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BOND VALUATION, YIELD MEASURES AND THE TERM STRUCTURE

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To
my parents
Savitri Parameswaran
and
Late A.S. Parameswaran

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PREFACE

This is the third volume of the series on Securities Markets. It covers the fundamentals of fixed income securities. The first chapter explains the key features of plain vanilla bonds, and briefly, the principles of zero coupon bonds, and bonds with embedded options, such as callable and convertible bonds.

Chapter 2 is devoted to the valuation of bonds between coupon dates. Unlike most textbooks, this book covers the issues of day-count conventions and accrued interest in elaborate detail.

Chapter 3 is dedicated to the important concept of yield measures. Starting with the simple concept of current yield, the book goes on to cover the concept of the yield to maturity and its variations. The concepts of yield to call and portfolio yield, are also explained in adequate detail.

The last chapter focuses on the term structure of interest rates. The various theories that have been expounded to explain the term structure are covered in depth. The estimation techniques for deriving the yield curve are also dwelt with in substantial detail.

The book is a concise summary of a standard textbook on fixed income securities. It is intended to be a stand alone resource from the standpoint of facilitating a study of this subject. Students wishing to study the subject in greater detail can refer to the books listed in Appendix-I which gives the sources and references used by me.

The contents of the book have been used at business schools as well as for corporate training programmes, and consequently are a blend of academic rigour and practical insights. It will appeal to market professionals, as well as students, who wish to build foundation in finance theory, in general, and securities markets in particular.

SUNIL K. PARAMESWARAN

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SUNIL K. PARAMESWARAN

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Volume–3

Bond Valuation, Yield Measures, and the Term Structure

by

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1

CHAPTER

An Introduction to Bond Valuation

Introduction

The concept of time value of money can be used to value any asset whose value is derived from future cash flows. We will see here how this principle is applied by investors to establish the value of a bond.

An ordinary bond, also known as a *Plain Vanilla Bond*, is a fairly simple instrument.[1] It will pay interest periodically, (typically every six months) and will repay the principal at maturity. *Zero Coupon* or *Deep Discount* bonds pay no interest. They are sold for less than the amount that is payable at maturity. The difference between the terminal payment and the issue price constitutes the interest.

Fundamentals of Bond Valuation

We will first take a close look at the pricing of plain vanilla bonds and attempt to understand the related principles and conventions. The following notations will be used in the course of this discussion. Additional variables will be defined as we progress.

Notations

- $M \equiv$ Face Value[2]

The *Face Value* or *Par Value* is the amount that the issuer promises to pay to the lender at maturity. It is also referred to as the *Redemption Value*, *Maturity Value* or *Principal Value*.

- $T \equiv$ Term to Maturity

A key feature of any bond is its Term to Maturity, which is the time remaining in the life of the bond. This is the period during which the borrower has to meet the conditions of the debt. A bond's Term to Maturity is the length of time after which the debt shall cease. The issuer can then redeem the issue by paying the face value or Principal back to the lender.[3] If we denote today as time *0* and the maturity date as time T, then T represents the term to maturity. The word Maturity, Term and Term to Maturity are used interchangeably to refer to the number of years remaining in the life of the bond.

The maturity of a bond indicates the expected life of the instrument, or the number of periods during which the holder can expect to receive the interest payments.[4] It also represents the number of periods before the principal will be repaid.

A plain vanilla bond is a *Term Bond*, that is, it has a single maturity date which is fixed at the outset. But situations can arise where a bond issue can be retired early by the issuer, either in full or part. We have already mentioned the case of Callable bonds (Endnote 4), where the issuer redeems the bonds by paying off the lender before the scheduled maturity date. The presence of *Sinking Fund* provisions can also lead to bonds being redeemed before maturity. In many cases, the Sinking Fund provisions require the issuer to retire a part of the debt according to a prespecified schedule during the life of the bond.

- $C \equiv$ Coupon

A bond's coupon is the periodic interest payment made to the owners during the bond's life. The coupon rate is the rate of interest that, when multiplied by the par value, provides the rupee value of the annual coupon payment (denoted by C). If the annual coupon rate is c, then the semi-annual coupon payment is $C/2$. The word 'coupon' is used because traditionally bonds were issued with a booklet of post-dated coupons. The bondholder was expected to redeem a coupon for cash at the end of every coupon period.

Example 1.1

Consider a bond with a face value of Rs. 1,000, and a coupon of 10% per annum paid semi-annually. Since $c = .10$, the semi-annual interest is given by

$$\frac{C}{2} = \frac{.10}{2} \times 1{,}000 = \text{Rs. } 50$$

- $y \equiv$ Yield to Maturity

The *Yield to Maturity* or the YTM is the rate of return that the buyer will get if he buys the bond at the prevailing price and holds it to maturity. This assumes that all intermediate coupon payments are reinvested at the YTM itself. Those who are familiar with capital budgeting should note that it is similar to the concept of the *Internal Rate of Return* (IRR) used in project appraisal.

- $N \equiv$ Number of coupons left in the life of the bond
- $k \equiv$ Time until the receipt of the first coupon expressed as a fraction of six months

If we are standing on a coupon date, then $k = 1$ else $k < 1$.

- $P_c \equiv$ Clean price of bond i in the spot market at time t.
- P or $P_d \equiv$ Dirty price
- $AI_{i,t_1,t_2} \equiv$ Accrued Interest on bond i from t_1 (the last coupon date) till t_2 (the valuation date).

Valuation of a Bond as on a Coupon Date

A plain vanilla bond, as mentioned earlier, is a fairly simple instrument. It pays interest periodically, typically every six months, and repays the principal at maturity.

The price of a bond as on a coupon date may be expressed by the following equation:

$$P_d = \sum_{t=1}^{N} \frac{\frac{C}{2}}{\left(1+\frac{y}{2}\right)^t} + \frac{M}{\left(1+\frac{y}{2}\right)^N}$$

A cursory glance will reveal that the price of a bond is nothing but the present value of the stream of coupon payments, which is an ordinary annuity, discounted using the YTM, plus the present value of the principal repayable at maturity.

Example 1.2

Let us assume that today is July 15, 2001. A Treasury bond is available that matures on July 15, 2021 and pays a coupon of 9% semi-annually on January 15 and July 15 every year.[5] The face value is \$ 1,000 and the Yield to Maturity is 8%.

The price of the bond may be calculated as follows. We know that $M = 1{,}000$, $c = 9\%$, $y = 8\%$, and $N = 40$. Therefore

$$P_d = \sum_{t=1}^{40} \frac{\dfrac{(.09 \times 1{,}000)}{2}}{\left(1+\dfrac{.08}{2}\right)^t} + \frac{1{,}000}{\left(1+\dfrac{.08}{2}\right)^{40}}$$

$$= 45 \times \text{PVIFA}\,(4, 40) + 1{,}000 \times \text{PVIF}(4, 40)$$

$$= 890.6748 + 208.2890 = \$\ 1{,}098.96$$

where PVIFA is the Present Value Interest Factor Annuity and PVIF is the Present Value Interest Factor.[6]

Zero Coupon Bonds and Cumulative Interest Debentures

A *Zero Coupon Bond* (ZCB) or a *Deep Discount Bond* pays no interest. In the case of Zero Coupon Bonds, you, as an investor, get interest by buying the security at a price less than the face value, and holding it to the maturity date. A *Cumulative Interest Debenture* is similar. It is issued at the face value, but the principal plus accumulated interest (compounded, of course) is payable at maturity.

Example 1.3

Larsen and Toubro has issued a ZCB with a face value of Rs. 1,000, and a yield of 10% per annum. It has a life of five years. What should be the price?

One issue while determining the price is whether the face value of the bond should be discounted at 10% for five years or at 5% for ten half-years. In practice, it is discounted at 5% for ten half-years. This is because while making an investment decision we will have a choice between coupon paying bonds and zero coupon bonds. To make a meaningful comparison between these, the method of discounting must be identical for both. Thus, since the cash flows from a coupon paying bond are discounted on a semi-annual basis, the same practice is followed to discount the cash flow from a zero coupon bond.

$$\text{Price} = \frac{1{,}000}{(1.05)^{10}} = \text{Rs. } 613.91$$

Thus, in a Zero Coupon Bond, you typically receive a round sum at redemption, while paying an odd amount at purchase.

Example 1.4

Assume that in the above case, you had a cumulative interest debenture instead of a ZCB.

The price paid at the outset will be Rs. 1,000. At redemption, you would receive

$$1{,}000\,(1.05)^{10} = 1{,}000 \times 1.6289 = \text{Rs. } 1{,}628.90$$

Thus in a cumulative interest debenture, you typically pay a round figure today, and receive an odd sum at redemption.

Premium and Discount Bonds

Example 1.5

On July 1, 2005 TELCO is borrowing Rs. 10MM by selling 10,000 individual bonds for Rs. 1,000 each. The coupon rate is 10% payable semi-annually, and the term to maturity is ten years. If the required annual yield is 10%, what should be the price of the bond?

$$P = 50 \text{ PVIFA}(5, 20) + 1{,}000 \text{ PVIF}(5, 20)$$

$$= 50 \times 12.4622 + 1{,}000 \times 0.3769$$

$$= 623.11 + 376.90 = 1{,}000.00$$

Thus, *if the required yield equals the coupon rate, the bond will sell at par.* Such bonds are referred to as *par bonds.*

Premium Bonds

Now let us assume that the required annual yield is 8% per annum. The price of the bond will be

$$P = 50 \text{ PVIFA}(4, 20) + 1{,}000 \text{ PVIF}(4, 20)$$

$$= 50 \times 13.5903 + 1{,}000 \times 0.4564$$

$$= 1{,}135.92$$

Hence, *if the required yield is less than the coupon rate, the bond will sell at a premium.* Such bonds are called *premium bonds.*

The question is, why is the bond selling at a premium? The bond is paying interest at the rate of 10%, when the required rate for investors in the market is only 8%. Thus, an investor will be willing to pay more than Rs. 1,000 (the face value) for it. Hence the price of the bond will be bid up. At the price of Rs. 1,135.92, the yield on the bond will be exactly 8%.

Discount Bonds

Let us assume that the required annual yield is 12%. The price of the bond will therefore be

$$P = 50 \text{ PVIFA}(6, 20) + 1{,}000 \text{ PVIF}(6, 20)$$
$$= 50 \times 11.4699 + 1{,}000 \times 0.3118$$
$$= 885.30$$

Thus, *if the required yield is more than the coupon rate, the bond will sell at a discount.* Such bonds are called *discount bonds.*

Why does the price decrease? The bond is paying interest at the rate of 10%, when the required rate in the market is 12%. Since the cash flows from the bond are fixed, the only way you can get a higher return is by paying a lower price. Hence the price of the bond will go down. At a price of 885.30, the yield on the bond will be exactly 12%.

Evolution of Price Over Time

We can see that the price of a bond is inversely related to its yield. As the yield changes, so will the price. However (with the exception of a par bond), the price of a bond will be different at every successive coupon date, even if the yield were to remain constant.

In Example 1.5, we assumed that the TELCO bond had ten years to maturity. Now let us move to a year after the date of issue. That is, there are nine years left to maturity.

If the required yield were to remain at 10%, the price of the bond one year after it is issued would be

$$
\begin{aligned}
P &= 50 \text{ PVIFA}(5, 18) + 1{,}000 \text{ PVIF}(5, 18) \\
&= 50 \times 11.6896 + 1{,}000 \times 0.4155 \\
&= 584.58 + 415.50 = \text{Rs. } 1{,}000
\end{aligned}
$$

Thus, the value of the bond will continue to be equal to the face value, as long as the yield equals the coupon rate.

Now let us consider the situation where the yield is 8% per annum. As we have already seen in this case, the bond will sell for Rs. 1,135.92 when there are ten years left to maturity. What would happen if the yield were to remain at 8% and there are nine years to maturity?

$$
\begin{aligned}
P &= 50 \text{ PVIFA}(4, 18) + 1{,}000 \text{ PVIF}(4, 18) \\
&= 50 \times 12.6593 + 1{,}000 \times 0.4936 \\
&= 1{,}126.57
\end{aligned}
$$

As can be seen, the price will fall, and will continue to do so over time until it approaches Rs. 1,000 at maturity. Thus, the price of a premium bond will change with the sheer passage of time, even if the yield remains unchanged, and *will gradually decline to approach the face value.*

As we have already seen, if the required yield is 12%, the bond will sell for Rs. 885.30 when there are ten years left to maturity. What would happen if the yield were to remain at 12% when there are nine years left to maturity?

$$
\begin{aligned}
P &= 50 \text{ PVIFA}(6, 18) + 1{,}000 \text{ PVIF}(6, 18) \\
&= 50 \times 10.8276 + 1{,}000 \times 0.3503 \\
&= 891.68
\end{aligned}
$$

As we can see, the price will rise. It will continue to rise with time till it approaches the face value at maturity. Thus, the price of a discount bond will change with time even if the yield remains unchanged, and will *gradually increase to approach the par value.*

Let us summarise the features that we have observed about bonds.

1. Whenever the going rate of interest, YTM, is equal to the coupon rate, a bond will sell at par.
2. When $y > c$, a bond will sell at below par, and such a bond is called a Discount bond.
3. When $y < c$, a bond will sell at above par, and such a bond is called a Premium bond.
4. An increase in interest rates will cause the price of an outstanding bond to fall, while a decrease in rates will cause the bond price to rise.

Proof of the evolution of price over time

The price of a bond when there are N coupons left is given by:

$$P = \frac{M \times \frac{c}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^{N}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N}}$$

The price when there are $N-1$ coupons left is given by:

$$P = \frac{M \times \frac{c}{2}}{\frac{y}{2}} \times \left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^{(N-1)}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{(N-1)}}$$

The price change between successive coupon periods is therefore given by

$$\Delta P = \frac{M \times \frac{c}{2}}{\frac{y}{2}} \times \left[\frac{1}{\left(1+\frac{y}{2}\right)^{N}} - \frac{1}{\left(1+\frac{y}{2}\right)^{(N-1)}} \right] +$$

$$M \times \left[\frac{1}{\left(1+\frac{y}{2}\right)^{(N-1)}} - \frac{1}{\left(1+\frac{y}{2}\right)^{N}} \right] = \frac{M}{\left(1+\frac{y}{2}\right)^{N}} \times \left[\frac{y}{2} - \frac{c}{2} \right]$$

If y is equal to c then the change in price will be zero. This is the case with par bonds. If $y > c$ then the change in price will be positive, as seen in discount bonds. These steadily increase in price with the passage of time. If $y < c$ then the change in price will be negative. In premium bonds we have seen this steady decrease in price with the passage of time.

Calculating the YTM

The bond valuation equation is a non-linear equation. How can the YTM be calculated, given other variables? One way is to use the IRR function in EXCEL.

We will demonstrate an alternative procedure. Let us first define the Approximate Yield to Maturity or AYM.

$$\text{AYM} = \frac{C + \frac{M-P}{N/2}}{\frac{M+P}{2}}$$

Rationale

The annual interest income is C. The capital gain/loss if the bond is held to maturity (N/2 years) on a straight line basis, is $\frac{M-P}{N/2}$. Thus, the annual income is $C+\frac{M-P}{N/2}$. The initial investment is P. An instant before redemption the money that is locked up in the bond is M. Thus the average investment is $\frac{M+P}{2}$.

The approximate YTM is the annual income divided by the average investment and is equal to

$$\frac{C+\dfrac{M-P}{N/2}}{\dfrac{M+P}{2}}$$

Example 1.6

Consider a 15% coupon bond with a face value of Rs. 1,000, a price of Rs. 860, and 14 years to maturity. Assume that the bond pays interest on an annual basis. What is the yield to maturity?

$$\text{AYM} = \frac{150+\dfrac{1{,}000-860}{14}}{\dfrac{1{,}000+860}{2}}$$

$$= \frac{150+10}{930} \equiv 17.20\%$$

Using the AYM as a starting point, we can then interpolate to get a more precise value for the YTM. The objective is to choose two interest rates that straddle 17.20% (that is, one should be below 17.20%, and the other above it), and give corresponding prices which straddle the price of Rs. 860.

Consider two rates, 17% and 18%.

Price at 17% = 150 PVIFA(17, 14) + 1,000 PVIF (17, 14)
= 895.4141.

This price is above 860.

Price at 18% = 150 PVIFA(18, 14) + 1,000 PVIF(18, 14)
= 849.7582.

This price is below 860.

Now let us interpolate. 18% – 17% corresponds to a price difference of 849.7582 – 895.4141. Thus 18% – y^* should correspond to a price difference of 849.7582 – 860, where y^* is the true YTM.

$$.18 - .17 \equiv 849.7582 - 895.4141$$

$$.18 - y^* \equiv 849.7582 - 860$$

Take the ratio of the two, and solve for y^*.

$$\frac{.01}{.18 - y^*} = \frac{-45.6559}{-10.2418}$$

$$\Rightarrow \qquad y^* = .1778 \equiv 17.78\%$$

You can verify the accuracy of our approximation using a financial calculator. Price at 17.78% = Rs. 859.46 ≅ Rs. 860.

Floating Rate Bonds

Floating rate instruments are securities whose coupons are adjusted periodically due to changes in a base or benchmark rate. The literature at times makes a distinction between *floating rate* notes and *adjustable-rate* notes. The coupons of the former are based on a short-term rate index, while the rates for adjustable-rate notes are based on long-term indices. In either case, as the benchmark rises, the coupon will rise, and vice versa.

One of the most widely used benchmark rates is the London Interbank Offer Rate or LIBOR. Banks will quote two rates when called. LIBID or the London Interbank Bid Rate is the rate at which the bank is willing to borrow. The LIBOR is the rate at which it is willing to lend. LIBID can be used as a benchmark, as can LIBOR. Sometimes, the average of the two, called LIMEAN is also used as a benchmark.

Options Present in Bonds

Bonds often include provisions that provide an option for the holder or the issuer, or both, to take certain measures. An option by definition is a *right,* and not an obligation.

Callable Bonds

These give the issuer the right to retire the debt before the maturity date. This benefits issuers because if interest rates decline, they can recall the old bonds, and replace them with a new issue carrying a lower coupon. The call provision allows the issuer to alter the maturity of the bond. This works against the investor, as it introduces cash flow uncertainty. Thus, callable bonds will offer a higher yield than otherwise similar but non-callable bonds. That is, they will sell for a lower price, compared to a non-callable bond with the same face value and coupon.

Certain bonds have a *Call Protection Period*. This is a time period during which the bonds cannot be recalled. Such bonds are referred to as *deferred callable bonds*. Normally, in order to sweeten the issue, the borrowers will offer a *Call Premium*, which is often equal to one year's coupon. In other words, if and when the bond is prematurely called, the holder will be paid the face value plus one year's coupon as a bonus.

Holders of callable bonds face two types of risk. Firstly, they are exposed to reinvestment risk. That is, since the bond will be called back when the rates, fall, there is a possibility of having to reinvest the proceeds at a lower rate of interest. Secondly, the potential for price appreciation is limited in an economic environment where interest rates are falling. This is because the market will expect the bonds to be called back and consequently will not offer the same price as it would for a plain vanilla bond. This aspect is referred to as *Price Compression*.

Puttable Bonds

These allow the holder to sell the bond back to the issuer at par on certain dates. Thus if interest rates have risen after the issue (which would lower the bond's price as explained earlier), the investor benefits by having the bond redeemed at face value. Compared to an otherwise similar plain vanilla bond, a puttable bond will offer a lower yield.

Convertible Bonds

These give the holder the right to exchange the bond for a specified number of shares of stock. The conversion option allows the holder to benefit from favourable movements in the share price. If the price at which the shares are being issued under the conversion option, is lower than the prevailing market price of the shares, then conversion is worthwhile.

The number of shares of common stock that the bondholder will receive if he exercises the conversion option is known as the *Conversion Ratio.* For instance take the case of a Rs. 1,000 face value bond that has a conversion ratio of 32. This means that each bond may be converted into 32 shares of stock. In this case, the conversion price is $\frac{1,000}{32}$ = Rs. 31.25. The *conversion value* of the bond is the value that the bondholder will get if he converts the bond immediately. Thus, the conversion value is the product of the current market price of the share and the conversion ratio.

Exercises

1. A plain vanilla bond with a face value of Rs. 1,000 pays coupons at a rate of 8% per annum on a semi-annual basis. The bond has ten years to maturity.
   (a) If the required yield in the market is 9% per annum, what should be the price of the bond?
   (b) If the required yield in the market is 7% per annum, what should be the price of the bond?
2. A zero coupon bond with a face value of Rs. 5,000 and ten years to maturity is available. The required yield is 8% per annum. What should its market price be?
3. A bond with a face value of Rs. 1,000 and ten years to maturity is trading at a price of Rs. 875. The coupon

rate is 8% per annum and coupons are paid on a semi-annual basis. Using the approximate yield to maturity technique, calculate the YTM of the bond.

Endnotes

1. The most basic form of any security is referred to as the *plain vanilla* version. More exotic versions are said to have *Bells and Whistles* attached.
2. The symbol ≡ stands for 'is equivalent to'.
3. Remember, the issuer of the bond is the borrower and the buyer of the bond is the lender.
4. We use the phrase *Expected Life*, because bonds other than those issued by the government are subject to *Default Risk*. This means that the borrower may cease to honour his obligations before the stated time to maturity. Secondly, in practice many bonds are *Callable*, that is they can be recalled by the issuer well before their maturity.
5. Unless otherwise stated the coupon rates given represent annual rates of interest.
6. $\text{PVIFA}(r, N) = \dfrac{\left[1 - \dfrac{1}{(1+r)^N}\right]}{r}$ and $\text{PVIF}(r, N) = \dfrac{1}{(1+r)^N}$, where r is the discount rate and N is the number of periods.

2

CHAPTER

Valuation Between Coupon Dates

Introduction

We have already seen how to compute the price of a bond on a coupon date. For a plain vanilla bond, the price on a coupon date is the present value of the coupons remaining in the life of the bond (which constitute an annuity) plus the present value of the terminal face value.

The price of a bond as on a coupon date may be expressed by the following equation:

$$P_d = \sum_{t=1}^{N} \frac{\frac{C}{2}}{\left(1+\frac{y}{2}\right)^t} + \frac{M}{\left(1+\frac{y}{2}\right)^N}$$

Example 2.1

Let us assume that today is July 15, 2001. A T-bond is available that matures on July 15, 2021 and pays a coupon of 9% semi-annually on January 15 and July 15 every year.[1] The face value is $ 1,000 and the yield to maturity is 8%.

The price of the bond as calculated earlier is $ 1,098.36.

Valuation of a Bond between Coupon Dates

Assume that we have a bond with a face value of M, paying a coupon of $C/2$ every six months, and with N coupons left to maturity. The semi-annual yield to maturity is $y/\mathbf{2}$. While valuing a bond between coupon dates, the difference is that the next coupon is not exactly one period away. Instead, it is k periods away where $k < 1$.

We will first value the bond at time **1**, or the next coupon date, and then discount the value so obtained back to time **0**, or the actual valuation date.

At time **1** we will receive a coupon of $\frac{C}{2}$. There will be $N - 1$ coupons left subsequently, whose value will be

$$\frac{\frac{C}{2}}{\frac{y}{2}}\left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^{N-1}}\right]$$

The present value of the face value will be $\frac{M}{\left(1+\frac{y}{2}\right)^{N-1}}$.

Thus the value of the bond at time **1** will be

$$\frac{C}{2} + \frac{\frac{C}{2}}{\frac{y}{2}}\left[1 - \frac{1}{\left(1+\frac{y}{2}\right)^{N-1}}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N-1}}$$

$$= \frac{\frac{C}{2}}{\frac{y}{2}} \left[\frac{\left(1+\frac{y}{2}\right)^{N} - 1}{\left(1+\frac{y}{2}\right)^{N-1}} \right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N-1}}$$

Discounting back to time **0**, we get a value of

$$\left[\frac{\frac{c \times M}{2}}{\left(1+\frac{y}{2}\right)^{k}} \times \frac{\left[\left(1+\frac{y}{2}\right)^{N} - 1\right]}{\left(\frac{y}{2}\right)\left(1+\frac{y}{2}\right)^{N-1}} \right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N-1+k}}$$

Day-Count Conventions

There is no standard approach to defining k, the fraction of a period left till the next coupon. Different markets use different approaches and consequently we need to be familiar with various day-count conventions. We will first look at the *Actual/Actual* method which is used for Treasury bonds in the US.

The Actual-Actual Approach

Given the data from Example 2.1 on the Treasury bond, assume that we are standing on July 25, 2001. In the Actual/Actual method we need to calculate the actual number of days between the date of valuation and the date of the next coupon. Let us call this period as N_1. In our case, the next coupon date is 174 days away as calculated below.

Table 2.1

Calculation of the Numerator

Month	*No. of Days*
July	6
August	31
September	30
October	31
November	30
December	31
January	15
Total	174

We have to then compute the number of days between the previous and the next coupon date. We will denote the period as N_2. In our example, the number of days between the last coupon date and the next coupon date is 184 days as shown in Table 2.2.

Table 2.2

Calculation of the Denominator

Month	*No. of Days*
July	16
August	31
September	30
October	31
November	30
December	31
January	15
Total	184

Hence the next coupon is $k = \frac{N_1}{N_2} = \frac{174}{184} = .9457$ semi-annual periods away.

The Actual/Actual method is often denoted as ACT/ACT and pronounced Ack/Ack by Wall Street traders. It is one of many daycount conventions used in the world of finance. The denominator under this convention will vary from 181 to 184 days, depending on the coupon payment dates of the bond that is under consideration. Note that the day on which we are valuing is not included, nor is the day on which the last coupon was paid.

Having determined the value of k, the price of the Treasury bond may be determined in either of the two ways discussed here.

The market method

Wall Street traders compute T-bond prices using the expression derived earlier.

$$P_d = \left[\frac{\frac{c \times M}{2}}{\left(1+\frac{y}{2}\right)^k} \times \frac{\left[\left(1+\frac{y}{2}\right)^N - 1\right]}{\left(\frac{y}{2}\right)\left(1+\frac{y}{2}\right)^{N-1}} \right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N-1+k}}$$

Continuing with the previous example, the price determined by this method will be

$$\frac{\frac{.09 \times 1,000}{2}}{\left(1+\frac{.08}{2}\right)^{.9457}} \times \frac{\left[\left(1+\frac{.08}{2}\right)^{40} - 1\right]}{\left(\frac{.08}{2}\right)\left(1+\frac{.08}{2}\right)^{40-1}} + \frac{1,000}{\left(1+\frac{.08}{2}\right)^{40-1+.9457}}$$

$= \$\ 1,101.3068$

The Treasury method

The difference between this and the market's approach is that the Treasury applies simple interest for the fractional first period. Hence, the Treasury will calculate the bond price as follows:

$$P_d = \left[\frac{\frac{c \times M}{2}}{\left(1 + k \times \frac{y}{2}\right)} \times \frac{\left[\left(1+\frac{y}{2}\right)^N - 1\right]}{\left(\frac{y}{2}\right)\left(1+\frac{y}{2}\right)^{N-1}} \right] + \frac{M}{\left(1 + k \times \frac{y}{2}\right)\left(1+\frac{y}{2}\right)^{N-1}}$$

Then the price will be

$$\frac{\frac{.09 \times 1{,}000}{2}}{\left(1 + \frac{.9457 \times .08}{2}\right)} \times \frac{\left[\left(1+\frac{.08}{2}\right)^{40} - 1\right]}{\left(\frac{.08}{2}\right)\left(1+\frac{.08}{2}\right)^{40-1}}$$

$$+ \frac{1{,}000}{\left(1 + \frac{.9457 \times .08}{2}\right)\left(1+\frac{.08}{2}\right)^{40-1}} = \$\ 1{,}101.2638$$

For a given value of k, the Treasury's approach will always give a lower price. This is because the rate of discount for a fractional period is higher when the simple interest technique is used.

Accrued interest

Consider the period between two coupon dates t_1 and t_2. Let us denote the ex-dividend date for the bond by t_d, where $t_1 < t_d < t_2$. Between t_1 and t_d, the bond will be traded cum dividend. This means that the person who buys the bond at any point during this period will be entitled to the coupon that will be paid on t_2. This facet is captured by the bond pricing equation discussed earlier because the dirty price includes the present value of the next coupon.

The dirty price also includes the compensation to the seller for parting with the entire next coupon payment, although he has held the bond for a part of the current coupon period. This compensation is called Accrued Interest and is calculated as follows for Treasury bonds.

$$\text{Accrued Interest} = AI = \frac{c \times M}{2} \times \frac{(t - t_1)}{(t_2 - t_1)}$$

where t is the date on which the bond is being sold. Continuing with Example 2.1, accrued interest as of July 25,2001 is

$$\frac{.09 \times 1{,}000}{2} \times \frac{10}{184} = \$\ 2.4457$$

The bond prices that are quoted in practice are called *clean prices*. They are computed by subtracting the accrued interest from the dirty price. Therefore,

$$P_c = P_d - AI$$

The rationale for computing clean prices is given here. As we can see from the example the passage of ten days time leads to an increase of \$ 2.3429 in the dirty price, (that is from \$ 1,098.9639 to \$ 1,101.3068) even though the YTM has remained unchanged. The clean price is a measure that stays relatively constant for short periods, unless there is a change in the YTM.

On July 15, 2001 the clean price is \$ 1,098.9639,[2] whereas on July 25, 2001 the clean price is

$$1{,}101.3068 - 2.4457 = \$\ 1{,}098.8611$$

For a bond market analyst, it is important to monitor the changes in the required market yields. If the price data were to consist of dirty prices, it would be difficult to separate the effect of yield changes from the impact of the accrued interest. However if clean prices are used in analysis, any price changes in the short run will be primarily induced by yield changes. It is for this reason that the reported bond prices are invariably clean prices.

One question that may strike you is, can the accrued interest be negative? In other words, can there be cases where the seller of the bond has to pay accrued interest to the buyer? The answer is yes. In some bond markets, the bond begins to trade ex-dividend after a certain date; that is, from this date onwards, the sale of a bond will result in the next coupon going to the seller rather than to the buyer. On the ex-dividend date, the dirty price will fall by the present value of the next coupon and the dirty price will be less than the clean price.

Example 2.2

Consider the bond that matures on July 15, 2021. We will assume that we are standing on January 5, 2002, which is the ex-dividend date. All the other variables have the same values as before.

The cum dividend price of the bond is:

$$\frac{\frac{.09\times 1,000}{2}}{\left(1+\frac{.08}{2}\right)^{.0543}}\times\frac{\left[\left(1+\frac{.08}{2}\right)^{40}-1\right]}{\left(\frac{.08}{2}\right)\left(1+\frac{08}{2}\right)^{40-1}}+\frac{1,000}{\left(1+\frac{.08}{2}\right)^{40-1+.0543}}$$

$= \$ \ 1,140.4910$

The moment the bond goes ex-dividend the dirty price will fall by the present value of the forthcoming coupon, because the buyer will no longer be entitled to it. Thus, the ex-dividend dirty price is

$$1,140.4910-\frac{\frac{.09\times 1,000}{2}}{\left(1+\frac{.08}{2}\right)^{.0543}}$$

$$= \$ \ 1,095.5867$$

This is the amount payable by the person who buys the bond after it goes ex-dividend.

The accrued interest an instant before the bond goes ex-dividend is

$$\frac{.09\times 1,000}{2}\times\frac{174}{184}= \$ \ 42.5543$$

Thus the clean price at the time of the bond going ex-dividend is

$1,140.4910-42.5543= \$ \ 1,097.9367$

As you can see the clean price is greater than the ex-dividend dirty price. The fraction of the next coupon that is payable to the buyer is

$$\frac{.09\times 1,000}{2}\times\frac{10}{184}= \$ \ 2.4457$$

Hence the buyer has to pay 1,097.9367 – 2.4457 = $ 1,095.4910, which is nothing but the ex-dividend dirty price.

Other Day-Count Conventions

The 30/360 PSA Approach

This is the method used for corporate bonds in the US. Here, the denominator, that is, the length between successive coupon dates, is always taken to be 180. Each month is therefore considered to be of 30 days. The numerator is then calculated as follows:

Define the start and end dates, D_1 and D_2 as:

$$D_1 = (month_1, day_1, year_1)$$

$$D_2 = (month_2, day_2, year_2)$$

The numerator is then calculated in this manner

$360(year_2 - year_1) + 30(month_2 - month_1) + (day_2 - day_1)$

There are some additional rules as well:

1. If $day_1 = 31$ then set $day_1 = 30$
2. If day_1 is the last day of February, then set $day_1 = 30$.
3. If $day_1 = 30$ or has been set to 30 using the above rule, then if $day_2 = 31$, set $day_2 = 30$

We will illustrate the calculation of the number of days using the 30/360 PSA approach for various start dates and end dates.

- Illustration 1

  Fractional period from March 15, 2006 to June 15, 2006.

$N_1 = 360 \times (2006 - 2006) + 30 \times (06 - 03) + (15 - 15) = 90$

- Illustration 2

  Fractional period from March 31 2006 to July 30 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 03) + (30 - 30) = 120$

- Illustration 3

  Fractional period from March 31 2006 to July 31 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 03) + (30 - 30) = 120$

- Illustration 4

  Fractional period from March 30 2006 to July 30 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 03) + (30 - 30) = 120$

- Illustration 5

  Fractional period from March 30 2006 to July 31 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 03) + (30 - 30) = 120$

- Illustration 6

  Fractional period from March 29 2006 to July 30 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 03) + (30 - 29) = 121$

- Illustration 7

  Fractional period from March 29 2006 to July 31 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 03) + (31 - 29) = 122$

- Illustration 8

  Fractional period from February 28 2006 to July 29 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 02) + (29 - 30) = 149$

- Illustration 9

  Fractional period from February 28 2006 to July 31 2006.

  $N_1 = 360 \times (2006 - 2006) + 30 \times (07 - 02) + (30 - 30)$
  $= 150$

Valuation of a corporate bond

Example 2.3

Consider a corporate bond that pays interest on July 15 and January 15 every year. Assume that we are standing on September 15, 2002 and that the bond matures on January 15, 2022. What should be the dirty price?

$$k = \frac{120}{180} = 0.6667$$

$P =$

$$\left[\frac{\frac{80}{2}}{\left(1 + \frac{.10}{2}\right)^{.6667}} \times \frac{\left[\left(1 + \frac{.10}{2}\right)^{39} - 1\right]}{\left(\frac{.10}{2}\right)\left(1 + \frac{.10}{2}\right)^{39-1}} \right] + \frac{1,000}{\left(1 + \frac{.10}{2}\right)^{39-1+.6667}}$$

$$= \$\ 843.4379$$

The 30/360 ISDA

The difference between 30/360 PSA and 30/360 ISDA is that the additional rule pertaining to the last day of February is not applicable here.

Example 2.4

Fractional period from February 28, 2006 to July 31, 2006.

$k = 360 \times (2006 - 2006) + 30 \times (07 - 02) + (31 - 28) = 153$

The 30/360 SIA

The additional rules for this convention are

1. If $day_1 = 31$ then set $day_1 = 30$
2. If day_1 is the last day of February and the bond pays a coupon on the last day of February then set $day_1 = 30$
3. If $day_1 = 30$ or has been set equal to 30 then if $day_2 = 31$, set $day_2 = 30$

The 30/360 European Convention

Here if $D_2 = 31$, then it is always set equal to 30. The additional rules may therefore be stated as

1. If $day_1 = 31$ then set $\text{day}_1 = 30$
2. If $day_2 = 31$ then set $\text{day}_2 = 30$

Example 2.5

Fractional period from March 29, 2006 to July 31, 2006.

$k = 360 \times (2006 - 2006) + 30 \times (07 - 03) + (30 - 29) = 121$

Actual/365 Convention

The difference between this and the Actual/Actual method is that the denominator will consist of 365 days even in leap years. For instance, consider a 10% coupon paying bond which pays interest on November 15 and May 15 every year. Assume that we are standing on January 15, 2000. The accrued interest can be calculated as follows:

$$AI = 1{,}000 \times 0.10 \times \frac{61}{365} = 16.7123$$

Notice that while calculating the accrued interest we multiply M by c and not by $\frac{c}{2}$. This is because the denominator of this day-count fraction represents the number of days in an entire year and not in a coupon period.

Any Actual/365 bond that pays periodic interest will usually accrue interest at a rate such that interest accrued over a full coupon period will not equal the periodic coupon payment. Thus, for an Actual/365 issue, day-count functions and interest accruals over a full coupon period need not generate the actual coupon payment for the period. Here is an illustration from Stigum and Robinson.

Example 2.6

Consider the period November 15, 1991 to May 15, 1992. The number of days is 182. The day count fraction for the full coupon period is

$$\frac{182}{365} = .499$$

and not .500. Since the semi-annual coupon payment must equal exactly half the coupon, the interest accrued on a given Actual/365 security over a full coupon period need not equal the exact coupon payment at the end of the period. This anomalous result cannot occur for an Actual/Actual security.

Actual/365 ISDA

This convention is identical to the Actual/365 convention for a coupon period that does not include days falling in a leap year. However for a coupon period that includes such days, the day count convention is given by:

Number of days falling within the leap year/366 + Number of days not falling within the leap year/365.

Actual/365 Japanese

This is the convention used to calculate accrued interest on Japanese Government Bonds (JGBs). It is similar to the Actual/365 method, the only difference being that in leap years, the extra day in February is ignored.

Actual/360

This is a simple variant of Actual/365, and is the interest payment convention used for money market instruments in most countries.

Exercises

1. A bond with a face value of Rs. 1,000 is available. It pays coupons on March 15 and September 15 every year. Assume that today is March 21, 2006 and that the bond matures on March 15, 2016. The coupon rate is 8% per annum and the YTM is 10% per annum. Compute the dirty price using:

   (a) The Actual/Actual day-count convention

   (b) The 30/360 PSA day-count convention

2. A US T-bond with a face value of $ 1,000, paying a semi-annual coupon at the rate of 6% per annum is available. The coupon dates are May 15 and November 15 every year. Assume that today is June 30, 2006 and that the bond matures on May 15, 2026. The YTM is 8% per annum. Calculate the dirty price using:
   (a) The Market method
   (b) The Treasury method
3. A bond pays a coupon of 10% per annum semi-annually on a face value of $ 1,000. The coupon dates are December 31 and June 30. Today is March 28, 2006. What is the accrued interest on applying:
   (a) The 30/360 PSA method
   (b) The Actual/Actual method
   (c) The Actual/365 method
   (d) The Actual/360 method

ENDNOTES

1. Unless otherwise stated, the coupon rates given represent annual rates of interest.
2. On a coupon payment date, the clean price and the dirty price will be identical.

3

CHAPTER

Yield Measures

Introduction

Given the market's required rate of return from a bond, we can calculate its price through the present values of the cash flows generated. On the other hand, with information about the price of a bond, we can find the rate of return that equates the present value of the cash flows to the price. This is the *Internal Rate of Return* (*IRR*) of the bond. The IRR of a bond is termed its *Yield to Maturity* (*YTM*) or *Redemption Yield.* For a bond which pays **N** coupons, the YTM is a solution to a polynomial of degree **N**. The easiest way to compute the YTM of a bond is by using the IRR function in EXCEL. Alternatively, one can use the approximate yield to maturity demonstrated in Chapter 1 as a starting point to obtain a more precise value using interpolation.

In practice, an investor can choose from a wide variety of bonds with different coupons and terms to maturity. The creditworthiness of the issuer will also vary from bond to bond. Thus, although data about bonds is primarily provided in the form of prices, it is the yield measure that facilitates comparison between different instruments.

Yield Measures

The yield to maturity is one of the key measures of the yield from a bond. However, in practice other measures of yield are computed as well. So a student of bonds ought to be well versed with them. We will now examine the various yield measures used in the market.

Current Yield

Although this is perhaps the most unsatisfactory one, it is often reported. It is also called the *flat yield, interest yield, income yield,* or *running yield.*

The current yield relates the annual coupon payment to the current market price.

$$CY = \frac{\text{Annual Coupon Interest}}{\text{Price}}$$

One of the issues in computing this is whether the price should be the clean price or the dirty price. The advantage of using the clean price is that the current yield will stay constant, provided the yield does not change. If the dirty price were used, then the current yield would be higher in the period between the ex-dividend date and the coupon date, when the dirty price is lower than the clean price. Similarly, the current yield would be lower between the coupon date and the ex-dividend date, when the dirty price is more than the clean price. This would result in a sawtooth pattern.

The current yield will be above the coupon rate if the bond is trading at a discount, and will be below the coupon rate if it is trading at a premium.

The current yield suffers from two major technical deficiencies. Firstly, it ignores any capital gain or loss experienced by the bondholder. Secondly, it fails to consider the time value of money.

Nevertheless, it is used to estimate the cost of or profit from holding a bond. The difference between the current yield and the cost of funding the bond is known as the *net carry*. If the short-term funding rates in the market are higher than the current yield, the bond is said to involve a running cost. This is also known as *negative carry* or *negative funding*.

Example 3.1

A bond which pays a coupon of 8% per annum is currently trading at \$ 95. A bond holder buys the bond by borrowing at the rate of 8.25% per annum. What is the current yield and what is the net carry?

$$CY = \frac{8}{95} = 0.0842 \equiv 8.42\%$$

The cost of funding is 8.25%. So the net carry or return is 0.17%.

Simple Yield to Maturity

This yield measure attempts to rectify the shortcomings of the current yield by taking into account capital gains and losses. The assumption is that capital gains and losses accrue evenly over the life of the bond, or in other words, on a straight line basis.

The formula is:

$$\text{Simple YTM} = \frac{C}{P} + \frac{M - P}{\frac{N}{2} \times P}$$

One of the problems with simple yield to maturity is that it does not take cognizance of the fact that an investor in a bond can earn compound interest. As coupons are paid, they can be reinvested and hence can earn interest. This increases the overall return from holding the bond.

Besides, the assumption that capital gains and losses arise evenly over the life of the bond is an over-simplification.

Simple yield to maturity is also known as the *Japanese Yield*, for it is the main yield measure used in the Japanese Government Bond (JGB) market.

Example 3.2

Assume that a bond with ten years to maturity and a coupon of 8% per annum is trading at $ 95. The face value is $ 100.

$$\text{SYTM} = \frac{8}{95} + \frac{(100 - 95)}{10 \times 95}$$

$$= 0.0842 + 0.0053 = 0.0895 \equiv 8.95\%$$

Yield to Maturity

The YTM is the interest rate that equates the present value of the cash flows from the bond to the price of the bond. It assumes that the bond is held to maturity.

Consider a bond that makes an annual coupon payment of C on a semi-annual basis. The face value is M, the price is P, and the number of coupons remaining is N. The YTM is that value of y, which satisfies the following equation.

$$P = \sum_{t=1}^{N}\left[\frac{\frac{C}{2}}{\left(1+\frac{y}{2}\right)}\right] + \frac{M}{\left(1+\frac{y}{2}\right)^{N}}$$

The value of y computed here is the nominal annual yield, and is also called the *Bond Equivalent Yield* (*BEY*). From the principles of time value of money, the effective annual yield is found as $\left(1+\frac{y}{2}\right)^{2} - 1$.

As discussed earlier, we require a computer program to calculate the YTM. A less precise, but very often an effective method, is the use of AYM with interpolation. The calculation is fairly simple for a coupon bond with two periods left to maturity, and for zero coupon bonds.

Example 3.3

A Reliance bond with a face value of Rs. 1,000, and a coupon rate of 10% per annum, payable semi-annually, has one year left to maturity. It is currently selling at Rs. 900. What is the YTM?

$$900 = \frac{50}{\left(1+\frac{y}{2}\right)} + \frac{50}{\left(1+\frac{y}{2}\right)^{2}} + \frac{1{,}000}{\left(1+\frac{y}{2}\right)^{2}}$$

$$\equiv \frac{50}{(1+i)} + \frac{1{,}050}{(1+i)^{2}}$$

where, we have denoted $\frac{y}{2}$ by i.

Therefore, $900\,(1 + i)^2 = 50(1 + i) + 1{,}050$

$\Rightarrow \qquad 900(1 + i)^2 - 50(1 + i) - 1{,}050 = 0$

This is a quadratic equation of the form

$$ax^2 + bx + c = 0,$$

where x in this case is $(1 + i)$.

The equation has two roots,

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$\text{Therefore, } (1 + i) = \frac{50 \pm \sqrt{\left[(-50)^2 - 4 \times (900) \times (-1{,}050)\right]}}{1800}$$

$$= \frac{50 \pm 1{,}944.87}{1{,}800}$$

$$= 1.1083 \quad \text{or} -1.0527$$

$$\text{Therefore,} \qquad i = \frac{y}{2} = 0.1083 \quad \text{or} -2.0527$$

We discard the negative root since the yield will always be positive. Thus, the nominal annual YTM = 0.1083 × 2 = 0.2166 ≡ 21.66%

Example 3.4

Consider a zero coupon bond with a face value of Rs. 1,000, maturing five years from now. The current price is Rs. 500. What is the YTM?

It is possible to use

$$500 = \frac{1{,}000}{(1+y)^{5}}\text{, or}$$

$$500 = \frac{1{,}000}{\left(1+\frac{y}{2}\right)^{10}}$$

The second approach is preferred, because with this we can compare our results with coupon paying bonds, which typically pay interest on a semi-annual basis.

Therefore, $\left(1+\frac{y}{2}\right)^{10} = \frac{1{,}000}{500} = 2$

That is, $\frac{y}{2} = (2)^{.1} - 1$

$$= 1.0718 - 1 = .0718$$

Thus

$$y = .1436 \equiv 14.36\%$$

The YTM calculation takes into account the coupon payments, as well as any capital gains/losses that accrue to an investor who buys and holds a bond to maturity.

Before analysing the YTM in detail, let us consider the various sources which contribute to the returns received by a bondholder. A bondholder can expect to receive income from:

- Coupon payments, which are typically paid every six months.
- A capital gain/loss obtained when the bond matures, or is called before maturity, or is sold before maturity.

(In the YTM equation, the assumption is that the bond is held to maturity).

- Reinvestment of coupon payments, from the time each coupon is paid till the time the bond matures, is sold, or is called. This income is the *Interest on Interest* income.

A satisfactory measure of the yield should take into account all three sources of income, which the YTM does. However, it makes two key assumptions:

- The bond is held until maturity
- The intermediate coupon payments are reinvested at the YTM itself

The second assumption is built into the mathematics of the YTM calculation, as shall be seen shortly.

The YTM is called a *Promised Yield*. The word *promised* is used because, in order to realise it the bondholder has to satisfy the two conditions discussed. If either of them is violated, he may not get the yield. Remember, the IRR calculation also assumes that intermediate cash flows are reinvested at the IRR itself.

The assumption that the bond is held to maturity is fairly easy to comprehend. Let us now focus on the reinvestment assumption.

Consider a bond with a face value of M, annual coupon of C, and number of coupons left N. Let the bond pay interest on a semi-annual basis, i.e. it pays $C/2$ every six months. Let r be the rate at which one can reinvest the coupon payments till maturity. r would depend on the prevailing rate of interest when the coupon is received, and need not be equal to y, the YTM, or c, the coupon rate. For ease of exposition, we will assume that r is a constant for the life of the bond.

Thus each coupon payment is reinvested at $\frac{r}{2}$ (for six monthly periods). The coupon stream is obviously an annuity. The final payoff from reinvesting the coupons is therefore given by the future value of this annuity, using a rate of $\frac{r}{2}$ for six monthly periods.

The future value is,

$$\frac{\frac{C}{2}}{\frac{r}{2}}\left[\left(1+\frac{r}{2}\right)^{N}-1\right]$$

Note, this amount represents the sum of all coupons which are reinvested (which in this case is the principal), plus interest earned on reinvesting the coupons.

The total value of the coupons $= \frac{C}{2} \times N = \frac{NC}{2}$

Thus, interest on interest is equal to

$$\frac{\frac{C}{2}}{\frac{r}{2}}\left[\left(1+\frac{r}{2}\right)^{N}-1\right]-\frac{NC}{2}$$

The YTM calculation assumes that $\frac{r}{2} = \frac{y}{2}$

An investor can actually get a rate of return equal to the YTM only if he manages to reinvest all the coupons at the YTM. This is shown by the following example.

Example 3.5

Consider an L & T bond that has ten years to maturity. The face value is Rs. 1,000. It pays a semi-annual coupon at the rate of 10% per annum. The YTM is 12% per annum.

Let us first calculate the price.

$$P = 50 \times \text{PVIFA}(6, 20) + 1{,}000 \times \text{PVIF}(6, 20)$$

$$= 50 \times 11.4699 + 1{,}000 \times .3118 = 885.295$$

We will assume that the semi-annual interest payments can be reinvested at a six monthly rate of 6%, which corresponds to a nominal annual rate of 12%.

The sources of income for a bondholder, assuming that he holds the bond till maturity, are:

1. Total coupon received = 50 × 20 = Rs. 1,000.
2. Interest on interest got by reinvesting the coupons

$$= \frac{50\left[(1.06)^{20} - 1\right]}{.06} - 1{,}000$$

$$= 50 \times 36.786 - 1{,}000 = \text{Rs. } 839.3$$

Notice that if we do not deduct 1,000 $\left(\text{or } \frac{NC}{2}\right)$ in the above equation, we can calculate the income from both the sources together. These have been separated for easy understanding.

3. Finally, the bondholder will get back the face value of Rs. 1,000.

Thus, in the end, the bondholder will have 1,000 + 839.3 + 1,000 = Rs. 2,839.3.

To get this income he has to pay Rs. 885.295 today, which is his investment. So what is the rate of return that he has earned? It is that value of *i*, which satisfies the following equation:

$$885.295\,(1+i)^{20} = 2{,}839.3$$

$$\Rightarrow \qquad (1+i) = \left[\frac{2839.3}{885.295}\right]^{.05} = 1.0600$$

$\Rightarrow$ $i = .06 \equiv 6\%$ on a semi-annual basis, or 12% on a nominal annual basis, which is exactly the same as the YTM.

So, how did the bondholder realise the YTM? Only by being able to reinvest the coupons at a nominal annual rate of 12%, compounded on a semi-annual basis. Note that the reinvestment rate affects only the interest on interest income. The other two sources are unaffected. If $r > y$, the interest on interest would have been higher, and i would have been greater than y. On the contrary, if $r < y$, the interest on interest would have been lower, and i would have been less than y.

So, if an investor buys a bond by paying a price which corresponds to a given YTM, he will realize that YTM only if,

1. He holds the bond till maturity, and
2. He is able to reinvest the coupons at the YTM.

In the above scenario, the risk that the investor faces is that future reinvestment rates may be less than the YTM at the time of purchasing the bond. This risk is called *Reinvestment Risk*, and its degree depends on two factors, namely, the time to maturity, and the quantum of the coupon.

For a bond with a given YTM and coupon rate, the greater the time to maturity, the more dependent is the bond's total return on the reinvestment income. Thus, everything else remaining constant, the longer the time to maturity, the greater is the reinvestment risk.

Secondly, for a bond with a given maturity and YTM, the greater the quantum of the coupon, or, in other words, the higher the coupon rate, the more dependent is the bond's total return on income from reinvestment. So with everything else held constant, the larger the coupon rate, the greater is the reinvestment risk. For bonds selling at a premium, i.e. $c > y$, vulnerability to reinvestment risk is higher than that for a bond selling at par. Correspondingly, discount bonds will be less vulnerable than a bond selling at par.

The important thing to note is that for a zero coupon bond, if it is held to maturity, there is absolutely no reinvestment risk, because there are no intermediate coupon payments. Hence, if a ZCB is held to maturity, the yield actually earned will equal the promised YTM. If the risk is lower or absent, the corresponding return should also be less. Thus a ZCB will command a higher price than an otherwise comparable coupon bond.

Reinvestment assumption behind the YTM calculation

We can claim that we have received a YTM of y% per annum, if the compounded semi-annual return on our initial investment is $\frac{y}{2}$.

The initial investment is

$$P = \frac{\frac{C}{2}}{\frac{y}{2}}\left[1 - \frac{1}{\left(1 + \frac{y}{2}\right)^N}\right] + \frac{M}{\left(1 + \frac{y}{2}\right)^N}$$

The compounded value of the investment is

$$P \times \left(1 + \frac{y}{2}\right)^N$$

$$= \frac{\frac{C}{2}}{\frac{y}{2}}\left[\left(1 + \frac{y}{2}\right)^N - 1\right] + M$$

The terminal cash flow from holding the bond, assuming that each coupon is reinvested at $\frac{r}{2}$ per semi-annual period, is

$$= \frac{\frac{C}{2}}{\frac{r}{2}}\left[\left(1 + \frac{r}{2}\right)^N - 1\right] + M$$

Equating the two, we get

$$\frac{r}{2} = \frac{y}{2}$$

Thus in order to get an annual YTM of y %, every intermediate cash flow must be reinvested at $\frac{r}{2}$% per six monthly period.

The Realised Compound Yield

To calculate the RCY, we once again assume that the bond is held until maturity. But we make an explicit assumption about the rate at which the coupons can be reinvested. Unlike the YTM, we no longer take it for granted that intermediate cash flows can be reinvested at the YTM.

Example 3.6

Consider the L & T bond. Assume that intermediate coupons can be reinvested at 7% for six months, or at a nominal annual rate of 14%.

Compared to the earlier example, the coupon income and the final face value payment will remain the same, but the reinvestment income will change.

$$\text{Interest on Interest} = \frac{50}{0.07}[(1.07)^{20} - 1] - 1{,}000$$

$$= 50 \times 40.995 - 1{,}000 = 1{,}049.75$$

So, the final amount received = 1,000 + 1,049.75 + 1, 000 = Rs. 3,049.75

The initial investment is Rs. 885.295

Therefore, the rate of return is given by:

$$885.295\,(1 + i)^{20} = 3{,}049.75$$

$$\Rightarrow \qquad (1 + i) = \left[\frac{3049.75}{885.295}\right]^{.05} = 1.0638$$

$$\Rightarrow \qquad i = 6.38\%$$

This is the return for six months. The nominal annual return is 6.38 × 2 = 12.76%. 12.76% is greater than the YTM of 12%. Thus, the realised compound yield will be greater than the YTM if the reinvestment rate is greater than the YTM. If the reinvestment rate were less than the YTM, the RCY would have been less.

The RCY can be an ex-ante measure if one makes an assumption about the reinvestment rate. It can also be an ex-post measure, if one uses the rate at which he is actually able to reinvest.

The Horizon or Holding Period Return

Now we will relax both the assumptions of the YTM. Firstly, the investor may not hold the bond till maturity, and, secondly, he may not be able to reinvest the coupons at the YTM.

Let us suppose that the investor has an investment horizon which is less than the time to maturity. His return will depend on three sources, namely, the coupons received, the reinvestment income, and the price at which he expects to sell the bond.[1]

Example 3.7

An investor has a seven year investment horizon. He wants to buy the L & T bond discussed in Examples 3.5 and 3.6.

The current price is Rs. 885.295 He will get coupons for seven years or 14 periods. The total coupon income $= 50 \times 14 =$ Rs. 700.

The investor believes that coupons can be reinvested at 7% per six monthly period. He also believes that the YTM when he is ready to sell the bond will be 12% per annum on a nominal annual basis.

The first step is to calculate the expected price at the time of sale. Remember, that the bond will at that time have three years to maturity.

$$
\begin{aligned}
P(\text{after seven years}) &= 50 \text{ PVIFA}(6, 6) + 1{,}000 \text{ PVIF}(6, 6) \\
&= 50 \times 4.9173 + 1{,}000 \times .705 \\
&= \text{Rs. } 950.865
\end{aligned}
$$

$$\text{Interest on Interest} = \frac{50\left[(1.07)^{14} - 1\right]}{.07} - 700$$

$$= 50 \times 22.55 - 700 = 427.5$$

The terminal value = 700 + 427.5 + 950.865

= Rs. 2,078.365

The rate of return is given by

$$885.295(1 + i)^{14} = 2,078.365$$

$$\Rightarrow \quad (1 + i) = \left[\frac{2078.365}{885.295}\right]^{\frac{1}{14}} = 1.0629$$

$$\Rightarrow \quad i = 6.29\%$$

The nominal annual return = 6.29 × 2 = 12.58%

This is the horizon yield. Once again, this can be calculated on either an ex-ante, or an ex-post basis.

Yield to Call

For a callable bond, it is a common practice to calculate the *Yield to Call* (*YTC*). The cash flows are usually taken to the first call date, although in principle they can be taken to any subsequent call date. The yield to call is the yield that will make the present values of the cash flows equal to the price of the bond, assuming that the bond is held to the call date.

$$P = \sum_{t=1}^{N^*}\left[\frac{\frac{C}{2}}{\left(1 + \frac{y}{2}\right)^t}\right] + \frac{M^*}{\left(1 + \frac{y}{2}\right)^{N^*}}$$

N^* is the number of coupons till the call date. M^* is the price at which the bond is recalled. It need not equal the face value. Often companies pay as much as one year's coupon as a *Call Premium* at the time of recall. If so, $M^* = M + C$.

Example 3.8

Let us now assume that the L & T bond is a callable bond. The first call date is seven years away, and a call premium of Rs. 100 has to be paid if the bond is recalled.

The YTC, which we will denote as y_c has to satisfy

$$885.295 = \sum_{t=1}^{14}\left[\frac{50}{\left(1+\frac{y_c}{2}\right)^t}\right] + \frac{1{,}100}{\left(1+\frac{y_c}{2}\right)^{14}}$$

The solution comes out to be 6.74%. The annual YTC = 6.74 × 2 = 13.48%.

The YTC is a very important measure for premium bonds. The very fact that a bond is selling at a premium, indicates that $c > y$, and thus there is a greater chance of it being recalled. Investors compute the YTC for every possible call date, and also calculate the YTM. The lowest of these values is called the *Yield to Worst.*

Portfolio Yield

Suppose you hold a collection of bonds. A simple way to calculate the yield is as a weighted average of the yields of the individual bonds in the portfolio. From a technically more accurate standpoint, you should first compute the cash flows for the portfolio, and then find that interest rate which will make the present value of the cash flows equal to the sum of the prices of the components of the portfolio. This is nothing but the computation of the portfolio IRR. We will illustrate both the techniques.

Example 3.9

Prashant Baliga buys a Ranbaxy bond and a CRB bond. The Ranbaxy bond has a time to maturity of five years, face value of Rs. 1,000, and pays coupons semi-annually at the rate of 10% per annum. The YTM is 12% per annum. The CRB bond has a face value of Rs. 1,000, time to maturity of four years, and pays a coupon of 10% per annum, every six months. The YTM is 16% per annum. If he buys one bond each, what is the yield on his portfolio?

The first step is to calculate the two prices.

$$P(\text{Ranbaxy}) = 50 \text{ PVIFA}(6, 10) + 1{,}000 \text{ PVIF}(6, 10)$$

$$= 50 \times 7.3601 + 1{,}000 \times .5584 = 926.405$$

$$P(\text{CRB}) = 50 \text{ PVIFA } (8, 8) + 1{,}000 \text{ PVIF}(8, 8)$$

$$= 50 \times 5.7466 + 1{,}000 \times .5403 = 827.63$$

The initial investment = 926.405 + 827.63 = Rs. 1,754.035

The weighted average approach

$$y_P = 0.12 \times \frac{926.405}{1{,}754.035} + 0.16 \times \frac{827.63}{1{,}754.035}$$

$$= 0.0634 + 0.0755 = 0.1389 \equiv 13.89\%$$

The IRR approach

Let us set up the cash flow table.

Table 3.1

Cash Flows from the Bond Portfolio

Period	*Investment*	*Inflow from Ranbaxy*	*Inflow from CRB*	*Total*
0	(1754.035)			(1.754.035)
1		50	50	100
2		50	50	100
3		50	50	100
4		50	50	100
5		50	50	100
6		50	50	100
7		50	50	100
8		50	1050	1,100
9		50		50
10		1,050		1,050

Calculate the IRR using a spread sheet or a financial calculator. It comes out to be 13.76% on a nominal annual basis.

Exercises

1. A bond with a clean price of Rs. 900 pays an annual coupon of Rs. 95. The cost of funding the bond is 10.25% per annum.
   (a) What is the current yield?
   (b) Calculate the net carry.

2. A bond with a face value of Rs. 1,000 is trading at Rs. 900. It has a time to maturity of ten years and pays a coupon of 10% per annum on a semi-annual basis. What is the simple yield to maturity?
3. A bond with a face value of Rs. 1,000 is currently trading at Rs. 950. It pays a coupon at the rate of 8% per annum on a semi-annual basis and has one year left to maturity. What is the yield to maturity?
4. A zero coupon bond with a face value of Rs. 1,000 is currently trading at Rs. 600. The bond has ten years to maturity. What is the yield to maturity?
5. A bond with a face value of Rs. 1,000, pays a coupon of 8% per annum on a semi-annual basis. It has ten years to maturity and the current YTM is 10% per annum.

   A bond holder buys and holds the bond till maturity. If he is able to reinvest every coupon at the rate of 4% per six-monthly period, what is his realised compound yield?
6. In the previous question, assume that the bond is sold when there are four years left to maturity. The yield to maturity at this point in time is 9% per annum. What is the holding period yield?
7. A bond with a face value of Rs. 1,000 pays a coupon of 8% per annum on a semi-annual basis. It has 20 years to maturity and the current YTM is 10% per annum. If the bond is callable after 12 years, what is the yield to call? Assume that a call premium equal to one year's coupon will be paid if the bond is recalled.

8. An investor has one each of the following bonds.

Table 3.2

Bond	*Face Value*	*Coupon Per Annum*	*Maturity*	*YTM Per Annum*
BASF	Rs. 1,000	8%	8 years	10%
NOVARTIS	Rs. 1,000	10%	10 Years	12%
CASTROL	Rs. 1,000	10%	12 years	8%
RANBAXY	Rs. 1,000	8%	9 years	12%

(a) Calculate the weighted average portfolio yield. Assume that coupons are paid on a semi-annual basis.

(b) What is the portfolio IRR?

Endnotes

1. The selling price may not equal the face value, because it would depend on the YTM at the time of sale.

4
CHAPTER

THE TERM STRUCTURE

Introduction

At any time, an investor will typically have access to a large number of bonds with different yields and varying times to maturity. It is common for investors and traders to examine the relationship between the yields on bonds belonging to a particular risk class. A plot of the yields of bonds that differ only with respect to their time to maturity versus their respective times to maturity is called a *Yield Curve*. The curve is an important indicator of the state of the bond market, and provides valuable information.

While constructing the yield curve it is very important that the data pertain to bonds of the same risk class, having comparable degrees of liquidity. For example, a curve may be constructed for government securities or AAA rated corporate bonds, but not for a mixture of both. The primary yield curve in any domestic capital market is the government bond yield curve, for these instruments are free of default risk. In the US debt market for instance, the primary yield curve is the US Treasury yield curve.

Analysing the Yield Curve

The yield curve is an indication of where the bond market is trading currently. It also has implications for the level of trading for the future or at least for what the market thinks will happen in the future.

The yield curve sets the yield for all debt market instruments. First, it fixes the price of money over the maturity structure. In practice, the yields of government bonds (from the shortest to the longest maturity instrument) set the benchmark for yields on all other debt instruments. This would mean that if a five-year government security is trading at a yield of 5%, all other five-year bonds, irrespective of the issuer, will be trading at yields over 5%. The excess over the yield on the corresponding government security is called the *spread*.

As mentioned earlier, the yield curve also acts as an indicator of future yield levels. It assumes certain shapes in response to market expectations of future interest rates. Market participants therefore analyse the current shape of the curve in order to determine the direction of future interest rates. These include bond traders and fund managers, as well as corporate finance personnel who need such information as a part of project appraisal.

Central banks and government treasury departments also analyse the curve for its information content. This information is then used to set rates for the economy as a whole.

Portfolio managers use the curve to assess the relative values of investments across the maturity spectrum. The curve indicates the returns that are available at different points of time and is therefore important for fixed income fund managers; they use this information to assist them in assessing the points along the curve that offer the best returns. The curve can also be analysed to determine which bonds are relatively cheap or costly.

Spot Rates

The spot rate of interest for a particular time period is the discount rate that is applicable for a zero coupon instrument maturing at the end of the period.

Example 4.1

For instance, assume that the price of a six-month zero coupon bond with a face value of Rs. 1,000 is Rs. 961.54. If we consider six months to be equivalent to one period, then the one period spot rate is given by

$$961.54 = \frac{1,000}{(1+s_1)} \Rightarrow s_1 = 0.04 \equiv 4\%$$

Similarly if a one year or a two period zero coupon bond has a price of Rs. 873.44, then the two period spot rate is given by

$$873.44 = \frac{1,000}{(1+s_2)^2} \Rightarrow s_2 = 0.07 \equiv 7\%$$

Relationship between Spot Rates and the YTM

A plain vanilla bond consists of a series of cash flows arising at six monthly intervals. Thus such a bond is equivalent to a portfolio of zero coupon bonds, where each cash flow represents the face value of a zero coupon bond maturing at that particular instant.

The correct way to price a bond is by discounting each cash flow at the spot rate for the corresponding period.

Example 4.2

Let us take the case of a bond with a face value of Rs. 1,000 and one year to maturity. Assume that it pays a coupon of 7% per annum on a semi-annual basis. Using the spot rates derived in the previous section, we can calculate the price of the bond to be

$$P = \frac{35}{(1.04)} + \frac{1,035}{(1.07)^2} = \text{Rs. } 937.66$$

The yield to maturity of this bond is given by

$$937.66 = \frac{35}{\left(1 + \frac{y}{2}\right)} + \frac{1,035}{\left(1 + \frac{y}{2}\right)^2}$$

$$\Rightarrow \qquad \frac{y}{2} = 0.069454 \equiv 6.9454\%$$

The yield to maturity is therefore a complex average of the spot rates. The problem with the yield to maturity is that it is a function of the coupon rate for bonds with identical terms to maturity but with different coupons.

For instance let us take a 12% coupon bond with a face value of Rs. 1,000 and one year to maturity. Its price is given by

$$P = \frac{60}{(1.04)} + \frac{1,060}{(1.07)^2} = \text{Rs. } 983.54$$

The yield to maturity of this bond is given by

$$983.54 = \frac{60}{\left(1+\frac{y}{2}\right)} + \frac{1{,}060}{\left(1+\frac{y}{2}\right)^2}$$

$$\Rightarrow \qquad \frac{y}{2} = 0.069092 \equiv 6.9092\%$$

Why is there a difference in the yields to maturity of the two bonds? After all, they both have one year to maturity.

Let us take the 7% bond first. It has

$$\frac{\frac{35}{(1.04)}}{937.66} = 0.035891 \equiv 3.5891\%$$

of its value tied up in one period money and the balance 96.4109% tied up in two period money.

However, in the case of the 12% bond,

$$\frac{\frac{60}{(1.04)}}{983.54} = 0.058658 \equiv 5.8658\%$$

of its value is tied up in one period money whereas the balance 94.1342% is tied up in two period money.

The one period spot rate is less than the two period spot rate, which implies that one period money is cheaper than two period money. Since the second bond has a greater percentage of its value tied up in one period money, its yield to maturity is less.

This is a manifestation of the *coupon effect*.

Yield Curve versus the Term Structure

Technically speaking, the term 'Yield Curve' is a graph depicting the relationship between the yield to maturity, which is plotted along the Y-axis, and the time to maturity, which is plotted along the X-axis. For the purpose of constructing the yield curve, it is imperative that the bonds being compared belong to the same credit risk class. This is the most commonly used version of the yield curve for the simple reason that the YTM is the most commonly used measure of the yield from a bond.

The expression 'Term Structure of Interest Rates' on the other hand, refers to a graph depicting the relationship between spot rates of interest as shown along the Y-axis, and the corresponding time to maturity, which is plotted along the X-axis. Once again, to facilitate meaningful inferences, the data used to construct the graph should be applicable to bonds of the same risk class. The term structure of interest rates is also referred to as the 'Zero Coupon Yield Curve' for obvious reasons. The zero coupon yield curve is considered to be the true term structure of interest rates because there is no reinvestment risk. The curve can be obtained from coupon bonds using the bootstrapping procedure, which will be discussed below.

The YTM yield curve does not distinguish between different payment patterns that may result from bonds with different coupons—that is, the fact that low coupon bonds pay a higher portion of their cash flows in present value terms at a later date as compared to higher coupon bonds of the same maturity. To compensate for this, bond analysts sometimes construct a coupon yield curve which plots the YTM against the term to maturity for a group of bonds with the same coupon.

The yield curve will be equivalent to the term structure if the term structure is flat, or in other words the spot rates are the same for all maturities. This is because when the term structure is flat, the YTM (which is a complex average of spot rates) will be equal to the observed spot rate.

Bootstrapping

In practice, we are unlikely to have data for the prices of zero coupon bonds maturing at regularly spaced intervals of time. Bootstrapping is a technique for determining the term structure of interest rates, given the price data for a series of coupon paying bonds.

Example 4.3

Assume that we have the following data for four bonds, each of which matures at the end of the stated period of time. For ease of exposition, we will also assume that the bonds pay coupons on an annual basis.

Table 4.1

Inputs for Determining the Zero Coupon Yield Curve

Time to Maturity	*Price in Dollars*	*Coupon*
1 Year	1,000	6%
2 Years	975	8%
3 Years	950	9%
4 Years	925	10%

The one year spot rate is obviously 6%. Using this information, the two-year spot rate can be determined as follows:

$$975 = \frac{80}{(1.06)} + \frac{1080}{(1+s_2)^2}$$

$$\Rightarrow \qquad s_2 = 9.57\%$$

Similarly, the three-year spot rate can be determined as:

$$950 = \frac{90}{(1.06)} + \frac{90}{(1.0957)^2} + \frac{1090}{(1+s_3)^3}$$

$$\Rightarrow \qquad s_3 = 11.32\%$$

And finally, using the same logic,

$$925 = \frac{100}{(1.06)} + \frac{100}{(1.0957)^2} + \frac{100}{(1.1132)^3} + \frac{1{,}100}{(1+s_4)^4}$$

$$\Rightarrow \qquad s_4 = 12.99\%$$

Practical difficulties with bootstrapping

In Example 4.3, we used the prices of four bonds, maturing after one, two, three and four years respectively. In some cases, there may be several bonds of the same risk class, maturing at a given time. Usually, each will have its own coupon. While estimating spot rates for a given maturity, obviously the coupon rates of the bonds being used is a factor. One of the other major issues while estimating the term structure using bootstrapping, is that a bond may not exist, or else it may not actively trade, for a particular maturity. And it is not necessary that we will always have access to a set of bonds whose maturity dates are conveniently spaced exactly one period apart.

Finally, all traded bonds may not be plain vanilla by nature. The US Treasury has issued bonds which can be recalled after a point. This, too, has implications for the bootstrapping procedure.

Coupon Yield Curves and Par Bond Yield Curves

One of the problems with bootstrapping is that in practice, we receive data in the form of prices of bonds with different coupons. This makes estimation difficult. One way to overcome it is by using data for bonds having the same coupon. The yield curve so obtained is called the *Coupon Yield Curve*. If we construct such a curve, it is seen that in general higher coupon bonds trade at a discount (have higher yields) relative to low coupon bonds. This is due to reinvestment risk.

The *Par Bond Yield Curve* on the other hand, is an estimate of the yield curve obtained from using data for bonds with different coupons, but all of which trade at par. In this case, the coupon for each of these bonds is nothing but its yield to maturity. The method of bootstrapping can then be applied to such a data set in order to derive the vector of spot rates as before.

Example 4.4

Table 4.2

The Par Bond Approach to Bootstrapping

Time to Maturity	*Price in dollars*	*YTM = Coupon*
1 Year	1,000	6%
2 Years	1,000	8%
3 Years	1,000	9%
4 Years	1,000	10%

The one year spot rate is obviously 6%. Using this information, the two-year spot rate can be determined as follows:

$$1{,}000 = \frac{80}{(1.06)} + \frac{1{,}080}{(1+s_2)^2}$$

$$\Rightarrow \qquad s_2 = 8.08\%$$

Similarly, the three-year spot rate can be determined as:

$$1{,}000 = \frac{90}{(1.06)} + \frac{90}{(1.0808)^2} + \frac{1{,}090}{(1+s_3)^3}$$

$$\Rightarrow \qquad s_3 = 9.16\%$$

And finally, using the same logic,

$$1{,}000 = \frac{100}{(1.06)} + \frac{100}{(1.0808)^2} + \frac{100}{(1.0916)^3} + \frac{1{,}100}{(1+s_4)^4}$$

$$\Rightarrow \qquad s_4 = 10.30\%$$

The par bond yield curve is not commonly encountered in secondary market trading. However it is often constructed and used by people in corporate finance departments and those involved with issues in the primary market. Investment bankers use par bond yield curves to determine the required coupon for a new bond. This is because new issues are typically issued at par and the banker needs to know the coupon that needs to be offered in order to ensure this. In practice the market uses data from non-par plain vanilla bonds to first derive the zero coupon yield curve. This information is then used to deduce the hypothetical par yields that would be observed if traded par bonds were to be available.

Deducing a par bond yield curve

The par bond yield curve can be derived using a vector of spot rates. We have already obtained the following information while analysing the data given in Example 4.3.

Table 4.3

Using Spot Rates to Infer a Par Bond Yield Curve

Time to Maturity	*Spot Rate*
1 Year	6%
2 Years	9.57%
3 Years	11.32%
4 Years	12.99%

The yield for a one year par bond is obviously 6%. The yield or equivalently the coupon for the two-year par bond can be deduced as follows.

$$1{,}000 = \frac{C}{(1.06)} + \frac{1{,}000 + C}{(1.0957)^2}$$

$$\Rightarrow \qquad C = \$\ 94.0441 \quad \Rightarrow \quad c = 9.4044\%$$

Similarly,

$$1{,}000 = \frac{C}{(1.06)} + \frac{C}{(1.0957)^2} + \frac{1{,}000 + C}{(1.1132)^3}$$

$$\Rightarrow \qquad \text{C} = \$\ 109.9837 \quad \Rightarrow \quad c = 10.9984\%$$

and

$$1{,}000 = \frac{C}{(1.06)} + \frac{C}{(1.0957)^2} + \frac{C}{(1.1132)^3} + \frac{1{,}000 + C}{(1.1299)^4}$$

$$\Rightarrow \qquad \text{C} = \$\ 124.0742 \quad \Rightarrow \quad c = 12.4074\%$$

Thus, a two-year bond of this risk class ought to be issued with a coupon of 9.4044% if it is to be sold at par. Similarly three-year and four-year bonds should carry coupons of 10.9984% and 12.4074% respectively.

Implied Forward Rates

Consider an investor who is contemplating a two period loan. He will be indifferent about choosing between a two period spot rate of s_2 and a one period spot rate of s_1 with a forward contract to rollover his one period loan at maturity for one more period at a rate f_1^1, provided

$$(1 + s_2)^2 = (1 + s_1)\,(1 + f_1^1)$$

f_1^1 is the one period forward rate or the rate for a one period loan to be made one period later. It is known as the implied forward rate that is contained in the term structure. In general, if we have an n period spot rate and an m period spot rate, where $m > n$, then

$$(1 + s_m)^m = (1 + s_n)^n\,(1 + f_n^{m-n})^{m-n}$$

where f_n^{m-n} is the m–n period implied forward rate for a loan to be made after n periods.

Forward rates are believed to convey information about the expected interest rate structure. In fact, one school of thought called the *unbiased expectations hypothesis* believes that such rates are nothing but current expectations of future interest rates.

Example 4.5

The one year spot rate is 8%; the two-year spot rate is 10%, and the three-year spot rate is 11.25%. Using this information, we can deduce this about implied forward rates:

$$(1 + s_2)^2 = (1 + s_1)\,(1 + f_1^1) \quad \Rightarrow \quad f_1^1 = 0.1204 = 12.04\%$$

Similarly,

$$(1 + s_3)^3 = (1 + s_1)\,(1 + f_1^2)^2 \Rightarrow f_1^2 = 0.1291 \equiv 12.91\%$$

$$(1 + s_3)^3 = (1 + s_1)\,(1 + f_1^1)\,(1 + f_2^1) \Rightarrow f_2^1 = 0.1379 \equiv 13.79\%$$

Fitting the Yield Curve

When plotting a yield curve, we fit a series of discrete points of yield against maturity. Similarly for the term structure of interest rates, we plot spot rates for a fixed time period against the time period. The yield curve itself is however a smooth curve drawn through these discrete points. We require a method that allows us to fit the curve as accurately as possible. This aspect of yield curve analysis is known as *yield curve modelling* or *estimating the term structure.*

The yield curve is derived from coupon bond prices and yields. In attempting to model the curve from bond yield data we need to be aware of two fundamental issues. First, there is the problem of gaps in the maturity spectrum as in reality there will not be a bond maturing at regular intervals along the term structure. Second, the term structure is defined in terms of spot rates or zero coupon interest rates. But in many markets, there is no zero coupon bond market.

We will study four methods for modelling the yield curve.

Interpolation

The simplest method that can be used to fit the yield curve is *linear interpolation.* For example assume that we are given the following data.

$$s_5 = 8\% \text{ and } s_{10} = 9\%$$

We can then calculate the eight-year spot rate as:

$$s_8 = 8.00 + \frac{(8-5)}{(10-5)} \times (9.00 - 8.00) = 8.60\%$$

Polynomial models

A polynomial of degree k may be expressed as

$$y_i = \alpha + \beta_1 t + \beta_2 t^2 + \ldots + \beta_k t^k + u_i$$

where y_i is the YTM of bond i; t is its term to maturity; and u_i is the residual error. To determine the coefficients of the polynomial we minimize the sum of the squared residual errors given by:

$$\sum_{i=1}^{N} u_i^2$$

where N is the number of bonds used.

Regression models

This is a variation of the polynomial approach. It uses bond prices as the dependent variable and the coupons and face values of the bonds as the independent variables. The standard format is given by:

$$P_i = \beta_1 C_{1i} + \beta_2 C_{2i} + \ldots + \beta_N (C_{Ni} + M) + u_i$$

In the above expression, P_i is the dirty price of bond i; C_{ji} is the coupon of bond i in period j; and u_i is the residual error. The spot rates can be derived from the estimated relationships using the expression

$$\beta_n = \frac{1}{(1+s_n)^n}$$

The Nelson-Siegel model

Before understanding the Nelson-Siegel technique, we need to be familiar with bond pricing in a continuous time framework.

Take a zero coupon bond which pays \$ 1 after n periods. In a discrete time setting, we would express the bond price as:

$$P(0,\ n) = \frac{1}{(1+s_n)^n}$$

where s_n is the n period spot rate at time 0. If we were to compound interest 'm' times per period where $m > 1$, then we would express the bond price as:

$$P(0,\ n) = \frac{1}{\left(1+\frac{s_n}{m}\right)^{mn}}$$

For instance, if we were to compound four times every period then m would be equal to 4. In the limit as $m \to \infty$ we get the case of continuous compounding. In the limit we can express the price of the bond as:

$$P(0,\ n) = e^{-s_n \times n}$$

We know that in the discrete time framework, the n period spot rate can be expressed as:

$$(1 + s_n)^n = (1 + s_1)\,(1 + f_1^1)\,(1 + f_2^1) \ldots (1 + f_{n-1}^1)$$

In continuous compounding, the equivalent representation is:

$$s_n \times n = \int_0^n f_s ds$$

where f_s is the instantaneous forward rate at time s as perceived at time zero.

Nelson-Siegel proposed the following representation for the instantaneous forward rate.

$$f_s = \beta_0 + \beta_1 e^{\frac{-s}{\theta}} + \beta_2 \times \frac{s}{\theta} e^{\frac{-s}{\theta}}$$

Integrating this, we get the following expression for the n period spot rate.

$$s_n = \beta_0 + \beta_1 \times \left[\frac{1 - e^{\frac{-n}{\theta}}}{\frac{n}{\theta}} \right] + \beta_2 \times \left[\frac{1 - e^{\frac{-n}{\theta}}}{\frac{n}{\theta}} - e^{\frac{-n}{\theta}} \right]$$

The parameters β_0, β_1, β_2, and θ have to be empirically estimated.

The Nelson-Siegel method for estimating the term structure has a number of advantages. Firstly, its functional form can handle a variety of shapes of the term structure that are observed in the market. Secondly, the model avoids the need to introduce other assumptions for interpolation between intermediate points. For instance, the bootstrapping approach will give us a vector of spot rates spaced six months apart. To value a bond whose life is not an integer multiple of semi-annual periods, we would obviously need to interpolate. On the contrary, using the Nelson-Siegel approach we can derive the spot rate at any point in time and not just at certain discrete points.

Theories of the Term Structure

From observing yield curves in different markets at various points in time, an individual who studies the bond market will notice that the yield curve tends to adopt one of these three basic shapes.

- **Upward Sloping:** In the case of yield curve with a positive slope (also termed as a rising yield curve) short term yields will be lower than long term yields
- **Downward Sloping:** In a yield curve with a negative slope (also termed as an inverted yield curve,) long term rates will be substantially lower than short term rates
- **Humped:** A humped yield curve is characterised by lower rates at the short end of the spectrum; the curve then rises, reaching a peak at the middle of the maturity spectrum, and then gradually slopes downward at longer maturities

A great deal of effort is expended by analysts and economists in analysing and interpreting the yield curve, for there is substantial information that is associated with the curve at any point in time. Various theories have been advanced that purport to explain the observed shapes of the curve. However, no theory by itself is able to explain all aspects observed in reality. So analysts seek to explain specific shapes of the curve using a combination of the accepted theories.

The Pure or Unbiased Expectations Hypothesis

This hypothesis states that current implied forward rates are unbiased estimators of future spot rates. Accordingly, long-term rates are geometric averages of expected future short-term rates. Thus a positively sloped yield curve would be consistent with the argument that the market expects spot interest rates to rise. If rates are expected to rise then investors in long-term bonds will be perturbed for they face the spectre of a capital loss. This is because rising interest rates will lead to declining bond prices, and long-term bonds are more sensitive to rising interest rates than short-term bonds. In such a situation, investors will start selling long dated securities and buying short dated securities. This will lead to an increase in yields on long-term bonds and a decline in yields on short-term bonds. The overall result will be an upward sloping yield curve. On the contrary, an inverted yield curve would indicate that the market expects future spot rates to fall.

The hypothesis can be used to explain any shape of the yield curve. For instance, a humped yield curve would be consistent with the explanation that investors expect short-term rates to rise and long-term rates to fall.

Expectations or views on the future direction of the market are a function mainly of the expected rate of inflation. If the market expects inflationary pressures in the future, the yield curve will be positively sloped, whereas if inflation is expected to decline then the yield curve will be negatively sloped.

The Liquidity Preference Theory (LPT)

Intuitively, most of us would feel that longer maturity instruments are more risky than shorter maturity ones. An investor lending money for five years will usually demand a higher rate of interest than if he were to lend money to the same customer for a year. This is because the borrower may not be able to repay the loan over a longer term period. For this reason, long dated yields should be higher than short dated yields.

Take the case where the market expects inflation to remain fairly stable over time. The expectations hypothesis would postulate that this scenario would be characterised by a flat yield curve. However, the liquidity preference hypothesis would predict a positively sloping yield curve. The argument would be as follows. Generally a borrower would like to borrow over as long a term as possible while a lender would like to lend over as short a term as possible. Thus, lenders have to be suitably rewarded if they are to be induced to lend for longer periods of time. This compensation may be considered as a premium for the loss of liquidity from the standpoint of the lender. The premium may be expected to increase further the investor lends across the term structure, so that the longest dated instruments will (all else being equal) have the highest yield.

As per this hypothesis, the yield curve should almost always be upward sloping, reflecting the bondholders' preference for liquidity. However inverted yield curves can still be explained in this theory by postulating that interest rates are likely to decline in the future, as a consequence of which, despite the liquidity premium, long-term rates are lower than short-term rates.

The expectations hypothesis versus the LPT: A mathematical analysis

As per the expectations hypothesis, forward rates are unbiased expectations of future spot rates. Thus

$$f_n^{m-n} = E_0[{}_nS_{m-n}]$$

In other words, the $(m - n)$ period forward rate, n periods from now, is the current expectation of the $(m - n)$ period spot rate that is expected to prevail n periods from now. The expectations hypothesis can explain any shape of the term structure. For instance, an expectation that future short-term interest rates will be above the current level would lead to an upward sloping term structure.

Example 4.6

Assume that $s_1 = 5.50\%$; $E[{}_1s_1] = 6.0\%$; $E[{}_2s_1] = 7.5\%$, and that $E[{}_3s_1] = 8.5\%$. If so then

$$s_2 = [(1.055)(1.06)]^{\frac{1}{2}} - 1 = 5.75\%$$

$$s_3 = [(1.055)(1.06)(1.075)]^{\frac{1}{3}} - 1 = 6.33\%$$

$$s_4 = [(1.055)(1.06)(1.075)(1.085)]^{\frac{1}{4}} - 1 = 6.87\%$$

According to the expectations hypothesis, investors care only about expected returns and not about risk. Let us take the case of an investor who chooses to invest for two periods. He can buy a two period bond yielding a rate of s_2. Or else he can buy a one period bond that yields s_1 and then roll over into another one period bond at maturity. As per the hypothesis, the investor will be indifferent choosing between the two strategies if the expected returns in both cases are equal.

In other words, the hypothesis predicts that the market will be in equilibrium if

$$(1 + s_2)^2 = E[(1 + s_1)(1 + {}_1s_1)]$$

But we know that if arbitrage is to be ruled out, then

$$(1 + s_2)^2 = (1 + s_1)\,(1 + f_1^1)$$

Thus as per the expectations hypothesis $f_1^1 = E({}_1s_1)$

Now let us focus on the liquidity preference theory. Take the case of an investor who has a one period investment horizon. He can buy a one period bond and lock in a rate of s_1. Or else he can buy a two period bond and sell it after one year. In the second case, the rate of return will be uncertain at the outset, for it will depend on the one period rate that will prevail one period from now.

Consider a two period zero coupon bond with a face value of \$ 1,000. Its current price will be

$$\frac{1{,}000}{(1+s_2)^2} = \frac{1{,}000}{(1+s_2)(1+f_1^1)}$$

The expected price of the bond after one period is

$$E\left[\frac{1{,}000}{(1+{}_1s_1)}\right] \geq \frac{1{,}000}{1+E({}_1s_1)}$$

We know that the expectation of the price after one year will be greater than or equal to the face value discounted by the expected one period spot rate one period from now from a result called Jensen's Inequality. It states that for a convex function

$$E[f(X)] \geq f[E(X)]$$

In this case, $\dfrac{1{,}000}{1+{}_1s_1}$ is a convex function because the second derivative is positive.

The expected rate of return from the two period bond over the first year will be:

$$\frac{E\left[\frac{1{,}000}{(1+{}_1s_1)}\right]-\frac{1{,}000}{(1+s_1)(1+f_1^1)}}{\frac{1{,}000}{(1+s_1)(1+f_1^1)}}$$

$$\geq \frac{\frac{1{,}000}{1+E({}_1s_1)}-\frac{1{,}000}{(1+s_1)(1+f_1^1)}}{\frac{1{,}000}{(1+s_1)(1+f_1^1)}}$$

$$\Rightarrow \quad \frac{E\left[\frac{1{,}000}{(1+{}_1s_1)}\right]-\frac{1{,}000}{(1+s_1)(1+f_1^1)}}{\frac{1{,}000}{(1+s_1)(1+f_1^1)}}$$

$$\geq \frac{(1+s_1)(1+f_1^1)}{1+E({}_1s_1)}-1$$

Obviously, the expected one period return from the two period bond will be greater than s_1 only if $f_1^1 > E({}_1s_1)$.

Now an investor with a one period investment horizon will obviously choose to hold a two period bond only if its expected return is greater than the assured return on a one period bond. This is because if he chooses to hold the two period bond, he will have to sell it after one period at a price that is presently unknown. From the above analysis, this would imply that the forward rate must be higher than the expected one period spot rate. Thus if investors are risk averse, which is the normal assumption made in Finance theory, the forward rate will exceed the expected spot rate by an amount equal to the risk premium or what may be termed as the *liquidity* premium.

We know that

$$(1 + s_2)\,(1 + s_2) = (1 + s_1)\,(1 + f_1^1)$$

As per the LPT

$$(1 + s_1)\,(1 + f_1^1) > (1 + s_1)\,[1 + E({}_1s_1)]$$

Therefore

$$(1 + s_2)\,(1 + s_2) > (1 + s_1)\,[1 + E({}_1s_1)]$$

Consider a downward sloping yield curve. This would imply that $s_1 > s_2$. Therefore, it must be the case that $E({}_1s_1)$ is substantially less than s_1. In other words, the market expects spot rates to decline substantially. For instance if $s_1 = 7\%$ and $s_2 = 6\%$ then $f_1^1 = 5.01\%$. As per the expectations hypothesis, $E({}_1s_1) = 5.01\%$. However as per the LPT, $E({}_1s_1) < 5.01\%$. If we assume that the liquidity premium is 0.50%, then $E({}_1s_1) = 4.51\%$.

Now let us take the case of a flat term structure. As per the expectations hypothesis,

$$s_1 = s_2 = f_1^1 = E({}_1s_1)$$

However, according to the LPT, $E({}_1s_1) < s_1 = s_2$. Thus, while the expectations hypothesis would imply that the market expects spot rates to remain unchanged, the prediction according to the liquidity preference theory is that the market expects spot rates to decline. For instance if $s_1 = s_2 = 7\%$ then according to the expectations hypothesis $E({}_1s_1) = 7\%$. However according to the LPT, $E({}_1s_1) = 7 - 0.50 = 6.50\%$.

Finally, let us take the case of an upward sloping yield curve. If $s_1 < s_2$ then for a slightly upward sloping yield curve the LPT would be consistent with the expectation that rates are going to marginally decline. However, if the curves were to be steeply upward sloping then the LPT would be consistent with the expectation that short-term rates are going to rise. For instance, assume that $s_1 = 7\%$ and that $s_2 = 7.1\%$. If so

$$f_1^1 = 7.2\% \Rightarrow E({}_1s_1) = 7.20 - 0.50 = 6.70\%$$

However, if $s_2 = 7.3\%$ then

$$f_1^1 = 7.6\% \Rightarrow E({}_1s_1) = 7.60 - 0.50 = 7.10\%$$

In both cases however, the expectations hypothesis would predict that spot rates are likely to rise. In the first scenario, as per the expectations hypothesis, $E({}_1s_1) = 7.20\%$, whereas in the second case $E({}_1s_1) = 7.60\%$.

The Money Substitute Hypothesis

According to this hypothesis, short-term bonds are substitutes for holding cash. Investors hold only short dated bonds because they are viewed as having low or negligible risk. As a result the yields on short dated bonds are depressed due to increased demand, and consequently, long-term yields are greater than short-term yields. Borrowers, on the other hand, prefer to issue debt for long maturities and on as few occasions as possible to minimise costs. Thus the yield on long-term securities are driven upward due to increased supply and lower liquidity.[1]

The Market Segmentation Hypothesis

This theory states that the capital market is made up of a wide variety of issuers, each with different requirements. Certain classes of investors will prefer short dated bonds while others will prefer long dated bonds. The theory argues that activity is concentrated in certain specific areas of the market, and that there are no interrelationships between these segments of the market. The relative amount of funds invested in each market segment causes differentials in supply and demand which leads to humps in the yield curve.

Thus, according to this theory the observed shape of the yield curve is determined by the supply and demand for specific maturity investments, and the dynamics in a particular market segment has no relevance for any other part of the curve. For example, banks concentrate a large part of their activity at the short end of the curve as a part of daily cash management (known as asset-liability management) and for regulatory purposes (known as liquidity requirements). On the other hand, fund managers such as pension funds and insurance companies are active at the long end of the market.[2] Few institutions however have a preference for medium dated bonds. This behaviour leads to high prices and low yields at both the short and long ends of the maturity spectrum and to high yields in the middle of the term structure.

The Preferred Habitat Theory

This is a slightly modified version of the segmentation hypothesis. This suggests that different market participants have an interest in specified areas of the yield curve but can be persuaded to hold bonds from other parts of the maturity spectrum if they are provided with sufficient incentives. Hence, banks which typically operate at the short-end of the spectrum may at times hold long dated bonds once the price of these bonds falls to a certain level, thereby ensuring that the returns from holding such bonds is commensurate with the attendant risk. Similar considerations may persuade long-term investors to hold short-term debt. So the incentive for an investor to shift out of his preferred habitat is the inducement offered by way of a higher rate of interest.

Exercises

1. The one year spot rate is 6% per annum and the two-year spot rate is 8% per annum.
   (a) Consider a bond with face value of Rs. 1,000, with two years to maturity, and paying a coupon of 7% per annum. What is the YTM?
   (b) Suppose that the bond were to offer a coupon of 8% per annum, what will be the YTM?
   (c) Why is the YTM different in the two cases?
2. Consider the following data. Assume that all the bonds have a face value of Rs. 1,000, and pay interest on an annual basis. Compute the following:

Table 4.1

Time to Maturity	*Price*	*Coupon*
1 Year	Rs. 950	6%
2 Years	Rs. 1,000	8%
3 Years	Rs. 1,050	10%
4 Years	Rs. 1,100	12%

   (a) s_1; s_2; s_3; and s_4.
   (b) f_1^1; f_2^1; f_1^2; f_1^3; f_2^2; and f_3^1.
3. Consider the following data. Assume that all bonds have a face value of Rs. 1,000 and pay interest on an annual basis.

Table 4.1

Time to Maturity	*Spot Rate*
1 Year	6%
2 Years	7%
3 Years	8%
4 Years	10%

What should be the coupons of 2, 3, and 4 year par bonds?

ENDNOTES

1. See Choudhry(2004).
2. See Choudhry(2004).

Appendix 1

Sources and References

1. Choudhry M., *An Introduction to Bond Markets*, Securities Institute, 2001
2. Choudhry M., *Analysing & Interpreting The Yield Curve*, John Wiley, 2004
3. Fabozzi F.J., *Bond Markets, Analysis, and Strategies*, Prentice-Hall, 1996
4. Garbade K.D., *Fixed Income Analytics*, The MIT Press, 1996
5. Stigum M. and Robinson, F.L., *Money Market* & *Bond Calculations*, Irwin, 1996.

Appendix 2

Solutions to End-of-Chapter Exercises

Chapter 1

Question–1

(a) Rs. 934.9604

(b) Rs. 1,071.0620

Question–2

Rs. 2,281.9347

Question-3

AYM = 9.8667%

YTM = 10.0068%

Chapter 2

Question–1

(a) Rs. 876.7724

(b) Rs. 876.8058

Question–2

(a) $ 809.9773

(b) $ 809.8593

Question–3

(a) $ 24.4441

(b) $ 24.0331

(c) $ 23.8356

(d) $ 24.1667

Chapter 3

Question–1

(a) 10.5555%

(b) 0.3055%

Question–2

12.22%

Question–3

13.5122%

Question–4

5.1741%

Question–5

9.3889%

Question–6

9.9554%

Question–7

10.9427%

Question–8

(a) 10.2781%

(b) 10.2657%

Chapter 4

Question–1

(a) YTM = 7.9305%

(b) YTM = 7.9217%

(c) In the first case the bond has 6.7153% tied up in one period money whereas in the second case it has 7.5366% tied up in one period money. And one period money is cheaper than two period money.

Question–2

(a) s_1 = 11.5789%; s_2 = 7.8617%; s_3 = 7.9502%; s_4 = 8.4937%

(b) f_1^1 = 4.2683%; f_2^1 = 8.1275%; f_1^2 = 6.1803%; f_1^3 = 7.4844%; f_2^2 = 9.1294%; f_3^1 = 10.1407%

Question–3

Coupon for a two year par bond = 6.9662%

Coupon for a three year par bond = 7.8970%

Coupon for a four year par bond = 9.6240%

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