water at 120 °C and compare with the value given in the steam table. The specific heat of liquid water and water vapour as $c_{pf} = 4.23$ kJ/kg K and $c_{pg} = 1.55$ K, respectively.

Solution

The Kirchoff equation for the liquid to vapour transition is given by,

$$\left(\frac{\partial h_{fg}}{\partial T}\right) \quad = \quad c_{pg} - c_{pf}$$

On integrating the above equation, we get

$$h_{fg2} - h_{fg1} = (c_{pg} - c_{pf}) \times (T_2 - T_1) \quad = \quad (1.55 - 4.23) \times 20 = -53.6 \text{ kJ/kg}$$

$$\therefore h_{fg} \text{ at } 120\ ^\circ\text{C} \quad = \quad 2257 - 53.6 = 2203.4 \text{ kJ/kg} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

The steam table value is 2202.3 kJ/kg.

14.8 One kg of liquid water at −10 °C as converted into ice at the same temperature. Calculate the entropy change taking place in the process. Take latent heat of fusion of ice at 0 °C = 335 kJ.kg. The specific heat of liquid water and ice as 4.2 kJ/kg K and 2.1 kJ/kg K, respectively.

Solution

The Kirchhoff relation can be used to determine the latent heat of fusion of ice at −10 °C which in turn can be used to find the entropy change associated with this process. Kirchhoff relation for solid to liquid transition is given by,

$$\frac{\partial\left(\frac{h_{fg}}{T}\right)}{\partial T} \quad = \quad \frac{c_{pf} - c_{pg}}{T}$$

The above equation can be integrated to obtain:

$$\left(\frac{h_{sf}}{T}\right)_2 \quad = \quad \left(\frac{h_{sf}}{T}\right)_1 + (c_{pf} - c_{ps}) \times \ln\frac{T_2}{T_1} = \left(\frac{335}{273}\right) + (4.2 - 2.1) \times \ln\frac{263}{273} = 1.1487$$

$$\Delta S \quad = \quad -\frac{h_{sf}}{T} = -1.1487 \text{ kJ/kg K} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

14.9 Derive using thermodynamic relations and show that:

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 V}{\partial T^2}\right)_p$$

(b) Prove that c_p of an ideal gas is a function of T only.

(c) In the case of a gas obeying the equation of state:

$$\frac{pv}{RT} = 1 + B'p$$

where B' is a function of T only, show that

$$c_p = \overline{R}Tp\frac{d^2}{dT^2}(B') + (c_p)_0$$

where $(c_p)_0$ is the value at very low pressure.

Solution

$$\begin{aligned}
c_p &= T\left(\frac{\partial S}{\partial T}\right)_p \\
\left(\frac{\partial c_p}{\partial p}\right)_T &= T\left(\frac{\partial^2 S}{\partial T.\partial p}\right) \\
\left(\frac{\partial S}{\partial p}\right)_T &= -\left(\frac{\partial V}{\partial T}\right)_p,
\end{aligned}$$

By Maxwells relation:

$$\begin{aligned}
\frac{\partial^2 S}{\partial p.\partial T} &= -\left(\frac{\partial^2 V}{\partial T^2}\right)_p \\
\therefore \left(\frac{\partial c_p}{\partial p}\right)_T &= -T\left(\frac{\partial^2 V}{\partial T^2}\right)_p
\end{aligned}$$

(b) For an ideal gas:

$$\begin{aligned}
V &= \frac{n\overline{R}T}{p} \\
\left(\frac{\partial V}{\partial T}\right)_p &= \frac{n\overline{R}}{p} \quad \text{and} \quad \left(\frac{\partial^2 V}{\partial T^2}\right)_p = 0 \\
\therefore \left(\frac{\partial \overline{C}_p}{\partial p}\right)_T &= 0
\end{aligned}$$

, i.e. c_p is a function of T alone.

(c) As obeying the equation of state:

$$\begin{aligned}
\frac{p\overline{v}}{\overline{\overline{R}}T} &= 1 + B'p \\
B'p &= \frac{p\overline{v}}{\overline{\overline{R}}T} - 1 \\
B'T &= \frac{T}{p}\left(\frac{p\overline{v}}{\overline{\overline{R}}T} - 1\right) = \left(\frac{\overline{v}}{\overline{\overline{R}}} - \frac{T}{p}\right) \\
\left[\frac{\partial}{\partial T}(B'T)\right]_p &= +\frac{1}{\overline{\overline{R}}}\left(\frac{\partial \overline{v}}{\partial T}\right)_p - \frac{1}{p}
\end{aligned}$$

$$\left[\frac{\partial^2(B'T)}{\partial T^2}\right]_p = \frac{1}{\overline{R}}\left(\frac{\partial^2\overline{v}}{\partial T^2}\right)_p = -\frac{1}{\overline{R}T}\left(\frac{\partial \overline{C}_p}{\partial p}\right)_T$$

Therefore, on integration,

$$\overline{C}_p = -\overline{R}Tp\frac{d^2}{dT^2}(B'T) + \overline{C}_{p0}$$

where $\overline{C}_{p0}$ (integration constant) is the value of $\overline{C}_p$ at very low values of pressure.

14.10 During a throttling process, μ_J Joule-Kelvin coefficient is a function of the temperature change. A similar measure of the temperature change produced by an isentropic change of pressure is provided by the coefficient μ_s, where,

$$\mu_S = \left(\frac{\partial T}{\partial p}\right)_S$$

Prove that, $\mu_S - \mu_J = \frac{V}{c_p}$.

Solution

The Joule-Kelvin coefficient, μ_j, is given by,

$$\mu_j = \frac{T\left(\frac{\partial V}{\partial T}\right)_p - V}{c_p}$$

Since $c_p = T\left(\frac{\partial S}{\partial T}\right)_p$ and by Maxwells relation,

$$\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial p}\right)_T$$

$$\mu_J = \frac{-T\left(\frac{\partial S}{\partial p}\right)_T}{T\left(\frac{\partial S}{\partial T}\right)_p} - \frac{V}{c_p}$$

$$\therefore \mu_J = -\left(\frac{\partial S}{\partial p}\right)_T\left(\frac{\partial T}{\partial S}\right)_p - \frac{V}{c_p}$$

Since $\left(\frac{\partial S}{\partial p}\right)_T\left(\frac{\partial T}{\partial S}\right)_p\left(\frac{\partial p}{\partial T}\right)_s = -1$

$$\mu_J = +\mu_s - \frac{V}{c_p}$$

$$\therefore \mu_s - \mu_J = \frac{V}{c_p} \quad \text{Proved.}$$

Ans $\Longleftarrow$

14.11 If the boiling point of benzene at 1 atm pressure is 353 K, estimate the approximate value of the vapour pressure of benzene at 300 K.

Solution

Using Clapeyrons equation and Troutons rule,

$$\begin{aligned} p &= 101.325 \times \exp\left\{\frac{88}{\overline{R}}\left(1 - \frac{T_B}{T}\right)\right\} \\ &= 101.325 \times \exp\left\{\frac{88}{8.3143} \times \left(1 - \frac{353}{300}\right)\right\} = 15.62\text{ kPa} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

14.12 The vapour pressure of solid ammonia in mm mercury is given by,

$$\ln p = 23.03 - \frac{3754}{T}$$

and that of liquid ammonia by $\ln p = 19.49 - \dfrac{3063}{T}$

What is the temperature of the triple point? What is the pressure?

Solution

At the triple point, the saturated solid and saturated liquid lines meet.

$$\begin{aligned} 23.03 - \frac{3754}{T} &= 19.49 - \frac{3063}{T} \\ T &= 195.2\text{ K} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \\ \ln p &= 23.03 - \frac{3754}{195.2} \\ \ln p &= 3.80 \\ p &= 44.67\text{ mm hg} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

14.13 Show that the van der Waals gas:

(i) $\left(\dfrac{\partial c_V}{\partial v}\right)_T = 0$

(ii) $(s_2 - s_1)_T = R \times \ln\dfrac{v_2 - b}{v_1 - b}$

(iii) $T(v-b)^{\frac{R}{cv}}$ = constant, for an isentropic

(iv) $c_p - c_V = \dfrac{R}{\dfrac{1 - 2a(v-b)^2}{RTv^3}}$

(v) $(h_2 - h_1)_T = (p_2v_2 - p_1v_1) + a\left(\dfrac{1}{v_1} - \dfrac{1}{v_2}\right)$

Solution

From the energy Eq.14.92,

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

$$\frac{\partial^2 U}{\partial V \partial T} = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V + \left(\frac{\partial p}{\partial T}\right)_V - \left(\frac{\partial p}{\partial T}\right)_V$$

$$\frac{\partial^2 U}{\partial V \partial T} = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V$$

$$c_V = \left(\frac{\partial U}{\partial T}\right)_V$$

$$\left(\frac{\partial c_V}{\partial V}\right)_T = \frac{\partial^2 U}{\partial T\ \partial V} = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V = \left(\frac{\partial c_V}{\partial v}\right)_T$$

For a van der Waals gas equation:

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

$$\because \qquad \left(\frac{\partial^2 p}{\partial T^2}\right)_V = 0$$

and $$\left(\frac{\partial c_V}{\partial v}\right)_T = 0 \quad \text{(Proved)} \qquad (i)$$

Therefore, c_V is independent of volume.

(ii) From the first Tds equation, Eq.14.54:

$$Tds = c_V dT + T\left(\frac{\partial p}{\partial T}\right)_V dv$$

and energy Eq.14.92,

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$$

$$ds = c_V \frac{dT}{T} + \frac{1}{T}\left[p + \left(\frac{\partial U}{\partial V}\right)_T\right] dv$$

For van der Waals gas equation:

$$\left(\frac{\partial U}{\partial V}\right)_T = \frac{a}{v^2}$$

$$\because \qquad ds = c_V\frac{dT}{T} + \frac{1}{T}\left(p + \frac{a}{v^2}\right)dv = c_V\frac{dT}{T} + \frac{R}{v-b}dv$$

$$\because \qquad (s_2 - s_1)_T = R\ln\frac{v_2 - b}{v_1 - b} \quad \text{(Proved)} \qquad (ii)$$

(c) At constant entropy:

$$c_V\frac{dT}{T} + \frac{R}{v-b}dv = 0$$

or
$$\frac{dT}{T} + \frac{R}{c_V}\frac{dv}{v-b} = 0$$

On integration,

$$T(v-b)^{\frac{R}{Cv}} = \text{constant} \quad \text{(Proved)} \qquad (iii)$$

(iv)
$$c_p - c_V = T\left(\frac{\partial p}{\partial T}\right)_V\left(\frac{\partial v}{\partial T}\right)_p = \left[\left(\frac{\partial U}{\partial V}\right)_T + p\right]\left(\frac{\partial v}{\partial T}\right)_p$$

$$= \left(\frac{a}{v^2} + p\right)\left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{RT}{v-b}\right)\left(\frac{\partial v}{\partial T}\right)_p$$

From the equation:

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

Therefore, $(v-b)\left(-2av^{-3}\right)\left(\frac{\partial v}{\partial T}\right)_p + \left(p + \frac{a}{v^2}\right)\left(\frac{\partial v}{\partial T}\right)_p = R$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{\dfrac{R}{(v-b)}}{\dfrac{RT}{(v-b)} - \dfrac{2a}{v^3}}$$

$$\therefore \qquad c_p - c_V = \frac{R}{1 - \dfrac{2a(v-b)^2}{RTv^3}} \quad \text{(Proved)} \qquad (iv)$$

(e)
$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p = \frac{a}{v^2}$$

$$du_T = \frac{a}{v^2}dv_T$$

$$\therefore \qquad (u_2 - u_1)_T = a\left(\frac{1}{v_1} - \frac{1}{v_2}\right)$$

$$\therefore \qquad (h_2 - h_1)_T = (p_2v_2 - p_1v_1) + a\left(\frac{1}{v_1} - \frac{1}{v_2}\right) \quad \text{(Proved)} \qquad (v)$$

14.14 The virial equation of state of a gas is given by $pv = RT\left(1 + B'p + C'p^2 + \ldots\right)$. Show that $\lim_{p\to 0}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = RT^2\frac{dB'}{dT}$. Hence, prove that the inversion temperature of a van der Waals gas is twice the Boyle temperature.

Solution

$$pv = RT\left(1 + B'p + C'p^2 + \ldots\right)$$

$$v = \frac{RT}{p} + RTB' + RTpC' + \ldots$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + RT\frac{dB'}{dT} + RB' + RTp\frac{dC'}{dT} + RpC' + \ldots$$

$$\therefore \qquad T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} + RT^2\frac{dB'}{dT} + RTB' + RT^2p\frac{dC'}{dT} + RTpC' + \ldots$$

$$T\left(\frac{\partial v}{\partial T}\right) - v = RT^2\frac{dB'}{dT} + RT^2p\frac{dC'}{dT} + \ldots$$

$$\mu_J = \frac{1}{c_p}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = \frac{RT^2}{c_p}\left[\frac{dB'}{dT} + p\frac{dC'}{dT} + \ldots\right]$$

$$\lim_{p\to 0}\mu_J = \frac{RT^2}{c_p}\frac{dB'}{dT}$$

or
$$\lim_{p\to 0}\left[T\left(\frac{\partial v}{\partial T}\right)_p - v\right] = RT^2\frac{dB'}{dT} \qquad \text{Proved}$$

For a van der Waals gas, to find Boyle temperature T_B,

$$B = b - \frac{a}{RT} = 0$$

$$\therefore \qquad T_B = \frac{a}{bR}$$

$$B' = \frac{B}{RT} = \frac{b}{RT} - \frac{a}{R^2T^2}$$

$$\frac{dB'}{dT} = -\frac{b}{RT^2} + \frac{2a}{R^2T^3}$$

$$\therefore \qquad \lim_{p\to 0}\mu_J = \frac{RT^2}{c_p}\left(-\frac{b}{RT^2}+\frac{2a}{R^2T^3}\right) = 0$$

$$\therefore \qquad \frac{b}{RT^2} = \frac{2a}{R^2T^3}$$

$$T_1 = \frac{2a}{bR}$$

$$\therefore \qquad T_1 = 2T_B$$

Inversion temperature $= 2 \times$ Boyles temperature Proved

14.15 Over a certain range of pressures and temperatures, the equation of a certain substance is given by the relation:

$$v = \frac{RT}{p} - \frac{C}{T^3}$$

where C is a constant. Derive an expression for (a) the change of enthalpy and (b) the change of entropy of this substance in an isothermal process.

Solution

From Eq.14.103,

$$dh = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$\therefore (h_2 - h_1)_T = \int_1^2 \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$v = \frac{RT}{p} - \frac{C}{T^3}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3C}{T^4}$$

$$T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} + \frac{3C}{T^3}$$

$$v - T\left(\frac{\partial v}{\partial T}\right)_p = \frac{RT}{p} - \frac{C}{T^3} - \frac{RT}{p} - \frac{3C}{T^3} = \frac{4C}{T^3}$$

$$\text{on substitution, } (h_2 - h_1)_T = \int_{p_1}^{p_2} -\frac{4C}{T^3} dp = \frac{4C}{T^3}(p_1 - p_2)T$$

(b) Using second Tds equation:

$$Tds = c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$

$$\begin{aligned} ds_T &= -\left(\frac{\partial v}{\partial T}\right)_p dp_T = \left(\frac{R}{p}+\frac{3C}{T^4}\right)dp_T \\ (s_2-s_1)_T &= R\ln\frac{p_1}{p_2}+\frac{3C}{T^4}(p_1-p_2)T \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

14.16 For a gas which obeys the van der Waals equation of state, determine the relations between T and v and between p and v in a reversible adiabatic process, assuming $c_V =$ constant. *Hint:* For a gas obeying the van der Waals equation, we have $s = s(T_1, v)$.

Solution

$$\begin{aligned} ds &= \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv = \frac{c_V}{T}dT + \left(\frac{\partial p}{\partial T}\right)_v dv \\ &= \frac{c_V}{T}dT + \frac{R}{v-b}dv \end{aligned}$$

For a reversible adiabatic process, $ds = 0$:

$$\begin{aligned} c_V\ln T + R\ln(v-b) &= \text{Constant} \\ T(v-1)^{\frac{R}{c_V}} &= \text{Constant} \\ \left[\left(p+\frac{a}{v^2}\right)\frac{v-b}{R}\right](v-b)^{\frac{R}{c_V}} &= \text{Constant} \\ \text{Hence,}\left(p+\frac{a}{v^2}\right)(v-b)^{\frac{1+R}{c_V}} &= \text{Constant} \qquad \overset{\textbf{Ans}}{\Longleftarrow} \end{aligned}$$

14.17 Derive the expression for the Joule-Thomas coefficient of a gas obeying the van der Waals equation of state and determine the equation of the inversion curve.

Hint: Volume expansivity, $\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p$.

Solution

For van der Waals equation of state:

$$\begin{aligned} p &= \frac{RT}{v-b}-\frac{a}{v^2} \\ \left(\frac{\partial p}{\partial T}\right)_v &= \frac{R}{v-b} \text{ and } \left(\frac{\partial p}{\partial v}\right)_T = \frac{2a}{v^3}-\frac{RT}{(v-b)^2} \\ \left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_v\left(\frac{\partial p}{\partial v}\right)_T &= -1 \\ \left(\frac{\partial v}{\partial T}\right)_p &= -\frac{\left(\frac{\partial p}{\partial T}\right)_v}{\left(\frac{\partial p}{\partial T}\right)_T} = \frac{R(v-b)}{\left(\frac{2a}{v^3}\right)-\frac{RT}{(v-b)^2}} = \frac{Rv^3(v-b)}{RTv^3-2a(v-b)^3} \\ \beta &= \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \frac{Rv^2(v-b)}{RTv^3-2a(v-b)^2} \end{aligned}$$

Joule-Thompson coefficient, μ_J is given by,

$$\begin{aligned}\mu_J &= \left(\frac{\partial T}{\partial p}\right)_h = \frac{v}{c_p}[\beta T - 1] = \frac{v}{c_p}\left[\frac{RTv^2(v-b)}{RTv^3 - 2a(v-b)^2} - 1\right] \\ &= \frac{v}{c_p}\left[\frac{v-b}{v - \dfrac{2a}{RTv^2}(v-b)^2} - 1\right] = \frac{v}{c_p}\left[\frac{2a(v-b)^2 - RTbv^2}{RTv^3 - 2a(v-b)^2}\right]\end{aligned}$$

The equation of the inversion curve is obtained by setting $\mu_J = 0$, which gives,

$$T = \frac{a}{bv}\left(1 - \frac{b}{v}\right)^2$$

Substituting this equation into the van der Waals equation gives,

$$p = \frac{a}{bv}\left(2 - \frac{3b}{v}\right)$$

The above two equations are the defining equations for the inversion curve in parametric form. The equation connecting the inversion temperature and the corresponding pressure can be obtained by eliminating v between the preceding two equations.

14.18 Show that for a perfect gas the difference in specific heats is given by

$$c_p - c_V = \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right]\left(\frac{\partial v}{\partial T}\right)_p = pv\beta + v\beta\left(\frac{\partial u}{\partial v}\right)_T$$

where β is the coefficient of cubical/volume expansion.

Solution

The first law of thermodynamics applied to a closed system undergoing a reversible process states as follows:

$$dQ = du + pdv \tag{i}$$

As per second law of thermodynamics,

$$ds = \left(\frac{dQ}{T}\right)_{rev} \tag{ii}$$

Combining these Eqs (i) and (ii), we have

$$Tds = du + pdv$$

Since $h = u + pv$

$$dh = du + pdv + vdp = Tds + vdp$$

$$Tds = du + pdv = dh - vdp$$

Writing the relation for u taking T and v as independent, we have

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv = c_V dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

Similarly, writing relation for h taking T and p as independent, we have

$$dh = \left(\frac{\partial h}{\partial T}\right)_p dT + \left(\frac{\partial h}{\partial p}\right)_T dv = c_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp$$

Substituting the value of du and dh in the equation for TdS, we have

$$c_V dT + \left(\frac{\partial u}{\partial v}\right)_T dv + pdv = c_p dT + \left(\frac{\partial h}{\partial p}\right)_T dp - vdp$$

or $$c_V dT + \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right] dv = c_p dT - \left[v - \left(\frac{\partial h}{\partial p}\right)_T\right] dp$$

Since the above equation is true for any process, it will also be true for the case when $dp = 0$ and hence:

$$(c_p - c_V)(dT)_p = \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right](dv)_p$$

Or $$(c_p - c_V) = \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right]\left(\frac{\partial v}{\partial T}\right)_p$$

By definition, the above equation becomes

$$c_p - c_V = \left[p + \left(\frac{\partial u}{\partial v}\right)_T\right] v\beta = pv\beta + v\beta\left(\frac{\partial u}{\partial v}\right)_T$$

14.19 For a gas obeying van der Waals equation, find the value of coefficient of volume expansion β and isothermal compressibility K.

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

Solution

van der Waals equation is given by,

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

Rearranging this equation, we can write

$$p = \frac{RT}{v - b} - \frac{a}{v^2}$$

Now, for β, we require $\left(\frac{\partial u}{\partial T}\right)_p$. This can be found by writing the cyclic relation,

$$\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_v\left(\frac{\partial p}{\partial v}\right)_T = -1$$

Hence,
$$\left(\frac{\partial v}{\partial T}\right)_p = -\frac{\left(\frac{\partial p}{\partial T}\right)_v}{\left(\frac{\partial p}{\partial v}\right)_T}$$

From the van der Waals equation:

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

Also,
$$\left(\frac{\partial p}{\partial v}\right)_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3}$$

Hence,
$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p = \frac{1}{v}\left[-\frac{\left(\frac{\partial p}{\partial T}\right)_v}{\left(\frac{\partial p}{\partial v}\right)_T}\right]$$

or
$$\beta = \frac{1}{v}\left[-\frac{\frac{R}{v-b}}{\frac{-RT}{(v-b)^2}+\frac{2a}{v^3}}\right]\frac{Rv^2(v-b)}{RTv^3-2a(v-b)^2}$$

Also,
$$K = -\frac{1}{v}\left(\frac{\partial u}{\partial p}\right)_T = -\frac{1}{v}\left[\frac{1}{\frac{2a}{v^3}-\frac{RT}{(v-b)^2}}\right] = \frac{v^2(v-b)^2}{RTv^3-2a(v-b)^2} \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

14.20 Show that the internal energy of an ideal gas is a function of only temperature.
Solution

The equation of state for an ideal gas is given by,

$$p = \frac{RT}{v}$$

$$\text{But } \left(\frac{\partial u}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p = T\frac{R}{v} - p = p - p = 0$$

Thus, if the temperature remains constant, there is no change in internal energy with volume (and therefore, also with pressure). Hence, internal energy (u) is a function of temperature (T) only. $\overset{\textbf{Ans}}{\Longleftarrow}$

14.21 Show that specific heat at constant volume (c_V) of a van der Waals gas is only a function of temperature.

Solution

The van der Waals equation of state is given by,

$$p = \frac{RT}{v-b} - \frac{a}{v^2}$$

or
$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

or
$$\left(\frac{\partial^2 p}{\partial T^2}\right)_v = 0$$

Now,
$$\left(\frac{dc_V}{dv}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Hence,
$$\left(\frac{\partial c_V}{\partial v}\right)_T = 0$$

Thus, c_V of a van der Waals gas is independent of volume (and therefore, of pressure also). Hence, it is a function of temperature only. $\overset{\textbf{Ans}}{\Longleftarrow}$

14.22 Derive expression for the following when a gas obeys van der Waals equation,

$$\left(p + \frac{a}{v^2}\right)(v-b) = RT$$

(i) Change in internal energy; (ii) Change in enthalpy; and (iii) Change in entropy.

Solution

(i) Change in internal energy is given by:

$$du = c_V dT + \left[T\left(\frac{\partial p}{\partial T}\right)_v - p\right]dv$$

But,
$$\left(\frac{\partial p}{\partial T}\right)_v = \left[\frac{\partial}{\partial T}\left(\frac{RT}{v-b} - \frac{a}{v^2}\right)\right]_v = \frac{R}{v-b}$$

$\therefore$
$$\int_1^2 du = c_V \int_1^2 dT + \int_1^2 \left[T\left(\frac{R}{v-b}\right) - p\right]dv$$

$$= c_V \int_1^2 dT + \int_1^2 \left[T\left(\frac{R}{v-b}\right) - \left(\frac{RT}{v-b} - \frac{a}{v^2}\right)\right]dv$$

$$= c_V \int_1^2 dT + \int_1^2 \left[\frac{RT}{v-b} - \frac{RT}{v-b} + \frac{a}{v^2}\right]dv$$

$$= c_V \int_1^2 dT + \int_1^2 \frac{a}{v^2}dv$$

$$\therefore \qquad u_2 - u_1 = c_V(T_2 - T_1) + a\left(\frac{1}{v_1} - \frac{1}{v_2}\right) \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

(ii) Change in enthalpy:

$$dh = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$\left(\frac{\partial h}{\partial p}\right)_T = 0 + v - T\left(\frac{\partial v}{\partial T}\right)_p \qquad \text{(i)}$$

Let us consider $p = f(v, T)$:

$$\therefore \qquad dp = \left(\frac{\partial p}{\partial v}\right)_T dv + \left(\frac{\partial p}{\partial T}\right)_v dT$$

$$\therefore \qquad (dp)_T = \left(\frac{\partial p}{\partial v}\right)_T dv + 0 \quad (\text{as } dT = 0) \qquad \text{(ii)}$$

From Eq.(i),

$$(dh)_T = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right](dp)_T$$

Substituting the value of $(dp)_T$ from Eq.(ii), we get

$$(dh)_T = \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right]\left(\frac{\partial p}{\partial v}\right)_T dv$$

$$= \left[v\left(\frac{\partial p}{\partial v}\right)_T - T\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T\right] dv \qquad \text{(iii)}$$

Using the cyclic relation for p, v, T which is:

$$\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial T}{\partial p}\right)_T\left(\frac{\partial p}{\partial v}\right)_T = -1$$

$$\left(\frac{\partial v}{\partial T}\right)_p\left(\frac{\partial p}{\partial v}\right)_T = -\left(\frac{\partial p}{\partial T}\right)_v$$

Substituting this value in Eq.(iii), we get

$$(dh)_T = \left[v\left(\frac{\partial p}{\partial v}\right)_T + T\left(\frac{\partial p}{\partial T}\right)_v\right] dv \qquad \text{(iv)}$$

For van der Waals equation:

$$\left(\frac{\partial p}{\partial v}\right)_T = \frac{\partial}{\partial v}\left[\left(\frac{RT}{v-b}\right) - \frac{a}{v^2}\right]_T = -\frac{RT}{(v-b)^2} + \frac{2a}{v^3} \qquad \text{(v)}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{\partial}{\partial T}\left[\left(\frac{RT}{v-b}\right) - \frac{a}{v^2}\right]_v = \frac{R}{v-b} \qquad \text{(vi)}$$

Substituting the values of Eqs (v) and (vi) in Eq.(i), we get

$$(dh)_T = \left[v\left(-\frac{RT}{(v-b)^2} + \frac{2a}{v^3}\right) + T\left(\frac{R}{v-b}\right)\right]dv$$

$$\int_1^2 (dh)_T = -RT\int_1^2 \frac{v}{(v-b)^2}dv + 2a\int_1^2 \frac{dv}{v^2} + RT\int_1^2 \frac{dv}{(v-b)}$$

$$(h_2-h_1)_T = -RT\left[\log_e\left(\frac{v_2-b}{v_1-b}\right) - b\left(\frac{1}{v_2-b} - \frac{1}{v_1-b}\right)\right] - 2a\left(\frac{1}{v_2} - \frac{1}{v_1}\right) + RT\log_e\left(\frac{v_2-b}{v_1-b}\right)$$

$$= bRT\left[\frac{1}{(v_2-b)} - \frac{1}{(v_1-b)}\right] - 2a\left[\frac{1}{v_2} - \frac{1}{v_1}\right] \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

(iii) The change in entropy is given by:

$$ds = c_p\frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_v dv$$

For van der Waals equation:

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v-b}$$

$$ds = c_V\frac{dT}{T} + \frac{R}{v-b}dv$$

$$\therefore \qquad \int_1^2 ds = c_V\int_1^2\left[\frac{dT}{T}\right] + R\int_1^2 \frac{dv}{(v-b)}$$

$$s_2 - s_1 = c_V\log_e\left[\frac{T_2}{T_1}\right] + R\log_e\left[\frac{v_2-b}{v_1-b}\right] \qquad \overset{\textbf{Ans}}{\Longleftarrow}$$

14.23 For a given range of pressure and temperature, the equation of state is given by

$$v = \frac{RT}{p} - \frac{C}{T^3}$$

where C is constant. Derive an expression for change of enthalpy and entropy for this substance during an isothermal process.

Solution

The general equation for finding dh is given by,

$$dh = c_p dT + \left[v - T\left(\frac{\partial v}{\partial T}\right)_p\right] dp$$

$$\int_1^2 dh = 0 + \int_1^2 \left[\left(v - T\left(\frac{\partial v}{\partial T}\right)_p\right)\right]_T$$

As $dT = 0$ for isothermal change, from the given equation of state, we have

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p} + \frac{3C}{T^4} \qquad \text{(i)}$$

$$h_2 - h_1 = \int_1^2 \left[\left(\left(\frac{RT}{p} - \frac{C}{T^3}\right) - \frac{RT}{p} - \frac{3C}{T^3}\right) dp\right]_T$$

$$= \left[\int_1^2 \left(-\frac{4C}{T^3}\right) dp\right]_T = -\frac{4C}{T^3}[p_2 - p_1]_T$$

The general equation for finding ds is given by,

$$ds = c_p \frac{dT}{T} - \left(\frac{\partial v}{\partial T}\right)_p dp$$

$$\int_1^2 ds = \left[-\int_1^2 \left(\frac{\partial v}{\partial T}\right)_p dp\right]_T$$

as $dT = 0$ for isothermal change.

Substituting the value from Eq.(i), we get

$$(s_2 - s_1) = \left[\int_1^2 -\left(\frac{R}{p} + \frac{3C}{T^4}\right) dp\right]_T$$

$$= -R\log_e\left(\frac{p_2}{p_1}\right) - \left(\frac{3C}{T^4}\right)(p_2 - p_1) \qquad \underleftarrow{\textbf{Ans}}$$

14.24 Show that c_V and c_p are independent of pressure for a perfect gas obeying $pv = RT$.

Solution

Let $s = f(T, v)$. Then, $ds = \left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv$

Also, $u = f(T, v)$. Then, $du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv = c_V dT + \left(\frac{\partial u}{\partial v}\right)_T dv$

Also, $du = Tds - pdv$

$$Tds - pdv = c_V dT + \left(\frac{\partial u}{\partial v}\right)_T dv$$

$$ds = c_V \frac{dT}{T} + \frac{1}{T}\left[\left(\frac{\partial u}{\partial v}\right)_T + p\right] dv$$

Equating the coefficient of dT in the two equations of ds, we have

$$\frac{c_V}{T} = \left(\frac{\partial s}{\partial T}\right)_v$$

$$c_V = T\left(\frac{\partial s}{\partial T}\right)_v$$

$$\left(\frac{\partial c_V}{\partial v}\right)_T = T\frac{\partial^2 s}{\partial T \partial v}$$

From Maxwell equation, we have

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

$$\frac{\partial^2 s}{\partial v \partial T} = \left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

$$\left(\frac{\partial c_V}{\partial v}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v$$

Also $p = \dfrac{RT}{v}$ (given):

$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v}$$

$$\left(\frac{\partial^2 p}{\partial T^2}\right)_v = 0 \quad \text{or} \quad \left(\frac{\partial c_V}{\partial v}\right)_T = 0$$

This shows that c_V is a function of T alone or c_V is independent of pressure. Also,

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p$$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = T\left(\frac{\partial^2 s}{\partial T \partial p}\right)$$

From Maxwell equation, we have

$$\left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial v}{\partial T}_p\right)$$

$$\frac{\partial^2 s}{\partial p \partial T} = -\left(\frac{\partial^2 v}{\partial T^2}_p\right)$$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p \text{ (Given)}$$

Again,

$$v = \frac{R}{p}$$

$$\left(\frac{\partial v}{\partial T}\right)_p = \frac{R}{p}$$

And,

$$\left(\frac{\partial^2 v}{\partial T^2}\right)_p = 0$$

$$\left(\frac{\partial c_p}{\partial p}\right)_T = 0$$

This shows that c_p is a function of T alone or c_p is independent of pressure. $\overset{\textbf{Ans}}{\Longleftarrow}$

14.25 Derive the remaining three using the first Maxwell equation.
Solution

The first Maxwell relation is as follows:

$$\left(\frac{\partial T}{\partial v}\right)_s = -\left(\frac{\partial p}{\partial s}\right)_v \qquad \text{(i)}$$

(1) Using the cyclic relation:

$$\left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial s}\right)_T \left(\frac{\partial s}{\partial T}\right)_v = -1$$

$$\left(\frac{\partial s}{\partial v}\right)_T = -\left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial s}{\partial T}\right)_v \qquad \text{(ii)}$$

Substituting the value from Eq.(i) in Eq.(ii), we get

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial s}\right)_v \left(\frac{\partial s}{\partial T}\right)_v \qquad \text{(iii)}$$

Using the chain rule,

$$\left(\frac{\partial p}{\partial s}\right)_v \left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial T}{\partial p}\right)_v = 1 \qquad \text{(iv)}$$

Substituting the value of Eq. (iv) in Eq.(iii), we get

$$\left(\frac{\partial s}{\partial v}\right)_T = \left(\frac{\partial p}{\partial T}\right)_v$$

This is Maxwell's third relation.

(2) Again using the cyclic relation:

$$\left(\frac{\partial s}{\partial p}\right)_v \left(\frac{\partial p}{\partial v}\right)_s \left(\frac{\partial v}{\partial s}\right)_p = -1$$

$$\left(\frac{\partial v}{\partial s}\right)_p = -\left(\frac{\partial p}{\partial s}\right)_v \left(\frac{\partial v}{\partial s}\right)_s \tag{v}$$

Substituting the value from Eq.(i) into Eq. (v),

$$\left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial p}\right)_s \tag{vi}$$

Again using chain rule,

$$\left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial p}\right)_s \left(\frac{\partial p}{\partial T}\right)_s = 1$$

Substituting the value from Eq.(vi) into Eq. (v), we get

$$\left(\frac{\partial v}{\partial s}\right)_p = \left(\frac{\partial T}{\partial p}\right)_s$$

This is Maxwell's second relation.

(3) $$\left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v \left(\frac{\partial p}{\partial v}\right)_T = -1$$

$$\begin{aligned}\left(\frac{\partial v}{\partial T}\right)_p &= -\left(\frac{\partial p}{\partial T}\right)_v \left(\frac{\partial v}{\partial p}\right)_T \\ &= -\left(\frac{\partial p}{\partial s}\right)_v \left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial s}{\partial p}\right)_T \left(\frac{\partial v}{\partial s}\right)_T\end{aligned}$$

Substituting the value from Eq.(i), we get

$$\left(\frac{\partial v}{\partial T}\right)_p = \left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial s}{\partial T}\right)_v \left(\frac{\partial s}{\partial p}\right)_T \left(\frac{\partial v}{\partial s}\right)_T$$

$$= \left(\left(\frac{\partial T}{\partial v}\right)_s \left(\frac{\partial v}{\partial s}\right)_T \left(\frac{\partial s}{\partial T}\right)_v\right) \left(\frac{\partial s}{\partial p}\right)_T = -\left(\frac{\partial s}{\partial p}\right)_T$$

$$\therefore \left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T$$

This is Maxwell's fourth relation. $\overset{\textbf{Ans}}{\Longleftarrow}$

Review Questions

14.1 Define the coefficient of volume and isothermal compressibility.

14.2 Mention the condition for a differential equation to be exact.

14.3 Derive Maxwell relation and explain their importance in thermodynamics.

14.4 Derive the expressions for c_p *and* c_V *based on first and second* Tds *equations and explain the significance of the above equations.*

14.5 Define the following: (a) isothermal compressibility (b) volume expansivity.

14.6 Prove that the slope of an isentrope is greater than that of an isotherm.

14.7 What is energy equation? Explain.

14.8 Show that enthalpy and internal energy of an ideal gas is a function of temperature only.

14.9 Why are $dU = c_V dT$ *and* $dH = c_p dT$ *true for any process of an ideal gas, whereas at constant, only these are true for any other substance?*

14.10 Write a short note on Joule-Kelvin effect.

14.11 What do you understand by inversion temperature?

14.12 What is Joule-Thomson coefficient?

14.13 What is the value of Joule-Thomson for an ideal gas?

14.14 Explain why hydrogen need to be precooled before throttling to get the cooling effect.

14.15 Explain Clapeyron equation in thermodynamics?

14.16 Explain the reasons for the maximum temperature drop when the state before throttling lies on the inversion curve.

14.17 What do you understand by phase-change?

14.18 Why Gibbs function remains constant during phase-change?

14.19 What are the characteristics of phase-change?

14.20 Write down the representative equation for phase-change.

14.21 Why is the fusion line of water have negative slope on p-T variation?

14.22 From equation of state, how thermodynamic properties are evaluated?

14.23 Why is the slope of the sublimation curve at triple point on the p-T diagram is greater than that of vaporisation curve at the same point?

14.24 How enthalpy change and entropy change of a gas are estimated from the equation of state? Explain.

14.25 Mention some important thermodynamic criteria which an equation of state must satisfy.

Exercise

14.1 Derive the following:

(i) $u = a - T\left(\frac{\partial a}{\partial T}\right)_v$ (ii) $h = g - T\left(\frac{\partial g}{\partial T}\right)_p$

(iii) $c_V = -T\left(\frac{\partial^2 a}{\partial T^2}\right)_v$ (iv) $c_p = -T\left(\frac{\partial^2 g}{\partial T^2}\right)_p$

where a is Helmholtz function (per unit mass) and g is Gibbs function (per unit mass).

14.2 Derive an expression for ds in terms of dT and dp.

14.3 Derive the following relations:

(i) $\left(\frac{\partial T}{\partial p}\right)_s = \frac{Tv\beta}{c_p}$ (ii) $\left(\frac{\partial T}{\partial v}\right)_s = -\frac{T\beta}{c_V K}$

where β is coefficient of cubical expansion and K is isothermal compressibility.

14.4 Show that

$$Tds = c_V\left(\frac{\partial T}{\partial p}\right)_v dp + c_p\left(\frac{\partial T}{\partial v}\right)_p dv$$

and also show that this may be written as:

$$Tds = \frac{c_V}{\beta}Kdp + \frac{c_p}{\beta v}dv$$

14.5 Using the Maxwell relation derive the following Tds equation:

$$Tds = c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp$$

14.6 Show for an ideal gas:

$$\left(\frac{\partial T}{\partial v}\right)_u = \frac{T\left(\frac{\partial p}{\partial T}\right)_v - p}{c_V}$$

14.7 Prove that for any fluid:

(i) $\left(\dfrac{\partial h}{\partial v}\right)_T = v\left(\dfrac{\partial p}{\partial v}\right)_T + T\left(\dfrac{\partial p}{\partial T}\right)_v$

(ii) $\left(\dfrac{\partial h}{\partial p}\right)_T = v - T\left(\dfrac{\partial v}{\partial T}\right)_p$

(iii) Show that for a fluid obeying van der Waals equation $p = \dfrac{RT}{v-b} - \dfrac{a}{v^2}$, where R, a and b are constants.

(iv) $h(enthalpy) = \dfrac{RTb}{v-b} - \dfrac{2a}{v} + f(T)$, where $f(T)$ is an arbitrary function.

14.8 Derive the following relations:

(i) $\left(\dfrac{\partial h}{\partial p}\right)_T = v - T\left(\dfrac{\partial v}{\partial T}\right)_p = -c_p\left(\dfrac{\partial T}{\partial p}\right)_h$

(ii) $\left(\dfrac{\partial u}{\partial v}\right)_T = T\left(\dfrac{\partial p}{\partial T}\right)_v - p$

(iii) Using (ii) show that $\left(\dfrac{\partial u}{\partial p}\right)_T = -T\left(\dfrac{\partial v}{\partial T}\right)_p - p\left(\dfrac{\partial v}{\partial p}\right)_T$

(iv) $c_p\left(\dfrac{\partial T}{\partial p}\right)_h$ is called *Joule-Thomson cooling effect.* Show that this cooling effect for a gas obeying the equation of state $(v-b) = \dfrac{RT}{p} - \dfrac{C}{T^2}$ is equal to $\left(\dfrac{3C}{T^2}\right) - b$.

14.9 The pressure on a solid copper of 1 kg is increased from 2000 kPa to 80000 kPa in a reversible process maintaining the temperature constant at 15 °C. Calculate (i) work done on the copper during the process, (ii) change in entropy, (iii) heat transfer, (iv) change in internal energy and (v) $(c_p - c_V)$ for this change of state. Given: β (Volume expansibility) $= 5 \times 10^{-5}$ K, K (thermal compressibility) $= 8.6 \times 10^{-12}$ m^{2}N and v (specific volume) $= 0.114 \times 10^{-3}$ m^3/kg.

Ans: (i) −3.135 J/kg; (ii) −0.446 J/kg K; (iii) −128 J/kg;
(iv) −124.8 J/kg; (v) 9.54 J/kg K

14.10 Using Clausius-Claperyon's equation, estimate the enthalpy of vaporisation. The following data is given: At 200 °C: $v_g = 0.1274$ m^3/kg; $v_f = 0.001157$ m^3/kg; $\left(\dfrac{dp}{dT}\right) = 32$ kPa/K.

Ans: 1910.8 kJ/kg

14.11 An ice skate is able to glide over the ice because the skate blade exerts sufficient pressure on the ice that a thin layer of ice is melted. The skate blade then glides over this thin melted water layer. Determine the pressure an ice skate blade must exert to allow smooth ice skate at −10 °C. The following data are given for the range of temperature and pressures involved: $h_{fg(ice)} = 334$ kJ/kg; $v_{liq} = 1 \times 10$ m^3/kg; $v_{ice} = 1.01 \times 10^3$ m^3/kg.

Ans: 13.47 bar

14.12 The following relation hold good for mercury between saturation pressure in bar and saturation temperature in K:

$$\log_{10} p = 7.0323 - \frac{3276.6}{T} - 0.652 \log_{10} T$$

Calculate the specific volume v_g of saturation mercury vapour at 0.1 bar. Given that the latent heat of vaporisation at 0.1 bar is 294.54 kJ/kg. Neglect the specific volume of saturated mercury liquid. *Ans:* 2.139 m^3/kg

14.13 For a pure substance, the entropy can be expressed as a function of the two independent variables p and v. Derive the following relation using the method of partial derivatives:

$$Tds = \left(\frac{\kappa c_V}{\beta}\right)dp + \left(\frac{c_p}{v\beta}\right)dv$$

14.14 Following the method of partial derivatives, derive the following partial derivatives of the internal energy:

$$\begin{aligned}\left(\frac{\partial u}{\partial T}\right)_v &= c_p - \frac{Tv\beta^2}{\kappa}\\ \left(\frac{\partial u}{\partial p}\right)_v &= \left(\frac{c_p\kappa}{\beta}\right) - Tv\beta\end{aligned}$$

14.15 Derive the relations to estimate the values of $\left(\frac{\partial u}{\partial p}\right)_T$ and $\left(\frac{\partial u}{\partial v}\right)_T$ in terms of measurable quantities.

14.16 The enthalpy of a pure substance can be expressed as a function of any two of the independent variables p, v and T. Using the method of partial derivatives, derive the following partial derivatives of enthalpy in terms of the measurable properties, whose knowledge enables us to estimate the change in the enthalpy associated with the change in the independent variables: $\left(\frac{\partial h}{\partial v}\right)_T$; $\left(\frac{\partial h}{\partial p}\right)_T$; $\left(\frac{\partial h}{\partial p}\right)_v$; $\left(\frac{\partial h}{\partial T}\right)_v$.

14.17 Using the method of partial derivatives, derive the relations to estimate the values of $\left(\frac{\partial h}{\partial v}\right)_p$ and $\left(\frac{\partial h}{\partial T}\right)_p$.

14.18 Derive the following relations by following the method of partial derivatives.

$$\text{(i)}\ \left(\frac{\partial h}{\partial s}\right)_p = T \quad \text{(ii)}\ \left(\frac{\partial h}{\partial s}\right)_v = T\left(1 + \frac{v\beta}{c_V\kappa}\right) \quad \text{(iii)}\ \left(\frac{\partial h}{\partial v}\right)_s = -\frac{\gamma}{\kappa} \quad \text{(iv)}\ \left(\frac{\partial h}{\partial s}\right)_T = T - \frac{1}{\beta}$$

14.19 Employing the method of partial derivatives, show that

$$\text{(i)}\quad a - T\left(\frac{\partial a}{\partial T}\right)_v = u \qquad \text{(ii)}\quad g - T\left(\frac{\partial g}{\partial T}\right)_p = h$$

14.20 Using the method of partial derivatives, prove that

$$\text{(i)}\quad \left(\frac{\partial u}{\partial p}\right)_s = \frac{pv\kappa}{\gamma} \qquad \text{(ii)}\quad \left(\frac{\partial u}{\partial T}\right)_s = \frac{pc_V\kappa}{T\beta}$$

14.21 Using the method of partial derivatives, prove that

$$\text{(i)}\quad \left(\frac{\partial T}{\partial p}\right)_h = \frac{v}{c_p}(T\beta - 1) \qquad \text{(ii)}\quad \left(\frac{\partial T}{\partial s}\right)_u = \frac{T}{c_V}\left(1 - \frac{T\beta}{p\kappa}\right)$$

14.22 Suppose an ideal gas is compressed reversibly and adiabatically, what is the relation between the temperature ratio and the pressure ratio?

14.23 Prove that $\left(\frac{\partial c_V}{\partial v}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_v$

14.24 Prove that $\left(\frac{\partial c_p}{\partial p}\right)_T = -T\left(\frac{\partial^2 v}{\partial T^2}\right)_p$

14.25 Determine the isothermal compressibility of an ideal gas, a Clausis gas and a van der Waals gas, respectively.

Multiple Choice Questions (choose the most appropriate answer)

1. If $Z = f(x, y)$ with suitable substitution we can write
   (a) $\left(\frac{\partial m}{\partial y}\right)_x = \left(\frac{\partial^2 Z}{\partial x \partial y}\right)$
   (b) $\left(\frac{\partial N}{\partial x}\right)_y = \left(\frac{\partial^2 Z}{\partial y \partial x}\right)$
   (c) $\left(\frac{\partial m}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$
   (d) All the above

2. If f is a function of x, y and z, then
   (a) $\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = 1$
   (b) $\left(\frac{\partial x}{\partial y}\right)_f + \left(\frac{\partial y}{\partial z}\right)_f + \left(\frac{\partial z}{\partial x}\right)_f = 1$
   (c) $\left(\frac{\partial x}{\partial y}\right)_f - \left(\frac{\partial y}{\partial z}\right)_f - \left(\frac{\partial z}{\partial x}\right)_f = 1$
   (d) $\left(\frac{\partial x}{\partial y}\right)_f \left(\frac{\partial y}{\partial z}\right)_f \left(\frac{\partial z}{\partial x}\right)_f = -1$

3. If $x = x(y, z)$, then
   (a) $\left(\frac{\partial y}{\partial z}\right)_y \left(\frac{\partial z}{\partial x}\right)_x \left(\frac{\partial x}{\partial y}\right)_z = -1$
   (b) $\left(\frac{\partial z}{\partial x}\right)_y \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial x}{\partial y}\right)_z = 1$
   (c) $\left(\frac{\partial y}{\partial z}\right)_y \left(\frac{\partial z}{\partial x}\right)_x \left(\frac{\partial x}{\partial y}\right)_z = 1$
   (d) $\left(\frac{\partial y}{\partial z}\right)_y + \left(\frac{\partial z}{\partial x}\right)_x + \left(\frac{\partial x}{\partial y}\right)_z = 1$

4. For a single-phase pure substance undergoing an infinitesimal reversible process, the change in entropy dH is given by

   (a) $dH = Tds - vdp$ (c) $dH = Tds + pdv$
   (b) $dH = Tds + vdp$ (d) $dH = Tds - pdv$

5. For a single-phase substance undergoing an infinitesimal reversible process, the change in $\ldots dF$ is given by

   (a) $dF = pdv - SdT$ (c) $dF = -pdv + SdT$
   (b) $dF = pdv + SdT$ (d) $dF = -pdv - SdT$

6. For a single-phase substance undergoing an infinitesimal reversible process, the change in Gibbs energy du is given by

   (a) $dH + SdT$ (c) $dH - SdT$
   (b) $Vdp + SdT$ (d) $vdp + pdV$

7. According to Maxwell relation

   (a) $\left(\frac{\partial T}{\partial V}\right)_s - \left(\frac{\partial p}{\partial S}\right)_V = 0$ (c) $\left(\frac{\partial V}{\partial T}\right)_s - \left(\frac{\partial S}{\partial p}\right)_V = 0$

   (b) $\left(\frac{\partial T}{\partial V}\right)_s + \left(\frac{\partial p}{\partial S}\right)_V = 0$ (d) $\left(\frac{\partial V}{\partial T}\right)_s + \left(\frac{\partial S}{\partial p}\right)_V = 0$

8. According to Maxwell relation

   (a) $\left(\frac{\partial p}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$ (c) $\left(\frac{\partial p}{\partial T}\right)_s = \left(\frac{\partial S}{\partial V}\right)_T$

   (b) $\left(\frac{\partial p}{\partial T}\right)_p = \left(\frac{\partial S}{\partial V}\right)_V$ (d) $\left(\frac{\partial p}{\partial T}\right)_p = \left(\frac{\partial S}{\partial V}\right)_p$

9. According to Maxwell relation

   (a) $\left(\frac{\partial V}{\partial T}\right)_p - \left(\frac{\partial s}{\partial p}\right)_T = 0$ (c) $\left(\frac{\partial V}{\partial T}\right)_V - \left(\frac{\partial S}{\partial p}\right)_p = 0$

   (b) $\left(\frac{\partial V}{\partial T}\right)_p + \left(\frac{\partial S}{\partial p}\right)_T = 0$ (d) $\left(\frac{\partial V}{\partial T}\right)_p - \left(\frac{\partial S}{\partial p}\right)_s = 0$

10. The coefficient of volume expansion is given by

    (a) $\beta = \frac{1}{v}\left(\frac{\partial V}{\partial T}\right)_p$ (c) $\beta = \frac{1}{T}\left(\frac{\partial V}{\partial T}\right)_p$

    (b) $\beta = \frac{1}{p}\left(\frac{\partial V}{\partial T}\right)_v$ (d) $\beta = \frac{1}{S}\left(\frac{\partial V}{\partial T}\right)_S$

11. Isothermal compressibility is given by

    (a) $K_i = \frac{1}{v}\left(\frac{\partial V}{\partial p}\right)_T$ (c) $K_i = \frac{1}{T}\left(\frac{\partial V}{\partial p}\right)_T$

    (b) $K_i = \frac{1}{p}\left(\frac{\partial V}{\partial p}\right)_T$ (d) $K_i = -\frac{1}{v}\left(\frac{\partial V}{\partial p}\right)_T$

12. The ratio of β and K_i is given by

(a) $\dfrac{\beta}{K_i} = \left(\dfrac{\partial p}{\partial T}\right)_v$ (c) $\dfrac{\beta}{K_i} = \left(\dfrac{\partial p}{\partial V}\right)_T$

(b) $\dfrac{\beta}{K_i} = \left(\dfrac{\partial T}{\partial p}\right)_v$ (d) $\dfrac{\beta}{K_i} = \left(\dfrac{\partial V}{\partial p}\right)_T$

13. Choose the correct Tds relation.

(a) $Tds = c_V dT - \left(\dfrac{\partial p}{\partial T}\right)_T dv$

(b) $Tds = c_V dT + \left(\dfrac{\partial p}{\partial T}\right)_T dv$

(c) $Tds = c_V dT - \left(\dfrac{\partial T}{\partial p}\right)_T dv$

(d) $Tds = c_V dT + \left(\dfrac{\partial T}{\partial p}\right)_T dv$

14. Tds in terms of s, T and p is given by

(a) $Tds = T\left(\dfrac{\partial s}{\partial T}\right)_v dT + T\left(\dfrac{\partial s}{\partial p}\right)_T dp$

(b) $Tds = T\left(\dfrac{\partial s}{\partial T}\right)_v dT - T\left(\dfrac{\partial s}{\partial p}\right)_T dp$

(c) $Tds = T\left(\dfrac{\partial s}{\partial T}\right)_p dT + T\left(\dfrac{\partial s}{\partial p}\right)_T dp$

(d) $Tds = T\left(\dfrac{\partial s}{\partial T}\right)_p dT - T\left(\dfrac{\partial s}{\partial p}\right)_T dp$

15. Tds in terms of $c_p dT$ is given by

(a) $Tds = c_p dT - \left(\dfrac{\partial p}{\partial T}\right)_T dp$ (c) $Tds = c_p dT + T\left(\dfrac{\partial p}{\partial T}\right)_T dp$

(b) $Tds = c_p dT - T\left(\dfrac{\partial p}{\partial T}\right)_T dp$ (d) $Tds = c_p dT + \left(\dfrac{\partial T}{\partial p}\right)_T dp$

16. The difference in specific heats can be expressed as

(a) $c_p - c_V = T\left[\left(\dfrac{\partial s}{\partial T}\right)_p - \left(\dfrac{\partial s}{\partial T}\right)_v\right]$

(b) $c_p - c_V = T\left[\left(\dfrac{\partial s}{\partial T}\right)_v - \left(\dfrac{\partial s}{\partial T}\right)_p\right]$

(c) $c_p - c_V = T\left[\left(\dfrac{\partial s}{\partial T}\right)_v + \left(\dfrac{\partial s}{\partial T}\right)_v\right]$

(d) $c_p - c_V = T\left[\left(\dfrac{\partial s}{\partial T}\right)_p + \left(\dfrac{\partial s}{\partial T}\right)_p\right]$

17. The difference in specific heats can be given by $c_p - c_V = -T\left(\frac{\partial v}{\partial T}\right)_T\left(\frac{\partial p}{\partial v}\right)_T$. Choose the correct implication of the above equation.
    (a) $c_p - C - v$ is always positive
    (b) At absolute zero, $c_p = c_V$
    (c) When specific volume is minimum $c_p = c_V$
    (d) All of the above

18. The difference in specific heat in terms of β, K, T and v can be expressed as
    (a) $c_p - c_V = \dfrac{T\beta^2 v}{K}$
    (c) $c_p - c_V = \dfrac{T\beta^3 v}{K}$
    (b) $c_p - c_V = \dfrac{T\beta v}{K}$
    (d) $c_p - c_V = \dfrac{T\beta^3 v}{v}$

19. The entropy relation is given by
    (a) $dH = \left[c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp\right]$
    (b) $dH = \left[c_p dT + T\left(\frac{\partial v}{\partial T}\right)_p dp\right]$
    (c) $dH = \left[c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp\right] + vdp$
    (d) $dH = \left[c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp\right] - vdp$

20. Clapton equation is given by
    (a) $\dfrac{dp}{dT} = \dfrac{s_g - s_f}{v_g - v_f}$
    (c) $\dfrac{dp}{dT} = \dfrac{h_{fg}}{Tv_{fg}}$
    (b) $\dfrac{dp}{dT} = \dfrac{s_{fg}}{v_{fg}}$
    (d) All of the above

21. Joule-Thomson coefficient is given by
    (a) $\mu = \left(\dfrac{\partial T}{\partial p}\right)_H$
    (c) $\mu = \left(\dfrac{\partial T}{\partial p}\right)_T$
    (b) $\mu = \left(\dfrac{\partial T}{\partial p}\right)_V$
    (d) $\mu = \left(\dfrac{\partial T}{\partial p}\right)_s$

22. Joule-Thomson coefficient is given by
    (a) $\mu = \dfrac{1}{c_p}\left[T\left(\dfrac{\partial v}{\partial T}\right) - p\right]$
    (b) $\mu = \dfrac{1}{c_p}\left[T\left(\dfrac{\partial v}{\partial T}\right) + p\right]$

(c) $\mu = \dfrac{1}{c_p}\left[T\left(\dfrac{\partial v}{\partial T}\right) - v\right]$

(d) $\mu = \dfrac{1}{c_p}\left[T\left(\dfrac{\partial v}{\partial T}\right) + v\right]$

23. If $h = f(p, T)$ then

(a) $dH = \left[\left(\frac{\partial h}{\partial p}\right)_T dp + \left(\frac{\partial h}{\partial T}\right)_p dT\right]$

(b) $dH = \left[\left(\frac{\partial h}{\partial p}\right)_p dp + \left(\frac{\partial h}{\partial T}\right)_T dp\right]$

(c) $dH = \left[\left(\frac{\partial h}{\partial p}\right)_v dp + \left(\frac{\partial h}{\partial T}\right)_T dT\right]$

(d) $dH = \left[\left(\frac{\partial h}{\partial p}\right)_p dv + \left(\frac{\partial T}{\partial h}\right)_T dT\right]$

24. Maximum work that can be obtained from a system during a given process in which the initial and final temperatures of the system are equal to the surrounding temperature, is equal to

(a) decrease in Helmholtz function
(b) increase in Helmholtz function
(c) decrease in Gibbs free energy
(d) increase in Gibbs free energy

25. The maximum power that can be obtained from a flow process, in which the initial and final temperature of the flowing fluid are the same as the surroundings, is equal to

(a) the rate of decrease of Gibbs free energy

(b) the rate of increase of Gibbs free energy

(c) the rate of decrease of Helmholtz potential

(d) the rate of increase of Helmholtz potential

Ans: 1. – (d) 2. – (a) 3. – (a) 4. – (b) 5. – (d)
6. – (c) 7. – (b) 8. – (a) 9. – (b) 10. – (a)
11. – (d) 12. – (a) 13. – (b) 14. – (c) 15. – (b)
16. – (a) 17. – (d) 18. – (a) 19. – (c) 20. – (d)
21. – (a) 22. – (c) 23. – (a) 24. – (a) 25. – (a)

15

SUMMARY OF ALL CHAPTERS

15.1 INTRODUCTION TO BASICS

- Study of thermodynamics can be undertaken either from a macroscopic or a microscopic point of view. For the great majority of engineering applications, macroscopic approach is more appropriate.
- A system is defined as a fixed quantity of matter or a region in space chosen for study. A system may be a closed, open or isolated system.
- A phase is a quantity of matter which is homogeneous throughout both in chemical composition and physical structure.
- A property is any macroscopic characteristic associated with a system. Thermodynamic properties, can be classified as intensive, extensive or specific.
- State is the unique condition of the system at an instant of time. It can be described or measured by its properties.
- State postulate is stated as 'the state of a simple compressible substance can be specified by two independent intensive properties'.
- The loci of various intermediate states passed through, by a system during a process is called path.
- A process is that, which causes the system to change its state.
- A quasi-static process denotes a process that is almost static. Infinite slowness is the characteristic feature of the quasi-static process.
- A cycle is the one in which the processes occur one after the other so as to finally bring the system to the same initial state.
- A phase is defined as the quantity of matter which is homogeneous throughout, both in chemical composition and physical structure.
- Broadly there are three types of equilibriums, viz chemical equilibrium, mechanical equilibrium, thermal equilibrium. A system is said to be in thermodynamic equilibrium when all the above three equilibrium conditions are simultaneously satisfied.
- In macroscopic thermodynamics, where the gross effect is of importance for a system and can be assigned with averaged properties then that system is called continuum.

- Pressure is defined as a normal force exerted by a fluid per unit area. There are four types of pressures, viz atmospheric pressure, absolute pressure; gauge pressure and vacuum pressure.

- Pressure is usually measured either by indicating gauges or recorders. These instruments may be mechanical, electromechanical, electrical or electronic in operation.

- The specific volume of a substance is defined as the volume per unit mass and is designated by the symbol v.

- Temperature is actually a measure of molecular activity. It is the tendency of a body to spontaneously give up energy to its surroundings.

- Zeroth law of thermodynamics states that if two systems are each equal in temperature to a third, they are equal in temperature to each other.

15.2 ENERGY, WORK AND HEAT

- Energy is normally viewed as the capacity to do work and has the ability to cause changes. Normally, energy is in the stored form.

- Work is nothing but an energy interaction between a system and surroundings. Heat is also a form of energy. Heat can be transferred between two systems or a system and its surrounding by virtue of a temperature difference between them.

- Only work and heat energy can cross the boundaries of a system. Neither heat nor work can exist as stored energy.

- Internal energy can be viewed as the energy needed to create the system. It involves energy on the microscopic scale. The internal energy is an extensive property.

- Work can be defined in two ways, viz by using the principle of mechanics and by using the principle of thermodynamics.

- Any quantity whose change is independent of the path is a point function and hence, it is a property. Because of this, characteristic, the state is a point function.

- The moving boundary or displacement work is given by $\delta W = \text{Force} \times \text{Distance} = pA(ds) = pdV$.

- For a constant volume process, $W = \int_1^2 pdV$ and for a constant pressure process, $W_d = \int_1^2 pdV = p\int_1^2 dV = p(V_2 - V_1)$

- For a constant pressure process, $W_d = p_1V_1 \ln\left(\frac{V_2}{V_1}\right) = p_2V_2 \ln\left(\frac{V_2}{V_1}\right) (\because p_1V_1 = p_2V_2)$

- For a polytropic process, $W_d = \frac{(p_2V_2 - p_1V_1)}{1-n} = \frac{(p_1V_1 - p_2V_2)}{n-1}$

- Mean effective pressure is that pressure which, when multiplied by the displacement volume gives the same net work as actually produced with the varying pressure.

- Displacement work for an unresisted process, $W_d = 0$.

- Gravitational work is given by $W_g = mg(z_2 - z_1)$.
- Shaft work is given by $W_s = \tau\theta$ and $\dot{W} = \omega\tau = 2\pi N\tau$.
- Spring work is given by $W_{\text{spring}} = \frac{1}{2}K\left(L_2^2 - L_1^2\right)$.
- Electric work done by battery is given by $\delta W_{bat} = -EIdt$.
- Work done during the extension of an elastic rod with both ends free is given by $\delta W = -V\sigma AdL$.
- Work done during the extension of an elastic rod with one end fixed: $W_{\text{elastic}} = -\int_1^2 \sigma dV = -\int_1^2 \sigma AdL$
- Work done during the stirring of fluid, $W = \int_1^2 mgdL = \int_1^2 WdL = \int_1^2 \tau d\theta$
- Thermodynamic definition of work: work is said to be done by a system on its surroundings, if, the sole effect, external to the system, could be reduced to the raising of a weight through a distance.
- Sign convention for work: Work done by a system (work out) is positive (+ve). Work done on a system (work in) is negative (−ve).
- Sign convention for heat: Heat input to the system is positive (+ve) and heat rejection is negative (−ve).

15.3 FIRST LAW OF THERMODYNAMICS AND CLOSED SYSTEMS

- On the whole there are four laws of thermodynamics, viz Zeroth law, First law, Second law and Third law of thermodynamics.
- First law of thermodynamics is the embodiment of conservation of energy which can be stated as: 'Energy can neither be created nor be destroyed during a process; it can only change forms'.
- According to Joule's experiment the ratio W/Q is given by 4.12 J/calorie and its ratio is called mechanical equivalent of heat.
- a new unit of heat transfer, viz Joule, denoted by J has a magnitude of 1/4.189 of a calorie. Then, the mechanical equivalent of heat turns out to be 4.189 Nm/4.189 J. = 1. In SI system of units, both heat and work are measured in the derived unit of energy, viz the Joule. The constant of proportionality, J, is therefore, unity, i.e. 1 Nm/J.
- For a cyclic process, $(\sum W)_{cycle} = J(\sum Q)_{cycle}$ and in integral form, $\oint \delta W = J \oint \delta Q$.
- Energy (E) is a point function and thereby it is the property of the system. E is an extensive property and specific energy, $e = E/m$, is an intensive property. The cyclic integral of any property is zero, since, the final state is exactly identical with the initial state.
- The three principles embedded in the first law are (i) law of conservation of energy (ii) the flow of heat is a form of energy transfer and (iii) performing work is also a form of energy transfer.

- Clausius statement of first law: In a thermodynamic process involving a closed system, the change in the internal energy is equal to the difference between the heat accumulated by the system and the work done by it.

- The energy of an isolated system is always constant, $dE = 0$ or $E =$ constant.

- The characteristic equation of state of an ideal gas is given by $pV = mRT$.

- Specific heat at constant volume c_V is defined as energy required to raise the temperature of the unit mass of a substance by one degree keeping the volume constant.

- Specific heat at constant pressure c_p is defined as energy required to raise the temperature of the unit mass of a substance by one degree keeping the pressure constant.

- The enthalpy of a substance, h, is defined as $h = u + pV$.

- According to Joules law, the internal energy of an ideal gas is a function of the absolute temperature only, i.e. $u = f(T)$.

- Relationship between two specific heats is given by $c_p - c_V = R$

- The equations for heat transfer and work transfer according to first law are given in Table 3.1 for various processes of a perfect gas.

15.4 FIRST LAW OF THERMODYNAMICS FOR OPEN SYSTEMS

- When there is a mass transfer across the system boundary, it is called an open system or control volume.

- For an open system while applying first law, we should take it into account both conservation of mass and conservation of energy.

- According to conservation of mass net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt. That is to say $m_{in} - m_{out} = \Delta m_{cv}$.

- For steady flows, the total rate of mass entering a control volume is equal to the total rate of mass leaving it, $\sum_{in} \dot{m} = \sum_{out} \dot{m}$.

- The equation for conservation of energy for an open system is given by:
  $Q - W + \sum E_{in} - \sum E_{out} = \Delta E_{CV}$ and is given by $W_{flow} = FL = pAL = pV$

- Total energy of a simple compressible system can be written as:
  $$e = e_{PE} + e_{KE} + e_{IE} = gz + \left(\frac{\mathbb{V}^2}{2}\right) + u$$

- The steady flow energy equation is given by
  $$\dot{Q} - \dot{W} = \sum \dot{m}_e\left(gz_e + \frac{\mathbb{V}_e^2}{2} + h_e\right) - \sum \dot{m}_i\left(gz_i + \frac{\mathbb{V}_i^2}{2} + h_i\right)$$

- According to second law, expressions for work for the following are:

(i)	Reciprocating compressor	$W = Q + (h_2 - h_1)$
(ii)	Rotary compressor,	$W = h_2 - h_1$
(iii)	Boiler, condenser, evaporator	$Q = h_2 - h_1$
(iv)	Centrifugal water pump	$W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (p_2v_2 - p_1v_1)$
(v)	Combustion chamber	$\dot{m}_f CV + (\dot{m}_a h_a + \dot{m}_f h_f) = \dot{m}_p h_p$
(vi)	Subsonic diffuser	$\frac{\mathbb{V}_2^2}{2} - \frac{\mathbb{V}_1^2}{2} = h_1 - h_2$
(vii)	Mixing chamber	$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$
(viii)	Steam nozzle	$\mathbb{V}_2 = \sqrt{2(h_1 - h_2)}$
(ix)	Steam de-superheater	$m_1h_1 + m_2h_2 = m_3h_3$, where $m_3 = m_1 + m_2$
(x)	Throttling valve	$u_1 + p_1V_1 = u_2 + p_2V_2$
(xi)	Steam turbine	$h_1 + \frac{\mathbb{V}_1^2}{2} - Q = h_2 + \frac{\mathbb{V}_2^2}{2} + W$
(xii)	Water turbine	$W = gz_1 + \frac{\vec{\mathbb{V}}_1^2 - \vec{\mathbb{V}}_2^2}{2} + V(p_1 - p_2)$

(i) Reciprocating compressor $W = Q + (h_2 - h_1)$
(ii) Rotary compressor, $W = h_2 - h_1$
(iii) Boiler, condenser, evaporator $Q = h_2 - h_1$
(iv) Centrifugal water pump $W = g(z_2 - z_1) + \frac{\mathbb{V}_2^2 - \mathbb{V}_1^2}{2} + (p_2v_2 - p_1v_1)$
(v) Combustion chamber $\dot{m}_f CV + (\dot{m}_a h_a + \dot{m}_f h_f) = \dot{m}_p h_p$
(vi) Subsonic diffuser $\frac{\mathbb{V}_2^2}{2} - \frac{\mathbb{V}_1^2}{2} = h_1 - h_2$
(vii) Mixing chamber $\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3$
(viii) Steam nozzle $\mathbb{V}_2 = \sqrt{2(h_1 - h_2)}$
(ix) Steam de-superheater $m_1h_1 + m_2h_2 = m_3h_3$ where $m_3 = m_1 + m_2$
(x) Throttling valve $u_1 + p_1V_1 = u_2 + p_2V_2$
(xi) Steam turbine $h_1 + \frac{\mathbb{V}_1^2}{2} - Q = h_2 + \frac{\mathbb{V}_2^2}{2} + W$
(xii) Water turbine $W = gz_1 + \frac{\vec{\mathbb{V}}_1^2 - \vec{\mathbb{V}}_2^2}{2} + V(p_1 - p_2)$

- First law of thermodynamics does not differentiate between heat and work. It assures full convertibility of one into other whereas full conversion of work into heat is possible but the vice-versa is not possible.

- First law of thermodynamics does not explain the direction of a process. Theoretically, it permits even heat transfer from low-temperature body to high-temperature body which is not practically feasible. Spontaneity of the process is not taken care of by the first law of thermodynamics.

15.5 SECOND LAW OF THERMODYNAMICS

- A body that can supply or absorb finite amounts of heat without undergoing any perceptible change in temperature is usually called a thermal energy reservoir or just a reservoir. Typical examples are oceans, lakes and rivers as well as the atmospheric air.

- While undergoing a cycle heat engines and other cyclic devices usually involve heat transfer to and from a fluid. This fluid is called the working fluid. The thermal efficiency of a heat engine can be expressed as

$$\text{Thermal efficiency} = \frac{\text{Net work output}}{\text{Total heat input}}$$

$$\eta_{th} = \frac{W_{net,out}}{Q_{in}} = \frac{Q_{in} - Q_{out}}{Q_{in}} = 1 - \frac{Q_{out}}{Q_{in}}$$

- Every heat engine must waste some amount of energy by transferring it to a low-temperature reservoir in order to complete the cycle. The most important requirement of a heat engine is to exchange heat with at least two reservoirs for continuous operation.

- Normally heat transfer takes place from a higher temperature source to lower temperature sink. For the reverse process to occur, we require some special device. Such a device is called refrigerator.

- The working fluid used in the refrigeration cycle is called a refrigerant. The most frequently used refrigeration cycle is the vapour-compression.

- The efficiency of a refrigerator is expressed in terms of the coefficient of performance (COP), which is expressed as
$$COP_{ref} = \frac{\text{Desired output}}{\text{Required input}} = \frac{Q_L}{W_{net,in}} = \frac{Q_L}{Q_H - Q_L} = \frac{1}{Q_H/Q_L - 1}$$

- Heat pump is another device that transfers heat from a low-temperature medium to a high- temperature one.

- COP of a heat pump is given by $COP_{HP} = \dfrac{Q_H}{Q_H - Q_L} = \dfrac{1}{1 - Q_L/Q_H} = COP_{ref} + 1$

- *Kelvin-Planck Statement*: It is impossible for any device that operates on a cycle to receive heat from a single reservoir and produce an equivalent amount of work. In other words, no process is possible whose sole result is the absorption of heat from a single reservoir and the conversion of this heat into equivalent amount of work.

- *Clausius Statement*: It is impossible to construct a device that operates in a thermodynamic cycle and produces no effect other than the transfer of heat from a lower temperature body to a higher temperature body. In other words, 'no process is possible whose sole result is the transfer of heat from a cooler to a hotter body'.

- A process is called internally reversible if no irreversibilities occur within the boundaries of the system during the process and a process is called externally reversible if no irreversibilities occur outside the system boundaries during the process.

- Thermal efficiency of a Carnot cycle is given by
$$\eta_{th} = \frac{\text{Work done}}{\text{Heat supplied by the source}} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H} = \left(1 - \frac{T_L}{T_H}\right)$$

- A temperature scale that is independent of the properties of the substances that are used to measure temperature is called a thermodynamic temperature scale.

- Temperature at which the energy is available determines the quality of energy. Higher the temperature of the thermal energy, higher is quality.

15.6 ENTROPY

- Entropy can be viewed as a measure of the energy that is dissipated away and is not available for work during a thermodynamic process. This unavailability of the energy is called entropy.

- Unlike energy, entropy is a non-conserved property. There is no such thing as conservation of entropy.

- it is always possible to replace any reversible process by a series of reversible processes such as reversible adiabatic and reversible isothermal. However, the heat interaction and work involved should remain the same.

- δQ is a thermodynamic property. For internally reversible engines, $\oint \left(\frac{\delta Q}{T}\right)_{int,rev} = 0$

- For a irreversible cycle engine, $\oint \left(\frac{\delta Q}{T}\right)_{\text{irrev}} < 0$

- For reversible and irreversible engines, it can be written as: $\oint \left(\frac{\delta Q}{T}\right)_{\text{irrev}} \leq 0$. This is called Clausius inequality.

- $\int \frac{\delta Q}{T}$ is the same for all reversible paths between states 1 and 2, i.e. it is independent of the path and is a function of end states only. Therefore, it is a property and named as entropy and designated as S.

- Entropy is an extensive property of a system and sometimes is referred to as total entropy, S and has the unit kJ/K. Entropy per unit mass, designated s, is an intensive property and given by kJ/kg K. The term, 'entropy' is generally used to refer to both total entropy and entropy per unit mass since the context usually clarifies which one is meant.

- The entropy, S, of a system can be defined as the property of a system whose change between the states 1 and 2 is given by $\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T}\right)_{\text{int, rev}}$. In differential form, $dS = \left(\frac{\delta Q}{T}\right)$.

- If during a process, the value of S, the change in entropy, is equal to $\int \frac{\delta Q}{T}$ such a process is reversible. If the change in entropy during a process is greater than $\int \frac{\delta Q}{T}$ such a process is irreversible. If the change in entropy during a process is less than $\int \frac{\delta Q}{T}$ such a process is impossible.

- A process shall always occur in such a direction in which either entropy does not change or increase. In general, almost all real processes are of irreversible in nature and therefore, entropy tends to increase. Entropy cannot be defined by an absolute value and the change in entropy should always have a positive or zero value.

- Entropy transfer can take place in two forms: (i) heat transfer and (ii) mass flow. Entropy can cross the boundary.

- The increase in entropy of any closed system can be achieved (i) by heat interaction in which there is entropy transfer (ii) due to irreversibilities or dissipative effects.

- By the second law, for open system $\dot{S}_{\text{gen}} \geq 0$. If the process is reversible, $\dot{S}_{\text{gen}} = 0$. For an irreversible process, $\dot{S}_{\text{gen}} > 0$.

- The two Tds relations, viz $Tds = du + pdv$ and $Tds = dh - vdp$ are very important and useful. They are very frequently used in calculating the entropy changes in majority of the cases.

- For a constant volume process, $\left(\frac{dT}{ds}\right)_{v=\text{const}} = \frac{T}{c_V}$ and for a constant pressure process, $\left(\frac{dT}{ds}\right)_{p=\text{const}} = \frac{T}{c_p}$. From the two expressions, it is clear that the slope of constant volume lines on a T-s plane are steeper than the constant pressure line as $c_V < c_p$.

- dS in terms of volume and temperature is given by: $dS = mR\ln\frac{V_2}{V_1} + mc_V\ln\frac{T_2}{T_1}$

- dS in terms of temperature and pressure:
  $dS = mR\ln\left(\frac{p_1}{p_2}\right) + mc_p\ln\left(\frac{T_2}{T_1}\right) = mc_p\ln\left(\frac{T_2}{T_1}\right) - mR\ln\left(\frac{p_2}{p_1}\right)$

- dS in terms of pressure and volume: $dS = mc_V\ln\left(\frac{p_2}{p_1}\right) + mc_p\ln\left(\frac{V_2}{V_1}\right)$

- dS in terms of constant volume process: $dS = mc_V\ln\left(\frac{T_2}{T_1}\right)$ and $dS = mc_V\ln\left(\frac{p_2}{p_1}\right)$

- dS in terms of constant pressure process: $dS = mc_p\ln\left(\frac{V_2}{V_1}\right)$ and $dS = mc_p\ln\left(\frac{T_2}{T_1}\right)$

- dS in terms of constant temperature process: $dS = mR\ln\left(\frac{V_2}{V_1}\right)$ and $dS = \frac{Q}{T}$

- dS in isentropic process $dS = 0$

- dS in terms of polytropic process: $dS = \left(\frac{\gamma - n}{\gamma - 1}\right)mR\ln\left(\frac{V_2}{V_1}\right)$

- Entropy of universe when there is transfer of heat through a finite temperature difference is given by $\Delta S_{\text{univ}} = \Delta S_1 + \Delta S_2 = \frac{-Q}{T_1} + \frac{Q}{T_2} = Q\left(\frac{T_1 - T_2}{T_1T_2}\right)$

- Entropy of universe during the mixing of two fluids is given by
  $\Delta S_{\text{univ}} = 2mC\ln\frac{(T_1 + T_2)/2}{\sqrt{T_1T_2}}$

- Maximum work from two finite bodies at temperatures T_H and $T_L$is given by $W_{\max} = c_p\left(\sqrt{T_1} - \sqrt{T_2}\right)^2$
- The third law of thermodynamics is the entropy of all perfect crystalline solids is zero at absolute zero temperature and is often referred to as Nernst law.

15.7 AVAILABLE AND UNAVAILABLE ENERGY

- The law of the loss for achieving maximum work states that the work obtained in real-life situation is always less than the maximum obtainable work due to the irreversibility in thermal processes.
- Energy can be conveniently categorised as low-grade energy and high-grade energy.
- Energy has two parts: available part and unavailable part. Available energy is the maximum portion of energy which could be converted into useful work by ideal processes. After the conversion, the system reduces to a dead state. A dead state is one which is in equilibrium with the earth and its atmosphere.
- If the first law is the law of conservation of energy, the second law is called the law of degradation of energy. Energy is always conserved but its quality is always degraded.
- The useful work per unit mass is given by $(W_u)_{max} = (u_1 + p_0 v_1 - T_0 s_1) - (u_2 + p_0 v_2 - T_0 s_2)$
- Effectiveness is defined as the ratio of actual useful work to the maximum useful work. Mathematically, it is written as: $\epsilon = \dfrac{W_{\text{useful}}}{W_{\text{max useful}}}$
- As per the first law of thermodynamics, the energy in the form of heat can be converted into work without losses. Then, the efficiency, η_I, is given by $\eta_I = \dfrac{\text{Work output}}{\text{Heat input}} = \dfrac{W}{Q}$
- Work is high-grade energy whereas heat is a low-grade energy. When $Q = W, \eta_I = 100\%$.
- As per second law of thermodynamics, all the energy absorbed as heat by an engine cannot be completely converted into work. Some part of the energy must be rejected to sink. $Q_1 = W + Q_2$, where W is useful work, $Q_1$and Q_2 are heat absorption and heat rejection, respectively. According to second law, the efficiency $\eta_{II} < 100\%$.
- Using availability analysis, we can show that for a closed system $AE = Q - T_0 \Delta S$ and $UE = Q - AE = T_0 \Delta S$.
- For a constant volume process, $Q = mc_V(T_2 - T_1)$ and $\Delta S = mc_V \ln\left(\dfrac{T_2}{T_1}\right)$
- For a constant pressure process, $Q = mc_p(T_2 - T_1)$ and $\Delta S = mc_p \ln\left(\dfrac{T_2}{T_1}\right)$
- For a constant temperature process, $Q = p_1 V_1 \ln\left(\dfrac{p_2}{p_1}\right)$ and $\Delta S = \dfrac{Q}{T_1}$ or $\frac{Q}{T_2}$

- For a polytropic process, $Q = (\gamma - n)\left(\dfrac{p_1V_1 - p_2V_2}{n-1}\right) = (\gamma - n)\left(\dfrac{mR[T_1 - T_2]}{n-1}\right)$ and
  $\Delta S = c_p \ln\left(\dfrac{T_2}{T_1}\right) - R\ln\left(\dfrac{p_2}{p_1}\right) = c_p \ln\left(\dfrac{T_2}{T_1}\right) + R\ln\left(\dfrac{V_2}{V_1}\right)$

- The second law efficiency for an open system is given by $\eta_{II} = \dfrac{W_{\text{act}}}{W_{max}}$

- For a compressor or pump, $W_{\text{act}} = m(h_2 - h_1) = Q$ and
  $W_{max} = m(h_2 - h_1) - T_0(s_2 - s_1)$

- For a heat exchanger, $\eta_{II} = \dfrac{\text{Availability of cold fluid}}{\text{Availability of hot fluid}}$, where availability of cold fluid,
  $B_c = m_2c_2(T_4 - T_3) - T_0(s_4 - s_3)$ and availability of hot fluid,
  $B_h = m_1c_1(T_1 - T_2) - T_0(s_4 - s_3)$

15.8 PROPERTIES OF PURE SUBSTANCES

- A pure substance is one which has a fixed chemical composition throughout. Air is a mixture of several gases and is most often considered as a pure substance because it has a uniform chemical composition.

- A phase is defined as having a distinct molecular arrangement that is homogenous

- *Gibbs phase rule*: It is expressed by the equation as $n = C - \phi + 2$, where n is the number of independent variable, C, the number of components, ϕ, the number of phases present in equilibrium.

- For water at very low pressure of 0.1 kPa, sublimation takes place when the phase-change takes place at constant pressure the temperature remains constant. At the pressure of 1 bar during the melting process and between 0 and 4 °C, there is a decrease in volume. This is peculiar to water.

- Dryness fraction is defined as the ratio of the mass of the dry steam (m_g) actually present to the mass of the total steam. It is denoted by x. The value of x varies between 0 and 1.

- Wetness fraction is defined as the ratio of the mass of water vapour in suspension to the total steam. Wetness fraction $1 - x$. The wetness fraction expressed in percentage, i.e. $100 \times (1 - x)$ is called Priming.

- Triple point is that state at which all the three phases of a pure substance coexist in equilibrium. For water, the triple point pressure and temperature are, 0.006117 bar and 0.01 °C, respectively.

- A critical point is that state beyond which it is difficult to distinguish between liquid and vapour phases. It can also be defined as that state at which liquid can be converted to vapour completely without any heat transfer.

- *For some important definitions, refer Section 8.8.*

- There are standard methods available for dryness fraction measurement such as throttling calorimeter, separating calorimeter, separating and throttling calorimeter and electrical calorimeter. *For details, refer Section 8.11.*

- Enthalpy of steam (h): It is the amount of heat added to the water from freezing point to till the water becomes wet or dry or superheated steam.

- Specific volume of steam (v): It is defined as the volume occupied by the unit mass of the steam at the given pressure and temperature.

- Density of steam (ρ): It is defined as the ratio of mass to the unit volume of the steam at given pressure and temperature. Its value for wet, dry and superheated steam is the reciprocal of the specific volume of the steam.

- Internal energy of steam (U): Internal energy of steam is defined as the actual heat energy stored in the steam above the freezing point of water at the given conditions. It is the difference between enthalpy of steam and the external work done.

- Entropy of steam (s): It is the property of the steam which increases with increase in temperature and vice versa.

- For various property relation of steam, refer Table 8.7.

15.9 PSYCHROMETRY

- Psychrometry is a general term used to describe physical and thermodynamic properties of gas-vapour mixtures. Other names for psychrometry is hygrometry or psychometrics. The principle of psychrometry is used in the field of air-conditioning.

- Air usually means moist air. However, in 1949, a standard composition of dry air was fixed by the International Joint Committee on Psychrometric data and is given in Table 9.1.

- The term, 'dry air' is used to indicate the water free contents of air having no degree of moisture.

- When the moisture content is maximum, the air is known as saturated air. For a given temperature, only a given quantity of air can be saturated with a fixed quantity of moisture.

- *Dry-Bulb Temperature* (t_{db}): It is the temperature of the moist air registered by a standard thermometer or any other temperature measuring instruments. It is abbreviated as DBT.

- *Wet-Bulb Temperature* (t_{wb}): It is the temperature registered by a thermometer when the thermometer bulb is covered by a wetted wick and is exposed to a current of rapidly moving air. It is abbreviated as WBT.

- *Saturated Vapour Pressure* (p_{sat}): It is the saturated partial pressure of water vapour at the corresponding dry-bulb temperature.

- *Adiabatic Saturation Temperature* (t_{wb}): It is the temperature at which the water or ice can saturate air by adiabatic evaporation into it. It is numerically equivalent to the measured thermodynamic wet-bulb temperature.
- *Wet-Bulb Depression* ($t_{db} - t_{wb}$): The difference between dry-bulb and wet-bulb temperature is called wet-bulb depression.
- *Dew Point Temperature* (t_{dp}): If unsaturated moist air is cooled at constant pressure, then the temperature at which the moisture in the air begins to condense is known as dew point.
- *Dew Point Depression* ($t_{db} - t_{dp}$): It is the difference between the dry-bulb and dew point temperatures.
- *Degree of Saturation, μ*: The degree of saturation is the ratio of the humidity ratio W to the humidity ratio of a saturated mixture W_s at the same temperature and pressure.
- *Sensible Heat*: It is the heat that changes the temperature of a substance when added to or removed from it.
- *Latent Heat*: It is the heat that does not affect the temperature but changes the state of substance when added to or removed from it.
- *Enthalpy* (h): It is the combination energy which represents the sum of internal and flow energy in a steady flow process.
- *Absolute Humidity*: Absolute humidity is the measure of water vapour (moisture) in the air, regardless of temperature. It is expressed as grams of moisture per cubic meter of air (g/m^3).
- *Specific Humidity or Humidity Ratio,* (W): It is defined as the ratio of the amount of water vapour in the air to the amount of dry air in the area.
- *Relative Humidity (ϕ)*: It is the ratio of the partial pressure of water vapour in the mixture to the saturated partial pressure at the dry-bulb temperature, expressed as percentage.
- *Sensible Cooling*: Air undergoes sensible cooling whenever it passes over a surface that is at a temperature lower than the dry-bulb temperature of the air but greater than the dew point temperature.
- *Sensible Heating*: When air passes over a dry surface which is at a temperature higher than its dry-bulb temperature of the flowing air, it is considered as sensible heating process.

15.10 PROPERTIES OF GASES AND GAS MIXTURES

- From the thermodynamic point of view, gases can be broadly divided into two categories. Ideal gas and Real gas.
- An ideal gas is an imaginary substance that obeys the equation of state at all pressures and temperatures. The equation of state is given by $pV = mRT$ or $pv = RT$.

- When the two ideal gases are mixed together, the state of components before mixing is same as the states of both the mixture of individual components after mixing.

- The total mass of gas after mixing = the sum of masses of different gases before mixing. $m_m = m_A + m_B + m_C$, where m_m is total mass of gas mixture and m_A, m_B, m_C is masses of gas A, B and C, respectively. Similarly, the total volume of gas mixture after mixing is the sum of volume of different gases before mixing, $V_m = V_A + V_B + V_C$, where V_A, V_B, V_C is volume of gas A, B and C respectively.

- Temperature T_m corresponds to the temperature of the mixture after thorough mixing: $T_m = \dfrac{(m_A c_{V_A} T_A + m_B c_{V_B} T_B + m_C c_{V_C} T_C)}{(m_A c_{V_A} + m_B c_{V_B} + m_C c_{V_C})}$

- The pressure of the mixture is given by $p_m = \dfrac{m_m R_m T_m}{V_m}$

- The change in enthalpy can be written as $\Delta H_m = \Delta H_A + \Delta H_B + \Delta H_C$

- In the case of mixture the temperature T_m is given by
  $T_m = \dfrac{(m_A c_{P_A} T_A + m_B c_{P_B} T_B + m_C c_{P_C} T_C)}{(m_A c_{P_A} + m_B c_{P_B} + m_C c_{P_C})}$

- Total entropy of gas mixture, $\Delta S_m = \Delta S_A + \Delta S_B + \Delta S_C$

- For irreversible process, $\Delta s > 0, \Delta S_m = \Delta S_A + \Delta S_B + \Delta S_C > 0$

- The compressibility factor is given by $Z = \dfrac{pv}{RT}$ It can also be expressed as $X = \dfrac{v_{actual}}{v_{ideal}}$

- According to van der Waals, the equation of state for real gases is given by
  $\left(p + \dfrac{a}{\overline{v}^2}\right)(\overline{v} - b) = \overline{R}T$

- Beattie-bridgeman equation is given by $p = \dfrac{R_u T}{\overline{v}^2}\left(1 - \dfrac{c}{\overline{v}T^3}\right)(\overline{v} + B) - \dfrac{A}{\overline{v}^2}$
  The constants are defined in Section 10.6.

- Benedict-Webb-Rubin equation of state is given by

  $$p = \frac{R_u T}{\overline{v}} + \left(B_0 R_u T - A_0 - \frac{c_0}{T^2}\right)\frac{1}{\overline{v}^2} + \frac{bR_u T - a}{\overline{v}^3} + \frac{a\alpha}{\overline{v}^6} + \frac{c}{\overline{v}^3 T^2}\left(1 + \frac{\gamma}{\overline{v}^2}\right)e^{\left(\frac{-\gamma}{\overline{v}^2}\right)}$$

  For details refer Section 10.7.

- The Virial equation of state of a substance is given by
  $p = \dfrac{RT}{V} + \dfrac{a(T)}{V^2} + \dfrac{b(T)}{V^3} + \dfrac{c(T)}{V^4} + \dfrac{d(T)}{V^5} + \ldots$

- The mass fraction of any component is defined as the ratio of mass of a component to the mass of the mixture Mathematically, $x_i = \dfrac{m_i}{m_m}$,

  where $m_m = m_1 + m_2 + m_3 + \ldots m_i = \sum_{i=1}^{n} m_i$

- The mole fraction is given by $y_i = \dfrac{N_i}{N_m}$ where $N_m = N_1 + N_2 + N_3 + \ldots N_i = \sum_{i=1}^{K} N_i$
- In molar analysis, moles of each component are specified. The number of moles N, the mass m and the molar mass M of a component and mixture is related by $m_i = N_i M_i; \quad m_m = N_m M_m; \quad m_m = \sum m_i = \sum N_i m_i$ and
  $M_m = \dfrac{m_m}{N_m} = \dfrac{\sum N_i M_i}{N_m} = \sum y_i M_i; \quad y_i = x_i \dfrac{M_m}{M_i}$
- The partial pressure, p_i of a component in a gas mixture is given by $p_i = y_i p_m$, where y_i is mole fraction and p_m is mixture pressure $\sum p_i = \sum y_i p_m = p_m \sum y_i = p_m$.
- The sum of *partial volumes* of the components of a gas mixture is equal to the volume of the mixture. The partial volume V_i of a component i in a gas mixture is given by $V_i = y_i V_m$
- For any uniform mixture, the temperature is the same for component of the mixture $T_m = T_A = T_B = T_C$
- Internal energy, $U_m = \sum_{i=1}^{k} x_i u_i$
- Enthalpy, $h_m = \sum_{i=1}^{k} x_i h_i$
- Entropy, $s_m = \sum_{i=1}^{k} x_i s_i$
- Specific heats, $c_{P_m} = \sum_{i=1}^{k} x_i c_{P_i}$
- Gas constant, $R_m = \dfrac{\overline{R}}{M_m} c$

15.11 STEAM POWER CYCLES

- Thermal power plants generate electricity by using fuels like coal, oil or natural gas and works on Rankine cycle using steam as working fluid.
- A vapour power cycle can theoretically work on Carnot cycle but practically it is not.
- Efficiency of the Rankine cycle is given by
  $\eta = \dfrac{W}{Q_S} = \dfrac{(h_1 - h_2) - W_P}{(h_1 - h_4)} = \dfrac{(h_1 - h_2) - W_p}{h_1 - (h_{f2} + W_p)}$
- Specific steam consumption (SSC) is defined as the mass flow of steam required to develop unit power output and is given by $SSC = \dfrac{3600}{W}$ kg/kW h.

- Specific steam flow rate (SSF) is defined as the steam flow rate in kg/s required to develop unit power output and is given by $SSF = \dfrac{1}{W}$ kg/kW.

- Work ratio is defined as the ratio of net work to the gross work and is given by

$$\text{Work ratio} = \frac{\text{Net work}}{\text{Turbine Work}} = \frac{W_T - W_P}{W_T}$$

- For actual Rankine cycle work output is given by $W = (h_1 - h_2)$ for cycle without pump work.

- For actual Rankine cycle, the various parameters are given by (Refer Fig.11.8):

$$\text{Isentropic efficiency} = \frac{\text{Actual work}}{\text{Isentropic work}} \quad \text{for an expansion process}$$

$$\text{Turbine efficiency, } \eta_T = \frac{h_1 - h_{2'}}{h_1 - h_2}$$

$$\text{Isentropic efficiency} = \frac{\text{Isentropic work input}}{\text{Actual work input}} \quad \text{for a compression process}$$

$$\text{Pump efficiency, } \eta_P = \frac{h_4 - h_3}{h_{4'} - h_3}$$

- Modified Rankine cycle is mainly for reciprocating steam engines. Modified Rankine efficiency is given by (Refer Fig.11.9):

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{p_1 v_1 + (u_1 - u_2) - p_3 v_2}{h_1 - h_{f3}}$$

- The efficiency of reheat Rankine cycle is given by (Refer Fig.11.10):

$$\eta_{reheat} = \frac{(h_1 - h_2) + (h_3 - h_4) - W_P}{h_1 - (h_{f_4} - W_P) + (h_3 - h_2)}$$

- The efficiency of regenerative Rankine cycle is given by (Refer Fig.11.11)

$$\eta_{regen} = \frac{Q_S - Q_R}{Q_S} = \frac{(h_1 - h_7) - (1 - m)(h_3 - h_{f_3})}{h_1 - h_7} \text{ where } m = \frac{h_6 - h_5}{h_3 - h_5}.$$

- For two-stage Rankine cycle, refer Fig.11.13 and Section 11.8.3.

- For details on binary vapour cycle, refer Section 11.9.

15.12 REFRIGERATION CYCLES

- Refrigeration is a process of moving heat from one location to another under controlled conditions. It can be defined as producing and maintaining temperatures below that of the surrounding atmosphere.

- Basic units of a refrigeration systems consists of (i) a low-temperature thermal ‘sink’ to which heat will flow from the space to be cooled. (ii) provision for extracting energy from the sink. (iii) a receiver to absorb heat from the high temperature high-pressure refrigerant. (iv) provision for reducing pressure and temperature of the refrigerant from the receiver to the ‘sink’.

- Coefficient of performance (COP) is defined as the ratio of heat absorbed by the refrigerant while passing through the evaporator to the work input required to compress the refrigerant in the compressor.

- In simple terms, it may be defined as the ratio of heat extracted and work done (in heat units). $COP = \dfrac{R_n}{W}$ and Relative $COP = \dfrac{\text{Actual } COP}{\text{Theoretical } COP}$

- One tonne of refrigeration is defined as the refrigerating effect produced by the melting of one tonne of ice at 0 °C in 24 hours. Since, the latent heat of fusion of ice is 336 kJ/kg, the refrigerating effect of 336×1000 kJ in 24 hours is rated as one tonne, i.e. 1 tonne of refrigeration (TR) $= \dfrac{336 \times 1000}{24} = 14000$ kJ/h.

- Tonne of refrigeration (TR) is basically an American unit. It is calculated as the rate at which heat is required to be removed to freeze 1 tonne of water from 0 °C to ice at 0 °C. Using American units this is equal to removal of 200 BTU of heat per minute and MKS unit it is adopted as 50 kcal/min or 3000 kcal/hour. In SI units, its conversion is rounded of to 3.5 kJ/s (kW) or 210 kJ/min.

- Coefficient of performance (COP) means the ratio of the desired effect in kJ/kg to the energy supplied in kJ/kg, therefore, (COP) in case of Carnot cycle runs either as a refrigerating machine or a heat pump or as a heat engine is given by $COP_{ref} = \dfrac{T_2}{T_1 - T_2}$

- For a Carnot cycle heat pump, $COP_{heatpump} = \dfrac{T_1}{T_1 - T_2} = \dfrac{T_1 - T_2 + T_2}{T_1 - T_2} = 1 + \dfrac{T_2}{T_1 - T_2}$

- For a Carnot cycle heat engine, $COP_{heatengine} = \dfrac{T_1 - T_2}{T_1}$

- Air refrigeration system works on reversed Brayton cycle. Assuming polytropic compression and expansion, coefficient of performance is given by (Refer Fig.12.3) $COP = \dfrac{T_3 - T_2}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)(T_4 - T_3 + T_2 - T_1)}$

- The COP of vapour compression cycle when the vapour is dry and saturated after compression is given by (refer Fig.12.5) $COP = \dfrac{h_2 - h_1}{h_3 - h_2} = \dfrac{h_2 - h_4}{h_3 - h_2}$

- The COP of vapour compression cycle when the vapour is superheated after compression is given by (refer Fig.12.6) $COP = \dfrac{h_2 - h_1}{h_3 - h_2}$ where $h_3 = h_{3'} + c_p(T_{sup} - T_{sat})$ and $h_{3'} =$ total heat of dry and saturated vapour at the point 3.

- The COP of vapour compression cycle when the vapour is wet after compression is given by (refer Fig.12.7) $COP = \dfrac{h_2 - h_1}{h_3 - h_2}$

- Factors affecting the performance of a vapour compression system are : suction pressure, delivery pressure, superheating, subcooling of liquid, suction temperature and condenser temperature.

- Volumetric efficiency is defined as the ratio of actual volume of gas drawn into the compressor (at evaporator temperature and pressure) on each stroke to the piston displacement.

- The ratio of actual volume of vapour sucked to the swept volume is defined as clearance volumetric efficiency. Thus, clearance volumetric efficiency is given by (Refer Fig.12.15):
$$\eta_{CV} = \frac{V_1 - V_4}{V_1 - V_3} = 1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}} \text{ where } C = \frac{V_3}{V_1 - V_3} = \frac{\text{Clearance volume}}{\text{Swept volume}}.$$

- Total volumetric efficiency is usually calculated by using the following equation:
$$\eta_{tv} = \left[1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}\right] \times \frac{p_c}{p_s} \times \frac{T_s}{T_c}$$

- If the refrigerant is absorbed on leaving the evaporator, it is called vapour absorption system. The absorbing medium can be either a liquid or a solid.

- Primary refrigerants are those working mediums or heat carriers which directly take part in the refrigeration system and cool the substance by the absorption of latent heat.

- Secondary refrigerants are those circulating substances which are first cooled with the help of the primary refrigerants and are then employed for cooling purposes.

15.13 GAS POWER CYCLE

- Carnot cycle is represented as a standard of perfection and engines can be compared with it to judge the degree of perfection and Carnot efficiency is given by $\eta_{Carnot} = 1 - \frac{T_1}{T_3}$ where T_H is higher temperature and T_L is lower temperature The details have already been discussed in Section 13.2.

- The Carnot cycle has a low mean effective pressure because of its very low work output.

- Stirling cycle efficiency is given by $\eta_{Stirling} = \frac{T_L - T_H}{T_L}$ same as Carnot efficiency when heat exchanger effectiveness is 100%.

- The efficiency of Otto cycle is given by $\eta_{Otto} = 1 - \frac{1}{r^{(\gamma-1)}}$

- Work output of the Otto cycle is given by $W = \frac{p_1 V_1}{\gamma - 1}(r_p - 1)\big(r^{\gamma-1} - 1\big)$, where r_p is the pressure ratio and r is the compression ratio.

- Mean effective pressure of the Otto cycle is given by $p_m = \frac{p_1 r (r_p - 1)\big(r^{(\gamma-1)} - 1\big)}{(\gamma - 1)(r - 1)}$

- The ideal Diesel cycle consists of two isentropic and one constant pressure and one constant volume process.

- The efficiency of Diesel cycle is given by $\eta_{Diesel} = 1 - \frac{1}{r^{(\gamma-1)}}\left[\frac{r_c^\gamma - 1}{\gamma(r_c-1)}\right]$ where r_c is the cut off ratio.
- Work output of the Diesel cycle is given by $W = \frac{p_1V_1r^{(\gamma-1)}\left[\gamma(r_c-1) - r^{(1-\gamma)}(r_c^\gamma - 1)\right]}{\gamma-1}$
- Mean effective pressure of the Diesel cycle is given by $p_m = \frac{p_1[\gamma r^\gamma(r_c-1) - r(r_c^\gamma - 1)]}{(\gamma-1)(r-1)}$
- The efficiency of Dual cycle is given by $\eta_{dual} = 1 - \frac{1}{r^{(\gamma-1)}}\left[\frac{r_pr_c^\gamma - 1}{(r_p-1) + r_p\gamma(r_c-1)}\right]$
- Work output of the Diesel cycle is given by:
  $W = \frac{p_1V_1}{\gamma-1}\left[\gamma r_pr^{\gamma-1}(r_c-1) + r^{\gamma-1}(r_p-1) - (r_pr_c^\gamma - 1)\right]$
- Mean effective pressure of the Diesel cycle is given by:
  $p_m = p_1\frac{[\gamma r_pr^\gamma(r_c-1) + r^\gamma(r_p-1) - r(r_pr_c^\gamma - 1)]}{(\gamma-1)(r-1)}$
- For the same compression ratio and heat addition, $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$.
- For the same compression ratio and heat rejection, $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$.
- For the same peak pressure, peak temperature and heat rejection,
  $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$.
- For the same maximum pressure and heat input rejection, $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$.
- For the same maximum pressure and work output, $\eta_{Otto} > \eta_{Dual} > \eta_{Diesel}$.
- The efficiency of Lenoir cycle is given by $\eta_{Lenoir} = 1 - \gamma\left(\frac{r_p^{(1/\gamma)} - 1}{r_p - 1}\right)$
- The efficiency of the Atkinson cycle is given by $\eta_{Atkinson} = 1 - \gamma\left[\frac{e-r}{e^\gamma - r^\gamma}\right]$
- The efficiency of Brayton cycle is given by $\eta_{Brayton} = 1 - \frac{1}{r^{(\gamma-1)}}$ where r_p is the pressure ratio.

15.14 THERMODYNAMIC RELATIONS

- Theorem 1: If a relation exists among the variables say, x, y and z, then z may be expressed as a function of x and y, then, z can be expressed as $z = f(x, y)$ and if dz is expressed as Ndy and if $\left(\frac{\partial M}{\partial y}\right)_x = \left(\frac{\partial N}{\partial x}\right)_y$, then it is a perfect differential.
- Theorem 2: If a quantity f is a function of the variables say, x, y and z and a relation exists among the variables say, x, y and z then f is a function of any two of x, y and z. Similarly, any one of x, y and z may be regarded as a function of f. Thus, if $x = x(f, y)$.
  $\left(\frac{\partial x}{\partial y}\right)_f\left(\frac{\partial y}{\partial z}\right)_f\left(\frac{\partial z}{\partial x}\right)_f = 1$

- Theorem 3: Among the variables say, x, y and z, any one variable may be considered as a function of the other two. Thus, if $x = x(y, z)$, then $\left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_V = -1$

- Maxwell relations are given by

$$\begin{aligned}
\left(\frac{\partial T}{\partial V}\right)_S &= -\left(\frac{\partial p}{\partial S}\right)_V \\
\left(\frac{\partial T}{\partial p}\right)_S &= \left(\frac{\partial V}{\partial S}\right)_p \\
\left(\frac{\partial p}{\partial T}\right)_V &= \left(\frac{\partial S}{\partial V}\right)_T \\
\left(\frac{\partial S}{\partial p}\right)_T &= -\left(\frac{\partial V}{\partial T}\right)_p
\end{aligned}$$

- Coefficient of volume expansion is given by $\beta = \frac{1}{v}\left(\frac{\partial V}{\partial T}\right)_p$

- Isothermal compressibility is given by $K \quad = \quad -\frac{1}{v}\left(\frac{\partial V}{\partial p}\right)_T$

- The two Tds relations are given by

$$\begin{aligned}
Tds &= c_V dT + T\left(\frac{\partial p}{\partial T}\right)_T dv \\
Tds &= c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp
\end{aligned}$$

- c_p is given by $c_p = T\left(\frac{\partial s}{\partial T}\right)_p$

- c_V is given by $c_V = T\left(\frac{\partial s}{\partial T}\right)_V$

- The ratio of specific heats is given by $\gamma = \left(\frac{\partial s}{\partial T}\right)_p \left(\frac{\partial T}{\partial S}\right)_v$

- The internal energy relation is given by $du = T\left[\left(\frac{\partial s}{\partial T}\right)_v dT + \left(\frac{\partial s}{\partial v}\right)_T dv\right] - pdv$

- The enthalpy relation is given by $dh = \left[c_p dT - T\left(\frac{\partial v}{\partial T}\right)_p dp\right] + vdp$

- Stefan-Boltzman law is given by $u = bT^4$, where b is a constant.

- Clapeyron equation is expressed as $\dfrac{dp}{dT} = \dfrac{[s_g -\ s_f]}{[v_g -\ v_f]} = \dfrac{s_{fg}}{v_{fg}} = \dfrac{h_{fg}}{Tv_{fg}}$

- Joule-Thomson coefficient is given by $\mu \quad = \quad \left(\dfrac{\partial T}{\partial p}\right)_H$

- Throttling process is defined as the process when the fluid expansion takes place through a minute orifice or slightly opened valve. During the throttling process pressure and velocity are reduced. However, there is no heat transfer and no work done by the system. In this process, enthalpy remains constant.

- *Inversion Curve*: The maximum point on each curve on T-p plane is called inversion point and the locus of the inversion point is called inversion curve.

- Joule-Thomson coefficient for ideal gas $\mu = \dfrac{1}{c_p}\left[T\left(\dfrac{\partial v}{\partial T}\right)_p - v\right]$ $\mu = 0$ implies that the Joule-Thompson coefficient is zero for ideal gas.

- Constant temperature coefficient: $c_p = \left(\dfrac{\partial h}{\partial p}\right)_T$ is known as *constant temperature coefficient.*

- *Gibbs free energy*: Gibbs free energy represents the potential to do work, it is also a thermodynamic potential.

- The Kirchoff equation is given by $\left(\dfrac{\partial h_{fg}}{\partial T}\right) = c_{pg} - c_{pf}$

- Gibbs phase rule is given by $P + F = C + 2$, where C is number of components which are distributed among all the P phases. F represents degree of freedom.

APPENDIX A

A.1 STEAM TABLES*

Table A.1.1 *Saturates steam : Temperature table*

	Specific Volume, m^3/kg			*Internal Energy, kJ/kg*				*Enthalpy, kJ/kg*		*Entropy, kJ/kg K*		
Temp. °C T	*Pressure kPa, MPa* P	*Sat. Liquid* v_f	*Sat. Vapour* v_g	*Sat. Liquid* u_f	*Evap.* u_{fg}	*Sat. Vapour* u_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Evap.* s_{fg}	*Sat Vapour* s_g
0.01	0.6113	0.001000	206.132	0.00	2375.3	2375.3	0.00	2501.3	2501.3	0.0000	9.1562	9.1562
5	0.8721	0.001000	147.118	20.97	2361.3	2382.2	20.98	2489.6	2510.5	0.0761	8.9496	9.0257
10	1.2276	0.001000	106.377	41.99	2347.2	2389.2	41.99	2477.7	2519.7	0.1510	8.7498	8.9007
15	1.7051	0.001001	77.925	62.98	2333.1	2396.0	62.98	2465.9	2528.9	0.2245	8.5569	8.7813
20	2.3385	0.001002	57.790	83.94	2319.0	2402.9	83.94	2454.1	2538.1	0.2966	8.3706	8.6671
25	3.1691	0.001003	43.359	104.86	2304.9	2409.8	104.87	2442.3	2547.2	0.3673	8.1905	8.5579
30	4.2461	0.001004	32.893	125.77	2290.8	2416.6	125.77	2430.5	2556.2	0.4369	8.0164	8.4533
35	5.6280	0.001006	25.216	146.65	2276.7	2423.4	146.66	2418.6	2565.3	0.5052	7.8478	8.3530
40	7.3837	0.001008	19.523	167.53	2262.6	2430.1	167.54	2406.7	2574.3	0.5724	7.6845	8.2569
45	9.5934	0.001010	15.258	188.41	2248.4	2436.8	188.42	2394.8	2583.2	0.6386	7.5261	8.1647
50	12.350	0.001012	12.032	209.30	2234.2	2443.5	209.31	2382.7	2592.1	0.7037	7.3725	8.0762
55	15.758	0.001015	9.568	230.19	2219.9	2450.1	230.20	2370.7	2600.9	0.7679	7.2234	7.9912
60	19.941	0.001017	7.671	251.09	2205.5	2456.6	251.11	2358.5	2609.6	0.8311	7.0784	7.9095
65	25.033	0.001020	6.197	272.00	2191.1	2463.1	272.03	2346.2	2618.2	0.8934	6.9375	7.8309
70	31.188	0.001023	5.042	292.93	2176.6	2469.5	292.96	2333.8	2626.8	0.9548	6.8004	7.7552
75	38.578	0.001026	4.131	313.87	2162.0	2475.9	313.91	2321.4	2635.3	1.0154	6.6670	7.6824
80	47.390	0.001029	3.407	334.84	2147.4	2482.2	334.88	2308.8	2643.7	1.0752	6.5369	7.6121
85	57.834	0.001032	2.828	355.82	2132.6	2488.4	355.88	2296.0	2651.9	1.1342	6.4102	7.5444
90	70.139	0.001036	2.361	376.82	2117.7	2494.5	376.90	2283.2	2660.1	1.1924	6.2866	7.4790
95	84.554	0.001040	1.982	397.86	2102.7	2500.6	397.94	2270.2	2668.1	1.2500	6.1659	7.4158
100	0.10135	0.001044	1.6729	418.91	2087.6	2506.5	419.02	2257.0	2676.0	1.3068	6.0480	7.3548
105	0.12082	0.001047	1.4194	440.00	2072.3	2512.3	440.13	2243.7	2683.8	1.3629	5.9328	7.2958
110	0.14328	0.001052	1.2102	461.12	2057.0	2518.1	461.27	2230.2	2691.5	1.4184	5.8202	7.2386
115	0.16906	0.001056	1.0366	482.28	2041.4	2523.7	482.46	2216.5	2699.0	1.4733	5.7100	7.1832
120	0.19853	0.001060	0.8919	503.48	2025.8	2529.2	503.69	2202.6	2706.3	1.5275	5.6020	7.1295
125	0.2321	0.001065	0.77059	524.72	2009.9	2534.6	524.96	2188.5	2713.5	1.5812	5.4962	7.0774

(Contd.)

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* Adapted from Joseph H. Keenan, Frederick G.Keyes, Philip G. Hill, and Joan G. Moore, *Steam Tables*, John Wiley and Sons, New York, 1969.

Table A.1.1 *(Contd.)*

130	0.2701	0.001070	0.66850	546.00	1993.9	2539.9	546.29	2174.2	2720.5	1.6343	5.3925	7.0269
135	0.3130	0.001075	0.58217	567.34	1977.7	2545.0	567.67	2159.6	2727.3	1.6869	5.2907	6.9777
140	0.3613	0.001080	0.50885	588.72	1961.3	2550.0	589.11	2144.8	2733.9	1.7390	5.1908	6.9298
145	0.4154	0.001085	0.44632	610.16	1944.7	2554.9	610.61	2129.6	2740.3	1.7906	5.0926	6.8832
150	0.4759	0.001090	0.39278	631.66	1927.9	2559.5	632.18	2114.3	2746.4	1.8417	4.9960	6.8378
155	0.5431	0.001096	0.34676	653.23	1910.8	2564.0	653.82	2098.6	2752.4	1.8924	4.9010	6.7934
160	0.6178	0.001102	0.30706	674.85	1893.5	2568.4	675.53	2082.6	2758.1	1.9426	4.8075	6.7501
165	0.7005	0.001108	0.27269	696.55	1876.0	2572.5	697.32	2066.2	2763.5	1.9924	4.7153	6.7078
170	0.7917	0.001114	0.24283	718.31	1858.1	2576.5	719.20	2049.5	2768.7	2.0418	4.6244	6.6663
175	0.8920	0.001121	0.21680	740.16	1840.0	2580.2	741.16	2032.4	2773.6	2.0909	4.5347	6.6256
180	1.0022	0.001127	0.19405	762.08	1821.6	2583.7	763.21	2015.0	2778.2	2.1395	4.4461	6.5857
185	1.1227	0.001134	0.17409	784.08	1802.9	2587.0	785.36	1997.1	2782.4	2.1878	4.3586	6.5464
190	1.2544	0.001141	0.15654	806.17	1783.8	2590.0	807.61	1978.8	2786.4	2.2358	4.2720	6.5078
195	1.3978	0.001149	0.14105	828.36	1764.4	2592.8	829.96	1960.0	2790.0	2.2835	4.1863	6.4697
200	1.5538	0.001156	0.12736	850.64	1744.7	2595.3	852.43	1940.7	2793.2	2.3308	4.1014	6.4322
205	1.7230	0.001164	0.11521	873.02	1724.5	2597.5	875.03	1921.0	2796.0	2.3779	4.0172	6.3951
210	1.9063	0.001173	0.10441	895.51	1703.9	2599.4	897.75	1900.7	2798.5	2.4247	3.9337	6.3584
215	2.1042	0.001181	0.09479	918.12	1682.9	2601.1	920.61	1879.9	2800.5	2.4713	3.8507	6.3221
220	2.3178	0.001190	0.08619	940.85	1661.5	2602.3	943.61	1858.5	2802.1	2.5177	3.7683	6.2860
225	2.5477	0.001199	0.7849	963.72	1639.6	2603.3	966.77	1836.5	2803.3	2.5639	3.6863	6.2502
230	2.7949	0.001209	0.07158	986.72	1617.2	2603.9	990.10	1813.8	2803.9	2.6099	3.6047	6.2146
235	3.0601	0.001219	0.06536	1009.88	1594.2	2604.1	1013.61	1790.5	2804.1	2.6557	3.5233	6.1791
240	3.3442	0.001229	0.05976	1033.19	1570.8	2603.9	1037.31	1766.5	2803.8	2.7015	3.4422	6.1436
245	3.6482	0.001240	0.05470	1056.69	1546.7	2603.4	1061.21	1741.7	2802.9	2.7471	3.3612	6.1083
250	3.9730	0.001251	0.05013	1080.37	1522.0	2602.4	1085.34	1716.2	2801.5	2.7927	3.2802	6.0729
255	4.3195	0.001263	0.04598	1104.26	1496.7	2600.9	1109.72	1689.8	2799.5	2.8382	3.1992	6.0374
260	4.6886	0.001276	0.04220	1128.37	1470.6	2599.0	1134.35	1662.5	2796.9	2.8837	3.1181	6.0018
265	5.0813	0.001289	0.03877	1152.72	1443.9	2596.6	1159.27	1634.3	2793.6	2.9293	3.0368	5.9661
270	5.4987	0.001302	0.03564	1177.33	1416.3	2593.7	1184.49	1605.2	2789.7	2.9750	2.9551	5.9301
275	5.9418	0.001317	0.03279	1202.23	1387.9	2590.2	1210.05	1574.9	2785.0	3.0208	2.8730	5.8937
280	6.4117	0.001332	0.03017	1227.43	1358.7	2586.1	1235.97	1543.6	2779.5	3.0667	2.7903	5.8570
285	6.9094	0.001348	0.02777	1252.98	1328.4	2581.4	1262.29	1511.0	2773.3	3.1129	2.7069	5.8198
290	7.4360	0.001366	0.02557	1278.89	1297.1	2576.0	1289.04	1477.1	2766.1	3.1593	2.6227	5.7821
295	7.9928	0.001384	0.02354	1305.21	1264.7	2569.9	1316.27	1441.8	2758.0	3.2061	2.5375	5.7436
300	8.5810	0.001404	0.02167	1331.97	1231.0	2563.0	1344.01	1404.9	2748.9	3.2533	2.4511	5.7044
305	9.2018	0.001425	0.01995	1359.22	1195.9	2555.2	1372.33	1366.4	2738.7	3.3009	2.3633	5.6642
310	9.8566	0.001447	0.01835	1387.03	1159.4	2546.4	1401.29	1326.0	2727.3	3.3492	2.2737	5.6229
315	10.547	0.001472	0.01687	1415.44	1121.1	2536.6	1430.97	1283.5	2714.4	3.3981	2.1821	5.5803
320	11.274	0.001499	0.01549	1444.55	1080.9	2525.5	1461.45	1238.6	2700.1	3.4479	2.0882	5.5361
330	12.845	0.001561	0.012996	1505.24	993.7	2498.9	1525.29	1140.6	2665.8	3.5506	1.8909	5.4416
340	14.586	0.001638	0.010797	1570.26	894.3	2464.5	1594.15	1027.9	2622.0	3.6593	1.6763	5.3356
350	16.514	0.001740	0.008813	1641.81	776.6	2418.4	1670.54	893.4	2563.9	3.7776	1.4336	5.2111
360	18.651	0.001892	0.006945	1725.19	626.3	2351.5	1760.48	720.5	2481.0	3.9146	1.1379	5.0525
370	21.028	0.002213	0.004926	1843.84	384.7	2228.5	1890.37	441.8	2332.1	4.1104	0.6868	4.7972
374.14	22.089	0.003155	0.003155	2029.58	0	2029.6	2099.26	0	2099.3	4.4297	0	4.4297

Table. A.1.2 *Saturated water: Pressure table*

Pressure mPa P	Tem. °C T	Specific Volume, m^3/kg		Internal Energy, kJ/kg			Entropy, kJ/kg			Entropy, kJ/kg K		
		Sat. Liquid v_f	Sat. Vapour v_g	Sat. Liquid u_f	Evap. u_{fg}	Sat. Vapour u_g	Sat. Liquid h_f	Evap. h_{fg}	Sat. Vapour h_g	Sat. Liquid s_f	Evap. s_{fg}	Sat. Vapour s_g
0.6113	0.01	0.001000	206.132	0	2375.3	2375.3	0.00	2501.3	2501.3	0	9.1562	9.1562
1.0	6.98	0.001000	129.208	29.29	2355.7	2385.0	29.29	2484.9	2514.2	0.1059	8.8697	8.9756
1.5	13.03	0.001001	87.980	54.70	2338.6	2393.3	54.70	2470.6	2525.3	0.1956	8.6322	8.8278
2.0	17.50	0.001001	67.004	73.47	2326.0	2399.5	73.47	2460.0	2533.5	0.2607	8.4629	8.7236
2.5	21.08	0.001002	54.254	88.47	2315.9	2404.4	88.47	2451.6	2540.0	0.3120	8.3311	8.6431
3.0	24.08	0.001003	45.665	101.03	2307.5	2408.5	101.03	2444.5	2545.5	0.3545	8.2231	8.5775
4.0	28.96	0.001004	34.800	121.44	2293.7	2415.2	121.44	2432.9	2554.4	0.4226	8.0520	8.4746
5.0	32.88	0.001005	28.193	137.79	2282.7	2420.5	137.79	2423.7	2561.4	0.4763	7.9187	8.3950
7.5	40.29	0.001008	19.238	168.76	2261.7	2430.5	168.77	2406.0	2574.8	0.5763	7.6751	8.2514
10.0	45.81	0.001010	14.674	191.79	2246.1	2437.9	191.81	2392.8	2584.6	0.6492	7.5010	8.1501
15.0	53.97	0.001014	10.022	225.90	2222.8	2448.7	225.91	2373.1	2599.1	0.7548	7.2536	8.0084
20.0	60.06	0.001017	7.649	251.35	2205.4	2456.7	251.38	2358.3	2609.7	0.8319	7.0766	7.9085
25.0	64.97	0.001020	6.204	271.88	2191.2	2463.1	271.90	2346.3	2618.2	0.8930	6.9383	7.8313
30.0	69.10	0.001022	5.229	289.18	2179.2	2468.4	289.21	2336.1	2626.3	0.9439	6.8247	7.7686
40.0	75.87	0.001026	3.993	317.51	2159.5	2477.0	317.55	2319.2	2636.7	1.0258	6.6441	7.6700
50.0	81.33	0.001030	3.240	340.42	2143.4	2483.8	340.47	2305.4	2645.9	1.0910	6.5029	7.5939
75.0	91.77	0.001037	2.217	384.29	2112.4	2496.7	384.36	2278.6	2663.0	1.2129	6.2434	7.4563
MPa												
0.100	99.62	0.001043	1.6940	417.33	2088.7	2506.1	417.44	2258.0	2675.5	1.3025	6.0568	7.3593
0.125	105.99	0.001048	1.3749	444.16	2069.3	2513.5	444.30	2241.1	2685.3	1.3739	5.9104	7.2843
0.150	111.37	0.001053	1.1593	466.92	2052.7	2519.6	467.08	2226.5	2693.5	1.4335	5.7897	7.2232
0.175	116.06	0.001057	1.0036	486.78	2038.1	2524.9	486.97	2213.6	2700.5	1.4848	5.6868	7.1717
0.200	120.23	0.001061	0.8857	504.47	2025.0	2529.5	504.68	2202.0	2706.6	1.5300	5.5970	7.1271
0.225	124.00	0.001064	0.7933	520.45	2013.1	2533.6	520.69	2191.3	2712.0	1.5705	5.5173	7.0878
0.250	127.43	0.001067	0.7187	535.08	2002.1	2537.2	535.34	2181.5	2716.9	1.6072	5.4455	7.0526
0.275	130.60	0.001070	0.6573	548.57	1992.0	2540.5	548.87	2172.4	2721.3	1.6407	5.3801	7.0208
0.300	133.55	0.001073	0.6058	561.13	1982.4	2543.6	561.45	2163.9	2725.3	1.6717	5.3201	6.9918
0.350	138.88	0.001079	0.5243	583.93	1965.0	2548.9	584.31	2148.1	2732.4	1.7274	5.2130	6.9404
0.375	141.32	0.001081	0.4914	594.38	1956.9	2255.13	594.79	2140.8	2735.6	1.7527	5.1647	6.9174
0.40	143.63	0.001084	0.4625	604.29	1949.3	2553.6	604.73	2133.8	2738.5	1.7766	5.1193	6.8958
0.45	147.93	0.001088	0.4140	622.75	1934.9	2557.6	623.24	2120.7	2743.9	1.8206	5.0359	6.8565
0.50	151.86	0.001093	0.3749	639.66	1921.6	2561.2	640.21	2108.5	2748.7	1.8606	4.9606	6.8212
0.55	155.48	0.001097	0.3427	655.30	1909.2	2564.5	655.91	2097.0	2752.9	1.8972	4.8920	6.7892
0.60	158.85	0.001101	0.3157	669.88	1897.5	2567.4	670.54	2086.3	2756.8	1.9311	4.8289	6.7600

(Contd.)

Table A.1.2 *(Contd.)*

0.65	162.01	0.001104	0.2927	683.55	1886.5	2570.1	684.26	2076.0	2760.3	1.9627	4.7704	6.7330
0.70	164.97	0.001108	0.2729	696.43	1876.1	2572.5	697.20	2066.3	2763.5	1.9922	4.7158	6.7080
0.75	167.77	0.001111	0.2556	708.62	1866.1	2574.7	709.45	2057.0	2766.4	2.0199	4.6647	6.6846
0.80	170.43	0.001115	0.2404	720.20	1856.6	2576.8	721.10	2048.0	2769.1	2.0461	4.6166	6.6627
0.85	172.96	0.001118	0.2270	731.25	1847.4	2578.7	732.20	2039.4	2771.6	2.0709	4.5711	6.6421
0.90	175.38	0.001121	0.2150	741.81	1838.7	2580.5	742.82	2031.1	2773.9	2.0946	4.5280	6.6225
0.95	177.69	0.001124	0.2042	751.94	1830.2	2582.1	753.00	2023.1	2776.1	2.1171	4.4869	6.6040
1.00	179.91	0.001127	0.19444	761.67	1822.0	2583.6	762.79	2015.3	2778.1	2.1386	4.4478	6.5864
1.10	184.09	0.001133	0.17753	780.08	1806.3	2586.4	781.32	2000.4	2781.7	2.1791	4.3744	6.5535
1.20	187.99	0.001139	0.16333	797.27	1791.6	2588.8	798.64	1986.2	2784.8	2.2165	4.3067	6.5233
1.30	191.64	0.001144	0.15125	813.42	1777.5	2590.9	814.91	1972.7	2787.6	2.2514	4.2438	6.4953
1.40	195.07	0.001149	0.14084	828.68	1764.1	2592.8	830.29	1959.7	2790.0	2.2842	4.1850	6.4692
1.50	198.32	0.001154	0.13177	843.14	1751.3	2594.5	844.87	1947.3	2792.1	2.3150	4.2198	6.4448
1.75	205.76	0.001166	0.11349	876.44	1721.4	2597.8	878.48	1918.0	2796.4	2.3851	4.0044	6.3895
2.00	212.42	0.001177	0.09963	906.42	1693.8	2600.3	908.77	1890.7	2799.5	2.4473	3.8935	6.3408
2.25	218.45	0.001187	0.08875	933.81	1668.2	2602.0	936.48	1865.2	2801.7	2.5034	3.7938	6.2971
2.50	223.99	0.001197	0.07998	959.09	1644.0	2603.1	962.09	1841.0	2803.1	2.5546	3.7028	6.2574
2.75	229.12	0.001207	0.07275	982.65	1621.2	2603.8	985.97	1817.9	2803.9	2.6018	3.6190	6.2208
3.00	233.90	0.001216	0.06668	1004.76	1599.3	2604.1	1008.41	1795.7	2804.1	2.6456	3.5412	6.1869
3.25	238.38	0.001226	0.06152	1025.62	1578.4	2604.0	1029.60	1774.4	2804.0	2.6866	3.4685	6.1551
3.50	242.60	0.001235	0.05707	1045.41	1558.3	2603.7	1049.73	1753.7	2803.4	2.7252	3.4000	6.1252
4.0	250.40	0.001252	0.049778	1082.28	1520.0	2602.3	1087.29	1714.1	2801.4	2.7963	3.2737	6.0700
5.0	263.99	0.001286	0.039441	1147.78	1449.3	2597.1	1154.21	1640.1	2794.3	2.9201	3.0532	5.9733
6.0	275.64	0.001319	0.032440	1205.41	1384.3	2589.7	1213.32	1571.0	2784.3	3.0266	2.8625	5.8891
7.0	285.88	0.001351	0.027370	1257.51	1323.0	2580.5	1266.97	1505.1	2772.1	3.1210	2.6922	5.8132
8.0	295.06	0.001384	0.023518	1305.54	1264.3	2569.8	1316.61	1441.3	2757.9	3.2067	2.5365	5.7431
9.0	303.40	0.001418	0.020484	1350.47	1207.3	2557.8	1363.23	1378.9	2742.1	3.2857	2.3915	5.6771
10.0	311.06	0.001452	0.018026	1393.00	1151.4	2544.4	1407.53	1317.1	2724.7	3.3595	2.2545	5.6140
11.0	318.15	0.001489	0.015987	1433.68	1096.1	2529.7	1450.05	1255.5	2705.6	3.4294	2.1233	5.5527
12.0	324.75	0.001527	0.014263	1472.92	1040.8	2513.7	1491.24	1193.6	2684.8	3.4961	1.9962	5.4923
13.0	330.93	0.001567	0.012780	1511.09	985.0	2496.1	1531.46	1130.8	2662.2	3.5604	1.8718	5.4323
14.0	336.75	0.001611	0.011485	1548.53	928.2	2476.8	1571.08	1066.5	2637.5	3.6231	1.7485	5.3716
15.0	342.24	0.001658	0.010338	1585.58	869.8	2455.4	1610.45	1000.0	2610.5	3.6847	1.6250	5.3097
16.0	347.43	0.001711	0.009306	1622.63	809.1	2431.7	1650.00	930.6	2580.6	3.4760	1.4995	5.2454
17.0	352.37	0.001770	0.008365	1660.16	744.8	2405.0	1690.25	856.9	2547.2	3.8078	1.3698	5.1776
18.0	357.06	0.001840	0.007490	1698.86	675.4	2374.3	1731.97	777.1	2509.1	3.8713	1.2330	5.1044
19.0	361.54	0.001924	0.006657	1739.87	598.2	2338.1	1776.43	688.1	2464.5	3.9387	1.0841	5.0227
20.0	365.81	0.002035	0.005834	1785.47	507.6	2293.1	1826.18	583.6	2409.7	4.0137	0.9132	4.9269
21.0	369.89	0.002206	0.004953	1841.97	388.7	2230.7	1888.30	446.4	2334.7	4.1073	0.6942	4.8015
22.0	373.80	0.002808	0.003526	1973.16	108.2	2081.4	2034.92	124.0	2159.0	4.3307	0.1917	4.5224
22.09	374.14	0.003155	0.003155	2029.58	0	2029.6	2099.26	0	2099.3	4.4297	0	4.4297

Table A.1.3 *Superheated vapour*

	P = 10 kPa (45.81)				*P = 50 kPa, (81.33)*				*P = 100 kPa (99.62)*			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
Sat.	14.674	2437.9	2584.6	8.1501	3.240	2483.8	2645.9	7.5939	1.6940	2506.1	2675.5	7.3593
50	14.869	2443.9	2592.6	8.1749	—	—	—	—	—	—	—	—
100	17.196	2515.5	2687.5	8.4479	3.418	2511.6	2682.5	7.6947	1.6958	2506.6	2676.2	7.3614
150	19.513	2587.9	2783.0	8.6881	3.889	2585.6	2780.1	7.9400	1.9364	2582.7	2776.4	7.6133
200	21.825	2661.3	2879.5	8.9037	4.356	2659.8	2877.6	8.1579	2.1723	2658.0	2875.3	7.8342
250	24.136	2736.0	2977.3	9.1002	4.821	2735.0	2976.0	8.3555	2.4060	2733.7	2974.3	8.0332
300	26.445	2812.1	3076.5	9.2812	5.284	2811.3	3075.5	8.5372	2.6388	2810.4	3074.3	8.2157
400	31.063	2968.9	3279.5	9.6076	6.209	2968.4	3278.9	8.8641	3.1026	2967.8	3278.1	8.5434
500	35.679	3132.3	3489.0	9.8977	7.134	3131.9	3488.6	9.1545	3.5655	3131.5	3488.1	8.8341
600	40.295	3302.5	3705.4	10.1608	8.058	3302.2	3705.1	9.4177	4.0278	3301.9	3704.7	9.0975
700	44.911	3479.6	3928.7	10.4028	8.981	3479.5	3928.5	9.6599	4.4899	3479.2	3928.2	9.3398
800	49.526	3663.8	4159.1	10.6281	9.904	3663.7	4158.9	9.8852	4.9517	3663.5	4158.7	9.5652
900	54.141	3855.0	4396.4	10.8395	10.828	3854.9	4396.3	10.0967	5.4135	3854.8	4396.1	9.7767
1000	58.757	4053.0	4640.6	11.0392	11.751	4052.9	4640.5	10.2964	5.8753	4052.8	4640.3	9.9764
1100	63.372	4257.5	4891.2	11.2287	12.674	4257.4	4891.1	10.4858	6.3370	4257.3	4890.9	10.1658
1200	67.987	4467.9	5147.8	11.4090	13.597	4467.8	5147.7	10.6662	6.7986	4467.7	5147.6	10.3462
1300	72.603	4683.7	4409.7	11.5810	14.521	4683.6	5409.6	10.8382	7.2603	4683.5	5409.5	10.5182
	P = 200 kPa (120.23)				*P = 300 kPa (133.55)*				*P = 400 kPa (143.65)*			
Sat.	0.88573	2529.5	2706.6	7.1271	0.60582	2543.6	2725.3	6.9918	0.46246	2553.6	2738.5	6.8958
150	0.95964	2576.9	2768.8	7.2795	0.63388	2570.8	2761.0	7.0778	0.47084	2564.5	2752.8	6.9299
200	1.08034	2654.4	2870.5	7.5066	0.71629	2650.7	2865.5	7.3115	0.53422	2646.8	2860.5	7.1706
250	1.19880	2731.2	2971.0	7.7085	0.79636	2728.7	2967.6	7.5165	0.59512	2726.1	2964.2	7.3788
300	1.31616	2808.6	3071.8	7.8926	0.87529	2806.7	3069.3	7.7022	0.65484	2804.8	3066.7	7.5661
400	1.54930	2966.7	3276.5	8.2217	1.03151	2965.5	3275.0	8.0329	0.77262	2964.4	3273.4	7.8984
500	1.78139	3130.7	3487.0	8.5132	1.18669	3130.0	3486.0	8.3250	0.88934	3129.2	3484.9	8.1912
600	2.01297	3301.4	3704.0	8.7769	1.34136	3300.8	3703.2	8.5892	1.00555	3300.2	3702.4	8.4557
700	2.24426	3478.8	3927.7	9.0194	1.49573	3478.4	3927.1	8.8319	1.12147	3477.9	3926.5	8.6987
800	2.47539	3663.2	4158.3	9.2450	1.64994	3662.9	4157.8	9.0575	1.23722	3662.5	4157.4	8.9244
900	2.70643	3854.5	4395.8	9.4565	1.80406	3854.2	4395.4	9.2691	1.35288	3853.9	4395.1	9.1361
1000	2.93740	4052.5	4640.0	9.6563	1.95812	4052.3	4639.7	9.4689	1.46847	4052.0	4639.4	9.3360
1100	3.16834	4257.0	4890.7	9.8458	2.11214	4256.8	4890.4	9.6585	1.58404	4256.5	4890.1	9.5255
1200	3.39927	4467.5	5147.3	10.0262	2.26614	4467.2	5147.1	9.8389	1.69958	4467.0	5146.8	9.7059
1300	3.63018	4683.2	5409.3	10.1982	2.42013	4683.0	5409.0	10.0109	1.81511	4682.8	5408.8	9.8780

(Contd.)

Table A.1.3 *(Contd.)*

	P = 500 kPa (151.86)				*P* = 600 kPa (158.85)				*P* = 800 kPa (170.43)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
Sat.	0.37489	2561.2	2748.7	6.8212	0.31567	2567.4	2756.8	6.7600	0.24043	2576.8	2769.1	6.6627
200	0.42492	2642.9	2855.4	7.0592	0.35202	2638.9	2850.1	6.9665	0.26080	2630.6	2839.2	6.8158
250	0.47436	2723.5	2960.7	7.2708	0.39383	2720.9	2957.2	7.1816	0.29314	2715.5	2950.0	7.0384
300	0.52256	2802.9	3064.2	7.4598	0.43437	2801.0	3061.6	7.3723	0.32411	2797.1	3056.4	7.2372
350	0.57012	2882.6	3167.6	7.6328	0.47424	2881.1	3165.7	7.5463	0.35439	2878.2	3161.7	7.4088
400	0.61728	2963.2	3271.8	7.7937	0.51372	2962.0	3270.2	7.7078	0.38426	2959.7	3267.1	7.5715
500	0.71093	3128.4	3483.8	8.0872	0.59199	3127.6	3482.7	8.0020	0.44331	3125.9	3480.6	7.8672
600	0.80406	3299.6	3701.7	8.3521	0.66974	3299.1	3700.9	8.2673	0.50184	3297.9	3699.4	8.1332
700	0.89691	3477.5	3926.0	8.5952	0.74720	3477.1	3925.4	8.5107	0.56007	3476.2	3924.3	8.3770
800	0.98959	3662.2	4157.0	8.8211	0.82450	3661.8	4156.5	8.7367	0.61813	3661.1	4155.7	8.6033
900	1.08217	3853.6	4394.7	9.0329	0.90169	3853.3	4394.4	8.9485	0.67610	3852.8	4393.6	8.8153
1000	1.17469	4051.8	4639.1	9.2328	0.97883	4051.5	4638.8	9.1484	0.73401	4051.0	4638.2	9.0153
1100	1.26718	4256.3	4889.9	9.4224	1.05594	4256.1	4889.6	9.3381	0.79188	4255.6	4889.1	9.2049
1200	1.35964	4466.8	5146.6	9.6028	1.13302	4466.5	5146.3	9.5185	0.84974	4466.1	5145.8	9.3854
1300	1.45210	4682.5	5408.6	9.7749	1.21009	4682.3	5408.3	9.6906	0.90758	4681.8	5407.9	9.5575
	P = 1.00 MPa (179.91)				*P* = 1.20 MPa (187.99)				*P* = 1.40 MPa (195.07)			
Sat.	0.19444	2583.6	2778.1	6.5864	0.16333	2588.8	2784.8	6.5233	0.14084	2592.8	2790.0	6.4692
200	0.20596	2621.9	2827.9	6.6939	0.16930	2612.7	2815.9	6.5898	0.14302	2603.1	2803.3	6.4975
250	0.23268	2709.9	2942.6	6.9246	0.19235	2704.2	2935.0	6.8293	0.16350	2698.3	2927.2	6.7467
300	0.25794	2793.2	3051.2	7.1228	0.21382	2789.2	3045.8	7.0316	0.18228	2785.2	3040.4	6.9533
350	0.28247	2875.2	3157.7	7.3010	0.23452	2872.2	3153.6	7.2120	0.20026	2869.1	3149.5	7.1359
400	0.30659	2957.3	3263.9	7.4650	0.25480	2954.9	3260.7	7.3773	0.21780	2952.5	3257.4	7.3025
500	0.35411	3124.3	3478.4	7.7621	0.29463	3122.7	3476.3	7.6758	0.25215	3121.1	3474.1	7.6026
600	0.40109	3296.8	3697.9	8.0289	0.33393	3295.6	3696.3	7.9434	0.28596	3294.4	3694.8	7.8710
700	0.44779	3475.4	3923.1	8.2731	0.37294	3474.5	3922.0	8.1881	0.31947	3473.6	3920.9	8.1160
800	0.49432	3660.5	4154.8	8.4996	0.41177	3659.8	4153.9	8.4149	0.35281	3659.1	4153.0	8.3431
900	0.54075	3852.5	4392.9	8.7118	0.45051	3851.6	4392.2	8.6272	0.38606	3851.0	4391.5	8.5555
1000	0.58712	4050.5	4637.6	8.9119	0.48919	4050.0	4637.0	8.8274	0.41924	4049.5	4636.4	8.7558
1100	0.63345	4255.1	4888.5	9.1016	0.52783	4254.6	4888.0	9.0171	0.45239	4254.1	4887.5	8.9456
1200	0.67977	4465.6	5145.4	9.2821	0.56646	4465.1	5144.9	9.1977	0.48552	4464.6	5144.4	9.1262
1300	0.72608	4681.3	5407.4	9.4542	0.60507	4680.9	5406.9	9.3698	0.51864	4680.4	5406.5	9.2983
	P = 1.60 MPa (201.40)				*P* = 1.80 MPa (207.15)				*P* = 2.00 MPa (212.42)			
Sat.	0.12380	2595.9	2794.0	6.4217	0.11042	2598.4	2797.1	6.3793	0.09963	2600.3	2799.5	6.3408
225	0.13287	2644.6	2857.2	6.5518	0.11673	2636.6	2846.7	6.4807	0.10377	2628.3	2835.8	6.4146
250	0.14184	2692.3	2919.2	6.6732	0.12497	2686.0	2911.0	6.6066	0.11144	2679.6	2902.5	6.5452
300	0.15862	2781.0	3034.8	6.8844	0.14021	2776.8	3029.2	6.8226	0.12547	2772.6	3023.5	6.7663
350	0.17456	2866.0	3145.4	7.0693	0.15457	2862.9	3141.2	7.0099	0.13857	2859.8	3137.0	6.9562
400	0.19005	2950.1	3254.2	7.2373	0.16847	2947.7	3250.9	7.1793	0.15120	2945.2	3247.6	7.1270

(Contd.)

Table A.1.3 *(Contd.)*

	P = 1.60 MPa (201.40)				*P* = 1.80 MPa (207.15)				*P* = 2.00 MPa (212.42)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
500	0.22029	3119.5	3471.9	7.5389	0.19550	3117.8	3469.7	7.4824	0.17568	3116.2	3467.6	7.4316
600	0.24998	3293.3	3693.2	7.8080	0.22199	3292.1	3691.7	7.7523	0.19960	3290.9	3690.1	7.7023
700	0.27937	3472.7	3919.7	8.0535	0.24818	3471.9	3918.6	7.9983	0.22323	3471.0	3917.5	7.9487
800	0.30859	3658.4	4152.1	8.2808	0.27420	3657.7	4151.3	8.2258	0.24668	3657.0	4150.4	8.1766
900	0.33772	3850.5	4390.8	8.4934	0.30012	3849.9	4390.1	8.4386	0.27004	3849.3	4389.4	8.3895
1000	0.36678	4049.0	4635.8	8.6938	0.32598	4048.4	4635.2	8.6390	0.29333	4047.9	4634.6	8.5900
1100	0.39581	4253.7	4887.0	8.8837	0.35180	4253.2	4886.4	8.8290	0.31659	4252.7	4885.9	8.7800
1200	0.42482	4464.2	5143.9	9.0642	0.37761	4663.7	5143.4	9.0096	0.33984	4463.2	5142.9	8.9606
1300	0.45382	4679.9	5406.0	9.2364	0.40340	4679.4	5405.6	9.1817	0.36306	4679.0	5405.1	9.1382
	P = 2.50 MPa (223.99)				*P* = 3.00 MPa (233.90)				*P* = 3.50 MPa (242.66)			
Sat.	0.07998	2603.1	2803.1	6.2574	0.06668	2604.1	2804.1	6.1869	0.05707	2603.7	2803.4	6.1252
225	0.08027	2605.6	2806.3	6.2638	—	—	—	—	—	—	—	—
250	0.08700	2662.5	2880.1	6.4084	0.07058	2644.0	2855.8	6.2871	0.05873	2623.7	2829.2	6.1748
300	0.09890	2761.6	3008.8	6.6437	0.08114	2750.0	2993.5	6.5389	0.06842	2738.0	2977.5	6.4460
350	0.10976	2851.8	3126.2	6.8402	0.09053	2843.7	3115.3	6.7427	0.07678	2835.3	3104.0	6.6578
400	0.12010	2939.0	3239.3	7.0147	0.09936	2932.7	3230.8	6.9211	0.08453	2926.4	3222.2	6.8404
450	0.13014	3025.4	3350.8	7.1745	0.10787	3020.4	3344.0	7.0833	0.09196	3015.3	3337.2	7.0051
500	0.13998	3112.1	3462.0	7.3233	0.11619	3107.9	3456.5	7.2337	0.09918	3103.7	3450.9	7.1571
600	0.15930	3288.0	3686.2	7.5960	0.13243	3285.0	3682.3	7.5084	0.11324	3282.1	3678.4	7.4338
700	0.17832	3468.8	3914.6	7.8435	0.14838	3466.0	3911.7	7.7571	0.12699	3464.4	3908.8	7.6837
800	0.19716	3655.3	4148.2	8.0720	0.16414	3653.6	4146.0	7.9862	0.14056	3651.8	4143.8	7.9135
900	0.21590	3847.9	4387.6	8.2853	0.17980	3846.5	4385.9	8.1999	0.15402	3845.0	4384.1	8.1275
1000	0.23458	4046.7	4633.1	8.4860	0.19541	4045.4	4631.6	8.4009	0.16743	4044.1	4630.1	8.3288
1100	0.25322	4251.5	4884.6	8.6761	0.21098	4250.3	4883.3	8.5911	0.18080	4249.1	4881.9	8.5191
1200	0.27185	4462.1	5141.7	8.8569	0.22652	4460.9	5140.5	8.7719	0.19415	4459.8	5139.3	8.7000
1300	0.29046	4677.8	5404.0	9.0291	0.24206	4676.6	5402.8	8.9442	0.20749	4675.5	5401.7	8.8723
	P = 4.00 MPa (250.40)				*P* = 4.50 MPa (257.48)				*P* = 5.00 MPa (263.99)			
Sat.	0.04978	2602.3	2801.4	6.0700	0.04406	2600.0	2798.3	6.0198	0.03944	2597.1	2794.3	5.9733
275	0.05457	2667.9	2886.2	6.2284	0.04730	2650.3	2863.1	6.1401	0.04141	2631.2	2838.3	6.0543
300	0.05884	2725.3	2960.7	6.3614	0.05135	2712.0	2943.1	6.2827	0.04532	2697.9	2924.5	6.2083
350	0.06645	2826.6	3092.4	6.5820	0.05840	2817.8	3080.6	6.5130	0.05194	2808.7	3068.4	6.4492
400	0.07341	2919.9	3213.5	6.7689	0.06475	2913.3	3204.7	6.7046	0.05781	2906.6	3195.6	6.6458
450	0.08003	3010.1	3330.2	6.9362	0.07074	3004.9	3323.2	6.8745	0.06330	2999.6	3316.1	6.8185
500	0.08643	3099.5	3445.2	7.0900	0.07651	3095.2	3439.5	7.0300	0.06857	3090.9	3433.8	6.9758
600	0.09885	3279.1	3674.4	7.3688	0.08765	3276.0	3670.5	7.3109	0.07869	3273.0	3666.5	7.2588
700	0.11095	3462.1	3905.9	7.6198	0.09847	3459.9	3903.0	7.5631	0.08849	3457.7	3900.1	7.5122
800	0.12287	3650.1	4141.6	7.8502	0.10911	3648.4	4139.4	7.7942	0.09811	3646.6	4137.2	7.7440
900	0.13469	3843.6	4382.3	8.0647	0.11965	3842.1	4380.6	8.0091	0.10762	3840.7	4378.8	7.9593
1000	0.14645	4042.9	4628.7	8.2661	0.13013	4041.6	4627.2	8.2108	0.11707	4040.3	4625.7	8.1612
1100	0.15817	4248.0	4880.6	8.4566	0.14056	4246.8	4879.3	8.4014	0.12648	4245.6	4878.0	8.3519
1200	0.16987	4458.6	5138.1	8.6376	0.15098	4457.4	5136.9	8.5824	0.13587	4456.3	5135.7	8.5330
1300	0.18156	4674.3	5400.5	8.8099	0.16139	4673.1	5399.4	8.7548	0.14526	4672.0	5398.2	8.7055

(Contd.)

Table A.1.3 *(Contd.)*

	P = 6.00 MPa (275.64)				P = 7.00 MPa (285.88)				P = 8.00 MPa (295.06)			
T	v	u	h	s	v	u	h	s	v	u	h	s
Sat.	0.03244	2589.7	2784.3	5.8891	0.02737	2580.5	2772.1	5.8132	0.02352	2569.8	2757.9	5.7431
300	0.03616	2667.2	2884.2	6.0673	0.02947	2632.1	2838.4	5.9304	0.02426	2590.9	2785.0	5.7905
350	0.04223	2789.6	3043.0	6.3334	0.03524	2769.3	3016.0	6.2282	0.02995	2747.7	2987.3	6.1300
400	0.04739	2892.8	3177.2	6.5407	0.03993	2878.6	3158.1	6.4477	0.03432	2863.8	3138.3	6.3633
450	0.05214	2988.9	3301.8	6.7192	0.04416	2977.9	3287.0	6.6326	0.03817	2966.7	3272.0	6.5550
500	0.05665	3082.2	3422.1	6.8802	0.04814	3073.3	3410.3	6.7974	0.04175	3064.3	3398.3	6.7239
550	0.06101	3174.6	3540.6	7.0287	0.05195	3167.2	3530.9	6.9486	0.04516	3159.8	3521.0	6.8778
600	0.06525	3266.9	3658.4	7.1676	0.05565	3260.7	3650.3	7.0894	0.04845	3254.4	3642.0	7.0205
700	0.07352	3453.2	3894.3	7.4234	0.06283	3448.6	3888.4	7.3476	0.05481	3444.0	3882.5	7.2812
800	0.08160	3643.1	4132.7	7.6566	0.06981	3639.6	4128.3	7.5822	0.06097	3636.1	4123.8	7.5173
900	0.08958	3837.8	4375.3	7.8727	0.07669	3835.0	4371.8	7.7991	0.06702	3832.1	4368.3	7.7350
1000	0.09749	4037.8	4622.7	8.0751	0.08350	4035.3	4619.8	8.0020	0.07301	4032.8	4616.9	7.9384
1100	0.10536	4243.3	4875.4	8.2661	0.09027	4240.9	4872.8	8.1933	0.07896	4238.6	4870.3	8.1299
1200	0.11321	4454.0	5133.3	8.4473	0.09703	4451.7	5130.9	8.3747	0.08489	4449.4	5128.5	8.3115
1300	0.12106	4669.6	5396.0	8.6199	0.10377	4667.3	5393.7	8.5472	0.09080	4665.0	5391.5	8.4842
	P = 9.00 MPa (303.40)				P = 10.00 MPa (311.06)				P = 12.50 MPa (327.89)			
Sat.	0.02048	2557.8	2742.1	5.6771	0.01803	2544.4	2724.7	5.6140	0.01350	2505.1	2673.8	5.4623
325	0.02327	2646.5	2855.9	5.8711	0.01986	2610.4	2809.0	5.7568	—	—	—	—
350	0.02580	2724.4	2956.5	6.0361	0.02242	2699.2	2923.4	5.9442	0.01613	2624.6	2826.2	5.7117
400	0.02993	2848.4	3117.8	6.2853	0.02641	2832.4	3096.5	6.2119	0.02000	2789.3	3039.3	6.0416
450	0.03350	2955.1	3256.6	6.4843	0.02975	2943.3	3240.8	6.4189	0.02299	3912.4	3199.8	6.2718
500	0.03677	3055.1	3386.1	6.6575	0.03279	3045.8	3373.6	6.5965	0.02560	3021.7	3341.7	6.4617
550	0.03987	3152.2	3511.0	6.8141	0.03564	3144.5	3500.9	6.7561	0.02801	3124.9	3475.1	6.6289
600	0.04285	3248.1	3633.7	6.9588	0.03837	3241.7	3625.3	6.9028	0.03029	3225.4	3604.0	6.7810
650	0.04574	3343.7	3755.3	7.0943	0.04101	3338.2	3748.3	7.0397	0.03248	3324.4	3730.4	6.9218
700	0.04857	3439.4	3876.5	7.2221	0.04358	3434.7	3870.5	7.1687	0.03460	3422.9	3855.4	7.0536
800	0.05409	3632.5	4199.4	7.4597	0.04859	3629.0	4114.9	7.4077	0.03869	3620.0	4103.7	7.2965
900	0.5950	3829.2	4364.7	7.6782	0.05349	3826.3	4361.2	7.6272	0.04267	3819.1	4352.5	7.5181
1000	0.06485	4030.3	4613.9	7.8821	0.05832	4027.8	4611.0	7.8315	0.04658	4021.6	4603.8	7.7237
1100	0.07016	4236.3	4867.7	8.0739	0.06312	4234.0	4865.1	8.0236	0.05045	4228.2	4858.8	7.9165
1200	0.07544	4447.2	5126.2	8.2556	0.06789	4444.9	5123.8	8.2054	0.05430	4439.3	5118.0	8.0987
1300	0.08072	4662.7	5389.2	8.4283	0.07265	4660.4	5387.0	8.3783	0.05813	4654.8	5381.4	8.2717
	P = 15 MPa (342.24)				P = 17.5 MPa (354.75)				P = 20 MPa (365.81)			
Sat.	.010338	2455.4	2610.5	5.3097	.0079204	2390.2	2528.8	5.1418	.0058342	2293.1	2409.7	4.9269
350	.011470	2520.4	2692.4	5.4420	.0017139	1632.0	1662.0	3.7612	.0016640	1612.3	1645.6	3.7280
400	.015649	2740.7	2975.4	5.8810	.0124477	2685.0	2902.8	5.7212	.0099423	2619.2	2818.1	5.5539
450	.018446	2879.5	3156.2	6.1403	.0151740	2844.2	3109.7	6.0182	.0126953	2806.2	3060.1	5.9016
500	.020800	2996.5	3308.5	6.3442	.0173585	2970.3	3274.0	6.2382	.0147683	2942.8	3238.2	6.1400
550	.022927	3104.7	3448.6	6.5198	.0192877	3083.8	3421.4	6.4229	.0165553	3062.3	3393.5	6.3347

(Contd.)

Table A.1.3 *(Contd.)*

	P = 15 MPa (342.24)				*P* = 17.5 MPa (354.75)				*P* = 20 MPa (365.81)			
T	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*	*v*	*u*	*h*	*s*
600	.024911	3208.6	3582.3	6.6775	.0210640	3191.5	3560.1	6.5866	.0181781	3174.0	3537.6	6.5048
650	.026797	3310.4	3712.3	6.8223	.0227372	3296.0	3693.9	6.7356	.0196929	3281.5	3675.3	6.6582
700	.028612	3410.9	3840.1	6.9572	.0243365	3398.8	3824.7	6.8736	.0211311	3386.5	3809.1	6.7993
800	.032096	3611.0	4092.4	7.2040	.0273849	3601.9	4081.1	7.1245	.0238532	3592.7	4069.8	7.0544
900	.035457	3811.9	4343.8	7.4279	.0303071	3804.7	4335.1	7.3507	.0264463	3797.4	4326.4	7.2830
1000	.038748	4015.4	4596.6	7.6347	.0331580	4009.3	4589.5	7.5588	.0289666	4003.1	4582.5	7.4925
1100	.042001	4222.6	4852.6	7.8282	.0359695	4216.9	4846.4	7.7530	.0314471	4211.3	4840.2	7.6874
1200	.045233	4433.8	5112.3	8.0108	.0387605	4428.3	5106.6	7.9359	.0339071	4422.8	5101.0	7.8706
1300	.048455	4649.1	5375.9	8.1839	.0415417	4643.5	5370.5	8.1093	.0363574	4638.0	5365.1	8.0441
	P =25 MPa				*P* = 30 MPa				*P* = 35 MPa			
375	.001973	1798.6	1847.9	4.0319	.001789	1737.8	1791.4	3.9303	.001700	1702.9	1762.4	3.8721
400	.006004	2430.1	2580.2	5.1418	.002790	2067.3	2151.0	4.4728	.002100	1914.0	1987.5	4.2124
425	.007882	2609.2	2806.3	5.4722	.005304	2455.1	2614.2	5.1503	.003428	2253.4	2373.4	4.7747
450	.009162	2720.7	2949.7	5.6743	.006735	2619.3	2821.4	5.4423	.004962	2498.7	2672.4	5.1962
500	.011124	2884.3	3162.4	5.9592	.008679	2820.7	3081.0	5.7904	.006927	2751.9	2994.3	5.6281
550	.012724	3017.5	3335.6	6.1764	.010168	2970.3	3275.4	6.0342	.008345	2920.9	3213.0	5.9025
600	.014138	3137.9	3491.4	6.3602	.011446	3100.5	3443.9	6.2330	.009527	3062.0	3395.5	6.1178
650	.015433	3251.6	3637.5	6.5229	.012596	3221.0	3598.9	6.4057	.010575	3189.8	3559.9	6.3010
700	.016647	3361.4	3777.6	6.6707	.013661	3335.8	3745.7	6.5606	.011533	3309.9	3713.5	6.4631
800	.018913	3574.3	4047.1	6.9345	.015623	3555.6	4024.3	6.8332	.013278	3536.8	4001.5	6.7450
900	.021045	3783.0	4309.1	7.1679	.017448	3768.5	4291.9	7.0717	.014883	3754.0	4274.9	6.9886
1000	.023102	3990.9	4568.5	7.3801	.019196	3978.8	4554.7	7.2867	.016410	3966.7	4541.1	7.2063
1100	.025119	4200.2	4828.2	7.5765	.020903	4189.2	4816.3	7.4845	.017895	4178.3	4804.6	7.4056
1200	.027115	4412.0	5089.9	7.7604	.022589	4410.3	5079.0	7.6691	.019360	4390.7	5068.4	7.5910
1300	.029101	4626.9	5354.4	7.9342	.024266	4616.0	5344.0	7.8432	.020815	4605.1	5333.6	7.7652
	P = 40 MPa				*P* = 50 MPa				*P* = 60 MPa			
375	.0016406	1677.1	1742.7	3.8289	.0015593	1638.6	1716.5	3.7638	.0015027	1609.3	1699.5	3.1740
400	.0019077	1854.5	1930.8	4.1134	.0017309	1788.0	1874.6	4.0030	.0016335	1745.3	1843.4	3.9317
425	.0025319	2096.8	2198.1	4.5028	.0020071	1959.6	2060.0	4.2733	.0018165	1892.7	2001.7	4.1625
450	.0036931	2365.1	2512.8	4.9459	.0024862	2159.6	2283.9	4.5883	.0020850	2053.9	2179.0	4.4119
500	.0056225	2678.4	2903.3	5.4699	.0038924	2525.5	2720.1	5.1725	.0029557	2390.5	2567.9	4.9320
600	.0080943	3022.6	3346.4	6.0113	.0061123	2942.0	3247.6	5.8177	.0048345	2861.1	3151.2	5.6451
650	.0090636	3158.0	3520.6	6.2054	.0069657	3093.6	3441.8	6.0342	.0055953	3028.8	3364.6	5.8829
700	.0099415	3283.6	3681.3	6.3750	.0077274	3230.5	3616.9	6.2189	.0062719	3177.3	3553.6	6.0824
800	.0115228	3517.9	3978.8	6.6662	.0090761	3479.8	3933.6	6.5290	.0074588	3441.6	3889.1	6.4110
900	.0129626	3739.4	4257.9	6.9150	.0102831	3710.3	4224.4	6.7882	.0085083	3681.0	4191.5	6.6805
1000	.0143238	3954.6	4527.6	7.1356	.0114113	3930.5	4501.1	7.0146	.0094800	3906.4	4475.2	6.9126
1100	.0156426	4167.4	4793.1	7.3364	.0124966	4145.7	4770.6	7.2183	.0104091	4124.1	4748.6	7.1194
1200	.0169403	4380.1	5057.7	7.5224	.0135606	4359.1	5037.2	7.4058	.0113167	4338.2	5017.2	7.3082
1300	.0182292	4594.3	5323.5	7.6969	.0146159	4572.8	5303.6	7.5807	.0122155	4551.4	5284.3	7.4837

A.2 THERMODYNAMIC PROPERTIES OF REFRIGERANT-12* (DICHLORODIFLUOROMETHANE)

Table A.2.1 *Saturated refrigerant-12*

		Specific Volume		*Enthalpy*			*Entropy*	
Temperature °C t	*Presssure* MPa p	*Sat. Liquid* v_f Cm^3/g	*Sat. Vapour* v_g m^3/kg	*Sat. Liquid* h_f	*Evap.* h_{fg} kJ/kg	*Sat. Vapour* h_g	*Sat. Liquid* s_f	*Sat. Vapour* s_g kJ/kg K
– 90	0.0028	0.608	4.415545	– 43.243	189.618	146.375	– 0.2084	0.8268
– 85	0.0042	0.612	3.037316	– 38.968	187.608	148.640	– 0.1854	0.8116
– 80	0.0062	0.617	2.038345	– 34.688	185.612	150.924	– 0.1630	0.7979
– 75	0.0088	0.622	1.537651	– 30.401	183.625	153.224	– 0.1411	0.7855
– 70	0.0123	0.627	1.127280	– 26.103	181.640	155.536	– 0.1197	0.7744
– 65	0.0168	0.632	0.841166	– 21.793	179.651	157.857	– 0.0987	0.7643
– 60	0.0226	0.637	0.637910	– 17.469	177.653	160.184	– 0.0782	0.7552
– 55	0.0300	0.642	0.491000	– 13.129	175.641	162.512	– 0.0581	0.7470
– 50	0.0391	0.648	0.383105	– 8.772	173.611	164.840	– 0.0384	0.7396
– 45	0.0504	0.654	0.302682	– 4.396	171.558	167.163	– 0.0190	0.7329
– 40	0.0642	0.659	0.241910	– 0.000	169.479	169.479	– 0.0000	0.7269
– 35	0.0807	0.666	0.195398	4.416	167.368	171.784	0.0187	0.7214
– 30	0.1004	0.672	0.159375	8.854	165.222	174.076	0.0371	0.7165
– 25	0.1237	0.679	0.131166	13.315	163.037	176.352	0.0552	0.7121
– 20	0.1509	0.685	0.108847	17.800	160.810	178.610	0.0730	0.7082
– 15	0.1826	0.693	0.091018	22.312	158.534	180.846	0.0906	0.7046
– 10	0.2191	0.700	0.076646	26.851	156.207	183.058	0.1079	0.7014
– 5	0.2610	0.708	0.064963	31.420	153.823	185.243	0.1250	0.6986
0	0.3086	0.716	0.055389	36.022	151.376	187.397	0.1418	0.6960
5	0.3626	0.724	0.047485	40.659	148.859	189.518	0.1585	0.6937
10	0.4233	0.733	0.040914	45.337	146.265	191.602	0.1750	0.6916
15	0.4914	0.743	0.035413	50.058	143.586	193.644	0.1914	0.6897
20	0.5673	0.752	0.030780	54.828	140.812	195.641	0.2076	0.6879
25	0.6516	0.763	0.026854	59.653	137.933	197.586	0.2237	0.6863
30	0.7449	0.774	0.023508	64.539	134.936	199.475	0.2397	0.6848
35	0.8477	0.786	0.020641	69.494	131.805	201.299	0.2557	0.6834
40	0.9607	0.798	0.018171	74.527	128.525	203.051	0.2716	0.6820
45	1.0843	0.811	0.016032	79.647	125.074	204.722	0.2875	0.6806
50	1.2193	0.826	0.014170	84.868	121.430	206.298	0.3034	0.6792
55	1.3663	0.841	0.012542	90.201	117.565	207.766	0.3194	0.6777
60	1.5259	0.858	0.01111	95.665	113.443	209.109	0.3355	0.6760
65	1.6988	0.877	0.009847	101.279	109.024	210.303	0.3518	0.6742
70	1.8858	0.897	0.008725	107.067	104.255	211.321	0.3683	0.6721
75	2.0874	0.920	0.007723	113.058	99.068	212.126	0.3851	0.6697
80	2.3046	0.946	0.006821	119.291	93.373	212.665	0.4023	0.6667
85	2.5380	0.976	0.006005	125.818	87.047	212.865	0.4201	0.6631
90	2.7885	1.012	0.005258	132.708	79.907	212.614	0.4385	0.6585
95	3.0569	1.056	0.004563	140.068	71.658	211.726	0.4579	0.6526
100	3.3440	1.113	0.003903	148.076	61.768	209.843	0.4788	0.6444
105	3.6509	1.197	0.003242	157.085	49.014	206.099	0.5023	0.6319
110	3.9784	1.364	0.002462	168.059	28.425	196.484	0.5322	0.6064

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* Repeated from *Fundamentals of Classical Thermodynamics* by G.J. Van Wylen and R. Sonntag. John Wiley, New York 1976, P. 667–673 (with kind permission of the publishers, John Wiley & Sons, Inc, New York).

Table A.2.2 *Saturated refrigerant-12*

t °C	*v* m^3kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m/kg	*h* kJ/kg	*s* kJ/kg K
		0.05 MPa			*0.10 MPa*			*0.15 MPa*	
– 20.00	0.341857	181.042	0.7912	0.167701	179.861	0.7401			
– 10.00	0.356228	186.757	0.8133	0.175222	185.707	0.7628	0.114716	184.619	0.7318
0.00	0.370508	192.567	0.8350	0.182647	191.628	0.7849	0.119866	190.660	0.7543
10.00	0.384716	198.471	0.8562	0.189994	197.628	0.8064	0.124932	196.762	0.7763
20.00	0.398863	204.469	0.8770	0.197277	203.707	0.8275	0.129930	202.927	0.7977
30.00	0.412959	210.557	0.8974	0.204506	209.866	0.8482	0.134873	209.160	0.8186
40.00	0.427012	216.733	0.9175	0.211691	216.104	0.8684	0.139768	215.463	0.8390
50.00	0.441030	222.997	0.9372	0.218839	222.421	0.8883	0.144625	221.835	0.8591
60.00	0.455017	229.344	0.9565	0.225955	228.815	0.9078	0.149450	228.277	0.8787
70.00	0.458978	235.774	0.9755	0.233044	235.285	0.9269	0.154247	234.789	0.8980
80.00	0.482917	242.282	0.9942	0.240111	241.829	0.9457	0.159020	241.371	0.9169
90.00	0.496838	248.868	1.0126	0.247159	248.446	0.9642	0.163774	248.020	0.9354
		0.20 MPa			*0.25 MPa*			*0.30 MPa*	
00.0	0.088608	189.669	0.7320	0.069752	188.644	0.7139	0.057150	187.583	0.6984
10.00	0.092550	195.878	0.7543	0.073024	194.969	0.7366	0.059984	194.034	0.7216
20.00	0.096418	202.135	0.7760	0.076218	201.322	0.7587	0.062734	200.490	0.7440
30.00	0.100228	208.446	0.7972	0.079350	207.715	0.7801	0.065418	206.969	0.7658
40.00	0.103989	214.814	0.8178	0.082431	214.153	0.8010	0.068049	213.480	0.7869
50.00	0.107710	221.243	0.8381	0.085470	220.642	0.8214	0.070635	220.030	0.8075
60.00	0.111397	227.735	0.8578	0.088474	227.185	0.8413	0.073185	226.627	0.8276
70.00	0.115055	134.291	0.8772	0.091449	233.785	0.8608	0.075750	233.273	0.8473
80.00	0.118690	240.910	0.8962	0.094398	240.443	0.8800	0.078200	239.271	0.8665
90.00	0.122304	247.593	0.9149	0.097327	247.160	0.8987	0.080673	246.723	0.8853
100.0	0.125901	254.339	0.9332	0.100238	253.936	0.9171	0.83127	253.530	0.9038
110.0	0.129483	261.147	0.9512	0.103134	260.770	0.9352	0.085566	260.391	0.9220

(Contd.)

Table A.2.2 *(Contd.)*

t	*v*	*h*	*s*	*v*	*h*	*s*	*v*	*h*	*s*
		0.40 MPa			*0.50 MPa*			*0.60 MPa*	
20.0	0.045836	198.762	0.7199	0.035646	196.935	0.6999			
30.0	0.047971	205.428	0.7423	0.037464	203.814	0.7230	0.030422	202.116	0.7063
40.0	0.050046	212.095	0.7639	0.039214	210.656	0.7452	0.031966	209.154	0.7291
50.0	0.052072	218.779	0.7849	0.040911	217.484	0.7667	0.033450	216.141	0.7511
60.0	0.054059	225.488	0.8054	0.042565	224.315	0.7875	0.034887	223.104	0.7723
70.0	0.056014	232.230	0.8253	0.044184	232.161	0.8077	0.036285	230.062	0.7929
80.0	0.057941	239.012	0.8448	0.045774	238.031	0.8275	0.037653	237.027	0.8129
90.0	0.059846	245.837	0.8638	0.047340	244.932	0.8467	0.038995	244.009	0.8324
100.0	0.061731	252.707	0.8825	0.048886	251.869	0.8656	0.040316	251.016	0.8514
110.0	0.063600	259.624	0.9008	0.050415	258.845	0.8840	0.041619	258.053	0.8700
		0.70 MPa			*0.80 MPa*			*0.90 MPa*	
40.0	0.026761	207.580	0.7148	0.022830	205.924	0.7016	0.019744	204.170	0.6982
50.0	0.028100	214.745	0.7373	0.024068	213.290	0.7248	0.020912	211.765	0.7131
60.0	0.029387	221.854	0.7590	0.025247	220.558	0.7469	0.022012	218.212	0.7358
70.0	0.030632	228.931	0.7799	0.026380	227.766	0.7682	0.023062	226.564	0.7575
80.0	0.031843	235.997	0.8002	0.027477	234.941	0.7888	0.024072	233.856	0.7785
90.0	0.033027	243.066	0.8199	0.028545	242.101	0.8088	0.025051	141.113	0.7987
100.0	0.034189	250.146	0.8392	0.029588	249.260	0.8283	0.026005	248.355	0.8184
110.0	0.035332	257.247	0.8579	0.030612	256.428	0.8472	0.026937	255.593	0.8376
		1.00 MPa			*1.20 MPa*			*1.40 MPa*	
50.0	0.018366	210.162	0.7021	0.014483	206.661	0.6812			
60.0	0.019410	217.810	0.7254	0.015463	214.805	0.7060	0.012579	211.457	0.6876
70.0	0.020397	225.319	0.7476	0.016368	222.687	0.7293	0.013448	219.822	0.7123
80.0	0.021341	232.739	0.7689	0.017221	230.398	0.7514	0.014247	227.891	0.7355
90.0	0.022251	240.101	0.7895	0.018032	237.995	0.7727	0.014997	235.766	0.7575
100.0	0.023133	247.430	0.8094	0.018812	245.518	0.7931	0.015710	243.512	0.7785
110.10	0.023993	254.743	0.8287	0.019567	252.993	0.8129	0.016393	251.170	0.7988
		1.60 MPa			*1.80 MPa*			*2.00 MPa*	
70.0	0.011208	216.650	0.6959	0.009406	213.049	0.6794			
80.0	0.011984	225.177	0.7204	0.010187	222.198	0.7057	0.008704	218.859	0.6909
90.0	0.012698	233.390	0.7433	0.010884	230.835	0.7298	0.009406	228.056	0.7166
100.0	0.013366	241.397	0.7651	0.011526	239.155	0.7524	0.010035	263.760	0.7402
110.0	0.014000	249.264	0.7859	0.012126	2467.264	0.7739	0.010615	245.154	0.7624

A.3 THERMODYNAMIC PROPERTIES OF REFRIGERENT-12 (MONOCHLORODIFLUOROMETHANE)

Table A.3.1 *Saturated refrigerant-22*

		Specific volume (m^3/kg)			*Enthalpy (kJ/kg)*			*Entropy (kJ/kg K)*		
Temp. °C	*Abs. Press. MPa* P	*Sat. Liquid* v_f	*Evap.* v_{fg}	*Sat. Vapour* v_g	*Sat. Liquid* h_f	*Evap.* h_{fg}	*Vapour* h_g	*Sat. Liquid* s_f	*Evap.* s_{fg}	*Sat. Vapour* s_g
70	0.0205	0.000670	0.940268	0.94093	– 30.607	249.425	218.180	– 0.1401	1.2277	1.0876
65	0.0280	0.000676	0.704796	0.705478	– 25.658	246.925	221.267	– 0.1161	1.1862	1.0701
60	0.0375	0.000682	0.536470	0.537152	– 20.652	244.354	223.702	– 0.0924	1.1463	1.0540
55	0.0495	0.000689	0.414138	0.414827	– 15.585	241.703	226.117	– 0.0689	1.1079	1.0390
50	0.0644	0.000695	0.323862	0.324557	– 10.456	238.965	228.509	– 0.0457	1.0708	1.0251
45	0.0827	0.000702	0.256288	0.256990	– 5.262	236.132	230.870	– 0.0227	1.0349	1.0122
40	0.1049	0.000709	0.205036	0.205745	0	233.198	233.197	0	1.0002	1.0002
35	0.1317	0.000717	0.165583	0.166400	5.328	230.156	235.484	0.0225	0.9664	0.9889
30	0.1635	0.000725	0.135120	0.135844	10.725	227.001	237.726	0.0449	0.9335	0.9784
25	0.2010	0.000733	0.111126	0.111859	16.191	223.727	239.918	0.0670	0.9015	0.9685
20	0.2448	0.000741	0.092102	0.092843	21.728	220.327	242.055	0.0890	0.8703	0.9593
15	0.2957	0.000750	0.076876	0.077625	27.334	216.798	244.132	0.1107	0.8398	0.9505
10	0.3543	0.000759	0.064581	0.065340	33.012	213.132	246.144	0.1324	0.8099	0.9422
5	0.4213	0.000768	0.054571	0.055339	38.762	209.323	248.085	0.1538	0.7806	0.9344
0	0.4976	0.000778	0.046357	0.047135	44.586	205.364	249.949	0.1751	0.7518	0.9269
5	0.5838	0.000789	0.039567	0.040356	50.485	201.246	251.731	0.1963	0.7235	0.9197
10	0.6807	0.000800	0.033914	0.034714	56.463	196.960	253.423	0.2173	0.6956	0.9129
15	0.7891	0.000812	0.029176	0.029987	62.523	192.495	255.018	0.2382	0.6680	0.9062
20	0.9099	0.000824	0.025179	0.026003	68.670	187.836	256.506	0.2590	0.6407	0.8997
25	1.0439	0.000838	0.021787	0.022624	74.910	182.968	257.877	0.2797	0.6137	0.8934
30	1.1919	0.000852	0.018890	0.019742	81.250	177.869	259.119	0.3004	0.5867	0.8871
35	1.3548	0.000867	0.016401	0.017269	87.700	172.516	260.216	0.3210	0.5598	0.8809
40	1.5335	0.000884	0.014251	0.015135	94.272	166.877	261.149	0.3417	0.5329	0.8746
45	1.7290	0.000902	0.012382	0.013284	100.982	160.914	261.896	0.3624	0.5058	0.8682
50	1.9423	0.000922	0.010747	0.011669	107.851	154.576	262.428	0.3832	0.4783	0.8615
55	2.1744	0.000944	0.009308	0.010252	114.905	147.800	262.705	0.4042	0.4504	0.8546
60	2.4266	0.000969	0.008032	0.009001	122.180	140.497	262.678	0.4255	0.4217	0.8472
65	2.6999	0.000997	0.006890	0.007887	129.729	132.547	262.276	0.4472	0.3920	0.8391
70	2.9959	0.001030	0.005859	0.006889	137.625	123.772	261.397	0.4695	0.3607	0.8302
75	3.3161	0.001069	0.004914	0.005983	145.986	113.902	259.888	0.4927	0.3272	0.8198
80	3.6623	0.001118	0.004031	0.005149	155.011	102.475	257.486	0.5173	0.2902	0.8075
85	4.0368	0.001183	0.003175	0.004358	165.092	88.598	253.690	0.5445	0.2474	0.7918
90	4.4425	0.001282	0.002282	0.003564	177.204	70.037	247.241	0.5767	0.1929	0.7695
95	4.8835	0.001521	0.001030	0.002551	196.359	34.925	231.284	0.6273	0.0949	0.7222
96.006	4.9773	0.001906	0	0.001906	212.546	0	212.546	0.6708	0	0.6708

Table A.3.2 *Superheated refrigerant-22*

Temp. °C	v m^3/kg	h kJ/kg	s. kJ/kg K	v m^3/kg	h kJ/kg	s. kJ/kg K	v m^3/kg	h kJ/kg	s kJ/kg K
		0.05 MPa			0.10 MPa			0.15 MPa	
– 40	0.440633	234.724	1.07616	0.216331	233.337	1.00523	—	—	—
– 30	0.460641	240.602	1.10084	0.226754	239.359	1.03052	0.148723	238.078	0.98773
– 20	0.480543	246.586	1.12495	0.237064	245.466	1.05513	0.155851	244.319	1.01288
– 10	0.500357	252.676	1.14855	0.247279	251.665	1.07914	0.162879	250.631	1.03733
0	0.520095	258.874	1.17166	0.257415	257.956	1.10261	0.169823	257.022	1.06116
10	0.539771	265.180	1.19433	0.267485	264.345	1.12558	0.176699	263.496	1.08444
20	0.559393	271.594	1.21659	0.277500	270.831	1.14809	0.183516	270.057	1.10721
30	0.578970	278.115	1.23846	0.287467	277.416	1.17017	0.190284	276.709	1.12952
40	0.598507	284.743	1.25998	0.297394	284.101	1.19187	0.197011	283.452	1.15140
50	0.618011	291.478	1.28114	0.307287	290.887	1.21320	0.203702	290.289	1.17289
60	0.637485	298.319	1.30199	0.317149	297.772	1.23418	0.210362	297.220	1.19402
70	0.656935	305.265	1.32253	0.326986	304.757	1.25484	0.216997	304.246	1.21479
80	0.676362	312.314	1.34278	0.336801	311.842	1.27519	0.223608	311.368	1.23525
90	0.695771	319.465	1.36275	0.346596	319.026	1.29524	0.230200	318.584	1.25540
		0.20 MPa			0.25 MPa			0.30 MPa	
– 20	0.115203	243.140	0.98184	—	—	—	—	—	—
– 10	0.120647	249.574	1.00676	0.95280	248.492	0.98231	0.078344	247.382	0.96170
0	0.126003	256.069	1.03098	0.099689	255.097	1.00695	0.082128	254.104	0.98677
10	0.131286	262.633	1.05458	0.104022	261.755	1.03089	0.085832	260.861	1.01106
20	0.136509	269.273	1.07763	0.108292	268.476	1.05421	0.089469	267.667	1.03468
30	0.141681	275.992	1.10016	0.112508	275.267	1.07699	0.093051	274.531	1.05771
40	0.146809	282.796	1.12224	0.116681	282.132	1.09927	0.096588	281.460	1.08019
50	0.151902	289.686	1.14390	0.120815	289.076	1.12109	0.100085	288.460	1.10220
60	0.156963	296.664	1.16516	0.124918	296.102	1.14250	0.103550	295.535	1.12376
70	0.161997	303.731	1.18607	0.128993	303.212	1.16353	0.106986	302.689	1.14491
80	0.167008	310.890	1.20663	0.133044	310.409	1.18420	0.110399	309.924	1.16569
90	0.171999	318.139	1.22687	0.137075	317.692	1.20454	0.113790	317.241	1.18612
100	0.176972	325.480	1.24681	0.141089	325.063	1.22456	0.117164	324.643	1.20623
110	0.181931	332.912	1.26646	0.145086	332.522	1.24428	0.120522	332.129	1.22603

(Contd.)

Table A.3.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* kJ/kg	*s.* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K	*v* m^3/kg	*h* kJ/kg	*s* kJ/kg K
		0.40 MPa			*0.50 MPa*			*0.60 MPa*	
0	0.060131	252.051	0.95359	—	—	—	—	—	—
10	0.063060	259.023	0.97866	0.049355	257.108	0.95223	0.040180	255.109	0.92945
20	0.065915	266.010	1.00291	0.051751	264.295	0.97717	0.042280	262.517	0.95517
30	0.068710	273.029	1.02646	0.054081	271.483	1.00128	0.044307	269.888	0.97989
40	0.071455	280.092	1.04938	0.056358	278.690	1.02467	0.046276	277.250	1.00378
50	0.074160	287.209	1.07175	0.058590	285.930	1.04743	0.048198	284.622	1.02695
60	0.076830	294.386	1.09362	0.060786	293.215	1.06963	0.050081	292.020	1.04950
70	0.079470	301.630	1.11504	0.062951	300.552	1.09133	0.051931	299.456	1.07149
80	0.082085	308.944	1.13605	0.065090	307.949	1.11257	0.053754	306.938	1.09298
90	0.084679	316.332	1.15668	0.067206	315.410	1.13340	0.055553	314.475	1.11403
100	0.087254	323.796	1.17695	0.069303	322.939	1.15386	0.057332	322.071	1.13466
110	0.089813	331.339	1.19690	0.071384	330.539	1.17395	0.059094	329.731	1.15492
120	0.092358	338.961	1.21654	0.073450	338.213	1.19373	0.060842	337.458	1.17482
130	0.094890	346.664	1.23588	0.075503	345.963	1.21319	0.062576	345.255	1.19441
		0.70 MPa			*0.80 MPa*			*0.90 MPa*	
20	0.035487	260.667	0.93565	0.030366	258.737	0.91787	0.026355	256.713	0.90132
30	0.037305	268.240	0.96105	0.032034	266.533	0.94402	0.027915	264.760	0.92831
40	0.039059	275.769	0.98549	0.033632	274.243	0.96905	0.029397	272.670	0.95398
50	0.040763	283.282	1.00910	0.035175	281.907	0.99314	0.030819	280.497	0.97859
60	0.042424	290.800	1.03201	0.036674	289.553	1.01644	0.032193	288.278	1.00230
70	0.044052	298.339	1.05431	0.038136	297.202	1.03906	0.033528	296.042	1.02526
80	0.045650	305.912	1.07606	0.039568	304.868	1.06108	0.034832	303.807	1.04757
90	0.047224	313.527	1.09732	0.040974	312.565	1.08257	0.036108	311.590	1.06930
100	0.048778	321.192	1.11815	0.042359	320.303	1.10359	0.037363	319.401	1.09052
110	0.050313	328.914	1.13856	0.043725	328.087	1.12417	0.038598	327.251	1.11128
120	0.051834	336.696	1.15861	0.045076	335.925	1.14437	0.039817	335.147	1.13162
130	0.053341	344.541	1.17832	0.046413	343.821	1.16420	0.041022	343.094	1.15158
140	0.054836	352.454	1.19770	0.047738	351.778	1.18369	0.042215	351.097	1.17119
150	0.056321	360.435	1.21679	0.049052	359.799	1.20288	0.043398	359.159	1.19047

(Contd.)

Table A.3.2 *(Contd.)*

Temp. °C	*v* m^3/kg	*h* *kJ/kg*	*s.* *kJ/kg K*	*v* m^3/kg	*h* *kJ/k g*	*s.* *kJ/kg K*	*v* m^3/kg	*h* *kJ/kg*	*s* *kJ/kg K*
	1.00 MPa			*1.20 MPa*			*1.40 MPa*		
30	0.024600	262.912	0.91358	—	—	—	—	—	—
40	0.025995	271.042	0.93996	0.020851	267.602	0.91411	0.017120	263.861	0.89010
50	0.027323	279.046	0.96512	0.022051	276.011	0.94055	0.018247	272.766	0.91809
60	0.028601	286.973	0.98928	0.023191	284.263	0.96570	0.019299	281.401	0.94441
70	0.029836	294.859	1.01260	0.024282	292.413	0.98981	0.020295	289.858	0.96942
80	0.031038	302.727	1.03520	0.025336	330.508	1.01305	0.021248	298.202	0.99339
90	0.032213	310.599	1.05718	0.026359	308.570	1.03556	0.022167	306.473	1.01649
100	0.033364	318.488	1.07861	0.027357	316.623	1.05744	0.023058	314.703	1.03884
110	0.034495	326.405	1.09955	0.028334	324.682	1.07875	0.023926	322.916	1.06056
120	0.035609	334.360	1.12004	0.029292	332.762	1.09957	0.024775	331.128	1.08172
130	0.036709	342.360	1.14014	0.030236	340.871	1.11994	0.025608	339.354	1.10238
140	0.037797	350.410	1.15986	0.031166	349.019	1.13990	0.026426	347.603	1.12259
150	0.038873	358.514	1.17924	0.032084	357.210	1.15949	0.027233	355.885	1.14240
160	0.039940	366.677	1.19831	0.032993	365.450	1.17873	0.028029	364.206	1.16183
	1.60 MPa			*1.80 MPa*			*2.00 MPa*		
50	0.015351	269.262	0.89689	0.013052	265.423	0.87625	—	—	—
60	0.016351	278.358	0.92461	0.014028	275.097	0.90573	0.012135	271.563	0.88729
70	0.017284	287.171	0.95068	0.014921	284.331	0.93304	0.013008	281.310	0.91612
80	0.018167	295.797	0.97546	0.015755	293.282	0.95876	0.013811	290.640	0.94292
90	0.019011	304.301	0.99920	0.016546	302.046	0.98323	0.014563	299.697	0.96821
100	0.019825	312.725	1.02209	0.017303	310.683	1.00669	0.015277	308.571	0.99232
110	0.020614	321.103	1.04424	0.018032	319.239	1.02932	0.015960	317.322	1.01546
120	0.021382	329.457	1.06576	0.018738	327.745	1.05123	0.016619	325.991	1.03780
130	0.022133	337.805	1.08673	0.019427	336.224	1.07253	0.017258	334.610	1.05944
140	0.022869	346.162	1.10721	0.020099	344.695	1.09329	0.017881	343.201	1.08049
150	0.023592	354.540	1.12724	0.020759	353.172	1.11356	0.018490	351.783	1.10102
160	0.024305	362.945	1.14688	0.021407	361.666	1.13340	0.019087	360.369	1.12107
170	0.025008	371.386	1.16614	0.022045	370.186	1.15284	0.019673	368.970	1.14070
180	0.25703	379.869	1.18507	0.022675	378.738	1.17193	0.020251	377.595	1.15995

(Contd.)

Table A.3.2 *(Contd.)*

Temp. °C	v m³/kg	h kJ/kg	s. kJ/kg K	v m³/kg	h kJ/kg	s. kJ/kg K	v m³/kg	h kJ/kg	s kJ/kg K
	2.50 MPa			3.00 MPa			3.50 MPa		
70	0.009459	272.677	0.87476	—	—	—	—	—	—
80	0.010243	283.332	0.90537	0.007747	274.530	0.86780	0.005765	262.739	0.82489
90	0.010948	293.338	0.93332	0.008465	286.042	0.89995	0.006597	277.268	0.86548
100	0.011598	302.935	0.95939	0.009098	296.663	0.92881	0.007257	289.504	0.89872
110	0.012208	312.261	0.98405	0.009674	306.744	0.95547	0.007829	300.640	0.92818
120	0.012788	321.400	1.00760	0.010211	316.470	0.98053	0.008346	311.129	0.95520
130	0.013343	330.412	1.03023	0.010717	325.955	1.00435	0.008825	321.196	0.98047
140	0.013880	339.336	1.05210	0.011200	335.270	1.02718	0.009276	330.976	1.00445
150	0.014400	348.205	1.07331	0.011665	344.467	1.04918	0.009704	340.554	1.02736
160	0.014907	357.040	1.09395	0.012114	353.584	1.07047	0.010114	349.989	1.04940
170	0.015402	365.860	1.11408	0.012550	362.647	1.09116	0.010510	359.324	1.07071
180	0.015887	374.679	1.13376	0.012976	371.679	1.11131	0.010894	368.590	1.09138
190	0.016364	383.508	1.15303	0.013392	380.695	1.13099	0.011268	377.810	1.11151
200	0.016834	392.354	1.17912	0.013801	389.708	1.15024	0.011634	387.004	1.13115
	4.00 MPa			5.00 MPa			6.00 MPa		
90	0.005037	265.629	0.82544	—	—	—	—	—	—
100	0.005804	280.997	0.86721	0.003334	253.042	0.78005	—	—	—
110	0.006405	293.748	0.90094	0.004255	275.919	0.84064	0.002432	243.278	0.74674
120	0.006924	305.273	0.93064	0.004851	291.362	0.88045	0.003333	272.385	0.82185
130	0.007391	316.080	0.95778	0.005335	304.469	0.91337	0.003899	290.253	0.86675
140	0.007822	326.422	0.98312	0.005757	316.379	0.94256	0.004345	304.757	0.90230
150	0.008226	336.446	1.00710	0.006139	327.563	0.96931	0.004728	317.633	0.93310
160	0.008610	346.246	1.02999	0.006493	338.266	0.99431	0.005071	329.553	0.96094
170	0.008978	355.885	1.05199	0.006826	348.633	1.01797	0.005386	340.849	0.98673
180	0.009332	365.409	1.07324	0.007142	358.760	1.04057	0.005680	351.715	1.01098
190	0.009675	374.853	1.09386	0.007444	368.713	1.06230	0.005958	362.271	1.03402
200	0.010009	384.240	1.11391	0.007735	378.537	1.08328	0.006222	372.602	1.05609
210	0.010335	393.593	1.13347	0.008018	388.268	1.10363	0.006477	382.764	1.07734
220	0.010654	402.925	1.15259	0.008292	397.932	1.12343	0.006722	392.801	1.09790

Table A.4 *Variation of $\bar{c}_p$ with temperature of various ideal gases*

$$\frac{\bar{c}_p}{R} = \alpha + \beta T + \gamma T^2 + \delta T^3 + \epsilon T^4$$

T is in K, equations valid from 300 to 1000 K

Gas	α	$\beta \times 10^3$	$\gamma \times 10^6$	$\gamma \times 10^9$	$\varepsilon \times 10^{12}$
CO	3.710	–1.619	3.692	–2.032	0.240
CO_2	2.401	8.735	–6.607	2.002	0
K	3.057	2.677	–5.180	5.521	–1.812
H_2O	4.070	–1.108	4.152	–2.964	0.807
O_2	3.626	–1.878	7.056	–6.764	2.156
N_2	3.675	–1.208	2.324	–0.632	–0.226
Air	3.653	–1.337	3.294	–1.913	0.2763
SO_2	3.267	5.324	0.684	–5.281	2.559
CH_4	3.826	–3.979	24.558	–22.733	6.963
C_2H_2	1.410	19.057	24.501	16.391	–4.135
C_2H_4	1.426	11.383	7.989	–16.254	6.749
Monatomic gases*	2.5	0	0	0	0

*For monatomic gases, such as He, Ne, and Ar, $\bar{c}_p$ is constant over a wide temperature range and is very nearly equal to 5/2 $\bar{R}$.

Source: Adapted from K. Wark, *Thermodynamics*, 4th ed. McGraw-Hill, New York, 1983, as based on NASA SP-273, U.S. Government Printing Office, Washington, DC, 1971.

Table A.5 *Ideal gas specific heats of some common gases (kJ/kg.K)*

Temp. K	c_p	c_v	k	c_p	c_v	k	c_p	c_v	k	*Temp. K*
	Air			*Nitrogen,* N_2			*Oxygen,* O_2			
250	1.003	0.716	1.401	1.039	0.742	1.400	0.913	0.653	1.398	250
300	1.005	0.718	1.400	1.039	0.743	1.400	0.918	0.658	1.395	300
350	1.008	0.721	1.398	1.041	0.744	1.399	0.928	0.668	1.389	350
400	1.013	0.726	1.395	1.044	0.747	1.397	0.941	0.681	1.382	400
450	1.020	0.733	1.391	1.049	0.752	1.395	0.956	0.696	1.373	450
500	1.029	0.742	1.387	1.056	0.759	1.391	0.972	0.712	1.365	500
550	1.040	0.753	1.381	1.065	0.768	1.387	0.988	0.728	1.358	550
600	1.051	0.764	1.376	1.075	0.778	1.382	1.003	0.743	1.350	600
650	1.063	0.776	1.370	1.086	0.789	1.376	1.017	0.758	1.343	650
700	1.075	0.788	1.364	1.098	0.801	1.371	1.031	0.771	1.337	700
750	1.087	0.800	1.359	1.110	0.813	1.365	1.043	0.783	1.332	750
800	1.099	0.812	1.354	1.121	0.825	1.360	1.054	0.794	1.327	800
900	1.121	0.834	1.344	1.145	0.849	1.349	1.074	0.814	1.319	900
1000	1.142	0.855	1.336	1.167	0.870	1.341	1.090	0.830	1.313	1000
250	0.791	0.602	1.314	1.039	0.743	1.400	14.051	9.927	1.416	250
300	0.846	0.657	1.288	1.040	0.744	1.399	14.307	10.183	1.045	300
350	0.895	0.706	1.268	1.043	0.746	1.398	14.427	10.302	1.400	350
400	0.939	0.750	1.252	1.047	0.751	1.395	14.476	10.352	1.398	400
450	0.978	0.790	1.239	1.054	0.757	1.392	14.501	10.377	1.398	450
500	1.014	0.825	1.229	1.063	0.767	1.387	14.513	10.389	1.397	500
550	1.046	0.857	1.220	1.075	0.778	1.382	14.530	10.405	1.396	550
600	1.075	0.886	1.213	1.087	0.790	1.376	14.546	10.422	1.396	600
650	1.102	0.913	1.207	1.100	0.803	1.370	14.571	10.447	1.395	650
700	1.126	0.937	1.202	1.113	0.816	1.364	14.604	10.480	1.394	700
750	1.148	0.959	1.197	1.126	0.829	1.358	14.645	10.521	1.392	750
800	1.169	0.980	1.193	1.139	0.842	1.353	14.695	10.570	1.390	800
900	1.204	1.015	1.186	1.163	0.866	1.343	14.822	10.698	1.385	900
1000	1.234	1.045	1.181	1.185	0.888	1.335	14.983	10.859	1.380	1000

Source: Adapted from K. Wark, Thermodynamics, 4th ed., McGraw-Hill, New York, 1983, as based on "*Tables of Thermal Properties of Gases*," NBS Circular 564, 1955.

APPENDIX B

Appendix B1 shows the Mollier diagram for steam and pressure-enthalpy diagrams for refrigerants R-12, R-11, R-134A given in figures B–2 to B–5.

Psychrometric chart is shown in Fig.B–6.

Refer extended sheets for figures.

APPENDIX C

Table C1 – SI supplementary units

Quantity	Name	Symbol
Plane angle	radian	rad
Solid angle	steradian	sr

Table C2 – Examples of SI derived units formed by using supplementary units

Quantity	Name	Symbol
Angular velocity	radian per second	rad/s
Angular acceleration	radian per second squared	rad/s^2
Radiant intensity	watt per steradian	W/sr
Radiance	watt per square metre steradian	W/(m^2 sr)

Table C3 – Metric prefixes

Factor	Prefix	Symbol	Factor	Prefix	Symbol
10^{24}	yotta	Y	10^{-1}	deci	d
10^{21}	zetta	Z	10^{-2}	centi	c
10^{18}	exa	E	10^{-3}	milli	m
10^{15}	peta	P	10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M	10^{-15}	femto	f
10^{3}	kilo	k	10^{-18}	atto	a
10^{2}	hecto	h	10^{-21}	zepto	z
10^{1}	deca	da	10^{-24}	yocto	y

Table C4 – SI derived units with special names

Quantity	Name	Symbol	SI units
Capacitance	farad	F	m^{-2} kg^{-1} s^4 A^2
Dynamic viscosity	pascal second	Pa s	m^{-1} kg s^{-1}
Electric charge amount of electricity	coloumb	C	s A
Electric charge density	coloumb/metre^3	C/m^3	m^{-3} s A
Electric conductance	siemens	S	m^{-2} kg^{-1} s^3 A^2
Electric field strength	volt/metre	V/m	m kg s^{-3} A^{-1}
Electric flux density, electric displacement	coloumb/metre^2	C/m^2	m^{-2} s A
Electric potential difference, electromotive force,	volt	V	m^2 kg s^{-3} A^{-1}
Electric resistance	ohm	Ω	m^2 kg s^{-3} A^{-2}
Energy density	joule/metre^3	J/m^3	m^{-1} kg s^{-2}
Energy, work, amount of heat	joule	J	m^2 kg s^{-2}
Force	newton	N	m kg s^{-2}
Frequency	hertz	Hz	s^{-1}
Heat capacity, entropy	joule/kelvin	J/K	m^2 kg s^{-2} K^{-1}
Heat flux density, irradiance	watt/metre^2	W/m^2	kg s^{-2}
Illuminance	lux	lx	m^{-2} cd sr
Inductance	henry	H	m^2 kg s^{-2} A^{-2}
Luminous flux	lumen	lm	Cd
Magnetic flux	weber	Wb	m^2 kg s^{-2} A^{-1}
Magnetic flux density	tesla	T	kg s^{-2} A^{-1}
Molar energy	joule/mole	J/mol	m^2 kg s^{-2} mol^{-1}
Molar entropy, molar heat capacity	joule/ mole kelvin	J/mol K	m^2 kg $s^{-2}K^{-1}mol^{-1}$
Moment of force	newton metre	N m	m^2 kg s^{-2}
Permeability	henry/metre	H/m	m kg s^{-2} A^2
Permittivity	farad/metre	F/m	m^{-3} kg^{-1} s^4 A^{-2}
Power, radiant flux	watt	W	m^2 kg s^{-3}
Pressure	pascal	Pa	m^{-1} kg s^{-2}
Specific energy	joule/kilogram	J/kg	m^2 s^{-2}
Specific heat capacity, specific entropy	joule/kg kelvin	J/(kg K)	m^2 s^{-2} K^{-1}
Surface tension	newton/metre	N/m	kg s^{-2}
Thermal conductivity	watt/metre kelvin	W/(m K)	m kg s^{-3} K^{-1}

Note: In SI system of units in the above table, you would notice some units are abbreviated using small letters and others with capital letters letters. Capital letter units are named after the famous scientists such as couloumb (C), henry (H), hertz (Hz), newton (N), watt (W) etc.

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